University of Cincinnati

Date: 7/14/2016

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It is entitled:

Synthesizing Uncorrelated Drive Files for MIMO Transmissibility Measurements on Road Simulators

Student's name: **Shounak Deshmukh**

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Synthesizing Uncorrelated Drive Files for MIMO Transmissibility Measurements on Road Simulators

A thesis submitted to the
Division of Research and Advanced Studies
of the University of Cincinnati

in partial fulfilment of the
requirements for the degree of

Master of Science

in the Department of Mechanical and Materials Engineering
of the College of Engineering & Applied Sciences

July 2016

by

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Abstract

Traditionally, road simulators have been used for durability and NVH testing of automobiles which involve estimation of simulated road inputs. The responses (ride files) are measured during actual road testing at a number of locations over different terrains which lead to the generation of the simulated road inputs by utilizing a process involving an inverse calculation and may require an iterative solution so that the response measured in the actual road test of the vehicle for a given type of terrain can be replicated very closely by estimating the inputs to the four axes of the road simulator. The numerical estimation procedure of the inputs is based upon the characteristics and limitations of the specific four axis road simulator being used but it does not constrain these inputs (drive files) to the individual hydraulic actuators to be uncorrelated. The drive files to the four axis road simulators more often than not are significantly correlated which is a big hindrance to MIMO transmissibility measurements. Thus synthesis of uncorrelated drive files using the estimated correlated drive files is what this research provides by the way of a MATLAB™ based interface. The analysis shows that the auto power spectra is equivalent for the synthesized uncorrelated drive files and the original correlated drive files which are further corroborated by the experimental results obtained from the measurements done on the MTS four axis road simulator at UC-SDRL. These same methods should also be useful in estimating FRFs from drive files if the input forces can be measured.

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Nomenclature

Abbreviations

<i>NVH</i>	Noise, Vibration and Harshness
<i>MIMO</i>	Multiple Input, Multiple Output
<i>FRF</i>	Frequency Response Function
<i>SVD</i>	Singular Value Decomposition
<i>PCA</i>	Principal Component Analysis
<i>DSP</i>	Digital Signal Processing
<i>LVDT</i>	Linear Variable Displacement Transducer
<i>SDOF</i>	Single Degree of Freedom
<i>MDOF</i>	Multiple Degrees of Freedom
<i>OEM</i>	Original Equipment Manufacturer
<i>EVD</i>	Eigen Value Decomposition
<i>DF</i>	Drive Files
<i>ODF</i>	Original Drive Files
<i>SDF</i>	Synthesized Drive Files
<i>VR</i>	Virtual References

Symbols

<i>F_{EQ}</i>	Equivalent or Driving Force
<i>F_T</i>	Transmitted Force
<i>F_{samp}</i>	Sampling frequency of the DSA
<i>F_{max}</i>	Maximum frequency that can be observed in a spectrum
$\hat{X}_p$	Spectrum of p-th Output, measured
$\hat{R}_q$	Spectrum of q-th Reference, measured
ν_q	Spectrum of Noise portion of Reference
η_p	Spectrum of Noise portion of Output
<i>R</i>	Actual Input (Reference)
<i>X</i>	Actual Output
<i>T_{pq}</i>	Transmissibility Function of Output p with respect to Reference q
$[G_{XR}]$	Output Reference Cross Spectra Matrix
$[G_{XX}]$	Output Output Cross Spectra Matrix
$[G_{RR}]$	Reference Reference Cross Spectra Matrix
$[A]$	Base matrix
$[U]$	Left singular matrix
$[V]$	Right singular matrix
$[\Sigma]$	Singular value matrix
$[I]$	Identity matrix
$[Q]$	Eigen vector matrix
$[\Lambda]$	Eigen value matrix

1 Introduction

1.1 Motivation

The automobile market today is highly competitive and has a number of vehicles available to the customer each having its own unique selling points. Amongst them ride comfort is and always will be a major criteria for a customer buying a vehicle. Thus a lot of automotive research has been dedicated to reducing cabin noise and vibrations in vehicles. Modal analysis and frequency response functions (FRFs) have been used to quantify these vibrations relating this information to road tests on various road surfaces. This approach provides accurate results for further analysis but is conducted on a limited number of vehicles. Thus in order to compare similar vehicles with different configurations and structural sizes, the number of tests becomes excessive.

Thus to compare with different vehicles and attain benchmark levels, simulation of vehicle road tests has been of huge interest to researchers. Recreating the responses on the vehicles that are the same as that obtained during the actual road test is what the simulation is aimed at. Creating custom input signals to get desired responses in frequency or time domain is an added capability so that different vehicles can be tested with the same reference. Four axis road simulators are used for different quality considerations but mostly for durability and NVH. Traditionally, these road simulators make use of LVDTs (linear variable displacement transformers) which quantifies input to vehicles in terms of displacement. Having vehicle response measured to this kind of input motion is quite convenient. FRF estimation requires input forces be measured which can be realized by using load cells at the four axes. But this instrument set up is uncommon, very expensive and not available at the University of Cincinnati, Structural Dynamics Research Lab (UC-SDRL).

The transmissibility concept is therefore useful here. The ratio of the force transmitted to the foundation (F_T) referenced to the equivalent or driving force (F_{EQ}) is defined as transmissibility [1]. Transmissibility can also be described as the ratio of structure displacement to the base displacement if the base is of moving type. Traditionally transmissibility is defined as a single input, single output (SISO), single degree of freedom (SDOF) concept, but a multiple input, multiple output (MIMO) approach, involving multiple degree of freedoms (MDOF) has to be used for a four axis road simulator owing to the inputs at four wheels. By utilizing the displacements measured from the LVDTs at the four shakers, the transmissibility can be estimated by taking responses at various locations on the vehicle with respect to the four inputs. Similar to MIMO FRF estimation, the four inputs have to be conditioned to allow for numerical estimation of MIMO transmissibility. With the inputs being simulated to recreate the responses obtained in the actual road test, the conditioning for the four inputs more often than not becomes poor and thus numerical estimation of MIMO transmissibility is not possible. Therefore, inputs have to be synthesized such that the numerical conditioning is improved and the synthesized inputs are still equivalent to the estimated inputs used to recreate the responses.

1.2 Literature Review

This thesis serves the purpose of providing for inputs to the four axis road simulator such that conditions are placed on them to estimate MIMO transmissibility while still satisfying typical road input characteristics. MIMO transmissibility concept has been developed in the MS theses by Yee [2] and Allemang [3]. Yee and Allemang developed MIMO displacement transmissibility as an extrapolation of the MIMO FRFs that was initially developed by Bendat and Piersol in 1971 [4] and proven experimentally by Allemang in 1979 [5].

The typical inputs to the four axis road simulator are numerically estimated from the responses obtained at different locations of a vehicle during an actual road test using an iterative process. This numerical

estimate of inputs is then used to simulate the actual inputs at the four axes so as to recreate the actual responses on the vehicle. This iterative numerical process has been developed by Uday Godbole in his MS thesis [6] specifically for the road simulator at UC-SDRL. The iterative process is also specified by OEMs for their specific road simulators in SAE papers by Cryer et al [7] and Kim et al [8] to be used for different quality testing. Balaji Sharma [9] in his MS thesis has discussed the feasibility of how these inputs can be used for automotive modal testing using different techniques. MIMO transmissibility is used as a technique for automotive modal testing and using the simulated inputs estimated from the actual responses leads to ill-conditioning of the reference matrix which will be discussed further in Section 3.1. SVD (singular value decomposition) as a tool is used to evaluate the number of contributing references (principal or virtual references) which is discussed in detail in references [10] [11] [12].

In the area of structural dynamics, SVD and the associated principal component analysis (PCA) have been found to be very useful. Even though the PCA techniques were first developed in the early 1900s as documented by Jolliffe [13], Deprettere [14], and others, use of the PCA techniques in structural dynamics was not common up until the 1970s and 80s. The application of PCA to structural dynamics (and automotive examples in particular) is highlighted in an International Modal Analysis Conference (IMAC) paper by Allemang et al [15].

1.3 Thesis Goals

Summarizing, the goal is to provide for a MATLAB™ based interface which synthesizes input signals using the numerically estimated input signals. The numerically estimated input signals are in turn derived from the response files available within UC-SDRL database for four different vehicles driven on four different road surfaces. Principal component analysis of the synthesized input signals is to be compared with the numerically estimated input signals to confirm that the inputs are no longer correlated and that the reference matrix will be well conditioned for the required matrix inversion procedure. Additionally, auto

power of the signals being compared remaining similar is of importance. The analytical proof is to be corroborated with experimental data which is to be measured using both the synthesized input signals and the numerically estimated input signals on the four axis road simulator.

2 MTS 320 Road Simulator

For this particular thesis, the MTS 320 Road Simulator, which is capable of simulating actual road tests conducted on vehicles was used. It is housed in the Non-linear Dynamics Test Facility which is a part of UC-SDRL.

Figure 2-1: MTS 320 Road Simulator at UC-SDRL

2.1 Overview

The components that essentially comprise the simulator are:

- Actuators
- Hydraulic Pump Assembly
- MTS 320 Road Simulator Controller

2.1.1 Actuators

There are four hydraulic actuators, one for each wheel of the vehicle. The hydraulic actuators can impart motion to the vehicle tires in the vertical direction, similar to road inputs to the vehicle, and have a maximum stroke of ± 3 inches from the mean position. The displacements correspond to voltage signals between ± 10 volts. Controlling the voltage signals controls the position of the actuator in time, and can be used to generate various drive signals to the vehicle. The actuators cannot simulate very low frequencies, below 2 Hz, due to the limited stroke of the wheel pan and piston assembly. On the higher frequency side, the maximum allowable frequency is around 80 Hz. This is limited by the natural frequency of the actuator and the oil column in the actuator.

2.1.2 Hydraulic Pump Assembly

The MTS 506 hydraulic pump assembly controls the actuators. The specifications for the pump assembly in the UC-SDRL facility are as follows:

Model Number	506.52C
Supply Rating	460 V, 60 Hz, 3 Phase
Working Pressure	3000 psi, 207 bar
Flow	55 gpm, 209 lpm

2.1.3 MTS 320 Road Simulator Controller

The MTS 320 Road Simulator Controller consists of the MTS 498.22 Test Processor and the MTS 497.05 Hydraulic Control Unit. The test rig is defined as the four hydraulic actuators with the test vehicle mounted on it. The test processor handles the actual control of this rig, the digitizing of the drive signals, communications between the controller and the test rig, and the data acquisition system. The hydraulic control handles the hydraulic lines and flow valves. A software user interface (Flex Test II®) provided with the controller enables control of the test rig using a commercially available PC.

2.2 Advantages of using Road Simulator

The aim of conducting a vehicle road test is to understand the dynamic behavior of the vehicle, and to study and characterize its vibration response to road inputs, during actual operating conditions. With this aim in mind, simulating road tests offers certain advantages over conducting actual road tests. Most laboratory tests focus on testing the automobile structure as an essentially linear structure when some nonlinearities are present. The road testing and the road simulator testing is an attempt to understand the effect of the nonlinearities to the overall response and performance of the vehicle.

2.2.1 Instrumentation

During an actual road test, only a limited amount of instrumentation of the vehicle may be possible due to constraints imposed by available measurement locations and the portability of the data acquisition system used. The larger the number of response sensors, the larger the data acquisition channels required, and consequently, the bulkier the data acquisition system. These limitations can be overcome in a laboratory test, where a more comprehensive setup can be used, as the data acquisition system does not have to be portable. Also, more response measurement locations may be available, thus enabling collection of spatial response data across non-linearities to better characterize the system.

Even if a large number of sensors are utilized in the actual road test, the ability to utilize a large number of sensor outputs in the road simulator is somewhat limited. Since there are four inputs to the road simulator, the four responses at the wheel spindles are most often the focus since collocated control is more robust. If sensors from other locations are utilized, the control algorithm moves to non-collocated control which is more difficult. If more than four sensors are involved, the control is normally performed in a least squares sense. In general, collocated or not, the solution is performed in a least squares sense meaning that the actual response is a similar response to that seen in actual road testing. Getting the exact, same response as that for the actual road response is generally not possible.

2.2.2 Repeatability

Each test on a test track will differ from the previous one, due to changes in the test conditions, namely changes in vehicle driving speeds, road surface conditions, and driver choice of path on test track. These can be eliminated in a laboratory setup, where the difference between two tests using the same input drive signals can be negligible, and the test is practically repeatable.

2.2.3 Safety aspects

Certain tests may involve risks of failure of vehicle components. It would be undesirable to have a test driver perform such tests, and a road simulator would be well utilized in such cases. Some tests are conducted for long periods of time (durability, for example) and use of a road simulator reduces the problems associated with driver fatigue.

2.2.4 Deterministic Testing

Certain modes of vibration may be of particular interest to the test engineer, and it may be possible to excite that particular mode using suitable drive signals on the simulator. This would particularly find

applications in the troubleshooting of problems caused by excessive vibration, or in the study of the suspension system characteristics at specific frequencies.

2.2.5 Flexibility

The inputs to the vehicle can be changed at will, which can enable testing of the vehicle by uniformly increasing or decreasing the input amplitudes of the drive signal, or by varying individual channels (tire inputs) while keeping the others the same, and other desired variations. This is obviously not possible on a test track, and thus simulation offers more flexibility as regards the inputs that can be given to the vehicle.

2.3 Disadvantages of using Road Simulator

Simulation on such a test rig has certain limitations which need to be understood while interpreting the results obtained for a simulated test. They are as follows:

2.3.1 Effects of Vehicle Dynamics

Since the vehicle has no driving velocity on the test rig, any effects that the vehicle drive speed has on its dynamic behavior are not simulated, for example the conditions of sudden acceleration or braking may not be accurately represented. The engine and the wheels are idle, and the inputs to the vehicle due to their operation (namely engine vibrations and tire rotations) are not simulated.

2.3.2 Tire-Road surface contact

The tires are assumed to be in contact with the road at all times. This assumption would normally be true on a highway drive. However, there may be times during the actual road test where the tire may momentarily lose contact with the road. At the UC-SDRL facility, more often than not, the tires are tied down to the actuators, so the loss of contact case is not simulated.

2.3.3 Input Axes

The inputs to the vehicle on a four axis road simulator are always vertical. Other angles are not possible, so oblique inputs to the vehicle cannot be simulated without adding other actuators such as those in a six axis road simulator.

Despite these limitations, the road simulator can be considered to be sufficiently accurate for the purpose of simulating various highway driving conditions and for many dynamic simulation goals.

3 Theory and background

To achieve the goal of this research it is important to understand how three different theories work. First would be how MIMO transmissibility is estimated. Second, the application of PCA to evaluate correlated inputs that give rise to ill-conditioned matrices in the estimation of MIMO transmissibility and, third the numerical procedure to estimate input signals from the responses obtained in actual road tests so that they can be recreated on the four axis road simulator.

