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I, Botao Zhang, hereby submit this original work as part of the requirements for the degree of Master of Science in Mechanical Engineering.

It is entitled:

**DESIGN OF VARIABLE-DENSITY STRUCTURES FOR ADDITIVE
MANUFACTURING USING GYROID LATTICES**

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Design of Variable-Density Structures for Additive Manufacturing using Gyroid Lattices

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Abstract

Additive manufacturing (AM) processes enable the creation of structures having complex geometry. Among such structures are lattice structures which offer great potential for designing lightweight structures. The combination of AM and cellular lattice structures provide promising design solutions in terms of material usage, cost and part weight. However, the geometric complexity of the structures calls for a robust methodology to incorporate the lattices in parts designs and create optimum lightweight designs. This thesis proposes a novel method for designing lightweight variable-density lattice structures using gyroids. The parametric 3D implicit function of gyroids has been used to control the shape and volume fraction of the lattice. The proposed method is then combined with the density distribution information from topology optimization algorithm. A density mapping and interpolation approach is proposed to map the output of topology optimization into the parametric gyroids structures which results in an optimum light-weight lattice structure with uniformly varying densities across the design space. The proposed methodology has been validated with two test cases.

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1. INTRODUCTION

Additive manufacturing (AM) is a layer-based manufacturing approach that creates parts by depositing slices on top of each other. Lattice structures, also referred as cellular structures, have received considerable attention in additive manufacturing field in recent years. Lattice structures are microporous solid structures having a variety of unit cell sizes and shapes. Honeycomb, gyroids, and octet truss are examples of such structures [1-6]. Such unit cell lattice geometries provide an effective way to reduce the total weight of a structure and maintain the required structural strength. For such complex geometries, additive manufacturing is preferred over traditional manufacturing processes due to the inherent flexibility and freedom in its manufacturing approach.

Truss-like lattices such as body centered cubic (BCC), octet truss and parametric surface based lattices such as gyroids and diamond are some of the frequently used cellular lattice structures in designs. Manufacture of truss-like lattices having large unit cell size may prove challenging due to features such as overhanging struts [5]. The gyroid structure, as opposed to strut-based lattices, has a smooth circular surface. When sliced, the angle between two adjacent layers of the gyroid structure gradually changes. This allows the lower layer to effectively support the upper layer making the gyroids self-supporting structures [5]. A gyroid is a triply periodic minimal surface [7] and can be observed throughout nature in various structures and objects such as the butterfly wing scales [8] (shown in Fig. 1.). The gyroid structure can be trigonometrically approximated [9] by a parametric equation, which defines the structure in a mathematical form. By changing the parameters in the equation, the volume fraction and unit cell size can be precisely controlled.

Figure 1. (a) Gyroid lattice structures. (b) TEM micrograph of a butterfly wing scale. [8]

This research presents an approach to design gyroid structures of uniformly varying density based on previously established relationship between the parametric equation of gyroids and the volume fraction. The method helps to achieve control over location based variation of the cellular density as well as unit cell size without losing any surface continuity of the gyroids. A topology optimization based methodology is used as the basis for generating variable-density gyroid lattice structures within a given design space under predefined loading conditions. Topology optimization is the method of obtaining optimum material distribution within a design space to achieve desired weight reduction while minimizing part compliance [10, 11]. In this paper, Solid Isotropic Material with Penalization (SIMP) approach which is a density-based approach is used for topology optimization [12, 13]. In SIMP, the material distribution is defined in terms of the value of the fractional density of each element from 0 (void) to 1 (solid). In this work, an interpolation method is applied to these element densities in order to obtain gradational density distribution throughout the design space. Using these density values and the parametric equation of gyroids, optimum lattice structures are generated. A test case of cantilever beam and an L-shaped beam are used to validate the methodology.

1.1 Motivation for Research

Over the decades, Additive Manufacturing processes have been growing rapidly in several areas in the industry. The one great advantage of AM is that it provides increased design flexibility which allows designer to create complex geometries. Lattice structures, which have already been observed in many cases in nature are drawing a lot of attention in the AM field due to its strength-to-weight efficiency. It is very important for the researchers to study the structural strength and manufacturability of a variety of lattice structures to effectively design and manufacture high performance parts.

