Non-Foster Impedance Matching and Loading Networks for Electrically Small Antennas

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of the Ohio State University

By

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2011

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ABSTRACT

The demand for wide-band small antennas is steadily increasing for both civilian and military applications due to the explosive growth of wireless communications systems.

Linearly polarized electrically small antennas can be generally classified as $TM_{10}$ and $TE_{10}$ mode antennas. For a $TM_{10}$ mode antenna, the input impedance of the antenna is considerably reactive with a small real part. In contrast, the input admittance of a $TE_{10}$ mode antenna is characterized by a high susceptance and a small conductance, i.e. the input impedance is almost a short. It is therefore critical to match the antenna to a receiver (or transmitter) to optimize the transfer of power in the frequency range of interest. With conventional passive matching networks, the antennas can be only matched over narrow frequency bands. However, Non-Foster matching networks composed of negative capacitors and/or inductors can in principle match the antenna over wide frequency bands because Non-Foster matching networks can overcome the gain-bandwidth restrictions derived by Bode-Fano.

In this dissertation, the design, implementation, and measurement of two Non-Foster matching networks for a $TM_{10}$ mode antenna and a Non-Foster loading network for a $TE_{10}$ mode antenna are the topics to be discussed, which improve performance of both types of electrically small antennas over broad frequency ranges. These devices take advantage of the unique property of Non-Foster impedances, counter-clock wise rotation on the Smith chart as the frequency increases.
First, a systematic methodology is introduced to design a Non-Foster matching network for an electrically small antenna. Key steps in the proposed methodology are presented to demonstrate how to realize a fabricated Non-Foster capacitor for a 3” electrically small monopole receiver antenna. Based on experimental results, it is verified that Non-Foster matching networks will improve both the antenna gain and the Signal to Noise Ratio (SNR).

Second, a Non-Foster matching network with a series connection of negative capacitor-inductor for the same 3” small monopole antenna is fabricated and tested. This Non-Foster matching network improves the performance of the antenna to higher frequencies. Through measured and simulated data, it is shown that the antenna with a negative capacitor-inductor has an advantage over both the antenna with and without a negative capacitor. To the best of our knowledge, it is the first time that a series combination of a negative capacitor-inductor has been demonstrated with measured data when connected to an actual antenna.

Lastly, this dissertation discusses a method to improve the performance of a small loop antenna by using a Non-Foster inductor loading. The Non-Foster loading network (located away from the input port of the antenna) is employed to improve the impedance mismatch and also maintain an omni-directional antenna pattern at higher frequencies than the case of a loop antenna with a Non-Foster matching network and a short loading (stands for a small loop antenna). Although the experimental radiation patterns are somewhat different from the simulated data, it is found that the measured antenna gain and SNR with the Non-Foster inductor loading are improved when compared to case with a short loading. To the best of our knowledge, it is the first time that a fabricated Non-Foster impedance loading network was applied to an actual antenna.
To God and my parents
ACKNOWLEDGMENTS

I would like to sincerely express my gratitude to my advisor, Professor Roberto G. Rojas for his guidance and support during my years at The Ohio State University. I would also like to thank Professor Fernando L. Teixeira and Professor Patrick Roblin for their time and effort to serve on my dissertation committee.

I would also like to thank all my friends at the ElectroScience Laboratory (ESL) for their helpful discussion and support during my doctoral studies, especially Jun Seok, Niru, Khaled, Shadi, Brandan, Renaud, and Bryan.

Finally, I thank God and my parents for giving me endless love and affection.
VITA

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PUBLICATIONS


Keum-Su Song and Roberto G. Rojas, “A Loop Antenna with Loading a negative inductor,” *IEEE Trans. Antennas Propagat.* (To be submitted)

**FIELDS OF STUDY**

Major Field: Electrical and Computer Engineering

Studies in: Electromagnetics
             RF Circuits
# TABLE OF CONTENTS

Abstract ........................................................................................................... ii
Dedication ........................................................................................................ iv
Acknowledgments ............................................................................................ v
Vita .................................................................................................................... vi
List of Figures .................................................................................................. xi

<table>
<thead>
<tr>
<th>CHAPTER</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Introduction ................................................................. 1</td>
</tr>
<tr>
<td>1.1</td>
<td>Motivation and Approach .................................................. 1</td>
</tr>
<tr>
<td>1.2</td>
<td>Organization of the Dissertation ...................................... 4</td>
</tr>
<tr>
<td>2</td>
<td>Background And Review ..................................................... 6</td>
</tr>
<tr>
<td>2.1</td>
<td>Electrically Small Antennas .............................................. 6</td>
</tr>
<tr>
<td>2.1.1</td>
<td>Definition of an electrically small antenna ........................ 6</td>
</tr>
<tr>
<td>2.1.2</td>
<td>Fundamental limitations of electrically small antennas ....... 7</td>
</tr>
<tr>
<td>2.2</td>
<td>Gain-Bandwidth Limitation of Lossless Passive Matching Networks ............................................. 14</td>
</tr>
<tr>
<td>2.3</td>
<td>Realization of Non-Foster impedances ................................ 16</td>
</tr>
<tr>
<td>2.3.1</td>
<td>Foster Impedances ............................................................... 18</td>
</tr>
<tr>
<td>2.3.2</td>
<td>Non-Foster Impedances ...................................................... 19</td>
</tr>
<tr>
<td>2.3.3</td>
<td>Negative Impedance Converters (NICs) ............................. 20</td>
</tr>
<tr>
<td>2.3.4</td>
<td>Negative Impedance Inverters (NIVs) ............................... 25</td>
</tr>
<tr>
<td>2.3.5</td>
<td>Stability ................................................................. 28</td>
</tr>
<tr>
<td>2.4</td>
<td>Literature Review : Fabricated Non-Foster impedance matching networks for electrically small antennas ............................................. 29</td>
</tr>
<tr>
<td>2.4.1</td>
<td>Harris’ work ................................................................. 30</td>
</tr>
<tr>
<td>2.4.2</td>
<td>Perry’s work ................................................................. 32</td>
</tr>
<tr>
<td>2.4.3</td>
<td>Sussman-Fort’s work ...................................................... 34</td>
</tr>
<tr>
<td>Section</td>
<td>Title</td>
</tr>
<tr>
<td>---------</td>
<td>----------------------------------------------------------------------</td>
</tr>
<tr>
<td>3</td>
<td>Non-Foster Matching Network with a Negative Capacitor for An Electrically Small Antenna</td>
</tr>
<tr>
<td>3.1</td>
<td>Introduction</td>
</tr>
<tr>
<td>3.2</td>
<td>Methodology to design Non-Foster impedance matching networks for electrically small antennas</td>
</tr>
<tr>
<td>3.3</td>
<td>Overall antenna gain with a Non-Foster impedance matching network</td>
</tr>
<tr>
<td>3.4</td>
<td>Fabrication of a stabilized Non-Foster impedance matching network for an electrically small antenna</td>
</tr>
<tr>
<td>3.5</td>
<td>Measured and Simulated Results</td>
</tr>
<tr>
<td>3.5.1</td>
<td>Input Impedance</td>
</tr>
<tr>
<td>3.5.2</td>
<td>$S_{11}$ as a function of input power</td>
</tr>
<tr>
<td>3.5.3</td>
<td>Antenna Gain and Signal to Noise Ratio</td>
</tr>
<tr>
<td>3.6</td>
<td>Summary</td>
</tr>
<tr>
<td>4</td>
<td>Non-Foster Matching Network with a Negative Capacitor-Inductor for An Electrically Small Antenna</td>
</tr>
<tr>
<td>4.1</td>
<td>Introduction</td>
</tr>
<tr>
<td>4.2</td>
<td>Fabricated Non-Foster matching network comprised of a negative capacitor</td>
</tr>
<tr>
<td>4.3</td>
<td>Fabrication of Non-Foster matching network consisting of series connected negative capacitor-inductor</td>
</tr>
<tr>
<td>4.4</td>
<td>Measured and Simulated results</td>
</tr>
<tr>
<td>4.4.1</td>
<td>Input Impedance</td>
</tr>
<tr>
<td>4.4.2</td>
<td>Antenna Gain and Signal to Noise Ratio</td>
</tr>
<tr>
<td>4.5</td>
<td>Summary</td>
</tr>
<tr>
<td>5</td>
<td>Non-Foster Impedance Loading Network for An Electrically Small Loop Antenna</td>
</tr>
<tr>
<td>5.1</td>
<td>Introduction</td>
</tr>
<tr>
<td>5.2</td>
<td>Limitation of Non-Foster matching networks with a single element for electrically small loop antennas</td>
</tr>
<tr>
<td>5.3</td>
<td>Design a Non-Foster impedance loading network for an electrically small loop antenna</td>
</tr>
<tr>
<td>5.3.1</td>
<td>Design of loading network for a small loop antenna based on the equivalent model</td>
</tr>
<tr>
<td>5.3.2</td>
<td>Design a loading network for a small loop antenna based on multi-port characteristic mode theory</td>
</tr>
<tr>
<td>5.4</td>
<td>Design and Fabrication of stabilized Non-Foster inductor</td>
</tr>
<tr>
<td>5.5</td>
<td>Measured and Simulated Results</td>
</tr>
<tr>
<td>5.5.1</td>
<td>Input Impedances</td>
</tr>
<tr>
<td>5.5.2</td>
<td>Reflection coefficient versus input power level</td>
</tr>
<tr>
<td>5.5.3</td>
<td>Antenna Pattern</td>
</tr>
</tbody>
</table>
## 5.5.4 Antenna Signal to Noise Ratio

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.5.4</td>
<td>121</td>
</tr>
</tbody>
</table>

## 5.6 Summary

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.6</td>
<td>124</td>
</tr>
</tbody>
</table>

## 6 Conclusions and Future Work

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>126</td>
</tr>
</tbody>
</table>

### 6.1 Summary and Conclusion

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.1</td>
<td>126</td>
</tr>
</tbody>
</table>

### 6.2 Future Work

<table>
<thead>
<tr>
<th>Subsection</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.2.1</td>
<td>129</td>
</tr>
<tr>
<td>6.2.2</td>
<td>130</td>
</tr>
<tr>
<td>6.2.3</td>
<td>131</td>
</tr>
</tbody>
</table>

### 6.2.1 Non-Foster impedance application to transmitter antennas

<table>
<thead>
<tr>
<th>Subsection</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.2.1.1</td>
<td>129</td>
</tr>
</tbody>
</table>

### 6.2.2 Integrated Non-Foster impedances

<table>
<thead>
<tr>
<th>Subsection</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.2.2.1</td>
<td>130</td>
</tr>
</tbody>
</table>

### 6.2.3 Low power Non-Foster impedances

<table>
<thead>
<tr>
<th>Subsection</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.2.3.1</td>
<td>131</td>
</tr>
</tbody>
</table>

## APPENDICES

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>132</td>
</tr>
</tbody>
</table>

### A Realization of a Negative Resistor

<table>
<thead>
<tr>
<th>Subsection</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>A.1</td>
<td>132</td>
</tr>
</tbody>
</table>

### B Schematics of Non-Foster impedances

<table>
<thead>
<tr>
<th>Subsection</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>B.1</td>
<td>138</td>
</tr>
</tbody>
</table>

### Bibliography

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bibliography</td>
<td>140</td>
</tr>
</tbody>
</table>
## LIST OF FIGURES

<table>
<thead>
<tr>
<th>FIGURE</th>
<th>DESCRIPTION</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>A dipole-type antenna (with the maximum dimension 2a) circumscribed by a hypothetical sphere with radius a [24]</td>
<td>7</td>
</tr>
<tr>
<td>2.2</td>
<td>Electric antenna represented by (a) a capacitor and Magnetic antenna represented by (b) an inductor enclosed by an equal cylindrical volume [1]</td>
<td>9</td>
</tr>
<tr>
<td>2.3</td>
<td>Equivalent lumped models for (a) an electric antenna and (b) a magnetic antenna [1]</td>
<td>10</td>
</tr>
<tr>
<td>2.4</td>
<td>Equivalent lumped model for a $TM_{10}$ mode antenna ($r$ is the maximum radius of a sphere circumscribing a radiator. $\epsilon_0$ and $\mu_0$ are the permittivity and permeability in free space.) [2, 27]</td>
<td>12</td>
</tr>
<tr>
<td>2.5</td>
<td>Equivalent lumped model for a $TE_{10}$ mode antenna [2, 27]</td>
<td>13</td>
</tr>
<tr>
<td>2.6</td>
<td>Lossless two-port matching network for a series passive RC [31]</td>
<td>15</td>
</tr>
<tr>
<td>2.7</td>
<td>A possible reflection coefficient response with a lossless 2-port matching network for a series passive RC [31]</td>
<td>16</td>
</tr>
<tr>
<td>2.8</td>
<td>Net-reactance of an electrically small antenna with (a) conventional lossless passive (Foster impedance) matching network (b) Non-Foster impedance matching network [20, 21]</td>
<td>17</td>
</tr>
<tr>
<td>2.9</td>
<td>Reactance or susceptance function of a Foster element along the frequency axis [27]</td>
<td>19</td>
</tr>
<tr>
<td>2.10</td>
<td>Reflection coefficient of (a) an ideal negative capacitor (b) an ideal negative inductor as frequencies increase</td>
<td>20</td>
</tr>
<tr>
<td>2.11</td>
<td>Overview of an ideal NIC [21]</td>
<td>21</td>
</tr>
<tr>
<td>2.12</td>
<td>Equivalent model for a general two-port $h$-parameter network with an arbitrary passive load $Z_L$ at port 2 [42]</td>
<td>22</td>
</tr>
</tbody>
</table>
Fabricated Non-Foster impedance matching network comprised of a negative capacitor for a 3'' receiving wire monopole on a 3'' × 3'' metal ground plane ........................................................................ 46

Loop Gain Analysis of Non-Foster impedance matching network at low frequencies without stabilization .................................................. 48

Loop Gain Analysis of Non-Foster impedance matching network at high frequencies without considering stabilization .......................... 48

Harmonic Balance Analysis of Non-Foster impedance matching network without stabilization ............................................................ 49

Schematic of a stabilized system including both a Non-Foster impedance matching network comprised of a negative capacitor and the test antenna (biasing circuits not shown) ............................................. 50

Measured low-frequency oscillations using a spectrum analyzer ........ 50

Comparison between measured and simulated data for the imaginary part of the input impedance of the test antenna with and without a Non-Foster impedance matching network (−6.8pF) ............................................. 52

Comparison between measured and simulated data for the real part of the input impedance of the test antenna with and without a Non-Foster impedance matching network (−6.8pF) ............................................. 53

Comparison between measured and simulated magnitude of $S_{11}$ of the test antenna with Non-Foster impedance matching network (−6.8pF) for low input powers (−5dBm, 0dBm, and 4dBm) ......................... 54

Comparison between measured and simulated magnitude of $S_{11}$ of the test antenna with Non-Foster impedance matching network (−6.8pF) for high input powers (4dBm, 8dBm, and 12dBm) ......................... 55

Transducer Gains at different input powers relative to input power of −5dBm .................................................................................. 56

Illustration of the outdoor range setup for antenna gain and SNR measurements .................................................................................. 58

Picture of the outdoor range setup for antenna gain and SNR measurements .................................................................................. 58

Comparison between measured antenna gains (with/without the matching network) before and after applying time-gating over 100MHz to 300MHz ........................................................................ 59
3.23 Measurement of realized antenna gains for frequencies 100MHz to 300MHz .......................................................... 62
3.24 Improvement of measured antenna gain with the Non-Foster matching network relative to the case without a matching network over 100MHz to 300MHz .......................................................... 62
3.25 Measured Noise Floor of a spectrum analyzer at outdoor range .... 64
3.26 Improvement of SNR with a Non-Foster impedance matching network relative to case without a matching network over 100MHz to 300MHz .......................................................... 65
3.27 Comparison between measured antenna gains (with/without the matching network) before and after applying time-gating over 300MHz to 550MHz .......................................................... 66
3.28 Measurement of realized antenna gains over 300MHz to 550MHz ... 67
3.29 Improvement of measured antenna gain with a Non-Foster impedance matching network relative to the case without a matching network over 300MHz to 550MHz .......................................................... 67
3.30 Improvement of SNR with a Non-Foster impedance matching network relative to case without a matching network over 300MHz to 550MHz .......................................................... 68
4.1 Loop Gain Analysis at low frequencies for Non-Foster impedance matching network (−5.6pF) without considering stabilization .......... 74
4.2 Loop Gain Analysis at high frequencies for the Non-Foster impedance matching network (−5.6pF) without considering stabilization ...... 74
4.3 Schematic of a stabilized system including both a Non-Foster impedance matching network comprised of a negative capacitor (−5.6pF) for the test antenna (biasing circuits not shown) ................. 75
4.4 Fabricated Non-Foster impedance matching network comprised of a negative capacitor (−5.6pF) ......................................................... 75
4.5 Equivalent lumped models for the input impedance of an antenna with the \( TM_{10} \) mode ................................................................. 76
4.6 A Non-Foster impedance matching with a series combination of negative capacitor-inductor for a 3” wire monopole antenna \[46,58,67,68\] 77
4.7 Schematic of a stabilized system including both a Non-Foster impedance matching network comprised of a series combination of negative capacitor-inductor for the test antenna (biasing circuits not shown) ........ 78
4.8 Fabricated Non-Foster impedance matching network comprised of a series combination of negative capacitor ($-5.6\text{pF}$)-inductor ($-10\text{nH}$) for the test antenna 79

4.9 Comparisons between measured and simulated results for the imaginary part of the input impedance of the test antenna with/without the Non-Foster impedance matching network (Blue Colors: Ant. with the negative capacitor ($-5.6\text{pF}$), Red Colors: Ant. with the series combination of negative capacitor ($-5.6\text{pF}$)-inductor ($-10\text{nH}$)) 80

4.10 Comparisons between measured and simulated results for the real part of the input impedance of the test antenna with/without the Non-Foster impedance matching network (Blue Colors: Ant. with the negative capacitor ($-5.6\text{pF}$), Red Colors: Ant. with the series combination of negative capacitor ($-5.6\text{pF}$)-inductor ($-10\text{nH}$)) 81

4.11 Comparison between measured antenna gains (with/without the matching network) before and after applying time-gating 83

4.12 Measured realized antenna gain at outdoor range 84

4.13 Measured antenna gain at outdoor range: Improvement of antenna gain with a Non-Foster impedance matching network relative to the case without a matching network 85

4.14 Improvement of SNR with a Non-Foster impedance matching network relative to case without a matching network at outdoor range 86

4.15 Comparison between measured antenna gains (with/without the matching network) before and after applying time-gating 87

4.16 Measured realized antenna gain at indoor range 88

4.17 Measured antenna gain at Indoor range: Improvement of antenna gain with a Non-Foster impedance matching network relative to the case without a matching network 88

4.18 Improvement of SNR with a Non-Foster impedance matching network relative to case without a matching network at indoor range 89

5.1 Circular loop antenna 93

5.2 Circular two-port loop antenna 93

5.3 Equivalent lumped model for the input impedance of an electrically small loop antenna [22] 95
<table>
<thead>
<tr>
<th>Section</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.4</td>
<td>3&quot; wire loop antenna on a octagonal metal ground plane (a) Top view (b) Side view</td>
<td>96</td>
</tr>
<tr>
<td>5.5</td>
<td>Measured $S_{11}$ of the 2-port antenna (Loading port is terminated with a short) in Fig. 5.4 for 10MHz $\sim$ 500MHz</td>
<td>97</td>
</tr>
<tr>
<td>5.6</td>
<td>Simulated $S_{11}$ of a loop antenna with/without a Non-Foster impedance matching network (a) Magnitude of $S_{11}$ (b) Phase of $S_{11}$</td>
<td>98</td>
</tr>
<tr>
<td>5.7</td>
<td>Simulated Input impedance of a loop antenna with/without a Non-Foster impedance matching network (a) Imaginary part (b) Real part</td>
<td>99</td>
</tr>
<tr>
<td>5.8</td>
<td>Equivalent lumped model (Left figure) for input impedance of a small loop antenna with a single negative inductor load (Right figure) located at the opposite of the feeding port</td>
<td>101</td>
</tr>
<tr>
<td>5.9</td>
<td>Linvill’s unbalanced short circuit stable NIC comprised of Bipolar Junction Transistors [38]</td>
<td>103</td>
</tr>
<tr>
<td>5.10</td>
<td>Loop Gain Analysis at high frequency when the Non-Foster impedance loading network without considering a stabilization</td>
<td>105</td>
</tr>
<tr>
<td>5.11</td>
<td>Schematic layout of a loading network comprised of a stabilized Non-Foster inductor for the 3&quot; half loop antenna in Fig. 5.4 (biasing circuits not shown)</td>
<td>105</td>
</tr>
<tr>
<td>5.12</td>
<td>Fabricated loading network comprised of a stabilized Non-Foster inductor for the 3&quot; half loop antenna in Fig. 5.4</td>
<td>106</td>
</tr>
<tr>
<td>5.13</td>
<td>Comparison between measured and simulated imaginary part of the input impedance of the test antenna with a short loading and with a Non-Foster inductor loading network: (a) and (b) use different scales on the y-axis</td>
<td>107</td>
</tr>
<tr>
<td>5.14</td>
<td>Comparison between measured and simulated real part of the input impedance of the test antenna with a short loading and with a Non-Foster inductor loading network: (a) and (b) use different scales on the y-axis</td>
<td>107</td>
</tr>
<tr>
<td>5.15</td>
<td>Comparison between measured and simulated magnitude of $S_{11}$ of the test antenna with the Non-Foster inductor load for low input power levels ($-10$dBm, $-5$dBm, and 0dBm)</td>
<td>109</td>
</tr>
<tr>
<td>5.16</td>
<td>Comparison between measured and simulated magnitude of $S_{11}$ of the test antenna with the Non-Foster inductor load for high input power levels (0dBm, 5dBm, and 10dBm)</td>
<td>110</td>
</tr>
</tbody>
</table>
5.17 Illustration of the outdoor range setup for antenna pattern measurements