3.1 MIMO Multi-Reference Transmissibility

The MIMO multi-reference transmissibility model for measurement estimation is unique in terms of each output since the outputs (responses) are not coupled in any way with respect to the inputs. The following development of the MIMO multi-reference transmissibility exactly parallels the development of the MIMO FRF estimation algorithms [5] [16] [17].

3.1.1 Transmissibility Function Estimation

The following schematic (Figure 3-1) is representative of the multi-reference transmissibility as a general case.

Figure 3-1 : Multiple Input Transmissibility Measurement Model

The following equation represents the transmissibility measurement case involving multiple references and a single output. This linear equation is simply replicated for each additional output.

$$\hat{X}_p - \eta_p = \sum_{q=1}^4 T_{pq} \times (\hat{R}_q - v_q) \quad (1)$$

Where,

- $R = \hat{R} - v$ Actual Input (Reference)
- $X = \hat{X} - \eta$ Actual Output
- $\hat{X}_p$ -- Spectrum of p-th output, measured
- $\hat{R}_q$ -- Spectrum of q-th reference, measured
- T_{pq} -- Transmissibility function of output p with respect to reference q
- v_q -- Spectrum of the noise portion of the reference
- η_p -- Spectrum of the noise portion of the output

3.1.2 Transmissibility Function Estimation Models

The following nomenclature was used to define the MIMO transmissibility measurement case for the least squares estimation of transmissibility, exactly paralleling the least squares estimation of MIMO frequency response functions [16].

$$[G_{XR}] = \{X\}\{R\}^H \quad \text{-- Output Reference Cross-Spectra Matrix}$$

$$[G_{XR}] = \begin{Bmatrix} X_1 \\ X_2 \\ \vdots \\ X_{N_o} \end{Bmatrix} [R_1^* \quad R_2^* \quad \cdot \quad \cdot \quad R_{N_r}^*] \quad (2)$$

$$[G_{XR}] = \begin{bmatrix} G_{XR_{11}} & \cdot & G_{XR_{1N_r}} \\ \cdot & \cdot & \cdot \\ G_{XR_{N_o1}} & \cdot & G_{XR_{N_oN_r}} \end{bmatrix}$$

R^* -- Complex Conjugate

$$[G_{XX}] = \{X\}\{X\}^H \quad \text{-- Output Output Cross-Spectra Matrix}$$

$$[G_{XX}] = \begin{Bmatrix} X_1 \\ X_2 \\ \vdots \\ X_{N_o} \end{Bmatrix} [X_1^* \quad X_2^* \quad \cdot \quad \cdot \quad X_{N_o}^*] \quad (3)$$

$$[G_{XX}] = \begin{bmatrix} G_{XX_{11}} & \cdot & G_{XX_{1N_o}} \\ \cdot & \cdot & \cdot \\ G_{XX_{N_o1}} & \cdot & G_{XX_{N_oN_o}} \end{bmatrix}$$

$$G_{XX_{ik}} = G_{XX_{ik}}^* \quad \text{-- Hermitian Matrix}$$

$$[G_{RR}] = \{R\}\{R\}^H \quad \text{-- Reference Reference Cross-Spectra Matrix}$$

$$[G_{RR}] = \begin{Bmatrix} R_1 \\ R_2 \\ \vdots \\ R_{N_r} \end{Bmatrix} [R_1^* \quad R_2^* \quad \cdot \quad \cdot \quad R_{N_r}^*] \quad (4)$$

$$[G_{RR}] = \begin{bmatrix} G_{RR_{11}} & \cdot & G_{RR_{1N_r}} \\ \cdot & \cdot & \cdot \\ G_{RR_{N_r1}} & \cdot & G_{RR_{N_rN_r}} \end{bmatrix}$$

$$G_{RR_{ik}} = G_{RR_{ik}}^* \quad \text{-- Hermitian Matrix}$$

Note that the subscript 'R' in the matrices above refers to 'reference' and is not to be confused with 'response'. Outputs (responses) are denoted with subscript 'X'. The G_{RR} matrix is analogous to the G_{FF} matrix in FRF estimation. As with the G_{FF} matrix, the G_{RR} matrix must be well conditioned (invertible) for the transmissibility to be computed for the MIMO case. Highly correlated inputs causes ill-conditioning of the G_{RR} matrix. The simulated input drive signals obtained from the numerical procedure using response signals acquired during actual road test, more often than not, are highly correlated. This has been discussed further in Chapter 4. The computation of this correlation parallels the principal force (virtual force) computation for MIMO FRF estimation and is used in the same way. For the case of four input transmissibility, this is referred to as principal reference (or virtual reference) analysis. For two or more references, a principal response analysis using singular value or eigenvalue decomposition of the G_{RR} matrix is needed to determine that the inputs are not highly correlated.

The MIMO transmissibility estimation process parallels the MIMO, least squares, FRF estimation process. The notations are same as the least squares FRF estimation techniques namely H_1 , H_2 , H_v [16] [17] and are denoted as T_1 , T_2 and T_v [2] [3]. In general, for all three techniques, the four inputs (references) used in

the four axis road simulator must not be highly correlated. This means that the actual measured motions at the four hydraulic actuators can be partially correlated but must not be completely correlated at any frequency in the analysis band. This evaluation is done using singular value decomposition of the G_{RR} matrix, in the form of virtual references, in the analytical as well experimental parts of this thesis.

3.2 Singular Value Decomposition

3.2.1 SVD Overview

Singular value decomposition (SVD) is a factorization technique similar (and related) to eigen-value decomposition. Its equation is as follows.

$$[A] = [U] \times [\Sigma] \times [V]^H \quad (5)$$

Note that $[A]$ is any matrix of size $m \times n$ and is in general complex-valued, $[U]$ is a square matrix of size $m \times m$, $[\Sigma]$ is a rectangular diagonal matrix of size $m \times n$ and $[V]$ is a square matrix of size $n \times n$. The columns of $[U]$ are called left singular vectors and the columns of $[V]$ are called right singular vectors. The diagonal entries in the diagonal matrix $[\Sigma]$, are scalar quantities called “singular values”. They are denoted by $\sigma_1, \sigma_2, \dots, \sigma_m$ and so on. The matrix is called, the singular value matrix.

$[U]$ and $[V]$ matrices are orthonormal matrices. That is, $[U] \times [U]^H = [I]$ and $[V] \times [V]^H = [I]$

The singular values, are arranged in descending order. Hence, the first few singular values, which are significantly larger in magnitude than the rest of the singular values, reveal the rank of the base $[A]$ matrix. They also denote the effective number of independent rows and columns of the base matrix.

When discussing SVD, it is important to understand its relationship with another important factorization method called the eigen-value decomposition (ED). ED is an important numerical technique, which is also used in principal component analysis [18].

The relationship between ED and SVD is as follows.

$$[A][A]^H = ([U][\Sigma][V]^H) \times ([V][\Sigma]^H[U]^H) = [Q](\Sigma[\Sigma]^H)[Q]^H = [Q] \times [\Lambda] \times [Q]^H$$

Hence, it can be observed that the singular values on the diagonal of the $[\Sigma]$ matrix are the square roots of the non-zero eigenvalues of both $[A][A]^H$ and $[A]^H[A]$ matrices. They are always real, scalar constants. Another notable point is that, the columns of the left singular matrix $[U]$ are eigenvectors of $[A][A]^H$ and the columns of right singular matrix $[V]$ are eigenvectors of $[A]^H[A]$.

For real-valued matrices, the 'H' operator, denoting Hermitian of the complex matrix is replaced by 'T' operator, denoting the transpose of the real-valued matrix. $[U]$ and $[V]$ matrices, in case of real-valued matrices, become unitary. That is, $[U] \times [U]^T = [I]$ and $[V] \times [V]^T = [I]$.

The singular value matrix indicates the magnitude of important information in the base $[A]$ matrix. To further illustrate the rank determination by the *number of significant singular values*, it can be said that, the number of first few significant singular values in the $[\Sigma]$ matrix, denotes the number of dominant, linearly independent characteristics present in the base $[A]$ matrix [19].

With respect to vibration response analysis, the dominant response is captured by the largest (first) singular value, since the right and left vectors are unitary, i.e. they are normalized to orthogonal with unit length.

Singular value decomposition is an increasingly popular tool in structural analysis and, in particular, modal analysis. The singular value matrix, and the left and right singular matrices have a significant amount of information which can be used to analyze various aspects of modal parameter estimation.

3.2.2 Principal (Virtual) Reference Analysis

The principal or virtual reference analysis involves the singular value decomposition of the G_{RR} matrix (Equation 4) at each frequency of the power spectra of the references. Since the singular vectors of such a decomposition are unitary, the singular values should all be of approximately the same size if each of the inputs or references is contributing to the excitation of the structure equally. If one of the singular values is much smaller at a particular frequency, it can be concluded that one of the inputs is not present or one of the inputs is correlated with the other input(s) at that frequency.

$$[G_{RR}(\omega)] = [U(\omega)] \times [\Sigma(\omega)] \times [V(\omega)]^H \quad (6)$$

$[\Sigma]$ in the above equation represents the singular values of the $[G_{RR}]$ matrix. If any of the singular values of the $[G_{RR}]$ matrix are zero or insignificant, then the $[G_{RR}]$ matrix is singular. Therefore, for a four input test like the road simulator, the $[G_{RR}]$ matrix should have four singular values at each frequency. (The number of significant singular values is the number of uncorrelated inputs). Figure 3-2 shows the principal reference plots for a three input case [15]. These principal reference curves are no longer linked to a specific physical exciter location due to the linear transformation involved. These curves are sometimes referred to as virtual references. Note that the overall difference in the three curves is typically one or two orders of magnitudes and there will be some fluctuation in the curves in the frequency region where there are lightly damped modes due to the exciter-structure interaction. Figure 3-3 shows a case where one exciter is not turned on (but the LVDT is still active and measuring displacement). The distinct change in amplitude of the third curve indicates that there are only two independent references contributing to the reference power spectra matrix.

Figure 3-2 : Principal or Virtual References

Figure 3-3 : Principal or Virtual References - One Exciter Off

3.3 Road Simulation Procedure

Road simulation essentially means recreating desired responses at predetermined locations on the vehicle. These locations may be either collocated or non-collocated with respect to the input locations. Collocated systems can be conveniently modeled by the prevalent method of using a linear estimate of the system, and minimal successive iterations of the drive file are needed to achieve the desired response. However, as the configuration becomes a non-collocated one, this approach may not yield the desired

results, chiefly due to the effects of the increased system dynamics on the measured response. As the path between the response measurement location and the input location gets more complicated, the accuracy of the simulation correspondingly decreases and excessive numbers of iterations may be required. In some configurations, no satisfactory solution may be possible.

The standard configurations used have four response locations corresponding to four inputs to the vehicle at the wheels. The input locations are thus, fixed. For the collocated case, the response locations are near the inputs (e.g. the spindles), whereas for the non-collocated case, the response locations may be elsewhere on the vehicle (e.g. the chassis frame).

To estimate the drive signals required at the input locations corresponding to a particular known response, an estimate of the system's dynamic characteristics is first required. The goal here is to determine a frequency domain relationship, between input drive signals (in the form of input voltage levels) and the output acceleration response (measured by the accelerometers), over the frequency band of interest. The estimate of the system's dynamic characteristics takes the form of the frequency response function (FRF) [20], which is a linear estimate of the system. The drive signals required to generate a specific response can be estimated using a convolution of the known (desired) response signals with the inverse system model derived from this FRF matrix, and transforming the calculated drive signals back into the time domain.

A schematic diagram of the control algorithm that is followed for the simulation process is shown below [21], [22].

Figure 3-4 : Control Algorithm for Simulation Process

where:

X_d -- Desired Response

X_a -- Measured Response

H^{-1} -- System Model

k -- Gain

F_n -- Previous Drive

F_{n+1} -- New (Corrected) Drive

The procedure for simulation of desired responses can be outlined as follows:

- Determine the system matrices using a white noise excitation over the frequency range of interest. As a linear system matrix is to be derived, a low level excitation is employed.

- Measure system response to the white noise excitation, at the desired response locations, either collocated or non-collocated with respect to the inputs.
- Generate system matrices based on the known input voltages (drive) and measured output accelerations (response). These system matrices are linear estimates of the system.
- Using the desired response at the measurement locations and the generated system matrices, calculate the drive signal required. This is the initial estimate of the drive signal.
- Measure the system response to the initial drive signal estimate. For a purely linear system, the measured response should exactly match the desired response. However, as most practical systems have varying degrees of nonlinearities, this is almost never true, and the desired response can only be obtained through successive iterations of the drive signal.
- Compare measured response with the desired response. The difference is the error, which needs to be reduced iteratively. Calculate the differential drive required to minimize the error, and add this to the initial estimate of the drive, to obtain a modified/corrected drive signal.
- Measure the system response to this modified drive signal.
- Repeat until the error lies within acceptable limits. The acceptable error may be a specified allowable maximum error over the entire measurement time for the time domain, or over the frequency range of interest for the frequency domain.
- Write out the final drive signal to file. This file contains the input signals required to be given to the system, in order to get the desired response. Input signals are in the form of time based input voltage levels that are to be fed to the four hydraulic actuators of the MTS 320 Road Simulator. Since this file can be played out any number of times, it can serve as a repeatable input to the system.

Figure 3-5 : Flowchart – Generating System Matrices

Figure 3-6 : Flowchart – Iteratively Generating Drive Files

4 Analytical Results

All the data was provided by UC-SDRL from their four axis road simulator database. The data primarily are response files generated during actual road tests on different surfaces for a number of vehicles. Data was consolidated for four different type of vehicles over four different types of road surfaces. Due to proprietary nature of data, the vehicles are denoted as Vehicle A, B, C and D. Similarly the road surfaces are designated as NVH 1, NVH 2, NVH 3 and NVH 4. Drive files for these 16 cases were generated using the MATLAB™ based SimTest® software and was provided by the database. The software follows the steps discussed in Chapter 3.3 to generate the drive files. These 16 drive files are the original drive files and are then utilized for further analysis and synthesis of uncorrelated drive files using a custom MATLAB™ script. The script is included in Appendix A.

4.1 Original Drive Files

4.1.1 Overview

A few illustrative examples of the original drive files estimated from response or ride files are as follows. These have been generated using the iterative process to recreate the responses of the actual road test. These have been generated using collocated responses at the axle closest to the wheels. Ride files having four responses were used to create these drive files. These drive files are the base for all the further analysis done.

