Planar Cable Direct Driven Robot: Hardware Implementation

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ABSTRACT

The planar cable-direct-driven-robot was designed, constructed, simulated and controlled in this thesis. A new design of cross cable configuration was implemented. The kinematics, statics and dynamics modeling of the proposed design were derived. The static workspace was determined for the new design. Only the translational CDDR whose end-effector may be fitted with a traditional serial wrist mechanism to provide rotational freedom was considered in this thesis. The robot was simulated using Simulink and Matlab software. The hardware of the cable-direct-driven-robot was designed and constructed. The hardware was interfaced with the computer. Wincon software and Quanser control boards were used for real time implementation. The inverse kinematics of the robot was implemented for generating linear and circular trajectory in real time control. The independent cable length PD controller was implemented for the Cartesian coordinated control. The repeatability of the CDDR was evaluated.

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1. INTRODUCTION

1.1. Statement of Problem

The problem addressed in the study was to build and evaluate a planar translational 4-cable direct driven robot with two translational degrees of freedom and one rotational degree of freedom to generate a linear and a circular trajectory by controlling rotational angle to zero.

1.2. Background and Literature Review

Cable-direct-driven robots (CDDRs) are a type of parallel manipulator wherein the end-effector link is supported in parallel by n cables with n tensioning motors. In addition to the well-known advantages of parallel robots relative to serial robots, CDDRs can have lower mass and better stiffness than other parallel robots. Several CDDRs and cable-direct-driven haptic interfaces (CDDHIs) have been studied in the past.

An early CDDR is the Robocrane (Albus, et al., 1993) developed by NIST for use in shipping ports (see Figure 1-1). They have been experimenting for several years with new concept for robot cranes. These concepts utilize the basic idea of the Stewart platform like manipulator. The unique feature of the NIST approach is to use cables as the parallel links and to use winches as the actuators instead of hydraulic-cylinder legs. In this system, gravity ensures that cable tension is maintained at all times.
The “Charlotte” was developed by McDonnel-Douglas for use on the International Space Station and is shown in Figure 1-2. It was a rectangular box driven in parallel by eight cables, with eight tensioning motors mounted onboard (one on each other).
Arai, et al. (2002) presented another CDDR which is a hybrid drive parallel arm driven by cables and cylinders in order to design a compact handling arm with larger workspace. They developed a dexterous arm capable of controlling six degree of freedom (DOF) motion of heavy materials, presented a basic concept of parallel mechanisms with hybrid actuation and presented comparisons of various hybrid mechanisms based on their kinematics and statics. They also discussed workspace analysis of the arm based on the required specifications.

Barrette and Gosselin (2000) presented a systematic analysis of planer parallel mechanisms. They presented equations for the velocities and obtained forces in the cables by the principle of virtual work, a detailed analysis of the workspace and proposed a method for the determination of the boundaries of a two dimensional subset. They
introduced a new notion of dynamic workspace as its shape depends on the accelerations at the end-effector and demonstrated that any subset of the workspace could be considered of three-cable sub-workspaces, with boundaries being of two kinds: two-cable equilibrium loci and three cable singularity loci. They presented using a parametric representation that for the x-y workspace of a simple no-spring mechanism, the two-cable equilibrium loci represent a hyperbolic section, degenerating, in some particular cases, to one or two linear segments. They also presented a procedure for the determination of the x-y workspace boundaries using quadratic programming with examples.

Most proposed CDDRs and CDDHIs involve both translational and rotational motion of the end-effector link guided by cables. Williams (1998) presented a concept of cable-suspended haptic interface. He presented an interface to provide six degree of freedom (DOF) wrench (force and moment) feedback to a human operator in virtual reality. He presented mathematical transformations for forward pose kinematics, inverse jacobian matrix, and statics modeling. He also presented a method for tension optimization.

CDDRs and CDDHIs can be made lighter, stiffer, safer, and more economical than traditional serial robots and haptic interfaces since their primary structure consists of lightweight, high load-bearing cables. On the other hand, one major disadvantage is that cables can only exert tension and cannot push on the end-effector. Proper design is required to ensure that the translational CDDR end-effector has sufficient stiffness in all directions to resist the rotational moments.
Williams and Gallina (2001) presented a hybrid parallel/serial manipulator architecture where translational freedoms are provided by a cable direct driven robot (CDDR) and the rotational freedoms are provided by a serial wrist mechanism. They presented 3 cables and 4 cables CDDR with two and one degree of actuation redundancy respectively. They presented kinematics and statics modeling of CDDR. In kinematics modeling they presented a pose kinematics and velocity kinematics. They also presented a method to maintain positive cable tension. Williams and Gallina (2001) also presented a dynamics modeling of hybrid parallel/serial manipulator. Dynamic modeling is required for improved control as compared to using kinematics and statics modeling, when CDDR are to provide high velocities and accelerations in translational motion. They presented a Cartesian Dynamics model of the end-effector and actuator. They also presented a dynamics model of the whole system. They presented control architecture of the CDDR and a method to calculate optimal actuator torques. They also presented dynamics and control examples for the planer 4-cable CDDR with two degree of actuation redundancy. The hybrid parallel/serial manipulator architecture presented by Williams and Gallina is shown in Figure 1-3.
Figure 1-3: Hybrid Parallel / Serial Manipulator Presented by Williams and Gallina

Cable-suspended robots have some of the inherent workspace limitations that result from the robot being cable actuated. Roberts, et al. (1998) presented some issues concerning the inverse kinematics and statics of cable-suspended robots. They presented necessary and sufficient conditions for a cable-suspended robot to stay in a given configuration. Another important issue was the extent to which the cables constrain the robot. They presented conditions for completely constraining the robot and formulated the problems of achieving static equilibrium and fully constraining the robot in terms of the left null space of a manipulator inverse Jacobian. This null space formulation is used to study the fault tolerance of cable-suspended robots that are redundantly actuated.
In many kinds of industrial fields, necessity of high-speed robots has been increasing. Choe, et al. (1996) developed an ultrahigh speed robot manipulator with 6-DOF based on the parallel wire mechanism. They developed a robot, which had four wires to realize 3-DOF motions. They analyzed the internal stiffness of the four-wire robot and revealed that stiffness of some degrees of freedom cannot be effectively increased by internal force and hence vibrations problems are anticipated. They proposed a new type of wire driven robot whose motion is mechanically constrained in order to reduce vibration coming from elasticity of wires and demonstrated the effectiveness of the proposed robot through some experimental results.

Walairacht, et al. (2001) proposed a new string-based haptic interface device that allowed a user to use both hands and multi-fingers to directly manipulate the virtual objects in the computer simulated virtual environment. One of the advantages of the device was to allow the user to use both hands to perform the cooperative works of hands, such as holding a large object that cannot be grasped or held by a single hand. The haptic feedback sensation provided by the device at the fingertips made possible for the user to perform dexterous manipulation, such as manipulating small sized objects. They discussed the design of the proposed device and elaborated on the methods of fingertip poses measurement and force feedback generation.

Kawamura, et al. (1993) proposed a new type of master robot for tele-operation in which several wires are stretched from a handle which is moved by a human operator. They explained how many wires are necessary to control a handle with six degrees of
freedom in a three-dimensional space. They also proposed an algorithm to determine for each wire in tension to feed forces from a slave robot.

Shen, et al. (1994) discussed the manipulability for wire suspended mechanisms. They presented the sufficient and necessary mechanical conditions corresponding to the force constraints (such as wire tension should always be greater than or equal to zero) and also obtained a manipulability of a wire driven mechanisms under these conditions. They introduced a more practical evaluation index ‘set of manipulating forces’ and also introduced the way to calculate this index. They also derived both manipulability indexes for wire suspended mechanisms in the gravity field by modeling the gravitational force as a wire having gravitational tension. They also presented numerical examples to explain the proposed indexes and to show their validity.

1.3. Purpose of the Study

The purpose of this thesis was to carry out an action research by proposing a modified design of a planar cable-direct-driven-robot introduced by Williams and Gallina (2001), to derive kinematics, statics, and dynamics modeling of the redesign, to simulate the robot using Matlab and Simulink software packages, to design and build the hardware of a planar cable-direct-driven-robot for experimental verification and evaluate the design of the robot by experiments.
1.4. Thesis Organization

This thesis presents modeling, controls simulation, design and construction of the planar cable-direct-driven-robot. Chapter 2 describes a model of the proposed planer cable-direct-driven-robot and also covers kinematics, statics, and dynamics. Chapter 3 covers controls simulation of the robot. The control law, control architecture, calculation of optimal actuator torque and an example are included. Chapter 4 covers design and construction of the robot. Hardware designs and component selections are included here. Chapter 5 contains the experimental results. Following the conclusion are the references, appendix, and an abstract.
2. MODELING

2.1. Model Description

In this thesis a CDDR consists of a single end-effector rigid body supported in parallel by $n$ cables controlled by $n$ tensioning actuators. Figure 2-1 shows the planar 4-cable CDDR kinematics diagram. The concept of hybrid CDDRs is introduced, where the translational freedoms are provided by the $n=4$ cables and the rotational freedoms can be provided by a serial wrist mechanism mounted to the translational CDDR end-effector. Two possible cable configurations were considered: uncrossed-cables, shown in Figure 2-2 and crossed-cables, shown in Figure 2-3.

![Figure 2-1: Planar 4-Cable CDDR Diagram](image-url)
Figure 2-2: Uncross Cable Configuration

Figure 2-3: Cross Cable Configuration

The uncrossed-cable configuration can provide translational motion but it is not capable of resisting moments about the Z-axis from a serial wrist mechanism consisting
of a single revolute joint rotating about the Z-axis, mounted to the end-effector centroid.

In addition to that, for uncross-cable configuration if the end-effector is supported by a base plate in the XY plane, end-effector/base plate symmetry is not desirable as it causes uncertainty in the forward pose solution (R.L. Williams, 1998).

The crossed-cables configuration is sufficient in general to resist moments about the Z-axis from a serial wrist mechanism consisting of a single revolute joint rotating about the Z-axis, mounted to the end-effector centroid. The crossed-cables configuration is shown in detail in Figure 2-1 and Figure 2-3.

The translational portion of the problem is considered here; an attempt has been made to keep zero orientation by controlling \( \phi = 0 \) for all motion; \( \phi \), not shown in Figure 2-1, the angle between the horizontal end-effector side \( a \) and the horizontal ground link \( L_B \).

For 3-dof planar motions (2 translations \( XY \) and 1 rotation about \( Z \)) there must be at least three cables. Since cables can only exert tension on the end-effector, there must be more cables to avoid configurations where the robot cables can be slack and lose control. Figure 2-1 represents one degree of actuation redundancy, i.e. four cables to achieve the three Cartesian degrees-of-freedom \( \mathbf{X} = \{x \ y \ \phi = 0\}^T \). This scenario represents actuation redundancy but not kinematic redundancy. That is, there is an extra motor which provides infinite choices for applying 3-dof Cartesian wrench vectors, but the moving rigid body has only three Cartesian degrees-of-freedom \( \mathbf{X} = \{x \ y \ \phi\}^T \).

