OPTICAL VORTEX BEAMS: GENERATION,
PROPAGATION AND APPLICATIONS

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ABSTRACT

OPTICAL VORTEX BEAMS: GENERATION, PROPAGATION AND APPLICATIONS

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An optical vortex (also known as a screw dislocation or phase singularity) is one type of optical singularity that has a spiral phase wave front around a singularity point where the phase is undefined. Optical vortex beams have a lot of applications in areas such as optical communications, LADAR (laser detection and ranging) system, optical tweezers, optical trapping and laser beam shaping. The concepts of optical vortex beams and methods of generation are briefly discussed.

The properties of optical vortex beams propagating through atmospheric turbulence have been studied. A numerical modeling is developed and validated which has been applied to study the high order properties of optical vortex beams propagating though a turbulent atmosphere. The simulation results demonstrate the advantage that vectorial
vortex beams may be more stable and maintain beam integrity better when they propagate through turbulent atmosphere.

As one important application of optical vortex beams, the laser beam shaping is introduced and studied. We propose and demonstrate a method to generate a 2D flat-top beam profile using the second order full Poincaré beams. Its applications in two-dimensional flat-top beam shaping with spatially variant polarization under low numerical aperture focusing have been studied both theoretically and experimentally.

A novel compact flat-top beam shaper based on the proposed method has been designed, fabricated and tested. Experimental results show that high quality flat-top profile can be obtained with steep edge roll-off. The tolerance to different input beam sizes of the beam shaper is also verified in the experimental demonstration. The proposed and experimentally verified LC beam shaper has the potential to become a promising candidate for compact and low-cost flat-top beam shaping in areas such as laser processing/machining, lithography and medical treatment.
Dedicated to my husband and my parents
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LIST OF ABBREVIATIONS AND NOTATIONS

OAM: Orbital Angular Momentum
CVB: Cylindrical vector beam
SOP: State of polarization
DOE: Diffractive optical element
SLM: Spatial light modulator
CGH: Computer generated hologram
LC: Liquid crystal
NA: Numerical aperture
FP: Full Poincaré (FP) beams
RCP: Right-hand circular polarization
LCP: Left-hand circular polarization
HW: Half-wave plate
LP: Linear polarizer
ITO: Indium tin oxide
CHAPTER 1

INTRODUCTION

An optical vortex (also known as a screw dislocation or phase singularity) is one type of optical singularity that has a spiral phase wave front. The spiral phase of vortex beams rotates about the optical axis that causes the wavefront of the light to twist like a corkscrew as it propagates. Topological charge is defined as how many twists the light experiences in one wavelength of propagation. The number of topological charge can be positive or negative, depending on the handedness of the twist. The higher the number of the twist, the faster the light is spinning around the axis.

Optical vortex beams have a lot of potential applications in areas such as optical communications, LADAR (laser detection and ranging) system, optical tweezers, optical trapping and laser beam shaping. Optical vortex beam can be applied as an information carrier because it carries orbital
angular momentum (OAM). Vortex beam may propagate through turbulence with less distortion than conventional Gaussian beams thanks to orbital angular momentum leading to an improved data encryption. Vortex beams also have shown great potential in laser beam shaping due to the fact that the dark hollow intensity distribution can be compensated by superimposing Gaussian beam. The resulting flat-top profile can be applied in areas such as laser weapons, optical data processing, lithography, medical surgery and national security.

In this dissertation, I will briefly introduce the definition of optical vortex beams and show several types of methods to generate this type of beams in Chapter 2. Also, propagation of vortex beams through turbulent atmosphere is studied. The simulation results are then compared with conventional Gaussian beams with the same propagation distance and the same turbulence condition. The advantages of using optical vortex beams in communication system are demonstrated. As a critical part in the detection of optical communication system, sensing of optical vortices is discussed in Chapter 3. In Chapter 4, laser beam shaping technique using optical vortex beams is discussed and developed. The successful demonstration of flat-top generation based on the simulation and laboratory results leads to the design of a compact laser beam shaper which is discussed in Chapter 5.
CHAPTER 2

OPTICAL VORTEX BEAMS

2.1 Scalar vortex beams

An optical vortex (also known as a screw dislocation or phase singularity) is one type of optical singularity which has a spiral phase wave front around a singularity point where the phase is undefined. Research into the properties of vortices has thrived since a comprehensive paper by Nye and Berry, in 1974 [1], described the basic properties of dislocations in wave trains.

The spiral phase of vortex beams rotates about the optical axis which causes the wavefront of the light to twist like a corkscrew as it propagates. Topological charge is defined as how many twists the light experiences in one wavelength of propagation. The number of topological charge can be positive or negative, depending on the handedness of the twist [2-6]. The higher the
number of the twist, the faster the light is spinning around the axis. The vortex beam that carries a topological charge of \( l \) is said to have an extrinsic orbital angular momentum (OAM) of \( lh \).

A well-known class of optical scalar vortex is the Laguerre-Gauss solution \( LG_{nl} \) modes:

\[
    u(r, \varphi, z) = E_0 \left( \frac{\sqrt{2}r}{w} \right)^l L_p^l \left( \frac{2r^2}{w^2} \right) \frac{w_0}{w(z)} \exp[-i\phi_{pl}(z)] \cdot \exp \left[ i \frac{kr^2}{2q(z)} \right] \exp(i\varphi),
\]

where

\( p \): Number of nodes in radial direction;

\( l \): Topological charge;

\( l^l_p(x) \): Associated Laguerre polynomials;

\( w(z) \): Gaussian beam size;

\( \phi_{pl}(z) = (2p + l + 1) \tan^{-1} \left( \frac{z}{z_0} \right) \): Gouy phase shift;

\( z_0 = \pi w_0^2 / \lambda \): Rayleigh range;

For \( l = p = 0 \), the solution reduces to the fundamental Gaussian beam solution. For \( l \neq 0 \), the LG mode has a vortex phase term \( e^{il\varphi} \). \( \varphi \) represents the distinctive, transverse vortex phase profile, impressing a linear phase increase in the azimuthal direction to the field. The charge of a vortex can be
an integer or fraction, and also be positive or negative, depending on the
cardedness of the twist.

Figure 2.1 shows the helical phase pattern and corresponding intensity
distributions at focal plane for $LG_{01}$, $LG_{11}$ and $LG_{02}$ mode of vortex beams. For $LG_{01}$ mode, a hollow center is shown in the intensity profile (Figure 2.1 (a))
with the phase pattern jumping from 0 to $2\pi$ around the singularity point. For $LG_{11}$ mode, an additional outer ring is shown besides the hollow center
with the phase pattern jumping from 0 to $2\pi$ for both the inner and outer ring. If the topological charge increases to 2 which is $LG_{02}$ mode, the size of the
dark hollow center is increased while the phase jumping from 0 to $4\pi$ around
the singularity point.
Figure 2.1 Intensity distributions (a,c,e) and corresponding helical phase pattern (b,d,f) at focal plane for $LG_{01}$, $LG_{11}$ and $LG_{02}$ mode of vortex beams.
2.2 Vectorial vortex beams

Vectorial vortex beams are optical beams with polarization singularities [7]. The vector Bessel-Gauss mode is one class of modes that has such properties with cylindrical polarization symmetry. The vector Bessel-Gauss mode with azimuthal polarization symmetry has the formula:

$$\vec{E}(r,z) = E_0 J_1 \left( \frac{\beta r}{1 + iz/z_0} \right) \exp \left[ - \frac{i\beta^2 z}{2k} \right] u(r,z) \exp[i(kz - \omega t)] \vec{e}_\phi, \quad (2.2)$$

where $\beta$ is a scale constant, $u(r,z)$ is the fundamental Gaussian solution, $J_1(x)$ is the 1st order Bessel function of the first kind. Similarly there should exist a transverse magnetic field solution:

$$\vec{H}(r,z) = -H_0 J_1 \left( \frac{\beta r}{1 + iz/z_0} \right) \exp \left[ - \frac{i\beta^2 z}{2k} \right] u(r,z) \exp[i(kz - \omega t)] \vec{h}_\phi, \quad (2.3)$$

where $H_0$ is a constant magnetic field amplitude and $\vec{h}_\phi$ is the unit vector in the azimuthal direction.

Cylindrical vector beams (CVBs) are the most well-known group of examples of vectorial vortex beams whose state of polarizations (SOPs) possess rotational symmetry [7]. Radial and azimuthal polarization is the most common CVBs (Figure 2.2 (a) and (b)) where the SOP follows radial and azimuthal direction at any point on the beam, respectively. Due to the orthogonality, radial and azimuthal polarizations form the basis for CVBs.
And for any generalized CVB as shown in Figure 2.2 (c), it can always be represented as a combination of radial and azimuthal modes.

![Image](image_url)

Figure 2.2 Cylindrical vector beams: (a) radial polarization; (b) azimuthal polarization; (c) generalized CVB.

### 2.3 Generation of optical vortex beams

Optical vortex beams can be obtained by imposing a spiral phase distribution onto the input beam, which can be readily realized by various ways such as spiral phase plate [8-11], deformable mirror [12], diffractive optical elements (DOEs) [13], spatial light modulators (SLMs) [14] and optic fiber [15].

Spiral phase plate is an optical element with helical surface whose optical thickness increases with azimuthal positions. The phase difference of starting point and ending point will determine the generated topological charge of vortex beams. It is a straightforward approach. However, once the spiral phase plate is fabricated, the generated vortex beam would be fixed which is not convenient. People also tried to use a segmented deformable mirror to realize the same function with a spiral phase plate. The tilt of
every segment is controlled to get a helical surface in order to generate the vortex beam. Yet, it is still difficult to control and not convenient.

![Image of spiral phase plate sample](image)

Figure 2.3 One spiral phase plate sample used in our experiment discussed in Chapter 4.

