I, Marcia Gail Headley, hereby submit this original work as part of the requirements for the degree of Doctor of Philosophy in Educational Studies.

It is entitled:
What is Symbolic Mathematics Language Literacy? A Multilevel Mixed Methods Study of Adolescents in a Middle School

Student’s name: Marcia Gail Headley

This work and its defense approved by:

Committee chair: Vicki Plano Clark, Ph.D.
Committee member: Rhonda Douglas Brown, Ph.D.
Committee member: Sarah Sitzlein, Ph.D.
Committee member: Christopher Swoboda, Ph.D.
What is Symbolic Mathematics Language Literacy? A Concurrent Mixed Methods Study of Adolescents in a Middle School

A dissertation submitted to the Graduate School of the University of Cincinnati in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Educational Studies in the College of Education, Criminal Justice, and Human Services

by Marcia Gail Headley

June 8, 2016

M. A. Mathematics Education, University of North Carolina, Charlotte May 1994

B. S. Mathematics, Virginia Polytechnic Institute and State University August 1990

Committee Chair: Vicki L. Plano Clark, Ph. D.

Committee Members:

Dr. Christopher M. Swoboda

Dr. Rhonda Douglas Brown

Dr. Sarah M. Stitzlein
Abstract

The language of mathematics may be the most influential language ever codified in a writing system. Despite the ubiquity of the language, literacy for mathematics remains something of a curiosity. It is not defined in critical educational policy documents such as the Common Core State Standards. The emphasis on disciplinary literacy has renewed interest in distinguishing literacy for mathematics from mathematics proficiency and from English language literacy. I conceptualized symbolic mathematics language literacy (SMaLL) as the ability to read and write symbolic mathematics using the conventions of the writing system for the language of mathematics. SMaLL, by this definition, is scarcely acknowledged or explored in educational research. My objective in conducting this exploratory study was to gain novel insights into SMaLL among middle school students learning under the Common Core State Standards with implications for instructional practice.

Guided by the theory of developmental bio-cultural co-constructivism, I adapted data collection tools from English language literacy research and implemented a multilevel concurrent mixed methods research design. In sum, I integrated quantitative and qualitative methodological approaches to understand SMaLL as a complex component of human development shaped by interactions within and across three levels of change: cultural, behavioral, and neurobiological. The results of the quantitative analysis of cognitive measures of students’ ability to read symbolic mathematics indicated that SMaLL is systematically related to measures of mathematics achievement. The results of the qualitative analysis of students’ metacognitive reflections on SMaLL indicated that assessing the symbolic mathematics features is critical to determining whether or not mathematical text was readable. In addition, decoding
mathematical symbols appears to be a critical SMaLL reading strategy. The mixed methods integrated results yielded a multilevel model of SMaLL describing how the in-class use of the language of mathematics, metacognitive strategies, and cognitive processing interact and contribute to the ways middle school students read symbolic mathematics in selections of typical academic texts. The model reveals three important aspects, one for each level, of SMaLL. First, developing SMaLL, or learning to read symbolic mathematics, requires assistance and depends on social interaction. Second, reading the language of mathematics is a process of navigating the ambiguity of the symbolic mathematics. It involves metacognitive behavior aimed at perceiving the meaning of symbols and producing an English language representation of the written text. Third, reading the language of mathematics involves the coordination of automated cognitive processes: orthographic processing, phonological processing, semantic processing, and emotional regulation.

The substantive findings have implications for future studies dedicated to refining a theory of SMaLL, developing methods and measurement tools to further investigate SMaLL, and tailoring instruction to support SMaLL development. In addition, this study advances the field of mixed methods research by providing an exemplar of a multilevel concurrent mixed methods study and introducing a novel conceptualization of multilevel variants of mixed methods research designs for use with theoretically defined levels and within-person investigations.

*Keywords: Symbolic Mathematics Language Literacy (SMaLL), multilevel research design, mixed methods, mathematics education, disciplinary literacy*
Acknowledgements

*Mathematics is the language with which God wrote the universe.*
— Galileo

Like Galileo, I think of mathematics as a language with which to explore the universe in practical and fantastic ways. In my experience, finding joy in learning mathematics is as much about being enamored with words and reading as it is about being enamored with numbers and calculations. This is a perspective of some singularity in a world where false dichotomies pervade our thinking: reading and math, arts and sciences, quantitative and qualitative. It took me quite some time to find a way to express my ideas. To those who helped me find the words, I am grateful.

I want to offer special thanks to the professors who helped me find my voice. Mary Brydon-Miller and Miriam Raider-Roth, thank you for introducing me to professional practice with patience and care. Beth O’Brien and Rhonda Brown, thank you for inspiring me with the cognitive literature in reading and mathematics. Connie Kendall Theado and Sarah Stitzlein, thank you teaching me to play the believing game as a reader and encouraging me as a writer. Chris Swoboda, thank you for introducing me to R and trusting me with your statistics students. Finally, Vicki Plano Clark, thank you for bringing mixed methods research to UC and helping me become a scholarly researcher. You are all exemplary educators and researchers. You helped me step out of my comfort zone to see new ways of thinking about what it means to learn and what it means to conduct research. You modeled kindness and commitment to your students, colleagues, and fellow researchers. You helped me to weather personal and professional trials and achieve more than I imagined just a few years ago. With your help, I became more like the person I want
to be: a learning teacher, a motherly professional, a practicing researcher, and a writing mathematician.

I am grateful to the graduate students and classmates who surrounded me. Ryan Hart, Kyle Cox, Jiaqi Zhang, and Zuchao “William” Shen, thank you for making our graduate office a place where colleagues become friends. Lori Foote, thank you for being a determined coordinator and planner. Laura Kelley, thank you for being my first true friend on campus — and for coming back. Rachael Clark, thank you for being my mixed methods role model and showing me how to dissertate. And, Sinem Toraman, thank you for bringing me good fortune and Turkish poetry just when I needed it.

Also, I would like to thank the friends and family who did not sign on for doctoral work, but willingly lived the process with me. I cannot name you all one by one, but I want to thank everyone from my mother to my hairdresser and from my brother to my allergist. Whether you offered constant prayers or occasional words of encouragement, you contributed to a well of support that kept me afloat. Christine, thank you for being a lover of words and walks. Challen, thank you for being my partner in adulting, sistering, and cousining. Emma, thank you for loving Donald Duck in Mathmagic Land, puzzles, and all of the other math toys I gave you before you were old enough to count. Savannah, thank you for laughing at anything — and everything — with me. Olivia, thank you for helping me rethink what it really means to live a life of purpose. Jeffrey, my steadfast and always-proud-of-me husband, thank you for sharing your good luck with me.

Finally, Daddy, thank you for being my first and forever hero. This dissertation is dedicated to you in honor of your service as a soldier and a father.
Grant and Scholarship Support

This research was supported by a College of Education, Criminal Justice, and Human Services Instructional Research and Development Technology Grant. In addition, this dissertation was supported by University Graduate Assistantships and Graduate Incentive Scholarship Awards offered by the School of Education.
Abstract
Acknowledgements
Table of Contents
List of Tables
List of Figures
Chapter 1
Introduction
Problem Statement
Purpose Statement
Research Questions
Mixed methods research questions.
Quantitative research questions.
Qualitative research questions.
Integrative research questions.
Significance of the Study
Substantive contributions.
Methodological contributions.
Practical contributions.
Definitions of Terms
Substantive terms.
Behavior.
Cognition.
Culture.
Lifespan development (Ontogenesis).
Metacognition.
Orthographic awareness.
Orthographic knowledge.
Orthographic process.
Plasticity.
Symbol.
Symbolic mathematics.
Symbolic Mathematics Language Literacy (SMaLL).
Methodological terms.
Concurrent design.
Constructivist paradigm.
Dialectical stance.
Initiation.
Mixed methods research.
Multilevel mixed methods research.
<table>
<thead>
<tr>
<th>Chapter 2</th>
<th>Literature Review</th>
<th>37</th>
</tr>
</thead>
<tbody>
<tr>
<td>Educational Policies</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>Curriculum Theories</td>
<td>43</td>
<td></td>
</tr>
<tr>
<td>A portrait of literacy</td>
<td>44</td>
<td></td>
</tr>
<tr>
<td>Mathematical practices</td>
<td>45</td>
<td></td>
</tr>
<tr>
<td>Grade level standards</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>Literacy Theories</td>
<td>53</td>
<td></td>
</tr>
<tr>
<td>Reading Models</td>
<td>58</td>
<td></td>
</tr>
<tr>
<td>SMaLL as a Literacy</td>
<td>62</td>
<td></td>
</tr>
<tr>
<td>The Language of Mathematics</td>
<td>64</td>
<td></td>
</tr>
<tr>
<td>Writing systems</td>
<td>64</td>
<td></td>
</tr>
<tr>
<td>Symbol theories</td>
<td>68</td>
<td></td>
</tr>
<tr>
<td>Multilevel Research Methods and Empirical Evidence</td>
<td>73</td>
<td></td>
</tr>
<tr>
<td>Multilevel issues in mixed methods research</td>
<td>73</td>
<td></td>
</tr>
<tr>
<td>Qualitative conceptions of multilevel designs</td>
<td>80</td>
<td></td>
</tr>
<tr>
<td>Quantitative conceptions of multilevel designs</td>
<td>83</td>
<td></td>
</tr>
<tr>
<td>Multilevel mixed methods research designs</td>
<td>87</td>
<td></td>
</tr>
<tr>
<td>Empirical research related to SMaLL</td>
<td>90</td>
<td></td>
</tr>
<tr>
<td>Summary of Chapter 2</td>
<td>98</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Chapter 3</th>
<th>Research Methods</th>
<th>99</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design Process</td>
<td>99</td>
<td></td>
</tr>
<tr>
<td>Characteristics of mixed methods research designs</td>
<td>99</td>
<td></td>
</tr>
<tr>
<td>Justification for a mixed methods design</td>
<td>104</td>
<td></td>
</tr>
<tr>
<td>Mixed methods rationale</td>
<td>104</td>
<td></td>
</tr>
<tr>
<td>Mixed methods in education</td>
<td>106</td>
<td></td>
</tr>
<tr>
<td>Mixed methods as a dialectical argument</td>
<td>107</td>
<td></td>
</tr>
<tr>
<td>Crafting a multilevel concurrent mixed methods design</td>
<td>108</td>
<td></td>
</tr>
<tr>
<td>Research Site</td>
<td>116</td>
<td></td>
</tr>
<tr>
<td>Recruitment, Consent, and Sampling</td>
<td>118</td>
<td></td>
</tr>
<tr>
<td>Consent and assent</td>
<td>118</td>
<td></td>
</tr>
<tr>
<td>Section</td>
<td>Page</td>
<td></td>
</tr>
<tr>
<td>------------------------------------------------------------------------</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>Quantitative sampling.</td>
<td>122</td>
<td></td>
</tr>
<tr>
<td>Qualitative sampling.</td>
<td>124</td>
<td></td>
</tr>
<tr>
<td>Instrumentation, Data Collection, and Data Products.</td>
<td>128</td>
<td></td>
</tr>
<tr>
<td>Quantitative data,</td>
<td>130</td>
<td></td>
</tr>
<tr>
<td>SMaLL Conventional Decision Task (S-CDT).</td>
<td>132</td>
<td></td>
</tr>
<tr>
<td>Math Print Exposure Survey (MPES).</td>
<td>137</td>
<td></td>
</tr>
<tr>
<td>Math Reading Habits Survey (MRHS).</td>
<td>139</td>
<td></td>
</tr>
<tr>
<td>Math Anxiety Survey (MAS).</td>
<td>140</td>
<td></td>
</tr>
<tr>
<td>Other variables.</td>
<td>141</td>
<td></td>
</tr>
<tr>
<td>Data collection process.</td>
<td>142</td>
<td></td>
</tr>
<tr>
<td>Qualitative data,</td>
<td>145</td>
<td></td>
</tr>
<tr>
<td>Analysis and Validation</td>
<td>149</td>
<td></td>
</tr>
<tr>
<td>Quantitative analysis.</td>
<td>150</td>
<td></td>
</tr>
<tr>
<td>Mixed methods interactive analysis.</td>
<td>155</td>
<td></td>
</tr>
<tr>
<td>Qualitative analysis.</td>
<td>156</td>
<td></td>
</tr>
<tr>
<td>Mixed methods integrative analysis.</td>
<td>160</td>
<td></td>
</tr>
<tr>
<td>Methodological reflections</td>
<td>167</td>
<td></td>
</tr>
<tr>
<td>Summary of Chapter 3</td>
<td>168</td>
<td></td>
</tr>
</tbody>
</table>

**Chapter 4**

**Results**                                                                 | 170  |
| Reporting conventions                                                   | 170  |
| Sample Descriptions                                                     | 171  |
| Quantitative sample                                                     | 171  |
| Qualitative sample                                                      | 174  |
| Quantitative Results                                                    | 182  |
| Performance of S-CDT                                                    | 184  |
| Group difference analysis                                               | 193  |
| Performance of surveys                                                  | 199  |
| MPES interactive analysis                                              | 199  |
| MRHS interactive analysis                                               | 208  |
| MAS confirmatory factor analysis                                        | 215  |
| Correlational analysis                                                  | 218  |
| Multiple regression analysis                                            | 222  |
| Qualitative Results                                                     | 225  |
|Readable and unreadable text selections.                                  | 225  |
| Readability criteria                                                    | 228  |
| Print features,                                                         | 229  |
| Familiarity,                                                            | 232  |
| Translation,                                                            | 235  |
| Math self-efficacy                                                     | 237  |
| Reading the language of mathematics.                                    | 239  |
| Perception strategies                                                  | 240  |
Chapter 4

Discussion of Results
Quantitative inferences.
Qualitative inferences.
Mixed methods meta-inferences.

Implications
Theoretical implications.
Methodological implications.
Practical implications.

Limitations
Reflections
Conclusion

References
Appendices
List of Tables

Chapter 2 Tables
Table 2.1 Symbolic mathematics in mathematical practices .......................... 48
Table 2.2 Symbolic mathematics by grade and domain .............................. 52

Chapter 3 Tables
Table 3.1 Summary of quantitative measurement tools ............................... 145
Table 3.2 Common data cleaning practices .................................................. 153
Table 3.3 Predicted direction of correlations ................................................. 154

Chapter 4 Tables
Table 4.1 Quantitative sample: Descriptive statistics .................................. 173
Table 4.2 Enrollment comparison of quantitative and qualitative samples .......... 174
Table 4.3 Qualitative sample: Descriptive statistics ...................................... 175
Table 4.4 Qualitative sample by pseudonym and order of interview ................ 177
Table 4.5 S-CDT measures across grades ..................................................... 194
Table 4.6 S-CDT measures across courses ................................................. 197
Table 4.7 Summary of missingness for MPES ............................................. 201
Table 4.8 Factor loadings for MPES ............................................................. 207
Table 4.9 Factor loadings for MRHS ............................................................ 214
Table 4.10 Correlation tables by grade ......................................................... 221
Table 4.11 Multiple regression analysis by grade .......................................... 224

Chapter 5 Tables
Table 5.1 Multilevel model of SMaLL ......................................................... 308
List of Figures

Chapter 1 Figures
Figure 1.1 Conceptualization of dialectical stance .......................................................... 24
Figure 1.2 DBCCC diagram .............................................................................................. 27

Chapter 2 Figures
Figure 2.1 Map of guiding questions ................................................................................ 38
Figure 2.2 Literature review map ...................................................................................... 39
Figure 2.3 Venn diagram of literacy habits and mathematical practices ......................... 49
Figure 2.4 Visualization of Knoblauch's literacy categories .............................................. 56
Figure 2.5 Qualitative view of SMaLL as a multilevel phenomenon ............................... 82
Figure 2.6 Quantitative view of SMaLL as a multilevel phenomenon ............................ 86

Chapter 3 Figures
Figure 3.1 Procedural diagram of research design ........................................................... 113
Figure 3.2 Multilevel concurrent mixed methods research design ................................. 114
Figure 3.3 Quantitative sampling diagram ...................................................................... 123
Figure 3.4 Qualitative sampling diagram ........................................................................ 128
Figure 3.5 Variable map ................................................................................................... 130
Figure 3.6 Selected SMaLL-CDT instructions .................................................................. 136
Figure 3.7 Iterative qualitative analysis process .............................................................. 157
Figure 3.8 Timeline summary of integration .................................................................... 162

Chapter 4 Figures
Figure 4.1 Reading interests across qualitative sample .................................................... 178
Figure 4.2 Future plans for math coursework .................................................................. 180
Figure 4.3 Career interests across qualitative sample ..................................................... 181
Figure 4.4 Data cleaning process for SMaLL-CDT ......................................................... 185
Figure 4.5 SMaLL-CDT: Conventional items ................................................................. 186
Figure 4.6 SMaLL-CDT: Unconventional items .............................................................. 187
Figure 4.7 SMaLL-CDT: Item pairs .................................................................................. 187
Figure 4.8 Distribution of SMaLL-CDT summary metrics ............................................ 192
Figure 4.9 Comparison across grades: Density graphs .................................................... 195
Figure 4.10 Comparison across courses: Density graphs ............................................... 198
Figure 4.11 Bar graph of MPES responses by item ......................................................... 203
Figure 4.12 Distribution of MPES scores ......................................................................... 204
Figure 4.13 Scree plot for MPES ..................................................................................... 205
Figure 4.14 Bar graph of MRHS responses by item ......................................................... 210
Figure 4.15 Distribution of MRHS scores 211
Figure 4.16 Scree plot for MRHS 212
Figure 4.17 Bar graph of MAS responses by item 216
Figure 4.18 Distribution of MAS summary scores 217
Figure 4.19 Frequency of text selections 227
Figure 4.20 Other text selections discussed 228
Figure 4.21 Readability criteria concept map 229
Figure 4.22 Reading strategy concept map 240
Figure 4.23 Joint display: Selection 2 259
Figure 4.24 Joint display: Selection 3 262
Figure 4.25 Joint display: Selection 4 265
Figure 4.26 Joint display: Selection 5 269
Figure 4.27 Spectrum analysis 273
Figure 4.28 Peter's quantitative profile 275
Figure 4.29 Jackson's quantitative profile 279
Figure 4.30 Katrina's quantitative profile 283
Figure 4.31 Elena's quantitative profile 287
Figure 4.32 Loren's quantitative profile 290
Figure 4.33 Model of SMaLL 294
Chapter 1: Introduction

The language of mathematics is arguably the most influential global language ever codified in a writing system (Guillen, 1995). The language of mathematics underlies the basic accoutrements of modern life — electricity, cars, and phones — and many of the most memorable moments in modern history — the launching of the A-bomb and the landing of a man on the moon. For the novice, the language of mathematics looks like a foreign language (National Research Council, 2001), bewildering ink marks on the page. For the expert, it is more like poetry conveying deep meaning through structure and patterns in written and spoken language (Guillen, 1995). Like poetry, the language of mathematics makes it possible to describe the complexity of what, at first glance, seemed patently simple and to simply express what, at first glance, seemed irreconcilably complex. In Five Equations that Changed the World: The Power and Poetry of Mathematics, the analogy is explained with the help of renowned poet, Robert Frost:

In attempting to distinguish between prose and poetry, Robert Frost once suggested that a poem, by definition, is a pithy form of expression that can never be accurately translated. The same can be said about mathematics: It is impossible to understand the true meaning of an equation, or to appreciate its beauty, unless it is read in the delightfully quirky language in which it was penned. (Guillen, 1995, pp. 3)

The analogy to poetry is limited, however. Where poetry is appreciated for the sake of its beauty, mathematics is appreciated for its practical potential. If mathematics is necessary to sustain modern life, then it is no wonder that mathematics literacy is commonly viewed as a

Despite its apparent import, mathematics literacy remains something of a curiosity. The Common Core State Standards (CCSS; National Governors Association Center for Best Practices [NGA], 2016a), adopted in 42 of the 50 states, does not define the term. Yet, as the CCSS have been adopted across the nation, there has been a renewed interest in distinguishing mathematics literacy, in the general sense, from English language literacy for mathematics and from literacy in the language of mathematics (Goldman, 2012; Hillman, 2014; Johnson, Watson, Delahunty, McSwiggen, & Smith, 2011). Because changes in policy and practice have required students to engage with increasingly complex informational texts, understanding how students develop the ability to use the writing system of mathematics has become especially important. Despite an extensive search, I have not found evidence that the question of how students learn to read symbolic mathematics, the written language of mathematics classrooms, has been asked or answered. For example, how do students read symbolic mathematics such as this simple equation: \( y = \sqrt{x} \)? Thus, it is not clear how students learn to interact with text written in the unique language of mathematics. The process of reading symbolic mathematics, which involves
decoding combinations of alphabetic characters, Arabic numerals, and discipline-specific symbols to produce a representation of the text in English, appears to be ignored in practice and in research. As a result, little is understood about how the ability to read and write symbolic mathematics supports formal mathematics education. Equally important, little is understood about how a lack of those skills constrains mathematical learning. As a first step towards addressing this gap, I have conceptualized *symbolic mathematics language literacy* (SMaLL) as the ability to read and write symbolic mathematics.

Current policies and practices demand that students learn mathematics, interacting with increasingly complex symbolic mathematics text, at a prescribed rate with respect to prescribed curriculum. Local, state, and federal documents outline annual learning goals in pacing guides, curricula, and standards (e.g., NGA, 2016a; U.S. Department of Education, 2010). Students’ scores on mandated periodic assessments determine progress towards those goals and play a role in high stakes decisions such as student eligibility for graduation, teacher accountability, and district funding formulas (e.g., U. S. Department of Education, 2014; Escue, 2012; Thompson & Crampton, 2014). Given the central role of mathematics in high-stakes testing, students’ ability to negotiate symbolic mathematics to read questions and write responses on mandatory assessments impacts schools’ access to funds and students’ access to future opportunities (e.g., Au, 2013; Thomas, 2013).

In contrast to educational policies, which are primarily concerned with producing particular outcomes of education, much of educational research is concerned with the inputs of education. For example, research in curriculum and instruction often focuses on the contribution of teachers and classroom practice to students’ learning while research in developmental and
learning sciences often focuses on the contribution of the learner. Although interdisciplinary 
research promises to make a valuable contribution to policy and practice, the current research 
connecting literacy and mathematics leaves many unanswered questions about the role of literacy 
in mathematics achievement (Friedland, McMillen, & del Prado Hill, 2011; Johnson et al., 2011). 
Despite the high stakes associated with SMaLL, current research conflates mathematics 
cognition with numeracy (e.g., Geary, 1993; Grabner, Saalbach, & Eckstein, 2012) and content 
area literacy with the ability to read word problems written in English (e.g., Fang & 
Schleppegrell, 2008).

**Problem Statement**

The role of symbolic mathematics, as a language and writing system, in mathematics 
education is largely uninspected. Policies presume SMaLL among students, yet the CCSS 
frames literacy skills as distinct from mathematical practices. In terms of literacy theory and 
reading models, English and its alphabetic writing system are the oft presumed tool of instruction 
and object of literacy learning. A vast body of literature suggests English language literacy has 
been explored at multiple levels – cultural, behavioral, and neurobiological (e.g., Hall, Smith, & 
Wicaksono, 2011; Wolf, 2007). However, SMaLL, as it is conceptualized for this study, is rarely 
acknowledged and scarcely explored. Despite an interdisciplinary search, I have found no extant 
model of SMaLL in the literature. Thus, this dissertation embarks on a novel research agenda to 
understand SMaLL using techniques adapted from literacy literature to answer questions about 
how symbolic mathematics language is experienced by students in the current academic culture, 
employed by learners as a metacognitive tool, and processed by the brain as a cognitive artifact 
(Heersmink, 2013).
Purpose Statement

The objective of this exploratory study was to gain novel insights into SMaLL skills among middle school students learning under the CCSS with implications for instructional practice. Using a multilevel concurrent mixed methods design and techniques adapted from studies in English language literature, I integrated two approaches — quantitative and qualitative — to conduct a feasible study of SMaLL in a middle school setting. The quantitative aim was to use measurement tools adapted from studies of reading acquisition to investigate SMaLL in terms of cognitive measures to explore the degree to which SMaLL is similar to English language literacy and distinguishable from mathematics achievement and other related constructs. The qualitative aim was to investigate SMaLL in terms of metacognitive reflections to explore how students intentionally and explicitly employ SMaLL skills in the context of academic classroom culture. Assuming componential theories of reading, a model of SMaLL needs to account for the orthographic nature of the writing system as well as print exposure and reading habits (e.g., Apel & Apel, 2011). Assuming constructivist theories of mathematical development, a model of SMaLL needs to account for math achievement, math anxiety, and value systems. The multilevel aim of the study was to use data collected and analyzed at different levels using different methodological traditions (Teddlie & Tashakkori, 2009) to understand SMaLL as a complex component of human development shaped by interactions within and across three levels — cultural, behavioral, and neurobiological — of change.

The mixed methods rationale was initiation (Greene, 2007). That is, the goal was to generate a new perspective for understanding the nature of SMaLL by using mixed methods to assess multiple facets of the phenomenon. While a review of interdisciplinary literature
suggested some tentative hypotheses regarding the relationship among some facets, the design of the study allowed for – even invited – the possibility of unexpected results. The study was grounded in developmental theories and designed with the potential to produce an initial model of SMaLL. The study was designed to have implications for future studies dedicated to refining a theory of SMaLL, developing methods and measurement tools to further investigate SMaLL, and tailoring instruction to support SMaLL development.

Finally, the overarching purpose of this study was to take a first step towards informing a wide variety of stakeholders about SMaLL. The immediate audience of this dissertation includes researchers in mathematics education, mathematics cognition, educational psychology, and cognitive psychology and educators at the research site. This dissertation is of interest to those conducting interdisciplinary research in developmental and learning sciences. This acknowledgment of SMaLL as a pervasive and hidden element of mathematics curriculum has implications for thinking about the process and purpose of mathematics education. Thus, I plan to reach the extended audience — including policymakers, practitioners, parents, and students — using other pathways to dissemination such as journal publications, conference presentations, and instruction. The staff at the research site are particularly important stakeholders. In collaboration, we have planned to share the results as a form of professional development to extend teachers’ pedagogical knowledge and understanding of the literacy issues experienced by their students. The model of SMaLL has the potential to reframe discussions about the intersection of literacy and mathematics by offering a fresh perspective.
Research Questions

The research questions, which are critical to shaping every aspect of this dissertation, are articulated in a manner that reveals the anticipated facets of SMaLL (see Chapter 2 for a review of the literature) as well as the methods necessary to explore SMaLL as a multilevel phenomenon (see Chapter 3 for a justification of the methods). In this section, I begin by highlighting the mixed methods research questions that reflect the overarching boundaries and essential purpose of the research project (Plano Clark & Badiee, 2010). Then, I itemize the quantitative, qualitative, and integrative research questions that focus the study in order to render SMaLL, despite its complexity, a researchable topic (Tashakkori & Teddlie, 2010).

Mixed methods research questions.

◦ What is SMaLL for adolescent students in middle grades learning mathematics under the Common Core State Standards?

◦ How do students reading typical academic texts with symbolic mathematics experience SMaLL?

Quantitative research questions.

◦ To what degree are cognitive processes of SMaLL and English language literacy analogous?

▪ Does SMaLL differ across academic classroom cultures?

▪ Do differences in orthographic processing exist across grade levels?

▪ Do differences in orthographic processing exist across mathematics courses?

▪ To what degree is SMaLL related to measures of classroom culture, student behavior, student affect, and student ability?
- What are the relationships between measures of orthographic processing, math print exposure, math reading habits, math anxiety, and math achievement?

**Qualitative research questions.**

- In what ways are metacognitive processes of SMaLL and English language literacy analogous?
  - What kinds of texts with symbolic mathematics do students identify as readable (or unreadable)?
  - What kinds of strategies do students use to read texts with symbolic mathematics?
  - How do students describe the relationships among classroom culture, their reading behaviors, their cognitive abilities, and their developing SMaLL?

**Integrative research questions.**

- How does integrating quantitative results and qualitative findings generate novel insights into SMaLL?
  - In what ways is SMaLL analogous to English language literacy?
  - What connections, if any, emerge across cultural, behavioral, and neurobiological levels of SMaLL?
  - What theories or models, if any, of SMaLL emerge?

**Significance of the Study**

This study is significant, and has implications, for multiple communities of research and practice. First, it makes a significant substantive contribution to the field of mathematics education. Second, it makes a significant methodological contribution to the community of
mixed methods research methodologists. And, finally, this study raises new questions for practitioners and policy makers.

**Substantive contributions.** This study makes a significant contribution to the field of mathematics education. Developing a model of SMaLL is an important first step towards bringing attention to its role in formal education, mathematical development, and mathematics cognition. Understanding SMaLL in relationship to English language literacy, mathematics achievement, and other theoretically related dimensions has implications for mathematics education and other related fields of study in developmental and learning sciences. In particular, a model of SMaLL has the potential to create fertile ground for interdisciplinary researchers interested in the intersection of language literacy and mathematical development. A model can be used as a framework to investigate SMaLL in various contexts.

In addition, the novel quantitative measurement tools will be of interest to mathematics cognition and mathematics education researchers. The tools are adaptations of tasks and surveys from the fields of cognitive psychology and literacy education. Although a rigorous inspection of psychometric properties is not a primary goal, this work provides empirical evidence to suggest next steps in the development of tools to understand literacy in mathematics education. With further development of a model of SMaLL and measurement tools for SMaLL, it may be possible to develop intervention materials tailored to individual differences in SMaLL.

**Methodological contributions.** This study also makes multiple methodological contributions to the community of mixed methods research methodologists. First, this concurrent mixed methods study illustrates how mixed methods can be employed to understand a phenomenon at three levels: cultural, behavioral, and neurobiological. The use of mixed
methods is not unprecedented in education (Greene, 2007); however, in applying both quantitative and qualitative methods to answer research questions in mathematics education, researchers have not capitalized on the vast body of methodological expertise in the field of mixed methods (Ross & Onwuegbuzie, 2012). By example, this study suggests new approaches for pursuing novel research agendas that account for the complexity of human development in varying contexts. Specifically, this study extends the existing conceptualization of multilevel mixed methods by presenting an example of a within-person multilevel mixed methods study.

Second, this study makes a contribution to the paradigm discussion by illustrating how differing paradigms can be employed to generate warranted conclusions and advance theory using a dialectic argument. In particular, this work is an exemplar for methodologists interested in mixed methods for the purpose of initiation or as an approach to theory building. In general, qualitative research is understood as the process of theory building while quantitative research is understood as the process of theory testing (e.g., Johnson & Christensen, 2012). This study illustrates how to use both quantitative and qualitative data for theory building. This study of SMaLL, which accounts for language in the classroom, students’ metacognitive reflections, and students’ cognitive processes justifies the need for multilevel mixed methods research to generate models of learning in education that account for the roles of culture, behavior, and neurobiology.

**Practical contributions.** This research was conducted within a politically charged era of educational change. High-stakes testing mandates and accountability policies, presented as a promise to improve the quality of education, have been perceived as a threat by many (e.g., Au, 2011). Stakeholders, especially the teachers responsible for implementing the CCSS in classrooms, continue to debate the aims and means of mathematics education (Hlebowitsh, 2012;
Noddings, 2009). This study offers systematic evidence about how adolescents experience SMaLL. Thus, this study makes an important contribution, in the form of pedagogical content knowledge, to middle school mathematics educators in general and to the teachers at the research site in particular. Considering SMaLL as they engage in reflective practice and interrogate their pedagogical assumptions about how students interact with mathematical texts may be useful in tailoring instruction to particular classrooms and particular students.

Finally, this dissertation has important implications for policy and curriculum development. Policymakers may find it useful to consider what is known – and what is not known – about SMaLL in the process of debating how to frame mathematics as connected to or distinct from English language arts and other disciplinary literacies. The results may help policymakers question what constitutes ethical curriculum and funding practice in the absence of the assumption that all students can acquire SMaLL, or the ability to read and write symbolic mathematics, in the same way. The CCSS fail to give careful attention to how SMaLL develops despite introducing standards that demand progressively complex mathematics texts from kindergarten to graduation. One of the greatest promises of the CCSS is that it “will allow research on learning progressions to inform and improve the design of standards to a much greater extent than is possible today” (NGA, 2010b, p. 5). Although the current study may not be sufficient to warrant immediate changes in policy or practice, it makes a significant contribution to substantiate this pledge by setting the stage for further research on SMaLL development.
Definitions of Terms

As with any rigorous research, this study required the careful definition and delineation of terms. Due to the interdisciplinary nature of this study, some terms may have different connotations for various readers in the audience. In addition, some terms may be unfamiliar to readers. Therefore, this section is intended to clarify terms as they are used in this study and provide the reader with a glossary for reference during reading. The terms are organized in two groups. First, the substantive terms section provides a guide to words and phrases associated with the content of the study. Then, the methodological terms section provides a guide to words associated with the research design and analysis.

Substantive terms.

The substantive terms, defined to clarify the subject of this study, are organized in alphabetical order.

Behavior. Agent-driven and self-initiated action, reaction, and interaction (Baltes, Reuter-Lorenz, & Rosler, 2006). Behavior includes introverted and extraverted acts. Behavioral development across the lifespan is characterized by three subprocesses: selection, optimization, and compensation (Baltes, 1997).

Cognition. A neurobiological process associated with the acquisition of new knowledge and/or the extension of learning (Bjorklund, 2012). Cognitive processes (e.g., memory retrieval or executive function) are not directly apparent to an observer or the individual completing the process (Bruning, Schraw, & Norby, 2011; Geary, 1993, 2011). The nature of cognition is inferred from observing brain activity, consequent behavior, or through metacognitive reflection.
**Culture.** The context in which human development occurs including all psychological, social, material, and knowledge-based resources that humans use, produce, and/or transmit across generations (Baltes, 1997).

**English language literacy.** The ability to read and write using academic English.

English language literacy entails the ability to use the English language alphabetic writing system and academic conventions for writing English to exchange ideas in print.

**Lifespan development (Ontogenesis).** Development over the lifespan, from conception to death, that accounts for cumulative processes and exceptional events (Baltes, 1987). Development can be supported and/or constrained by cultural plasticity, behavioral plasticity, and neurobiological plasticity (Baltes et al., 2006).

**Metacognition.** An individual’s knowledge of his/her own cognitive processes.

Metacognition may entail an awareness of the component mental processes required for a task, an assessment of the degree to which those processes can be (or have been) successfully applied, and value-judgments about those processes and/or the task (Bjorklund, 2012; Schneider & Artelt, 2010). *Declarative metacognition* refers to the conscious awareness of the nature or demands of a given task; in contrast, *procedural metacognition* refers to the conscious awareness of strategy selection and process monitoring (Bjorklund, 2012).

**Orthographic awareness.** The ability to distinguish between conventional, or plausible, patterns of text and irregular, or implausible, patterns of text in a given language (Apel, 2011). For example, *tosk* is a plausible word in the English language given...
the relationship of the vowels and consonants. In other words, it is readable although not meaningful in English. However, *tdsk* is not a plausible word in the English language because it lacks a vowel. Thus, it is neither readable nor meaningful.

**Orthographic knowledge.** The ability to recognize a particular pattern of text in a given language as a meaningful unit of the language (Apel, 2011). For example, compare *cat* and *kat*. Orthographic knowledge allows an English speaker to recognize the former as a word and identify the latter as a misspelling.

**Orthographic process.** An automatic neurobiological process that begins with the visual perception of a written character(s) and ends with the recognition of a word (or awareness of a meaningful unit of text). Orthographic processing does not entail comprehension of the word, merely recognition of a word as something with the potential for meaning. Furthermore, orthographic processing does not entail the ability to articulate a rationale for understanding the pattern or to comprehend the meaning of the symbol.

**Plasticity.** The degree of malleability. *Cultural plasticity* refers to the degree to which the resources (e.g., environmental settings, material goods, social structures, knowledge structures) transmitted across generations are malleable (Baltes, 1997). *Behavioral plasticity* refers to the degree to which agent-driven (observable or unobservable) behaviors (e.g., physical activities, language skills, reactions to and interactions with others) are malleable. *Neurobiological plasticity* refers to the degree to which neural and biological structures (e.g., synaptic development,
myelination, availability of neurotransmitters) are malleable.

**Reading.** “The process of extracting and constructing meaning from text” (Faust & Kandelshine-Waldman, 2011, p. 546).

**Symbol.** A written sign imbued with meaning through enculturation and logical rules.

**Symbolic mathematics.** Vernacular term for the formal writing system of mathematics. Symbolic mathematics is written as in-line text using discipline specific symbols, or characters, and syntax to express quantities, expressions, equations, mathematical object and statements. It is a representational system that acquires its meaning through “logical rules and convention” (Heersmink, 2013, p. 475). It is a cognitive artifact in that it is manmade and designed to “functionally contribute to performing a cognitive task” (Heersmink, 2013, p. 465). The system is a cultural artifact in that it evolved over millennia and has become the sine qua non of modern mathematics and science (Van Dyck & Heeffer, 2012).

**Symbolic mathematics language literacy (SMaLL).** The ability to read and write using the unique symbols and language of mathematics. In other words, the ability to use the writing system of the language of mathematics and academic conventions for writing symbolic mathematics to exchange ideas in print.

**Methodological terms.** The methodological terms, defined to clarify how this study was conducted, are organized in alphabetical order.

**Concurrent design.** A mixed methods research design characterized by the simultaneous (or nearly so) and independent collection and analysis of both quantitative and qualitative data (Creswell & Plano Clark, 2011; Morse, 1991). The quantitative
and qualitative analyses are succeeded by a final stage of integration.

**Constructivist paradigm.** A research paradigm grounded in the ontological assumption that multiple realities exist. Although some realities may be shared among groups, different realities are constructed by individuals who view the world from their unique vantage points (Hatch, 2002).

**Dialectical stance.** The willingness to respectfully and intentionally negotiate multiple traditions of inquiry in the process of considering what constitutes knowledge (Greene, 2007). In the context of this study, a dialectical stance entails negotiating post-positivist traditions and constructivist traditions.

**Initiation.** A rationale for using a mixed methods research design motivated by the need to generate novel insights into a multi-faceted phenomenon (Greene, 2007).

**Mixed methods research.** An inquiry enterprise that intentionally incorporates both quantitative and qualitative data and analytical tools for the purpose of transcending what can be discovered and understood from a traditionally quantitative or traditionally qualitative approach alone (Creswell & Plano Clark, 2011).

**Multilevel mixed methods research.** A multilevel mixed methods research design is a variant of a mixed methods research design driven by an assumption about the nature of a system for the purpose of better understanding these elements of the system: the structure of the system; the components that emerge from, give rise to, or evolve in tandem with the structure; and the mechanisms at work between the components and the structure. As a matter of convention, the components are
conceived as a lower level while the structure is conceived as an upper level. The mechanisms at work within the system may be top-down (with the structure causing the components to take on particular characteristics), bottom-up (with the components causing the structure to take on particular characteristics), or reciprocal (with the structure and components imposing transformation at both levels).

According to this definition, a multilevel mixed methods research design is defined primarily by its purpose. As a variant of a mixed methods research design, a multilevel mixed methods research design must incorporate both quantitative and qualitative data and analytical tools for the purpose of transcending what can be discovered and understood from a traditionally quantitative or traditionally qualitative approach alone. A thorough high-quality multilevel mixed methods research designs should entail the following:

- A quantitative strand designed to investigate one or more levels or a mechanism within the system
- A qualitative strand designed to investigate one or more levels or a mechanism within the system
- A sampling strategy that involves more than one level of the system
- Data collections tools that generate evidence about more than one level and/or a mechanism within the system
- Data analysis techniques for the quantitative and qualitative strands that support across-levels inferences independently or generate inferences
sufficient for supporting across-level meta-inferences during integration

- Integration techniques that support across-level meta-inferences by explaining at least two of the following: the nature of the system, the nature of the levels, the nature of the mechanisms

**Post-positivist paradigm.** A research paradigm grounded in the ontological assumption that there exists an absolute reality which cannot be fully known by any individual or collective of individuals. Although knowledge about reality can be generated, accumulated, and updated, it is necessarily incomplete given that it is limited by the confines of human perception and comprehension (Hatch, 2002).

**Research paradigm.** A socially negotiated framework for making compelling arguments about what knowledge is, how to do research, and what inquiries and results have value (Kuhn, 2012).

**Conceptual Foundations**

Conceptual foundations are critical to every aspect of research from inception and design to conclusion and dissemination. Currently, philosophical assumptions establishing the grounds for what constitutes knowledge and an adequate argument for new knowledge are particularly important in the field of mixed methods. The assumptions associated with mixed methods cannot be assumed in the absence of explication on the part of the researcher. The paradigmatic underpinnings of mixed methods research are still a subject of debate (e.g., Johnson, 2011; Maxwell, 2011). Theoretical frameworks are necessary to establish an assumptive starting point regarding what is important and what is known in a given substantive field of study (Creswell & Plano Clark, 2011). Finally, a personal position statement is necessary to provide insight into the
researcher’s intentions and perspectives on how to engage in the practice of research (Greene, 2007). In this section, I lay out the philosophical foundations that undergird this mixed methods study. Then, I describe the overarching theory that guided my initial assumptions about SMaLL development. Finally, I provide a personal statement to explain what brings me to this field of study and these methods.

Philosophical assumptions. Rather than adopting a prescribed research paradigm position, I took a dialectical stance to conduct this research. A dialectical stance does not reject commonly accepted research paradigms; rather it entails the respectful and intentional negotiation of multiple traditions of inquiry (Greene, 2007). The term research paradigm, usually traced back to Kuhn’s 1962 seminal work, *The Structure of Scientific Revolutions*, describes a socially negotiated infrastructure for grounding research practice and building compelling arguments for knowledge (Kuhn, 2012; Morgan, 2007). With repeated use and frequent acceptance, research paradigms are named (e.g. post-positivism, constructivism), reified, and, thus, presumed to be sufficiently well-defined and adequate axiomatic systems for conducting research. Conversely, with disuse and frequent repudiations, research paradigms are subject to debate and deposition (Kuhn, 2012).

Research paradigms have the potential to be useful and productive in that they can facilitate the identification and justification of knowledge. In other words, outlining a philosophy of knowledge that explains what knowledge is and how knowledge is generated makes research and its implications identifiable. Many social scientists agree that a research paradigm must entail agreement on the presumed requirements and implications of holding particular ontological, epistemological, and methodological positions (Greene & Hall, 2010;
Guba, 1990; Johnson, 2011; Morgan, 2007). Some agree that a research paradigm requires agreement on an axiological perspective (i.e., beliefs about what knowledge is valuable and what courses of action are good) (Mertens, 2012; Morgan, 2007). Others suggest that a research paradigm entails innumerable dimensions and, thus, are more aptly described as socially situated, unique worldviews of researchers (Koltko-Rivera, 2004; Morgan, 2007).

Taking a dialectical stance to combine quantitative and qualitative approaches demands vigilant attention to the components and structure of prevailing paradigms. However, it does not demand total reconciliation of paradigms in terms of ontology, epistemology, methodology, or axiology as suggested by the incompatibility thesis. Popular during the paradigm wars, the incompatibility thesis required taking an “either-or position” (Johnson & Christensen, 2012, p. 31). In other words, some argued that quantitative and qualitative paradigms could never be combined in a defensible research design on the grounds that underlying assumptions of quantitative and qualitative research approaches dictated the application of incommensurable rules of logic (Bergman, 2011; Guba, 1990; Morgan, 2007). However, the paradigm wars have subsided (Bryman, 2006b); although philosophical challenges remain, mixing approaches is often recognized as not only legitimate, but necessary (e.g., Guba, 1990; Maxwell, 2004). Morgan (2007, p. 55) issued a call to “Re-Kuhnify” any debate about paradigms by returning to Kuhn’s conception of a research paradigm. According to Kuhn, radical discoveries depend on a “piecemeal process” (Kuhn, 2012, p. 2) in which researchers and research communities occasionally re-negotiate paradigms by reexamining and restructuring values, beliefs, assumptions, and parameters for generating knowledge.

Because the notion of SMaLL is novel — perhaps even revolutionary — a dialectical
stance, or piecemeal approach, was an important consideration. Greene (2007) suggests that the greatest “generative potential” (p. 79) arises from tailoring paradigmatic perspectives to the research problem. A dialectical stance allows the researcher to be responsive to the context of the study and give attention to stakeholders’ philosophical foundations and sociopolitical values (Greene, 2007; Johnson, 2011, 2012).

In addition, a dialectical stance allows the researcher to design a research study that will support an effective argument regarding the findings. Crafting a dialectical argument entails two steps (Vokey, 2009). First, one must identify the constraints or blind spots of prior frameworks. Second, one must effectively argue that a new way of proceeding has all of the necessary advantages of alternative approaches and sufficiently resolves the complication. It follows that crafting a dialectical argument is not inherently superior to making an argument using a common approach; but, there are circumstances under which a common approach (e.g., a traditional quantitative approach or a traditional qualitative approach) is insufficient for making sound arguments for answers to significant questions.

For this study, I determined it was acceptable and essential to mix quantitative approaches and qualitative approaches in order to generate data and make reasonably compelling claims in answering novel research questions. I did not ignore concerns raised by incompatibility arguments. Instead, I took a dialectical stance for the purpose of intentionally managing paradigmatic differences in the process of collecting evidence with the most potential to justify inferences and meta-inferences.

Because I presumed SMaLL to be a phenomenon subject to cultural, behavioral, and neurobiological differences (see Theoretical framework section), this initial study of SMaLL
required multiple ontological and epistemological perspectives. To employ a post-positivist perspective is to adopt the assumption that there exists an objective, though not necessarily observable, reality to which everyone is subject and which can be summarized in terms of magnitude, direction, central tendency, and dispersion (Creswell & Plano Clark, 2011; Greene, 2007; Morgan, 2014). From a post-positivist paradigmatic position, a researcher can use counterfactual thinking, formal logic and mathematics, and operationally pre-defined measurements to generate quantitative descriptions of latent variables. Therefore, a post-positivist lens was appropriate to address the research questions directed at understanding cognitive mechanisms (i.e., neurobiological processes beyond human perception).

In contrast, to employ a constructivist paradigm is to adopt the assumption that multiple realities exist, each constructed by an individual which can only be subjectively interpreted or reconstructed by another (Creswell & Plano Clark, 2011; Greene, 2007; Morgan, 2014). From a constructivist paradigmatic perspective, a researcher can use observation, dialogue, and other semi and unstructured forms of data to reconstruct a vicarious understanding of another’s reality. Therefore, a constructivist lens was appropriate to address the research questions directed at understanding metacognitive thinking (i.e., an individual’s suppositions about his/her own cognitive processes).

Although the paradigm wars have subsided, the mixed methods community is still debating whether a single paradigm does — or should — exist to undergird all mixed methods research (Shannon-Baker, 2015). Because two different well-established paradigms could be reasonably applied to the study, I adopted a dialectical stance and intentionally negotiated the post-positivist and constructivist traditions of inquiry. This tack allowed me to answer questions
easily addressed from an existing paradigm using traditional approaches and enabled me to answer the remaining research questions using mixed methods techniques to integrate the data and findings and generate meta-inferences.

For clarification, Figure 1.1 shows a conceptualization of my understanding of what it means to take a dialectical stance. The outermost frame illustrates my view that the purpose of taking a dialectical stance is to craft a defensible dialectical argument. The overlapping circle and triangle illustrate two different paradigms in negotiation. The nodes on the circle and triangle show that, during the design process and throughout the study, a dialectical stance requires a researcher to acknowledge the tension between paradigms and carefully fit the paradigmatic components of distinct research traditions together. The dotted lines show flexible boundaries that influence the way in which the paradigms can be negotiated. The innermost circle represents the unique characteristics and perspective of the researcher that may influence how paradigms are mixed. The oval represents the surrounding research communities that may set axiological boundaries to influence what can be accepted as good practice and valuable knowledge. Ultimately, taking a dialectical stance means customizing a research design and presenting a novel form of argument for what knowledge is, how to do research, and what inquiries and results have value.

**Theoretical framework.** In this section, I discuss the multilevel theory of human development that guided this study. I explain how the theory was adapted and applied to this study of SMaLL. Then, I discuss the theoretical assumptions at each of the three levels: cultural, behavioral, and neurobiological.
Figure 1.1. Conceptualization of using a dialectical stance to navigate multiple research paradigms to construct a defensible argument.
This study was designed in keeping with the developmental bio-cultural co-
constructivism (DBCCC) theory of ontogenetic, or lifespan, human development. DBCCC
claims the “brain and culture are in a continuous, interdependent, co-productive transaction and
reciprocal determination” (Baltes et al., 2006, p. 3). Thus, the theory holds that the architecture
of learners’ brains has an effect on the nature of culture. Furthermore, the theory holds that the
constraints and affordances of the cultural environment impact the development of learners’
neurological structures and cognitive processing. Notably, DBCCC implies more than mere
interaction between nature and nurture. It suggests both collaborative production and reciprocal
modification. According to DBCCC, the development and evolution of culture, behavior, and
neurobiology are inseparable. To explain one, it is necessary to account for the others;
furthermore, to change one is to impact the others.

The central concern of DBCCC is understanding the affordances and constraints on the
lifespan development of the individual. The theory guards against claims of biological
determinism and social determinism in two ways. First, it holds that “the brain and culture are
independent sources and full-fledged partners and reciprocal modifiers” (Baltes et al., 2006, p.
8). Second, it holds that individuals are active agents in their development (Bjorklund, 2012). In
other words, as behaving organisms, humans are constantly adapting and interacting in ways that
bring about change in their neurobiology and culture (Baltes et al., 2006).

Importantly, DBCCC presumes that lifespan development is limited in plasticity (Baltes
et al., 2006). More specifically, the degree to which society can change in terms of culture, the
degree to which an individual can change in terms of behavior, and the degree to which the
human brain can change in terms of function and structure constrain the potential for human
development. In addition to providing a framework for understanding how development occurs, DBCCC is useful for inspecting the boundary conditions of neurobiological plasticity, behavioral plasticity, and cultural plasticity.

Figure 1.2 illustrates SMaLL as a multilevel phenomenon through the lens of DBCCC. The large triangle at the center of the diagram shows that DBCCC is a framework for understanding human development. The inner triangle shows that SMaLL is conceptualized as a dimension of human development. The nodes on the ring illustrate the three levels of plasticity that influence development: cultural, behavioral, and neurobiological. Finally, the text on the outer ring illustrates malleable aspects of SMaLL related to each of those levels: language, metacognition, and cognition. Applied to SMaLL, DBCCC holds that the ability to read and write the language of mathematics develops as a result of complex, ongoing reciprocal interactions between classroom context, students’ attempts to learn, and students’ brains.

Symbolic mathematics, the written language of mathematics, is a cultural construction. The writing system evolved with the study of mathematics and science. In the 21st century, modern mathematics is completely dependent on symbolic mathematics (Van Dyck & Heeffer, 2012). Like other languages, the ways in which symbolic mathematics is habitually used — transcribed or translated — in various contexts can differ (Hall et al., 2011). In addition, scientific advances and events can change the language itself (Devlin, 2000; Guillen, 1995; Kuhn, 2012; Schoenfeld, 2004; Van Dyck & Heeffer, 2012). For this study, the cultural plasticity of the language of mathematics was also presumed to be related to educational policy and curriculum with the potential to change the environment in mathematics classrooms.
Figure 1.2. Conceptualization of SMaLL development illustrating the relationships between language, metacognition, and cognition given the theory of developmental bio-cultural co-constructivism (DBCCC). Adapted from Baltes et al. (2006)
Metacognition is an introspective behavior that entails reflection on mental processes involved in a task (Bjorklund, 2012). Metacognition may entail value judgments about the task, an awareness of the strategies that can be applied to the task, or an assessment of the degree to which various strategies can be (or have been) successfully applied (Bjorklund, 2012; Schneider & Artelt, 2010). Metacognition, then, appears to mirror the three subprocesses of lifespan development: selection, optimization, compensation (Baltes, 1997; Lindenberger & Lövdén, 2006). According to Baltes (1997), development requires selecting opportunities and developmental targets. In addition, development is a process of coordinating resources to optimize development “as a movement toward increased efficacy and higher levels of functioning” (Baltes, 1997, p. 371). And, when resources are overextended, development involves compensatory measures to avoid regression (Lindenberger & Lövdén, 2006). For this study, the behavioral plasticity of SMaLL development was presumed to be related to learners’ goals and stance towards using the language of mathematics as a tool to extend mathematical knowledge, communication, and learning.

Cognition is a manifestation of neurobiological activity in the brain. Although the brain has long been associated with learning, the study of its structure and function as a basis for cognitive development is relatively new due to the methodological challenges of inspecting the brain during cognition (Casey, Giedd, & Thomas, 2000). In the current literature, plasticity in brain development is commonly classified in three categories: experience-independent, experience-expectant and experience-dependent (Baltes et al., 2006; Kolb & Gibb, 2014). Experience-independent development depends on genetic instructions and not sensory input. Most experience-independent neurobiological development occurs during prenatal development
when the basic structure of the brain is forming. Experience-expectant development depends on the coincidence of genetic instructions and sufficiently typical conditions (Bjorklund, 2012). For example, children with limited auditory input (e.g., due to chronic fluid in the middle ear) during sensitive periods may not develop typical neurobiological organization in the auditory cortex.

Most academic learning involves experience-dependent neurobiological development. In other words, it changes the brain by co-opting structures of the brain and recycling neurons with evolutionarily similar functioning (Dehaene, 2005).

Geary (1995) suggests that experience-dependent development can be thought of as biologically primary or biologically secondary depending on the nature of the experience necessary for change. Biologically primary development occurs similarly across cultures as a consequence of routine rewarding experiences. For example, counting, which develops similarly and relatively effortlessly across cultures, can be understood as biologically primary and experience-dependent (Gallistel & Gelman, 2005). In contrast, biologically secondary development is culture-specific, laborious, and requires sustained instruction and practice. Geary (1995) cites reading, which has only become commonplace with the advent of formal education (Wolf, 2007), as a biologically secondary cognitive domain. Likewise, for the purpose of this study, SMaLL was presumed to develop as a biologically secondary, experience-dependent cognitive domain.

**Personal position.** I engaged in reflective practice to repeatedly challenge myself to rely on the philosophical assumptions and theoretical frameworks previously described to design and conduct this work. However, I acknowledge that the whole of my experiences both professional and personal shaped, at least to some degree, this study in ways that are not accounted for by the
previously discussed foundations. Here, I tell my “mixed methods story”, as recommended by Greene (2007, p. 60), as a means to offer insight into the unique perspective I brought to bear on the project. My story focuses on the salient experiences that brought me to this particular study of SMaLL and the conceptualization of integrity that guided me. Both contributed to my interest in pursuing these research questions and my ability and willingness to use mixed methods to answer them.

The earliest conceptions of this research project grew out of my experiences as a mathematician and mathematics teacher. The language of mathematics is, I think, beautiful, eloquent, and robust. In addition to being a source of recreation for me, the language is an indispensable tool for expressing abstract ideas and rendering complex problems solvable. I do not know exactly how I learned the language of mathematics. I do know, however, that I have skills that some of my peers do not. Looking back, I can recall moments when I recognized a contrast between my own literacy and that of others.

As an undergraduate majoring in mathematics, as I began to study the foundations of higher mathematics and modern algebra (Durbin, 1985; Fletcher & Patty, 1988), I witnessed peers in my cohort change their majors as we entered our junior year: *What happened to plugging in numbers and solving for* \( x \)? *I don’t know how to read and write mathematical proofs.*

As a master’s student, my decision to pursue mathematics education despite my success in an applied mathematics program was met with disappointment: *Why don’t you stay? Women with your skills — the ability to navigate the languages of advanced mathematics and computer science — are hard to find.* As a doctoral student, I was surprised to hear a student in an advanced statistics course commiserating with classmates: *Did you complete the reading?*
Surely, the professor didn’t expect us to read those equations. I know now that my ability to persist in reading and writing in the language of mathematics has allowed me to read-to-learn and, in turn, given me opportunities I might not have had otherwise.

As a mathematics teacher, I noticed many students did not experience the language of mathematics as I did. For some it seemed effortful — and on occasion traumatic — to engage with the written, or symbolic, form of mathematics. Whenever I was teaching or tutoring students — adults in community colleges and 4-year institutions, adolescents in high schools and middles schools, and children in elementary school — it seemed we were negotiating language as much as we were algorithms and calculations. My successes and failures in teaching and inspiring an appreciation for the language of mathematics among my students motivated me to question how students learn to read and write symbolic mathematics.

As a doctoral student, taking a comprehensive approach to coursework helped me to appreciate the magnitude of my question and the necessity of refining my focus and assessing the significance of my research agenda. The literature took me on an interdisciplinary journey through, among other things, mathematics, cognition, curriculum, literacy, linguistics, and research methods. At times, I was frustrated by the challenge of inspecting the connection between math and reading from a novel perspective. However, taking opportunities to share my progress allowed me to see how my questions were important to others. For example, a mathematics professor at UC followed up on a poster presentation to ask if a measurement tool was ready for use to inform students’ mathematics course enrollment decisions. Also, students at an interdisciplinary paper presentation described reading symbols as a recognizable obstacle to learning mathematics. Finally, interacting with pre- and in-service teachers in professional
development workshops confirmed that practitioners have critical questions about the role of literacy and cognitive theories in mathematics classrooms. Despite dedicated efforts, I have not yet found any extant theory of SMaLL, as it is defined for this study, to address my questions. The combination of the gap in the literature and anecdotal evidence that stakeholders agree SMaLL is critical to mathematical learning provided the inspiration for the substantive focus of this work.

As my substantive expertise developed, my identity as a methodologist also evolved. When I set out to draft a plan for my coursework, a professor asked me whether I was interested in methods or methodologies. Seeing my confusion, he offered an explanation of the difference. Since then my interest in methodologies, or the process of choosing and designing methods for particular tasks, has grown for two reasons. First, calls for evidence-based practices in classrooms have created demand for educational research, particularly in mathematics education. As reliance on research has increased, educational research has escalated as a high stakes activity. In turn, it is necessary to question the sufficiency of research designs and the ways results are (or are not) used to inform instructional practice. Second, inspecting methodologies presents interesting logical and ethical issues. The logical issues interest me as a mathematician who conducts thought experiments by adopting a set of assumptions and mapping the consequences. The ethical issues interest me as an educator who aspires to contribute to improving mathematics education in a meaningful way. Wrestling with how to negotiate logic and ethics for practical purposes is a compelling issue for me as a researcher and methodologist.

It is not uncommon, in the course of conducting social sciences research, for unexpected circumstances to arise. There are “contested spaces in which philosophy, perspective, and
practice encounter one another — and either join together in reciprocal, respectful, and mutually beneficial conversation or walk right by without noticing one another — or perhaps pretending not to notice” (Greene, 2007, p. 59). Those contested spaces present ethical and logical dilemmas for researchers. Here, I provide some insight into my understanding of integrity and my habits of thinking to give the reader a general sense of my approach to research dilemmas with ethical and logical dimensions. In Chapter 3, I describe some of the contested spaces that arose during this study and my responses to them in detail.

As an ethical dilemma, contested spaces in research call for a practical response to this question keeping the well-being of others in mind: What should be done? I believe Calhoun (1995) makes a convincing argument that three common interpretations of integrity reduce it to “something else: to the conditions for unified agency, to the conditions for continuing as the same self, and to the conditions for having a reason to refuse cooperating” (p. 252). Furthermore, I was convinced to agree that “integrity calls us simultaneously to stand behind our convictions and to take seriously others’ doubts about them” (Calhoun, 1995, p. 260). With this view, I do not view contested spaces as threats to integrity. Instead, contested spaces are an opportunity to deliberate with myself and others and, ultimately, reevaluate and reaffirm the nature of my integrity.

As a logical dilemma, contested spaces in research call for a practical response to this question keeping the possibility of addressing the research problem in mind: What should be done? As a mathematician, I was trained to impose or remove assumptions for the sake of determining what was necessary or sufficient to draw conclusions. Because statistics is grounded in the study of mathematics and calculus, I began my research career with reasoning
habits suitable for quantitative research. With additional training, I cultivated the ability to adopt
the assumptions and reasoning habits necessary for action research and qualitative research. It is
my ability to reason in multiple ways (e.g., deductive, inductive, abductive) from various
paradigmatic perspectives that makes it possible for me to adopt a mixed methods way of
thinking and take on the dialectical stance required to explore SMaLL as a multilevel
phenomenon.

When I became a doctoral student, I imagined myself a bridge builder. I wanted to build
bridges between mathematics and literacy, and I wanted to build bridges between research and
practice. To make inroads on both, I designed this study to generate new ways of thinking about
the nature of literacy in mathematics classrooms using an approach that allowed me to explore
multiple facets of SMaLL among adolescents in a middle school. In each contested space, I took
an action that I defend and believe others can also endorse as ethical behavior towards others and
reasonable action in the quest to better understand SMaLL.

**Delimitations and Limitations**

This study was delimited by the definition of SMaLL. Taking the perspective that
interacting with symbolic mathematics entails specialized reading and writing abilities shaped
the scope of this study. Although SMaLL involves reading and writing in theory, this study was
delimited by research questions focused on reading. These delimitations influenced the
population of interest, the mathematical content area of interest, and the measurement tool
selection.

Although SMaLL can conceivably be investigated among multiple populations and
across multiple subfields of mathematics, this study was delimited in terms of population and
content by the approach to sampling. The sample included participants attending a single middle school in an exemplary district operating under the CCSS. Thus, this study was restricted to an investigation of adolescents of middle school age. In addition, this study was restricted to an investigation of SMaLL as it relates to the mathematics prescribed for adolescents in seventh and eighth grades by the CCSS.

SMaLL is presumed to be influenced by a host of variables. However, the variables under consideration were delimited based on a review of variables related to English language literacy development in general and reading acquisition in particular. In addition, some variable delimitations were imposed to make the study feasible. For example, although a more direct inspection of neurobiological aspects of SMaLL is possible given modern advances in equipment, the budget of this study did not permit the use of sophisticated equipment. Also, minimizing the time investment for participants limited the possibility of collecting data related to additional variables of interest.