Figure 2-1 shows the inertially fixed reference frame \( \{0\} \) whose origin is the centroid of the base square. The base square has sides of fixed length \( L_B \). Each cable is
passed through the ground link at the fixed points \( A_i = \{ A_{ix}, A_{iy} \}^T \). The length of each cable is denoted as \( L_i \), and the cable angles designations are \( \theta_i \) \((i = 1, \cdots, 4)\). The moving end-effector frame \( \{ H \} \) is also shown in Figure 2-1. Note vector \( \{ x, y \}^T \) gives the pose of \( \{ H \} \) with respect to the \( \{ 0 \} \) origin, expressed in \( \{ 0 \} \) coordinates. The end-effector rigid body mass and mass moment of inertia are \( m \) and \( I \), and the lumped motor shaft/cable pulley rotational inertias for each actuator are \( J_i \) \((i = 1, \cdots, 4)\). The viscous damping coefficients \( c_i \) \((i = 1, \cdots, 4)\) at each motor shaft are also included to provide a linear model for the system friction. The cable pulley radius for each actuator is \( r_i \) \((i = 1, \cdots, 4\); not shown in Figure 2-1).

Theoretically the end-effector center can reach any \( x\)-\( y \) point within the base square (reduced on all sides by half the end-effector dimension, \( a/2 \)), if cable lengths can go to zero. A singular condition exists when the edge of the square end-effector aligns with an edge of the base square. In this case two adjacent cables align with the base plate edge; infinite force is required in the two adjacent cables to move the end-effector normal to the aligned cables. The other two cables cannot push so motion is restricted to this reduced base square; large cable tensions will be required as this edge of motion is approached.

Cable interference is a potential problem in CDDRs. Using crossed cables as shown in Figure 2-1, there will always be cable/cable contact for all motion; to avoid this problem either select low-friction cable materials to allow cables to slide freely over each other, or mount the cables in different planes, if the base plate sufficiently supports the
end-effector. In the design of Figure 2-1, cable/end-effector interference is non-existent in the useful motion range if orientation $\phi = 0$ is maintained by control. In the singularities at the edge of the useful motion range, two cables will have just touched the square end-effector side, even with $\phi = 0$. The potential exists for interference between cables and workspace items and/or humans, but this problem can be minimized by design in the case of planar CDDRs.

2.2. CDDR Kinematics Modeling

Kinematics modeling is concerned with relating the active joint variables and rates to the Cartesian pose and rate variables of the end-effector. The intermediate, passive cable angles and rates are also involved. Assuming all cables always remain in tension, CDDR kinematics is similar to in-parallel-actuated robot kinematics (e.g. L.W.Tsai, 1999; C.M.Gosselin, 1996); however, with CDDRs the joint space is over constrained with respect to the Cartesian space.

![Figure 2-4: Planar CDDR Kinematic Diagram](image-url)
2.2.1. Pose Kinematics

The inverse position kinematics problem is stated: given the Cartesian position \( \mathbf{x} = [x \ y \ \phi]^T \) calculate the cable lengths \( L_i \). The solution is simply calculating the Euclidean norm between the moving point \( \mathbf{x} = [x \ y \ \phi]^T \) and each fixed ground link vertex \( A_i \):

\[
L_i = \sqrt{\left(x - A_{ix} + h_{ix}c\phi - h_{iy}s\phi\right)^2 + \left(y - A_{iy} + h_{ix}s\phi + h_{iy}c\phi\right)^2} \quad i = 1, \ldots, n \tag{1}
\]

For use in velocity kinematics and statics, we require the cable angles:

\[
\theta_i = \tan^{-1}\left(\frac{y - A_{iy} + h_{ix}s\phi + h_{iy}c\phi}{x - A_{ix} + h_{ix}c\phi - h_{iy}s\phi}\right) \quad i = 1, \ldots, n \tag{2}
\]

The quadrant-specific inverse tangent function must be used in Eq. 2.

The forward pose kinematics problem requires the solution of over constrained coupled nonlinear equations and is more difficult. A Newton-Raphson numerical solution has been employed, where the over constrained Moore-Penrose pseudoinverse is used in the iteration.

The end-effector pose is described by \( {}^H_T \) (position \( {}^H_x \) and orientation \( {}^H_R \), using Z-Y-X \( \alpha, \beta, \gamma \) Euler convention (Craig, 1989):

\[
{}^H_T = \begin{bmatrix}
{}^H_R & \{{}^H_x\}
\end{bmatrix}
\]

where $c\alpha = \cos \alpha$, $s\alpha = \sin \alpha$, etc. A vector loop-closure equation is written for each cable (Figure 2-4 shows details for the third cable):

$$0A_i + 0L_i = 0x_H + 0R_Hh_i - 0H_i$$

(4)

The cable lengths $L_i$ are related to the Cartesian pose variables via the lengths (Euclidean norms) of vectors $0L_i$:

$$L_i = \|0L_i\| = \|0H_i - 0A_i\|$$

$$L_i = \|0x_H + 0R_Hh_i - 0A_i\|$$

(5)

Given the cable lengths $L_i$, the forward pose solution calculates the end-effector pose $0T_H$. This problem is not straightforward because it requires the solution of coupled non-linear equations and generally results in multiple solutions. "Analytical" solutions have been presented for similar problems in the past (e.g. Nanua, et al., 1990). However, these techniques involve highly complicated symbolic terms and the final result requires finding the roots of a high-order polynomial, which must be performed numerically (for order greater than 4). Therefore, the approach in this thesis is to solve the forward pose
problem numerically, applying the Newton-Raphson method to the vector loop closure
Eq. (4). This method yields only one solution. Using the previous solution as the initial
guess generally yields convergence to the proper solution.

For CDDRs with actuation redundancy the system of equations is over
constrained. Assuming consistent cable length inputs (i.e. at least one valid forward pose
solution exists), these over constrained equations are beneficial because generally only
one solution exists. This eases the problem of multiple solutions. There is an exception
to this rule that will be discussed at the end of this section.

Here the forward pose equations are derived and solved for the planar case
(Figure 2-4). The same method can be extended to the spatial case. Given \( L_{1,2,3,4} \) we
must calculate end-effector pose unknowns \( x, y, \phi \). Eq. 5 is written four times, one for
each cable. Each equation is squared to yield four scalar equations in the three unknowns
\( X = \{x \ y \ \phi\}^T \):

\[
F_i(X) = x^2 + y^2 + h_{ix}^2 + h_{iy}^2 + A_{ix}^2 + A_{iy}^2 + 2h_{ix}(x - A_{ix})c\phi + (y - A_{iy})s\phi \\
+ 2h_{iy}(y - A_{iy})c\phi - (x - A_{ix})s\phi - 2(xA_{ix} + yA_{iy}) - L_i^2 = 0
\]

\( i = 1,2,3,4 \) \hspace{10cm} (6)

where \( ^0x_H = \{ x \ y \}^T \), \( ^Hh_i = \{ h_{ix} \ h_{iy} \}^T \), \( ^0A_i = \{ A_{ix} \ A_{iy} \}^T \), and the planar orientation
matrix is:
(6) are solved numerically using Newton-Raphson iteration (e.g. see Mabie and Reinholtz, 1987, Section 2.3), modified to handle over constrained equations. Starting from an initial guess \( X_0 \), the process for step \( k+1 \) is summarized below:

- Solve \( J_{NR} \delta X_k = -F(X) \) for \( \delta X_k = -J_{NR}^+ F(X) \)
- Then \( X_{k+1} = X_k + \delta X_k \)
- Iterate until \( \| \delta X_k \| < \epsilon \)

where \( J_{NR}^+ = \left( J_{NR}^T J_{NR} \right)^{-1} J_{NR}^T \) is the over constrained Jacobian matrix pseudo inverse, \( F(X) = \{ F_i(X) \} \), \( \delta X = \{ \delta x \ \delta y \ \delta \phi \}^T \), \( \epsilon \) is a user-defined tolerance (\( \epsilon \) can be different for translational and rotational terms), and the Newton-Raphson Jacobian matrix \( J_{NR} \) is:

\[
J_{NR} = \begin{bmatrix}
\frac{\partial F_i}{\partial x} & \frac{\partial F_i}{\partial y} & \frac{\partial F_i}{\partial \phi}
\end{bmatrix}
\]

where \( (i = 1,2,3,4) \):
\[
\frac{\partial F_i}{\partial \chi} = 2\left(x + h_{ix} c \phi - h_{iy} s \phi - A_{ix}\right)
\]
\[
\frac{\partial F_i}{\partial \chi} = 2\left(y + h_{ix} s \phi + h_{iy} c \phi - A_{iy}\right)
\]
\[
\frac{\partial F_i}{\partial \phi} = 2\left(h_{ix} \left[-(x - A_{ix}) s \phi + (y - A_{iy}) c \phi\right] + h_{iy} \left[-(y - A_{iy}) s \phi - (x - A_{ix}) c \phi\right]\right)
\]

This forward pose problem is equivalent to finding the assembly configurations of a four-bar linkage with input/output links \(L_1, L_2\) and two \(RR\) constraining dyads of lengths \(L_3\) and \(L_4\). By itself the four-bar linkage has infinite assembly configurations because it has \(1\)-\textit{dof}. \(RR\) dyad \(A_1H_3\) constrains the mechanism to a statically-determinate structure of \(0\)-\textit{dof}. Point \(H_3\) defines a four-bar coupler curve that is a tri-circular sextic (sixth-degree algebraic curve) that has a maximum of six intersections with the circle of radius \(L_3\) centered at \(A_3\) (Hunt, 1990). \(RR\) dyad \(A_4H_4\) further constrains the structure to a statically-indeterminate structure of freedom \(-1\). Assuming consistent input cable lengths, the circle of radius \(L_4\) centered at \(A_4\) must intersect point \(H_4\) in one of these possible six solutions. \textit{Generally} only one of the six possible solutions will assemble with \(L_4\). The exception to this rule (alluded to above) is that two possible valid solutions exist when the cable connections to ground \(A_4\) lie on vertices of a square and the end-effector is a square. The same is true if the ground link and end-effector are similar rectangles. Therefore, end-effector/ground link symmetry is not desirable as it causes uncertainty in the forward pose solution.
2.2.2. Inverse Jacobian Matrix

The CDDR inverse velocity Jacobian matrix is closely related to the Newton-Raphson Jacobian matrix and the statics Jacobian matrix.

The inverse rate kinematics problem for CDDRs is expressed by:

\[ \dot{L} = {}^j M \dot{X} \]  

where \( \dot{L} = \{ \dot{L}_1, \dot{L}_2, \ldots, \dot{L}_n \}^T \) is the vector of cable rates, \(^j M\) is the \( n \times 6 \) inverse Jacobian matrix expressed with respect to \( \{j\} \) coordinates, and \(^j \dot{X} = \{ v, \omega \}^T = \{ \dot{x}, \dot{y}, \dot{z}, \omega_x, \omega_y, \omega_z \}^T\) is the vector of translational and rotational Cartesian rates, also in \( \{j\} \). For the planar case, \(^j M\) is \( n \times 3 \) and \(^j \dot{X} = \{ \dot{x}, \dot{y}, \omega_z \}^T\).