5.18 Illustration of (a) the azimuth and (b) elevation plane coordinate for antenna pattern measurements

5.19 Comparison between measured and simulated of the antenna azimuth patterns with a short loading and with a Non-Foster inductor loading at (a) 100MHz (b) 120MHz (c) 140MHz (d) 160MHz

5.20 Comparison between measured and simulated of the antenna azimuth patterns with a short loading and with a Non-Foster inductor loading at (a) 180MHz (b) 200MHz (c) 220MHz (d) 240MHz

5.21 Comparison between measured and simulated of the antenna azimuth patterns with a short loading and with a Non-Foster inductor loading at (a) 260MHz (b) 280MHz

5.22 Maximum gains of measured antenna patterns in the azimuth plane

5.23 Comparison between measured and simulated of the antenna elevation patterns with a short loading and with a Non-Foster inductor loading at (a) 100MHz (b) 120MHz (c) 140MHz (d) 160MHz

5.24 Comparison between measured and simulated of the antenna elevation patterns with a short loading and with a Non-Foster inductor loading at (a) 180MHz (b) 200MHz (c) 220MHz (d) 240MHz

5.25 Comparison between measured and simulated of the antenna elevation patterns with a short loading and with a Non-Foster inductor loading at (a) 260MHz (b) 280MHz

5.26 Maximum gains of measured antenna patterns in the elevation plane

5.27 Measured Noise Floor of a spectrum analyzer at outdoor range

5.28 Improvement of SNR with the Non-Foster loading relative to the case with a short loading for $\theta = 0^\circ$ in elevation plane

5.29 Improvement of SNR with the Non-Foster loading relative to the case with a short loading for $\theta = 90^\circ$ in elevation plane

5.30 Improvement of SNR with the Non-Foster loading relative to the case with a short loading for $\theta = 270^\circ$ in elevation plane

A.1 Test setup for assessing the performance of a negative resistor

A.2 Measured high-frequency oscillations using a spectrum analyzer
A.3 Fabricated test setup in Fig. A.1 including a negative resistor . . . . 134

A.4 Comparison between measured and simulated input impedance \((Z_{in} \text{ in Fig. A.1})\) with the negative resistor (a) Real part (b) Imaginary part 135

A.5 Comparison between measured and simulated magnitude of the reflection coefficient (seen at the input port) with the negative resistor . . 135

A.6 (a) Open (b) Short for de-embedding of both the pad for a SMA connector and the two 50Ω resistors in Fig. A.3 . . . . . . . . . . . . 136

A.7 Comparison between measured (Red curves) and simulated (Blue curves) input impedance \((Z_{in,L} \text{ in Fig. A.1})\) of the negative resistor (a) Real part (b) Imaginary part . . . . . . . . . . . . . . . . . . . . . . . . . . . . 137

B.1 Schematic layout of a stabilized Non-Foster impedance \((-6.8 \text{pF})\) matching network for the 3'' wire monopole antenna in Fig. 3.13 . . . . . . . . 138

B.2 Schematic layout of a loading network comprised of a stabilized Non-Foster inductor for the 3'' half loop antenna in Fig. 5.4 . . . . . . . . 139
CHAPTER 1
INTRODUCTION

1.1 Motivation and Approach

The demand for wide-band small antennas is steadily increasing for wireless communication systems in both civilian and military applications because of the need for compact multi-function devices. In particular, in many of these applications, electrically small antennas are required due to the limited space available in the structures such as electronics mobile devices, medical equipment, unmanned aerial vehicles, etc.

For example, 4G LTE (Long Term Evolution) services will be operated at 700MHz. To seamlessly integrate an antenna into a compact 4G LTE mobile device, an electrically small antenna is required. Although the need of wide-band small antennas is booming, the realization of wide-band electrically small antennas is one of the most challenging problems in antenna engineering.

Various authors [1–6, 8, 9] have investigated the fundamental limitation of small antennas over the half-century, especially focusing on the lower bounds of the radiation quality factor ($Q_{\text{rad}}$) along with the electrical size of the radiators (involving both the antenna and a finite ground plane). For instance, it is shown in [2] that the $Q_{\text{rad}}$ of a small antenna is proportional to the third power of the reciprocal of the electrical size of antenna. It can be interpreted that an electrically small antenna has extremely high $Q_{\text{rad}}$ because most of the input power (reactive energy) is stored in
the near-field region and little power is radiated in the far-field region [10]. For this reason, it is necessary to increase the radiated power (radiation resistance) and/or reduce the stored energy in the near-field to decrease the $Q_{rad}$ in the frequency range of interest. Two methods are generally used to decrease the $Q_{rad}$: namely, modify the radiator using as much of the available volume [11–13] as well as making use of lumped elements [14,15] or materials [16] with appropriate permittivity/permeability values. However, it is difficult to design an antenna satisfying two requirements (a high gain and a wide-bandwidth) at the same time with the aforementioned two methods.

Linearly polarized electrically small antennas (ESAs) can be generally classified into two types based on their excited modes: One is a dipole-type antenna with a $TM_{10}$ mode and the other is a loop-type with a $TE_{10}$ mode. For the antenna with $TM_{10}$ mode, the input impedance of the antenna is considerably reactive (capacitive) with a small real part. In contrast, the input admittance of the antenna with $TE_{10}$ mode is characterized by a high susceptance and a small conductance, i.e. the input impedance is almost a short. It is critical to reduce the input reactance/susceptance of the antenna in order to improve impedance matching in the frequency range of interest. Impedance matching networks are incorporated to the input port of an antenna to increase the transfer of power from a transmitter to the highly reactive/susceptive antenna (transmitting case) or from the antenna to the receiver (receiving case) in a given frequency range. However, when lossless passive matching networks are used, there is a fundamental Gain-Bandwidth restriction derived by Bode in [17] and Fano in [18].

The work considered in this dissertation develops methods to improve performance of both types of electrically small antennas over broad frequency ranges. The
improvement was achieved by means of a Non-Foster impedance matching and loading network for a dipole-type and a loop-type antenna ESAs, respectively. These methods take advantage of the unique property of Non-Foster impedances; namely, counter-clockwise rotation on the Smith chart with increasing frequency.

**Major contributions** of this dissertation:

- Introduced a systematic methodology to design a Non-Foster impedance matching network for monopole or dipole-type small receiving antennas. Key steps of the method are shown how to realize a fabricated Non-Foster impedance matching network for the antenna. The advantage of Non-Foster matching network relative to the case without a matching network is maintained up to the highest frequency among published papers. The schematic presented in this dissertation can be applied to other monopole or dipole-type small antennas.

- Designed, fabricated, and tested a Non-Foster impedance loading network for a small loop antenna to both increase the antenna gain and maintain the omni-directional radiation pattern at higher frequencies.

- Employed a fabricated Non-Foster impedance composed of a series connection of negative capacitor and negative inductor to a small monopole antenna in order to maintain the advantage of Non-Foster matching network for higher frequencies.

- Measured and discussed one of non-linear properties of a Non-Foster impedance matching and loading network.

- Showed completely measurement results. (Since previous fabricated Non-Foster impedance matching networks for electrically small antennas were worked for the military, the publications show restrictively measured data.)
1.2 Organization of the Dissertation

The rest of this dissertation is organized in the following manner.

Chapter 2 addresses the background behind electrically small antennas and Non-Foster impedances, and reviews previous literature on fabricated Non-Foster matching network for electrically small antennas. First it starts with a review of electrically small antennas, i.e. the definition of electrically small antennas and their fundamental limitation. Electrically small antennas are neither efficient nor good radiators because their radiation quality factor is considerably high. It is therefore critical to add appropriate matching networks to the antenna to enhance its realized gain and maintain its performance over a large frequency range. However, when passive matching networks are used, there is a fundamental Gain-Bandwidth restriction. Hence, the Gain-Bandwidth limitation of conventional passive matching networks is investigated. Fortunately, there is a way to overcome this limitation by using matching networks implemented with Non-Foster impedances (negative capacitors and/or inductors). Therefore, we will review the properties of Non-Foster impedances. Since Non-Foster impedances can be implemented with Negative Impedance Converters or Inverters, these active circuits are investigated. Lastly, previous literature of fabricated Non-Foster impedance matching networks for electrically small antennas is reviewed as well.

Chapter 3 introduces a systematic methodology to design a Non-Foster impedance matching network for an electrically small monopole antenna. Each key step of the proposed methodology is presented to show how to fabricate a Non-Foster matching network comprised of a negative capacitor for a 3″ electrically small receiving monopole antenna and how to stabilize the Non-Foster impedance matching network. After fabricating a stabilized system comprised of a negative capacitor and an antenna, we measured its performance (input impedance, antenna gain, and Signal
to Noise Ratio (SNR)). The changes of the reflection coefficient ($S_{11}$) to the input power levels were also measured and simulated at an input port of the antenna with the Non-Foster matching network. Through both measured and simulated data, it is shown that a Non-Foster impedance matching network can be used to match an electrically small antenna to a generator with a 50 Ω internal impedance over a fairly large frequency band.

Chapter 4 introduces a fabricated Non-Foster impedance matching network with a series connection of negative capacitor-inductor for an electrically small monopole antenna to extend the performance of the antenna to higher frequencies. To the best of our knowledge, all fabricated Non-Foster impedance matching networks for monopole or dipole-type small antennas have been implemented with single elements only like the active networks in Chapter 3 and in [19–21]. With a single negative capacitor, the reactance of the antennas input impedance can only be reduced at low frequencies where the reactance of the input impedance is characterized by a large capacitive reactance. Through measured and simulated data, it is shown that the antenna with a negative capacitor-inductor performs better at higher frequencies than an antenna with/without a single negative capacitor.

Chapter 5 introduces a method to improve the performance of a small loop antenna. The proposed method is to use a Non-Foster inductor as a loading in the antenna. The advantage of a loading network for a loop antenna over the case with a Non-Foster impedance matching network is an improvement of the antenna gain and keeps an omni-directional pattern at higher frequencies. After fabricating the antenna with a Non-Foster inductor loading, we measured its performance as was done Chapter 3. To the best of our knowledge, it is the first time that a fabricated Non-Foster impedance loading network is applied to an actual antenna.

Finally, a summary and suggestions for future work are given in Chapter 6.
CHAPTER 2
BACKGROUND AND REVIEW

This chapter first reviews the fundamental limitations of electrically small antennas and Gain-Bandwidth restriction of lossless passive matching networks. Second, a brief review of Non-Foster impedances and previous literature on fabricated Non-Foster impedances for electrically small antennas is given. Since Non-Foster impedances can be implemented with Negative Impedance Converters (NICs) or Inverters (NIVs), the active circuits are investigated in this chapter as well.

2.1 Electrically Small Antennas

2.1.1 Definition of an electrically small antenna

Although there is no strict definition of electrically small antennas, they are usually specified by the value of \( ka \) where \( k \) is the wave-number \((2\pi/\lambda)\) in free space and \( a \) is the minimum radius of a hypothetical sphere enclosing the radiator, including a finite ground plane (when applicable) [22]. This definition is based on the concept of a radian-length \((\lambda/2\pi)\) introduced by Wheeler in [1]. When the maximum dimension of an antenna is less than the radian-length, it can be considered an electrically small antenna [1]. Note that the sphere with a radius \( \lambda/2\pi \) is referred to a radian-sphere which is a boundary of the transition between the near-field and far-field of a small antenna [23]. Fig. 2.1 illustrates a dipole-type electrically small antenna with
Figure 2.1: A dipole-type antenna (with the maximum dimension 2a) circumscribed by a hypothetical sphere with radius $a$ [24]

a maximum linear dimension 2a fitted within a hypothetical sphere with radius $a$. When the criterion by Wheeler is applied to the antenna in Fig. 2.1, the definition of an electrically small antenna can be given by

$$2a < \frac{\lambda}{2\pi}. \tag{2.1}$$

In terms of a wave-number ($k$), (2.1) can be rewritten as

$$\frac{2\pi}{\lambda}a = ka < \frac{1}{2}. \tag{2.2}$$

Therefore, an antenna, either a dipole-type or a loop-type, is considered to be an electrically small when $ka < 0.5$.

### 2.1.2 Fundamental limitations of electrically small antennas

An electrically small antenna with an open-ended side such as a dipole or a monopole antenna could be characterized by an open circuit on the Smith chart while the input impedance of a small antenna with a short-ended side such as a loop antenna could be a short circuit on the Smith chart.
The electrically small antenna has extremely high radiation quality factor because most of the input power (reactive energy) is stored in the near-field regions and little power is radiated in the far-field regions [10]. The radiation quality factor ($Q_{rad}$) of such a small antenna is strongly dependent on the electric height of the antenna [22]. Hence, various authors [1–9] have studied the fundamental limitations of small antennas for over a half-century, especially focusing on the lower bounds of the radiation quality factor as a function of the electrical size of the antenna. In the following two subsections, earlier literature about the limitations of electrically small antennas written by major contributors (Wheeler and Chu) will be reviewed.

**Wheeler’s small antenna**

The fundamental limitation of small antennas was first addressed by Wheeler in 1947 [1]. He assumed that either a lossy capacitor or a lossy inductor could behave as a small antenna, where the dissipated power (loss) stands for only the radiated power. Hence, a capacitor and an inductor in an equal cylindrical volume were considered to be an electric antenna and a magnetic antenna, respectively. The shape of a cylindrical volume was intentionally used because it could make an identical volume occupied by either the capacitor or the inductor. Fig. 2.2 illustrates the two different antennas [1]: (a) an electric antenna is represented by a capacitor, (b) a magnetic antenna represented by a solenoidal inductor. The antennas in Fig. 2.2 (a) and (b) can be modeled as a capacitor in shunt with a conductor for an electric antenna and as an inductor in series with a resistor for a magnetic antenna, respectively. The equivalent lumped models are shown in Fig. 2.3. Wheeler calculated the capacitance ($C$) and the inductance ($L$) in Fig. 2.3 (a) and (b) by using the following formulas [1]:

\[ C = \frac{\epsilon_0 k_a A}{b} \]  \hspace{1cm} (2.3)

\[ L = \mu_0 n^2 \frac{A}{kb} \]  \hspace{1cm} (2.4)
Figure 2.2: Electric antenna represented by (a) a capacitor and Magnetic antenna represented by (b) an inductor enclosed by an equal cylindrical volume [1]

where $k_a$ and $k_b$ are the shape factors of the capacitor and the inductor, respectively, and $n$ is the number of turns in a solenoidal inductor. The constants $\epsilon_0$ and $\mu_0$ denote the permittivity and permeability in free space, respectively. The shape factor $k_a$ in (2.3) was used to obtain the effective area with consideration of the electric field outside the cylinder shown in Fig. 2.2 (a). Similarly, the $k_b$ in (2.4) was used to attain the effective length of the magnetic path, considering the external magnetic field of the inductor in Fig. 2.2 (b).

The radiation shunt conductance ($G_e$) in Fig. 2.3 (a) and series resistance ($R_m$) in Fig. 2.3 (b) were given by [1, 23]

\[ G_e = \frac{1}{6\pi Z_0} \left( \frac{k_a A}{(\lambda/2\pi)^2} \right)^2 \]  
(2.5)

\[ R_m = 20 \left( \frac{n A}{(\lambda/2\pi)^2} \right)^2, \]  
(2.6)

where $Z_0$ is the wave impedance ($120\pi$) in free space.

Wheeler used the Radiation Power Factor (RPF) rather than the radiation quality factor ($Q_{rad}$) to account for the ratio of radiated power to reactive power by a small
antenna [1, 25]. Based on these lumped models in Fig. 2.3, the RPF for each small antenna was evaluated in [1].

For an electric antenna, the RPF ($p_e$) is

$$p_e = \frac{G_e}{\omega C} = \frac{1}{6\pi} \frac{k_a A b}{(\lambda/2\pi)^3} = \frac{1}{6\pi} k_a A b k^3,$$  \hspace{1cm} (2.7)

where $\omega = 2\pi f$ is the radian frequency.

For a magnetic antenna, the RPF ($p_m$) is

$$p_m = \frac{R_m}{\omega L} = \frac{1}{6\pi} \frac{k_b A b}{(\lambda/2\pi)^3} = \frac{1}{6\pi} k_b A b k^3.$$  \hspace{1cm} (2.8)

From (2.7) and (2.8), it can be found that the RPFs are proportional to $k^3$ ($k$ wave-number in free space). Both RPFs are always less than unity for an electrically small size [1].

Since the most common quantity of a small antenna is the $Q_{rad}$ rather than the RPF, the $Q_{rad}$ of antennas in Fig. 2.2 can be derived by using the lumped models in Fig. 2.3 [26].

The radiation quality factor ($Q_{rad}$) is generally defined as [6]

$$Q_{rad} = \frac{2\omega_0 Max(\tilde{W}_e \text{ or } \tilde{W}_m)}{P_{rad}},$$  \hspace{1cm} (2.9)
where \( \bar{W}_e \) and \( \bar{W}_m \) are the time-average stored electric and magnetic energy, and \( P_{rad} \) is the radiated power. \( \omega_0 \) is the resonant frequency of a tuned antenna.

The \( Q_{rad} \) for the electric antenna in Fig. 2.2 (a) can be expressed as \([26]\)

\[
Q_{rad} = \frac{2\omega_0 \bar{W}_e}{P_{rad}} = \frac{2\omega_0 (1/4Cv^2)}{1/2G_e v^2} = \frac{\omega_0 C_e}{G_e},
\]

(2.10)

The \( Q_{rad} \) for the magnetic antenna in Fig. 2.2 (b) can be \([26]\)

\[
Q_{rad} = \frac{2\omega_0 \bar{W}_m}{P_{rad}} = \frac{2\omega_0 (1/4Li^2)}{1/2R_m i^2} = \frac{\omega_0 L}{R_m}.
\]

(2.11)

It can be found that the \( Q_{rad} \) in (2.10) and (2.11) are proportional to the reciprocal of the PRFs in (2.7) and (2.8), respectively.

**Chu’s Small Antennas**

The theoretical limitation of omni-directional antennas was first introduced by Chu in 1948 \([2]\). He used spherical wave functions and applied the recurrence relation of spherical Bessel functions to determine the equivalent lumped element models for the wave impedances of spherical TM and TE modes. Based on these equivalent circuits, theoretical lower bounds for the \( Q_{rad} \) of small antennas as a function of \( ka \) were developed. The derivation of Chu’s \( Q_{rad} \) presented in the following is based on \([2,27,28]\).

The wave impedance of \( r \)-directed (outward) \( TM_{n0} \) modes in a spherical coordinate system can be given by

\[
Z_{r}^{TM} = \frac{E_\phi}{H_\phi} = j\eta \left( k r h_n^{(2)}(k r) \right)' = j\eta \left( \frac{1}{k r} + \left( h_n^{(2)}(k r) \right)' \right),
\]

(2.12)

where \( n \) and \( \eta \) are the mode number and wave impedance in free space, and \( h_n^{(2)} \) is a \( n^{th} \) order spherical Hankel function of the second kind. Applying recurrence relations \([29]\) of the spherical Bessel functions \( (f_n(x)) \), shown in both (2.13) and (2.14), into (2.12),

\[
\frac{n+1}{x} f_n(x) + f_n'(x) = f_{n-1}(x),
\]

(2.13)
\[ f_{n-1}(x) + f_{n+1}(x) = \frac{2n+1}{x}f_n(x) \]  

(2.14)

then the wave impedance (2.12) can be rewritten as

\[ Z_{TM}^r = \eta \frac{n}{jkr} + \frac{\eta}{2n-1} \frac{1}{jkr} - \frac{h_{n-2}^{(2)}(kr)}{j h_{n-1}^{(2)}(kr)} \]  

(2.15)

The expanded equation of (2.15) is given by [27]

\[ Z_{TM}^r = \eta \frac{n}{jkr} + \frac{\eta}{2n-1} \frac{1}{jkr} + \frac{\eta}{2n-3} \frac{1}{jkr} + \cdots + \frac{\eta}{3} \frac{1}{jkr} + \frac{1}{jkr} + 1 \]  

(2.16)

Chu used (2.16) to build an equivalent lumped model for \( TM_{n0} \) modes. For instance, the wave impedance \( Z_{TM}^{10} \) of the \( TM_{10} \) mode is

\[ Z_{TM}^{10} = \eta \frac{n}{jkr} + \frac{\eta}{jkr} + 1 \]  

(2.17)

With the impedance (2.17), the equivalent lumped model for \( Z_{TM}^{10} \) can be modeled as shown in Fig. 2.4.

