DESIGN, MODELING AND CONTROL OF MAGNETORHEOLOGICAL FLUID-BASED FORCE FEEDBACK DAMPERS FOR TELEROBOTIC SYSTEMS

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

By

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* * * * *

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The overall goal of the research done in this dissertation is to develop next generation force feedback systems by combining novel Magnetorheological (MR) fluid based electromechanical systems with microstructural analysis and advanced control system design. Four MR fluid based systems are designed, prototyped and tested with medical applications: A two degree of freedom (2-DOF) force feedback joystick and a 5-DOF force feedback manipulator for telerobotic surgery application, a passive and a semi-active orthopedic knee brace for rehabilitation application. Furthermore, a force feedback steering wheel is modified using MR damper with application to steer-by-wire automobiles. The test results show the appropriate performance of MR fluid based systems used in haptic and force feedback applications.
Dedicated to my dear parents
and my beloved wife, Bahareh
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CHAPTER 1

INTRODUCTION

1.1 Problem Statement

A Recently, force feedback techniques have been in use to increase the effectiveness of human-machine interfaces. The applications range from biomedicine to computer games and virtual reality devices. The goal of this research is to combine novel MR fluid based electromechanical devices with microstructural analysis and control system design to advance the capabilities of modern force feedback systems (Figure 1.1).

A teleoperation system consists of a pair of manipulators: master and slave. The operator handling the master directs control signals to the slave which acts on an external object. The performance of teleoperation systems can be improved if force feedback is employed. In force feedback systems the slave receives motion commands from the master and transmits information from its environment back to the master when comes into contact with an external object. Most traditional force feedback systems employ DC motors or hydraulic actuators in the master to simulate the feedback forces. When the operator controlling the master has the feeling of direct interaction with the remote
environment, the system is called transparent. Recent technology in teleoperation systems has the following limitations: (1) These systems have relatively higher inertias for the slave than master which lead to the transmission of large signals sent by the slave to the master. These large signals may result in system instability and if attenuated, may cause nontransparent systems. This is one of the main reasons why the Food and Drug Administration (FDA) has not adopted bilateral telerobotic systems for laparoscopic and thoracoscopic surgery. (2) The large forces needed by some of these systems require large active components that limit their use in some applications. (3) Due to complexity of force feedback systems, cost associated with these systems is another big issue.

![Diagram](image.png)

**Figure 1.1: Dissertation objective**

The primary aim of this dissertation is to address the aforementioned problems with traditional force feedback systems by the application of magnetorheological fluid based
force feedback systems. For the consequent novel applications, areas of telerobotic surgery, orthopedic knee braces, and drive-by-wire vehicles are addressed.

1.2 Motivation and Significance

1.2.1 Telerobotic Surgery

The researchers at the Intelligent Structures and Systems Lab (ISSL) and associates at the Ohio State University (OSU) medical school have been working on the development of novel haptic and force feedback devices aimed at improvements in telerobotic surgical systems. For minimally invasive cardio-thoracoscopic (MICT) surgery, the present state of the art is a telerobotic surgical system where the surgeon sits at workstation and controls a robot, which conducts the surgery (Figure 1.2). Some of the benefits of using telerobotic MICT surgery are smaller incisions, less pain, lower risk of infection, and less scaring. In addition, the recovery time for a person is reduced by at least a factor of 4 when minimally invasive surgery is used. In MICT surgery only three small incisions are needed and stopping the heart is optional (Figure 1.3).

While these systems are highly advanced, the surgeon only has visual feedback from a camera (scope) inserted in the patient. Although the lack of tactile feedback has been cited by surgeons as one of the main reasons for limited adoption of telerobotic machines, the FDA has not approved telerobotic systems including force feedback. In fact, because of the limited field of view from the scope, there have been reports of bones being broken because the surgeons just don’t see them.
Figure 1.2: Ohio State University’s MICT Surgical System (Da Vinci)

Figure 1.3: MICT Surgical System
The force feedback discussed and investigated in this research is associated with the instrument encountering or running into dense muscle, bone, etc. (and not with grasping, cutting or holding).

1.2.2 Orthopedic Knee Brace

Resistance exercise has a wide range of benefits. Aged adults are more liable to support themselves when standing. Children who have neurological problems can enhance their motor skills. Stroke victims can enhance their muscles strength. People who suffer from hemiparesis can counteract their muscle weakness. Osteoarthritis pain can be managed. Additionally, resistance exercise can moderately restore bone density losses due to bone disease as well as slow bone loss from aging.

According to the American Academy of Orthopaedic Surgeons, there are four categories of knee braces: prophylactic, patellofemoral, rehabilitative and functional. Prophylactic knee braces are intended to prevent or reduce the severity of knee injuries in contact sports. Patellofemoral knee braces are designed to improve patellar tracking and relieve anterior knee pain. Rehabilitative knee braces are designed to allow protected and controlled motion during the rehabilitation of injured knees. Functional knee braces are designed to provide stability for unstable knees (Figure 1.4).

The rehabilitative and functional knee braces are the only ones that have been proven by physicians to be effective. Rehabilitative knee brace are limited in the sense that they
only stabilize the knee to prevent movement that could result in injury. Mobility is sacrificed for effective stabilization which could decrease the work that muscles do during a given activity. The functional knee brace, which is used after injury, also decreases mobility and only protects the knee from further injury by stabilization. Although stabilization is very important it cannot compensate muscle strength during rehabilitation. The disadvantage of these types of knee braces ranges from high cost for customization to a loss in physical capability. One approach to solving this problem is to use a Magnetorheological MR fluid (MRF)-based device to make the knee brace have a controllable resistance.

Figure 1.4: Knee braces (a) Prophylactic, (b) Functional, (c) Rehabilitative, and (d) Patellofemoral knee braces (Photos courtesy of Restorative Care of America, Pass Bracing and Knee Shop)
Dong et al designed and tested a rehabilitative MRF device. Their patient-specific device operates according to the patient’s natural strength. The device can be adjusted to vary its resistive forces according to pre-determined strength-to-position measurements made by a physical therapist. Another device under research is the variable resistance exercise device. The device is programmed such that it simulates a resistance equal to a percentage of static torques measured at certain positions of knee movement. Its modes of operation are test mode and exercise mode. In addition to the two-mode device, research is being done on a three-mode, versatile rehabilitation device. These modes are isometric, variable controlled resistance and variable controlled velocity mode.

Our design solution is to incorporate an MR fluid into a three-plate joint and replace the manufactured knee brace joint with our “active” joint. The MR fluid is soaked into sponges to prevent leakage from the plates. The device is designed based on a maximum yield stress, a corresponding magnetic field, a torque and the MR fluid viscosity. With proper knowledge of these parameters the dimensions of the device components can be determined.

Braces are functionally versatile because they are applicable to all of the moveable joints of the human body. However, braces do not prevent injuries nor do they heal the injured. They only provide stability for people who are already injured. While this may sound beneficial it has its downsides. For instance, patients in rehabilitation would be protecting the injured area with a brace by stabilization but there are no means of developing and improving the skeletal and muscular areas that the injured area cooperates
with. Over time this could lead to an asymmetrical body strength and physique if there is no prescribed exercise regime to incorporate into the rehabilitation. Although these things can be employed they can be costly and time consuming. Here is where an “MR Fluid” based active brace would be advantageous. An active knee brace, for example, allows the injured individual to control the resistive torque against knee flexion and extension to develop the quadriceps and hamstrings. Hence, an active knee brace would be a means of muscular development as well as stabilization. The National Institutes of Health Osteoporosis and Related Bone Disease say that exercise “has been shown to help maintain or even modestly increase bone density in adulthood and; can assist in minimizing age related bone loss in older adults.” Thus, contrary to a passive knee brace, an active knee brace could also improve skeletal conditions of the affected area.

1.2.3 Drive-By-Wire

Another novel application of this research is the area of automobile design. Recently the idea of replacing the hydraulic and mechanical systems of automobiles by proven aerospace technology has been under consideration. Systems that operate automobiles by means of computer-controlled electronic signals instead of direct action on control devices are called drive-by-wire systems. The term sounds like fly-by-wire, which is a method of controlling commercial aircraft that has been in use for more than a decade. For more than a century, drivers controlled vehicles under direct actuation by mechanical, hydraulic or pneumatic systems. If such a system fails the consequences could affect the safety of the vehicle or passengers. There is considerable interest in increasing
functionality and safety by developing drive-by-wire systems where electronic controls are used to supplement the driver controls. In a drive-by-wire system, the driver controls are simply inputs to a computerized system rather than directly commanding the vehicle functions. The idea is to remove the mechanical linkages between the controls of a car and replace it with electronic devices. Instead of operating the steering, accelerator, and brake directly, the electronic systems will send commands to a central computer, which will control the vehicle (Figure 1.5).

Figure 1.5: The Hy-wire's X-drive, Photo courtesy General Motors.

Steer-by-wire offers improvements in safety with the removal of the steering column, therefore improves the crash performance of the vehicle. Other benefits include corrective steering to improve stability control, tunable steering feel, reduction in vehicle weight, fine-tune vehicle handling by just new computer software, better fuel
consumption, to react to emergencies faster than a human driver, and therefore decrease the number of road accidents. Automakers are planning to move beyond the conventional car sometime soon, toward a computerized, environmentally friendly alternative.

As operational information is conveyed by means of electronic signals, there is no mechanism for tactile feedback to the driver since there is no direct link to the steering wheel. This is the most important problem with computer controlled steering systems. Such deficiency could be eliminated by the capabilities of haptic technology. By employing force feedback technology, the drive-by-wire system conveys required information to the driver in tactile form. Feedback would certainly be required in order to give the driver meaningful information about what is happening at the road wheels. For instance, feedback on irregularities in the surface of an unpaved road is transmitted via movement of the steering wheel. Furthermore, the force feedback technology allows the communication of cautionary information to the driver through vibration or cessation of operation, such as a warning when the driver is deviating from lane, is following too closely, or has become drowsy.

1.3 Background

Smart Materials are materials that have one or more properties that can be significantly altered in a controlled fashion by external stimuli; such has electrical fields, magnetic fields, stress, moisture etc. Smart Materials convert one form of energy to another, so it can be said that they are a kind of transducers. Some of the most common and popular
types of smart materials are: Piezoelectric, Shape Memory Alloys (SMA), Electrostrictive, Magnetostrictive, Electrorheological (ER) fluids, and Magnetorheological (MR) fluids.

1.3.1 Magnetorheological fluids

Magnetorheological (MR) fluids are a special class of rheological fluid whose yield stress varies with an applied magnetic field (Figure 1.6 and Figure 1.7). The fluids consist of micron-sized ferrous particles suspended in a Newtonian fluid.

MR devices have been under development since Rabinow employed them in the late 1940s. Researchers have utilized the controllable variation in yield stress to develop various ‘smart’ devices like vibration-dampers, transmission-clutches and brakes. Research work performed on MR fluids had been minimal until recently mainly due to
the lack of commercially available MR fluids. Once commercialized MR fluids were available, research on MR fluid devices has increased dramatically.

![Graph of shear stress as a function of magnetic field](image)

Figure 1.7: Shear stress as a function of magnetic field

### 1.3.2 Telerobotic/Haptic systems

The use of telerobotic systems was started in the 1950’s with applications to radioactive materials handling and space teleoperation. The problem of transparency has been around since the conception of these devices and a number of researchers have contributed to the development of techniques to achieve varying degrees of transparency. While novel to minimally invasive telerobotic surgery, other researchers have studied the use of smart materials in haptic systems. Recently, researchers have explored providing force feedback using Electrorheological (ER) fluid devices. Mavroidis et al. have
developed a ‘smart glove’ system named MEMICA (Mechanical Mirroring using Controlled stiffness and Actuators) using ER devices. The primary application area involves providing users with tactile feeling in a virtual reality environment. Sakaguchi et al. developed simple two-dimensional force display systems that demonstrate the use of ER devices in gaming applications. Two outcomes of the Sakaguchi work relate directly to this research. One is the fact that they had difficulty replicating anything but hard obstructions. The second lies in the discovery of the “sticky wall” phenomenon. This phenomenon occurs when one encounters a rigid obstruction and then wants to move away from it. Since the damper is in the “on” position, it doesn’t move and the user feels the system being attracted or ‘stuck’ to the obstruction.

MR fluids however have certain advantages over their ER counterparts including an order of magnitude higher yield stress, broader operating temperature range, and lower sensitivity to impurities. These significant advantages make MR fluids more attractive in force feedback applications. Another major factor in favor of MR fluids is that ER fluids require very high voltages (on the order of a few kilovolts). This makes it hazardous to use ER fluids in force feedback systems where the user is in close contact with the device in operation. There has been two MR fluid based research studies in the literature somewhat related to this research. Scilingo et al. developed simple haptic devices using MR fluids to mimic the compressional compliance of different biological tissues and Jolly et al. developed MR based haptic devices, neither addressed the issue of transparency.
1.3.3 MR sponge based force feedback systems

Carlson et al. have recently developed MR sponge based dampers to solve vibration-related problems in high-performance washing machines. MR sponge devices reduce the complexity of conventional shear mode MR devices in the sense that the actual fluid is replaced with a sponge saturated in the fluid, thereby retaining the shear mode operation but also reducing the problem of sealing. As a part of a preliminary study the ISSL researchers have adapted the MR sponge concept for use in providing force feedback in robotic systems. The study consisted of the development of simple 1-D linear and rotary magnetorheological fluid based force feedback devices. The goal of the study was to demonstrate the feasibility of these MR sponge devices in force feedback applications. In the linear study, the user was able to command the movement of a remote linear actuator using a hand-held MR sponge device (with force feedback). Placing different stiffness elements like coil springs and hard stops in the remote actuator’s path varied the force encountered by the remote linear actuator during its motion. During each encounter a load cell recorded the forces and a controller was used to command an appropriate current to the MR device, thereby making the user ‘virtually’ feel that he/she was pushing against a soft or a hard spring.

1.3.4 Preliminary Studies

One important application of MR fluid-based force feedback devices in telerobotic surgery systems has been explored specifically with the help of collaborators of the OSU’s Medical Center. The results of the preliminary studies at the ISSL were the
following:  (a) Cost effective MR fluid based force feedback (MRFF) systems could be
developed.  (b) Surgeon accuracy can be increased by a factor of 3 over systems without
force feedback.  (c) While there is initial success a number of important problems persist,
namely the “sticky wall” phenomenon, reduced transparency, increased responsiveness at
very low levels of touch and better overall responsiveness when moving abruptly from
highly compliant regions (muscle) to highly stiff regions (bone).  Since the devices used
in the preliminary studies were purely passive, there was no way of replicating the
restoring force that the external actuator encounters due to the stiffness of whatever the
actuator is in contact with.  For example when the external actuator pushes on a spring,
the spring pushes back.  The MRFF preliminary system could not provide the user with
that restoring force.

1.4 Research Objectives

The overall vision is to develop next generation force feedback systems using MR fluid
devices.  These new systems can be used in a variety of applications including
biomedical, light industrial, military training applications, aerospace, entertainment, and
gaming.  Three important applications in tele robotic surgery, orthopedic knee braces, and
steer-by-wire vehicles are explored.  In order to realize the conceptual vision of MR fluid-
based force feedback devices and to address the problems encountered in the preliminary
study, three objectives must be met.  1) The development of first principle,
microstructurally-based models of MR Fluids, 2) The development of novel MR device
designs.  3) The development of measurement and control techniques that adequately
couple the system dynamics to the human interface. In order to meet the stated objectives the research is split into four main phases, namely microstructural analysis, MR fluid characterization, MR device design and mechatronics, and measurement and control techniques.

1.5 Thesis Organization

The dissertation is structured as follows: In chapter 2 kinetic-based microstructural model for MR is derived. Chapter 3 details the MR fluids being assessed and their preparation. The application of MR fluid-based force feedback systems in a 2-DOF and a 5-DOF systems are investigated in chapters 4 and 5. Chapter 6 discusses the stability and transparency issues associated with haptic and force feedback systems. The application of MR fluid-based dampers in orthopedic knee braces is presented in chapter 7. The application of MR fluid-based systems in steer-by-wire automobiles is investigated in chapter 8. Chapter 9 introduces a developed software package for modeling, analysis, and design of MR fluids. Chapter 10 is discussion and conclusions.
CHAPTER 2

MICROSTRUCTURAL ANALYSIS

2.1 Phenomenological 1-D models

Magnetorheological (MR) fluids exhibit a controllable yield stress-like behavior in shear, whereby the application of a magnetic field transverse to the flow creates a resistance to flow which increases with an increasing magnetic field. One degree of freedom piston-like devices exploiting this controllable variation of resistance in simple shear have been under development since Rabinow in the late 1940s. Practical devices currently incorporating MR fluids include vibration-dampers, transmission-clutches and brakes. For these one-degree of freedom, rectilinear devices, 1-D macroscale phenomenological MR fluid models like the Bingham and Herschel-Buckley models suffice. The Bingham model relates the total shear stress $\tau$ in simple shear to the shear rate $\dot{\gamma}$ and magnitude $H$ of a transverse applied magnetic field according to the equation

$$\tau = \left[ \tau_y(H) + \eta |\dot{\gamma}| \right] \text{sgn}(\dot{\gamma}),$$

(2.1)

where $\tau_y(H)$ is a yield stress, assumed to be a function of the magnitude of the transverse magnetic field, and the constant $\eta$ is the effective bulk viscosity of the
composite system. The Herschel-Buckley model generalizes the Bingham model to accommodate the shearing thinning observed in MR fluids,

\[ \tau = \left[ \tau_y(H) + K \left| \dot{\gamma} \right|^\frac{1}{2} \right] \text{sgn}(\dot{\gamma}), \]  

(2.2)

where \( m, K \) are constants. We note that both models describe fluids which exhibit a strict yield stress \( \tau_y(H) \) in shear where there is no flow (i.e. \( \dot{\gamma} \) identically zero) until \( \tau \) exceeds \( \tau_y(H) \). In the Herschel Buckley model, the constants \( m, K \) and the function \( \tau_y(H) \) are empirically determined from experiments.