With the development of computer generated hologram (CGH), diffractive optical element (DOE) and liquid crystal display (LCD), the generation of vortex beams using spatial light modulators (SLMs) is more common. Both reflective and transmissive SLMs are commercially available in the market. Liquid crystal (LC) SLMs are either optically or electrically addressed and can modulate the amplitude, the phase, or both, for the input field. For phase only LC SLMs, birefringent LC molecules can be viewed as individual wave plates whose fast axis are modeled as directors. By applying electric field across LC molecules, the angle between the wave vector $k$ of the input electric field and the director, or fast axis of the wave plate can be changed. Thus the birefringence of the LC molecules can be adjusted by varying the voltage applied. A look up table is used for SLM driver software.
to address required phase with a prescribed voltage. The two types of SLMs used in my dissertation research are shown in Figure 2.4. One example of generated charge 1 vortex beam is shown in Figure 2.5.

![Figure 2.4 Two types of SLMs: (a) Boulder Nonlinear Systems XY Series, resolution of 512×512 with pixel size 15 μm; (b) Holoeye HEO 1080P, resolution of 1920×1080 with pixel size 8 μm.](image)
Besides the phase modulation, polarization modulation is also involved for the generation of vectorial vortex beams. Take the radial polarization for an example. The initial electric field distribution is taken to have the same amplitude distribution at the beam waist as the scalar vortex:

\[
\vec{E}(r, \varphi, z = 0) = E_0 \left( \sqrt{2} \frac{r}{w_0} \right)^l L_p \left( 2 \frac{r^2}{w_0^2} \right) \exp \left[ -\frac{r^2}{w_0^2} \right] \vec{e}_r ,
\]

where \( \vec{e}_r \) is the unit vector along radial direction. Mathematically, a radial polarization can be decomposed into a linear superposition of two orthogonally polarized components shown in Figure 2.6.
Figure 2.6 Radial polarized beam as superimposition of two orthogonally linearly polarized HG modes.

Vector vortex beams can be readily generated with a variety of methods such as laser intra-cavity devices and optic fiber \[^7\]. One example of the generated radial polarization is shown in Figure 2.7 \[^{15}\]. The intensity pictures of the generated mode passing through a linear polarizer at different orientation angles are shown to confirm the polarization symmetry.

Figure 2.7 Generated optical vortex beam with radial polarization in our group by using few-mode optic fiber. [Ref. 15]
2.4 Propagation of optical vortex beams through turbulence

Propagation of laser beams through turbulent atmosphere has important impacts in many applications such as free-space optical communications, remote sensing, Laser Radar (LADAR), Light Detection and Ranging (LIDAR).

Previously, spherical wave, plane wave or fundamental Gaussian beams were treated in most of the studies [16]. Higher order Gaussian or various other modified Gaussian beams have been recently studied [17-24]. It has been found that the propagation properties of a laser beam in turbulent atmosphere are strongly affected by its initial beam properties such as the beam profile and coherence. With the unique properties of optical scalar and vectorial vortex beams [25], it is of great interest to look at the propagation of vortex beams through turbulent atmosphere [26-31].

In this section, the numerical modelling capabilities is developed to investigate higher order properties of a $l = 1$ scalar vortex beam (LG$_{01}$) and one typical vectorial vortex beam (Radially Polarized Beam) propagating through turbulent atmosphere and make comparisons with the fundamental Gaussian beam under the same conditions. Specifically, our group studied the scintillation index of the vectorial vortex beams for the first time.
We adopted the multiple thin phase screen method by Martin and Flatte \cite{Martin, Flatte}. The random phase screen is generated using the method of filtering white Gaussian noise to obtain the random field with the desired second-order statistics \cite{Flatte}. A $512 \times 512$ field of pseudorandom complex numbers $A+jB$ are generated with Matlab. This array of complex numbers is then multiplied by $\Delta K^{-1} = \sqrt{\Phi_\theta(K)}$, where $K$ is the transverse wave number, $\Delta K^{-1} = 2\pi/N\Delta$ is the wavenumber increment, $N=512$ is number of sampling points and $\Delta$ is spatial sampling interval; $\Phi_\theta(K)$ is the random phase power spectrum that is given by $\Phi_\theta(K) = 2\pi K^2 \delta_z \Phi_n(K)$, with $\delta_z$ being the propagation distance between two consecutive screens and $\Phi_n(K)$ being the refractive-index power spectrum density. This result is then inverted Fourier transformed to yield the desired random phase field $\theta_1 + j\theta_2$. In this case, two random screens will be generated after this procedure. In our simulation, we choose the real part $\theta_1$ as the random screen.

In this work, a von Kármán type index power spectrum of the turbulence is used:

$$\Phi_n(K) = 0.033 \, C_n^2 \times \exp \left[ - \left( \frac{K l_0}{2\pi} \right)^2 \left( K^2 + \left( \frac{2\pi L_0}{L_0} \right)^2 \right)^{-\frac{11}{6}} \right],$$

\hspace{1cm} (2.5)

where $l_0$ and $L_0$ are the inner and outer scales of the turbulence, $C_n^2$ is refractive index structure constant that represents the atmosphere turbulence level. In this study, the inner and outer scales are chosen to be 1 cm and 3 m respectively. $C_n^2$ with values range from $10^{-14}$ m$^{-2/3}$ to $10^{-12}$ m$^{-2/3}$
are used for atmosphere turbulence from weak to strong. A sample random phase screen generated this way is shown in Figure 2.8 as an example.

Figure 2.8 One example of the generated Random Phase Screen with a grid of 512x512.

The number of screens needed in this method strongly depends on the strength of the turbulence measured by the index of refraction structure parameter $C_n^2$ and the propagation distance. For each propagation distance and turbulence, 20 equally spaced random screens are generated and used. For a given initial laser beam, the beam is propagated to the next plane using angular plane wave spectrum method \cite{34}. Then the random phase screen representing the accumulated turbulence effect is multiplied to the resulted field and continues to propagate to the next screen until the final distance is reached. For each propagation distance, 500 independent realizations were
generated to provide the necessary statistics for the calculations of average irradiance, polarization properties and scintillation index. The word “realization” represents a specific sample function of a stochastic process which means that the field propagates through this sequence of pseudorandom complex numbers mentioned in above will yield different phase-screen realizations and therefore different realizations of received optical field [35].

Below is the sketch of our simulation model for propagation distance of 2000 meters based on the platform of Matlab. For each propagation distance, a total of 500 realizations of the beam’s evolution are captured to yield results of the average irradiance and polarization distribution, and an annular range with plus minus 5% of beam size is picked to evaluate the beam coherence. The 500 realizations take 2-3 days on a personal computer to finish all the calculations for one type of beam propagating through one specific turbulent atmosphere. Due to memory limitation in Matlab on a normal personal computer, it is not possible to store all the fields for each propagation distance in the first place and then analyze other parameters later with this “database”. As a result, the simulation process for different situations was accomplished by three computers to accelerate the computations.
Figure 2.9 Multiple phase screen model for propagation distance of 2000 meters.
The scintillation index is calculated here to quantitatively characterize the irradiance fluctuation which can be used as one parameter to evaluate the beam quality after the propagation [16]:

\[ \sigma_I^2 = \frac{\langle I^2 \rangle - \langle I \rangle^2}{\langle I \rangle^2}, \]  

(2.6)

where \( I \) is the irradiance and the angle brackets \( <> \) denote an ensemble average. The value of scintillation index represents degree of coherence of optical wave through atmosphere turbulence. With increasing path distance,
the scintillation index also increases until reaching the saturation level when the scattering effect gradually weakens the focusing effect. This scattering effect causes the beam losing coherence as it propagates. In a word, lower value of scintillation index means that the beam can keep relative higher beam coherence which would be more desirable in communications.

Furthermore, for the radially polarized vector vortex, the propagation of the radially polarized beam is done by decomposing the field into x- and y-polarized components and let them propagate individually, then assemble them at the observation distance. I record the beam irradiance ($S_0$) as well as the rest three components of the Stokes Parameters ($S_1$, $S_2$ and $S_3$) at each distance that can be used to investigate the polarization properties.

The numerical grid I use is 512×512 points with window size of 3 m. The initial beam radius $w_0$ is 7 cm and the wavelength is 2 μm. The index of refraction structure parameter is chosen to be $C_n^2=10^{-12}$ m$^{-2/3}$. All the simulation is from 500 realizations for each propagation distance. More simulation results can be found in Ref. 36 and 37. More simulation results can be found in Ref. 38.

The irradiance pattern for propagation distance of 100m, 700m and 1500m for $C_n^2=10^{-12}$ m$^{-2/3}$ are summarized in Table 1. For the radially polarized beam, the average values for Stokes Parameters ($S_1$ and $S_3$) of 500 independent realizations are also shown in addition to the average irradiance.
Comparison of scintillation index calculations for these three types of beams is shown in Figure 2.11 [36-37].

Table 1 Average distributions of fundamental Gaussian beam, scalar vortex beam and vectorial vortex beam with $C_n^2=10^{-12} \text{ m}^{-2/3}$ at propagation distance 100m, 700m, 1500m and 2000m.
Figure 2.11 Scintillation Index versus Propagation Distance with $C_n^2=10^{-12} \, m^{2/3}$.

Scalar vortex beam has been proposed for optical communications as information channel. One of the challenges is to detect the existence of the singularity points. Consider the irradiance pattern under the strong turbulence case (Table 1), after 700 m propagation, the beam profiles of both scalar and vectorial vortex beams appear to break down and no characteristic of vortex can be observed due to the de-coherence effect from turbulence. Thus the “information” is lost. However, the image of the Stokes parameter $S_1$ still clearly shows the vectorial vortex beam polarization singularity. This polarization characteristic can still be observed until 1500 m, more than
twice the 700 m propagation distance at which the irradiance characteristic disappears. Eventually, the polarization characteristic also diminishes after further propagation. By examining these Stokes images, it is possible to identify the existence of the vectorial vortex, even though the characteristic vortex structure has disappeared in the irradiance images. This indicates that the vectorial vortex beam may enable a more robust communication channel than the scalar vortex or fundamental Gaussian channels with improved link distance and signal-to-noise-ratio (SNR).