Figure 2.4: Equivalent lumped model for a \( TM_{10} \) mode antenna (\( r \) is the maximum radius of a sphere circumscribing a radiator. \( \epsilon_0 \) and \( \mu_0 \) are the permittivity and permeability in free space.) [2, 27]
For $r$-directed (outward) $TE_{n0}$ modes [28],

$$Z_{r}^{TE} = \frac{E_\theta}{H_\phi} = -j\eta \frac{kr h_n^{(2)}(kr)}{\left( kr h_n^{(2)}(kr) \right)} = -j\eta \frac{kr h_n^{(2)}(kr)}{h_n^{(2)}(kr) + \left( h_n^{(2)}(kr) \right)}.$$  \hspace{1cm} (2.18)

Similarly, the wave admittance $Y_{r}^{TE}$ can be given by [28]

$$Y_{r}^{TE} = -\frac{1}{\eta kr} \left( \frac{1}{kr} \right) + \frac{1}{kr} \left( \frac{1}{kr} \right) + \cdots + \frac{1}{kr} \left( \frac{1}{kr} \right) + 1$$  \hspace{1cm} (2.19)

The equivalent lumped model for $TE_{10}$ mode is shown in Fig. 2.5.

![Figure 2.5: Equivalent lumped model for a $TE_{10}$ mode antenna [2,27]](image)

Using the wave impedance (2.17) the minimum $Q_{rad}$ (for $TM_{10}$) can be calculated. The minimum $Q_{rad}$ [4] is

$$Q_{chu} = \frac{1 + 2k^2a^2}{(ka)^3[1 + k^2a^2]}.$$  \hspace{1cm} (2.20)

For $ka \ll 1$, the $Q_{chu}$ was shown to be proportional to the third power of the reciprocal of $ka$.

Given a small antenna ($ka \ll 1$), it can also be shown that the Radiation Power Factors in (2.7) and (2.8) are proportional to the reciprocal of $Q_{chu}$. Note that Chu’s
$Q_{\text{rad}}$ does not consider the stored energy inside ($a < r$) of the sphere in Fig. 2.1 and there is no mismatch between an antenna and an input port. Later, Mclean [4] improved the lower bound of the $Q_{\text{chu}}$. For a $TM_{10}$ or $TE_{10}$ mode, the theoretical lower bound for the $Q_{\text{rad}}$ [4] is

$$Q_{\text{rad}} = \frac{1}{(ka)^3} + \frac{1}{ka}. \quad (2.21)$$

When $Q_{\text{rad}}$ is quite large, the fractional half-power bandwidth of the antenna can be approximated by the reciprocal of $Q_{\text{rad}}$ [30]. From (2.21), it can be deduced that the frequency bandwidth of small antennas is strongly affected with their electrical size.

### 2.2 Gain-Bandwidth Limitation of Lossless Passive Matching Networks

As mentioned in section 2.1, the input impedance (or admittance) of an electrically small antenna can be characterized by a large reactance (or susceptance) with a small real part. It is critical to add external impedance matching networks to increase the transfer of power from a transmitter with a real internal impedance, usually 50 Ω, to the highly reactive/susceptive antenna (transmitting case) or from the antenna to the receiver (receiving case) and therefore enhance its realized gain over a large frequency range.

When lossless passive matching networks are employed, there is a fundamental Gain-Bandwidth restriction\(^1\) (or the Gain-Bandwidth product) derived by Bode [17] and Fano [18] between a resistive generator and a complex passive load. Given the minimum reflection coefficient magnitude, Bode-Fano integral criterion on a complex

---

\(^1\)Bode initiated the limitation of lossless matching networks between a resistive generator and a shunt RC load. Later, Fano extended Bode’s work to an arbitrary passive load (using Darlington’s synthesis in [32]) in generality [33, 34].
passive load can be expressed as the Gain-Bandwidth restriction on the load within a flat pass-band [18, 31]. According to the Bode-Fano criterion, there is a tradeoff between the bandwidth and the minimum tolerable reflection coefficient magnitude; once the minimum reflection coefficient magnitude is determined, the bandwidth is limited, and vice versa.

When a dipole-type antenna is electrically small, the input impedance can simply be modeled as a series combination of a resistor and a capacitor. From [18, 31] the Gain-Bandwidth restriction of lossless passive matching networks for a series RC is given by

$$\int_{0}^{\infty} \frac{1}{\omega^2} \ln \left| \frac{1}{\Gamma(\omega)} \right| d\omega < \pi RC, \quad (2.22)$$

where $\Gamma(\omega)$ as shown in Fig. 2.6 is the reflection coefficient seen looking into the lossless two-port matching network. Let the minimum reflection coefficient magnitude be a constant ($\Gamma_m$) over the frequency $\omega_1$ to $\omega_2$ as shown in Fig. 2.7.

Then, (2.22) can be rewritten as [31]

$$\int_{\omega_1}^{\omega_2} \frac{1}{\omega^2} \ln \frac{1}{|\Gamma_m|} d\omega = \Delta \omega \ln \frac{1}{|\Gamma_m|} \leq \pi \omega_0^2 RC, \quad (2.23)$$

![Figure 2.6: Lossless two-port matching network for a series passive RC [31](image)
where $\omega_0 = \sqrt{\omega_1 \omega_2}$ is the geometric mean of the frequency band of interest.

When an infinite number of matching sections is used in the matching network, the equality in (2.23) is satisfied. Hence, the magnitude of the threshold reflection coefficient in (2.23) can be written as

$$\exp \left( -\pi \omega_0^2 \frac{RC}{\Delta \omega} \right) \leq |\Gamma_m|.$$  \hspace{2cm} (2.24)

(2.24) demonstrates the threshold reflection coefficient can be lowered at the expense of the smaller bandwidth.

### 2.3 Realization of Non-Foster impedances

In previous section 2.2, it was mentioned that small antennas could not be matched over wide bandwidth frequencies with conventional passive matching networks due to Gain-Bandwidth limitation. However, Non-Foster impedance matching networks are affected by this limitation and are able to match the antenna for wider bandwidth frequencies.

Fig. 2.8 illustrates the advantage of a Non-Foster impedance matching network
for an electrically small antenna over a conventional passive matching network. The dashed black curves in Fig. 2.8 stand for the reactance of a $TM_{10}$ mode small antenna such as a dipole. It features a highly capacitive reactance. When a first-order conventional lossless passive matching network is applied to the antenna, an inductor can be usually used and is connected in series with the antenna. As shown in Fig. 2.8 (a), it makes the net-reactance of the antenna zero at a single frequency. However, when an ideal negative capacitor (one of Non-Foster impedances), an negating the reactance of the antenna model, is applied and connected to the antenna in series, it can completely cancel out the net-reactance over a large frequency in Fig. 2.8 (b), and subsequently increase the antenna gain over the wide frequencies.

![Diagram](image)

Figure 2.8: Net-reactance of an electrically small antenna with (a) conventional lossless passive (Foster impedance) matching network (b) Non-Foster impedance matching network [20, 21]

When an ideal Non-Foster matching network is applied to an electrically small
antenna to cancel out the net-reactance of the antenna, the reflection coefficient of the antenna at the driving port is mapped into the horizontal axis on the Smith chart because Non-Foster impedances move in the counterclockwise direction on the Smith chart as frequency increases. Note that, when the complex conjugate of a passive load is equal to the complex generator source impedance, it can transfer the maximum power to the passive load \[56\]. Although it cannot be completed the complex conjugate matching of an electrically small antenna by ideal Non-Foster matching networks, it can be partially realized while there is a mismatch between two different resistances (the antenna with the real portion of the impedance and the generator with a resistor). Hence, the efficiency and the antenna gain with ideal Non-foster matching networks can be eventually increased.

### 2.3.1 Foster Impedances

Before addressing Non-Foster impedances, it is necessary to review what Foster impedances are. If a lossless one-port network obeys the Foster Reactance Theorem \[35\], then it is referred to as a Foster element or impedance. This is a conventional lossless passive element such as an inductor and a capacitor. According to the Foster Reactance Theorem, the frequency derivatives of both the reactance and the susceptance are related to the total stored energy and are therefore positive \[27, 36\]. In other words, the slope versus frequency of both the reactance and the susceptance are always positive, i.e.

\[
\frac{\partial X(\omega)}{\partial \omega} > 0 \quad \text{and} \quad \frac{\partial B(\omega)}{\partial \omega} > 0.
\]  

(2.25)

Fig. 2.9 demonstrates the reactance or susceptance function of a Foster element along the frequency axis. In Fig. 2.9, it can be observed that the poles and zeros of the reactance or susceptance alternate as the frequency increases. Since the frequency
derivatives of both the reactance and the susceptance are always positive, the reflection coefficient ($\Gamma$) of Foster elements (positive inductor and/or capacitor) rotates in the clockwise direction on the Smith chart with increasing frequency.

2.3.2 Non-Foster Impedances

If a lossless one-port network disobeys the Foster Reactance Theorem, then it is referred to as a Non-Foster element or impedance. Non-Foster elements such as a negative capacitor and a negative inductor have completely opposite characteristics than conventional Foster elements as follows:

- The Non-Foster element is characterized by the negative slope of its reactance or susceptance as frequency increases.

- The reflection coefficient ($\Gamma$) of a Non-Foster element rotates in the counterclockwise direction on the Smith chart with increasing frequency.

As frequency increases, the reflection coefficients of an ideal negative capacitor and an ideal negative inductor are shown on the Smith chart in Fig. 2.10 (a) and (b), respectively.
Therefore, Non-Foster elements must produce rather than consume energy in order to make the frequency derivative of the reactance (or the susceptance) negative. In this sense, Non-Foster elements, both negative capacitors and negative inductors, should be implemented with active circuits referred to as Negative Impedance Converters (NICs) and Inverters (NIVs).

2.3.3 Negative Impedance Converters (NICs)

According to two earlier papers [20,37], Marius Latour devised the conceptual circuit of a negative impedance converter in the 1920s. Merrill [37] introduced the realization of a negative impedance converter (NIC) comprised of vacuum tube circuits in 1951. He made a negative resistor, called negative impedance repeater, to increase transmission gains on telephone lines. In 1953, Linvill [38,39] presented the first practical negative impedance converters comprised of transistors. Linvill designed two balanced voltage inversion NICs to implement negative resistances and showed
measured data as well. He also introduced two unbalanced voltage inversion NICs. Each of these two types of NICs are either open circuit stable or short circuit stable. In 1957, Larky [40] and Yanagisawa [43] presented the current inversion NICs comprised of transistors and also showed experimental results. Although the topologies of NICs have been developed a long while ago, it is arduous to realize stable Non-Foster impedances for antennas in a large frequency range. It is worthy to note that there are a variety of NIC topologies in [44,45].

A NIC is defined as an active two-port network where the driving port impedance is converted to the negative of a load impedance connected to the other port [37,38,40]. An ideal NIC is shown in Fig. 2.11, where the input impedance $Z_{in}$, seen at port 1,

$$Z_{in} = -KZ_L \quad (K > 0)$$

is the negative of $K$ multiplied by the corresponding load impedance, $Z_L$, at port 2, where $K$ is the impedance converter coefficient of the NIC. For an ideal NIC, $K$ is a positive real constant. An ideal NIC can be expressed in terms of conventional two-port parameters. Herein, the hybrid $h$-parameters are used to represent ideal NICs [20,46,47]. Fig. 2.12 shows the equivalent model for a general two-port $h$-parameter network with an arbitrary passive load $Z_L$ at port 2.

![Figure 2.11: Overview of an ideal NIC [21]](image-url)
Figure 2.12: Equivalent model for a general two-port $h$-parameter network with an arbitrary passive load $Z_L$ at port 2 [42]

The defining equations for a two-port $h$-parameter are as follows:

\[ V_1 = h_{11}I_1 + h_{12}V_2 \]  
\[ I_2 = h_{21}I_1 + h_{22}V_2 \]

where,

\[ h_{11} = \left. \frac{V_1}{I_1} \right|_{V_2=0} \quad h_{12} = \left. \frac{V_1}{V_2} \right|_{I_1=0} \quad h_{21} = \left. \frac{I_2}{I_1} \right|_{V_2=0} \quad h_{22} = \left. \frac{I_2}{V_2} \right|_{I_1=0} . \]

Given an arbitrary load passive $Z_L$ at port 2 in Fig. 2.12, the input impedance ($Z_{in} = V_1/I_1$) at the driving port is [20]

\[ Z_{in} = h_{11} \left[ 1 - \frac{h_{12}h_{21}Z_L}{h_{22}Z_L + 1} \right] . \]  

The necessary and sufficient conditions to realize a NIC with the ideal property ($Z_{in} = -KZ_L$ in Fig. 2.11) can be obtained from (2.28) [41]. For an ideal NIC with impedance converter coefficient $K$, it should be $h_{11} = 0$, $h_{22} = 0$, and $h_{12}h_{21} = K$.

Since the impedance converter coefficient, $h_{12}h_{21} = K$, should be positive to convert a positive impedance into a negative one, both $h_{12}$ and $h_{21}$ can be either positive or negative. Depending on the sign of both $h_{12}$ and $h_{21}$, the properties of a NIC are different [20]. In the following two subsections, two different types of ideal
NICs will be investigated. For simplicity, it is assumed the impedance converter coefficient is unity.

**Current Inversion NICs**

When both $h_{12}$ and $h_{21}$ of an ideal NIC ($h_{11} = 0$, $h_{22} = 0$) are positive and unity, (2.26) and (2.27) leads to

$$V_1 = V_2 \quad I_1 = I_2.$$  \hspace{1cm} (2.29)

This makes the following relationship between the current at port 1 and at the load:

$$I_1 = -I_L.$$  \hspace{1cm} (2.30)

Hence, the impedance at the port 1 is

$$\frac{V_1}{I_1} = \frac{I_L Z_L}{-I_L} = -Z_L.$$  \hspace{1cm} (2.31)

From (2.29) and (2.30), it can be observed that the voltage at port 1 is the same as the voltage across a load $Z_L$ in Fig. 2.12 while the direction of currents are opposed. If a NIC has the features indicated by (2.29) and (2.30), it is called a current inversion NIC. For an ideal current inversion NIC, the $h$-parameter matrix is

$$h = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$  \hspace{1cm} (2.32)

As one of example, Fig. 2.13 shows Laky’s current inversion NIC [40]. When terminating $Z_L$ at port 2 in the NIC, the input impedance at port 1 is approximated by $-\frac{R_1}{R_2} Z_L$. Larky demonstrated experimentally that this circuit was stable as long as the port 2 was open-circuited or both ports were shorted [40].

**Voltage Inversion NICs**

Consider an ideal NIC where both $h_{12}$ and $h_{21}$ are equal to negative one. Then, (2.26) and (2.27) become

$$V_1 = -V_2 \quad I_1 = -I_2.$$  \hspace{1cm} (2.33)
The voltage and current at the port 1 have the following relations:

\[ V_1 = -I_L Z_L \quad I_1 = I_L. \]  

(2.34)

Hence, the driving port impedance is given by

\[ \frac{V_1}{I_1} = \frac{-I_L Z_L}{I_L} = -Z_L. \]  

(2.35)

It can be found that the direction of the current at the input terminal is the same as that at the load terminal. However, the voltage across the input port is inverted with respect to that at the load port. Hence, it is called a voltage inversion NIC. For an ideal voltage inversion NIC, the \( h \)-parameter matrix is

\[
h = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}.
\]  

(2.36)

Fig. 2.14 shows a unbalanced Linvill’s open circuit stable voltage inversion NIC with terminating impedance \( Z_L \). The input impedance of the NIC is ideally \( -\frac{R_2}{R_1} Z_L \).
2.3.4 Negative Impedance Inverters (NIVs)

It was noted that for an ideal NIC, the driving port impedance at one port is the negative of a load impedance connected at the other port. However, the negative impedance inverter (NIV) transforms the driving port impedance to the negative of the load admittance [48–50]. An ideal NIV comprised of a two-port network is shown in Fig. 2.15, where the input impedance $Z_{in}$, seen at port 1, is the negative of $K$
multiplied by the corresponding load admittance, \( Y_L = 1/Z_L \), at port 2. Note that \( K \) is the impedance inverter coefficient of the NIV. For an ideal NIV, \( K \) is a positive real constant.

\[ I_1 \quad Z_{11} \quad Z_{12}I_2 \quad Z_{21}I_1 \quad I_2 \quad Z_{22} \]

\[ V_1 \quad - \quad Port 1 \]
\[ + \]
\[ Z_L \]
\[ V_2 \quad - \quad Port 2 \]
\[ + \]
\[ I_L \]

Figure 2.16: Equivalent model for a general two-port \( z \)-parameter network with an arbitrary passive load \( Z_L \) at port 2 [42]

Similarly to an ideal NIC, an ideal NIV can be expressed in terms of a two-port network parameter. Fig. 2.16 shows the equivalent model for a general two-port \( z \)-parameter network with an arbitrary passive load \( Z_L \) at port 2. The defining equation of a two-port \( z \)-parameter is as follows:

\[ V_1 = z_{11}I_1 + z_{12}I_2 \quad (2.37) \]
\[ V_2 = z_{21}I_1 + z_{22}I_2 \quad (2.38) \]

where,

\[ z_{11} = \frac{V_1}{I_1} \bigg|_{I_2=0} \quad z_{12} = \frac{V_1}{I_2} \bigg|_{I_1=0} \quad z_{21} = \frac{V_2}{I_1} \bigg|_{I_2=0} \quad z_{22} = \frac{V_2}{I_2} \bigg|_{I_1=0} . \]

The driving port impedance can be written in terms of \( z \)-parameter as [49]

\[ Z_{in} = z_{11} - \frac{z_{12}z_{21}}{Z_L + z_{22}}. \quad (2.39) \]
From (2.39), the following necessary and sufficient conditions to realize a NIV with the ideal property \( Z_{in} = -KY_L \) in Fig. 2.15 can be found: \( z_{11} = 0, z_{22} = 0, \) and \( z_{12}z_{21} = K \) [49].

Since the impedance converter coefficient, \( z_{12}z_{21} = K \), should be positive to transform the driving port impedance into the negative of the load admittance, both \( z_{12} \) and \( z_{21} \) can be either positive or negative. Unlike NICs, the NIV cannot be generally divided into a voltage or a current inversion because the property of a NIV is different depending on the passive load and the sign of \( z_{12} \) and \( z_{21} \). For simplicity, the impedance converter coefficient is assumed unity.

![Figure 2.17: Negative capacitor by means of a NIV [51]](image)

When a resistor terminated at the port 2 in Fig. 2.15, it is possible to separate two type NIVs as the sign of \( z_{12} \) and \( z_{21} \) are either positive or negative. When both \( z_{12} \) and \( z_{21} \) are positive unity, the phase of voltage at the driving port (port 1) is shifted to 180° relative to that at the port 2 while the currents are in phase. Hence, it makes a voltage inversion NIV. In the other case, when both the sign of \( z_{12} \) and
$z_{21}$ are negative, the current at the port 1 is out of phase with that at port 2 while the voltages are in phase. It leads to a current inversion NIV.

However, when a reactive load terminated at the port 2, the sign of the impedance phase at the port 1 is not changed relative to that at port 2 because the NIV negates the corresponding load admittance. Note that the frequency derivative of the input reactance is always negative.

Fig. 2.17 shows a negative capacitor by means of a NIV in [51,52] and the input impedance, $Z_{in}$, is approximated by $-\frac{1}{g_{m}R_{ds}} - j\omega \frac{C_{gs}}{g_{m}R_{ds}} - \frac{1}{j\omega Lg_{m}}$.

### 2.3.5 Stability

Negative impedance converters or inverters are always potentially unstable because of the positive feedback present in the circuits. It is therefore necessary to analyze the stability of the whole system including both NICs/NIVs and internal/external loads connected to the NIC/NIVs. However, it is difficult to generalize the stability issue because the stability of NICs/NIVs depends on many factors, including the biasing circuits (points) of the active devices, the geometry of the layout, the passive load $Z_L$ in the NICs/NIVs, and the external loads connected to the NICs/NIVs.

Fortunately, the Nyquist stability criterion [54] can be applied to this problem while using commercial circuit simulators because it would be practically impossible to calculate analytically an accurate transfer function of the whole network over a wide frequency range.

Based on [47,54–56], the Nyquist stability criterion is briefly reviewed here. Fig. 2.18 shows a positive feedback system where $A(s)$ is an open-loop transfer function and $\beta(s)$ is a feedback transfer function. From Fig. 2.18, a closed-loop transfer function can be given by

$$\frac{Y(s)}{X(s)} = \frac{A(s)}{1 - \beta(s)A(s)}. \quad (2.40)$$
The denominator of the transfer function is known as the characteristic equation of the closed-loop system [47]. Zeros of the characteristic equation indicate unstable positions in the system. Since it is practically difficult to represent the transfer function of a whole system as a rational polynomial, it is usually done by means of a graphical approach with the Nyquist stability criterion, generally referred to as the Nyquist plot. The Nyquist plot is valid only if the open-loop transfer function, \( A(s) \) in Fig. 2.18, have no poles in the right half-plane of the complex plane [47]. When plotting the graph in polar coordinates, it is necessary to check if the curve rotates in the clockwise direction enclosing the point \((1, 0)\). If so, the system is unstable.

2.4 Literature Review: Fabricated Non-Foster impedance matching networks for electrically small antennas

Previous research on Non-Foster impedance matching networks for electrically small receiver antennas is summarized well in [21, 61]. However, there are few papers [19–21] where measured data with fabricated Non-Foster impedance matching networks for electrically small antennas are presented. Note that negative resistors are not considered Non-Foster impedances. In this section, previous work on fabricated Non-Foster impedance matching networks for electrically small antennas are presented.
and arranged in chronological order. It is worthy to note that according to [61] in 2007, the previous papers [19, 20] written by Harris and Perry have been unclassified recently.

2.4.1 Harris’ work

The earliest work among successfully fabricated Non-Foster impedance matching networks for electrically small antennas was achieved by A. D. Harris et al. in 1968 [19]. Their test antennas were a 2\( \frac{1}{2} \)" monopole with a 2\( \frac{1}{2} \)" diameter top hat and a 10" monopole with a 10" diameter top hat. The improvement of the measured antenna gains with negative capacitors with respect to a 16 foot untuned whip antenna over 0.5MHz ~ 10MHz was shown in [19].