Many lattice designs have been proposed and explored over the last few years, but the performance of the lattice structures is far from optimal. Currently, many designers have generated uniform density lattice structures with a variety of unit cells such as BCC or octet truss [1-6]. Even though the design and manufacture process are simple to achieve, it is almost certain that the structural performance is not optimal since the material distribution is uniformly distributed throughout the design space. The goal is to obtain optimum material distribution structures for the design space while reducing weight. While designers can use variable density lattice structure designs to fill the design space, one challenge is to sew those lattices with different densities together. Since the cross section between two densities lattices may not exactly match, additional algorithms are required to generate smooth connections between variable density lattices. In addition, filling the design space with variable density lattice requires some sort of intelligent guideline to distribute material based on loading conditions. The topology optimization method has developed rapidly in the recent years and it provide an optimum solution to placing material within a given design space to obtain best structural performance [12, 13].

This thesis lies in the intersection of these research areas. A SIMP-based density topology optimization is used as the guideline for intelligently incorporating variable density gyroid lattices in the design space to generate optimal lightweight lattice based design.

1.2 Objectives of Research

The primary objective of this research is to develop a methodology to create lightweight part designs using variable density gyroid lattices and topology optimization for additive manufacturing. This methodology helps optimize the material distribution of a structure by mapping the SIMP-based topology optimized density distribution to the implicit function of gyroid structures. For a given design space, an efficient MATLAB code which uses the SIMP-based topology optimization algorithm, developed by Liu and Tovar [22], is first applied to provides a coarse mesh of density distribution. Then the coarse density mesh is refined using the linear interpolation method and the fine density mesh is mapped into the 3D implicit parametric function of the gyroid structure to generate the weight optimized lattice design. A MATLAB algorithm has been developed that adapts the SIMP-based topology optimization to perform density mapping and generates gyroid structure in the form of a STL file. The research methodology is validated by performing finite element analysis on two test cases of cantilever beam and L-shaped beam.

1.3 Thesis Outline

This thesis is divided into 5 chapters. The first chapter briefly discusses additive manufacturing technology, motivation of this research and the objective of this research. The second chapter presents a brief literature review of the research related to structure performance

and manufacturability of the gyroid structure, the applications of topology optimization method in additive manufacturing field, and the current studies on variable density lattice structure design. The third chapter presents the design methodology of generating variable density gyroid lattice structure and density mapping algorithm which assigns physical density data from the output of topology optimization to implicit function of the gyroid structure. In the fourth chapter, the design methodology is validated by performing Finite Element Analysis on two case studies: a cantilever beam and a L-shaped beam. The conclusions and future scopes for this research are highlighted in the last chapter.

2. LITERATURE REVIEW

The available literature on gyroid structure, topology optimization, and variable density lattice structure is presented in the following sub-sections:

2.1 Gyroid structure

Over the last few decades, various studies have been conducted to investigate the structural performance and buildability of the gyroid lattice structure using AM. Yan et al. [5, 6] explored the manufacturability and performance of gyroid lattice structure having different unit cell sizes from 2 mm to 8 mm using selective laser melting (SLM). Khaderi et al. [14] explored the plastic collapse response of an ideal rigid plastic gyroid structure and investigated the stiffness and strength of the gyroid lattice by using finite element calculations. Yáñez et al. [15] performed the compression test on gyroid structures to determine their elastic modulus and overall strength. Hussein et al. [16] and Strano et al. [17] designed and manufactured optimized gyroid lattice structures as support structures that sustain overhang parts.