From Figure 2.11, the scintillation index for vectorial vortex beam shows consistent lower value than scalar vortex case. The two curves are separated by a considerable difference. This demonstrates one advantage of using vectorial vortex with respect to the scalar vortex beams.

This advantage of vectorial vortex beam is due to the fact that a vectorial vortex beam can be decomposed into two orthogonally polarized LG modes. These two LG modes are only partially spatially overlapping. As they propagate through the turbulence, they are spatially uncorrelated and will experience different randomness of the atmosphere to some extent, which leads to better retention of the information. Although radial polarization is used as one example to illustrate the unique characteristics of vectorial vortex propagation through turbulence in this study, the same effect is expected for azimuthally polarized laser beams. Explorations into even more complicated spatially variant polarization states may enable further
mitigation of the turbulence effect on their propagation and allow the development of better FSOC systems.

2.5 Summary

In this chapter, the definitions of scalar vortex and vectorial vortex beam are introduced. Methods of generating such types of beams are generally discussed. The properties of optical vortex beams propagating through atmosphere have been studied by developing a numerical modeling tool. This model has been applied to study the high order properties of optical vortex beams propagating though a turbulent atmosphere and calculate scintillation index for vortex beams. The simulation results demonstrated vectorial vortex beams generally have lower scintillation index than scalar vortex beams and fundamental Gaussian beams. In addition, by inspecting the spatial polarization distribution, the results of stokes parameters proved that vectorial vortex beam can keep polarization state much further after the beam breaking down. These advantages indicate that vectorial vortex beams may be more stable and maintain beam integrity better when they propagate through turbulent atmosphere. As a result, the vectorial vortex beams may provide more robust free space communication channels with longer link distance.
CHAPTER 3
SENSING AND DETECTION OF ORBITAL ANGULAR MOMENTUM STATES

From results shown in Chapter 2, optical vortex beams can be good candidates to be as information carriers for optical communication systems. The topological charge of optical vortex beam represents a specific eigenstate of OAM. Every eigenstate is orthogonal to each other so that every optical vortex beam with individual topological charge would be orthogonal to each other. Since the number of eigenstates is unlimited in theory, there would be a potential to provide unlimited channels using optical vortex beam in optical communication systems. In order to build a complete system, the sensing of multiple vortex beams becomes important.

As a fundamental nature of light, the OAM detection cannot be measured directly. Here, we employed the concept of diffraction grating to realize the detection. When the SLM loaded with vortex sensing diffraction grating (the “fork”) is illuminated with a vortex beam, the sign and order of
the topological charge of the incident beam can be detected.

A 1D linear phase diffraction grating can be represented as:

\[ g(x) = \exp\left(\frac{j2\pi}{\Lambda} x \right), \quad (3.1) \]

where \(\Lambda\) is the grating period and \(x \gg \Lambda\) is the horizontal coordinate. Then the linear diffraction pattern is superimposed with a spiral phase of topological charge \(l\).

\[ V_{sp}(\phi) = \exp(il\phi), \quad (3.2) \]

The phase combination of diffraction grating and vortex charge would be:

\[ \varphi(x) = \frac{2\pi}{\Lambda} x + l\phi, \quad (3.3) \]

Grating structure is achieved when the phase is quantized to two-level values, i.e., 0 and \(\pi\). The phase after quantization can be represented as:

\[ \varphi_{qu}(x) = \begin{cases} 
0, & -\frac{1}{2} \varphi_{th} < \varphi(x) < \frac{1}{2} \varphi_{th} \\
\pi, & \frac{1}{2} \varphi_{th} < \varphi(x) < 2\pi - \frac{1}{2} \varphi_{th} \end{cases}, \quad (3.4) \]

where \(\varphi_{th}\) is the quantization threshold, which is used to change the weighting among diffraction orders. And the quantized E-field (level 2) after performing a Fourier expansion along x direction can be represented as:

\[ e^{i\varphi_{qu}(x)} = \sum_{n=-\infty}^{+\infty} A_n e^{in\varphi(x)}, \quad (3.5) \]
With a vortex beam with unknown topological charge \( L \) incident on the grating, the far-field at the focal plane of a lens can be given as a Fourier transform of the product:

\[
E_{\text{far}}(x) = F(k_x)\big|_{k_x = \frac{k_0x}{f}} = F\{ \sum_{n=-\infty}^{+\infty} A_n e^{i(nl+L)\phi} e^{i2\pi n\frac{x}{\Lambda}} \}
\]

\[
= \sum_{n=-\infty}^{+\infty} A_n F\{ e^{i(nl+L)\phi} \} \otimes \delta(k_x - n \frac{2\pi}{\Lambda}), \quad (3.6)
\]

Therefore, at the focal plane, the electric field will become an array of LG beams with different orders. Note that Dirac-Delta function is achieved at \( L = -nl \), where a bright spot is observed. In other words, if a Delta function is observed at the \( n \)th order from the center, then the unknown topological charge \( L \) can be detected to be \( L = -nl \).

**Table 2 Examples of detection of unknown topological charge \( L \) with spiral phase of topological charge \( l \) embedded in the diffraction grating.**

<table>
<thead>
<tr>
<th>( l = 1 )</th>
<th>The order number where Delta function is spotted: ( n )</th>
<th>The unknown incident topological charge: ( L )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>0 (Gaussian)</td>
</tr>
<tr>
<td></td>
<td>+1</td>
<td>-1</td>
</tr>
<tr>
<td></td>
<td>-1</td>
<td>+1</td>
</tr>
<tr>
<td>( l = 2 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>+1</td>
<td>-2</td>
</tr>
<tr>
<td></td>
<td>-1</td>
<td>+2</td>
</tr>
<tr>
<td></td>
<td>+2</td>
<td>-4</td>
</tr>
</tbody>
</table>
The quantized phase of the 1D linear diffraction grating structure with a spiral phase of topological charge \( l = 1 \) is shown in Figure 3.1. The experimental demonstration is shown in Figure 3.2. One linearly polarized HeNe laser with 632.8nm wavelength is collimated to generate the fundamental Gaussian beam. A charge 1 spiral phase plate is used to generate charge 1 vortex beam. The phase pattern shown in Figure 3.1 is loaded into a liquid crystal spatial light modulator (LC-SLM) (BNS XY series). A lens with 250 mm focal length is used to bring the beam into focus and a CCD camera is located at the focal plane for detection.

Figure 3.1 1D vortex sensing diffraction grating with “fork” shape.
Figure 3.2 Experimental setup of vortex sensing using diffraction grating.

If the incident is Gaussian, a delta function would be expected at the 0th order (Figure 3.3). With the +1 order being spotted as a delta function (Figure 3.4 (a)), the incident beam can be detected as a vortex beam carrying topological charge -1. On the contrary, if the -1 order being spotted as a delta function (Figure 3.4 (b)), then the incident beam would be a vortex beam carrying topological charge +1. The color bars on the side of the Figure 3.3 and 3.4 show the relative intensity magnitudes.

This type of vortex sensing diffraction grating can be further extended to two dimensional. If the incident wave is superimposed with multiple
different charges of vortex beams, these different charges can be separated spatially and projected to be as an array. This advantage makes vortex sensing diffraction grating very useful at the receiver end to do de-multiplexing for free space communication system which has been widely developed recently [39-46]. Other types of sensing techniques using specially designed polygon-shaped aperture such as triangular aperture [47-48] and square aperture have also been studied. By breaking the rotational symmetry using polygon-shaped aperture, it is possible to separate and visualize the phase singularities with topological charge \( l \) in the far-field.
Figure 3.3 Experimental results at the focal plane with Gaussian beam input.
Figure 3.4 Experimental results at the focal plane with LG beams input: charge +1 (a) and charge -1 (b) vortex beam illumination.
CHAPTER 4
LASER BEAM SHAPING WITH VORTEX BEAMS

4.1 Introduction to laser beam shaping

Beam shaping is generally understood as the manipulation of the intensity distribution and the wave front of a laser beam \[49\]. A typical and important beam shaping problem is the conversion of a Gaussian-profile laser beam into a beam of uniform amplitude and phase \[50-52\]. This is the Gaussian to flat-top beam shaping problem achieved by implementing an appropriate phase function which would produce the shaped beam into optical design process. Applications of beam shaping include laser/material processing, semiconductor processing, LEDs, solar technology, flat-panel display structuring, laser weapons, optical data processing, lithography, printing and LADAR system.

In these applications, beam profile with uniform intensity and steep edge roll-off is preferred due to the improved energy efficiency. In
semiconductor lithography processing, a flat-top beam profile is often desired across the imaging field of the optical systems which can improve the laser energy processing window dramatically as well as the efficiency; In industrial laser machining/welding/cutting systems, the evenly distributed energy output is preferred for efficiency and precision; in laser radar and coherent imaging systems, uniform illumination is desired in the far field of the source. However, common Gaussian beams do not provide uniform intensity distributions and suffer considerable loss of optical power if expanded to obtain uniform illumination. Therefore the efficient shaping of a Gaussian beam into a uniform beam is of great significance.

4.2 Review of related beam shaping research

Traditional methods such as reflective/ refractive methods and diffractive methods have been extensively developed to generate flat-top beam profile by modifying the amplitude and/or the phase of the incident field. These techniques are designed with laser beams of linear or spatially homogeneous polarization. Most of those techniques work fine, but they still suffer from complicated system design and low energy efficiency. Polarization related methods have also been discovered to add a new degree of freedom to the beam shaping system design.

One of the accepted parameter to evaluate the quality of laser beam shaping system is $\beta$ \cite{49}. The $\beta$ value is defined to provide a relationship
between the desired flat-top beam size with the diffraction-limited beam size which is a measure of the contribution of diffractive effects over geometric optics effects. Optical design of laser beam shaping systems can be achieved using either physical or geometrical optics. For a single-mode Gaussian beam system, a pre-calculation of the parameter $\beta$ can help to determine whether geometrical optics or diffraction would dominate the system design so that appropriate beam shaping methods can be decided.

$$\beta = \frac{2\sqrt{2\pi} r_0 y_0}{\lambda f},$$

(4.1)

where

$r_0$: $1/e^2$ intensity radius of the incident Gaussian beam;

$y_0$: half width of the desired output flat-top profile;

$\lambda$: wavelength of the laser beam;

$f$: effective focal length of the focusing optics.

