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I, Pranjal Makarand Vinze, hereby submit this original work as part of the requirements for the degree of Master of Science in Mechanical Engineering.

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Developing a correlation criterion(spaceMAC) for repeated and pseudo-repeated modes

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DEVELOPING A CORRELATION CRITERION(SPACEMAC) FOR REPEATED AND PSEUDO-REPEATED MODES

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by

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Abstract

One of the most important factors in the validation of a finite element model is whether the modal vectors obtained from the finite element solution match sufficiently with the modal analysis results of the part or a test prototype. Modal assurance criterion(MAC) is usually a very effective way to check this condition. However, in case of repeated modes, MAC can give misleading results. A low MAC value could indicate that the finite element model suffers from some deficiencies or in case of closely spaced modes it could mean that these modes are repeated modes. Hence, there is a need for a method that could indicate how well the finite element method based estimates for the repeated modes correlate with the modal analysis modes. This thesis work is an attempt to develop a correlation criterion between a set of repeated roots from a finite element method solution and a experimental modal analysis solution.

Building on a low dimensional modal vector example a vector subspace based approach was identified to help properly define the solution to a characteristic equation with repeated roots. This analogy was extended to higher dimension modal vector cases and vector subspace or hyper planes were identified as a way to model a repeated mode case.

Similar to MAC, consistency of the solution was considered an ideal way to establish correlation. But in this case the consistency of the solution subspace was found to be more important than that of normalized modal vectors. The smallest principal angle between the two solution subspace was identified as a way of measuring the consistency. The criterion, called spaceMAC, was developed as a function of this angle such that the range of the criterion is 0 to 1, similar to MAC. SpaceMAC was defined as $1 - \sin(\theta)$ where θ is the principal angle between the two solution subspaces.

The performance of this criterion was tested with experimental data sets from two different symmetric structures known to exhibit the repeated mode condition.

Preface

To thank everyone who contributed to this research is a task of which I will surely fall short. However, to not do it altogether would not be drawing the correct lessons from that predicament. So, destined for failure as it is and with apologies to anyone I miss out, I will attempt it.

I would like to thank Dr. Randall J Allemang for agreeing to be my advisor. He has been a pillar of support, academically and otherwise, for the past two years. Despite his busy schedule he made sure he discussed all of my questions and concerns in detail. It was during one of his classes and a discussion that followed afterwards, that the idea for this thesis came up. I would like to thank Dr. Allyn Phillips for helping me out not only with general research related problems but especially with X-Modal related questions and also for being on my thesis committee. I would like to thank Dr. Yonfeng Xu for agreeing to be on the defense committee and for his valuable suggestions for this thesis work. I would also thank Michael Mains for helping me with the geometry mapping part of this thesis.

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Chapter 1

Introduction

1.1 Motivation

As the modern design process moves further towards computer based modeling and finite element computations, there is a need for higher and higher certainty about the accuracy of these models. In case of dynamic modeling of systems, comparisons of modal parameter estimates from the model and a prototype is a great way to ascertain the validity of the model. The modal frequencies and modal vectors are usually used for this purpose.

Modal Assurance Criterion(MAC) is the most common way of measuring the consistency of estimates of modal vectors when the lengths of the modal vectors are same. It is also a very convenient method because it can be used in a 'black-box' manner where the user can input the modal vectors and get a direct indicator of correlation between various modal vectors. It is often used to check the validity of an analytic model by using the cross-MAC values between analytic and experimental solution. If the diagonal terms in the cross-MAC are close to 1, it means that the model *models* the dynamic behavior very well. However, there is a drawback in this approach. MAC can give misleading results in case of repeated and psuedo-repeated modes. This can jeopardize the amount of certainty that can be had in a model.

A usual workaround that can be used in these cases is to observe a wireframe animation or to observe the mode shape for the modes in question, that is the modes with low cross MAC values, and check if any shape can be rotated in space to get the shape for some other mode.

1.2 Existing methods

Apart from using the wireframe animation or mode shape to judge whether a set of modes are a repeated set or not, some methods exist to detect and correlate repeated roots. For example, it has been shown that by taking the singular value decomposition(SVD) of unity scale factor(USF), which is defined similarly to modal scale factor(MSF), the number of repeated roots can be found^[1]. A method developed for correlating repeated modes is shown in^[2], where the test mode is expressed as a linear combination of all possible combinations of analytic modes. The cross-MAC values for all these combinations and the test mode shape are calculated. From these cross-MAC values it is found which combination is a repeated mode combination.

1.3 New method

A new way of correlating repeated modes is proposed in this thesis. Since repeated modes do not exist in unique modal vectors but form a unique set of modal vectors, the proposed correlation factor evaluates the consistency of the sets obtained from a FEM solution and experimental analysis. Vector subspace theory is used to identify repeated modal vector sets as vector subspace. The concept of principal or canonical angles is then implemented to compare the two vector subspaces.

1.4 Summary of contents

The remainder of this thesis consists of four more chapters. The necessary theoretical background can be found in Chapter 2. Some basic information about the repeated root condition is followed by the theoretical background for vector subspaces. This is followed by a discussion on the geometry mapping required to downsample the analytical modal vector to the size of the experimental modal vector so that MAC and spaceMAC can be applied. Then a brief discussion on the existing methods to identify and correlate repeated root conditions is done.

Chapter 3 describes the process undertaken to extract and condition the modal vectors for both the analytical and experimental methods so that the resulting modal vectors are accurate and describe the same system. A description of the two case studies and the experimental data and analytical models for the two case studies is also present in this chapter. Then the results from an attempt to implement existing methods and spaceMAC method are discussed in Chapter 4. Chapter 5 conclude this research work with the conclusions from the results and with a discussion on the scope of future work.

Chapter 2

Theoretical background

2.1 Repeated root condition

A repeated modal frequency condition is said to exist when the modal frequencies of two or more modes occur at exactly same frequency. This condition exhibits equal complex modal frequency, that is both the modal frequency and the modal damping are equal. This condition can also be referred to as a repeated root condition or a repeated pole condition. Repeated modes are usually found in structures with some symmetry but it is not a necessary for the object to be symmetric to have repeated modes. It is important to correctly estimate repeated modes to get an accurate model of the system. Therefore, it is important to have a method to estimate the certainty in the estimates of repeated modes. Analytically, repeated modes are calculated from the characteristic equation in the same way as non repeated modes. The characteristic equation for damped and an undamped systems are

$$(s^2[M] + s[C] + [K])\{X\} = \{0\} \quad (2.1a)$$

$$(s^2[M] + [K])\{X\} = \{0\} \quad (2.1b)$$

The equation is first solved for the eigenvalues, that is, s . Each eigenvalue is then substituted in the Equation 2.1b to get the eigenvector corresponding to the eigenvalue. For analytical solutions, only Equation 2.1b is considered and not Equation 2.1a. However, for experimental solutions it is the real system that is tested and solutions represent solution to Equation 2.1a. This is why the analytical modal vectors are real values normal modal vectors while the experimental modal vectors are complex valued. In the case

of repeated roots when the analytically evaluated eigenvalue is substituted in Equation 2.1b, the equation is rank deficient by the number of repeated roots at that eigenvalue. If a set of N_r repeated roots are present at the eigenvalue λ_r , N_r physical coordinates have to be assumed in the calculation of the eigenvector. This has to be repeated N_r times, once for each repeated eigenvalue. A number of N_r physical coordinates can be selected and they can be assigned infinite number of arbitrary combinations of constants. Based on our selection of sets of these coordinates and their values, different eigenvectors are obtained. This is why normalized eigenvectors for repeated modes are not unique like regular eigenvectors.