Given a small monopole antenna, the equivalent lumped circuit for the input impedance of the antenna can be modeled as a capacitance connected in parallel to a conductance as shown in the dashed-black box of Fig. 2.19. \( C \) and \( G_r \) in Fig. 2.19 are represented by the total capacitances and the radiation conductance of the antenna, respectively.

![Parallel equivalent lumped model for a small monopole with a shunt negative capacitor](image)

Figure 2.19: Parallel equivalent lumped model for a small monopole with a shunt negative capacitor [19,21]
To cancel out the capacitance in Fig. 2.19, a corresponding negative capacitance (a red negative capacitance) was connected in shunt with the input port of the antenna. Based on a conceptually ideal voltage inversion NIC with a voltage gain ($A_v$) of 2 in Fig. 2.20, a negative capacitor was realized by using multiple amplifiers [19]. The $V_1$ at the input port in Fig. 2.20 can be written as

$$V_1 = I_1 Z_L + A_v V_1.$$  \hspace{1cm} (2.41)

From (2.41), the input impedance ($Z_{in}$) can be given by

$$Z_{in} = \frac{V_1}{I_1} = \frac{Z_L}{1 - A_v}.$$  \hspace{1cm} (2.42)

When $A_v$, the amplifier gain, is 2, the input impedance can be $-Z_L$. However, the performance of this NIC is limited at high frequencies because of the finite input and output impedances [19]. The gain of the amplifier with the factor of two cannot be sustained at higher frequencies.

Figure 2.20: Conceptually ideal voltage inversion negative impedance converter based on an amplifier (When $A_v = 2$, $Z_{in} = -Z_L$) [19, 21]
Although the noise of the antenna with a negative capacitor was not measured, he mentioned that the performance was restricted by external noise rather than that of the negative capacitor in the measured frequency range (0.5MHz ∼ 10MHz).

Harris also discussed the dynamic range of Non-Foster impedances, and measured the non-linear performance of a Non-Foster impedance. For the measurement setup, two transmitter antennas operating at different frequencies (200kHz and 230kHz) were placed nearby the test antenna with a Non-Foster impedance. Received signals were measured for three different cases of transmitting power levels: (1) two large signals, (2) large and small signals, (3) two small signals. As larger power was received at the antenna with a Non-Foster impedance, the non-linear effects of the active circuit become severe [19]. Although the non-linear effects of a Non-Foster impedance were identified through measured results, there were not enough cases to determine the dynamic range of the input power for the Non-Foster impedance.

2.4.2 Perry’s work

Perry elaborated the topic of Non-Foster impedance matching for small receiver antennas further in [20]. He used the term conjugate impedance matching rather than Non-Foster impedance matching to emphasize the partial realization of complex conjugate matching by means of active circuits. As mentioned above, the maximum transfer of power between an antenna and a generator with an internal impedance can be achieved when the complex conjugate of the input impedance of the antenna is equal to the internal impedance of the generator.

He fabricated a 3” monopole receiver antenna (2” monopole antenna with a 1” × 1” top mounted load) with three different Non-Foster impedance matching networks and showed the improvement of the antenna gain relative to a 12 foot untuned Tricor whip antenna over 0.3MHz ∼ 2.5MHz. Like Harris, all negative capacitors were connected
in shunt with the antenna. Among three different fabricated active networks, two of them were comprised of operational amplifiers (OP-amps) and the last one was implemented with transistors.

One of the active networks implemented with OP-amps was based on the ideal generator version of the voltage inversion NIC (VNIC) model in Fig. 2.20. Fig. 2.21 shows an ideal generator VNIC configuration based on two ideal OP-amps. Since two OP-amp networks in the red box consist of two inverting amplifiers, the ratio of the output voltage at the second OP-amp to the input voltage at the first OP-amp is

\[
\frac{V_{\text{out}2}}{V_{\text{in},1}} = \frac{\sqrt{2}R}{R} \times \frac{\sqrt{2}R}{R} = 2.
\]

Hence, this configuration enables to realize a generator with a voltage gain of 2 (\(V_2 = 2V_1\) in Fig. 2.21).

The second fabricated active network based on OP-amps was derived from nodal admittance synthesis. The second VNIC design was identical with that in [53]. The
last one was a current inversion NIC (INIC) based on BJT transistors in [43]. The configuration is shown in Fig. 2.22.

Based on a simple small signal T-model for a BJT ($g_m \gg 1$ and $r_e \ll 1$), the input impedance ($Z_{in}$) in Fig. 2.22 can be expressed as

$$Z_{in} = -\frac{R_1}{R_2}Z_L.$$  \hspace{1cm}(2.44)

Although Perry achieved outstanding improvement of the antenna gain with Non-Foster impedance matching networks, the active networks only worked below 5MHz and no noise measurements was performed.

### 2.4.3 Sussman-Fort’s work

In the most recent publication [21], Sussman-Fort summarized his previous work [57, 58, 60, 61] on fabricated Non-Foster impedance matching networks for a 6” monopole and a 12” dipole antenna. Unlike the previous two researchers, the Non-Foster capacitors in [57, 58, 60, 61] were connected to the antennas in series. A negative capacitor connected in parallel with a monopole type small antenna, as shown in Fig. 2.19,
can reduce its net-reactance for low frequencies because the equivalent model for the input impedance of the antenna (a parallel combination of a resistor and a capacitor) is valid for only low frequencies. It is therefore necessary to use a negative capacitor connected to the antenna in series to maintain the improvement of the antenna with the Non-Foster impedance matching network for higher frequencies. In his papers, he used balanced Linvill’s open circuit stable NICs in [38], shown in Fig. 3.5, comprised of two bipolar junction transistors for receiver antennas. Sussman-Fort showed improvement of both antenna gain and signal to noise ratio (SNR) from 20MHz to 120MHz, when compared to antennas without matching networks. To assess the SNR of a receiver antenna with a Non-Foster matching network, he introduced and applied the following method [21, 58]: Received signals of the antenna with and without a Non-Foster matching network were first recorded. Then, the difference between received signals with the matching network ($S_1$) and one without a matching network ($S_0$) were taken. The difference ($S_1 - S_0$ in dB scale) is the gain improvement of the matching network. For noise measurements one can perform the same procedure while the transmitter antenna is turned off. The difference ($N_1 - N_0$ in dB scale) is the added noise of the matching network, where $N_1$ and $N_0$ are received signals with and without the matching network, respectively.

Hence, the improved SNR with a matching network is given by

$$SNR_{imp} = (S_1 - S_0) - (N_1 - N_0).$$ (2.45)

In [61], the highest effective frequency range obtained by applying a Non-Foster impedance matching circuit to a 20" × 2" lossy dipole antenna is from 60MHz to 400MHz. It is also shown that there is the improvement of antenna gain; however, the electrically size of the antenna is around $\lambda/2$ at 286MHz.

Sussman-Fort also presented Non-Foster matching networks for transmitter electrically small antennas [21, 58, 59, 62]; however, the Non-Foster matching networks
were applied to surrogate antenna models (series connections of a resistor and a capacitor). In these papers, he pointed out that the power efficiency (including DC bias for activating NICs) should be considered in applications of Non-Foster matching networks to transmitter antennas. For the high voltage problem in a transmitter antenna, it was suggested that a negative LC matching could be applied to an electrically small antenna. He also showed a measured data with a surrogated antenna model to prove the concept. It was shown that the resonance of the antenna caused by a negative LC matching forces to transfer the maximum power to the antenna (for a narrow bandwidth) [21, 58, 59, 62]. With this concept, class-B and -C biased NICs were introduced to improve the power efficiency of transmitter antennas. Note that the linearity of circuits, another important factor in transmitter application, was not considered in his papers.

Although measured data of antennas with Non-Foster matching networks were shown in [21, 57, 58, 60, 61], none of this previous work shows in detail how to design and make the whole network stable.
CHAPTER 3
NON-FOSTER MATCHING NETWORK WITH A NEGATIVE CAPACITOR FOR AN ELECTRICALLY SMALL ANTENNA

3.1 Introduction

In the previous chapter, it was shown that a Non-Foster impedance matching network for an electrically small antenna has an advantage over a conventional passive matching network. In this chapter, we introduce a systematic methodology to design a Non-Foster impedance matching network for a monopole-type ($TM_{10}$ mode) small receiving antenna. Each key step in the methodology is shown, including how to fabricate a Non-Foster impedance matching network comprised of a negative capacitor for a thin $3''$ wire monopole on a $3'' \times 3''$ metal ground plane for a receiver antenna as shown in Fig. 3.1. This antenna, characterized by a very high input reactance, is chosen in order to assess the performance of the Non-Foster impedance matching network. Note that, based on the definition ($ka < 0.5$) of a small antenna, the antenna is an electrically small antenna up to $314\text{MHz}$. The measured reflection coefficient, $S_{11}$, of the antenna over $10\text{MHz} \sim 1\text{GHz}$ is shown in Fig. 3.2. From this measured data, it can be confirmed that the input impedance of the antenna starts with a highly capacitive reactance and a small resistance.
To demonstrate the potential benefits of a Non-Foster impedance matching network, consider the antenna in Fig. 3.1 whose measured $S_{11}$ is depicted in Fig. 3.2. When an ideal Non-Foster impedance matching network is designed to completely compensate the reactance of the antenna, the resulting $S_{11}$ (red curve) is shown in Fig. 3.3. For convenience, the measured complex $S_{11}$ of this antenna is also shown (black curve).
Figure 3.3: Measured real part of input impedance (red line) of antenna in Fig. 3.1 (100MHz ~ 200MHz). It corresponds to case when antenna is connected to an ideal Non-Foster impedance matching network. Black curve is the measured input impedance (real and imaginary parts) of the antenna without a matching network.

A Non-Foster capacitor (−6.8pF) is fabricated with a voltage inversion negative impedance converter (NIC) in [38] comprised of two discrete bipolar junction transistors (BJTs), and then the Non-Foster impedance matching network is connected in series with the test antenna to reduce the high capacitive reactance of the antenna [63].

With the stabilized system composed of a negative capacitor and the antenna, we took a variety of measurements such as input impedance, realized antenna gain and signal to noise ratio (SNR). Based on measured data, the observed improvement of antenna gain and SNR with the Non-Foster matching network is from 100MHz to 550MHz, when compared to the same antenna without a matching network. It is also demonstrated, through measurements and simulations, a non-linear behavior of a Non-Foster matching network occurs depending on the input power level of the active circuit.
3.2 Methodology to design Non-Foster impedance matching networks for electrically small antennas

The proposed methodology to design Non-Foster impedance matching networks for electrically small antennas is graphically summarized in Fig. 3.4. As the first step, the topology of a NIC should be determined. In this chapter, Linvill’s balanced Open Circuit Stable (OCS) NIC [38] comprised of BJTs in Fig. 3.5 is used to build a Non-Foster impedance matching network with a negative capacitor for the test antenna shown in Fig. 3.1.

Figure 3.4: Flowchart summarizing the proposed method to design Non-Foster impedance matching networks for electrically small antennas
Figure 3.5: Linvill’s balanced open circuit stable NIC with Bipolar Junction Transistors [38]

Figure 3.6: Small signal π-model for a Bipolar Junction Transistor [55]

Fig. 3.6 (top figure) depicts a small signal π-model for a BJT. Without loss of generality, $r_0$ and $C_\mu$ can be neglected for frequencies within the range of interest in this chapter. After replacing each BJT in Fig. 3.5 with the small signal model shown
at the bottom of Fig. 3.6, the input impedance \( Z_{in} \) at the driving port of the NIC in Fig. 3.5 with a passive load \( Z_L \) terminated at the other port is given by

\[
Z_{in} = -Z_L + \frac{2(Z_{\pi} + Z_L)}{Z_{\pi}g_m + 1} = Z_L - \frac{2r_{\pi}(Z_Lg_m - 1)}{C_{\pi}r_{\pi}S + r_{\pi}g_m + 1},
\]

(3.1)

where \( S \) is the Laplace variable \((j\omega)\).

Furthermore, for low frequency approximations \((r_{\pi} \gg 1 \text{ and } g_m \gg C_{\pi}S)\), (3.1) can be approximated by

\[
Z_{in} \approx -Z_L + \frac{2}{g_m}.
\]

(3.2)

With ideal transistors in the NIC, it can be observed in (3.2) that the impedance converter coefficient is unity. The second term in the right hand side of (3.2) is a parasitic element due to the non-ideal transistors.

Once the topology of the NIC has been selected, an appropriate passive load \( Z_L \) in Fig. 2.11 should be chosen. To cancel out or reduce the reactance of the input impedance of the antenna, it is important to first obtain an equivalent lumped model for the input reactance of the antenna over the frequency range of interest. Based on this equivalent circuit, the passive load of the NIC can be determined.

It is well known that the input impedance of a monopole or dipole-type \((TM_{10} \text{ mode})\) electrically small antenna can be simply modeled by a capacitor in series with the parallel combination of an inductor and a resistor, as shown in Fig. 2.4. This model usually works well up to the first series resonance of the antenna. Ideally, it would be necessary to design a negative capacitor connected in series to the input port of the antenna and then a negative inductor connected in parallel. This combination would greatly reduce the reactance of the antenna considered here up to the first series resonant frequency of the small antenna. In this chapter, it will be shown that
a single negative capacitor, which is much simpler to build than a combination of a negative capacitor and a negative inductor, is very effective in reducing the reactance of the small antenna because the dominant reactance of the input impedance of the test antenna is capacitive in the frequency range of interest. Fig. 3.7 shows a Non-Foster impedance matching network with a negative capacitor connected in series with an electrically small monopole antenna [21, 63].

![Diagram of Non-Foster impedance matching network with a negative capacitor](image)

Figure 3.7: Non-Foster impedance matching network with a negative capacitor connected in series with an electrically small monopole antenna [21, 63]

The NIC in Fig. 3.5 is potentially unstable due to the presence of positive feedback loops. Once the initial design of the NIC is completed, the stability of the whole network including the antenna should be investigated. The stability of NICs depends on many factors, including the biasing points of the active devices, the configuration of the layout, the passive load $Z_L$ in the NICs, and the external loads connected to the NICs. To rigorously scrutinize the stability, both loop gain and harmonic balance analyses for the whole system should be performed. Based on these results, the active
network should be modified to make the whole system stable but making sure it still performs as a Non-Foster element. To confirm whether the system is stable, a time domain analysis, namely, transient analysis can be carried out as well.

### 3.3 Overall antenna gain with a Non-Foster impedance matching network

It is important to briefly discuss how the overall gain of an antenna is obtained when a Non-Foster impedance matching network is connected to its terminals [63].

Consider an antenna connected to an arbitrary two-port matching network, as shown in Fig. 3.8. The realized antenna gain can generally be written as [10]

\[
G_{\text{ant}}(\theta, \phi) = \frac{4\pi U(\theta, \phi)}{P_{\text{inc}}}, \tag{3.3}
\]

where \(U(\theta, \phi)\) and \(P_{\text{inc}}\) are a radiation intensity of the antenna and the incident power from a source, respectively. (3.3) can be rewritten as

\[
G_{\text{ant}}(\theta, \phi) = \frac{4\pi U(\theta, \phi)}{P_{\text{deliver}}} \frac{P_{\text{deliver}}}{P_{\text{inc}}} = \frac{P_{\text{rad}}}{P_{\text{inc}}} \frac{4\pi U(\theta, \phi)}{P_{\text{rad}}} \frac{P_{\text{deliver}}}{P_{\text{inc}}}
\]

\[
= e_r D(\theta, \phi) \frac{P_{\text{deliver}}}{P_{\text{inc}}}, \tag{3.4}
\]

Figure 3.8: Two-port network representation of a matching network connected to an antenna
where $D(\theta, \phi)$ and $e_r$ are the directivity and radiation efficiency of an antenna, respectively. Considering the antenna as a load in the two-port network, the last factor, $\frac{P_{\text{deliver}}}{P_{\text{inc}}}$, in the right hand side in (3.4) is similar to the definition of a transducer gain ($G_T$) in the two-port network [31], except that in the definition of $G_T$. $P_{\text{inc}}$ would be replaced by the available power $P_{av}$ from the source in [31]. In this dissertation, the ratio $\frac{P_{\text{deliver}}}{P_{\text{inc}}}$ could be referred to as the transducer gain.

When taking in account the case of an overall antenna gain with a matching network, it is assumed there is minor crosstalk (unwanted coupling) between an antenna and the matching network. Based on (3.4), the realized antenna gain with the matching network can be expressed as

$$G_{\text{ant. w/ MN}} = e_r D(\theta, \phi) G_T = e_r D(\theta, \phi) G_{\text{ieee}} \left( \frac{|S_{21}|^2}{|1 - S_{22} \Gamma_L|^2} \right) (1 - |\Gamma_L|^2). \quad (3.5)$$

Note that the IEEE antenna gain ($G_{\text{ieee}}$) is given by [10]

$$G_{\text{ieee}} = e_r D(\theta, \phi). \quad (3.6)$$

In the case of the antenna without a matching network, $G_T = \frac{P_{\text{deliver}}}{P_{\text{inc}}}$ is equal to $(1 - |\Gamma_L|^2)$. The realized antenna gain without a matching network can be represented as

$$G_{\text{ant. w/o MN}}(\theta, \phi) = e_r D(\theta, \phi)(1 - |\Gamma_L|^2) = G_{\text{ieee}}(1 - |\Gamma_L|^2). \quad (3.7)$$

Therefore, the gain improvement ($G_{\text{imp}}$), defined as the ratio of the antenna gain with the matching network over the gain without a matching network, is given by

$$G_{\text{imp}} = \frac{G_{\text{ant. w/ MN}}}{G_{\text{ant. w/o MN}}} = \frac{|S_{21}|^2}{|1 - S_{22} \Gamma_L|^2}. \quad (3.8)$$

As the $|S_{21}|$ of the matching network becomes larger and the $|S_{22}|$ is minimized, the realized antenna gain improvement becomes higher.
3.4 Fabrication of a stabilized Non-Foster impedance matching network for an electrically small antenna

A Non-Foster matching network with a negative capacitor comprised of two discrete NPN BJTs (NE68133) with Class-A type biasing was fabricated on a RT/duroid 5880 substrate with $\epsilon_r = 2.2$ and thickness = 62 mil to compensate the large capacitive reactance of the test antenna in Fig. 3.1. As mentioned above, the Non-Foster impedance matching network is designed on the basis of a balanced OCS NIC shown in Fig. 3.5.

![Fabricated Non-Foster impedance matching network comprised of a negative capacitor for a 3'' receiving wire monopole on a 3'' x 3'' metal ground plane](image)

**Figure 3.9:** Fabricated Non-Foster impedance matching network comprised of a negative capacitor for a 3'' receiving wire monopole on a 3'' x 3'' metal ground plane

Fig. 3.9 shows the fabricated Non-Foster impedance matching network comprised of a negative capacitor connected in series with the test antenna. Note that a male-to-male connector, shown at the left hand side in Fig. 3.9, is needed to connect the fabricated Non-Foster matching network to the antenna.
Herein the measured reflection coefficient ($S_{11}$ in Fig. 3.2) of the 3" wire monopole antenna is used to determine the equivalent lumped model of the input reactance up to the first series resonant frequency of the antenna. A simple model, consisting of a series connection of a capacitor (4.48pF) and an inductor (7.52nH) that accounts for the input reactance of the antenna including the connector, is used at the frequencies. Since the ideal impedance converter coefficient of the NIC used in this chapter is unity, it is assumed that the coefficient is valid in low frequencies. To cancel out the large capacitive reactance of test antenna in the frequencies, it would be desired to use a capacitor (4.48pF), based on the equivalent model, as the load of the NIC. However, the residual capacitance should be positive in an actual Non-Foster matching network for an antenna due to the stability of the whole network [21]. Therefore, a capacitor (6.8pF) is chosen as the load of the NIC to realize a negative capacitor ($-6.8pF$) in the frequencies.

When the Non-Foster impedance matching network with a negative capacitor is connected to the antenna without a stabilization circuit, it can be observed by performing a loop gain analysis\(^1\) that there are two potentially unstable points (The criterion of the loop gain: Magnitude[dB] $> 0$ and Phase $= 0$) at low (around 16MHz, as shown in Fig. 3.10) and high (around 1.55GHz, as shown in Fig. 3.11) frequencies.

Furthermore, an oscillation at 1.547GHz is also seen in Fig. 3.12 when a harmonic balance analysis\(^2\) is performed.

To eliminate the oscillation at 1.547GHz, an inductor can be placed in the branch of the positive feedback loop as suggested in [21]. This should result in the reduction

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\(^1\)To investigate potential unstable points in the whole network, the conventional method to design a oscillator can be used by a commercial simulator. In this paper, OscTest component in Advanced Design System by Agilent was performed to analyze the loop gain.

\(^2\)Harmonic balance simulation in Advanced Design System by Agilent was performed to check the steady state oscillation.
Figure 3.10: Loop Gain Analysis of Non-Foster impedance matching network at low frequencies without stabilization

Figure 3.11: Loop Gain Analysis of Non-Foster impedance matching network at high frequencies without considering stabilization
of the magnitude of the loop gain at high frequencies. However, the high-frequency oscillation in this circuit cannot be eliminated, even when a large valued inductor is used. It should be kept in mind that the performance of the Non-Foster impedance matching network must be maintained at the higher frequency range of interest while the oscillations are eliminated. A pair of inductors/resistors connected in parallel ($L_1 = 30\,\text{nH}$ and $R_1 = 150\,\Omega$, $L_2 = 33\,\text{nH}$ and $R_2 = 150\,\Omega$) are placed in two different branches of the positive feedback loop as depicted in Fig. 3.13.