Once the method is fixed, the larger beta value is, the better quality of flat-top profile will achieve.
4.2.1 Geometrical optical method for laser beam shaper design

Initial design of beam shaping systems started from the use of geometrical methods. Reflective and refractive optical systems have been intensively developed to shape the laser beam intensity profile. Lenses and mirrors are the major components in these systems. The laws of reflection and refraction, ray tracing techniques and constant optical path length will help to design the geometrical surfaces of each optical component.

Malyak,\textsuperscript{[53]} Shealy and Chao\textsuperscript{[54-55]} have designed a two-mirror laser profile shaping system with rectangular symmetry and no central obscuration. Consider the two-mirror beam shaping system illustrated in Figure 4.1. The authors collimated the input and output beams and made them parallel to the optical axis. Then the two geometrical surfaces were solved by combining two dimensional ray tracing equation and energy conservation equation. The optical analysis software ZEMAX has been used for performance modeling and tolerance analysis of this system.
Besides the optical surface shapes, index of refraction profiles may also be used as design variables. A set of differential equations using intensity mapping and the constant optical path length condition had been derived by Rhodes and Shealy\cite{56} to calculate the shape of two-aspherical surfaces of a lens system that expands and converts a Gaussian laser beam profile into a collimated, uniform irradiance output beam. A two-plano-aspherical lenses based on the method above have then been designed, fabricated and tested\cite{57-58}.

A typical two-lens optical beam shaping system design is shown below in Figure 4.2. Generally speaking, a two-lens system with the two curved
surfaces should satisfy the design condition of the beam shaping system. Important parameters which need to be carefully designed are:

- Wavelength of laser; Most of the case, we are dealing with monochromatic laser beam system. No chromatic aberration is considered.

- Radius of curvature of the primary and secondary lens;

- Thickness of the primary and secondary lens;

- Spacing between lenses;

- Glass choice of the lenses (Index of refraction of the lenses);

\[ \text{Figure 4.2 Configuration of two-lens shaping system.} \]

Reflection or refractive techniques are energy efficient. There are several mature software to design the proposed system. However, this type of
technique generally requires complicated aspheric surfaced which are sometimes difficult to fabricate, therefore limited in applications. Furthermore, reflective/refractive techniques do no preserve polarization.

### 4.2.2 Diffraction theory for laser beam shaper design

Diffraction-based method for converting single-mode Gaussian beams into beams with uniform irradiance profiles had been widely developed during this decade. The diffraction approach is based on a Fourier transform relation between the input and output beam functions. As described in Figure 4.3, the black box in the system is used to diffract the input beam to the designed output irradiance pattern. Either a single optical element or a set of optical components of differing types such as lenses, mirrors, prisms, diffractive optics and holograms can do the trick depending on the product requirements. Iterative algorithm is the most popular approach to solving diffraction-based beam shaping problem. Using these algorithms, a general solution for a shaping function, amplitude and phase can be obtained and then optimized in the optical design realizing the shaping function.
Figure 4.3 Configuration of diffraction theory beam shaping system.

\textit{a) Iterative technique to design binary gratings for profile shaping}

In 1982, W. B. Veldkamp\textsuperscript{[59]} presented an efficient flat-top beam profile shaper which used a binary grating to modulate the amplitude and phase of the incident beam, and converted incident Gaussian laser beam profile to an approximate flat-top distribution at the focal plane. An iterative phase-amplitude interdependent solution by using a least-mean-square-error criterion was provided.

\textit{b) Holographic beam shaping system}

Compared to the design of aspherical lenses in previous beam shaping system, computer-generated hologram (CGH) can easily generate a required wave front. Moreover, the diffraction efficiency of volume phase holograms can be high, so that high diffraction efficiency could be obtained. Han, Yukihir, and Kazumi\textsuperscript{[60]} designed two holograms to reshape collimated laser beams with Gaussian profile to uniform profiles. The first hologram deflects
the incident rays so that the intensity distribution is converted to a uniform one. The second hologram deflects the rays so that the uniform distribution leaves as a collimated beam.

![Diagram of holographic beam shaping system](image)

**Figure 4.4** Configuration of holographic beam shaping system. [Ref 60]

The advantage of using computer-generated hologram design is the hologram is very easy to generate. But the major problem here is that the diffraction efficiency is relatively low.

c) **Diffractive phase elements design for beam shaping system**

The most straightforward approach, apodization and truncation, is often avoided because of its low transfer efficiency in energy. For energy efficiency, optical-phase filters that are refractive, reflective, diffractive, or a combination of all may be implemented. Because the reflective and
refractive surfaced (which are usually in the form of aspherical lenses) are
difficult to fabricate and are thus limited in application, diffractive
techniques are more general and practical in beam shaping. Diffractive
optics in the form of computer-generated holograms, diffraction gratings
have found wide applications. However, they still suffer low diffraction
efficiency. Therefore the diffractive optical phase element stands out as an
ideal choice for beam shaping because of high diffraction efficiency, a coaxial
transformation feature, compact configuration, and low cost of production
and replication.

A typical design method based on the Yang-Gu algorithm\textsuperscript{[61]} is
proposed for computing the phase distributions of an optical system
composed of diffractive phase elements that achieve beam shaping with a
high transfer efficiency in energy. Again, the diagram of a typical compact
optical system for beam shaping is illustrated in Figure 4.5. This optical
system consists of two diffractive phase elements with a separate distance
under which the paraxial approximation is valid. The first element locates
close to the input plane P1, converts the perpendicularly incident Gaussian
beam into a beam that is of uniform intensity but has phase distribution on
the output plane P2. The second element locates close to the output plane,
compensates the phase nonuniformity so as to retain the desired uniform
intensity distribution. For fabrication to the diffractive phase element the
phase distribution should be quantized in the interval $[-\pi, \pi]$ and encoded
as a thickness distribution onto the surface relief material using etching or lithography methods.

Figure 4.5 Configuration of DPE design for beam shaping system. [Ref 61]

Advantages of using diffractive phase elements design is energy efficient, simple and flexible. The resultant product is compact and low cost. But unfortunately, phase quantization leads to a degradation of output uniformity.

4.2.3 **Polarization related laser beam shaper design**

Polarization has been considered as a new degree of freedom in beam shaping system design recently. Flat-top has been achieved by manipulating the polarization of input optical beam such as focusing cylindrical vector beams under high numerical aperture (NA) and spatial engineering of polarization under low NA.
a) *High NA beam shaping techniques*

Flat-top is achieved by properly balancing radial and azimuthal polarized component \[^{62}\]. Under high NA condition, the longitudinal component of radial polarized beam has a central peak which makes the combination of radial and azimuthal component flat-top profile. Figure 4.6 shows the theoretical calculation of the superposition of radial and azimuthal polarized beam.

![Figure 4.6 Theoretical calculation of the superposition of radial and azimuthal polarized beam. [Ref 62]](image)

This method suggested a new approach for beam shaping technique because it is polarization involved. Flat-top is achieved simply by adjusting the weight (Ex: half-wave plate) two orthogonally polarized components. Furthermore, phase distribution for these two components is uniform which
made the flat-top generation much easier. The drawback of this technique is that the high NA may not be preferable for some applications such as laser machining, and it is not easy to generate a well-shaped radial (azimuthal) polarized beam at that time.

b) *Polarization Plate Design*

Followed by the high NA beam shaping design thought, people start to think about techniques for low NA beam shaping design. One example is from Bing Hao (University of Minnesota) \[63-64\]. Spatial engineering of polarization is proposed as a novel method of beam shaping. A flat-top shaped focus can be obtained in the far field by changing the polarization in the pupil plane in a spatially inhomogeneous manner. The polarization distribution is designed using numerical global optimization algorithms. Compared to phase modulation, polarization modulation is non-absorbing in nature, thus retaining the advantage of high conversion efficiency. In addition, the polarization beam shaping method appears to be particularly well suited to tightly-focused laser beams, where the overall size of the focused spot is to be minimized.
Figure 4.7 Polarization plate design beam shaping system (left) and comparison of theoretical calculation with experimental results (right). [Ref 64]

In this technique, a polarization plate is designed as an optical element that modifies only the polarization in a spatially varying way across the wavefront. Figure 4.7 suggests that in order to achieve a flat-top intensity distribution in the focal plane, the polarization of the incident beam needs to change linearly with location from one side of the beam to the other. The simple linear dependence of the polarization angle with position greatly simplifies the optimization process. Once the linear relationship is revealed, it is straightforward to optimize the polarization plate by adjusting the starting and ending points of the linear segment along with its slope. Figure 4.7 (right) shows the comparison of theoretical results with experimental results. The $\beta$-value for this technique is 1.8, which makes sense for their minimum focus spot purpose.
c) **Superposition of two orthogonally polarized components—our initial work**

Our initial design for flat-top profile is to superimpose two orthogonally polarized beams which are x-polarized Gaussian component and y-polarized charge 1 vortex beam \[65\]. Under low NA lens, Gaussian component gives rise to a focal intensity distribution with a solid center while the vortex component gives rise to a donut distribution with hollow dark center. With appropriate weight adjustment between these two components, flat-top profile focusing can be obtained.

Vortex beam can be easily generated using spatial light modulator (SLM) by loading appropriate spiral phase pattern. Because the SLM only responses to one polarization (y-polarized vortex beam), the Gaussian component and vortex component can be in the same optical path without interfering with each other. The experimental result shows good agreement with the theoretical prediction. But the problem here is that the edge of flat-top profile is not steep which is not preferable in many applications. This can be improved through the using of LG modes with higher topological charges which will be discussed in detail in section 4.3.
Figure 4.8 (a) superposition of x-polarized Gaussian component and y-polarized charge 1 vortex beam (b) resultant profile after superposition (c) experimental results of flat-top profile (d) linescan of the experimental results.