The inverse Jacobian matrix \(^j M\) with \( j = 0 \) may be derived directly as follows (Roberts et al., 1997). First the \( i^{th} \) cable rate is expressed and related to Cartesian variables using Eq. 5 (where leading superscripts were dropped for brevity):

\[
\frac{d}{dt}\|L_i\| = \frac{1}{2} \frac{d}{dt} \left[ \frac{1}{2} \|L_i\|^2 \right] = \frac{1}{2} \frac{d}{dt} \left[ \|x_{ii} + \dot{\omega}R_{ii} - A_i\|^2 \right] \\
= \frac{1}{2} \frac{d}{dt} \left[ \|x_{ii} + \dot{\omega}R_{ii}\|^2 + \|A_i\|^2 + 2x_{ii}^T \dot{\omega}R_{ii} - 2A_i^T \dot{\omega}R_{ii} \right] \\
= \frac{1}{\|L_i\|} \left[ (x_{ii} + \dot{\omega}R_{ii} - A_i)v + (x_{ii} - A_i)^T \dot{\omega}R_{ii} \right] \\
= \frac{1}{\|L_i\|} \left[ L_i \cdot v + (x_{ii} - A_i)^T \dot{\omega}R_{ii} \right]
\]

(10)

Working on the rotational term, since \( \dot{\omega}R_{ii} = \omega \times \dot{\omega}R_{ii} \), we have:
(11)

\[(x_H - A_i) \cdot R h_i = (x_H - A_i) \cdot \omega \times R h_i = \omega \cdot R h_i \times (x_H - A_i) = \omega \cdot (L_i \times R h_i)\]

Where we have used \(a \cdot (b \times c) = b \cdot (c \times a)\) and \(a \times b = a \times (b + a)\). Now the relationship between one cable rate and the Cartesian rates is:

\[
\dot{L}_i = \frac{1}{\|L_i\|} \left[ L_i \cdot v + \omega \cdot \left( L_i \times R h_i \right) \right] = \left[ \hat{\dot{L}}_i^{T} \ - \left( \hat{L}_i \times R h_i \right)^{T} \right] \left[ \begin{array}{c} v \\ \omega \end{array} \right]
\]

where \(\hat{L}_i = \frac{L_i}{\|L_i\|}\) is a unit vector in the \(L_i\) direction. Therefore, the inverse Jacobian matrix is:

\[
^0 M = \begin{bmatrix}
\hat{L}_1^T \\
\hat{L}_2^T \\
\vdots \\
\hat{L}_n^T
\end{bmatrix}
- \begin{bmatrix}
\left( \hat{L}_1 \times R h_1 \right)^T \\
\left( \hat{L}_2 \times R h_2 \right)^T \\
\vdots \\
\left( \hat{L}_n \times R h_n \right)^T
\end{bmatrix}
\]

(13)

This matrix is closely related to the Newton-Raphson Jacobian matrix (given for the planar case in Eq. 8) divide each row of \(J_{NR}\) by \(2L_i\) to yield \(^0 M\); care must be taken to convert Euler rotational rates to Cartesian rotational rates for the spatial case. To apply Eq. 13 to the planar case, the translational \(z\) and rotational \(x\) and \(y\) terms are dropped because they are zero, resulting in an \(n\) by 3 matrix \(^0 M\).

Kinematic singularities occur at CDDR configurations where the inverse Jacobian matrix \(M\) has less than full rank. Singularities result when \(\text{rank}(M) < m\) where \(m\) is the dimension of Cartesian space. In the neighborhood of kinematic singularities, the
determinant \( |M^T M| \) approaches zero. In a serial-chain robotic arm, kinematic singularities physically correspond to configurations where the robot loses one or more degrees-of-freedom. For in-parallel-actuated devices, kinematic singularities physically correspond to configurations where the device gains additional, unwanted, uncontrollable degrees-of-freedom.

For planar CDDRs, when the ground link and end-effector are both squares (or similar rectangles) and \( \phi = 0 \), all configurations are kinematically singular, regardless of translations \( x, y \). For cases where the ground link and end-effector are not similar geometric shapes, the only singular configurations are when the end-effector center is placed at \( A_i; i = 1,2,3,4 \) with \( \phi = \pm 90^\circ \), which is out of the workspace. The minimum \( |M^T M| \) is always greater than zero for non-similar geometric shapes, so there are no singularities in practical operation. Thus, end-effector/ground link symmetry is not desirable in terms of singularities (this symmetry is also undesirable in the forward pose solution).

Forward rate kinematics can be obtained by simply inverting Eq. 9. Forward rate solution exists only if inverse jacobian matrix \( M \) has full rank.

\[
\dot{X} = M^{-1} \dot{L}
\]  

(14)

2.3. CDDR Statics Modeling

In this thesis, the workspace wherein all cables are under positive tension while exerting all possible Cartesian wrenches is called the statics workspace. Statics modeling
and an attempt to maintain positive cable tension for all wrenches are presented in this section. A simple method is used to determine the extent of the statics workspace, i.e. the workspace wherein all possible end-effector wrenches can be resisted with only positive cable tensions.

2.3.1. Statics Modeling

This section presents statics modeling for planar CDDRs. For static equilibrium the sum of external forces and moments exerted on the end-effector by the cables must equal the resultant external wrench exerted on the environment (or, the wrench exerted by a serial wrist mechanism acting on the environment must react on the CDDR end-effector). Figure 2-5 shows the statics free-body diagram for the planar 4-cable CDDR.

![Figure 2-5: Planar 4-Cable CDDR Statics Diagram](image)
The statics equations are:

$$\sum_{i=1}^{4} t_i = -\sum_{i=1}^{4} t_i \hat{L}_i = F_R$$
$$\sum_{i=1}^{4} m_i = \sum_{i=1}^{4} \left(0_H R h_i \right) \times t_i = M_R$$  \hspace{1cm} (15)$$

In this thesis gravity is ignored because it is assumed to be perpendicular to the CDDR plane; it is assumed that the end-effector is supported on a base plate with negligible friction. The definition of frames \{0\} and \{H\} are given in Figure 2-1. In (15), \(t_i\) is the cable tension applied to the \(i^{th}\) cable (in the negative cable length unit direction \(\hat{L}_i\) because \(t_i\) must be in tension); \(0_H R\) is the orthonormal rotation matrix relating the orientation of \{H\} to \{0\} (nominally, \(0_H R = I_3\) since orientation is controlled to zero, \(\phi = 0\)); \(h_i\) is the pose vector from the origin of \{H\} to the \(i^{th}\) cable connection, expressed in \{H\} coordinates (only \(h_3\) is shown in Figure 2-5); and \(F_R\) and \(M_R\) are the resultant vector force and moment (taken together, wrench) exerted on the environment. Substituting the above terms into (15) yields:

$$ST = W_R$$  \hspace{1cm} (16)$$

\(T = \{t_1 \ t_2 \ t_3 \ t_4\}_T\) is the vector of scalar cable forces, \(W_R = \{F_R \ M_R\}_T = \{F_{Rx} \ F_{Ry} \ M_{Rz}\}_T\) is the resultant external end-effector wrench vector (expressed in \{0\} coordinates but felt at the origin of \{H\}), and the \(3\times4\) Statics Jacobian matrix \(S\) (expressed in \{0\} coordinates) is:

$$S = \begin{bmatrix} -\hat{L}_1 & -\hat{L}_2 & -\hat{L}_3 & -\hat{L}_4 \\ \hat{L}_1 \times 0_H R h_1 & \hat{L}_2 \times 0_H R h_2 & \hat{L}_3 \times 0_H R h_3 & \hat{L}_4 \times 0_H R h_4 \end{bmatrix}$$  \hspace{1cm} (17)$$
The specific (17) expressions for the Figure 2-4 CDDR are:

\[
S = \begin{bmatrix}
-c\theta_1 & -c\theta_2 & -c\theta_3 & -c\theta_4 \\
-s\theta_1 & -s\theta_2 & -s\theta_3 & -s\theta_4 \\
c\theta_1 h_{1y} - s\theta_1 h_{1x} & c\theta_2 h_{2y} - s\theta_2 h_{2x} & c\theta_3 h_{3y} - s\theta_3 h_{3x} & c\theta_4 h_{4y} - s\theta_4 h_{4x}
\end{bmatrix}
\]  

(18)

Where \( h_i = \begin{bmatrix} h_{ix} & h_{iy} \end{bmatrix}^T \), \( c\theta_i = \cos\theta_i \), and \( s\theta_i = \sin\theta_i \). Eq. 18 assumes that the orientation is \( \phi = 0 \) for all pseudo static motion; otherwise each third row term of (18) is:

\[c\theta_1 (h_{ix} s\phi + h_{iy} c\phi) - s\theta_1 (h_{ix} c\phi - h_{iy} s\phi)\].

This assumption is fine for pseudo static motion, but in the dynamics modeling section the general form is used where \( \phi \neq 0 \) since dynamics can cause small errors in \( \phi \) despite control attempts to make \( \phi = 0 \). The statics Eq. (16) can be inverted in an attempt to resist general (in this thesis, planar) Cartesian wrenches while maintaining positive cable tension. This work is presented in the next subsection.

2.3.2. Maintaining Positive Cable Tension

For CDDRs with actuation redundancy, (16) is under constrained which means that there are infinite solutions to the cable tension vector \( T \) to exert the required Cartesian wrench \( W_R \). To invert (16) (solving the required cable tensions \( T \) given wrench \( W_R \)), the well-known particular and homogeneous solution from rate control of kinematically redundant serial manipulators is adapted:

\[T = S^+ W_R + (I_n - S^+ S) z\]  

(19)
Where $I_n$ is the $n \times n$ identity matrix, $z$ is an arbitrary $n$-vector, and $S^+ = S^T \left( S S^T \right)^{-1}$ is the $n \times 3$ under constrained Moore-Penrose pseudo inverse of $S$. The first term of (19) is the particular solution to achieve the desired wrench, and the second term is the homogeneous solution that projects $z$ into the null space of $S$.

For CDDRs with one degree of actuation redundancy (the case in this thesis), the positive cable tension method (Shen et al., 1994) is adapted to determine the extent of the statics workspace. For actuation redundancy of degree one, an equivalent expression for (19) is:

$$\mathbf{T} = \begin{pmatrix} t_{p_1} \\ t_{p_2} \\ t_{p_3} \\ t_{p_4} \end{pmatrix} + \alpha \begin{pmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \end{pmatrix}$$

(20)

Where the particular solution $S^+ W_R$ is the first term in (20) and the homogeneous solution is expressed as the kernel vector $N$ of $S$ ($N = \{n_1, n_2, n_3, n_4\}^T$) multiplied by arbitrary scalar $\alpha$.

This method (Shen et al., 1994) to determine if a given point lies within the statics workspace for a given CDDR is simple. To ensure positive tensions $t_i$ on all cables $i = 1, \cdots, 4$, for all possible exerted forces and moments, it is necessary and sufficient that all kernel vector components ($n_i$, $i = 1, \cdots, 4$) have the same sign. That is, for a given point to lie within the statics workspace, all $n_i > 0$ OR all $n_i < 0$ ($i = 1, \cdots, 4$). If one of these two conditions is satisfied, regardless of the particular solution, a scalar $\alpha$ can be found in (20) which guarantees that all cable tensions $\mathbf{T}$ are positive by adding (or subtracting)
enough homogeneous solution. Note that a strict inequality is required; if one or more
\( n_i = 0 \), the CDDR configuration in question does not lie within the statics workspace.