For multi-degree of freedom applications, such as the one shown in Figure 2.1, the Bingham, Herschel-Buckley, and similar models are lacking in three aspects: (i) The models are macroscale (treating the fluid as single continuum rather than a composite system) and phenomenological (fitting the coefficients \( m, K \) and function \( \tau_y(H) \) in an assumed form to experimental measurements of bulk properties, rather than deducing the model from fundamental physics). The magneto-mechanical coupling that is the cause of the macroscale properties of MR fluids takes place at the particle level and is governed by fundamental first principles (conservation of momentum, Maxwell equations, etc.) at that level. The empirical macroscale Bingham/Herschel-Buckley type models have no particle level, are incapable differentiating between particles and carrier fluid, and have no explicit connection to fundamental first principles. The use of empirical, macroscale modeling for the design and control of MR fluid-based devices limits fidelity to a narrow range of applicability in the vicinity of the conditions used in the fit of the coefficients \( m, \)
$K$ and function $\tau_y(H)$ to experimental measurements. (ii) They only model 1-D simple shear flow with a transverse applied magnetic field. The MR fluids in multi-degree of freedom devices are subjected to flow and magnetic fields in all directions, so that the models employed must be fully 3-D. (iii) Because the Bingham plastic/Herschel-Buckley-type models, with their notion of a strict yield stress, idealize what actually are small nonzero flows at low values of shear stress to be identically zero, they are inaccurate at low values of stress. These low values of stress are important in haptic and force feedback systems because they correspond to low levels of touch.

Figure 2.1: 2-D force feedback setup using an MR fluid; the MR fluid is in the gap between the ball and socket
Instead of Bingham plastic/Herschel-Buckley-type models, what is needed to design and control advanced multi-degree of freedom MR fluid-based devices are models of MR fluids which (i) are microstructure-based and derived from first principles, resulting in fidelity beyond the vicinity of current practice; (ii) are applicable to general 3-D motions, and (iii) remove the idealization of a strict yield stress so as to recognize that there is flow (albeit small) below large-scale yielding.

2.2 Kinetic theory-based 3-D model

In our kinetic theory-based models, we consider the iron particles as dumbbells: two beads each of mass $m/2$ with position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$, respectively, joined by a connector $\mathbf{q} = \mathbf{r}_2 - \mathbf{r}_1$. The mass center of the bead-spring pair is $\mathbf{x} = (\mathbf{r}_1 + \mathbf{r}_2)/2$. This connector $\mathbf{q}$ between two beads represents the orientation and length of the iron particle in the carrier fluid.

The kinetic (or Smoluchowski) equation that describes the rate of change of the orientation vector $\mathbf{q}$ with time is

$$
\dot{\mathbf{q}}_i = \left( L_{\mathbf{y}} - \mu D_{\mathbf{y}} \right) q_j + a_y \frac{2kT}{\psi} \frac{\partial}{\partial q_k} \left( \frac{1}{\xi_{ij}^{\text{int}}} \psi \right) + 2a_y f_{ij}^{\text{(int)}} + a_y \left( f_{ij}^{\text{(ext)}} - f_{kj}^{\text{(ext)}} \right) \tag{2.3}
$$

The left hand side of equation (2.3) is the Lagrangian time derivative of particle orientation, and the terms on the right hand side arise from four physical effects that contribute to this change of orientation.
The first term in equation (2.3) models the effect of carrier fluid to a change in particle orientation. $L_{ij} = \frac{\partial}{\partial x_j} v_i$ is the velocity gradient tensor, with symmetric part $D_{ij} = \frac{1}{2} (L_{ij} + L_{ji})$, where $v_i(x,t)$ is the velocity of the carrier fluid at location $x$ and time $t$. The scalar parameter $\mu$ measures the magnitude of the non-affine motion of the dumbbell. The second term $a_{ij} \frac{2kT}{\psi} \frac{\partial}{\partial q_k} (\xi_{ij}^{-1} \psi)$ is the effect of the Brownian motion caused by thermal fluctuation; $k$ is the Boltzmann constant, $T$ is absolute temperature, $\psi(q,x,t)$ is the probability distribution function of the particle orientation, $\xi_{ij}$ is the anisotropic tensor responsible for the anisotropic Brownian motion, and $a_{ij}$ is given by

$$a_{ij} = -\left(\delta_{im} - \xi_{mn} \Omega_{nm}\right) \xi_{nj}^{-1}$$  \hspace{1cm} (2.4)
with \( \zeta_{ij} \) defined as the friction tensor. The Oseen-Burgers tensor \( \Omega_{ij} \) accounting for the hydrodynamic interaction is

\[
\Omega_{ij} = \frac{1}{8\nu \eta_s} \left[ \delta_{ij} + \frac{1}{|\mathbf{q}|^2} q_i q_j \right]
\]  

(2.5)

where \( \eta_s \) is the solvent viscosity. The probability distribution function (PDF) \( \psi(q, x, t) \) is the probability of the particle at place \( x \) and time \( t \) having a specific orientation \( \mathbf{q} \). The PDF is governed by the evolution equation

\[
\frac{\partial}{\partial t} \psi = -\frac{\partial (\psi \dot{\mathbf{q}})}{\partial \mathbf{q}} = -\frac{\partial (\dot{q}_i \psi)}{\partial q_i}
\]  

(2.6)

The third term in equation (2.3), \( 2 a_{ij} f_{j}^{(in)} \), is the effect of intraparticle forces such as elasticity on the particle orientation, with \( f_{j}^{(in)} \) the force in the connector. The fourth term \( a_{ij} (f_{1,j}^{(ex)} - f_{2,j}^{(ex)}) \) is the effect of external forces, in this case the magnetic field, so that \( f_{1,j}^{(ex)} \) is the magnetic force on bead 1 and \( f_{2,j}^{(ex)} \) is the magnetic force on bead 2.

The second order orientation tensor \( \langle \mathbf{q} \otimes \mathbf{q} \rangle \), with components \( \langle q_i q_j \rangle \), defined as the dyadic product of \( \mathbf{q} \) averaged over orientation space

\[
\langle q_i q_j \rangle = \int q_i q_j \psi(q, x, t) dq_1 dq_2 dq_3
\]  

(2.7)

provides a concise interpretation of the mesoscale orientation state, namely the orientation of the particles averaged over a suitably large region, and is the measure of orientation which influences the stress in the material. The governing equation for
\( \langle q \otimes q \rangle \) is obtained by multiplying evolution equation (2.6) of the distribution function by \( q_i q_j \) and integrating over Euclidean 3-space:

\[
\int \frac{\partial \psi(q, x, t)}{\partial t} q_i q_j dq_i dq_j = -\int \frac{\partial}{\partial q_k} [(\dot{q}_k \psi) q_i q_j dq_i dq_j + \int (\dot{q}_k \psi) (\frac{\partial}{\partial q_k}) (q_i q_j) dq_i dq_j dq_k,
\]

(2.8)

\[
= 0 + \int \psi (\dot{q}_i q_j + q_i \dot{q}_j) dq_i dq_j dq_k
\]

\[
= \langle \dot{q}_i q_j + q_i \dot{q}_j \rangle = \frac{d}{dt} \langle q_i q_j \rangle,
\]

using the divergence theorem and the condition \( \psi \to 0 \) as \( |q| \to \infty \). The evolution equation of the orientation tensor \( \langle q \otimes q \rangle \) then follows from equation (2.3):

\[
\langle \dot{q}_i q_j + q_i \dot{q}_j \rangle = \frac{d}{dt} \langle q_i q_j \rangle = \langle L_{im} - \mu D_{im} \rangle \langle q_m q_j \rangle + \langle q_i q_m \rangle \langle L_{jm} - \mu D_{mj} \rangle + 2kT \left( \frac{\partial}{\partial q_k} \left[ \left( (\delta_{im} - \zeta_{im} \Omega_{nm}) \zeta_{ml}^{-1} \right) q_j + q_i \left[ \left( (\delta_{jm} - \zeta_{jm} \Omega_{nm}) \zeta_{ml}^{-1} \right) \zeta_{kl}^{-1} \right] \right] - \left[ \left( (\delta_{ik} - \zeta_{im} \Omega_{nk}) \zeta_{kl}^{-1} \right) (2f_{1}^{\text{(mech)}} + f_{1}^{\text{(mag)}} - f_{2}^{\text{(mag)}}) \right] q_j, \right)
\]

(2.9)

or

\[
\frac{D}{Dt} \langle q_i q_j \rangle = 2kT \left( \frac{\partial}{\partial q_k} \left[ \left( (\delta_{im} - \zeta_{im} \Omega_{nm}) \zeta_{ml}^{-1} \right) q_j + q_i \left[ \left( (\delta_{jm} - \zeta_{jm} \Omega_{nm}) \zeta_{ml}^{-1} \right) \zeta_{kl}^{-1} \right] \right] \right)
\]

\[
- \left[ q_i \left[ (\delta_{ij} - \zeta_{jm} \Omega_{nk}) \zeta_{kl}^{-1} \right] \left( 2f_{1}^{\text{mech}} + f_{1}^{\text{mag}} - f_{2}^{\text{mag}} \right) \right] q_j,
\]

(2.10)

where the Gordon-Schowalter derivative is defined by

\[
\frac{D}{Dt} \langle q_i q_j \rangle = \frac{d}{dt} \langle q_i q_j \rangle - W_{ik} \langle q_k q_j \rangle + \langle q_i q_k \rangle W_{kj} - a \left[ D_{ik} \langle q_i q_j \rangle + \langle q_i q_k \rangle D_{kj} \right]
\]

(2.11)
with \( \frac{d}{dt} \langle q_i q_j \rangle(x,t) = \frac{\partial}{\partial t} \langle q_i q_j \rangle(x,t) + \frac{\partial}{\partial x_k} \langle q_i q_j \rangle \hat{x}_k \), \( W_j \) the skew part of the velocity gradient \( \frac{\partial V}{\partial x_j} \), and \( a = 1 - \mu \); \( a = 1, 0, -1 \) correspond to the upper convected, corotational, and lower convected derivative, respectively.

We investigate kinetic theory-based models with the constitutive assumption that anisotropic effects in the Brownian motion and hydrodynamic interaction are negligible (i.e. \( \xi_{ij} = \delta_{ij} \) and \( \Omega_{ij} = 0 \)), and the friction is isotropic (i.e. \( \zeta_{ij} = \zeta \delta_{ij} \)). With these specializations the evolution equation of the orientation tensor simplifies to:

\[
\frac{D}{Dt} \langle q_i q_j \rangle = \frac{4kT}{\zeta} \delta_{ij} - \frac{2}{\zeta} \left\{ q_i \left( f_{i}^{\text{int}} - f_{i}^{\text{ext}} \right) + \left( f_{1}^{\text{ext}} - f_{2}^{\text{ext}} \right) q_j \right\} (2.12)
\]

We assume the total Cauchy stress tensor \( \tau_{ij} \) of the composite system is given by the sum of the constraint pressure \( p \) maintaining incompressibility, the viscous stress \( 2\eta_s D_{ij} \) due to the solvent, a mechanical stress \( \tau_{ij}^{\text{mech}} \), and the stress \( \tau_{ij}^{\text{mag}} \) due to the magnetic field:

\[
\tau_{ij} = -p \delta_{ij} + 2\eta_s D_{ij} + \tau_{ij}^{\text{mech}} + \tau_{ij}^{\text{mag}} \quad (2.13)
\]

The mechanical stress \( \tau_{ij}^{\text{mech}} \) consists of contributions from the intraparticle mechanical force \( f_i^{\text{int}} \) and the bead motion:

\[
\tau_{ij}^{\text{mech}} = \frac{n}{2} \left( q_i f_j^{\text{int}} + f_i^{\text{int}} q_j \right) - nm \sum_{\nu=1}^{2} \left\langle \left( \hat{r}_{\nu} - \nu \right) \left( \hat{r}_{\nu} - \nu \right) \right\rangle + nkT \delta_{ij} \quad (2.14)
\]
where \( n \) is the particle number density, \( m \) the mass of each particle, \( \vec{r}_v \) the velocity of the bead \((v=1,2)\), and \( \vec{v} \) the velocity of the solvent particle at \( \vec{x} \). With a Maxwellian velocity distribution, the mechanical stress \( \tau_{ij}^{\text{mech}} \) simplifies to:

\[
\tau_{ij}^{\text{mech}} = \frac{n}{2} \left( q_i f_j^{(\text{int})} + f_i^{(\text{int})} q_j \right) - nkT \delta_{ij}
\]

(2.15)

The magnetic stress \( \tau_{ij}^{\text{mag}} \) is related to magnetic forces in eq. (3) through

\[
\tau_{ij}^{\text{mag}} = \frac{1}{2} n \left( q_i \left( f_i^{(\text{ext})} - f_j^{(\text{ext})} \right) \right)
\]

(2.16)

All the equations thus far are derived from first principles. To complete a kinetic theory-based MR fluid model there remains to specify the dependence of the intraparticle force \( f^{(\text{int})} \) and magnetic force \( f_1^{(\text{ext})} - f_2^{(\text{ext})} \) appearing in equations (2.12), (2.15), and (2.16). In this model, the expressions for the intraparticle force and magnetic force are phenomenological.

To investigate the accuracy of the 3-D models, the 1-D form of each model is computed and compared with experimental results obtained from tensile testing machine test. This comparison consists of shear stress versus shear rate (Figure 2.4) and yield stress versus shear magnetic field (Figure 2.5) graphs.

We propose a MR fluid model in which the ferrous particles are deformable and linearly elastic, so that the dumbbell idealization of each particle is an initially unstretched linear spring with spring constant \( \beta \) connecting the two beads, with internal force
\[ f_i^{(\text{int})} = \beta q_i \cdot \]  

(2.17)

and, using equation (2.15), the mechanical stress is given by

\[ \tau_{ij}^{\text{mech}} = n\beta \langle q_i q_j \rangle - nkT\delta_{ij} \]  

(2.18)

In addition we assume

\[ f_{ij}^{(\text{ext})} - f_{2j}^{(\text{ext})} = \frac{c\dot{\gamma}}{(1 + \chi)(H_k H_k)^{0.5}} q_j \]  

(2.19)

where \( \chi \) is the magnetic susceptibility of the particle. With this choice, magnetic stress is

\[ \tau_{ij}^{\text{mag}} = \frac{c\dot{\gamma} n \langle q_i q_j \rangle}{2(1 + \chi)(H_k H_k)^{0.5}} \]  

(2.20)

Substituting the constitutive equations (2.17)-(2.20) into equations (2.12) and (2.13) gives

\[ \frac{D}{Dt} \langle q_i q_j \rangle = -\frac{4kT}{\xi} \delta_{ij} - \frac{4\beta}{\xi} \langle q_i q_j \rangle - \frac{c\dot{\gamma}}{\zeta (1 + \chi)(H_k H_k)^{0.5}} \left[ \langle q_i q_j \rangle + \langle q_i q_j \rangle \right] \]  

(2.21)

\[ \tau_{ij} = -p\delta_{ij} + 2\eta_s D_{ij} + n\beta \langle q_i q_j \rangle + \frac{c\dot{\gamma} n}{2(1 + \chi)(H_k H_k)^{0.5}} \langle q_i q_j \rangle - nkT\delta_{ij} \]  

(2.22)

This kinetic theory-based model for MR fluids is now complete. It consists of the evolution equation (2.21) for the orientation tensor \( \langle q_i q_j \rangle \), constitutive equation (2.22) for Cauchy stress tensor \( \tau_{ij} \) and the balance of momentum.