2.2 Topology optimization

Topology optimization method developed rapidly since its first introduction by Bendsøe and Kikuchi [18], and has been explored for many different applications. Zhou and Rozvany [12], and Mlejnek [13] developed a density-based approach which parameterizes the material distribution problem into a density distribution problem where the material property of each element is determined by using the power-law interpolation function between 0 and 1. The power-law approach, which is often referred as Solid Isotropic Material with Penalization (SIMP) method, simplifies the penalization procedure for intermediate densities. Bendsøe [19] analyzed various approaches of penalization concept on the properties of composite materials in order to derive optimal designs. In recent years, the application of topology optimization method

has been widely studied for additive manufacturing. Brackett et al. [20] discussed the challenges associated with manufacturing of the optimized designs using AM while achieving the maximum geometric resolution and also proposed the use of small scale lattice structures for intermediate density regions. Leary et al. [21] proposed an approach to modify the topology optimization model to increase the manufacturability in AM without requiring additional support material. Based on regularized SIMP interpolation density-based approach, an efficient MATLAB code for solving (3D) topology optimization problems was presented by Liu and Tovar [22].

2.3 Variable density lattice structures

Pasko et al. and Fryazinova et al. [23, 24] proposed a function-based approach using periodic functions for modeling irregularly shaped microstructures. The basic pore shape is subtracted using pseudo-random variations of pore parameters. Li et al. [25] developed a shape model method using density-variables that first estimates the density distribution based on the cross-sectional stress analysis and subsequently fills the design space with porous structure having corresponding densities. A size matching and scaling method was developed by Chang and Rosen [26] which uses a combination of pre-defined lattice configuration and finite element analysis to create lattice topology. Alzahrani et al. [27] proposed a design method that generates truss-like cellular structures using the topology optimization density data to govern the diameter of the individual struts in the structure. Zhang et al. [28] proposed an efficient variable-density cellular structure design by mapping topology optimization density results to a hexagonal cellular structure with variable strut diameter.

3. METHODOLOGY

This section presents the various steps (Fig.2.) that are involved in generating optimally designed variable density gyroid structures. The steps can be briefly explained as follows:

Figure 4. Methodology of generating variable density gyroid structure.

- (1) Preprocessing: design space, constraints and loading conditions are defined in preparation for topology optimization algorithm.
- (2) Topology optimization: SIMP-based topology optimization is performed in MATLAB and the physical density of each element is derived from the result.
- (3) Density mapping: physical density data from topology optimization is interpolated through the design space in order to map a finer density distribution in the structure.

- (4) Gyroid structure generation: The interpolated density data is imported into the implicit function of gyroid and a variable density lattice structure is constructed.
- (5) Lattice Refinement: Unit cells with low volume fraction are removed to further reduce the weight.
- (6) Export: The results are exported as a STL file.

Section 3.1 presents the properties of gyroid structure and introduces the method of creating non-uniform gyroid structures. In section 3.2, topology optimization approach for density distribution is explained. In section 3.3 and section 3.4, a density mapping algorithm is introduced, followed by the process of constructing optimized variable density gyroid lattices.

3.1 Variable density gyroid structure

Cellular structures such as foams, octet truss, gyroid, and honeycomb can be broadly classified as open, closed-cell, random or periodic lattice structures [1-6]. The focus of this research is on the parametric gyroid lattice structures. The gyroid structure is based on gyroid surface which can be represented by the implicit function $F: R^3 \rightarrow R$ at point $(x, y, z) \in R^3$, that satisfies the Eq. (1). [9]:

$$F(x, y, z) = t \quad (1)$$

where, t is a constant.

The surface of a unit sphere is a good example of an implicit function which is represented by:

$$F(x, y, z) = x^2 + y^2 + z^2 = 1 \quad (2)$$

The gyroid surface is defined by the parametric equation [9]:

$$F(x, y, z) = \sin\left(\frac{2\pi x}{L}\right) \cos\left(\frac{2\pi y}{L}\right) + \sin\left(\frac{2\pi y}{L}\right) \cos\left(\frac{2\pi z}{L}\right) + \sin\left(\frac{2\pi z}{L}\right) \cos\left(\frac{2\pi x}{L}\right) = t \quad (3)$$

Where, ‘L’ is the size of the unit cell and ‘t’ is a constant parameter that governs the volume fraction of the unit cell [9, 29]. Figure 3. shows an example of a gyroid surface generated by the implicit function (Eq. (3)). It is observed that the parametric surface of the gyroid splits the space into two continuous regions.

Figure 7. Gyroid surface expressed by Eq. (3). with $t=0$.