Although the high-frequency oscillations are eliminated, there is still another low-frequency oscillation in the circuit. Fig. 3.14 shows measured low-frequency oscillations, including the fundamental (at 7.139MHz) and higher order harmonics. This measurement was performed using a spectrum analyzer (Agilent E4440A).

To eliminate these low frequency oscillations, an inductor ($L_3 = 2.2\,\mu\text{H}$) is connected in parallel with the passive load (6.8pF). This reduces the magnitude of the
Figure 3.13: Schematic of a stabilized system including both a Non-Foster impedance matching network comprised of a negative capacitor and the test antenna (biasing circuits not shown)

Figure 3.14: Measured low-frequency oscillations using a spectrum analyzer loop gain at low frequencies. Fig. 3.13 shows the schematic of the stabilized negative capacitor without biasing circuits. Note that Fig. B.1 shows a full schematic layout
(including biasing circuits) of a stabilized Non-Foster impedance (−6.8pF) matching network for the 3” wire monopole antenna in Fig. 3.13.

It is worthy to note, based on both loop gain and harmonic balance analyses, a 3.3µH is required for stabilizing the circuit instead of 2.2µH. This discrepancy comes from the transistor model because the model is only valid above 50MHz. The inductor for stabilizing the low frequency oscillation should have a large inductance value as much as possible in order to maintain the Non-Foster behavior (negative capacitor) at even lower frequencies.

### 3.5 Measured and Simulated Results

After fabricating the stabilized whole network, including the test antenna, its performance such as input impedance, reflection coefficient ($S_{11}$) of the antenna versus frequency and input power level, antenna gain, and SNR is measured. Note that the SNR is an important figure of merit for a receiving antenna.

#### 3.5.1 Input Impedance

The measured and simulated input impedance (based on the $S_{11}$ at 0dBm input power) of the antenna with and without the Non-Foster impedance matching network over the frequency range 100MHz to 650MHz are shown in Fig. 3.15 for the imaginary part and Fig. 3.16 for the real part. The black curves in both Fig. 3.15 and Fig. 3.16 represent the measured results for the antenna without a matching network. The blue and red curves are the simulated and measured impedances of the antenna with the matching network, respectively. The solid blue curves are the simulated results with an ideal lumped inductor (2.2µH) added to eliminate the low-frequency oscillation, while the dashed blue curves are simulated results where the measured $S$-parameters of the 2.2µH inductor are used in the model.
Figure 3.15: Comparison between measured and simulated data for the imaginary part of the input impedance of the test antenna with and without a Non-Foster impedance matching network (−6.8pF)

Since the 2.2μH inductor has a low self-resonant frequency (around 131MHz), the measured real part of the input impedance of the antenna with the Non-Foster impedance matching network shows negative resistances from 474MHz to 542MHz where the inductor no longer functions as a desired inductor. We were not able to find a surface mounted 2.2μH chip inductor with a self-resonant frequency higher than 200MHz. It can be confirmed through simulations that this inductor causes the real part to be negative from 474MHz to 542MHz. The simulated resistance, where the measured S-parameters of the inductor are used, is shown Fig. 3.16 (dashed blue curved). This curve shows negative values over a similar frequency range as the measured input resistance results (red curve).

As shown in Fig. 3.15, the measured reactance (red curve) agrees very well with the simulated results (two blue curves), especially at low frequencies. It can also be observed that the measured reactance of the antenna with the Non-Foster impedance
matching network is significantly reduced (in magnitude), compared to the case of the antenna without a matching network.

### 3.5.2 $S_{11}$ as a function of input power

Since a Non-Foster impedance matching network includes active devices such as transistors, it has inherently non-linear properties. One of the features is an oscillation due to the positive feedback loop in the active matching network.

The other non-linear features are harmonics, intermodulation, and gain compression [64]; however, this subsection focuses on the $S_{11}$ response as a function of input power. This behavior is related to the effect on the dynamic range due to gain compression in the transistor. As mentioned in section 2.4.1, Harris discussed non-linear
behavior of an antenna with a Non-Foster matching network and measured non-linear properties in [19].

Herein, the response of the $|S_{11}|$ of the antenna with a Non-Foster impedance matching network to different input power levels is simulated and measured.

![Simulated and Measured $|S_{11}|$ of the antenna with the active matching network](image)

**Figure 3.17:** Comparison between measured and simulated magnitude of $S_{11}$ of the test antenna with Non-Foster impedance matching network ($-6.8$ pF) for low input powers ($-5$ dBm, 0 dBm, and 4 dBm)

The measured and simulated magnitudes of $S_{11}$ with the Non-Foster matching network for different input powers over 100 MHz to 650 MHz are shown in both Fig. 3.17 for low input power ($-5$ dBm, 0 dBm, 4 dBm) and Fig. 3.18 for high input power (4 dBm, 8 dBm, 12 dBm). Note that the measured and simulated $|S_{11}|$ at 4 dBm input power are intentionally plotted in both Fig. 3.17 and Fig. 3.18 to clearly compare the two cases. For convenience, comparison between measured $|S_{11}|$ of antenna with and without the matching network, the measured $|S_{11}|$ (when 0 dBm input power) of the
antenna without a matching network is also plotted in Fig. 3.17. It can be observed that the matching network enables the reflection coefficient to decrease in the range over 100MHz to around 460MHz, when compared to the case without a matching network.

The solid and dashed curves in both Fig. 3.17 and Fig. 3.18 represent measured and simulated data for different input power levels, respectively. For convenience, the color of the solid and dashed curves are the same for identical input power.

Note that the simulated data is obtained through a Large-Signal S-parameter simulation in Advanced Design System (ADS) by Agilent. While performing the analyses, an ideal 3.3µH for the stabilization at low frequencies is used. However, as mentioned in section 3.4, a 2.2µH is used to fabricate the network.

When low input power levels are used in Fig. 3.7, small variation of both measured and simulated $|S_{11}|$ to different input powers can be observed in Fig. 3.17. It should
be pointed out that the transistors in the active network have a linear response to up to around 4dBm input power. However, as shown in Fig. 3.18 the larger the input power, the bigger change in $|S_{11}|$ can be observed. Although the simulated and measured magnitude of $S_{11}$ at 12dBm input power (red curves in Fig. 3.18) decreases dramatically, it does not imply improvement of the realized antenna gain. Through simulation of the transducer gain ($G_T$) in (3.5), it can be confirmed that the decreased of $|S_{11}|$ at higher input power levels is associated with the increased of loss resistance (increased dissipated power) in the active circuit. Fig. 3.19 shows the transducer gain at different input powers relative to an input power of $-5$dBm. It can be observed that the relative transducer gain decreases for larger input powers because of the higher losses in the transistors.

![Graph showing transducer gains at different input powers](image)

Figure 3.19: Transducer Gains at different input powers relative to input power of $-5$dBm

As mentioned above, this characteristic is caused by the gain compression of the
transistors. This behavior should be specially considered when Non-Foster impedance matching networks are designed for transmitter antennas. Although the author in [21, 58, 59, 62] applied a Non-Foster impedance matching network to the surrogate model for a transmitter electrically small antenna, it is indicated in [62] that it is better to apply an active network based on Class-C biasing to a transmitter antenna rather than that Class-A or -B biasing.

3.5.3 Antenna Gain and Signal to Noise Ratio

The antenna gain and SNR measurements with and without the Non-Foster matching network were performed in an outdoor range for 100MHz to 550MHz. Note that the antenna’s electrical length including the finite ground plane is about $\lambda/39$ at 100MHz. Since two different calibration antennas are used in the outdoor measurement, each set of measured results is intentionally addressed in two different subsections (Lower and higher frequencies).

Lower Frequencies Measurement : 100MHz to 300MHz

Fig. 3.20 illustrates the outdoor range setup for the antenna gain and SNR measurements with and without the fabricated Non-Foster matching network. The red dashed line shows a signal reflected by the ground (multi-path). Note that in order to perform an outdoor measurement with high quality, the reflected wave path should be much longer than the direct signal. The setup in Fig. 3.20 was the best outdoor measurement setup we could have. For the reader’s convenience, a picture of the actual range is provided in Fig. 3.21 as well.

The antenna, inset in Fig. 3.21, is placed on the top of a plastic pole perpendicular to the ground. When the pole is upright, the monopole antenna is intentionally directed parallel to the ground (horizontal plane) to minimize common mode currents on
the outer conductor of a long coaxial cable vertically directed along the pole [65]. Additionally, we used ferrite beads (by Fair-Rite Products, the part number: 0461164951)
on a cable at the input of the antenna with and without the matching network to prevent common mode currents. At the input of a transmitter antenna shown in Fig. 3.21, a power amplifier is used to clearly distinguish noises and received signals at the test antenna (receiving antenna). After turning on the transmitter antenna, received signals ($S_{21}$) at the receiving antenna with and without the active matching network are recorded with a Network Analyzer (Agilent N5230A) to measure the antenna gain.

![Graph](image)

Figure 3.22: Comparison between measured antenna gains (with/without the matching network) before and after applying time-gating over 100MHz to 300MHz

For the antenna gain measurements, time-gating is applied to minimize the effects of multi-path reflections from the ground, walls, etc. After applying time-gating to the raw measured data, and then it is necessary to calibrate the time-gated data with a reference calibration antenna. Fig. 3.22 shows the measured antenna gains
(with/without the matching network) before and after applying time-gating. The solid curves represent measured antenna gain before applying time-gating, while calibrating a long cable between the test antenna and the Network Analyzer. After applying time-gating to the solid curves, the dashed curves can be achieved. The red and black curves stand for measured antenna gains with and without the matching network, respectively. It can be definitely observed that the antenna gain with the matching network (the solid red curve in Fig. 3.22) is higher than that without a matching network (the solid black curve) over 50MHz to 300MHz. However, it can also be seen that there are fluctuation on the raw measured antenna gain (both solid curves), which are mainly caused by multi-path reflections from the ground.

To identify the fluctuations of the curves, let us assume that the ground in Fig. 3.20 is considered to be a flat PEC (Perfect Electric Conductor). As mentioned above, in the measurement setup, both transmitter and receiver antennas are in parallel to the ground (horizontal plane). Note that the polarization of both antennas is linear. According to Image Theory [10], when the difference in length of the direct and reflected paths becomes odd multiples of $\lambda/2$, the reflected-path and the directed-path signals at the receiver antenna are in phase. Under this condition, the received signal achieves its maximum value. On the contrary, the signals are out of phase when the difference is multiples of $\lambda$.

Based on the illustration in Fig. 3.20, the difference in length of the directed and reflected paths is about 7.87m. It can be expected that the max peak of a received signal (the sum of directed and multi-path reflected signals) starts with 19MHz and the next peak is 57MHz. Hence, a $\lambda$ ($\simeq 38$MHz) periodic fluctuation at the raw measured antenna gains in Fig. 3.22 can be expected. It can be observed that the raw measured antenna gains in Fig. 3.22 have around 40MHz periodic fluctuations from 50MHz to 130MHz. As the frequency increases, the received signals can be

60
severely deteriorated by ambient environment so that the fluctuations deviate from
the predicted values.

After calibrating the time-gated data with a reference calibration antenna, the
measured realized gains are plotted in Fig. 3.23. The solid red and black curves
in Fig. 3.23 are measured data for the realized antenna gain with and without the
matching network, respectively. To confirm whether the measurement is correct, the
simulated antenna gain without a matching network (dashed black curve) generated
by using Ansoft HFSS v12.0 is also plotted in Fig. 3.23. When comparing the solid and
dashed black curves, it can be observed that the discrepancy is large, especially over
the frequency range 100MHz to 120MHz and 220MHz to 290MHz. This discrepancy
could be caused by two reasons; common mode currents on the cable and multi-path
reflections by ground. It can be observed that the measured gain is higher than
the simulated data in the lower frequencies. This behavior could be caused by the
common mode effect. When an electrically small ground plane is used, it could make
the common mode effect severe. For multi-path reflections, time-gating was applied
to the received signal (the sum of directed and multi-path reflected signals), but it is
difficult to completely recover the direct received signal.

To investigate the improvement of the measured antenna gain with the matching
network, the difference (in dBs) between the gains with and without the matching
network is plotted in Fig. 3.24. It can be observed that the measured antenna gain
with the matching network is clearly improved in the range 100MHz to 300MHz.
For instance, the improvement with the matching network is around 10.45dB at
100MHz. At the lower frequencies (below 160MHz), the measured gain improvement
with the matching network (red curve) is similar to the simulated result (blue curve
in Fig. 3.24) which is evaluated using (3.8). As the frequency increases, the deviation
between measured and simulated results becomes larger. In the derivation of the
Figure 3.23: Measurement of realized antenna gains for frequencies 100MHz to 300MHz

Figure 3.24: Improvement of measured antenna gain with the Non-Foster matching network relative to the case without a matching network over 100MHz to 300MHz
improvement gain, (3.8), in section 3.3, it was assumed that there was minor crosstalk between the antenna and matching network. However, this assumption may not be valid at higher frequencies. This could be one of the reasons for the disagreement between measured and simulated data.

To assess the SNR of the antenna with the matching network, the change of SNR with the matching network included relative to the case without a matching network is used in this chapter. To measure the change of the SNR, we use the same procedure suggested in [21], except that time-gating is applied to the received signals to reduce multi-path effects when the transmitter antenna is turned on. When the transmitter is on, both received signals with and without the matching network are recorded with a Network Analyzer (Agilent N5230A). The difference (in dBs) of these two time-gated received signals is calculated. Namely, \( S_1 - S_0 \) where \( S_1 \) is the received signal with the matching network present, while \( S_0 \) is the received signal without a matching network. This difference in dBs is identical to the improvement of antenna gain depicted by the red curve in Fig. 3.24.

To measure received noise, the signal at the antenna input terminal with and without the matching network is measured with a spectrum analyzer (Agilent E4440A) while the transmitter antenna is turned off. When performing noise measurements, it is very important to be aware of the sensitivity of the receiver, namely, its noise floor. Depending on the noise floor, the noise at the terminals of the antenna with and without a matching network may not be detected by the spectrum analyzer. In this measurement, the noise floor of a spectrum analyzer is around \(-110\)dBm as shown in Fig. 3.25. Note that, in order to measure the noise floor, the power is recorded at the spectrum analyzer where nothing is connected.

In dB scale, the improvement of SNR is evaluated by taking the difference (in dBs) between the received signals (with/without the matching network) while the
transmitter is on, and the difference in noise (with/without the matching network), including external RF interference signals while the transmitter is turned off. In equation form, the improvement of SNR (in dBs) can be expressed as [21]

\[
\text{Improvement of } SNR_{dB} = (S_1 - S_0) - (N_1 - N_0).
\]  \hspace{1cm} (3.9)

where \(N_1\) and \(N_0\) are the signals at the antenna terminals (transmitter off) with and without the matching network, respectively.

Fig. 3.26 shows the measured improvement of SNR with the matching network relative to the case without a matching network. Since the original measured result (dashed black curve) in Fig. 3.26 has several fluctuations, a smoothed version\(^3\) (red curve) is also plotted on the same graph. Based on the smoothed curve, it can be observed that the overall improvement of SNR is up to 20.3dB in the range 100MHz to 550MHz.

\(^3\)Used smooth (a function in Matlab by MathWorks) with a lowess method and 0.05MHz span.
Figure 3.26: Improvement of SNR with a Non-Foster impedance matching network relative to case without a matching network over 100MHz to 300MHz

300MHz. The fluctuations in the black curve around FM radio (87 ∼ 108MHz), Airband (108 ∼ 137MHz), Amateur Radio (144 ∼ 147MHz) and TV broadcast (174 ∼ 216MHz) frequencies (in U.S.A) [66] have been investigated. Keeping in mind that this measurement was done in an outdoor range, it was concluded that the fluctuations could be due to interference from FM, Airplane, Amateur radio and TV transmitters.

**Higher Frequencies Measurement : 300MHz to 550MHz**

For frequencies above 300MHz, a different reference antenna was used in the same outdoor range. For convenience, measured antenna gains and the improved SNRs from 300MHz to 550MHz are shown in this subsection. The measured raw and the time-gated antenna gains (with/without the matching network) are presented in Fig. 3.27. The solid curves stand for the measured raw antenna gains before applying
time-gating, while only calibrating a long cable between the receiver antenna and the Network Analyzer. The dashed curves are the time-gated antenna gains. Based on the raw antenna gains (the solid curves in Fig. 3.27), it can be observed that the measured antenna gain with the matching network (the red solid curve) is higher up to 536.5MHz when compared to the antenna without a matching network (the black solid curve), except for a notch around 512MHz. After calibrating the time-gated antenna gains in Fig. 3.27 with a reference calibration antenna, the measured realized antenna gains are plotted in Fig. 3.28. To clearly verify the improvement of the antenna gain with the matching network relative to the case without a matching network, the difference between both gains with and without the matching network is plotted in Fig. 3.29. It can be observed that the discrepancy between measured and simulated data in Fig. 3.29 is large. As mentioned above, the crosstalk between the antenna and matching network could be of the reasons for the disagreement.
Figure 3.28: Measurement of realized antenna gains over 300MHz to 550MHz

Figure 3.29: Improvement of measured antenna gain with a Non-Foster impedance matching network relative to the case without a matching network over 300MHz to 550MHz
For frequencies up to 550MHz, the antenna gain with the matching network is higher than the case without a matching network. However, the improvement decreases as the frequency increases. A Non-Foster impedance matching network comprised of a single negative capacitor makes the net-reactance larger instead of reducing it as the electrical size of the antenna becomes larger. Finally, measured results for

![Figure 3.30: Improvement of SNR with a Non-Foster impedance matching network relative to case without a matching network over 300MHz to 550MHz](image)

the improvement of SNR with the Non-Foster impedance matching network relative to the case without a matching network is plotted in Fig. 3.30. Based on the dashed black curve, it can be observed that the improved SNR is highly fluctuated over 450MHz. This behavior could be caused by a negative resistance of the antenna input impedance with the active matching network in the range 474MHz to 542MHz. Note that, in a one-port network, a negative input resistance indicates that the device can generate power. It is also be observed that the overall SNR with the Non-Foster
impedance matching network (smoothed curve) is higher than the SNR without a matching network over the frequency range of 300MHz to 550MHz.

3.6 Summary

This chapter first introduced a systematic methodology to design a Non-Foster matching network for an electrically small receiver antenna. Describing each key step in the methodology, a stabilized Non-Foster impedance matching network with a negative capacitor was fabricated to compensate the considerably high reactance of a thin 3″ wire monopole antenna on a 3″ × 3″ metal ground plane. The schematic of the negative capacitor shown in this chapter can be employed for monopole and dipole-type electrically small antennas after adjusting the values of the stabilization elements and the load of the NIC. Through measured and simulated data, it was sequentially shown that a Non-Foster impedance matching network can reduce the input reactance of an electrically small antenna, making the transducer gain between the antenna and a receiver increased. Consequently, the Non-Foster matching network can enhance the realized gain and SNR of an antenna, comparing to the case without a matching network.

Based on measured results, the Non-Foster impedance matching network improves the antenna gain (or SNR) up to 20.9dB (or 20.3dB) with respect to the case without a matching network over the frequency band 100MHz to 550MHz. To the best of our knowledge, the implemented here Non-Foster matching network operates at higher frequencies than previously published papers (the improvement of antenna gain over 60MHz to 400MHz [61]).

Measured and simulated data for the reflection coefficient of the antenna with a Non-Foster matching network included was obtained for different input power levels. At high power level (> 4dBm), the non-linear behavior of the Non-Foster matching
network was observed. When applying a Non-Foster matching network to transmitter antenna, this non-linear behavior should be considered.
CHAPTER 4

NON-FOSTER MATCHING NETWORK WITH A NEGATIVE CAPACITOR-INDUCTOR FOR AN ELECTRICALLY SMALL ANTENNA

4.1 Introduction

As mentioned in Chapter 2, Non-Foster impedance matching networks have an advantage over conventional passive matching networks because the former can overcome the limitation of Gain-Bandwidth while the latter cannot. To the best of our knowledge, all fabricated Non-Foster impedance matching networks for monopole or dipole-type ($TM_{10}$ mode) small antennas have been made up of only single negative capacitor connected in a series or a parallel to the antenna similar to the active network described in Chapter 3 and in [19–21]. A single negative capacitor can only reduce the reactance of the antenna input impedance at low frequencies where the reactance of the input impedance is characterized by a large capacitive reactance. However, as mentioned in Chapter 3, it is necessary to design a negative capacitor connected in series to the input port of the $TM_{10}$ mode small antenna and then a negative inductor connected in parallel to cancel out the reactance of the antenna up to the first series resonance. However, it is difficult to realize both a series and a shunt Non-Foster impedances at the same time.

In this chapter, it will be shown that an alternative is to use a series combination
of negative capacitor and negative inductor because it is much simpler to build than a pair of negative series capacitor and shunt inductor combination. The test antenna (a 3” monopole antenna on a 3” square metal ground plane) used in this chapter is the same as that in Chapter 3.