Experimentally, repeated roots occur as closely spaced modal frequencies. The multi-degree of freedom(MDOF) FRF is expressed as a superposition of many single degree of freedom systems, each belonging to a mode, as in the equation below

$$H_{pq}(\omega) = \sum_{r=1}^{2N} \frac{A_{pqr}}{j\omega - \lambda_r} \quad (2.2)$$

In case of repeated roots, the Equation 2.2 still holds true. It can be expressed for a column q of the frequency response function as,

$$\{H\}_q = \frac{Q_1\psi_{q1}\{\psi\}_1}{j\omega - \lambda_1} + \frac{Q_2\psi_{q2}\{\psi\}_2}{j\omega - \lambda_2} + \dots + \sum_{r=1}^{2N} \frac{Q_s\psi_{qs}\{\psi\}_s}{j\omega - \lambda_s} + \dots + \frac{Q_r\psi_{qr}\{\psi\}_r}{j\omega - \lambda_r} \quad (2.3)$$

where λ_s is the repeated modal frequency. It can be seen from the above equation that if there is only one column of frequency response function, the residue estimated at λ_s would be the sum of all terms belonging to each modal frequencies. To be able to detect repeated modes experimentally, it is important to have at a minimum the number of independent columns in the FRF matrix equal to the number of repeated modes in the largest repeated mode set that is to be identified. It is usually better to have more references than this number.

A modal vector that belongs to a repeated mode set is orthogonal to other modal vectors but not necessarily to the modal vectors that are from the same repeated mode set. Although, the condition of orthogonality can be enforced when estimating the analytical

modes, it is not necessary. Experimentally determined repeated modal vectors are very often not orthogonal to each other. In the case of a repeated root of multiplicity 2, two independent modes will be required to describe the motion of the plate at that frequency but these two modes are not unique. An infinite number of pairs exist such that their linear combination can describe that motion. Therefore, the exact vector of each mode is not unique. Since the complex valued frequencies may not be perfectly equal in the case of repeated roots in experimental analysis, it can be problematic when regular modes occur at closely spaced frequencies. This condition gives the appearance of being a repeated mode condition. It is called a psuedo-repeated root. Usually checking the wireframe animation can clarify this and confirm which modes belong to a repeated set but with a repeated mode correlation method this can be done in an automatic and more convenient way.

2.2 Vector subspace theory

A *vector space* is a non empty set of vectors, on which the operations of vector addition and scalar multiplication are defined, such that they follow ten basic axioms stated below^[3]. Given that $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in set V and a and b are scalars, the axioms can be stated as

1. $\mathbf{u} + \mathbf{v}$ is in V
2. $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$
3. $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{v} + (\mathbf{u} + \mathbf{w})$
4. a $\mathbf{0}$ vector exists in V such that $\mathbf{u} + \mathbf{0} = \mathbf{u}$
5. for each vector $\mathbf{u}$ a vector $-\mathbf{u}$ exists in V such that $\mathbf{u} + -\mathbf{u} = \mathbf{0}$
6. $a\mathbf{u}$ belongs to V
7. $a(\mathbf{u} + \mathbf{v}) = a\mathbf{u} + a\mathbf{v}$
8. $(a + b)\mathbf{u} = a\mathbf{u} + b\mathbf{u}$

$$9. (ab)\mathbf{u} = a(b\mathbf{u})$$

$$10. 1(\mathbf{u}) = \mathbf{u}$$

Vector spaces can contain within them other vector spaces. Such a vector space is said to be a *vector subspace*. A vector space H , which is a subset of a vector space V , is said to be a subspace of V if,

1. the zero vector of V is in H
2. for all vectors $\mathbf{u}$ and $\mathbf{v}$ in H , $\mathbf{u} + \mathbf{v}$ is in H
3. for all vectors $\mathbf{u}$ in H and all scalars a , $a\mathbf{u}$ belongs to H

When a set satisfies the second condition it is said to be *closed* in vector addition and the when a set satisfies the third condition it is said to be *closed* in scalar multiplication.

2.2.1 Modal vectors as vector spaces and subspaces

The set of all solutions to an eigenvalue equation is called an eigenspace. As can be checked with the axioms stated in the previous section, an eigenspace is also a vector space. It is also closed in vector addition and scalar multiplication. So it can be said that the eigenspace is basically a subspace of $\mathbf{C}^n$, where n is the length of the eigenvectors.

Because the characteristic equation for a multi degree of freedom system is also a eigenvalue equation, the solution to that equation belongs to an eigenspace. In case of repeated modes, the modes are not unique and any linear combination of the modes of a repeated set can be said to be a mode in the repeated set. Also any mode multiplied by a scalar still describes the same mode. These two considerations mean that a repeated mode set is closed in addition and in scalar multiplication. Since each repeated root set is a set contained completely in the set containing all possible solutions to the characteristic equation, each set of repeated root would also be a vector subspace of the eigenspace corresponding to the characteristic equation. This is because the repeated roots satisfy all three condition laid down in Section 2.2 for a vector space to be called a vector subspace of another vector space.

2.2.2 Principal angles between subspaces

The angle between vector subspaces for a vector dimensionality greater than 3 is a difficult concept to visualize. However, when a $\mathbf{R}^3$ space and the eigenvectors belonging to this space are considered, an analogy can be extended to general concept of angles between subspaces. In a general eigenvector solution belonging to $\mathbf{R}^3$ space, the normalized eigenvectors would be three unit vectors pointing in mutually perpendicular directions. All vectors in any one of these directions form a subspace of the complete solution set. In case of an eigenspace belonging to a $\mathbf{R}^3$ space with a repeated eigenvalue of multiplicity two, the subspace described by the repeated root is a plane in the 3 dimensional space. All possible eigenvectors in the direction of the one orthogonal eigenvector form one subspace of the complete set of solution, whereas the other two vectors form a separate subspace, that is the plane. The line and the plane are *complementary subspaces*. The angle between the planes is essentially the principal angle between subspaces for this case. The principal angles between subspaces of complex-valued larger dimension modal vectors are just an extension to this idea. This angle can be viewed as the angle between the normals to the planes in a $\mathbf{C}^3$ space. It is also convenient that the eigenvectors belonging to the non-repeated set are the normal vectors to the plane or subspace of repeated modes. Hence a dot product of the other normalized eigenvectors should give the cosine of the angle. The principal angles for higher dimensional subspace are defined in a similar way. The minimal angle between subspaces X and Y is given by

$$\cos(\theta) = \max_{x \in X} (\max_{y \in Y} (|x^H y|)) \quad (2.4)$$

where x and y are unit vectors in the X and Y subspaces respectively. The computation of the minimal angle using Equation 2.4 is possible but sometimes not very accurate. The angles can be calculated as shown by Wedin^[4]. Various methods have been developed over time to improve the calculation. One such method is developed by Bjorck et al.^[5]. This method is used in the *subspace()* function in MATLAB that has been used in this thesis. The *subspace()* function first finds the orthonormal bases for both sets of vectors. The

orthonormal bases are swapped to maintain an order of rank. It is shown in^[5] that small principal angles can be calculated more accurately through the use of sine projections which can be calculated through singular value decomposition(SVD) of a matrix C given by,

$$C = A' - (A' * B) * B' \quad (2.5)$$

where, A and B are the orthonormal bases of the two sets of vectors. The 2-norm of C gives the sine of the principal angle.

2.3 Analytical model and experimental wireframe

To be able to correlate modal vector results from experimental and analytical solutions, it is necessary to map the geometries that are used to arrive at those respective solutions. In case of a finite element method modal solution, the modal vector contains modal information from all nodes in the meshed geometry. This modal vector is usually of a very high dimension. Figure 2.1 shows the meshed circular plate model and the experimental wireframe corresponding to the data used in case studies in Chapter 4.

In this research work, since the two case studies belong to relatively simple planar objects, it was easy to identify the common plane between the ANSYS model and X-Modal(experimental modal analysis program) wireframe. This is usually a difficult task for more complicated geometries. If the model and wireframe coordinate systems are scaled differently and at an angle other than $90^\circ, 180^\circ, 270^\circ$ or 0° it becomes a further complicated task. Horn and Hilden show a way of finding a transformation matrix that can reorient and rescale one of the geometries so that nodes and wireframe points can be matched^[6]. Once that is done, the points on the mesh corresponding to the points on the wireframe can be found out by calculating the euclidean distance of the wireframe point from each mesh node. The node with the minimum distance can then be selected as the matching node to the wireframe point. The analytical nodes(ANSYS) can then be truncated to the nodes that best match the experimental nodes.