4.2.4 Summary

In this section, beam shaping techniques and applications are introduced. Basic beam shaping metrics are explained. Four different beam shaping techniques: reflective/refractive method, diffractive method, polarization
related method and resonator design related method are discussed in detail. Several examples are interpreted for each method. Advantage and disadvantage of each method are discussed and compared. Further study will be combining appropriate DOE design with polarization related method which makes the system more compact and flexible.

4.3 Vectorial vortex beam shaper design

We propose and experimentally demonstrate a method using Full Poincaré (FP) beams \cite{66} to generate two-dimensional (2D) flat-top with spatially variant polarization under low NA focusing \cite{67-68}. One key feature of FP beams is that the states of polarization within the beam cross-section span the entire surface of the Poincaré sphere [Appendix I]. FP beams can be generated through superimposing orthogonally polarized beams with spatially different intensity distributions. Generation of the first order FP beams has been previously studied using fundamental Gaussian (LG$_{00}$) and charge 1 vortex beam (first order Laguerre Gaussian (LG$_{01}$) beams) of right hand circular polarization (RCP) and left hand circular polarization (LCP), respectively.

As mentioned in section 4.2, the edge roll-off of the flat-top generated by first order FP beams is not steep due to the gentle roll-off of the transverse profile for the focused LG$_{01}$ component. This can be improved through the using of LG modes with higher topological charges. Our proposed design is to
achieve the flat-top focusing with the second order FP beams generated through linear combination of horizontally (x) polarized fundamental Gaussian ($LG_{00}$) beam and vertically (y) polarized charge 2 vortex beam (second order Laguerre Gaussian ($LG_{02}$) beam).

The fundamental Gaussian beam can be represented as:

$$LG_{00}(r,z) = A_0 \frac{w_0}{w(z)} \exp \left(-j(kz - \varphi(z)) - r^2 \left(\frac{1}{w(z)^2} + \frac{jk}{2R(z)}\right)\right), \quad (4.2)$$

where $A_0$ is a constant amplitude, $w_0$ is the beam waist size, $\varphi(z) = \tan^{-1}\left(\frac{z}{z_R}\right)$ is the Gouy phase and $z_R$ is the Rayleigh range. Similarly, the second order Laguerre Gaussian ($LG_{02}$) beam can be written as:

$$LG_{02}(r,z) = 2A_0 \frac{r^2}{w(z)^2} \frac{w_0}{w(z)} \exp \left(-j(kz - 3\varphi(z)) - r^2 \left(\frac{1}{w(z)^2} + \frac{jk}{2R(z)}\right)\right)$$

$$- j2\phi \right), \quad (4.3)$$

where $\varphi(z)$ is the Gouy phase for fundamental Gaussian and $\phi$ is the azimuthal angle.

A superposition of orthogonally polarized $LG_{00}$ and $LG_{02}$ can be expressed as the following:

$$E_{FP}(r,z) = cosy LG_{00} \hat{x} + siny LG_{02} \hat{y} = \{\hat{x} + 2tany \frac{r^2}{w(z)^2} \expj(2\varphi(z) - 2\phi)\hat{y}\}$$

* $cosyA_0 \frac{w_0}{w(z)} \exp\left\{-j(kz - \varphi(z)) - r^2 \left(\frac{1}{w(z)^2} + \frac{jk}{2R(z)}\right)\right\}, \quad (4.4)$
where $\gamma$ is the angle between the incident polarization and horizontal (x) polarization. Equation 4.4 can be rewritten in the form of Jones vector, yielding,

$$E_{FP}(r,z) = C \left( \frac{1}{2 \tan \gamma \frac{r^2}{w(z)^2}} \exp(j(2\varphi(z) - 2\phi)) \hat{y} \right), \quad (4.5)$$

Here we can denote the second term in Jones vector as $\rho = \rho_0 e^{j\delta}$, where $\rho_0$ is the ratio between the amplitudes of the y and x components and $\delta$ is the phase difference between the two components. Then we have the following relationship:

$$\rho_0 = 2 \tan \gamma \frac{r^2}{w(z)^2}, \quad (4.6)$$

$$\delta = 2\varphi(z) - 2\phi, \quad (4.7)$$
Simple algebra shows that close to optical axis (z-axis), the polarization is mainly along x-axis while away from axis the polarization evolves into y-polarization. The phase delay is twice the difference between the azimuthal angle and the Gouy phase shift. Therefore at any cross section of the superimposed beam, the phase delay between two components will range from 0 to 4π. Thus, the state of polarization within the superimposed beam cross section will span the entire surface of Poincaré sphere twice. Hence, we call this the second order FP beam. The intensity profile superimposed with polarization map of the second order FP beam at the origin (z=0) is shown in Figure 4.9. Within the Rayleigh range, the evolution of the Gouy phase along propagation will cause a rotation of the polarization pattern. The Gouy phase approaches a constant when the propagation distance is far outside the
Rayleigh range and the polarization pattern will be maintained upon propagation.

When this second order FP beam is focused with low NA lens, the two orthogonally polarized components are spatially separated. The x-polarized fundamental Gaussian produces a solid spot and the y-polarized LG$_{02}$ produces a donut distribution around the x-polarized solid spot. With appropriate relative weighting between these two orthogonally polarized components, the overall intensity becomes a flat-top.

One potential problem of using higher order LG beam is that the size of the dark center area of LG$_{02}$ mode is slightly larger than the size of the LG$_{00}$ mode. This size mismatch causes ripples in the resultant intensity profile instead of the ideal flat-top. This problem can be solved by defocusing the y-polarized LG$_{02}$ beam slightly with respect to the x-polarized component to axially separate the focal planes of two orthogonally polarized components. Therefore we can find a location between the two focal planes where the two orthogonally polarized components produce comparable intensity and size to generate a flat-top profile. A theoretical calculation is shown in Figure 4.10 (a). The input beam diameter is 7mm. Due to the use of the second order FP beam, the edge roll-off is much steeper compared with the case of using the first order FP beam.
Figure 4.10 Theoretical calculation (a) and experimental result (b) of flat-top profile produced by superimposing fundamental Gaussian beam and second order FP beam.
4.4 **Experimental Demonstration**

The schematic of experimental verification is shown in Figure 4.12. One linearly polarized HeNe laser with 632.8 nm wavelength is collimated to generate fundamental Gaussian beam. The pinhole right after the laser is used to control the input beam size. The half-wave plate (HW) and the linear polarizer (LP) are used to change the beam intensity from the linearly polarized input laser beam and to conveniently rotate the linear polarization of the laser beam, which essentially adjusts the ratio between the two orthogonally polarized components, x and y polarizations. A phase pattern to generate $LG_{02}$ mode with defocusing power of 0.167 diopter (Figure 4.11) is loaded onto a liquid crystal spatial light modulator (LC-SLM) (HOLOEYE 1080P). This reflective phase-only LC-SLM only provides phase modulation to the y-polarized component of the incoming beam and reflects off the x-polarized component without phase modulation. After reflected off from the SLM, the x-polarized component maintains Gaussian profile with a planar phase while the y-polarized component becomes $LG_{02}$ mode. A lens with 250 mm focal length is used to bring the FP beam into focus. The defocusing power in the pattern separates the focus of the two orthogonally polarized components axially. A CCD camera is translated between the two focal planes in order to find the best position for high quality flat-top profile. A 2D flat-top beam profile with steep edge roll-off which is 14mm away from the
focal plane of the fundamental Gaussian component has been obtained (Figure 4.10(b)).

Figure 4.11 Phase pattern to generate LG_{02} mode with defocusing power of 0.167 diopter.
Figure 4.12 Experimental setup for flat-top profile generation. HeNe Laser: Linearly polarized HeNe Laser; Pinhole: adjust the input beam size; LP: linear polarizer; HW: half-wave plate; BS: beam splitter; SLM: liquid crystal spatial light modulator (HOLOEYE 1080P); CCD: Spiricon camera. (Inset) Topological charge of +2 phase pattern with lens phase pattern superimposed displayed on the SLM.

Due to the nature of orthogonality, the x- and y-polarized components will not interfere at the observation plane, which would cut off high frequency interference noise. This is one of the most important advantages of this spatially variant polarization based technique. It is also worth noting that the phase pattern loaded to SLM does not require any information about input beam size. Different input beam sizes can be accommodated to generate flat-top by conveniently adjusting the second half-wave plate to get
appropriate relative weighting and searching for an optimal working distance. Flat-top profiles of two different input beam sizes are shown in Figure 4.13 to illustrate this feature. The input beam diameters are 4.5mm and 5.0mm, respectively. This result represents a significant advantage over many of the previous reported flat-top beam shaping techniques that have strict requirement of the input beam size.
Figure 4.13 Flat-top profile of three different input beam diameters: (a) 2D profile with diameter 4.5 mm ($\beta = 4.099$); (b) 3D profile with diameter 4.5 mm; (c) 2D profile with diameter 5 mm ($\beta = 4.71$); (d) 3D profile with diameter 5 mm.
4.5 Phase quantization for potential beam shaper design

In order to practically implement the flat-top generation technique, the phase (Figure 4.11) has to be quantized for potential diffractive optical element (DOE) design\cite{68}. The continuous phase pattern and its corresponding phase through quantization process are shown in Figure 4.14, respectively. For further simplification of the fabrication process, a 4-level quantization has been chosen and verified, both theoretically and experimentally. With the quantized phase pattern, the following flat-top profile can be obtained at 13mm defocus away from the focal plane by adjusting the ratio between the y-polarized LG_{02} mode and x-polarized fundamental Gaussian beam with input beam radius 7mm. A 2D profile and its line scan are shown in Figure 4.15. To experimentally demonstrate the feasibility of this concept, a quantized phase of topological charge 2 with defocusing power 0.167 is loaded onto the SLM. The intensity and its line scan captured at the image plane 15mm away from focus is shown in Figure 4.16. A steep edge roll-off is observed as expected from theoretical prediction. The successful demonstration of quantized phase would lead to the fabrication of a compact flat-top beam shaper.
Figure 4.14 Phase pattern loaded on SLM (a) continuous phase; (b) 4-level quantized phase.
Figure 4.15 Theoretical prediction for flat-top generation: (a) 2D profile; (b) 3D profile; (c) line scan.
Figure 4.16 Experimental results for flat-top generation: (a) 2D profile; (b) 3D profile; (c) line scan.
4.6 Summary