We specialize the 3-D theory to the 1-D application of steady simple shear in the presence of a transverse magnetic field (Figure 2.3) by inserting the special velocity and magnetic fields.
In the steady state, \( \frac{\partial}{\partial t} (q_i q_j) = 0 \), \( \frac{\partial}{\partial x_i} (q_i q_j) = 0 \), \( \frac{\partial}{\partial x_3} (q_i q_j) = 0 \) and for simplicity we assume the viscous carrier fluid is Newtonian (so that \( \eta_s \) is a constant). From the evolution equation (2.10) we solve for \( \langle q_i q_j \rangle \), \((i=1 \text{ and } j=2)\), substitute the solution into the equation for the total stress tensor (2.22) and compute the 1-D shear stress.

\[
\tau_{12} = \eta_s \dot{\gamma} + \frac{nkT \zeta \dot{\gamma}}{4 \left( \beta + \frac{c \dot{\gamma}}{2(1 + \chi)H} \right)}
\]  

(2.24)
This equation is in proper agreement with the experimental results described in chapter 3 and shown in Figure 2.4 and Figure 2.5.

Figure 2.4: Shear stress as a function of shear rate and magnetic field strength: comparison of measurements to model predictions, for sinusoidal displacement

The shear stress $\tau_{12}$ is plotted as a function of shear rate $\dot{\gamma}$ and magnetic field strength $H$ according to equation (2.24). The parameters $\eta_s = 10$, $n = 4.0 \times 10^{21}$, $k = 1.3807 \times 10^{-23}$, $T = 313$, and $\chi = 4.5$ are based on the physical properties of MR fluid components,
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$c = 1.1 \times 10^7$ and $\beta = 0.06$ are empirical constants fit to data, and the parameter $\zeta$ is assumed to be a function of magnetic field $H$, determined by fit to experimental data. The details are described in chapter 3.

![Yield stress vs. magnetic field](image)

Figure 2.5: Yield stress vs. magnetic field

### 2.3 Kinetic theory-based model 2: Ciocanel et al

In the kinetic theory-based MR fluid model of Ciocanel et al., the particle pair model is developed considering two magnetizable particles connected by a vector $q$. The time
evolution equation for the moments of the distribution function and the total stress tensor are given by

\[
\frac{D}{Dt} \langle q_i q_j \rangle = \frac{4}{\zeta} \frac{\Delta}{(2r)^5} \left( H_i H_k \langle q_k q_j \rangle + \langle q_i q_k \rangle H_k H_j \right) + \frac{\partial}{\partial x_k} \langle q_i q_j \rangle x_k
\]

\[
- \frac{\lambda D_{kj}}{2\alpha^2} \langle q_i q_k \rangle \langle q_i q_j \rangle + \frac{4}{\zeta} \frac{\Delta H^2}{(2r)^5} u_3(t) \langle q_i q_j \rangle \\
- \frac{4}{\zeta} \frac{\Delta}{(2r)^7} (3u_3(t) + 2) H_k H_3 \langle q_i q_k \rangle \langle q_i q_j \rangle,
\]  

(2.25)

where

\[
\tau_{ij} = 2\eta D_{ij} + \phi \mu_i \langle q_i q_j \rangle + \phi \frac{2\alpha}{(2r)^2} H_i H_k \langle q_k q_j \rangle \\
+ \phi \left[ \mu_2 D_{kj} \langle q_k q_j \rangle \langle q_i q_j \rangle + 2\mu_3 \left( D_{ij} \langle q_k q_j \rangle + \langle q_i q_k \rangle D_{ij} \right) \right] \\
+ \phi \left[ \alpha u_s(t) H^2 \langle q_i q_j \rangle - \frac{\alpha (3u_3(t) - 2)}{(2r)^2} H_k H_3 \langle q_i q_k \rangle \langle q_i q_j \rangle \right],
\]  

(2.26)

\[
\Delta = \frac{4\mu_0 \pi r^3 \chi_{\text{eff}}^2}{3}, \quad \alpha = \mu_0 \chi_{\text{eff}}^2 r^3 / 2, \quad u_s(t) \text{ is the Heaviside function to prevent particle overlap, } \phi \text{ is the particle volume fraction, } \mu_\text{i} \ (i = 1, 2, 3) \text{ are material constants, and } r \text{ is particle radius. Comparing these constitutive equations with corresponding equations (2.21) and (2.22) of kinetic theory-based model 1, we observe the following.}

Their equation neglects the Brownian motion and elasticity effects (the first two terms on the right hand side of equation (2.21)), and the convected term \( \frac{\partial}{\partial x_k} \langle q_i q_j \rangle x_k \). It also neglects the incompressibility constraint \(-\rho \delta_{ij}\) present in equation (2.21). The Ciocanel model includes the coupled anisotropic hydrodynamic drag/magnetization terms (last three terms of equation (2.25)) that we neglect when we set the Oseen-Burgers tensor \( \Omega_{ij} \).
equal to zero. Their constitutive assumption (2.26) on stress includes the same Newtonian viscosity, elasticity, and leading order magnetic/mechanical coupling (the first, second, and third terms on the right hand side) as our assumption (2.22).

In this model, the hydrodynamic force and interaction potential between two particles, and consequently the magnetic force are postulated and the others are based on first principles. The 1-D shear stress is given by

$$
\tau_{12} = \eta \dot{\gamma} + \\
\phi \left[ \beta_1 \langle q_1 q_2 \rangle - \beta_2 \langle q_2 q_2 \rangle + 2 \mu_2 \ddot{\gamma} \langle q_1 q_2 \rangle \rangle + \mu_3 \ddot{\gamma} \langle q_1 q_1 \rangle + \langle q_2 q_2 \rangle \right]
$$

(2.27)

where \( \beta_1 = 3 \mu_0 \chi^2 H^2 / 16 \), \( \beta_2 = 5 \mu_0 \chi^2 H^2 / 16 \), \( \mu_1 = 0 \), \( \mu_2 = 0.3 \), \( \mu_3 = 1 \), and \( \phi = 0.32 \).

Equation (2.25) was solved by the authors (Ahmadkhanlou et. al) to yield

$$
\langle q_2 q_2 \rangle = \frac{4 \gamma^2 (1 + \lambda) (u_2 (t) + 1)}{2 \lambda - (3 u_2 (t) + 2) (\lambda - 1)}
$$

(2.28)

$$
\langle q_1 q_1 \rangle = \frac{4 \gamma^2 (1 - \lambda) (u_1 (t) + 1)}{2 \lambda - (3 u_1 (t) + 2) (\lambda - 1)}
$$

(2.29)

$$
\langle q_1 q_2 \rangle = \frac{\Delta H^2 (u_2 (t) + 1)}{\dot{\gamma} z 2 \gamma^3 \left[ 2 \lambda - (3 u_2 (t) + 2) (\lambda - 1) \right]}
$$

(2.30)

This model's prediction of shear stress is compared with experimental results in Figure 2.4 and Figure 2.5.
2.4 Other 3-D models for MR fluids

2.4.1 Model 3: Chen and Yeh’s model

Chen and Yeh model an MR fluid as a co-existing fluid and solid continua and use a framework of irreversible thermodynamics to formulate the general dynamic equations.

The stress tensor $\tau^F_{ij}$ in the fluid phase is assumed to be symmetric, with deviatoric part, $\tau^{FS}_{ij}$, spherical part, $\tau^{FB}_{ij}$, and incompressibility response, $-p^F \delta_{ij}$,

$$\tau^F_{ij} = \tau^F_{ji} = -p^F \delta_{ij} + \tau^{FB}_{ij} + \tau^{FS}_{ij} + \tau^{FS}_{ii} = 0.$$  \hspace{1cm} (2.31)

The stress tensor $\tau^S_{ij}$ in the fluid phase, not necessarily symmetric due to the application of an external magnetic field, is hypothesized to be of the form

$$\tau^S_{ij} = \tau^{SR}_{ij} + \tau^{SD}_{ij} = \tau^{SR}_{ij} + \tau^{SR}_{ji} + \tau^{SD}_{ij}, \tau^{SD}_{ij} = -\tau^{SD}_{ji} \hspace{1cm} (2.32)$$

where the superscripts “FB”, “FS”, “SR” and “SD” stand for the bulk of fluid, the shear of fluid, the reversible part and the dissipative part, respectively. The authors define a generalized entropy density $\eta$ and generalized statement of the second law, and postulate that the free energy $F$ depends on the conserved quantities $\theta, \rho^F, e^{SE}_{ij}, B^S_i$ and fluxes $q_i, M^{SD}_i, v^R_i, \tau^{FS}_{ij}, \tau^{FB}$

$$F = F(\theta, \rho^F, e^{SE}_{ij}, B^S_i, q_i, M^{SD}_i, v^R_i, \tau^{FS}_{ij}, \tau^{FB})$$ \hspace{1cm} (2.33)

where $\theta, \rho^F, e^{SE}_{ij}, B^S_i, M^{SD}_i$, and $v^R_i$ are the temperature, density of the fluid, the Eulerian elastic strain tensor, the magnetic field, dissipative part of magnetization, and relative velocity vector of the fluid to the solid respectively. With this list of independent
internal variables, the second law expresses the conjugate dependent internal variables as partial derivatives of the potential $F(\theta, \rho^F, e^{SE}_{ij}, B^i, M^S, \nu^F)$ i.e.

$$\eta = -\frac{\partial F}{\partial \theta}, \quad p^F = \rho \rho^F \frac{\partial F}{\partial \rho^F}, \quad \tau_{ij}^{SR} = \rho \left( \frac{\partial F}{\partial e^{SE}_{ij}} - 2 \frac{\partial F}{\partial e^{SE}_{ik}} e^{SE}_{kj} \right), \quad M^S_i = -\rho \frac{\partial F}{\partial B^S_i} \quad (2.34)$$

where $\rho$ is density of the whole mixture. Chen and Yeh select a polynomial expression for the reversible part $F^R$ of the free energy,

$$\rho F^R = F^R_o + \frac{1}{2} \Lambda^{(1)} B^i_1 B^j_1 + \Lambda^{(2)} B^i_1 e^{SE}_{ij} B^j_1 + \Lambda^{(3)} B^i_1 e^{SE}_{ij} e^{SE}_{ij} B^j_1$$

$$+ \frac{1}{2} \Lambda^{(4)} u^F_{i,i}, u^F_{j,j} + \Lambda^{(5)} u^F_{k,k} B^i_1 e^{SE}_{ij} B^j_1 \quad (2.35)$$

where $F^R_o$ is a reference energy, $u$ is displacement and the parameters $\Lambda^{(i)}$, $\Lambda^{(2)}$, $\Lambda^{(3)}$, $\Lambda^{(4)}$, and $\Lambda^{(5)}$ are functions of temperature. The 3D constitutive equation is given by

$$\tau_{ij}^{SR} = \Lambda^{(2)} B^S_1 B^S_1 + \left( \Lambda^{(3)} - 2 \Lambda^{(2)} \right) B^S_1 B^S_1 u^S_{(j,k)} + \Lambda^{(3)} B^S_1 B^S_1 u^S_{(i,k)} + O(2) \quad (2.36)$$

In this model, as well as the model to follow, the constitutive modeling lies in the choice of which internal variables are treated as independent variables (and which are not), and the form of the second law of thermodynamics. For Yeh and Chen’s assumption, $M = M_1 e_1 + M_2 e_2$, for simple shear flow (equation (2.23)) the 1-D shear stress $\tau_{21}^{SR}$ can be found by the following equation

$$\tau_{21}^{SR} = \Lambda^{(2)} B^S_2 B^S_1 + \left( \Lambda^{(3)} - 2 \Lambda^{(2)} \right) \left( B^S_2 \right)^2 \dot{\gamma} \quad (2.37)$$

This model's prediction of shear stress is compared with experimental results in Figure 2.4 and Figure 2.5. The coefficients $\Lambda^{(i)}$s are found by fit to data (Ahmadkhanlou et al).
2.4.2 Model 4: Brigadnov and Dorfmann’s model

Brigadnov and Dorfmann define the generalized vector of unknown variables as

\[ F = F(\theta, \rho, \textbf{D}, \textbf{B}) \]  

(2.38)

where \( \textbf{B} \) and \( \theta \) are magnetic flux density and absolute temperature respectively. Based on the general models of Maxwell field equations by Pao, the mechanical balance laws, the laws of thermodynamics, introducing specific Helmholtz free-energy as

\[ F := U - \theta S - \frac{1}{\rho} \textbf{E}^T \textbf{P} \]

using the Rivlin-Ericksen representation theorem and applying some general results of the invariant theory, from (2.38), the constitutive relations for isotropic MR fluids is found to be

\[ \textbf{n} = -p\mathbf{I} + \alpha_{21} |\textbf{D}|^{q-1} \textbf{B} \cdot \textbf{B} + \left( \alpha_{30} + \alpha_{32} |\textbf{B}|^2 \right) \textbf{D} + \left( \alpha_{31} + \alpha_{33} |\textbf{B}|^2 \right) |\textbf{D}|^{q-2} \textbf{D} \]

(2.39)

\[ + \left( \alpha_{40} + \alpha_{41} |\textbf{D}|^{q-2} \right) (\textbf{D} \cdot \textbf{B} \textbf{B} + \textbf{B} \textbf{B} \cdot \textbf{D}) \]

where \( \alpha_i \ (i = 1, \ldots, 6) \) are functions of \( \theta, \rho, \text{tr}\textbf{D}, |\textbf{D}|, \text{det}\textbf{D}, |\textbf{B}|, \textbf{D} \cdot \textbf{B} \textbf{B}, \textbf{D}^2 : \textbf{B} \textbf{B} \). For isotropic non-Newtonian incompressible MR fluid the 3D constitutive equation in indicial notation is given by

\[ \sigma_{ij} = -p\delta_{ij} + \alpha_{21} \left( D_{kl} D_{kl} \right)^{q-1} B_i B_j + \left( \alpha_{30} + \alpha_{32} B_k B_k \right) D_{ij} \]

(2.40)

\[ + \left( \alpha_{31} + \alpha_{33} B_k B_k \right) \left( D_{kl} D_{kl} \right)^{q-2} D_{ij} + \left( \alpha_{40} + \alpha_{41} \left( D_{kl} D_{kl} \right)^{q-2} \right) \left( D_{kl} B_k B_j + B_j B_k D_{kl} \right) \]

Where the coefficients \( \alpha_{ij} \) depend on absolute temperature only and have to satisfy the following conditions
\[\alpha_{30} \geq 0, \quad \alpha_{31} \geq 0, \quad \alpha_{32} \geq 0, \quad \alpha_{33} \geq 0, \quad \alpha_{32} + \frac{4}{3} \alpha_{40} \geq 0, \quad \alpha_{33} + \frac{4}{3} \alpha_{41} \geq 0 \]

\[|\alpha_{21}| \leq \frac{3}{2} \left[ \alpha_{32} + \alpha_{33} + \frac{4}{3} \left( \alpha_{40} + \alpha_{41} \right) \right] \] (2.41)

The constitutive equation is function of magnetic field B, the strain rate, and coefficients \(\alpha_{ij}\). This model does not explicitly include the properties and characteristics of the particles and carrier fluid such as viscosity, susceptibility, and weight ratio of magnetizable particles to carrier fluid as our kinetic theory based models do.

In this constitutive equation, there are some assumptions based on experimental data:

Electric polarization is negligible \((P = 0)\), the electric charge and electric conductivity are very small \((q = 0, \eta = 0)\) therefore the influence of an electric field on the constitutive equation is non-essential, the Helmholtz free-energy \(F = F(T, \rho, B)\) is a smooth function, the MR fluid is isotropic and the stress tensor is symmetric, the MR fluid is non-Newtonian incompressible, and the Cauchy stress tensor of the MR fluid is linear in D and quadratic in B. The remaining equations are based on first principals. For \(q=1\), the simplified 1-D form of shear stress is found by

\[\sigma_{12} = \gamma \left\{ \left[ \alpha_{30} + \frac{\alpha_{31}}{\sqrt{2}} \right] + \left[ \alpha_{32} + \frac{\alpha_{33}}{\sqrt{2}} + \alpha_{40} + \alpha_{41} \left( \frac{2}{\sqrt{2} \dot{\gamma}} \right) \right] B^2 \right\} \] (2.42)

The values of \(\alpha\)'s are not explicitly defined in this model. Based on our investigation, we found out if some of these coefficients are functions of magnetic susceptibility (or other descending function of magnetic field H) then it satisfies the saturation of MR fluids.
This model's prediction of shear stress is compared with experimental results in Figure 2.4 and Figure 2.5. The coefficients $\alpha$'s are found by fit to data (Ahmadkhanlou et al).

### 2.5 Conclusions

A methodology for deriving generalized 3-D tensorial expressions relating forces, flow, and applied magnetic field in MR fluid is developed. In the special case of simple shear the dependence of $\tau_{12}$ on H presented by equation (2.24) is in agreement with observed behavior.