This method is simple but powerful since there is no need to consider specific wrenches: it works for all possible wrenches. It should also be noted that this method is applicable to any planar and spatial CDDR with one degree of actuation redundancy.

A symbolic expression for the kernel vector (null space basis) of the 4-cable CDDR (with \( \phi = 0 \)) is:

\[
N = \begin{bmatrix}
  n_1 \\
  n_2 \\
  n_3 \\
  n_4
\end{bmatrix} = \begin{bmatrix}
  \cos(\theta_2 - \theta_3 + \theta_4) - \cos(\theta_2 - \theta_3 - \theta_4) + \sin(\theta_2 - \theta_3 + \theta_4) - \sin(\theta_2 + \theta_3 - \theta_4) \\
  \cos(\theta_1 - \theta_3 - \theta_4) - \cos(\theta_1 + \theta_3 - \theta_4) + \sin(\theta_1 + \theta_3 - \theta_4) - \sin(\theta_1 - \theta_3 + \theta_4) \\
  \cos(\theta_1 + \theta_2 - \theta_4) - \cos(\theta_1 - \theta_2 - \theta_4) + \sin(\theta_1 - \theta_2 - \theta_4) + \sin(\theta_1 - \theta_2 + \theta_4) \\
  \cos(\theta_1 - \theta_2 + \theta_3) - \cos(\theta_1 + \theta_2 - \theta_3) - \sin(\theta_1 - \theta_2 + \theta_3) - \sin(\theta_1 - \theta_2 - \theta_3)
\end{bmatrix}
\]  

(21)

Now, the allowable cable angle ranges are \( 0 < \theta_1 < 90^\circ \), \( 90^\circ < \theta_2 < 180^\circ \), \( 180^\circ < \theta_3 < 270^\circ \), and \( 270^\circ < \theta_4 < 360^\circ \). The analysis based on these allowable angles ranges, by careful consideration of sums/differences of the three distinct angle combinations in each row of (21) is shown in Table 2-1. By substitution of these results in (21), it can be proved easily that the sign of ALL \( n_i \) components is always the same (-ve in this case), \( i = 1, \ldots, 4 \).

<table>
<thead>
<tr>
<th>( n_1 )</th>
<th>( \cos )</th>
<th>( \sin )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 180 &lt; \theta_2 - \theta_3 + \theta_4 &lt; 270 )</td>
<td>-Ve</td>
<td>-Ve</td>
</tr>
<tr>
<td>( 0 &lt; \theta_2 + \theta_3 - \theta_4 &lt; 90 )</td>
<td>+Ve</td>
<td>+Ve</td>
</tr>
<tr>
<td>( -360 &lt; \theta_2 - \theta_3 - \theta_4 &lt; -450 )</td>
<td>+Ve</td>
<td>-Ve</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( n_2 )</th>
<th>( \cos )</th>
<th>( \sin )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 90 &lt; \theta_1 - \theta_3 + \theta_4 &lt; 180 )</td>
<td>-Ve</td>
<td>+Ve</td>
</tr>
</tbody>
</table>
Therefore, the entire allowable kinematic workspace of the base square is also the statics workspace! Now, when the edge of the end-effector square is aligned with an edge of the base square, two components \( n_i = 0 \) and thus the allowable statics workspace is the base square, reduced by \( a/2 \) (half the end-effector side) on all sides. This edge singularity condition was discussed earlier in Section 2.1. At all points outside of the base square, all components of the kernel vector \( N \) do not have the same sign so outside the useful region of the base square is also outside of the statics workspace. This statics workspace discussion holds only for \( \phi = 0 \), the nominal case of the planar translational CDDR. In previous work, it was discovered that the statics workspace is extremely limited when considering general \( \phi \) rotations (R.L.Williams, P. Gallina, 2001).

For real-time pseudostatic control of a planar CDDR with one degree of actuation redundancy, the cable tensions for control are calculated by (20) and (21), choosing \( \alpha \) so

| \(-450 < \theta_1 - \theta_3 - \theta_4 < -540\) | \(-Ve\) | \(-Ve\) |
| \(-90 < \theta_1 + \theta_3 - \theta_4 < 0\) | \(+Ve\) | \(-Ve\) |

| \(n3\) | \(\cos\) | \(\sin\) |
| \(-360 < \theta_1 - \theta_2 - \theta_4 < -450\) | \(+Ve\) | \(-Ve\) |
| \(180 < \theta_1 - \theta_2 + \theta_4 < 270\) | \(-Ve\) | \(-Ve\) |
| \(-180 < \theta_1 + \theta_2 - \theta_4 < -90\) | \(-Ve\) | \(-Ve\) |

| \(n4\) | \(\cos\) | \(\sin\) |
| \(90 < \theta_1 - \theta_2 + \theta_3 < 180\) | \(-Ve\) | \(+Ve\) |
| \(-270 < \theta_1 - \theta_2 - \theta_3 < -360\) | \(+Ve\) | \(+Ve\) |
| \(-90 < \theta_1 + \theta_2 - \theta_3 < 0\) | \(+Ve\) | \(-Ve\) |

Table 2-1: Kernel Vectors Analysis
that one component of \( T \) is zero (or, a small positive tension value) and the remaining terms are positive. Since the pseudo static condition is a limiting subset of the general dynamics case, especially for high velocities and accelerations, dynamics modeling is discussed in next section.

### 2.4. CDDR Dynamics Modeling

This section presents dynamics modeling for planar CDDRs. The 4-cable planar translational CDDR is shown in Figure 2-1. Dynamics modeling is required for improved control (compared to using kinematics and statics modeling only) when CDDRs are to provide high velocities and accelerations in translational motion. Dynamics modeling is concerned with relating the Cartesian translational motion of the moving CDDR end-effector to the required active joint torques. Due to the cable actuation, CDDR dynamics modeling is not very similar to in-parallel-actuated robot dynamics modeling (e.g. L. W. Tsai, 1999; C. M. Gosselin, 1996). Another complicating factor is that the joint space is over constrained with respect to the Cartesian space due to redundant actuation. Also, though only translational CDDR is presented in this thesis, the rotational motion is included in the dynamics equations to evaluate effectiveness of \( \phi = 0 \) control in simulation.

For the dynamics model derived in this section it is assumed that the CDDR cables are massless and perfectly stiff so their inertias or spring stiffnesses are not considered. The Coulomb friction is further ignored and instead linear viscous friction is modeled to account for the frictional losses. Despite these simplifications, the resulting
model is coupled and nonlinear. The Cartesian, actuator, and overall system dynamics models are presented in the next section.

2.4.1. Cartesian Dynamics Model

The 3-dof Cartesian dynamic model for the planar CDDR end-effector is given by

\[ M \ddot{\mathbf{X}} = \mathbf{W}_R \text{ :} \]

\[
\begin{bmatrix}
  m & 0 & 0 \\
  0 & m & 0 \\
  0 & 0 & I
\end{bmatrix}
\begin{bmatrix}
  \ddot{x} \\
  \ddot{y} \\
  \ddot{\phi}
\end{bmatrix}
= 
\begin{bmatrix}
  F_{Rx} \\
  F_{Ry} \\
  M_{Rz}
\end{bmatrix}
\] (22)

Where \( M \) is the Cartesian inertia matrix (\( m \) is the end-effector mass and \( I \) is the end-effector mass moment of inertia about the \( z \) axis through the center of mass), \( \ddot{\mathbf{X}} = \begin{bmatrix} \ddot{x} & \ddot{y} & \ddot{\phi} \end{bmatrix}^T \) is the end-effector acceleration (acceleration of the \( \{H\} \) frame with respect to the inertial frame \( \{0\} \), expressed in \( \{0\} \) coordinates), and \( \mathbf{W}_R = \begin{bmatrix} F_{Rx} & F_{Ry} & M_{Rz} \end{bmatrix}^T \) is the force and moment (about the center of mass) resultant of all \( n \) cable tensions acting on the end-effector; \( \mathbf{F}_R = \begin{bmatrix} F_{Rx} & F_{Ry} \end{bmatrix}^T = -\sum_{i=1}^{4} t_i \mathbf{L}_i \) and

\[ M_R \hat{k} = \sum_{i=1}^{4} \left( 0_{H} \mathbf{R}_i \right) \times t_i. \]
2.4.2. Actuator Dynamics Model

The dynamic behavior of the lumped motor shaft/cable pulley is also considered; the free-body diagram for the \( i^{th} \) motor shaft/cable pulley subsystem is shown in Figure 2-6.

![Free-Body Diagram For the ith Pulley/Shaft](diagram)

The combined motor shaft/cable pulley dynamics equations are expressed by the relationship:

\[
J\ddot{\beta} + C\dot{\beta} = \tau - rT
\]  \hspace{1cm} (23)

Where: 
\[
J = \begin{bmatrix}
J_1 & & 0 \\
& \ddots & \\
0 & & J_n
\end{bmatrix}
\]
\[
C = \begin{bmatrix}
c_1 & & 0 \\
& \ddots & \\
0 & & c_n
\end{bmatrix}
\]

are diagonal matrices with rotational inertia and rotational viscous damping coefficients on the diagonal, all cable pulley radii (\( r_i \) in Figure 2-6) are identical (\( r_i = r ; i = 1, \cdots, n \)),
τ ∈ R^n is the vector of torques exerted by the motors, T ∈ R^n is the vector of cable tensions t_i, and β ∈ R^n is the vector of pulley angles. Since the cables can only exert positive tensions (they cannot push), to express the cable tensions as a function of the motor torques and angular motion from (23):

$$T = \text{pos}\left(\frac{1}{r}(\tau - J\ddot{\beta} - C\dot{\beta})\right)$$  \hspace{1cm} (24)

Where the symbol \(\text{pos}()\) means the value of each vector component that is positive is taken and those components that were originally negative are set to zero. Assuming the torque on each motor is large enough to make all cables remain in tension at all times. Under this assumption:

$$T = \frac{1}{r}(\tau - J\ddot{\beta} - C\dot{\beta})$$  \hspace{1cm} (25)

2.4.3. System Dynamics Model

The overall system dynamics model is derived by combining the Cartesian and actuator dynamics equations of motion. The statics relationship \(W_R = ST\) between forces on the end-effector and cable tensions was derived in Section 2.3, where the 3xn (n=4) statics Jacobian matrix \(S\) is given in (18). However, as mentioned in Section 2.3, the pseudo static assumption that \(\phi = 0\) for all motion cannot be applied and hence the third row of (18) must be replaced with the terms:

\[c \theta_i(h_{ix}s\phi + h_{iy}c\phi) - s \theta_i(h_{ix}c\phi - h_{iy}s\phi);\]

\(i = 1, \cdots, 4\), one for each column of the statics Jacobian matrix.