Figure 4-1 : Original Drive File for Car A on Surface NVH 1

Figure 4-2 : Original Drive File for Car B on Surface NVH 2

Figure 4-3 : Original Drive File for Car C on Surface NVH 3

Figure 4-4 : Original Drive File for Car D on surface NVH 4

4.1.2 Correlation Due to Time Lag

Upon close inspection of the time domain data it can be seen clearly that the rear wheels lags the front wheels by a constant amount of time. This is expected as the drive files are numerically estimated from actual road test response or ride files. After the front wheels move over a particular road surface irregularity, the rear wheel will also be subjected to that irregularity, most of the times leading to the constant time lag and subsequently the correlation of the inputs estimated. The following figures illustrate the time lag between the front and rear wheels. Specific time intervals have been zoomed in to clearly show the illustration. The example is for Car D on road surface NVH 2. The time lag is measured to be 0.1367 sec. The overlap explains how the front and rear wheels line up almost perfectly after the time lag has been removed thus resulting in correlation of input references.

Figure 4-5 : Time Interval 7.5-8.5 seconds zoomed in from Original Drive File

Figure 4-6 : Time Interval 25-26 seconds zoomed in from Original Drive File

4.1.3 PCA to Yield Virtual References

The use of PCA using the SVD function in MATLAB™ further corroborates the high correlation of the input drive signals. The DSP (digital signal processing) parameters used for this analysis were:

- Sampling frequency- 204.8 Hz
- Window- Hanning
- Averaging- 50% Overlap
- Block Size- 1024

The original drive files generated from the SimTest® software were bandpass filtered from 4 to 40 Hz using a Butterworth filter. The filtering capability is built in the software. The limits are selected taking into consideration that the discomfort in the cabin is generally caused by road surface inputs in between these frequencies. Again, a few illustrative examples of principal or virtual references of the original drive files are provided as follows.

Figure 4-7 : Virtual References of Original Drive File for Car A on surface NVH 1

Figure 4-8 : Virtual References of Original Drive File for Car B on surface NVH 2

Figure 4-9 : Virtual References of Original Drive File for Car C on Surface NVH 3

Figure 4-10 : Virtual References of Original Drive File for Car D on surface NVH 4

4.2 Synthesized Drive Files

Synthesizing drive files was done using a custom made script on MATLAB™. Since the synthesized files needed to be comparable to the simulated original input files, different randomizations were applied to the simulated drive files. Physically that would correspond to the wheels moving independently relative to each other leading to removal of correlation between the inputs.

4.2.1 Fixed Time Shifting

The first randomization technique applied was shifting time by a fixed duration for all the four wheel inputs. But as expected, it was correlated as the time lag was still constant between the wheels. The illustrative examples for this randomization technique are below. The virtual references plot is also included.

Figure 4-11 : Fixed Shifted Synthesized Inputs with Virtual References for Car A on surface NVH 1

Figure 4-12 : Fixed Shifted Synthesized Inputs with Virtual References for Car B on surface NVH 2

Figure 4-13 : Fixed Shifted Synthesized Inputs with Virtual References for Car C on surface NVH 3

Figure 4-14 : Fixed Shifted Synthesized Inputs with Virtual References for Car D on surface NVH 4

4.2.2 Random Time Shifting

The second technique applied was moving the input to the wheels by random time durations to get over the constant time lag that the rear wheels have over the front wheels. The random time durations were applied to all wheels with them either moving forward or backward relative to each other. This technique helped in removal of the correlation between the inputs. The illustrative examples are as below.

Figure 4-15 : Random Shifted Synthesized Inputs with Virtual References for Car A on surface NVH 1

Figure 4-16 : Random Shifted Synthesized Inputs with Virtual References for Car B on surface NVH 2

Figure 4-17 : Random Shifted Synthesized Inputs with Virtual References for Car C on surface NVH 3

Figure 4-18 : Random Shifted Synthesized Inputs with Virtual References for Car D on surface NVH 4

4.2.3 Flipped Time History

Next technique applied was flipping the complete time history for the rear wheels as they are the ones with the constant time lag. This helps in keeping the front wheel inputs same as the simulated original drive files and removing the correlation between the inputs. Illustrative examples are as below with the virtual references also included.

Figure 4-19 : Flipped Synthesized Inputs with Virtual References for Car A on surface NVH 1

Figure 4-20 : Flipped Synthesized Inputs with Virtual References for Car B on surface NVH 2

Figure 4-21 : Flipped Synthesized Inputs with Virtual References for Car C on surface NVH 3

Figure 4-22 : Flipped Synthesized Inputs with Virtual References for Car D on surface NVH 4

4.2.4 Half Flipped Time History

One final technique used was a variation of the flipped technique used earlier. In this technique, only half of the time data for the rear wheels was flipped. This helps in keeping half of the rear wheel inputs same as the simulated original drive files. The front wheel drive files are same as the original drive files. Thus one more technique which helps in removing correlation between the inputs. The illustrative examples are as follows with the virtual references plot included.

Figure 4-23 : Half Flipped Synthesized Inputs with Virtual References for Car A on surface NVH 1

Figure 4-24 : Half Flipped Synthesized Inputs with Virtual References for Car B on surface NVH 2

Figure 4-25 : Half Flipped Synthesized Inputs with Virtual References for Car C on surface NVH 3

Figure 4-26 : Half Flipped Synthesized Inputs with Virtual References for Car D on surface NVH 4

4.3 Extended Drive Files

The synthesized drive files and original drive files were extended for use on the four axis road simulator as there was no opportunity to retake longer data records for these specific vehicles and road surfaces for this research. It was done to get a bigger time stream of data for a better analysis and to have enough confidence on the data acquired. The original drive files were extended end to end by 20 times the original length of the time data. The synthesized drive files were created using a conjunction of the three randomizing techniques discussed earlier which lead to uncorrelated inputs. Each of the 20 streams of original time data was divided into quarters and with each quarter having one of the techniques applied to it. They were then added end to end to get the big time stream of synthesized data. Even the selection of which technique was applied to one time stream was chosen randomly. This resulted in drive files which even though extended are same as the original drive files and the synthesized drive files in its properties.

Illustrative examples of the extended drive files are given below along with the virtual references.

Figure 4-27 : Extended Original and Synthesized Drive Files for Car A on surface NVH 1

Figure 4-28 : Extended Original and Synthesized Drive Files for Car B on surface NVH 2

Figure 4-29 : Extended Original and Synthesized Drive Files for Car C on surface NVH 3

Figure 4-30 : Extended Original and Synthesized Drive Files for Car D on surface NVH 4

4.4 Auto Power Comparison

The synthesized data was then checked for their auto power spectra. This was done to compare the energy content between the synthesized drive files with the original simulated drive files. It was imperative that they are similar so that it can safely be assumed that the synthesized drive files excite the vehicle quite similar to the original simulated drive files. The following illustrative examples show how they are equivalent and form the basis for further experimentation on the four axis road simulator using the analytical data.

Figure 4-31 : Auto Power Comparison for Car A on surface NVH 1

Figure 4-32 : Auto Power Comparison for Car B on surface NVH 2

Figure 4-33 : Auto Power Comparison for Car C on surface NVH 3

Figure 4-34 : Auto Power Comparison for Car D on surface NVH 4

5 Experimental Results

5.1 Data Acquisition

A readily available pick-up truck frame was used for testing on the four axis road simulator. Responses were measured at the axle closest to wheel hub connection for all the four wheels to simulate collocated response measurement. The inputs were given to the actuators using a combination of softwares. The X-Modal software (software internal to UC-SDRL) allows the user to input a drive file using a commercially available PC. The drive files need to be in the MATLAB™ .mat format. The X-Modal software interfaced with HP VXi Mainframe having four VXi VT1435 8 channel cards. Each card had one input interfaced with the MTS 320 road simulator controller. The FlexTest II® software interfaces with the MTS 320 controller to enable the user to control the four axis road simulator using a commercially available PC.

Figure 5-1 : Experimental Test Setup using Pick-up Truck Frame

Figure 5-2 : HP Mainframe with four VXI VT1435 8 Channel Cards with one Input each

The data acquisition was done using X-Modal software interfaced with one of the four VXI VT1435 eight channel cards. Four channels on the card were used to measure the actual input to the pick-up truck frame by using the measurements from the LVDT fitted in the actuators. The other four channels were used to measure response from the PCB tear drop accelerometers attached on the axle of the frame. The sampling frequency for the analytical results was 204.8 Hz which was governed by the F_{samp} of the response data in the information bank at UC-SDRL. X-Modal has the capability to resample this input data to match the F_{samp} available for data acquisition. The digital signal processing (DSP) parameters used for data acquisition are as follows.

- F_{samp} - 256 Hz, F_{max} - 100 Hz, frequency lines- 500 Hz
- Window- Hanning
- 75 ensembles, each 5 seconds

- Accelerometer calibration data: FL- 98.4 mV/g, FR- 100.6 mV/g, RL- 99.4 mV/g & RR- 103.2 mV/g
- LVDT calibration data: All actuators- 3.34 V/in

5.2 Extended Drive Files

The drive files which were extended were used as an input to the actuators with the data acquisition system in place. The data acquired was auto power data for the reference and response channels. MATLAB™ based script was used to analyze these measurements and is included in Appendix A. The virtual references of the original drive files and synthesized drive files showed expected results. The original drive files were still correlated and the synthesized drive files had no correlation of inputs. Illustrative examples of the virtual references of the measured data from the four axis road simulator are as below.

Figure 5-3 : Virtual References of Measured Data for Car A on surface NVH 1

Figure 5-4 : Virtual References of Measured Data for Car B on surface NVH 2

Figure 5-5 : Virtual References of Measured Data for Car C on surface NVH 3

Figure 5-6 : Virtual References of Measured Data for Car D on surface NVH 4

5.3 Auto Power Comparison

Again it was imperative to have equivalent auto power from the synthesized drive files and original drive files as measured from the actuators of the four axis road simulator with the pick-up truck frame. The energy content exciting the frame should be quite similar so that the subsequent analysis provides results which are equivalent to what are expected from an actual road test. Since the original drive files are simulated from actual road test responses. Illustrative examples are provided below clearly showing that the auto power of the measured data, both for original drive files and synthesized drive files are equivalent.

Figure 5-7 : Auto Power Comparison of Measured Data for Car A on surface NVH 1

Figure 5-8 : Auto Power Comparison of Measured Data for Car B on surface NVH 2

Figure 5-9 : Auto Power Comparison of Measured Data for Car C on surface NVH 3

Figure 5-10 : Auto Power Comparison of Measured Data for Car D on surface NVH 4

6 Summary

6.1 Conclusions

Recreating response obtained during actual road test using simulated input signals has a lot advantages. These simulated input signals or drive files will be quite useful in durability, NVH testing of vehicles as the vehicle is exposed to the actual testing conditions with all the limitations of an actual road test are eliminated. But to do any kind of analysis, the input has to suffice the condition of not being correlated. The MIMO transmissibility is one such effective analysis tool which relies on the condition of inputs not being correlated.

More often than not and as seen from the analytical results, the drive files are correlated. There is a constant time lag between rear and front wheels of a vehicle moving over a particular surface during an actual road test. The simulated inputs are generated using the actual road test responses which have the constant time lag effect. This translates to correlation between the simulated inputs.

The MATLAB™ based interface which has been developed helps to synthesize drive files using different randomization techniques leading to removal of correlation of these input signals. The randomization technique are mostly aimed at removing the time lag effect. It effectively makes the wheels move differently relative to each other such that a particular irregularity of a surface which was acting on the wheels with a constant time lag now acts on the wheel without the constant time lag.

The randomization techniques used in conjunction with each other provides with synthesized drive files which now are uncorrelated. The uncorrelation between the inputs was corroborated by the experimental results. Furthermore these signals carry the equivalent energy as the original drive files as seen from the analytical as well as experimental results. These signals can now be directly used for analysis as the

condition of uncorrelation has been met and it will excite a structure with an equivalent energy content as the original drive files.

6.2 Recommendations for Future Work

Response files obtained from the UC-SDRL database are only applicable to the vehicle on which the actual road test was performed. So, the drive files generated using these response files correspond only to that particular vehicle. Moreover, the synthesized drive files use the original drive files as a base. So, any experimental measurements done using these synthesized drive files can be used for meaningful analysis only if the actual vehicle on which the actual road test was performed are used on the four axis road simulator. Thus, the application of these methods to analyze the vehicles using MIMO transmissibility was not possible due to the unavailability of the actual vehicles.

The application of these methods can also be useful in estimating FRFs if the input forces acting on the wheels could be measured. Availability of load plates or load cells during testing are necessary for measuring the input forces. Development of an interface which does start to end on the MIMO transmissibility and FRF measurements with the using the interface provided by this research looks like having some commercial viability.