In Figure 2.4 and Figure 2.5 the behavior predicted by expression (2.24) is plotted against experimental measurements and the predictions of the Herschel-Buckley, Ciocanel et al, Chen-Yeh, and Brigadnov-Dorfmann models. From these two figures we observe that expression (2.24) replaces the strict yield stress of the Bingham/Herschel-Buckley-type visco-plastic models (i.e. absolutely no flow until the shear stress reaches a specified value) with a steep gradient, which is in better agreement with experimental observation. Importantly, we emphasize that while we have reduced the general equations (2.21) and (2.22) to a 1-D form similar to the Bingham plastic model, we now have a clear path for incorporating 3-D flow and magnetic field conditions, when velocity and magnetic fields other than those of equations (2.23) are investigated.
3.1 Preparation of MR Fluids

We investigated MR fluids by mixing carbonyl iron powder in one of two size ranges (2 to 5μm or 4 to 7μm) with silicone oil of variable densities (907, 920, 955, 953, and 967 kg m$^3$). The carbonyl iron powder to carrier fluid weight ratios varied from 3, 3.5, 4, 5 or 6 percent, a total of 30 MR fluids. The corresponding steady shear viscosities of the five carrier fluids were measured with a MCR300 rheometer fitted with a 50 mm diameter parallel plate geometry (Figure 3.1 and Figure 3.2).

The MR fluid samples were also tested and characterized in a linear sponge-based damper. In this design, the MR fluid occupies two gaps between the moving piston and a stationary sleeve. The damper incorporates open-celled polyurethane foam to retain the MR fluid in the gaps. The use foam restricts the range of allowable MR fluids to those,
which can saturate the foam (Figure 3.3). The linear MR fluid-based damper then is tested in a MTS tensile testing machine (Figure 3.4).

Figure 3.1: MCR300 rheometer fitted with a 50 mm diameter parallel plate geometry.

We found that the ability to saturate the foam was a function of the viscosity of the silicone oil: the 18 MR fluids with silicone oil viscosity 0.00935, 0.01906, and 0.3406 Pa s successfully saturated the foam, and the 12 MR fluids with viscosities 0.984 and 12.2 Pa s failed to saturate the foam, regardless of iron particle size and concentration. Because of the low magnetic permeability of free space \( \mu_0 = 4\pi \times 10^{-7} \text{H/m} \), the air in the unsaturated foam increases the reluctance of the magnetic system dramatically, thereby decreasing the magnetic flux density so much that the magnetic flux density in the MR fluid cannot be accurately measured. Hence, the highly viscous silicone oils, which do not saturate the foam, were not investigated further.
Figure 3.2: Testing MR fluid sample with rheometer.

Figure 3.3 Sections of the rectilinear foam-based damper
The shear stress measured as a function of shear rate for the remaining carrier fluids are shown in Figure 3.5. All were found to be Newtonian (shear stress linear over the range of shear rates encountered in the experiment). Table 1 gives the labeling scheme of the 18 different MR fluids we investigated. This broad array of fluids allows us to measure the dynamic range of the MR fluid (the ratio of maximum viscosity in on-state to minimum viscosity in off-state) as a function of the MR fluid ingredients and design.
3.2 Experimental Apparatus and Measurements

This section assesses the ability of three-dimensional MR fluid models to qualitatively and quantitatively predict the measured mechanical response to specified magnetic fields of the rectilinear MR damper.

We measured the mechanical response to specified magnetic fields of the MR damper designed and constructed by the authors (Figure 3.4a). The damper and its circuitry were designed to exert a resistive axial force of 100 N in response to a magnetic field emanating from an applied 4 amp current to a solenoid. The copper solenoid (350 turns of 24 AWG with 3.2 ohms of resistance) is positioned on the shaft of the piston so that in
the MR fluid-filled gaps the applied magnetic field is in the radial direction, transverse to the flow of the MR fluid in the gaps induced by the piston motion.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Carbonyl iron powder</th>
<th>Silicone Oil (carrier fluid)</th>
<th>weight ratio of iron powder to oil</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Particle size (μm)</td>
<td>Density (kg/m³)</td>
<td>Viscosity (Pa s)</td>
</tr>
<tr>
<td>1</td>
<td>2-5</td>
<td>907</td>
<td>0.01</td>
</tr>
<tr>
<td>2</td>
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<td>907</td>
<td>0.01</td>
</tr>
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<td>4-7</td>
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<td>0.01</td>
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<td>4-7</td>
<td>907</td>
<td>0.01</td>
</tr>
<tr>
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<td>2-5</td>
<td>921</td>
<td>0.0192</td>
</tr>
<tr>
<td>8</td>
<td>2-5</td>
<td>921</td>
<td>0.0192</td>
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<tr>
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<td>2-5</td>
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<tr>
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<td>4-7</td>
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</tr>
<tr>
<td>18</td>
<td>4-7</td>
<td>955</td>
<td>0.334</td>
</tr>
</tbody>
</table>

Table 3.1 MR fluids investigated in the research

A key feature of the damper design is its incorporation of an open-celled polyurethane foam to retain the MR fluid at both ends of the piston (Figure 3.3 and Figure 3.4a). The confining of the MR fluid in foam has the following benefits: (1) The design eliminates
the need for seals and reducing the off-state shear resistance\(^1\). (2) Confining the iron particles in the foams reduces the settling of the particles in the carrier fluid; thereby maintaining the performance of the device. (3) The design minimizes the amount of MR fluid by placing and holding the fluid only where it is needed to modify the resistance of the damper. Figure 3.6 displays Scanning Electron Microscopy (SEM) of the foam, together with the length scale of the carbonyl iron particles, which is seen to be much less than the length scale of the foam cells.

![Figure 3.6: SEM images of open-celled polyurethane foam, with comparative length scale of iron particles: (a) magnification=50, (b) magnification=150, showing relative size of the iron particles](image)

\(^1\) Conventional MR fluid-based dampers have a reservoir of MR fluid sealed by an o-ring or gasket; these mechanical seals create sizable off-state flow resistance (i.e. resistance when there is no applied magnetic field)
In the construction of the damper the foam is stretched and then glued to the piston using a heat-resistant epoxy. There is an air gap of approximately 0.1 mm between the stretched foam moving with the piston and the inner surface of the stationary sleeve. The MR fluid both saturates the foam and fills the 0.1 mm air gap (Figure 3.7).

![Diagram of linear sponge-based MR damper](image)

**Figure 3.7: Cross section of linear sponge-based MR damper**

The primitive measurements collected are

- the electric current supplied to the solenoid
- the displacement of the piston shaft relative to the stationary sleeve
- the accompanying axial force transferred through the MR fluid from the piston to the sleeve.
Specifically, the damper is installed in a tensile testing machine. The tensile testing machine used in this research is an MTS® machine which has three components: SilentFlo™ hydraulic power unit, 858 Table Top System load frame, and a TestStar™ IIs control unit, which simultaneously controls the piston displacement and measures the force on the stationary sleeve from the moving piston (Figure 3.8). The current supplied to the solenoid is controlled with a power supply. In our experiments, piston displacement is controlled to be a specified function of time, current is controlled to be constant, and the axial force is a measured function of time.

![MTS tensile testing machine](image)

Figure 3.8: MTS tensile testing machine

Each of the five MR fluid models listed in chapter 2 describes the coupled magnetomechanical behavior of an MR fluid as a surface in the stress/strain-rate/magnetic-field space experienced by the fluid. To validate these model predictions
of MR fluids against experimental measurements, we deduce the strain rate, stress, and magnetic field in the fluid from the primitive measurements described above as sketched in Figure 3.9

![Figure 3.9 Construction of the 1-D shear stress $\tau$ vs. magnetic field $H$ and shear stress $\tau$ vs. strain rate $\dot{\gamma}$ of the general constitutive relation $\tau = \tau(H, \dot{\gamma})$ from the primitive measurements of sleeve force, piston displacement, and coil current](image)

The strain rate is deduced by numerically differentiating the controlled displacement of the piston to get the velocity of the piston, which combined with the no-slip boundary conditions on the surfaces of the piston and the stationary sleeve deduces the rate of
strain in the MR fluid as a function of time. The stress in the MR fluid is deduced from the measured force at the base of the sleeve transmitted through the load cell. The magnetic flux is deduced from the applied electric current in the solenoid and this flux combined with the magnetization properties and the geometry of the magnetic path determines the magnetic induction in the MR fluid, which from experimental measurement and constitutive assumption gives the magnetic field.

Given the rectilinear motion of the piston and smallness of the thickness-to-radius ratio of the annular volumes containing the MR fluid, the flow of the MR fluid in these volumes can be regarded as planar shear with a transverse magnetic field. For such a flow, the five 3-D constitutive models reduce to scalar equations relating the shear stress $\tau$ in simple shear to the shear rate $\dot{\gamma}$ and magnitude $H$ of a transverse applied magnetic field. These equations are comparable to the 1-D Bingham and Herschel Bulkley (equations (2.1) and (2.2)) discussed in chapter 2.

The hypothesized flow profile of the MR fluid in the gap between piston and sleeve is shown in Figure 3.10. Recall that the gap is completely filled with MR fluid and partially filled with foam. The foam is affixed to and moves with the piston. The MR fluid impregnating this foam moves with the speed of the piston (and foam) except near the foamless gap where, because of the small length scale of the ferrous particles in the MR fluid relative to the foam pore size (see Figure 3.6.b), viscous drag acts through the foam to slow down the fluid. In the gap between foam and sleeve the MR fluid velocity decreases linearly to zero at the stationary sleeve surface.
We model this flow simple shear penetrating into the foam a distance \((\alpha - 1)h\) into the foam, i.e. a distance \(\alpha h = h_{\text{eff}}\) from the sleeve, where \(h\) is the gap between the sleeve and the foam, and \(0 < \alpha \leq 1\) is a coefficient that takes into account the relative motion of fluid in the foam (Figure 3.10). The coefficient \(\alpha\) is determined by comparing the test results in the absence of magnetic field \((H=0)\) using the tensile testing machine and a rheometer with and without foam. The comparison results in \(\alpha = 1.08\).

The scalar strain rate \(\dot{\gamma}\) in these special 1-D forms of the 3-D model is deduced by filtering the displacement signal measured by the tensile testing machine using a low-pass filter and taking a numerical derivative with respect to time. The strain rate is then
computed by dividing the velocity difference between the moving piston and stationary sleeve bounding the shear flow by the effective width $a h$: With the axial velocity $v$ of the piston (known from measurements) and $a$ determined as described above we calculate the shear strain rate vs. time from

$$\dot{\gamma}(t) = \frac{v(t)}{a h}$$  \hspace{1cm} (3.1)

To deduce the magnetic field $H$ in the fluid as a function of the measured constant DC current $I$ supplied to the solenoid, the magnetic flux $\phi$ is calculated by $\phi = \frac{I N}{R_T}$, where $R_T$ is the total reluctance within solenoid circuit and $N$ is the number of turns in the solenoid. The magnetic induction $B$ is then known by $B = \frac{\phi}{A}$, where $A = 2(2\pi RL)$ is the surface area of the sleeve (and piston) wetted by the MR fluid, through which the magnetic flux $\phi$ acts (see Figure 3.11 for the dimensions $R$ and $L$). Finally, the magnetic field intensity $H$ is calculated from the implicit equation $H = \frac{B}{\mu(H)}$, where the magnetic permeability $\mu$ of the of MR fluid is $\mu = 1 + \chi = 1 + \frac{dM}{dH}$, with magnetization $M$ given as a function of magnetic field $H$ by the Frohlich-Kennelly law for magnetization,

$$M(H) = \frac{M_s \chi_i H}{M_s + \chi_i H}$$  \hspace{1cm} (3.2)

where $\chi_i = 131$ is the initial susceptibility and $M_s = 1990$ kAmp/m is the saturation magnetization (Figure 3.12). In simple shear flow the magnetic field strength vector $H$ has one component (equation (2.23)), however, due to tilting of the branch chains, the
magnetization $\mathbf{M}$ may have two components, $\mathbf{M} = M_1 \mathbf{e}_1 + M_2 \mathbf{e}_2$. We assumed the $M_1$ component is negligible and used 1-D magnetization. The shear stress $\tau$ in the MR fluid is found from a force balance on the stationary sleeve, $\tau(t) = F(t) / A$, where $F(t)$ is axial force measured by the load cell on the tensile testing machine, and $A = 2(2\pi RL)$ is the surface area of the sleeve wetted by the MR fluid.

![Diagram](image)

Figure 3.11: Views of the rectilinear MR damper: (a) piston showing the applied load and shear stress from the sleeve transferred through the MR fluid-saturated foam, (b) section of the movable piston in the fixed sleeve showing the magnetic flux path
Figure 3.12: Frohlich-Kennelly law

In Figure 3.13 we display a representative set of measurements. The displacement of the piston is imposed by the tensile testing machine to be a quarter cycle of a sine wave (Figure 3.13.a), at ten different constant solenoid currents from zero to four amps. The force necessary to support the piston displacement is measured as a function of time by the load cell at each current (Figure 3.13.b). Differentiating the displacement $x$ and dividing by the effective width of simple shear flow $h_{eff}$, which is shown in Figure 3.10.b, gives strain rate $\dot{\gamma}$ as a function of time (Figure 3.13.c). The force divided by the MR fluid contact area gives shear stress in the fluid as a function of time for each constant current (Figure 3.13.d). In Figure 3.14 the shear stress versus strain rate predictions at different values of applied magnetic field strength are obtained. This is done by combining shear stress versus time (Figure 3.13.d) and shear rate versus time (Figure 3.13.c) graphs for each constant applied magnetic field strength and eliminating time.
Figure 3.13: Test measurements: (a) imposed displacement of damper piston relative to the fixed sleeve; (b) measured force applied to the piston required to produce this displacement, for ten specified currents in the damper coil from 0.0 Amp (bottom) to 4.0 Amp (top); five of the plots left off for clarity; (c) strain rate versus time; (d) shear stress versus time
Figure 3.14: Shear stress as a function of shear rate and magnetic field strength: comparison of measurements to model predictions, for sinusoidal displacement

We define the yield stress $\tau_y$ to be the point on the shear stress versus shear rate curve where the tangent to curve has a slope of one (Figure 3.15). To find the yield stress $\tau_y$ analytically the following approach is studied:

a) The slope of shear stress-shear rate curve is computed by

$$m = \frac{\partial \tau_{12}}{\partial \dot{\gamma}} = \eta_s + \frac{n k T \zeta 4 \left( \beta + \frac{c \dot{\gamma}}{2(1 + \chi)H} \right) - n k T \zeta \frac{4 c}{2(1 + \chi)H}}{16 \left( \beta + \frac{c \dot{\gamma}}{2(1 + \chi)H} \right)^2}$$

(3.3)
Therefore, the following quadratic equation can be obtained

\[
\left( \frac{c}{2(1 + \chi)H} \right)^2 \dot{\gamma}^2 + \frac{c \beta}{(1 + \chi)H} \dot{\gamma} + \left( \beta^2 + \frac{n k T \zeta \beta}{4(\eta_2 - m)} \right) = 0 \tag{3.4}
\]

The solution to equation (3.4) is the yield shear rate \( \dot{\gamma}_y \) corresponding to yield shear stress.

b) The parameter \( \zeta \) in equation (3.4) is assumed to be a function of magnetic field \( H \), determined by fit to experimental data which is found to be

\[
\zeta(H) = 96402e^{(-4\times10^5)H} \tag{3.5}
\]

Figure 3.15: Definition of yield stress based on the derivative of normalized shear stress with respect to shear rate
The yield stress $\tau_y$ is found by substituting $\dot{\gamma}$ with $\dot{\gamma}_y$, and $\zeta$ with $\zeta(H)$ in equation (2.24). The resulting $\tau_y$ versus corresponding magnetic field strength $H$ for each applied constant current is plotted in Figure 3.16.

Figure 3.16: Yield stress vs. magnetic field
CHAPTER 4

ONE AND TWO DEGREE OF FREEDOM FORCE FEEDBACK SYSTEMS

This chapter describes the design and control of a force feedback system utilizing the MR fluid devices. It discusses how the haptic devices, microstructural modeling, MR fluid preparation, and MR fluid characterization combine with a DC motor and a force sensor to create a nearly transparent haptic system (Figure 4.1).

Figure 4.1: MR fluid-based haptic system

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As stated previously, the ultimate goal for control of MR fluid based force feedback systems is to provide haptic feedback to the user to improve system transparency and tracking performance. There are two distinct types of force feedback devices: rectilinear and rotary. In this chapter we investigate the application of MR fluid-based rotary dampers in single degree of freedom (SDOF) and two degree of freedom (2DOF) rotary force feedback systems. The two systems apply different control algorithms for the same equation of motion.