To derive an inverse kinematics mapping relating the pulley angles \(\beta_i\) (\(i = 1, \cdots, n\)) expressed as functions of the end-effector pose and orientation
\( X = \{x, y, \phi\}^T \), define all \( \beta_i \) to be zero when the end-effector centroid is located at the origin of frame \( \{0\} \), with zero orientation \( (\phi = 0) \). From this configuration, a right-handed positive angle \( \beta_i \) on one pulley will cause a negative change \( \Delta L_i \) in cable length \( i \):

\[
\beta_i r = -\Delta L_i. 
\]

The change in cable length \( i \) is \( \Delta L_i = L_i - L_{0i} \) where \( L_i \) is the general length for cable \( i \) from the inverse pose solution and \( L_{0i} \) is the initial length for cable \( i \):

\[
L_i = \sqrt{(x - A_{ix} + h_{ix}c\phi - h_{iy}s\phi)^2 + (y - A_{iy} + h_{ix}s\phi + h_{iy}c\phi)^2}
\]

\[
L_{0i} = \sqrt{(h_{ix} - A_{ix})^2 + (h_{iy} - A_{iy})^2}
\]  \hspace{1cm} (26)

\[
\beta = \begin{bmatrix} \beta_1(X) \\ \vdots \\ \beta_n(X) \end{bmatrix} = \frac{1}{r} \begin{bmatrix} L_{01} - L_1 \\ \vdots \\ L_{0n} - L_n \end{bmatrix}
\]  \hspace{1cm} (27)

Successive time derivatives of (27) yield:

\[
\dot{\beta} = \frac{\partial \beta}{\partial X} \dot{X}
\]

\[
\ddot{\beta} = \frac{d}{dt} \left( \frac{\partial \beta}{\partial X} \dot{X} \right) + \frac{\partial \beta}{\partial X} \ddot{X}
\]  \hspace{1cm} (28)

Where \( \frac{\partial \beta}{\partial X} \) may easily be derived; it is a function of the Cartesian pose kinematics terms.

By substituting (28) into (25):

\[
T = \frac{1}{r} \left( \tau - J \left( \frac{d}{dt} \left( \frac{\partial \beta}{\partial X} \dot{X} \right) + \frac{\partial \beta}{\partial X} \ddot{X} \right) - C \frac{\partial \beta}{\partial X} \dot{X} \right)
\]  \hspace{1cm} (29)

Finally, by combining (22), \( W_R = ST \), and (29), the overall dynamics equations of motion can be obtained, expressed in a standard Cartesian form for robotic systems (F.L.Lewis et al., 1993):
\[ M_{eq}(X)\dddot{X} + N(X, \dot{X}) = S(X)\tau \] (30)

Where the equivalent inertia matrix \( M_{eq}(X) \) and nonlinear terms \( N(X, \dot{X}) \) are:

\[ M_{eq}(X) = rM + S(X)J \frac{\partial \beta}{\partial X} \] (31)

\[ N(X, \dot{X}) = S(X)\left( J \frac{\partial \beta}{\partial X} + C \frac{\partial \beta}{\partial X} \right) \dot{X} \] (32)

Note the statics Jacobian matrix \( S = S(X) \) from (18) is a function of Cartesian pose \( X = \{x \quad y \quad \phi\}^T \) through the cable angles \( \theta_i = \tan^{-1}\left( \frac{y - A_{iy} + h_{ix}s\phi + h_{ix}c\phi}{x - A_{ix} + h_{ix}c\phi - h_{ix}s\phi} \right) \) (see Figure 2-1).
3. CDDR CONTROLS SIMULATION

This section presents CDDR control architecture and control law development, followed by the method for resolving the actuation redundancy, including an algorithm for on-line estimation of minimum actuator torques in order to maintain cable tension despite CDDR dynamics.

3.1. Control Law and Architecture

This sub-section presents control architecture and control law development for planar CDDRs based on the overall system Cartesian dynamics equations of motion (30). The input to the plant was the vector of actuator torques $\tau$. Each component of $\tau$ had to be positive or zero at the minimum (in practice, a small positive value). In order to facilitate this problem, a virtual generalized Cartesian wrench input $W_V$ (units $N, N, Nm$) was introduced:

$$W_V = S(X)\tau$$  \hspace{1cm} (33)

Since the statics Jacobian matrix $S(X)$ had dimension $3 \times n$, this virtual generalized wrench input $W_V$ had the dimension of the Cartesian space 3. Though a planar translational CDDR was presented, there was a need to attempt to control $\phi = 0$ for all motion. The components of $W_V$ were not restricted to be positive. A control law was developed for the virtual Cartesian wrench input $W_V$ to find a real controls torque input vector $\tau$ with all positive components that satisfy (33), if the CDDR pose was within the statics workspace. In Section 2.3.2 it was described that the base square was the statics...
workspace for planar translational CDDR, reduced by half the end-effector side, \( a/2 \), on all sides. Therefore, for control law development, the following dynamics equation was considered:

\[ \mathbf{M}_{eq}(\mathbf{X})\ddot{\mathbf{X}} + \mathbf{N}(\mathbf{X}, \dot{\mathbf{X}}) = \mathbf{W}_V \]  \hspace{1cm} (34)

The effects of the nonlinear dynamics terms \( \mathbf{N}(\mathbf{X}, \dot{\mathbf{X}}) \) were canceled and inertial terms were counted by using the well-known computed-torque (or feedback linearization) technique (L. W. Tsai, 1999). A Cartesian PD controller was implemented to reduce the tracking error \( \mathbf{e} = \mathbf{X}_R - \mathbf{X} \). The commanded (reference) Cartesian pose was \( \mathbf{X}_R = \{x_R \ y_R \ \phi_R = 0\}^T \). The computed-torque control law for the virtual Cartesian wrench input \( \mathbf{W}_V \) is:

\[ \mathbf{W}_V = \mathbf{M}_{eq}(\mathbf{X}_R)(\ddot{\mathbf{X}}_R + \mathbf{K}_P \mathbf{e} + \mathbf{K}_D \dot{\mathbf{e}}) + \mathbf{N}(\mathbf{X}_R, \dot{\mathbf{X}}_R) \]  \hspace{1cm} (35)

The reference Cartesian values \( \mathbf{X}_R \) were used in (35). The actual feedback values from actuator encoder sensors and forward pose kinematics could be used alternatively. Due to this uncertainty, plus sensor noise problems, and the problem of digitally twice differentiating the sensor feedback \( \mathbf{X}, \mathbf{X}_R \) was chosen instead.

The controller architecture (shown in the block diagram of Figure 3-1) is made up of three different parts: the Cartesian PD controller, the computed-torque terms, and the virtual-Cartesian-wrench-input-to-real-actuator-torque calculation, with dynamic minimum torque estimation to ensure cable tension was maintained at all times despite the CDDR dynamics. The PD controller gains were determined via pole placement for the resulting effective unit inertia plant, specifying desired settling time and percent
overshoot for a unit step input. The matrix gains $K_p, K_D$ were 3x3 diagonal matrices, which means that the PD control was accomplished independently for the $x$, $y$, and $\phi$ motions, even though the dynamics model was coupled. The same settling time and percent overshoot were specified for all Cartesian motions (see Section 3.3). The inertial terms $M_{eq}(X_R)\ddot{X}_R$ were composed of the overall pose-dependent Cartesian inertia matrix $M_{eq}(X_R)$ (31) and the reference Cartesian acceleration components $\ddot{X}_R$; the nonlinear terms were $N(X_R, \dot{X}_R)$, given in (32). The virtual-to-real calculation has the problem to invert non-square matrix $S(X)$, such that only positive actuator torques result given the virtual Cartesian wrench input $W_V$. This problem is solved in Section 2.3.2 for CDDRs assuming the pseudostatic condition; this is adapted in Section 3.2 below for CDDR dynamics.

Direct access of Cartesian pose $X$ feedback via sensors is not feasible generally. Instead, this feedback needs to be calculated using the encoder feedback for each cable pulley angle $\beta_i$ to determine the cable lengths $L_i$; these lengths are then used as the inputs to the forward pose kinematics solution to calculate Cartesian pose $X$ for feedback in the control architecture. This feedback scheme will work well only if sufficient tension is maintained on all cables at all times.
3.2. Calculation of Optimal Actuator Torques

This sub-section presents a method for determining the optimal actuator torques for the controller architecture of Figure 3-1. This sub-section presents the “Virtual-to-Real Calculation” block of Figure 3-1. Also presented is an algorithm for on-line estimation of minimum actuator torques required to maintain positive cable tensions despite the CDDR dynamics; this is the “Min. Torque Estimation” block in Figure 3-1.

The Jacobian matrix relationship between torques and virtual generalized Cartesian wrenches was under constrained for CDDRs, given by (33), $W_V = S(X)\tau$. For control the real actuator torques $\tau$ was calculated given the virtual Cartesian control wrenches $W_V$. For actuation redundancy of degree one, the pseudo statics solution of Section 2.3.2 (to this method the on-line, dynamic minimum torque estimation algorithm, developed below in the current sub-section was added) was adapted. The difference from (20) was that pseudo statics assumption $\tau_i = rt_i$ ($\tau_i$ is the $i^{th}$ actuator torque, $r$ is the cable pulley radius, and $t_i$ is the $i^{th}$ cable tension) couldn’t be made. Due to dynamics
this assumption no longer holds; the required actuator torques were calculated for control while attempting to maintain positive cable tensions dynamically. The solution of the under constrained system $W_v = S(X)\tau$ was similar to (20), given in (36).

$$\tau_{\text{opt}} = \begin{bmatrix} \tau_{p1} \\ \tau_{p2} \\ \tau_{p3} \\ \tau_{p4} \end{bmatrix} + \alpha \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \end{bmatrix} > \tau_{\text{min}} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

(36)

The particular solution $S^+W_v$ is the first term in (36) and the homogeneous solution is expressed as the kernel vector of $S$ ($N = \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \end{bmatrix}^T$) multiplied by arbitrary scalar $\alpha$. The goal of torque calculation in finding an optimal solution $\tau_{\text{opt}}$ consisted of the following features: 1) The Euclidean norm of $\tau$ was minimized with the particular solution wherein 2) The homogeneous solution ensured that each component of $\tau_{\text{opt}}$ must be greater than or equal to a specified minimum torque $\tau_{\text{min}}$. $\alpha$ was calculated at each control cycle to ensure, in the worst component case, that this minimum positive torque was satisfied for all four actuators. This was an easy calculation for actuation redundancy of degree one; at each control cycle, not the most negative particular torque component dominated for all time, but the combination of particular torque and kernel vector component determined the dominant $\alpha$. For each particular torque component that was negative, $\alpha_i = (\tau_{\text{min}} - \tau_{pi})/n_i$ was calculated; then the largest magnitude of these was selected to be the $\alpha$ at each control cycle. Further, $\alpha$ had to be positive if all $n_i$ were positive and $\alpha$ had to be negative if all $n_i$ were negative. This single $\alpha$ value of course
had to be then used for all four components in (36). Since the entire useful CDDR workspace was equal to the statics workspace, this $\alpha$ could always be found for all possible motions (i.e. the signs of $n_i$ were always the same for all four components).