Methodologically, this study was limited by the lack of measurement tools. Specifically, there were no instruments available to assess print exposure to symbolic mathematics, reading habits related to symbolic mathematics, or the ability to read symbolic mathematics. Because instrument development was not a primary goal of the study, the study employed measurement tools adapted from the fields of English language literacy and cognitive psychology. As a byproduct of this study, empirical evidence for the efficacy of the tools is available; however, the psychometric properties of the tools used in this study are not fully understood.

Finally, this study was limited by nature of the research site. The data were collected at a public school in collaboration with administrators, teachers, and students. Educational goals and
policy influenced time lines and access to prospective participants. In addition, the demographics of the district constrained the range of diversity among participants.

**Summary of Chapter 1**

This chapter described the emergence of my conception of SMaLL and a rationale for exploring what SMaLL is for adolescent students in middle grades learning mathematics under the CCSS. I highlighted the significance of understanding what SMaLL is and the implications this study has for researchers in mathematics education and related fields, mixed methods methodologists interested in multilevel developmental phenomena, and practitioners and policymakers with the power to influence the way adolescents experience SMaLL. I explained how I took a dialectical stance to navigate post-positivist and constructivist traditions to develop a multilevel concurrent mixed methods research design. I discussed the theoretical frameworks that guided the study and the conceptualization of SMaLL as a multilevel phenomenon that entails language, metacognition, and cognition. Finally, I itemized the delimitations of this study to call attention to questions and issues this study does not attempt to answer or address.
Chapter 2: Literature Review

This chapter is a summary of available research that provided insights into, and evoked questions about, symbolic mathematics language literacy (SMaLL) and how to explore SMaLL. In general, I used the developmental bio-cultural co-constructivism (DBCCC) framework to guide this review of the literature and select research relevant to the primary research question: What is SMaLL for adolescent students in middle grades learning under the Common Core State Standards (CCSS)? DBCCC, which claims human development entails reciprocal and interdependent relationships between variables at multiple levels, did not dictate a linear approach to reviewing the literature. This, coupled with initiation as the rationale for this mixed methods study, demanded a relatively broad literature review of topics and methods connecting cultural, behavioral, and neurobiological aspects of development.

In order to make connections within and across fields of study and levels of human development, I developed guiding questions. I use these guiding questions (see Figures 2.1 and 2.2) to facilitate this discussion of the literature by answering the apparently disparate questions in cohesive units and then making explicit the connections between those units. Figure 2.1 shows the relationship between the guiding questions and DBCCC. In the figure, the guiding questions are placed around the DBCCC model to be suggestive of connections between questions and levels. Because cultural, behavioral, and neurobiological plasticity are woven together, the placement of the questions is not intended to convey a fixed mapping onto DBCCC, but rather illustrate the complexity and fluidity of relationships.
Figure 2.1. Map of guiding questions for a literature review revolving around developmental bio-cultural co-constructivism (DBCCC) as a framework to determine what is known about symbolic mathematics language literacy (SMaLL).
Figure 2.2. Literature review map: Guiding questions arranged to illustrate the literature review process and generation of research questions and methods.
In contrast to Figure 2.1, Figure 2.2 shows a mapping of the order in which the questions are addressed in this chapter and how they worked together to refine the research questions and inform the research design process. First, I review educational policies and curricular theories that guide the implementation of formal education. Next, I review issues in literacy development with implications for instructional practice. With that context established, I return to the definition of SMaLL to clarify what it is, and what it is not, for the purpose of this study. Then, I discuss how the nature of the language of mathematics, differences in languages, and various symbol theories informed this study. Then, I review methods with implications for developing this study of a multifaceted multilevel human development phenomenon. In conclusion, I draw on empirical research to highlight how this study of SMaLL is situated in the current literature and makes important interdisciplinary contributions — substantive and methodological — to educational research.

**Educational Policies**

The question of how education should be administered dates back to Plato. Although — or perhaps because — disputes about what a quality education entails have endured across the centuries, the mission to administer education to all children is a relatively modern phenomena. In the United States, formal education was only available to a select few before the rise of compulsory public schooling at the turn of the 20th century (Flinders & Thornton, 2009). A century after committing to educate all children, what constitutes a valuable education and the means for providing such an education are still vigorously contested. Those debates give rise to educational policy. *Educational policy* comes into being, as a process and a product, in response to educational problems (Fowler, 2003). As a process it entails reflection on and deliberation
about values and practices. As a product it entails formally “expressed intentions and official enactments as well as its consistent patterns of activity and inactivity” (Fowler, 2003, p. 9). Policy change typically occurs in response to economic changes, demographic trends, or ideological shifts.

During the 1980s, the *Nation at Risk* report (National Commission on Excellence in Education, 1983) framed academic excellence and accountability as critical and urgent problems of education. In the wake of the report, educational policy changed dramatically in terms of product and process. Authority to draft various forms of educational policy shifted; and curriculum and standardized tests became essential elements of policy implementation (Fowler, 2003; Spring, 2009a, 2009b).

As of 2015, the majority of states had adopted the CCSS as a curricular guide (National Governors Association Center for Best Practices [NGA], 2016a). Standardized tests, which became a federal requirement for funding in the No Child Left Behind Act of 2001, remain the primary measuring stick for educational progress. For more than a decade, education has been delivered in the context of educational policy in which test scores are associated with high-stakes decisions (Au, 2011) and high-stakes goals. Recently *A Blueprint for Reform* stated the goal of compulsory education this way: “Every student should graduate from high school ready for college and a career, regardless of their income, race, ethnic or language background, or disability status” (U.S. Department of Education, 2010, p. 3). Public schools have been required, under threat of penalty, to report statistics demonstrating forward progress towards this goal (Spring, 2009a). And, students are required, sometimes under threat of penalty, to achieve test scores to demonstrate sufficient mastery for annual advancement and graduation (e.g., Ohio

Policy changes, including curricular shifts and accountability measures, in the new millennium have been grounded in concern for the future of American culture and the future of individual students. For example, changes associated with No Child Left Behind were intended to be a compassionate and fair approach to returning the United States to its former status as the best educated, most competitive nation in the world while attending to the particular needs of a diverse student population (U.S. Department of Education, 2002). Although some may agree that a dedication to making it possible for every student to go to college demonstrates care, some submit that requiring achievement in prescribed ways at prescribed rates is neither compassionate nor fair (Noddings, 2003). Two complaints can be made about recent educational policy change: First, policy has been formalized in the absence of evidence that all students are biologically capable of sustaining the necessary learning trajectory (e.g., Brady, Braze, & Fowler, 2011; Cirino, 2010; Geary, 2011). Second, and equally important for the purpose of this study, policy has been formalized without careful consideration of differences in students’ aspirations and goals for education (e.g., Noddings, 2013). In other words, it is not clear whether every student can, even with theoretically ideal instruction, meet the achievement requirements; nor, is it clear that all students wish to pursue the comprehensive college-ready goals outlined by policy makers (Noddings, 2013).

Understanding educational policy is not a specific goal of this study per se. However, to understand SMaLL as a multilevel phenomenon through the lens of DBCCC requires some background knowledge of the cultural context in which students learn mathematics. This brief review of current educational policy, and the grounds for administering education in particular
ways, suggests that adolescents in middle schools in the United States receive, at least implicitly, these pervasive messages: The purpose of K-12 education is to lay the groundwork for financial security upon graduation from high school or college. All students, regardless of any kind of individual difference, are capable of acquiring basic content knowledge and developing basic skills at the same rate. As a corollary, when goals are not met, the blame lays with the students and the teachers.

**Curriculum Theories**

The question of what should be taught in schools has also long been debated. In popular culture in the United States, early schools are often portrayed as simple and dedicated to the three R’s: reading, ‘riting, and ‘rithmetic. However, evidence suggests that early American curriculum theorists, such as Franklin Bobbitt and John Dewey, discussed quite sophisticated ideas about the content of schooling as well as the purpose of acquiring particular knowledge and skills (Flinders & Thornton, 2009). Mathematics has a long-standing history of contention. Rationales for what mathematics should be taught shift as theories about why mathematics should be taught evolve. For example, in the wake of World War II, as arguments were being made about how to usher education into the scientific age, theorists claimed the mathematics curriculum written by the National Education Association in the previous century was unnecessarily rigorous (Flinders & Thornton, 2009).

The CCSS remain, as of this writing in 2016, the prevailing instantiation of curriculum theory. Framed as an inert educational policy of intent, the CCSS does not describe how instruction should be delivered. However, the CCSS is a critical policy document in that it provides a description of standards, or learning goals, for English language arts and mathematics
from kindergarten to high school graduation. Given its role in the context of legislated educational policy, the CCSS functions, in practice in the states that have adopted it, as a description of what should be taught and what will be tested and monitored for accountability purposes (Au, 2011). In theory, adherence to the curricular progression of the CCSS will ensure that, by the end of high school, all students are "college and career ready" (NGA, 2010a, p.3; NGA, 2010b, p. 57).

Although, the CCSS does not describe instructional practice, it does offers insight into the aims of instruction practice. In addition, it offers insight into the ways that literacy education and mathematics education are framed. Specifically, the CCSS clearly delineate the differences between English language arts curriculum and mathematics curriculum for K-12 instruction. Thus, the CCSS provides a more clear picture of the national academic culture within which this study was conceived and conducted.

**A portrait of literacy.** The CCSS for English Language Arts (CCSS-ELA) presents “an integrated model of literacy” (NGA, 2010a, p. 4). In other words, reading, writing, speaking, listening, and language skills are conceptualized as inseparable components of literacy. As such the standards are accompanied by a “portrait of literacy” (p. 7) to illustrate how literate individuals seamlessly employ the seven habits of literacy. First and foremost, the literate read complex texts as self-directed learners. They develop content knowledge through purposeful reading and writing. In the course of expressing oneself orally or in writing, they adjust language to contextual and discipline-specific demands. Likewise, in the course of listening or reading, the literate comprehend the precise language and, in addition, discern the assumptions and claims of the speaker. Literate readers identify supporting evidence (or lack there of); and
literate writers present evidence to support their claims. Finally, the literate select appropriate tools to apply these skills. In sum, the literate use multiple media to communicate effectively and constructively across cultures and despite differences in perspectives.

According to the CCSS-ELA, this vision of integrated literacy is not intended to extend across all educational content areas. Discipline-specific reading and writing standards for grades 6 to 12 for history/social studies, science and technical subjects are stated independently of the English language literacy standards. Importantly, the CCSS-ELA explicitly excludes mathematics from the literacy standards in a statement of “what is not covered by the standards” (NGA, 2010a, p. 6). Other than a brief mention of symbols and equations as potential features of discipline-specific texts, the ability to read and write symbolic mathematics is not addressed in the CCSS-ELA. This suggests that, in middle schools operating under the CCSS, adolescents are being taught under the assumption that English language literacy development has little, if anything, in common with mathematical development.

**Mathematical practices.** In contrast to the CCSS-ELA portrait of literacy, the CCSS for Mathematics (CCSS-M) portrays college- and career-ready mathematics proficiency. Specifically, it describes a functional set of practical skills presumed to be sufficient for success in math-related endeavors upon graduation. The CCSS-M claims to provide coherence and organization — within and across grades — tailored to how students learn mathematics. Given the fact that literacy as it applies to mathematics is not specified explicitly, I analyzed the CCSS-M to determine the degree to which SMaLL (i.e., the ability to read and write using the writing system of the language of mathematics) is embedded in the standards. First, I present the analysis of the mathematical practices. Then, I present the analysis of the grade-level
standards.

The mathematical practices, or habits associated with doing mathematics across grade levels and domains, were adapted from the National Council of Teachers of Mathematics’ (2000) process standards and the National Research Council’s *Adding It Up* report (National Research Council, 2001). As such, experts agree that these practices are central to the essence of doing mathematics. Each of the eight practices suggest that reading and writing symbolic mathematics are critical skills (see Table 2.1). Making sense of problems and persevering in solving them means repeatedly returning to the text features associated with the problem to re-assess comprehension. Reasoning abstractly and quantitatively, as it is described by the CCSS-M, is not intended to convey simple cognitive processes. Rather, it entails a habit of contextualizing and decontextualizing abstracted mathematical symbols to make sense of relationships. Although arguments can be made and critiques can be presented orally or visually, constructing arguments in mathematics is traditionally done by writing proofs, identifying examples, and drawing attention to counterexamples using the conventions of the writing system of the language of mathematics. Modeling with mathematics entails understanding a problem (often presented as text), writing symbolic mathematics to represent the problem, and manipulating the symbols to solve the problem. Using technology strategically requires the ability to read and write symbolic mathematics. For example, students may use the internet to learn more about the Pythagorean theorem (commonly presented in the form, $a^2 + b^2 = c^2$) or use the auto-sum feature of Excel (identified with by a summation symbol, $\sum$) to make calculations. Attending to precision refers to the careful use of oral and written mathematical language to convey particular meaning and limit ambiguity. Looking for and making use of patterns requires reading
and/or re-reading to the point of noticing similarities. In contrast, looking for and expressing regularity in reasoning entails writing and/or re-writing to the point of noticing similarities. In summary, the mathematical practices cannot be reduced to reading and writing skills; yet, the mathematical practices clearly indicate that reading and writing texts with discipline-specific features, including symbolic mathematics, is inherent to learning and doing mathematics.

For further insights into the way the CCSS frames English language arts and mathematics as distinct content areas, I compared and contrasted the visions of college- and career-readiness for English language literacy and mathematics proficiency. Framing, or classifying, curriculum has implications for communicating who and what different kinds of knowledge are for (Au, 2012). I drew on Cheuk’s (2012) comparison of the CCSS-M mathematical practices, the CCSS-ELA portrait of literacy, and Next Generation Science Standards (Next Generation Science Standards Lead States, 2013). To better understand the relationships across content areas and make recommendations for accommodating English language learners, Cheuk (2012) devised a Venn diagram to present the intersections and distinctions. Adaptations of the diagram appear on numerous websites (e.g., http://betterscienceteaching.com/2013/07/31/ccss-and-ngss-a-venn-diagram/ and http://www.classroomscience.org/the-power-of-linking-science-to-common-core). Recently, Cheuk’s current version was featured on Education Week’s blog (Heitin, 2014). Figure 2.3 shows a simplified version of the diagram to draw attention to the repeatedly endorsed understanding of the relationship between mathematical practices and literacy habits as they are described in the CCSS. Notice that there are four commonalities: building a strong knowledge base, constructing and critiquing arguments, using evidence to support claims, and using media and tools strategically.
Table 2.1

*Symbolic mathematics language literacy skill demands by CCSS-M Mathematical Practice*

<table>
<thead>
<tr>
<th>Mathematical Practice</th>
<th>Summary of evidence of SMaLL demand</th>
<th>Skill</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Make sense of problems and persevere in solving them.</td>
<td>Students return to the problem to re-read and review as necessary. Examples suggest this entails reading algebraic expressions and equations and assessing their relationships to other text features.</td>
<td>R</td>
</tr>
<tr>
<td>2. Reason abstractly and quantitatively.</td>
<td>Students use writing as a means to symbolically represent and reason in the process of practicing mathematics. Examples suggest that this entails inspection of the meaning of the referents of symbols and flexible application of symbolic mathematics.</td>
<td>W</td>
</tr>
<tr>
<td>3. Construct viable arguments and critique the reasoning of others.</td>
<td>Students read and write to create and critique arguments. References to analysis of data, cases, counterexamples, and logical progression of statements imply that symbolic mathematics is expected to be a common feature of the relevant texts and products.</td>
<td>R, W</td>
</tr>
<tr>
<td>4. Model with mathematics.</td>
<td>Students write equations and functions to represent real-world situations and solve problems.</td>
<td>W</td>
</tr>
<tr>
<td>5. Use appropriate tools strategically.</td>
<td>Students use technology appropriately to explore concepts. Examples suggest this entails reading (e.g., using the internet as a resource) as well as writing (e.g., using the function feature of Excel).</td>
<td>R, W</td>
</tr>
<tr>
<td>6. Attend to precision.</td>
<td>Students use precise language to communicate orally and in writing. Examples clarify that this is not limited to attending to vocabulary in speech and narrative writing. In writing, students use symbolic mathematics consistently, conventionally, and unambiguously.</td>
<td>W</td>
</tr>
<tr>
<td>7. Look for and make use of structure.</td>
<td>Students become aware of patterns in the process of reading. Examples referencing the distributive property and algebraic expressions confirm that this practice is particularly relevant to recognizing patterns in text written using symbolic mathematics.</td>
<td>R</td>
</tr>
<tr>
<td>8. Look for and express regularity in repeated reasoning.</td>
<td>Students become aware of patterns in the process of writing. Examples referencing the Cartesian coordinates, the equation of a slope, and the expansion of polynomial expressions confirm that this practice is particularly relevant to recognizing orthographic patterns while writing symbolic mathematics.</td>
<td>W</td>
</tr>
</tbody>
</table>

*Note:* R = Reading; W = writing; Adapted from CCSS-M (NGA, 2010)
Figure 2.3. Relationship of English language literacy habits (ELA1-7) and mathematical practices (M1-8) of a college and career ready student as defined by the Common Core State Standards. Adapted from Cheuk (2012).

For the purposes of this study, I am interested in the perception that independent reading of complex texts is unique to English language arts and not applicable to the mathematical practices. According to the previous analysis (summarized in Table 2.1), independent reading of
complex text is inherent to four out of eight mathematical practices. In other words, independent reading is not unique to English language arts. Similarly, it is not the case that adjusting to the demands of audience, task, purpose, and discipline is irrelevant to mathematical practice. On the contrary, attending to precision, constructing arguments, and critiquing the reasoning of others demands adjustments in vocabulary, format, and media to account for the background knowledge of peer learners as well as discipline-specific conventions. This comparison suggests that attending to varying perspectives and cultures may be the only literacy habit that is not critically essential to any of mathematical practices defined by the CCSS-M. This review of the CCSS-ELA and CCSS-M illustrates the degree to which literacy and mathematics are framed as distinct. Notably, this framing suggests a culture in which reading and writing are not conveyed as valuable in the context of learning mathematics.

**Grade level standards.** A review of the grade-level standards indicates that students are faced with reading and writing an increasing number of symbols as well as increasingly complex combinations of symbolic mathematics (see Table 2.2). In the early grades, students use simple symbols. For example, in kindergarten students might read or write Arabic numerals and $+$ or $-$ symbols as they learn to decompose numbers. In first grade, students might read or write more complicated combinations of symbols to apply the commutative and associative properties of addition as a strategy for learning the relationships between math facts. By the end of first grade, students see a variety of non-alphabetic symbols (e.g., $+$, $-$, $>$, $<$, and $=$) that cannot be read, or translated into speech, by applying English language reading strategies such as phonics. During second and third grade, the symbolic mathematics presented to students increases to include new operations, $\times$ and $\div$, as well as characters required for measuring money, time,
lengths, weights, and volumes (e.g., \(\varepsilon, \$, \text{cm, m, g, kg, l}\)). The introduction of fractions in the third grade introduces the vinculum, the line used to separate the numerator and the denominator. Although the symbol does not have a unique literal translation in English, recognizing it as an orthographic pattern is critical to translating the text to speech. The following year, the decimal is introduced to represent fractional amounts without the vinculum. By fifth grade, the standards state explicitly that students are expected to read and write symbolic mathematics. In addition, the minus sign, \(-\), is introduced with a new mathematical meaning and translations such as *opposite* or *negative*. Similarly, nearly every standard that calls for calculating, comparing, interpreting, evaluating, applying, or understanding a new idea requires learning to read and write symbolic mathematics in a novel way.

Where Cheuk’s (2012) comparison of the portrait of literacy and mathematical practices suggested that literacy is of limited consequence in mathematics, my analyses of the standards suggested quite the opposite. The CCSS-M clearly calls for specialized literacy skills to interact with the written language of mathematics. Reading and writing symbolic mathematics is embedded in the CCSS-M across grade levels, content domains, and mathematical practices. Thus, SMaLL is a hidden curriculum at work in CCSS academic culture. This study is not aimed at understanding SMaLL as a hidden curricular element of the CCSS. However, given the theory of DBCCC, it is informative to note that hidden curriculum can contribute to insidious differences in classroom culture and students’ mathematical development (e.g., Anyon, 1980; Martin, 2012). This suggests that understanding how the language of mathematics operates at the classroom level has implications for developing a model of SMaLL.
Table 2.2

*Progressive introduction of symbolic mathematics language by grade and domain*

<table>
<thead>
<tr>
<th>Grade</th>
<th>Domain</th>
<th>Context</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>OA</td>
<td>Decomposing numbers.</td>
</tr>
<tr>
<td></td>
<td>NBT</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>OA</td>
<td>Applying commutative, associative, and transitive properties. Applying addition and subtraction strategies. Identifying true and false sums and differences in the form of an equation. Determining the missing number. Comparing 2-digit numbers.</td>
</tr>
<tr>
<td></td>
<td>NBT</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>NBT</td>
<td>Comparing 3-digit numbers, working with time and money.</td>
</tr>
<tr>
<td></td>
<td>MD</td>
<td></td>
</tr>
<tr>
<td></td>
<td>NF</td>
<td></td>
</tr>
<tr>
<td></td>
<td>MD</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>OA</td>
<td>Decomposing fractions. Interpreting fractions as a product. Using decimal notation. Converting to larger or smaller units.</td>
</tr>
<tr>
<td></td>
<td>NBT</td>
<td></td>
</tr>
<tr>
<td></td>
<td>NF</td>
<td></td>
</tr>
<tr>
<td></td>
<td>MD</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>OA</td>
<td>Writing expressions. Comparing decimals. Applying operations to fractions with unlike denominators. Applying formulas to solve problems.</td>
</tr>
<tr>
<td></td>
<td>NF</td>
<td></td>
</tr>
<tr>
<td></td>
<td>MD</td>
<td></td>
</tr>
<tr>
<td></td>
<td>NS</td>
<td></td>
</tr>
<tr>
<td></td>
<td>EE</td>
<td></td>
</tr>
<tr>
<td></td>
<td>G</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>RP</td>
<td>Applying complex fractions. Interpreting coordinates on a graph. Extending properties to integers. Applying inequalities to real world problems.</td>
</tr>
<tr>
<td></td>
<td>NS</td>
<td></td>
</tr>
<tr>
<td></td>
<td>EE</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>NS</td>
<td>Using a symbol to refer to a constant. Applying properties of exponents, expressing very large and very small numbers using scientific notation. Applying the equation of a line. Interpreting a slope and the unit rates.</td>
</tr>
<tr>
<td></td>
<td>EE</td>
<td></td>
</tr>
<tr>
<td></td>
<td>F</td>
<td></td>
</tr>
<tr>
<td></td>
<td>SP</td>
<td></td>
</tr>
<tr>
<td>HS</td>
<td>N</td>
<td>Using irrational and complex numbers. Applying operations to vectors.</td>
</tr>
<tr>
<td></td>
<td>F</td>
<td></td>
</tr>
<tr>
<td></td>
<td>G</td>
<td></td>
</tr>
<tr>
<td></td>
<td>SP</td>
<td></td>
</tr>
</tbody>
</table>

*Note: OA = operations & algebraic thinking; NBT = number & operations in base ten; MD = measurement & data; NF = number & operations – fractions; RP = ratios & proportional relationships; NS = number system; EE = expressions & equations; G = geometry; N = number & quantity; A = algebra; F = functions; SP = statistics & probability; Adapted from CCSS-M (NGA, 2010)*
Literacy Theories

Up to this point, I have defined SMaLL parsimoniously as the ability to read and write using the language of mathematics and established that the standards and practices in the CCSS entail those abilities. In this section, I explore the question of what it means to define a literacy.

Originally meaning learned or educated, literacy was redefined as “the ability to read and write” (Literacy [Def. 1]) during the 1920s with the rise of compulsory public schooling (Kendall Theado, 2010). Since then, literacy is often defined with respect to skills required to communicate using the writing system of a language as a medium (e.g., Calfee & Sperling, 2010). Sometimes, literacy is defined more broadly as a set of skills, presumably including some reading and writing skills, related to a particular subject (Literacy [Def. 2]). In practice, definitions of literacy — whether specified in curriculum or merely enacted as hidden curriculum — have a profound influence on educational opportunity and outcomes (Keefe & Copeland, 2011). This inspection of conceptions of literacy is useful for thinking about what it means to define SMaLL as a literacy in theory and contrasting SMaLL with other conceptions of literacy that are relevant to mathematics education.

Literacy is a relatively modern concept. The word did not appear in dictionaries until the late 1800s, about 200 years after the word illiteracy (Kendall Theado, 2010). Despite its origin and use as the antonym of illiteracy, common definitions of literacy do not identify a threshold for distinguishing the literate from the illiterate. Based on dictionary definitions alone, one might suppose that to be illiterate is to lack any literacy skill whatsoever and that to be literate is to possess all possible literacy skills. However, that logic would lead to paradoxical conclusions. For example, a person who had some reading and writing skills, but not all reading and writing
skills, would be neither illiterate nor literate. Ironically, literacy would be an unachievable ideal because spoken and written languages are ever-evolving (Hall et al., 2011).

To distinguish between illiterate and literate requires delimiting necessary or sufficient conditions. In other words, it is only possible to establish criteria for being literate by reframing literacy with respect to a coherent subset of reading, writing, and related skills. Unfortunately, attempts to distinguish between literacy and illiteracy give rise to sociopolitical complications (e.g., Kendall Theado, 2010). For example, multiple literacies may be defined, raising the question of whether one variant of literacy should be privileged over another. Literacies may be conceptualized with differences in the degree to which language skills (as opposed to subject area skill) determine the difference between literate and illiterate, raising the question of whether a core set of reading and writing skills should be a minimal requirement to define any literacy.

Using a critical lens, Knoblauch (1990) touches on these issues in the process of identifying four variations of literacy definitions. Functional definitions of literacy describe language as a utilitarian tool. A functional literacy professes to delineate the minimum requisite reading and writing skills to cope with daily life. Other definitions of literacy describe language in the context of its cultural construction. A cultural literacy claims to describe the language skills necessary to develop strong connections within a given society (e.g., Hirsch, 1987). In contrast to cultural definitions, liberal definitions of literacy view language as a vehicle to personal growth. A liberal literacy accommodates — and encourages — the use language as a means of self-expression and exploration. Finally, critical definitions claim literacy is an instantiation of political power. A critical literacy promises literacy is a means to establish oneself as an empowered citizen within a society.
Although Knoblauch’s classification of literacy definitions is not exhaustive (Keefe & Copeland, 2011; Knoblauch, 1990), it highlights important sociopolitical aspects of a literacy defined. First, a literacy defined is necessarily situated in time and place. Second, a literacy defined entails assumptions about its value for society and the individual. To illustrate, Figure 2.4 shows Knoblauch’s literacies in terms of the degree to which each claims to support social progress and self-actualization. Shown near the intersection of the axes, functional definitions of literacy do not purport to serve society or the individual beyond necessity. Shown at the extreme of the horizontal axis, cultural definitions prioritize the unification and perpetuation of culture through literacy. Shown at the extreme of the vertical axis, liberal definitions prioritize the realization of individual potential through literacy. Shown at the extremes of both axes, critical definitions suggest that literacy should be mutually beneficial to the advancement of both society and the individual. In short, a literacy defined has the potential to profoundly impact cultural evolution and individual growth.

The adoption of the CCSS, which gives unprecedented attention to complex texts and informational texts in the content areas, has given rise to new conceptions of literacy with implications for mathematics education. **Content area literacy** is conceived as the application of reading and writing strategies used by English language arts experts to other fields of study. Thus, content area literacy is predicated on the understanding that reading and writing instruction remains largely under the purview of English language arts educators (Wilson, 2011). On the contrary, **disciplinary literacy** supposes that experts in the discipline, having developed specialized literacy skills within the discipline, should have a central role in developing appropriate literacy instruction (Johnson et al., 2011). The purpose of the former is to give
students access to discipline-specific content via English language literacy (Fang & Schleppegrell, 2008) while the purpose of the latter is to immerse students in the language of the discipline.

*Figure 2.4.* A visualization of Knoblauch’s (1990) categories of literacy definitions.
Research studies from the literature illustrate how the conceptions of content area literacy and disciplinary literacy have been applied to mathematics education. Haltiwanger and Simpson (2013) drew on content area literacy theory in recommending four strategies to incorporate writing in math instruction. Likewise, Dunston and Tyminski (2013) recommended applying research-based English language vocabulary strategies to incorporate reading in math instruction. The student work presented in both publications to illustrate the strategies in practice highlight the complication with a content area literacy approach to reading and writing in mathematics classrooms. Haltiwanger & Simpson (2013) shared the work of a student who applied one of the writing strategies to compare these terms: distance formula, coordinate plane, and Pythagorean theorem. Although these concepts are commonly and concisely described with symbolic mathematics, the illustration showed no evidence of symbolic mathematics. The student work was completely alphabetic, using English language conventions. Dunston and Tyminski (2013) showed how a student using a graphic organizer defined the word function by identifying essential and non-essential characteristics, examples and non-examples, definitions, and mnemonic words and pictures. Again, the symbolic characters most commonly written to refer to a function, $f(x)$, did not appear in the student work. These examples illustrate that content area literacy in mathematics is understood as the ability to use English language to talk about (rather than in) mathematics. As a result, content area literacy privileges the discourse practices of English language arts over the discipline-specific discourse of mathematics.

Recognizing that a content area literacy approach fosters communicating about a discipline rather than in the language of the discipline, Johnson et al. (2011) asked this question: “What does it mean to be literate in particular disciplines and how do we begin to shift to
disciplinary literacy?” (p. 101). The field of mathematics has its own language with unique grammatical and symbolic forms. The very nature of the language allows one to recognize patterns, define abstract concepts, make concise arguments, and solve problems that would not otherwise be possible (Devlin, 2000). Proponents of disciplinary literacy argue that mathematics literacy entails becoming conversant within the language of mathematics as it is understood by the leaders of the community of discourse. That is, a disciplinary literacy approach invites mathematicians and mathematics teachers to lead the community of discourse and invites learners to enter into the community as apprentices. Current conceptions of disciplinary literacy attend to the meaning of English language vocabulary words and habits of grammar as they are used by mathematicians such as let for identifying variables and the inclusive interpretation of or (Johnson et al., 2011). However, disciplinary literacy is not specifically concerned with the skills necessary to interact with symbolic mathematics in mathematical texts.

I envision SMaLL as a reading and writing literacy in relationship to the symbolic writing system of mathematics. This distinguishes SMaLL from content area literacy which is focused on using English language literacy in the service of learning mathematics. This distinguishes SMaLL from disciplinary literacy for mathematics which entails a broad conceptualization of text as anything with the potential for meaning (Johnson et al., 2011). For this study, I have not defined SMaLL as a literacy for the purpose of rationalizing a divide between the literate and the illiterate with respect to the language of mathematics. With caution, I have defined SMaLL with concern knowing that defining a literacy may be tantamount to defining an illiteracy.
Although SMaLL is defined broadly as the ability to read and write using the language of
mathematics, this study was focused on the reading component of SMaLL. I drew on research
describing English language reading and reading acquisition to gain insight into whether SMaLL
may be analogous to English language literacy with respect to reading. Experts agree that there
is no single key to reading acquisition in any language (Brady et al., 2011; Polk & Hamilton,
requires the coordination of many processes. The phonological deficit hypothesis, which
suggests differences in reading ability can be explained by differences in the ability to map
sounds to printed characters, was revolutionary when it was proposed in the 1970s (Brady et al.,
2011). Since then, constant debate and the quest to understand why some students struggle to
read has generated new insights into the processes that contribute to reading and reading
acquisition (Brady et al., 2011; Headley, 2012).

Reading is “the process of extracting and constructing meaning from text” (Faust &
Kandelshine-Waldman, 2011, p. 546). Reading models attempts to explain the top-down
processes and bottom-up processes involved in reading. Top-down processing involves
extracting meaning from whole units of text (e.g., words) and progressively making connections
to smaller units of text (e.g., letters). The strictly whole language approach to reading instruction
embodies the theory that reading can be reduced to a top-down process and, as a consequence,
holds that learners acquire the alphabetic principle independently (Faust & Kandelshine-
Waldman, 2011). Bottom-up processing involves constructing meaning by collating small units
of sensory input. A strictly phonetic approach to reading instruction embodies the theory that
reading can be reduced to a bottom-up process of mapping letters to sounds until semantic information can be associated with the word as a whole. Simple models of reading — along with simple approaches to reading instruction — have been discredited in favor of more sophisticated reading models that account for the integration of multiple reading processes.

Based on a meta-analysis of studies focused on reading development among children in grades K to 12, the National Reading Panel summarized what was known about the impact of instruction on these aspects of reading: phonemic awareness, phonics, fluency, vocabulary, and comprehension (National Institute of Child Health and Human Development, 2000). Other components of reading theories (e.g., semantics, morphology, and syntax) have also been the subject of research. Orthographic processing, or the ability to recognize meaningful patterns of characters, has emerged as an important — and malleable — component of reading. Research in English language literacy suggests that instructional interventions and print exposure contribute to orthographic development (Cunningham, Perry, & Stanovich, 2001; Wolf, Barzillai, Gottwald, & Miller, 2009). Orthographic processing has also been investigated in multiple alphabetic languages as well as logographic languages such as Chinese (e.g., Tan, Spinks, Eden, Perfetti, & Siok, 2005). Together, these studies suggest that orthography plays a critical role in reading any language, but raise the question of whether the role of orthography depends on the nature of the writing system.

Technological advances have made it possible to address more specific questions about how the brain processes text. There is general agreement that the visual word form area (located in the fusiform gyrus region) plays a critical role in reading (Wolf, 2007); however, there are competing theories regarding its function (Polk & Hamilton, 2006). Some argue that the visual
word form area processes words sublexically, using combinations of a few characters at a time, while others argue that the visual word form area houses a lexicon of known words. For instance, the results of two functional magnetic resonance imaging (fMRI) experiments support the former hypothesis (Braet, Wagemans, & Op de Beeck, 2012). If that is the case, the visual word form area is likely organized by orthography rather than semantics. That hypothesis is in line with behavioral research suggesting that reading is more dependent on the ability to parse ink marks than the ability to associate meaning with word-length chunks of text as a whole (Faust & Kandelshine-Waldman, 2011).

In summary, the literature suggests that understanding reading requires attention to both top-down processes and bottom-up processes. Thus, the debates about whether reading acquisition is a consequence of nature or nurture have abated in favor of theories that account for the roles of both nature and nurture. In light of the reading literature, understanding the reading component of SMaLL requires accounting for the nature of the writing system of the language of mathematics, the ways students engage with symbolic mathematics, and the neurobiological processes involved in interacting with text written in the language of mathematics.

Given the theoretical framework of this study, the roles of cultural context in top-down processes and neurobiology in bottom-up processes are of special interest. According to current educational policy, every student is presumed to be biologically capable of matching the learning trajectories outlined by the CCSS “regardless of their income, race, ethnic or language background, or disability status” (U.S. Department of Education, 2010, p. 3). Viewing this brief review of the reading literature through the lens of DBCCC suggests this assumption is at least suspect if not unreasonable. As an analogy consider the possibility of suggesting every child
could be a college- or career-ready athlete by the end of high school. Many would agree, on the grounds that it requires self-evident biological conditions, that the suggestion is preposterous. I argue similarly that, given how little is known about the neurobiological conditions required, it may well be preposterous to suggest that every child can be college- and career-ready in terms of SMaLL by the end of high school. It is, I think, laudable that American society increasingly rejects the oppressive notion that some children are incapable of learning by virtue of race, gender, or other group membership (e.g. Halpern et al., 2007; Martin, 2012). However, insisting that all students are capable of similar literacy without regard for unique neurobiological conditions may be equally oppressive (Keefe & Copeland, 2011). In this study, I attempted to gain insight into both the cultural and neurobiological demands on adolescents developing SMaLL in mathematics classrooms guided by the CCSS.

**SMaLL as a Literacy**

SMaLL, as it is defined for this research, entails the ability to speak, read, and write using symbolic mathematics, the writing system of the language of mathematics. Thus, SMaLL is distinct from numeracy, mathematical literacy, and mathematical proficiency as they are commonly used in the field of education. Collectively, those three terms are often conceptualized as encompassing a wide range of competencies necessary to employ mathematics for particular purposes (Diezmann & Lowrie, 2009). In contrast, SMaLL does not encompass number sense, facility with the calculations, efficient problem solving, or connections between mathematical concepts and graphical representations. Given this definition, mathematical development cannot be reduced to SMaLL development; however, SMaLL development is central to doing mathematics and, thus, differences in SMaLL may offer affordances or impose
constraints on mathematics achievement.

For this study, SMaLL is conceptualized relative to the scope of the language of mathematics. The written language of mathematics has evolved over the last two millennia to precisely express words, make statements, ask questions, and argue the validity of claims in mathematics. From this perspective, the potential for SMaLL development is as deep and broad as the writing system allows. Given the review of the role of symbolic mathematics in current curriculum, a formidable degree of SMaLL is the sine qua non of mathematics achievement as it is conceived in formal mathematics education. This definition makes no claim about what depth or breadth of SMaLL is adequate or preferred for any particular purpose, only that it is possible for one individual to have quantitatively more or fewer SMaLL skills or qualitatively different SMaLL skills than another.

This definition of SMaLL has the potential to become yet another one of those “mischievous concepts” (Knoblauch, 1990, p. 1) used to create an arbitrary, or perhaps political, division between the literate and illiterate. Because some degree of SMaLL is a necessary, but not sufficient, component of mathematics achievement, there may be a temptation to determine some practical point by which to differentiate between the literate and illiterate. For this study, SMaLL is intentionally defined in a manner that allows it to be viewed as a possible pathway to liberation as in liberal and critical literacies. However, in practice, especially in the context of educational policies that link curriculum and high stakes assessments, future conceptualizations of SMaLL may ultimately be used to exercise power over teachers and students as in cultural literacies (Au, 2011).
If SMaLL is analogous to English language literacy, then supporting SMaLL development through similarly explicit instruction — helping students learn to read — has the potential to release captive students from frustration on their compulsory march towards mathematics achievement. In addition, supporting SMaLL development — encouraging students to read to learn — has the potential to emancipate students eager to experience more than is required of them. The degree to which SMaLL imprisons or enfranchises students depends on its purposeful employment in the future. The ethical application of SMaLL will demand a return to the debate of what is worth knowing and doing, reflecting on our personal convictions and giving careful consideration to others’ doubts as we move forward in educational research and practice (Calhoun, 1995).

**The Language of Mathematics**

At this point I turn the literature review from the general — establishing SMaLL as an unacknowledged but essential academic literacy — towards the specific. For the remainder of the literature, I draw on literature with the potential to highlight similarities and contrasts between SMaLL skills and English language literacy skills.

Mathematics is often called the universal language. In defining SMaLL as a literacy, I presume that the language of mathematics, although similar in terms of the writing system used around the world, is not universally accessible to readers (Sharon, 2009; Whiteford, 2009). I take the view that every person is an in-progress mathematics language learner (Thompson & Rubenstein, 2014). In this section, I establish the language of mathematics as a unique writing system. In addition, I discuss prevailing symbol theories with implications for understanding what it means to read symbolic mathematics.
Writing systems. In the mathematics education literature, reading in mathematics is conflated with reading word problems, written in English, solvable with a mathematical approach. In contrast to SMaLL, which entails reading symbolic mathematics, reading word problems entails reading text written with the alphabetic writing system and conventions of English. This conflation may be due to a lack of awareness that two different writing systems are operating in mathematics classrooms (see Curriculum Theories discussion). This brief review of the use of terms in empirical studies and comparison of the writing systems serves two purposes: to clarify the distinction between the writing systems and to inform suppositions about the similarities between SMaLL and English language literacy.

Studies investigating the relationship between reading and symbolic mathematics exist in the literature. The examples presented here illustrate how those studies typically address English language reading comprehension rather than reading symbolic mathematics. Bernardo and Okagaki (1994) explored the “effects of symbolic knowledge” (p. 212), and Capraro and Joffrion (2006) investigated students’ “facility with translating English language into mathematical symbols and vice versa” (p. 147). Both researchers presented participants with statements in English. Students were tasked with generating an equation (i.e., a statement written in symbolic mathematics) consistent with a scenario such as “There are six times as many students as professors” (Bernardo & Okagaki, 1994, p. 212) or “Julie has 3 times as many trading cards as Mary” (Capraro and Joffrion, 2006, p. 156). Although such tasks involve symbolic mathematics, they do not involve reading symbolic mathematics and demand more than translating English into written symbolic mathematics. Such tasks demand multiple mathematical practices: making sense of a problem, reasoning abstractly, and modeling with mathematics. For comparison,
consider the task of generating symbolic mathematics consistent with this statement: “the function, \( F \) of \( X \), is equal to \( X \) squared plus one.” In contrast to the tasks in the Bernardo and Okagaki (1994) and Capraro and Joffrion (2006) studies, this task need not require mathematical problem solving; on the contrary, the ability to negotiate the spoken language of mathematics and the symbolic writing system of mathematics would suffice. Arguably, reading in the latter sense is, at most, a component skill of mathematics achievement. However, the distinction is meaningful.

SMaLL is inherently distinct from traditionally conceptualized English language literacy in important ways. Though based in the same spoken language, the writing systems of mathematics and English differ dramatically in terms of the way ink marks are used to convey meaning and speech. English is a phonemic language in that its written form is a mapping of sounds to alphabetic letters (Hirshorn & Fiez, 2014). In contrast, the written form of the language of mathematics includes special characters and logographic symbols that map onto whole words or phrases. For example, \( \pi \), \( \perp \), and \( \angle \) represent \( \text{pi, perpendicular, and angle} \) respectively. In addition, visual units such as \( f(x) \) are constructed using common English language characters and punctuation yet require the production of novel vocabulary. In this example, a meaningful reading, or oral translation of the text, might be \text{the function } F \text{ of } X. \) Also, meaning can be indicated via changes in font, size, and location. For example, \( x^2 \) and \( x_2 \) are visually similar except for the location of the numeral for two. A reasonable reading of the former is \text{ } X \text{ squared} while a reasonable reading of the latter is \text{the second } X \text{ value or } X \text{ sub two}.

The nature of written symbolic mathematics has evolved as abstract thinking has been applied to understand the world around us (e.g. Devlin, 2000; Eves, 1990; Lakoff & Núñez,
2000; Sfard, 2009, Van Dyck & Heeffer, 2012). Because it is intended to convey complex abstract ideas, the language is precise and dense with meaning. To read symbolic mathematics with comprehension and write symbolic mathematics to express particular meaning may have different demands than understanding the mathematical ideas represented. As an analogy, consider the word *idiosyncratic*. Given the written form, and without knowing its meaning, an English literate student may be able draw on phonological, orthographic, and morphological knowledge to render a reasonable oral reading by sounding out the letters and recognizable groups of letters. Conversely, given the oral form, and without knowing how to spell the word, an English literate student may be able to draw on phonological, orthographic, and morphological knowledge to render a reasonable spelling by matching the sounds to known letters or words. Likewise, SMaLL may depend on a lexicon of novel symbols as well as expertise in the structure of – and mapping between – the oral and written forms of the language of mathematics.

Given the differences in their written forms, a shared spoken language is insufficient evidence to assume the processes underlying reading and writing are the same for SMaLL and English language literacy. Minimally, reading symbolic mathematics must entail visual perceptual abilities. However, it is an open question as to whether the visual units are processed through semantic or orthographic cognitive processes. The difference in grain size of the writing systems, or mapping from visual unit to sound unit (Ziegler & Goswami, 2005), suggests that phonological processing may play a markedly different role in SMaLL than English language literacy. The writing system for English has a smaller grain size with letters mapped onto sounds. In contrast, the writing system for mathematics has a larger grain size with characters
mapped onto words and phrases. In general, larger grain size systems demand a larger inventory of characters and require more time to learn (Hirshorn & Fiez, 2014). While there may be some similarities in reading symbolic mathematics and English, the differences in writing systems suggest that SMaLL and English language literacy are distinct in some ways.

**Symbol theories.** In the social sciences literature, the term symbol is used with a wide range of meanings. In addition, multiple theories of symbolizing exist in the field of mathematics education. Here I review both to draw a connection between letters and symbols and reading and symbolizing. The discussion has implications for understanding symbolic mathematics as a writing system and conceptualizing the process of interpreting symbolic mathematics as reading.

Broadly speaking, a symbol is an external referent for an object, event, or abstract idea (Bjorklund, 2012). For example, a portrait of a woman might be used to represent a mother or mothering or motherhood. Piaget, the nominal father of developmental psychology, carefully delineated a model of symbols. For example, he distinguished between symbols and signs (Piaget & Inhelder, 2000). The former term he used to specify a referent created by its user. The latter he used to specify conventional referents shared within a culture.

From the perspective of situated cognition theory, a symbol can be understood as *cognitive artifact* (Heersmink, 2013). Through this lens, symbols are thought of a means of scaffolding cognition. This view supports Devlin’s (2000) claim that the nature of the symbolic mathematics writing system makes it possible to have mathematical thoughts that are not otherwise possible. Heersmink (2013) builds on prevailing general definitions of cognitive artifacts as physical or man-made objects with the potential to support a representation to argue
for a new classification of cognitive artifacts. Heersmink’s classification is artifact-centered in that it depends on the features of the artifacts itself rather than the ways in which any particular agent employs them. His work draws on the model of representation proposed by pragmatist Charles Saunders Pierce to identify symbolic cognitive artifacts. Pierce’s model was based on a simple system including a sign (physical representation), an object (represented entity), and interpretant (understanding of the object). In contrast to Piaget’s usage, the term symbol was reserved to describe representations imbued with meaning through enculturation and logical rules. For example, Arabic numerals and characters that make up writings systems are examples of symbols. This understanding of symbol is in line with the commonly accepted use of the term symbol in the disciplinary literacy of the mathematics education community: a symbol is a written referent to a mathematical object, event, or abstract idea. For example, the symbol $x^2$ might be used to refer to a quantity, the process of squaring, or the area of a square.

In the field of English language literacy, researchers have inspected the use of finely grained symbols (e.g. letters, sounds) as well as coarsely grained symbols (e.g. sentences, paragraphs, gestures) to better understand the developing ability to transition between symbolizing meaning with written English and symbolizing meaning with spoken English (e.g. Bruning et al., 2011; Wolf, 2007). The result has been a deeper understanding of how symbolizing in various units contributes to the overarching goals of reading: fluency and comprehension. SMaLL has not been carefully inspected in a similar manner; and, thus, there is little evidence to draw conclusions about how learners use symbols or meaningful combinations of symbols to represent mathematical meaning.
A plausible explanation for this gap in the literature may be related to theories that symbolic mathematics express a virtual reality (Sfard, 2009) or arise from epistemic action (De Cruz & De Smedt, 2013). Using slightly different terms, both Sfard (2009) and De Cruz and De Smedt (2013) acknowledge that mathematical ideas can develop in the absence of a concrete referent. In such a case, symbolizing mathematics serves the purpose of making an abstract concept concrete or transparent. From this perspective, to symbolize mathematics is to bring an abstraction to fruition. Consider this example: One cannot, in any literal sense, perceive that which is commonly referred to as 0. It is not visible to the eye. It can neither be felt nor heard. It is merely conceived of as a notion of absence or void in contrast to something imagined or previously present. It was the introduction of the symbol named zero, 0, as a placeholder that made it possible to think about nothing as a quantity and manipulate nothing as if it existed tangibly.

Similarly, the advent of making abstract mathematical ideas concrete via an evolving writing system has made it possible for mathematicians to notice new patterns and connections that were previously limited by the boundaries of human cognition (De Cruz & De Smedt, 2013; Devlin, 2000). Notably, the virtual reality and epistemic action theories presume the ability to symbolize occurs as a consequence of or, at least, in transaction with mathematical comprehension. Understanding symbolizing in this way appears to support theories of constructivist theories of learning (e.g. Bruning et al., 2011). Practitioners may draw on these theories to justify the practice of avoiding the use of symbolic mathematics until after students have demonstrated evidence of conceptual understanding via some other means (Sfard, 2009). Furthermore, viewing symbolic mathematics through these lenses may make it difficult to
conceive of merely reading symbols as an instance of using symbols.

It may be reasonable to suppose that SMaLL and English language literacy may develop in ways that mirror the historical development of their respective writing system: one framed as symbolizing and the other framed as reading. The historical evolution of the written English language does not make a compelling case for viewing alphabetic symbols in the same light as mathematical symbols. The written English language, at the word level, is fundamentally a sophisticated mapping of distinguishable sounds to arbitrary letters (Hall et al., 2011). Sound-letter correspondences are arranged in order from left to right to reproduce the spoken word. Similarly, phrases and sentences and paragraphs are ordered to mirror the temporal order of speech.

The nature of the mapping between symbol and sound in the language of mathematics limits the expectation of similarities between SMaLL and English language literacy. The mapping differences suggests the possibility of different cognitive demands in terms of both process and knowledge. Nemirovsky and Monk (2009) provide a framework that may be useful for understanding SMaLL as a symbol reading process. They propose fusion as way of understanding symbol use. According to their theory of fusion, it is not necessary for the symbol user to understand a symbol’s referent in order to engage with the symbol itself. That is, the nature of a symbol need not be fully transparent for the act of symbolizing to be productive. Fusion is novel in that, as a metaphor for symbolizing, it acknowledges symbols as inherently ambiguous and reframes symbolizing as the act of managing ambiguity as a process to determine meaning. Using fusion as a lens, engaging in interpreting symbolic mathematics need not be reserved for the learned, but is also available to learners. Thus, fusion is not framed as an
immature approach to symbolizing that resolves with expertise. “Rather, it is a pervasive quality of symbol use found at any age level” (Nemirovsky & Monk, 2009, p. 195). The authors go on to discuss the distinctions between and intersections of path-following and trail-making as means of symbolizing. *Path-following* can be understood as an algorithmic approach to decoding symbols. In contrast, *trail-making* can be thought of as a open-ended process guided by the purpose of understanding a symbol, but shaped by details, circumstances, and revisions along the way. Neither path-following nor trail-making is privileged as a way symbolizing. Instead, they are both a means for conceptualizing the experience of symbolizing from the perspective of the symbolizer.

Considering symbolizing theories in light of Faust and Kandelshine-Waldman’s (2011) definition of reading — “the process of extracting and constructing meaning from text” (p. 546) — suggests multiple ways of thinking about SMaLL. Reading symbolic mathematics is a receptive form of symbolizing that entails interpreting characters, and combinations of characters, written in the language of mathematics. The three theories — virtual reality, epistemic action, and fusion — have different implications for understanding the relationship between reading symbolic mathematics and developing mathematical expertise. An epistemic action framework is directional in the sense that it suggests SMaLL would not develop in the absence of mathematical expertise. A virtual reality framework allows for a greater degree of flexibility on that account; however, it is intended to be applied to an extended conception of discourse that can not be reduced to language use (Cobb, 2009). Fusion, on the other hand, does not demand mathematical expertise a priori. The central focus of the theory lies in understanding the experience of the symbolizer given that symbolizers bring different knowledge to the process.
of symbolizing. Fusion is sufficiently flexible to allow for the possibility that SMaLL may develop in advance of, in tandem with, or in response to comprehension of symbolic referents.

**Multilevel Research Methods and Empirical Evidence**

In this section, I transition from a substantive review of the literature to a methodological review to answer the question of how knowledge about SMaLL might be generated. I begin by presenting a review of methodological literature with implications for studying SMaLL among adolescents in a middle school learning under the CCSS as a multilevel — cultural, behavioral, and neurobiological — phenomenon. Having previously discussed my mixed methods mindset (Greene, 2007) in Chapter 1, I begin by identifying the reigning definition of a multilevel mixed methods research design. Then, taking a dialectical stance, I discuss the qualitative and quantitative conceptions of multilevel research designs. After specifying the definition of multilevel mixed methods research design for this study, I briefly review empirical studies that informed the research design of this study and the methods used to investigate each level.

**Multilevel issues in mixed methods research.** In the mixed methods literature, the first explicit discussion of multilevel variants appeared in the literature in 1990s (Tashakkori & Teddlie, 1998). Discussions of multilevel issues are scarce in handbooks written by leading mixed methods methodologists. In some texts, multilevel designs are not included as critical mixed methods design variants (e.g., Greene, 2007; Morgan, 2014). Other authors acknowledge the existence of multilevel studies in mixed methods but provide little context or explanation. For example, Creswell and Plano Clark (2011) reference a multilevel statewide study as an example of a multiphase design variant, and Plano Clark and Ivankova (2016) list multilevel mixed designs as a family of designs that mix across multiple data levels. Teddlie and
Tashakkori (2010; 2009) define a multilevel mixed methods design; however, the complexities of the design are under-theorized leaving questions about the foundations and requirements for conducting multilevel research using mixed methods.

The Teddlie and Tashakkori definition appears to be the prevailing conceptualization of a multilevel mixed method design. They describe multilevel mixed designs as “multistrand designs in which QUAL [qualitative] data are collected at one level of analysis (e.g., child) and QUAN [quantitative] data are collected at another (e.g., family) in a parallel or sequential manner” (Teddlie & Tashakkori, 2009, p. 156, emphasis in original). According to their conceptualization, multilevel designs are applied to answer research questions about hierarchical organizations such as schools (with students) or hospitals (with patients) or states (with voters). Furthermore, each strand is designed to answer either quantitative or qualitative research questions at a given level and analyzed accordingly. Whether conducted sequentially or in parallel, the implementation of each strand is intended to yield inferences drawn in the quantitative or qualitative tradition at the corresponding level of the design. The implication of the definition is that multilevel research questions are answered by integrating the data and inferences to generate meta-inferences about the role of a particular level, the relationships between the levels, and the organization, or system, as a whole.

The Teddlie and Tashakkori multilevel mixed methods design is grounded in an examplar using an extended mixed methods research design: the Louisiana School Effectiveness Study (Tashakkori & Teddlie, 1998). The longitudinal study included five phases conducted over more than a decade to understand micro and macro levels. The definition continues to guide multilevel mixed methods studies. For example, Papadimitriou (2010) conducted a
contemporary multilevel mixed methods design based on this definition to investigate quality management at Greek universities. Although other definitions of multilevel mixed methods designs may exist, this definition serves as a point of departure to inspect the complexity of conducting multilevel mixed methods research.

Contrasting the Teddlie and Tashakkori definition with a selection of recent mixed methods publications suggests that multilevel mixed methods research designs entail accounting for levels at five different stages of the research process: theoretical grounding, sampling, data collection, data analysis, and integration. First, multilevel mixed methods designs are grounded in a theory of levels. Teddlie and Tashakkori restrict the application of their multilevel mixed methods design to “hierarchically organized social institutions” (2009, p. 279). This restriction ensures a theory of nested levels of analysis that is self-evident because the levels are, having been socially constructed in the process of institution building, “naturally occurring” (Teddlie & Tashakkori, 2009, p. 156).

In practice, multilevel mixed methods studies are grounded in theories of levels that vary from physical to meta-physical. For example, de Meij, van der Wal, van Mechelen, and Chinapaw (2013) designed a process evaluation study to investigate the adoption of a school-based physical activity program with four levels: the intervention, the user, the organization, and the sociopolitical context. The authors argued that, in order to understand the delivery and reach of the program, it was necessary to inspect the nature of the intervention as it was enacted by different users within different organizations within different sociopolitical contexts. Thus, the four-level theory that guided the study differed from the putative hierarchal organization of students, physical education teachers, school staff, school directors, and city district sports
coordinators. Eastwood, Jalaludin, and Kemp (2014) used a paradigmatically driven conceptualization of levels to conduct a multilevel mixed methods study aimed at building a theory of social epidemiology. The researchers set out to develop a theory of contextual levels related to neighborhood environment and social networks that impact postnatal depression. The protocol for their study illustrates how critical realism, which entails the ontological assumption that “reality consists of hierarchically ordered levels where a lower level creates the conditions for a higher level” (Eastwood et al., 2014, p. 7), can serve as a theoretical foundation for a multilevel mixed methods design in the absence of institutionally defined levels.

Second, sampling strategies for multilevel mixed methods designs are driven by the theory of levels. The Teddlie and Tashakkori definition implies that multilevel mixed methods designs require that at least one sampling strategy for each level be tailored to the planned analysis (quantitative or qualitative) of that level. However, there need not be a one-to-one correspondence between sampling strategies and levels of analysis (see the discussion in the next paragraph about how sampling relates to data collection considerations). In the Louisiana School Effectiveness study, five levels (students, classrooms, schools, districts, states) entailed eight different sampling procedures (Teddlie & Tashakkori, 2009; Teddlie & Yu, 2007). Onwuegbuzie and Collins (2007) describe multilevel sampling as a process in which samples are “extracted from different levels of the study (i.e., different populations)” (p. 292) for the quantitative and qualitative strands. In their prevalence study of published mixed methods studies comparing multilevel sampling, identical sampling (i.e., using the same sample for both quantitative and qualitative strands), nested sampling (i.e., using a subset of the sample for one strand as a sample for the other strand), and parallel sampling (i.e., using different samples drawn from the same
population for each strand), they found that 25% of the articles used multilevel sampling. Interestingly, multilevel sampling is prevalent in mixed methods designs, such as concurrent or sequential designs, that cannot be characterized as multilevel mixed methods designs according to a strict interpretation of the Teddlie and Tashakkori definition (Collins, Onwuegbuzie, & Jiao, 2007). Thus, in current practice, multilevel sampling is not necessarily restricted to multilevel mixed methods designs.

Third, data collection procedures for multilevel mixed methods designs are intended to assess characteristics at each level. Although this is implied in the definition, Teddlie and Tashakkori offer no particular guidance for making data collection choices in the context of a multilevel mixed methods design. Historically, data describing characteristics at one level may be collected from another level and are sometimes collected from multiple levels. For example, in the Louisiana School Effectiveness Study, teachers (conceptualized as classroom-level participants) provided data about both school-level characteristics (e.g., school climate, principal leadership) and classroom-level characteristics (e.g., teacher curricular choices, teacher self-concept) (Stringfield & Teddlie, 1991). This practice continues in contemporary multilevel mixed methods studies. For example, in a study of therapist effects, Green, Barkham, Kellett, and Saxon, (2014) collected data from three different levels (patients, therapists, and supervisors) to describe characteristics of only two levels (patients and therapists). The patients generated data related to patient-level characteristics, therapists generated data related to therapist-level characteristics, and supervisors generated data related to therapist-level characteristics. In another study, which investigated students’ engagement in classrooms with different teaching practices, Cooper (2014) collected data from students to explore engagement (a student-level
characteristic) and the prevalence of teaching practices (a classroom-level characteristic). In practice, the data collection procedures are justified for each level in terms of feasibility constraints, availability of measurement tools, and methodological considerations.

Fourth, data analysis for multilevel mixed methods designs are planned to draw credible inferences at each level or for each strand with the potential to facilitate the generation of metainferences across levels. According to the Teddlie and Tashakkori definition, the data analysis techniques for the quantitative and qualitative strands are only restricted by the standards of the quantitative and qualitative research communities, respectively. Exemplars for data analysis in the context of multilevel mixed methods designs exist in multiple fields including education and counseling psychology (Teddlie & Tashakkori, 2009). The quantitative data analysis used in recent multilevel mixed methods studies ranges from descriptive statistics (e.g., de Meij et al., 2013) to multilevel regression analysis (e.g., Cooper, 2014; Green et al., 2014). The qualitative data analysis used in recent multilevel mixed methods studies includes both a priori coding techniques (e.g., Green et al., 2014) and emergent coding techniques (e.g., Cooper, 2014; de Meij et al., 2013). The differences in data analysis techniques suggest that multilevel mixed methods research studies can differ in terms of priority, or the degree to which the quantitative or the qualitative strand dominate or drive the study (Creswell & Plano Clark, 2011; Morse, 1991).

Finally, integration techniques for multilevel mixed methods designs are used in complex ways to draw meta-inferences across levels. Although Teddlie and Tashakkori offer no particular instruction, a “longitudinal, sequential, integrated multilevel [mixed methods] research design” (Tashakkori & Teddlie, 2010, p. 708) exemplar suggests that standard mixed methods integration techniques such as data transformation (Caracelli & Greene, 1993), well-integrated
discussion (Bronstein & Kovacs, 2013), and joint displays (Plano Clark & Sanders, 2015) can be
tailored to meet the challenge of drawing inferences across levels. For example, in the Effective
Provision of Preschool Education study (Sammons, 2010), researchers qualitized the quantitative
data to create school profiles at a critical point of interface. Integrating through data
transformation created a means to connect the quantitative data to the qualitative data and
explicitly compare inferences drawn from multilevel value-added statistics to inferences drawn
from the qualitative case studies. The researchers used a “dialogic approach” (Sammons, 2010,
p. 709) to synthesize the quantitative and qualitative data. The well-integrated discussion
generated a robust picture of effective pedagogy in early childhood education based on a wide
range of evidence. The researchers also used a joint display of the integrated results to
summarize the findings in a conceptual diagram. The joint display illustrated their updated
understanding of how the levels were related in terms of structure and process.

The Teddlie and Tashakkori conceptualization of a multilevel mixed methods design is
useful. It guided me, explicitly or by way of exemplar, to an understanding of the roles of
theoretical grounding, sampling, data collection, data analysis, and integration. It has been
sufficient to guide some high-quality multilevel mixed methods research. However, a major
complication with the Teddlie and Tashakkori conceptualization of multilevel mixed methods
design is the implication that strand analysis be restricted to an investigation of a single level of
analysis. Because multilevel designs exist in some form in both the quantitative and qualitative
traditions, this provision raises questions about how multilevel qualitative or multilevel
quantitative methods can – or should – contribute to conceptualizing and conducting multilevel
mixed methods research designs. Therefore, I reviewed multilevel designs from the qualitative
and quantitative perspectives as a means to better understand how conceptualizations in those traditions might contribute to a more robust definition of multilevel mixed methods.

**Qualitative conceptions of multilevel designs.** Qualitative research, which began to emerge as a formal approach to research in the social sciences in the late 1800s (Hatch, 2002), encompasses a wide range of methods and, thus, is difficult to define. An extensive discussion of various qualitative methods is beyond the scope of this dissertation. However, the shared characteristics of qualitative research suggest that, historically, qualitative researchers were concerned with multilevel research designs long before the term *multilevel* came into vogue. In the social sciences, qualitative research is intended to investigate people within their natural environment. Qualitative researchers endeavor to hear the voice and see the perspective of individuals to better understand how people engage in particular settings (Hatch, 2002). At the same time, qualitative researchers assume that “social contexts can be systematically examined as a whole” (Hatch, 2002, p. 9). Given the dual interest in the individual and society as a whole, it is not surprising that some qualitative studies are decidedly multilevel undertakings.