This chapter will address the usage of a fabricated Non-Foster impedance matching network consisting of a series connection of negative capacitor (−5.6pF) and negative inductor (−10nH) to improve the antenna’s performance at higher frequencies [67]. The measured performance of a thin 3” monopole antenna with this matching network will be compared with a matching network consisting of a negative capacitor (−5.6pF) as well as the antenna by itself. Based on measured antenna gain, it is shown that the improved antenna gain (or SNR) with the negative capacitor-inductor relative to that without a matching network is up to 739MHz (or 771.8MHz), while the case with the negative capacitor is up to only 558MHz (or 482.8MHz). To the best of our knowledge, it is the first time that a pair of series connected negative capacitor-inductor matching network is applied to an actual antenna.

Before we discuss the design of the series connected negative capacitor-inductor pair, we will start with the design of a single negative capacitor. Once this device is designed, fabricated and tested, a passive inductor, connected in series with the passive capacitor (in the load of the NIC in Fig. 3.5), will be added. This is done to demonstrate the improvement in the antenna performance when the negative inductor is added to the matching network.
4.2 Fabricated Non-Foster matching network comprised of a negative capacitor

Based on the same procedure developed in Chapter 3, a second Non-Foster impedance matching network comprised of a negative capacitor for the test antenna is designed and fabricated. The Non-Foster impedance matching network is connected to the antenna in series. Moreover, a Linvill’s balanced OCS NIC [38] comprised of two BJTs is used to realize a negative capacitor for the antenna.

As mentioned in Chapter 3, the equivalent lumped model for the measured input reactance of the antenna including a connector is a series connection of a capacitor (4.48pF) and an inductor (7.52nH) up to the first series resonant frequency of the antenna. Based on this model, a passive load (5.6pF) of the NIC in this chapter is chosen to better cancel the effect of the equivalent capacitor model than the passive load (6.8pF) used in Chapter 3. It should be kept in mind that with ideal transistors, the impedance converter coefficient of a Linvill’s balanced OCS NIC is unity.

For an initial stability analysis of the whole network including the antenna, a loop gain simulation is performed. It can be observed that there are two potential unstable points at both low (13.5MHz, as shown in Fig. 4.1) and high (2.085GHz, as shown in Fig. 4.2) frequencies. It shows the similar results (two potential unstable points at low and high frequencies) to those in Chapter 3. It can be expected that there can be two oscillation (low and high) frequencies. Hence, the scheme, developed in Chapter 3, to eliminate these oscillations is applied to this circuit, as shown in Fig. 4.3. To eliminate low frequency oscillations, an inductor (4.7µH) is connected to the passive load of the fabricated NIC in parallel. For a stability at high frequencies, two parallel-connected inductor (33nH) and resistor (150Ω) are placed on two branches of the positive feedback loop in the NIC. Finally, a stable Non-Foster
Figure 4.1: Loop Gain Analysis at low frequencies for Non-Foster impedance matching network (−5.6pF) without considering stabilization

Figure 4.2: Loop Gain Analysis at high frequencies for the Non-Foster impedance matching network (−5.6pF) without considering stabilization
Figure 4.3: Schematic of a stabilized system including both a Non-Foster impedance matching network comprised of a negative capacitor (−5.6pF) for the test antenna (biasing circuits not shown).

Figure 4.4: Fabricated Non-Foster impedance matching network comprised of a negative capacitor (−5.6pF), shown in Fig. 4.4, has been fabricated.
4.3 Fabrication of Non-Foster matching network consisting of series connected negative capacitor-inductor

Using Chu’s equivalent lumped models for the wave impedances of small antennas [2,27], the input impedance of an antenna with a $TM_{10}$ mode can be modeled as an equivalent lumped circuit shown in Fig. 4.5 (top figure). Through a simple algebraic manipulation of the function representing the input impedance, $Z_{in}$, in the top figure, an alternative equivalent lumped model shown in Fig. 4.5 (bottom figure) can be obtained. Note that this model includes a frequency dependent inductor. However, $L_{eq} \approx L$ and $R_{eq} \approx \omega^2 L^2 / R$, when $\omega^2 L^2 \ll R^2$. In this chapter, the topology of

![Equivalent lumped models](image)

Figure 4.5: Equivalent lumped models for the input impedance of an antenna with the $TM_{10}$ mode

the equivalent model at the bottom in Fig. 4.5 is used to design a series connection of negative capacitor and negative inductor for the 3" wire monopole antenna. As mentioned before, it is much simpler to fabricate a series connected negative capacitor
and negative inductor than a pair of negative series capacitor and shunt inductor combination for the model at the top figure in Fig. 4.5. Note that two different NICs are needed to realize the latter circuit, whereas, the former needs only one NIC. Fig. 4.6 shows a Non-Foster impedance matching network consisting of a series connection of negative capacitor-inductor, connected to an antenna in series [46, 58, 67, 68]. As in the case with a negative capacitor in section 4.2, a Linvill’s balanced OCS NIC comprised of BJTs is used for a series combination of Non-Foster impedances. To compare the antenna performance between a negative capacitor and a series connection of negative capacitor-inductor, we used the same capacitor (5.6pF) in the loads (C and L in the dashed red box of Fig. 4.6) of the NIC as that in the single negative capacitor designed in section 4.2. Then, through simulation of the input reactance of an antenna with a series connection of negative capacitor-inductor (with different inductors), an inductor in the loads of the NIC in Fig. 4.6 is determined.
to reduce the reactance to a minimum in the frequency range of interest. In this case, that goal is achieved with $L = 10\, \text{nH}$. As mentioned above, the whole network including an antenna is potentially unstable. To stabilize the active circuit, the same methods as the case with a negative capacitor in section 4.2 are employed in this matching network. Fig. 4.7 shows a schematic (not including biasing circuits) of a stabilized Non-Foster impedance matching network comprised of a series combination of negative capacitor($-5.6\, \text{pF}$)-inductor($-10\, \text{nH}$) for the $3''$ wire monopole antenna.

Figure 4.7: Schematic of a stabilized system including both a Non-Foster impedance matching network comprised of a series combination of negative capacitor-inductor for the test antenna (biasing circuits not shown)

After considering the stabilization, a Non-Foster impedance matching network comprised of a series combination of negative capacitor($-5.6\, \text{pF}$)-inductor($-10\, \text{nH}$), shown in Fig. 4.8, for the test antenna is fabricated.
4.4 Measured and Simulated results

After fabricating both the negative capacitor and the series combination of negative capacitor-inductor for the 3” wire monopole, the antenna performance such as input impedance, antenna gain, and signal to noise ratio (SNR) were measured.

4.4.1 Input Impedance

The imaginary part of both the measured and simulated input impedances of the antenna with/without the Non-Foster impedance matching networks for the frequency 100MHz to 800MHz are shown in Fig. 4.9. The real part of the input impedance is shown in Fig. 4.10. All black curves in both Fig. 4.9 and Fig. 4.10 represent the measured results of the antenna without a matching networks. The solid blue and red curves are the measured input impedances of the antenna with the negative capacitor (−5.6pF) and with the series combination of negative capacitor (−5.6pF)-inductor (−10nH), respectively. The red (or blue) star marker is the simulated result for an ideal lumped 4.7μH inductor in the negative capacitor-inductor (or the negative
Figure 4.9: Comparisons between measured and simulated results for the imaginary part of the input impedance of the test antenna with/without the Non-Foster impedance matching network (Blue Colors: Ant. with the negative capacitor (−5.6pF), Red Colors: Ant. with the series combination of negative capacitor (−5.6pF)-inductor (−10nH))

capacitor) used to stabilize the low frequency oscillations, while dashed red (or blue) curve is the simulated result where the measured S-parameter of the inductor is used in the model.

As shown in Fig. 4.9, both measured reactances (solid blue and red curves) of the antennas with Non-Foster impedance matching networks agreed well with the simulated reactances, especially at lower frequencies. It can also be observed that both measured reactances are considerably decreased compared to that of the antenna without a matching network at the lower frequencies. For the measured resistance in Fig. 4.10, it can be observed that both measured results (solid blue and red curves) are in good agreement with the simulated results, and the measured resistance with
Figure 4.10: Comparisons between measured and simulated results for the real part of the input impedance of the test antenna with/without the Non-Foster impedance matching network (Blue Colors: Ant. with the negative capacitor (−5.6pF), Red Colors: Ant. with the series combination of negative capacitor (−5.6pF)-inductor (−10nH)).

The negative capacitor-inductor is almost same as that with the negative capacitor up to 300MHz.

Like Chapter 3, the measured real part of the antenna input impedance with the Non-Foster impedance matching networks show negative resistance from 301MHz to 428MHz for the negative capacitor and from 307MHz to 396MHz for the negative capacitor-inductor. This behavior is due to the 4.7µH inductor used low frequency stabilization which has a low self-resonant frequency (around 58MHz). It can be confirmed through simulations (dashed blue and red curves in Fig. 4.10) that this inductor causes the real part to be negative. Note that we were not able to find a surface mounted 4.7µH inductor with a self resonant frequency higher than 200MHz. The input impedance with the negative capacitor-inductor shows another negative
resistance starting at 514MHz. When an inductor is dominant in the load of a Linvill’s balanced OCS NIC and $\omega$ is larger than $g_m/C_\pi$ in Fig. 3.6, it can be shown that a negative resistance appears in the input impedance of the antenna with the matching network combination.

Since both measured input impedances with both Non-Foster impedance matching networks are almost the same over 100MHz to 300MHz, it can be expected that the improvement of antenna gain relative to the case without a matching network is similar at low frequencies for both cases. However, when comparing the measured reactances with the negative capacitor and the negative capacitor-inductor at higher frequencies, it can be observed that a pair of series negative capacitor-inductor has better performance in reducing the reactance of the antenna above 365MHz. Due to the reactance’s reduction at higher frequencies, the antenna gain with the negative capacitor-inductor improves even at higher frequencies when compared to the case with the negative capacitor.

4.4.2 Antenna Gain and Signal to Noise Ratio

At low frequencies (100MHz to 400MHz), the antenna gain and the SNR measurements with and without the Non-Foster impedance matching networks were performed in an outdoor range. Note that the antenna’s electrical length including the finite ground plane is about $\lambda/39$ and $\lambda/4$ at 100MHz and 900MHz, respectively. For frequencies above 400MHz, an indoor anechoic chamber was used.

Outdoor Measurement

Note that the outdoor range setup and procedure for this measurement were the same as that (including time-gating) in section 3.5.3. Measured antenna gains (with/without the matching network) before and after employing time-gating are plotted in Fig. 4.11.
Figure 4.11: Comparison between measured antenna gains (with/without the matching network) before and after applying time-gating

The solid curves represent measured antenna gains before applying time-gating, but calibrating a long cable between the receiver antenna and a network analyzer. The dashed curves are achieved after employing time-gating to the solid curves. The black curves stand for measured antenna gains without a matching network. The red and blue curves are measured antenna gains with the negative capacitor-inductor and the negative capacitor, respectively. When comparing the raw and time-gated antenna gains, it can be confirmed that the time-gated results become smooth but follow the same trend as the raw data. Based on the raw (the solid curves) and time-gated (the dashed curves) antenna gains, it can definitely be observed that both measured antenna gains with the matching networks are higher than the case without a matching network for frequencies 50MHz to 400MHz. It can also be observed that both the time-gated antenna gains, even raw antenna gains, with the negative capacitor-inductor and with the negative capacitor are similar at lower frequencies.
Figure 4.12: Measured realized antenna gain at outdoor range

After calibrating the time-gated data with a reference calibration antenna, the measured realized gains are plotted in Fig. 4.12. Note that two different reference calibration antennas are used in the outdoor measurements: One is up to 300MHz while the other covers 300MHz to 400MHz. That is why discontinuities appear in Fig. 4.12 at 300MHz. The solid blue and red curves in Fig. 4.12 represent measured data for the realized antenna gain with the negative capacitor and the negative capacitor-inductor, respectively. The solid (or dashed) black curve represents measured (or simulated) realized antenna gain without a matching network. Note that the solid (or dashed) black curve are identical to those black curves in Fig. 3.23 and Fig. 3.28. As mentioned in section 4.4.1, when comparing the antenna gains between the negative capacitor and the negative capacitor-inductor, it can be observed that both realized antenna gains are almost the same over 100MHz to 230MHz because the negative capacitors dominate both Non-Foster impedance matching networks at...
the lower frequencies. Hence, both improvements of antenna gain with the matching networks relative to the antenna without a matching network are nearly identical.

Figure 4.13: Measured antenna gain at outdoor range: Improvement of antenna gain with a Non-Foster impedance matching network relative to the case without a matching network.

The similarity in improvement of the antenna gain with the matching network relative to the case without a matching network can be confirmed in Fig. 4.13 where both improvement of measured antenna gain (solid curves) with both matching networks are almost the same up to 230MHz. It can also be observed that the measured antenna gain with the matching network is clearly improved in the range 100MHz to 400MHz. By using (3.8), the simulated improvements (dashed curves) are also plotted in Fig. 4.13. It can be observed that the difference between measured and simulated data becomes larger as the frequency increases. As mentioned in section...
3.5.3, there is one potential reason for this disagreement; Crosstalk between antenna and matching network as the frequency increases.

![Figure 4.14: Improvement of SNR with a Non-Foster impedance matching network relative to case without a matching network at outdoor range](image)

Herein, the same procedure of section 3.5.3 is used to assess the improvement of the SNR with the matching network. When comparing the improvement (smoothed version) of SNR with the negative capacitor and with the negative capacitor-inductor relative to the case without a matching network, the overall improvement is nearly identical in the range 100MHz to 230MHz. Based on the smoothed curves, the improvement with the negative capacitor-inductor and with the negative capacitor is up to 20.5dB and 19.2dB over 100MHz to 400MHz, respectively. Note that the noise floor of the spectrum analyzer used in this measurement is around $-110$dBm.
Indoor Measurement

Measurements at frequencies over 400MHz to 900MHz were performed in an indoor anechoic chamber. The measurement procedure is the same as that of the outdoor measurements, including time-gating. Fig. 4.15 shows measured antenna gains before and after time-gating.

![Measured raw antenna gain before and after time-gating](image)

Figure 4.15: Comparison between measured antenna gains (with/without the matching network) before and after applying time-gating

It can be observed that the raw gains (solid blue and red curves) with both Non-Foster matching networks have strong fluctuations at higher frequencies, when compared to the case (solid black curve) without a matching network. As mentioned in section 3.5.3, this could be caused by coupling between the antenna and the matching network at higher frequencies. Simulated and measured antenna realized gains are plotted in Fig. 4.16 and the improvement of antenna gain with the matching network relative to the case without a matching network is shown in Fig. 4.17.
Figure 4.16: Measured realized antenna gain at indoor range

Figure 4.17: Measured antenna gain at Indoor range: Improvement of antenna gain with a Non-Foster impedance matching network relative to the case without a matching network
Based on the improvement of measured antenna gains (solid curves) in Fig. 4.17, the antenna gain with the negative capacitor becomes worse than that of the antenna without a matching network after 558MHz and it has a dip around 496MHz. In the case of the negative capacitor-inductor, the antenna gain is maintained up to 793MHz but it has two dips around 512MHz and 695MHz due to crosstalk between the fabricated antenna and the matching network. It can definitely be observed that the antenna gain with the negative capacitor-inductor is kept higher than the case with the negative capacitor at higher frequencies.

![Improved Signal to Noise Ratio](image)

**Figure 4.18:** Improvement of SNR with a Non-Foster impedance matching network relative to case without a matching network at indoor range

When comparing the improved SNR (smooth version) with the matching network relative to the antenna without a matching network over 400MHz to 900MHz, the overall improvement with the negative capacitor-inductor is maintained at higher frequencies than the SNR with the negative capacitor.
The improvement of SNR (in dBs) with the negative capacitor-inductor is lower than the improved antenna gain (in dBs) because negative resistances of the input impedance with the negative capacitor-inductor over 514MHz can increase more noises in the active circuit.

4.5 Summary

We introduced a Non-Foster impedance matching network with a series combination of negative capacitor-inductor for an electrically small monopole antenna to maintain the performance of the Non-Foster matching network for higher frequencies.

Based on measured data, it is shown that the antenna with the negative capacitor-inductor has an advantage over both the antenna with/without a negative capacitor in terms of antenna gain and SNR.

Although both Non-Foster impedance matching networks (a series combination of negative capacitor-inductor and a negative capacitor) relative to the case without a matching network improved the antenna gain and SNR at low frequencies, the antenna with the negative capacitor had a poor antenna gain after 558MHz when comparing the case without a matching network. (the improved SNR with the negative capacitor relative to the antenna without a matching network was up to 482.8MHz) However, the Non-Foster impedance with the negative capacitor-inductor made the antenna gain (or SNR) improved up to 793MHz (or 771.8MHz), comparing to that of the antenna without a matching network. However, there are several dips caused by what we believe to be coupling between the antenna and the matching network.

Although the antenna with the negative capacitor-inductor provided improvement of antenna performance up to higher frequencies than the single negative capacitor, the former could result in a negative resistance as the frequency becomes higher.
Thus, it may be necessary to use two different NICs (series capacitor and shunt inductor) rather than a single NIC to prevent negative resistance in the series connection of a negative capacitor and a negative inductor at higher frequencies.
CHAPTER 5
NON-FOSTER IMPEDANCE LOADING NETWORK FOR
AN ELECTRICALLY SMALL LOOP ANTENNA

5.1 Introduction

To date, most Non-Foster circuits have been used as impedance matching network connected the antenna at its input terminal. Furthermore, all fabricated Non-Foster impedances matching networks, like circuits in Chapters 3, 4, and [19–21], have been implemented for monopole or dipole-type (TM$_{10}$ mode) antennas because the input impedance of these antennas is considerably reactive.

This chapter will address another application, namely Non-Foster impedance loading networks. The concept of loading an antenna with Non-Foster impedances has recently been extensively by Prof. Rojas’ group [72–76]. In this study, these type of load is applied to an electrically small loop (TE$_{10}$ mode) antenna. The proposed method is to employ a Non-Foster inductor loading to a two-port antenna [69]. Note that, Albee in [69] showed a similar concept, inclusion a negative inductor in a loop antenna to match the antenna over a broad frequency range, but did not mention about the pattern bandwidth. Although [69] presents a schematic layout, no simulated or measured results are provided in this patent. A typical circular loop antenna is shown in Fig. 5.1. The equivalent lumped circuit for the input impedance of this antenna when is electrically small is shown in Fig. 5.3. A load is added to this
Figure 5.1: Circular loop antenna

Figure 5.2: Circular two-port loop antenna

antenna at the opposite side of the input terminals as depicted in Fig. 5.2. This antenna can be analyzed as a two-port network where one port is the input terminal (port 1) and the second port is where the load is added. The advantage of adding a load to the antenna away from the input terminals is two-fold as discussed in [72, 75]. Unlike the matching network added at the input terminals, the load can modify the current distribution on the antenna structure itself. This implies that in addition to modifying the input impedance of the antenna, its radiation pattern can also be better controlled as will be shown in this chapter.
After fabricating a Non-Foster inductor loading for a small loop antenna, we measured its performance such as input impedance, realized antenna gain, and signal to noise ratio (SNR), and antenna pattern. Furthermore, the reflection coefficient of the antenna with the Non-Foster loading network were measured with various input power levels. Based on measured data, the antenna with the Non-Foster inductor loading improves the maximum gain in the azimuth plane up to around 6.97dB over 100MHz to 267MHz, compared to the case with a short loading. The improved SNR with the Non-Foster loading relative the case with a short loading is up to 8.3dB over 100MHz to 277MHz. To the best of our knowledge, it is the first time that a Non-Foster impedance loading network was implemented, applied to an actual antenna, and tested.

5.2 Limitation of Non-Foster matching networks with a single element for electrically small loop antennas

It is well known that the input impedance of an electrically small loop antenna can be simply modeled by a capacitor in parallel with a series combination of an inductor and a radiation resistor [22], as illustrated in Fig. 5.3. This model is valid up to the first parallel resonance of the antenna.

In the sense of compensating the reactance of such an antenna to improve the impedance matching between the antenna and an input port with a 50Ω, and therefore increase the antenna gain up to the resonant frequency, it is required to design a negative capacitor connected in parallel to the input port of the antenna and then a negative inductor connected in series. However, it is difficult to realize a pair of Non-Foster impedances (a series and a shunt type) at the same time.

An alternative is to use a Non-Foster impedance matching network comprised
of only a negative inductor because the input reactance of a small loop antenna is inductive up to the first parallel resonance of the antenna. In considering the equivalent lumped model in Fig. 5.3, a negative inductor would be connected in series with the loop antenna. For frequencies far below the first parallel resonance, it can be expected that the input reactance of a small loop antenna with a Non-Foster inductor matching network would be smaller relative to the case without a matching network. However, as the frequency becomes closer to the resonant frequency, the series negative has little impact on the inductor input reactance of the antenna.

If a Non-Foster impedance matching network comprised of a negative capacitor is connected in parallel with the antenna, the resonant frequency can be shifted higher while the input reactance of the antenna remains the same as that without a matching network at frequencies far below the first parallel resonance. Hence, the shunt negative capacitor is not a method considered here to improve the impedance matching of an electrically small loop antenna.

It can be confirmed, through simple simulations with the measured input impedance of a small loop antenna, that a Non-Foster impedance matching network comprised of a single element (either a series negative inductor or a shunt negative capacitor) for an electrically small loop antenna does not have a significant impact on the input
Figure 5.4: 3” wire loop antenna on a octagonal metal ground plane (a) Top view (b) Side view

impedance of the antenna. The test antenna used here is a 3” wire circular loop antenna on a octagonal metal ground plane (with 8.49” the longest diagonal) shown in Fig. 5.4. This antenna has two symmetrical ports: One is a feeding port, while the second is a loading port. When the loading port is terminated with a short, the antenna performs like a normal loop antenna. Note that a male-to-male connector is needed to connect the antenna to a load at a loading port. Although the metal plate is electrically too small to function as a ground plane, this antenna behaves similar, but not the same, to a circular loop antenna. The radiation pattern is distorted in the upper hemisphere and is somewhat different from the pattern of a circular loop antenna.