If dynamic effects are considered, the tension in each cable turns out to be greater than $r \tau_{\text{min}}$. Under this pseudo static condition, if $\tau_{\text{min}}$ is set sufficiently high, the cables will never go slack. Unfortunately, when high speed is employed, because of dynamic effects, one or more cable can become slack despite a positive $\tau_{\text{min}}$. In this dynamic case the minimum value for $\tau_{\text{min}}$ must be estimated on-line for each cable in real-time. The on-line cable tension estimation algorithm came from forcing each tension component to be positive at all times in the dynamics model (29):

$$\{T\}_i = \left\{ \frac{1}{r} \left( \tau - \left\{ \frac{d}{dt} \left( \frac{\partial \beta}{\partial \mathbf{X}} \right) \mathbf{X} + \frac{\partial \beta}{\partial \mathbf{X}} \dot{\mathbf{X}} \right) - \mathbf{C} \frac{\partial \beta}{\partial \mathbf{X}} \dot{\mathbf{X}} \right) \right\} \geq 0 \quad i = 1, \cdots, n \quad (37)$$

The estimated minimum torque solution for each actuator from (37) to maintain cable tension based on CDDR dynamics was:

$$\tau_{\text{min}} = \max \left\{ \left\{ \mathbf{J} \left( \frac{d}{dt} \left( \frac{\partial \beta}{\partial \mathbf{X}} \right) \mathbf{X} + \frac{\partial \beta}{\partial \mathbf{X}} \dot{\mathbf{X}} \right) + \mathbf{C} \frac{\partial \beta}{\partial \mathbf{X}} \dot{\mathbf{X}} \right\} \right\} \quad i = 1, \cdots, n \quad (38)$$

The reason for the max function in (38) was that when CDDR dynamics were taken into account, the minimum torque required to ensure the corresponding cable was in tension could be negative for one or more components. In (38) all torque components were forced to be zero at the minimum. In practice and in the examples of the next section, a small positive value was chosen.
3.3. Examples

This section presents dynamics and control examples for the planar 4-cable translational CDDR with one degree of actuation redundancy. These examples are intended to demonstrate the CDDR control architecture including feedback linearization and on-line dynamic torque estimation to maintain positive cable tensions despite the dynamics, plus optimal torque calculation for a CDDR with one degree of actuation redundancy. In this section, a given planar 4-cable CDDR performs a simulated task twice, the first time without and the second time with the on-line minimum torque estimation algorithm.

The planar translational 4-cable CDDR model was shown in Figure 2-1. The following parameters are taken from a hardware system designed for experimental verification that is covered in section 4.2. The base square has side $L_B = 0.70 \text{ m}$ and the end-effector square has side $a = 0.10 \text{ m}$. The simulated dynamic task is for the CDDR end-effector point $\mathbf{X} = [x \ y]^T$ to trace a circle in the plane, while attempting to maintain $\phi = 0$ for all motion. The circle is centered at the base square centroid (the origin of $\{0\}$) and the circle radius is $r = 0.15 \text{ m}$. Figure 3-2 shows the simulated task to scale for the 4-cable CDDR at the starting (and ending) point.
In the simulated example the 4-cable CDDR was commanded to trace the given circle in 1 sec (zero external Cartesian wrench was specified). Polar angle $\gamma$ was defined as the independent parameter for the circle; it was measured using the right-hand from the right horizontal to the circle radius; $\gamma$ is shown at 0 (and 360°) in Figure 3-2. For ‘smooth’ motion starting and ending at rest, trajectory generation techniques were adopted (Y. Shen, et al., 1994): Angle $\gamma$ started at zero and ended at 360° during the 1 sec motion; $\dot{\gamma} = \ddot{\gamma} = 0$ at the start and end of motion, for ‘smoothness’. These conditions yielded a 5th order polynomial for angle $\gamma$: $\gamma(t) = 2160t^5 - 5400t^4 + 3600t^3$ (deg). The associated commanded (reference) Cartesian pose $X_R$, velocity $\dot{X}_R$, and acceleration
for use in the controller architecture were easy to determine. The commanded Cartesian angular values were $\phi = \dot{\phi} = \ddot{\phi} = 0$ for all motion since only a translational CDDR is considered.

The parameters for the dynamics equations of motion (16) for the 4-cable CDDR were (again, from hardware design): point mass $m = 0.91 \text{ kg}$; end-effector mass moment of inertia $0.00150 \text{ kgm}^2$; rotational shaft/pulley inertias $J_i = 0.00026 \text{ kgm}^2$ (for all $i = 1, \cdots, 4$); shaft rotational viscous damping coefficients $c_i = 0.01 \text{ Nms}$ (for all $i = 1, \cdots, 4$); and $r_i = r = 3.81 \text{ cm}$ (for all $i = 1, \cdots, 4$).

The Cartesian PD controller was found by standard pole placement techniques, settling time of a 0.2 second and overshoot of 5% was specified (the feedback linearization approach made the plant appear linear, as unit inertias in the $x$, $y$, and $\phi$ directions; units kg, kg, and kgm$^2$, respectively). It was designed for Cartesian $x$, $y$, and $\phi$ directions independently, with the same settling time and percent overshoot specifications. Gain $K_P$ was a 3x3 diagonal matrix with equal gains of $K_P = 839.9$ on the diagonal, and gain $K_D$ was a 3x3 diagonal matrix with equal gains of $K_D = 40$ on the diagonal.

A Matlab Simulink simulation based on the controller architecture of Figure 3-1 and the methods of this section was developed to produce the results given in this section. Two control simulations of the dynamics model are presented in this section for the 4-cable CDDR, first without and then with the on-line minimum torque estimation algorithm of Figure 3-1. Figure 3-3 shows the minimum actuator torques, Figure 3-4, the
simulated actuator torques, and Figure 3-5, the simulated cable tensions, for the circle task, \textit{without} the on-line minimum torque estimation algorithm. As shown in Figure 3-3, for the simulation \textit{without} the on-line minimum torque estimation algorithm, the minimum torques are constant and identical for all four actuators, taking a user-specified value of 0.05 \textit{Nm} in this example.

![Figure 3-3: Minimum Actuator Torques (Nm)](image)

The control torques, calculated by the optimal method in the Virtual to Real Calculation block of Figure 3-1, but without the minimum torque estimation algorithm, are shown in Figure 3-4. Actuator torques 1 and 4 peak near the center of motion time, due to the maximum velocity occurring at this point; torque 3 peaks twice, nearer the start
and end of motion. During motion all four torques at different times yield negative values in the Virtual to Real Calculation, but these are limited to a small positive value, the constant specified minimum torque of 0.05 Nm, using the actuation redundancy. The associated simulated cable tensions resulting from the simulated motion considering the dynamics model are shown in Figure 3-5. Without considering the dynamic minimum torque estimation block of Figure 3-1, cable tensions 1 and 3 become negative and thus slack at different times during the simulated motion. Clearly this is unacceptable as control would be lost in these ranges of motion and large Cartesian errors result. Thus, the constant minimum actuator torque specification is suitable only for pseudo static motions, not for dynamic motions with high velocities and accelerations.

![Figure 3-4: Simulated Actuator Torques (Nm)](image-url)
Figure 3-5: Simulated Cable Tensions (N)

Figure 3-6 shows the minimum actuator torques, Figure 3-7, the simulated actuator torques, and Figure 3-8, the simulated cable tensions, for the simulated circle task, with the on-line minimum torque estimation algorithm. As shown in Figure 3-6, for the simulation with the on-line minimum torque estimation algorithm, the minimum torques are no longer constant, but vary (greater than the minimum positive torque) so that no cable tensions will go negative in the dynamics model.
Figure 3-6: Minimum Actuator Torques (Nm)

The control torque values, obtained by including the on-line minimum torque estimation algorithm in the Virtual to Real Calculation, are shown in Figure 3-7. Again, during motion all four torques yield negative values but these are limited to the minimum torque of 0.05 \( Nm \) in these ranges, using the actuation redundancy. Though Figure 3-3 and Figure 3-6 are quite different (minimum torques without and with the minimum torque estimation), the resulting control torques of Figure 3-4 and Figure 3-7 are similar, though not identical. The simulated cable tensions considering the dynamics model are shown in Figure 3-8. By including the on-line, dynamic minimum tension estimation block, each cable tension never becomes negative and thus control is maintained at all times. In Figure 3-8 zero cable tension is allowed as a minimum; in practice a small positive value should be used instead. Figure 3-5 and Figure 3-8 are very similar in shape and magnitude; however, Figure 3-8 is a big improvement over Figure 3-5, where two
cable tensions were negative for two portions of the motion. Thus, on-line minimum tension estimation algorithm for dynamic motions must be included.

![Figure 3-7: Simulated Actuator Torques (Nm)]

![Figure 3-8: Simulated Cable Tensions (N)]
For the latter example only (i.e. including the on-line minimum tension estimation algorithm), Figure 3-9 shows the simulated $\phi$ tracking error. Since the 4-dof CDDR was used for translations only, maintaining $\phi = 0$ for all motion.

![Figure 3-9: Simulated $\phi$ Tracking Error (rad)](image)

Again, this was done to avoid the cable interference and limited statics workspace problems inherent in CDDRs with large rotational motions. In the simulation, the error remains very low, less than $4.6 \times 10^{-6}$ deg. The error appears to be increasing with time, but with such a low magnitude, this error can be accepted until experimental validation is performed in the future.
4. TEST BED DESCRIPTION

4.1. Project Funding and Objective

A grant of $10,000 was awarded from the Stocker Research Fund to the project for design and construction of the novel cable-direct-driven-robot. The main objective of the development was experimental verification of the theory.

4.2. Design Specifications

Budget and size were two factors considered while selecting components. A firm $10,000 budget was in place for all components and materials. Since the robot was intended for research and demonstration, the size of the robot was kept small enough to fit on a tabletop; however, it should be large enough for experimental verification and demonstration. Looking at all factors, the robot was designed for a workspace area of 0.7 square meters. Motors were selected considering the maximum speed of 1 meter in 0.3 second and maximum continuous torque of 1.8 N-m based on the suggestion of Paolo Gallina, visiting faculty from University of Trieste, Italy.

4.3. Component Selection

The first step involved in selecting the components was the drawing of an early conceptual CAD model shown in Figure 4-1. This model was helpful in determining the overall size and placement of components for the robot.
Component selection started with a base plate. Mild steel and aluminum were considered for the base plate. One factor that influenced the material selection was the weight of the robot. Specifically aluminum was selected due to lighter weight. Various sizes of an aluminum plate were considered. A 3’ x 3’ aluminum plate with 0.25” thickness was selected considering the available tabletop workspace in lab and weight of the plate.