Figure 2.1: ANSYS mesh model and wireframe, 2001 Circular plate Dataset

2.4 Existing correlation methods

Cross-MAC is the most common way of correlating modal vectors obtained from two different methods. It can be calculated as^[7],

$$MAC_{qr} = \frac{\{\phi_r\}^H \{\psi_q\} \{\psi_q\}^H \{\phi_r\}}{\{\phi_r\}^H \{\phi_r\} \{\psi_q\}^H \{\psi_q\}} \quad (2.6)$$

where ϕ and ψ are the modal vectors obtained from two different methods. The modal vectors in Equation 2.6 are complex-valued in general. Other methods like the shape difference indicator (SDI) are methods that focus on measuring the difference between

two modes. The SDI is calculated as^[13],

$$SDI = \left(\frac{2\text{real}(\{\phi_r\}^H \{\psi_r\})}{\{\phi_r\}^H \{\phi_r\} + \{\psi_r\}^H \{\phi_r\}} \right)^2 \quad (2.7)$$

But these criteria do not succeed in correlating repeated modes. Figure 2.2 shows the cross-MAC values for modal vectors obtained from the Circular Plate 2001 Dataset and its analytical model. As can be seen in the figure, low cross-MAC values are observed at diagonal points for repeated modes. In some cases cross diagonal correlation is also observed. In case of the 12th and 13th modes it can be seen that the cross-MAC values are not close to unity on the diagonal or on the cross diagonal. Because of the symmetry

Figure 2.2: Cross-MAC between experimental and analytical modes, 2001 Circular Plate Dataset

that usually goes with repeated roots, it can be easy to observe that mode shapes of repeated modes are actually rotated versions of each other. As can be seen in Figure 2.3, the modeshape on the left is the same as the modeshape on the right rotated about an axis

normal to the plate by 45° . A similar observation about modeshapes 12 and 13 can be made in Figure 2.4. But this is not always a feasible solution. In industrial applications this can time be consuming when complicated geometries and higher frequency modes are present.

Figure 2.3: Repeated roots - saddle mode shape in 2001 Circular Plate Dataset

Figure 2.4: Repeated roots - 12th and 13th mode 2001 Circular Plate dataset

Apart from using the wireframe animation or mode shape to judge whether a set of modes are a repeated set or not, some methods exist to detect and correlate repeated roots. For example, it has been shown^[1] that by taking the singular value decomposition(SVD) of unity scale factor(USF), which is defined as

$$\{\hat{\psi}_{cr}\}^H \{\hat{\psi}_{dr}\} = USF_{cdr} \{\hat{\psi}_{dr}\}^H \{\hat{\psi}_{dr}\} \quad (2.8)$$

where $\{\hat{\psi}_{dr}\}$ and $\{\hat{\psi}_{cr}\}$ are the modal vectors scaled to unity length, the number of repeated roots can be found^[1]. To get to the exact repeated root, this process has to be applied repeatedly. First, it should be applied to a larger set of modes, which would show how many independent modes are present in the set. If this is more than the number of

frequencies it can be inferred that repeated mode are present. Then smaller groups of modes can be selected and put through the same process and a group of modes that are a repeated set can be arrived at. Though, this does not help define a way of correlating repeated modes.

A method developed for correlating repeated modes is shown in [2], where a combined mode is first set up as

$$[\psi_{Ak}] = [\psi_k]\{\beta\} \quad (2.9)$$

where $\{\beta\}$ is the linear coefficients matrix, $[\psi_k]$ is the matrix made up of analytical modal vectors for a set of modes and $[\psi_{Ak}]$ is the combined modal vector for k^{th} mode. The combined mode is substituted for k^{th} test mode and β values are calculated. The combined mode is reconstructed from these β values and MAC is calculated between the combined mode and test mode. This is repeated for all combinations for the combined mode. The method has some drawbacks that are discussed in Section 4.1.

Chapter 3

Extracting and conditioning modal vectors

Before a modal correlation criterion can be implemented, the modal vectors from an analytical and experimental modal analysis have to be extracted. These extracted modes have to be conditioned so that they can be made of same dimension. The processes for extracting modal vectors for experimental and FEM analyses are different. The modes may also be normalized so that the different modal vector sets are similar. Some other changes might also be required depending on the kind of modal vectors obtained.

3.1 Description of case studies

To develop and test a method of correlation for modal vectors from repeated modes, two circular plates were chosen. The data for these plates has been obtained from previous experiments at UC-SDRL. Circular plates are very symmetric objects and yield many repeated root combinations. The circular plates are referred to as the 2001 Circular Plate Dataset and the 2014 Circular Plate Dataset. The circular plates tested can be seen in Figure 3.1 and Figure 3.2. In case of the 2001 Circular Plate Dataset, a case where a pair of repeated roots exist very close to a non-repeated mode can be observed. This is a useful condition as it will be helpful in checking if the repeated set can be distinguished from the non-repeated mode when the frequencies are close.

Different experimental results were obtained by varying experimental factors such as forcing methods, number of reference points and modal parameter estimation methods. This was done because it was expected that repeated mode modal vectors for different

experimental results would be different and it would be important to see if spaceMAC would be able to establish a correlation between them. Comparison with results from the correlation method implemented in Section 4.1 were also done. Different sets of results were obtained with different MAC thresholds in spaceMAC implementation. Observations were made about whether having a high or low MAC threshold affects the spaceMAC results and if it does, the nature of changes to the results.

Figure 3.1: 2001 plate

Figure 3.2: 2014 plate

3.2 Analytical modal vectors

For the analytical solution, FEM based modal analysis using ANSYS Workbench was selected as the analysis method. The geometries were created on the ANSYS Design-Modeler. The analysis was done for both the circular plates models. It was done on the modal analysis module in ANSYS Workbench. The circular plates can be seen in Figure 3.1 and Figure 3.2. Measurements were taken on the plates and the plates were modelled in ANSYS. The models can be seen in Figure 3.3 and Figure 3.4. To ensure that the model was sufficiently accurate, the modal frequencies and modal vectors were compared. The modal vectors comparison was done using cross-MAC. The cross-MAC values that were significant for this comparison were the ones corresponding to modes that were known to be non repeated. Figure 3.5 shows the cross-MAC values between modal vectors from analysis and experiment. The modal frequencies were compared numerically. Table 3.1 shows the modal frequency for both analytical and experimental

modes for 2001 Circular Plate Dataset. The material assigned to the model was Steel for 2014 plate and Aluminium for 2001 Circular Plate Dataset.

Figure 3.3: ANSYS model with mesh for 2001 plate

Figure 3.4: ANSYS model with mesh for 2014 plate

Once the modal frequencies were close, that is experimental modal frequencies are within $\pm 5\%$ of analytical modal frequencies and cross-MAC values were close to unity, a method to extract the modal vectors could be developed. As can be seen from the cross-MAC values, modes that can be suspected to be repeated modes are 1 and 2, 6 and 7, and 12 and 13. To be able to extract the modal vector, a piece of ANSYS script was used to get modal vectors as an output of the modal analysis in the form of txt files. The script can be seen in Appendix A.6. These .txt files could then be accessed in MATLAB. The format of the files obtained can be seen in Table 3.2. The Table only shows the first

Table 3.1: ANSYS and experimental frequencies

No.	Ansyz Frequencies	Experimental Frequencies
1	341.9	362.33
2	341.99	363.64
3	533.51	557.04
4	730.49	761.10
5	730.54	764.10
6	1184.2	1223.01
7	1184.3	1224.10
8	1267.3	1328.05
9	1267.5	1328.76
10	1927.6	2019.25
12	1928.1	2023.74
13	2195.4	2322.18
14	2196.1	2322.55
15	2270.0	2338.21

Figure 3.5: Cross-MAC between experimental and analytical modes, Circular Plate 2001 Dataset

few rows but the files were usually 6000 rows long. As can be seen the columns two to four contain the information about the undeformed body whereas the columns five to seven contain information about deformation for that particular modeshape.