With a short at the loading port of the antenna in Fig. 5.4, the reflection coefficient ($S_{11}$) at the feeding port of the antenna is measured over 10MHz ~ 500MHz, and the measured data is plotted in Fig. 5.5. Based on this measured data, it can be confirmed that the input impedance of the two-port antenna with a short loading
starts with inductive reactance and small resistance. It can also be seen that the first parallel resonant frequency of the antenna is about 224MHz.

An equivalent lumped model of the input impedance of the 2-port antenna with a short loading is made up of \( C = 2.3869 \text{pF}, L = 210.75 \text{nH}, \) and \( R = 6.9 \Omega \) in Fig. 5.3. Based on this equivalent model, the input impedance and reflection coefficient (\( S_{11} \)) of the antenna with three different Non-Foster impedance matching networks are investigated: One is a negative inductor \((-210.75 \text{nH})\) connected to the antenna in series. The other is a negative capacitor \((-2.3869 \text{pF})\) connected to the antenna in parallel. The last is a negative capacitor \((-2.3869 \text{pF})\) connected in parallel to the input port of the antenna and then a negative inductor \((-210.75 \text{nH})\) connected in series. Both simulated \( S_{11} \) and input impedance of the antenna with and without the Non-Foster impedance matching network over 10MHz to 500MHz are shown in Fig. 5.6 and Fig. 5.7, respectively. It is important to keep in mind that the measured \( S_{11} \) of the antenna with a short loading, shown in Fig. 5.5, is used to simulate both \( S_{11} \) and
input impedance of the antenna with the active matching network. In other words, these results are a combination of measured and simulated data (matching network).

All black curves in both Fig. 5.6 and Fig. 5.7 represent the measured results of the antenna without a matching network. The blue, red, and green curves are the measured data of the antenna with a simulated Non-Foster impedance matching network comprised of a negative capacitor, a negative inductor, and a combination of a negative capacitor-inductor, respectively. It can be observed that the input impedance and $|S_{11}|$ with a combination of a negative capacitor-inductor (green curves in Fig. 5.6 and Fig. 5.7) has the best performance in the frequency range.

As expected, the magnitude of $S_{11}$ (blue curve in Fig. 5.6 (a)) with the negative capacitor is the same as that without a matching network (black curve) up to 310MHz. Hence there is no advantage of the negative capacitor connected to the antenna in parallel over the case without a matching network up to 310MHz.

Figure 5.6: Simulated $S_{11}$ of a loop antenna with/without a Non-Foster impedance matching network (a) Magnitude of $S_{11}$ (b) Phase of $S_{11}$
In the case with the negative inductor, it can be observed that the magnitude of $S_{11}$ is lowered up to 224MHz than that without a matching network. It can also be observed that the reactance (red curve in Fig. 5.7 (a)) is greatly reduced at low frequencies, while the improvement of the reactance with the negative inductor relative to that without a matching network becomes smaller for frequencies close to the first parallel resonant frequency. Hence a negative inductor connected in series with a small loop antenna can be only used to decrease the reactance of the antenna at low frequencies. Note that a series resonant frequency of the antenna with the negative inductor in Fig. 5.6 (b) is found around 50MHz. This is because the input reactance with the negative inductor starts with very small capacitance (less than $|X_c| = 1.3\Omega$).
5.3 Design a Non-Foster impedance loading network for an electrically small loop antenna

This section addresses a method to modify the current in the loop antenna such that the input reactance of an electrically small loop antenna (therefore improve the antenna gain) is reduced and shift up the parallel resonant frequency of the antenna. The proposed method is to employ a Non-Foster impedance loading network comprised of a single element to the antenna. As another advantage of the loading network over a matching network is that it enables the antenna to maintain an omni-directional pattern at higher frequencies than the case without a loading network. This is a direct result of controlling the current on the antenna by adding a load.

5.3.1 Design of loading network for a small loop antenna based on the equivalent model

The method used here is basically to modify the current in the antenna that result in the reduction of the input reactance looking into the feeding port of an antenna by using Non-Foster impedances inside the antenna (at loading ports) [52,69–76]. It is worthy to note that to the best of our knowledge, no published work has shown a fabricated Non-Foster impedance loading network for an actual antenna.

Although the load modifies the current in the antenna and therefore changes the input impedance and pattern versus frequency, an equivalent model to explain the behavior of the input impedance is shown in Fig. 5.8. To avert cross-talk between the feeding and loading ports is to place the loading port as far as possible from the feeding port of the antenna. [72,75]
5.3.2 Design a loading network for a small loop antenna based on multipport characteristic mode theory

An impedance \([Z]_{N \times N}\) matrix of an antenna with \(N\)-port network can be extracted using a variety of electromagnetic (EM) commercial simulators. Let us first assume that we define \(N\) ports in the antenna under study. Based on a generalized eigenvalue problem with the \(n^{th}\) eigenvector \((\vec{I}_n)\), the \(n^{th}\) eigenvalue \((\lambda_n)\) is represented the ratio of the stored reactive power to the radiated power \([73]\).

\[
\lambda_n = \frac{P_{\text{stored}}}{P_{\text{rad}}} = \frac{\langle \vec{I}_n, [X] \vec{I}_n \rangle}{\langle \vec{I}_n, [R] \vec{I}_n \rangle},
\]  

(5.1)

where, \([R]\) and \([X]\) are the real and imaginary parts of the \([Z]_{N \times N}\) matrix, respectively. Using the systematic methodology introduced in \([72]\), it is possible to design a multipport loading antenna with desired currents at the \(N\) ports of the antenna. In this case, the desired current would be the mode that generates an omni-directional pattern and also yields an antenna with a large input impedance and pattern bandwidth. The lossless impedance \(N \times N\) matrix loaded at a \(N\)-port antenna can be expressed as \([Z_L] = [jX_L]\). Since any mutual coupling between the loads is not desired to keep the loading network as simple as possible, the \([Z_L]\) matrix has only diagonal elements.
Therefore, assuming the desired multi-port current is $\bar{I}_d$, (5.1) can be re-written as:

$$\lambda_d = \frac{\langle \bar{I}_d, [X_t] \bar{I}_d \rangle}{\langle \bar{I}_d, [R] \bar{I}_d \rangle}, \quad (5.2)$$

where $X_t = X + X_L$. $\lambda_d$ and $\bar{I}_d$ in (5.2) are the desired eigenvalue and eigenvector (eigencurrent), respectively. For this desired current to be the dominant current in the antenna, $\lambda_d$ should be equal to zero. The desired load $X_{L_n}$ at the $n^{th}$ port of the antenna is therefore be given by [72]

$$X_{L_n} = \frac{\left\{ (\lambda_d [R] - [X]) \bar{I}_d \right\}_n}{\bar{I}_{d_n}} = -\frac{([X] \bar{I}_d)_n}{\bar{I}_{d_n}}, \quad (5.3)$$

where $I_{d_n}$ is the $n^{th}$ element of the eigenvector $\bar{I}_d$. Note that the numerator in the right hand side of (5.3) is the $n^{th}$ element of the $N \times 1$ matrix. The denominator is the desired current at the $n^{th}$ port of the antenna. Using this methodology, the loading network for the 2-port antenna in Fig. 5.4 can be computed. If we consider only one port to be the loading port, (5.3) becomes a single equation, where $[X]$ is a scalar (a $1 \times 1$ matrix) and equal to the input reactance seen looking into a loading port of the antenna. Although using one port makes the control of the current more difficult and decreases the bandwidth of the antenna, it turns out that the desired load ($X_L$) at the loading port is just the negative ($-X$) of the input reactance looking into a loading port of the antenna. As shown in [72], adding additional ports will allow us to obtain the desired current over a larger frequency range.

### 5.4 Design and Fabrication of stabilized Non-Foster inductor

Based on the procedure to design Non-Foster impedances in Fig. 3.4, a loading network comprised of a stabilized Non-Foster inductor for the 3" half loop antenna in Fig. 5.4 is designed. Herein, Linvill’s unbalanced Short Circuit Stable (SCS) NIC [38] comprised of Bipolar Junction Transistors (BJTs) shown in Fig. 5.9 is used to build a Non-Foster inductor for the test antenna.
With a small signal $\pi$-model for a BJT shown at the bottom of Fig. 3.6, the driving input impedance ($Z_{in}$) of the NIC in Fig. 5.9 with a passive load $Z_L$ at the other port can be given by

$$Z_{in} = - \frac{(1 + g_m Z_\pi) Z_L + Z_\pi + R_2) (Z_\pi + g_m R_1 Z_\pi + R_1)}{(-g_m Z_\pi - 1) Z_L + g_m^2 Z_\pi^2 R_2 - Z_\pi - R_2}.$$  \hfill (5.4)

When $Z_\pi$ at the bottom of Fig. 3.6 is larger than unity; when the angular frequency ($\omega = 2\pi f$) is

$$\omega \ll \frac{r_\pi - 1}{C_\pi r_\pi} \approx \frac{1}{C_\pi}.$$  

The driving input impedance ($Z_{in}$) in (5.4) can be approximated by

$$Z_{in} \approx - \frac{R_1}{R_2} Z_L - \frac{g_m Z_L + g_m R_1 + 1}{g_m^2 R_2}.$$  \hfill (5.5)

From terms on the right hand side in (5.5), it can be observed that the impedance converter coefficient is $R_1/R_2$ with ideal transistors. The second term can be interpreted as a parasitic element due to the non-ideal transistors. It can also be observed that the parasitic effects can be varied by both resistors ($R_1, R_2$) and the passive load ($Z_L$) in the NIC. Once the topology of the NIC has been selected, the a passive load ($Z_L$) and two ratio resistors ($R_1, R_2$) in the NIC should be chosen.

To determine both a passive load and the two resistors in the NIC shown in Fig. 5.9, measured two-port S-parameters of the 3" half loop antenna in Fig. 5.4 are
used. Note that a male-to-male connector is needed to connect the antenna to a load at the loading port of the antenna. It is desired to determine the load and the two ratio resistors in the NIC such that the input reactance at the feeding port is minimal over as wide of a frequency range as possible. At the same time, the loss from the NIC should be minimized over the same frequency range. Based on simulated results, an inductor (270nH) and two resistors \( R_1 = R_2 = 50\Omega \) are chosen as the load of the NIC and the ratio resistors, respectively. Having chosen load and resistors in the NIC, the simulated input impedance looking into the NIC in Fig. 5.9 is a negative inductor \((-229\text{nH at } 100\text{MHz})\) with a small resistance \((5.62\Omega \text{ at } 100\text{MHz})\). Note that, at 100MHz, the input reactance seen looking into the loading port of the test antenna including the connector is about 155.83\(\Omega\) (Ideally, a \(-248\text{nH is needed at the frequency}\) while the feeding port terminated with a 50\(\Omega\). Although a NIC comprised of both ideal transistors and unity ratio resistors would convert the load \(Z_L\) in the NIC converted into \(-Z_L\), it is impossible for an actual (fabricated) NIC to perform ideally over wide frequencies.

Similar to the stability analysis in section 3.4, a loop gain simulation of the whole network including the antenna is performed. It can be found that there is one potential unstable point at 3.468GHz shown in Fig. 5.10. For stability at high frequencies, two parallel connections of an inductor \((L_1 = 7.5\text{nH})\) and a resistor \((R_3 = 50\Omega)\) are placed on two branches of the positive feedback loop in the NIC, as depicted in Fig. 5.11. These extra loads reduced the magnitude of the loop gain at high frequencies while the performance of the NIC is not changed at low frequencies.

Fig. 5.11 shows a schematic layout of a stabilized loading network comprised of a Non-Foster inductor for the 3" half loop antenna in Fig. 5.4. Note that Fig. B.2 shows a full schematic layout (including biasing circuits) of a loading network comprised of a stabilized Non-Foster inductor for the 3" half loop antenna in Fig. 5.4. Once loads
Figure 5.10: Loop Gain Analysis at high frequency when the Non-Foster impedance loading network without considering a stabilization.

Figure 5.11: Schematic layout of a loading network comprised of a stabilized Non-Foster inductor for the 3" half loop antenna in Fig. 5.4 (biasing circuits not shown).
were determined, the Non-Foster inductor for the 3″ half loop antenna was fabricated. The fabricated Non-Foster inductor on the basis of a unbalanced SCS NIC in Fig. 5.9 is shown in Fig. 5.12. The circuit is comprised of two discrete NPN BJTs (NE68133) transistors with Class-A type biasing on a RT/duroid 5880 substrate with $\epsilon_r = 2.2$ and thickness = 62 mil.

Figure 5.12: Fabricated loading network comprised of a stabilized Non-Foster inductor for the 3″ half loop antenna in Fig. 5.4

5.5 Measured and Simulated Results

After fabricating the stabilized system composed of both a loading network comprised of a Non-Foster inductor and a 3″ half loop antenna, its performance such as input impedance, frequency response ($S_{11}$) to levels of input power, antenna pattern, and SNR are measured.
Figure 5.13: Comparison between measured and simulated imaginary part of the input impedance of the test antenna with a short loading and with a Non-Foster inductor loading network: (a) and (b) use different scales on the y-axis.

Figure 5.14: Comparison between measured and simulated real part of the input impedance of the test antenna with a short loading and with a Non-Foster inductor loading network: (a) and (b) use different scales on the y-axis.
5.5.1 Input Impedances

The measured and simulated input impedance (based on the $S_{11}$ at 0dBm input power) of the loop antenna with a short loading and with a stabilized Non-Foster inductor loading over 100MHz to 400MHz is shown in both Fig. 5.13 for the imaginary part and Fig. 5.14 for the real part. The black curves in both Fig. 5.13 and Fig. 5.14 represent the measured results for the antenna with a short loading. The blue and red curves are the simulated\textsuperscript{1} and measured impedances of the test antenna with the Non-Foster inductor loading, respectively.

As shown in Fig. 5.13, the measured reactance of the antenna with the Non-Foster inductor loading (red curve) agrees very well with the simulated result (blue curve) except near the parallel resonance at 260MHz. The disagreement around the parallel resonance frequency is due to the large losses in the fabricated load than the losses included in the model used to obtain the simulated data. The simulated data shows a higher quality factor ($Q$) than what is obtained with the fabricated device.

It can be observed that the measured reactance with the Non-Foster inductor loading is significantly reduced (in magnitude) and the first parallel resonant frequency of both the measured and simulated with the Non-Foster inductor loading is shifted to higher frequency, compared to the case of the antenna with a short loading. The resonant frequency of both the measured (262.4MHz) and simulated (265MHz) with the Non-Foster inductor loading are very close to each other. The decreased in reactance and the shifted resonant frequency contributes to the improvement of antenna gain and the maintenance of the antenna pattern at higher frequencies as shown in section 5.5.3.

\textsuperscript{1}When simulating the input impedance of the antenna with the Non-Foster inductor loading, the measured two-port $S$-parameters of the antenna was used.
5.5.2 Reflection coefficient versus input power level

As done in Chapter 3, it is necessary to investigate the changes of the reflection coefficients ($S_{11}$) looking into the input port of the antenna with a Non-Foster impedance loading for varying levels of the input power. As mentioned in section 3.5.2, it is associated with the dynamic range of a Non-Foster impedance circuit. As in section 3.5.2, the Large-Signal S-parameter simulation in Advanced Design System by Agilent is used to simulate the response of $S_{11}$ for different input power levels.

![Graph showing simulated and measured $S_{11}$](image)

**Figure 5.15:** Comparison between measured and simulated magnitude of $S_{11}$ of the test antenna with the Non-Foster inductor load for low input power levels ($-10$dBm, $-5$dBm, and 0dBm)

The measured and simulated magnitude of $S_{11}$ with the Non-Foster inductor loading over 100MHz to 400MHz is shown in both Fig. 5.15 for low input powers ($-10$dBm, $-5$dBm, 0dBm) and Fig. 5.16 for high input powers (0dBm, 5dBm, 10dBm). The measured $S_{11}$ of antenna with a short loading is also plotted in Fig. 5.15 to easily
identify the improvement of the antenna with the Non-Foster inductor loading. It can be observed that the measured $|S_{11}|$ (blue curve at 0dBm input power) with the Non-Foster inductor loading decreases in the range of 100MHz to around 310MHz, when compared to the case with short loading.

The solid and dashed curves in both Fig. 5.15 and Fig. 5.16 represent the measured and the simulated data at different input powers, respectively. For convenience, the color of solid and dashed curves are same at exciting an identical input power.

When using low input powers ($-10$dBm, $-5$dBm, 0dBm), both measured and simulated data in Fig. 5.15 are shown little changes response to the different input powers. The transistors in the active circuit still would be the linear respond along to an input power up to 0dBm. The measured magnitude of $S_{11}$ (solid black curve) with the Non-Foster inductor load at $-10$dBm input power shows small fluctuations because the input power is not high enough to overcome the noise in the measurement
environment. For high input powers (0dBm, 5dBm, 10dBm), it can be observed that the measured and simulated data in Fig. 5.16 shows significantly impact to different input power levels: The higher input power, the smaller $|S_{11}|$. The simulated and measured magnitude of $S_{11}$ at 10dBm input power (red curves in Fig. 5.16) decreases dramatically, due to the improvement of mismatch loss between the antenna and active network with the 50Ω input port; however this improvement is achieved with a large increase in the loss produced by the active network. As mentioned in section 3.5.2, this characteristic is caused by the gain compression of the transistors in the Non-Foster impedance.

5.5.3 Antenna Pattern

The antenna pattern measurements were performed in an outdoor range over 100MHz to 280MHz. Note that the antenna’s electrical length in terms of a 3” radius (or the 8.49” the longest diagonal direction of the ground plane) of the half loop is about $\lambda/39$ (or $\lambda/29$) at 100MHz. Fig. 5.17 illustrates the outdoor range setup for the antenna pattern measurements. The measurement setup is the same as that in section 3.5.3, except for the location of the transmitting antenna and the usage of the rotator for the radiation pattern measurements. Note that, in order to set easily the reference angle in the antenna pattern measurements, the transmitter antenna is placed to make the directed path in Fig. 5.17 perpendicular to the ESL building. Furthermore, identical ferrite beads in section 3.5.3 are used on a cable at the input of the antenna to prevent the common mode currents on the cable.

The procedure used here to measure antenna patterns is the same as the gain measurements in section 3.5.3, except that gain of the test antenna is measured at multiple-angles. In the antenna pattern measurements, the received signal ($S_{21}$) of the antenna with a short and Non-Foster inductor loading are recorded with a
Network Analyzer (Agilent N5230A), and then time-gating is applied to the raw received data. With the measured gain of a reference calibration antenna, the time-gated data can be calibrated. Fig. 5.18 (a) and (b) show the azimuth and elevation plane coordinates for the antenna pattern measurements, respectively. In the azimuth pattern measurements, the transmitter antenna (a linear polarization) is positioned perpendicular to the earth ground and the finite ground of the test antenna in Fig. 5.18 (a) is also placed parallel to the ground. Therefore, the polarizations of the both antennas are vertical with respect to the ground. In the elevation measurements, the transmitter antenna and the loop are positioned in parallel to the ground.

Since an electrically small loop antenna can be thought of a hypothetical magnetic dipole antenna perpendicular to the plane of the loop [10], the radiation pattern of the antenna is an omni-directional (donut shaped pattern). However, the omni-direction pattern of loop antenna changes as the frequency gets closer to the first parallel
resonant frequency of the antenna. Since the finite metal plate for this antenna is electrically small (radius < λ/4), the pattern of this antenna can be distorted.

**Azimuth Antenna Pattern**

When it comes to the azimuth plane (0° ≤ φ ≤ 360° at θ = 90°) in Fig. 5.18 (a), we can expect that the azimuth radiation pattern of the antenna with a short loading and the Non-Foster inductor loading will have two null points at φ = 90° and 270° for frequencies lower than the first parallel resonance. The measured and simulated azimuth antenna patterns with a short and the Non-Foster inductor loading are shown in Fig. 5.19, 5.20, and 5.21 over 100MHz to 280MHz (with 20MHz steps). The simulated antenna patterns with the loading networks are generated by using Ansoft HFSS v12.0, and an ideal negative inductor (−230nH) was applied to simulate the antenna pattern with a Non-Foster inductor loading. The dashed and solid curves in Fig. 5.19, 5.20, and 5.21 represent the simulated and measured antenna patterns,
Figure 5.19: Comparison between measured and simulated of the antenna azimuth patterns with a short loading and with a Non-Foster inductor loading at (a) 100MHz (b) 120MHz (c) 140MHz (d) 160MHz respectively. The red (blue) curves stand for the results with the Non-Foster inductor loading (the short loading).
When comparing the measured (solid curves) and simulated (dashed curves) results, it can be found that the discrepancy is large, especially at the lower frequencies.
Figure 5.21: Comparison between measured and simulated of the antenna azimuth patterns with a short loading and with a Non-Foster inductor loading at (a) 260MHz (b) 280MHz

However, as the frequency becomes higher, it can also be observed that the difference becomes smaller. This discrepancy could be caused by two reasons: One is the model of the Non-Foster inductor in the simulation. As mentioned above, an ideal negative inductor ($-230\text{nH}$) was used in the simulation because it was difficult to obtain the model of the actual Non-Foster load. The other is common mode currents on the cable. When using an electrically small ground plane for an antenna, it could make the common mode effect severe.