A gyroid structure can be obtained by filling the region on one side of the surface as solid. The parameter ‘t’ has a range from -1.5 to 1.5, and it is observed to have an approximately linear relationship with the volume fraction as given by Eq. (4). [9]. We can infer from the equation that at $t = -1.5$ the relative density of the gyroid is equal to 0 and at $t = 1.5$ the relative density of the gyroid is equal to 1.

$$\rho = 0.5 + \frac{t}{3} \quad (4)$$

Since gyroid lattice structures are purely defined by mathematical expressions, the unit cell size and volume fraction can be precisely controlled to generate a network of periodic cellular structures. If the parameter ‘t’ is constant throughout the structure, then the lattice gets uniform unit cell size as well as uniform volume fraction. Han et al. [5] have successfully manufactured models of gyroid structures having volume fraction of 15% and different unit cell sizes varying from 2 mm to 8 mm made by SLM. Figure 4. shows three gyroid lattice structures with 8-unit-cells which are constructed using the Eq. (3). with parameter t equal to 0, -0.5 and -1, respectively for (A), (B) and (C). The volume fraction of the structures is 0.5, 0.33 and 0.17, respectively.

Figure 8. Variable volume fraction gyroids cells with volume fraction of (A) 50%, (B) 33% and (C) 17%.

In this work, non-uniform gyroid lattices are generated by replacing the constant parameter ‘t’ with a variable parameter ‘t (x, y, z)’. Eq. (3). then becomes,

$$F'(x, y, z) = \sin\left(\frac{2\pi x}{L}\right) \cos\left(\frac{2\pi y}{L}\right) + \sin\left(\frac{2\pi y}{L}\right) \cos\left(\frac{2\pi z}{L}\right) + \sin\left(\frac{2\pi z}{L}\right) \cos\left(\frac{2\pi x}{L}\right) - t(x, y, z) = 0 \quad (5)$$

where ‘ t ’ is a variable that controls the density at the location (x, y, z) . This gives the structure a non-uniform density which depends on the function of $t(x, y, z) \in [-1.5, 1.5]$. For example, by giving the unit cell size $L=1$, $x, y \in [0, 3]$, $z \in [0, 15]$, and defining the density function: $t = 1.5 - 0.2z$ in Eq. (5), the implicit function generates a gyroid structure with parameter t linearly decreasing from 1.5 to -1.5 along the z -axis, as shown in Fig. 5. It is clear from the figure that the volume fraction of the cross-section also decreases along the z -axis.

As presented in Fig. 5., different cases for the variation in the volume fraction with respect to parameter t could be further described as follows:

- (1) When $t \geq 1.5$, the gyroid structure becomes a solid body which has the volume fraction of 1.
- (2) When $t=0$, the gyroid surface divides the space equally into two identical regions which have the same volume fraction of 50%.
- (3) When $t < -1.41$ (volume fraction $< 3\%$), it is observed that the gyroid structure loses connectivity and such values should be avoided in generating the gyroid lattice.
- (4) When $t \leq -1.5$, the gyroid disappears, since there is no point in space that satisfies the Eq. (5).

For the example in Fig. 5., the equation governing the parameter ‘ $t(x, y, z)$ ’ is linear.

However, in certain situations, the parameter ‘ t ’ can be represented with an expression of higher

Figure 11. Variable density Gyroid structure with t from 1.5 to -1.5.

degree to achieve more complex density distribution. However, it is important to note that the value of density parameter 't' should not be changed abruptly in a short interval to avoid sharp geometry changes in the gyroid lattice.

In the next step the proposed parametric gyroid generation approach is combined with conventional density based topology optimization techniques. This is discussed in the next section.

3.2 Topology optimization approach with lattice structures

Topology optimization provides an optimum solution for placing available material within a given design space to obtain best structural performance [10, 11]. A density-based approach was developed by Zhou and Rozvany [12], and Mlejnek [13] that defines the material distribution problem as material density distribution problem using the SIMP method. This approach allows the elements to possess partial densities and uses a penalization based power-law to obtain a structure with continuously varying partial densities between 0 and 1. Based on the density-based approach and SIMP method, Liu and Tovar [22] have provided an efficient 3D topology optimization algorithm using 0 code, which is adapted and further developed in this paper.