We proposed and demonstrated a method to generate 2D flat-top beam profile using the second order full Poincaré beams. Its applications in two-dimensional flat-top beam shaping with spatially variant polarization under low numerical aperture focusing have been experimentally studied. Experimental results show that the flat-top profile with good quality and steep edge roll-off can be realized. Further experimental results verified that flat-top profile can be conveniently achieved with respect to different input beam sizes, which represents one key advantage of this technique compared with existing techniques. The concept of quantizing the phase pattern has been proposed and theoretically and experimentally verified, which makes a potential candidate to a compact, low-cost and versatile beam shaper. The design, fabrication and test of the proposed beam shaper will be discussed in detail in Chapter 5.
CHAPTER 5

COMPACT VECTORIAL VORTEX BEAM SHAPER

5.1 Introduction

Laser beams with flat-top profile are often desired in many applications including lithography, laser/material processing, medical treatment and national security. One common way to generate such uniform power distribution is the conversion from a Gaussian laser source by the use of a flat-top laser beam shaper. Several types of beam shaper have been proposed and manufactured that utilized refractive or diffractive optical devices with spatially homogeneous polarization as the input. Flat-top shaping techniques with spatially variant polarization that requires appropriate phase retardation modulation also have been proposed and demonstrated recently. As mentioned in Chapter 4, we proposed and experimentally demonstrated the method of generating two-dimensional flat-top focusing with the second order full Poincaré (FP) beams by using a LC spatial light modulator (SLM). The second order FP beam is a beam whose states of polarization within the beam cross-section span the entire surface of the Poincaré sphere twice. This
type of beams can be generated through linear superposition of horizontally (x-) polarized fundamental Gaussian (LG\(_{00}\)) and vertically (y-) polarized second order Laguerre Gaussian (LG\(_{02}\)) beams. When focused with low numerical aperture (NA) lens, the x-polarized LG\(_{00}\) produces a solid spot while the y-polarized LG\(_{02}\) contributes a concentric donut shape distribution. A flat-top profile can be obtained with appropriate relative weighting between these two orthogonally polarized components and optimal imaging distance. The SLM served well for the purpose of proof-of-principle demonstration. However, the LC SLM itself is expensive and bulky, making it unrealistic for practical beam shaping applications. The LC based cell structure would be a good alternative in this situation because of its compactness, low cost and high efficiency.

### 5.2 Liquid crystal cell structure

Liquid crystal (LC) material\(^{[70-72]}\), which has been widely used in electro-optics devices such as spiral phase plate and liquid crystal cell structures\(^{[73-74]}\), is a popular choice to provide such a phase modulation. Liquid crystals (LCs) are matter in a state whose molecule order is between the crystalline solid state and the liquid state. Because of such an intermediate state, the LC may flow like a liquid and also show dielectric and optical anisotropies (such as birefringence) which are typical for crystals. One of the most common LC phases is the nematic. In a nematic phase, the molecules have a characteristic linear alignment. They are free to flow and their center of mass
positions are randomly distributed as in a liquid, but still maintain their long-range directional order. For practical use such as liquid crystal display, LCs are arranged in spatially separated cells with carefully chosen dimensions and the optically properties of LC cell can be manipulated by application of an external electric field. Twisted nematic LC cell is commonly used in LCD application whose top and bottom cover perpendicular alignment structures for the molecules. However, in our beam shaper design, untwisted LC cell is preferred whose molecule orientation of entrance and exit facet is parallel. The molecule configurations of the twisted and untwisted LC cell are shown in Figure 5.1.

![Figure 5.1 Molecule configurations of the twisted (a) and untwisted (b) LC cell.](image)

The nematic liquid crystal (LC) cell structure can be treated as a succession of lots of very thin wave plates whose orientation of their optical
axis can be altered with respect to the change of molecular axis direction. The 
Jones matrix of the LC cell structure may be described as the multiplication 
of all the matrices of those very thin wave plates: [Ref. 75]

\[
W_{LC} = R(\alpha)e^{-i(\beta + \Phi_o)} \begin{pmatrix}
\cos \gamma - i\left(\frac{\beta}{\gamma}\right)\sin \gamma & -\left(\frac{\alpha}{\gamma}\right)\sin \gamma \\
\left(\frac{\alpha}{\gamma}\right)\sin \gamma & \cos \gamma + i\left(\frac{\Gamma}{\gamma}\right)\sin \gamma
\end{pmatrix}, \tag{5.1}
\]

where

- \(\alpha\): twist angle of the molecules between the entrance and exit facets of the cell;
- \(R(\alpha) = \begin{pmatrix}
\cos \alpha & \sin \alpha \\
-\sin \alpha & \cos \alpha
\end{pmatrix}\): the rotational matrix;
- \(\Phi_o\): the voltage independent phase which will be a constant phase in our 
  derivation;
- \(\gamma\) is given by

\[
\gamma = \sqrt{\alpha^2 + \beta^2}, \tag{5.2}
\]

and \(\beta\) is the birefringence according to the specification the liquid crystal 
material and wavelength of the incident light:

\[
\beta(V) = \frac{\Gamma(V)}{2} = \frac{\pi d}{\lambda}(n_e - n_o), \tag{5.3}
\]

Since the extraordinary index of refraction \(n_e\) is dependent on the 
orientation of the LC molecules which is controlled by the external applied 
voltage, the birefringence \(\beta\) then can be modified by choosing appropriate
voltage. In our case, we’d like the molecule orientation of entrance and exit facet to be parallel. Thus, the twist angle $\alpha$ and quantity $\gamma$ can be written as:

\[
\alpha = 0; \quad \gamma = \beta, \tag{5.4}
\]

\[
W_{LC} = e^{-i(\beta + \Phi_0)} \begin{pmatrix}
\cos \beta - i \sin \beta & 0 \\
0 & \cos \beta + i \sin \beta
\end{pmatrix}
\]

\[
= e^{-i(\beta + \Phi_0)} \begin{pmatrix}
e^{-i\beta} & 0 \\
0 & e^{i\beta}
\end{pmatrix}
\]

\[
= e^{-i\Phi_0} \begin{pmatrix}
e^{-i2\beta} & 0 \\
0 & 1
\end{pmatrix}, \tag{5.5}
\]

Now the Equation 6.5 is multiplied by the Jones matrix of incident light

\[
W = W_{LC} \cdot W_{inc} = e^{-i\Phi_0} \begin{pmatrix}
e^{-i2\beta} & 0 \\
0 & 1
\end{pmatrix} \cdot \begin{pmatrix} E_x \\ E_y \end{pmatrix}
\]

\[
= e^{-i\Phi_0} \begin{pmatrix}
e^{-i2\beta} E_x & 0 \\
0 & E_y
\end{pmatrix}, \tag{5.6}
\]

If the incident is consist of two orthogonally polarized components and only one of them is aligned parallel with the LC molecule orientation, the phase change induced by voltage will be imposed on this polarized component. The orthogonally polarized component will pass through the entire LC cell structure without any change.
5.3 Working principles of LC beam shaper

In our proposed technique, LC SLM was used for the second order FP beam generation. The phase pattern that was loaded onto SLM is shown in Figure 5.2(a). For practical implementation purpose, the phase has to be quantized for potential beam shaper design. A 4-level phase quantization process (Figure 5.2(b)) has been successfully verified, both theoretically and experimentally using LC SLM.

Figure 5.2 Phase pattern loaded into SLM for second order FP beam generation: (a) continuous phase; and (b) 4-level quantized phase.

This 4-level quantized phase pattern can be realized with a LC based birefringent wave plate whose optic axis depends on an externally applied electric field. The LC beam shaper cell structure we propose consists of a LC layer sandwiched by two pieces of indium tin oxide (ITO) coated glass (25 x 25
x 1.1 mm from Cytodiagnostics). The thickness of the coated ITO layer is around 30nm. The illustration of the proposed cell structure is shown in Figure 5.3. In order to obtain a pure phase modulation and avoid amplitude or polarization modulation, the directors of LC molecules need to be homogeneously aligned (untwisted). In other words, the rubbing directions of two ITO glasses are parallel. The LC device becomes a wave plate which features a voltage-dependent birefringence. Therefore, the incident light would experience ordinary and extraordinary refractive indices for orthogonally polarized components. For light polarized along the directors of LC molecules (extraordinary light), the desired phase pattern would be imposed onto the beam through engineering the voltage distribution on the ITO glass. Meantime, for light polarized in the orthogonal direction (ordinary light), the wave front does not change in distribution except a piston phase due to propagation.

As mentioned above, the desired second order FP beam can be generated through superposition of orthogonally polarized $LG_{00}$ and $LG_{02}$ beam. As in the proposed LC device, the second order FP beam will be generated by the superposition of extraordinary field and ordinary field. After being focused by a lens, the extraordinary field would become a donut-like Laguerre-Gaussian beam while the ordinary field remains a Gaussian shape. By introducing a defocus function to match the sizes of intensity distribution
of both beams, flat-top can be achieved by directly superimposing the intensity patterns at optimal imaging plane.