Table 3.2: Ansys modal vector data format

Node Num	X	Y	Z	UX	UY	UZ
1	-0.10128	0.00764	-0.17541	0.14173	0.18986	0.02061
2	-0.11084	0.00764	-0.17798	0.14171	0.18539	0.02068
3	-0.10827	0.00764	-0.18754	0.14164	0.20177	0.02066
4	-0.09871	0.00764	-0.18498	0.14166	0.20624	0.02059
5	-0.10659	0.00764	-0.17471	0.14174	0.18423	0.02065
6	-0.11154	0.00764	-0.18329	0.14167	0.19266	0.02068
7	-0.10296	0.00764	-0.18824	0.14163	0.20740	0.02062
8	-0.09801	0.00764	-0.17966	0.14170	0.19898	0.02058
9	-0.10128	-0.00506	-0.17541	0.15271	0.18986	0.00179
10	-0.11084	-0.00506	-0.17798	0.15269	0.18539	0.00186

3.3 Experimental modal vectors

For the experimental modal analysis, data from two circular plate tests was used. 30 accelerometers were placed on the circular plate shown in Figure 3.1 to obtain the 2014 Circular Plate Dataset whereas 2001 plate data was obtained from an older data set obtained from impact testing of a circular plate at 36 locations. This dataset comes as a demonstration dataset with the X-Modal experimental modal analysis program. The distribution of the accelerometers on the 2014 plate was according to the the wireframe shown in Figure 3.8. Similarly the impact points were according to the wireframe shown in Figure 3.7. The modal analysis was carried on out on X-Modal. The modal parameter settings for both sets of experimental data can be seen in Figure 3.6. For 2014 Circular Plate Dataset, a typical shaker testing procedure was followed in order to obtain extra experimental data to compare different experimental cases for repeated mode correlation. The MPE settings were varied to get a set of results that could be compared for different experimental cases for both the plates. Figure 3.6 shows the basic settings for 2001 plate.

Method	PTD
Domain	time
Basis	short
Iteration	[A] order
A Orders	2:20
Residuals	none
Dataset	252 [259]
Inputs	36 [36]
Outputs	7 [7]
Freq Lines	454
Time Pts	461
Start Time %	0

Figure 3.6: MPE settings for 2001 Circular Plate Dataset

Figure 3.7: 2001 plate wireframe

For extraction of modal information from the test results in X-Modal experimental modal analysis software, the 'Swiss army knife' option was used. This option can be accessed by clicking on the Swiss army icon on the menu bar at the top. The DGMR Extras windows opens up. This window can be seen in Figure 3.9. The MAT file utility option was selected. This opens the MAT file utility window where the 'Measurements' and 'Results' check-boxes were selected. This window can be seen in Figure 3.10. Once the 'Export' button is clicked, a .mat format file with the name 'matlab' can be found exported to the X-Modal folder. A matlab script was written to extract the modal

Figure 3.8: 2014 plate wireframe

information from this extracted file. This script can be found in Appendices A.1 and A.2.

Figure 3.9: DGMR Extras window in X-Modal

Figure 3.10: MAT file utility window in X-Modal

Chapter 4

Repeated mode correlation

4.1 Implementing existing methods

Repeated mode correlation as shown by Walther et al^[2] can be easily implemented in MATLAB. The modal vectors from both experimental and analytical analyses were extracted. A set of modes was selected from the complete set of analytical modal vectors. One vector was then selected from the complete set of experimental modal vectors. Preferably, the selected experimental mode should be a modal vector such that the corresponding analytical mode exists in the selected set of analytical modes. This is to check if a linear combination of the analytical set correlates well with the selected experimental mode. Selecting an experimental mode that does not belong to the set of selected analytical modes would not give meaningful results even if high correlation could be obtained. The selected analytical set can then be used to get a combined analytical mode as in the following equation,

$$[\psi_{Ak}] = [\psi_k]\{\beta\} \quad (4.1)$$

where $\{\beta\}$ is the linear coefficient matrix, $[\psi_k]$ is the matrix made up of selected analytical modal vectors and $[\psi_{Ak}]$ is the combined modal vector for k^{th} mode.

The combined mode is then substituted with k^{th} test mode $[\psi_{Tk}]$. The coefficient vector $\{\beta\}$ is then calculated for the selected $[\psi_k]$ according to the equation,

$$\{\beta\} = [\psi_k]^T \{\psi_{Tk}\} \quad (4.2)$$

The combined mode was then reconstructed from Equation 4.1 by utilizing the $\{\beta\}$ matrix. The MAC between the k^{th} test mode $[\psi_{Tk}]$ and the corresponding combined mode was calculated. This calculation was then repeated for all sets of $[\psi_k]$. MAC values close to unity suggests the selected set of analytic modes used for that combined mode may be a repeated mode set. The drawback with this method is that only one modal vector from the experimental modal vectors is being used for correlation at a time(one by one). This can lead to high MAC values with a set of modes that are not repeated modes with the selected experimental mode. If the 12th and 13th modes for the 2001 Circular Plate Dataset are taken as an example this can be illustrated. Table 4.1 shows results for various combinations of analytical modes with the 12th experimental mode. The combinations are [12 and 13], [12,13,14], [12,10] and [12,10,14]. As can be seen from some of the MAC values in the table, there are two problems with this approach

- The high correlation between the combination [12,13,14] and the 12th experimental mode suggests that the repeated root set is [12,13,14]. This is clearly not true as can be seen from the Figure 4.1, where 12th and 13th can be seen to be similar modes which can be rotated to look the same, whereas the 14th mode is a very different modal shape as can be seen in Figure 4.2. The rotation test works only for objects with circular symmetry and is not a good general approach. Although this is an issue, it can be addressed by considering the smallest set that has a high correlation as a repeated mode.

Figure 4.1: Repeated roots - 12th and 13th mode 2001 Circular Plate Dataset

- The high correlation between the combination [12,10,14] and the 12th experimental mode is more problematic as it suggests that high correlation may be obtained when-

Figure 4.2: 14th mode in 2001 Circular Plate Dataset

ever the analytical set contains the one experimental mode it is being correlated with. High correlation with the [12,10] also presents the same problem. Table 4.1 shows that a similar problem exists when analytical combinations are tested with the 13th experimental mode.

Table 4.1: MAC between combined and experimental modes

Experimental mode number	Analytical modes	MAC
12	12,13	0.972
12	12,13,14	0.9778
12	12,10	0.867
12	12,11	0.867
12	12,10,14	0.8707
13	13,12	0.972
13	13,11	0.8481

The first issue occurs because once a repeated mode set has been identified, the presence of noise causes some part of the experimental modal vector to be modeled by a linear combination of some other modes. This is why a good correlation between the 12th experimental mode with the set [12,13,14] of analytical modes is obtained. The 14th mode models some part of the leftover noise based result, in the experimental modal vector, that is not modelled by the set [12,13] modes. This second problem is essentially because this method compares a set of analytical modes with one experimental mode, which is a comparison between an analytical vector space and a experimental vector. This is why the sets [12,10,14] and [12, 10] give a high MAC with 12th experimental mode. Once either of these two problems are observed, a judgment call needs to be made based

on observing the mode shape or wireframe animation. This is why this method is not a acceptable solution and it would be advantageous to have a criterion that works towards correlating experimental vector space and analytical vector space.

4.2 SpaceMAC

For comparing vector spaces, a criterion was developed. The concept of principal angle between subspaces, θ , was used to estimate the similarity of the vector subspaces. The criterion was calculated as $1 - |\sin(\theta)|$. This is also known as a coversine function. The choice of function of θ that should be selected was based on the the preference of having a 0 to 1 scale such that an output of 0 would mean no spatial correlation while that of 1 would mean full spatial correlation. This is similar and consistent with the original definition of MAC. The first function of choice was $\cos(\theta)$ but the cosine function shows very little sensitivity to the angle at low angular values so $1 - |\sin(\theta)|$ was selected instead. This criterion was used as part of a process which can be seen in the flowchart below.