The design space for topology optimization is first discretized into finite cubic elements and the iterative optimization algorithm is executed. The output is represented in terms of physical density variables between 0 and 1, where the density variable represents the relative density (or volume fraction) of each element. Figure 6. shows an example output of voxel-based topology optimization, where gray elements show low density region and darker elements represent higher density or solid region.

While converting the variable density based output of topology optimization into a 3D solid model, the elements with partial density are often approximated as solids. The non-solid

Figure 12. Topology optimization results of a 3D cantilever beam.

elements with partial volume fraction are often pushed to be a solid element of complete density. In such a case, that particular element does not necessarily need to be completely solid to withstand the applied forces [10, 11, 19]. Periodic cellular lattice structures are a plausible solution to physically represent the elements with partial density. While converting into a solid model, if these elements are replaced with lattice structure cells of corresponding density instead of solid elements, the structure could be made lighter while maintaining its structural strength. Traditional lattice structures such as body centered cubic (BCC) and face centered cubic (FCC) have a problem with sewing or connectivity when unit cells with different volume fraction come together. On the other hand, the parametric gyroid structures presented in section 3.1 are ideally suited for a topology optimized design space with variable density. The parametric form of the gyroid lattice structure allows accurate control on the density in different regions of the structure. This enables a gradual variation in the volume fraction of gyroid lattice unit cells. As a result, the structure maintains smooth continuity through the entire body.

To achieve the integration of gyroid lattice structure with topology optimization, the element-wise density data from topology optimization is used to determine the value of parameter ‘t’,

using Eq. (4). However, this density data cannot be directly applied to parameter ‘t’, because it pertains to the entire volumetric element and cannot be converted into the implicit function of t (x, y, z). Furthermore, in order to create smoothly varying gyroids from this density data, a smooth transition in the densities is required across the design space, which calls for a finer distribution of the densities. The problem could potentially be solved by decreasing the size of each finite element, however, this will unrealistically increase the processing time and memory requirement. Therefore, to obtain a fine density distribution, a density mapping and interpolation approach is developed.

3.3 Density mapping

The steps involved in the density mapping and interpolation approach are illustrated in Fig. 7. First, the density of each element from the output of topology optimization is assigned to all the nodes corresponding to that element. The resultant density at each node is then calculated by taking the average of densities of all the elements that share the node using Eq. (6).

$$\rho_{ijk} = \frac{\sum X_{ijk}^*}{N} \quad (6)$$

Where, ρ_{ijk} is the resultant density of the node point (i, j, k) and X_{ijk}^* are the densities of the elements which share the node point. N is the number of the elements which share that node (N

Figure 16. Schematic illustration of interpolation method for density mapping.

can be 1, 2, 4 or 8). Equation (6). thus provides aggregate projected densities at each element node.

Once the densities at element nodes are obtained, those are linearly interpolated throughout the element volume using the MATLAB algorithm given in [30], which use linear interpolation along three dimensions. For example, for two adjacent points in x axis (x_a, y, z) and (x_b, y, z) with density data ρ_{x_a} and ρ_{x_b} , respectively, first the distance between x_a and x_b is equally divided into n points $[x_1, x_2, \dots, x_n]$, then for each point (x_i, y, z) , the density data is giving by:

$$\rho_{x_i} = \rho_{x_b} - (\rho_{x_b} - \rho_{x_a}) \frac{x_b - x_i}{x_b - x_a} \quad i = 1, 2, \dots, n, \quad (7)$$

Simultaneously using the same approach along Y and Z directions gives a fine distribution of interpolated densities throughout the entire design space. Density at each interpolated point defines the density of gyroid structure at that location. The number of the interpolated points can be manually controlled, which affects the smoothness of the gyroid surface. In this study, 10 interpolated density points are generated between every two adjacent points. This provides a considerable number of density gradient for the smooth surfaces while maintaining a moderate file size. Then, the fine grid of density $\rho(x, y, z)$ distribution is applied to gyroid structure parameter $t(x, y, z)$ using Eq. (4). Finally, using the Eq. (5)., the gyroid lattice surface is generated in the form of a triangulated mesh defined by faces and vertices. The open boundaries of the meshed gyroid surface are closed with planar surfaces. The facet and vertex data for the closing surfaces is added to the mesh data of gyroids giving a closed surface body, as shown in Fig. 8(b). Finally, the mesh is converted and exported as STL file, which can be used as input for additive manufacturing.