For example, the framework proposed by Urie Bronfenbrenner in the 1970s continues to play an important role in thinking about qualitative research (e.g., Hong & Garbarino, 2012; Lau & Ng, 2014). Bronfenbrenner’s ecological theory suggests that human development can be explained in terms of the how the individual interacts within environmental sub-systems: microsystem (of interpersonal relationships), mesosystem (of interconnected microsystems), exosystem (or context that influences microsystems and, indirectly, individuals), and macrosystem (of cultural contexts). For example, given this theory one can imagine an individual who is developing within a microsystem (as a student in a classroom), a mesosystem
(as a friend in a clique or an athlete on an sports team), an exosystem (as a child influenced by a
parent’s work life) and a macrosystem (as a learner in a national school system). Bronfenbrenner
added a chronosystem level to the theory to account for changes in environments over time. For
example, in a study of education, the chronosystem might account for how curricular shifts
change the nature of the classroom over time.

Figure 2.5 shows how a study of SMaLL could have been conceptualized in the
qualitative tradition. Adapting the style of an ecological theory diagram, the outer circle shows
the classroom culture of language as a macrolevel of investigation. The white ring in the circle
shows the student metacognitive behavior as a mesolevel. The inner circle shows the within-
student neurobiological cognition as a microlevel. The red arrow shows a chronolevel or time
period of investigation. Finally, the double-headed black arrow represents mechanisms within
and across levels that contribute to SMaLL and define the way the system operates. The kinds of
questions about SMaLL that could be answered using this qualitative approach would differ from
the kinds of questions that could be answered using other approaches.

Qualitative researchers have been leaders in the effort to investigate individual behavior
within the context of structured systems (Lau & Ng, 2014). When qualitative studies are
grounded in theories about society as a system of structured levels and planned to generate
inferences about more than one level or the mechanisms across levels, they are multilevel
research designs even if they are not labeled as such. Qualitative research designs (e.g., case
studies and participatory action research) intended to explain the nature of hierarchical structures
in terms of “the meanings individuals construct in order to participate in their social
lives” (Hatch, 2002, p. 9) are, in some sense, the original multilevel research designs. Notably,
qualitative research designs are not intended to produce generalizable results beyond the scope of the sample. Thus, there are limitations to the application of multilevel qualitative research designs.

Figure 2.5. A view of SMaLL as a multilevel phenomenon from a qualitative perspective.
**Quantitative conceptions of multilevel designs.** Multilevel modeling in the quantitative tradition began to emerge in the 1980s in response to pressure to account for context in educational studies (Goldstein, 2003). In the 21st century, these methods are being further developed for use in multiple social science fields including sociology, demography, and epidemiology (Courgeau, 2003; Diez Roux, 2002; Sun & Pan, 2014). Multilevel modeling methods have made it possible to craft multilevel quantitative research designs (e.g., experimental or quasi-experimental) intended to explain the nature of hierarchical data structures. In contrast to multilevel qualitative designs, multilevel quantitative designs explain numerical data in terms of variances and covariances within and across levels of structured data sets. The methods, which involve calculation intensive procedures, are becoming increasingly popular as software packages (e.g., HLM, MLwiN, SAS, R) become more accessible (Goldstein, 2003; Sun & Pan, 2014). Multilevel quantitative research designs are also becoming more sophisticated as extensions of the basic quantitative multilevel modeling methods make it possible to account for dynamic structure within the system (e.g., multiple membership or changing membership). A discussion of the computational demands of quantitative multilevel modeling methods is beyond the scope of this dissertation. However, a discussion centered on the theoretical grounding of multilevel research designs from a quantitative perspective has implications for this study.

Multilevel quantitative research designs have emerged as critical tools in the social sciences, at least in part, as a means to resolve the complications that arise from assuming either holism or individualism (Courgeau, 2003). Mathematically, many common approaches to quantitative analysis require the assumption of independent measures (i.e., measures do not
covary). Thus, in the absence of modeling techniques that account for covariation, the assumptions of quantitative analysis require that a social system be approached as one of two things: “either as a totality endowed with specific properties, irreducible to those of its members, or as a set of individuals, such that all social phenomena resolve into individual decisions and actions, without involving any supra-individual factors” (Courgeau, 2003, p. 4). Unfortunately, to take either a holistic or an individualistic stance is to ignore — or deny the possibility of — mechanisms between the whole and its parts that are the essence of a system. Incorporating multilevel modeling into quantitative research designs is novel in that it allows for the existence of some mechanism between the whole and its parts and the calculation of an intraclass correlation coefficient to describe the magnitude, or degree of influence, of each level.

Ultimately, multilevel quantitative research designs attempt to avoid two common logical fallacies. First, there is the ecological fallacy in which one assumes that the characteristics of a system apply to its components. In the context of social sciences this amounts to failing to recognize that a community may be comprised of unique individuals. For example, given a study that compares schools in terms of effectiveness, it would be an error to presume that every teacher in a highly effective school is a highly effective teacher. Second, and conversely, there is the atomistic fallacy in which one assumes that the characteristics of components apply to the system (Diez Roux, 2002). In the context of social sciences this amounts to failing to recognize that individuals may comprise unique communities. For example, given a study that finds that students with higher after-school participation have higher academic achievement, it would be an error to conclude that schools with high after-school participation rates will have higher academic achievement rates. Using multilevel quantitative research designs to account for the
relationship between the system as whole and its components is one way to avoid committing these fallacies.

The analyses entailed in multilevel quantitative designs suggest that multilevel data can be reconceptualized in terms of dependencies rather than in terms of institutional or physical nesting. As previously discussed, multilevel data that arise from social institutions are easily conceived of as units nested within levels because of the way institutions are intentionally constructed (e.g., students within classrooms or teachers within schools). However, conceptualizing multilevel data as dependent, or covarying, data makes it possible to conceptualize data that are not intentionally organized as multilevel (e.g., friends within cliques or workers within industries). Importantly, thinking of multilevel data as dependent, or covarying, data suggests that data without a recognizable hierarchical social structure may be, mathematically speaking, multilevel data. For example, in longitudinal studies repeated measures for each participant covary and, thus, amount to data nested within participants (Tan, 2008).

Figure 2.6 shows how a study of SMaLL could have been conceptualized in the quantitative tradition as a three-level phenomenon within the context of a school. The diagram is drawn in the style of hierarchical linear modeling, a quantitative multilevel modeling technique for nested data. The outermost level is shown at the top of the diagram while the innermost level, called level 1 by convention, is shown across the bottom of the diagram. The cells at level 1 show that the within-student level could consist of repeated measures collected over time (e.g., sequentially collected responses to the same cognitive task). The cells at level 2 show that the student level could consist of static and one-time measures (e.g., gender and mathematics
achievement score). The cells at level 3 show that the classroom level could consist of variables specific to the class (e.g., expertise of the teacher, class size). The lines connecting the cells illustrate that the student level variables may covary with measures of classroom culture or within-student repeated measures. The top of the diagram illustrates that variables under consideration for each of the levels might, ultimately, be related to unexamined dependencies at a higher level. The kinds of questions about SMaLL that could be answered using this quantitative approach would differ from the kinds of questions that could be answered using other approaches.

Figure 2.6. A view of SMaLL as a multilevel phenomenon from a quantitative perspective
Despite the advances in quantitative multilevel modeling, there are a number of unresolved methodological issues. One interesting problem that quantitative methodologists have recognized is this: rates of change differ across levels of a system (Courgeau, 2003). For example, student characteristics may evolve more quickly than school level characteristics. Another remaining challenge is the interpretation of the statistics that describe covariances across levels. Like effect sizes, the intraclass correlation coefficients that describe the degree of interdependence across levels only take on practical meaning in the context of relevant empirical evidence (Hedges & Hedberg, 2007; Kelcey & Phelps, 2013). For example, intraclass correlations coefficients that describe the extent to which student outcomes are related to classroom characteristics can differ across content areas (e.g., reading vs. math) and across grade-level. As a result, the implications of using multilevel quantitative designs are yet to be fully understood.

**Multilevel mixed methods research designs.** Having reviewed conceptualizations of multilevel research designs from the qualitative and quantitative perspectives, I return to problem of refining the conceptualization that guides the field of mixed methods. Although the Teddlie and Tashakkori definition of multilevel mixed methods design is useful in many ways, it may be insufficient to carry the field forward. Researchers are looking to mixed methodologists for guidance to answer consequential questions about multilevel systems. For example, Van Cleve and Mayes (2015) identify a “mixed-methods moment” (p. 424) of clarity as that moment when a researcher recognizes that a mixed methods design is the best hope for addressing an important research question because neither qualitative nor quantitative research methods alone can adequately address the multilevel complexity of a critical issue (e.g., systemic racial inequality).
A new conceptualization of multilevel mixed methods research designs needs to be robust enough to incorporate the latest advances in both quantitative and qualitative research methods to address sophisticated new problems and generate valid and trustworthy inferences. Based on this review of the literature, I developed this refined definition of multilevel mixed methods research design:

A multilevel mixed methods research design is a variant of a mixed methods research design driven by an assumption about the nature of a system for the purpose of better understanding these elements of the system: the structure of the system; the components that emerge from, give rise to, or evolve in tandem with the structure; and the mechanisms at work between the components and the structure. As a matter of convention, the components are conceived as a lower level while the structure is conceived as an upper level. The mechanisms at work within the system may be top-down (with the structure causing the components to take on particular characteristics), bottom-up (with the components causing the structure to take on particular characteristics), or reciprocal (with the structure and components imposing transformation at both levels).

According to this definition, a multilevel mixed methods research design is defined primarily by its purpose. As a variant of a mixed methods research design, a multilevel mixed methods research design must incorporate both quantitative and qualitative data and analytical tools for the purpose of transcending what can be discovered and understood from a traditionally qualitative or traditionally quantitative approach alone. A thorough high-quality multilevel mixed methods research designs should entail the following:
• A quantitative strand designed to investigate one or more levels or a mechanism within the system
• A qualitative strand designed to investigate one or more levels or a mechanism within the system
• A sampling strategy that involves more than one level of the system
• Data collections tools that generate evidence about more than one level and/or a mechanism within the system
• Data analysis techniques for the quantitative and qualitative strands that support across-levels inferences independently or generate inferences sufficient for supporting across-level meta-inferences during integration
• Integration techniques that support across-level meta-inferences by explaining at least two of the following: the nature of the system, the nature of levels, the nature of the mechanisms

By this definition, the assumptions about the levels have implications for every other design element of a multilevel mixed methods study. In order to design a multilevel mixed methods study, it is necessary to have an a priori theory about what constitutes a level, what levels exist, how levels are related either structurally or operationally, and which levels are relevant to the research problem of interest. Multilevel sampling should be a process of tailoring sampling techniques to the research problem in four ways (Onwuegbuzie & Collins, 2007). First, the samples should, to a degree commensurate with the purpose, represent the intended population. Second, the data generated from the samples should be valid and trustworthy. Third, the data generated from the samples should facilitate integration. Finally, the data generated
from the samples should have the potential to support compelling arguments to stakeholders and consumers of the research. Data collection tools and procedures should be carefully justified to explain the variables of interest and the level or mechanism with which the variable is associated. The analysis of the quantitative and qualitative strands of multilevel mixed methods designs should be guided by the purpose of understanding the system and the specific research questions about the system. The research questions for each strand may vary in accordance with the paradigmatic perspective and priority. Ultimately, the integration phase of a multilevel mixed methods research design should lead to meta-inferences about the system that further illuminate the structure, the components, and mechanism of the system.

**Empirical research related to SMaLL.** In this section, I return to the substantive literature. I have established that there is a cultural demand for SMaLL in the CCSS era. However, the overall purpose of the study and the integrative research questions require methods that allow for a closer inspection of SMaLL as it exists within mathematics classrooms and is experienced by students. In addition, I have established that reading entails multiple cognitive processes. However, the quantitative and integrative research questions require a closer inspection of the orthographic processing associated with SMaLL. In this section, I discuss substantively relevant empirical studies for the purpose of highlighting the range of methods with potential for generating data about the three levels of SMaLL under inspection — cultural, behavioral, and neurobiological — based on data collected from students.

Metacognition refers to an individual’s knowledge of his/her own cognitive processes (Bjorklund, 2012). Metacognition may entail an awareness of the component mental processes and/or the degree to which those processes are successfully applied. *Declarative metacognition*
refers to the conscious awareness of the nature or demands of a given task. In other words, reflections on declarative metacognition generate data about the context in which strategies or behaviors are initiated to respond to a task. In contrast, *procedural metacognition* refers to the conscious awareness of strategy selection and process monitoring (Bjorklund, 2012).

Reflections on procedural metacognition generate data about thinking behavior that can only be observed directly by the thinker. In addition, data generated by reflections on procedural metacognition can lead to inferences about cognition. Metacognition can be studied using quantitative methods. For example, surveys or questionnaires could be developed to draw conclusions about typical metacognitive habits. However, qualitative methods allow the researcher to gain an understanding of the extent of variations in metacognitive habits used in particular contexts.

Vlassis’ (2008) study of misconceptions about negative numbers provides a methodologically and substantively relevant example of using interviews to elicit metacognitive reflections for qualitative analysis. Questioning the traditional notion that difficulties with negative numbers are due to a fundamental misunderstanding of the concept of negative numbers, Vlassis drew on Vygotsky’s theories to hypothesize that the multiple functions (*minus, negative, and opposite*) associated with the ambiguous symbol, $-$, give rise to the confusion.

The interviews revolved around a set of simple linear equations designed to require the use of $-$ in multiple different ways. Having already attempted to solve the equations, students discussed the thinking process related to their written arguments for each solution. For example, in response to $4 - x = 5$, one student wrote (incorrectly): $x = 5 - 4$. The student explained that he was unable to imagine the legitimacy of a $-$ written to the left of a variable and, thus, reasoned
that it should be omitted when subtracting four from both sides of the equation. Altogether, the interviews provided insights about context and students’ thinking behaviors. That is, the interviews suggested that the differences in equations contributed to differences in responses and that the students’ difficulties were related to misconceptions about the syntax of the text rather than conceptual misunderstandings about negative numbers.

Cognition refers to the mental processes that yield new knowledge and/or extend learning in some way (Bjorklund, 2012). Cognition is, by definition, imperceptible to both observers and the individual completing the process (Bruning et al., 2011). The nature of cognition can be inferred from neurobiological change or, through the lens of theory, from observable behavior. In addition, an individual can make inferences about his/her own cognition, or lack thereof, through metacognitive reflection. This important caution applies to the study of both metacognition and cognition: A lack of overt communication (e.g., written or oral response) implies neither a lack of metacognition nor a lack of cognition.

Technological and methodological advances have made it increasingly feasible to collect quantitative neurobiological data to make inferences about cognition. For example, neurobiological data suggests there are differences between the cognitive processes related to mathematical text and alphabetic text. Grabner, Reishofer, Koschutnig, and Ebner (2011) used functional magnetic resonance imaging (fMRI) to analyze brain activity during a task involving bar graphs and simple equations. Differences in brain activity showed that cognitive processing among more mathematically competent adults differed from their less mathematically competent peers in ways that were not be detected by the behavioral data. In light of the task, the location of the increased activation appears to support the symbol-referent mapping hypothesis that the
angular gyrus is responsible for semantic mapping of arithmetic symbols and equations.

In a similar study, Grabner, Saalbach, and Eckstein (2012) studied the impact of presenting arithmetic tasks in a recently trained second language. The results suggest familiarity influences the angular gyrus activation. In response to numbers written in alphabetic first language form, the angular gyrus was activated as expected. However, the brain regions associated with executive function and working memory were activated in response to the alphabetic second language forms. The results suggest a caveat to the theory that the angular gyrus region is involved in mapping symbols to semantics: The degree of familiarity with the symbols appears to determine whether the angular gyrus activates a known semantic mapping or whether other channels work together to search for a mapping. Exploring cognition using technology often involves the analysis and interpretation of response times. In their study, Grabner et al. (2012) inferred that the increased response time associated with unfamiliar symbols indicated a cognitive cost of searching for unfamiliar symbols.

Quantitative data collection tools and analytical approaches can be combined in novel ways to generate inferences. For example, quantitative neurobiological data also suggests some hypotheses about reading symbolic mathematics expressions with operation and Arabic numerals. Using both fMRI and magneto-encephalography equipment to monitor a matching task, Maruyama, Pallier, Jobert, Sigman, and Dehaene (2012) required adults to distinguish between well-formed and poorly-formed complex expressions built with these symbols: operations (+, −, ×, ÷), parentheses, and Arabic numerals. Stimuli with differences located on the right of the character string were associated with longer response times, suggesting a tendency to read symbolic mathematics left to right in a manner similar to reading English. In
addition, recall of the location of operation symbols was more error-prone than recall of the location of Arabic numerals, suggesting that some symbols are more difficult to read than others. Interestingly, the brain activation appeared close to, but not in, the visual word form area usually associated with processing alphabetic words. These are intriguing findings given that Braet et al. (2012) concluded that the visual word form area was organized by orthography. In light of the neuronal recycling theory hypothesis that cortical areas specialize in response to repeated recruitment for functions sufficiently similar to the region’s original evolutionary purpose (Dehaene & Cohen, 2007), this suggests that, although there are may be some differences in SMaLL and English language literacy at the cognitive level, the differences are not extreme enough to require recruitment from distant brain regions.

Studies using various forms of quantitative neurobiological data are compelling because they offer insight into individual differences in cognition that cannot be determined by self-report or behavioral data. However, it is important to note that using specialized equipment can be limiting in that the tasks are restricted by the nature of the technology. For instance, fMRI studies necessarily involve tasks that limit physical movement. In contrast, quantitative and qualitative approaches to observing behavior without advance technology support some inferences about cognition while allowing a wider range of tasks and analyses. For example, a study by Tan et al. (2005) highlights the role of writing in developing literacy skills. They used time-limited and timed tasks to compare the contributions of listening, reading, speaking, and writing to the language development of Chinese speaking children. Writing, as determined by a timed copying task, emerged as a critical skill suggesting that the relative importance of phonological awareness (a cognitive ability) may depend on the structure of the written
language. This behavioral study contributes to the mounting evidence that reading may depend on writing and vice versa (Perfetti, 2011; Share, 2011).

Another methodologically unique study draws attention to the impact of oral reading fluency on mathematics language development. The practical goal of the study by Isaacson, Srinivasan, and Lloyd (2010) was to improve the usefulness of MathSpeak as a tool for the visually impaired. The algorithm for generating auditory renderings of complex expressions (e.g. $a + \frac{b}{c} + d$) was adjusted to include context specific pauses matching the natural pauses produced by experienced teachers reading similarly structured expressions aloud as if presenting them to a class. A test of the new renderings demonstrated that the adjustments offered substantial improvements in terms of accurate comprehension among the visually impaired users. The results of this behavioral study suggest phonological awareness (a cognitive ability) plays a role in SMaLL. A body of methodologically diverse behavioral studies suggests the ways learners negotiating language features, metacognitively and cognitively, may constrain or foster mathematical development (e.g. Göbel, Shaki, & Fischer, 2011; Han & Ginsburg, 2001; Paik & Mix, 2003).

Qualitative analyses of students’ discourse may generate inferences about metacognitive and cognitive processes. For example, Staats and Batteen (2010) analyzed the use of indexical language in a collaborative community of learners discussing mathematics text. Indexical language takes on meaning only in context. Pronouns are indexical terms inherently. However, many other relatively concrete words are indexical as they are applied in classroom conversation. For instance, indexical verbs such as move or slide might be used by students to convey a variety of steps in working with an equation. In classroom discussion, revoicing is common as students
and teachers refer repeatedly to a word to establish or confirm a referent-index relationship. Poetic structures, or patterned use of language, can also emerge as indexicals that set boundaries on interpretations. In their study of low-performing young adults enrolled in an introductory algebra class, a speaker struggling to discuss a common denominator repeatedly used the term *that*. Other students used descriptions of motion such as *line up, cross out, and drop through* to explain algebraic procedures. With respect to this study of SMaLL, the results highlight the possibility that students’ development of a mapping of oral language onto symbolic mathematics may be impacted by the quality of and use of indexical references during instruction. The authors concluded that the degree to which students and teachers coordinate written and spoken representations using indexical language may be more important than the indexicals themselves in classrooms that emphasize “reasoning over recitation” (Staats & Batteen, 2010, p. 55).

In terms of methodology, the Staats and Batteen (2010) study, which was framed as a sociocultural study of students’ collaborative learning process, highlights complications associated with distinguishing levels and whether inferences can be drawn about particular levels (e.g., cultural, behavioral, or neurobiological). Their inspection of collaborative discourse incorporated data related to classroom culture and social interactions. In addition, the data included students’ descriptions of their inner-thoughts and the behaviors involved in their mathematical process. Finally, that analytical process involved hypotheses about the students’ cognition grounded in theories related to social cognition. Despite, its relevance to multiple levels, as they are defined by DBCCC, the Staats and Batteen (2010) study was not designed or conducted as a multilevel study. Together, the theoretical frameworks and methodological approaches determine what inferences can be drawn and whether those inferences can be
ascribed to any particular level.

In a literature review, Ansari (2010) used comparisons of behavioral data and neurobiological data to highlight additional cautions in interpreting behavioral data. Similar behavior does not guarantee similar cognitive processing. Also, cognitive processing in developing brains is not necessarily a slower, weaker, or noisier version of mature processing of adult brains. On the contrary, the cognitive processing among children may take place along entirely different neurobiological pathways. In other words, developing children and adults are dissimilar in important ways. Children’s thinking, at any given stage of development, may not resemble adult thinking even when the conclusions they draw are similar. For example, Ansari reported that the neurobiological nature of the Arabic symbols mappings appears to continue to develop in substantial ways even after behavioral evidence suggests a functional ability to perceive the symbols as the intended referents. This evidence appears to support the overlapping waves model which describes learning as a process of adopting multiple strategies and applying them in strategic ways until a strategy that is consistently effective emerges as a preferred strategy for the particular task (Siegler, 2000).

In summary, quantitative and qualitative methods have been used effectively to study phenomenon with implications for studying SMaLL. More to the point, both quantitative and qualitative methods can be used to draw inferences about the use of the language of mathematics in the classroom, students’ rationales and strategies for reading symbolic mathematics, and students’ cognitive responses to reading symbolic mathematics. In the literature, quantitative approaches, using behavioral or neurobiological data, appear to be common for making the most direct inferences regarding cognition. In the literature, qualitative approaches appear to be
common for making inferences about context and metacognition.

Summary of Chapter 2

In this chapter I discussed a broad range of interdisciplinary topics. I established the role of SMaLL in formal education using educational policy and curriculum theory. The prevailing view, codified in the CCSS, is that learning to read and reading to learn is associated with the study of English language arts and not mathematics. However, there is substantial evidence that SMaLL is hidden in the curriculum. I also explored literacy theories, reading models, writing systems, and symbol theories to consider the implications of defining SMaLL as a literacy, contrast SMaLL with English language literacy, and question what it means to read symbolic mathematics. The review suggests reading is an instance of symbolizing and, more specifically, reading symbolic mathematics is an instance of symbolizing using the conventions of the language of mathematics. Finally, I discussed multilevel mixed methods research designs and described empirical research with implications for selecting methods for the multilevel study (see Chapter 3 for further discussion). In general, the literature suggests that, given their shared oral language, students may experience some similarities between SMaLL and English language literacy. However, the disparities between their writing systems suggest that students may experience SMaLL and English language literacy in different ways.
Chapter 3: Research Methods

This chapter describes the research design process and the research methods as they were implemented to conduct this study of symbolic mathematics language literacy (SMaLL). It is organized according to the critical considerations for the study design. First, the justification for using mixed methods is established. Then, these aspects of the research process are described: recruitment, consent, and sampling; instrumentation, data collection, and data products; and analysis and validation of the quantitative, qualitative and mixed methods analyses. Finally, I share my methodological reflections.

Design Process

Given the interdisciplinary nature of this study, the design process of this study was extensive, culminating in the development of a multilevel concurrent mixed methods research design. In this section, I explain in detail the research design process. First, I review the definition and characteristics of mixed methods research designs in general. Then, I justify the rationale for using a mixed methods research approach for this study of SMaLL. Finally, I describe how I modified the basic concurrent mixed methods research design to develop a multilevel variant appropriate to study SMaLL in a school setting through the lens of developmental bio-cultural co-constructivism (DBCCC).

Characteristics of mixed methods research designs. I define \textit{mixed methods research} as an inquiry enterprise that intentionally incorporates both quantitative and qualitative data and analytical tools for the purpose of transcending what can be discovered and understood from a traditionally quantitative or traditionally qualitative paradigmatic perspective alone. This definition includes three common criteria for an adequate characterization of mixed methods
research: a description of what to mix, when to mix, and why to mix (Johnson, Onwuegbuzie, & Turner, 2007). A brief analysis of the definition according to these criteria is useful for understanding the fundamental features of mixed methods research designs. Furthermore, the analysis scaffolds the introduction of vocabulary that holds particular meaning in the field of mixed methods research.

The specification of what to mix — quantitative and qualitative data — clarifies that, minimally, a data set consisting of operationally predefined measures and a data set consisting of open-ended or unstructured information are collected. In the quantitative tradition, the former data set, usually relatively large, includes the precise measurement of a discrete number of specific constructs (Johnson & Christensen, 2012). In contrast, in the qualitative tradition, the latter data set, usually relatively small, is comprised of purposefully imprecise products (e.g., field notes, interviews, photographs) with the researcher as the primary collection instrument (Johnson & Christensen, 2012). The former allows for the estimation of representative or average characteristics in terms of multiple variables of interest. In contrast, the latter allows for the thick, rich descriptions of a phenomenon of interest as it uniquely exists in a particular context (Johnson & Christensen, 2012; Patton, 2002).

In mixed methods research, the collection of each kind of data and the basic analysis of each data set within its own research tradition is conceptualized as a strand (Creswell & Plano Clark, 2011). A mixed methods research design requires at least two distinct strands of inquiry (Plano Clark & Ivankova, 2016). The strands must be deliberately mixed, in what is referred to as a process of integration, to produce insights beyond those generated by either strand independently (Plano Clark & Ivankova, 2016). To use a colloquial expression, mixing
quantitative and qualitative data makes it possible to investigate the nature of both the forests and the trees in multiple ways. For example, mixing different kinds of data may be used to study what is unique about a tree given what is typical of trees. Or, as in the case of this study, mixing different kinds of data may be used for a multilevel study of the reciprocal influences between the forest and the trees. The benefit of mixing both quantitative and qualitative data is the potential to generate complex knowledge such as “structure and process, outside-researcher and inside-participant, macro and micro levels, and cause and meaning” in ways that connects research and practice (Greene, 2007, p. 46).

Defining mixed methods research as an intentional practice addresses the issue of when to mix. The requirement of deliberate engagement with both quantitative and qualitative data and analysis conveys the idea that mixed methods research is not the addition of an optional or reactive phase in the context of traditional quantitative or qualitative research. Instead, mixing is planned before data collection with mental models and thought experiments. In mixed methods parlance, mixing occurs at points of interface (Creswell & Plano Clark, 2011). It is not clear that there is a consensus among mixed methodologies whether there is a necessary degree and quality of mixing required to justify a point of interface. However, in some sense, the first point of interface takes place during the design phase in the form of imagining how sampling, data collection, and analyses from different research traditions can work together to yield defensible answers to particular research questions.

As the research unfolds, additional points of interface depend on the timing, or the relative order of implementation of the strands (Creswell & Plano Clark, 2011). The names of some common mixed methods research designs are based on the timing of the strands. For
example, concurrent mixed methods research designs in which the quantitative and qualitative strands are implemented in tandem are named for their simultaneous timing. In contrast, sequential designs in which one strand is implemented first and, then, influences the implementation of the other strand are named for their sequential timing. The timing of a mixed methods research design influences when (and how many) points of interface occur and the nature of the mixing techniques at those points.

Finally, the definition answers the question of why to mix. In general, mixed methods research is used to generate complex forms of knowledge that would not be possible given the restrictions inherent to relatively narrowly prescribed paradigms and methods associated with quantitative and qualitative research (Morgan, 2007; Rossman & Wilson, 1985). Because the purpose of using mixed methods research is to transcend, in some way, the inferences drawn from each individual strand, an inference drawn from integration is referred to as a *meta-inference* (Creswell & Plano Clark, 2011). As mixed methods research has become more prevalent, mixed methods research methodologists have classified the specific reasons cited for conducting mixed methods research (e.g., Bryman, 2006a; Greene, Caracelli, & Graham, 1989, Plano Clark & Ivankova, 2016).

The specific reason, or *rationale*, for addressing a problem using mixed methods (rather than either quantitative methods or qualitative methods) influences the research design and the nature of potential meta-inferences (Plano Clark & Ivankova, 2016). If triangulation is the rationale for a mixed methods study, for instance, the meta-inferences will be related to the convergence (or divergence) of conclusions draw from each strand. Alternatively, if development of a survey is the rationale for a mixed methods study, the meta-inferences will be
related to how both strands serve to understand the affordances and constraints of the measurement tool. As with the case of this study, if initiation is the rationale for using mixed methods, the analysis will “turn ideas around” (Rossman & Wilson, 1985, p. 637) in a way that generates meta-inferences with the potential to “initiate new interpretations, suggest areas for further analysis, or recast the entire research question” (Rossman & Wilson, 1985, p. 633).

Notably, mixed methods research, by this definition, requires neither a prescribed paradigmatic position nor a prescribed analytic technique. Instead, mixed methods research is an approach to conducting a study to address sophisticated questions that cannot be fully addressed by either a quantitative or a qualitative research design. Some mixed methods studies are conducted in such a way that the quantitative data and techniques are privileged as more critical and informative, and others are conducted in such a way that the qualitative data and techniques are privileged as more critical and informative. In mixed methods terminology, the former is said to have a quantitative priority and the latter is said to have a qualitative priority. The priority of a mixed methods research design may be driven by the paradigmatic perspective, the research question, or the analytic technique (Creswell & Plano Clark, 2011).

Importantly, mixed methods research does not provide a logical resolution to concerns about incommensurability of quantitative and qualitative perspectives. Rather, mixed methods research is a process of carefully mixing data types and analytic techniques to maximize commensurability and draw reasonable and useful meta-inferences (Maxwell, 2011; Morgan, 2007; Rossman & Wilson, 1985). Mixed methods research is sometimes the only sufficiently complex and customizable approach to generating meaningful and usable knowledge about a world that is simultaneously ever-expanding and increasingly particularized.
Justification for a mixed methods design. Mixed methods research is only advisable in the case that it is necessary to generate thorough and trustworthy answers to a complex research question (Creswell & Plano Clark, 2011). Conducting interdisciplinary mixed methods research to answer such a question demands a range of expertise across content areas and methodologies. With this admonition in mind, the use of mixed methods as a means to answer a research question is only warranted when it is necessary to realize the study’s goals (Plano Clark & Badiee, 2010). The purpose of this study is to generate novel insights into the nature of SMaLL in the context of formal education with implications for instructional practice. In order to justify the use of mixed methods as necessary, I looked to the mixed methods research community and the educational research community for guidance and considered whether mixed methods could support an effective argument for answers to the research questions.

Mixed methods rationale. Applied purposefully, a mixed methods research approach has unique potential compared to quantitative and qualitative research approaches (Johnson & Onwuegbuzie, 2004). Traditional quantitative research studies are typically designed to confirm or deny specific hypotheses (Johnson & Christensen, 2012). They are well-suited to exposing patterns of regularity under specified circumstances (Lomax & Hahs-Vaughn, 2012) and, in the context of select designs, have the potential to substantiate cause-effect relationships (Shadish, Cook, & Campbell, 2002). In contrast, traditional qualitative research studies are typically designed to generate rich description (Patton, 2002). They are well-suited to exposing complexity under natural conditions and, in the context of select designs, have the potential to generate causal explanations (Maxwell, 2012). There is a tradition of justifying mixed methods as both necessary and sufficient for a given research problem and purpose (Plano Clark &
The rationale for this study is described as *initiation* in the mixed methods research literature (Greene, 2007; Greene et al., 1989; Johnson et al., 2007, Rossman & Wilson, 1985). The aim of this study was to expose points of convergence, points of divergence, and novel themes as a means to launch a new research agenda with the potential to inform instructional practice in mathematics education. In short, this study was intended to reveal the “provocative” (Greene, 2007; Greene et al., 1989). Using DBCCC as a guiding framework, I conceived of SMaLL as a multilevel phenomenon shaped by cultural change, behavioral change, and neurobiological change. Given the limited substantive research with relevance, I presumed SMaLL to be multifaceted, influenced by multiple variables on each of the three levels. The purpose of conducting a study with implications for instructional practice required gaining insights into SMaLL culture in the classroom. The review of the literature suggested that, minimally, an investigation of metacognitive and cognitive responses to mathematical text were necessary to initiate and situate a productive SMaLL research agenda within interdisciplinary research communities.

Language was the cultural facet of SMaLL of interest. Restricting the study to a middle school in a district operating under the Common Core State Standards (CCSS) made it possible to gain insight into the ways SMaLL operates in classrooms that exist in an idealized academic culture. The study of classroom culture does not demand a particular paradigmatic perspective and, for the purpose of initiation, may be better understood when studied from multiple perspectives. Metacognition, the behavioral facet of SMaLL of interest, was presumed to be accessible only to an individual. It involves knowledge about thinking patterns and uniquely
constructed ideas about how cognition works and, thus, demands a qualitative inspection. Cognition, the neurobiological facet of SMaLL of interest, was presumed to be inaccessible to an individual. It involves theoretically driven measurements of operationalized latent variables and, thus, demands a quantitative inspection. In summary, this study required the integration of quantitative and qualitative approaches to venture into uncharted territory using DBCCC to explore SMaLL as a multilevel phenomena with the potential to invite new ways of thinking about how students develop the literacy skills necessary to meet the demands of mathematics achievement.

*Mixed methods in education.* The history of curriculum theory suggests that both quantitative and qualitative data play critical roles in education in the United States. Curriculum is concerned with what is learnable and what is worth knowing. Some argue that curriculum should entail what is typically learnable and typically valued as knowledge (Bobbitt, 2009; Hirsch, 1987). Others argue that curriculum should be un-standardized and tailored to individual students’ instructional needs and learning goals (Au, 2011; Dewey, 1998; Noddings, 2009; Sleeter, 2005). Curriculum can be both the object of research and the subject of research (Dillon, 2009). On one hand, the written curriculum, when informed by statistics such as students’ test scores or predicted employment trends, is an object of quantitative inquiry. On the other hand, when researchers seek to understand how the enacted curriculum is unintentionally shaped by the demands of teachers and students in particular classrooms, it becomes the subject of qualitative inquiry. Both modes of inquiry have implications for understanding the relationship between formal education and learning. The centrality of curriculum in formal education — and the tradition of drawing on both quantitative and qualitative research to debate what students can and
should learn — may be among the reasons that mixed methods research is encouraged (Johnson & Onwuegbuzie, 2004) and commonly accepted as a productive approach to studying complex phenomena in the field of education (e.g., Agirdag, Van Avermaet, & Van Houtte, 2013; Fitzpatrick, 2011; Calfee & Sperling, 2010; Kyei-Blankson, 2014; Smith, Swars, Smith, Hart, & Haardörfer, 2012; Sondergeld & Koskey, 2011; Teague, Anfara, Wilson, Gaines, & Beavers, 2012; Wei, Chesnut, Barnard-Brak, & Schmidt, 2014).

This study was not designed to answer curricular questions of what degree of SMaLL is learnable or worth learning. Nor was this study designed to answer the question of how much or in what ways SMaLL ought to be taught. This study was conducted under the assumption that SMaLL is an experience-dependent skill and that the CCSS demand a biological secondary degree of SMaLL development. In order to set the stage for future research with the potential to inform instructional practice, this study was designed to be a first step towards answering the questions of how SMaLL develops typically among students and how individual students experience it in classrooms during the CCSS era. The former demanded a quantitative approach while the latter demanded a qualitative approach.

**Mixed methods as a dialectical argument.** As previously discussed in Chapter 1, I approached this study with a dialectical stance. In essence, that means I embarked on the project with a mixed methods mindset and a willingness to negotiate paradigmatic positions as needed to craft a research design and present a novel form of argument to answer the research questions.

According to Vokey’s (2009) framework for *dialectical argument*, these four criteria can be used to assess educational inquiry: intelligibility, internal coherence, plausibility, and success in practice. Intelligibility requires that the approach be described sufficiently to avoid ambiguity.
Internal coherence requires that the approach be employed in a manner that minimizes logical contradictions. Plausibility requires that the approach be reasonably acceptable in relevant research communities. Finally, success in practice requires that the productivity of the approach bear out over time. I defined mixed methods using standards and terminology commonly accepted in the mixed methods research community. I established that both quantitative and qualitative approaches are necessary to answer the research questions. Furthermore, I established that mixed methods can be applied to the study of SMaLL in a way that minimizes logical contradictions and capitalizes on the affordances of quantitative and qualitative approaches. Finally, I confirmed that mixed methods research is prevalent and considered plausible and successful in educational research including research in the field of mathematics education (Hart, Smith, Swars, & Smith, 2009; Ross & Onwuegbuzie, 2012).

**Crafting a multilevel concurrent mixed methods design.** In this section, I describe the design development process and explain how the features of the research design worked together to allow for inferences about SMaLL as a multilevel DBCCC system with reciprocal and interactive mechanisms. First, I considered a number of factors in the process of selecting an appropriate mixed methods research design. I compared the sufficiency, feasibility, and potentiality of two basic mixed methods designs, selecting the concurrent mixed methods research design as the best methodological choice. Then, I tailored that design to meet the criteria for a multilevel design in terms of sampling, data collection tools, data analysis, and integration.

Initially, I considered using a sequential design. A sequential design, an ordered two-phase design with the results of one strand being used to inform the implementation of the other
strand phase, is relatively easy to conduct, particularly for a single researcher, because it follows a stepwise progression with a somewhat uniform workload (Creswell & Plano Clark, 2011; Morse, 1991). An explanatory sequential design (denoted as QUAN → QUAL) would have made it possible to capitalize on quantitative results in the selection of particularly salient cases to develop a qualitative explanation of those results. An exploratory sequential design (denoted as QUAL → QUAN) would have made it possible to conduct a qualitative exploration to inform the selection or development of quantitative measurement tools and analyses. Although a sequential design could have been used to conduct a useful study of SMaLL, the option was excluded due to validity and feasibility concerns for this particular study.

A sequential design was inappropriate for this study of SMaLL on two accounts. First, additional time to analyze data and conduct a second data collection phase would have reduced the likelihood of collecting sufficient and appropriate data. Engaging with adolescents for research purposes in a school setting is a challenge in terms of ethics and scheduling. Planning to schedule adolescents for a second phase of data collection after the end of the school year would have required an initial collection of personal contact information and increased the chances of attrition in the subsequent qualitative phase. Second, sequential designs impose a temporal gap for analysis between data collection for the strands. For example, in a mixed methods study of teaching practices related to eliciting student engagement, Cooper (2014) collected students’ responses to quantitative surveys, then conducted quantitative analyses, and then used those quantitative results to inform the selection of classrooms for the qualitative strand. As a result of the process, the qualitative data collection did not take place immediately after the quantitative. Because SMaLL is presumed to be a developmental phenomenon subject
to change, a temporal gap between data collection for each strand would introduce a methodological complication. For the quantitative and qualitative data sets to reflect the same developmental stage, data collection for both strands needed to be nearly simultaneous to reduce the likelihood of significant and unaccounted for development in the interim.

With the considerations related to the sequential design in mind, I evaluated whether a concurrent parallel design was sufficient to address the research questions. The concurrent design (denoted QUAN + QUAL) is characterized by the simultaneous (or nearly so) collection and analysis of both quantitative and qualitative data (Creswell & Plano Clark, 2011; Morse, 1991). In contrast to sequential designs, the quantitative and qualitative strands of the research occur simultaneously in the sense that neither strand necessarily depends on the data or analysis of the other. Thus, a concurrent parallel design eliminated both the feasibility and validity concerns associated with using a sequential design for this study.

The concurrent design is a common mixed methods design for multiple reasons (Creswell & Plano Clark, 2011). It is generally recognized as the first mixed methods design formally described in methodological literature (Jick, 1979). The design is intuitive in that each of the independent strands are often easily understood in terms of a prevalent research paradigm. Finally, the design reduces the investment required of collaborators and participants by consolidating the data collection for both strands into a single phase. This last feature was an important consideration in the design of this study. In brokering an agreement with the research site, I was cognizant of the complex relationships between administrators and teachers (e.g., Minckler, 2014) and, therefore, careful to avoid increasing the demands on teachers and students. The concurrent design limited the potential loss of teacher productivity and classroom
instructional time. Thus, the concurrent mixed methods design emerged as a more reasonable choice than a sequential design for conducting this study of SMaLL among adolescents in a middle school setting.

In addition, concurrent designs are acceptable in educational research communities. For example, the design has been successfully applied in mathematics education (e.g., Eli, Mohr-Schroeder, & Lee, 2013; Kyei-Blankson, 2014) and literacy education (e.g., Huang, Capps, Blacklock, & Garza, 2014; Sanger, Ritzman, Schaefer, & Belau, 2010). Although one strand may take priority over the other in a concurrent design (Creswell & Plano Clark, 2011; Morse, 1991), neither the DBCCC theoretical framework nor the extant literature suggested that one strand should be identified a priori as more important or influential than the other. Ultimately, I identified the concurrent mixed methods design with quantitative and qualitative strands of equal priority as sufficient given the nature of the research questions, feasible given the practical concerns related to the population of interest and research site, and defensible given the demands of making arguments for inferences and meta-inferences.

Selecting the concurrent mixed methods research design served the purpose of framing the logic for the overall study (Plano Clark & Ivankova, 2016, p. 108). However, it did not dictate any particular decisions related to data collection, strand analysis, or integrative analysis. Those decisions were made through constant negotiation of sufficiency, feasibility, and potentiality issues from a dialectical stance. In other words, the detailed decisions about how to conduct this exploratory study with a concurrent approach entailed returning repeatedly to the purpose (i.e., to gain novel insights into SMaLL skills among middle school students learning under the CCSS with implications for instructional practice) and rationale (i.e., to “turn ideas
around” (Rossman & Wilson, 1985, p. 637) and initiate a new perspective by considering SMaLL as a multilevel developmental phenomenon) to select best approaches from among possible approaches. Ultimately my approach to this study led to a research design in line with recommendations for initiation studies: different methods guided by different paradigms integrated with a relatively high degree of interaction (Greene et al., 1989).

The procedural diagram for this study (see Figure 3.1) illustrates the overall concurrent design logic and research methods (Plano Clark & Ivankova, 2016). The shape of the diagram highlights the principal features of a concurrent mixed methods design. At the top, the gray cell summarizes the design phase which was the first point of interface. The parallel paths illustrate the concurrent phase which consisted of a quantitative strand (shown in red on the left) and a qualitative data strand (shown in black on the right). The parallel paths indicate that, for this study, the design of each strand entailed data collection and analysis and neither strand was necessarily dependent on the analysis of the other. Finally, the diagram shows that mixed methods analytic integration techniques were used to generate meta-inferences related to the research questions.

Although Figure 3.1 summarizes the research design and methods for this study, it fails to highlight how the basic concurrent mixed methods research design was adapted to accommodate the rationale of initiation and the purpose of exploring an multilevel system. I designed Figure 3.2 to mirror Figure 3.1 and draw attention to the initiation and multilevel features of the design. The design phase, shown at the top of the diagram entails a process of negotiating two paradigms (quantitative and qualitative) to insert links of potentiality into each strand. For example, in designing the sampling procedures I satisfied the sampling requirements for each strand and
inserted a link by the selecting a quantitative sample to cover the qualitative needs and then selecting the qualitative participants from the quantitative sample.

Figure 3.1. Procedural diagram for the multilevel concurrent mixed methods research design
Figure 3.2. Multilevel concurrent mixed methods research design indicating interaction adaptations.
In order to adapt the concurrent framework to the multilevel purpose, I inserted links to each level in each strand. For example, the quantitative strand, although primarily focused on drawing inferences about automated reading at the neurobiological level, included a measure of print exposure and a measure of reading habits to allow for the exploration of the cultural and behavioral levels, respectively. Although the qualitative strand was primarily focused on drawing inferences about intentional approaches to reading at the behavioral level, I included questions that allowed participants to spontaneously attribute behavior to classroom expectations or automatic processes. This made it possible to explore the cultural and neurobiological levels, respectively, with the participants. As previously illustrated in Figure 3.1, both strands were sufficient to produce useful results alone. However, inserting links allowed for a high degree of interaction during the concurrent analysis (as shown at the intersection of the dotted lines). In the mixed methods tradition, particularly from a dialectical stance, integration during the concurrent phase is not only permissible but advisable (Greene et al., 1989; Ivankova & Kawamura, 2010). Figure 3.2 shows that, through interaction during the concurrent phase, I was able to notice and craft links for the final integration process. During the finale integration process, I linked the data and findings together to produce a connected, full picture of SMaLL and draw meta-inferences about SMaLL as a multilevel phenomenon.

The remainder of this chapter provides a detailed explanation of how the multilevel concurrent mixed methods research design was implemented to answer the research questions. The procedural diagram (see Figure 3.1) and interaction diagram (see Figure 3.2) discussed in this section are intended to serve as tools to facilitate the explanation. As the research process unfolded, methodological decisions reified the design, tailoring it to the demands of the data.
This chapter presents the methods used and describes the methodological turning points that shaped the implementation of the design.

**Research Site**

For privacy, pseudonyms are used to refer to organizations and all individuals recruited to facilitate or participate in this study outside of the research team. The study took place at Holland Middle School in the Brookover Public Schools district located in Ohio. In preparation for the study, I worked with the principal at Holland Middle School to ensure that the research study and process would be feasible and useful given their needs. Two administrators and three mathematics teachers at the research site collaborated with respect to setting logistical parameters and providing access to potential participants. However, the staff declined to act as co-researchers as defined by the University of Cincinnati (UC) Institutional Review Board (IRB) and, thus, did not design the study, recruit or obtain consent from participants, collect data, access the raw data, or analyze the data. The staff influenced the design of this study in that they imposed reasonable restrictions on access to potential participants, the school building, and computer equipment. In accordance with the research design, the teachers acted as local experts, giving their assessments upon request.

At the time of the study, the state of Ohio had designated Brookover Public Schools as a high performing district annually for more than a decade. The Brookover Public Schools district is politically important, in part, because it met academic goals despite a relatively large student body and relatively low per pupil expenditure. The district enrolled more students than the average district yet kept per pupil expenditures near the state-wide average. The district committed to and achieved the goal of fully aligning classroom practice with CCSS by the
2014-2015 academic year.

Given the district’s status and efforts to stay current with curricular trends, it was reasonable to make some presumptions about the nature of the educational environment in grade-level core mathematics courses at Holland Middle School. For example, educators and students likely understood mathematics as a relatively high-stakes course. In addition, the typical student would likely be preparing to transition from math as arithmetic to math as algebra. In addition to offering core mathematics courses aligned with the CCSS, Holland Middle School offered two mixed-grade elective courses in mathematics. Thus, Holland Middle School as a research site offered two benefits: an idealized CCSS environment and access to mathematics students in 7th and 8th grade without disrupting instructional time in required courses.

The overall demographics of the district limits the population to which inferences may be made. In terms of ethnicity, the majority of the student body was White (Ohio Department of Education, 2014). Together the students who were American Indian, Black, Hispanic, and multiracial made up less than 20% of the student body. There were no migrant students. Less than 10% of the students were identified as members of each of these subgroups: students with disabilities, economic disadvantage, and limits English proficiency. All subgroups had similar rates of attendance. The student body in the district may be lacking in terms of demographic diversity as compared to other schools in the United States. However, the research site offered access to district-wide diversity because all middle school students in Brookover City Schools attended the same school. With those limitations in mind, Holland Middle School was determined to be an appropriate research site for this study because it reflected the kind of learning environment that is commonly regarded as ideal in terms of performance, curriculum,
Recruitment, Consent, and Sampling

The concurrent design made it possible to manage recruitment and consent in a single stage despite differences in the rationales for sampling and sample size between the quantitative and qualitative strands. The recruitment process was determined in collaboration with the staff at the research site to minimize disruption to the learning environment and maximize the validity of the study. To explain the process, I begin by discussing the procedures used for the quantitative strand. Then, I explain the procedures used for the qualitative strand. Although the quantitative strand was not prioritized over the qualitative strand, this approach mirrors the temporal process used to implement the procedures and makes for a more comprehensible explanation of the relationship between the quantitative and qualitative samples.

Consent and assent. Recruitment, consent, and sampling for this study was guided by the mission to design a high quality mixed methods research study dedicated to the ethical treatment of participants and sufficient to address the research questions. This goal indicated that a waiver of parental consent was appropriate on four accounts. First, the study was expected to have a trivial immediate and long-term impact on participants as the activities required for participation were not substantially different from typical school activities. Second, the risk of harm was at a minimum. Participation in the study was expected to be less cognitively and psychologically demanding than activities typical of mathematics classrooms that require problem solving. Third, if a problem did arise, I had a procedure to inform the school staff, parent, and/or student about the concern as quickly as possible. In particular, I was prepared to draw on my experiences as a mathematics educator and an emerging developmental learning
sciences expert to be vigilant with respect to math anxiety. Finally, the requirement of obtaining explicit consent from parents of adolescents was a potential obstacle to achieving the purpose of this study because it could have constrained the variation among the participants in terms of relevant dimensions.

The widest possible range of diversity among the student body was necessary to fulfill the purpose of the study and inform the faculty at Holland Middle School about the range of abilities and experiences among their students. Requiring explicit consent from parents would have limited the diversity of the participants in ways that might have precluded the possibility of finding important similarities, differences, or themes across the full spectrum of students. Even in cases that permission would have been granted, the explicit consent might not have been received. Students with parental permission to participate might fail to submit appropriate forms of explicit consent on time, or at all, due to inadequate organizational skills, limited technical resources, or difficult family relations. In summary, an explicit consent process was likely to undermine the purpose of the study by producing an unnecessarily homogenous sample excluding students on the basis of potentially relevant factors (e.g., cultural, behavioral, or neurobiological factors) rather than an objection on the part of the parent.

In lieu of requiring explicit parental consent, recruitment entailed a two phase process intended to inform parents, provide parents with multiple means of communicating objections, and ascertain student assent. In the first phase, I sent a letter of introduction (see Appendix A) and an information sheet (see Appendix B) to parents of students enrolled in the courses identified for recruitment. Parents of those students, designated as the participant pool by the district administrators, received both a hard copy and an electronic copy. The former was
delivered via students in an anonymous opaque envelope with a label indicating two time-sensitive documents were enclosed. The latter was distributed by the classroom teacher using a typical electronic classroom announcement procedure.

The recruitment materials, approved by Holland Middle School and the UC IRB, described what is known, in laymen’s terms, as a passive consent process. In addition to describing the nature of the research study and the parents’ and participants’ rights, the letter provided instructions and a deadline (seven or more days after the distribution date) for revoking permission to recruit their student. After the window for parental revocation closed, the remaining students were considered eligible to provide their assent independently. Students who were not also enrolled in a mainstream mathematics course were excluded from eligibility to ensure that all students invited to participate were free of atypical impairment and, thus, reasonably capable of informed assent. The school staff verified that there were no students who met the exclusion criterion.

On the day of the scheduled data collection, I went to the classes identified for recruitment. All eligible students accompanied me to the lab for the assent process. Clipboards with two copies of the assent form (one with a participant identification label affixed and one without) and a handout with information about how to cope with mathematics anxiety were randomly distributed to the students. After guiding the students through the assent form with an oral explanation, I offered wait time for independent reading and questions about the assent form (see Appendix C). The mathematics anxiety handout (see Appendix D) was provided in an abundance of caution to alert students and parents to the symptoms of math anxiety and provide contact information for in-school and professional resources. All students were instructed to take
both the assent form without a label and the mathematics anxiety handout home to discuss with a parent.

Students who assented were given instructions to complete and sign the assent form with the participant identification label affixed. Assenting participants used the form to indicate whether they agreed to participate in only the quantitative portion of the study or both the quantitative and qualitative portion of the study. The completed forms served as the master list of participants linking participants’ names to their participant identification number (ID). The process made it possible to randomly and efficiently assign participant IDs within each class without gathering any information from students who elected not to participate. Students who declined to participate were instructed to return the clipboard, leaving the unsigned assent form with the participant identification label affixed, and return to class.

According to the enrollment numbers provided by the staff, a total of 217 students were eligible for recruitment. Parental permission was revoked for nine students via phone or email before the study began. Parents were not asked to indicate their rationale for declining to participate. Most parents did not make a statement about their decision process. One parent, however, suggested the decision was related to beliefs about the detrimental effects of statewide standardized testing which were scheduled to be administered soon after the data collection for this study. Initially, 161 students were assented; therefore, 56 students were either absent during recruitment or declined to assent. Students were also not asked to indicate their rationale for declining to participate. My field notes suggest that students who declined to assent considered these factors in making their decision: stance towards mathematics; beliefs about parental, teacher, and/or peer preferences; interest in alternative classroom activity. For example, some
students consulted with classmates during the wait time to discuss the benefits of returning to class to collaborate on an in-progress group project. After the assent process, parental permission was revoked for one student. In addition, two students decided to revoke their assent after completing a portion of the quantitative data collection process. Data generated by those three students were disregarded. The number of students who assented and fully participated in the quantitative portion of the study was 158. Eighteen of those students went on to participate in the qualitative portion of the study.

**Quantitative sampling.** For the quantitative strand, the goal was to generate a relatively large sample of adolescent 7th and 8th graders (ages 12 to 15) enrolled in different mathematics courses. Notably, a large quantitative sample was necessary to create a sufficient pool of participants to cover the variation requirement of the qualitative strand (for more details see Qualitative sampling). For the quantitative strand, the sample needed to be large enough to support t-tests and correlational analysis. Also, a large sample was required for a factor analyses to empirically inspect the nature of the adapted survey tools. The sample size recommendations for this procedure differ in the literature. Although some suggest that a ratio of 10 participants per item is sufficient, others claim that estimates of latent variables will be inaccurate in samples of less than 200 (Kelloway, 2015; Tabachnick & Fidell, 2013). A sample size of approximately 200 was determined to satisfy both heuristic minimum sample size recommendations for exploratory purposes.

Given the nature of collaborating with a public school and ethical considerations of recruiting adolescents, a clustered convenience sampling was the only feasible option (Teddlie & Tashakkori, 2009). As illustrated in Figure 3.3, the sampling began with the selection of a school
delivering instruction aligned with the CCSS. Within that school, I had permission to recruit students enrolled in elective classes taught by three mathematics teachers. Two of the teachers were responsible for teaching multiple sections of Math Extension, an elective designed for students who wanted to extend their mathematical practice beyond the scope of their required mathematics course. The remaining teacher was responsible for teaching multiple sections of Math Intervention, an elective designed for students who wanted support to meet challenges in their required mathematics course. This process made it possible to fix the academic culture of the school and allow for the possibility of variations in SMaLL culture across classrooms.

Figure 3.3. Illustration of the quantitative sampling process from research site selection to participant assent.
Students in the initial participant pool were enrolled in a total of 13 classrooms. There were eight Math Extension classrooms with a total of 180 students taught by either Mr. Rowe or Ms. Davis. There were five Math Intervention classes taught by Ms. Kennedy with a total of 37 students. In sum, the participant pool included 217 students. Although students were not required to participate, the recruitment process (as described in the previous section) was designed to increase the likelihood of generating a sample of sufficient size for the quantitative analysis. In addition, this process increased the likelihood of generating an appropriately varied subset of the quantitative sample for the qualitative sample. The sampling process yielded a reasonably large and diverse quantitative sample ($N_{QUAN} = 158$) which is described in further detail in Chapter 4.

**Qualitative sampling.** In order to answer the qualitative research questions, I planned to generate a sample size of 15 to 20 students of maximum variation (Teddlie & Tashakkori, 2009) for the qualitative sample. A homogeneous qualitative sample would have undermined the overarching purpose of initiation by limiting the likelihood of generating novel results. In order to answer the mixed methods research questions, I used a nested sampling design (Collins et al., 2007), which entailed selecting a subset of quantitative participants as a qualitative sample. This approach created an intentional link between the qualitative and quantitative strands to facilitate integration.

The concurrent design did not allow for analyzing the quantitative data before selecting a subset for the qualitative sample. Therefore, I used a purposive reputational sampling process for generating a sample of maximum variation (Teddlie & Tashakkori, 2009) with the classroom teachers acting as local experts. Using this approach, a pool of students who varied in terms of
math achievement and reading skills was created. Logistically, the sampling for the qualitative strand involved two post-assent processes: referral collection and schedule development. The purpose of the referral collection was to identify where potential qualitative participants fell along the maximum variation dimensions. I began by compiling a list of all quantitative participants who indicated interest in an interview by marking the box labeled “I want to do the computer part AND the interview part” on the assent form. Then, I prepared three different Maximum Variation Matrix Worksheets (see Appendix E), one for each teacher. The worksheet included a list of eligible students organized by class period. In addition, the worksheet included a 4x4 matrix labeled with two dimensions: math achievement and reader type. Teachers were asked to make referrals by sorting each name on the list into the cell that most closely described both the student’s math achievement and reader type.

The referral collection process was not intended to be a reliable (in the classical test theory tradition) classification of the students, but a pragmatic process for generating a sample that varied as much as possible without requiring a rigorous analysis phase. To complete the matrix, the teachers estimated whether each student fell into the 1st, 2nd, 3rd, or 4th quartile in terms of mathematics achievement. In addition, the teachers characterized each student as resistant (unwilling to read even when asked), compliant (willing to read when asked), independent (reads without being asked), or exceptional (exhibits some unique reading habit). Evidence from communication with the three teachers suggested that either the student populations differed across teachers or the teachers used the scales in slightly different ways. After receiving the completed Maximum Variation Matrix Worksheets, I used the teachers’ responses to compile a master maximum variation pool of students organized by math
achievement and reader type. The Maximum Variation Matrix Worksheets were not used as data for further analysis.

The purpose of the schedule development was to avoid instructional loss. The interviews were all scheduled near the end of the spring semester during the week of federally mandated testing for the school district. For five calendar days, students were engaged in testing for half of the school day. The other half of the school day, students attended classes on rotating schedules that varied by grade. Because teachers and students were engaged in relatively light activities and did not cover any new content during the testing period, I was granted access to participants for interviews during the time in which their elective course met. I began by creating a master schedule matrix for the week, accounting for both the 7th and 8th grade rotations. Then, I inserted the referred students’ names to indicate their availability over the course of the week. Finally, I ordered names in each cell to create a call list for each period in which students were available for an interview. That is, I ordered the students in such a way that I increased the likelihood of interviewing students from as many cells of the maximum variation pool as possible.

The qualitative data collection took place over the course of five days during May of 2015. During the qualitative data collection, I used the schedule to determine which classroom to visit during each class period of each day. When I arrived at the classroom, I requested permission to interview the first student on the call list. In the event the student was absent or engaged in critical educational activity, I moved to the next name on the call list. In response to the few students who asked about the process, I explained the two step process this way: First, I would compile the names of the participants who agreed to interview into lists by class. Then, during each class, I would go to the class and, starting at the top of the list, call names until I
found a someone on the list who was present and still agreeable to an interview. I explained that it would essentially be a random process that would allow me to talk with a variety of students from a variety of classes.

There are no decisive sample size guidelines for qualitative research (Patton, 2002). Thus, the sample size for the qualitative strand could not be precisely determined a priori. In qualitative research, the sample size is guided by an attempt to balance the breadth and depth of the data collected (Hatch, 2002). Ideally, interviews should be conducted and analyzed iteratively until saturation. That is, the final sample size should be determined when additional interviews fail to produce novel themes. However, given schedule constraints it was not feasible to analyze each interview before conducting the next. Instead, all interviews were conducted prior to any systematic qualitative analysis. Given the goal of maximum variation in terms of math achievement and reader type, a minimum of 15 participants was estimated to be sufficient for saturation. Given feasibility considerations, no more than 20 interviews were planned. Ultimately, the qualitative sample size \( N_{\text{qual}} = 18 \) was determined by the availability of participants during the time I had approval to conduct interviews. Figure 3.4 illustrates the qualitative sample as a maximum variation subset of the quantitative sample. The grey cells indicate that no participant fitting the description was assented for participation. The white cells indicate that at least one participant fitting the description was assented. The colored shapes in the cells represent the classroom membership of each participant in relationship to the quantitative sample shown in Figure 3.3.
Instrumentation, Data Collection, and Data Products

This mixed methods research study entailed two distinct data collection processes, one for the quantitative strand and another for the qualitative strand. Given the concurrent design, the data were collected, in methodological terms, simultaneously. Although the data collection took place over a number of days and weeks with quantitative data accumulating first and qualitative data accumulating second, the order of data collection were designed to be effectively arbitrary in terms of analysis. That is, the quantitative data was not analyzed before the qualitative data was collected; and, thus, the results of the quantitative strand did not drive the qualitative data collection process.

Figure 3.4. Illustration of the qualitative sample as a maximum variation subset of the quantitative sample. Matching shapes indicate enrollment in the same elective with the same teacher. The red triangle indicates enrollment in the intervention elective.
To provide context to the discussion, Figure 3.5 provides a variable map for the study. At the center of the map is a representation of the multilevel guiding theory, DBCCC. The variables are arranged around the theory, placed to be suggestive of the levels for which the variable has implications. The variables are identifiable by color and shape with the red rounded shapes indicating variables generated in the context of the quantitative strand and the black rectangles indicating variables generated in the context of the qualitative stand. Although a host of other variables might have been productive in terms of understanding SMaLL more broadly or more deeply, the number of variables under consideration were limited due to two ethical concerns. It was important to keep instructional loss to a minimum and protect the privacy of the participants. The variables selected were those most closely aligned with the research questions and the multilevel theoretical framework for the study.