The matlab script implementation of the SpaceMAC can be seen in Appendix A.5. As can be seen in the flowchart, first both sets of modal vectors(analytical and experimental) have to be extracted. This process is described in detail in Chapter 3. This also includes conditioning the modal vector sets so that both the sets are of same size and at the same degree of freedom(DOFs). Then the user is asked how many rigid body modes are present in both sets respectively. In case there are any rigid body modes, they are removed from the set. This is so that modal vectors that correspond to a particular mode have the same index in both modal vector sets. This makes it easier to find modes that do not have a high cross-MAC value by just comparing the diagonal of the cross-MAC. The next step is to map the geometry of the analytical model with experimental model(wireframe). The method for this process is detailed in Chapter 2. An important part of mapping the geometry is comparing the error in the mapping process. This can be done by calculating the distance between each point on the experimental wireframe and the corresponding point on the downsampled analytical node geometry. Table 4.2 shows the error for 2001

Figure 4.3: Flowchart for implementing space MAC

Circular Plate Dataset geometry. As can be seen, the errors in this case are very small compared to the dimension of the system. Once a low error mapping is achieved, the next step is to downsize the analytical modal vector according to the mapped geometry. The modal deformation information for the nodes selected in the mapping process is kept and the data for the remaining nodes is discarded. Analytical modal vectors of the same size as that of the experimental modal vectors are obtained. Then a cross-MAC is evaluated between these two sets of modal vectors. A user defined MAC threshold

is then chosen. All modal vectors with a cross-MAC higher than the threshold value are removed from the selection for spaceMAC calculation. The multiplicity of repeated modes to be detected and correlated is then selected. The spaceMAC with multiplicity n is denoted as $spaceMAC_n$. Based on this selection, a set of combinations of all modes of the selected multiplicity is created. That is, if multiplicity selected is 2, all possible pair combinations for the selected modes are created. Then, spaceMAC is calculated for all of these calculation using θ values obtained from the *subspace()* function in MATLAB. This procedure was applied for both circular plate data case studies. The procedure was also applied for experimental to experimental results comparison.

Table 4.2: Error in mapping geometry normalized by plate diameter

number	Error
1	0.0001479
2	0.0001306
3	0.0001277
4	0.0001321
5	0.0001427
6	0.0001632
7	0.0001427
8	0.0001321
9	0.0001277
10	0.0001321
11	0.0001427
12	0.0001632
13	0.0001443
14	0.0001413
15	0.0001454
16	0.0001583
17	0.0001614
18	0.0001507
19	0.0001458
20	0.0001506
21	0.0001740
22	0.0001540
23	0.0001553
24	0.0001458
25	0.0001712
26	0.0001441
27	0.0001346
28	0.0002052
29	0.0001758
30	0.0001324
31	0.0001420
32	0.0001881
33	0.0001661
34	0.0001737
35	0.0001744
36	0.0001776

4.3 Discussion of Results

When two different sets of results were compared for both the circular plates, one important observation was that the repeated modal vectors were numerically consistent to begin with. The cross-MAC between two different modal parameter estimation results is shown in Figure 4.4. As is evident from the cross-MAC values, repeated modal vectors obtained from different experimental methods also correlate very well. This is an interesting observation because historically it is expected that repeated modes would be numerically inconsistent. However, this consistency was observed for all different cases that were tried with experimental results. For all the different experimental results, similar cross-MAC was observed. The cross-MAC for all these comparisons can be seen in Appendix B. Because of the consistency of the modal vectors in different experimental settings, the question of correlating vector space becomes trivial. Also, if the space-MAC process is applied to these two sets of vectors even a fairly high cross-MAC threshold will remove all of the vectors. This is by design because if a space correlation was attempted on modes that are possibly repeated and have a high cross-MAC, good space correlation would be obtained. This is because the modal vectors correlate with each other well already, so the space that is constructed with those vectors would be highly correlated too. The high cross-MAC values between different experimental results could be because of various factors. One factor that might be affecting this is the real dominant normalization procedure that was applied to the modal vectors during residue estimation. Historically the cross-MAC values have been low for repeated modes between modal vectors from two different experimental datasets for the same system but this might have been before the real dominant normalization procedure was introduced or the modal parameter estimation procedures may have improved. The cause for the high correlation could also be that the real system is not perfectly symmetric like the model created in ANSYS and the repeated modes occur as closely spaced non-repeated modes with modal vectors that are unique when normalized. The vector subspace created out of these vectors would still be very close to the vector subspace of analytical solution where it is a repeated mode set.

Figure 4.4: Cross MAC between two experimental results for 2014 Circular Plate Dataset

The spaceMAC method correlates one modal vector space of a selected multiplicity and another modal vector space of the same multiplicity. This is important as the drawback of the correlation method described in Section 4.1 is that it correlates a space with a vector. This gives space-MAC an advantage over this method. For example, if the 12th, 13th and 14th modes for 2001 Circular Plate Dataset are taken as in Section 4.1 it is easier to detect and correlate repeated modes with spaceMAC. But it has to be made sure that the search for repeated modes must be made in an increasing order of multiplicity of repeated modes. Table 4.3 shows the space-MAC values of multiplicity equal to 2. The cross MAC threshold was selected as 0.9. The cross-MAC values can be seen in Figure 3.5 for 2001 Circular Plate Dataset.

Space MAC values above 0.8 can be considered a good enough correlation. It can

be seen in the above TABLE that there are some space-MAC values around 0.6. Another observation was that the frequency comparison for these values was such that the frequencies were far apart. Also two modes that have a high space-MAC between them had same sets of vector with which values of around 0.6 space-MAC were obtained. For example the mode at 761.104 and 764.163 Hz had a 0.924 spaceMAC between them and both these modes showed a very low spaceMAC with the modes at 1223.011 and 1224.080 Hz while they showed a space-MAC value of around 0.6 for the modes at 1328.055 and 1328.769 Hz. This can be clarified if we see actual modeshapes. Figure 4.5, 4.6, 4.7, 4.8, 4.9 and 4.10 show the modeshapes at these six frequencies. As can be seen from these figures, the spaceMAC values around 0.6 seems to be because of spatial aliasing phenomenon. But clearly the spatial aliasing was not strong enough to affect the cross-MAC values. SpaceMAC was observed to have higher sensitivity to aliasing effects.

Table 4.3: $spaceMAC_2$ values for 2001 Circular Plate Dataset

number	Frequency1	Frequency2	Principal Angle	spaceMAC
1	362.337	363.648	0.044	0.956
2	362.337	761.104	1.255	0.049
3	362.337	764.163	1.255	0.049
4	362.337	1223.011	1.257	0.049
5	362.337	1224.080	1.255	0.049
6	362.337	1328.055	1.256	0.049
7	362.337	1328.769	1.255	0.049
8	362.337	2322.186	1.266	0.046
9	362.337	2322.555	1.195	0.070
10	363.648	761.104	1.239	0.054
11	363.648	764.163	1.239	0.054
12	363.648	1223.011	1.254	0.050
13	363.648	1224.080	1.239	0.054
14	363.648	1328.055	1.239	0.054
15	363.648	1328.769	1.239	0.054
16	363.648	2322.186	1.183	0.074
17	363.648	2322.555	1.247	0.052
18	761.104	764.163	0.077	0.924
19	761.104	1223.011	1.253	0.050
20	761.104	1224.080	1.173	0.078
21	761.104	1328.055	0.385	0.624
22	761.104	1328.769	0.388	0.622
23	761.104	2322.186	0.387	0.623
24	761.104	2322.555	0.405	0.606
25	764.163	1223.011	1.253	0.050
26	764.163	1224.080	1.173	0.078
27	764.163	1328.055	0.395	0.615
28	764.163	1328.769	0.395	0.615
29	764.163	2322.186	0.397	0.614
30	764.163	2322.555	0.402	0.609
31	1223.011	1224.080	0.119	0.881
32	1223.011	1328.055	1.253	0.050
33	1223.011	1328.769	1.253	0.050
34	1223.011	2322.186	1.253	0.050
35	1223.011	2322.555	1.253	0.050
36	1224.080	1328.055	1.173	0.078
37	1224.080	1328.769	1.174	0.078
38	1224.080	2322.186	1.173	0.078
39	1224.080	2322.555	1.174	0.078
40	1328.055	1328.769	0.079	0.922
41	1328.055	2322.186	0.375	0.634
42	1328.055	2322.555	0.402	0.609
43	1328.769	2322.186	0.377	0.632
44	1328.769	2322.555	0.400	0.611
45	2322.186	2322.555	0.176	0.825