3.4 Removal of low volume fraction lattices

As shown in Fig. 8(a), the interpolation method unavoidably generates gyroids with low volume fraction (very close to 0). As discussed in section 3.1, those locations are where gyroid regions lose connectivity and need to be removed. In reality, for volume fractions that are less than 10%, the struts in the gyroid are too fragile to be manufactured consistently and accurately due to the constraints in AM processes [17]. Therefore, those gyroid lattices need to be removed.

Figure 19. Density mapping results of Gyroid structure.

The low volume fraction regions are removed by filtering the interpolated density data. After interpolation method, every density data point is checked and if the density value is less than a user defined threshold, it is changed to 0. In this work the threshold is taken as 0.26 ($t = -0.7$). Suppressing the density value to zero avoids generation of gyroid lattice in that location. The sample result is shown in Fig. 9(a).

Figure 21. Two ways of lattice refinement.

Another option of avoiding the potential manufacturing failure of low volume fraction gyroids is to increase the density of the low volume fraction regions above the threshold, as shown in the example in Fig. 9(b). This theoretically increases the strength of the structure, but also increases the total weight of the part.

4. CASE STUDIES AND RESULTS

To validate the strength of the variable density gyroid structures, Finite Element Analysis (FEA) is performed on the two test cases. The goal of the validation is to demonstrate that the gyroid lattice design can further reduce weight while maintaining good structural performance. The first case is a simple cantilever beam loaded at one edge and the second example is an L-shape beam that is fixed at the top with a vertical load applied to the lower free edge. The designs obtained from topology optimization as well as the gyroid lattice design are meshed and analyzed using Altair Hypermesh and Optistruct Software. The results for maximum displacement and maximum Von Mises stresses are tabulated and compared. For both the design examples, the original geometry and topology optimization results are symmetric about the z axis. However, because of the trigonometric definition of gyroid lattices, the unit cells of gyroid are not symmetric about x , y , z axes. It is noted that this may result in non-symmetric properties of the gyroid lattice structures.

4.1 Simple cantilever beam

Figure 22. Cantilever beam example. (a) boundary and loading condition, (b) topology optimization results.

Table 1. Initial properties of the 3D cantilever beam example.

Length (mm)	100
Width (mm)	5
Height (mm)	50
Target Volume (mm ³)	7500
Force (N)	100
Young's Modulus (Pa)	2E11
Poisson's ratio	0.3
Total FEA elements count	200

In the first example, the beam is fixed at one end and 100 N load is applied on the lower free edge as shown in Fig. 10(a). The purpose of this optimization is to reduce the weight of the structure while minimizing the overall compliance. The design parameters of this example are summarized in Table 1. The conventional topology optimization is performed on this design space first. The variable density gyroid structure is then generated using the density data from topology optimization results. The size of the unit gyroid cell is 3.3mm, as shown in Fig.11. The displacement and Von Mises stress results from the finite element analysis are shown in Fig. 12. and Table 2. The first column represents the results for topology-optimized beam, and the second column represent the results for the design with variable density gyroid structure. It is

Figure 11. Gyroid lattice structure results with unit cell size of 3.3mm.

observed that the gyroid lattice design has moderately more displacement but slightly less max stress. This may be caused by the asymmetric form of gyroid which has a small amount displacement in the Z direction. The final volume of the gyroid structures is 26.7% of volume of the design space whereas, the volume of the topology optimized beam is 37.5% of volume of the design space. Therefore, a further reduction of 30% in the volume is observed in case of gyroid structure as compared to the solid topology optimized beam.

Table 4. Results of the FEA of the 3D cantilever beam.