4.1 Single degree of freedom (SDOF) system

Details of our single degree of freedom force feedback system are shown in Figure 5c. The user operates the rotary MR damper joystick to command the motion of a rotary actuator (Figure 5a). In a telerobotic system the part of the slave, which contacts and manipulates the external object is called the end effector. In our SDOF force feedback system the end effector for the force feedback system is a flexible 1-DOF rotary probe (Figure 4.2b). The probe consists of a rigid arm connected to the housing through a pin joint and two springs. The housing is then supported by a stationery base that is attached to a DC motor. The shaft of DC motor rotates the housing. A flexible force sensor is attached to the end of the arm. A joystick commands the DC motor, which, due to the stiffness of the springs, causes the probe to rotate. The position of the joystick is sensed by a digital encoder. The encoder signal is sent to a data acquisition system, which in turn sends a voltage to the slave DC motor. When the probe contacts a solid object or barrier it
pivots (i.e. the respective springs connected to the pivot point compress and extend). The angular change is sensed by an encoder that is coupled to the pivot point. Based on the arm position, a DC power supply sends a current to the solenoid within the MR damper joystick. As the angular change increases, the current sent to the MR damper in the joystick increases and the user feels the resistance from the MR joystick equivalent to the resistance encountered by the probe.

![Figure 4.2: Single degree of freedom rotary force feedback system: (a) Rotary force feedback system setup, (b) Rotary end-effector](image)

The equation of motion (EOM) for the single degree of freedom rotary force feedback system couples the governing equations for the MR damper/joystick (master), the end effector (slave), and the force feedback sensor (See Figure 4.3).
4.1.1 Equation of motion of MR damper/master

The rotary damper shares the design of the linear damper used to characterized the MR fluids, with two MR fluid-soaked sponges occupying thin gaps between rotor and housing, except the piston executes a rotary motion in response to an applied resistive torque, as opposed to axial motion and an applied force. The second order differential equation relating the angular motion of rotary damper to the applied torque and resistance from the MR fluid is

\[ J_D \ddot{\theta} = T_D - 2(2\pi R L) \tau R - k_D \theta \]

(4.1)

where \( R \) is the radius of the damper rotor, \( L \) is the length of its wetted area, \( J_D \) is the rotational inertia of damper rotor (kg-m\(^2\)), \( k_D \) is the spring constant of the centering spring (N.m/rad), \( \theta \) is the angle rotation of damper rotor (rad), \( \tau \) is shear stress from the MR fluid as modified by force feedback command from the sensor, and \( T_D \) is the applied torque to the MR damper (N.m) (Figure 4.4).
The state space form of the EOM for the master can be written as

\[
\begin{bmatrix}
\dot{\theta} \\
\dot{\theta}
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
-K_D/J_D & 0
\end{bmatrix}
\begin{bmatrix}
\theta \\
\dot{\theta}
\end{bmatrix} +
\begin{bmatrix}
0 \\
1/J_D
\end{bmatrix}
\begin{bmatrix}
0 \\
-4\pi R^2 L/J_D
\end{bmatrix} u 
\] \quad u = \begin{bmatrix} T_D \\ \tau \end{bmatrix} \tag{4.2}

Figure 4.4: MR damper rotor with nominal dimensions

The next task is to express the shear stress as a function of the input voltage command \( V_m \) from the sensor. The magnetic field strength \( H \) (Amp/m) can be expressed in terms of field \( B \) (Tesla) as

\[
H(B) = \ln \left( \frac{2B_{\text{max}}}{B + B_{\text{max}}} - 1 \right)^{-B_1} \tag{4.3}
\]

where \( B_1 \) is the time constant. The magnetic field \( B \) can be found by

\[
B = \frac{\phi}{A} \tag{4.4}
\]

where \( A = (2\pi RL) \) is the wetted area (m\(^2\)), and \( \phi \) is the magnetic flux (Wb) defined by
\[ \phi = I \frac{N}{R_T} ; I = \frac{V_m}{R_{coil}} \quad (4.5) \]

with \( R_T \) defined as the sum of reluctance elements within the circuit, \( I \) is the current sent to the solenoid (Amp), \( V_m \) the solenoid voltage, \( R_{coil} \) the coil resistance (Ohms).

Substituting equations (4.4) and (4.5) in equation (4.3) results in

\[ H(B) = \ln \left[ \left( \frac{2B_{\text{max}}}{NV_m + B_{\text{max}}} - 1 \right)^{-\frac{1}{2}} \right] ; N_1 = \frac{N}{AR_T R_{coil}} \quad (4.6) \]

Rewriting equation (2.24) with respect to the input voltage \( V_m \) results in

\[ \tau = \eta_0 \dot{\gamma} + \frac{nkT \zeta \dot{\gamma}}{4(\beta + c\dot{\gamma} / (1 + \chi))} \times \left[ c + 2 \frac{c\dot{\gamma} / (1 + \chi) B_i N_i V_m}{(\beta + c\dot{\gamma} / (1 + \chi)) B_{\text{max}}} - 4 \frac{c\dot{\gamma} / (1 + \chi) B_i^2 N_i^2 \beta}{(\beta + c\dot{\gamma} / (1 + \chi))^2 B_{\text{max}}^2} V_m^2 \right] + O(V_m^3) \quad (4.7) \]

The shear rate \( \dot{\gamma} \) in the MR-filled gap can be expressed as

\[ \dot{\gamma} = R \dot{\theta} / h \quad (4.8) \]

where \( h \) is the width of the gap. Inserting equation (4.8) into equation (4.7), linearizing the input voltage \( V_m \) and angular velocity \( \dot{\theta} \), assuming the elastic term \( \beta \) is negligible with respect to the other term \( \frac{c}{2(\chi + 1)|H|} \) in equation (2.24), and using Taylor series expansion about equilibrium results in

\[ \tau = \left( \frac{2nkT \zeta B_i N_i h V_m + nkT \zeta c B_{\text{max}} h + 4R\eta(1 + \chi)^{-1} B_{\text{max}} \dot{\theta}}{4(1 + \chi)^{-1} B_{\text{max}} h} \right) \quad (4.9) \]

Substituting equation (4.9) into equation (4.1) results in
The state space form of equation of motion can be written as

\[
f(\theta, \dot{\theta}, \ddot{\theta}, V_m) \approx \ddot{\theta} + \left( \frac{4\pi R^3 L_n}{\hbar J_D} \right) \dot{\theta} + \left( \frac{K_D}{J_D} \right) \theta + \left( \frac{2(1 + \chi) \pi R^2 L_n T \zeta B I N_I}{B_{max} J_D} \right) V_m - \left( \frac{1}{J_D} \right) T_D. \tag{4.10}
\]

The state space form of equation of motion can be written as

\[
\begin{bmatrix}
\dot{\theta} \\
\ddot{\theta}
\end{bmatrix} = 
\begin{bmatrix}
0 & 1 \\
-K_D & -\frac{4\pi R^3 L_n}{\hbar J_D}
\end{bmatrix}
\begin{bmatrix}
\theta \\
\dot{\theta}
\end{bmatrix} +
\begin{bmatrix}
0 & 0 \\
\frac{1}{J_D} & -2(1 + \chi) \frac{\pi R^2 L_n T \zeta B I N_I}{B_{max} J_D}
\end{bmatrix}
\begin{bmatrix}
u \\
V_m
\end{bmatrix}; \quad u = \begin{bmatrix} T_D \\
v_m \end{bmatrix}. \tag{4.11}
\]

this equation relates the inputs to the master, \( T_D \) and \( V_m \), to the output position \( \theta \) which is sent to the slave. The following subsection clarifies how the position command \( \theta \) from the master is related to the vector position \( x \) of the slave and how to find the control command.

### 4.1.2 Control logic for slave/end effector

The control logic for the motion control of the slave commanded by master is explained in this sub-section. Full state feedback control is implemented to control the motion of the robotic arm. The state space model of the system can be expressed as follows

\[
\begin{cases}
\dot{x} = A\dot{x} + B\ddot{u} \\
\dot{y} = C\dot{x} + Du \\
\ddot{u} = -K\ddot{x} + \ddot{\nu}_d
\end{cases} \quad \begin{bmatrix} \theta_1 \\
\theta_2 \end{bmatrix}, \tag{4.12}
\]

where \( \theta_1 \) = angle of slave housing relative to base (rad) and \( \theta_2 \) = angle of probe relative to housing (rad). The State matrices are computed by
where $k$ is the spring constant (N/m), $J_1$ is the rotational inertia of body (kg–m$^2$), $J_2$ is the rotation inertia of probe (kg–m$^2$), $K_g$ is the gear ratio, $K_m$ is the motor voltage constant (V-s/rad), and $R_m$ is defined as the motor resistance (Ohms). In order to account for steady state errors, we employ the well-known N-bar technique, with $N$ as defined in reference

$$N = N_u + KN_x,$$  \hspace{1cm} (4.14)

where,

$$\begin{bmatrix} N_x \\ N_u \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$  \hspace{1cm} (4.15)

Proportional – Derivative (PD) control is employed through a matrix gain to give an output voltage to the arm that moves the base to the desired position. A 4th order Bessel function is used to place the gain values of $K$ such that the system has the quickest response possible while reducing overshoot. The encoder input from the joystick is used as the reference input, $r_d$. An encoder is attached to the base provides position and velocity of the body relative to the base. Another encoder is attached to the body to
provide feedback of the probe relative to the body. Furthermore, a saturation limit of +/- 5 volts is added to protect the system hardware.

4.2 Two degree of freedom (2DOF) system

This system provides the user with tactile information in a two-degree-of-freedom environment. The end-effector for this system is a probe that can sense deflections in two planar directions. This probe is connected to a five bar linkage that is able to move in the horizontal plane (Figure 4.5). The motion of the five bar linkage is controlled by an innovative MR force feedback joystick (MRFFJ) (Figure 4.6). Three single rotary dampers are designed for this system: the smaller Y-axis dampers are energized when the probe hits a barrier in the x direction and similarly the larger X-axis damper is energized when the probe hits a barrier in y-direction. A 24 gauge copper coil with 1400 and 900 coil turns is used for the small and large dampers respectively. There are two encoders on the x and y shafts that send the signals, related to joystick movement, to the 5-link bar system. The small and large dampers are designed to generate resistance torques of 2.5 and 5 N.m. respectively.
The 2DOF system is designed to trace a two-dimensional curved structure whose path is shown in (Figure 4.5b). As the user moves the MRFFJ handle, the x and y-axis digital encoders sense the position of the joystick. The encoder signals are sent to a data acquisition system that, in turn, sends voltage to the five-bar linkage actuators. When the probe comes into contact with the boundaries of a curved path, the probe pivots and the angular changes are sensed by a second set of encoders, which are directly connected to rotary springs on the probe. This coupling allows one to infer the force that the probe experiences. This calibrated signal is then sent to data acquisition board and the corresponding control commands are sent to a DC power supply and as a result the current sent to MRFFJ dampers is increased. This activates the MR fluid within the dampers and the resistance the user feels simulates the boundaries of the curved path. The control block diagram for the 2DOF system is illustrated in Figure 4.7 and the control command is found using a control algorithm similar to the one used in the SDOF system.
Figure 4.6: MR force feedback joystick (MRFFJ)
Figure 4.7: (a), (b) [136], and (c) Block diagram of 2DOF system used for tracking
4.3 EXPERIMENTAL RESULTS

Among all the MR fluid samples, sample number six generates the largest shear stress in the on-state mode while maintaining a low off-state shear stress. This fluid mixture is used for saturating the sponge based MR dampers used in the SDOF and 2DOF studies. A sponge is an effective way to prevent leakage while not providing excessive friction caused by sealing.

In a SDOF system the user controls the motion of a rotary arm by a rotary MR joystick. As the end effector comes in contact with an object, the joystick motion is impeded as a result of an increased shear stress in the MR fluid within the damper proportional to the stiffness of the object. This is illustrated in Figure 4.8 in which the output torques generated by the rotary MR damper are compared for rigid and compliant surfaces.

The SDOF rotary system was used to test five different objects with different stiffness/compliance. Three different reference inputs were applied to the system: a human controlled reference input and two automated sinusoidal reference inputs of 0.1 and 1 Hz respectively. These inputs were used to model the quasi-static and low frequency dynamic cases. The force-deflection graphs of these objects were investigated. The sensed forces were also larger for stiffer objects. Using the collected data on displacement and forces, the stiffness of the different objects were computed and simulated in the master. The results show that one can simulate the stiffness/compliance and restoring force of the environment by using the force and deflection sensor in the EE and a DC motor in the master.
Figure 4.8: Torque outputs for SDOF system

Figure 4.9: Tracking the 2-D profile by MRFFJ (2DOF system)
For tracking purposes, the 2DOF force feedback system is used to track the profile shown in Figure 4.5b. The force-feedback system allows for much greater accuracy in tracking the profile compared to visual only feedback (i.e. force feedback disabled) (Figure 4.9). The mean squared error for visual only feedback is 7.3 mm$^2$ and for force feedback is 1.8 mm$^2$.

One problem with the 2DOF system is the slight delay between the end-effector contacting an object and the user feeling it. The dynamics of activation coupled with the regulator response can cause a delay of about ten milliseconds between the end-effector contacting an object and the system responding to the contact. Furthermore, the slave probe is flexible due to the springs used in the end effector probe. The probe flexibility along with the delay in the system causes the slave end effector to pass the rigid limits of the tracking profile at some locations as shown in Figure 4.9. There are two ways to overcome this phenomenon: (i) Increasing the gain for motion feedback control; this causes the slave (5-bar linkage) to track the motion of master (2-D joystick) faster and consequently decreases the delay. If the gain is increased too much then it causes system instability (chattering) in motion control. (ii) Increasing the gain for force feedback control; this generates higher resistive torque in the master. The higher gain for force feedback control is not a sufficient condition for transparency and should be accompanied with a higher gain for motion control to give better performance. Furthermore, higher force feedback gain may cause sticky wall phenomenon. Therefore there will be a trade off between higher stability and more accurate tracking of the solid profile.
Another problem experienced by the dampers used in SDOF and 2DOF systems is they are unable to create a torque as large as their theoretical counterparts. One of the problems could be the inability to accurately predict the number of wire turns a piston could accommodate. Furthermore, the physical dampers saturate at lower currents than the theory predicts. Nevertheless, the sponge damper outputs a torque a few times greater when fully activated, which is enough range for the user to experience a range of forces associated with force feedback. Friction also served to be a problem, even with compensation, as the amount of friction present was dependent on the position of the machine. The steady state error in the five bar linkage could be further reduced by lighter linkages and higher quality bearings and joints. However, even with these technical difficulties, the usability of the system increased by having force feedback available. Thus, the primary goal of the research was achieved [136].
CHAPTER 5

FIVE DEGREE OF FREEDOM TELEROBOTIC FORCE FEEDBACK SYSTEM

MR fluid is used in the design of a novel five degree of freedom (DOF) MR sponge-based haptic system (master) for controlling a telerobotic arm used in telerobotic surgery. The master 5-DOF joystick controls the movement of a 5-DOF slave (Lynx 5 by LynxMotion). A novel multi-axis force sensor is designed by the authors and used at the end effector (EE) of the slave for force feedback control. Force and displacement sensors in the slave sense the environment conditions along which the end effector moves. When the EE contacts a solid object or barrier, the exerted force is sensed by the force sensor and the signals are sent to the 5-DOF MR based master to activate the MR dampers accordingly to replicate the force. The user feels the force from master joystick proportional to the force encountered at the slave. For example if the slave encounters soft tissue (low force, small to no deceleration) a relatively small current signal is sent to the MR damper which will give the user the slight resistance associated with soft tissue.
Likewise if the slave encounters bone (high force, large deceleration) a large signal is sent to the MR damper, which will give the user a larger resistance.

![Figure 5.1: (a) Five DOF MR fluid-based telerobotic haptic system; (b) Master (right) and Slave (left)](image)

**5.1 Control, Transparency, and Stability of the System**

The main objective of any force feedback system is to reproduce the forces encountered by the actual or virtual system at the user’s end. This can be seen as a tracking problem where the goal is to track the force at the ‘virtual end’ and regenerate it at the user end. The development of the controller must be comprehensive enough that it can handle
models that will grow in complexity from both an analytical and experimental point of view.

The control methodology in this research is categorized in two sub-sections: motion control of the slave and force feedback control. Figure 5.2 shows the block diagram of the telerobotic system with motion and force feedback control. Vector $X_m$ is the commanded motion by the operator hand, $F_m$ is the force applied to the master by the operator, $X_s$ is the motion of the slave end-effector, and $F_e$ is the force exerted to the end-effector by the external environment.

**Figure 5.2: Control block diagram of the telerobotic force feedback system**
5.1.1 Motion control

An encoder is placed on each joint of the 5-DOF MR-based master robot. The encoders send the signals to a data acquisition board. This data is stored in shared memory through a compiled Simulink file (Figure 5.3). Another Simulink file is used to read the data from the shared memory (Figure 5.4). A *mex c-file s-function* is used to send the required command to the slave robot joints through a RS232 (COM) port to follow the master motion. The data is sent to all the servo-motors in the slave.