7 References

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Appendices

Appendix A – MATLAB scripts

A.1 Synthesis of Different Drive Files and its Virtual References

```
function [] = Virtual_Reference(xw)
close all;% function needs the original input drive file for further analysis
choice=menu('Choose the time history','Provided Time History','Flipped time
history for rear wheels','Fixed Shifted Time History','Randomly Shifted Time
History','Half Flipped time history for rear wheels','Extended Time
History','Fixed Shifted Extended Time History','Randomly Shifted Extended
Time History');
if choice==2
% time flipping of input forces
xw1=(xw(:,1));xw2=(xw(:,2));
xw3=flipud(xw(:,3));xw4=flipud(xw(:,4));
xw=[xw1 xw2 xw3 xw4];
elseif choice==1
    fprintf('You chose provided time history \n');
elseif choice==3
% fixed Time Shifting of input forces
yw1=zeros(randi(200),1);yw2=zeros(randi(200),1);yw3=zeros(randi(200),1);yw4=z
eros(randi(200),1);
xw1=[yw1;(xw(1:length(xw)-length(yw1),1))];xw2=[yw2; xw(1:length(xw)-
length(yw2),2)];
xw3=[yw3; xw(1:length(xw)-length(yw3),3)];xw4=[yw4; xw(1:length(xw)-
length(yw4),4)];
xw=[xw1 xw2 xw3 xw4];
elseif choice==4
% Randomly time shifting of input forces
yw11=randi([-200 200],1);yw12=randi([-200 200],1);yw13=randi([-200
200],1);yw14=randi([-200 200],1);
yw21=randi([-200 200],1);yw22=randi([-200 200],1);yw23=randi([-200
200],1);yw24=randi([-200 200],1);
yw31=randi([-200 200],1);yw32=randi([-200 200],1);yw33=randi([-200
200],1);yw34=randi([-200 200],1);
yw41=randi([-200 200],1);yw42=randi([-200 200],1);yw43=randi([-200
200],1);yw44=randi([-200 200],1);
% Shifting first quarter block
if yw11<=0
    xw11=xw(-1*yw11+1:length(xw)/4-yw11,1);
else
    xw11=[zeros(yw11,1);xw(1:(length(xw)/4)-yw11,1)];
end
if yw21<=0
    xw21=xw(-1*yw21+1:length(xw)/4-yw21,2);
else
    xw21=[zeros(yw21,1);xw(1:(length(xw)/4)-yw21,2)];
end
if yw31<=0
    xw31=xw(-1*yw31+1:length(xw)/4-yw31,3);
```



```

xw3=[xw(:,3);xw(:,3);xw(:,3);xw(:,3);xw(:,3);xw(:,3);xw(:,3);xw(:,3);xw(:,3);
xw(:,3);xw(:,3);xw(:,3);xw(:,3);xw(:,3);xw(:,3);xw(:,3);xw(:,3);xw(:,3);
3);xw(:,3)];

xw4=[xw(:,4);xw(:,4);xw(:,4);xw(:,4);xw(:,4);xw(:,4);xw(:,4);xw(:,4);xw(:,4);
xw(:,4);xw(:,4);xw(:,4);xw(:,4);xw(:,4);xw(:,4);xw(:,4);xw(:,4);xw(:,4);
4);xw(:,4)];
xw=[xw1 xw2 xw3 xw4];
elseif choice==7
% fixed shifted Extended Provided Time history
% 1st quarter
for k=1:20
    sell=randi(4);
    if sell==1
        yw11=zeros(randi(200),1);
        xw11(:,k)=[yw11;xw(1:length(xw)/4-length(yw11),1)];
        xw21(:,k)=[yw11;xw(1:length(xw)/4-length(yw11),2)];
        xw31(:,k)=[yw11;xw(1:length(xw)/4-length(yw11),3)];
        xw41(:,k)=[yw11;xw(1:length(xw)/4-length(yw11),4)];
    elseif sell==2
        yw11=randi([-200 -1],1);
        xw11(:,k)=xw(-yw11+1:length(xw)/4-yw11,1);
        xw21(:,k)=xw(-yw11+1:length(xw)/4-yw11,2);
        xw31(:,k)=xw(-yw11+1:length(xw)/4-yw11,3);
        xw41(:,k)=xw(-yw11+1:length(xw)/4-yw11,4);
    elseif sell==3
        xw11(:,k)=flipud(xw(1:length(xw)/4,1));
        xw21(:,k)=flipud(xw(1:length(xw)/4,2));
        xw31(:,k)=flipud(xw(1:length(xw)/4,3));
        xw41(:,k)=flipud(xw(1:length(xw)/4,4));
    elseif sell==4
        xw11(:,k)=xw(1:length(xw)/4,1);
        xw21(:,k)=xw(1:length(xw)/4,2);
        xw31(:,k)=xw(1:length(xw)/4,3);
        xw41(:,k)=xw(1:length(xw)/4,4);
    end
end
% 2nd Quarter
for k=1:20
    sel2=randi(4);
    if sel2==1
        yw12=randi(200);
        xw12(:,k)=xw(length(xw)/4-yw12+1:length(xw)/2-yw12,1);
        xw22(:,k)=xw(length(xw)/4-yw12+1:length(xw)/2-yw12,2);
        xw32(:,k)=xw(length(xw)/4-yw12+1:length(xw)/2-yw12,3);
        xw42(:,k)=xw(length(xw)/4-yw12+1:length(xw)/2-yw12,4);
    elseif sel2==2
        yw12=randi([-200 -1],1);
        xw12(:,k)=xw(length(xw)/4-yw12+1:length(xw)/2-yw12,1);
        xw22(:,k)=xw(length(xw)/4-yw12+1:length(xw)/2-yw12,2);
        xw32(:,k)=xw(length(xw)/4-yw12+1:length(xw)/2-yw12,3);
        xw42(:,k)=xw(length(xw)/4-yw12+1:length(xw)/2-yw12,4);
    elseif sel2==3
        xw12(:,k)=flipud(xw(length(xw)/4+1:length(xw)/2,1));
        xw22(:,k)=flipud(xw(length(xw)/4+1:length(xw)/2,2));
        xw32(:,k)=flipud(xw(length(xw)/4+1:length(xw)/2,3));

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```

        xw42(:,k)=flipud(xw(length(xw)/4+1:length(xw)/2,4));
elseif sel2==4
    xw12(:,k)=xw(length(xw)/4+1:length(xw)/2,1);
    xw22(:,k)=xw(length(xw)/4+1:length(xw)/2,2);
    xw32(:,k)=xw(length(xw)/4+1:length(xw)/2,3);
    xw42(:,k)=xw(length(xw)/4+1:length(xw)/2,4);
end
end
% 3rd Quarter
for k=1:20
    sel3=randi(4);
    if sel3==1
        yw13=randi(200);
        xw13(:,k)=xw(length(xw)/2-yw13+1:3*length(xw)/4-yw13,1);
        xw23(:,k)=xw(length(xw)/2-yw13+1:3*length(xw)/4-yw13,2);
        xw33(:,k)=xw(length(xw)/2-yw13+1:3*length(xw)/4-yw13,3);
        xw43(:,k)=xw(length(xw)/2-yw13+1:3*length(xw)/4-yw13,4);
    elseif sel3==2
        yw13=randi([-200 -1],1);
        xw13(:,k)=xw(length(xw)/2-yw13+1:3*length(xw)/4-yw13,1);
        xw23(:,k)=xw(length(xw)/2-yw13+1:3*length(xw)/4-yw13,2);
        xw33(:,k)=xw(length(xw)/2-yw13+1:3*length(xw)/4-yw13,3);
        xw43(:,k)=xw(length(xw)/2-yw13+1:3*length(xw)/4-yw13,4);
    elseif sel3==3
        xw13(:,k)=flipud(xw(length(xw)/2+1:3*length(xw)/4,1));
        xw23(:,k)=flipud(xw(length(xw)/2+1:3*length(xw)/4,2));
        xw33(:,k)=flipud(xw(length(xw)/2+1:3*length(xw)/4,3));
        xw43(:,k)=flipud(xw(length(xw)/2+1:3*length(xw)/4,4));
    elseif sel3==4
        xw13(:,k)=xw(length(xw)/2+1:3*length(xw)/4,1);
        xw23(:,k)=xw(length(xw)/2+1:3*length(xw)/4,2);
        xw33(:,k)=xw(length(xw)/2+1:3*length(xw)/4,3);
        xw43(:,k)=xw(length(xw)/2+1:3*length(xw)/4,4);
    end
end
% 4th Quarter
for k=1:20
    sel4=randi(4);
    if sel4==1
        yw14=randi(200);
        xw14(:,k)=xw(3*length(xw)/4-yw14+1:length(xw)-yw14,1); %#ok<*AGROW>
        xw24(:,k)=xw(3*length(xw)/4-yw14+1:length(xw)-yw14,2);
        xw34(:,k)=xw(3*length(xw)/4-yw14+1:length(xw)-yw14,3);
        xw44(:,k)=xw(3*length(xw)/4-yw14+1:length(xw)-yw14,4);
    elseif sel4==2
        yw14=randi([-200 -1],1);
        xw14(:,k)=[xw(3*length(xw)/4-yw14+1:length(xw),1);zeros(-yw14,1)];
        xw24(:,k)=[xw(3*length(xw)/4-yw14+1:length(xw),2);zeros(-yw14,1)];
        xw34(:,k)=[xw(3*length(xw)/4-yw14+1:length(xw),3);zeros(-yw14,1)];
        xw44(:,k)=[xw(3*length(xw)/4-yw14+1:length(xw),4);zeros(-yw14,1)];
    elseif sel4==3
        xw14(:,k)=flipud(xw(3*length(xw)/4+1:length(xw),1));
        xw24(:,k)=flipud(xw(3*length(xw)/4+1:length(xw),2));
        xw34(:,k)=flipud(xw(3*length(xw)/4+1:length(xw),3));
        xw44(:,k)=flipud(xw(3*length(xw)/4+1:length(xw),4));
    elseif sel4==4
        xw14(:,k)=xw(3*length(xw)/4+1:length(xw),1);

```



```

        xw24(:,k)=xw(3*length(xw)/4+1:length(xw),2);
        xw34(:,k)=xw(3*length(xw)/4+1:length(xw),3);
        xw44(:,k)=xw(3*length(xw)/4+1:length(xw),4);
    end
end

xwa=[xw11;xw12;xw13;xw14];xwb=[xw21;xw22;xw23;xw24];xwc=[xw31;xw32;xw33;xw34];
xwd=[xw41;xw42;xw43;xw44];

xw1=[xwa(:,1);xwa(:,2);xwa(:,3);xwa(:,4);xwa(:,5);xwa(:,6);xwa(:,7);xwa(:,8);
xwa(:,9);xwa(:,10);xwa(:,11);xwa(:,12);xwa(:,13);xwa(:,14);xwa(:,15);xwa(:,16);
xwa(:,17);xwa(:,18);xwa(:,19);xwa(:,20)];

xw2=[xwb(:,1);xwb(:,2);xwb(:,3);xwb(:,4);xwb(:,5);xwb(:,6);xwb(:,7);xwb(:,8);
xwb(:,9);xwb(:,10);xwb(:,11);xwb(:,12);xwb(:,13);xwb(:,14);xwb(:,15);xwb(:,16);
xwb(:,17);xwb(:,18);xwb(:,19);xwb(:,20)];

xw3=[xwc(:,1);xwc(:,2);xwc(:,3);xwc(:,4);xwc(:,5);xwc(:,6);xwc(:,7);xwc(:,8);
xwc(:,9);xwc(:,10);xwc(:,11);xwc(:,12);xwc(:,13);xwc(:,14);xwc(:,15);xwc(:,16);
xwc(:,17);xwc(:,18);xwc(:,19);xwc(:,20)];

xw4=[xwd(:,1);xwd(:,2);xwd(:,3);xwd(:,4);xwd(:,5);xwd(:,6);xwd(:,7);xwd(:,8);
xwd(:,9);xwd(:,10);xwd(:,11);xwd(:,12);xwd(:,13);xwd(:,14);xwd(:,15);xwd(:,16);
xwd(:,17);xwd(:,18);xwd(:,19);xwd(:,20)];
xw=[xw1 xw2 xw3 xw4];
elseif choice==8
% randomly extended time history
% 1st quarter
for k=1:20
    sell=randi(4);
    if sell==1

yw11=zeros(randi(200),1);yw21=zeros(randi(200),1);yw31=zeros(randi(200),1);yw
41=zeros(randi(200),1);
        xw11(:,k)=[yw11;xw(1:length(xw)/4-length(yw11),1)];
        xw21(:,k)=[yw21;xw(1:length(xw)/4-length(yw21),2)];
        xw31(:,k)=[yw31;xw(1:length(xw)/4-length(yw31),3)];
        xw41(:,k)=[yw41;xw(1:length(xw)/4-length(yw41),4)];
    elseif sell==2
        yw11=randi([-200 -1],1);yw21=randi([-200 -1],1);yw31=randi([-200 -
1],1);yw41=randi([-200 -1],1);
        xw11(:,k)=xw(-yw11+1:length(xw)/4-yw11,1);
        xw21(:,k)=xw(-yw21+1:length(xw)/4-yw21,2);
        xw31(:,k)=xw(-yw31+1:length(xw)/4-yw31,3);
        xw41(:,k)=xw(-yw41+1:length(xw)/4-yw41,4);
    elseif sell==3
        xw11(:,k)=xw(1:length(xw)/4,1);
        xw21(:,k)=xw(1:length(xw)/4,2);
        xw31(:,k)=flipud(xw(1:length(xw)/4,3));
        xw41(:,k)=flipud(xw(1:length(xw)/4,4));
    elseif sell==4
        xw11(:,k)=flipud(xw(1:length(xw)/4,1));
        xw21(:,k)=flipud(xw(1:length(xw)/4,2));
        xw31(:,k)=xw(1:length(xw)/4,3);
        xw41(:,k)=xw(1:length(xw)/4,4);
    end
end
end