To explain the how the data related to each variable were generated, I begin by describing the quantitative measurement tools and data collection process. Then, I describe the interview protocol and the qualitative data collection process. This approach reflects the temporal process of data collection which was dictated by logistical concerns.
Quantitative data. Quantitative data collection was constrained by feasibility and lack of measurement tools appropriate to an investigation of SMaLL. In order to address the essential question of the research, a cognitive task which required reading symbolic mathematics, but did not demand problem solving or arithmetic, was necessary. At the time this study was planned, no such measurement tool was available. A modified lexical decision task emerged as a reasonable candidate for adaptation. Lexical decision tasks measure orthographic processing which is a critical cognitive component of reading (see Literature Review) (O’Brien, Wolf,
Miller, Lovett, & Morris, 2011; Wolf, Miller, & Donnelly, 2000). Lexical decision tasks have been created for a number of languages including languages with logographic writing systems (e.g., Sze, Liow, & Yap, 2014; Yap, Liow, Jalil, & Faizal, 2010). In addition, tools for creating and implementing lexical decision tasks are readily available.

According to current reading theories, orthographic development entails two components: orthographic representations and orthographic rules (Apel, 2011). Orthographic representations are stored mental images of specific words or text units and, thus, are reasonably conceived as static orthographic knowledge. In contrast, orthographic rules are conventions for representing speech in writing. Orthographic rules allow for the creation of novel written forms yet prohibit some combinations. Thus, in theory, to use orthographic rules demands dynamic orthographic awareness. For example, keeping the English language in mind, consider these combinations of letters: brain, brane, brne. The first is a correct spelling of a meaningful word that might be stored as orthographic knowledge. The second, a pseudoword, is not currently defined as an English language word, however, according to the rules that dictate restrictions on character combinations and relative position, it is permissible in the English writing system. Finally, the last, a nonword, is neither defined nor permissible given the arrangement of the letters. In current literature, orthographic knowledge and orthographic awareness are often used interchangeably. For the purpose of this study, I use the term orthographic processing.

For this study, the orthographic processing task producing cognitive measures of reading was the primary quantitative variable of interest. Variables of interest for the purpose of understanding how the cognitive task related to other SMaLL-relevant experiences, behaviors, and abilities were: print exposure to symbolic mathematics, reading habits associated with
mathematical text, mathematics anxiety, and mathematics achievement. Next, I explain the data collection tools, data sources, and data collection processes for each quantitative variable of interest.

**SMaLL Conventional Decision Task (S-CDT).** The SMaLL Conventional Decision Task (S-CDT) used in this study was a modified version of a publicly available lexical decision task downloaded from http://www.millisecond.com/download/library/LexicalDecisionTask/ (Lepore & Brown, 2002). As a user with access via the UC Inquisit 4 Web License, I had permission to adapt and create new tasks (K. Borchert, personal communication, February 27, 2015). I programmed the task using Inquisit 4.0 (2014) software and ran the task via Inquisit’s online platform. The purpose of the S-CDT, designed to avoid conflating number sense with SMaLL, was to produce measures of orthographic processing in response to symbolic mathematics.

Like other lexical decision tasks (e.g., Naples, Katz & Grigorenko, 2012), the S-CDT presented participants with a series of images of text. Their task was to indicate a dichotomous decision — readable or unreadable — by pressing one of two keys. Their goal was to be as accurate as possible as quickly as possible. Each response to each image generated two measures of orthographic processing: response time (RT) and response decision (RD). To give a more clear picture of the task, I explain the nature of the instructions and the nature of the items. Then I discuss the data generated by the task in more detail.

I programmed the task to guide participants through instructions and a practice round of a lexical decision task in English before continuing on to the S-CDT task. During the practice round, the lexical task followed a focus/stimuli/response/feedback pattern. Participants focused on a * at the center of the screen until the stimuli appeared. When the participants saw a chunk
of text (at most six distinct ink marks) composed of alphabetic characters, they responded by pressing one of two keys on the keyboard. The keys indicated their answer, yes or no, to this question: Is the text readable? The stimuli for the practice round included correctly spelled English language word, pseudowords, and nonwords (e.g., plain vs. plin, and skit vs. skti). After each response, the participant saw feedback describing their selection (i.e., “You marked YES” or "You marked NO"). The practice round was intended to give participants an opportunity to become familiar with the nature of the task and the meaning of the keys. Data were not recorded for the practice round.

During the S-CDT round, the task followed a similar focus/stimuli/response pattern, but did not include feedback. As illustrated by the screen shots in Figure 3.6, participants were instructed to respond to each prompt accurately and quickly. For both the practice round and the S-CDT round, the * appeared as a focus point at the center of the screen for 500ms before each stimulus. As quickly as possible after the * was replaced with the stimuli, the participant answered “yes” or “no” to indicate a readable or unreadable decision by pressing the K or D key, respectively, to terminate the stimuli. In order to make the task more user friendly, a green and red sticker was applied to the K and D keys of each keyboard as a visual clue to their meaning during the task. If a response was not made within 1500ms after the stimuli appeared, the screen returned to blank until “yes” or “no” was selected. After each response, another * appeared to begin the next focus/stimuli/response round. During the task, the samples of text were randomly presented as stimuli. As a result, it is very unlikely that any two participants responded to the items in the same order. For each response during the S-CDT round, Inquisit recorded the following to a data file: date, time, participant ID, stimulus number, response decision (0/1),
response time (in milliseconds).

For the S-CDT, I created 30 item pairs (for a total of 60 items) designed to identify orthographic awareness or orthographic knowledge of symbolic mathematics. See Appendix F for an overview and classification of the item pairs presented in the SMaLL-CDT task. I developed all of the stimuli through analysis of grade-level requirements of the CCSS-M. Half of the items were written conventionally (i.e. well-formed according to the conventions of symbolic mathematics). The remaining half were written unconventionally. The unconventional stimuli were paired to the conventional items in such a way that each unconventional items had one of the following mistakes: invalid character substitution, invalid character order, invalid character font or location. Paired items were matched for the number of distinct characters. Both conventional and unconventional stimuli were created in MathType Version6.7e using a size 16 font. All of the stimuli were presented in the center of a white screen using 20% of the vertical span of the screen and 20% of the horizontal span of the screen.

I conducted a pilot phase to inspect the SMaLL-CDT before the study. Volunteers including adolescents and adults completed the task and provided general comments regarding their experience. Specifically, I asked them to comment on the comprehensibility of the instructions and identify any items of concern. In addition, I asked the three local experts to provide an assessment of the face validity of the item pairs. In particular, I asked them to comment on whether the conventional item in each pair was text their students might be expected to read prior to middle school, in grade-level middle school required courses, or in post-middle school mathematics courses according to the district curriculum. In addition, I asked them to comment on whether the unconventional item in each pair was identifiable as
unconventional. As a result of the pilot phase, a few changes were made to the task. The instructions were distributed across multiple screens and visual cues were provided to help the participants understand when the timed task would begin (see Figure 3.6). No changes were made to the items as the expert assessments verified that, for each pair, one was meaningful and conventionally-written symbolic mathematics while the other was neither meaningful nor conventionally-written text according to the K-12 mathematics curriculum.

The responses to the items were used to generate two summary measures of orthographic processing for each participant. The response decision accuracy (RD) score was the sum of all correct responses to only conventional items after data cleaning; and the mean response time (RT) was the average of all responses times to only conventional items after data cleaning. Because the instructions for the task instructed students to prioritize accuracy over speed in making each response, the RD accuracy score was prioritized over the mean RT. The data cleaning process is discussed in detail in Chapter 4 as it entailed analysis to guide the decision-making process.
Figure 3.6. Selected SMaLL-CDT instruction screens.
Math Print Exposure Survey (MPES). The Math Print Exposure Survey (MPES) was designed to measure how often a student has access to common mathematical text. In the absence of an established measurement tool, the survey was loosely designed based on the Title Recognition Test used to determine children’s access to print in the early grades (Cunningham et al., 2001). For that test, students were given a list of children’s book titles and pseudotitle foils to identify as real or made-up. Popular book titles that did not have a prominent role in classroom activities were used as real titles. The foils were included to account for guessing and limit social desirability bias. Scores, which had a split-half reliability of 0.67 on the real titles, were determined by subtracting the proportion of foils identified as book titles from the proportion of actual titles that were correctly identified. The test was used to capture a measure of out-of-school print exposure and literacy environment.

Given the presumption that students do not read a variety of math books outside of school, I designed the MPES to ascertain the frequency with which participants had contact with commonplace symbolic mathematics such as $a^2 + b^2 = c^2$. In other words, the purpose of the MPES was to generate a self-reported measure of print exposure to common mathematics formulas. In this study, the MPES served a dual purpose. First, it allowed for an assessment of the relationship between print exposure and orthographic processing for the quantitative strand. Second, it allowed for connections to the qualitative data generated by the reading samples during the integration phase. In a study of English language literacy, print exposure was related to orthographic processing even after statistically controlling for phonological processing (Cunningham et al., 2001). Suspecting some similarity between SMaLL and English language literacy development with respect to the influence of culture, I expected to see a positive
relationship between S-CDT measures and MPES measures. Because the MPES included links to the qualitative interview protocol, it had the potential to offer insights into the nature of the SMaLL culture surrounding the student.

The MPES incorporated formulas that students were likely to have seen or used prior to middle school as well as formulas not likely to be introduced during formal education until after completing Algebra I. I reviewed publicly available online reference sheets with relevance to CCSS to select items (e.g., http://education.ohio.gov/Topics/Ohios-Learning-Standards/Mathematics) and drew on my pedagogical content knowledge to select the items. See Appendix G for an overview of the MPES. The instructions were designed to reduce the likelihood of guessing and social desirability bias effects. By alerting participants to the inclusion of formulas introduced in earlier grades, the instructions were expected to help students look for familiar formulas. By alerting participants to the inclusion of formulas beyond their grade level, the instructions were expected to alleviate anxiety associated with seeing unfamiliar formulas. As a whole the instructions were expected to frame the survey as a means to describe how often each of them came in contact with text rather than how much each of them knew about mathematics, thereby, reducing the tendency to present themselves as more knowledgeable or more capable.

Response options used a completely labeled clear Likert scale (Borgers, Hox, Sikkel, 2003): Never, A few times, Only one or two times, and Many times. Before conducting the study, I solicited feedback from volunteer adolescents. The pilot study phase confirmed the importance of using completely labeled clear options. In pilot versions I tried Always as well as specific time-frames such as Less than once a month. Volunteers noted that Always did not make sense given that mathematics is commonly taught in distinct units; thus, Always presented a dilemma.
in responding to a very familiar formula not used in a current unit. Volunteers noted that options with specific time frames presented a calculation challenge. In collaboration with pilot study participants, the four-level Likert scale was the most sensible way to describe life-time exposure to each formula. The local experts verified a reasonable degree of face validity by confirming that the items included some formulas every student was likely to have seen frequently and some formulas students were not likely to have seen at all based on the curriculum. The items were scored on a scale of 1 to 4. All of the items were positively scored. Thus, a higher total score indicated more frequent exposure and more extensive exposure.

**Math Reading Habits Survey (MRHS).** In contrast to the MPES, which was intended to measure print exposure, the Math Reading Habits Survey (MRHS) was designed to measure how often students intentionally engage in reading mathematical text in different settings and for different purposes. That is, the MRHS was a self-report of situated behaviors. Although math reading habits were not a central focus of the quantitative strand, I included this measure as a first step towards understanding what habits are related to SMaLL. Studies in English language literacy show a positive relationship between literacy and at-home or leisure reading habits (e.g., Hughes-Hassell & Rodge, 2007; Verhoeven, Reitsma & Siegel, 2011). I expected to see a positive quantitative relationship between orthographic processing and math reading habits. Because the MRHS included links to the qualitative interview protocol, it had the potential to offer insights into the situated behavior of the student. See Appendix H for an overview of the MRHS instructions and items.

Each item presented a scenario in which a student might encounter symbolic mathematics associated with an “I” statement describing a possible behavioral response. The survey
instructions prepared the participants by explaining that each sentence would describe a common setting/situation and something a person might do in that situation. Participants were instructed to select the response that best explained how often they would engage in the same behavior in that setting/situation. Response options used a completely labeled vague Likert scale (Borgers et al., 2003): Never, Rarely, Sometimes, Frequently, Always. During a pilot phase, I consulted with adolescent volunteers to finalize the phrasing for the items and the response options. For an informal assessment of face validity, the local experts verified that all 10 items were related to reading habits with implications for learning mathematics. The Flesch-Kincaid Grade Level Readability score of the items was 3.1, well below the grade level of the participants. The items were scored on a scale of 1 to 5. Nine of the ten items were supportive habits generally considered to foster mathematical development. One item was an avoidance habit presumed to constrain mathematical development and was, therefore, reverse scored. The score was totaled so that a higher total score indicated more positive habits related to reading math text.

**Math Anxiety Survey (MAS).** Finally, a Math Anxiety Survey (MAS) was used to understand the role of affect towards mathematics and doing mathematics. See Appendix I for an overview of the MAS. Although math anxiety was not a central focus of the quantitative strand, I included this measure because affect has been linked to mathematics achievement and avoidance strategies in mathematics (e.g., Turner et al., 2002). Although math anxiety is typically assumed to be associated with doing mathematics, I hypothesized that, for some students, seeing symbolic mathematics could trigger math anxiety. Therefore, I expected to see a negative quantitative relationship between orthographic processing and math anxiety. The MAS did not include any specific links to the qualitative interview protocol; however, during
integration I used the scale to rank order participants to explore whether math anxiety was related to any patterns in the qualitative data.

The MAS used in this study was an adaptation of the Math Anxiety Scale-Revised (MAS-R) which has 14 items and exhibits sound psychometric properties when used to assess college students and adolescents (Bai, 2011; Bai, Wang, Pan, & Frey, 2009). The MAS-R is bidimensional including positive affect items and negative affect items. Because the central concern of this research study was reading, the MAS included modifications designed to reduce the reading level and increase the likelihood that all participants would find the items and options comprehensible. After the items were modified, the Flesch-Kincaid Grade Level Readability score of the items was 1.8, well below the grade level of the participants. To maintain the psychometric properties of the original MAS-R, the items were modified with an effort to maintain the essential meaning, and the items were presented in the same order as the original pencil-paper version.

During the pilot phase, more than one participant reported being confused by the Neutral option in the five-point Likert scale in the original MAS-R. Therefore, the MAS was modified to use a Likert scale with only four options (Strongly disagree, Disagree, Agree, Strongly agree) to indicate the degree of agreement with each statement. Like the original survey, positive affect items were reverse scored. Thus, a higher total score indicated higher math anxiety.

Other variables. Secondary data and demographic variables were collected to describe the sample. Grade, course, and teacher are indicators of academic culture and experiences with mathematics. Ethnicity, second language experience, and musical training offer additional insight into cultural factors with implications for orthographic processing and literacy.
development. Gender, age, and handedness are neurobiological markers relevant to human
development and cognition.

Holland Middle School provided data related to these descriptive variables in a secondary
data set: gender, age, ethnicity, grade, teacher, elective enrollment, and standardized measure of
mathematics achievement. Participants self-reported data related to these variables a part of the
quantitative data collection: ability to speak a second language, ability to read a second language,
ability to play music, ability to read sheet music, handedness. According to my field notes, the
measures related to second language and music ability are limited in that more than one student
asked for clarification. For example, in response to the question “Do you speak a language other
than English well?” some students may have answered yes rather than no on the grounds that
they were receiving satisfactory grades in their first your of an academic Spanish course.

The principal at Holland Middle School selected the 2014 Ohio Achievement Assessment
(OAA) scores as the standardized measure of mathematics achievement. The scores were from
the previous academic year because, in comparison to the scores associated with newer
standardized test delivered electronically, the older OAA scores were considered more reliable
and interpretable. The OAA tests for math differ by grade. Given the scoring, the OAA scores
can be compared across grades as a measure of grade-level proficiency; however, the OAA
scores cannot be compared across grades as a measure of overall mathematics proficiency. This
measure was included in the study because mathematics achievement is the critical variable of
interest to schools in the CCSS era. I expected to see a positive quantitative relationship between
orthographic processing and math achievement.
Data collection process. I combined all of the quantitative measurement tools in a batched script to create a single online data collection tool using Inquisit. In the weeks before the data collection, I reserved the computer lab for one day for each collaborating teacher and worked with a systems administrator at Holland Middle School to ensure the setup of 30 stations in a computer lab. Due to unexpected differences in software/hardware, additional changes to Inquisit were necessary to fit the instructions to the screen. Before the data collection, I conducted a final test run of the batched script on every station to be used in the study. A few stations were excluded due to technical difficulties (e.g., streaks on the monitor and unusual default settings for the browser). The quantitative data collection took place over the course of three days during April of 2015.

On the day of each quantitative data collection, I arrived at the school early to log on and test each of the stations again. Using the teacher’s daily schedule as a reference, I met the teacher in their classroom at the beginning of each bell. The classroom teacher and I reviewed the list of students for whom permission was revoked and asked the eligible students to accompany me to the lab. As previously described, I guided students through the assent process (see Recruitment, Consent, and Sampling section) as a group. For each class, I printed and stapled a copy of the script for the assent. I used those pages to guide the explanation and record field notes. Students who declined returned to their classroom and remained under the supervision of the classroom teacher there. Students who agreed to participate remained under my supervision in the computer lab.

The computer lab was arranged with stations along three walls of the room allowing me to see the students’ monitors and guide them through the data collection as a group. For each
class, I printed and stapled images of each screen the students would see. I used those pages to guide the data collection and record field notes. I began the data collection by instructing students to key in the randomly assigned research ID on their completed assent form. Because the research ID was critical for linking the quantitative and qualitative data by participant during integration, I customized the Inquisit script to require students to verify the research ID by entering it as second time. After all of the students successfully initiated a data collection session in Inquisit, I guided the group through an explanation designed to distinguish between instructional pages and data collection pages. For the remainder of the quantitative data collection, I read the instructional pages aloud as students read along silently. At the end of each instructional page, I gave students an opportunity to ask questions. On data collection pages, students worked silently and independently. At the end of each section, a screen with a stop sign alerted students to wait for me to guide the group through the next set of instructions.

The batched script presented the tasks and surveys in this order: demographic data, lexical decision task (practice round for SMaLL-CDT), SMaLL-CDT, MAS, MRHS, MPES. The quantitative data collection process took approximately 30 minutes and did not extend beyond the length of the class period. Table 3.1 gives an overview of each measurement tool and the nature of the data generated for each item. Inquisit recorded all responses to a downloadable tab-delimited file on a password protected website.
To obtain the secondary data, I provided the first and last names of assented participants to a designated staff member at Holland Middle School. After receiving an Excel file containing the secondary data, I immediately used a multi-step process to create a de-identified data set organized by research ID. Then, I conducted a quality control check and eliminated the identifiable version of the data.

**Qualitative data.** I collected the qualitative data using a semi-structured interview protocol (Rubin & Rubin, 2012). I used this format, characterized by a framework of essential interview questions and tentative follow-up questions, because it centered the interview on the research questions while allowing for probing questions to follow up on novel responses. Furthermore, the semi-structured format allowed me to interact with the adolescent participants.
using the familiar demeanor of a tutor or classroom volunteer.

The framework of essential questions provided similarity across interviews. The emergent probing questions, developed to explore participants responses, provided the flexibility necessary given the variation across participants and the purpose of the study. Because I was interviewing adolescents who differed in terms of reading and math achievement, I anticipated the need to follow-up on unpredictable responses. In addition, because the purpose of the study was initiation, I anticipated the need to modify ancillary interview questions across participants. Over the course of the interviews, the questions remained focused on the primary questions designed to answer the qualitative research questions. The secondary probing questions were asked to explore the essence of responses and emerging themes. For example, questions like “Can you give an example?” and “Can you tell me more about that?” were intended to prompt the student to elaborate. Questions of the form “I am going to read… Can you compare the way I read the selection to the way you read it?” were intended to make connections across participants or to salient features of the reading selection. Finally, questions of the form “I think you are saying… Can you correct me if I misunderstood?” were intended to elicit a real-time member check. I used professional discretion to restrict the follow-up questions to those that pertained to the primary interview questions and these research questions: What kinds of texts with symbolic mathematics do students identify as readable (or unreadable)? What kinds of strategies do students use to read texts with symbolic mathematics?

The interviews were completed, as planned, in approximately 30 minutes. None of the interviews took more than one class period. Before each interview began, I reminded the participants of their right to reverse their decision to participate. In addition, each participant
selected a pseudonym they believed would protect their privacy to alleviate any concern that a
teacher or peer or anyone outside the research team might find out what they said.
Approximately five minutes were dedicated to introductory questions designed to focus the
interview on the topic of reading and put the student at ease. Essential questions, math text
prompts, and probing questions designed to generate data relevant to the research questions took
approximately 15 to 20 minutes. The final five minutes were dedicated to discussing future
academic and career plans and inviting the student to provide any additional comments about
reading symbolic mathematics and how it compares or contrasts with other reading.

Appendix J shows the semi-structured protocol guide and reading selections. To conduct
the interviews, I used the following single-use materials per participant: interview protocol/data
recording guide and sample math text prompts. For each interview, I printed and stapled a copy
of pages 1 to 7 to use as a guide to conduct and pace the interview. I also used those pages as a
space for collecting field notes. The reading selections (on pages 8 to 10) were printed, cut to
size using the borders, and stored in a bag. During each interview, I drew the reading selections
from the bag in random order and presented them in a random physical arrangement. After
asking participants to make a selection, I gave them approximately 10 seconds, by a show of
counting on my fingers, to choose a text selection.

The mathematical text presented in the reading selections were designed to create an
authentic reading scenario typical of formal mathematics education settings. To ensure the text
was typical and accessible, I created each prompt by adapting samples of text from open-access
websites that purported to support mathematical learning. The samples were aligned with the
CCSS grade-level standards in such a way that each sample was similar to text the participants
might reasonably be faced with in the course of learning mathematics between grade 6 and Algebra. This ensured that the reading selections were appropriate to the academic culture of the school.

The reading selections were chosen to ensure that at least one prompt would be perceived as familiar in some way by every participant and, conversely, that at least one prompt would be perceived as unfamiliar by every participant. The prompts varied in terms of the degree of vocabulary support available in the surrounding English text. In addition, the prompts varied in terms of the degree of abstraction. That is, some prompts were contextualized with concrete features such as definitions, examples, and Arabic numerals while others were more formal mathematical text with limited English language contextual support. The prompts also varied in the degree of mathematical complexity. For example, one selection presented the absolute value as a step-wise function whereas another used absolute value notation in the context of explaining how to simplify the absolute value of a known quantity. Each of the prompts had a symbolic mathematics language feature in common with the items in the SMaLL-CDT such as an exponent, a subscript, a square root, a vinculum, or an inequality. I used the variations to initiate discussions about readable and unreadable characteristics of the selections. In addition, I used the variations to initiate discussions about the difference between the process of reading a readable selection and the process of reading an unreadable selection. In total there were 10 reading selections from which students could select.

All of the interviews were audio recorded. Over the course of the interviews, I developed strategies for coordinating my oral language with written notations on the reading selections to create a means of identifying which textual elements were being discussed. In addition, I asked
some students to write on the reading selections to draw a character and/or identify math text. Thus, the reading selections were also retained as artifacts. At the end of the interview, the audio recordings were transcribed by a professional transcriptionist. Upon receiving the transcriptions, I reviewed the transcription and made corrections as needed. The data products were field notes, transcripts, and reading selection artifacts.

**Analysis and Validation**

This mixed methods research study entailed traditionally quantitative analyses as well as traditionally qualitative analyses. Given the multilevel concurrent mixed methods research design, the quantitative and qualitative data were available for analysis at the same time. Neither the quantitative nor the qualitative data analysis was prioritized. In the absence of priority as a rationale for analyzing one data set before the other, I reflected carefully on the research paradigms associated with the quantitative and qualitative methods. I reasoned that the quantitative methods, which entailed well-established assumptions and logical procedures in the post-positivist tradition, would not be unduly influenced by qualitative analysis. I reasoned that the qualitative methods, which entailed constructivist thinking, would not be unduly influenced by the quantitative analysis so long as I did not review quantitative results for any individual student in the qualitative sample until after completing the initial analysis to answer the qualitative research questions.

Integration is the hallmark of any mixed methods analysis. As described earlier in the chapter, and illustrated in Figure 3.2, the multilevel concurrent mixed methods research design entailed unique opportunities for a high degree of integration during the concurrent phase. For example, it was possible for a theme emerging from the qualitative data set to suggest a new
exploratory analysis of the quantitative sample. In addition, it was possible for a statistic summarizing the quantitative sample to suggest a new lens for analysis of the qualitative data set. Thus, during the strand analysis, I remained open to integrating the quantitative sample statistics with the qualitative sample data and, conversely, allowing the qualitative themes to inform my understanding of the quantitative sample statistics during the concurrent analysis. I returned to and used the links I inserted into each strand during the design phase to drive the interaction. After conducting the strand analysis to answer the quantitative and qualitative research questions, I conducted a final mixed methods integrative phase of analysis.

Although the analysis of the concurrent strands was interactive, I describe the methods linearly. First, I describe the quantitative analysis. Then, I describe the interactive analysis. Then, I describe the qualitative analysis. Finally, I describe the mixed methods analysis.

**Quantitative analysis.** The quantitative analysis was intended to address the overarching quantitative research question: To what degree are cognitive processes of SMaLL and English language literacy analogous? The nature of the quantitative sample and measurement tools were critical to understanding the nature of the data and the implications for inferences. Therefore, much of the analysis was dedicated to describing the sample and inspecting the surveys. Given the purpose of this study and the novelty of the measurement tools, the analyses were intended to be exploratory (rather than confirmatory) first steps towards understanding SMaLL.

I began the analysis with an overall review of the data. The Inquisit data collection process was designed to force responses in order to proceed for all but the last survey, the MPES. Therefore, the missingness at the item level was expected to be minimal. Missingness in the
secondary data was also expected to be minimal as the school maintained records as required by law. Missing data was reviewed and reported, but not eliminated using any statistical techniques.

To summarize the sample, I generated descriptive statistics appropriate to the data type for each of the sample descriptor variables. For each survey, I calculated a summary score for each participant and assessed the statistical reliability of the measure for the sample. In addition, I reviewed the distribution and measures of central tendency for each survey and each item of each survey.

The SMaLL-CDT, a timed cognitive task generating two data points simultaneously, could not be analyzed for reliability using a classical test theory approach. Exploring the nature of the task through a comparison of experimental conditions of similar tasks with variations in instructions and ratios of conventional/unconventional items was beyond the scope of this study. A review of the literature suggested that concerns regarding the use of general linear models to analyze RD and RT pairs generated by linguistic tasks are well-founded (e.g., Balota & Yap, 2011; Balota, Yap, Cortese, & Watson, 2008; Loeys, Rosseel, & Baten, 2011; Voss, Nagler, & Lerche, 2013). Common practice entails data reduction during data cleaning and violations of statistical assumptions. Because it is an ethical imperative to attend to concerns from the research community about analyses with implications for educational assessment and practice (American Education Research Association, 2011), I considered more sophisticated approaches such as Ratcliff’s diffusion model (Ratcliff & McKoon, 2007; Voss et al., 2013). However, those approaches are grounded in theory and statistical assumptions that were tenuous given the novelty of the task and theory of SMaLL. For example, Ratcliff’s diffusion model is only applicable to tasks that entail dichotomous decisions that are made cognitively as evidenced by
response times under 1000 to 1500ms (Ratcliff & McKoon, 2007). Given the exploratory nature of this study, and the low risk of the results being associated with high-stakes decisions, I analyzed the RD and RT data in accordance common practice. With this approach, I was able to produce results in terms that allowed for comparison to other studies and, as a result, assess the evidence of the degree to which the S-CDT is similar to lexical decision tasks in other languages.

Before cleaning the data, I explored the RDs and RTs for each item in terms of central tendency and dispersion. Then, I proceeded with data cleaning using common data cleaning practices (see Table 3.2) as a guidelines for my decisions process (see Chapter 4 for more details). After cleaning the data at the item level, I generated an RD accuracy score and a mean RT for each participant as summary measures of orthographic processing. Then I conducted t-tests to determine if there was sufficient evidence to suggest group differences existed across grade levels and elective courses in terms of either measure of orthographic processing. I reasoned that, if SMaLL develops cumulatively across grades, students in higher grades should differ from students in lower grades. Furthermore, I reasoned that, if SMaLL is related to mathematics achievement, then students enrolled in a course for additional support should differ from students enrolled in a course designed to extend beyond the required mathematics curriculum. Because students are not assigned to core mathematics courses or elective courses by grade level during middle school, analysis of the former and the latter group differences provided different insights.
Table 3.2

*Common data cleaning practices for cognitive tasks with response decisions and response times*

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Exclusion Rule</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Participant</td>
<td>10% of RTs &lt; 300 ms</td>
<td>Cvencek et al., 2011</td>
</tr>
<tr>
<td></td>
<td>Error rate of RDs &gt; 35%</td>
<td>Cvencek et al., 2011</td>
</tr>
<tr>
<td></td>
<td>Mean RT &gt; 3SD above sample mean RT</td>
<td>Cvencek et al., 2011</td>
</tr>
<tr>
<td>Item</td>
<td>Items with pseudo targets</td>
<td>Elgort &amp; Warren, 2014</td>
</tr>
<tr>
<td></td>
<td>Items with low accuracy rate (&lt; 60%)</td>
<td>Elgort &amp; Warren, 2014</td>
</tr>
<tr>
<td>Response Time</td>
<td>RTs associated with incorrect responses</td>
<td>Elgort &amp; Warren, 2014</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Yap et al., 2010</td>
</tr>
<tr>
<td></td>
<td>RTs &lt; 200-400 ms</td>
<td>Elgort &amp; Warren, 2014</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Naples et al., 2012</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Van Zandt &amp; Townsend, 2013</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Yap et al., 2010</td>
</tr>
<tr>
<td></td>
<td>RTs &gt; 3000-5000ms</td>
<td>Elgort &amp; Warren, 2014</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Van Zandt &amp; Townsend, 2013</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Yap et al., 2010</td>
</tr>
<tr>
<td></td>
<td>RTs &gt; 95th percentile for participant</td>
<td>Naples et al., 2012</td>
</tr>
<tr>
<td></td>
<td>RTs &gt; 2.5SD above participant mean RT</td>
<td>Yap et al., 2010</td>
</tr>
<tr>
<td></td>
<td>RTs &gt; 3.5-4SD above mean RT</td>
<td>Naples et al., 2012</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Van Zandt &amp; Townsend, 2013</td>
</tr>
</tbody>
</table>

Then, I generated correlations between measures of orthographic processing, the survey scores, and the math achievement scores. I reasoned that, if processing of symbolic mathematics is analogous to English language orthographic processing, there would be a positive relationship
between orthographic processing and each of these measures: math print exposure, math reading habits, and math achievement. That is, higher scores on math print exposure, math reading habits, and math achievement should be associated with faster response times (lower mean RTs) and higher RD accuracy scores. I reasoned that math anxiety might inhibit SMaLL as it does mathematics achievement and, thus, expected to find a negative relationship between math anxiety and SMaLL-CDT measures. A summary of the anticipated direction of the correlations between orthographic processing and the other quantitative measures is described in Table 3.3.

Table 3.3

Predicted direction of correlations among quantitative measurements

<table>
<thead>
<tr>
<th></th>
<th>RT</th>
<th>RD</th>
<th>MPES</th>
<th>MRHS</th>
<th>MAS</th>
<th>OAA</th>
</tr>
</thead>
<tbody>
<tr>
<td>RT</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RD</td>
<td>−</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MPES</td>
<td>−</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MRHS</td>
<td>−</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MAS</td>
<td>•</td>
<td>−</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>OAA</td>
<td>−</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note. RT = SMaLL-CDT Mean Reaction Time; RD = SMaLL-CDT Reaction Decision Score; MPES = Math Print Exposure Survey Score; MRHS = Math Reading Habits Survey Score; MAS = Math Anxiety Survey Score; OAA = Math Achievement Score
Correlations are commonly calculated under the assumption of an interval scale. However, such correlations may be suspect when total scores for each survey are based on ordinal responses to items. While the practice of treating Likert responses, and summary scores based on Likert responses, as interval data is common, it remains controversial (Jamieson, 2004; Pell, 2005). Therefore, the correlational analysis was conducted twice: once with the survey scores treated as interval and again with the survey scores treated as ordinal. Given the results of correlation analysis, I also ran exploratory regression analyses to determine which measures, if any, could be used to mathematically model (i.e., estimate or statistically predict) the mathematics achievement scores.

The MAS was adapted from the Math Anxiety Scale – Revised (Bai, 2011; Bai et al., 2009) in order to reduce the reading level. I conducted a confirmatory factor analysis of the MAS to establish whether the modified survey exhibited the same bi-dimensional structure as the MAS-R. In addition, I calculated similar reliability statistics for comparison.

The primary analysis was conducted in R with the use of R Markdown, SPSS, and Excel for occasional quality control comparison. After the analysis was complete, a full quality control review was conducted. Processing records were maintained to provide an audit trail. The audit trail provides a means of validating the use of appropriately rigorous methods and making the limitations of the analysis transparent.

**Mixed methods interactive analysis.** During the concurrent phase, I engaged in the dialectical practice of negotiating the post-positivist and constructivist perspectives. This entailed repeatedly challenging myself to question the results emerging from one perspective by viewing them through the lens of the other perspective. This required what Peter Elbow named
the believing game and the doubting game (Maxwell, 2011). I repeatedly asked myself “for each conceptual model or assumption, what believing this model or assumption enables us to see, and also in what ways this model or assumption is misleading, incomplete, or unhelpful” (Maxwell, 2011. p. 29). In some cases, this was enacted in a seamless process as I brought my emerging knowledge to bear on subsequent reasoning and vice versa. The influence of the many minor interactions are documented in my written reflections and across the various tables, diagrams, and concepts maps I created during analysis.

In this study, I also conducted an overtly dialectical and interactive analysis before the final integration phase. In order to understand the nature of the surveys, I used factor analysis techniques in a novel way. Although it was not a goal of this study to establish the psychometric properties of the surveys, I used quantitative factor analysis to gain insight into the nature of the surveys and suggest qualitative coding themes. At the same time, I used emerging qualitative coding themes to interpret the quantitative factor analyses. Specifically, I conducted an exploratory factor analysis of the MPES and the MRHS in a manner that brought both the quantitative and qualitative perspectives to bear on the process. Thus, the results cannot be interpreted as evidence of a factor structure in the traditional quantitative sense. However, taking a dialectical stance, the results offer some insight into the tools that have implications for further development. In addition, the results offered convergent and divergent evidence across the strands with relevance to the research questions.

**Qualitative analysis.** I conducted the analysis of the qualitative data using MAXQDA 11 for Mac (2014). MAXQDA provides a user-friendly interface for writing memos and coding data in multiple stages. Thus, the use of MAXQDA facilitated thematic analysis of students’
descriptions of what, how, and why they read symbolic mathematics. As shown in Figure 3.7, the analysis entailed multiple cycles of coding. Reading the diagram from left to right, the diagram shows that process began with all of the qualitative data collected. In each iteration, I probed and parsed the data, breaking it down into chunks of meaning. After applying codes, I visualized and summarized the data using concept maps, tables, and narrative writing. At the end of each cycle, I reviewed the emerging codes, categories, and themes. This process continued until the meaning distilled into saturated themes.

*Figure 3.7. Visualization of iterative qualitative analysis process.*

During the first round of coding, I coded reading selections as readable and unreadable based on manifest content (Cho & Lee, 2014), or the explicit declaration of a selection as *most readable*, *next most readable*, or *most unreadable*, in response to the corresponding interview questions. In addition, I made contextual observations, noted vocabulary and symbols of
interest, and generated tentative themes (e.g., familiarity, text exposure, text length). Given the purpose of initiation, themes related to the nature of readable and unreadable text were not identified a priori; rather, themes were refined through repeated immersion in the data. Both the similarity and contrast principle were be employed (Teddlie & Tashakkori, 2009). That is, initial stages of analysis generated tentative themes based on emerging understandings of meaningful similarities. Later stages of analysis were characterized by a return to the data to challenge themes until a set of distinct themes emerged. In order to challenge the themes, I revisited the data to search for disconfirming evidence (Creswell & Miller, 2000) to test whether the emerging themes held the meaning of the data and whether the relationships between the themes matched the relationships in the data. Although I did not engage in grounded theory practice strictly speaking, I did approach the coding using grounded theory in principle. Thus, my interpretations of the interviews “indicate the possible range of empirical meanings, actions, and process” (Charmaz, 2015, p. 1616, emphasis in original).

To answer the question of what kinds of text students identify as readable, I challenged themes that emerged from the participants’ explanations of their selection choice and their oral rending of the text. For example, I revisited the data to challenge an emerging no words theme associated with students’ descriptions of why a selection was unreadable. To answer the question of how students read mathematical text, I challenged themes related to participants’ descriptions of the reading strategies they apply to reading math text. For example, I revisited the data to carefully discern between the rough read and decode themes. To answer the question of why students read mathematical text, I challenged themes related to participants’ explicit and implicit reasons for reading and not reading mathematical text. For example, I revisited the data to
question whether I was justified in the interpretation that students’ primary purpose for reading was to solve an assigned problem. Throughout the analysis, I was vigilant for opportunities to make connections between quantitative items to qualitative statements. For example, when I noticed that students identified teachers as a resource for help with reading, I made a connection to Item 5 on the MRHS. Appendix K and Appendix L show additional detail and make the coding process transparent.

Making a claim of validity, with respect to qualitative analysis, refers to making an argument that results are complete and accurate (Creswell & Plano Clark, 2011). I argue that the analysis is valid on the grounds that I used a systematic approach to reach saturation or the point at which I gained no new information by returning to the data (Creswell & Miller, 2000). I used multiple procedures to make the case for validity: real-time member checks; researcher reflexivity; thick, rich description; triangulation across participants; and peer debriefing (Creswell & Miller, 2000).

I used real-time member checking to resolve points of confusion while I had access to the participant. To do this I alerted the participant that I was going to summarize and asked them to correct me if my summary did not match their intended meaning. For example, at one point, I said to Loren, “I'm going to sometimes try to summarize what I think you're telling me. But, if I'm getting it wrong, your job is to tell me, ‘that's not what I meant’, okay?” After she responded in the affirmative, I asked, “So are you saying that when you see a symbol that you don't know, you skip it?” I used memoing, journaling, and mapping to make my reflexive practice transparent and create an audit trail. I endeavored to create thick, rich description in the form of narratives summarizing the data sufficiently to provide the reader with a vicarious experience. I
carefully distinguished between themes holding across participants and cases of interest (Creswell & Miller, 2000). For example, ‘curiosity’ was an emerging theme; however, because it appears to exist in a different form across a limited number of participants, I did not give the theme the same status as other themes on the concept map. Finally, I engaged in peer debriefing, or the process of submitting artifacts of my systematic process for review to peers. For example, in regular meetings with fellow mixed methods researchers I provided summaries of my process and findings. In addition, I presented portions of the study at conferences. The questions posed by peers and audiences challenged me to reflect on and refine my reasoning in new ways. Using these validity measures in conjunction helped me to improved the accuracy, completeness, and validity of the results.

**Mixed methods integrative analysis.** Integration, or the process of drawing meta-inferences from both quantitative and qualitative data, is the hallmark of mixed methods research (e.g., Bryman, 2006a; Teddlie & Tashakkori, 2009). Effective integration is crucial to developing understandings that are not apparent given the independent analysis of a single strand (Teddlie & Tashakkori, 2009). The understandings generated by integration, called meta-inferences, are overall conclusions drawn from both the quantitative and qualitative strands (Tashakkori & Teddlie, 2008). Given the multilevel concurrent mixed methods design, integration consisted of interactive analysis during the concurrent phase and merged data analysis techniques (Creswell & Plano Clark, 2011) during the final integrative analysis phase. Merged data analysis techniques (e.g., side-by-side comparisons, joint displays, and data transformation) are intended to address the overall rationale for using a mixed methods design. For this study, the integration techniques extended beyond highlighting convergence or
divergence across datasets to gain insight into the nature of SMaLL as a multilevel phenomenon.

A timeline summary of the integration is shown in Figure 3.8. The illustration shows how the order of data collection and analysis occurred in practice and contributed to integration in ways that may not be apparent given the isolated explanations of the quantitative and qualitative strands offered earlier in this chapter. Reading the figure from left to right, it shows that the interactive analysis began organically with data collection. In other words, as an independent researcher, I began to accumulate dialectical insights as soon as data were available. The strands were analyzed in a to-and-fro process that facilitated workflow and integration without prioritizing one strand over the other or undermining the methods planned for each strand. Although the analysis to answer the quantitative and qualitative research questions was explained sequentially in the Analysis and Validation section, the analysis for the quantitative strand was not, strictly speaking, conducted before or independently of the analysis for the qualitative strand. The diagram highlights the mixed methods integration in gray. The continuous gray line indicates that integration was ongoing. The gray circle in the center indicates the mixed methods interaction that occurred during the concurrent phase. Finally, the gray circle at the end represents the final integrative analysis.
Side-by-side, or weaving, discussion were used for integration. I looked for points of convergence suggesting consistency across the strands. For example, both quantitative and qualitative evidence suggested that \( A = \pi r^2 \) is readable. I looked for points of divergence suggesting discrepancies across the strands. For example, the quantitative data suggested students do not ask other for help with reading; however, the qualitative reports suggested teachers, classmates, and family members are primary resources. I looked for novel themes as well. For example, the comparison of the MPES and the readability selections suggest that plug and chug formulas such as \( A = \pi r^2 \) are more readable than equations expressing a generalizable pattern in mathematics such as \( a(b + c) = ab + ac \).
Joint displays are tables or figures that allow the researcher to juxtapose datasets for comparison from a new perspective (Creswell & Plano Clark, 2011; Plano Clark & Sanders, 2015). The effective use of joint displays entails careful selection of dimensions and information with the potential to answer the mixed methods research questions. Given the nature of mixed methods research, I was unable to determine the joint displays a priori. In practice, some of the joint displays I had envisioned during the design phase proved to be impossible or ineffective. However, during the concurrent phase, the interaction between the strands suggested alternative joint displays, each with a unique arrangement, with potential to generate meta-inferences. I worked dialectically, negotiating paradigms and questioning the meaning of the data from multiple perspectives to develop new displays with the potential to generate new findings or strengthen the validity of the meta-inferences.

I created quantitative × qualitative displays illustrating quantitative descriptive statistics summarizing all participants juxtaposed with convergent, divergent, and novel exemplars of the qualitative data; and, I created qualitative × quantitative displays illustrating qualitative themes juxtaposed with the quantitative statistics. In other words, given the equal priority of this study, I arranged some displays in a manner that fostered thinking about the quantitative in light of the qualitative data, and I arranged others in a manner that fostered thinking about the qualitative data in light of the quantitative data. The distinction between the quantitative × qualitative and qualitative × quantitative arrangements had implications for understanding the results as well as assigning value to the strands and datasets. For example, the joint displays showing the qualitative participants in relationship to the range and mean of each quantitative measure used a quantitative lens to review the qualitative data. Conversely, the joint displays showing each
reading selection with the quantitative results of related items used a qualitative lens to review the quantitative data.

I determined that data transformation was not appropriate for this study. The technique requires either qualitizing the quantitative data or systematic quantitizing of the qualitative data for the purpose of direct comparison of variables. It was not an appropriate integration approach for this study because the sample sizes for the quantitative and qualitative strands were unequal (Bazeley & Kemp, 2012; Morse, 2010). In particular, I determined that, although I used manifest coding to identify readability selections, the counts should not be viewed as quantitative data. For example, some participants indicated they were caught between one of two selections as a choice. That rich data, which would be lost in the process of quantizing, informed the findings.

In a concurrent mixed methods design, integration can occur at the design level, the methods level, and the reporting level (Fetters, Curry, & Creswell, 2013). I wove an explanation of the design-level integration, which entailed arguing that a mixed methods study was feasible and well-suited to answer the research questions, into the Design process section. The design level integration also included the description of the links inserted into the research design to create the potential for downstream interaction and integration. I wove an explanation of how the quantitative and qualitative sampling procedures and data collection tools were connected to integration at the methods level into the Recruitment, Consent, and Sampling section and the Instrumentation, Data Collection, and Data Products section. Integration at each of those levels had implications for the reporting level of integration, which is presented Chapter 4 using side-by-side discussion and joint displays (Fetters et al., 2013; Plano Clark & Sanders, 2015).
Because this study was designed as a multilevel mixed methods design, the final integration techniques were aimed at generating across-level meta-inferences. Thus, I drew on the DBCCC theoretical framework to guide integration. Through integration, I was able to bring a clearer picture of SMaLL, as a multilevel phenomenon, into focus. I used integration techniques to understand SMaLL as a system: SMaLL as whole; the essence of the cultural, behavioral, and neurobiological levels of SMaLL; and the reciprocal mechanisms between those levels.

I continued the integrative process of reviewing the data until I had sufficient and thorough answers to the research questions. The goal of initiation was realized when I was able to see how meta-inferences drawn in the context of the multilevel concurrent research design supported a model of SMaLL. Given the rich quantitative and qualitative data generated during data collection, I suspect that the data may yield additional interesting meta-inferences that are not of critical relevance to the research questions of this study.

To assess the quality of mixed methods integration, I first considered within strand quality issues (described at the end of the Quantitative analysis and Qualitative analysis sections). To assess the quality of the mixed methods meta-inferences, I drew heavily on Dellinger and Leech’s (2007) validation framework, or construct validity as it is defined from the mixed methods perspective (Plano Clark & Ivankova, 2016). This framework suggests that validity exists in the continuous negotiation of the meaning generated by the implementation of the research design. This entails the believing game and the doubting game (Maxwell, 2011) as described in the Interactive analysis section. It can be described as an iterative process of trying on new lenses and exploring the meaning of the measures and themes as a means to inspect “the
quality and stability of the generated inferences” (Plano Clark & Ivankova, 2016, p. 168). In the tradition of multiplism, as it is described by Greene (2007), I did not consider convergence, consonance, or consensus as better evidence of validity than the puzzles and paradoxes that arose. Rather, I considered the clashes and conflicts, some of which were not resolved, as evidence of validity in the sense that it indicated I had moved into meaningful engagement with the data.

In addition, I established the quality of this study using Teddlie and Tashakkori’s (2009) integrative framework for inference quality (Plano Clark & Ivankova, 2016). From this perspective, the quality of a mixed methods study depends on the extent to which the meta-inferences address the purpose of using mixed methods. Thus, this study emerged as an exemplar of a multilevel concurrent mixed methods research study because it met the goal of initiation and produced a multilevel model describing how students in middle grades learning under the CCSS experience SMaLL. In other words, the study generated cultural, behavioral, and neurobiological insights into SMaLL as it exists in classrooms in the Common Core era. In addition, the study revealed the strategies students employ to read symbolic mathematics and the automated processes associated with SMaLL. Finally, the study suggested that, although SMaLL is similar to English language literacy in some ways, it it very unique.

Finally, I addressed quality as it is described by Curry and Nunez-Smith (2015). From their critical appraisal perspective, quality lies in overall quality from conception to conclusion (Plano Clark & Ivankova, 2016). Furthermore, quality is determined by a review of the dissemination product. Thus, this dissertation, which includes a thorough explanation of every aspect of the study, exists as evidence of the quality of this multilevel concurrent mixed methods
research study of SMaLL.

**Methodological reflections.** As I developed and conducted this multilevel concurrent mixed methods research study, one of my favorite quotes about mathematics education came to mind. It is widely credited to Edward Griffith Begle (Silver, 1985) and referred to as the second law of mathematics education: *Mathematics education is much more complicated than you expected, even though you expected it to be more complicated than you expected.*

For this study, all the complexity of the multilevel concurrent mixed methods design was required to match the complexity of the research problem and the address the gap in the mathematics education research literature. It was necessary to think quantitatively and qualitatively. It was necessary to think top-down and bottom-up. It was necessary to think in circles of reciprocal interaction. The development of the research design and the measurement tools to implement it required me to deepen my pedagogical content knowledge and strengthen my methodological skills. It was more complicated than I expected even though I expected it to be more complicated than I expected.

DBCCC was an ideal multilevel framework for conceptualizing SMaLL. Its three levels of change — cultural, behavioral, and neurobiological — apply broadly to human development and have been used to guide thinking about academic development in particular (Baltes et al., 2006). However, the nature of the theory lead to unanticipated methodological complications. Most multilevel research studies are designed with social institutions in mind (see *Multilevel issues in mixed methods research* in Chapter 2). For example, many multilevel studies in education inspect schools, teachers, and students. Those are all commonplace units of analysis. In contrast, DBCCC entails three levels of plasticity, or change. The levels were necessarily
indistinct and intertwined. At times, I was unsure what constituted a level. I sought out quantitative researchers, qualitative researchers, mixed methods researchers, and action researchers and asked them all the same question: What constitutes a level in multilevel research? The answers were all different. Although I always knew I was conducting a multilevel study, my conceptualization of levels evolved over time. As I conducted this research, I periodically recorded my methodological reflections in the form of memos in my research journal and diagrams illustrating my thinking. A final reflection is presented at the end of Chapter 5.

Summary of Chapter 3

This chapter described the multilevel concurrent mixed methods research design used for this exploratory study. The study was designed to initiate a better understanding of how adolescent students in middle grades learning under the CCSS experience SMaLL. The concurrent design made the study feasible and allowed for the relatively high degree of interaction necessary for initiation. The multilevel adaptations supported an investigation of SMaLL as an aspect of human development shaped by cultural, behavioral, and neurobiological change. The sample was comprised of adolescents attending the same school, but different classes. As a result, the participants shared the same academic culture at school, but experienced different classroom cultures. The sampling made it possible to focus the study on the language of mathematics in classrooms, students’ metacognitive reflection on reading, and students’ cognitive reactions to symbolic mathematics. The quantitative strand entailed a novel cognitive task and adapted surveys analyzed with common statistical techniques. The qualitative strand entailed metacognitive reflections analyzed with a thematic approach. After the concurrent
phase, the data and inferences were integrated using merged data analysis techniques to develop a multilevel model of SMaLL with implications for understanding how SMaLL compares to English language literacy. In conclusion, this chapter justified the methodological decisions made during the design and implementation of this study.
Chapter 4: Results

This chapter describes the characteristics of the sample and the results of the analytic procedures described in Chapter 3. I begin by presenting the results of the quantitative analysis and addressing the quantitative research questions. Next, I present the results of the qualitative analysis and address the qualitative research questions. Although I present the quantitative results first, I do not intend to give them priority over the qualitative results. Finally, I present the meta-inferences drawn from the integration process to answer the mixed methods research questions about symbolic mathematics language literacy (SMaLL).

For this study, the distinction between what was written and what was spoken is critical. However, the common English language writing conventions uses quotes to identify both written and spoken recitations of communication in similar fashion. Researchers in literacy and linguistics have established conventions with various brackets to address this complication (Hall et al., 2011; Sampson, 2015). In the field of linguists, paired angle brackets are used to clarify references to the appearance of written, not spoken, forms of communication (Sampson, 2015). For example, \( < g > \) and \( < G > \) can be used to illustrate and distinguish between two substantially different (from the perspective of writing systems) instances of the same written letter, one lowercase and the other uppercase. Unfortunately, angle brackets have particular meaning in symbolic mathematics. In this study, what linguists call angle brackets are called inequalities. Therefore, I ruled angle brackets out as a means to indicate a reference to the appearance of written symbolic mathematics. I found a solution to the problem of distinguishing between the appearance of symbolic mathematics and the possible English translation of a symbol in an equation editor, MathType Version6.7e. These are the conventions used in the
remainder of this chapter:

- Boxes are used to clarify printed symbolic mathematics. For example, $A = \pi r^2$ refers to the printed text with the understanding that the box did not appear around the symbolic mathematics.

- Quoted qualitative data are evidence of words spoken. Utterances such as “um” or “like” and pauses that did not contribute to the meaning are, for readability, omitted without any indication. Ellipses are used to indicate the omission of stutters and words with meaning that were not necessary to understanding the meaning of the statement and could be removed without changing the meaning.

**Sample Description**

The sampling procedures were designed to generate a sample large enough to support the quantitative analyses and varied enough to support the qualitative analyses. The nature of the data influences the possibilities for describing the samples. The nested sampling process allows me to describe both the quantitative sample and the qualitative sample using descriptive statistics. Although the portrait of the quantitative sample will be restricted to those aggregate statistics, the portrait of the qualitative sample also includes narrative description.

**Quantitative sample.** The quantitative sample ($N_{QUAN} = 158$) included students from both elective courses and students from classes taught by all three teachers. The pool of potential participants was not proportionate across courses or teachers, nor was the sample. There were 75 participants enrolled in Mr. Rowe’s Math Extension sections, 59 participants enrolled in Mrs. Davis’ Math Extension sections, and 24 participants enrolled in Mrs. Kennedy’s Math Intervention sections. The majority of the students were enrolled in the extension elective. The
discrepancy between the number of participants enrolled in the extension elective and the number of participants enrolled in the intervention elective may be attributable to differences in total enrollment in the courses; however, systematic differences in the populations in terms of willingness to participate in the study cannot be ruled out. Of the 217 students eligible for recruitment, 73% participated in the study (see Consent and assent in Chapter 3 for more detail).

Descriptive statistics generated from the demographic data provided in the secondary data set are shown in the left column of Table 4.1. The sample had more 7th graders than 8th graders and more males than females. The participants ranged in age from 12 to 15 with approximately half of the participants being 13 years old. The majority of the participants were White, almost a third were Asian or Pacific Islander, and the remaining were Black, Hispanic, or Multiracial.

Descriptive statistics generated from the self-reported survey data are shown in the right column of Table 4.1. Just over a third of the participants claimed to speak a language other than English well; and just under a third claimed to read a language other than English well. The nature of the data did not make it possible to distinguish participants who learned English as a second language from native English-speaking participants. For example, participants who were satisfied with their progress in an introductory Spanish elective at the school may be included in the counts of who reads or speaks a language other than English. More than two-thirds of the participants reported the ability to play a musical instrument well or read sheet music well. The number of students reporting musical skills may reflect the Holland Middle School policy requiring students to enroll in one of four fine arts courses: band, choir, orchestra, or visual arts. Finally, the ratio of left to right handers reflects the proportion in the general population with 9%
of the sample reporting left-handedness (Llaurens, Raymond, & Faurie, 2009). Therefore, any extant SNARC (spatial numerical association of response codes) effects, or systematic differences in response time associated with handedness, were assumed to reflect the effect in the target population proportionately (Hesse, Fiehler & Bremmer, 2016; Shaki & Fischer, 2008).

Table 4.1

Quantitative sample: Descriptive statistics ($N_{QUAN} = 158$)

<table>
<thead>
<tr>
<th>Secondary data</th>
<th>$n$</th>
<th>%</th>
<th>Self-reported data</th>
<th>$n$</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Grade</strong></td>
<td></td>
<td></td>
<td><strong>Speak language other than English</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7th</td>
<td>88</td>
<td>56</td>
<td>No</td>
<td>98</td>
<td>62</td>
</tr>
<tr>
<td>8th</td>
<td>70</td>
<td>44</td>
<td>Yes</td>
<td>60</td>
<td>38</td>
</tr>
<tr>
<td><strong>Gender</strong></td>
<td></td>
<td></td>
<td><strong>Read language other than English</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Female</td>
<td>65</td>
<td>41</td>
<td>No</td>
<td>109</td>
<td>69</td>
</tr>
<tr>
<td>Male</td>
<td>93</td>
<td>59</td>
<td>Yes</td>
<td>49</td>
<td>31</td>
</tr>
<tr>
<td><strong>Age</strong></td>
<td></td>
<td></td>
<td><strong>Play musical instrument</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>31</td>
<td>20</td>
<td>No</td>
<td>51</td>
<td>32</td>
</tr>
<tr>
<td>13</td>
<td>84</td>
<td>53</td>
<td>Yes</td>
<td>107</td>
<td>68</td>
</tr>
<tr>
<td>14</td>
<td>42</td>
<td>27</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Ethnicity</strong></td>
<td></td>
<td></td>
<td><strong>Read sheet music</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Asian or Pacific Islander</td>
<td>42</td>
<td>27</td>
<td>No</td>
<td>36</td>
<td>23</td>
</tr>
<tr>
<td>Black (Non Hispanic)</td>
<td>9</td>
<td>6</td>
<td>Yes</td>
<td>122</td>
<td>77</td>
</tr>
<tr>
<td>Hispanic</td>
<td>4</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Multiracial</td>
<td>6</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>White</td>
<td>97</td>
<td>61</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Right-handed

No 14 9
Yes 144 91
Qualitative sample. The qualitative sample was a subset of the quantitative sample \( (N_{\text{QUAL}} = 18) \) selected from among the participants who, during the initial assent process, had agreed to an interview. By design, using a maximum variation sampling technique, the qualitative sample was intended to vary along two dimensions, math ability and reading habits. However, any variation in terms of other variables was unintended. The qualitative sample was similar to the quantitative sample in terms of classroom enrollment (see Table 4.2), secondary data, and self-reported survey data (see Table 4.3).

Table 4.2

Enrollment comparison of quantitative and qualitative samples

<table>
<thead>
<tr>
<th>Quantitative sample ((N = 158))</th>
<th>(n)</th>
<th>%</th>
<th>Qualitative sample ((N = 18))</th>
<th>(n)</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Teacher</strong></td>
<td></td>
<td></td>
<td><strong>Teacher</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rowe</td>
<td>75</td>
<td>47</td>
<td>Rowe</td>
<td>8</td>
<td>44</td>
</tr>
<tr>
<td>Davis</td>
<td>59</td>
<td>37</td>
<td>Davis</td>
<td>7</td>
<td>39</td>
</tr>
<tr>
<td>Kennedy</td>
<td>24</td>
<td>15</td>
<td>Kennedy</td>
<td>3</td>
<td>17</td>
</tr>
<tr>
<td><strong>Elective</strong></td>
<td></td>
<td></td>
<td><strong>Elective</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Math Extension</td>
<td>134</td>
<td>85</td>
<td>Math Extension</td>
<td>15</td>
<td>83</td>
</tr>
<tr>
<td>Math Intervention</td>
<td>24</td>
<td>15</td>
<td>Math Intervention</td>
<td>3</td>
<td>17</td>
</tr>
</tbody>
</table>
Table 4.3

Qualitative sample: Descriptive statistics

<table>
<thead>
<tr>
<th>Secondary data</th>
<th>n</th>
<th>%</th>
<th>Self-reported data</th>
<th>n</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Grade</strong></td>
<td></td>
<td></td>
<td><strong>Speak language other than English</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7th</td>
<td>10</td>
<td>56</td>
<td>No</td>
<td>12</td>
<td>67</td>
</tr>
<tr>
<td>8th</td>
<td>8</td>
<td>44</td>
<td>Yes</td>
<td>6</td>
<td>33</td>
</tr>
<tr>
<td><strong>Gender</strong></td>
<td></td>
<td></td>
<td><strong>Read language other than English</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Female</td>
<td>10</td>
<td>56</td>
<td>No</td>
<td>13</td>
<td>72</td>
</tr>
<tr>
<td>Male</td>
<td>8</td>
<td>44</td>
<td>Yes</td>
<td>5</td>
<td>28</td>
</tr>
<tr>
<td><strong>Age</strong></td>
<td></td>
<td></td>
<td><strong>Play musical instrument</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>3</td>
<td>17</td>
<td>No</td>
<td>6</td>
<td>33</td>
</tr>
<tr>
<td>13</td>
<td>9</td>
<td>50</td>
<td>Yes</td>
<td>12</td>
<td>67</td>
</tr>
<tr>
<td>14</td>
<td>5</td>
<td>28</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>1</td>
<td>6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Ethnicity</strong></td>
<td></td>
<td></td>
<td><strong>Read sheet music</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Asian or Pacific Islander</td>
<td>5</td>
<td>28</td>
<td>No</td>
<td>2</td>
<td>11</td>
</tr>
<tr>
<td>Black (Non Hispanic)</td>
<td>1</td>
<td>6</td>
<td>Yes</td>
<td>16</td>
<td>89</td>
</tr>
<tr>
<td>Hispanic</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Multiracial</td>
<td>1</td>
<td>6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>White</td>
<td>11</td>
<td>61</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The proportion of participants taught by each teacher and enrolled in each elective was similar in both the quantitative and qualitative samples. The qualitative sample included both 7th graders and 8th graders, ranging in age from 12 to 15 years old, in the same proportion as the quantitative sample. The qualitative sample included both males and females in inverse proportion as compared to the quantitative sample; the qualitative sample included more females than males. Most of the interviewees were White. In contrast to the quantitative sample, the qualitative sample did not include any Hispanic participants. The qualitative sample was similar
to the quantitative sample in terms of the self-reported data with the biggest difference in the proportion of students reporting the ability to read sheet music well; all but two of the interviewees reported the ability to read sheet music well.

The qualitative sample can be described more richly than the quantitative sample using themes that emerged from field notes, introductory and closing questions, and probing questions. Table 4.4 shows the pseudonyms of the 18 interviewees with a summary of their grade level, course enrollment, and gender. The pseudonyms, which I assigned, were intended to disguise both the participants' real name as well as the names selected by the participants for the interview process (some of which may have been identifiable as nicknames or favorite characters). The pseudonyms were selected in alphabetical order to facilitate the analysis.