	Topology Optimization result	Gyroid structure beam
Unit cell size (mm)	-	3.33
Volume Fraction	37.5%	26.7%
Max Displacement Magnitude (mm)	1.289E-2	1.727 E-2
Max Von Mises stress (MPa)	0.948 E2	0.660 E2

Figure 12. (a) Displacement plot and (b) Von Mises stress plot of the cantilever beam gyroid lattice structure with unit cell size of 3.3mm.

4.2 L-shape beam

The second test case is a 3-dimensional L-shape beam as shown in Fig. 13. The beam has the same dimension along the x-axis and y-axis and has uniform square cross-section of 25mm X 25mm. It is fixed on the top face and a 100N uniformly distributed vertical load is applied on the bottom edge of the free end. Table 3 summarizes the geometry parameters of the beam.

Figure 13. Boundary and load condition of an L-shape beam.

Figure 14(a). presents the designs using topology optimization method. Gyroid lattice structure created using the densities from topology optimization result is shown in Fig. 14(b). The finite element analysis results are summarized in Table 4. Figure 15. shows the displacement and Von Mises stress results from FEA. The two designs have approximately the same amount of displacement, however, the gyroid lattice appears to show more stress than the topology

Table 5. Initial properties of the 3D L-shape beam example.

Length (mm)	75
Height (mm)	75
Width (mm)	25
Cross section (mm x mm)	25 X 25
Target Volume (mm ³)	56250
Force (N)	100
Young's Modulus (Pa)	2E11
Poisson's ratio	0.3
Total element count	5000
Element number along length	30
Element number along width	10
Element number along height	30

Figure 14. (a) Topology optimization results with element size of 2.5mm. (b) Corresponding gyroid lattice structure with unit cell size of 2.5mm.

optimization design. The high stress concentration is observed on certain edges where the gyroids are trimmed. The gyroid lattice could be further refined to reduce the max stress.

Table 4. shows that the Gyroid structures has the volume fraction of 32.7% as compared to the design space, while the conventional topology optimization result has a volume fraction of 42.7%. A 23% reduction in total volume is observed from conventional design to the gyroid structure design.

Table 6. Results of the FEA of the 3D L-shape beam.

	Topology Optimization result	Gyroid structure beam
Unit cell size (mm)	-	2.5
Volume Fraction	42.7%	32.7%
Max Displacement Magnitude (mm)	1.124 E-2	1.003 E-2
Max Von Mises stress (MPa)	2.019 E1	3.456 E1

Figure 15. (a) Displacement plot and (b) Von Mises stress plot of the L-shape beam gyroid lattice structure with unit cell size of 2.5mm.

5. CONCLUSIONS AND FUTURE SCOPE

This paper presents a novel approach towards design using variable density gyroid lattice structures and its integration with topology optimization. The methodology helps minimize the weight of a structure by mapping the topology optimized density distribution to the implicit function of gyroid structures. A MATLAB algorithm has been developed that performs density mapping and generates gyroid structure in STL file. To start with, the SIMP-based topology optimization algorithm provides a coarse mesh of density distribution in a defined design space. The coarse density mesh is then refined using an interpolation method. Finally, the fine density distribution is used in the 3D implicit parametric function of the gyroid structure to generate the gyroid based design. The design is exported as STL format and is validated using two test cases of cantilever beam and L-shaped beam.

Future work could include additional mechanical properties of gyroid lattices such as Young's Modulus and overall stiffness in the topology optimization process to obtain more accurate results. This method can be further explored to generate self-supporting topologically optimized gyroid structures as optimum support structures. Although a gyroid structure is self-supporting, in the test cases presented in this work, when the low density gyroids are suppressed, the resultant part may require additional support structures. However, if these low density gyroids are bumped up above a threshold value, those could potentially act as support structures. Further investigations can be pursued on this topic to validate the robustness of this claim. Also, the current study considers cubic elements for topology optimization. By modifying and refining the interpolation approach, the proposed design methodology can be adapted to non-cubic elements, such as tetrahedrons. Future study may also include generation of conformal gyroid lattices with variable cell size for parts with complex geometry contour.

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