![Simulink block diagram used to write data to shared memory](image)

Figure 5.3: Simulink block diagram used to write data to shared memory
5.1.2 Force feedback control

A 2-D force sensor is designed and placed at the slave end-effector (Figure 5.5). Two strain-gages are used for this purpose. Each strain gage is placed on a plate and the plate is connected to the end-effector with a special designed connection. The connection is fixed in one direction and hinged in the other direction. The plates and connections are designed and made such that when a force is applied perpendicular to each strain gage the other strain gage does not get activated.
Figure 5.5: Prototyped 2-D strain gage placed on the slave end-effector

5.1.3 Transparency

When the operator controlling the master has the feeling of direct interaction with the remote environment, the system is `called “transparent” (Figure 5.6). It physically means that the impedance of the environment (external object) is equal to the transferred impedance from slave to the master or operator’s hand. In this case for the same force we will have the same motion. There are two variables to change in order to match the impedance of master to the impedance of slave: damping and stiffness/compliance. The damping coefficient is a function of shear stress developed in the MR damper and the stiffness/compliance is a function of MR damper spring.
5.2 Experimental Results

The MR-based joints of the master robot were tested separately in the tensile testing machine (Figure 5.7). The resulting resistive torque-current for each joint is found and plotted in Figure 5.8. Then the joints are assembled to make a 5-DOF robot. The MR-based master robot is used to control the motion of a slave. The test setup is shown in Figure 5.9. Wheatstone bridges and strain gage amplifiers are used to amplify the output signals of strain gages. DC power supplies are used to send the required currents to the MR damper in the master to generate desired resistive torque at each joint.

Figure 5.6: Transparent telerobotic system
Figure 5.7: MTS machine test of one of the 5-DOF joints and
Figure 5.8: the resulting torque-current graph of MTS test

Figure 5.9: Test setup of the 5-DOF master-slave telerobotic system
A cutter is placed at the end effector. The slave is used to cut different objects with different stiffness and compliance. Based on the forces sensed at the end effector through the 2-D designed force sensor, the MR fluid-based dampers in the joints of the master are activated to replicate the resistive forces/torques encountered at the slave. Although there is a little coupling between the strain gages, the resulting sensed forces are acceptable and the coupling is negligible. Figure 5.10 shows the forces sensed at the end effector for two perpendicular applied forced to the 2-D force sensor.

![Figure 5.10: The signals of the 2-D force sensor for applied forces in x and y directions](image)

Two different external objects with different stiffness were tested. Figure 5.11 shows the results of the test. By the forces sensed at the slave end-effector and replicated at the
master joints the operator can distinctly determine which object is soft and which one is hard.

![Graph showing signals of 2-D force sensor exerted by soft and hard external objects](image)

Figure 5.11: signals of the 2-D force sensor exerted by soft and hard external objects

5.3 Conclusions

Haptic systems greatly increase the effectiveness of a human machine interface. However, these systems use passive devices in force feedback systems that are unable to recreate a reactive force that helps in disengaging an object. The relative, simplicity and quick dynamics of an MR damper makes it a viable option for a myriad of haptic applications. This chapter presents the design and implementation of a 5DOF MR force feedback system. Carbonyl iron powder and silicone oil was used to make the required MR fluid. MR sponge based dampers were designed using an interactive MR damper toolbox to simultaneously meet torque and magnetic circuit requirements. The results show good achievements in force feedback control of the system. The operator can sense

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the stiffness of external objects and distinguish hard and soft objects. These systems can naturally be extended to those appropriate for telerobotic surgery.
CHAPTER 6

STABILITY AND TRANSPARENCY

The goal of a haptic system is to achieve transparency while keeping the system stable. The stability and transparency of a haptic system are closely related to one another. The stability of a teleoperation system and its transparency are discussed in this chapter.

6.1 Stability

Passivity is one of the tools to verify the stability of teleoperation systems under specific criteria. A system is called passive when it consumes energy and the following inequality holds

$$\int_{t_0}^{t} y^T(\tau)u(\tau)d\tau \geq V(x(t))-V(x(t_0)) \quad (6.1)$$

where $x$, $y^T$, and $u$ are the states, output, and input to the dynamic system. The continuous function $V(x) \geq 0$ which satisfies $V(0)=0$ is called the storage function. Colgate et al investigated the stability of sampled data systems (continuous time plant and discrete time controller) through passivity for virtual wall (virtual stiffness and
damper in mechanical parallel) as shown in Figure 6.1a. Using the Small Gain Theorem
they derived a necessary and sufficient condition to be as follows

\[ b > \frac{KT}{2} + B \]  

(6.2)

where K, B, b, and T are virtual stiffness, virtual damping, inherent physical damping of
haptic system, and sampling time. For the SDOF system shown in Figure 4.2, these
parameters can be found as follows (See section 4.1.1)

\[ b = \frac{4\pi R^3 L \eta}{h}, \quad K = K_D, \quad B = 2\pi R^3 L n k T \zeta B_i N_1 \chi B_{\text{max}} \]  

(6.3)

An and Kwon showed the stability condition for a hybrid system consisting of a MR
brake and a DC motor in mechanical parallel (shown in Figure 6.1b) to be

\[ b + B > \frac{KT}{2} \]  

(6.4)

The damping of the DC motor is neglected and the damping of MR brake is modeled as
Coulomb friction. Introducing two dimensionless parameters \( \alpha = \frac{B}{b}, \beta = \frac{KT}{b} \) and
applying these to equations (6.2) and (6.4) the stability conditions for active and hybrid
systems can be shown by the following equations

\[ 0.5 \beta + \alpha < 1; \ (\alpha \geq 0, \ \beta \geq 0) \]  

(6.5)

\[ 0.5 \beta - \alpha < 1; \ (\alpha \geq 0, \ \beta \geq 0) \]  

(6.6)

The regions of stability for active and hybrid systems can be shown in Figure 6.2. As
\( \alpha \) and \( \beta \) are positive, all the systems (stable and unstable) will fall within region I of \( \alpha-\beta \)
plane; i.e. right hand side of \( \beta \) axis and upper side of \( \alpha \) axis. Equation (6.5) defines a
triangular region within which the active system is stable. Equation (6.6) defines a stable region for hybrid systems that is the area constrained by lines $O\alpha$, $OB$, and $B\lambda$.

Figure 6.1: Block diagram of haptic interface: (a) active system (b) hybrid system
Figure 6.2: Stability regions for active and hybrid systems: The system is stable if it is within the marked region and unstable otherwise.

In the case of the active haptic system when there is no damper in the system and where the damping of the DC motor is neglected, the parameter \( B \) and subsequently \( \alpha \) will be close to zero and the stability condition, from equation (6.2), will be \( \frac{K}{b}T < 2 \) (or \( \beta < 2 \)). This can be shown as line OB in Figure 6.2. This condition implies that the active system with negligible virtual damping \( B \) will be unstable if the inherent physical damping \( b \) is small or the virtual stiffness \( K \) is large (in case of large voltage applied to DC motor), or the sampling time \( T \) is large (slow control algorithm or implementation).

In the case of the semi-active haptic system when there is a MR based damper or brake and in the absence of any active device such as DC motor in the system, the parameter \( K \)
will be close to zero and the stability condition, from equation (6.4), will be $b + B > 0$ (or $\alpha > 0$). This can be shown as line O\(\alpha\) in Figure 6.2. This condition implies that the semi-active system is stable as long as $b + B > 0$ or $\alpha > 0$. This condition holds for all physical semi-active systems and therefore the system is always stable. Even in the case of negative damping the condition $\alpha > 0$ is always satisfied.

Thus the semi-active haptic system consisting of an MR brake/damper is always stable whereas the active haptic system may go unstable if the applied voltage to the DC motor is large, the inherent physical damping of the haptic system is small, or the controller sampling time is large (slow controller). The work in this chapter extends the work done by Colgate et al and An et al in two distinct ways. First, the system has the ability to capture low levels of touch by utilizing an accurate microstructural model for MR fluid behavior. Second, the slave in this research is an actual electromechanical system while the slave in the previous work was a computer simulation in virtual environment.

### 6.2 Transparency

When the operator controlling the master has the feeling of “direct” interaction with the remote environment, i.e. the mechanical impedance of the environment (external object) is equal to the transferred impedance from the slave to the master, the system is called “transparent” (Figure 6.3a).
Figure 6.3b shows an end effector (EE) at the moment of touching the environment. In this figure the EE is connected to the slave through a linear spring. A force sensor is attached to the EE, at the point where it comes in contact with the external object. As the operator commands the slave to move further, a compression force $F$ is exerted to the EE spring and force sensor.

The equation of motion (EOM) for the EE is given by

$$m_s \ddot{x} + b_s (\dot{x} - \dot{x}_0) + k_s (x - x_0) = -F$$

(6.7)

where $m_s$, $b_s$, $k_s$, $x_0$, and $F$ are the mass, damping, and stiffness of the end effector, the position of the slave (where the end effector is attached), and the force sensed by the
force sensor respectively. All the parameters in this differential equation are known so we can solve for x. The impedance of the environment can be calculated from $Z_e = \frac{F}{\dot{x}}$. In case of the SDOF rotary system shown in Figure 4.2, an encoder will give the required information for the deflections in the EE spring and the environment. The equation of motion (EOM) for the EE shown in Figure 6.4 is given by

$$J_2\ddot{\theta} + k'(\theta - \theta_0) = -T$$  \hspace{1cm} (6.8)

The parameters are defined in 4.1.2. The impedance of the master is thus defined by

$$Z_m(s) = J_m s^2 + B_m s + \frac{K_m}{s}$$  \hspace{1cm} (6.9)
Therefore there are two variables to change in order to match the impedance of master to the impedance of slave: damping \((B_m)\) and stiffness/compliance \((K_m)\) of the master. The damping coefficient is a function of shear stress developed in the MR damper \(B_m = B_m(\tau)\) and the stiffness/compliance is a function of MR damper spring \(K_m = K_m(\theta_s)\) and the current applied to the DC motor. The torsion spring is used to simulate the restoring force and the DC motor is used to adjust the initial angle of rotation in the spring.

The block diagram for motion and force feedback control of a MR based telerobotic operation is shown in Figure 6.5. The function \(G(H, \gamma)\) is a second order equation in current \(I\) as a function of magnetic field strength \(H\) and shear rate \(\gamma\). The transfer functions \(Z_o(s)\) and \(Z_e(s)\) are the impedances of the operator hand and the external object in the environment respectively. The block diagram makes more sense when one realizes that there are essentially two interrelated aspects of this system: (1) Position control of the end effector (EE) and (2) replication of the force.

For the position control component, the operator applies the force \(F_m\) to the joystick handle and turns it. The commanded angle of rotation \(\theta_m\) is sensed through an encoder mounted on the joystick axis of rotation. This signal is sent to the input channel of data acquisition board. The required current \(I_s\) for generating the same angle of rotation in the slave is calculated through a motion control algorithm, which is state feedback control in this case. The current \(I_s\) is applied to the DC motor of slave through the output channel of data acquisition board and a power amplifier. The actual angle of rotation of the slave
\( \theta_i \) is sensed through an encoder mounted on the slave axis of rotation and the signal is sent to the input channel of data acquisition board. The difference of the commanded motion \( \theta_m \) and the actual motion \( \theta_i \), \( E_{\theta} \), modifies the gain in the motion control and consequently the calculated current sent to the slave DC motor.

Figure 6.5: Block diagram of telerobotic system

For the force replication component, once the slave encounters an external object the generated force is sensed by a force sensor mounted on the slave arm. This force is sent to the input channel of data acquisition board. The required current \( I_m \) to generate the same amount of torque, exerted to the slave, in the master device is computed through mathematical equations based on kinetic theory. Consequently the operator will feel the generated resistive torque in the master as he/she moves the joystick handle. The torque
applied by the operator’s hand to the joystick handle is measured using a torque sensor mounted on the master. The signal is sent back to the data acquisition board. The difference between the required torque and the generated torque $E_F$ is fed back to the force feedback control algorithm to adjust the gain and consequently the applied current to the MR damper in the master. The system is transparent if the control errors with respect to the motion, $E_\theta$, and force, $E_F$, are negligible for different working frequencies, the frequencies that can be applied by the operator to the master joystick. The system can be called transparent if the control errors $E_\theta$ and $E_F$ diminish to zero rapidly.

To investigate transparency, a reference input with three different frequencies is applied to the SDOF (slave) system (Figure 6.6a). The Forces, $F_s$, generated when the probe of the slave comes in contact with the environment are transmitted by a force transducer to the data acquisition system. Concurrently a signal is sent back from the potentiometer, which records the displacement of the end effector. These signals together with the model of MR damper in the master combine to deduce the current in the coils that will generate similar forces to the operator as encountered by the slave. If the current sent to the MR damper and the consequent generated force, $F_m$, in the master are computed exclusively as functions of the displacement signal, the frequency of the reference input has no effect on the force. This is highlighted by the differences in the $F_m(\theta)$ and $F_m(F_s)$ plot in Figure 6.6b. The root mean squared error for $F_m(\theta)$ is 2.86 N and for
is 0.212 N. The addition of the displacement signal allows one to infer if the end effector is getting closer to the object or moving away from it.

Figure 6.6: Transparency result of SDOF system: (a) Reference input; (b) Measured force at the slave ($F_s$) and generated force in the master ($F_m$) as a function of encoder signal ($\theta$) and force transducer signal ($F_s$) from slave probe.
6.3 Conclusions

The work developed in this chapter examines the stability and transparency of single and two degree of freedom haptic systems which utilize MR fluid dampers to provide force feedback. The relative simplicity and quick dynamics of an MR damper makes it a viable option for a myriad of haptic applications. This chapter presents the design and implementation of a SDOF rotary MR force feedback system, and a 2-DOF MR force feedback joystick. Carbonyl iron powder and silicone oil was used to make the required MR fluids. MR sponge based dampers were designed using an interactive MR damper design toolbox developed by the authors to simultaneously meet torque and magnetic circuit requirements. The results show good achievement in force feedback based control of both SDOF and 2DOF systems. The model and control algorithm we developed is shown to replicate the forces encountered by the slave in the master. An additional feature of this research is that it highlights the main advantage of an MR-based haptic system over an active one, its inherent stability. By coupling a force transducer with an encoder in a rotary slave (end effector) along with a spring and DC motor in the master, the restoring force exerted by the environment to the end effector is simulated and generated in the joystick. With the general issues of modeling, control and stability resolved, these systems can naturally be extended to multi degree of freedom force feedback systems that are appropriate for telerobotic surgical systems currently being utilized.
CHAPTER 7

ORTHOPEDIC KNEE BRACE

To address the muscle strengthening problem of knee braces, we investigated several MR fluid-based devices. The first approach is to use permanent magnets for a passive MRF-based knee brace. Since magnets do not have a variable magnetic field, the user would have to change the number or size of magnets. Consumer magnets are inexpensive but do not produce large magnetic fields as the more expensive Neodymium magnets. The experimental results show the feasibility of using MR fluid in rehabilitative knee braces.

In this chapter, a passive and a novel MR fluid-based active knee brace is modeled, designed, prototyped, and tested. The experimental results show better performance and larger resistive torque and dynamic range comparing to the passive design. The new device is continuously and smoothly controllable which results in an easily controllable variable stiffness system.

According to United States Food and Drug Administration, designers must consider the following human factors: (1) leg length, (2) reachable controls, (3) portability, (4) structure compatibility, and (6) “compactness”. The device must accommodate for range
of users of different heights. The controls must be readily accessible and visible to the user. The structure, compatibility and compactness of the device must be such that the user can take, use and store it in different environments. These are important for determining if the device is “safe and efficient”. The scheme for employing an MR fluid to transform a passive knee brace into an active knee brace is discussed in the topics below.

7.1 Passive Knee Brace

7.1.1 Analysis and Design

To investigate the feasibility of MR fluid based knee braces, the first knee brace designed is a passive MR fluid knee brace. It consists of three low carbon steel plates separated by MR fluid sponges and strapped arms to attach to the users leg for torque transmission as seen in Figure 7.1.

The device is designed based on minimum active fluid volume (MAFV) technique. The MAFV for the device can be quantified by relating its fluid constant $k$, dynamic range $\lambda$, yield stress $\tau_y$, viscosity $\eta$, and mechanical power dissipation $W_m$ in equation (7.1)

$$V = k \left( \frac{\eta}{\tau_y} \right) \lambda W_m$$

(7.1)
Figure 7.1: Schematic of the passive MRF-based knee brace
Figure 7.2: Prototyped passive MRF-based knee brace; (a) side view; (b) top view

The resistive torque $T$ can be determined by relating it to the geometry of the device and its yield stress as shown in equation (7.2). The maximum resistive torque is achieved by attaching two large permanent magnets to the outer and inner plates (Figure 7.2a).

$$T = \frac{2\pi r_y}{3} \left( r_o^3 - r_i^3 \right)$$  \hspace{1cm} (7.2)

where $r_o$ and $r_i$ are outer radius and inner radius respectively.