Figure 4.5: Analytical modeshape corresponding to experimental mode at 761.104Hz

Figure 4.6: Analytical modeshape corresponding to experimental mode at 764.163Hz

Figure 4.7: Analytical modeshape corresponding to experimental mode at 1223.011Hz

Figure 4.8: Analytical modeshape corresponding to experimental mode at 1224.080Hz

Figure 4.9: Analytical modeshape corresponding to experimental mode at 1328.055Hz

Figure 4.10: Analytical modeshape corresponding to experimental mode at 1328.769Hz

Following an increasing order of selected multiplicity is an important factor in the spaceMAC implementation. SpaceMAC calculations for multiplicity 3 were done. Some sample values can be seen in Table 4.4 below. As is already known this set of modal vectors does not contain any repeated modes of multiplicity 3. But some high values of $spaceMAC_3$ can be seen. Some of these high values were observed to be around 0.6 and the set of modes corresponding to these values are the same as those that showed high values with multiplicity 2 due to spatial aliasing. These can be put aside as spatial aliasing cases. However, in some cases there were some mode sets that show very high

spaceMAC with three multiplicity. It was observed that these sets are made up of one pair that shows high $spaceMAC_2$ and a normal mode. This means that out of three mode, two made a repeated pair and one was orthogonal to the space described by the repeated pair. Similar effect can be observed in Table 4.5 where the first 20 rows of $spaceMAC_4$ calculation have been shown. In this, apart from the aliasing effects, high $spaceMAC_4$ can be observed between quadruples formed by combining two repeated mode pairs. This can be misleading if $spaceMAC_2$ calculation have not been run beforehand. This is why spaceMAC calculations should be run with an increasing multiplicity.

Table 4.4: $spaceMAC_3$ values

number	Frequency1	Frequency2	Frequency3	spaceMAC
1	362.337	363.648	761.104	0.624
2	362.337	363.648	764.163	0.615
3	362.337	363.648	1223.011	0.050
4	362.337	363.648	1224.080	0.078
5	362.337	363.648	1328.055	0.656
6	362.337	363.648	1328.769	0.656
7	362.337	363.648	2322.186	0.647
8	362.337	363.648	2322.555	0.631
9	362.337	761.104	764.163	0.049
10	362.337	761.104	1223.011	0.049
11	362.337	761.104	1224.080	0.049
12	362.337	761.104	1328.055	0.049
13	362.337	761.104	1328.769	0.049
14	362.337	761.104	2322.186	0.046
15	362.337	761.104	2322.555	0.070
16	362.337	764.163	1223.011	0.049
17	362.337	764.163	1224.080	0.049
18	362.337	764.163	1328.055	0.049
19	362.337	764.163	1328.769	0.049
20	362.337	764.163	2322.186	0.046

Table 4.5: $spaceMAC_4$ values

number	Frequency1	Frequency2	Frequency3	Frequency4	spaceMAC
1	362.337	363.648	761.104	764.163	0.924
2	362.337	363.648	761.104	1223.011	0.050
3	362.337	363.648	761.104	1224.080	0.078
4	362.337	363.648	761.104	1328.055	0.625
5	362.337	363.648	761.104	1328.769	0.622
6	362.337	363.648	761.104	2322.186	0.623
7	362.337	363.648	761.104	2322.555	0.621
8	362.337	363.648	764.163	1223.011	0.050
9	362.337	363.648	764.163	1224.080	0.078
10	362.337	363.648	764.163	1328.055	0.615
11	362.337	363.648	764.163	1328.769	0.615
12	362.337	363.648	764.163	2322.186	0.615
13	362.337	363.648	764.163	2322.555	0.615
14	362.337	363.648	1223.011	1224.080	0.882
15	362.337	363.648	1223.011	1328.055	0.050
16	362.337	363.648	1223.011	1328.769	0.050
17	362.337	363.648	1223.011	2322.186	0.050
18	362.337	363.648	1223.011	2322.555	0.050
19	362.337	363.648	1224.080	1328.055	0.078
20	362.337	363.648	1224.080	1328.769	0.078

The MAC threshold selection was also varied from 0.7 to 0.95. This was done to check how much it affected the spaceMAC values. The selected threshold mainly affected the modes that were removed from the complete set of mode. A high threshold meant that more modes remained in the selected set than would have remained if lower threshold was selected. More modes meant more combinations of selected multiplicity could be created. Table 4.6 shows how the number of combinations increase as we increase the threshold value. Largely the spaceMAC values remained unchanged, expect for the fact that more combinations meant more spaceMAC values. But the spaceMAC values for individual combinations did not change with the threshold value. Table 4.6 also shows the spaceMAC values for the mode at 362 and 363 Hz(experimental) for threshold values from 0.7 to 0.95. However, with a higher threshold, more pairs were made possible that could have aliasing effects. It was observed that a low MAC threshold could eliminate some repeated modes that had a higher MAC value. So it is necessary to set the MAC

threshold depending on which repeated modes needs to be checked and correlated.

Table 4.6: MAC threshold variation

number	MAC threshold	number of pairs	spaceMAC for 362 and 363 Hz mode pair
1	0.7	6	0.956
2	0.75	6	0.956
3	0.8	6	0.956
4	0.85	10	0.956
5	0.9	45	0.956
6	0.95	45	0.956

In comparison to the method implemented in Section 4.1, the main advantage spaceMAC had is that it compares two modal vector spaces instead of a modal vector space with a modal vector. This means that if an increasing order of multiplicity is followed when searching and correlating repeated mode sets, a more clear indication of which sets of modes are repeated sets can be obtained. Once the sets of modal vectors that belong to a repeated set have been identified and the vector spaces have been shown to have a high correlation, the $\{\beta\}$ coefficient matrix method can be used to get experimental modal vectors that have a high MAC value with the analytical modal vectors.

Chapter 5

Conclusion

A vector subspace based approach, called spaceMAC, has been developed to identify and correlate modal vectors belonging to repeated mode sets. Analytical models and experimental data pertaining to two circular plates have been used as case studies to test the implementation of the developed metric. An existing method that compared analytical modal vector space with experimental modal vector using MAC has been implemented on the same case studies.

A high sensitivity to spatial aliasing has been observed in the developed correlation method. However, the spaceMAC values obtained due to spatial aliasing were around 0.6 whereas the spaceMAC values obtained for repeated modes were higher than 0.8. Also since the modal frequencies are normally quite different, it is easy to eliminate those cases as being possible repeated root cases based upon frequency alone. It has been observed that MAC values between two different experimental results belonging to the same experimental setup were close to unity even for repeated modes. This observation was contrary to expectation. It was the same for different force functions used, signal processing settings and number of reference points.

When compared to another method for correlation of closely spaced modes, the results obtained with spaceMAC were clearer as long as an increasing order of multiplicity of repeated modes was followed. This was largely because of the fact that spaceMAC works with correlation of two modal vector spaces whereas the other method compares a space with a vector. The effect of variation of MAC threshold has been explored. It has been observed that it did not directly affect the spaceMAC calculation for a particular set of modes. It did however affect if a particular combination of modes is selected or not.

5.1 Future Work

One of the main subject of curiosity during this research, apart from the main subject of repeated mode correlation, was that repeated modes for different experiments performed on the same setup were observed to have high correlation even when . Usually it is expected that repeated mode should vary and not show cross-MAC values close to unity. Although some exploration was done to find out the cause of this, it would be interesting to do a more thorough research work to examine the possible reason for this anomaly. This observation is mostly based upon historical parameter estimation methods and current modal parameter estimation methods may just be that much better.