```

```

% 2nd Quarter
for k=1:20
    sel2=randi(4);
    if sel2==1
        yw12=randi(200);yw22=randi(200);yw32=randi(200);yw42=randi(200);
        xw12(:,k)=xw(length(xw)/4-yw12+1:length(xw)/2-yw12,1);
        xw22(:,k)=xw(length(xw)/4-yw22+1:length(xw)/2-yw22,2);
        xw32(:,k)=xw(length(xw)/4-yw32+1:length(xw)/2-yw32,3);
        xw42(:,k)=xw(length(xw)/4-yw42+1:length(xw)/2-yw42,4);
    elseif sel2==2
        yw12=randi([-200 -1],1);yw22=randi([-200 -1],1);yw32=randi([-200 -
1],1);yw42=randi([-200 -1],1);
        xw12(:,k)=xw(length(xw)/4-yw12+1:length(xw)/2-yw12,1);
        xw22(:,k)=xw(length(xw)/4-yw22+1:length(xw)/2-yw22,2);
        xw32(:,k)=xw(length(xw)/4-yw32+1:length(xw)/2-yw32,3);
        xw42(:,k)=xw(length(xw)/4-yw42+1:length(xw)/2-yw42,4);
    elseif sel2==3
        xw12(:,k)=xw(length(xw)/4+1:length(xw)/2,1);
        xw22(:,k)=xw(length(xw)/4+1:length(xw)/2,2);
        xw32(:,k)=flipud(xw(length(xw)/4+1:length(xw)/2,3));
        xw42(:,k)=flipud(xw(length(xw)/4+1:length(xw)/2,4));
    elseif sel2==4
        xw12(:,k)=flipud(xw(length(xw)/4+1:length(xw)/2,1));
        xw22(:,k)=flipud(xw(length(xw)/4+1:length(xw)/2,2));
        xw32(:,k)=xw(length(xw)/4+1:length(xw)/2,3);
        xw42(:,k)=xw(length(xw)/4+1:length(xw)/2,4);
    end
end
% 3rd Quarter
for k=1:20
    sel3=randi(4);
    if sel3==1
        yw13=randi(200);yw23=randi(200);yw33=randi(200);yw43=randi(200);
        xw13(:,k)=xw(length(xw)/2-yw13+1:3*length(xw)/4-yw13,1);
        xw23(:,k)=xw(length(xw)/2-yw23+1:3*length(xw)/4-yw23,2);
        xw33(:,k)=xw(length(xw)/2-yw33+1:3*length(xw)/4-yw33,3);
        xw43(:,k)=xw(length(xw)/2-yw43+1:3*length(xw)/4-yw43,4);
    elseif sel3==2
        yw13=randi([-200 -1],1);yw23=randi([-200 -1],1);yw33=randi([-200 -
1],1);yw43=randi([-200 -1],1);
        xw13(:,k)=xw(length(xw)/2-yw13+1:3*length(xw)/4-yw13,1);
        xw23(:,k)=xw(length(xw)/2-yw23+1:3*length(xw)/4-yw23,2);
        xw33(:,k)=xw(length(xw)/2-yw33+1:3*length(xw)/4-yw33,3);
        xw43(:,k)=xw(length(xw)/2-yw43+1:3*length(xw)/4-yw43,4);
    elseif sel3==3
        xw13(:,k)=xw(length(xw)/2+1:3*length(xw)/4,1);
        xw23(:,k)=xw(length(xw)/2+1:3*length(xw)/4,2);
        xw33(:,k)=flipud(xw(length(xw)/2+1:3*length(xw)/4,3));
        xw43(:,k)=flipud(xw(length(xw)/2+1:3*length(xw)/4,4));
    elseif sel3==4
        xw13(:,k)=flipud(xw(length(xw)/2+1:3*length(xw)/4,1));
        xw23(:,k)=flipud(xw(length(xw)/2+1:3*length(xw)/4,2));
        xw33(:,k)=xw(length(xw)/2+1:3*length(xw)/4,3);
        xw43(:,k)=xw(length(xw)/2+1:3*length(xw)/4,4);
    end
end
% 4th Quarter

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for k=1:20
    sel4=randi(4);
    if sel4==1
        yw14=randi(200);yw24=randi(200);yw34=randi(200);yw44=randi(200);
        xw14(:,k)=xw(3*length(xw)/4-yw14+1:length(xw)-yw14,1);
        xw24(:,k)=xw(3*length(xw)/4-yw24+1:length(xw)-yw24,2);
        xw34(:,k)=xw(3*length(xw)/4-yw34+1:length(xw)-yw34,3);
        xw44(:,k)=xw(3*length(xw)/4-yw44+1:length(xw)-yw44,4);
    elseif sel4==2
        yw14=randi([-200 -1],1);yw24=randi([-200 -1],1);yw34=randi([-200 -
1],1);yw44=randi([-200 -1],1);
        xw14(:,k)=[xw(3*length(xw)/4-yw14+1:length(xw),1);zeros(-yw14,1)];
        xw24(:,k)=[xw(3*length(xw)/4-yw24+1:length(xw),2);zeros(-yw24,1)];
        xw34(:,k)=[xw(3*length(xw)/4-yw34+1:length(xw),3);zeros(-yw34,1)];
        xw44(:,k)=[xw(3*length(xw)/4-yw44+1:length(xw),4);zeros(-yw44,1)];
    elseif sel4==3
        xw14(:,k)=xw(3*length(xw)/4+1:length(xw),1);
        xw24(:,k)=xw(3*length(xw)/4+1:length(xw),2);
        xw34(:,k)=flipud(xw(3*length(xw)/4+1:length(xw),3));
        xw44(:,k)=flipud(xw(3*length(xw)/4+1:length(xw),4));
    elseif sel4==4
        xw14(:,k)=flipud(xw(3*length(xw)/4+1:length(xw),1));
        xw24(:,k)=flipud(xw(3*length(xw)/4+1:length(xw),2));
        xw34(:,k)=xw(3*length(xw)/4+1:length(xw),3);
        xw44(:,k)=xw(3*length(xw)/4+1:length(xw),4);
    end
end

xwa=[xw11;xw12;xw13;xw14];xwb=[xw21;xw22;xw23;xw24];xwc=[xw31;xw32;xw33;xw34];
xwd=[xw41;xw42;xw43;xw44];

xw1=[xwa(:,1);xwa(:,2);xwa(:,3);xwa(:,4);xwa(:,5);xwa(:,6);xwa(:,7);xwa(:,8);
xwa(:,9);xwa(:,10);xwa(:,11);xwa(:,12);xwa(:,13);xwa(:,14);xwa(:,15);xwa(:,16)
);xwa(:,17);xwa(:,18);xwa(:,19);xwa(:,20)];

xw2=[xwb(:,1);xwb(:,2);xwb(:,3);xwb(:,4);xwb(:,5);xwb(:,6);xwb(:,7);xwb(:,8);
xwb(:,9);xwb(:,10);xwb(:,11);xwb(:,12);xwb(:,13);xwb(:,14);xwb(:,15);xwb(:,16)
);xwb(:,17);xwb(:,18);xwb(:,19);xwb(:,20)];

xw3=[xwc(:,1);xwc(:,2);xwc(:,3);xwc(:,4);xwc(:,5);xwc(:,6);xwc(:,7);xwc(:,8);
xwc(:,9);xwc(:,10);xwc(:,11);xwc(:,12);xwc(:,13);xwc(:,14);xwc(:,15);xwc(:,16)
);xwc(:,17);xwc(:,18);xwc(:,19);xwc(:,20)];

xw4=[xwd(:,1);xwd(:,2);xwd(:,3);xwd(:,4);xwd(:,5);xwd(:,6);xwd(:,7);xwd(:,8);
xwd(:,9);xwd(:,10);xwd(:,11);xwd(:,12);xwd(:,13);xwd(:,14);xwd(:,15);xwd(:,16)
);xwd(:,17);xwd(:,18);xwd(:,19);xwd(:,20)];
xw=[xw1 xw2 xw3 xw4];
end
% Plotting of drive files in time domain
dt=1/204.8;% This is sampling frequency dependent, please change accordingly
T=length(xw)*dt;
dT=0:dt:T-dt;
figure(1)
subplot(2,2,1)
plot(dT,xw(:,1));
xlabel('Time (Sec)'),ylabel('Amplitude (in)'),title('Drive on Front Left')
subplot(2,2,2)

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plot(dT,xw(:,2));
xlabel('Time (Sec)'),ylabel('Amplitude (in)'),title('Drive on Front Right')
subplot(2,2,3)
plot(dT,xw(:,3));
xlabel('Time (Sec)'),ylabel('Amplitude (in)'),title('Drive on Rear Left')
subplot(2,2,4)
plot(dT,xw(:,4));
xlabel('Time (Sec)'),ylabel('Amplitude (in)'),title('Drive on Rear Right')
% Plotting of drive files in frequency domain
a0=1;a1=0.5; pi_x=2;P110=(1/pi_x)*(a0-
(2*a1*cos(2*pi*(dT/T))));W=[P110;P110;P110;P110].';
Fnyq=1/(2*dt);
df=1/T;
w=0:df:Fnyq;
xw1=W.*xw;
Xw1=(fft(xw1)*pi_x)/length(xw1);
Xw2=2*Xw1(2:Fnyq*T,:);
Xw=[Xw1(1,:); Xw2; Xw1(Fnyq*T+1,:)];
figure(2)
subplot(2,2,1)
semilogy(w((4/df):(40/df)),abs(Xw((4/df):(40/df),1)));
xlabel('Frequency (Hz)'),ylabel('Amplitude (in)'),title('Drive on Front Left
in freq domain')
subplot(2,2,2)
semilogy(w((4/df):(40/df)),abs(Xw((4/df):(40/df),2)));
xlabel('Frequency (Hz)'),ylabel('Amplitude (in)'),title('Drive on Front Right
in freq domain')
subplot(2,2,3)
semilogy(w((4/df):(40/df)),abs(Xw((4/df):(40/df),3)));
xlabel('Frequency (Hz)'),ylabel('Amplitude (in)'),title('Drive on Rear Left
in freq domain')
subplot(2,2,4)
semilogy(w((4/df):(40/df)),abs(Xw((4/df):(40/df),4)));
xlabel('Frequency (Hz)'),ylabel('Amplitude (in)'),title('Drive on Rear Right
in freq domain')
% Plotting of autopower spectrum of drive files
BS=1024; % BS can also be chosen as needed
AvgT=dt*BS;AvgdT=(0:BS-1)*dt;
a0=1;a1=0.5; pi_x=2;P110=(1/pi_x)*(a0-
(2*a1*cos(2*pi*(AvgdT/AvgT))));W=[P110;P110;P110;P110].';
w1=0:1/AvgT:Fnyq;
GFF11avg=zeros(BS/2+1,1);GFF12avg=zeros(BS/2+1,1);GFF13avg=zeros(BS/2+1,1);GF
F14avg=zeros(BS/2+1,1);
GFF21avg=zeros(BS/2+1,1);GFF22avg=zeros(BS/2+1,1);GFF23avg=zeros(BS/2+1,1);GF
F24avg=zeros(BS/2+1,1);
GFF31avg=zeros(BS/2+1,1);GFF32avg=zeros(BS/2+1,1);GFF33avg=zeros(BS/2+1,1);GF
F34avg=zeros(BS/2+1,1);
GFF41avg=zeros(BS/2+1,1);GFF42avg=zeros(BS/2+1,1);GFF43avg=zeros(BS/2+1,1);GF
F44avg=zeros(BS/2+1,1);
for N=1:BS/2:length(xw)-BS/2
    F1=W.*xw(N:N+BS-1,:);
    F1=(fft(F1)*pi_x)/BS;
    F3=2*F1(2:Fnyq*AvgT,:);
    F1=[F1(1,:); F3; F1(Fnyq*AvgT+1,:)];
    % Auto power in all force data channels

```

```

GFF11=F1(:,1).*conj(F1(:,1));GFF12=F1(:,1).*conj(F1(:,2));GFF13=F1(:,1).*conj
(F1(:,3));GFF14=F1(:,1).*conj(F1(:,4));

GFF21=F1(:,2).*conj(F1(:,1));GFF22=F1(:,2).*conj(F1(:,2));GFF23=F1(:,2).*conj
(F1(:,3));GFF24=F1(:,2).*conj(F1(:,4));

GFF31=F1(:,3).*conj(F1(:,1));GFF32=F1(:,3).*conj(F1(:,2));GFF33=F1(:,3).*conj
(F1(:,3));GFF34=F1(:,3).*conj(F1(:,4));

GFF41=F1(:,4).*conj(F1(:,1));GFF42=F1(:,4).*conj(F1(:,2));GFF43=F1(:,4).*conj
(F1(:,3));GFF44=F1(:,4).*conj(F1(:,4));

GFF11avg=GFF11avg+GFF11;GFF12avg=GFF12avg+GFF12;GFF13avg=GFF13avg+GFF13;GFF14
avg=GFF14avg+GFF14;

GFF21avg=GFF21avg+GFF21;GFF22avg=GFF22avg+GFF22;GFF23avg=GFF23avg+GFF23;GFF24
avg=GFF24avg+GFF24;

GFF31avg=GFF31avg+GFF31;GFF32avg=GFF32avg+GFF32;GFF33avg=GFF33avg+GFF33;GFF34
avg=GFF34avg+GFF34;

GFF41avg=GFF41avg+GFF41;GFF42avg=GFF42avg+GFF42;GFF43avg=GFF43avg+GFF43;GFF44
avg=GFF44avg+GFF44;
end
GFF11avg=GFF11avg/((length(xw)/(BS/2))-
1);GFF12avg=GFF12avg/((length(xw)/(BS/2))-
1);GFF13avg=GFF13avg/((length(xw)/(BS/2))-
1);GFF14avg=GFF14avg/((length(xw)/(BS/2))-1);
GFF21avg=GFF21avg/((length(xw)/(BS/2))-
1);GFF22avg=GFF22avg/((length(xw)/(BS/2))-
1);GFF23avg=GFF23avg/((length(xw)/(BS/2))-
1);GFF24avg=GFF24avg/((length(xw)/(BS/2))-1);
GFF31avg=GFF31avg/((length(xw)/(BS/2))-
1);GFF32avg=GFF32avg/((length(xw)/(BS/2))-
1);GFF33avg=GFF33avg/((length(xw)/(BS/2))-
1);GFF34avg=GFF34avg/((length(xw)/(BS/2))-1);
GFF41avg=GFF41avg/((length(xw)/(BS/2))-
1);GFF42avg=GFF42avg/((length(xw)/(BS/2))-
1);GFF43avg=GFF43avg/((length(xw)/(BS/2))-
1);GFF44avg=GFF44avg/((length(xw)/(BS/2))-1);
figure(3)
subplot(2,2,1)
semilogy(w1(4*AvgT+1:40*AvgT+1),(GFF11avg(4*AvgT+1:40*AvgT+1)));
xlabel('Frequency (Hz)'),ylabel('Power (in^2)'),title('Autopower on Front
Left F11')
subplot(2,2,2)
semilogy(w1(4*AvgT+1:40*AvgT+1),(GFF22avg(4*AvgT+1:40*AvgT+1)));
xlabel('Frequency (Hz)'),ylabel('Power (in^2)'),title('Autopower on Front
Right F22')
subplot(2,2,3)
semilogy(w1(4*AvgT+1:40*AvgT+1),(GFF33avg(4*AvgT+1:40*AvgT+1)));
xlabel('Frequency (Hz)'),ylabel('Power (in^2)'),title('Autopower on Rear Left
F33')
subplot(2,2,4)
semilogy(w1(4*AvgT+1:40*AvgT+1),(GFF44avg(4*AvgT+1:40*AvgT+1)));

```

```

xlabel('Frequency (Hz)'),ylabel('Power (in^2)'),title('Autopower on Rear
Right F44')
% Plotting of virtual forces using PCA
for i=1:length(GFF11avg)
    GFF=[GFF11avg(i) GFF12avg(i) GFF13avg(i) GFF14avg(i);GFF21avg(i)
GFF22avg(i) GFF23avg(i) GFF24avg(i);GFF31avg(i) GFF32avg(i) GFF33avg(i)
GFF34avg(i);GFF41avg(i) GFF42avg(i) GFF43avg(i) GFF44avg(i)];
    GFFdecomp=sort(svd(GFF));
    GFF11PF(i)=GFFdecomp(1); %#ok<*SAGROW>
    GFF22PF(i)=GFFdecomp(2);
    GFF33PF(i)=GFFdecomp(3);
    GFF44PF(i)=GFFdecomp(4);
end
figure(4)
semilogy(w1(4*AvgT+1:40*AvgT+1),(GFF11PF(4*AvgT+1:40*AvgT+1)),w1(4*AvgT+1:40*
AvgT+1),(GFF22PF(4*AvgT+1:40*AvgT+1)),w1(4*AvgT+1:40*AvgT+1),(GFF33PF(4*AvgT+
1:40*AvgT+1)),w1(4*AvgT+1:40*AvgT+1),(GFF44PF(4*AvgT+1:40*AvgT+1)));
xlabel('Frequency (Hz)'),ylabel('Magnitude'),title('Virtual references of the
Input Drive Forces')
end