When asked about their current reading interests, the interviewees described their preferred media and genre in general terms and provided authors and titles as examples. In order to provide a sense of the responses for the qualitative sample, I created a word cloud, using http://www.wordle.net, of words and phrases in the participants’ responses. The word cloud shows a random arrangement of the words and phrases using font size to give an indication of their frequency across the sample. Word clouds provide an illustration of the range and mode of the data in a way that evokes a sense of meaning and prevalence while preventing unjustified quantitative comparisons. Figure 4.1 shows that some students preferred books, and others preferred to read using electronic devices or apps such as www.wattpad.com. Many preferred popular fiction genres such as fantasy and mystery, and some expressed interest in nonfiction books about history and travel.
Table 4.4

Qualitative sample by pseudonym and order of interview

<table>
<thead>
<tr>
<th>Name</th>
<th>Grade</th>
<th>Elective</th>
<th>Gender</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>7th</td>
<td>8th</td>
<td>Extension</td>
</tr>
<tr>
<td>Amelia</td>
<td>7</td>
<td></td>
<td>E</td>
</tr>
<tr>
<td>Bobby</td>
<td>8</td>
<td></td>
<td>E</td>
</tr>
<tr>
<td>Claire</td>
<td>7</td>
<td></td>
<td>E</td>
</tr>
<tr>
<td>Delaney</td>
<td>8</td>
<td></td>
<td>E</td>
</tr>
<tr>
<td>Elena</td>
<td>8</td>
<td></td>
<td>E</td>
</tr>
<tr>
<td>Freddie</td>
<td>7</td>
<td></td>
<td>E</td>
</tr>
<tr>
<td>Gil</td>
<td>8</td>
<td></td>
<td>E</td>
</tr>
<tr>
<td>Harper</td>
<td>8</td>
<td></td>
<td>I</td>
</tr>
<tr>
<td>India</td>
<td>7</td>
<td></td>
<td>E</td>
</tr>
<tr>
<td>Jackson</td>
<td>7</td>
<td></td>
<td>E</td>
</tr>
<tr>
<td>Katrina</td>
<td>7</td>
<td></td>
<td>I</td>
</tr>
<tr>
<td>Loren</td>
<td>7</td>
<td></td>
<td>I</td>
</tr>
<tr>
<td>Martin</td>
<td>8</td>
<td></td>
<td>E</td>
</tr>
<tr>
<td>Nicki</td>
<td>8</td>
<td></td>
<td>E</td>
</tr>
<tr>
<td>Owen</td>
<td>7</td>
<td></td>
<td>E</td>
</tr>
<tr>
<td>Peter</td>
<td>8</td>
<td></td>
<td>E</td>
</tr>
<tr>
<td>Quincy</td>
<td>7</td>
<td></td>
<td>E</td>
</tr>
<tr>
<td>Robin</td>
<td>7</td>
<td></td>
<td>E</td>
</tr>
<tr>
<td>Totals</td>
<td>10</td>
<td>8</td>
<td>15</td>
</tr>
</tbody>
</table>
None of the participants could recall seeing symbolic mathematics in the books they read for fun; however, a number of participants indicated the books they like occasionally include mathematical words, maps, or scenarios that include mathematical elements. For example, Quincy, who liked fantasy and adventure books, remarked that his favorite book series had “elements of calculating because there's basically a big war going on. And so there are very subtle elements — numbers of how many people are in the army and maybe the percentage or the likelihood of winning.” Amelia, an avid reader who liked nonfiction and several genres of fiction, noted that mystery books are inherently mathematical: “That's kind of how it works. You have the missing variables and then throughout the story, you can fill them in and then by the end
of it,… [you] figure out what it is.” Amelia, Delaney, Freddie, Martin, and Robin made remarks such as “reading is one of my favorite hobbies” indicating that they choose to read beyond what was required for school. In contrast, Bobby and Loren made remarks such as “only when I have to” to indicate that they would not choose to read, even for fun, unless required to do so for an academic assignment.

Interviewees’ responses to the question of what math coursework they would like to take in the future suggested differences in knowledge about course offerings and beliefs about their role in choosing classes at the high school level. Although some student referred to Algebra, Geometry, and Calculus, some students did not use the common names to refer to on-level math high school mathematics classes: Quincy and Robin used “normal math”, Katrina and Loren used “regular math”, and Freddie used “standard math”. Harper appeared to believe that she would have no choice about which math courses to take: “You can't really pick your math classes. They pick the basic ones for you.” Peter, on the other hand, had developed a 4-year plan for integrating his math coursework with his science coursework:

So in high school, I'm going to be taking honors and then I'll do honors geometry, and I'm going to go up all the way through honors calculus, I'm pretty sure. And then… I'm probably going to take some computer programming classes. And then if science classes count… I'm taking up to AP physics or whatever in science. I sort of planned a lot of that stuff.

Figure 4.2 is a word cloud illustrating the terms interviewees used to describe the math classes they planned to take during high school.
Figure 4.2. Future plans for math coursework \( (N_{QUAL} = 18) \).

Figure 4.3 shows that interviewees also had a wide range of career goals. Some described specific jobs they had gravitated towards at an early age. For example, Jackson wanted to be a policeman, Claire wanted to be a singer, and Loren wanted to be a basketball player. Some described careers grounded in the language arts: Delaney an editor and Freddie a writer. Several were interested in fields requiring science, technology, engineering, and mathematics (STEM) skills. Students interested in STEM careers readily agreed that it was important to get better at reading mathematical text. For example, Peter wanted to get better at reading symbols so he could be more efficient at his job:
I think it's going to be really important because if I were to go into programming, you obviously have to read math every day and all that. And then if I were to go into engineering, I'd have to be able to read the math well and understand it the first or second time through because it's more efficient to work if you're able to understand and you don't have to keep going back and reading it.

Amelia, who wanted to work for NASA, exuberantly noted that her ability to read symbols might have serious consequences for others: “It's really important because if you don't know how to do something… if I’m working on a shuttle or something, that could end somebody's life. You've got to be really spot on about that.”

Among the students who were not yet able to name a career or field, there was not agreement on whether improving SMaLL skills would be important. Some believed SMaLL would benefit them in the future. Others seemed to be unable to envision the role or value of reading math in their future. For example, Bobby said,
It's probably going to be really important because it can differentiate against a good job and maybe a lower paying job because if I know the symbols more, I could probably get into a better college and get a better job that pays more.

Claire’s response summarizes the conflicted responses to this question: “Do you think it is important to get better at reading math?” She said, “Possibly. Maybe not. I'm not sure. Maybe, itty bitty.”

**Quantitative Results**

The quantitative strand of the research design was intended to provide exploratory evidence of the degree of similarity between the cognitive processes of SMaLL and the cognitive processes of English language literacy. Measures of orthographic processing, operationally defined by a task analogous to an English language literacy task, were the primary interest of the quantitative analysis. Therefore, I begin with a review of the data generated by the SMaLL Conventional Decision Task (S-CDT) and then report the results of the group differences. Next, I describe the nature of the data produced by each of the surveys before reporting the results of the correlative analyses and regression analysis.

This study is exploratory in the sense that the rationale is initiation and the measurement tools are novel. In addition, this study is exploratory in that the quantitative analyses are not intended to confirm particular statistical hypotheses about SMaLL. In confirmatory quantitative analysis, power analysis is an important research consideration (Connelly, 2008). A power analysis is a statistical process that is used to consider sufficient or optimal combinations of sample size (usually denoted as \( N \)), effect size (usually denoted as \( d \)), statistical power threshold (usually denoted as \( 1 - \beta \)), and statistical significance threshold (usually denoted as


\( \alpha \). Often, statistical power thresholds and statistical significance thresholds are set within a research community. For example, power thresholds, or the likelihood of finding an effect if it exists, may be set as low as 0.80 for relatively low risk research (e.g., educational research) and 0.90 for higher risk research (e.g., clinical medical research) (Connelly, 2008). Similarly, significance thresholds, or the probability of erroneously concluding an effect exists when it does not, may be set as low as 0.01 for clinical medical research but are more commonly set at 0.05 for educational research. A power analysis often involves using thresholds set by the research community in combination with an expected effect size to calculate a necessary sample size. In general, a power analysis is intended to determine a combination of the four variables that suggest the study can be feasibly conducted and generate interpretable results with an appropriate degree of risk in light of the seriousness of the research problem.

Although a power analysis is generally recommended within the quantitative research committee. Research communities still debate rationales and best practices. For example, Hill, Bloom, Black, and Lipsey (2008) argued that expected effect sizes should be grounded in empirical findings. Aguinis and Harden (2008) suggested that \( \alpha \) should not be set arbitrarily by the field. Rather, they argue that researchers should estimate and compare the practical risks of making a Type I error (i.e., rejecting the null hypothesis when it is true) versus a Type II error (i.e., failing to reject the null hypothesis when it is false). In the literature, a numerical method has been proposed for estimating \( \alpha \) taking into account the desired relative serious of making Type I error or a Type II error (Aguinis & Harden, 2008).

In short, good arguments exist for grounding decisions about power in empirical evidence related to expected effect sizes and risks associated with the Type I and Type II errors. Given
that SMaLL is a new area of research, no such empirical evidence exists. A calculation based on
the desired relative seriousness suggests a significance threshold as high as \( \alpha = .20 \) may be
reasonable considering the low risk and the potential of this novel study (Aguinis & Harden,
2008). However, the quantitative results will be reported using the commonly accepted power
threshold of 95% and significance threshold of \( \alpha = .05 \) to facilitate comparison to other results in
educational research and establish empirical evidence. Importantly, these analyses are conducted
as exploratory, and should be interpreted as exploratory, rather than confirmatory.

**Performance of S-CDT.** The SMaLL Conventional Decision Task (S-CDT) used in this
study was a modified English language lexical decision task designed to produce analogous
measures of orthographic processing in response to the language of mathematics. The initial
analysis of the S-CDT data included three steps. First, I reviewed the response decision and
response times for each item. Then, I cleaned the data using conservative decisions in line with
common data cleaning practices as described in Chapter 3. Finally, I generated a mean response
time and total accuracy score for each participant. Figure 4.4 illustrates the data review process
and data cleaning process and summarizes the nature of the intentionally omitted data.

Figure 4.5 summarizes the raw data points, response times (RTs) and response decisions
(RDs), generated for each response to each conventional (odd numbered) item. Similarly, Figure
4.6 summarizes the two data points generated for each response to each unconventional (even
numbered) item. Because each participant responded to items in a uniquely random order, there
is no reason to believe the order of the items systematically influenced the RTs or RDs. Figure
4.7 shows the item pairs in columns for quick reference. For a more detailed description of the
item pairs, refer to Appendix F.
Set criteria for in-range RTs for each response
- Limits determined based on
  - Literature review
  - Inspection of histograms of RTs for each item
- 200 ms < RT < 5000 ms

Identify out-of-range RTs

<table>
<thead>
<tr>
<th>Low RTs</th>
<th>High RTs</th>
</tr>
</thead>
<tbody>
<tr>
<td>(RT &lt; 200 ms)</td>
<td>(RT &gt; 5000 ms)</td>
</tr>
<tr>
<td>n&lt;sub&gt;RTs&lt;/sub&gt; = 25</td>
<td>n&lt;sub&gt;RTs&lt;/sub&gt; = 32</td>
</tr>
</tbody>
</table>

Calculate mean (in-range) RTs for each participant
- Means of means across participants
  - $\overline{RT}_{all\ items} = 1119, SD = 240$ (no outliers)
  - $\overline{RT}_{conventional\ items} = 1059, SD = 236$ (1 outlier)
  - $\overline{RT}_{unconventional\ items} = 1179, SD = 279$ (1 outlier)

Identify participant outlier(s)
- ID: 3112
  - Above +3SD (Conventional items)
- ID: 3113
  - Above +3SD (Unconventional items)

Calculate error rates for each item
- Items with high error rate (accuracy rate < 60%)
  - Conventional items ($n_{items} = 0$)
  - Unconventional items ($n_{items} = 20$)

Identify high error rate items:
- Items:
  - 2, 6, 8, 10, 14, 18, 20, 22, 28, 36, 38, 40, 44, 48, 50, 52, 54, 56, 58, 60

Calculate error rates for each participant (conventional items only)
- Error rate > 35% (or accuracy >= 65%)
  - Relative to responses associated with all RTs ($n = 12$)
  - Relative to responses associated with in-range RTs ($n = 11$)

Identify participants with high error rates:
- Participant IDs:
  - 1117, 1509, 1702, 2102, 2205, 2504, 2506, 2602, 2605, 2606, 3509, 3519

Summary of data cleaning
- Omit RDs and RTs associated with out-of-range RTs
- Omit RDs and RTs associated with unconventional items
- Omit 1 participant with outlying $RT_{conventional\ items}$
- Omit 11 participants with high error rate on conventional items

*Figure 4.4. Data cleaning process for SMaLL-CDT*
Figure 4.5. Summary of response times (in milliseconds) and accuracy rate of response decisions for each conventional item. Note that some items have extreme outliers that do not appear on this view of the data. For example, the maximum response times for items 11, 19, and 37 exceed 7000ms.
Figure 4.6. Summary of response times (in milliseconds) and accuracy rate of response decisions for each unconventional item. Note that some items have extreme outliers that do not appear on this view of the data. For example, the maximum response times for items 10, 16, 20, 22, 30, 48, 54, and 60 exceed 7000ms.
Figure 4.7. S-CDT items pairs. Odd numbered items are conventional items and even numbered items are unconventional items.
In my initial review of the data, my intent was to judge whether the data performed as expected given the results of similar cognitive tasks. The box plots in the top half of Figure 4.5 show that, for each of the conventional items, the mean, the median, and the inter-quartile (i.e., middle 50%) of the RTs fell within a 200 to 1500 ms range, suggesting that most responses were made using a cognitive process without the benefit of intentional metacognitive reflection (Ratcliff & McKoon, 2007). A few response were made before the 200 ms mark suggesting guessing before the possibility of visual recognition. Many students responded after the text disappeared (when the screen returned to blank at 1500ms). Some of the responses were not made until well past the 3000ms and 5000ms cut-off times used for similar cognitive tasks (e.g., Elgort & Warren, 2014; Van Zandt & Townsend, 2013; Yap et al., 2012). Three items (11, 19, and 37) had maximum response rates that exceeded 7000ms. The bar graphs in the bottom half of Figure 4.5 show that, for each of the conventional items, the accuracy of the RDs surpassed the 60% criteria used by Elgort and Warren (2014). In summary, the data generated in response to the conventional items suggested that the task was sufficiently analogous to theoretically similar cognitive tasks.

The box plots and bar graphs in Figure 4.6 suggest the unconventional items varied more than the conventional items in terms of RTs and RDs. Four of the items (22, 26, 48, and 60) had either inter-quartile RTs or mean RTs outside of the 200 to 1500 ms range. Eight items (10, 16, 20, 22, 30, 48, 54, and 60) had maximum response rates that exceeded 7000ms. Only one-third of the items had an RD accuracy rate of 60% or better. These results indicate that the unconventional items performed differently as compared to the conventional items. This was expected and, in conjunction with the review of the conventional items, suggested proceeding
with the common practice of omitting unconventional items from further analysis.

Given the exploratory nature of the study, I elected to define in-range RTs using the most conservative range that was consistent with the literature. After the initial review of the raw data, I identified in-range RTs as those with responses made between 200ms and 5000ms. Among the 9480 responses made by all participants to all items, 25 responses (<1%) were made in less than 200ms and 32 responses (<1%) were made after 5000ms. Altogether, a total of 57 responses (<1%) were omitted.

Using the in-range RTs, I calculated three different mean RTs for each participant: all items, conventional items, and unconventional items. Then, I calculated an aggregate mean, across the sample, of each. The mean RTs for conventional items had the lowest sample mean and the least variation ($M = 1059, SD = 236$). The mean RTs for unconventional items had the highest sample mean and the most variation ($M = 1179, SD = 279$). The three different mean RTs lead to different conclusions about which participants were outliers (using $3SD$ above or below the sample mean as a cut-off). Using the means calculated with all items suggested there were no participants with an outlying mean RT. Using the means calculated with conventional items suggested that one participant had an outlying mean RT. Using the means calculated with unconventional items suggested that a different participant had an outlying mean RT. These differences offered further insight into the complexity of the data cleaning process and the common practice of analyzing response to conventional (i.e., real words) and omitting unconventional (i.e., pseudowords or invalid words) from analysis (see Table 3.2 in Chapter 3 for more detail). I omitted the unconventional items from further analysis.
The remainder of the analysis proceeded using data associated with conventional items and in-range RTs. For each participant, I calculated the two summary metrics of interest: mean RT and RD accuracy score (e.g., Balota & Yap, 2011; Sze et al., 2014; Yap et al., 2010). The mean RT was determined using the in-range RTs associated with conventional items. One participant was omitted due to an outlying mean RT more than 3SD above the sample mean. The RD accuracy score was calculated as the sum of the correct RDs associated with in-range RTs and conventional items. The review of the RD scores showed that 11 participants had high error rates with more than 35% of their responses being errors. Those 11 participants, who identified more than 35% of the conventionally written text samples as unreadable, were omitted from further analysis. In total, 12 participants were omitted.

Figure 4.8 shows the distribution of the summary metrics used for further analysis. The distribution of the RD accuracy scores, which ranged from 17 to 30, shows the anticipated ceiling effect on the conventional items with 13 students (nearly 10% of the sample) able to respond accurately to all 30 items between 200 and 5000ms. The distribution of mean RTs, which ranged from 642 to 1721ms, shows that the distribution is skewed slightly to the right. Interestingly, it shows that, after data cleaning, there is an outlier with respect to the cleaned mean response times.
Figure 4.8. Distribution of SMaLL Conventional Decision Task (S-CDT) summary metrics after data cleaning \((n = 146)\). RD range: 17-30. RT range: 641-1721 ms.
**Group difference analysis.** Analyses of group differences were conducted to answer these two research questions: Do differences in orthographic processing exist across grade levels? Do differences in orthographic processing exist across mathematics courses? I assessed the comparison in terms of the mean RTs and the RD accuracy scores. I used tables, visual inspection, and t-tests to draw conclusions.

**Across grade levels.** Table 4.5 shows statistical measures of dispersion and central tendency of the mean RTs and RD accuracy scores for both 7th and 8th grades side by side. Inspection of the table suggested there was little difference between the mean RTs and RD accuracy scores across grade levels. To assess normality, I used QQ-plots to compare the distribution of mean RTs and RD accuracy scores to the distribution of the normal curve. The review of QQ-plots suggested the assumption of normality was reasonable for both grades on both measures. For more insight, I created density plots for both measures (see Figure 4.9). The density plots, a practical tool for illustrating an estimation of the distribution of the data with a smooth curve, showed some differences in the shapes of the data across grades. A comparison of box plots, however, suggested there was no difference in mean of the mean RTs or the mean of the RD accuracy across grades.

Because the assumption of normality was not in question for the mean RTs or the RD accuracy scores, I ran the Welch two sample t-test. The Welch two sample t-test, which adjusts for unequal sample size, is a parametric test of differences in sample means appropriate for use with normally distributed data. The results indicate there was insufficient evidence to conclude there is a difference in the mean of the mean RTs \( (t = 0.96, df = 128.74, p = 0.34) \) or the mean of the RD accuracy scores \( (t = -0.41, df = 142.19, p = 0.68) \) across grades.
Table 4.5

Comparison of S-CDT measures across grades

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Mean RT</th>
<th>RD Accuracy</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>7\textsuperscript{th} Grade</td>
<td>8\textsuperscript{th} Grade</td>
<td>7\textsuperscript{th} Grade</td>
<td>8\textsuperscript{th} Grade</td>
</tr>
<tr>
<td>n</td>
<td>80</td>
<td>66</td>
<td>80</td>
<td>66</td>
</tr>
<tr>
<td>Minimum</td>
<td>664.13</td>
<td>641.63</td>
<td>17</td>
<td>20</td>
</tr>
<tr>
<td>Maximum</td>
<td>1630.10</td>
<td>1721.23</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>Range</td>
<td>965.97</td>
<td>1079.60</td>
<td>13</td>
<td>10</td>
</tr>
<tr>
<td>Median</td>
<td>1045.48</td>
<td>1015.93</td>
<td>26</td>
<td>26</td>
</tr>
<tr>
<td>Mean ( $\bar{X}$ )</td>
<td>1067.58</td>
<td>1031.31</td>
<td>26.05</td>
<td>26.24</td>
</tr>
<tr>
<td>Standard Error ( $\sigma_{\bar{X}}$ )</td>
<td>23.10</td>
<td>29.68</td>
<td>0.33</td>
<td>0.33</td>
</tr>
<tr>
<td>95% CI</td>
<td>45.98</td>
<td>59.27</td>
<td>0.65</td>
<td>0.66</td>
</tr>
<tr>
<td>Variance ( $s^2$ )</td>
<td>42685.52</td>
<td>58127.56</td>
<td>8.50</td>
<td>7.23</td>
</tr>
<tr>
<td>Standard Deviation (s)</td>
<td>206.60</td>
<td>241.10</td>
<td>2.92</td>
<td>2.69</td>
</tr>
<tr>
<td>Coefficient of Variation ( $CV = \frac{\bar{X}}{s}$ )</td>
<td>0.19</td>
<td>0.23</td>
<td>0.11</td>
<td>0.10</td>
</tr>
</tbody>
</table>
Figure 4.9. Comparisons across grades: Density graphs of mean response time (RT) and response decision (RD) accuracy score.
Across mathematics courses. Table 4.6 shows statistical measures of dispersion and central tendency of the mean RTs and RD accuracy scores for both courses, Math Intervention and Math Extension, side by side. Inspection of the table suggested there was more variation across electives than across grades. Notably, there was a large discrepancy between sample sizes for each course. The review of QQ-plots suggested the assumption of normality was reasonable for each subset of the data except for the RD accuracy scores among students enrolled in the Math Intervention elective. The comparison of the density graphs for both measures (see Figure 4.10) suggested the shape of RD accuracy data differed more across courses than did the shape of the mean RT data. Likewise, a comparison of box plots for suggested there was a difference in the mean of the RD accuracy scores across courses, but no difference in mean of the mean RTs.

The Welch two sample t-test \( t = 5.21, df = 22.53, p < 0.05 \) indicated there was sufficient evidence to conclude there was a difference between the mean of the RD accuracy scores across courses. Because the assumption of normality was in question for the RD accuracy scores, I also conducted the Wilcoxon rank sum test, a nonparametric test which does not require an assumption of normality, for comparison. The Wilcoxon rank sum test \( W = 1810, p < 0.05 \) suggested there was sufficient evidence to conclude that distribution of the differences between the ranked pairs of RD accuracy scores was not symmetrically distributed. Because the assumption of normality was not in question for the mean RTs, I ran Welch two sample t-test which suggested a lack of evidence to conclude there is a difference between mean RT scores across courses.
Table 4.6

*Comparison of S-CDT measures across elective mathematics courses*

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Mean RT</th>
<th></th>
<th>RD Accuracy</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Intervention</td>
<td>Extension</td>
<td>Intervention</td>
<td>Extension</td>
</tr>
<tr>
<td><em>n</em></td>
<td>17</td>
<td>129</td>
<td>17</td>
<td>129</td>
</tr>
<tr>
<td>Minimum</td>
<td>697.10</td>
<td>641.63</td>
<td>20</td>
<td>17</td>
</tr>
<tr>
<td>Maximum</td>
<td>1232.13</td>
<td>1721.23</td>
<td>27</td>
<td>30</td>
</tr>
<tr>
<td>Range</td>
<td>535.03</td>
<td>1079.60</td>
<td>7</td>
<td>13</td>
</tr>
<tr>
<td>Median</td>
<td>1026.40</td>
<td>1025.20</td>
<td>24</td>
<td>27</td>
</tr>
<tr>
<td>Mean ( $\bar{x}$ )</td>
<td>1021.87</td>
<td>1055.05</td>
<td>23.41</td>
<td>26.50</td>
</tr>
<tr>
<td>Standard Error ($\sigma_{\bar{x}}$)</td>
<td>34.69</td>
<td>20.37</td>
<td>0.54</td>
<td>0.24</td>
</tr>
<tr>
<td>95% CI</td>
<td>73.54</td>
<td>40.31</td>
<td>1.15</td>
<td>0.47</td>
</tr>
<tr>
<td>Variance ($s^2$)</td>
<td>20457.61</td>
<td>53548.25</td>
<td>5.01</td>
<td>7.19</td>
</tr>
<tr>
<td>Standard Deviation ($s$)</td>
<td>143.03</td>
<td>231.41</td>
<td>2.24</td>
<td>2.68</td>
</tr>
<tr>
<td>Coefficient of Variation ($CV = \frac{\bar{x}}{s}$)</td>
<td>0.14</td>
<td>0.22</td>
<td>0.10</td>
<td>0.10</td>
</tr>
</tbody>
</table>
Figure 4.10. Comparisons across Math Extension (ME) and Math Intervention (MI) electives:
Density graphs of mean response time (RT) and response decision (RD) accuracy score.
In summary, the RD accuracy scores of students in the Math Intervention elective differed from the RD accuracy scores of student in the Math Extension elective. However, the mean RTs did not differ based on the elective course. In addition, there was difference in orthographic processing across grades in terms of either RD accuracy or mean RT. The analysis does not support any causal claims, but it does indicate there is a relationship between elective course enrollment and orthographic processing. Because all students in this sample were enrolled in a mixed grade elective course, the similarity in orthographic processing across grades may be attributable to the nature of the sample; therefore, the results may not generalize to students not enrolled in an elective course.

**Performance of surveys.** Although it is not uncommon to refer to the psychometric properties of an instrument, the nature of a measurement tool is determined by both the instrument and the sample. In this section, I describe the results of the analyses used to understand the properties of the measurement tools given the sample described in the previous section. Because a thorough analysis of the properties of the measurement tools is beyond the scope of this research study, the presentation of these results are intended to describe the item-level data in sufficient detail to understand the affordances and limitations of the scores generated by each tool and suggest strategies for the future development of each tool. I discuss the properties of the four primary quantitative surveys in this order: Math Print Exposure Survey (MPES), Math Reading Habits Survey (MRHS), and Math Anxiety Survey (MAS).

**MPES interactive analysis.** The purpose of conducting an exploratory factors analysis of the Math Print Exposure Survey (MPES) with a dialectical stance, using emerging knowledge from the interviews and exploratory factor analysis, was two-fold. The first purpose was to
engage in interactive analysis with implications for the thematics analysis of the qualitative strand. The second purpose was to generate insights into the nature of the survey as a means to interpret the scores and subsequent analyses involving the scores. My analysis of the MPES data included three steps. First, I reviewed the data for missingness. Then, I generated summary scores for each participant and assessed the reliability of the scores. Finally, I conducted an exploratory factor analysis using an integrative approach.

The MPES was designed to be a self-reported measure of how often a student has access to common mathematical text. The items selected for this survey were not selected based on a theory about the structure of the formulas or students’ ability to read the formulas. Rather the items were selected based on a review of the Common Core State Standards for Mathematics (CCSS-M) curriculum and common testing materials.

While completing the MPES, participants viewed only one formula per page and were permitted to return to each item as needed to select — and correct, if necessary — a response that best described how often they had seen, read, written, or used the formula at school, home, or anywhere. Allowing participants to return to items was intended to encourage them to take time to inspect the text and reflect on their experiences. Programming the MPES to allow a return to each item had the effect of allowing participants to skip an item. Participants were instructed to respond to every item; however, unlike the other surveys, Inquisit did not provide alerts to omitted items for the MPES survey. There was a low rate of missing data for the MPES (see Table 4.7). Only nine participants omitted a total of 17 responses. Across the 15 items, less than 1% of the responses were missing.
The pattern in the rate of missingness across items suggests the rate of missingness may have been related to item order. Each participant responded to the MPES items in the same order (excluding the possibility of returning to items). Because the MPES survey was the last survey completed, participants may have experienced fatigue. Summary scores were calculated only for the participants who responded to all of the items. This prevented the analysis of artificially low MPES scores and excluded only nine participants from further quantitative analysis. Given the exploratory nature of this study, I determined it was more important to maintain data quality than sample size. A higher MPES score indicated more frequent exposure to more formulas.

I conducted a visual inspection of histograms for each item. The distribution of responses to each item were not expected to be normal given the nature of the items. For
example, Item 15 was negatively skewed and leptokurtic (Lomax & Hahs-Vaughn, 2012) as expected given that the formula for the area of a circle, $A = \pi r^2$, is in frequent use in CCSS-M. In contrast, Item 7 was positively skewed with 84 participants reporting they had never seen the principal square root defined using symbolic mathematics as $\sqrt{x^2} = |x|$. Figure 4.11 provides a summary of the responses, by item, to the 4-point Likert scale. The items are ordered by frequency of exposure. The percentage of participants responding *A few times or Many times* appears on the right. The percentage of participants responding *Never or Only one or two times* appears on the left.

I reviewed the total scores via visual inspection of a histogram (see Figure 4.12) and measures of central tendency and dispersion. The distribution of total scores, ranging from 24 to 60 (on a scale from 15 to 60), was approximately normal ($N = 149$, $M = 45.03$, $SD = 8.03$, $Mdn = 45$, skewness = -.5, kurtosis = -.17). The Cronbach’s alpha for the total scale suggests the reliability ($\alpha = .81$) was good. The split-half reliability of the Title Recognition Test, a measure of print exposure used in an English language study of orthographic processing, was reported at 0.67 (Cunningham et al., 2001).

Before conducting the exploratory factors analysis, I created a scree plot to estimate the number of factors. The scree plot (see Figure 4.13) illustrates the eigenvalues associated with the number of factors, or components. The total possible number of factors, one for each of the 15 items in the MPES, are shown along the x-axis. Eigenvalues, shown along the y-axis, illustrate the variance explained. Scree plots associated with a well-defined $n$-factor measurement tool, will show higher eigenvalues ($<1$) for the first $n$ factors and then a severe bend, or elbow, in the graph with the eigenvalues leveling off. Scree plots associated with a scale with indistinct
factors do not have the characteristic elbow shape. In general, the elbow of the scree plot provides some visual evidence of the number of factors and the quality of the factors.

Figure 4.11. Bar graph of responses to Math Print Exposure Survey (MPES) by item, ordered by frequency of exposure (N = 149).
Figure 4.12. Distribution of Math Print Exposure Survey (MPES) total scores. Scale: 15-60. Range: 24-60.

The visual inspection of eigenvalues (shown in black) on a scree plot for the MPES suggested a 3-factor or 4-factor solution would be a good fit. I compared 3-factor and 4-factor solutions using the lavaan package in R solutions. Given the exploratory nature of this study, I used an orthogonal rotation, varimax, to simplify the factor structure and facilitate interpretation (Tabachnick & Fidell, 2013). The chi-square statistic suggested the 4-factor solution fit the data ($\chi^2 = 67.54$, $df = 51$, $p = 0.06$). Three items had cross-loadings and only one item had a loading below .30. The 4-factor solution explained 46% of the variance; however, the four
factors were not interpretable. The chi-square statistic suggested the 3-factor solution did not fit the data as well ($\chi^2 = 103.14$, $df = 63$, $p = 0.001$) and only explained 39% of the variance. The quantitative literature suggests that $\chi^2$ is a rudimentary measure of model fit (Kelloway, 2015). For a number of reasons, a rigorous factor analysis for the purpose of establishing defensible psychometric properties or developing a more sound psychometric tool depends on other measures of fit such as root mean square error of approximation (RMSEA) or Akaike information criteria (AIC).

*Figure 4.13. Scree plot for Math Print Exposure Survey (MPES)*
However, the purpose of this analysis was to exploit the mathematical analysis of covariance among the items to glean some insight into the nature of the items. Therefore, I used the factor loadings for the 3-factor and 4-factor solutions to guide a qualitative exploration of the item groupings. Keeping in mind the themes emerging from participants during the interview, I tried to interpret the meaning of the item groupings for the 3-factor and 4-factor solutions. The groupings for the 4-factor solution did not make practical sense; but, I was able to interpret the 3-factor solution in light of what I learned from the students.

The factor loadings for the 3-factor solution are shown in Table 4.8. There were five items with cross-loadings and two items with loadings below .30. The first factor was comprised of ‘plug and chug’ reference formulas. The items associated with the first factor are central to the curriculum and are found on common reference sheets associated with assignments or assessment (e.g., GED Testing Service, 2014; Ohio Department of Education, 2015). The second factor included ‘textbook’ formulas with simple structure. That is, the items associated with the second factor included relatively simple symbolic mathematics characters in a relatively simple syntactic arrangement to represent formulas less common than those associated with factor one. The third factor included ‘complex’ formulas associated with mathematical definitions and theorems. Compared to second factor, these items have less common characters arranged in a more complex structure. The two items that did not load onto a factor appear on typical reference sheets for standardized tests, but, in my experience, are less commonly used in mathematics classrooms.
### Table 4.8

*Factor loadings for 3-factor interpretation of Math Print Exposure Survey (MPES)*

<table>
<thead>
<tr>
<th>Item</th>
<th>Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>2.</td>
<td>$V = \frac{1}{3} \pi r^2 h$ Volume of a cone</td>
</tr>
<tr>
<td>5.</td>
<td>$a^2 + b^2 = c^2$ Pythagorean Theorem</td>
</tr>
<tr>
<td>8.</td>
<td>$y = mx + b$ Equation of a line (slope-intercept)</td>
</tr>
<tr>
<td>9.</td>
<td>$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope of line between 2 points</td>
</tr>
<tr>
<td>10.</td>
<td>$A = \frac{1}{2} bh$ Area of a triangle</td>
</tr>
<tr>
<td>14.</td>
<td>$V = Bh$ Volume of a rectangular prism</td>
</tr>
<tr>
<td>15.</td>
<td>$A = \pi r^2$ Area of a circle</td>
</tr>
<tr>
<td>3.</td>
<td>$d = rt$ Distance formula</td>
</tr>
<tr>
<td>4.</td>
<td>$y = \frac{k}{x}$ Inverse proportion</td>
</tr>
<tr>
<td>1.</td>
<td>$a(b + c) = ab + ac$ Distributive Property</td>
</tr>
<tr>
<td>6.</td>
<td>$\frac{x^a}{x^b} = x^{a-b}$ Law of exponents</td>
</tr>
<tr>
<td>7.</td>
<td>$\sqrt{x^2} =</td>
</tr>
<tr>
<td>13.</td>
<td>$f(x) = a(x - h)^2 + k$ Parabola with vertex (h,k)</td>
</tr>
<tr>
<td>11.</td>
<td>1 in = 2.54 cm Inch to centimeter conversion</td>
</tr>
<tr>
<td>12.</td>
<td>$C = \pi d$ Circumference of a circle</td>
</tr>
</tbody>
</table>

*Note:* Factor loadings < 0.30 were omitted.
Although the MPES was intended to be a self-reported measure of exposure to common symbolic mathematics, the very premise of this study suggests that the MPES might have functioned as a test of ability to read symbolic mathematics rather than a report of exposure to particular formulas. The results of this analysis are inconclusive on that account. Further development and analyses are warranted. However, viewing the item response rates, the reliability, and the exploration of the covariances of the items in light of the curriculum review, suggested that it was reasonable to interpret the MPES as a measure of print exposure.

This analysis provided a framework for thinking about the nature of formulas during the subsequent qualitative analysis. Specifically, I attended to the nature of symbolic mathematics identified as readable and unreadable with these 3-factors in mind.

Given that the reliability was sufficient and instrument development was not a focus of this study, I proceeded with the planned analysis using the total score for the MPES. Because a high score on the MPES indicated frequent exposure, I expected to find positive correlations with measures of orthographic processing.

**MRHS interactive analysis.** Given the novelty of the Math Reading Habits Survey (MRHS), the purpose of conducting an exploratory factors analysis of the MRHS with a dialectical stance, using emerging knowledge from the interviews and exploratory factor analysis, was two-fold. The first purpose was to engage in interactive analysis with implications for the thematics analysis of the qualitative strand. The second purpose was to generate insights into the nature of the survey as a means to interpret the scores and subsequent analyses involving the scores. My analysis of the MRHS data included two steps. First, I generated summary scores for each participant and assessed the reliability of the scores. Then, I conducted an
exploratory analysis using an interactive approach.

The MRHS was designed to measure how often students intentionally engage in reading mathematical text in different settings and for different purposes. Many studies show a relationship between English language literacy and reading habits (e.g., Hughes-Hassell & Rodge, 2007; Verhoeven et al., 2011). However, the reading habits surveys and home literacy environment surveys I reviewed varied dramatically. Although they differed, they included items intended to assess the frequency of engaging in activities teachers commonly encourage as a means to support literacy development. The MRHS is analogous with one exception. One of the items, which was reverse coded, described a habit that, in my experience, is discouraged in many mathematics classrooms. Ultimately, the MPES was intended to be a self-reported measure of situated behaviors related to mathematical text and presumed, based on pedagogical content knowledge, to support (or undermine) mathematical learning.

There was no missing data for the MRHS as the Inquisit format required every participant to respond to every item in order to proceed. I conducted a review of the responses to each item. Figure 4.14 shows the responses, by item, to the 5-point Likert scale ordered by frequency of habit. The percentage of participants responding Frequently or Always appears on the right. The percentage of participants responding Sometimes appears in the middle. The percentage of participants responding Never or Rarely appears on the left. In addition, I visually inspected histograms for each item. With the exception of items 6 and 8, the histogram for each item were reasonably normal. The histograms for items 6 and 8 were monotonically increasing.
I calculated total scores as the sum of the numeric Likert scale responses (with item 5 reverse scored). The histogram of the MRHS scores (see Figure 4.15) illustrates the central tendency and dispersion. The distribution of total scores, ranging from 24 to 50 (on a scale from 10 to 50), was approximately normal \((N = 158, M = 36.03, SD = 4.79, Mdn = 36, \text{ skewness} = .11, \text{ kurtosis} = -.04)\). The Cronbach’s alpha for the total scale suggests the reliability \((\alpha = .72)\) was acceptable and in line with the reliability of another home literacy environment survey: Niklas and Schneider (2015) used a six question home literacy environment with a reliability of 0.72. A review of item-total statistic using SPSS suggested that deleting items 5 and 10 would result in a modest improvement in reliability, raising Cronbach’s total alpha to .74.
I conducted the interactive exploratory factor analysis of the MRHS using a process similar to the process used for the MPES. Before conducting the exploratory factors analysis, I created a scree plot to estimate the number of factors. The visual inspection of the eigenvalues (shown in black) on a scree plot (see Figure 4.16) suggested a 3-factor or 4-factor solution might be a best fit. Using varimax, an orthogonal rotation, I conducted an exploratory factor analysis by comparing the 2-factor, 3-factor, and 4-factor solutions using the lavaan package in R. The chi-square statistic suggested the 2-factor solution did not fit the data well ($\chi^2 = 62.1$, $df = 26$, $p < 0.01$).
The chi-square statistic suggested the 3-factor ($\chi^2 = 27.17, \ df = 18, \ p = 0.08$) and 4-factor ($\chi^2 = 10.12, \ df = 11, \ p = 0.52$) solutions fit the data explaining 46% and 51% of the variance, respectively. These results provided sufficient evidence to continue with the interactive analysis by bringing evidence from the interviews to bear on interpretation of the grouping of the items.

![Scree plot for Math Reading Habits Survey (MRHS)](image)

*Figure 4.16. Scree plot for Math Reading Habits Survey (MRHS)*
By comparing the item groupings, I determined the 4-factor solution was the most useful as it loaded all items onto a factor with a minimum loading of .33 and yielded an interpretable cross-loading (Costello & Osborne, 2005). The factor loadings are shown in Table 4.9. The first factor is comprised of items that describe where students have a habit of reading mathematics, the second factor is comprised of items that describe habits of attending to the language of mathematics (i.e., symbolic mathematics), and the third factor is comprised of items that describe common English language reading practices (i.e., independent reading, word reading, and repeated reading). The fourth factor included the reverse coded item intended to assess habitual responses associated with the difficulty reading symbolic mathematics. The Cronbach’s alpha for the subscale associated with the first factor was good ($\alpha = .84$), however, the reliability for the subscales related to the second and third factor were poor ($\alpha < .70$).

The results related to item 5 were of particular interest. Item 5 had a low loading onto the first factor in the 3-factor solution, and it had a cross loading with first factor in the 4-factor solution. During the qualitative data collection, several interviewees had indicated that their teacher was a primary resource for help reading. However, two-thirds of participants responded *Never* or *Rarely* to item 5. The results of the exploratory factor analysis, in conjunction with the qualitative results, suggest that item 5 should not be dropped. On the contrary, further item development is warranted to understand what students do when they have difficulty reading symbolic mathematics.
Table 4.9

Factor loadings for 4-factor interpretation of Math Reading Habits Survey (MRHS)

<table>
<thead>
<tr>
<th>Item</th>
<th>Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>1. When I am at school, I read things with math symbols.</td>
<td>0.59</td>
</tr>
<tr>
<td>2. When I am at home, I read things with math symbols.</td>
<td>0.82</td>
</tr>
<tr>
<td>3. When I use the computer, I read things with math symbols.</td>
<td>0.78</td>
</tr>
<tr>
<td>4. As part of my hobbies, I read things with math symbols.</td>
<td>0.77</td>
</tr>
<tr>
<td>5. When I see math symbols, I get someone else to read them to me.</td>
<td></td>
</tr>
<tr>
<td>6. Before I do a math problem, I read the problem to myself.</td>
<td>0.62</td>
</tr>
<tr>
<td>7. When I read a math problem, I focus on the words.</td>
<td>0.62</td>
</tr>
<tr>
<td>8. When I read a math problem, I focus on the numbers.</td>
<td></td>
</tr>
<tr>
<td>9. When I read a math problem, I focus on the symbols.</td>
<td>0.94</td>
</tr>
<tr>
<td>10. When I do a math problem, I read the problem more than once.</td>
<td></td>
</tr>
</tbody>
</table>

*Note:* Factor loadings < 0.30 were omitted.

The results of this analysis present a complicated picture of the MRHS. Although reliabilities of the subscales associated with the second and third factors were low, the results revealed interpretable subscales that suggested that habits of focusing on symbolic mathematics during reading may be distinguishable from habits commonly associated with the English language arts. The results suggest careful item development for each emerging factor may yield a more reliable and relevant measurement tool.

This analysis suggested a framework for the thematic analysis of the qualitative data. In particular, I was vigilant in attending to how participants described the differences between strategies for reading numbers and symbols and strategies for reading words in English.
Given that the reliability was sufficient and instrument development was not a focus of this study, I proceeded with the planned analysis using the total score for the MRHS. Because a high score on the MRHS indicated engagement in recommended habits, I expected to find a positive correlations with measures of orthographic processing.

**MAS confirmatory factor analysis.** The MAS was intended to be a reduced reading level version of the Math Anxiety Scale-Revised (MAS-R) (Bai, 2011; Bai et al., 2009). The purpose of the confirmatory analysis was to determine if the MAS used in this study had the same psychometric properties as the original despite the modifications and difference in population. The initial analysis of the MAS data included two steps. First, I generated summary scores for each participant. Then, I conducted a confirmatory analysis of the measurement tool using classical test theory.

There was no missing data for the MAS as the Inquisit format required every participant to respond to every item in order to proceed. I conducted a review of the response patterns to items associated with positive and negative affect. Figure 4.17 shows the responses, by item to the 4-point Likert scale ordered by degree of agreement. The percentage of participants responding *Agree* or *Strongly agree* appears on the right. The percentage of participants responding *Disagree* or *Strongly disagree* appears on the left. I also made a visual inspection of histograms for each item. After generating the summary score, I visually inspected a histogram (see Figure 4.18) and calculated measures of central tendency and dispersion. Both suggested that the distribution of scores, ranging from 14 to 52 (on a scale from 14 to 56), was slightly skewed to the right and platykurtic, but reasonably normal ($N = 158$, $M = 27.61$, $SD = 8.24$, $Mdn = 26.5$, skewness = .47, kurtosis = −.33). The floor effect and shape of the data suggests that the
sample included many students who do not suffer from math anxiety and a few students who suffered from an atypically high degree of math anxiety.

*Figure 4.17.* Bar graphs of responses to Math Anxiety Survey (MAS) by item. Items 1-6 are positive affect items. Items 7-14 are negative affect items.
The psychometric analysis of the MAS suggested the modifications to reduce the reading level of the MAS-R (Bai, 2011; Bai et al., 2009) did not change the reliability or factor structure of the scale. Cronbach’s alpha suggests the reliability was high for the 14-item total scale, the 6-item positive affect subscale, and the 8-item negative affect subscale (α = .93, α = .88, α = .9, respectively). The reliability measures were commensurate with the reliability of the MAS-R (α = .91) reported by Bai et al. (2009). A visual inspection of eigenvalues on a scree plot suggested a 2-factor solution would be a best fit. The confirmatory factor analysis using the \texttt{lavaan}
package in R indicated that the 2-factor structure was a reasonable fit given the small sample size ($\chi^2 = 173.088$, $df = 76$, $p = 0.06$; $CFI = .927$; $TLI = .913$; $RMSEA = .09$ [90% CI = .072, .108]; $SRMR = .058$).

For further comparison to the MAS-R, I used SPSS to generate item-total statistics similar to those reported by Bai (2011) and Bai et al. (2009). Two items on the MAS-R showed relatively low item-total correlations compared to the other items. “I think I will use math in the future.” and “Math relates to my life.” had item-total correlations of .31 and .26, respectively. For the MAS used in this study, these items were changed, as follows, to reduce the reading level: “I plan to use math in the future” (item 5) and “Math is part of my life” (item 4). In the MAS, these items had the lowest item-total correlation, however, the correlations were higher at .55 and .50, respectively. Because participants agreed with these two items more often than others (see Figure 4.17), it may be useful to further investigate whether these two items are susceptible to social desirability bias, particularly in a K-12 school setting influenced by CCSS.

The results of the analysis suggested the MAS had the expected psychometric properties and was a reliable measure given the sample for this study. Because a high MAS score indicated high math anxiety, I expected the MAS scores to be negatively correlated with other quantitative measures.

**Correlational analysis.** The correlational analysis was conducted to answer this research question: What are the relationships between measures of orthographic processing, math print exposure, math reading habits, math anxiety, and math achievement? To answer the question these summary quantitative measures were used: mean RT (in milliseconds) and RD accuracy score on the S-CDT; the MPES, MRHS, and MAS summary scores; and the 2014 Ohio
Achievement Assessment (OAA) score provided in the secondary data set. The analysis was conducted by grade because OAA assessments differed by grade level. Although the OAA scores are standardized so that similar scores, theoretically, indicate a commensurate degree proficiency with respect to grade level, the scores do not indicate similar knowledge and abilities overall.

Before conducting the correlational analysis, I considered the nature of the data. Because some of the summary scores were based on Likert scale, or ordinal, responses, it was possible that treating them as ordinal data would yield different results. For comparison, I conducted analyses using Pearson’s $r$ (i.e., Pearson’s produce-moment correlation coefficient), Spearman’s $\rho$ rank correlation coefficient, and Kendall’s $\tau$. Pearson’s $r$, a parametric test which assumes data is interval or ratio, can underestimate correlations when there are range restrictions, such as floor and ceiling effects, on the data (Lomax & Hahs-Vaughn, 2012). Two summary scores, the RD accuracy score on the S-CDT and the total score on the MPES, had ceiling effects. The MAS had a floor effect. Spearman’s $\rho$ rank correlation coefficient, which is appropriate when both variables are ordinal, is not well-suited to data with ties in the ranks of either variable (Lomax & Hahs-Vaughn, 2012). Kendall’s $\tau$, also a ranked correlation for ordinal data, accounts for ties in the ranking of one variable with the ranking of the other variable. All three procedures produce estimates between $-1.0$ and $+1.0$. Although the estimates produced differ, the signs are all interpreted similarly: A negative sign indicates that as the values of one variable increase, the values of the other decrease. A positive sign indicates that as the values of one variable increase, the values of the other also increase.

Because the data were not perfectly suited to any of the techniques, I compared the
results of all three. After the data cleaning, the sample size for the correlational analysis was reduced to $n = 146$, including 80 seventh graders and 66 eighth graders. I used the rcorr() function in the Hmisc package in R to compute both Pearson’s $r$ and Spearman’s $\rho$. I used the cor() function in the stats package in R to compute Kendall’s $\tau$. I used pairwise deletion to minimize the reduction of data due to the missing MPES ($n = 9$ (6%)) and OAA ($n = 11$ (7%)) scores.

The results showed that the three procedures yield similar results in that all of the estimates with statistical significance had the same sign. In general, Kendall’s $\tau$ produced lower estimates and Spearman’s $\rho$ produced higher estimates. Given the exploratory nature of this study, I proceeded with the analysis based on Pearson’s $r$ for these reasons: First, the literature suggests that applying Pearson’s $r$ to Likert data, even with small sample sizes, unequal variances, and non-normal distributions, produces valid results (Norman, 2010). Second, the mean RTs, a measure of orthographic processing of primary interest in this study, are not ordinal, calling into question the appropriateness of Spearman’s $\rho$ or Kendall’s $\tau$. Finally, the prevalence of Pearson’s $r$ suggests the meaning of the magnitude and direction of coefficient produced by this method will be interpretable with the caution appropriate to the limitations.

Table 4.10 shows the results of the correlational analysis in two tables. The analysis was conducted by grade because the mathematics achievement scores are associated with two different tests. The first table shows the relationships among the variables for 7th graders with respect to their 6th grade Ohio Achievement Assessment (OAA6) scores from the previous spring. The second table shows the relationships among the variables for 8th graders with respect to their 7th grade Ohio Achievement Assessment (OAA7) scores from the previous spring.
Table 4.10

Comparison of correlation tables by grade.

<table>
<thead>
<tr>
<th>7th graders</th>
<th>RT</th>
<th>RD</th>
<th>MPES</th>
<th>MRHS</th>
<th>MAS</th>
<th>OAA6</th>
</tr>
</thead>
<tbody>
<tr>
<td>RT</td>
<td>-0.06</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RD</td>
<td></td>
<td>-0.08</td>
<td>0.34</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MPES</td>
<td>-0.08</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MRHS</td>
<td>-0.09</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MAS</td>
<td>-0.02</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>OAA6</td>
<td>0.12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>8th graders</th>
<th>RT</th>
<th>RD</th>
<th>MPES</th>
<th>MRHS</th>
<th>MAS</th>
<th>OAA7</th>
</tr>
</thead>
<tbody>
<tr>
<td>RT</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RD</td>
<td>0.01</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MPES</td>
<td>-0.15</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MRHS</td>
<td>-0.35</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MAS</td>
<td>0.19</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>OAA7</td>
<td>-0.10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: The $p$-values are displayed in parentheses below the Pearson’s $r$ value. The circles above the diagonal illustrate the magnitude of correlations with statistical significance ($p < .05$). Black indicates a negative correlation, and red indicates a positive correlation. RT = mean response time; RD = response decision; MPES = Math Print Exposure Survey; MRHS = Math Reading Habits Survey; MAS = Math Anxiety Survey; OAA6 = 6th grade Ohio Achievement Assessment; OAA7 = 7th grade Ohio Achievement Assessment.
The results suggest that, in comparison to the predictions in Table 3.3, the direction of the correlations matched for all statistically significant results. The statistically significant relationships between RD accuracy score and measures of print exposure, math anxiety, and math achievement suggest that orthographic processing of the language of mathematics is relevant to mathematical development in systematic ways. Together, the results suggest that the magnitude of the relationship between between RD accuracy score and measures of print exposure and math achievement may change over time. The mean RTs did not have the expected relationships with other variables at the level of statistical significance except in one case: For 8th graders, there was a negative correlation between mean RTs and MRHS scores. This result suggests the possibility that reading habits are associated with automated reading processes.

Overall, the results indicate that further investigation is warranted. In particular, the patterns and magnitudes of the correlations across grades show that, despite the lack of unidimensional group differences found by the t-tests in the Group difference analysis, the relationships between measures related to SMaLL may evolve. The results also warranted a multiple regression analysis to gain insight into whether measures of orthographic processing can be used to mathematically model, or statistically predict, mathematics achievement and, if so, estimate the effect size.

**Multiple regression analysis.** I conducted an exploratory multiple regression analysis based on the correlational analysis. The magnitude and statistical significance of the correlations suggested that the RD accuracy score, a measure of orthographic processing of primary interest, might be a statistical predictor of math achievement, the measure of primary interest to the school. Because the OAA tests differ substantively across grades, I conducted two regression
analyses, one for each grade.

I used the step() function in the \texttt{stats} package in R to conduct a stepwise model selection. The function does not allow for missing data. Therefore, I began with the listwise deletion of participants with missing data on the MPES or OAA score. From the set of 80 seventh graders, six students were missing MPES scores and two students were missing OAA6 scores. After removing those 8 students, 72 participants remained in the sample used for the regression analysis for the 7\textsuperscript{th} graders. From the set of 66 eight graders, three students were missing MPES scores and seven students were missing OAA7 scores. After removing those 10 students, 56 participants remained in the sample used for the regression analysis for the 8\textsuperscript{th} graders.

Table 4.11 displays the results of the multiple regression analysis. For the seventh graders, the adjusted $R^2$ indicated that 33\% of the variance in the 6\textsuperscript{th} grade OAA scores could be explained by the RD accuracy scores, the MAS scores, and the MPES scores. The coefficients suggest that every one point increase in RD accuracy score is associated with an increase of more than two points in the math achievement while controlling for math anxiety and math print exposure. For the eighth graders, the adjusted $R^2$ indicated that 50\% of the variance in the 7\textsuperscript{th} grade OAA score could be explained by the RD accuracy scores and MAS scores. The coefficients suggest that every one point increase in RD accuracy score is associated with an increase of more than three points in the math achievement while controlling for math anxiety.

These results corroborate the correlational analysis in the sense that they suggest the relationships between measures relevant to SMaLL may change over time. In particular, the differences in the models identified by the multiple regression analyses suggest that math anxiety
and orthographic processing may play a larger role in mathematics achievement over time.

However, it is important to note that mathematically modeling (i.e., statistically predicting) the data, even with a statistically significant model fit, does not imply a cause-effect relationship in the context of a non-experimental study such as this one. Rather, these models establish a baseline effect size and raise questions about the possibility of mediating variables and cause-effect relationships.

Table 4.11

*Multiple regression analysis on OAA by grade.*

<table>
<thead>
<tr>
<th>Variable</th>
<th>B</th>
<th>SE(B)</th>
<th>t</th>
<th>Sig. (p)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>7th Grade</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Intercept)</td>
<td>403.83</td>
<td>44.34</td>
<td>9.109</td>
<td>&lt; 0.01</td>
</tr>
<tr>
<td>MAS</td>
<td>-1.47</td>
<td>0.56</td>
<td>-2.61</td>
<td>0.01</td>
</tr>
<tr>
<td>RD</td>
<td>2.67</td>
<td>1.22</td>
<td>2.19</td>
<td>0.03</td>
</tr>
<tr>
<td>MPES</td>
<td>0.95</td>
<td>0.46</td>
<td>2.06</td>
<td>0.04</td>
</tr>
<tr>
<td>$F(3,68) = 12.88, p &lt; 0.01$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adj $R^2 = .33$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>8th Grade</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Intercept)</td>
<td>410.27</td>
<td>43.32</td>
<td>9.47</td>
<td>&lt; 0.01</td>
</tr>
<tr>
<td>MAS</td>
<td>-2.08</td>
<td>.43</td>
<td>-4.87</td>
<td>&lt; 0.01</td>
</tr>
<tr>
<td>RD</td>
<td>3.78</td>
<td>1.37</td>
<td>2.75</td>
<td>&lt; 0.01</td>
</tr>
<tr>
<td>$F(2,53) = 28.79, p &lt; 0.01$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adj $R^2 = .50$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: $B =$ unstandardized regression coefficient; $SE(B) =$ standard error of the coefficient
Qualitative Results

The qualitative strand of the research design was intended to generate results with implications for understanding the similarity between the metacognitive processes of SMaLL and the metacognitive processes of English language literacy. In particular, the purpose was to understand what kinds of texts students characterize as readable and unreadable and what reading strategies students apply to texts with symbolic mathematics. Given the multilevel concurrent mixed methods research design, an overarching goal of the qualitative strand was to gain insight into interactions between classroom culture, reading behaviors, and cognitive abilities from the perspective of the student.

The qualitative results were dependent on how the semi-structured interviews unfolded. Each interview was uniquely determined by the participant’s choice of text selections from among the ten options. Because the participants used different reasoning to choose different text selections, the interviews varied substantially despite the fact that each interview was guided by the same essential questions. Therefore, I begin this section with an overview of the text selections that ultimately served as the platform for the qualitative analysis. Then, I discuss the themes related to criteria participants used to select those text selections as readable or unreadable. Next, I describe the procedural knowledge participants drew on to read and translate the symbolic mathematics in those text selections. Finally, I describe the themes related to the purpose of reading mathematical text.

Readable and unreadable text selections. I identified the readable and unreadable text selections based on manifest content, or explicit responses, to these primary interview questions: Which reading selection looks the most readable or the easiest to read? Which reading selection
looks the next most readable or the next most easy to read? Which reading selection looks the most unreadable or something that you would not want to read? Participants designated two of the text selections as readable in order: most readable and next most readable. Then a single selection was designated as not readable.

The text selection options are shown as they were presented to participants in Appendix J. Figure 4.19 summarizes the text selections in terms of the participants’ choices. When selected, selections 1, 2 and 10 were always identified as readable. Selection 2 (volume of a cylinder), which was identified as readable by 12 of the 18 participants, describe the relationship between two common formulas: the area of a circle and the volume of a cylinder. Selection 1 (surface area of a rectangular prism), which was selected as readable by 7 participants, was the shortest option in terms of total number of characters. In contrast, selection 10 (ratios and proportions) was a relatively long option, but the symbolic mathematics elements were brief and less complex compared to other options. When selected, selection 5 (definition of absolute value) was always identified as unreadable. Selection 5 provided a formal stepwise definition of the absolute value, a common topic in the middle grades, written in symbolic mathematics. The remaining selections were sometimes identified as readable and sometimes identified as unreadable. The conflicting designations highlight the need for qualitative analysis to understand what kinds of texts with symbolic mathematics students identify as readable and unreadable.

On a number of occasions, I prompted students to discuss portions of additional selections for the purpose of comparing and contrasting or discussing particular symbols. Those discussions, which allowed me to explore and clarify meaning during the interview, contributed substantially to the analysis. The frequency with which other selections were discussed for any
reason (shown in Figure 4.20) reflects the evolution of the interviews. In other words, as the interviews progressed, I attempted to engage participants in conversation about selections that appeared to be controversial or unreadable. The approach I used to sampling qualitative participants and collecting qualitative data was not intended to generate reliable, in the quantitative tradition, data about which selections were readable or unreadable. The qualitative analysis was intended to generate trustworthy data with the potential to illuminate the complexity of what adolescents view as readable and unreadable. Discussing additional text selections that emerged as readable and unreadable across students was critical to the process.

Figure 4.19. Frequency with which text selections were identified as readable and not readable.
Readability criteria. Four criteria for readability emerged during the analysis to answer this research question: What kinds of texts with symbolic mathematics do students identify as readable (or unreadable)? In making a readability choice without the benefit of pre-reading each selection in its entirety, the students used various criteria in different ways, giving them different weight in their decision, and using them to draw different conclusions about readability. Figure 4.21 illustrates the criteria themes (central gray boxes), the subthemes that define the criteria (red boxes), and overlapping themes (ovals) with the potential to influence how the criteria were employed. Appendix K shows additional detail about the meaning of the codes.
The remainder of this section is divided into four sections, one for each readability criteria: print features, familiarity, translation, math self-efficacy. I begin each section by clarifying the qualitative coding process that emerged. Then I describe, using the students’ voice, how those criteria were used to choose readable and not readable selections for discussion.

*Figure 4.21. Readability criteria concept map*

**Print features.** For this sample, print features emerged as central to determining readability. Only one student, Freddie, explained his readable selections without a clear reference to any print feature, and only one student, Amelia, explained her unreadable selection without a clear reference to any print feature. Print features were discussed in terms of the
appearance of symbolic mathematics, English language words, and structure features. During coding, references to equations, formulas, numbers, symbols or syntactic complexity of the written language of mathematics were coded as *symbolic mathematics print features*. References to words, paragraphs, and vocabulary were coded as *English language print features*. References to fonts, explanations, examples, offset text, or embedded text were coded as *structural print features*. Participants also compared the *length* of, or amount of, symbolic mathematics, English words, or structuring within the selection or across selections to explain readability.

For some students, the symbolic mathematics print features contributed to a determination of either readable or unreadable. Robin and Quincy gave some insight into how the symbolic mathematics print features of selection 2 (volume of a cylinder) made it readable. Robin chose the selection as most readable by comparing the symbolic mathematics to the English, “I like seeing both symbols and words. I kind of get overwhelmed with too many symbols because I'm trying to think… what type of formula is this?” Quincy also suggested that there is some point at which too many symbols makes text unreadable: “I recognize the formula and it looked really simple. It didn't look like a huge string of equations.”

Other students explained how symbolic mathematics made text selections unreadable. In explaining why selection 3 (distant between two points) was unreadable, Freddie and Katrina identified some symbols that are similar to the point of being confusable. The square root symbol, \( \sqrt{ } \), looks very much like the graphic organizer used for long division, \( \frac{\text{ }}{\text{}} \). Freddie was aware of the need to distinguish between the two, but found it difficult to do so. He said, “Well, I get that that's a square root, not a division sign, but the fact that it's division and
multiplication and exponents… and only variables inside… square roots is pretty complicated.”

It is not clear whether Katrina was aware of the difference as she explained how the complexity of the symbolic mathematics made the text unreadable: “That line — I don't know what it's called — like the division line thing. And then the parentheses in it and just the letter down there, the bottom letters and then the numbers and then - it's just the numbers and the letters inside of that.”

English language word print features also weighed heavily in students’ assessment of whether a selection was either readable or unreadable. For example, Jackson chose selection 9 (distance equals rate times time) as the most readable because, “it also has a long paragraph section, so which I know I could read that.” However, extended English language features did not contribute to readability for all students. When Loren was looking for a readable selection, she chose selection 6 (absolute value example) “because it had the least words.” Nicki seemed to identify with Loren as she said, “When I'm reading math, I don't think that I'd have to read words first, just numbers and variables and stuff.” Interestingly, Martin identified selection 9 (distance equals rate times time) as unreadable for the same reason Jackson found it readable. Martin did not think the text was readable because of “how many words it has and… also it sort of has some vocabulary.”

Several students pointed out that the way the symbolic mathematics and English language print features are integrated in the overall text is a factor in readability. For Peter, the “structure” of selection 7 (inequality explanation) made it more readable. He said,

So I noticed that it explained each rule or whatever and then gave an example. And I thought that that added, I guess, a better structure to the form…And the structure made it
more readable. It splits after the introduction part; it splits it up into the rules. And then it gives an example that's bolded.