Another avenue of improvement would be developing a general geometry mapping algorithm and combining it with a repeated mode correlation method like spaceMAC. This would be extremely helpful in making the whole process applicable to any generic test object and corresponding model. This could also involve a smoother transition from FEM analysis and experimental analysis to the application of spaceMAC.

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Appendices

Appendix A

Final codes

A.1 Experimental Modal Vector Extraction

```
%Experimental modal vector extraction
% This script extracts two sets of modal vectors from two experimental
% modal analysis results obtained from X-Modal in a universal file format
% and saves it in .mat format

%Author: Pranjali M Vinze
%Email :vinzepam@mail.uc.edu

% clear workspace
clear;
clc;

%load 1st mat file, the file contains two structure-
%1) Measurements
%2) Results
%Results contains modal vector data

load('2014results.mat');
numModes = size(Results,2)
for ii = 1:numModes
    modeStruct = Results(1,ii);
    xx(ii,:) = [modeStruct.Point.Z];
```

```

end

% load 2nd mat file
load('2014result_randompfdcyc.mat');
numModes = size(Results,2)
for ii = 1:numModes
    modeStruct = Results(1,ii);
    yy(ii,:) = [modeStruct.Point.Z];
end

xx = xx';
yy = yy';

%normalize the vectors obtained
for j=1:size(xx,2)/2
    xx1(:,j)=xx(:,2*j-1)./max(xx(:,2*j-1));
end
for j=1:size(yy,2)/2
    yy1(:,j)=yy(:,2*j-1)./max(yy(:,2*j-1));
end

%in some cases removing some rows might be desirable
numRows = input('how many rows of the vector are to be kept');
xx2=xx1(1:numRows,:);
yy2=yy1(1:numRows,:);

%plot a few vectors to see the results and compare the two sets of vectors
for i=1:3
    figure;
    plot(real(xx1(:,i)),'.');
    hold on
    plot(real(yy1(:,i)),'r*');
end

```

```

% MAC between yy2 and xx2
for i =1:size(xx2,2)
    for j=1:size(yy2,2)
        Mac_num=xx2(:,i)'*yy2(:,j)*yy2(:,j)'*xx2(:,i);
        Mac_den=xx2(:,i)'*xx2(:,i)*yy2(:,j)'*yy2(:,j);
        MAC(i,j)=Mac_num/Mac_den;
    end
end

%save the two sets of vectors
save('y_exp','yy2','xx2')

```


A.2 Analytical Modal Vector Extraction - 2001 dataset

```
%Ansys modal vector extraction and conditioning

%This script extracts modal vectors and coordinates for nodes for modes
%obtained from modal analysis done on ANSYS Workbench. The nodal
%coordinates are then mapped with experimental wireframe to find nodes
%closest to the experimental measurement points. The ANSYS modal vectors
%are then downsized to the size of the experimental modal vectors using the
%mapping. These vectors are then normalized and saved. This script works
%mainly for a aptricular circular plate.

%Author: Pranjal M Vinze
%Email :vinzepam@mail.uc.edu

%clear workspace
clear;
clc;

%Load wireframe and convert to SI units
frame = load('wireframe circle 2001.txt');
ConversionFactor = input('conversion factor for wireframe units to SI units(0.0254 for i
frame2 = frame(:,2:2:4)*ConversionFactor;%convert to SI units
FRAME = frame(:,2:4)*0.0762;
numModes = 20;
aa = frame2;

%For each mode extracting -
% 1) Coordinates of geommetry
% 2) modal information
%These are extracted from a selected plane on the circular plane
for i=2:numModes
    strr=strcat('Mode',num2str(i),'.txt');
```

```

dataStruct=importdata(strr);
data{i}=dataStruct.data;
textdata = dataStruct.textdata;
Data=data{i};
uniq=unique(Data(:,3));
A(:,i)=(Data(:,3)==uniq(1));
B(:,i)=(Data(:,3)==uniq(2));
TAB = tabulate(Data(:,3));
[tabval1(i), tabind] = max(TAB(:,3));
tabval2(i) = TAB(tabind,1);
[minVal minInd] = min(abs(uniq-tabval2(i)));% find the data corresponding to the top
%the idea being that in a plate geometry the most nodes are at the two
%planes TAB evaluates percentage of occurrence of a value
C(:,i)=(Data(:,3)==uniq(minInd));
X(:,i)=Data(C(:,i),5);% all x displacements on the selected plane
Y(:,i)=Data(C(:,i),6);% all y displacements on the selected plane
Z(:,i)=Data(C(:,i),7);% all z displacements on the selected plane
COORD = Data((C(:,i)),2:4);% planar geometry of the plate
XX = Data(C(:,i),2);
ZZ = Data(C(:,i),4);
AA = [XX ZZ];

end

lenA = size(aa,1);

%finding nodes closest to the experimental wireframe points

for jj = 1:size(FRAME,1)
vecD = COORD - repmat(FRAME(jj,:),size(COORD,1),size(COORD,2)/3);
Dist = (vecD(:,1).^2+vecD(:,2).^2+vecD(:,3).^2);
[j1(jj) h1(jj)] = min(Dist);
end

%downsizing by keeping only the modal vector data from the selected nodes
Y_filtered = Y(h1,:);

```

```

%Plotting the modal vectors with the geometry to get the mode shapes
for jj = 1:numModes
    figure;
    Y_filtered(:,jj) = Y_filtered(:,jj)/max(Y_filtered(:,jj));
    plot3(aa(:,1),aa(:,2),Y_filtered(:,jj));
end

%saving the modal vectors
save('y_ansys.mat','Y_filtered','aa')

```

A.3 Analytical Modal Vector Extraction - 2014 dataset

```
%Ansys modal vector extraction and conditioning
```

```
%This script extracts modal vectors and coordinates for nodes for modes  
%obtained from modal analysis done on ANSYS Workbench. The nodal  
%coordinates are then mapped with experimental wireframe to find nodes  
%closest to the experimental measurement points. The ANSYS modal vectors  
%are then downsized to the size of the experimental modal vectors using the  
%mapping. These vectors are then normalized and saved. This script works  
%mainly for a particular circular plate.
```

```
%Author: Pranjal M Vinze
```

```
%Email :vinzepam@mail.uc.edu
```

```
%clear workspace
```

```
clear;
```

```
clc;
```

```
%Load wireframe and convert to SI units
```

```
frame = load('wireframe circle 30.txt');
```

```
ConversionFactor = input('conversion factor for wireframe units to SI units(0.0254 for i
```

```
frame(:,2:3) = frame(:,2:3)*ConversionFactor;% convert to SI units
```

```
numModes = input('input number of modes expected');
```

```
aa = frame(:,2:3);
```

```
%For each mode extracting –
```

```
% 1) Coordinates of geometry
```

```
% 2) modal information
```

```
%These are extracted from a selected plane on the circular plane
```

```
for i=2:numModes
```

```

    strr=strcat('mode',num2str(i),'.txt');
    dataStruct=importdata(strr);
    data{i}=dataStruct.data;
    Data=data{i};
    uniq=unique(Data(:,3));
    A(:,i)=(Data(:,3)==uniq(1));
    B(:,i)=(Data(:,3)==uniq(2));

    [minVal minInd] = min(abs(uniq-0.0076));% find the data corresponding to the top pla
    %the idea being that in a plate geometry the most nodes are at the two
    %planes TAB evalutes percentage of occurance of a value
    C(:,i)=(Data(:,3)==uniq(minInd));
    X(:,i)=Data(C(:,i),5);% all x displacements on the selected plane
    Y(:,i)=Data(C(:,i),6);% all y displacements on the selected plane
    Z(:,i)=Data(C(:,i),7);% all z displacements on the selected plane
    XX = Data(C(:,i),2);
    ZZ = Data(C(:,i),4);
    AA = [XX ZZ];

end

lenA = size(aa,1)
for ii =1 : lenA
    loc1 = aa(ii,1);
    loc2 = aa(ii,2);
    distance(:,1) = AA(:,1) - loc1;
    distance(:,2) = AA(:,2) - loc2;
    d = sqrt(distance(:,1).^2 + distance(:,2).^2);
    [mindis(ii) minindex(ii)]= min(d);
end

%downsizing by keeping only the modal vector data from the selected nodes

%finding nodes closest to the experimental wireframe points
Y_filtered = Y(minindex',:);

%finding nodes closest to the experimental wireframe points