```

A.2 Auto Power Comparison for Analytical Data

```

function [] = Auto_power_comparison(xw)
close all;xz=xw; % needs only the correlated original drive file as the input

%% extended time history data
xz1=[xz(:,1);xz(:,1);xz(:,1);xz(:,1);xz(:,1);xz(:,1);xz(:,1);xz(:,1);xz(:,1);
xz(:,1);xz(:,1);xz(:,1);xz(:,1);xz(:,1);xz(:,1);xz(:,1);xz(:,1);xz(:,1);xz(:,1);xz(:,1)];

xz2=[xz(:,2);xz(:,2);xz(:,2);xz(:,2);xz(:,2);xz(:,2);xz(:,2);xz(:,2);xz(:,2);
xz(:,2);xz(:,2);xz(:,2);xz(:,2);xz(:,2);xz(:,2);xz(:,2);xz(:,2);xz(:,2);xz(:,2);xz(:,2)];

xz3=[xz(:,3);xz(:,3);xz(:,3);xz(:,3);xz(:,3);xz(:,3);xz(:,3);xz(:,3);xz(:,3);
xz(:,3);xz(:,3);xz(:,3);xz(:,3);xz(:,3);xz(:,3);xz(:,3);xz(:,3);xz(:,3);xz(:,3);xz(:,3)];

xz4=[xz(:,4);xz(:,4);xz(:,4);xz(:,4);xz(:,4);xz(:,4);xz(:,4);xz(:,4);xz(:,4);
xz(:,4);xz(:,4);xz(:,4);xz(:,4);xz(:,4);xz(:,4);xz(:,4);xz(:,4);xz(:,4);xz(:,4);xz(:,4)];

xzET=[xz1 xz2 xz3 xz4];
%% randomly extended time history
% 1st quarter
for k=1:20
    sell=randi(4);
    if sell==1
        yw11=zeros(randi(200),1);yw21=zeros(randi(200),1);yw31=zeros(randi(200),1);yw
41=zeros(randi(200),1);
        xw11(:,k)=[yw11;xw(1:length(xw)/4-length(yw11),1)]; %#ok<*AGROW>
        xw21(:,k)=[yw21;xw(1:length(xw)/4-length(yw21),2)];
        xw31(:,k)=[yw31;xw(1:length(xw)/4-length(yw31),3)];
        xw41(:,k)=[yw41;xw(1:length(xw)/4-length(yw41),4)];
    end
end

```

```

elseif sel1==2
    yw11=randi([-200 -1],1);yw21=randi([-200 -1],1);yw31=randi([-200 -
1],1);yw41=randi([-200 -1],1);
    xw11(:,k)=xw(-yw11+1:length(xw)/4-yw11,1);
    xw21(:,k)=xw(-yw21+1:length(xw)/4-yw21,2);
    xw31(:,k)=xw(-yw31+1:length(xw)/4-yw31,3);
    xw41(:,k)=xw(-yw41+1:length(xw)/4-yw41,4);
elseif sel1==3
    xw11(:,k)=xw(1:length(xw)/4,1);
    xw21(:,k)=xw(1:length(xw)/4,2);
    xw31(:,k)=flipud(xw(1:length(xw)/4,3));
    xw41(:,k)=flipud(xw(1:length(xw)/4,4));
elseif sel1==4
    xw11(:,k)=flipud(xw(1:length(xw)/4,1));
    xw21(:,k)=flipud(xw(1:length(xw)/4,2));
    xw31(:,k)=xw(1:length(xw)/4,3);
    xw41(:,k)=xw(1:length(xw)/4,4);
end
end
% 2nd Quarter
for k=1:20
    sel2=randi(4);
    if sel2==1
        yw12=randi(200);yw22=randi(200);yw32=randi(200);yw42=randi(200);
        xw12(:,k)=xw(length(xw)/4-yw12+1:length(xw)/2-yw12,1);
        xw22(:,k)=xw(length(xw)/4-yw22+1:length(xw)/2-yw22,2);
        xw32(:,k)=xw(length(xw)/4-yw32+1:length(xw)/2-yw32,3);
        xw42(:,k)=xw(length(xw)/4-yw42+1:length(xw)/2-yw42,4);
    elseif sel2==2
        yw12=randi([-200 -1],1);yw22=randi([-200 -1],1);yw32=randi([-200 -
1],1);yw42=randi([-200 -1],1);
        xw12(:,k)=xw(length(xw)/4-yw12+1:length(xw)/2-yw12,1);
        xw22(:,k)=xw(length(xw)/4-yw22+1:length(xw)/2-yw22,2);
        xw32(:,k)=xw(length(xw)/4-yw32+1:length(xw)/2-yw32,3);
        xw42(:,k)=xw(length(xw)/4-yw42+1:length(xw)/2-yw42,4);
    elseif sel2==3
        xw12(:,k)=xw(length(xw)/4+1:length(xw)/2,1);
        xw22(:,k)=xw(length(xw)/4+1:length(xw)/2,2);
        xw32(:,k)=flipud(xw(length(xw)/4+1:length(xw)/2,3));
        xw42(:,k)=flipud(xw(length(xw)/4+1:length(xw)/2,4));
    elseif sel2==4
        xw12(:,k)=flipud(xw(length(xw)/4+1:length(xw)/2,1));
        xw22(:,k)=flipud(xw(length(xw)/4+1:length(xw)/2,2));
        xw32(:,k)=xw(length(xw)/4+1:length(xw)/2,3);
        xw42(:,k)=xw(length(xw)/4+1:length(xw)/2,4);
    end
end
% 3rd Quarter
for k=1:20
    sel3=randi(4);
    if sel3==1
        yw13=randi(200);yw23=randi(200);yw33=randi(200);yw43=randi(200);
        xw13(:,k)=xw(length(xw)/2-yw13+1:3*length(xw)/4-yw13,1);
        xw23(:,k)=xw(length(xw)/2-yw23+1:3*length(xw)/4-yw23,2);
        xw33(:,k)=xw(length(xw)/2-yw33+1:3*length(xw)/4-yw33,3);
        xw43(:,k)=xw(length(xw)/2-yw43+1:3*length(xw)/4-yw43,4);
    elseif sel3==2

```

```

        yw13=randi([-200 -1],1);yw23=randi([-200 -1],1);yw33=randi([-200 -
1],1);yw43=randi([-200 -1],1);
        xw13(:,k)=xw(length(xw)/2-yw13+1:3*length(xw)/4-yw13,1);
        xw23(:,k)=xw(length(xw)/2-yw23+1:3*length(xw)/4-yw23,2);
        xw33(:,k)=xw(length(xw)/2-yw33+1:3*length(xw)/4-yw33,3);
        xw43(:,k)=xw(length(xw)/2-yw43+1:3*length(xw)/4-yw43,4);
    elseif sel3==3
        xw13(:,k)=xw(length(xw)/2+1:3*length(xw)/4,1);
        xw23(:,k)=xw(length(xw)/2+1:3*length(xw)/4,2);
        xw33(:,k)=flipud(xw(length(xw)/2+1:3*length(xw)/4,3));
        xw43(:,k)=flipud(xw(length(xw)/2+1:3*length(xw)/4,4));
    elseif sel3==4
        xw13(:,k)=flipud(xw(length(xw)/2+1:3*length(xw)/4,1));
        xw23(:,k)=flipud(xw(length(xw)/2+1:3*length(xw)/4,2));
        xw33(:,k)=xw(length(xw)/2+1:3*length(xw)/4,3);
        xw43(:,k)=xw(length(xw)/2+1:3*length(xw)/4,4);
    end
end
% 4th Quarter
for k=1:20
    sel4=randi(4);
    if sel4==1
        yw14=randi(200);yw24=randi(200);yw34=randi(200);yw44=randi(200);
        xw14(:,k)=xw(3*length(xw)/4-yw14+1:length(xw)-yw14,1);
        xw24(:,k)=xw(3*length(xw)/4-yw24+1:length(xw)-yw24,2);
        xw34(:,k)=xw(3*length(xw)/4-yw34+1:length(xw)-yw34,3);
        xw44(:,k)=xw(3*length(xw)/4-yw44+1:length(xw)-yw44,4);
    elseif sel4==2
        yw14=randi([-200 -1],1);yw24=randi([-200 -1],1);yw34=randi([-200 -
1],1);yw44=randi([-200 -1],1);
        xw14(:,k)=[xw(3*length(xw)/4-yw14+1:length(xw),1);zeros(-yw14,1)];
        xw24(:,k)=[xw(3*length(xw)/4-yw24+1:length(xw),2);zeros(-yw24,1)];
        xw34(:,k)=[xw(3*length(xw)/4-yw34+1:length(xw),3);zeros(-yw34,1)];
        xw44(:,k)=[xw(3*length(xw)/4-yw44+1:length(xw),4);zeros(-yw44,1)];
    elseif sel4==3
        xw14(:,k)=xw(3*length(xw)/4+1:length(xw),1);
        xw24(:,k)=xw(3*length(xw)/4+1:length(xw),2);
        xw34(:,k)=flipud(xw(3*length(xw)/4+1:length(xw),3));
        xw44(:,k)=flipud(xw(3*length(xw)/4+1:length(xw),4));
    elseif sel4==4
        xw14(:,k)=flipud(xw(3*length(xw)/4+1:length(xw),1));
        xw24(:,k)=flipud(xw(3*length(xw)/4+1:length(xw),2));
        xw34(:,k)=xw(3*length(xw)/4+1:length(xw),3);
        xw44(:,k)=xw(3*length(xw)/4+1:length(xw),4);
    end
end
xwa=[xw11;xw12;xw13;xw14];xwb=[xw21;xw22;xw23;xw24];xwc=[xw31;xw32;xw33;xw34];
xwd=[xw41;xw42;xw43;xw44];

xw1=[xwa(:,1);xwa(:,2);xwa(:,3);xwa(:,4);xwa(:,5);xwa(:,6);xwa(:,7);xwa(:,8);
xwa(:,9);xwa(:,10);xwa(:,11);xwa(:,12);xwa(:,13);xwa(:,14);xwa(:,15);xwa(:,16)
];xwa(:,17);xwa(:,18);xwa(:,19);xwa(:,20)];

xw2=[xwb(:,1);xwb(:,2);xwb(:,3);xwb(:,4);xwb(:,5);xwb(:,6);xwb(:,7);xwb(:,8);
xwb(:,9);xwb(:,10);xwb(:,11);xwb(:,12);xwb(:,13);xwb(:,14);xwb(:,15);xwb(:,16)
];xwb(:,17);xwb(:,18);xwb(:,19);xwb(:,20)];

```



```

xw3=[xwc(:,1);xwc(:,2);xwc(:,3);xwc(:,4);xwc(:,5);xwc(:,6);xwc(:,7);xwc(:,8);
xwc(:,9);xwc(:,10);xwc(:,11);xwc(:,12);xwc(:,13);xwc(:,14);xwc(:,15);xwc(:,16)
);xwc(:,17);xwc(:,18);xwc(:,19);xwc(:,20)];

xw4=[xwd(:,1);xwd(:,2);xwd(:,3);xwd(:,4);xwd(:,5);xwd(:,6);xwd(:,7);xwd(:,8);
xwd(:,9);xwd(:,10);xwd(:,11);xwd(:,12);xwd(:,13);xwd(:,14);xwd(:,15);xwd(:,16)
);xwd(:,17);xwd(:,18);xwd(:,19);xwd(:,20)];
xwRET=[xw1 xw2 xw3 xw4];
% Plotting of autopower spectrum of drive files
dt=1/204.8;Fnyq=1/(2*dt);
BS=1024;
AvgT=dt*BS;AvgdT=(0:BS-1)*dt;
a0=1;a1=0.5; pi_x=2;P110=(1/pi_x)*(a0-
(2*a1*cos(2*pi*(AvgdT/AvgT))));W=[P110;P110;P110;P110].';
w1=0:1/AvgT:Fnyq;
GFF11avg=zeros(BS/2+1,1);GFF12avg=zeros(BS/2+1,1);GFF13avg=zeros(BS/2+1,1);GF
F14avg=zeros(BS/2+1,1);
GFF21avg=zeros(BS/2+1,1);GFF22avg=zeros(BS/2+1,1);GFF23avg=zeros(BS/2+1,1);GF
F24avg=zeros(BS/2+1,1);
GFF31avg=zeros(BS/2+1,1);GFF32avg=zeros(BS/2+1,1);GFF33avg=zeros(BS/2+1,1);GF
F34avg=zeros(BS/2+1,1);
GFF41avg=zeros(BS/2+1,1);GFF42avg=zeros(BS/2+1,1);GFF43avg=zeros(BS/2+1,1);GF
F44avg=zeros(BS/2+1,1);
for N=1:BS/2:length(xwRET)-BS/2
    F1=W.*xwRET(N:N+BS-1,:);
    F1=(fft(F1)*pi_x)/BS;
    F3=2*F1(2:Fnyq*AvgT,:);
    F1=[F1(1,:); F3; F1(Fnyq*AvgT+1,:)];
    % Auto power in all force data channels

GFF11=F1(:,1).*conj(F1(:,1));GFF12=F1(:,1).*conj(F1(:,2));GFF13=F1(:,1).*conj
(F1(:,3));GFF14=F1(:,1).*conj(F1(:,4));

GFF21=F1(:,2).*conj(F1(:,1));GFF22=F1(:,2).*conj(F1(:,2));GFF23=F1(:,2).*conj
(F1(:,3));GFF24=F1(:,2).*conj(F1(:,4));

GFF31=F1(:,3).*conj(F1(:,1));GFF32=F1(:,3).*conj(F1(:,2));GFF33=F1(:,3).*conj
(F1(:,3));GFF34=F1(:,3).*conj(F1(:,4));

GFF41=F1(:,4).*conj(F1(:,1));GFF42=F1(:,4).*conj(F1(:,2));GFF43=F1(:,4).*conj
(F1(:,3));GFF44=F1(:,4).*conj(F1(:,4));

GFF11avg=GFF11avg+GFF11;GFF12avg=GFF12avg+GFF12;GFF13avg=GFF13avg+GFF13;GFF14
avg=GFF14avg+GFF14;

GFF21avg=GFF21avg+GFF21;GFF22avg=GFF22avg+GFF22;GFF23avg=GFF23avg+GFF23;GFF24
avg=GFF24avg+GFF24;

GFF31avg=GFF31avg+GFF31;GFF32avg=GFF32avg+GFF32;GFF33avg=GFF33avg+GFF33;GFF34
avg=GFF34avg+GFF34;

GFF41avg=GFF41avg+GFF41;GFF42avg=GFF42avg+GFF42;GFF43avg=GFF43avg+GFF43;GFF44
avg=GFF44avg+GFF44;
end
GFF11avg=GFF11avg/((length(xwRET)/(BS/2))-
1);GFF12avg=GFF12avg/((length(xwRET)/(BS/2))-