Figure 5.3 Illustration of the proposed beam shaper cell structure.
To realize the design proposed in Figure 5.3, the Indium tin oxide (ITO) electrode on one piece of the glass is lithographically patterned into 8 spiral-shaped regions that are shown in Figure 5.4. Every 4 contiguous regions realize a $2\pi$ linear phase change with properly chosen driving voltages. Therefore, the entire design will impose a $2 \times 2\pi$ phase modulation around the center of the design to generate LG$_{02}$ beam. Adjacent regions are
separated by a 100 um insulation gap (blue areas shown in Figure 5.4) where ITO is removed from glass substrate. The entire spiral pattern has a 10um narrow annular gap near the outer edge of the structure. Since the resistance of ITO is inversely proportional to the width, the narrow gap behaves like a large resistance. The width of each gap is carefully designed to be the same so that each spiral region of the ITO patterning is connected to the adjacent one through a fixed resistance. When external voltage is applied to the patterned ITO glass with the other un-patterned ITO coated glass grounded, a linear voltage drop will be present across each 4 spiral regions on the ITO layer. In other words, the design becomes a circuit with equal resistances in series so that evenly-spaced driving voltages can be obtained on the spiral regions. By choosing an appropriate input voltage range where the phase retardation change of $2\pi$ is linearly proportional to the driving voltage, the LC cell structure would be able to realize $2\pi$ phase change with 4 contiguous spiral regions or $2 \times 2\pi$ with the entire design.

A COMSOL simulation had been done to verify the voltage distribution across the proposed beam shaper structure with applied external voltage (Figure 5.5 and 5.6). Assuming the applied voltage of 10V, the shown color distribution presents the desired linear voltage drop across each 4 spiral areas (8 spiral areas across the entire pattern) which is calculated as 0V, 3.1V, 6.32V and 9.74V. Meanwhile, the 10um narrow annular gap shows significant color transition which plays the role of dividing most of the
voltages successfully. The 10V voltage is randomly picked for proof-of-principal, the actual voltage applied to the designed LC beam shaper would be determined according to the choice of LC material and the thickness of the LC cell structure.

Figure 5.5 COMSOL simulation of voltage distribution across the proposed beam shaper structure with applied external voltage.
Nematic LC mixture E7 from Merck is chosen in this design, and the birefringence of the LC material with 633 nm illumination is 0.21. The thickness of the LC cell structure is chosen as 12 μm. The birefringence of this specific LC cell is measured by a simple setup described below [76-80]. The schematic diagram of the experimental setup is shown in Figure 5.7. The LC cell is placed 45 degree with respect to two polarizers with fast axis perpendicular to each other. A liquid crystal voltage controller is used to apply external voltage to the LC cell. The relationship of output intensity and applied voltage is shown in Equation 5.7.

\[ I = \sin^2 \frac{sV}{2} \]  

(5.7)
where \( \delta(V) \) is the phase change of the LC cell induced by the applied voltage, 
\( d \) is the thickness of the LC cell:

\[
\delta(V) = \frac{2\pi}{\lambda} \Delta n(V) d = \frac{2\pi}{\lambda} [n_e(\theta, V) - n_o] d ,
\]

(5.8)

Figure 5.7 The schematic diagram of the experimental setup to measure the birefringence of this specific LC cell.
Based on the behavior of the liquid crystal, if the applied voltage is sufficient enough, the tilt of molecules will reach the maximum. The direction of molecule orientation (extraordinary index of refraction) would be along with the propagation direction (k direction). Therefore, extraordinary index of refraction now equals to ordinary index of refraction which means that there is no birefringence at this time. With the curve in Figure 5.8 continues, eventually the corresponding phase change will be zero. Following the curve backward, the corresponding phase change of the first peak is $\pi$, the corresponding phase change of the first valley is $2\pi$, etc...

Figure 5.8 Intensity measurements versus applied voltage of the design the LC cell structure of 12 $\mu$m.
The plot of the retardation $2\pi d\Delta n/\lambda$ of the 12-μm thick liquid crystal cell based on the measurement from Figure 5.8 with respect to the applied voltage of the LC cell is shown in Figure 5.9. $d$ is the thickness of the LC cell structure and $\lambda$ is the wavelength of the laser beam. $\Delta n$ is the birefringence of the chosen LC material which is 0.21 for 633 nm illumination. Since a linear voltage drop is required for a $2\pi$ phase change, voltage drops of 0.7 V, 0.9 V, 1.1 V and 1.3 V are chosen to be applied on each contiguous spiral region.

Figure 5.9 Phase retardation versus liquid crystal driving voltage with cell thickness of 12 μm.
5.4 Fabrication of LC beam shaper

Several lithography/nonlithography methods have been developed to fabricate diffractive optics\cite{81-85}. After multiple trials, proper photolithography and chemical etching procedures are developed according to our design in order to remove ITO from substrate for specific areas (blue gaps shown in Figure 5.4). The diagram of the fabrication procedures are shown in Figure 5.11. The mask for photolithography is designed using AutoCAD and CleWin and manufactured by Photo Science Ltd. The AutoCAD script of the mask is provided in Appendix II.

Figure 5.10 Designed mask for photolithography using AutoCAD and CleWin and manufactured by Photo Science Ltd.
**ITO patterning process procedure:**

Before starting the ITO patterning process, make sure that the ITO glass plates has been cleaned.

1) Photoresist spin coat
   a. Put a piece of ITO glass on the spinner with the ITO coated side up, and then fix it by vacuum;
   b. Pour a puddle of SPR-955CM (a positive photoresist) onto the center of the ITO substrate \([\text{Ref. 86]}\]
   c. The thickness of the coated photoresist is about 1.5μm

2) Prebake and clean mask

3) UV exposure
   a. Mask (shown in Figure 6.10) is used to realize spiral regions patterning;

4) Photoresist develop and Hard bake

5) ITO etching
   a. Prepare the etching solution: TE-100 \([\text{Ref. 87]}\):
   b. Heat the etching solution to 80℃;
   c. Immerse the glass plates into the etching solution for 2 minutes;
   d. Rinse with DI water and blow dry;

6) Photoresist removal
Figure 5.11 Fabrication steps of the designed ITO patterning process.

Four gold pads at the bottom of the sample with slim gold trails are deposited to apply external input voltage. The lithographically patterned piece (Figure 5.12) is then rubbed and combined with another piece of ITO coated glass to form the beam shaper cell structure. LC material E7 is then filled into the cell structure with two 12-µm spacers being placed between the two pieces to control the cell thickness.
Figure 5.12 The lithographically patterned ITO piece with deposited four gold pads at the bottom.

Figure 5.13 Experimental platform for rubbing and liquid crystal molecule process.
5.5 **Performance evaluation**

The Veeco Wyko NT9100 White light interferometer is used to produce a topographical image of the fabricated beam shaper sample. The detailed structure is shown in the left side of Figure 5.14. The experimental setup to test the performance of the beam shaper sample is schematically illustrated in Figure 5.15. One half-wave plate is placed in front of the sample in order to adjust the relative weighting between the two orthogonally polarized components. The beam shaper is properly mounted and connected to a designed circuit where the voltage drop from 1.3 V to 0.7 V is supplied for 4 contiguous spiral regions accurately. A lens with 400 mm focal length is used to bring the generated FP beam into focus. One CCD (Spiricon) camera is translated to find the optimal position for high quality flat-top profile.

![Image](image.png)

Figure 5.14 Fabricated ITO electrodes with detailed structure viewed under a white light interferometer. Dark slits are the areas where ITO layer (~30nm) is removed.
Figure 5.15 Experimental setup of testing the performance of the beam shaper. Laser: Linearly polarized HeNe Laser; Iris: adjust the input beam size; HW: half-wave plate; CCD: Spiricon camera.

Figure 5.16 Picture of mounted beam shaper sample in the experimental setup.
5.6 Results and discussions

A flat-top beam profile with input beam size of 6mm has been obtained with steep edge roll-off (Figure 5.17)[88]. The flat-top profile diameter is about 107 µm. The calculated β value is 3.18 which is slightly smaller than the common values in conventional beam shaping using phase modulation. In our case, the small β value indicates the significant role that diffraction plays. The conversion efficiency of the beam shaper is measured as 76% which is defined as the ratio of output power with and without the fabricated beam shaper. It is very close to the diffraction efficiency due to the quantization is 81% ($sinc^2(1/N)$) where N is the number of quantization steps. The loss of conversion efficiency may be due to the reflection caused by index mismatch between multiple interfaces such as air-to-glass, glass-to-ITO and ITO-to-LC. The patterned ITO electrode with complex fine features may cause scattering and diffraction loss as well. Further improvement can be done by putting index matching LC molecules for ITO layer and depositing anti-reflection (AR) coating layer onto the front and back sides of the beam shaper to increase transmission.

Beam intensity flatness and edge steepness are measured to provide the quality of generated flat-top profile. The flatness error at the flat-top area is calculated based on the standard deviation of the height at the flat-top area. And by letting $d_{20\%}$ and $d_{90\%}$ equal the beam diameters at which the
normalized intensity of the beam is 20% and 90%, respectively, the edge steepness $E$ can be defined as:

$$E = \frac{d_{90\%}}{d_{20\%}}$$  \hspace{1cm} (5.9)

The maximum value of $E$ is 1 which represents the best flat-top profile with the ideal rectangular shape. When superimposing the orthogonally polarized $\text{LG}_{00}$ and $\text{LG}_{02}$ modes to generate flat-top profile by using the designed beam shaper, the centers of $\text{LG}_{00}$ and $\text{LG}_{02}$ mode are slightly mismatched which cause a relatively larger base of the flat-top profile. This is why 20% was chosen as the threshold to evaluate the edge steepness. The measured flatness error and edge steepness of generated flat-top profile are shown in Table 3 compared with the values for results from Figure 4.10. The simulated flat-top got the best flatness and edge steepness. After quantization, the beam quality drops because of the diffraction loss. With the fabricated beam shaper, the loss from reflection, absorbing, fabrication and liquid crystal alignment error further decreases the flatness and edge steepness.
### Table 3 Measured flatness error and edge steepness of flat-top profiles