Although Peter thought the bold added to readability, Loren identified the bold as a specific factor in her decision to declare that same text selection unreadable. Neither Katrina nor Nicki found the structure beneficial as they were noticing that the symbolic mathematics was embedded within English sentences. Katrina said, “It has a lot of words in it and it's just really long and… signs!” She clarified, “I don't really like when [math symbols] are in the text and you have to search for them.” Nicki’s comment gave further insight into why what seemed like a well-stated and highly structured explanation to Peter was the most unreadable text selection for others. She explained her thinking,

I think it's the math is in between the words because it doesn't seem - I don't know if connected is the right word - but it seems out of place where math would be in between words. The explanation is interrupted. It just doesn't seem right where math is in between words.

**Familiarity.** Familiarity, or lack thereof, was also used as grounds to explain why selections were identified as readable or unreadable. Thirteen students described the degree of familiarity as a rationale for readability. In contrast, only three of those student described the degree of familiarity as a rationale for unreadability. References to recent, frequent, or repeated acquaintance with the topic and indications that a selection was so familiar that some elements were easily recognized or already memorized were coded as familiar. Comments indicating that similar text had never been “seen” or “covered” or had not been used recently, frequently, or in a long time were coded as unfamiliar. In contrast to the print features code, familiarity codes were
used for such statements made in the absence of or in addition to an explicit references to print features as described in the previous section. Curiosity emerged as a factor related to interestingly (un)familiar text.

Familiar text were often designated as readable. Some students described the nature of familiarity in concrete terms. There were three common rationales for identifying a text selection as familiar: It was similar to a recent or current lesson. It was related to a recent or upcoming assessment (i.e., homework, quiz, or test). Or, it was related to common mathematical practice in their current math class or their past learning experiences. For example, Robin chose selection 4 (point-slope form of a line) as most readable saying, “We're kind of redoing or reforming this unit, I guess. So I'm really used to seeing these right now.”

Selection 2 (volume of a cylinder) may have been the most readable across the sample because it was familiar in multiple ways. Nicki said selection 2 (volume of a cylinder) was the most readable because, “That's kind of the most familiar to me… I learned it in sixth grade and I've been using it a lot.” India explained, “I feel like it looked really familiar because we were using it almost every day in math class.” Amelia noted, “We have a 3D geometry quiz coming up.” And, Delaney said, “Well, it's something like we're reviewing for the end of the year final in my class.”

Notably, familiarity was not sufficient for readability. For example, Delaney chose a selection with a familiar topic as unreadable. She said,

Well, number four is about slope and lines and point-slope, and we did this… I think I did pretty well but I think the unit went kind of fast to me. So it was like I didn't get to learn it all so it will stay with me so to speak.
For her, the familiarity that comes from completing a unit on the topic some time during the academic year was not the same as being able to read the text.

Conversely, lack of familiarity did not necessarily make text unreadable. For instance, Owen chose selection 9 (distance equals rate times time), the text selection describing a uniform rate problem scenario, as the most readable. When asked if he could describe a time in class that he read something similar he paused and then said, “I haven't really seen this very much.”

Likewise, India indicated that, although she identified selection 1 (surface area of a prism) as readable, “this was a really long time ago…and I remember it barely.”

In explaining why she chose selection 7 (inequality explanation) as unreadable, Amelia gave a plausible explanation for why limited familiarity has the potential to make text unreadable. She noted,

Well, the thing is is that we haven't seen a lot of this. It's either new information or it's information that we learned a very long time ago and have to really think a lot harder just to understand where it's coming from, what it's doing.

In short, she hypothesized that lack of familiarity made reading taxing.

Some students elaborated on familiarity in a way that expressed a sort of curiosity about a text selection. In each case, curiosity lead to the identification of the selection as readable. For Amelia, curiosity emerged where familiarity and unfamiliarity created a conflict. She was intrigued by selection 3 (distance between two points), which she identified as the most readable. She explained that she was familiar with the topic, named using a proper noun in the English language portion of the text selection, and was surprised to see an unfamiliar equation in the symbolic mathematics text. She said, “Well, I was reading through it, and it said Pythagorean
Theorem, but it had that formula, and I was confused on why that was tagged with that.”  She had expected to see the familiar form of Pythagorean Theorem, \( a^2 + b^2 = c^2 \). For Claire, curiosity seemed to be more associated with a lack of familiarity. She was curious about selection 3 as well. She indicated that she was not familiar with the topic or the formula, and chose the selection as readable because “the formula just pops out at me.” In contrast, for Robin, curiosity seemed to be more associated with familiarity. After choosing selection 4 (point-slope form of a line), Robin explained why she thought it was readable, “It's really weird. But I like slope — slope intercept form – point-slope. So I like the idea of slope. So it's something in math that I like so I attract myself to it.”

**Translation.** About half of the students indicated that they considered whether they could translate all of the text into words. References to translation issues appeared, in roughly equal proportion, in explanations of both readable and not readable selections. Statements indicating that the student knew ‘how to say’ what was written or what the symbols ‘stood for’ were coded as *know words*. Statements indicating the student did not know ‘how to say’ what was written or what the symbols ‘stood for’ were coded as *no words*. Students’ comments about whether they knew words for translation or had no words for the translation were associated with the *ambiguity* of the language of mathematics. The role of ambiguity, which emerged as a combination of knowing and not knowing what to say, is described in more detail in the Reading the language of mathematics section.

Knowing words for a translation was always associated with identifying a selection as readable. For example, Bobby thought selection 1 (surface area of a prism) was readable because “I already guessed out all the words even though I didn't know the full words and I
could read everything okay.” Katrina also thought about whether she had words before identifying selection 1 as readable. She shared her thinking, “When I first saw it, I saw the L and the W, which I knew what that meant… And I was like ‘oh’ I know what that was… I just looked for the things I knew.” Freddie, the only student to identify selection 8 (product rule for exponents) as readable, did so “because I know how to say exponents most of the time.”

Knowing words may offer additional insight into why selection 2 (volume of a cylinder) was most often selected as readable and never selected as unreadable. Delaney said, “I was looking at the equations… I know what the symbols mean.” Jackson demonstrated that he was thinking about whether he had the words by explaining his thinking as he made his selection: “I saw pi and radius squared in here. And I think this is area and that's volume.” These students identified text selections as readable because they could associate meaningful words with the variables in the symbolic mathematics.

Statements indicating the student had no words, or were unable to identify reasonable words, to generate a translation were most often associated with selections identified as not readable. However, some students found that they had no words in the process of trying to read a selection they had previously identified as readable. For example, Bobby identified selection 10 (ratios and proportions) as readable on the grounds that it included familiar symbolic mathematics. However, he was unable to give a translation for the text written as $x : y$. He explained that the variables caught him by surprise, “I know how to put it into numbers but not really into variables. I just didn't really know how to say it.”

Other students were aware they had no words during the selection process. Freddie identified selection 3 (distance between two points) as unreadable saying, “It would be almost as
hard to say as it would be to translate.” In choosing selection 8 (product rule for exponents) as unreadable, Elena illustrated that the language of mathematics can be unreadable even in the absence of variables. Looking at the numeric example of simplifying exponents, she explained, “I don't know how I would read these [exponents]… I could just say four squared times four cubed. But I don’t… four to the power of five, I guess.”

When students identified a selection as unreadable it sometimes meant the student was unable to read any of the text selection and sometimes meant the students was not able to read the text selection in its entirety. For example, Harper suggested she might be able to get some information from a selection she identified as unreadable. She said, “I think I can read this part maybe, but… I wouldn't be confident with my answer if I read it out loud.” However, Gil was overwhelmed by selection 4 (point-slope form of a line), which he selected as most unreadable. His comments expressed the jumble of thoughts that occurred to him as he decided the text was unreadable:

It's that when they just give you a slope of a line and… just give you all these equations. And first of all, you don't really know what X and Y is! And it just give you all this in the first point, and then they just gave you all these things here. Because first of all, how do you even say this?!

**Math self-efficacy.** About half of the students indicated that they took their math abilities into consideration in the process of identifying text selections as readable or not readable. References to math self-efficacy appeared in connection to explanations for choosing both readable and not readable selections, but were more often mentioned in an explanation of choosing a readable selection. Notably, none of the reading selections included any math
problems. In fact, the participants were told during the assent process and again before the interview process that the data collection was designed to ensure that there were no math problems to do or solve. Thus, students who discussed their ability to do math related to the reading selections had an imagined problem in mind that, in most cases, was not articulated during the interview. Statements indicating that the student felt able to do or solve a relevant math problem or was satisfied that he/she had the ability to understand the mathematical content of the reading selection were coded as positive self-efficacy. Statements that conveyed a lack of ability or confusion to the opposite extreme were coded as poor self-efficacy.

Only one student, Elena, discussed her math self-efficacy to explain both a readable and not readable selection. She identified selection 10 (ratios and proportions) as the most readable “because it looks like something that we've studied and it looks like something that I understand.” Interestingly, when Elena read the selection she skipped over three symbolic mathematics elements of the selection: \( x : y \) and \( \frac{x}{y} \) and \( \frac{20}{1} = \frac{40}{2} \). She read, “A ratio can be written in three different ways. X to Y, X to - yeah, I guess. A simple proportional relationship, on the other hand…” She also brought up self-efficacy again to explain why she chose selection 8 (product rule for exponents) as unreadable. She said, “I probably wouldn't have chosen this one as unreadable if it didn't have all the little squared and cubes and stuffs… I’ve never really done super well with exponents.”

Some of the students explained how positive self-efficacy contributed to identifying a selection as readable. For example, Owen said selection 2 (volume of a cylinder) was readable because “I have those formulas memorized in my mind, and I can do them easily.” Other
students who mentioned poor self-efficacy did so in identifying a selection as not readable. For instance, Jackson noticed the absolute value in selection 5 (definition of absolute value) and said it was unreadable because “I somewhat understand but it’s sometimes confusing to me.”

**Reading the language of mathematics.** I asked students to read their readable selections aloud and discuss their reading process to answer this research question: How do students read texts with symbolic mathematics? In some cases, I asked students to read all or some of their unreadable selection aloud. The analyses indicated that reading, particularly reading aloud, entails *perception* strategies and *production* strategies. Perception strategies are ways of perceiving, or receiving, the text. Production strategies are ways of producing, or expressing, the meaning of the text. In other words, reading requires coordinating the visual examination of the written text with the formation of thoughts or speech in the English language to represent the text. Ultimately, navigating perception and production entails resolving ambiguity or deciding among various possible English language translations to represent the written text.

*Figure 4.22* shows a concept map of the reading strategies for the language of mathematics. The red diamond at the center illustrates reading strategies as a process of navigating the ambiguity of the text. The adjoining gray boxes show the two processes that contribute to reading: perception and production. As illustrated by the white rectangles, perception may happen automatically, entail intentional effort, or require assistance, and production may entail compressing the text or expanding the text.

The remainder of this section is divided into two sections to explain the concept map more fully. First, I focus on perception strategies, describing what students do to take in what
they see written on the page and begin to make sense of the ink marks. Then I focus on production strategies, describing how students weigh the demands of comprehension and communication to arrive at a reasonable facsimile or representation of the text in English. Appendix L shows more detail about the meaning of the codes.

**Figure 4.22.** Concept map of reading strategies for the language of mathematics

**Perception strategies.** For this sample, three discernible strategies for interacting with text written in the language of mathematics emerged. The coding for the perception strategies were based on a combination of students’ generalized summaries of their process and inferences based on participants’ description of a particular instance of reading. Statements describing reading as effortless cognition, in some cases explicitly attributed to the brain, were coded as
自动阅读。有时阅读中学数学对青少年来说是无压力的。在描述她阅读符号数学的方法时，安梅莉亚报告说，“它在这个阶段看起来很自然。”她没有解释它是如何变得“自然”的，但她确信她的大脑负责阅读任务，“[符号]看起来很奇怪，但在你的大脑中，它们会触发某些东西。”昆西也给出了类似的解释，“我看到符号，然后如果我大声朗读，我看到符号，我的头脑将其翻译成英语。但这是更非意识的过程，我就是这么做的…就像呼吸一样。”

德兰尼和卡特里娜指出，自动性可能来自于对公式的持续努力，将其牢记于心。德兰尼说，“我猜[我]认为最好记住[公式]，因为随着我们的发展…你必须知道它在你的脑海中。”与德兰尼不同，她指出她将公式作为文本块记住，卡特里娜，她参加了数学辅导，认为她努力在字符级别上发展自动性：“我只是记住π和半径的平方看起来是什么样子。”

对于一些学生来说，自动性似乎有机地发展。例如，印度认为，随着多次阅读机会，符号与词语的配对会逐渐发生。她说，“我认为开始时很重要的是配对符号和词语。”
translate math symbols into words but over time… you just look at a math symbol, just automatically know what it is.” Even for students who do not like math and worry about their self-efficacy, reading common symbolic mathematics text can, apparently, become automated through repeated experiences. For example, Elena said the way she read \( A = \pi r^2 \) and \( V = \pi r^2 h \) was “just like instinct.” She surmised that it was “because I've heard it so many times like that.”

Automated reading was not clearly associated with comprehension or lack of comprehension. Peter, who read fluently, explained how automated reading and comprehension were connected for him:

So since there are a lot of equations that I've had to remember in the past year taking physical science and taking algebra, … when I read equations, I'm thinking of the variables but I'm also subconsciously thinking about what each of the variables sort of means.

However, some students who indicated that reading worked automatically did not appear to be aware of their lack of comprehension. For example, Harper said, “I guess it just comes to me naturally,” to summarize how she read symbolic mathematics. However, earlier in the interview Harper was unable to read \( d = rt \) correctly. She read hesitantly, “D equals \textit{radius}, and I'm not sure what the T stands for - maybe time - where D stands for distance, R stands for constant or average rate of speed, and T stands for time” (italics added for emphasis on the error). She continued on without recognizing her error in reading \( r \) as “radius.”

\textit{Intentional reading}. During the interviews I used my tutoring and teaching expertise to lead each participant to a selection that required intentional effort to read. The purpose was to
create a situation in which participants could experience and then describe effortful reading. They shared strategies that fell into four subthemes: rough reading and re-reading, decoding symbolic mathematics language, making use of English language, and making meaning. The strategies were sometimes used in tandem. The choice of strategy appeared to depend on whether the student presumed the purpose of reading was to describe the text as it was written, gain lexical access to vocabulary, comprehend the text, or solve an imagined problem related to the text.

Some students used a rough read and re-read strategy. Students using this strategy began reading and continued, skipping or guessing, to the end. Some students reported re-reading to update and improve the translation and comprehension. Amelia explained it this way, “For some stuff, you don't know what it is and you figure it out as you're moving on.” Gil pointed out why he re-reads: “You just kind of have to stop and re-read it and just think through, try to get all what's going on.” Delaney described this process as a variation of “guess and checking.” Freddie explained that he uses this strategy when he is at a loss for vocabulary: “I had no idea what else to call it.” Amelia saw this as a way of tapping into her automatic reading “because it starts triggering stuff.”

Some students used a decoding symbolic mathematics language strategy in which they intentionally focused on each symbol and the arrangement of symbols. For Peter, this process was a variation of context clues. He said he would “use the context of the equations or whatever to figure out what things mean.” Quincy explained it more like using morphology and parts of words. He said, “It was almost like breaking down a word into word groups. I just broke it down.” For Quincy, this strategy entailed thinking about the order of reading. He used it to read
as “the absolute value of negative three with a negative value.” Reading it from the inside out, he said, made it harder to follow along. In fact, it was harder for me to follow along as I kept seeing it.

So I feel like [reading left to right] would be better because it's the same way. It's the same direction and process of reading a normal text.

For some students this strategy is not so much about trying to make sense of novel text as it is trying to produce a correct translation. For example, Bobby said he preferred to read the text “how it's printed out.” He explained,

I think my teacher wants me to read it like that instead of how I want to read it. And how I might think it is, maybe it's wrong. And so I just read it how it is so I don't want to take a chance on being wrong.

Students made use of English language text to support reading in different ways. Some students questioned whether reading the English language text was relevant. For example, Nicki paused and asked, “Oh, wait - do you want me to read the whole thing or just [this]?” I took those opportunities to ask the student what their teacher might expect and what they might do during independent reading. Nicki explained why she skipped the English language elements. She said, I was “supposed to be reading math and that was [the part] that first popped out to me because, first of all, it's an equation.” Robin explained that she would skip the English language elements “unless they were directions.” Freddie explained that he did not like to read “huge walls of text.” He liked to skim the English language elements looking for underlined things to determine if those elements were “optional.” Harper and India indicated that they would use the English language text as a resource as needed for clues about how to read the symbolic
mathematics elements of the text. India explained, “When I first look at a formula, I don't really understand it, so I will go back to English, and I will read again.” Quincy explained why he liked to use English language text as a resource: “If I don't understand something, it's about a hundred times easier to understand it if it's written in words below it.” Faced with a chunk of symbolic mathematics he did not know how to read, he indicated that he wished the English language text was a better resource. He said, “There's only one thing in there I don't understand. And if someone had written the entire equation in English, I could find that spot in English and translate it and figure out what it means.”

Finally, some students tried to make meaning of the text as a means to reading it. This strategy included techniques such as visualization, categorization, and writing. However, it most often entailed drawing on mathematical knowledge and attempting to solve an imagined problem associated with the text. Amelia described how she used this process: I “go through what [I] know about math and try and piece it in through there.” Although this strategy was useful in many cases, the data suggested that applying limited mathematical expertise to reading novel symbolic mathematics may not always be a productive strategy. For example, Martin could not determine how to read selection 6 (absolute value example). He noticed the novelty of the absolute value bars in $[-3]$ and said, “It’s looking like it's just to separate a negative from a negative… The double negative equals a positive.” He called this approach “common sense” but, ultimately, decided he could not read the selection because the solution shown in the text $[-3]$ did not match his expectations. Owen used a similar approach to try to read selection 5 (definition of absolute value). He said, “I know an absolute value is always equal to a positive number. Negative three has an absolute value of three. But I've never seen a negative absolute
value.” Attempting to read a portion of the text, \( x < 0 \), caused him to question his understanding that the absolute value function always produces a positive result.

**Assisted reading.** In the course of discussing text selections that students were unable to read, I asked them: What do you do when you need to read something like this? The data indicates that they turn to one of two resources: people or media. Most students reported they would ask a person for help. Students who were in class would ask a classmate or a teacher. Students at home would ask a parent or a sibling. Although some students preferred to seek help from the teacher, some students indicated that classroom etiquette dictated asking a classmate first. Quincy explained, “Usually it's better to ask someone else before you ask the teacher so that people aren't swarming the teacher.” However, some students suggested that their classmates were not likely to have the required reading expertise. For example, Harper, who told me she did not like to raise her hand in class, was willing to ask her friend but said, “She'd probably say ‘I don't know’ or something like that.” Although some students would ask a parent, their attitude towards using that strategy varied. Robin was confident in her parents, “because obviously in our family, math is our strong subject… So I'd first go to parents.” Amelia, however, was less confident that her father would give her the help she requested. She said, “He's a little too pushy. [He would] look through and see which ones are wrong and then hand it back to me.”

For some students media resources such as notes, binders, textbooks, online search engines and websites, and Khan Academy were considered resources. Several students indicated that they might use their classroom notes. For example, trying to think about what he did when he had difficulty reading, Jackson said, “I kind of looked at my notes somewhat if I got notes,
and I try to compare that to the formula.” Katrina used a similar process, “I either try to look it up or something or try to go in my binder and look for stuff that looks like that to see what it is.” Martin said, “I usually take notes on stuff,” suggesting, like Jackson, that he was aware he may not find what he needed in his class notes. Owen and Robin went to greater lengths to find resources. Owen explained that he sometimes types math equations into a search engine to find online help and practice problems. Robin explained avoided textbooks as a reference: “I would use a person and then later if I still needed - if I still didn't understand it, I'd use videos on YouTube.” Ironically, trying to use media as a resource for reading symbolic mathematics may entail some SMaLL skills.

**Production strategies.** During each interview, I used probing questions to understand how students decided which words to say during reading. This had the effect of generating discussion about how the student made choices when the text was ambiguous. Participants varied in terms of whether they tended to compress the text giving a minimalist translation or expand the text by translating lettered variables to words or inserting words that did not correspond to a written character. For example, given the equation, \( V = \pi r^2 h \), one student might read, \( V \) equals \( \pi \) \( R \) squared \( H \), while another might read, volume equals \( \pi \) times \( R \) squared times the height. The former was spontaneously described as “compressed” by several students including Martin, Owen, and Quincy. I adopted their term *compressed* to refer to translations like the former, and I used the term *expanded* to refer to translations like the latter.

Analysis of participants’ use of and preference for compressed and expanded translations suggested there are a number of considerations for using varying degrees of compression and expansion in the production process of reading. Some students considered their own learning or
others’ learning suggesting they engaged in self-teaching and had audience awareness. Some
students considered visual patterns or syntax. Others preferred particular phonological patterns.
Some students considered the ease of articulation. And, others imagined how it would
coordinate with their writing. Although rationale subthemes were discernible, the variations in
semi-structured interviews did not support conclusions about their prevalence in relationship to
participants or particular reading selections. In many interviews the subthemes emerged as a
consequence of the summary questions at the end of the interview.

*Expanded translations.* Although students differed in their particular reasons for
expanding the translation, they generally agreed that the primary purpose was related to adding
clarity and meaning. Sometimes clarity was intended to describe the written text. Amelia, who
read fluently with comprehension, did not have a habit of expanding text by pairing written
parentheses with words. However, when asked if she would read a selection 3 (distance between
two points) differently in class she said, “I might add the parentheses, it just makes sense in my
head just to kind of leave them off, even though I know they're there.” In other cases, the intent
was to describe the mathematical meaning more clearly for others who might have less
knowledge. Elena explained how she thinks about others, “If I was teaching it to somebody, I
would say ‘area equals pi times the radius squared’ so they'd know what R means.” Some
students add meaning as a form of self-teaching. For instance, Peter said, “The reason I prefer to
read it the [expanded] way is because it sort of reminds me of what all the variables mean.”
Some students preferred expanded translations despite being unable to produce them. For
example, after Loren read $d = rt$ as “D equals R T,” I read the text as “distance equals rate
times time” for comparison. She explained why she thought my reading was better, “I actually
know what it is. When I say D equals RT, I don't really know what I'm doing.”

Sometimes expanding text did not relate to adding clarity or meaning. Nicki explained that she expanded $h$ to “height” when she read $V = \pi r^2 h$ because of the visual “flow” of the formula. “Visually, it felt weird… I think it's because of the exponent. So I felt like it didn't really flow.” She gave a counterexample to explain what she meant by flow: “If the exponent — if the squared part of R wasn't there — I probably would have said V equals pi RH.”

**Compressed translations.** Although students differed in particular reasons for compressing the translation, they generally agreed that the overall purpose was related to efficiency. Amelia explained her hypothesis, “Once they get used to it, they start working really fast. If they have to think through the entire words. Instead of - you can just shorten them up and keep going.” For some, moving to compressed translations required a period of repetition and memorization. For example, Owen said

I sometimes use the words… When I am more like learning the equations and the variables, that's when I'd be more likely to use the words…But once I know what it is and I can just memorize, I can do it easily. I just do it like the shortened version.

Compressing text did not necessarily relate to efficiency. On the contrary, compressing text may be an indication of reading difficulty. Martin hesitated when he read $d = rt$ from selection 9 (distance equals rate times time). When I asked him about it he explained, “I was sort of deciding if I should use the word or the variable or just the letter. I used the letter.” When pressed to explain how he made the decision, he said, “I don't really know - I didn't know what they stand for until I read this part.” We discussed the location of the context clues that explained the meaning of the variables. He said he preferred to expand. Asked to re-read the
symbolic mathematics to illustrate, he expanded beyond the common reading: “distance equals rate of speed times time” (emphasis added to highlight the expansion).

**Reading in the classroom.** The data yielded insights into the culture of reading in middle school mathematics classrooms. Reading is a “have-to” in math classrooms. Teachers are the expert readers in the classroom. And, finally, reading for math in the language of mathematics is, for most students, different from reading English. In this section, I use the students’ voices to describe why they read, how they view their teachers as readers, and how they see “reading math” as a unique skill.

**Reading is a have-to.** The text selections used in the semi-structured interviews included typical academic text that a middle school student might reasonably read-to-learn. All of the selections were adaptations from publicly available online resources offering support for mathematics learners. All of the selections stopped short of posing questions or presenting problems to be solved. Despite taking care to ensure that participants would not feel pressed to solve any math problems, students often spoke about reading in relationship to an imagined problem. To use Jackson’s term, reading is not assigned as a learning activity, but reading symbolic mathematics is a “have to” in the service of solving problems.

The data suggested that students rarely, if at all, read-to-learn. Nicki explained that she prefers to avoid reading-to-learn: “I don't like reading and learning something, especially if it's math or science. I like it when someone teaches me because it just makes more sense. So I don't particularly like reading explanations.” Reading in the classroom primarily involves reading problems and solutions. Quincy explained how reading as a whole class worked. “If the question is asked… the whole class gets the benefit of hearing the question and the answer,” he
said. “And I feel like those are probably the benefits of reading [math] with the whole class. So that people can benefit from everyone else's thoughts on it.” I asked Jackson about how independent reading worked. I wanted to know whether he decided when to read or his teacher told him when to read. He replied, “I'm assigned to read. Actually, no. [The teacher] gives a paper so we kind of have to read it if we want to complete the problem.” Peter’s remarks confirmed that independent reading is necessary for assessments. After he described several strategies for reading difficult text, I asked him if he ever used any resources. He responded, “If we're allowed to, yeah. But I'm sort of thinking of [reading] as in the context of tests.”

The reported amount of time dedicated to reading activities differed. In Quincy’s experience reading was a typical activity in his class. He said, “I remember reading things independently and as a whole class. Both we do a lot.” Nicki explained that, as she has advanced, she seemed to be doing less reading in her math classes:

So last year and the year before that, we would read the whole thing and sometimes we'd read it independently if it was a homework assignment. So then you would first read all of it, including the explanations behind whatever you're learning.

Robin, an advanced student, suggested there was little reason to read in her class. She said, “There's never a time when we have to read… We'll just know. If she asks us for it, then we can just say it. We don't need to look down on a piece of paper and read it.” When they do read in her class, Robin said it included things “like what questions we’re on, like how to find the answer, what are the steps that you need to take.”

**Reading like a teacher.** The data suggested that teachers are the best readers in the classroom. Specifically, students viewed teachers as expert readers because they can read,
interpret, and explain the language of mathematics. Gil indicated that the way his teacher used reading in her teaching helped him learn through listening: “She will write this on the board, and she will say all the words and slowly so that all the words all sink in.” Amelia also suggested that noticing how her teacher reads helps her learn. Referring to selection 4 (point-slope form of a line), she explained, “For something like this — the slope — I know that because she read it out loud. My brain connected that with M, and I wrote it down because that's how I remember things, by repetition.” Claire especially liked the way her teacher read the longer texts with her class. Claire explained that she usually only needed to read “just two or three lines” by herself, but their weekly worksheet included some “really long problems that have a lot of information.” For those, the teacher helped with reading. I asked Claire how she felt about it when the teacher acted as the reader. She replied:

Helpful. Because sometimes she reads it in ways that I can understand. And when I go to read it by myself sometimes I don't understand it fully. And then when she reads it, she'll read a part and then stop to explain what it means and then she'll continue on.

**Reading math vs. reading English.** I used the interviews to explore the differences and similarities between SMaLL and English language literacy from the students’ perspective. The students had unique ideas about how “reading math” compared to “reading English”. Although most agreed that reading math is unique and somewhat different from reading English, they did not all see the differences in the same way.

Many students, like Amelia and Peter, attributed at least some of the differences to the nature of the writing system. Peter explained what he thinks of as reading math and what he thinks of as reading English this way:
If something either has a number in it or an equation or a greater than, less than sign just anywhere in there, I think it's more math. But then if it's just words formatted into a sentence, I feel like it's English.

Amelia echoed Peter’s description when she said, “Math, [the symbols] all stand for something… When you're reading English, it's all written out for you.” She went onto to explain how reading symbols rather than reading words “all written out” changed the nature of reading. She said,

When you're reading [English], if you don't know what a word means, you can usually figure it out after a while. But when it comes to math and you don't have enough stuff before it, it takes a lot longer to figure it out or you need some help just figuring it out.

Several students indicated an awareness that the language of mathematics does not map onto sound in the same way that English does. Jackson suggested that he relied on the alphabetic principle to read English, “Say like if you don't know a word, you sound it out.” Nicki explained how the mapping between symbol and sound made the symbols in equations seem less connected that letters in a word.

Like when I read English… each letter has a specific sound it makes, and you put them together to make a word, right? But when I read math, it's like equations and variables and… they're symbols that represent something but it's not like a word that flows. It's just like separate components put together.

Bobby noticed that the way whole words mapped onto symbols in the language of mathematics had the effect of hiding the vocabulary. He said, “Well, for the words, I just know how they sound and stuff. So that's easy… But for these [math symbols]… I couldn't see the words or
anything and how to say it.”

Some students described the differences in terms of what it was like to read the language of mathematics as compared to reading English. For example, Claire tried to explain how attentive she had to be when she was reading math:

Whenever I see math symbols, I feel like I have to concentrate more on what I'm reading. I'm not sure why… Normally, whenever I'm reading regular books, I'm just kind of into it. And then whenever I get to math symbols I'm like, ‘Oh! I need to pay attention to this because it's math.’

Harper seemed to agree. She pointed out that the language of mathematics can look deceptively simple but, generally, requires more effort than reading English.

I think at first we glance at it, the math looks probably easier because it's less. But then when we actually look at it and try to read it, it'd be easier to read the English part because you're more familiar with it and you know what to do while you're reading it. But for the math, sometimes you don't know what to do and you're confused about it.

As Freddie put it, sometimes trying to read symbolic mathematics is “like reading a foreign language.”

Several students specifically noted that there is more ambiguity in reading the language of mathematics than in reading English. Loren explained with an example, “unless someone doesn't know how to pronounce the word”, everyone reads English the same way. She continued, “But math has different terms of saying it. Sometimes the teachers will say L times W but sometimes they'll say length times width.” Elena explained in more general terms:

Reading just plain English with the alphabet is like you know what each letter means.
And you know how it's used and punctuation and stuff. You know how all that's used.

But with math symbols, there can be different ways to use different symbols.

Delaney noticed that, because of the ambiguity of the language of mathematics, there is more to reading math than knowing all of the symbols. She said,

I feel like in math even if you know [what a symbol] means,… it might not have the same context in a certain formula or equation, so it's kind of hard to comprehend math sometimes because it's not always the same thing. It's always changing. It's always different.

Not all students agreed that the writing system explained the difference between reading math and reading English. Robin told me, “It doesn't matter really to me if it's symbols or words. If it makes sense in my brain as mathematics, then [it is reading math].” She gave me an example by comparing selection 7 (inequality explanation) and selection 5 (definition of absolute value). The former she thought was reading math, but she was not convinced about the latter because the word magnitude made her think of a topic she did not relate to math. She tried to explain, “[Selection 7] says ‘inequalities’… immediately I think of math, not English because I use inequalities in math… Magnitude, I think of earthquakes because that's what I relate it to more. I don't really relate it to math.”

Not all students agreed that a notable difference existed; on the contrary, they saw similarities. When I asked Owen whether and how reading math and reading English were different he said, “They're not very different. I would just think of it as all just one big thing: reading. And then, the math is doing the problem.” He agreed to read part of selection 7 (inequality explanation) to demonstrate how he could easily switch gears between the two
languages. India discovered she was conflicted about whether reading math and reading English were more different or similar. She initially explained how they were different by saying, for math…

you are reading things with math symbols. You just find the things that you know about first…and then you just - actually, you just finish it… But for English you just read it from the start…. it will make more sense when you go along.

She quickly clarified that her processes were similar but her order for reading could differ:

“That's kind of the same but it's also different because you're thinking [about] the things that you know… But for English, you just read from the start.”

Quincy was aware of the differences in the writing systems. He noted that “With reading English, it spells out every word… With math, I think it compresses more meaning into one symbol. So I think that in math, the meaning is more compressed, while in English it's more dragged out.” However, he argued that processes of reading math and reading English were analogous:

I think they're mostly the same because in English, I'll break down the word to see the word parts. In math, I break down the equation to see what each letter or symbol means. So I think they're very similar, in the way of breaking them down.

Nicki was unique in that she explained the difference between reading math and reading English in terms of how she used them in math class. Early in the interview, Nicki had given me insight into what reading math meant to her. While explaining why selection 7 (inequality explanation) was unreadable she told me, “When I'm reading math, I don't think that I'd have to read words first, just numbers and variables and stuff.” During the summary questions, as Nicki
reflected again on the difference between reading math and reading English, she explained how the two kinds of reading are associated with doing mathematics:

If you're reading a word problem, you have to convert the English or the language into math. But then when you're reading math, you need to convert the symbols and numbers and stuff into what it actually means. And it's like converting into English so something you’re familiar with.

The former she thought of as reading English. The latter she thought of as reading math.

**Multilevel Mixed Methods Integrative Results**

The purpose of the multilevel mixed methods integrative analysis was to merge the data and findings to meet the objective of the research study: to gain novel insights into SMaLL skills among middle school students learning under the CCSS with implications for instructional practice. The integration techniques were guided by the integrative research questions with the aim of answering the mixed methods research questions: What is SMaLL for adolescent students in middle grades learning under the CCSS? How do students reading typical texts with symbolic mathematics experience SMaLL? This section is divided into three parts. First, I report the readability spectrum that emerged from the integrated analysis of responses to quantitative items and qualitative text selections. Then, I report the SMaLL spectrum that emerged from the integrated analysis of participants quantitative and qualitative profiles. Finally, I present the model of SMaLL that emerged as a culmination of these results.

**Readability spectrum.** I began the multilevel mixed methods integration at the cultural level with an analysis of the language of mathematics by examining the text selections. I created a multilevel joint display for each of the ten qualitative text selections. For each text selection, I
integrated the relevant quantitative data: cultural data (response rates for MPES items with similar features), behavioral data (quotes from participants’ discussion and reading samples), and neurobiological data (accuracy rates for S-CDT items with similar features). As a result of the process, I identified relationships between the quantitative data and the readable and unreadable designations (as shown in Figure 4.19) for the text selections. I used those relationships to rank the text selections from most readable to least readable.

In this section, I present the detailed analysis of four of the text selections. First, I discuss selection 2 (volume of a cylinder), which ranked 1st, as the most readable text selection. Then, I discuss two controversial selections: selection 3 (distance between two points) and selection 4 (point-slope form of a line). Finally, I discuss selection 5 (definition of absolute value), which ranked 10th, as the most unreadable text selection. I selected this subset of text selection to report for three reasons. First, they illustrate the spectrum of readability from readable to unreadable. Second, they illustrate the complexity of the relationships in the joint displays. Finally, they highlight the methodological benefits and challenges of the multilevel joint displays.

**Most readable.** Selection 2 (volume of a cylinder) emerged as the most readable during the interviews, with seven participants identifying it as *most readable*, five participants identifying it as *next most readable*, and no participants identifying it as *not readable* (see Figure 4.19). Those clear results from the qualitative data served as an aid to interpreting the integrated results. Figure 4.23 shows the joint display for selection 2 in a single page (with a reduction in size) for quick reference. Appendix M shows the joint display in its original format for readability.
Figure 4.23. Joint display for selection 2 (volume of a cylinder)

**Selection # 2**

Thinking about the similarities between a cylinder and a prism can help to determine the formula for the volume of a cylinder. Although a cylinder is not a prism, it is similar to a prism in some ways. Like prisms, the volume of a cylinder is found by multiplying the area of its base by its height.

Since the base of a cylinder is a circle, the area of its base is given by the formula:

\[ A = \pi r^2 \]

Therefore, the formula is:

\[ V = \pi r^2 h \]

<table>
<thead>
<tr>
<th>S-CDT Item Comparison</th>
<th>Item pair</th>
<th>Accuracy Rate</th>
<th>Item pair</th>
<th>Accuracy Rate</th>
<th>Item pair</th>
<th>Accuracy Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conventional</td>
<td>( \pi r^2 )</td>
<td>91%</td>
<td>( x^2 )</td>
<td>98%</td>
<td>( \pi )</td>
<td>98%</td>
</tr>
<tr>
<td>Unconventional</td>
<td>( \pi r )</td>
<td>62%</td>
<td>( x )</td>
<td>56%</td>
<td>( \pi )</td>
<td>59%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>EPS Item Comparison</th>
<th>Response rates by response option</th>
</tr>
</thead>
<tbody>
<tr>
<td>Item 15</td>
<td>( A = \pi r^2 )</td>
</tr>
<tr>
<td>Item 2</td>
<td>( V = \frac{1}{3} \pi r^2 h )</td>
</tr>
<tr>
<td>Item 10</td>
<td>( A = \frac{1}{2} bh )</td>
</tr>
<tr>
<td>Item 14</td>
<td>( V = Bh )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Grade</th>
<th>Name</th>
<th>Selected quotes</th>
<th>Inferences</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>Delaney</td>
<td>&quot;...the easiest to come in my brain to know&quot;</td>
<td>Readable factors</td>
</tr>
<tr>
<td>8</td>
<td>Elena</td>
<td>&quot;I think it was just like an instinct to say because I've heard it so many times like that.&quot;</td>
<td>- Multiple highly recognizable orthographic patterns</td>
</tr>
<tr>
<td>8</td>
<td>Gil</td>
<td>&quot;Because there's a description what's going on, and there's a formula on the bottom.&quot;</td>
<td>- High exposure to formulas</td>
</tr>
<tr>
<td>8</td>
<td>Martin</td>
<td>&quot;I'll remember area equals pi R squared instead of A equals pi R squared.&quot;</td>
<td>- Familiar symbols (familiar meaning)</td>
</tr>
<tr>
<td>8</td>
<td>Nicky</td>
<td>&quot;I learned it in sixth grade...&quot;</td>
<td>- Familiar topic</td>
</tr>
<tr>
<td>7</td>
<td>Amelia</td>
<td>&quot;We have a 3D geometry quiz coming up.&quot;</td>
<td>- Recurring topics</td>
</tr>
<tr>
<td>7</td>
<td>India</td>
<td>&quot;We were learning about cylinders and how to calculate the area of cylinders just really recently.&quot;</td>
<td>Unreadable factors</td>
</tr>
<tr>
<td>7</td>
<td>Jackson</td>
<td>&quot;I saw pi and radius squared in here. And I think this is area and that's volume.&quot;</td>
<td>- None noted</td>
</tr>
<tr>
<td>7</td>
<td>Owen</td>
<td>&quot;It's not really different from reading to myself. The formulas, I just said them out loud.&quot;</td>
<td>Emergent themes</td>
</tr>
<tr>
<td>7</td>
<td>Robin</td>
<td>&quot;Actually, it's kind of like how I'm used to hearing it...I probably started using this when I was eight.&quot;</td>
<td>- SMaLL has orthographic &amp; phonological components</td>
</tr>
<tr>
<td>7</td>
<td>Quinn</td>
<td>&quot;I have those formulas memorized.&quot;</td>
<td>- Memorization, repeated use, and repeated hearing support SMaLL</td>
</tr>
<tr>
<td>7</td>
<td>Katrina</td>
<td>&quot;I usually say height so I can get a visualization of just height&quot;</td>
<td>- Expanding variables supports comprehension</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
<td>- The ability to detect orthographic errors in patterns comprised of known symbols may impact readability selections</td>
</tr>
</tbody>
</table>
The joint display shows the text as it appeared to participants during the interview. It shows the accuracy rates of S-CDT items that were most similar in appearance or meaning. Those rates represent a quantitative measure of the readability at the neurobiological level. The joint display also shows the response rates of MPES items that were most similar in appearance or meaning. Those rates represent a quantitative measure of the degree of relevance to academic culture among adolescents. The joint display shows the names of the participants who discussed the text selection (ordered by grade and elective enrollment). Students enrolled in the Math Intervention course are identified in gray cells. Finally, the right-most column describes the inferences I drew based on my analysis of the text selection in comparison to the joint displays for other text selections.

The results suggest that readable mathematical texts include multiple highly recognizable orthographic patterns in the language of mathematics. For selection 2, there were three conventional items that were similar to the text selection. All of them were associated with a high accuracy rate across the sample \((RD > 90\%)\), highlighted with black cells. In addition, the unconventional items were associated with moderate accuracy rates \((55\% < RD < 90\%)\), highlighted with gray cells, suggesting that students were able to detect the errors in the unconventional items.

The results also indicate that readable mathematical text was associated with high exposure symbolic mathematics. More than 90% of the participants had seen item 15 (formula for the area of a circle) on the MPES, \(A = \pi r^2\), \textit{a few times or many times}. As show on Figure 4.11, item 15 ranked as the highest exposure item. Two other relevant items (10 and 14) on the MPES were relatively high exposure items. The MPES items relevant to this text selection were
all associated with the first factor, ‘plug and chug’ formulas, for the MPES (see Table 4.8).

The qualitative evidence corroborates this explanation of readability for selection 2. The participants remarks suggested that the symbolic mathematic elements in the text are common in classroom culture and learned to the point of automaticity at the cognitive level. The topic is familiar and the symbols are familiar because the topic is a recurring. Some students suggested they would agree with Elena who said that reading the symbolic mathematics associated with this text selection, particularly the formula for the area of a circle, was “like an instinct because I’ve heard it so many times.”

**Controversial with familiar symbols.** Selection 3 (distance between two points) was controversial with two participants identifying it as *most readable*, one participant identifying it as *next most readable*, and five participants identifying it as *not readable* (see Figure 4.19 and Figure 4.20). The conflicting results and rich qualitative data generated insights into the complexity of readability. Figure 4.24 shows the joint display for selection 3 (in the format that was used for selection 2) in a single page (with a reduction in size) for quick reference.

Appendix N shows the joint display in its original format for readability.
The formula for the distance between two points \((x_1, y_1)\) and \((x_2, y_2)\) can be derived by creating a triangle and using the Pythagorean Theorem to find the length of the hypotenuse. The hypotenuse of the triangle will be the distance between the two points. The distance formula is commonly written as:

\[
d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}
\]

### Table

<table>
<thead>
<tr>
<th>Item Pair</th>
<th>Accuracy</th>
<th>Item Pair</th>
<th>Accuracy</th>
<th>Item Pair</th>
<th>Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>(9 &amp; 10)</td>
<td>91%</td>
<td>(47 &amp; 48)</td>
<td>94%</td>
<td>(17 &amp; 18)</td>
<td>95%</td>
</tr>
<tr>
<td>Conventional</td>
<td>(\sqrt{x})</td>
<td>((x - y)^2)</td>
<td>((x - y)^2)</td>
<td>((x - y)^2)</td>
<td>((x - y)^2)</td>
</tr>
<tr>
<td>Unconventional</td>
<td>(\sqrt{x})</td>
<td>((x - y)^2)</td>
<td>((x - y)^2)</td>
<td>((x - y)^2)</td>
<td>((x - y)^2)</td>
</tr>
</tbody>
</table>

### Table

<table>
<thead>
<tr>
<th>Item</th>
<th>Response Rates by Response Option</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Never</td>
</tr>
<tr>
<td>Item 5</td>
<td>(a^2 + b^2 = c^2)</td>
</tr>
<tr>
<td>Item 7</td>
<td>(\sqrt{x^2} =</td>
</tr>
<tr>
<td>Item 13</td>
<td>(f(x) = a(x - h)^2 + k)</td>
</tr>
</tbody>
</table>

### Discussion

- **Loren**: Referring to \(\sqrt{x}\). “So I knew I had to click it, so I just clicked the button. I never knew what it was… Oh, I thought that was divided by.”
- **Delaney**:
- **Owen**:
- **Gil**: “Distance equals the square root of quantity of X power of one minus X squared - wait, no - X power of two and quantity of square…”
- **Harper**: “D – diameter - equals the square root of X one minus X two squared plus Y one minus Y two squared.”
- **Robin**: “I don’t like looking at big formula. I might understand what this is saying. I just don’t like things that are very long.”
- **Freddie**: “I get that that’s a square root, not a division sign, but the fact that it’s division and multiplication and exponents equal - and only variables inside, square roots is pretty complicated.”
- **Amelia**: “If I was reading it to anybody I might add the parentheses, it just makes sense in my head just to kind of leave them off.”
- **Claire**: “The formula for the distance between two points X. Whatever that is. It’s a subscript…” “The distance formula is commonly written as - I don’t know how to say that.”
- **Nicki**: “I recognize the points, X and Y. I know those are coordinate points… Finding distance, there’s a specific formula and I know this is one of them”
In light of the results related to selection 2, the data related to selection 3 suggest that mathematical texts with multiple highly recognizable orthographic patterns in the language of mathematics are not all similarly readable. For selection 3, all of the S-CDT conventional items that were similar in appearance or meaning were associated with high accuracy rates ($RD > 90\%$), highlighted with black cells. In contrast to the readable selection 2, the related unconventional items were associated with low accuracy rates ($RD < 55\%$), highlighted with red cells. This suggests that students were not able to detect the errors in the unconventional items.

The results suggest that the graphic organizer used for long division, $\div$, is easily confused with the square root sign, $\sqrt{}$. A comparison of the other S-CDT item pairs suggests that detecting errors in the location of sub/superscripts may be particularly difficult.

Item 5 on the MPES ranked 3rd in terms of high exposure suggesting that the topic, Pythagorean theorem, was familiar (see Figure 4.11). Thus, the MPES indicated that the topic of the text selection was common. However, the MPES items that were most similar in appearance to the text selection ranked low in print exposure and were associated with the third factor, ‘complex’ formulas. None of the items on the MPES appears in the text selection as a unit. Thus, any conclusions drawn from comparison to the MPES data are tentative.

The qualitative evidence suggests that, although the individual symbols in this text selection may be common, that distance formula as a whole is difficult for adolescents to read. Only one of the two students who identified the text selection as readable was able to read it fluently. Some students explained explicitly that the square root symbol and sub/superscripts were obstacles to readability. Three students, Freddie, Katrina, and Loren, made specific
remarks corroborating the conclusion that the square root $\sqrt{}$ is easily confused with the graphic organizer used for long division, $\overline{\text{}}$. For example, Katrina pointed to it and said, “Just that line… I don’t know what it’s called - the division line.” And, Gil, who was willing to try to read the text selection, translated the subscripts as if they were exponents. He began, “Distance equals the square root of quantity of $X$ power of one minus $X$ squared…”

In summary, selection 3 is related to multiple highly recognizable orthographic patterns. It includes familiar symbols and a familiar topic leading some students to identify it as readable despite being unable to read it. The low rate of error detection in unconventionally written items with confusable text features may explain the disagreement among students about whether this text selection is readable. Claire, who identified the selection is readable was unable to read it. She stopped when she reached the symbolic mathematics elements of the text selection: “The distance formula is commonly written as [PAUSE] I don’t know how to say that.”

**Controversial with familiar topic.** Selection 4 (point-slope form of a line) was unreadable with no participants identifying it as *most readable*, one participant identifying it as *next most readable*, and three participants identifying it as *not readable* (see Figure 4.19 and Figure 4.20). The results suggest that text central to the curriculum are not necessarily readable. Figure 4.25 shows the joint display for selection 4 (in the format that was used for selections 2 and 3) in a single page (with a reduction in size) for quick reference. Appendix O shows the joint display in its original format for readability.
Figure 4.25. Joint display for selection 4 (point-slope form of a line)

<table>
<thead>
<tr>
<th>Selection #4</th>
<th>Readability Rank: 9</th>
</tr>
</thead>
<tbody>
<tr>
<td>The slope of a line is ( m = \frac{y_2 - y_1}{x_2 - x_1} ) where ((x_1, y_1)) and ((x_2, y_2)) are two points on the line.</td>
<td></td>
</tr>
<tr>
<td>The point-slope form of the equation for the line is ( y - y_1 = m(x - x_1) )</td>
<td></td>
</tr>
<tr>
<td>Therefore, the two-point form of the equation of a line can be written as ( y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1) )</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>S-C-T Item Comparison</th>
<th>Item pair</th>
<th>Accuracy Rate</th>
<th>Item pair</th>
<th>Accuracy Rate</th>
<th>Item pair</th>
<th>Accuracy Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conventional</td>
<td>21 &amp; 22</td>
<td>66%</td>
<td>51 &amp; 52</td>
<td>75%</td>
<td>7 &amp; 8</td>
<td>84%</td>
</tr>
<tr>
<td>Unconventional</td>
<td>( x - x_1 )</td>
<td>59%</td>
<td>( y_2 - y_1 )</td>
<td>45%</td>
<td>( f(x) )</td>
<td>23%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MPES Item Comparison</th>
<th>Response rates by response option</th>
</tr>
</thead>
<tbody>
<tr>
<td>Item 9 ( m = \frac{y_2 - y_1}{x_2 - x_1} )</td>
<td>Never 1 or 2 times  A few times  Many times</td>
</tr>
<tr>
<td>Item 8 ( y = mx + b )</td>
<td>22% 13% 10% 55%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Grade</th>
<th>Name</th>
<th>Selected quotes</th>
<th>Inferences</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>Robin</td>
<td>“I literally can see the different formulas that we've learned, which kind of helps me. And especially in math, it's kind of redoing or reforming this unit, I guess. So I'm really used to seeing these right now.” It's really weird. But I like slope - slope intercept form - point-slope. So I like the idea of slope. So it's something in math that I like so I attract myself to it.”</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Delaney</td>
<td>“[This] is about slope and lines and point-slope, and we did this. I just didn't really like it as much... I think I did pretty well but I think the unit went kind of fast to me. So it was like I didn't get to learn it all so it will stay with me” “I've seen subscripts done. It's just that we did it a long time ago.”</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Gil</td>
<td>“They just give you a slope of a line and... all these equations. And first of all, you don't really know what X and Y is... And then they just gave you all these things here. Because first of all, how do you even say this?! I could [be] say it if I look at it longer but when you start doing this problem and you see all these... You might get the reaction and you just kind of have to stop and re-read it.”</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Claire</td>
<td>“There's subscripts and equations on top... like the numerator of a fraction but it's an equation. And then on the bottom it's the denominator of a fraction but it's an equation. And I don't know what that means... I know it means divide but it looks really complicated.”</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Peter</td>
<td>Instead of saying M, “I'd read slope equals.”</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Rocky</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Amelia</td>
<td>“I've learned this and know what that is. It's slope.”</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Katrina</td>
<td>“All the letters with the numbers and those two numbers. And then fraction thing and the parentheses after that.”</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Loren</td>
<td>“This line - that big line thing... I don't know what that is.”</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Readable factors</th>
<th>Unreadable factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Familiar symbols</td>
<td>Limited exposure to formula</td>
</tr>
<tr>
<td>Moderate exposure to related formula</td>
<td>Multiple SML elements</td>
</tr>
<tr>
<td>SML embedded in EL</td>
<td>Complex syntax</td>
</tr>
<tr>
<td>Vinculum</td>
<td>Parentheses used in multiple ways</td>
</tr>
<tr>
<td>Recognizable orthogonal patterns</td>
<td>Moderately recognizable orthogonal patterns</td>
</tr>
<tr>
<td>Undetectable errors in confusable patterns</td>
<td>Less familiar (or new) topic</td>
</tr>
<tr>
<td>Emergent themes</td>
<td>Adjustments in symbols (e.g., vinculum) may require unique SML skill</td>
</tr>
<tr>
<td>Additive effects of unreadable features can induce affective reaction</td>
<td>SML entails the ability to use context to assess the meaning of parentheses</td>
</tr>
<tr>
<td>Left-hand subscripts and superscripts are confusable orthogonal patterns</td>
<td></td>
</tr>
</tbody>
</table>

Discussion
The results suggest that mathematical texts with confusable orthographic patterns in the language of mathematics are not necessarily readable. For selection 4, all of the S-CDT conventional items that were similar in appearance or meaning were associated with a moderate accuracy rate across the sample ($55\% < RD < 90\%$), highlighted with gray or purple cells. One of the related unconventional items was associated with a moderate accuracy rate ($55\% < RD < 90\%$), highlighted with a purple cell. The others were associated with low accuracy rates ($RD < 55\%$), highlighted with red cells. The purple cells highlight a pair of S-CDT items both of which have moderate accuracy rates. This is an particularly interesting result because $x - x_i$, the conventional item, appears in the text selection as is. It suggests that readability may be related to the confusability of conventional items and unconventional items.

The results also suggest that parentheses, which are used in three different ways between the text selection and the S-CDT items, may be challenging to read. Furthermore, the evidence suggests subscripts, even in the absence of superscripts, are difficult to read. Specifically, detecting errors in the location of subscripts appears to be difficult.

The relevant items on the MPES suggest that this selection was associated with moderate exposure symbolic mathematics items. Item 9 (formula for the slope), which appears in the text as is, ranked 9th out of 15 in terms of exposure on the MPES with 85% of the sample indicating they had seen it a few times or many times (see Figure 4.11). Item 8 (slope-intercept form of a line) was more familiar, ranking 5th out of 15 suggesting that the topic was familiar. In general, the MPES indicated that the topic of the text selection was not uncommon. Both items, 8 and 9, were associated with the first factor, ‘plug and chug’ formulas, of the MPES.
The qualitative evidence supports the conclusion that familiarity is insufficient for readability. At least three students, Amelia, Delaney, and Robin, indicated that this text selection was related to a familiar topic and formulas they had seen. Robin, the only student who identified this text selection as readable, indicated that it was related to a topic she particularly liked even though some others did not. She said, “It’s really weird. But I like slope — slope-intercept form — point-slope.” The length and complex syntax of the symbolic mathematics elements reduced the readability. Katrina gave the most concise summary of the overwhelming complexity when she said, “All the letters with numbers and those two numbers. And then [the] fraction thing and parentheses after that!”

This was an unexpected result. As a mathematics educator, I recognize this text as ubiquitous and central to applied mathematics in Algebra and beyond. The analysis of this item suggests that readability is distinct from ability to do math. Students seemed to be familiar with the concept of slope and the use of \( m \) as the variable for slope. For example, Amelia said, “I’ve learned this and know what this is. It’s slope.” However, despite its familiarity and ‘plug and chug’ nature, the evidence suggests that the text selection is best presumed unreadable among adolescents in middle school due to the syntactic complexity of the symbolic mathematics.

**Most unreadable.** Selection 5 (definition of absolute value) was unreadable with no participants identifying it as *most readable* or *next most readable* and two participants identifying it as *not readable* (see Figure 4.19 and Figure 4.20). The integration of the qualitative and quantitative data and findings for this text selection provide additional insights into the complexity of readability, highlight the limitations of this analysis, and indicate the need
for further item development. Figure 4.26 shows the joint display for selection 5 (in the format that was used for the other selections) in a single page (with a reduction in size) for quick reference. Appendix P shows the joint display for selection 5 in its original format for readability.

The results suggest that mathematical texts with syntactic complexity may be unreadable even when multiple symbolic mathematics components of the text are recognizable at the cognitive level. For selection 5, all of the S-CDT items that were similar in appearance or meaning were associated with a moderate accuracy rates across the sample ($55\% < RD < 90\%$), highlighted with gray cells. Two of the related unconventional items was associated a moderate accuracy rate ($55\% < RD < 90\%$), highlighted with gray cells. The other was associated with high accuracy rates ($RD > 90\%$), highlighted with black cells. Two of the S-CDT conventional items shown for comparison, $|x|$ and $x \geq 0$, appear in the text selection as is. In this case, the text selection was unreadable despite the fact that the errors in the S-CDT unconventional items were detectable. This result indicates the limitations of using the S-CDT, particularly the unconventional items, as evidence. The S-CDT requires further development and empirical evidence to understand the nature of the unconventional items.
The absolute value of a number describes the magnitude of the number. That is, the absolute value is the distance from 0 on a number line.

The absolute value formula can be expressed as:

\[ |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases} \]

**Discussion**

<table>
<thead>
<tr>
<th>Grade</th>
<th>Name</th>
<th>Selected quotes</th>
<th>Inferences</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>Bobby</td>
<td>“I don’t think I’ve ever seen any of that, except for the absolute value. But I don’t know how you would figure that out with an X in it.”</td>
<td>Familiar topic, Unreadable factors</td>
</tr>
<tr>
<td>7</td>
<td>Jackson</td>
<td>“Minus X or negative X. And then X, that makes no sense… And it’s absolute value so if I was negative, it’s already negative. It would not be a negative anymore unless it is outside. And then it would be a positive if it’s already a negative.”</td>
<td>Low exposure to formula with similar SML feature, Complex syntax, Familiar symbols with unfamiliar syntax (e.g., unpaired bracket, commas, spacing), Inequalities, Limited EL context, Moderately recognizable orthographic patterns</td>
</tr>
<tr>
<td>8</td>
<td>Delaney</td>
<td>“The absolute value of X equals. There’s a bracket. X, and then X to the inequality. Greater than zero and then negative X, X is less than zero.”</td>
<td>Given novel text, some students draw on mathematical knowledge to read, Students request help from an expert reader when…</td>
</tr>
<tr>
<td>8</td>
<td>Elena</td>
<td>Referring to</td>
<td>x</td>
</tr>
<tr>
<td>8</td>
<td>Gil</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Peter</td>
<td>“I’m sort of being a little thrown off by the commas and the bracket there… While I’m reading it, I’m trying to figure out what all of the variables in each place mean.”</td>
<td>Given novel text, some students draw on mathematical knowledge to read, Students request help from an expert reader when…</td>
</tr>
<tr>
<td>8</td>
<td>Harper</td>
<td>“There’s the negatives and greater than and less than, and the absolute value next to it… I would honestly have no idea what to say.”</td>
<td>Familiar symbols with unfamiliar syntax (e.g., unpaired bracket, commas, spacing), Inequalities, Limited EL context, Moderately recognizable orthographic patterns</td>
</tr>
<tr>
<td>7</td>
<td>Arnelia</td>
<td>“I’m not sure how you’re supposed to word it”</td>
<td>Given novel text, some students draw on mathematical knowledge to read, Students request help from an expert reader when…</td>
</tr>
<tr>
<td>7</td>
<td>Claire</td>
<td>“Well, I know that that means the absolute value of X but I don’t know what it means afterwards… I don’t know why that is there. The bracket. I don’t know why the bracket’s there.”</td>
<td>Familiar symbols with unfamiliar syntax (e.g., unpaired bracket, commas, spacing), Inequalities, Limited EL context, Moderately recognizable orthographic patterns</td>
</tr>
<tr>
<td>7</td>
<td>India</td>
<td>“When I first look at a formula, I don’t really understand it so I will go back to English, and I will read again.” “I would just ask my teacher: How do you read this?”</td>
<td>Familiar symbols with unfamiliar syntax (e.g., unpaired bracket, commas, spacing), Inequalities, Limited EL context, Moderately recognizable orthographic patterns</td>
</tr>
<tr>
<td>7</td>
<td>Owen</td>
<td>“I know it’s the absolute value of X equals something and… I’ve never seen that sign before.”</td>
<td>Familiar symbols with unfamiliar syntax (e.g., unpaired bracket, commas, spacing), Inequalities, Limited EL context, Moderately recognizable orthographic patterns</td>
</tr>
<tr>
<td>7</td>
<td>Quincy</td>
<td>“It would be easier if you could translate that. There’s only one thing in there I don’t understand… If someone had written the entire equation in English, I could find that spot in the English and translate it and figure out what it means.” “The bracket is equaling ‘or’.” “The commas basically mean when. The opposite of X when X is less than zero.”</td>
<td>Familiar symbols with unfamiliar syntax (e.g., unpaired bracket, commas, spacing), Inequalities, Limited EL context, Moderately recognizable orthographic patterns</td>
</tr>
<tr>
<td>7</td>
<td>Robin</td>
<td>Referring to the commas: “I always think of that like points on graphs, but it doesn’t seem like that’s what it is because it’s X and X, not X and Y. But I would probably mess that up if I thought of it is points because it says magnitude.”</td>
<td>Familiar symbols with unfamiliar syntax (e.g., unpaired bracket, commas, spacing), Inequalities, Limited EL context, Moderately recognizable orthographic patterns</td>
</tr>
<tr>
<td>7</td>
<td>Loren</td>
<td>“Would you say whatever it was called? Absolute value?... Or bar X bar? ... Or if I should just say X”</td>
<td>Familiar symbols with unfamiliar syntax (e.g., unpaired bracket, commas, spacing), Inequalities, Limited EL context, Moderately recognizable orthographic patterns</td>
</tr>
</tbody>
</table>
The only MPES item with a relevant feature or meaning was item 7 (principal square root). As shown in Figure 4.11, item 7 ranked the lowest in terms of least print exposure. Item 7 was associated with the third factor, ‘complex’ formulas, in the factor analysis. The limited similarity makes it impossible to draw any particular conclusions. However, it has methodological implications for future studies. In addition to further instrument development, future studies should include additional links between quantitative measures and qualitative protocol to support stronger conclusions.

The qualitative data suggests that many of the students were familiar with the phrase “absolute value.” However, the unfamiliar syntax, including the one-sided bracket and the commas, made the symbolic mathematics text unreadable. Some students were unable to begin, struggling to read $|x|$. For example, Elena said, “If I was reading that, I would just say X.” Loren had several questions, “Would you say whatever it is called? Absolute value?.. Or ‘bar X bar’?…just say X?” For a few students, this text selection appeared to be unreadable because elements of the symbolic mathematics text conflicted with their knowledge about absolute value. For instance, Owen told me that, as he struggled with the text, he was thinking, “I know an absolute value is always equal to a positive number.” This suggested that the meaning he associated with a recognizable portion of the symbolic mathematics, $x < 0$, created cognitive dissonance he was unable to reconcile until I read the symbolic mathematics as a coherent statement.

**Assessing readability.** In summary, the results of integrating across levels for each text selection met the goal of initiation by revealing a pattern and new perspective I had not predicted. In addition to generating insights about how the students experienced the texts
selections, the analysis revealed a new possibility for future mixed methods studies of SMaLL with practical implications. The results in this section suggest that the MPES, designed to be a self-reported measure of access to symbolic mathematics, and the S-CDT, designed to be an cognitive assessment of the orthographic component of SMaLL, may be useful in combination with qualitative inquiry for understanding the readability, or reading level, of mathematical texts for particular populations.