4.3.2. Motors Selection

Motors were selected considering several factors like cost, availability, torque requirement and weight. Motors were required to produce a range of torques up to 1.8
N.m to generate varying forces when coupled with the pulleys and cables (See section 4.2). Encoders were also required in the chosen motors. Encoders provide a feedback in terms of motor angle that is used to determine cable lengths and then Cartesian pose of the end-effector using forward pose kinematics (See section 2.2.1). A small laser, a disk with slots and an optical sensor generate an encoder signal. The laser is pointed at discs mounted to the motor shaft. As a disc moves with the motor, the laser will pass through the slots in the disc and strike the optical sensor. When the laser hits the sensor, a circuit is completed and a signal is sent from the sensor to the data acquisition board and passed on to the computer for use in calculating the shaft’s rotational pose. When two sets of lasers, discs and sensors are used; direction and pose can be determined by a method known as quadrature (Collins, 2001). Each time the motor shaft moves enough to go from one slot to the next on one disc, there are four pulses. The direction is determined by which sensor signals first.

The motors were chosen for the CDDR considering a maximum speed of 1 meter in 0.3 second (see section 4.2).

\[ d = \frac{1}{2} at^2 \]

where \( d = \) distance (m), \( a = \) acceleration \( \left( \frac{m}{s^2} \right) \), \( t = \) time (second)

Therefore,

\[ a = \frac{2d}{t^2} \]

\[ = (2\times(1))/(0.3)^2 \]
As per the suggestion of Dr. Paolo Gallina- visiting faculty from Italy at Ohio University, acceleration was approximated to simplify further calculation.

\[
a = 20 \frac{m}{s^2}
\]

Considering end-effector mass- \(m\) of 2 Kg. Force \(F\) (N) can be expressed as:

\[
F = mA
\]

\[
= 2*20
\]

\[
F = 40 \text{ N}
\]

Pulley diameter was also approximated to 0.09 meter for derivation of torque as suggested by Dr. Gallina, though the diameter of the pulley was kept 0.0762 meter specifically as the pulley was designed based on the project of Josh Collins (Collins, 2001).

\[
T = Fr
\]

\[
= 40*0.09/2
\]

\[
= 1.8 \text{ Nm}
\]

where \(T\) is the motor torque and \(r\) is the pulley radius.

The Pittman gear motor GM14904S016 was chosen for the CDDR considering its maximum continuous torque of 2.64 N-m that was more than the design torque requirement of 1.8 N-m.
The GM14904S016 is a 24 VDC brush type motor equipped with an optical encoder with quadrature. It has a built-in 19.7:1 gear ratio, 179 rpm no load speed and a peak torque of 20.72 N-m. The encoder provides feedback resolution of 39,400 counts per revolution of output shaft. The GM14904S016 motor is shown in Figure 4-2.

The above-mentioned procedure for selection of motor was based on the assumption of the maximum speed of the end-effector however, the author found a better method for selection of motor in Mechatronics (ME-555) course later; this later method was not used since the motors had already been purchased.

4.3.3. Amplifiers and Power Supplies

The motors selected for use in the CDDR required a higher voltage and current supply than the data acquisition board could supply. In order to provide necessary driving
current and voltage source, amplifiers were used. Amplifiers required a 30VDC voltage source, so to provide them with the necessary voltage and current power supply model PS16L30 from advanced motion controls (AMC) was chosen for the CDDR that is shown in Figure 4-3. This model converts line voltage (120 VAC) to a 30 VDC supply.

![Figure 4-3: PS16L30 Transformer](image)

AMC’s model 25A8 servo amplifier was used for the CDDR that is shown in Figure 4-4. This servo amplifier is designed to drive brush-type DC motors. The amplifier is designed to use AMC models (including the PS16L30) as power sources and it can be mounted directly to the power supply platform. Each 25A8 unit can be used to command either voltage or current. The 25A8 is protected against over-voltage, over-current, over-heating and short circuits across the motor, ground and power leads. It
interfaces with digital controllers or can be used as stand-alone drive. It requires only a single unregulated DC power supply. The most important feature of the amplifier is that it offers current and voltage limiting. It allows users to set a maximum level of current or voltage supplied to the servomotors at any time and to protect them from damage due to excessive input. For the CDDR, the amplifier was set to supply variable current at a constant voltage to the motors. For servomotors, input current is linearly related to the output torque thus by commanding and controlling the current applied to the motors, the programmer can control the torque and therefore cable tension outputs.

![Figure 4-4: 25A8 Servo Amplifier](image)

4.3.4. Computer

A Compaq PC with 512 MB RAM and Pentium IV microprocessor was purchased for the CDDR project. The computer was configured with Matlab, Simulink and Wincon-3.2 software as well as a Quanser MultiQ-PCI board. The PC’s operating
system was Windows XP. The Operating system was changed to Windows 98 (Second Edition) due to Windows XP’s incompatibility with Matlab and Wincon 3.2.

4.3.5. Quanser Board

The MultiQ PCI data acquisition board has 8 analog input, 8 analog output, 4 digital input and 6-encoder input. It acts as a link between the computer and the CDDR hardware. The CDDR needs 4 encoder inputs and 4 analog outputs. The picture of the MultiQ PCI board and terminal board is shown in Figure 4-5.

Figure 4-5: MultiQ PCI Board and Terminal Board

Source: www.quanser.com

4.4. Design and Construction

4.4.1. Base Plate Design

First, an aluminum base plate was designed to carry the motors. Design was based on the workspace area of 0.7 square meters (section 4.2). Fixing aluminum strips on the
base plate defined the workspace bounds. Legs were placed on all corners of the base plate for sufficient clearance to accommodate the motors. Cables coming out from the pulleys are at different heights as they come out from the different threads of the pulley depending on their usage. Small guides were placed on all four corners of the workspace to keep cables at the same height and thereby in the same plane. A solid model of the guide and the base plate is shown in Figure 4-6 and Figure 4-7 respectively.

Figure 4-6: Guide
4.4.2. Pulley Design

The pulleys were made based on the pulley design from Josh Collins’ master’s project (Collins, 2002). Stocker technician made the pulleys. The depth of the pulley thread was kept lesser due to limitation of manufacturing facility at Stocker Workshop. It is made of aluminum to reduce the overall weight. A picture of the pulley is shown in Figure 4-8. The final design of the pulley hardware is shown in the Figure 4-9.
4.4.3. Cables

Two types of cables have been used in the CDDR to date. Initially fishing line with 40 pounds strength was used due to its high tensile strength. The author found that fishing line needed very high tension to move the end-effector due to its small diameter and less contact area with the pulley. The author found that bigger diameter would increase contact area of the cable with pulley that would make cable more effective. The bigger cross-sectional area of the cable also helped to reduce stress in the cable. A thicker nylon cable was used to achieve reduction in cable tension. After a number of experiments with the CDDR, the new cable has proven superior to the fishing line. The
new nylon cable also appeared to have less friction during spooling and when rubbing against other wires.

4.4.4. End-effector

The end-effector for the CDDR was designed considering only the cross cable case (See section 2.1). Two types of end-effector have been used to date.

The first version of the end-effector consists of an end-effector body made of plastic, supported completely on the tip of the marker pen as shown in the Figure 4-10. The author found by experiment that significant friction occurred with this design. The author also found that this design was not keeping the robot in the same plane.

![Figure 4-10: First Version of the End-effector](image)
The new end-effector was designed by placing ball transfers on the bottom to reduce friction and also to keep the end-effector in the same plane as all four cables. The marker plane was placed in such a way so that it just managed to touch the base plate to draw a profile. A picture of the new end-effector is shown in the Figure 4-11.

![Figure 4-11: Final Version of End-effector](image)

Motors were mounted on the base plate. Pulleys were fixed on the motor shafts using setscrews. The hardware was interfaced with the computer using the Quanser PCI data acquisition board and terminal board. The completed CDDR hardware is shown in Figure 4-12.
Figure 4-12: CDDR Hardware
5. EXPERIMENTAL RESULTS AND FINDINGS

5.1. Experimental Setup

The schematic diagram of the CDDR experimental set-up is shown in Figure 5-1. The Quanser MultiQ-PCI board was integrated with the Simulink and Matlab software to create a real-time control system. A power supply was used to convert 120 VAC line supply to 30VDC to provide power to the amplifiers. Amplifiers were used for the amplification of the current input of the MultiQ-PCI board because motors needed higher current than MultiQ-PCI board could supply. Encoders provide rotational feedback in pulses to the MultiQ-PCI board. The MultiQ-PCI board is fitted in the PCI slot of the computer to interface the computer with the hardware for real time control.

![Figure 5-1: Experimental Set-up](image)

5.2. Control Architecture

The control architecture of the CDDR system is shown in Figure 5-2. X is commanded pose \((x,y,\phi)\), L is the commanded leg lengths, \(\Upsilon\) is the vector of cable angles, \(N\) is kernel vector as given by equation 21 and \(\alpha\) is a scalar quantity. In the control
architecture, the inverse kinematics transforms the commanded pose into commanded leg
lengths and cable angles. Kernel vectors can be found using equation-21. The scalar $\alpha$
can be found by dividing the minimum allowable cable tension $T_{\text{min}}$ by the minimum
kernel vector.

![Diagram of Control Architecture]

**Figure 5-2: Control Architecture**

Cable lengths $L$ and kernel vectors $N$ are provided to the 4-independent length PD
controllers. The Simulink program that was used to implement this control architecture is
shown in Figure 5-4. The block diagram of the PD controller is shown in Figure 5-3. The
simulink program for one independent length PD-controller is shown in Figure 5-5.
Length errors were calculated by using feedback from the system. The output from the
PD block was provided to the motor when error was positive only. Kernel vector output
was provided to motors all the time to maintain positive tension in all cables. Encoders
provided motor rotation angles as feedback, which were mapped to cable lengths using
equation 27.
Figure 5-3: PD Controller Block Diagram

Figure 5-4: Simulink Program for Control Architecture Implementation
A detailed view of a PD block is shown in Figure 5-6. The PD block is a part of the Simulink library of functions.
5.3. Results

The experiments aimed at generating two linear profiles: move the end-effector from the origin (center of the plate) to (0.10,0) and (0.10,0.10) and one circular profile: trace a circle of 0.15 meter radius starting from (0.15,0) keeping center of the base plate as origin. The Simulink program as shown in Figure 5-4 was used for both experiments with the only change in the commanded trajectory.