```

```
for jj = 1:numModes
    figure;
    Y_filtered(:,jj) = Y_filtered(:,jj)/max(Y_filtered(:,jj));
    plot3(aa(:,1),aa(:,2),Y_filtered(:,jj));
end

%saving the modal vectors
save('y_ansys.mat','Y_filtered','aa')
```

A.4 Implementing Existing Correlation Method

```
%Correlation for closely spaced modes

%This script loads a set of experimental and analytical modes, then asks
%user to input indices for experimental mode and set of indices for
%combined analytical mode. A crossMAC is calculated between the combined
%analytical mode and experimental mode

load('y_modes.mat')
femmode = input('input the mode numbers for FEM modes');
expmode = input('input the mode number for experimental mode');
phi_1 = [Y_FEM(:, femmode)];
phi_2 = [Y_EXP(:, expmode)];
beta = phi_1.'*phi_2;
phi_3 = phi_1*beta;
Mac_num = phi_2'*phi_3*phi_3'*phi_2;
Mac_den = phi_2'*phi_2*phi_3'*phi_3;
MAC = Mac_num/Mac_den;
```

A.5 Implementing spaceMAC

```
% Correlation of modal vector spaces
```

```
%This script loads the saved experimental and analytical modal vector and  
%frequencies. The experimental and analytical frequencies are compared are  
%matched. The
```

```
%Author: Pranjali M Vinze
```

```
%Email: vinzepam@mail.uc.edu
```

```
clear;
```

```
clc;
```

```
%Load experimental and analytical modal vectors
```

```
load('y_exp.mat');
```

```
load('y_ansys.mat');
```

```
%load frequencies from experimental and analytical solutions
```

```
freqAnsys = load('modal_freq.txt');
```

```
freqExp = load('exp_freq.txt');
```

```
freqExp = freqExp(freqExp(:,1)>0,:);
```

```
Frequencies = freqExp;
```

```
%Match frequencies
```

```
rigidModesAnsys = input('number of rigid body modes expected in the Ansys modal vectors');
```

```
rigidModesExp = input('number of rigid body modes expected in the experimental modal vectors');
```

```
model = input('match the first non rigid frequency?')
```

```
if model == 1
```

```
sizeexp = size(freqExp,1) - rigidModesExp ;
```

```
sizeAnsys = size(freqAnsys,1) - rigidModesAnsys ;
```

```
Size = min(sizeexp,sizeAnsys);
```



```

sizeexpvec = size(yy2,2) - rigidModesExp;
sizeAnsysvec = size(Y_filtered,2) - rigidModesAnsys;
for ii =1: Size
    Expmin(ii,:) = freqExp(rigidModesExp+ii,:);
    Ansysmin(ii,:) = freqAnsys(rigidModesAnsys+ii,:);
end
fr = [Expmin Ansysmin];
numModes = min([sizeexpvec,sizeAnsysvec]);
else
for ii = 1: size(freqAnsys,1)
    success = false;
    [fr(ii) fr_in(ii)] = min(abs(freqAnsys(ii,2) - freqExp(:,1)));
    if fr(ii) < 0.15 * freqAnsys(ii,2)
        success = true;
        freqExp(fr_in(ii),1) = 0;
    else
        success = false;
        fr_in(ii) = 0;
    end
end
end

counter = 0;
for ii = 1:size(fr_in,2)
    if fr_in(1,ii) > 0
        counter = counter + 1;
        AnsysFreq(counter,:) = freqAnsys(ii,2);
    end
end

%remove rigid body modes
numModes = max(fr_in);
end

% numModes = input('input the number of modes to be analyzed ');
if numModes <= size(yy2,2) && numModes <= size(Y_filtered,2);
    repOrder = input('input the order of repeatedness for which to search');

```

```

Y_FEM = Y_filtered(:,rigidModesAnsys+1:rigidModesAnsys+numModes);
Y_EXP = yy2(:,rigidModesExp+1:rigidModesExp+numModes);

%crossMAC evaluation
for ii = 1:(size(Y_FEM,2))
    for jj = 1:(size(Y_FEM,2))
        Mac_num = Y_FEM(:,ii)'*yy2(:,jj)*yy2(:,jj)'*Y_FEM(:,ii);
        Mac_den = Y_FEM(:,ii)'*Y_FEM(:,ii)*yy2(:,jj)'*yy2(:,jj);
        MAC(ii,jj) = Mac_num/Mac_den;
    end
end
MAC = abs(MAC);
crosDiag = abs(diag(MAC));

%Filter modes based on crossMAC threshold values
minMAClimit = input('input the minimum acceptable diagonal crossMAC value');
lowMACindex = find(crosDiag<minMAClimit) ;

%Create all combinations of the multiplicity selected
combinations = combntns(lowMACindex,repOrder);

%Create sets of vectors of all combinations and evaluate principal angles
%for all these combinations

for pp = 1:length(combinations)
    A1 = Y_FEM(:,combinations(pp,:));
    B1 = Y_EXP(:,combinations(pp,:));
    angle(pp) = subspace(A1,B1)';
end
FreqCombs = Frequencies(combinations);
results = [FreqCombs angle'];
end

```

```

%Evaluate spaceMAC
results(:,repOrder+2)=1-abs(sin(results(:,repOrder+1)));
freqStr = num2cell(results(:,1:end-2));
freqStr = cellfun(@num2str, freqStr, 'UniformOutput', false);
for ii = 1: size(freqStr,1)
    Strings{ii,:} = strjoin(freqStr(ii,:), ', ');
end

%plot spaceMAC results
plot(results(:,end));

```

A.6 ANSYS script to extract modal vectors

```
allsel
*get,NumNd,node,,count ! total node number
TotMN=20! total mode number
*do,i,1,TotMN
allsel
*get,NumNd,node,,count !total node number
*dim,NodalInfo,,Numnd,7
set,1,i
allsel
!*del,MODALFREQ
!*get,MODALFREQ,ACTIVE,,i,FREQ
*vget,NodalInfo(1,1),node,,nlist
*vget,NodalInfo(1,2),node,,loc,x
*vget,NodalInfo(1,3),node,,loc,y
*vget,NodalInfo(1,4),node,,loc,z
*vget,NodalInfo(1,5),node,,u,x
*vget,NodalInfo(1,6),node,,u,y
*vget,NodalInfo(1,7),node,,u,z
*cfopen,C:\temp\Mode%i%,txt
*vwrite
('Node Num X Y Z UX UY UZ')
*vwrite,NodalInfo(1,1),NodalInfo(1,2),NodalInfo(1,3),NodalInfo(1,4),NodalInfo(1,5),NodalInfo(1,6),NodalInfo(1,7)
(f8.0,f11.5,f11.5,f11.5,f11.5,f11.5,f11.5)
*cfclose
*enddo
```


Appendix B

Cross MAC

X axis, Reference File: Mode Group 4
Y axis, Comparison File: Mode Group 3
Sieva File: <none>
Matching DOFs: 33

(a) Random forcing, cyclic averaging and Hanning window vs burst random

X axis, Reference File: Mode Group 15
Y axis, Comparison File: Mode Group 14
Sieva File: <none>
Matching DOFs: 33

(b) RFP vs PTD of burst random forced data
2014 Circular Plate Dataset

(a) RFP vs PTD of burst random forced data
2001 Circular Plate Dataset

(b) Experimental(PTD of burst random forced data 2014 Circular Plate Dataset) vs Analytical