The evidence in this study suggests that, for adolescent students in middle grades learning under the CCSS, readability of a text selection is related to quantitative measures of cultural and neurobiological data. More specifically, readability of a text selection was associated with high exposure to similar equations written in conventional symbolic mathematics, high accuracy in identifying conventional patterns in similar small chunks ($\leq 6$ characters) of text. In contrast, unreadable text selections was associated with lower exposure to similar equations written in symbolic mathematics and lower accuracy in identifying conventional patterns in similar small chunks of text. The role of accuracy in detecting errors in confusable small chunks of similar but unconventional text is unclear, but warrants further item development and investigation.

**SMaLL spectrum.** The maximum variation sampling strategy was intended to make it possible to explore the differences in SMaLL among adolescents. This was critical to the purpose of crafting a study with implications for instructional practice in the Common Core era. The assumption of current educational policy is that all children, not just typical children, can and should be college- and career-ready upon graduation.

As a part of my analytical process, I created a qualitative and quantitative profile of each participant to generate understanding about each participant. Then, I used the quantitative data
to guide an exploration of participants across the spectrums. The joint display show in Figure 4.27 shows where each of the interviewees falls along the spectrum for each quantitative variable. For this report, I selected five students to profile. Peter and Loren, who are at opposite end on multiple spectrums, illustrate the extremes. Katrina, Jackson, and Elena, who are closer to the middle on multiple spectrums, illustrate the variation among students who otherwise appear to be similar by quantitative measures.

When I interviewed each of the participants, I had not yet reviewed their quantitative data. I only knew their grade-level and elective course enrollment. In each section to follow, I will use narrative style to weave the quantitative and qualitative data together. Given the multilevel purpose, I did not take the approach of describing the participants by recounting the highlights of their interviews in temporal order as they occurred. Instead, I organized the side-by-side integrated discussion by level — cultural context, individual behavior, and within-person neurobiological processing — to share what I learned about SMaLL from participants across the spectrum.
Figure 4.27. Joint display spectrum analysis. Black indicates enrollment in Math Extension. Red indicates enrollment in Math Intervention.
Most developed SMaLL. Peter was an 8th grader enrolled in Ms. Davis’ Math Extension elective. His quantitative profile is summarized in Figure 4.28. His 7th grade OAA score (OAA7 > 400) indicated that he was an advanced student, well above the state standards for proficiency.

Cultural context. Peter was existing and thriving in a rigorous academic environment. Peter indicated that some of his math reading habits were related to the time allowed to complete his work. For example, he said, “When I [need to] finish it in a set period of time… I just skim [the English language text] because some of them are sort of long.” I asked for clarification to understand if he felt that he did not have enough time to read what was necessary in class. He explained, “Well, we have enough time because he's willing to give us extra class time to work on it, I think, in the next days. But I want to try to finish it in the day.” Peter particularly noticed pressure to read — and read quickly — in the context of tests. He explained the order in which he used reading strategies entirely in the context of a testing situation in which he did not want to “end up wasting a good amount of time.”

In addition to doing well in mathematics, Peter reported that he can speak and read a second language well. Also, he reported that he can play and read music well. Although Peter appeared to have many talents and interests, Peter already knew he wanted to go into either computer science or engineering. He had already created a 4-year plan to ensure he had plenty of honors and AP math and science class upon graduation. Peter clearly placed a high value on the ability to read math. He explained,
Figure 4.28. Peter’s quantitative profile
I feel like reading English is always important because you need to know how to read something correctly to be able to understand other people's opinions and other people's ideas… But since I really like math and science and I'm probably going to end up going into one of those fields, I feel like reading math is also very important because I need to be able to understand the math that there is when I'm reading something so that I can correctly convey my points and correctly understand what other people are trying to say.

*Individual behavior.* Peter chose selection 9 (distance equals rate times time) as the most readable choice “because it has the most words.” He read the selection fluently and expanded the symbolic mathematics text to “distance equals rate times time” without pausing. He explained, “The reason I prefer to read it the [expanded] way is because it sort of reminds me of what all the variables mean.”

Peter was the only student to identify selection 7 (inequality explanation) as readable. The in vivo code “structure” comes from his explanation, “The structure made [selection 7] more readable.” In contrast to other students, some of whom paused and asked questions when they reached the inequalities, he read the entire section with confidence. His ability to transition quickly between reading English and reading math makes sense in light of the fact that he ranked the highest in terms of orthographic processing \((RD = 29)\) and math print exposure \((MPES = 57)\) and ranked low on the math anxiety spectrum \((MAS = 15)\).

Peter identified selection 3 (distance between two points) as not readable, specifically identifying subscripts as the difficulty. He explained that it is hard to keep the subscripts in the correct order: “Sometimes if I'm reading subscripts… I’ve switched X1 and X2 around before, like Y2 and Y.”
Peter used multiple strategies to read symbolic mathematics including rough read & re-read, decoding symbolic mathematics, making use of English language elements, and making meaning. He demonstrated his decoding approach when he tried to read selection 5 (definition of absolute value). As he was working, he explained,

I'm sort of being a little thrown off by the commas and the bracket there… While I'm reading it, I'm trying to figure out what all of the variables in each place mean. And since there's just X with a comma there and then X is greater than or equal to zero or X is less than zero, it's sort of difficult for me to figure out what's the X means in that situation.

He explained that he used the English language text as needed to see “if there's any words or any pretense… to see if there's anything else I can catch onto.” He went on to say, “And then if there's still nothing, I try to start solving towards it” indicating that making meaning was his last-choice independent strategy. The implication is that he prefers to read the symbolic mathematics before attempting to solve a problem. His response to item 6, Before I do a math problem, I read the problem to myself, on the MRHS confirms that he always prefers to read a problem before attempting to do a problem.

Within-person neurobiological processing. Peter indicated that he attributes some of his reading to cognitive processes. For example, he explained that he prefers to compress text when he is reading very common symbolic mathematics text “because it’s faster.” When he does that, he is “subconsciously thinking about what each of the variables sort of means.” He prefers to expand the text for applied problems so that he can “easily just start plugging them in in my head.” His descriptions suggest that, at the cognitive level, Peter associates orthographic patterns with both lexical and semantic information. This may explain his relatively high orthographic
processing ability. However, it raises questions about assuming low mean RTs are associated with orthographic processing ability. He had a high mean RT ($RT = 1493$) near the 1500ms threshold that, theoretically, suggests a transition from cognitive to metacognitive processing. His high response times were associated with items with superscripts. Peter’s high mean RT cannot reasonably be explained by the cognitive consequences of math anxiety as he strongly disagreed with all of the negative items on the MAS, and his score was among the lowest.

**SMaLL for an accelerated math student.** Jackson was an 7th grader enrolled in Mr. Rowe’s Math Extension elective. His quantitative profile is summarized in Figure 4.28. His 6th grade OAA score ($OAA6 > 400$) indicated that he was an accelerated student, above the state standards for proficiency.

*Cultural context.* The data from my interview with Jackson suggested he felt like his school experiences were normal. He did not express any particular difficulties at school. He indicated that he viewed his father as a resource at home: “I sometimes would ask my dad. He helps me with my homework sometimes.” His MPES score (48) fell in the middle range compared to his peers. The only two equations he had never seen were associated with the ‘complex’ formula factor. He had seen several of the ‘plug and chug’ and ‘textbook’ formulas many times.
Figure 4.28. Jackson’s quantitative profile
Jackson viewed reading as a necessary first step to doing mathematics in his class. He is the source of the ‘have-to’ description of SMaLL culture in mathematics. I asked him how often, if ever, he had to read mathematical texts like the ones we were discussing. At first, he hesitated, “Somewhat. It - I don't have - we actually have to read this often.” I prompted him to clarify asking, “Are you deciding to read it or does your teacher tell you that you have to read it?” He responded, “I'm assigned to read. Actually, no, he gives a paper so we kind of have to read it if we want to complete the problem. But yeah, I can choose to read it, too.”

Compared to Peter, Jackson’s response patterns on the MRHS suggested he had less engagement with mathematical text outside of school. When asked about his future plans he said, “I like math and science so why not combine them together.” He had not yet chosen a career plan yet. He explained that his plans were “questionable.” He said, “When I was younger, I always wanted to be a policeman or something like that. But my mom said no… ‘Nooooooo, it's too dangerous’.”

*Individual behavior.* Jackson math reading habits (MRHS = 34) fell in the middle range as compared to his peers. The pattern of responses to the MPES suggest that he always reads math problems more than once and focuses on the symbolic mathematics more than the English language features.

For Jackson, reading independently and reading out loud are the same. After he read selection 2 (volume of a cylinder) with a combination of compression and expansion, I asked him if he might read it any differently if he were reading aloud to his class. He replied, “No, not really. I might speak louder – that’s it.” He went on to say, “Because [reading aloud is] not really different from reading to myself. The formulas, I just said them out loud.”
Jackson used a variety of strategies: rough read and re-read, decoding symbolic mathematics, and making meaning. He sees decoding symbolic mathematics as different from reading English. He explained that in reading English, “if you don't know a word, you sound it out.” But, reading math is “dealing with words in your brain.” He summarized his thinking this way:

English, I could just get off the bat. But math - you can also learn - but it's harder because it's got expressions. It's got formulas, which the alphabet does not have. And you also got to know what you're saying with those formulas.

He responded with *sometimes* to item 5 on the MPES (When I see math symbols, I get someone else to read them to me). This was inline with his remark in the interview, “If you don't know it, just ask for it. I just ask my teacher a lot of stuff… That’s why I say ask my teacher.” When I asked about whether he ever applied any English language arts strategies to reading math he said, “Just look around, find the resources. That's a strategy… look at your notes.”

*Within-person neurobiological processing.* Other than the one statement suggesting that reading was like “dealing with words in your brain,” Jackson did not attribute his reading to cognitive processes. This suggests that, for Jackson, reading symbolic mathematics is somewhat effortful. That inference was inline with his math anxiety score (MAS = 28) and response patterns. His agreement with item 12, *When I see a math problem, I worry about whether I can do it,* and strong disagreement with item 11, *I get frustrated when I do math problems,* may indicate that his anxiety manifests during the initial process of reading mathematics rather than the later stage of doing mathematics.
Jackson’s orthographic processing was on the low end compared to his peers in the Math Extension elective \((RD = 21)\). The patterns in his responses and response times suggests that √ and \(\sqrt{\)}\) are confusable for him. Although he was able to identify items with √ accurately, his response times to items with \(\sqrt{\)}\) were relatively high compared to his other RTs.

**SMaLL for a Math Intervention student.** Katrina was a 7th grader enrolled in Ms. Kennedy’s Math Intervention elective. Her quantitative profile is summarized in Figure 4.29. Her 6th grade OAA score \((OAA6 > 400)\) indicated that she was proficient by state standards.

**Cultural context.** Katrina’s experience of academic culture differed from Peter’s and Jackson’s. Unlike them, she was enrolled in the Math Intervention elective. Although her total MRHS score (39) was higher than Jackson’s, her responses indicated she spent less time engaging with math symbols outside of school. For her, weekly worksheets were a critical aspect of classroom learning. She described how they related to reading math,

Well, we get these [work]sheets every week… Some of them have problems that I've never really seen before or don't remember… When I come up on that, I either try to look it up or something or try to go in my binder and look for stuff that looks like that to see what it is. Or ask my teacher if I don't know.

She only recalled reading as a part of classroom instruction “if it’s a word problem.”

During the summary portion of the interview, Katrina elaborated in response to a question about whether reading math was important. She said,
Figure 4.30. Katrina’s quantitative profile
I think it's important because my math teacher…Well, she gets mad at my class easily because we're usually off topic or something. And then we don't know what was going on. So then she [explains how we] need to study even if we don't like it… [We need math] for our whole life. [We may] just not notice it, but we do.

I followed up with a question about whether she had become more aware of math in her life. Her response suggested that she has little math print exposure. She said,

Kind of, like when you pay someone or if someone pays you… You ask for a certain amount, but they give you a lot more than what you asked them for. Then you need to know how much to pay them back.

Her MPES score (37) and response patterns support that conclusion, indicating that she has low exposure to some common ‘plug and chug’ formulas.

She had not yet thought about her future high school coursework or how it might be related to a career in the future. She said, “I'm just taking regular math. And, in high school, I'm probably just going to take regular math, too.” She had, however, ruled out a career in STEM fields. She explained, “I don't really know, but… I think I want to do something to do with language arts.” This did not come as a surprise as she had told me, “I just never really like when you had to solve for X or [this] algebra stuff.”

*Individual behavior.* Katrina’s math reading habits (*MRHS* = 39) appear to be better than her peers in the Math Intervention elective. According to the response patterns, she reported engaging with math symbols in more situations and, in relationship to math problems, engaging in recommended reading habits more often. However, her MRHS may have been influence by social desirability bias. She responded *always* to item 10, *When I do a math problem, I read the*
problem more than once, but during the interview she said, “When I'm reading math, I can just - I usually just read it once and I get it.”

Her independent reading strategies included decoding symbolic mathematics and making meaning. For her, decoding amounted to “just remembering the words for it.” For her making meaning meant visualizing. For example, she explained that “I usually say height so I can get a visualization of just height.” Her reading was influenced by her memory of phonological patterns. She said that she did not expand [A] “because my teacher usually says A.”

Within-person neurobiological processing. In explaining her approach to reading, Katrina only made one reference to automated processes. When she explained why she had chosen selection 2 (volume of a cylinder) as readable, she said, “I just memorized what the pi and the radius and the squared looked like.” She did not explicitly attribute her reading to automaticity. However, her orthographic processing accuracy rate \(RD = 25\) was higher than her peers in the Math Intervention elective and similar to most of the other interviewees.

Like her peers in the Math Intervention elective, she had high math anxiety \(MAS = 39\). Her responses suggested that her anxiety likely impacts her cognition. She responded agree to these statements, My mind goes blank when I see a math problem and When I see a math problem, I worry about whether I can do it. This suggests that, for Katrina, the onset of the anxiety may coincide with the initial interaction with the symbolic mathematics text.

Her math anxiety may explain her reading style. She read selection 2 (volume of a cylinder) which she had selected as most readable, hesitantly. She read some of the symbolic mathematics as questions, “The area of its base is given by the formula A equals pi [HESITANTLY] radius squared? And then therefore, the formula equals [PAUSE] V equals pi
radius squared [HESITANTLY] then height? I think.”

**Small for a student with math anxiety.** Elena was a 8th grader enrolled in Mr. Rowe’s Math Extension elective. Her quantitative profile is summarized in Figure 4.29. Her 7th grade OAA score \((OAA7 > 400)\) indicated that she fell just above the state standards for proficiency.

*Cultural context.* Elena, who ranked high on math anxiety \((MAS = 40)\), attributed at least some of her negative affect to classroom experiences. Specifically, she said, “I never really liked math in the past… I think that was kind of because of my teachers.” However, she found her Math Extension elective to be a departure from her previous experiences. She noted, “I’ve never been good at [math] in the past, but I kind of like it this year… I like this elective right now.”

Elena had transferred to Brookover Public Schools. She indicated that it had been hard to adjust because she struggled with reading in math class. She explained,

> When we moved here, I had a lot of trouble with math. And I had a hard time keeping up with what the different symbols mean and how we would substitute the multiplication symbols for the dot, and I always got things confused like that.

The differences between her old school and her new school may have contributed to this observation: “We've learned how to do this [math], and we've learned what it's for and where to use it. But we've never really been taught how to read it. It's not something [teachers] really focus on.” When I asked her what she thought about that she replied frankly:
Figure 4.31. Elena’s quantitative profile
Honestly, I kind of think it's a bad idea… I think that they should teach us how to read things like this because the fact that I know this and I know where it can be used and how to do it and stuff, but I don't know how to read [it]. That seems a little weird. It just seems a little backwards, so I do think they should teach us more about that.

**Individual behavior.** When Elena was reading she relied heavily on decoding the symbolic mathematics with the intention of expanding the text. To explain, she described her reaction to a hearing a peer read $A = \pi r^2$: “My neighbor, she’s in seventh grade,…when we were first going over this, she said ‘A equals pi R squared.’ And, I just would never read it like that because I know that this A means area.” She indicated that she was able to adjust to her audience as needed, “If I was teaching it to somebody, I would say area equals pi times the radius squared so they'd know what R means.”

Elena pointed out that symbolic mathematics with parentheses are particularly hard to read. She referred to selection 8 (example of exponent rule) to explain the nature of the difficulty. “I don't know how I'd read it. I know in equations… we were always taught to solve what's in the parenthesis first, but I mean I don't know how I would read them.” In general, she worried when she read things differently than others. She explained, “Well, it makes me think well, ‘Am I saying this wrong?’ How is it really? How are we supposed to be reading this?”

**Within-person neurobiological processing.** Elena did attribute some of her reading process to automaticity. In particular, she alluded to the role of phonology when she said, “I think it was just like an instinct to say because I've heard it so many times like that.” Other remarks suggested that hearing other students also contributed to her incidental phonological learning. For example, she said, “I mean I've heard teachers and other students say R squared, so
that's just where I got it.”

Although Elena scored the highest among all of the interviewee in terms of math anxiety, her response pattern suggested that she may not agree that her math anxiety impedes her reading at the cognitive level. She marked disagree for all of the negative affect items, indicating that she did not feel nervous or uneasy and her mind does not go “blank.” Her high score was inflated because she marked strongly disagree to every positive affect item to indicate she does not like math.

**Least developed SMaLL.** Loren was an 7th grader enrolled in Ms. Kennedy’s Math Intervention elective. Her quantitative profile is summarized in Figure 4.29. Her 6th grade OAA score (OAA6 < 400) indicated that she fell just below the state standards for proficiency.

*Cultural context.* Loren, who was on the opposite end of the spectrum from Peter, seemed to experience school, in general, and math, in particular, quite differently than her peers. Although some of the other interviewees seemed less than eager to read, she was frank about her reading habits. When I asked about how often she read for fun, she said, “Only when I have to!” Loren indicated that she struggles in school and her low grades prevent her from participating in the activities she likes. I asked her, “Do you think that math classes are going to be important for planning for your future?” She replied,

I mean, I have to get good grades to play basketball. The only reason I didn't try out for the school team this year is because of my grades…if you get a C, below a C average, then you can't play in the game that week.
Figure 4.32. Loren’s quantitative profile
Loren really wants to be a basketball player, but she said, “my mom kind of told me I couldn't be a basketball player so I'd have to have something else. And I don't know what.”

In light of her preference to be on the court rather than reading, her low total math print exposure ($MPES = 25$) is not surprising. She reported that she had never seen 9 out of the 15 formulas. The only ‘plug and chug’ item she reported seeing many times was $A = \pi r^2$. Her MRHS (25) responses indicated that she rarely, if ever, sees math symbols outside of school and only sometimes sees them in school.

*Individual behavior.* For Loren, readable text is short in terms of both English language and symbolic mathematics. She looks for “small numbers.” In contrast to Peter, she identified selection 7 (inequality explanation) as *not readable* because “It had a lot more words and numbers stood out because… and then the bolded stuff.” Loren agreed to read a single sentence of selection 7, but had to guess at some of the symbols. She read, “Okay. The inequality $X$ minus two is *something to* five - same solutions as the inequality $X$ *something to* seven” (emphasis added to highlight translation corresponding to $\geq$). She explained, “I couldn't tell if they were greater than or less than.”

As demonstrated above, Loren’s primary independent reading strategy was rough reading. She indicated, however, that she did not always re-read when she noticed clues that might improve her comprehension. At one point she said, “I'd probably just keep reading.” This suggests that, despite Loren’s response, *frequently*, to item 10 (When I do a math problem, I read the problem more than once) on the MRHS, her re-reading habits may be inconsistent.

Loren took some time to explain to me how she approaches reading aloud in class. Imagining what she would do if she were reading aloud to the class and came to a symbol she
could not read, she said, “I would just stop and eventually the teacher would say the answer.” In other words, she counted on her teacher to fill the silence by starting to read where she left off.

Within-person neurobiological processing. Loren did indicate that she has some awareness of automated processes. In the process of reading part of selection 10 (ratios and proportions), she used two different phrases to interpret the vinculum. She read \[
\frac{20}{1} = \frac{40}{2}
\] as “20 over one equals 40 over two.” She read \[
\frac{x}{y}
\] as “X divided by Y.” I asked her if she knew how she decided when to say “over” and when to say “divided by.” She replied, “No, sometimes I say one and then the other time I'll say something else. I have both words in my head and it's just one comes out faster than the other.”

Some of Loren’s remarks insinuated that she took reading math and doing math to be innate cognitive abilities. She was astonished when I told her I had heard another student read \[
d = rt
\] as “distance equals rate times time.” She said, “That kid's smart! I would have never knew that!” Later during the interview, she explained that she did not have plans take anything other than “regular” math because “the other maths are too hard for my brain.” These remarks are in line with her responses to the MAS. She reported that she strongly agreed with item 7, I think math is hard. Both the interview data and Loren’s accuracy rate on the S-CDT (\(RD = 12\)) support the conclusion that reading symbolic mathematics is a major obstacle to her mathematical development.
Model of SMaLL. To better understand what SMaLL is for adolescents, I used a quantitative spectrum to guide the exploration of the qualitative data. I inspected variations at the extremes as well as variations among participants who seemed similar in terms of quantitative data. I used the DBCCC framework to guide the side-by-side discussion in order to make connections across the levels of SMaLL. The integrative spectrum analyses yielded a model of multilevel model of SMaLL.

Figure 4.33 shows the updated model of SMaLL superimposed on the original model of the DBCCC framework. The three gray circles represent the three general conclusions, one for each level of SMaLL. The white circles represent themes related to those general conclusions. Although the white circles are located with the level to which they are most associated, the red arrows reiterate that the mechanisms in this model should be understand as interactive and reciprocal.

The primary conclusion drawn from analyzing the cultural context is this: Learning to read symbolic mathematics requires assistance. Students view teachers as expert readers, but do not know how to interpret text like their teachers. When they are unable to read, their primary recourse is the help of a teacher. They view reading to themselves as a form of self-teaching, and, when reading aloud, presume that they are teaching others in the process. In math classrooms, students do not think about reading-to-learn mathematics. Rather, they read symbolic mathematics for the purpose of understanding a problem to be solved. Their long-term motivation for reading symbolic mathematics is academic and career success.
Figure 4.32. Multilevel model of symbolic mathematics language literacy among adolescents in a middle school in the Common Core era
The primary conclusion drawn from analyzing students’ metacognitive reflections and reported behaviors is this: Reading symbolic mathematics is a process of navigating ambiguous symbols. It necessarily entails the formation of a some perception of, or way of understanding, the symbols. To move from perception of the symbol to production of an English translation requires decisions about expanding the text (e.g., to add meaning) or compressing the text (e.g., to closely resemble the print).

The primary conclusion drawn from analyzing neurobiological data and students hypotheses about the role of their brain is this: Reading symbolic mathematics is a process of coordinating automatic processes. Students can develop automated responses to orthographic patterns. Students can automatically associate phonological patterns with orthographic patterns. Students can automatically associate semantic information with orthographic patterns. Importantly, students may have automatic responses to orthographic patters that impede cognitive processing (e.g., math anxiety).

Summary of Chapter 4

This chapter described the results from the quantitative strand, the qualitative strand, and the mixed methods integration. The quantitative strand entailed $t$-tests to assess group differences in orthographic processing, correlational analysis to assess relationships between orthographic processing and other relevant measures, and multiple regression analyses to determine whether measures of orthographic processing contribute to a model of mathematics achievement scores. The results suggest that, for this sample, there were no difference in orthographic processing across grades. However, there was a difference between the Math Extension and Math Intervention electives in terms of the accuracy rates associated with
orthographic processing. The correlational analysis suggested that orthographic processing was systematically related to a number of variables, but that those relationships might change across grades. The regression analyses indicated that measures of orthographic processing, math anxiety, and math print exposure can explain approximately 30% of the variance in mathematics achievement scores for 7th graders. For 8th grades, orthographic processing and math anxiety can explain approximately 50% of the variance in mathematics achievement scores.

The qualitative analyses entailed manifest and thematic approaches to determine the kinds of text students identify as readable (or unreadable) and the strategies they use to read them. The results show that, although students use different criteria to assess readability, most students apply some criteria related to the symbolic mathematics features. In addition, the results show that, although students use a variety of reading strategies, decoding symbolic mathematics is a critical strategy. Students who lack automaticity in reading symbolic mathematics report that they cannot apply common English language reading strategies (e.g., ‘sound it out’).

The model of SMaLL that emerged from the mixed methods integrative analysis included these three major findings: Adolescents seek assistance, primarily from teachers, to develop SMaLL. Reading symbolic mathematics is, at least until it is automated, an effortful process that entails navigating the ambiguity of the language of mathematics to produce and English translation of the text. Reading symbolic mathematics is dependent on the automated cognitive processes related to orthography, phonology, semantics, and affect.
Chapter 5: Discussion

This study was an exploration of symbolic mathematics language literacy (SMaLL) as a multilevel developmental phenomenon through the lens of developmental bio-cultural co-constructivism (DBCCC). Two research questions guided the design and implementation of the study: What is SMaLL for adolescent students in middle grades learning under the Common Core State Standards (CCSS)? How do students reading typical academic texts with symbolic mathematics experience SMaLL? This chapter describes the findings in relationship to extant theory and research (see Chapter 2) and the implications for the future. First, I discuss the inferences and meta-inferences generated by the analysis. Next, I highlight the implications for theory, methodology, and practice. Then, I provide a critical statement about the limitations of the study. Finally, I conclude the chapter with a final personal reflection.

Discussion of Results


> with this book I hope to push you gently toward reconsidering things you might long have taken for granted — such as how natural it is for a child to learn to read. In the evolution of our brain’s capacity to learn, the act of reading is not natural, with consequences both marvelous and tragic for many people. (p. x)

Likewise, this dissertation issues a call to reconsider what seems to be taken for granted in mathematics classrooms for the sake of the many students required to interact with texts written in the unique language of mathematics. Starting with the assumption that interacting with mathematical text necessarily involves reading, this study was an attempt to better understand
what SMaLL entails.

The organization of this discussion is guided by the major research questions. First, I take a post-positivist stance to discuss the quantitative strand and the inferences related to the quantitative research questions. Then, I take a constructivist stance to discuss the qualitative strand and the inferences related to the qualitative research questions. Finally, I take a dialectical stance to discuss the meta-inferences related to the integrative and mixed methods research questions.

**Quantitative inferences.** The quantitative strand was designed to answer these research questions: Does SMaLL differ across academic classroom cultures? What are the relationships between measures of orthographic processing, math print exposure, math reading habits, math anxiety, and math achievement? The overarching purpose of answering these questions was to draw inferences, from a post-positivist perspective, about the degree to which SMaLL and English language literacy are similar.

I had anticipated differences in SMaLL across grades because the analysis of the CCSS for mathematics (CCSS-M) suggested that the use of the language of mathematics changes annually with the curriculum (see Curriculum Theories). In addition, because I had presumed SMaLL to be an experience-dependent skill, I suspected differences across elective courses offering different academic experiences (see Theoretical framework). I used $t$-tests, correlational analysis, and multiple regression analyses to explore differences in orthographic processing as measured by the SMaLL Conventional Decision Task (S-CDT), the primary variables of interest, across grades ($7^{th}$ vs. $8^{th}$) and across electives (Math Extension vs. Math Intervention).
According to the *t*-tests, the only detectable group difference was found in the orthographic processing accuracy scores across electives. Although this suggests that orthographic ability may not evolve across grades, two plausible explanations for the insignificant across-grade results are related to design issues. First, the effect size may have been too small to detect in the context of this research design. Hill et al. (2008) suggest that normative annual change, in the absence of an intervention, declines in magnitude each year from kindergarten to 12th grade for both reading and mathematics. Their empirical benchmarks suggest an effect across grade levels at the middle school level could be, in terms of statistics, very small. Second, the nature of the sample may have influenced the results. All of the participants in this study were enrolled in a mixed-grade (including 7th and 8th grade students) mathematics elective. If a difference with a small effect size exists between 7th and 8th graders, a research design guided by a power analysis using a sample of students without a history of mixed-grade instruction may be necessary to detect it. The statistically significant result across electives suggests that, at the middle school level, curricular and cultural aspects of the classroom may be more relevant to SMaLL than grade level.

As anticipated, the correlational analysis provided evidence that accuracy in orthographic processing of symbolic mathematics was systematically related to math print exposure, math reading habits, math anxiety, and mathematics achievement. Like English language literacy, accuracy in orthographic processing of symbolic mathematics was positively correlated with print exposure (e.g., Cunningham, et al., 2001; Wolf et al., 2009). Like mathematics achievement, orthographic processing of symbolic mathematics was negatively correlated to math anxiety (Bai, 2011; Bai et al., 2009). Comparison of the correlation patterns across grades
suggest that the role of SMaLL, and the measures of orthographic processing in particular, changes in relationship to human development and/or curricular shifts. Typical neurobiological human development undoubtedly plays a role; however, given the theory of DBCCC and the assumption that SMaLL is biological secondary, differences are more likely related to the timing and nature of curricular shift towards the use of more sophisticated symbolic mathematics during instruction (Nelson, 2006). Frost (2012) claims that increasingly complex writing systems involve “some level of optimization aimed at providing their readers with maximal phonological and semantic information by the use of minimal orthographic units for processing” (p. 267). In light of Frost’s claim, the larger correlation between orthographic processing and mathematics achievement for 8th graders supports the hypothesis that the use of more sophisticated symbolic mathematics in the classroom increases the orthographic processing demand.

The patterns and strength of the correlations warranted multiple regression analyses to explore whether mathematics achievement, the primary variable of concern from the perspective of educational policy makers, could be modeled using variables related to SMaLL. The results indicated that, for 7th graders, accuracy in orthographic processing explained 33% of the variance in mathematics achievement after controlling for math anxiety and math print exposure. For 8th graders, accuracy in orthographic processing explained 50% of the variance in mathematics achievement after controlling for math anxiety. These models are particularly interesting in light of the fact more than a third of the participants agreed or strongly agreed with this statement related to math anxiety: *When I see a math problem, I worry about whether I can do it* (emphasis added). Together, these results raise the question of whether the onset of math anxiety, conceptualized as an affective neurobiological condition that interferes with the ability to do
mathematics (Bai, 2011; Bai et al., 2009), coincides with difficulty in orthographic processing of symbolic mathematics.

In general, the findings from the quantitative strand offered a proof of concept suggesting the S-CDT measures orthographic processing of symbolic mathematics, and, in addition, SMaLL is related to mathematics achievement. The pattern of response decisions and response times to items showed that the S-CDT performed, as designed, as a cognitive task producing data similar to that of a lexical decision task. However, the data and analysis fall short of supporting a convincing argument about the degree to which the task is semantic, lexical, phonological, and/or orthographic (Geary, 1993; Maruyama, et al., 2012; Rao & Singh, 2015). Using advanced equipment (e.g. fMRI, electroencephalography, or magneto-encephalography equipment) to monitor brain activity during the task would allow for a stronger argument about whether — and for whom — responses are associated with mapping text to meaning, mapping text to words, mapping text to sounds, or processing visuospatial complexity. Mapping brain activity has the potential to offer important clues about the nature of the S-CDT as well as individual differences in SMaLL that are not apparent from implementing the S-CDT as a behavioral task (e.g., Braet et al., 2012; Cirino, 2010; Maruyama, et al., 2012; Rao & Singh, 2015).

In this study, I conducted an exploratory factor analysis using a mixed methods approach — combining classical test theory, curriculum theory, and pedagogical content knowledge — to understand the Math Print Exposure Survey (MPES) and the Math Reading Habits Survey (MRHS) data. As they were implemented in this study, the surveys were sufficiently reliable to expose systematic relationships in the data. In general, the analysis suggests that, like English language literacy, SMaLL is related to print exposure; however, the differences in the
measurement tools limit the comparison. The Title Recognition Test (Cunningham et al., 2001) that inspired the MPES was designed to be a measure of only out-of-school print exposure. In contrast, given concerns that some adolescents have no exposure to symbolic mathematics outside of school, I designed the MPES to measure life-time math print exposure at school, at home, or anywhere else. The MRHS data and the qualitative results supported the supposition that out-of-school exposure to symbolic mathematics is rare among middle school students. Thus, the MPES, unlike the English language literacy print exposure assessment, may be tantamount to a measure of in-school print exposure. Thus, it remains unclear whether SMaLL and English language literacy are similarly related to in-school and out-of-school print exposure.

In conclusion, from a quantitative perspective, SMaLL and English language literacy appear to be somewhat analogous. Further measurement tool analysis and development is necessary for a deeper inspection of the similarities and differences between the two literacies. In the field of mathematics education, research in the area of measurement and assessment has long-been established as critical to informing research and practice (Crosswhite, 1979; Headley, Swoboda & Foote, 2016; Lesh et al., 2014). If the theory of SMaLL leads to the development of evidence-based hypotheses about what particular kinds of experiences and activities support SMaLL development, high-quality measurement tools will be critical to evaluating outcomes associated with implementing interventions, such as explicit SMaLL instruction, designed to foster SMaLL and, by extension, mathematics achievement.

**Qualitative inferences.** The qualitative strand was designed to answer these research questions: What kinds of texts with symbolic mathematics do students identify as readable (or unreadable)? What kinds of strategies do students use to read texts with symbolic mathematics?
The overarching purpose of answering these questions was to draw inferences, from a constructivist perspective, about the ways in which SMaLL and English language literacy are similar.

Given my conceptualization of SMaLL, I had expected participants to identify print features, particularly those features associated with the writing system of mathematics, as criteria for assessing the readability of a text selection. Consistent with my prediction, the thematic analysis of the semi-structured interviews revealed print features as the most prominent of four themes associated with the readability of the mathematical text selections. Participants discussed print features in terms of the appearance of symbolic mathematics elements, English language elements, and the overall structure of the text. Thus, the findings suggest that SMaLL makes a new contribution to current thinking about literacy in mathematics education. Both content area literacy (Fang & Schleppegrell, 2008), which is centered on bringing English language literacy to bear on the study of mathematics, and disciplinary literacy (Johnson et al., 2011), which is centered on bringing the literacy skills of disciplinary experts to bear on the study of mathematics, privilege the English language writing system (Wilson, 2011). However, the data showed that participants noticed mathematical symbols and syntax and weighed the demands of reading the symbolic mathematics elements against the demands of reading the English language elements to determine readability of mathematical text selections.

The students’ metacognitive reflections on reading strategies suggest that SMaLL involves neurobiological processes similar to those associated with literacy in English. In particular, they suggest SMaLL entails orthographic, phonological, semantic, and lexical cognitive processes (Wolf et al., 2000). For example, Elena described her reading as a
phonological process saying, “I've heard it so many times like that.” And, Peter described his reading as semantic process explaining that he was “subconsciously thinking about what each of the variables sort of means.” Jackson suggested that reading symbolic mathematics was a lexical process. In contrast to reading English, which he experienced as a phonological sound-it-out process, reading symbolic mathematics was a matter of “dealing with words in your brain.” Future studies of SMaLL may inform the debate about the relationship between cognitive processes and nature of writing systems (Tan et al., 2005).

The qualitative analysis revealed that students rarely, if ever, receive explicit instruction aimed at developing SMaLL skills. Instead, teachers act as readers. Yet, reading symbolic mathematics was clearly necessary for participation in commonplace independent classroom activities such as worksheets and tests. For example, Jackson noted, “We kind of have to read [what’s on the paper] if we want to complete the problem.” These data support the conclusion drawn from the analysis of the CCSS in Chapter 2 (see Grade level standards): SMaLL is hidden in the curriculum in that it is related to typical academic activities and necessary from mathematics achievement, but it is not explicitly acknowledged, named, or taught. Hidden curriculum can have a profound influence — sometimes deleterious — on educational opportunity and outcomes (Anyon, 1980; Au, 2012; Keefe & Copeland, 2011; Martin, 2012). Because curriculum is a “tool that structures the accessibility of knowledge” (Au, 2012, p. 44), hidden curriculum has the potential to deny some students and privilege others. In order to question whether students have appropriate educational opportunities related to SMaLL, it is necessary to first acknowledge its existence in theory and practice. This study was conducted at a school using an idealized curriculum. Studies in other schools are necessary to understand how
SMaLL manifests in other schools operating under other curricular policies.

In conclusion, from the constructivist perspective, SMaLL is analogous to English language literacy in that reading symbolic mathematics is fundamentally a process of translating written text to spoken English or, as Bobby’s explained, saying words that represent “how it’s printed out.” Like reading English, reading symbolic mathematics depends on knowledge about the conventions of the writing system. This is somewhat counterintuitive given theories in mathematics education that symbolizing emerges from conceptual understanding (e.g., Sfard, 2009; De Cruz & De Smedt, 2013). However, it is consistent with Nemirovsky and Monk’s (2009) fusion theory, which holds that that symbols need not be fully understood for the act of symbolizing to be useful for learning mathematics. Fusion theory describes symbolizing as an act of managing ambiguity that is similar to what I refer to as reading symbolic mathematics. For some students, SMaLL and strategies for reading symbolic mathematics may be the gateway to mathematical learning.

Mixed methods meta-inferences. The multilevel concurrent mixed methods research approach was designed to generate meta-inferences about SMaLL. Because the rationale for this study was initiation, the meta-inferences, or findings that extend or transcend the findings of each strand (Creswell & Plano Clark, 2011), were expected to “initiate new interpretations, suggest areas for further analysis, or recast the entire research question” (Rossman & Wilson, 1985, p. 633). The integrative research questions and integrative procedures were not expected to confirm generalizable hypotheses with any probabilistic degree of certainty (as is often the case in the quantitative research tradition). Nor were they expected to be limited strictly to the sample, ungeneralizable (as is often the case in the qualitative tradition). The questions were
provocative in ways that required me to “turn ideas around” (Rossman & Wilson, 1985, p. 633) but focused in ways that required me to compare SMaLL to English language literacy, explore SMaLL as a multilevel developmental phenomenon, and develop a model of SMaLL. In short, the meta-inferences are evidence-based answers, with implications for educational research and practice, to these open-ended research questions: In what ways is SMaLL analogous to English language literacy? What connections, if any, emerge across cultural, behavioral, and neurobiological levels of SMaLL? What theories or models, if any, of SMaLL emerge?

I drew meta-inferences from a spectrum analysis of the text selections and a spectrum analysis of SMaLL among the participants. I used joint displays to analyze each of the text selections and compare them as a means to understand readability in terms of both quantitative and qualitative data. I also use joint displays to analyze each participant and compare their SMaLL skills in terms of both quantitative and qualitative data.

The analysis revealed that readable text selections were associated with high print exposure to similar symbolic mathematics and high accuracy in identifying similar conventionally written orthographic patterns. In contrast, unreadable text selections were associated with lower print exposure to similar symbolic mathematics and lower accuracy in identifying similar conventionally written orthographic patterns. The results were consistent with current theories that suggest literacy entails both top-down cultural processes and bottom-up neurobiological processes (Baltes et al., 2006; Bjorklund, 2012; Faust & Kandelshine-Waldman, 2011). Links between reading and print exposure have been used to infer that cultural differences, such as varying learning environments, influence literacy development (e.g., Cunningham et al., 2001). Links between reading and orthographic processing have been used
to infer that neurobiological differences influence literacy development (e.g., Wolf, 2007). These integrated findings, which connect readability, print exposure, and orthographic processing, suggest that SMaLL is similar to English language literacy in that they both involve across-level cultural-neurobiological mechanisms.

I distilled all of the meta-inferences into a multilevel model of SMaLL. Table 5.1 describes the model in a table that shows the three levels of SMaLL — cultural, behavioral, and neurobiological — from top to bottom (see Figure 4.33 for a circular illustration of the model). As shown in the model, learning to read the language of mathematics is a cultural process that requires assistance. In other words, SMaLL develops through social interaction with the language. In the interviews, students indicated little, if any, encouragement to read-to-learn mathematics independently. On the contrary, students generally reported reading mathematics in the service of completing assigned problems. This is not surprising in light of the CCSS-M and current educational policy (see Chapter 2). At the cultural level, SMaLL appears to be somewhat different from English language literacy. To support English language literacy, students are encouraged to read outside of school, given time to read at school, and taught to read for multiple purposes (e.g., Hughes-Hassell & Rodge, 2007).
Table 5.1

*Multilevel model of SMaLL.*

<table>
<thead>
<tr>
<th>Level</th>
<th>Components and mechanisms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cultural</td>
<td></td>
</tr>
<tr>
<td>Language:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Learning to read requires assistance</td>
</tr>
<tr>
<td></td>
<td><strong>SMaLL is...</strong></td>
</tr>
<tr>
<td></td>
<td>• For problem solving</td>
</tr>
<tr>
<td></td>
<td>• For school and work</td>
</tr>
<tr>
<td></td>
<td>• Self-teaching and teaching others</td>
</tr>
<tr>
<td></td>
<td>• Interpreting text like a teacher</td>
</tr>
<tr>
<td>Behavioral</td>
<td></td>
</tr>
<tr>
<td>Metacognition:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Reading is navigating ambiguous mathematical symbols</td>
</tr>
<tr>
<td></td>
<td><strong>SMaLL is...</strong></td>
</tr>
<tr>
<td></td>
<td>• Perceiving symbols</td>
</tr>
<tr>
<td></td>
<td>• Producing an English translation</td>
</tr>
<tr>
<td></td>
<td>• Compressing meaning</td>
</tr>
<tr>
<td></td>
<td>• Expanding text</td>
</tr>
<tr>
<td>Neurobiological</td>
<td></td>
</tr>
<tr>
<td>Cognition:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Reading is coordinating automatic responses</td>
</tr>
<tr>
<td></td>
<td><strong>SMaLL is...</strong></td>
</tr>
<tr>
<td></td>
<td>• Orthographic processing</td>
</tr>
<tr>
<td></td>
<td>• Phonological processing</td>
</tr>
<tr>
<td></td>
<td>• Semantic processing</td>
</tr>
<tr>
<td></td>
<td>• Emotional regulation</td>
</tr>
</tbody>
</table>
Learning to read the language of mathematics is also a behavioral process that entails interacting with particular instances of text. It involves navigating the ambiguity of symbolic mathematics and making decisions about compressing the meaning or expanding text to produce an English language translation or representation of the text. The integrated mixed methods results indicated that all students, regardless of mathematical proficiency, had to read symbolic mathematics in the course of learning mathematics. This suggests that theories accounting for the strategies of novices, such as Nemirovsky and Monk’s (2009) fusion theory, Siegler’s overlapping waves (2000), and this model of SMaLL, are critical to research and practice.

Finally, learning to read the language of mathematics involves neurobiological processes. The findings indicate that SMaLL depends, minimally, on the ability to make orthographic, phonological, and semantic connections. Thus, at the neurobiological level reading symbolic mathematics appears to involve the basic processes associated with reading English (see Reading Models). Given the similarities, interventions adapted from successful reading interventions such as RAVE-O (Retrieval, Automaticity, Vocabulary, Engagement with language, and Orthography; Wolf et al., 2000; Wolf et al., 2009) may be useful to support mathematical learning among students with limited SMaLL. As I expected, SMaLL was also associated with math anxiety. This is an important finding given Lyons’ and Beilock’s (2012) recent recommendations for addressing the needs of students with high math anxiety. They cautioned against specialized mathematics classes for students with math anxiety. In addition, they cautioned against efforts to eliminate math-related anxiety responses altogether. Instead, they recommended helping “students learn how to marshal cognitive control resources and effectively check one’s math-related anxiety response once it occurs” (p. 2109) in order to intentionally
adopt an appropriate strategy to proceed. For students whose math anxiety is induced or exacerbated by interaction with symbolic mathematics, interventions aimed at SMaLL development may provide strategies that reduce anxiety-related math difficulties.

In conclusion, the multilevel model of SMaLL that emerged from the mixed methods integration suggests that SMaLL is similar to English language literacy to some degree. They are most similar at the neurobiological level where the two literacies appear to entail the same cognitive processes. Further research using equipment to map brain activity while reading symbolic mathematics is warranted to determine the extent to which this conclusion holds from other paradigmatic perspectives. The literacies appear to be least similar at the cultural level where reading symbolic mathematics is implicitly accepted as a have-to and English language literacy is explicitly encouraged as a leisure activity. Given this model of SMaLL, it is incumbent upon us to debate and investigate whether SMaLL, like English language literacy, demands explicit instruction to evolve from an effortful coordination of decoding strategies and cognitive subprocesses to a relatively effortless automated process that allows attention to be allocated to comprehension (Wolf et al., 2009).

Implications

This study was designed to take a first look at SMaLL as a multilevel phenomenon. In answering the question of what SMaLL is for adolescent students in middle grades learning under the CCSS, I intended to raise questions among communities of research and practice. In this section, I describe the implications for theory, methodology, and practice and make recommendations for the future.
**Theoretical implications.** This study was grounded in a definition of SMaLL and framed by the DBCCC theory of human development. It was aimed at initiating an evidence-based model of SMaLL development. Although this study did not entail a rigorous grounded theory approach in the qualitative tradition, it used a multilevel concurrent mixed methods research design to embody a central grounded theory principle. Specifically, the study was designed to yield “the possible range of empirical meanings, actions, and processes” (Charmaz, 2015, p. 1616, emphasis in original). The result was a robust multilevel theory of SMaLL, a new lens through which to view mathematics education and mathematics cognition. In the future, it will be important to locate this theory of SMaLL among theories in mathematics cognition that guide educational psychology, theories in mathematics education that guide instructional practice, and theories in curriculum studies that guide educational policy.

The current study situates SMaLL in relationship to theories of English language literacy and reading acquisition. The successful application of measurement tools adapted from those fields of study suggests that SMaLL, like English language literacy, is associated with orthographic knowledge and awareness. In other words, reading the language of mathematics requires the automated recognition of symbols as meaningful (or not) elements of the language. In addition, SMaLL requires an understanding of the permissible patterns or the syntax for combining the symbols. Future research should explore the possibility of comparing SMaLL to theories of second language literacy. This might entail a lexicon project similar to those being developed for languages around the globe (e.g., Sze et al., 2014; Tan et al., 2005, Yap et al., 2010).
For the students in this study, SMaLL developed as a result of incidental learning or implicit instruction. The image of reading (or lack thereof) in mathematics classrooms that emerged from the data suggests that the SMaLL demands of engaging in the mathematical practices (see Table 2.1) are not explicitly recognized or acknowledged. This may be a consequence of the way that curriculum and educational policies, specifically the CCSS and standardized testing policies, frame English language arts and mathematics as distinct (Au, 2012; Cheuk; 2012). This may be a consequence of prevailing symbolizing theories in mathematics education that suggest conceptual understanding should precede the use of symbols (De Cruz & De Smedt, 2013; Sfard, 2009). Whatever the reason, SMaLL appears to be hidden, existing primarily as a functional literacy, a utilitarian tool to be learned and used only as needed, rather than a critical literacy or tool of empowerment (Knoblauch, 1990). Ultimately, this study calls into question theories, policies, and practices that frame literacy, particularly SMaLL, as irrelevant or secondary rather than integral to learning mathematics.

**Methodological implications.** In Chapter 2, I discussed current issues in multilevel studies using mixed methods research designs, qualitative research designs, and quantitative research designs. Then I proposed a new conceptualization of multilevel mixed methods research design. In addition to meeting those design criteria, this study is methodologically novel in two respects. First, it uses a theoretically driven conceptualization of levels. Second, it entails a within-person level. I crafted the multilevel concurrent mixed methods research design, which specified particular ways of mixing quantitative and qualitative data collection and analysis techniques (Plano Clark & Ivankova; 2016), for the express purpose of exploring SMaLL within and across levels associated with the theoretical framework, DBCCC. In this
section, I describe this study as a methodological exemplar for multilevel mixed methods research designs.

A multilevel mixed methods design is grounded in assumptions about a multilevel system for the purpose of better understanding the system, its components, and its mechanisms. The research design for this study was driven by the assumption that SMaLL was a multilevel developmental phenomenon. SMaLL, viewed through the lens of DBCCC (see Figure 1.2), was presumed to entail change at three levels. Therefore, the multilevel concurrent mixed methods research design was crafted to allow for a robust exploration of SMaLL in terms of culture (related to classroom language), behavior (related to student metacognition) and neurobiology (related to within-person cognition). A similar multilevel approach can be applied to a wide range of developmental phenomenon.

The concurrent strands of the research design for this study were designed to address two different levels of the system. In addition, each strand was designed to yield insights into the third level. The quantitative strand was designed primarily to explore cognition (the phenomenon at the neurobiological level of SMaLL). The qualitative strand was designed primarily to explore metacognition (the phenomenon at the behavioral level of SMaLL). Both strands included elements designed to explore language in the classroom context (the phenomenon at the cultural level of SMaLL). For example, the math print exposure and math reading habits surveys implemented in the quantitative strand served the multiple purposes of supporting conclusions with regard to the quantitative questions, creating links for integration with the qualitative strand, and generating inferences about classroom context. In addition, asking interviewees questions about the resources they employed when independent reading
strategies failed served the multiple purposes of supporting conclusions with regard to the qualitative questions, creating links for integration with the quantitative strand, and generating inferences about classroom context. Thus, each strand independently supported a rigorous investigation of a single level of SMaLL and included sufficient links to support an investigation of the third level of SMaLL through integration. Conducting a multilevel concurrent mixed methods study requires careful planning at the design stage to ensure that each strand has the potential for high-quality inferences as well as links for integration.

Both the sampling strategies and data collection tools involved more than one level. For the quantitative strand, feasibility issues limited me to a convenience sample of students. To ensure variation in classroom context at the cultural level, I used a clustered sampling strategy. Specifically, I selected two courses (Math Intervention and Math Extension) taught by three different teachers during several different class periods. For the qualitative strand, I used a maximum variation sampling strategy. Generating a sample that varied as much as possible in terms of mathematics achievement and reading habits was intended to maximize variation at the behavioral and neurobiological levels. For the quantitative strand, I generated within-student cognitive data at the neurobiological level. In addition, items on the surveys solicited reports related to the other two levels. For the qualitative strand, the interview generated metacognitive data for each student at the behavioral level as well as reflections on classroom context and co-constructed inferences about the neurobiological processes. In designing a multilevel mixed methods research study, it is important to ensure that the sampling strategies and data collection procedures address the demands of each strand of inquiry as well as each level of the guiding theory.
The data analysis and integration techniques supported within-level and across-level inferences and meta-inferences. As a result, I developed a model of SMaLL as a developmental phenomenon that is shaped by language use in classrooms, students’ metacognitive approaches to reading, and students’ cognitive processes during reading. The study generated greater insight into the components (e.g., curriculum, students, brains) and the mechanisms (e.g., self-teaching, compressing, memorizing, etc.) that operate as a system of across-level change. Although this study was designed for theory-building, a multilevel mixed methods research design can also be used to refine or test a multilevel theory.

Together, the revised conceptualization of multilevel mixed methods research designs I proposed in Chapter 2 and this exemplar may serve mixed methods researchers who wish to pursue interdisciplinary work driven by a multilevel theory. The definition is flexible enough to allow for differences in paradigmatic perspectives. In addition, it is flexible enough to be used as a guide to craft multilevel studies from basic mixed methods research designs. Finally, it is flexible enough to allow for the incorporation of sophisticated analytic techniques from multiple methodological traditions. Importantly, the definition is specific enough to provide a guideline for assessing the quality of multilevel mixed methods research students. This dissertation, by example, illustrates how to use the definition of a multilevel mixed methods research design in conjunction with a basic mixed methods framework to design and implement a multilevel concurrent mixed methods research design for the purpose of initiating a theory of multilevel phenomenon of human development.
Practical implications. The results of this study suggested students differ in their SMaLL skills. In addition, the results suggested that SMaLL is related to factors that concern teachers such as exposure, habits, affect, and self-efficacy. This study does not describe ‘what works’ for helping students develop SMaLL skills. However, this study does describe ‘what is’ among a sample of adolescents in a high-performing middle school operating under an idealized curriculum. These insights into how middle school students read — or struggle to read — the language of mathematics has practical implications for pedagogical content knowledge among mathematics teachers and instructional design in mathematics classrooms.

Students reported that they read mathematics as needed as determined by their teacher. In general, they read mathematical text independently in order to complete assignments and assessments. On occasion, reading is a classroom activity guided by the teacher, usually for the purpose of introducing or reviewing mathematical problems. The value and purpose of reading implied by teachers’ enactment of curriculum and the nature of mathematics assignments is central to SMaLL development. If reading aloud is, as Wolf (2007) suggests, the sine qua non of reading acquisition, then it may be necessary to rethink explicit reading instruction (or lack thereof) in mathematics classrooms.

This study suggests that math self-efficacy play a role in SMaLL. However, it appears the role is minor in comparison to other factors. I had expected it to play an larger role in readability by way of contributing to persistence or comprehension. Quincy offered an alternative reason students may have discussed their ability in identifying selections as readable or unreadable. As he was looking for a readable selection he was, apparently, imagining problems related to each. He chose selection 2 (volume of a cylinder) as readable saying, “if I
wanted to get the best grade possible, if this were a challenge, I felt like the easiest challenge here would be this one because it's very simple and easy.” This suggests that, when considering the introduction of explicit reading activities, teachers need to be aware that classroom assessment practices can influence the ways students think about reading and doing mathematics. To develop SMaLL and value it as a skill that empowers them to read-to-learn, students may need more opportunities to reading mathematical text in the absence of a problem solving scenario.

Some students’ remarks indicated a belief that reading symbolic mathematics should be an automated process or, at least, could become an automated process. For example, Bobby said, “You can't really sound it out or do anything like that, and you can't just have an educated guess. You just have to know it.” The results suggest that adolescents, collectively, approach effortful reading with a variety of intentional strategies. Some students described multiple reading strategies. Others described only one or two. Thus, the ability to read symbolic mathematics may emerge in accordance with Siegler’s overlapping waves model (2000). That is, SMaLL development may entail adopting multiple strategies and applying them in strategic ways until automaticity develops. Further research should seek to identify those strategies and determine whether there is a natural progression of strategies among typical students. This study did not produce evidence of a single best strategy. Explicit instruction related to multiple strategies may be useful elements of intervention for struggling learners.

Importantly, typical classroom observation is insufficient to determine students’ reading strategies or ability to read symbolic mathematics with comprehension. Freddie’s response to selection 1 (surface area of a prism) provides an illustration. He read
$SA_{\text{rectangular prism}} = 2\times l \times w + 2\times l \times h + 2\times w \times h$ as “S times A rectangular prism equals two times L times W plus two times L times H plus two times W times H.” Although his prosody and fluency suggested he understood what he had read, a moment later he said, “I think I just missed out on something…That it wasn't just a bunch of variables but actually length and width.” Some students may not recognize reading or comprehension errors, despite engaging in metacognitive reflection, without feedback. For example, Jackson read $d = rt$ as “D equals RT.” Then he attempted to expand the text to clarify his meaning for me. He said, without recognizing his expansion error, “D stands for diameter. R stands for radius.” Students with dramatically different SMaLL skills may produce similar compressed readings. Students who struggle to read produce compressed translations because they do not associate the symbols with meaningful words whereas students who are good readers compress translations to save time. In summary, a compressed reading should not be interpreted as an indicator of reading comprehension.

This study suggests that further research and a deeper understanding of SMaLL development may contribute meaningfully to teachers’ knowledge of adolescent learners. As middle school students transition from learning mathematics as arithmetic to learning mathematics as algebra, the SMaLL demands increase in terms of the symbols and syntax. Classroom practice has the potential to send explicit and implicit messages about the role of reading in mathematics. Focusing on reading as a vital element of mathematical practice in its own right may be a means to help some students. Elena was clear in her hope that this research will inspire teachers to do just that.

As we co-constructed a summary of our interview, Elena remarked, “We've learned how to do this stuff and we've learned what it's for and where to use it. But we've never really been
taught how to read it. It's not something they really focus on.” When I asked, “So what's your opinion of that? Good idea, bad idea, somewhere in between?” She replied,

Honestly, I kind of think it's a bad idea. I think that they should teach us how to read things like this because the fact that I know this and I know where it can be used and how to do it and stuff, but I don't know how to read. That seems a little weird. It just seems a little backwards, so I do think they should teach us more about that.

Limitations

This dissertation describes a rigorous study of SMaLL among adolescents in a middle school. However, research is always limited in various ways. The conceptual foundations described in Chapter 1 highlight the ways in which this study was framed, and limited from the outset, by my perspectives. The methods described in Chapter 3 highlight the ways in which this study was planned and limited in scope by the research design. This section discusses additional limitations with implications for understanding the results.

The sampling process may have had the unintended consequence of excluding some students in the target population. First, it is plausible that students who did not elect to enroll (or who were not enrolled by a parent or teacher) in either the Math Extension elective or the Math Intervention elective have different SMaLL skills. Soliciting participants from elective courses made this study feasible and likely captured the spectrum and extremes of the target population in terms of mathematics achievement, reading habits, and academic experiences. However, the unavoidable exclusion of students enrolled in only required mathematics courses may have generated a sample that is not representative of (i.e. in statistical proportion to) typical middle school students. Second, field notes from the assent process suggest that rationales for
participation or nonparticipation may have been related to variables in the study. For example, some students appeared to be eager to participate in the study as a means to avoid their elective math class or math assignment. Others appeared to decline in order to stay in their elective math class and work on a math project. Because the results of this study are dependent on the samples, interpretations and generalizations should be made cautiously and conditioned on the nature of the sample.

Because I conducted this exploratory study using novel data collection tools, all inferences should be conditioned on the operational definitions of the variables. Further research is required to fully develop and understand the nature of the quantitative measurement instruments. Based on the results in this study, a longitudinal study following participants over time would be particularly useful for understanding the relationships between the measures. The results of this study do not support claims about what causes differences in any aspect of SMaLL. Nor do the results of this study support claims about what degree of SMaLL or aspects of SMaLL are important for students to have or acquire.

The S-CDT was designed to be analogous to lexical decisions tasks used in other languages. The results of this study suggest that this task, in particular, is worthy of further development and more advanced analysis. The S-CDT would benefit from item development. For example, participants’ responses to reading selection 6 (absolute value example) indicate that more items are important to explore responses to differences in the location of \[\text{relative to variables and other symbols. In addition, item development should also be focused on the location of subscripts and superscripts, which emerged as particularly difficult patterns to read. Further research should also include experimenting with variations in the instructions for the S-}

320
CDT. In this study, students were instructed to be accurate keeping speed in mind. The results may have differed had they been instructed to be fast keeping accuracy in mind. Further research should also include variations in the proportion of conventional to unconventional items. In this study, the ratio of conventional-unconventional items was 1:1. The results may have differed if the ratio of conventional to unconventional were, for instance, 2:1 or 1:2.

The S-CDT was analyzed using rudimentary statistical tools following practices established in published literature using similar tools. This entailed extensive data cleaning, aggregate scoring, and the separate analysis of response times and response decisions. Thus, the process entailed data reduction and treated dependent measures as independent measures. Further research should include a review of advanced statistical techniques with the potential to explore and explain the data set as a whole.

The surveys used in this study relied on self-reports of feelings, behaviors, and experiences. Thus, the responses may have been influenced by social desirability (Krumpal, 2013). From a post-positivist perspective, social desirability contributes to error in the data and, thus, impedes the interpretability of the results. From a DBCCC perspective, some degree of social desirability was to be expected as a consequence of cultural influences. The limitation arises from having no statistical or methodological means to assess the impact of social desirability or make an accounting of it. In this study, the mixed methods approach addressed social desirability by bringing qualitative data to bear on the interpretation of the survey data. Future research might include alternative theoretical frameworks, methodological approaches, and/or sources of data.
In general, the qualitative analysis was limited by the short engagement with participants. In the qualitative tradition, validity is enhanced by prolonged and repeated engagement (Hatch, 2002; Onwuegbuzine & Leech, 2007). However, I was only permitted to plan 30 minute interviews. The interviews I conducted during that small time frame depended on the text selections chosen by each participant. Thus, the qualitative data were not intended to be directly comparable in that the interviews entailed discussion about different texts for different reasons in different orders. Future research might include extended or multiple interviews that make it possible to discuss similar text with each of the participants. Although I included real-time member checks during the interviews, future research might also include member checks of summary reports.

The mixed methods inferences are relatively limited in scope compared to the potential of the data set. The extensive data generated by the quantitative instruments and qualitative interviews suggest a vast array of possibilities. As an independent researcher it was not possible to pursue all of the potential connections and emerging patterns. This dissertation explored SMaLL as a multilevel developmental phenomenon using summary quantitative scores and major qualitative themes. Bringing my pedagogical content knowledge to bear on all of the data was a strength of this study. However, working with a research team would have made it possible to include additional explorations of connections between quantitative items and minor qualitative themes.

**Reflections.** My natural interest in mixed methods stems from my eagerness to answer complicated questions. I am interested in methods with the potential to fill in big gaps rather than little cracks in the research literature. In addition, I am interested in doing research of consequence. From my point of view, all research is consequential. What I mean by doing
research of consequence is this: I want to conduct research that has some meaning for professional colleagues in my research community, for professional colleagues in my educational community, and, most importantly, for learners. Designing a multilevel concurrent mixed methods study was difficult precisely because it entailed making sophisticated arguments in a way that would press every person willing to give me an audience to take an interest or, at the very least, pause and ask the questions I now have a better answer for: Is SMaLL a thing? And how do you know?

In laymen’s terms, I took a mundane activity experienced by virtually everyone and made a big deal about it. I saw a significant omission in the literature where others saw a trivial distinction. As described in the previous chapters, the multilevel approach to this study was necessary for reasons that can be explained in terms of research questions, theoretical frameworks, and paradigmatic complications. However, in hindsight, the multilevel approach was necessary for a very simple reason: There was a discrepancy between the size of the gap I saw and the size of the gap others imagined. As an analogy, imagine, with me, trying to find a small fish in a big sea. For me, the tool that comes to mind is a net, a large net. The process that comes to mind is repeatedly dropping the net into the sea, lower and lower each time. Upon reflection, the multilevel approach gave me a tool and a process to look for this small thing that might be anywhere.

As I conducted the analysis, I became very aware that the “naturally occurring” institutionally defined levels (Teddle & Tashakkori, 2009, p. 156) used in many multilevel studies were fundamentally different than the fluid levels I envisioned. Institutionally defined levels are nouns. In other words, they are mapped to distinct people, places, or things. However,
for this study, the levels were adjectives. In other words, the levels were permitted to entail any number of things that could be justified as cultural, behavioral, neurobiological. Ironically, the theory of DBCC is complex because it is parsimonious with respect to assigning things to each level. In fact, in my view, the essence of the theory is that, despite the urge to simplify human development by sorting things into distinct levels, it is necessary to allow for across-level fluidity and complexity. In some sense, the theory demands remaining open to the idea that a level is not what it, at first, appear to be. Applying the theory of DBCCC to this novel conception of SMaLL required a willingness to allow the levels to be ill-defined and indistinct until the results defined them.