5.3.1. Linear Trajectory Generation

The end-effector was first placed manually to the center of the base plate. It was commanded to move from: The origin (center of the base plate) to (0.10, 0) and (0.10,0.10) in one second while attempting to maintain $\phi = 0$ for all motion. As commanded motion was symmetrical to the x-axis for the first case, desired cable lengths L1 and L4 as well as L2 and L3 were the same. Therefore in the Figure 5-7, desired lengths for cables L3 and L4 covers those of cables L2 and L1. Cables L2 and L3 were required to pull the cables while cables L1 and L4 were required to release the cables but keep minimum tension to avoid slack and hence maintain control of the robot. Figure 5-7 shows the commanded and actual length control of the robot. Cables L1 and L4 were required to release the cables and therefore increase in lengths of the cable L1 and L4, which can be observed in the Figure 5-7. Cables L2 and L3 were required to pull the cables and therefore decrease in the lengths of cables L2 and L3, which can also be observed in Figure 5-7.
For the second case, commanded motion was symmetrical to the xy-axis therefore all desired lengths were different unlike the first case. L1 and L2 were required to release the cables but keep minimum tension to avoid slack and hence maintain control of the robot. Cables L3 and L4 were required to pull the cables. L1 and L2 were required to release the cables therefore increase in their lengths can be observed in Figure 5-8. Cables L3 and L4 were required to pull the cables therefore decrease in their lengths can be observed. Desired cable length L4 is covered by the actual value in Figure 5-8 for most of the part. Figure 5-9 shows the commanded and actual Cartesian control of the robot for the linear trajectory (0,0) to (0.10,0). As desired motion in y-direction as well as commanded rotational angle phi are zero phi-desired in Figure 5-9 covers the y-desired. Figure 5-10 shows the commanded and actual Cartesian control of the robot for the linear trajectory (0,0) to (0.10,0.10). As desired motion is symmetrical to x-y axis, desired x and y motion are overlapping. There is a significant error in phi initially, which reduces gradually.
Figure 5-7: Length Control for the Linear Trajectory (0,0) to (0.10,0)

Figure 5-8: Length Control for Linear Trajectory (0.0,0) to (0.10,0.10)
Figure 5-9: Cartesian Control for Linear Trajectory (0,0) to (0.10,0)

Figure 5-10: Cartesian Control for Linear Trajectory (0,0) to (0.10,0.10)
The errors in the Cartesian control of the robot for both cases are shown in the Figure 5-11 and Figure 5-12. The errors in x and y are in meters while the Phi error is in radians. Figure 5-13 and Figure 5-14 shows the trajectory achieved by the CDDR hardware in real time compared to the desired trajectory.

As it can be seen in the Figure 5-13, the end-effector could not reach to the exact commanded x coordinate which explains the positive error \((X_{\text{commanded}} - X_{\text{actual}})\) in Figure 5-11. As the end-effector moved towards the workspace boundary it approached to singularity. The cables needed infinite force to pull the end-effector towards the boundary of the workspace. The end-effector moves in y direction while commanded trajectory in y-direction is zero for this trajectory, which explains the negative error \((Y_{\text{commanded}} - Y_{\text{actual}})\) for y coordinate in Figure 5-11. Figure 5-11 also shows the positive error for the rotational angle phi, which means that the end-effector rotated clockwise about the z-axis while attempting the linear trajectory. The error in phi can be reduced if the Cartesian controller could be implemented instead of independent length controller. Cartesian controller needs online Cartesian feedback from the CDDR, which can be achieved by introducing vision system in future.

For the second case, the end-effector is commanded to move from the origin to \((0.10,0.10)\). The end-effector movement in x-y direction is very close to the commanded motion, it can be seen in the Figure 5-14. During this motion the end-effector was approaching the corner of the workspace but it was not approaching to singularity because cable in that corner could pull the end-effector in the same direction.
Figure 5-11: Cartesian Control Errors for Linear Trajectory (0,0) to (0.10,0)

Figure 5-12: Cartesian Control Errors for Linear Trajectory (0,0) to (0.10,0.10)
Figure 5-13: Linear Trajectory Generation : (0,0) to (0.1,0)

Figure 5-14: Linear Trajectory Generation : (0,0) to (0.10,0.10)
5.3.2. Circular Movement

In this experiment the end-effector was first placed manually to its starting point on the base plate. The end-effector was commanded to trace a circle of \( r = 0.15 \) meter radius and centered at the origin (center of the base plate) while attempting to maintain \( \phi = 0 \) for all motion in 10 seconds. Angle \( \gamma \) was defined as the polar angle for the circle; it was measured using the right-hand from the right horizontal to the circle radius. For ‘smooth’ motion starting and ending at rest, trajectory generation techniques were adopted (Craig, 1989): We required that angle \( \gamma \) starts at zero and ends at 360° during the 10 sec motion; also, we required that \( \gamma = \dot{\gamma} = 0 \) at the start and end of motion, for ‘smoothness’. These conditions yielded a 5th order polynomial for angle \( \gamma \): \[ \gamma(t) = \frac{27}{1250}t^5 - \frac{27}{50}t^4 + \frac{18}{5}t^3 \text{(deg)}. \] The associated commanded (reference) Cartesian pose for use in the controller architecture were \( x = r \cos(\gamma) \) and \( y = r \sin(\gamma) \). The commanded Cartesian angular value was \( \phi = 0 \) for all motion. Figure 5-15 shows the commanded and actual length control for the circular trajectory. Figure 5-16 shows the commanded and actual Cartesian control of the circular trajectory. Figure 5-17 shows the Cartesian control errors and Figure 5-18 shows the simulation of the actual trajectory generated by the end-effector compared to the desired circle. Implementing dynamics controller can reduce errors. Introducing Cartesian controller can also reduce the errors in phi as already discussed in 5.3.1.
Figure 5-15: Length Control for Circular Trajectory

Figure 5-16: Cartesian Control for Circular Trajectory
Figure 5-17: Cartesian Control Errors for Circular Trajectory

Figure 5-18: Circular Trajectory Generation
5.3.3. **Repeatability**

Repeatability is the ability of the robot to repeatedly position itself when asked to perform a task multiple times. Repeatability of the CDDR was measured by generating a circular trajectory repeatedly (seven times here) keeping all parameters like gain, time for trajectory generation, minimum cable tensions unchanged. The end-effector was commanded to trace a circle of $r = 0.15$ meter radius and centered at the origin (center of the base plate) while attempting to maintain $\phi = 0$ for all motion in 10 seconds. Figure 5-19 shows the commanded trajectory (circle) with solid black line and actual trajectories generated by the CDDR shown with different colors.

![Figure 5-19: Repeatability of the CDDR](image)

Figure 5-19: Repeatability of the CDDR
5.4. Findings

The whole theoretical statics workspace could not be used in hardware as it was difficult to move the end-effector in the region close to the edges of the statics workspace as it was approaching to singularity on the edges of the workspace where cable needs infinite force to move the end-effector. Commanded trajectory was very close to the singularity region, so it was found that during most of the motion the actual trajectory was inside the desired trajectory, as cables needed very high force to move the end-effector close to the workspace boundary.

It was also found by simulation results that the on-line dynamic minimum torque estimation algorithm was required for dynamic CDDR motions with high velocities and accelerations. Otherwise, the simulation revealed that some cables become slack during motion and thus control is lost.

It was revealed that fishing line, which was used initially as a cable needed very high tension to move the end-effector due to its small diameter and less contact area with the pulley hence the author found that bigger diameter of the cable would increase contact area of the cable with pulley that would make cable more effective. The bigger cross-sectional area of the cable also helped to reduce stress in the cable.

It was found that repeatability of the CDDR was quite good. The major source of error could be the initial position and orientation of the end-effector as end-effector was placed manually to its initial position. It was also found that due to sliding of the cable knots on the eyebolts during trajectory generation could change the effective end-effector size and hence that could be one of the possible sources of error.
6. CONCLUSION

The kinematics, statics and dynamics modeling of the modified design were derived. The static workspace was determined for the new design. The robot was simulated using Simulink and Matlab software. The computed-torque, or feedback linearization technique performed perfectly in simulation, i.e. when it was assumed that the dynamic model is known perfectly, the inertial effects and nonlinear dynamics terms are cancelled perfectly.

The hardware of the cable-direct-driven-robot was designed and constructed. The hardware was interfaced with the computer. Wincon software and Quanser control boards were used for real time implementation. The inverse kinematics of the robot was implemented to generate linear and circular trajectories in real time control. Independent cable length PD controllers were implemented for the Cartesian coordinated control. It was found that repeatability of the CDDR was quite good.

A planar cable-direct-driven-robot was designed, constructed, simulated and controlled in this thesis. A new design of translational 4-cables CDDR with 3 degree of freedom (\(\phi\) commanded to 0) and one degree of actuation redundancy (4 cables- 3 d.o.f) with cross cable configuration was implemented successfully. Only the translational CDDR whose end-effector may be fitted with a traditional serial wrist mechanism to provide rotational freedom was considered in this thesis.
7. RECOMMENDATIONS

The results revealed the need to implement the dynamics controller in the future for better performance. It was also found that there is a further scope of improvement in the performance of the robot, if the coordinated Cartesian controller could be used, by introducing vision system for online feedback of the Cartesian coordinates instead of independent length controller.

The whole theoretical statics workspace could not be used in hardware as it was difficult to move the end-effector in the region close to the edges of the statics workspace as it was approaching to singularity on the edges of the workspace where cable needs infinite force to move the end-effector. The future cable robots should be designed considering this factor.

It was also found that due to sliding of the cable knots on the eyebolts during trajectory generation could change the effective end-effector size and hence that could be one of the possible sources of error. Hence author recommends redesigning the end-effector to fix this problem.

The author also recommends that future work should focus on the real time application as well as commercialization of this robot.
REFERENCES


APPENDIX - CDDR OPERATION

This section is intended as a guide for users to operate the CDDR hardware in the appropriate way. It will provide the user with a detailed procedure for setting up, operating and shutting down the CDDR.

Startup procedure

1. Start the computer if not already on. Use Stocker logon and password.

2. Check the MultiQ-PCI board for power. The board is connected with the computer so when the computer is on, the board should be powered. If the board light shows no power, try to change 1A fuse on the board just next to the board light. If the board light still shows no power, contact Dr. Williams to have the board inspected immediately.

Figure A-1: MultiQ PCI board
3. Verify all motor leads are secured in the appropriate amplifier. Each motor is numbered and it can be matched with its corresponding number on the amplifier as shown in the figure below. Each cable coming from the encoder is numbered and can be matched with the corresponding number on the amplifier as shown in the figure below. Each amplifier is numbered on its side.

4. Verify all encoders are in the matching numbered position on the MultiQ-PCI board. Each encoder cable is labeled with a number from 1-4 that matches the number on the encoder slot on the board, which is numbered from 0-3. (Example: Encoder 1 matches with 0)
5. Plug in the power supplies and verify each amplifier is receiving power. Each amplifier has a green light that will be lit when the power supply is being powered.

6. Open Matlab from the desktop.

7. Set path of the Matlab to C:\My Documents\research\Realsim\circletest and open the variables.m from the same folder and run that file. It will define all the variables required for the execution of the program. This program defines the workspace and coordinates for the CDDR. The Matlab workspace should have been cleared after running this program.

![Figure A-3: Matlab Window](image)

8. Open the circletest.mdl file from the same folder. You will see following window.
9. Choose *Wincon* from the menu choices and choose *build*. This step builds the appropriate code programmed in C for implementation of the program in real time. If compiling is successful, a message will be displayed in the Matlab window.

10. After a build, a Wincon server window will be opened as seen in figure below. Choose *Plot* from Wincon server window and select the scope. Select the parameter you want to plot. If you want to plot more than one parameter, follow the same procedure, as it will create only one plot at a time.
11. Verify that the end-effector is in the correct initial position in accordance with the value of the \textit{circ\_radius} in the \textit{variables.m} program. The correct position of the end-effector is \((\text{circ\_radius}, 0)\) with 0 rotation. If the end-effector is not in the right position, user needs to place it manually. The CDDR has no feedback initially to determine its initial position.

12. Press start in the Wincon window. The CDDR will follow the trajectory described in the program and will return to the starting position with some errors.

13. After motion is complete, press stop in the Wincon window and disable power to the amplifiers.


15. Logoff and shutdown the PC.