Determination of the dimensions of the outer plates was done by relating a desired torque to their radius. The process involved failure analysis. Below are the equations that were implemented to predict how the device components would perform under certain stress. From inspection we found that failure is likely to occur in section B-B (Figure 7.3). Strength of materials analysis shows that if the applied force on this material along the y-axis as designated in the picture is greater than $0.5F_y$ then the material will fail. This basically means that if for example, the material yield stress, $F_y$, is larger than 30.9 lbs
then the material exhibit some kind of failure. The critical section is at B-B with rectangular dimensions \( b=2.667\,mm \) and \( h = 30.175\,mm \). The average shear stress at section B-B turns out to be \( 0.37F \). The maximum shear stress at section B-B is \( 0.056\tau_{av} \). Further the normal stress turns out \( 1.21F \).

![Figure 7.3: Critical section for system design](image)

With \( S = b \frac{h^2}{6} = 420.36\,mm^3 \) and a factor of safety F.S.=1.65 we get

\[
\sigma_{\text{max}} \leq \sigma_{\text{allow}} \Rightarrow 0.403F \leq 0.6F_y \Rightarrow F \leq 1.5F_y \tag{7.3}
\]

\[
\tau_{\text{max}} \leq \tau_{\text{allow}} \Rightarrow 0.018F \leq 0.4F_y \Rightarrow F \leq 22.22F_y \tag{7.4}
\]

Where \( S, \sigma_{\text{max}}, \tau_{\text{max}} \) and are section modulus, maximum normal stress, and maximum shear stress respectively.

Therefore we should have the following constraint

\[
F \leq 1.5F_y \text{ (N)} \tag{7.5}
\]

Assuming the yield stress to be \( F_y = 275\,Mpa \) we conclude
\[ \Rightarrow F_1 \leq 412.5N \quad (F_1 \leq 92.7lb) \quad (7.6) \]

Similarly, bending about minor axis results in

\[ S_{xy} = h \frac{b^2}{6} = 36.45 \text{mm}^3 = \frac{S_{xx}}{11.53} \quad (7.7) \]

\[ \Rightarrow F_2 \leq \frac{412.5}{11.53} = 35.8N \quad (F_2 \leq 8.0lb) \quad (7.8) \]

And finally combination of stresses leads us to

\[ \frac{M_{xx}}{S_{xx}} + \frac{M_{yy}}{S_{yy}} = \frac{F_1 \times 169.5}{420.36} + \frac{F_2 \times 169.5}{36.45} \leq 0.6F_y \quad (7.9) \]

Or

\[ F_1 \leq 2.48 \left( 0.6F_y - 4.65F_2 \right) \quad (7.10) \]

Figure 7.4: Allowable applied forces to the device (the area shown with red arrows is the feasible area)
The resistant torque of the device can be computed by the following equations

\[
T = 2 \times \int_{0.025}^{0.04318} \tau_y \times R \times dA = 2 \times \int_{0.025}^{0.04318} \tau_y \times R \times 2 \times \pi \times R \times dR = 4 \times \tau_y \times \pi \times \frac{R^3}{3} \left|_{0.025}^{0.04318} \right.
\]

\[
= 3.37 \times 10^{-4} \times \tau_y \ (N \cdot m)
\]

Having a yield stress \( \tau_y = 45kPa \) results in \( T = 15.16N \cdot m \)

A Graphical User Interface (GUI) developed that calculates minimum active fluid volume, sponge thickness, plate radius, and sponge half length. This GUI is very useful if pertinent information (viscosity, dynamic range, etc.) is supplied.

### 7.1.2 Experimental Results

The empirical maximum resistive torque is determined using an MTS machine (Figure 7.5). A linear displacement is applied to the MTS machine and the compression force is measured using a force cell. The test is done in three stages: (1) With no magnets which represents zero magnetic field to measure the off-state resistance; (2) With one magnet on one side; (3) With two magnets on inner and outer sides. The corresponding resistive torque for each test is presented in Table 1. The results show the resistive torque increases as the number of magnets and consequently the magnetic field increases.
Figure 7.5: Test of passive knee brace in MTS machine

<table>
<thead>
<tr>
<th># of magnets</th>
<th>T (N.m)</th>
<th>dynamic range $\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.25</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0.95</td>
<td>3.8</td>
</tr>
<tr>
<td>2</td>
<td>1.8</td>
<td>7.2</td>
</tr>
</tbody>
</table>

Table 7.1: Maximum damper resistance torque for different number of magnets
Figure 7.6 shows that if there are no magnets attached then the maximum torque will be 0.25 N.m, when 1 magnet is attached then the maximum torque will be 0.95 N.m, and if there are 2 magnets attached then the maximum torque will be 1.8 N.m. This essentially implies that the amount of torque that can be produced increases with magnetic field strength.
7.2 Active Knee Brace

7.2.1 Analysis and Design

Having tested the feasibility of passive MR fluid-based knee brace, the research is extended to active knee brace. A novel MR fluid-based active knee brace is designed in such a way that a solenoid is incorporated into the device to generate a controllable magnetic field. This user controlled magnetic field activates the MR fluid and provides variable resistance to the rotary motion of the knee joint. The configuration of the device is as shown in Figure 7.8. The arms of the device are fastened to the upper and lower limbs of the leg. One arm is connected to the housing and remains stationary. The other arm is connected to a shaft and rotary piston that fits axially through the device. The
intended rotary motion will be supplied by the patient’s leg, which is connected to the device arms, when the knee is flexed or extended.

The device is designed based on analytical electromagnetic modeling and analysis and two dimensional stress distribution. The device geometry is modified in such a way to get stress ratio less than one along the device, minimize the total stress and corresponding weight of the device components, and to maximize the developed shear stress in the MR fluid.

Figure 7.8: Active knee brace; (a) Schematic (b) Prototyped (Saturated foam with MR fluid is placed between the rotary piston and housing)

The rotary piston and housing are made from low carbon steel 1018 and the rest of the parts from Aluminum 5051-T6. A 24 gauge copper coil with 500 turns, creating 9 ohms
of resistance, is used for the solenoid winded around the rotary piston. A magnetic field strength $H=500 \text{ kAmp/s}$ is produced when a current $I=4 \text{ Amps}$ is applied to the solenoid. A transmissive optical encoder is placed on the knee brace to measure the angle of leg limbs (or device arms).

The electromagnetic analysis and design of the system is based on the following calculations. The dimensions of the damper are shown in Figure 7.9.

![Figure 7.9: Dimensions of MR fluid-based damper](image)

The total reluctance of the device is given by

$$R_T = \sum_{i=1}^{n} \frac{L_i}{\mu_i A_i}$$

(7.12)
where \( L_i, A_i, \) and \( \mu_i \) are the length, cross sectional area, and permeability of element \( i \) in the magnetic flux path. The magnetic flux \( \phi \) and magnetic flux density \( B \) are given by

\[
\phi = \frac{NI}{R_T}, \quad B = \frac{\phi}{A}
\]  

(7.13)

where \( N \) is the number of coil turns and \( I \) is the applied current to the solenoid. The corresponding generated shear stress in MR fluid is found from the experimental results of MTS machine in the next sub-section. And finally the resistant torque is found by

\[
T = \tau AR
\]  

(7.14)

The damper geometry and dimensions shown in Figure 7.9 are found such that the maximum generated torque \( T \) in on state is equal to 10 N.m and the size of the damper is small enough to be easily portable.

The arms of the knee brace are designed such that the maximum stress ratio (ratio of existing stress at an infinitesimal point to allowable stress of the material) within the arms is approximately equal to 1. The maximum normal stress in the arm is computed by

\[
\sigma_{\text{max}} = \frac{M_x}{S_x} + \frac{M_y}{S_y} = \frac{F_y z}{S_x} + \frac{F_z}{S_y}
\]

(7.15)

Where \( S_x, S_y, \) and \( \sigma_{\text{max}} \) are section modulus about \( x \) and \( y \), and maximum normal stress respectively. \( F_x \) and \( F_y \) are forces exerted to the knee brace arm by limbs of the leg along \( x \) and \( y \) directions respectively. The stress ratio is given by

\[
S.R. = \frac{\sigma_{\text{max}}}{\sigma_a}
\]  

(7.16)
where $\sigma_a$ is the allowable stress of the material of the arm in normal stress (tension or compression). The maximum normal stress and the corresponding maximum stress ratio occurs at point A as shown in Figure 7.10a. The maximum stress ratio of point A as a function of forces $F_x$ and $F_y$ is plotted in Figure 7.10b. The red line shows the locus of forces that generate stress ratio equal to one at point A.

It should be noted that as a result of exerted forces $F_x$ and $F_y$, there are also shear stresses generated in two perpendicular directions in cross sectional area (x-y plane). But the corresponding stress ratios are low comparing to the stress ratio of normal. Also the maximum normal stress is occurred where the shear stress is zero so there is no need to calculate the Von Mises stress.

Figure 7.10: Plot of maximum stress ratio (S.R.) in the arm as a function of $F_x$ and $F_y$
7.2.2 Experimental Results

The prototyped knee brace is tested using an MTS machine, a commonly used tensile testing system. To measure the actual generated torque in the damper, the arms of the knee brace are attached to the MTS grippers using two pins to simulate simple supports (Figure 7.11a). This guarantees negligible resistant torque between the arms and the grippers; therefore, by writing the equilibrium equations the generated resistant torque can be computed.

![Figure 7.11: (a) MTS test setup; (b) Geometry of device; (c) Change in geometry while doing the test, the dash line shows the initial position and the solid line shows the current position](image)

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The test is conducted by applying a controlled linear displacement to the device. This causes linear decrease in length $l_3$ shown in Figure 7.11b and Figure 7.11c. The tests were done several times for different values of applied current to the knee brace solenoid using a variable power supply. The applied linear displacement and measured forces for different values of applied currents are shown in Figure 7.12. The generated torque in the knee brace damper can be computed by

$$T = F \cdot h_2$$  \hspace{1cm} (7.17)

where $F$ is the measured force using the MTS load cell (Figure 7.11a) and $h_2$ is found by the following equation (see Figure 7.11c)

$$h_2 = \frac{2}{l_3} \sqrt{s(s-l_1)(s-l_2)(s-l_3)}; \quad s = \frac{1}{2}(l_1 + l_2 + l_3)$$  \hspace{1cm} (7.18)
The corresponding torque is plotted versus time in Figure 7.13a. Though the measured force is decreasing after about 1 second, the length $h_2$ is increasing based on equation (7.18) such that the corresponding torque $T$ reached its steady state value and is constant in less than a second. The maximum torque for each test is plotted versus its corresponding applied current in Figure 7.13b. This curve does not show saturation of torque. The reason is the maximum applied current to the solenoid was limited to 4 amps to avoid heating up of the coil. For larger applied current the curve would show the saturation phenomenon.

Figure 7.13: Generated resistant torque of the knee brace

The knee brace generates a maximum torque of 6 N.m when the applied current is 4 Amps. In the absence of magnetic field, the off-state ($I=0$ Amps), the device exhibits a 0.8 N.m resistant torque; Therefore, the knee brace damper has a dynamic range (ratio of maximum generated resistant torque in on-state to that of off-state) of 7.5.
7.3 Conclusions

A passive MR fluid-based knee brace is designed and prototyped. Having tested the feasibility of the device, a MR fluid-based active knee brace is designed and prototyped. The experimental results show variable resistant torque that is continuous and controllable. The resistive torque can be varied to fit the requirements of the individuals using the device by controlling the applied current. The applied current and consequently the resistive torque can be function of the knee brace arms angle (or limbs of leg). The dynamic range of the device is 7.5. Based on the FDA regulations, this novel MR fluid-based knee brace may be considered “safe and efficient”.
CHAPTER 8

STEER-BY-WIRE SYSTEM

8.1 Force Feedback Control Strategy

As operational information is conveyed by means of electronic signals, there is no mechanism for information feedback to the driver on actual road-surface conditions, vehicle status, and cautionary information. This is the most important problem with computer control of steering systems. Such deficiency could be eliminated by capabilities of haptic technology. By employing force feedback technology, the drive-by-wire system conveys required information to the driver in tactile form. Feedback would certainly be required in order to give the driver meaningful information about what is happening at the road wheels. For instance, feedback on irregularities in the surface of an unpaved road is transmitted via movement of the steering wheel. Furthermore, the force feedback technology allows the communication of cautionary information to the driver through vibration or cessation of operation, such as a warning when the driver is deviating from lane, is following too closely, or has become drowsy. Sensors are required for steering
position and velocity both at the steering wheel and the road wheels. Torque measurements are required for the road wheels as well as the steering wheel when force feedback is applied.

Figure 8.1: Steer-by-wire control strategy

The steering wheel has an encoder measuring the steering wheel angle. For redundancy sake, another revolving digital sensor could be used. The magnetorheological fluid based damper is connected to the steering wheel shaft to give some feedback to the driver. By applying a current to this MR damper, it is possible to create a resistant torque when the driver rotates the steering wheel. The torque is dependent on the steering wheel angle or the difference between the steering wheel angle and the angle of vehicle wheels. There should also be a speed dependent behavior, restricting the turning angle at high speed. There are three main requirements for a steer by wire power supply: sufficient power for the steering actuators, activating MR damper, and redundancy which may require multiple power systems. The power wiring also has to be protected against single point
failure, which may be achieved by multiple wires from the power supplies to the critical components. The feedback control methodology is shown in Figure 8.1.

The equation of motion of MR damper can be written as

\[ J_D \ddot{\theta} = T_D - 2(2\pi RL)\tau R - k_D \theta \]  

(8.1)

### 8.2 Design of MR Fluid-Based Steer-By-Wire System

For this application of MR dampers we have the master (steering wheel) in the real world, while the slave (vehicle) is in a virtual reality environment. The steering wheel has an encoder measuring the steering wheel angle. For the sake of redundancy, another revolving digital sensor could be used. The MR damper is connected to the steering wheel shaft to give some feedback to the driver (Figure 8.2: Steer-by-wire system alternative one). By applying a current to the MR damper, it is possible to generate a resistance torque when the driver rotates the steering wheel. The torque is dependent on the steering wheel angle or the difference between the steering wheel angle and the angle of vehicle wheels. There should also be a speed dependent behavior, restricting the turning angle at high speed. There is an encoder on steering wheel shaft to measure the angle of rotation.

There are two alternatives for the design of dampers depending on the location of damper in the system. In the first alternative, the damper is installed on the axis of the steering wheel. This case leads to design of a large damper to resist a design torque of 15 Nm.
Figure 8.2: Steer-by-wire system alternative one

In the second alternative, the DC motor in the system is replaced with a MR damper. This results in the design of a smaller damper. In this case the resistance torque of the damper is multiplied by a factor of 40 because of use of two gears that are placed in series. Another advantage of the second design alternative is the minimum cost and lack of need for an external power supply. The damper is designed such that a current of 0.75 Amps will be enough to energize the damper to get the required shear stress and resistance.
torque out of system. The only disadvantage of this case is the off-state resistance torque of the damper which is obviously folded 40 times. For this reason the MR fluid is made and damper is designed such that we get the lowest possible off-state viscosity and shear stress when the damper is not activated. A 24 AWG gauge copper coil with 300 coil turns is used for the damper. The damper is designed to generate a resistance torque of 1.0 N.m (Figure 8.3).

![Figure 8.3: MR fluid based dampers](image)

The damper is used in a Logitech WingMan steering wheel game controller. The device has a built in force feedback system that is derived by a DC motor (Figure 8.4). This motor is removed and replaced by the MR damper (Figure 8.5). The most important advantage of doing this is replacing an active actuator with a semi-active damper which results in system stability. The control strategy that is used for driving the DC motor sometimes makes the system unstable. On the other hand the semi active MR damper will
never cause instability because it never adds energy to the overall system it just removes it.

Figure 8.4: Steer-by-wire system with DC motor

The steering wheel rotation angle, the accelerator, and brake are sensed by the built in encoder and potentiometers. As the user rotates the steering wheel the angle of rotation is sensed by an encoder and a signal is sent to the computer through a USB port. As the wheels of the car hit obstacles in virtual reality environment a proportional current is sent back to the MR damper which causes resistant torque sensed by the user. If the user now reverses the direction of the steering wheel there is no way of replicating the restoring force that the wheels encountered due to the stiffness of the depressed obstacle. In fact when one increases the current to the MR damper of the steering wheel during this movement the device will slow down or stop giving the user a “sticking like” sensation called the “sticky wall” phenomenon. To solve this problem two remedies have been studied: restoring linear springs to bring the steering wheel back to its original position
and a torque sensor on the shaft. As soon as the torque changes direction or actually changes sign then the MR damper gets deactivated.