In hindsight, I suspect that conflating levels with units of analysis may be a hazard in conducting theoretically driven multilevel mixed methods studies. As I conducted the study, I had to effortfully refuse to reduce a level to a unit of analysis and intentionally use the fluid conceptualization of levels associated with the theoretical framework. For example, it took some time to justify math anxiety as most closely associated with the neurobiological level. From the outset, math anxiety was most easily conceptualized as a student-level variable because, for this study, math anxiety was measured by a survey completed by the student to describe an attribute of the student. However, math anxiety can be conceptualized as an attitude towards math associated with social interactions (Gunderson, Ramirez, Levine & Beilock, 2012) or as a physiological and emotional reaction to doing math (Lyons & Beilock, 2012). In other words, math anxiety could be a cultural construct or a neurobiological construct, respectively. In the end, I used the guiding questions that emerged from my methodological reflections (see Variable map in Chapter 3) and the findings to conclude that, with respect to SMaLL, math anxiety is
most reasonably considered a neurobiological construct that influences reciprocal interactions with the cultural and behavioral levels. Ultimately, it was integrating the data — and repeatedly viewing the data through the lens of the theoretical framework — that made it possible to discern the interdependent and transactional levels of SMaLL.

**Conclusion**

This study produced a multilevel model of SMaLL based on the experiences of adolescent students in middle grades learning mathematics under the CCSS. The model was constructed of inferences and meta-inferences generated by a multilevel concurrent mixed methods research design grounded in the DBCCC theory of human development. It explains reading the language of mathematics as a system of cultural, behavioral, and neurobiological components and mechanisms. In addition, the model reveals three important aspects, one for each level, of SMaLL. First, developing SMaLL, or learning to read symbolic mathematics, requires assistance and depends on social interaction and shared purpose. Second, reading the language of mathematics is a process of navigating the ambiguity of the symbolic mathematics. It involves metacognitive behavior aimed at perceiving the meaning of symbols and producing an English language representation of the written text. Third, reading the language of mathematics involves the coordination of automated cognitive processes. In particular, it entails coordinating orthographic processing, phonological processing, and semantic processing. For students with math anxiety, it entails emotional regulation as well.

This study of SMaLL was developed and implemented in an exploratory manner. Specifically, I designed the study to generate a model of SMaLL as a point of initiation for future research. The concept, and grounding definition, of SMaLL as the ability to read and write in the
language of mathematics grew out of my experiences teaching a wide range of students. As a teacher, I was fascinated by my students’ efforts to read symbolic mathematics and wondered what role reading played in their mathematical development. The quantitative results in this study suggest that the ability to read symbolic mathematics with accuracy and automaticity is positively related to mathematics achievement. In addition, the results suggest, as I suspected, that math anxiety is related to reading symbolic mathematics. The qualitative results of the study suggest that adolescents bring different SMaLL skills to the practice of reading symbolic mathematics. Many recognize that the symbol-to-sound mapping of the language of mathematics differs from the letter-to-sound mapping of English. In the absence of explicit instruction, some students develop a number of independent reading strategies. Others have limited strategies and limited success in reading symbolic mathematics. When they are unable to read, most students depend on their teachers as readers. I used integration techniques to combine the quantitative and qualitative findings, culminating in the development of this initial multilevel model of SMaLL. My hope is that this study will inspire researchers, educators, and policy makers to re-imagine that challenge of learning mathematics through the lens of SMaLL.

This study is an exemplar of a multilevel concurrent mixed methods research design. It illustrates the entire research process — from design to implementation to reporting — necessary to conduct a mixed methods study framed by a multilevel theory for the purpose of generating knowledge about a multilevel system. The process required careful attention to the multilevel theory and multilevel purpose at every stage. The quality of this study is due, in part, to careful design of the quantitative and qualitative strands. The strands were designed to be sufficient as independent studies of a single level during the concurrent phase. In addition, the strands were
designed with links of potentiality for downstream integration. This dissertation provides insight into the demands of sampling, data collection, data analysis, and integration associated with conducting a complex study using both quantitative and qualitative techniques. It is my hope that this work will provide mixed methodologists and interdisciplinary researchers with a guide for conducting high-quality multilevel mixed methods research.
References


Routledge.


methods analysis. *Behaviour Research and Therapy, 63*, 43-54.


accountid=2909


finding_overlap_in_the_common.html


methods research. *Journal of Mixed Methods Research, 1*(2), 112-133.


Kyei-Blankson, L. (2014). Training math and science teacher-researchers in a collaborative


Nemirovsky, R., & Monk, S. (2009). "If you look at it the other way...": An exploration into the nature of symbolizing. In P. Cobb, E. Yackel & K. McClain (Eds.), *Symbolizing and
communicating in mathematics classrooms: Perspectives on discourse, tools, and instructional design (pp. 177-223). New Jersey: Lawrence Erlbaum.


Plano Clark, V. L., & Sanders, K. (2015). The use of visual displays in mixed methods research: Strategies for effectively integrating the quantitative and qualitative components of a study. In M. McCrudden, G. Schaw & C. Buckendahl (Eds.), *Use of visual displays in research and testing: Coding, interpreting, and reporting data* (pp. 177-206). Charlotte, NC: Information Age Publishing.


Spring, J. (2009b). The Republican education agenda for the twenty-first century: The culture wars. In Political agendas for education: From change we can believe in to putting
America first (4th ed.).


## Appendices

Appendix A: Parent Letter .................................................. 358  
Appendix B: Parent Information Sheet .................................. 359  
Appendix C: Youth Assent Form ........................................... 362  
Appendix D: Math Anxiety Handout ...................................... 364  
Appendix E: Maximum Variation Matrix Worksheet .................. 365  
Appendix F: SMaLL-Conventional Decision Task (SMaLL-CDT) 367  
Appendix G: Math Print Exposure Survey (MPES) .................... 370  
Appendix H: Math Reading Habits Survey (MRHS) .................... 371  
Appendix I: Math Anxiety Survey (MAS) ............................... 372  
Appendix J: Semi-Structured Interview Protocol ...................... 373  
Appendix K: Readability: Coding Concept Map ....................... 390  
Appendix L: Reading Strategies: Coding Concept Map ............... 391  
Appendix M: Joint Display for Selection 2 ............................ 392  
Appendix N: Joint Display for Selection 3 ............................ 394  
Appendix O: Joint Display for Selection 4 ............................ 396  
Appendix P: Joint Display for Selection 5 ............................ 398
April 10, 2015

Dear Parents or Guardians:

I am doing a research study called *How do students read in mathematics class?* at [Redacted]. I am writing to invite your student to participate. The purpose of the study is to understand how students read the special texts they encounter in math classrooms. I hope that this will lead to improvements in teacher education and classroom instruction that will help future students. More immediately, this study will help the teachers in [Redacted] understand students in the district.

If your student participates, he/she will work with me. I am a teacher-researcher with a background in math education. I have experience teaching math students of all ages. I often volunteer as a way to share my interests. For example, I have been a volunteer math tutor in [Redacted]. Currently, I am a doctoral candidate in the Educational Studies program at the University of Cincinnati. My advisor, Dr. Vicki Plano Clark, will be on the research team and will guide this study.

I have attached an information sheet to explain the research study in more detail. Please contact me if you have any questions.

*If you do not want your student to participate in this study,* leave a message for Gail Headley at marcia.gail.headley@uc.edu or [Redacted] by Wednesday, April 15th. If I do not get a message from you, your student will be considered eligible for the study, and your student will be given a choice about whether to participate in the study.

Even after the study begins, you can change your mind about allowing your student to participate. *To take your student out of the study,* leave a message for Gail Headley at marcia.gail.headley@uc.edu or [Redacted]. Your student may also change his/her mind and stop participating at any time. If your student wants to stop, he/she should tell Gail.

Thank you for giving this thoughtful consideration. While I have permission from the University of Cincinnati and Mason City Schools to conduct this research project, the final decision about participation belongs to you and your student. Please discuss this opportunity with your student. If you have any questions about the study, or if you would like additional information, please feel free to email or call me.

With thanks,

Gail Headley
Doctoral Candidate, Educational Studies, School of Education
College of Education, Criminal Justice, and Human Services
University of Cincinnati
marcia.gail.headley@uc.edu
phone: [Redacted]
Appendix B: Parent Information Sheet

IRB #: 2015-0701

University of Cincinnati
Department: CECH - Educational Studies
Principal Investigator: Marcia Gail Headley
Faculty Advisor: Dr. Vicki Plano Clark

Information Sheet for Research

Title of Study: How do students read in mathematics class?

Introduction:
You are being asked to allow your student to take part in a research study. Please read this paper carefully and ask questions about anything you do not understand.

Who is doing this research study?
The person in charge of this research study is Gail Headley. She is being guided in this research by Dr. Vicki Plano Clark. There may be other people on the research team helping at different times during the study.

What is the purpose of this research study?
The purpose of this study is to understand how students read math symbols. Also, the purpose of this study is to help teachers better understand math students at your school.

Who will be in this research study?
Up to 300 seventh and eighth graders from your student’s school will help with the computer part of the study. Less than 30 of those students will help with the interview part of the study. Only students who are enrolled in a mainstream core mathematics course will participate.

What will your student be asked to do in this research study, and how long will it take?
The school will give Gail information about all students who participate in the study. The information will include a math achievement index. It also will include general information such as grade level (7th or 8th), age, and your gender.

For the computer part of the study, your student will be asked to go to the computer lab with his/her class to complete a survey and reading task.
- The survey will take about 10 minutes. Your student will be asked to answer questions about his/her background, his/her study habits, and his/her likes and dislikes. This is expected to take less than 10 minutes.
- The reading task will take about 10 minutes. Your student will read text similar to what he/she sees in math class.
- Your student will not have to make any calculations or complete any math problems.
- Your student will spend about half an hour in the lab. No more than 1 class period will be spent in the lab.
- Your student will not miss any instructional time in the computer lab.
For the interview part of the study, your student may also be invited to talk to Gail about how he/she reads in math class.

- All interviews will be audio-recorded.
- Each interview is expected to take about 30 minutes. It will not take more than 1 class period.
- Your student will not have to make any calculations or complete any math problems.

Participants will do the computer part and interview part on different days. All research activities will be done school during normal school hours.

Are there any risks to being in this research study?
The risk of being hurt during this study is very low. If your student feels upset, he/she should tell Gail. If any unexpected harm results from participation, Gail will request that the school forward a message of explanation to inform parents.

Are there any benefits from being in this research study?
There is no direct benefit to being in this study. But, it is possible that thinking about how to read math will help your student learn math. If your student wants to learn more about what he/she reads in this study, he/she may ask Gail questions. This study may help math teachers and students in the future.

What will you get because of being in this research study?
Your student will not be given any reward for taking part in this study. He/she will be given a handout about math anxiety explaining what it is and how to get help if needed.

Do you have choices about taking part in this research study?
- If you do not want your student to take part in this research study, you and your student will not be treated any differently. If you do not want your student to participate in this study, leave a message for Gail Headley at marciagail.headley@uc.edu or by Wednesday, April 15th.
- School records will be collected for each participant. If you do not want the school information to be used in this study, leave a message for Gail Headley at marciagail.headley@uc.edu or by Wednesday, April 15th.
- All interviews will be audio-recorded. If you do not want your student to be audio-recorded, leave a message for Gail Headley at marciagail.headley@uc.edu or by Wednesday, April 15th.
How will your research information be kept confidential?
- Information about your student will be kept private. Only the research team will see your student’s name on this form and a master list of names and IDs. Even teachers and parents will not see participants’ data unless it is required by law. Your student’s real name will not appear on any other forms or recordings. If your student speaks to Gail in an interview, he/she will pick a false name. The results from this research study may be published; but your student will not be identified by name.
- All of your student’s information will be kept locked in a research office at University of Cincinnati for 5 years after the study is complete. After that, the master list with your student’s name will be shredded. The de-identified data may be used to explore other research questions in the future.

What are your legal rights in this research study?
Nothing in this Information Sheet waives any legal rights you or your student may have. This Information Sheet also does not release the investigator the institution, or its agents from liability for negligence.

What if you have questions about this research study?
- If you have any questions about this research study, you should contact Gail Headley at marciagail.headley@uc.edu or Dr. Vicki Plano Clark at planocvi@ucmail.uc.edu.
- The UC Institutional Review Board reviews all research projects that involve human participants to be sure the rights and welfare of participants are protected.
- If you have questions about your rights as a participant, complaints and/or suggestions about the study, you may contact the UC IRB at (513) 558-5259. Or, you may call the UC Research Compliance Hotline at (800) 889-1547, or write to the IRB, 300 University Hall, ML 0567, 51 Goodman Drive, Cincinnati, OH 45221-0567, or email the IRB office at irb@ucmail.uc.edu.

Do you HAVE to take part in this research study?
- No one has to be in this research study. Refusing to take part will NOT cause any penalty or loss of benefits that your student would otherwise have. Refusing to take part will not change your student’s grade in his/her math class.
- Your student may start and then change his/her mind and stop at any time. Also, you may change your mind and ask to have your student removed from the study. To remove your student from the study, you should tell Gail Headley (marciagail.headley@uc.edu) or Dr. Vicki Plano Clark (planocvi@ucmail.uc.edu).

PLEASE KEEP THIS INFORMATION SHEET FOR YOUR REFERENCE.
Appendix C: Youth Assent Form

IRB #: 2015-0701

Approved: 3/30/2015
Do Not Use After: 3/29/2016

Youth Assent Form for Research
University of Cincinnati
Department: CECH - Educational Studies
Principal Investigator: Marcia Gail Headley
Faculty Advisor: Dr. Vicki Plano Clark

Title of Study: How do students read in mathematics class?

Introduction: You are being asked to be in a research study. Please ask questions about anything you do not understand.

Who is doing this research study? The people in charge of this research study are Gail Headley and Dr. Vicki Plano Clark. Gail will collect the data. Vicki will act as an advisor. Other people may help as well.

What is the purpose of this research study? The purpose of this study is to understand how students read math symbols. Also, the purpose of this study is to help teachers better understand math students at your school.

Who will be in this research study? Up to 300 seventh and eighth graders from your school will help with the computer part of the study. Less than 30 of those students will help with the interview part of the study.

What will you be asked to do in this research study, and how long will it take? The school will give Gail information about you. The information will include a math achievement score. It will also include general information such as what grade you are in, your age, and your gender.

For the computer part of the study, you will be asked to go to the computer lab with your class to complete a survey and reading task.
- The survey will take about 10 minutes. You will answer questions about your background, your study habits, and your likes and dislikes. This is expected to take less than 10 minutes.
- The reading task will take about 10 minutes. You will read text similar to what you see in math class.
- You will not have to make any calculations or complete any math problems.
- You will spend about half an hour in the lab. No more than 1 class period will be spent in the lab.
- You will not miss any teaching time in the computer lab.

For the interview part of the study, you may also be invited to talk to Gail about how you read in math class.
- Each interview is expected to take about 30 minutes. It will not take more than 1 class period.
- If you interview, you will not have to make any calculations or complete any math problems.
- Interviews will be tape recorded.

The computer part and interview parts will be done on different days. All activities will be done during normal school hours.

Are there any risks to being in this research study? The risk of being hurt during this study is very low. If you feel upset, tell Gail.

Are there any benefits from being in this research study? There is no direct benefit to being in this study. It is possible that thinking about how you read math will help you learn math. If you want to learn more about what you read in this study, you may ask Gail questions. This study may help math teachers and students in the future.
What will you get because of being in this research study?
You will not be given any reward for taking part in this study. You will be given a handout about math anxiety so you will know what it is and how to get help if you need to.

Do you have choices about taking part in this research study?
• If you do not want to take part in this research study, you will not be treated any differently. If you do not want to take part, you should not sign this form.
• School records will be collected for each participant. If you do not want the school information to be used in this study, you should not sign this form.
• If you want to complete the computer part of the study and not the interview part, you may.
• All interviews will be audio-recorded. If you do not want to be audio-recorded, you should check the box that says “I want to do the computer part. But I do NOT want to do the interview part”.

How will your research information be kept confidential?
• Information about you will be kept private. Only the research team will see your name on this form and a master list of names and IDs. Even your teacher and parents will not see your data unless it is required by law. Your real name will not appear on any other forms or recordings. If you speak to Gail in an interview, you will pick a false name. The results from this research study may be published; but you will not be identified by name.
• All of your information will be kept locked in a research office at University of Cincinnati for 5 years after the study is complete. After that, the master list with your name will be shredded. The study information, with no names, may be used for other research in the future.

What are your legal rights in this research study?
Nothing in this assent form takes away your rights.

What if you have questions about this research study?
If you have any questions about this research study, you should contact Gail Headley at marciagail.headley@uc.edu or [redacted] or Dr. Vicki Plano Clark at planocvi@ucmail.uc.edu or [redacted].

Do you HAVE to take part in this research study?
No one has to be in this research study. You will not get in any trouble if you say no. Refusing to take part will not change your grade in your math class.

You may start and then change your mind and stop at any time. To stop being in the study, you should tell Gail Headley (marciagail.headley@uc.edu or [redacted]) or Dr. Vicki Plano Clark (planocvi@ucmail.uc.edu or [redacted]).

Agreement: I have read this information. I want to be in this research study.
Check 1 box: □ I want to do the computer part AND the interview part.
□ I want to do the computer part of the study. But I do NOT want to do the interview part.

Your Name (please print) ________________________________

Your Date of Birth ____________________ (Month / Day / Year)

Your Signature ___________________________ Date __________

Signature of Person Obtaining Assent ________________________ Date __________
Appendix D: Math Anxiety Handout

Coping with Math Anxiety?

Take on a growth mindset.
According to Carol Dweck, professor of psychology at Stanford and author of *Mindset: The New Psychology of Success* it is important to remember that with appropriate exercise the brain, just like your muscles, can be changed and strengthened. With time, effort, and practice suited to your unique needs, you *can* change the way you feel about and understand math. Mathphobia is *not* a permanent condition.

Value your success.
Congratulate yourself on your effort. Thank yourself for working towards productive study habits. Commend yourself for sticking to your exercise program -- repeated practice is key to success.

Try these tips.
- Focus on developing understanding of math rather than worrying about grades.
- Ask your teacher for feedback on the quality of your work and suggestions tailored to your specific needs.
- Do math exercises each day. Practice with a variety of problems and attend to the ways you use math in every day life.
- Refine your note taking habits. Consider pretending that you are writing for a younger student with less expertise. Or pretend you are preparing a lesson plan to teach others. Reading and writing as well as speaking and listening have an important impact on your learning.
- Always be on the look out for patterns. Look for the connections between various math problems and strategies. Learning a series of connected math concepts is much easier than trying to understand an infinite number of separate problems.
- Explore online resources. If reading math is difficult, there are many educational videos on YouTube and Khan Academy. Research can help you find the right tool for your problem (and notice some of those patterns!).
- Get your rest. Eat well. Exercise you other muscles and keep yourself in good health. Healthy habits support a healthy growing brain.

Reach out.
If you continue to experience symptoms of anxiety (such as warming skin, sweaty palms, nausea, increased heart rate, dizziness, etc.) in response to your efforts to learn math, seek professional help. Consider these resources:
- Guidance Counselors at
- Lindner Center at http://lindnercenterofhope.org or 513-536-HOPE (4673)
Appendix E: Maximum Variation Matrix Worksheet

The goal is to select students who vary in math achievement across each type of reader. The matrix below shows a 4×4 grid for generating a maximum variation sample across math achievement quartiles (M-Q1, M-Q2, M-Q3, M-Q4) and 4 different reader types (RR, CR, IR, ER).

NOTE: Students will not be informed about the sampling process for interviews. The sampling process is designed to appear random to students. If a student asks, they will be told that students were selected “at random”.

![Maximum Variation Matrix](image)

<table>
<thead>
<tr>
<th>Reader Category</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistant readers (RR)</td>
<td>Students who appear to be resistant to reading any kind of text – especially symbolic mathematics – in math class even when instructed to do so.</td>
</tr>
<tr>
<td>Compliant readers (CR)</td>
<td>Students who appear to read texts – including symbolic mathematics – in math class when instructed to do so.</td>
</tr>
<tr>
<td>Independent readers (IR)</td>
<td>Students who appear to read mathematics texts – including symbolic mathematics – as needed without being instructed to do so.</td>
</tr>
<tr>
<td>Exceptional readers (ER)</td>
<td>Students who appear to have unique reading habits/processes or novel ways of interacting with texts – especially symbolic mathematics – in math class.</td>
</tr>
</tbody>
</table>
**Instructions:** For each cell, write the name(s) of 1-3 students who fit the description. It is not necessary to know exactly which quartile a student falls into with respect to a particular achievement measure – rather use the concept of quartiles to consider how to place students on a math achievement scale. If you do not have a student who fits the description, leave the cell blank.

**Scheduling:** After I receive the responses from all teachers, I will compile a list of potential participants across teachers/classes. The goal is to interview 15 (and no more than 20) students across as many cells as possible. After I compile the recommendations, we can negotiate days to determine the most feasible and least disruptive way to interview a variety of students.

<table>
<thead>
<tr>
<th>Teacher Name:</th>
<th>RR</th>
<th>CR</th>
<th>IR</th>
<th>ER</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Student</td>
<td>Bell</td>
<td>Student</td>
<td>Bell</td>
</tr>
<tr>
<td>M-Q4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M-Q3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M-Q2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M-Q1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
## Appendix F: SMaLL - Conventional Decision Task (SMaLL-CDT)

<table>
<thead>
<tr>
<th>Item #s</th>
<th>Conventional</th>
<th>Unconventional</th>
<th>Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 2</td>
<td>$x^2$</td>
<td>$^2x$</td>
<td>Ink Marks 2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Orthographic change location(exp/sub)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>QUAL item alignment 2,3,4,8</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Curriculum topic Functions</td>
</tr>
<tr>
<td>3, 4</td>
<td>$</td>
<td>x</td>
<td>$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Orthographic change location(bracket)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>QUAL item alignment 5,6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Curriculum topic Functions</td>
</tr>
<tr>
<td>5, 6</td>
<td>$\frac{x}{y}$</td>
<td>$\sqrt[y]{x}$</td>
<td>Ink Marks 3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Orthographic change orientation(operator)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>QUAL item alignment 4,10</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Curriculum topic Bin. operation</td>
</tr>
<tr>
<td>7, 8</td>
<td>$f(x)$</td>
<td>$(f)x$</td>
<td>Ink Marks 3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Orthographic change location(bracket)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>QUAL item alignment 3,4,6,8</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Curriculum topic Functions</td>
</tr>
<tr>
<td>9, 10</td>
<td>$\sqrt{x}$</td>
<td>$\sqrt{y}$</td>
<td>Ink Marks 2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Orthographic change shape(function)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>QUAL item alignment 3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Curriculum topic Functions</td>
</tr>
<tr>
<td>11, 12</td>
<td>$l \times w$</td>
<td>$l \otimes w$</td>
<td>Ink Marks 3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Orthographic change shape(operator)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>QUAL item alignment 1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Curriculum topic Bin. operation</td>
</tr>
<tr>
<td>13, 14</td>
<td>$a &gt; b &gt; c$</td>
<td>$a &lt; b &lt; c$</td>
<td>Ink Marks 5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Orthographic change orientation(relationship)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>QUAL item alignment 5,7</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Curriculum topic Statements</td>
</tr>
<tr>
<td>15, 16</td>
<td>$x \leq y$</td>
<td>$x \ll y$</td>
<td>Ink Marks 4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Orthographic change shape(relationship)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>QUAL item alignment 5,7</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Curriculum topic Statements</td>
</tr>
<tr>
<td>17, 18</td>
<td>$(x - y)^2$</td>
<td>$(x - y)_2$</td>
<td>Ink Marks 6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Orthographic change location(exp/sub)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>QUAL item alignment 3,4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Curriculum topic Complex op.</td>
</tr>
<tr>
<td>19, 20</td>
<td>$x^{m+n}$</td>
<td>$xm^{n}$</td>
<td>Ink Marks 4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Orthographic change size(variable)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>QUAL item alignment 8</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Curriculum topic Complex op.</td>
</tr>
<tr>
<td>Items #</td>
<td>Conventional</td>
<td>Unconventional</td>
<td>Characteristics</td>
</tr>
<tr>
<td>-------</td>
<td>--------------</td>
<td>----------------</td>
<td>----------------</td>
</tr>
<tr>
<td>21, 22</td>
<td>$x - x_1$</td>
<td>$x - _1x$</td>
<td>Ink Marks 4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Orthographic change location(exp/sub)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>QUAL item alignment 3,4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Curriculum topic Bin. operation</td>
</tr>
<tr>
<td>23, 24</td>
<td>$a \cdot b$</td>
<td>$a\cdot b$</td>
<td>Ink Marks 3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Orthographic change location(operator)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>QUAL item alignment 8</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Curriculum topic Operations</td>
</tr>
<tr>
<td>25, 26</td>
<td>$\pi r^2$</td>
<td>$2\pi r$</td>
<td>Ink Marks 3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Orthographic change location(exp/sub)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>QUAL item alignment 2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Curriculum topic Complex op.</td>
</tr>
<tr>
<td>27, 28</td>
<td>$2x$</td>
<td>$2\times$</td>
<td>Ink Marks 2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Orthographic change shape(operator)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>QUAL item alignment 1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Curriculum topic Bin. operation</td>
</tr>
<tr>
<td>29, 30</td>
<td>$d = rt$</td>
<td>$d \parallel rt$</td>
<td>Ink Marks 5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Orthographic change orientation(relationship)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>QUAL item alignment 9</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Curriculum topic Statements</td>
</tr>
<tr>
<td>31, 32</td>
<td>$y^3$</td>
<td>$^3y$</td>
<td>Ink Marks 2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Orthographic change location(exp/sub)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>QUAL item alignment 2,3,4,8</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Curriculum topic Functions</td>
</tr>
<tr>
<td>33, 34</td>
<td>$</td>
<td>y</td>
<td>$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Orthographic change location(bracket)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>QUAL item alignment 5,6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Curriculum topic Functions</td>
</tr>
<tr>
<td>35, 36</td>
<td>$\frac{a}{b}$</td>
<td>$\sqrt[n]{a}$</td>
<td>Ink Marks 3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Orthographic change orientation(operator)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>QUAL item alignment 4,10</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Curriculum topic Bin. operation</td>
</tr>
<tr>
<td>37, 38</td>
<td>$g(x)$</td>
<td>$g(x)$</td>
<td>Ink Marks 3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Orthographic change location(bracket)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>QUAL item alignment 3,4,6,8</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Curriculum topic Functions</td>
</tr>
<tr>
<td>39, 40</td>
<td>$\sqrt{y}$</td>
<td>$\sqrt[y]{y}$</td>
<td>Ink Marks 2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Orthographic change shape(function)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>QUAL item alignment 3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Curriculum topic Functions</td>
</tr>
<tr>
<td>Items #</td>
<td>Conventional</td>
<td>Unconventional</td>
<td>Characteristics</td>
</tr>
<tr>
<td>---------</td>
<td>--------------</td>
<td>----------------</td>
<td>----------------</td>
</tr>
<tr>
<td>41, 42</td>
<td>$p \div q$</td>
<td>$p \div q$</td>
<td>Ink Marks 5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Orthographic change: shape(operator)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>QUAL item alignment: 1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Curriculum topic: Bin. operation</td>
</tr>
<tr>
<td>43, 44</td>
<td>$x &gt; y &gt; z$</td>
<td>$x &lt; y &lt; z$</td>
<td>Ink Marks 5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Orthographic change: orientation(relationship)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>QUAL item alignment: 5, 7</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Curriculum topic: Statements</td>
</tr>
<tr>
<td>45, 46</td>
<td>$x \geq y$</td>
<td>$x \gg y$</td>
<td>Ink Marks 4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Orthographic change: shape(relationship)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>QUAL item alignment: 5, 7</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Curriculum topic: Statements</td>
</tr>
<tr>
<td>47, 48</td>
<td>$(x - y)^3$</td>
<td>$3(x - y)$</td>
<td>Ink Marks 6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Orthographic change: location(exp/sub)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>QUAL item alignment: 3, 4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Curriculum topic: Complex op.</td>
</tr>
<tr>
<td>49, 50</td>
<td>$y^{a+b}$</td>
<td>$ya^b$</td>
<td>Ink Marks 4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Orthographic change: size(variable)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>QUAL item alignment: 8</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Curriculum topic: Complex op.</td>
</tr>
<tr>
<td>51, 52</td>
<td>$y_2 - y_1$</td>
<td>$y_2 - \gamma y$</td>
<td>Ink Marks 5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Orthographic change: location(exp/sub)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>QUAL item alignment: 3, 4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Curriculum topic: Bin. operation</td>
</tr>
<tr>
<td>53, 54</td>
<td>$x \cdot y$</td>
<td>$x \cdot y$</td>
<td>Ink Marks 3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Orthographic change: location(operator)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>QUAL item alignment: 8</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Curriculum topic: Operations</td>
</tr>
<tr>
<td>55, 56</td>
<td>$\pi^2$</td>
<td>$\pi^2$</td>
<td>Ink Marks 3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Orthographic change: location(exp/sub)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>QUAL item alignment: 2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Curriculum topic: Complex op.</td>
</tr>
<tr>
<td>57, 58</td>
<td>$2^x$</td>
<td>$2^x$</td>
<td>Ink Marks 2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Orthographic change: shape(operator)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>QUAL item alignment: 1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Curriculum topic: Bin. operation</td>
</tr>
<tr>
<td>59, 60</td>
<td>$y = \frac{c}{x}$</td>
<td>$y = \frac{c}{x}$</td>
<td>Ink Marks 6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Orthographic change: shape(operator)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>QUAL item alignment: 9</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Curriculum topic: Statements</td>
</tr>
</tbody>
</table>
## Appendix G: Math Print Exposure Survey (MPES)

<table>
<thead>
<tr>
<th>Formula for display</th>
<th>Common name/description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. (a(b + c) = ab + ac)</td>
<td>Distributive Property</td>
</tr>
<tr>
<td>2. (V = \frac{1}{3}\pi r^2 h)</td>
<td>Volume of a cone</td>
</tr>
<tr>
<td>3. (d = rt)</td>
<td>Distance formula</td>
</tr>
<tr>
<td>4. (y = \frac{k}{x})</td>
<td>Inverse proportion</td>
</tr>
<tr>
<td>5. (a^2 + b^2 = c^2)</td>
<td>Pythagorean Theorem</td>
</tr>
<tr>
<td>6. (\frac{x^a}{x^b} = x^{a-b})</td>
<td>Law of exponents</td>
</tr>
<tr>
<td>7. (\sqrt{x^2} =</td>
<td>x</td>
</tr>
<tr>
<td>8. (y = mx + b)</td>
<td>Equation of a line (slope-intercept)</td>
</tr>
<tr>
<td>9. (m = \frac{y_2 - y_1}{x_2 - x_1})</td>
<td>Slope of line between 2 points</td>
</tr>
<tr>
<td>10. (A = \frac{1}{2}bh)</td>
<td>Area of a triangle</td>
</tr>
<tr>
<td>11. (1 \text{ in} = 2.54 \text{ cm})</td>
<td>Inch to centimeter conversion</td>
</tr>
<tr>
<td>12. (C = \pi d)</td>
<td>Circumference of a circle</td>
</tr>
<tr>
<td>13. (f(x) = a(x - h)^2 + k)</td>
<td>Parabola with vertex ((h,k))</td>
</tr>
<tr>
<td>14. (V = Bh)</td>
<td>Volume of a rectangular prism</td>
</tr>
<tr>
<td>15. (A = \pi r^2)</td>
<td>Area of a circle</td>
</tr>
</tbody>
</table>

**Instructions:** The purpose of this part of the survey is to determine how often you have seen, read, written, or used different formulas. Some formulas are from elementary school books. Some formulas are from middle school books. Some formulas are from high school books. Your job is to describe how often these formulas show up in your life at school, at home, or anywhere else. Read each formula carefully. Click the word or phrase that indicates how often you have seen it, read it, written it, or used it in any way at school, at home, or anywhere else.

1 = Never
2 = Only one or two times
3 = A few times
4 = Many times
Appendix H: Math Reading Habits Survey (MRHS)

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>When I am at school, I read things with math symbols.</td>
</tr>
<tr>
<td>2.</td>
<td>When I am at home, I read things with math symbols.</td>
</tr>
<tr>
<td>3.</td>
<td>When I use the computer, I read things with math symbols.</td>
</tr>
<tr>
<td>4.</td>
<td>As part of my hobbies, I read things with math symbols.</td>
</tr>
<tr>
<td>5.</td>
<td>When I see math symbols, I get someone else to read them to me.</td>
</tr>
<tr>
<td>6.</td>
<td>Before I do a math problem, I read the problem to myself.</td>
</tr>
<tr>
<td>7.</td>
<td>When I read a math problem, I focus on the words.</td>
</tr>
<tr>
<td>8.</td>
<td>When I read a math problem, I focus on the numbers.</td>
</tr>
<tr>
<td>9.</td>
<td>When I read a math problem, I focus on the symbols.</td>
</tr>
<tr>
<td>10.</td>
<td>When I do a math problem, I read the problem more than once.</td>
</tr>
</tbody>
</table>

**Instructions:** Each sentence describes a situation and something a person might do in that situation. Read each sentence carefully. Then, select the response that best explains how often you do the same thing in that situation.

1 = Never  
2 = Rarely  
3 = Sometimes  
4 = Frequently  
5 = Always  

*This is a negative item and, thus, reverse scored.*
Appendix I: Math Anxiety Survey (MAS)

<table>
<thead>
<tr>
<th>Math Anxiety Survey</th>
<th>Adapted from Math Anxiety Scale - Revised (MAS-R; Bai, Wang, Pan, &amp; Frey, 2009)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. I think math is interesting. *</td>
<td></td>
</tr>
<tr>
<td>2. I enjoy learning math. *</td>
<td></td>
</tr>
<tr>
<td>3. Math is one of my favorite classes. *</td>
<td></td>
</tr>
<tr>
<td>4. Math is a part of my life. *</td>
<td></td>
</tr>
<tr>
<td>5. I plan to use math in the future. *</td>
<td></td>
</tr>
<tr>
<td>6. I would like to take more math classes. *†</td>
<td></td>
</tr>
<tr>
<td>7. I think math is hard.</td>
<td></td>
</tr>
<tr>
<td>8. Math makes me feel uneasy. †</td>
<td></td>
</tr>
<tr>
<td>9. Math makes me nervous.</td>
<td></td>
</tr>
<tr>
<td>10. Math is confusing.</td>
<td></td>
</tr>
<tr>
<td>11. I get frustrated when I do math problems.</td>
<td></td>
</tr>
<tr>
<td>12. When I see a math problem, I worry about whether I can do it.</td>
<td></td>
</tr>
<tr>
<td>13. I get upset when I am doing math.</td>
<td></td>
</tr>
<tr>
<td>14. My mind goes blank when I see a math problem.</td>
<td></td>
</tr>
</tbody>
</table>

**Instructions:**
Read each sentence. Then, select a button to indicate how much you agree with the statement.
Expected time to completion: <10 minutes

* Positive (reverse-scored) item
† Unmodified MAS-R item (Bai et al., 2009, pp. 189-190)

1 = Strongly Disagree
2 = Disagree
3 = Agree
4 = Strongly Agree
IRB#2015-0701 What is SMaLL

Participant ID:

Pseudonym:

Age:

Grade:

Current ELECTIVE math course:

Current REQUIRED math course:

Teacher:

Date:

Start time:

End time:

Interviewer:
Hello [insert pseudonym]. Thank you for taking the time to talk to me today about how you read math. Before we begin, I want to remind you that this interview is being conducted as part of a research project. If you change your mind about participating, you may ask to stop.

For my research project, I need to record our conversation to be sure I can remember what we said.
- Your privacy is important. I’d like for you to pick a fake name so you don’t have to worry about anyone knowing what you say.
  - What name would you like to use for the interview? 

OK, now your identity will be a secret, __________. I want you to feel free to share your thoughts and ideas without worrying about whether your teacher or your parents or your friends will know what you said.
- Do you have any questions or concerns?
- Is it still okay with you if I make an audio recording of our conversation?
- Are you ready to begin?

[Note: If the student declines to have the conversation recorded, the interview will not proceed. Instead the next student on the sampling list will be invited to interview.]
- [After an affirmative response] OK, __________, I’ll turn on the audio recorder now.

[Examples of the types of questions to be asked are on the next page. Additional questions may be asked and probing will occur in response to the interviewee’s comments, but all questions will focus on the participants’ metacognitive reflections of their experiences reading mathematics.]

1. [Ice breakers] First, tell me about the kinds of things you like to read for fun?

   **Possible probes:**
   - Can you tell me about a time when you were read something that had to do with MATH for fun?
   - How are the THINGS that you read for fun different from the THINGS you read for math?
     - What is different about the BOOKS or TEXT?
   - How does your APPROACH to reading things for fun differ from your approach to reading things for math?
     - What is different about what you actually DO when you read for math?
   - Is there a difference in WHEN or WHERE your read for fun or for math?

Participant ID: 

Pseudonym: 

Interview Initials: 

A9 QUAL INTERVIEW protocol V15.05.02.docx

5/2/15 10:09 PM
2. [Reading preferred math text] Now I want to show you some reading selections from math texts. Remember there are NO problems to solve. I’ve made sure that none of them have questions. I’d like for you to pick a sample to read to me.

- I’m going to turn them over and I want you to choose one as quickly as possible. Are you ready?

[RS1]
- Which one looks like something you can read aloud to me?
- Whenever you are ready, read it out loud the way you read it to yourself.....

Possible probes:
- Tell me about how you decide whether or not to read the parts with symbols when you are reading to yourself?
- If you were reading it aloud to your teacher or in front of the class, how might you read it differently?
  - Can you read it that way?
  - I’m going to read it another way I’ve heard it.
  - How are they different?
  - In what ways is one better or worse? More or less useful?
- How do you think your teacher would read it?
- Do you think it is a good or a bad idea to be able to read this in multiple ways? WHY?

- Can you tell me a little bit about why you chose that sample?

Possible probes:
- Can you give an example of a time when you read something like this?
- Can you tell me how you knew how to read [________]?
[RS2]

- Now I want you to choose another sample to read. I’m going to turn them over again and I want you to choose another one as quickly as possible. Are you ready?
- Whenever you are ready, read it out loud the way you read it to yourself. . . . Possible probes:
  - Tell me about how you decide whether or not to read the parts with symbols when you are reading to yourself?
  - If you were reading it aloud to your teacher or in front of the class, how might you read it differently?
    - Can you read it that way?
    - I’m going to read it another way I’ve heard it.
    - How are they different?
    - In what ways is one better or worse? More or less useful?
  - How do you think your teacher would read it?
  - Do you think it is a good or a bad idea to be able to read this is multiple ways? WHY?

- Can you tell me a little bit about why you chose that sample?
  Possible probes:
  - Can you give an example of a time when you read something like this?
  - Can you tell me how you knew how to read [_____]?
3. [Avoiding math text] Now I’m going to turn all of the samples over again. This time, can you tell me about any of the samples that look like something you would definitely NOT want to read?

- What do you think makes some math less readable and other math more readable?

Possible probes:

- When you are deciding quickly which to read what made you decide against these samples?
  - Can you tell me what you noticed about the numbers or the symbols or the length?
- I know you don’t want to read it, but I’d like to read it together.
  - How about if you got started and if you get stuck, I’ll take a turn?
- I noticed that there are some symbols on the page that you didn’t translate into words.
  - Can you tell me about what was going through your mind when you first saw them?
- Can you describe a time when you needed to read something with symbols like this but didn’t know how?
  - What went through your mind when you saw the symbols?
  - What did you do to figure out how to put the symbols into words?
  - Can you tell me about WHERE you were when it happened?
  - Can you tell me about how you felt when it happened?
- Why do you think nobody is picking this sample?
4. [How is SMALL different?] How would you describe the difference between reading things with math symbols and reading text that is written in plain English using the alphabet?

- In what ways does your strategy for reading – or putting the text into words – change when you see math symbols?
- In what ways is one harder than the other?
  - What makes one harder than the other?
- In what ways is one more important than the other?
  - What makes one more important than the other?
  - Do you think you need to be a good reader?
  - Do you think you need to be able to read math symbols well?

Possible probes:
- Can you point out some symbols that you rarely/never see when you are reading for fun?
- Do you think it is important for students to learn how to read things like that? Why or why not?
5. [Closing] Before you go, I have 2 quick questions.

- I’d love to know what kinds of math classes you are hoping to take [next year/in high school]
  o How important do you think it will be for you to get better at -- or learn to like -- at reading math symbols?
  o What do you want to be when you grow up?

- Is there anything else you’d like to tell me about you and reading math symbols?
  o How do you do translate math symbols into words?
  o Why do you do translate math symbols into words?
  o Why -- or why NOT -- is being able to translates math symbols into words important?

I really enjoyed getting to know you. Thank you talking to me about how you read math.

I’ll turn off the audio-recording now.
A rectangular prism has six sides, and each side is a rectangle. We can use this formula to determine the surface area:

\[ S_{A_{\text{rectangular prism}}} = 2 \times l \times w + 2 \times l \times h + 2 \times w \times h \]
Selection # 2

Thinking about the similarities between a cylinder and a prism, can help to determine the formula for the volume of a cylinder. Although a cylinder is not a prism, it is similar to a prism in some ways. Like prisms, the volume of a cylinder is found by multiplying the area of its base by its height.

Since the base of a cylinder is a circle, the area of its base is given by the formula:

\[ A = \pi r^2 \]

Therefore, the formula is:

\[ V = \pi r^2 h \]
Selection # 3

The formula for the distance between two points \((x_1, y_1)\) and \((x_2, y_2)\) can be derived by creating a triangle and using the Pythagorean Theorem to find the length of the hypotenuse. The hypotenuse of the triangle will be the distance between the two points. The distance formula is commonly written as:

\[
d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}
\]
The slope of a line is \( m = \frac{y_2 - y_1}{x_2 - x_1} \) where \((x_1, y_1)\) and \((x_2, y_2)\) are two points on the line.

The point–slope form of the equation for the line is

\[
y - y_1 = m(x - x_1)
\]

Therefore, the two-point form of the equation of a line can be written as

\[
y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)
\]
Selection # 5

The absolute value of a number describes the magnitude of the number. That is, the absolute value is the distance from 0 on a number line.

The absolute value formula can be expressed as

\[ |x| = \begin{cases} 
  x, & x \geq 0 \\
  -x, & x < 0 
\end{cases} \]
Selection # 6

Here is an example to show how I solved this problem:

Simplify \(-|-3|\).

Given \(-|-3|\), I first simplified \(-3\) to get +3.
So, I wrote \(-|-3| = -(+3)\).
Finally, I simplified \(-(+3)\).
I can demonstrate each step of my solution process by writing \(-|-3| = -(3) = -3\).
Selection # 7

There are rules for manipulating inequalities which help us avoid changing the solution set. Here is a list of manipulations that do not change the solution set for an inequality:

Rule 1. Adding or subtracting the same number on both sides.
   **Example:** The inequality \( x - 2 > 5 \) has the same solutions as the inequality \( x > 7 \).
   (Adding 2 to both sides does not change the solution set.)

Rule 2. Reversing the quantities and reversing the inequality sign.
   **Example:** The inequality \( 5 - x > 4 \) has the same solutions as the inequality \( 4 < 5 - x \).

Rule 3. Multiplying or dividing by the same *positive* number on both sides.
   **Example:** The inequality \( 2x \leq -6 \) has the same solutions as the inequality \( x \leq 3 \).
   (Dividing both sides by \(+2\)).

Rule 4. Multiplying or dividing by the same *negative* number on both sides AND reversing the inequality sign.
   **Example:** The inequality \( -2x \geq 4 \) has the same solutions as the inequality \( x \leq -2 \).
   (Dividing by \(-2\) on both sides and changing \(>\) to \(<\)).
Selection # 8

The product rule for exponents tells us that, when multiplying two powers that have the same base, you can add the exponents. In this example, you can see how it works. Adding the exponents is just a short cut!

Here is the product rule for exponents:

\[ x^m \cdot x^n = x^{m+n} \]

Here is an example:

\[ 4^2 \cdot 4^3 = (4 \cdot 4)(4 \cdot 4 \cdot 4) \]
\[ 4^{2+3} = 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \]
\[ 4^{2+3} = 4^5 \]
Distance word problems, often also called *uniform rate* problems, involve something traveling at some fixed and steady pace or moving at some average speed. Whenever you read a problem that involves *how fast*, *how far*, or *for how long*, you should think of the distance formula, $d = rt$, where $d$ stands for distance, $r$ stands for the constant or average rate of speed, and $t$ stands for time.

For example: Suppose an executive drove from home at an average speed of 30 mph to an airport where a helicopter was waiting. The executive boarded the helicopter and flew to the corporate offices at an average speed of 60 mph. The entire distance was 150 miles; the entire trip took 3 hours. The distance formula is a useful tool to determine the distance from the airport to the corporate offices.
Selection # 10

Let's talk about ratios and proportional relationships. When we talk about the speed of a car or an airplane we measure it in miles per hour. A rate, like the speed of a car, is a type of ratio. A ratio is a way to compare two quantities by using division as in miles per hour where we compare miles and hours.

A ratio can be written in three different ways: $x : y \quad \frac{x}{y} \quad x \text{ to } y$

A simple proportional relationship, on the other hand, is an equation that describes two equivalent ratios. For instance, if one package of cookie mix results in 20 cookies then two packages will result in 40 cookies.

$$\frac{20}{1} = \frac{40}{2}$$
Appendix K: Readability Coding Concept Map

**ENGLISH**
- References to appearance of the English language
  - words
  - paragraph
  - known vocabulary words

**LENGTH**
- References to length of
  - overall selection
  - EL elements
  - SML elements

**FAMILIAR**
- Statements suggesting familiarity:
  - Topic
  - Recent use
  - Frequent use
  - Use over a long period of time
  - Also references that do NOT clearly identify the print features but suggest something about the selection is
    - Memorable or memorized
    - Recognizable

**SYMBOLIC MATHEMATICS**
- References to appearance of the symbolic mathematics language
  - equations
  - symbols (without a statement about self-efficacy OR translation)

**STRUCTURE**
- References to the way the structure connects the elements of selection
  - fonts identify EL elements
  - italics identifying headers
  - bold identifying vocabulary
  - explanation
  - example
  - offset SML
  - embedded SML

**UNFAMILIAR**
- Statements suggest a lack of familiarity
  - Unfamiliar topic
  - NOT used or seen
  - recently
  - frequently
  - since a long time ago

**POSITIVE**
- Given the reading selection, the student claims to...
  - be able to do/solve relevant math problems
  - be satisfied with relevant math ability
  - have enough content understanding

**POOR**
- Given the reading selection, the student claims to...
  - be unable to do/solve relevant math problems
  - be dissatisfied with relevant math ability
  - have limited content understanding

**Curiosity**
- interestingly (un)familiar
  - contrast between (un)familiar

**Print features**

**Self-efficacy**

**Translation**
- Know words to say
  - how to “say” SML
  - what the letters & symbols “stand for”

**Ambiguity**
- Navigating multiple translations
  - “out loud” vs. “in my head”
  - multi-meaning symbols (e.g., –)
  - audience awareness
  - represent text (e.g., “over”)
  - represent meaning (e.g., “divided by”)
Appendix L: Reading Strategies Coding Concept Map

- **Rough read & re-read**
  - skip unknown symbols
  - guess unknown symbols
  - re-read (SML &/or EL)
  - guessing & checking

- **INTENTIONAL process**
  - Described as independent, intentional, metacognitive process
  - Decode SML
    - "break down" text
    - work left to right
    - character by character
    - work inside out
    - syntactic structure
    - analogous to "sound it out"
  - Make use of EL text
    - skim or skip
    - read as-needed
    - read after SML

- **ASSISTED process**
  - Resources when independent reading is unsuccessful

- **UNIDENTIFIED process**
  - Unable to describe
  - "I don’t know"

- **PRODs vs CONs**
  - Reflection
    - How does ambiguity help?
    - How does ambiguity complicate?

- **SML AMBIGUITY**
  - silent symbols
  - implied symbols
  - multi-meaning symbols
  - context specific meaning

- **RECEPTION strategies**

- **PRODUCTION strategies**

- **EXPAND**
  - Expand oral translation
    - Translate variables to words
    - e.g., read A as "area"
    - Insert words describing written text
    - e.g., read "parentheses"
    - Insert words for omitted ink marks
    - e.g., read "times" for juxtaposed multiplication

- **COMPRESS**
  - Minimize oral translation
    - Liberal or reduces translation of written text
    - Omit words for meaningful ink marks
    - e.g., read 1/x as "x"

- **Self awareness**
  - add meaning
  - give instructions

- **Audience awareness**
  - account for audience knowledge
  - describe what to write
  - "out loud" vs. "inner voice"

- **Articulation factors**
  - ease of articulation
  - time & efficiency
  - maintain order

- **Visual factors**
  - how it is written on the paper
  - flow of print
  - describe appearance of text

- **Phonologic factors**
  - how I’ve heard it
  - flow of sound
  - match the teacher

- **RATIONAL**

- **People**
  - teacher
  - classmates
  - family member

- **Media**
  - notes
  - binder
  - texts
  - websites
  - video
Appendix M: Joint Display for Selection 2 (Volume of a Cylinder)

<table>
<thead>
<tr>
<th>Selection # 2</th>
<th>Readability Rank: 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thinking about the similarities between a cylinder and a prism can help to determine the formula for the volume of a cylinder. Although a cylinder is not a prism, it is similar to a prism in some ways. Like prisms, the volume of a cylinder is found by multiplying the area of its base by its height.</td>
<td></td>
</tr>
<tr>
<td>Since the base of a cylinder is a circle, the area of its base is given by the formula:</td>
<td></td>
</tr>
<tr>
<td>[ A = \pi r^2 ]</td>
<td></td>
</tr>
<tr>
<td>Therefore, the formula is:</td>
<td></td>
</tr>
<tr>
<td>[ V = \pi r^2 h ]</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>S-CDT Item Comparison</th>
<th>Item pair</th>
<th>Accuracy Rate</th>
<th>Item pair</th>
<th>Accuracy Rate</th>
<th>Item pair</th>
<th>Accuracy Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conventional</td>
<td>( \pi r^2 )</td>
<td>91%</td>
<td>( x^2 )</td>
<td>98%</td>
<td>( \pi^2 )</td>
<td>98%</td>
</tr>
<tr>
<td>Unconventional</td>
<td>( 2\pi r )</td>
<td>62%</td>
<td>( 2x )</td>
<td>56%</td>
<td>( 2\pi )</td>
<td>59%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MPES Item Comparison</th>
<th>Response rates by response option</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Never</td>
</tr>
<tr>
<td>Item 15 [ A = \pi r^2 ]</td>
<td>3%</td>
</tr>
<tr>
<td>Item 2 [ V = \frac{1}{3} \pi r^2 h ]</td>
<td>22%</td>
</tr>
<tr>
<td>Item 10 [ A = \frac{1}{2} bh ]</td>
<td>9%</td>
</tr>
<tr>
<td>Item 14 [ V = Bh ]</td>
<td>8%</td>
</tr>
<tr>
<td>Grade</td>
<td>Name</td>
</tr>
<tr>
<td>-------</td>
<td>--------</td>
</tr>
<tr>
<td>8</td>
<td>Delaney</td>
</tr>
<tr>
<td>8</td>
<td>Elena</td>
</tr>
<tr>
<td>8</td>
<td>Gil</td>
</tr>
<tr>
<td>8</td>
<td>Martin</td>
</tr>
<tr>
<td>8</td>
<td>Nicki</td>
</tr>
<tr>
<td>7</td>
<td>Amelia</td>
</tr>
<tr>
<td>7</td>
<td>India</td>
</tr>
<tr>
<td>7</td>
<td>Jackson</td>
</tr>
<tr>
<td>7</td>
<td>Owen</td>
</tr>
<tr>
<td>7</td>
<td>Quincy</td>
</tr>
<tr>
<td>7</td>
<td>Robin</td>
</tr>
<tr>
<td>7</td>
<td>Katrina</td>
</tr>
</tbody>
</table>

**Readable factors**
- Multiple highly recognizable orthographic patterns
- High exposure to formulas
- High exposure to related formulas
- Familiar symbols (familiar meaning)
- Familiar topic
- Recurring topic

**Unreadable factors**
- None noted

**Emergent themes**
- SMaLL has orthographic & phonological components
- Memorization, repeated use, and repeated hearing support SMaLL
- Expanding variables supports comprehension
- The ability to detect orthographic errors in patterns comprised of known symbols may impact readability selections
Appendix N: Joint Display for Selection 3 (Distance Between Two Points)

The formula for the distance between two points \((x_1, y_1)\) and \((x_2, y_2)\) can be derived by creating a triangle and using the Pythagorean Theorem to find the length of the hypotenuse. The hypotenuse of the triangle will be the distance between the two points. The distance formula is commonly written as:

\[
d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}
\]

<table>
<thead>
<tr>
<th>S-CDT Item Comparison</th>
<th>Item pair</th>
<th>Accuracy Rate</th>
<th>Item pair</th>
<th>Accuracy Rate</th>
<th>Item pair</th>
<th>Accuracy Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conventional</td>
<td>(\sqrt{x})</td>
<td>91%</td>
<td>((x-y)^3)</td>
<td>94%</td>
<td>((x-y)^2)</td>
<td>95%</td>
</tr>
<tr>
<td>Unconventional</td>
<td>(\sqrt{x})</td>
<td>31%</td>
<td>((x-y)^3)</td>
<td>54%</td>
<td>((x-y)^2)</td>
<td>29%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MPES Item Comparison</th>
<th>Response rates by response option</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Never</td>
</tr>
<tr>
<td>Item 5 (a^2 + b^2 = c^2)</td>
<td>3%</td>
</tr>
<tr>
<td>Item 7 (\sqrt{x^2} =</td>
<td>x</td>
</tr>
<tr>
<td>Item 13 (f(x) = a(x - h)^2 + k)</td>
<td>28%</td>
</tr>
<tr>
<td>Grade</td>
<td>Name</td>
</tr>
<tr>
<td>-------</td>
<td>--------</td>
</tr>
<tr>
<td>8</td>
<td>Nicki</td>
</tr>
<tr>
<td>7</td>
<td>Amelia</td>
</tr>
<tr>
<td>7</td>
<td>Claire</td>
</tr>
<tr>
<td>8</td>
<td>Peter</td>
</tr>
<tr>
<td>7</td>
<td>Freddie</td>
</tr>
<tr>
<td>7</td>
<td>Robin</td>
</tr>
<tr>
<td>8</td>
<td>Harper</td>
</tr>
<tr>
<td>7</td>
<td>Katrina</td>
</tr>
<tr>
<td>8</td>
<td>Gil</td>
</tr>
<tr>
<td>8</td>
<td>Delaney</td>
</tr>
<tr>
<td>7</td>
<td>Owen</td>
</tr>
<tr>
<td>7</td>
<td>Loren</td>
</tr>
</tbody>
</table>
Appendix O: Joint Display for Selection 4 (Point-Slope Form of a Line)

The slope of a line is \( m = \frac{y_2 - y_1}{x_2 - x_1} \) where \((x_1, y_1)\) and \((x_2, y_2)\) are two points on the line.

The point-slope form of the equation for the line is

\[
y - y_1 = m(x - x_1)
\]

Therefore, the two-point form of the equation of a line can be written as

\[
y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)
\]

### S-CDT Item Comparison

<table>
<thead>
<tr>
<th>Item pair</th>
<th>Accuracy Rate</th>
<th>Item pair</th>
<th>Accuracy Rate</th>
<th>Item pair</th>
<th>Accuracy Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>21 &amp; 22</td>
<td>66%</td>
<td>51 &amp; 52</td>
<td>75%</td>
<td>7 &amp; 8</td>
<td>84%</td>
</tr>
<tr>
<td>Conventional</td>
<td>(x - x_1)</td>
<td>(y_2 - y_1)</td>
<td>(f(x))</td>
<td>(x_1 )</td>
<td>59%</td>
</tr>
<tr>
<td>Unconventional</td>
<td>(x - x_1)</td>
<td>(y_2 - y_1)</td>
<td>((f))</td>
<td>(x_1 )</td>
<td>59%</td>
</tr>
</tbody>
</table>

### MPES Item Comparison

<table>
<thead>
<tr>
<th>Response rates by response option</th>
<th>Never</th>
<th>Only 1 or 2 times</th>
<th>A few times</th>
<th>Many times</th>
</tr>
</thead>
<tbody>
<tr>
<td>Item 9 ( m = \frac{y_2 - y_1}{x_2 - x_1} )</td>
<td>22%</td>
<td>13%</td>
<td>10%</td>
<td>55%</td>
</tr>
<tr>
<td>Item 8 ( y = mx + b )</td>
<td>9%</td>
<td>8%</td>
<td>13%</td>
<td>71%</td>
</tr>
<tr>
<td>Grade</td>
<td>Name</td>
<td>Selected quotes</td>
<td>Inferences</td>
<td></td>
</tr>
<tr>
<td>-------</td>
<td>--------</td>
<td>---------------------------------------------------------------------------------</td>
<td>---------------------------------------------------------------------------</td>
<td></td>
</tr>
</tbody>
</table>
| 7     | Robin  | “I literally can see the different formulas that we've learned, which kind of helps me. And especially in math, we're kind of redoing or reforming this unit, I guess. So I'm really used to seeing these right now.” “It's really weird. But I like slope - slope intercept form – point-slope. So I like the idea of slope. So it's something in math that I like so I attract myself to it.” | **Readable factors**  
• Familiar symbols  
• Moderate exposure to related formula  
**Unreadable factors**  
• Limited exposure to formula  
• Multiple SML elements  
• SML embedded in EL  
• Complex syntax  
• Vinculum  
• Parentheses used in multiple ways  
• Moderately recognizable orthographic patterns  
• Undetectable errors in confusable orthographic patterns  
• Less familiar (or new) topic  
**Emergent themes**  
• Adjustable symbols (e.g., vinculum) may require unique SMaLL skills  
• Additive effects of unreadable features can induce affective reaction  
• SMaLL entails the ability to use context to assess the meaning of parentheses  
• Left-hand subscripts and superscripts are confusable orthographic patterns |
| 8     | Delaney| “[This] is about slope and lines and point-slope, and we did this. I just didn't really like it as much... I think I did pretty well but I think the unit went kind of fast to me. So it was like I didn't get to learn it all so it will stay with me” “I've seen subscripts done. It’s just that we did it a long time ago.” |
| 8     | Gil    | “They just give you a slope of a line and... all these equations. And first of all, you don't really know what X and Y is... And then they just gave you all these things here. Because first of all, how do you even say this?! I could may[be] say it if I look at it longer but when you start doing this problem and you see all these... You might get the reaction and you just kind of have to stop and re-read it..” |
| 7     | Claire | “There's subscripts and equations on top... like the numerator of a fraction but it's an equation. And then on the bottom it's the denominator of a fraction but it's an equation. And I don't know what that means... I know it means divide but it looks really complicated.” |
| 8     | Peter  | Instead of saying M, “I'd read slope equals.”                                      |
| 8     | Bobby  |                                                                                   |
| 7     | Amelia | “I've learned this and know what that is. It's slope.”                               |
| 7     | Katrina| “All the letters with the numbers and those two numbers. And then fraction thing and the parentheses after that.” |
| 7     | Loren  | “This line - that big line thing… I don't know what that is.”                       |
Appendix P: Joint Display for Selection 5 (Definition of Absolute Value)

### Selection # 5

The absolute value of a number describes the magnitude of the number. That is, the absolute value is the distance from 0 on a number line.

The absolute value formula can be expressed as

\[ |x| = \begin{cases} 
  x, & x \geq 0 \\
  -x, & x < 0 
\end{cases} \]

<table>
<thead>
<tr>
<th>S-CDT Item Comparison</th>
<th>Item pair</th>
<th>Accuracy Rate</th>
<th>Item pair</th>
<th>Accuracy Rate</th>
<th>Item pair</th>
<th>Accuracy Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3 &amp; 4</td>
<td>74%</td>
<td>33 &amp; 34</td>
<td>75%</td>
<td>45 &amp; 46</td>
<td>87%</td>
</tr>
<tr>
<td>Conventional</td>
<td>$</td>
<td>x</td>
<td>$</td>
<td>74%</td>
<td>$</td>
<td>y</td>
</tr>
<tr>
<td>Unconventional</td>
<td>$</td>
<td>x</td>
<td>$</td>
<td>85%</td>
<td>$</td>
<td>y</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MPES Item Comparison</th>
<th>Response rates by response option</th>
</tr>
</thead>
<tbody>
<tr>
<td>Item 7</td>
<td>Never</td>
</tr>
<tr>
<td>$\sqrt{x^2} =</td>
<td>x</td>
</tr>
<tr>
<td>Grade</td>
<td>Name</td>
</tr>
<tr>
<td>-------</td>
<td>--------</td>
</tr>
<tr>
<td>Unreadable</td>
<td>8 Bobby</td>
</tr>
<tr>
<td>Unreadable</td>
<td>7 Jackson</td>
</tr>
<tr>
<td>8</td>
<td>Delaney</td>
</tr>
<tr>
<td>8</td>
<td>Elena</td>
</tr>
<tr>
<td>8</td>
<td>Gil</td>
</tr>
<tr>
<td>8</td>
<td>Peter</td>
</tr>
<tr>
<td>8</td>
<td>Harper</td>
</tr>
<tr>
<td>7</td>
<td>Amelia</td>
</tr>
<tr>
<td>7</td>
<td>Claire</td>
</tr>
<tr>
<td>7</td>
<td>India</td>
</tr>
<tr>
<td>7</td>
<td>Owen</td>
</tr>
<tr>
<td>7</td>
<td>Quincy</td>
</tr>
<tr>
<td>7</td>
<td>Robin</td>
</tr>
<tr>
<td>7</td>
<td>Loren</td>
</tr>
</tbody>
</table>