<table>
<thead>
<tr>
<th>Description</th>
<th>Flatness Error</th>
<th>Edge Steepness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulation (Figure 4.10 (a))</td>
<td>0.93%</td>
<td>0.61</td>
</tr>
<tr>
<td>Experiment based on SLM (Figure 4.10 (b))</td>
<td>1.16%</td>
<td>0.58</td>
</tr>
<tr>
<td>Simulation with 4-level quantization (Figure 4.15)</td>
<td>1.24%</td>
<td>0.56</td>
</tr>
<tr>
<td>Experiment with 4-level quantization based on SLM (Figure 4.16)</td>
<td>1.29%</td>
<td>0.54</td>
</tr>
<tr>
<td>Experiment based on beam shaper with 6 mm input beam size</td>
<td>1.31%</td>
<td>0.48</td>
</tr>
<tr>
<td>Experiment based on beam shaper with 5 mm input beam size</td>
<td>1.44%</td>
<td>0.52</td>
</tr>
<tr>
<td>Experiment based on beam shaper with 6.5 mm input beam size</td>
<td>1.67%</td>
<td>0.45</td>
</tr>
</tbody>
</table>
Figure 5.17 The flat-top profile (a) 2D view and (b) 3D view obtained by testing the fabricated beam shaper.
Figure 5.18 Flat-top profile (a) and corresponding linescan (b) of input beam diameters: 5 mm.
Figure 5.19 Flat-top profile (a) and corresponding linescan (b) of input beam diameters: 6.5 mm.
In our proposed beam shaper design, spatially variant polarization-based technique is utilized. One advantage of this technique is that the two orthogonally polarized component of the incident laser beam would not interfere at the observation plane, which would cut off high frequency interference noise. Moreover, the desired phase modulation does not require any information about input beam size. As a result, flat-top profiles can be achieved with different input beam sizes by slightly adjusting the relative weighting between two orthogonal components and finding optimal image distance. Flat-top profiles of two different input beam sizes of 5 mm and 6.5 mm are shown in Figure 5.18 and 5.19 to illustrate this feature. The associated flat-top size is 137 µm and 98 µm respectively. And the \( \beta \) values are calculated to be 3.73 and 3.15, respectively.

5.7 Summary

In summary, we demonstrated a novel compact flat-top beam shaper to realize two-dimensional flat-top beam focusing through generating the second order Full Poincaré beam. A liquid crystal based beam shaper cell structure has been designed, fabricated and tested. Experimental results show that high quality flat-top profile can be obtained with steep edge roll-off. The tolerance to different input beam sizes of the beam shaper is also verified in the experimental demonstration. The proposed and experimentally verified
LC beam shaper has the potential to become a promising candidate for compact and low-cost flat-top beam shaping in areas such as laser processing/machining, lithography and medical treatment.
CHAPTER 6

CONCLUSIONS

In this dissertation, optical vortex beams have been introduced and studied. Concepts and methods of generation are briefly discussed. The properties of optical vortex beams propagating through atmospheric turbulence have been studied. A numerical modeling is developed and validated which has been applied to study the high order properties of optical vortex beams propagating through a turbulent atmosphere and calculate scintillation index for vortex beams. The simulation results demonstrate the advantage that vectorial vortex beams may be more stable and maintain beam integrity better when they propagate through turbulent atmosphere.

1D vortex sensing using specially designed diffraction gratings is simulated and experimentally demonstrated. The success of separating and detecting topological charges of input vortex beams may find important potential applications in optical communications.
We propose and demonstrate a method to generate 2D flat-top beam profile using the second order full Poincaré beams. Its applications in two-dimensional flat-top beam shaping with spatially variant polarization under low numerical aperture focusing have been experimentally studied. Experimental results show that the flat-top profile with good quality and steep edge roll-off can be realized. Further experimental results verified that flat-top profile can be conveniently achieved with respect to different input beam sizes, which represents one key advantage of this technique compared with existing techniques. The concept of quantizing the phase pattern has been proposed and theoretically and experimentally verified, which makes a potential candidate to a compact, low-cost and versatile beam shaper.

A novel compact flat-top beam shaper to realize two-dimensional flat-top beam focusing through generating the second order Full Poincaré beam has been designed, fabricated and tested. Experimental results show that high quality flat-top profile can be obtained with steep edge roll-off. The tolerance to different input beam sizes of the beam shaper is also verified in the experimental demonstration. The proposed and experimentally verified LC beam shaper has the potential to become a promising candidate for compact and low-cost flat-top beam shaping in areas such as laser processing/machining, lithography and medical treatment.


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87. Transene Company, INC. Tin Oxide Indium Tin Oxide Etchant TE-100 Etching datasheet.

Appendix I: Poincaré sphere and Full Poincaré beam

Poincaré Sphere is a useful tool to visualize the polarized light with stokes parameters $S_1$, $S_2$ and $S_3$ as the coordinates. Stokes parameters are all measurable quantities (intensities), which is an alternative of the state of polarization (SOP) representation. Stokes parameters are given as:

\[
\begin{align*}
S_0 &= I_x + I_y = E_{0x}^2 + E_{0y}^2 \\
S_1 &= I_x - I_y = E_{0x}^2 - E_{0y}^2 \\
S_2 &= I_{\frac{\pi}{4}} - I_{-\frac{\pi}{4}} = 2E_{0x}E_{0y}\cos\phi \\
S_3 &= I_r - I_l = 2E_{0x}E_{0y}\sin\phi
\end{align*}
\]

Any combination of $S_1$, $S_2$ and $S_3$ can be found on a sphere with constant radius using the coordinate $(S_1, S_2, S_3)$. In other words, the sphere defined by Stokes parameters contains all possible SOP on its surface. This is the so called Poincaré sphere, which is shown in Figure I below.

Full Poincaré (FP) beam is a new class of beams whose states of polarization within the cross-section span the entire surface of the Poincaré
sphere. FP beams can be generated through superimposing orthogonally polarized beams with spatially different intensity distributions.
Appendix II: AutoCAD script for mask design of the proposed beam shaper

;;; Full Poincare flattop mask is patterned in this script.
;;; This consists of a charge 2 LG and a defocus function with 6·meter focal length.

(defun myerror (s) ; If an error (such as CTRL·C) occurs
                  ; while this command is active...
    (if (/= s "Function cancelled")
        (princ (strcat "\nError: " s))
    )
    (setvar "cmdecho" ocmd) ; Restore saved modes
    (setvar "blipmode" obl)
    (setq *error* olderr) ; Restore old *error* handler
    (princ)
)

;;; This function is set to find the curve for FP flattop generation
;;; for \lambda equal to 0.543 micron.
;;; The expression is
;;; r=sqrt(12\lambda*(\theta/pi-N-C)) with the origin at `center', where N
;;; can take either 0 or 1 and a takes 0, 1/4, 1/2 and 3/4.
;; So 8 curves will be plotted.

;; 2D

(defun fpflattop (N a

   / x y x1 y1 slope xold yold width center theta_min theta_max dtheta delta_theta theta dist tmp_point)

   (setvar "blipmode" 0) ; turn blipmode off
   (setvar "cmdecho" 0) ; turn cmdecho off
   (setvar "osmode" 0) ; turn off Object snap
   (setq lambda0 0.543e-6) ; wavelength
   (setq center '(0 0 0))
   (setq theta_min (* pi (+ N a))) ; starting theta
   (setq delta_theta (/ (* pi 3.5e-3 3.5e-3) (* 12 lambda0))) ; 1.88pi rotation for 3.5mm radius
   (setq theta_max (+ theta_min delta_theta)) ; ending theta
   (setq dtheta (/ pi 180)) ; step size 1 degrees
   (setq theta theta_min)
   (setq dist (* 1000 (sqrt (* 12 lambda0 (- (/ theta pi) N a)))))
   (setq x (* dist (cos theta)))
   (setq y (* dist (sin theta)))
   (command ".pline" (polar center theta dist)) ; start polyline
   (while (< theta theta_max)
      (setq theta (+ dtheta theta))
      (setq dist (* 1000 (sqrt (* 12 lambda0 (- (/ theta pi) N a)))))
      (setq xold x)
      (setq yold y)
(setq x (* dist (cos theta)))
(setq y (* dist (sin theta)))
(setq tmp_point (polar center theta dist))
(command tmp_point)
(setq slope (atan (- y yold) (- x xold))) ; angle
(setq width 0.1)
(setq x1 (+ x (* width (cos (+ slope (/ pi 2))))))
(setq y1 (+ y (* width (sin (+ slope (/ pi 2))))))
(command (polar center (atan y1 x1) (sqrt (+ (* x1 x1) (* y1 y1)))))
(while (>= theta (+ theta_min dtheta))
  (setq theta (- theta dtheta))
  (setq dist (* 1000 (sqrt (* 12 lambda0 (* (/ theta pi) N a)))))
  (setq xold x)
  (setq yold y)
  (setq x (* dist (cos theta)))
  (setq y (* dist (sin theta)))
  (setq slope (atan (- yold y) (- xold x)))
  (setq x1 (+ x (* width (cos (+ slope (/ pi 2))))))
  (setq y1 (+ y (* width (sin (+ slope (/ pi 2))))))
  (command (polar center (atan y1 x1) (sqrt (+ (* x1 x1) (* y1 y1)))))
(command center)
(command "")
(princ) )
;;;

(princ "\n\sfpflat top loaded. ")

(princ)
VITA

EDUCATION

2007--Present  University of Dayton, Dayton OH  
• Ph.D. Electro-Optics Graduate Program  Advisor: Dr. Qiwen Zhan  
  Dissertation: Optical Vortex Beams: Generation, Propagation and Applications  
  • M.S. Electro-Optics Graduate Program  Advisor: Dr. Qiwen Zhan  
  Thesis: Propagation of Vortex Beams through a turbulent atmosphere  
  2003—2007  Nanjing University of Posts & Communications, Nanjing China  
  • B.S. Electro-Optics Engineering

PUBLICATIONS AND PRESENTATIONS

2. Wei Han, Yanfang Yang, Wen Cheng and Qiwen Zhan are preparing a manuscript “Vectorial optical field generator for the creation of arbitrarily complex fields” submitted to Opt. Express.
10. Presentation at Ohio Innovation Summit, April 2009 at Dayton OH.