```

```

1);GFF13avg=GFF13avg/((length(xwRET)/(BS/2))-1);
1);GFF14avg=GFF14avg/((length(xwRET)/(BS/2))-1);
GFF21avg=GFF21avg/((length(xwRET)/(BS/2))-1);
1);GFF22avg=GFF22avg/((length(xwRET)/(BS/2))-1);
1);GFF23avg=GFF23avg/((length(xwRET)/(BS/2))-1);
1);GFF24avg=GFF24avg/((length(xwRET)/(BS/2))-1);
GFF31avg=GFF31avg/((length(xwRET)/(BS/2))-1);
1);GFF32avg=GFF32avg/((length(xwRET)/(BS/2))-1);
1);GFF33avg=GFF33avg/((length(xwRET)/(BS/2))-1);
1);GFF34avg=GFF34avg/((length(xwRET)/(BS/2))-1);
GFF41avg=GFF41avg/((length(xwRET)/(BS/2))-1);
1);GFF42avg=GFF42avg/((length(xwRET)/(BS/2))-1);
1);GFF43avg=GFF43avg/((length(xwRET)/(BS/2))-1);
1);GFF44avg=GFF44avg/((length(xwRET)/(BS/2))-1);

GFF11avgz=zeros(BS/2+1,1);GFF12avgz=zeros(BS/2+1,1);GFF13avgz=zeros(BS/2+1,1);
;GFF14avgz=zeros(BS/2+1,1);
GFF21avgz=zeros(BS/2+1,1);GFF22avgz=zeros(BS/2+1,1);GFF23avgz=zeros(BS/2+1,1);
;GFF24avgz=zeros(BS/2+1,1);
GFF31avgz=zeros(BS/2+1,1);GFF32avgz=zeros(BS/2+1,1);GFF33avgz=zeros(BS/2+1,1);
;GFF34avgz=zeros(BS/2+1,1);
GFF41avgz=zeros(BS/2+1,1);GFF42avgz=zeros(BS/2+1,1);GFF43avgz=zeros(BS/2+1,1);
;GFF44avgz=zeros(BS/2+1,1);
for N=1:BS/2:length(xzET)-BS/2
    F1=W.*xzET(N:N+BS-1,:);
    F1=(fft(F1)*pi_x)/BS;
    F3=2*F1(2:Fnyq*AvgT,:);
    F1=[F1(1,:); F3; F1(Fnyq*AvgT+1,:)];
    % Auto power in all force data channels

GFF11z=F1(:,1).*conj(F1(:,1));GFF12z=F1(:,1).*conj(F1(:,2));GFF13z=F1(:,1).*c
onj(F1(:,3));GFF14z=F1(:,1).*conj(F1(:,4));

GFF21z=F1(:,2).*conj(F1(:,1));GFF22z=F1(:,2).*conj(F1(:,2));GFF23z=F1(:,2).*c
onj(F1(:,3));GFF24z=F1(:,2).*conj(F1(:,4));

GFF31z=F1(:,3).*conj(F1(:,1));GFF32z=F1(:,3).*conj(F1(:,2));GFF33z=F1(:,3).*c
onj(F1(:,3));GFF34z=F1(:,3).*conj(F1(:,4));

GFF41z=F1(:,4).*conj(F1(:,1));GFF42z=F1(:,4).*conj(F1(:,2));GFF43z=F1(:,4).*c
onj(F1(:,3));GFF44z=F1(:,4).*conj(F1(:,4));

GFF11avgz=GFF11avgz+GFF11z;GFF12avgz=GFF12avgz+GFF12z;GFF13avgz=GFF13avgz+GFF
13z;GFF14avgz=GFF14avgz+GFF14z;

GFF21avgz=GFF21avgz+GFF21z;GFF22avgz=GFF22avgz+GFF22z;GFF23avgz=GFF23avgz+GFF
23z;GFF24avgz=GFF24avgz+GFF24z;

GFF31avgz=GFF31avgz+GFF31z;GFF32avgz=GFF32avgz+GFF32z;GFF33avgz=GFF33avgz+GFF
33z;GFF34avgz=GFF34avgz+GFF34z;

GFF41avgz=GFF41avgz+GFF41z;GFF42avgz=GFF42avgz+GFF42z;GFF43avgz=GFF43avgz+GFF
43z;GFF44avgz=GFF44avgz+GFF44z;
end
GFF11avgz=GFF11avgz/((length(xzET)/(BS/2))-1);GFF12avgz=GFF12avgz/((length(xzET)/(BS/2))-1);

```

```

1);GFF13avgz=GFF13avgz/((length(xzET)/(BS/2))-
1);GFF14avgz=GFF14avgz/((length(xzET)/(BS/2))-1);
GFF21avgz=GFF21avgz/((length(xzET)/(BS/2))-
1);GFF22avgz=GFF22avgz/((length(xzET)/(BS/2))-
1);GFF23avgz=GFF23avgz/((length(xzET)/(BS/2))-
1);GFF24avgz=GFF24avgz/((length(xzET)/(BS/2))-1);
GFF31avgz=GFF31avgz/((length(xzET)/(BS/2))-
1);GFF32avgz=GFF32avgz/((length(xzET)/(BS/2))-
1);GFF33avgz=GFF33avgz/((length(xzET)/(BS/2))-
1);GFF34avgz=GFF34avgz/((length(xzET)/(BS/2))-1);
GFF41avgz=GFF41avgz/((length(xzET)/(BS/2))-
1);GFF42avgz=GFF42avgz/((length(xzET)/(BS/2))-
1);GFF43avgz=GFF43avgz/((length(xzET)/(BS/2))-
1);GFF44avgz=GFF44avgz/((length(xzET)/(BS/2))-1);

figure(1)
subplot(2,2,1)
semilogy(w1(4*AvgT+1:40*AvgT+1),(GFF11avgz(4*AvgT+1:40*AvgT+1)),w1(4*AvgT+1:40*AvgT+1),(GFF11avg(4*AvgT+1:40*AvgT+1)));
xlabel('Frequency (Hz)'),ylabel('Power (in^2)'),title('Autopower on Front Left F11'),legend('Time Extended','Random Time Extended')
subplot(2,2,2)
semilogy(w1(4*AvgT+1:40*AvgT+1),(GFF22avgz(4*AvgT+1:40*AvgT+1)),w1(4*AvgT+1:40*AvgT+1),(GFF22avg(4*AvgT+1:40*AvgT+1)));
xlabel('Frequency (Hz)'),ylabel('Power (in^2)'),title('Autopower on Front Right F22')
subplot(2,2,3)
semilogy(w1(4*AvgT+1:40*AvgT+1),(GFF33avgz(4*AvgT+1:40*AvgT+1)),w1(4*AvgT+1:40*AvgT+1),(GFF33avg(4*AvgT+1:40*AvgT+1)));
xlabel('Frequency (Hz)'),ylabel('Power (in^2)'),title('Autopower on Rear Left F33')
subplot(2,2,4)
semilogy(w1(4*AvgT+1:40*AvgT+1),(GFF44avgz(4*AvgT+1:40*AvgT+1)),w1(4*AvgT+1:40*AvgT+1),(GFF44avg(4*AvgT+1:40*AvgT+1)));
xlabel('Frequency (Hz)'),ylabel('Power (in^2)'),title('Autopower on Rear Right F44')
figure(2)
subplot(3,2,1)
semilogy(w1(4*AvgT+1:40*AvgT+1),abs(GFF12avgz(4*AvgT+1:40*AvgT+1)),w1(4*AvgT+1:40*AvgT+1),abs(GFF12avg(4*AvgT+1:40*AvgT+1)));
xlabel('Frequency (Hz)'),ylabel('Power (in^2)'),title('Crosspower F12,F21'),legend('Time Extended','Random Time Extended')
subplot(3,2,2)
semilogy(w1(4*AvgT+1:40*AvgT+1),(abs(GFF31avgz(4*AvgT+1:40*AvgT+1))),w1(4*AvgT+1:40*AvgT+1),(abs(GFF31avg(4*AvgT+1:40*AvgT+1))));
xlabel('Frequency (Hz)'),ylabel('Power (in^2)'),title('Crosspower F13,F31')
subplot(3,2,3)
semilogy(w1(4*AvgT+1:40*AvgT+1),(abs(GFF14avgz(4*AvgT+1:40*AvgT+1))),w1(4*AvgT+1:40*AvgT+1),(abs(GFF14avg(4*AvgT+1:40*AvgT+1))));
xlabel('Frequency (Hz)'),ylabel('Power (in^2)'),title('Crosspower F14,F41')
subplot(3,2,4)
semilogy(w1(4*AvgT+1:40*AvgT+1),abs(GFF23avgz(4*AvgT+1:40*AvgT+1)),w1(4*AvgT+1:40*AvgT+1),abs(GFF23avg(4*AvgT+1:40*AvgT+1)));
xlabel('Frequency (Hz)'),ylabel('Power (in^2)'),title('Crosspower F23,F32')
subplot(3,2,5)
semilogy(w1(4*AvgT+1:40*AvgT+1),abs(GFF42avgz(4*AvgT+1:40*AvgT+1)),w1(4*AvgT+1:40*AvgT+1),abs(GFF42avg(4*AvgT+1:40*AvgT+1)));

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xlabel('Frequency (Hz)'),ylabel('Power (in^2)'),title('Crosspower F24,F42')
subplot(3,2,6)
semilogy(wl(4*AvgT+1:40*AvgT+1),abs(GFF34avgz(4*AvgT+1:40*AvgT+1)),wl(4*AvgT+
1:40*AvgT+1),abs(GFF34avg(4*AvgT+1:40*AvgT+1)));
xlabel('Frequency (Hz)'),ylabel('Power (in^2)'),title('Crosspower F34,F43')
end

```

A.3 Virtual References and Auto Power Comparison of Measured Data

```

% Input is auto power spectrum of measured data from the four axis road
simulator

GFF=BB25_drive; % Name the file as it has been saved
for i=1:length(xdata)
    GFFdecomp=sort(svd(squeeze(GFF(:,:,i))));
    GFF11PF(i)=GFFdecomp(1); %#ok<*SAGROW>
    GFF22PF(i)=GFFdecomp(2);
    GFF33PF(i)=GFFdecomp(3);
    GFF44PF(i)=GFFdecomp(4);
end
figure(9)
semilogy(xdata(21:201),GFF11PF(21:201),xdata(21:201),GFF22PF(21:201),xdata(21
:201),GFF33PF(21:201),xdata(21:201),GFF44PF(21:201))
xlabel('Frequency (Hz)'),ylabel('Magnitude'),title('Virtual References of
Drive for NVH1 condition')
GFF=Harsh45_drive; % Different road surface
for i=1:length(xdata)
    GFFdecomp=sort(svd(squeeze(GFF(:,:,i))));
    GFF11PF(i)=GFFdecomp(1); %#ok<*SAGROW>
    GFF22PF(i)=GFFdecomp(2);
    GFF33PF(i)=GFFdecomp(3);
    GFF44PF(i)=GFFdecomp(4);
end
figure(10)
semilogy(xdata(21:201),GFF11PF(21:201),xdata(21:201),GFF22PF(21:201),xdata(21
:201),GFF33PF(21:201),xdata(21:201),GFF44PF(21:201))
xlabel('Frequency (Hz)'),ylabel('Magnitude'),title('Virtual References of
Drive for NVH2 condition')
GFF=RB25_drive;
for i=1:length(xdata)
    GFFdecomp=sort(svd(squeeze(GFF(:,:,i))));
    GFF11PF(i)=GFFdecomp(1); %#ok<*SAGROW>
    GFF22PF(i)=GFFdecomp(2);
    GFF33PF(i)=GFFdecomp(3);
    GFF44PF(i)=GFFdecomp(4);
end
figure(11)
semilogy(xdata(21:201),GFF11PF(21:201),xdata(21:201),GFF22PF(21:201),xdata(21
:201),GFF33PF(21:201),xdata(21:201),GFF44PF(21:201))
xlabel('Frequency (Hz)'),ylabel('Magnitude'),title('Virtual References of
Drive for NVH3 condition')
GFF=TP40_drive;
for i=1:length(xdata)
    GFFdecomp=sort(svd(squeeze(GFF(:,:,i))));
    GFF11PF(i)=GFFdecomp(1); %#ok<*SAGROW>
    GFF22PF(i)=GFFdecomp(2);