![MR Damper diagram](image)

Figure 8.5: New MR damper based steer-by-wire system

### 8.3 Virtual reality environment for steer-by-wire

A Graphical User Interface (GUI) has been developed for controlling the steer-by-wire device. The code is developed in Visual Basic and Visual .Net. The damper is activated when the car is very close to the curbs. The code used the DirectX for graphical and I/O communications with the device through a USB post. Microsoft® DirectX® is a set of low-level application programming interfaces (APIs) for creating games and other high-performance multimedia applications. It includes support for two-dimensional (2-D) and three-dimensional (3-D) graphics, sound effects, music, input devices, and networked applications.
applications such as multiplayer games. The end user controls the car in virtual reality using the steering wheel, brake, and accelerator in real world (Figure 8.6).

Figure 8.6: Virtual reality environment to simulate steer-by-wire system

### 8.4 Experimental Results

Among all the MR fluid samples, sample number three generates acceptable large shear stress in on-state mode while maintaining a very low off-state shear stress. This fluid sample is used for saturating the sponge based MR dampers used in this study. A sponge is an effective way to prevent MR fluid leakage while not providing excessive friction caused by sealing. The second alternative for MR damper design, small one, was finally chosen for the system (Figure 8.3). By applying a current of 0.75 Amps it generates the
requisite torque the damper was designed for. The steering wheel is used to steer the car in virtual reality world. As the car gets close to the curbs a voltage signal is computed by the controller and is sent to damper to energize the MR fluid and subsequently the steering wheel gets stiff and does not let the driver to rotate the wheel in that direction anymore. Two linear springs are used to generate the restoring force to bring the steering wheel back to its original position. Also to solve the sticky wall phenomenon the driver can easily press one of the buttons on the wheel to deactivate the MR dampers. A torque sensor can also be placed on the steering wheel to solve this problem.

The voltage to be sent to the MR damper was derived based on Force Editor Software (Figure 8.7). In this software the type of force (Constant, Ramp, Square, Sine, Triangle, Sawtoothup, Sawtoothdown), duration, magnitude, and direction can be defined by the user. Other features include definition of friction, damper, spring, and inertia which were used when the DC motor was installed on the system. When DC motor is replaced by MR damper the shape of force will not have much effect on the performance. It is just the duration and magnitude that sends the required output voltage and current to the damping device. The Force Editor is mainly developed for force feedback devices with two axes like joysticks. To apply it to a 1-D force feedback system, steer-by-wire device, the direction angle of 270° generates the largest effective force.
8.5 Conclusions

Haptic systems greatly increase the effectiveness of a human machine interface. However, these systems use passive devices in force feedback systems that are unable to recreate a reactive force that helps in disengaging an object. The relative, simplicity and quick dynamics of an MR damper makes it a viable option for a myriad of haptic applications. This chapter presents the design and implementation of a MR fluid based damper for steer-by-wire systems. Carbonyl iron powder and silicone oil is used to make the required MR fluid. MR sponge based dampers are designed using an interactive MR damper toolbox to simultaneously meet torque and magnetic circuit requirements. The MR damper is used in a Logitech Wingman steering wheel device. The most important
improvement of this system over the original one is the stability issue. Despite the original system, which used a DC motor for force feedback control, the new designed system does not enforce any external energy to the system and consequently there will be no instability. The results show good achievements in force feedback controlling of the system.
9.1 Graphical User Interface (GUI)

An interactive toolbox has been developed for the modeling, analysis, and design of MR fluid-based devices. This GUI provides unique and advanced analysis and design tools for students in academia as well as engineers and researchers in industry. The overall features of this toolbox are listed below:

- MR fluid modeling
- MR damper analysis and design
- MR brake analysis and design
- Sensitivity analysis
- Optimization of MR damper design
- Automatic report preparation
- MATLAB interface
• SI and English system of units

This software package is named MRFMAD® (Magneto-Rheological Fluid Modeling, Analysis, and Design) and is in the process of licensing under OSU license disclosure. Different parts and elements of the package are described in the following sub-sections.

9.1.1 MRFMAD® Output

The information for analysis and designing of MR fluid based devices can be stored and retrieved in text file format as well as database format. Furthermore the software produces a detailed report sheet with all the equations used and analysis/design process. The report can be saved as a rich text format (RTF) file. A sample of MRFMAD® report sheet is shown in Appendix B.

9.1.2 MR Fluid Modeling

This part of the MRFMAD® software package provides the user with different 1-D models of MR fluids. Four different versions of our kinetic-based models with four assumptions on applied magnetic forces are included in this software. The software package has the following models as well: Bingham Plastic, Herschel-Bulkley, Rosensweig, and Dorfmann. Based on the selected model, the resulting shear stress versus shear rate and shear stress versus magnetic field graphs are calculated and produced. This part also includes sensitivity analysis of MR fluid shear stress with respect to magnetic susceptibility, temperature, carrier fluid viscosity, constant C and friction $\xi$ in kinetic-based model.
In reality the magnetic susceptibility, $\chi = \frac{M}{H}$, decreases as the magnetic field increases. This is due to the saturation of magnetization $M$. Some of the MR fluid models consider this phenomenon and some does not. In calculating the shear stress or sensitivity analysis in MRFMAD®️, the user can use either constant or variable magnetic susceptibility assumptions.

![MRFMAD®️ Modeling](image)

Figure 9.1: MRFMAD®️ Modeling
9.1.3 MR Fluid Damper Analysis/Design

The MR fluid damper analysis and design is done in this part. Based on the geometry and dimensions of the damper, the material properties, wire gage and number of coils, and the MR fluid specifications, the magnetic circuit is analyzed and the maximum generated shear stress is calculated. Consequently the maximum resistive force or torque is computed for linear and rotary dampers respectively. The power consumption of the damper is calculated as well. Also the number of coil spools can be defined. A scaled version of the damper is updated and drawn automatically. This part includes the sensitivity analysis with respect to the damper dimensions.

Figure 9.2: MRFMAD® Damper

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9.1.4 MR Fluid Brake Analysis/Design

This part is very similar to the MR damper analysis/design part. The difference between a brake and damper is the placement of solenoid. In MR damper the solenoid wound around the inside piston while in MR brake it is wound inside the housing (sleeve). The MRFMAD® brake has all the features of MRFMAD® damper. In addition, it calculates the minimum active fluid volume. For more details please see section 7.1.1.

Figure 9.3: MRFMAD® Brake
9.1.5 Sensitivity Analysis

One of the useful features of this toolbox is sensitivity analysis, which can be used for the optimum design of MR fluid-based devices in general. Sensitivity analysis clarifies how the magnetic flux density and consequently the resistive force/torque generated by MR linear/rotary damper is changed with respect to changes in MR damper dimensions, wire gage size, and current.

![Figure 9.4: MRFMAD® Sensitivity Analysis](image)

Figure 9.4: MRFMAD® Sensitivity Analysis
9.1.6 Optimization

This part optimizes the MR damper dimensions based on the defined optimization methodology. The available options are: weight optimization, cost optimization, dimension optimization (length or diameter), power optimization, and force/torque optimization.

Figure 9.5: MRFMAD® Optimization
9.1.7 Flow Mode

This part calculates the generated force in flow mode (fixed plate) operation, in which the boundary plates are stationery and the MR fluid flows.

\[ F_r = c \frac{\tau_s L A_p}{d}, \quad c = (2 \rightarrow 3) \]  \hspace{1cm} (9.1)

Figure 9.6: MRFMAD® Flow
9.1.8 Stress Analysis

This part calculates the normal and shear stress of arms. The stress ratio graph based on interaction of shear forces $F_x$ and $F_y$ acting on the arm is calculated and generated. This tool can be used to optimize the size of the arm (width and thickness). The user can check the stresses and resulting stress ratio at each point of the arm.

Figure 9.7: MRFMAD® Stress
9.1.9 Gear

Sometimes it is required to use gears in the damper/brake design to reduce the overall size of the system. Such designed is done for the 5-DOF system described in chapter … of this dissertation. This part can read the data of worm gear and rack and pinion gears from vendor websites such as McMaster CAR. It draws the scaled schematic of the gears and shows the gear ratio. It is a very useful tool in finding the required gears for the design.

Figure 9.8: MRFMAD® Gear
9.1.10 MR Fluid Hinge

This part is used to design several types of MR fluid-based hinges and locking mechanisms. This part is designed specifically for designing door hinges and locking mechanism but can be developed and used for other types of hinges and locks. The idea behind the MR fluid-based door locking mechanism is adding one more security level to the mechanical locking of the door. In this case only a secured electronic/digital code can deactivate the locking to open the door. Of course a backup power source is required for safety issues.

![MRFMAD® Hinge](image)

Figure 9.9: MRFMAD® Hinge

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CHAPTER 10

CONCLUSIONS AND FUTURE WORK

10.1 Research Summary and Contributions

Haptic systems greatly increase the effectiveness of a human machine interface. However, these systems use passive devices in force feedback systems that are unable to recreate a reactive force that helps in disengaging an object. The relative, simplicity and quick dynamics of an MR damper makes it a viable option for a myriad of haptic applications.

The overall goal of the research done in this dissertation was to develop next generation force feedback systems by combining novel Magnetorheological (MR) fluid based electromechanical systems with microstructural analysis and advanced control system design. Four MR fluid based systems were designed, prototyped and tested with medical applications: A two degree of freedom (2-DOF) force feedback joystick and a 5-DOF force feedback manipulator for telerobotic surgery application, a passive and a semi-
active orthopedic knee brace for rehabilitation application. Furthermore, a force feedback steering wheel was modified using MR damper with application to steer-by-wire automobiles. The test results showed the appropriate performance of MR fluid based systems used in haptic and force feedback applications. The motivations, methodologies, and results of the dissertation are shown in Figure 10.1.

Figure 10.1: Motivations, methodologies, and results

A methodology for deriving generalized 3-D tensorial expressions relating forces, flow, and applied magnetic field in MR fluid is developed. In the special case of simple shear
the dependence of $\tau_{12}$ on $H$ presented by equation (2.24) is in agreement with observed behavior. This 1-D expression for shear stress replaces the strict yield stress of the Bingham/Herschel-Buckley-type visco-plastic models (i.e. absolutely no flow until the shear stress reaches a specified value) with a steep gradient, which is in better agreement with experimental observation.

Carbonyl iron powder and silicone oil was used to make the required MR fluid. MR sponge based dampers were designed using an interactive MR damper toolbox to simultaneously meet torque and magnetic circuit requirements. The results show good achievements in force feedback control of the system. The operator can sense the stiffness of external objects and distinguish hard and soft objects. These systems can naturally be extended to those appropriate for telerobotic surgery.

In rehabilitative knee brace application, a passive MR fluid-based knee brace is designed and prototyped. Having tested the feasibility of the device, a MR fluid-based active knee brace is designed and prototyped. The experimental results show variable resistant torque that is continuous and controllable. The resistive torque can be varied to fit the requirements of the individuals using the device by controlling the applied current. The applied current and consequently the resistive torque can be function of the knee brace arms angle (or limbs of leg). The dynamic range of the device is 7.5. Based on the FDA regulations, this novel MRF-based knee brace may be considered “safe and efficient”.

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In steer-by-wire application, an MR damper is designed and used in a Logitech Wingman steering wheel device. The most important improvement of this system over the original one is the stability issue. Despite the original system, which used a DC motor for force feedback control, the new designed system does not enforce any external energy to the system and consequently there will be no instability. The results show good achievements in force feedback controlling of the system.

### 10.2 Future Work

- Modify the microstructural model of MR fluid by considering the hysteresis of MR fluids and temperature effect.
- Adding force feedback to the 5th degree of freedom of the 5-DOF telerobotic system for grabbing.
- Adding force sensor to the knee brace arms for force feedback control
- Research on tactile feedback (texture recognition)
APPENDIX A

MACHINE DRAWINGS
A1- Two Degree of Freedom Joystick

Figure A.1: Two DOF haptic joystick
Figure A.2: Base Plate, Detailed Drawing
Figure A.3: Bracket, Detailed Drawing
Figure A.4: Support X axis, Detailed Drawing
Figure A.5: Support Y axis, Detailed Drawing
Figure A.6: Joystick handle, Detailed Drawing
Figure A.9: Y-Y housing, Detailed Drawing
Figure A.10: X-X damper, Detailed Drawing

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Figure A.11: Five DOF manipulator, Detailed Drawing
Figure A.13: Shaft-Damper, Detailed Drawing
Figure A.14: Shaft 1, Detailed Drawing
Figure A.16: Damper, Detailed Drawing
Figure A.17: Box-Front, Detailed Drawing
Figure A.18: Box-Base, Detailed Drawing
Figure A.19: Box-Back, Detailed Drawing
Figure A.21: Support-Encoder, Detailed Drawing
Figure A.22: Active knee brace, Detailed Drawing
Figure A.23: Arm, Detailed Drawing
Active Knee Brace
Part name: Housing

Designed by: Farzad Ahmadkhanlou
KSL - OBU
Material: Low Carbon Steel (AISI 1010)
Quantity: 1
Date: 06/28/06
All dimensions are in inches
Figure A.25: Left Sleeve, Detailed Drawing
Figure A.26: Right Sleeve, Detailed Drawing
Figure A.27: Rotor, Detailed Drawing
Figure A.28: Shaft, Detailed Drawing
APPENDIX B

SAMPLE OUTPUT OF MRFMAD TOOLBOX
Calculation Sheet for MRF-Based Damper Design

Dimensions:
L = 15.875 mm
l = 38.1 mm
R = 19.05 mm
r = 3.175 mm
H = 1.5875 mm
h = 3.175 mm
t = 3.175 mm
Damper overall dimensions: 69.85 × 50.8 mm

Material:
\[ \mu_0 = 4\pi \times 10^{-7} \frac{H}{m} \]
\[ \mu_{\text{steel}} = 2000 \mu_0 \]
\[ \mu_{\text{MRF}} = 2.5 \mu_0 \]

Reluctance:
\[ R_t = \frac{L_t}{\mu_i A_i} \]
\[ R_i = \frac{L_i}{\mu_i A_i}, \quad L_i = L / 2 + L / 2 + l = L + l, \quad A_i = \pi \left[ (r + H)^2 - r^2 \right], \quad \mu_i = \mu_{\text{steel}} \]
\[ R_2 = \frac{L_2}{\mu_2 A_2}, \quad L_2 = R - r - \frac{H}{2}, \quad A_2 = 2\pi \left[ \frac{R + (r + H / 2)}{2} \right] L, \quad \mu_2 = \mu_{\text{steel}} \]

\[ R_3 = \frac{L_3}{\mu_3 A_3}, \quad L_3 = h, \quad A_3 = 2\pi \left( R + h / 2 \right) L, \quad \mu_3 = \mu_{\text{MRF}} \]

\[ R_4 = \frac{L_4}{\mu_4 A_4}, \quad L_4 = l + L, \quad A_4 = \pi \left[ (R + h + t)^2 - (R + h)^2 \right], \quad \mu_4 = \mu_{\text{steel}} \]

\[ R_T = \sum R_i = (R_1 + 2R_2 + 2R_3 + R_4) \]

<table>
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<th>( R_1 ) (1/H)</th>
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<td>1580177</td>
</tr>
</tbody>
</table>

Coil:
26 (AWG)
d=4.0386E-04 m
Imax=2 Amps

Design based on saturated shear stress
\[ \tau_{\text{max}} \rightarrow H \rightarrow B \rightarrow \phi \rightarrow \left( NI \right)_{\text{required}} \]

\[ A_{\text{max}} = 0.7l(R+0.9h-H-r) = 3.810476E-04 \ m^2 \]

\[ N_{\text{max}} = \frac{A_{\text{max}}}{\pi d_{\text{wire}}^2 / 4} = 2975 \]

Check based on applied current:
\( (NI)_{\text{actual}} \rightarrow \phi \rightarrow B \rightarrow H \rightarrow \tau \)

\[ NI=4800 \ \text{Amp-turns} \]
\[ \phi = \frac{NI}{R_T} = 3.037635E-03 \ \text{T/m}^2 \ (\text{weber}) \]
\[ B = \frac{\phi}{A} = 1.598674 \ \text{T} \ (\text{Tesla}) \]
H=844.9268 kAmp/m
\( \sigma = 45.79985 \ \text{KPa} \)

\[ A=2(2\pi RL) = 1.900097E-03 \ \text{m}^2 \]

\[ F = \tau A = 174.0483 \ \text{N} \]
\[ T = \tau A R = 3.315621 \text{ N.m} \]

**Aluminium shaft (2014-T6):**

\[ \tau_y = 172\text{MPa} \]

\[ \tau_{allow} = 0.4\tau_y = 69\text{MPa} \]

\[ r_{(shaft)} = 0.003175 \text{ m} \]

\[ J = \pi r^4 / 2 = 0.1596 \times 10^{-9} \text{ m}^4 \]

\[ \tau_{max} = \frac{T_{max} r}{J} \implies T_{max} = \frac{\tau_{max} J}{r} = 3.52 \text{ N.m} \]
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