The maximum gain of the measured azimuth antenna patterns with the Non-Foster and short loading are plotted in Fig. 5.22 to inspect the improvement of the Non-Foster loading relative to the case with short loading. Based on the measured data in Fig. 5.22, it is shown that the Non-Foster loading improves the gain up to
Figure 5.22: Maximum gains of measured antenna patterns in the azimuth plane around 6.97dB (maximum improvement) with respect to the case with a short over the frequency band 100MHz to 267MHz.

The antenna patterns with the Non-Foster loading in Fig. 5.19, 5.20, and 5.21 have two nulls (characteristic of an omni-directional radiation pattern), while the measured results with a short loading start to change after 200MHz due to excitation of higher order mode currents. As expected from the measured reactance in section 5.5.1, the Non-Foster inductor loading enables the $TE_{10}$ mode to continue to be the dominant mode at higher frequencies, compared to the case of the antenna with a short loading.

**Elevation Antenna Pattern**

The elevation plane coordinate ($0^\circ \leq \theta \leq 360^\circ$ at $\phi = 0^\circ$) is shown in Fig. 5.18 (b). Similar to the azimuth patterns above, the measured and simulated elevation antenna patterns with the loading networks (short and Non-Foster inductor) are shown in
Figure 5.23: Comparison between measured and simulated of the antenna elevation patterns with a short loading and with a Non-Foster inductor loading at (a) 100MHz (b) 120MHz (c) 140MHz (d) 160MHz

Fig. 5.23, 5.24, and 5.25 over 100MHz to 280MHz. Ideally, the radiation pattern of a small loop antenna in the elevation cut is a constant for frequencies below the first
Figure 5.24: Comparison between measured and simulated of the antenna elevation patterns with a short loading and with a Non-Foster inductor loading at (a) 180MHz (b) 200MHz (c) 220MHz (d) 240MHz parallel resonance. Based on the simulated results (dashed curves) with the loading networks, omni-directional patterns can be seen at lower frequencies.
Realized Antenna Gains (Elevation) @260MHz

Realized Antenna Gains (Elevation) @280MHz

(a) (b)

Figure 5.25: Comparison between measured and simulated of the antenna elevation patterns with a short loading and with a Non-Foster inductor loading at (a) 260MHz (b) 280MHz

When comparing the measured and simulated results, it can be recognized that there is a large discrepancy. As mentioned above, this difference would be caused by two reasons (the model of the Non-Foster inductor in the simulation and the common mode effect).

Based on the simulated and measured results, the Non-Foster loading keeps the pattern omnidirectional up to higher frequencies, compared to the case with a short loading. As mentioned before, this is due to the dominance of the $TE_{10}$ current mode up to higher frequencies due to the addition of the Non-Foster load.

Fig. 5.26 shows the maximum measured gain of the antenna in the elevation plane with the loading networks. It can be observed that the maximum gain improvement with the Non-Foster loading is about 10.3dB at 100MHz with respect to the case with short loading over the frequency band 100MHz to 242MHz.
Figure 5.26: Maximum gains of measured antenna patterns in the elevation plane

5.5.4 Antenna Signal to Noise Ratio

The signal to noise ratio (SNR) with the loading networks was also measured in the outdoor range. The procedure to access the improved SNR is the same as that in section 3.5.3. Since it is difficult to investigate the improvement for every angle, the measured gains at $\theta = 0^\circ$, $90^\circ$ (at the feeding port), $270^\circ$ (at the loading port) in the elevation plane are used to determine the SNR. To measure noise at the angles, the received signal at the loop antenna with the loading networks is measured with a spectrum analyzer (Agilent E4440A) while the transmitter antenna is turned off. Fig. 5.27 shows the noise floor of the spectrum analyzer; it is around $-100$dBm. Note that the measured noise received by the antenna can change at different angles because the measured noise received signal can include external RF interference signals.

The improved SNRs with the Non-Foster loading relative to the case with a short loading at $\theta = 0^\circ$, $90^\circ$, $270^\circ$ in the elevation plane are plotted in Fig. 5.28, Fig. 5.29,
Figure 5.27: Measured Noise Floor of a spectrum analyzer at outdoor range

Figure 5.28: Improvement of SNR with the Non-Foster loading relative to the case with a short loading for $\theta = 0^\circ$ in elevation plane
<table>
<thead>
<tr>
<th>Freq [MHz]</th>
<th>Improved SNR [dB]</th>
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<tr>
<td>100</td>
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<td>120</td>
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</table>

**Figure 5.29:** Improvement of SNR with the Non-Foster loading relative to the case with a short loading for $\theta = 90^\circ$ in elevation plane.

<table>
<thead>
<tr>
<th>Freq [MHz]</th>
<th>Improved SNR [dB]</th>
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<tr>
<td>100</td>
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<td>280</td>
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**Figure 5.30:** Improvement of SNR with the Non-Foster loading relative to the case with a short loading for $\theta = 270^\circ$ in elevation plane.
and Fig. 5.30, respectively. It can be observed that the improvements (smoothed curves) at \( \theta = 0^\circ, 90^\circ, 270^\circ \) in the elevation plane are up to 13.5dB over 100MHz to 241MHz, 6.4dB over 103MHz to 243MHz, and 8.3dB over 100MHz to 277MHz, respectively. It can also be found that the noises with the Non-Foster loading are increased at around 100MHz, compared to that with a short loading. It is because the Non-Foster loading makes the test antenna received more signals from external RF interferences such as the FM radio (FM radio bands : 87MHz ∼ 108MHz) while the transmitter antenna is turned off. It can be confirmed through the drops of the improved SNR curves (smooth curves) in the FM frequencies, shown in Fig. 5.28, Fig. 5.29, and Fig. 5.30.

5.6 Summary

In this chapter, a fabricated Non-Foster inductor load for an electrically small two-port loop antenna was introduced to both increase the antenna gain and maintain an omni-direction radiation pattern at higher frequencies. To the best of our knowledge, it is the first time that a fabricated Non-Foster impedance loading network was applied to this antenna. The fabricated negative inductor shown in this chapter could be employed to enhance the performance of any small loop type antenna.

We showed the limitations of a Non-Foster impedance matching network comprised of a single element (either a negative inductor or a negative capacitor) for a small loop antenna.

When comparing the measured and simulated input impedances of the antenna with the loading networks, the reactance with the Non-Foster loading was reduced and the first parallel resonant frequency of the antenna with the Non-Foster loading was shifted to a higher frequency. Through both measured and simulated reflection
coefficients for different input powers, the Non-linear property of the Non-Foster loading was also shown in this chapter.

For the radiation patterns, the measured data did not agree well with the simulated results because of two effects: the model of the Non-Foster inductor in the simulation and common mode currents on the cable. However, based on both measured and simulated antenna gains, it could be confirmed that the antenna gain with the Non-Foster loading is improved. For instance, the Non-Foster inductor loading improved the maximum gain in the azimuth plane up to around $6.97\,\text{dB}$ over $100\,\text{MHz}$ to $267\,\text{MHz}$, compared to the case with a short loading. It was also demonstrated that the SNR with the Non-Foster loading relative to the case with a short loading could be improved as well. Based on measured SNRs, the improvement with the Non-Foster loading was up to $8.3\,\text{dB}$ over $100\,\text{MHz}$ to $277\,\text{MHz}$, when compared with the short loading case.
CHAPTER 6
CONCLUSIONS AND FUTURE WORK

6.1 Summary and Conclusion

Linearly polarized electrically small antennas can be generally classified into two types based on their excited modes: One is a $TM_{10}$ mode antenna and the other is a $TE_{10}$ mode antenna. Such antennas can be characterized by high radiation quality factors, in other words, the operational bandwidth of the antennas is considered to be narrow. The work considered in this dissertation was to enhance the performance of both types of electrically small antennas by using Non-Foster impedance devices over broad frequency ranges.

For a $TM_{10}$ antenna such as a dipole-type electrically small antenna, the input impedance of the antenna is significantly reactive with a small resistance. It is crucial to reduce the reactance of the antenna and/or increase the radiation resistance in the frequency range of interest. Herein, Non-Foster impedance matching networks for the antennas were applied to reduce the input reactance, and therefore improve the antenna gain and signal to noise ratio when comparing to the case without a matching network.

In the case of a $TE_{10}$ antenna such as a loop-type electrically small antenna, even though the input impedance of the antenna is not highly reactive but it has a small resistance, there is a huge mismatch between a generator with a 50Ω and the input
port of the antenna. Herein, a Non-Foster loading network was employed to improve the mismatch and maintain an omni-directional antenna pattern at higher frequencies than the case without a loading network. In other words, the current within the loop antenna is modified.

In Chapter 2, the fundamental limitations of electrically small antennas and Gain-Bandwidth product for lossless passive matching networks were first reviewed. Furthermore, a survey of Non-Foster impedances including Negative Impedance Converter or Inverters and previous literature on fabricated Non-Foster impedances for electrically small antennas was provided in this chapter.

Chapter 3 introduced a systematic methodology to design a Non-Foster impedance matching network for an electrically small antenna, and demonstrated how to apply the methodology to a Non-Foster capacitor for a 3” monopole small receiver antenna. After fabricating a stabilized Non-Foster capacitor and an antenna, it was shown that, through both measured and simulated results, the Non-Foster impedance matching network could be utilized to improve the electrically small antenna over a broad frequency range (100MHz to 550MHz). To the best of our knowledge, the improvement by adding a Non-Foster matching network relative to the case without a matching network was maintained up to the highest frequency among published papers (the previous highest effective frequency range was over 60MHz to 400MHz [61]). Furthermore, the changes in the reflection coefficient ($S_{11}$) seen at the input port of the antenna with the Non-Foster matching network as a function of the input power levels were measured and simulated. When higher input power is used with this network, the smaller $|S_{11}|$ is related with the increase of loss resistance in the active circuit. When applying a Non-Foster matching network to a transmitter antenna, this non-linear effect should be considered.
As the electrical size of the antenna becomes larger (when $ka > 0.5$), the net-reactance starts to increase rather than decrease because the single negative capacitor can no longer compensate the reactance of the antenna. Chapter 4 addressed a fabricated Non-Foster matching network with a series combination of a negative capacitor and a negative inductor for an electrically small monopole antenna to maintain the advantage of a Non-Foster matching network for higher frequencies. To the best of our knowledge, it is the first time that such a fabricated negative capacitor-inductor was implemented with an actual antenna. Through measured and simulated data, it was shown that the antenna with a series combination of a negative capacitor ($-5.6pF$) and a negative inductor ($-10nH$) had an advantage over both the antennas with and without the negative capacitor ($-5.6pF$). Based on measured antenna gains, the improved antenna gain (or SNR) with the negative capacitor-inductor relative that without a matching network is up to 739MHz (or 771.8MHz), while the case with the negative capacitor up to 558MHz (or 482.8MHz). Even if the antenna with the negative capacitor-inductor provides the improvement of antenna performance up to higher frequencies than the single negative capacitor, the former can result in a negative resistance as the frequency goes higher.

Chapter 5 discussed a method to improve the performance of an electrically small loop antenna by using a Non-Foster inductor loading for the antenna. The Non-Foster loading network improves the antenna gain and help the antenna maintain an omni-directional pattern at higher frequencies, when comparing to the case with a Non-Foster impedance matching network and the case without a matching/loading network. To the best of our knowledge, it is the first time that a fabricated Non-Foster impedance loading network was applied to an actual antenna. Through simulated and measured data, it was shown that the loop antenna with a Non-Foster inductor loading improved both the antenna gain and the omni-directional pattern was kept.
at higher frequencies, compared to the case with a short loading. Although the experimental radiation patterns did not agree as well with the simulated data, it could be observed that the measured antenna gain and SNR with the Non-Foster loading were improved, comparing to that with a short loading. For instance, the Non-Foster inductor loading improved the maximum gain in the azimuth plane up to around 6.97dB over 100MHz to 267MHz, compared to the case with a short loading. The improved SNR with the Non-Foster loading relative to the case with a short loading was up to 8.3dB over 100MHz to 277MHz.

6.2 Future Work

6.2.1 Non-Foster impedance application to transmitter antennas

RF systems can be generally divided into a transmitter and a receiver depending on their functions. Since Non-Foster impedance matching and loading networks for electrically small receiver antennas were considered and completed in this dissertation, it would be interesting to work on Non-Foster impedance devices for small transmitter antennas. For receiver antennas, the SNR of the antennas is the key figure of merit. However, the power efficiency is an important factor to design a transmitter antenna system.

As briefly summarized in section 2.4.3, Sussman-Fort has published Non-Foster matching networks for electrically small transmitter antennas [21, 58, 59, 62]. Although experimental results for the Non-Foster devices were shown in these papers, the equivalent lumped circuit of an antenna were used. In terms of measurements, Class-B and -C biased NICs comprised of bipolar junction transistors were designed to improve power efficiencies for transmitter antennas.
Another key factor when designing transmitters is a linearity. As shown in Chapters 3 and 5, a Non-Foster impedance has non-linear properties because of the active devices in the NIC. When applying a Non-Foster impedance to a transmitter antenna, this effect should be considered. To maintain linearity, linearization techniques for power amplifiers such as a feed-forward and a pre-distortion [77] could be employed in the design of Non-Foster impedances for transmitter antennas.

6.2.2 Integrated Non-Foster impedances

It is well known that active circuits composed of discrete transistors and components have a limitation to operate at high frequencies due to parasitics and the electrically large size of the layout. It would be highly recommended to design and fabricate integrated-version Non-Foster impedances for antenna applications, especially, loading Non-Foster impedances for compact antennas [72]. Therefore, the integrated-version of negative impedance converters or invertors are feasible solutions to extend the operation of Non-Foster impedances to much higher and wider operating frequencies.

A publication written by Kolev in [51] is an example of fabricated Non-Foster impedances using integrated circuits. With the basis of a NIV in Fig. 2.17, a negative capacitor was realized in order to increase the tuning range of a varactor diode and a negative resistance to compensate a series resistance of the varactor diode. The negative capacitor was fabricated using 0.25 \( \mu \)m gate length pHEMT process H40 of GEC-Marconi, Caswell, U.K. Based on measured data in [51], the circuit performed as a negative capacitor \((-1\text{pF})\) from 1GHz to 5GHz.
6.2.3 Low power Non-Foster impedances

Unlike passive elements, Non-Foster elements consume DC power because they rely on active devices such as transistors. It can be considered a drawback in using Non-Foster impedances, when comparing to passive elements. It is important to consider low power NICs or NIVs in designing Non-Foster impedances. The easiest way to solve the power issue is to use low power transistors. The solution comes back to the integrated version of NICs or NIVs because lower power is easier to achieve with integrated circuits relative to discrete circuits.

Sussman-Fort in [21, 62] used a figure of merit for the power efficiency, \( \eta \), to consider DC power biasing of active circuits in a transmitter application. In [21, 62] the efficiency of the Non-Foster impedance was expressed as

\[
\eta_{\text{active}} = \frac{P_{\text{ant}}}{P_{\text{DC-active}}},
\]  

(6.1)

where, \( P_{\text{DC-active}} = \text{DC power biasing NIC + DC power for a transmitter} \), and \( P_{\text{ant}} \) is the power delivered to the antenna.

According to [31], (6.1) could be overvalued because the RF input power is not considered in this definition. Hence, it could be better to use the power added efficiency instead of the power efficiency defined above. The power added efficiency is given by [31]

\[
\eta_{P\text{AE}} = \frac{P_{\text{out}} - P_{\text{in}}}{P_{\text{DC}}}.
\]  

(6.2)

When using a Non-Foster impedance in a transmitter antenna, the power added efficiency can be written as

\[
\eta_{P\text{AE}} = \frac{P_{\text{ant}} - P_{\text{in}}}{P_{\text{DC-active}}},
\]  

(6.3)

In a receiver application, the total (overall) efficiency could be used to measure the efficiency of an active circuit. The total (overall) efficiency is given by [78]

\[
\eta_{\text{total}} = \frac{P_{\text{out}}}{P_{\text{in}} + P_{\text{DC-active}}}.
\]  

(6.4)
Appendix A

REALIZATION OF A NEGATIVE RESISTOR

Although a negative resistor is not included in Non-Foster impedances, a negative resistor is fabricated and tested to complete the negative of all passive impedances (resistor, capacitor, and inductor). This could be used to increase the quality factor of an active inductor [79] or to compensate losses in a passive element [80].

A.1 Fabrication of a Negative resistor

Fig. A.1 shows a test setup [46] used to assess the performance of a negative resistor. To match two 50Ω resistors to a generator with a 50Ω in Fig. A.1, it is necessary to build a −50Ω by means of a NIC/NIV. Herein, a Linvill’s unbalanced open circuit stable NIC [38] comprised of BJTs, shown in the red box of Fig. A.1, is used to build a negative resistor.

With a small signal π-model for a BJT shown at the bottom of Fig. 3.6, the \(Z_{in,L}\) (seen at the NIC) in Fig. A.1 can be expressed as

\[
Z_{in,L} = -\frac{R_2}{R_1}Z_L + \frac{Z_\pi (Z_\pi + R_2 + Z_L)}{(g_mZ_\pi + 1)(Z_L + Z_\pi)} + \frac{Z_LR_2(R_1^2 + 2Z_LR_1 + 2Z_\pi R_1 + Z_\pi^2 + 2Z_\pi Z_L + Z_L^2)}{(Z_L + R_1 + g_mZ_\pi R_1 + Z_\pi)R_1(Z_L + Z_\pi)}, \tag{A.1}
\]

where \(Z_L\) is a passive load of the NIC. When \(Z_\pi\) at the bottom of Fig. 3.6 is much
larger than unity, the $Z_{in,L}$ in (A.1) can be approximated by

$$Z_{in,L} \approx -\frac{R_2}{R_1}Z_L + \frac{1}{g_m} + \frac{R_2Z_L}{R_1(g_mR_1 + 1)}. \quad (A.2)$$

From terms on the right hand side in (A.2), it can be observed that the impedance converter coefficient is $R_2/R_1$ with ideal transistors. The remaining terms can be interpreted as parasitic elements due to the non-ideal transistors. In this paper, a load ($Z_L = 50\,\Omega$) and two resistors ($R_1 = R_2 = 50\,\Omega$) are chosen as the load of the NIC and the ratio resistors to realize a negative resistor ($-50\,\Omega$ at 50MHz), respectively.

Without any stabilizations for the NIC, we could observe high-frequency oscillations (including the fundamental oscillation at 2GHz) in the NIC, as shown in Fig. A.2. Note, this measurement was performed using a spectrum analyzer (Agilent E4440A). To eliminate these high frequency oscillations, a resistor ($50\,\Omega$) was placed on the positive feedback loops in the NIC. After fabricating a stabilized whole network shown in Fig. A.3, an input impedance $Z_{in}$ (seen at the input port) in Fig. A.1 and the reflection coefficient were measured over the frequency range of 50MHz to 1GHz.

Fig. A.4 and Fig. A.5 show the comparison between measured and simulated
Figure A.2: Measured high-frequency oscillations using a spectrum analyzer

Figure A.3: Fabricated test setup in Fig. A.1 including a negative resistor

input impedance and magnitude of the reflection coefficient with the negative resistor, respectively. The red curves represent the measured results while the blue curves are the simulated one. It can be observed that the measured results agree well with the simulated data at low frequencies (up to around 200MHz). As the frequency
increases, the measured results would be deviated from the simulated data due to parasitics on layout.

Figure A.4: Comparison between measured and simulated input impedance ($Z_{in}$ in Fig. A.1) with the negative resistor (a) Real part (b) Imaginary part

Figure A.5: Comparison between measured and simulated magnitude of the reflection coefficient (seen at the input port) with the negative resistor
To measure the input impedance ($Z_{in,L}$) seen at the negative resistor in Fig. A.1, it is necessary to employ de-embedding to get rid of the parasitics included in the pad for a SMA connector and the two 50Ω resistors in Fig. A.3. With a open and a short (using via-hole) load termination at the end of the two 50Ω resistors, as shown in Fig. A.6, it can be de-embedded.

![Figure A.6: (a) Open (b) Short for de-embedding of both the pad for a SMA connector and the two 50Ω resistors in Fig. A.3](image)

Fig. A.7 shows the de-embedded results of measurement in comparison with those of simulation. Fig. A.7 (a) and (b) present the real and the imaginary part of de-embedded input impedance ($Z_{in,L}$ in Fig. A.1) of the negative resistor, respectively. It can be observed that the measured data are much closer to the simulated one, when comparing to the data before de-embedding in Fig. A.4. We can still see that the performance of the negative resistor decreases as the frequency becomes higher. Based on the measured data in Fig. A.7 (a) and (b), the impedance of the active circuit becomes dominantly inductive after 300MHz. This is because the electrical length of the signal path (including the feedback loop) in the active circuit becomes
longer at higher frequencies. By using integrated-version NIC, the issue could be improved.

Figure A.7: Comparison between measured (Red curves) and simulated (Blue curves) input impedance ($Z_{in.L}$ in Fig. A.1) of the negative resistor (a) Real part (b) Imaginary part
Appendix B

SCHEMATICS OF NON-FOSTER IMPEDANCES

Figure B.1: Schematic layout of a stabilized Non-Foster impedance (−6.8pF) matching network for the 3" wire monopole antenna in Fig. 3.13
Figure B.2: Schematic layout of a loading network comprised of a stabilized Non-Foster inductor for the 3” half loop antenna in Fig. 5.4
BIBLIOGRAPHY