```

```

        GFF33PF(i)=GFFdecomp(3);
        GFF44PF(i)=GFFdecomp(4);
end
figure(12)
semilogy(xdata(21:201),GFF11PF(21:201),xdata(21:201),GFF22PF(21:201),xdata(21:201),GFF33PF(21:201),xdata(21:201),GFF44PF(21:201))
xlabel('Frequency (Hz)'),ylabel('Magnitude'),title('Virtual References of Drive for NVH4 condition')
GFF=BB25_drive_redf;
for i=1:length(xdata)
    GFFdecomp=sort(svd(squeeze(GFF(:, :, i)))));
    GFF11PF(i)=GFFdecomp(1); %#ok<*SAGROW>
    GFF22PF(i)=GFFdecomp(2);
    GFF33PF(i)=GFFdecomp(3);
    GFF44PF(i)=GFFdecomp(4);
end
figure(13)
semilogy(xdata(21:201),GFF11PF(21:201),xdata(21:201),GFF22PF(21:201),xdata(21:201),GFF33PF(21:201),xdata(21:201),GFF44PF(21:201))
xlabel('Frequency (Hz)'),ylabel('Magnitude'),title('Virtual References of Drive for NVH1 condition')
GFF=Harsh45_drive_redf;
for i=1:length(xdata)
    GFFdecomp=sort(svd(squeeze(GFF(:, :, i)))));
    GFF11PF(i)=GFFdecomp(1); %#ok<*SAGROW>
    GFF22PF(i)=GFFdecomp(2);
    GFF33PF(i)=GFFdecomp(3);
    GFF44PF(i)=GFFdecomp(4);
end
figure(14)
semilogy(xdata(21:201),GFF11PF(21:201),xdata(21:201),GFF22PF(21:201),xdata(21:201),GFF33PF(21:201),xdata(21:201),GFF44PF(21:201))
xlabel('Frequency (Hz)'),ylabel('Magnitude'),title('Virtual References of Drive for NVH2 condition')
GFF=RB25_drive_redf;
for i=1:length(xdata)
    GFFdecomp=sort(svd(squeeze(GFF(:, :, i)))));
    GFF11PF(i)=GFFdecomp(1); %#ok<*SAGROW>
    GFF22PF(i)=GFFdecomp(2);
    GFF33PF(i)=GFFdecomp(3);
    GFF44PF(i)=GFFdecomp(4);
end
figure(15)
semilogy(xdata(21:201),GFF11PF(21:201),xdata(21:201),GFF22PF(21:201),xdata(21:201),GFF33PF(21:201),xdata(21:201),GFF44PF(21:201))
xlabel('Frequency (Hz)'),ylabel('Magnitude'),title('Virtual References of Drive for NVH3 condition')
GFF=TP40_drive_redf;
for i=1:length(xdata)
    GFFdecomp=sort(svd(squeeze(GFF(:, :, i)))));
    GFF11PF(i)=GFFdecomp(1); %#ok<*SAGROW>
    GFF22PF(i)=GFFdecomp(2);
    GFF33PF(i)=GFFdecomp(3);
    GFF44PF(i)=GFFdecomp(4);
end
figure(16)

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semilogy(xdata(21:201),GFF11PF(21:201),xdata(21:201),GFF22PF(21:201),xdata(21:201),GFF33PF(21:201),xdata(21:201),GFF44PF(21:201))
xlabel('Frequency (Hz)'),ylabel('Magnitude'),title('Virtual References of Drive for NVH4 condition')
% Plotting drive auto power from 4 poster testing
xdata=0:0.2:100;
figure(1)
subplot(2,2,1),semilogy(xdata(21:201),squeeze(abs(BB25_drive(1,1,21:201))),xdata(21:201),squeeze(abs(BB25_drive_redf(1,1,21:201))));
xlabel('Frequency (Hz)'),ylabel('Power (in^2)'),title('Drive Autopower on Front Left F11 for NVH1 condition'),legend('Time Extended','Random Time Extended')
subplot(2,2,2),semilogy(xdata(21:201),squeeze(abs(BB25_drive(2,2,21:201))),xdata(21:201),squeeze(abs(BB25_drive_redf(2,2,21:201))));
xlabel('Frequency (Hz)'),ylabel('Power (in^2)'),title('Drive Autopower on Front Right F22 for NVH1 condition')
subplot(2,2,3),semilogy(xdata(21:201),squeeze(abs(BB25_drive(3,3,21:201))),xdata(21:201),squeeze(abs(BB25_drive_redf(3,3,21:201))));
xlabel('Frequency (Hz)'),ylabel('Power (in^2)'),title('Drive Autopower on Rear Left F33 for NVH1 condition')
subplot(2,2,4),semilogy(xdata(21:201),squeeze(abs(BB25_drive(4,4,21:201))),xdata(21:201),squeeze(abs(BB25_drive_redf(4,4,21:201))));
xlabel('Frequency (Hz)'),ylabel('Power (in^2)'),title('Drive Autopower on Rear Right F44 for NVH1 condition')
figure(2)
subplot(2,2,1),semilogy(xdata(21:201),squeeze(abs(Harsh45_drive(1,1,21:201))),xdata(21:201),squeeze(abs(Harsh45_drive_redf(1,1,21:201))));
xlabel('Frequency (Hz)'),ylabel('Power (in^2)'),title('Drive Autopower on Front Left F11 for NVH2 condition'),legend('Time Extended','Random Time Extended')
subplot(2,2,2),semilogy(xdata(21:201),squeeze(abs(Harsh45_drive(2,2,21:201))),xdata(21:201),squeeze(abs(Harsh45_drive_redf(2,2,21:201))));
xlabel('Frequency (Hz)'),ylabel('Power (in^2)'),title('Drive Autopower on Front Right F22 for NVH2 condition')
subplot(2,2,3),semilogy(xdata(21:201),squeeze(abs(Harsh45_drive(3,3,21:201))),xdata(21:201),squeeze(abs(Harsh45_drive_redf(3,3,21:201))));
xlabel('Frequency (Hz)'),ylabel('Power (in^2)'),title('Drive Autopower on Rear Left F33 for NVH2 condition')
subplot(2,2,4),semilogy(xdata(21:201),squeeze(abs(Harsh45_drive(4,4,21:201))),xdata(21:201),squeeze(abs(Harsh45_drive_redf(4,4,21:201))));
xlabel('Frequency (Hz)'),ylabel('Power (in^2)'),title('Drive Autopower on Rear Right F44 for NVH2 condition')
figure(3)
subplot(2,2,1),semilogy(xdata(21:201),squeeze(abs(RB25_drive(1,1,21:201))),xdata(21:201),squeeze(abs(RB25_drive_redf(1,1,21:201))));
xlabel('Frequency (Hz)'),ylabel('Power (in^2)'),title('Drive Autopower on Front Left F11 for NVH3 condition'),legend('Time Extended','Random Time Extended')
subplot(2,2,2),semilogy(xdata(21:201),squeeze(abs(RB25_drive(2,2,21:201))),xdata(21:201),squeeze(abs(RB25_drive_redf(2,2,21:201))));
xlabel('Frequency (Hz)'),ylabel('Power (in^2)'),title('Drive Autopower on Front Right F22 for NVH3 condition')
subplot(2,2,3),semilogy(xdata(21:201),squeeze(abs(RB25_drive(3,3,21:201))),xdata(21:201),squeeze(abs(RB25_drive_redf(3,3,21:201))));
xlabel('Frequency (Hz)'),ylabel('Power (in^2)'),title('Drive Autopower on Rear Left F33 for NVH3 condition')

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subplot(2,2,4),semilogy(xdata(21:201),squeeze(abs(RB25_drive(4,4,21:201))),xdata(21:201),squeeze(abs(RB25_drive_redf(4,4,21:201))));
xlabel('Frequency (Hz)'),ylabel('Power (in^2)'),title('Drive Autopower on Rear Right F44 for NVH3 condition')
figure(4)
subplot(2,2,1),semilogy(xdata(21:201),squeeze(abs(TP40_drive(1,1,21:201))),xdata(21:201),squeeze(abs(TP40_drive_redf(1,1,21:201))));
xlabel('Frequency (Hz)'),ylabel('Power (in^2)'),title('Drive Autopower on Front Left F11 for NVH4 condition'),legend('Time Extended','Random Time Extended')
subplot(2,2,2),semilogy(xdata(21:201),squeeze(abs(TP40_drive(2,2,21:201))),xdata(21:201),squeeze(abs(TP40_drive_redf(2,2,21:201))));
xlabel('Frequency (Hz)'),ylabel('Power (in^2)'),title('Drive Autopower on Front Right F22 for NVH4 condition')
subplot(2,2,3),semilogy(xdata(21:201),squeeze(abs(TP40_drive(3,3,21:201))),xdata(21:201),squeeze(abs(TP40_drive_redf(3,3,21:201))));
xlabel('Frequency (Hz)'),ylabel('Power (in^2)'),title('Drive Autopower on Rear Left F33 for NVH4 condition')
subplot(2,2,4),semilogy(xdata(21:201),squeeze(abs(TP40_drive(4,4,21:201))),xdata(21:201),squeeze(abs(TP40_drive_redf(4,4,21:201))));
xlabel('Frequency (Hz)'),ylabel('Power (in^2)'),title('Drive Autopower on Rear Right F44 for NVH4 condition')
figure(5)
subplot(3,2,1)
semilogy(xdata(21:201),squeeze(abs(BB25_drive(1,2,21:201))),xdata(21:201),squeeze(abs(BB25_drive_redf(1,2,21:201))));
xlabel('Frequency (Hz)'),ylabel('Power (in^2)'),title('Drive Crosspower F12,F21'),legend('Time Extended','Random Time Extended')
subplot(3,2,2)
semilogy(xdata(21:201),squeeze(abs(BB25_drive(1,3,21:201))),xdata(21:201),squeeze(abs(BB25_drive_redf(1,3,21:201))));
xlabel('Frequency (Hz)'),ylabel('Power (in^2)'),title('Drive Crosspower F13,F31')
subplot(3,2,3)
semilogy(xdata(21:201),squeeze(abs(BB25_drive(1,4,21:201))),xdata(21:201),squeeze(abs(BB25_drive_redf(1,4,21:201))));
xlabel('Frequency (Hz)'),ylabel('Power (in^2)'),title('Drive Crosspower F14,F41')
subplot(3,2,4)
semilogy(xdata(21:201),squeeze(abs(BB25_drive(2,3,21:201))),xdata(21:201),squeeze(abs(BB25_drive_redf(2,3,21:201))));
xlabel('Frequency (Hz)'),ylabel('Power (in^2)'),title('Drive Crosspower F23,F32')
subplot(3,2,5)
semilogy(xdata(21:201),squeeze(abs(BB25_drive(2,4,21:201))),xdata(21:201),squeeze(abs(BB25_drive_redf(2,4,21:201))));
xlabel('Frequency (Hz)'),ylabel('Power (in^2)'),title('Drive Crosspower F24,F42')
subplot(3,2,6)
semilogy(xdata(21:201),squeeze(abs(BB25_drive(3,4,21:201))),xdata(21:201),squeeze(abs(BB25_drive_redf(3,4,21:201))));
xlabel('Frequency (Hz)'),ylabel('Power (in^2)'),title('Drive Crosspower F34,F43')
figure(6)
subplot(3,2,1)
semilogy(xdata(21:201),squeeze(abs(Harsh45_drive(1,2,21:201))),xdata(21:201),squeeze(abs(Harsh45_drive_redf(1,2,21:201))));

```

```

xlabel('Frequency (Hz)'),ylabel('Power (in^2)'),title('Drive Crosspower
F12,F21'),legend('Time Extended','Random Time Extended')
subplot(3,2,2)
semilogy(xdata(21:201),squeeze(abs(Harsh45_drive(1,3,21:201))),xdata(21:201),
squeeze(abs(Harsh45_drive_redf(1,3,21:201))));
xlabel('Frequency (Hz)'),ylabel('Power (in^2)'),title('Drive Crosspower
F13,F31')
subplot(3,2,3)
semilogy(xdata(21:201),squeeze(abs(Harsh45_drive(1,4,21:201))),xdata(21:201),
squeeze(abs(Harsh45_drive_redf(1,4,21:201))));
xlabel('Frequency (Hz)'),ylabel('Power (in^2)'),title('Drive Crosspower
F14,F41')
subplot(3,2,4)
semilogy(xdata(21:201),squeeze(abs(Harsh45_drive(2,3,21:201))),xdata(21:201),
squeeze(abs(Harsh45_drive_redf(2,3,21:201))));
xlabel('Frequency (Hz)'),ylabel('Power (in^2)'),title('Drive Crosspower
F23,F32')
subplot(3,2,5)
semilogy(xdata(21:201),squeeze(abs(Harsh45_drive(2,4,21:201))),xdata(21:201),
squeeze(abs(Harsh45_drive_redf(2,4,21:201))));
xlabel('Frequency (Hz)'),ylabel('Power (in^2)'),title('Drive Crosspower
F24,F42')
subplot(3,2,6)
semilogy(xdata(21:201),squeeze(abs(Harsh45_drive(3,4,21:201))),xdata(21:201),
squeeze(abs(Harsh45_drive_redf(3,4,21:201))));
xlabel('Frequency (Hz)'),ylabel('Power (in^2)'),title('Drive Crosspower
F34,F43')
figure(7)
subplot(3,2,1)
semilogy(xdata(21:201),squeeze(abs(RB25_drive(1,2,21:201))),xdata(21:201),squ
eeze(abs(RB25_drive_redf(1,2,21:201))));
xlabel('Frequency (Hz)'),ylabel('Power (in^2)'),title('Drive Crosspower
F12,F21'),legend('Time Extended','Random Time Extended')
subplot(3,2,2)
semilogy(xdata(21:201),squeeze(abs(RB25_drive(1,3,21:201))),xdata(21:201),squ
eeze(abs(RB25_drive_redf(1,3,21:201))));
xlabel('Frequency (Hz)'),ylabel('Power (in^2)'),title('Drive Crosspower
F13,F31')
subplot(3,2,3)
semilogy(xdata(21:201),squeeze(abs(RB25_drive(1,4,21:201))),xdata(21:201),squ
eeze(abs(RB25_drive_redf(1,4,21:201))));
xlabel('Frequency (Hz)'),ylabel('Power (in^2)'),title('Drive Crosspower
F14,F41')
subplot(3,2,4)
semilogy(xdata(21:201),squeeze(abs(RB25_drive(2,3,21:201))),xdata(21:201),squ
eeze(abs(RB25_drive_redf(2,3,21:201))));
xlabel('Frequency (Hz)'),ylabel('Power (in^2)'),title('Drive Crosspower
F23,F32')
subplot(3,2,5)
semilogy(xdata(21:201),squeeze(abs(RB25_drive(2,4,21:201))),xdata(21:201),squ
eeze(abs(RB25_drive_redf(2,4,21:201))));
xlabel('Frequency (Hz)'),ylabel('Power (in^2)'),title('Drive Crosspower
F24,F42')
subplot(3,2,6)
semilogy(xdata(21:201),squeeze(abs(RB25_drive(3,4,21:201))),xdata(21:201),squ
eeze(abs(RB25_drive_redf(3,4,21:201))));

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xlabel('Frequency (Hz)'),ylabel('Power (in^2)'),title('Drive Crosspower
F34,F43')
figure(8)
subplot(3,2,1)
semilogy(xdata(21:201),squeeze(abs(TP40_drive(1,2,21:201))),xdata(21:201),squ
eeze(abs(TP40_drive_redf(1,2,21:201))));
xlabel('Frequency (Hz)'),ylabel('Power (in^2)'),title('Drive Crosspower
F12,F21'),legend('Time Extended','Random Time Extended')
subplot(3,2,2)
semilogy(xdata(21:201),squeeze(abs(TP40_drive(1,3,21:201))),xdata(21:201),squ
eeze(abs(TP40_drive_redf(1,3,21:201))));
xlabel('Frequency (Hz)'),ylabel('Power (in^2)'),title('Drive Crosspower
F13,F31')
subplot(3,2,3)
semilogy(xdata(21:201),squeeze(abs(TP40_drive(1,4,21:201))),xdata(21:201),squ
eeze(abs(TP40_drive_redf(1,4,21:201))));
xlabel('Frequency (Hz)'),ylabel('Power (in^2)'),title('Drive Crosspower
F14,F41')
subplot(3,2,4)
semilogy(xdata(21:201),squeeze(abs(TP40_drive(2,3,21:201))),xdata(21:201),squ
eeze(abs(TP40_drive_redf(2,3,21:201))));
xlabel('Frequency (Hz)'),ylabel('Power (in^2)'),title('Drive Crosspower
F23,F32')
subplot(3,2,5)
semilogy(xdata(21:201),squeeze(abs(TP40_drive(2,4,21:201))),xdata(21:201),squ
eeze(abs(TP40_drive_redf(2,4,21:201))));
xlabel('Frequency (Hz)'),ylabel('Power (in^2)'),title('Drive Crosspower
F24,F42')
subplot(3,2,6)
semilogy(xdata(21:201),squeeze(abs(TP40_drive(3,4,21:201))),xdata(21:201),squ
eeze(abs(TP40_drive_redf(3,4,21:201))));
xlabel('Frequency (Hz)'),ylabel('Power (in^2)'),title('Drive Crosspower
F34,F43')

```

Appendix B – Figures for All Cases

B.1 Original Drive Files (ODF)

Figure 0-1 : ODF for Car B, C and D on Road Surface NVH 1

Figure 0-2 : ODF for Car A, C and D on Road Surface NVH 2

Figure 0-3 : ODF for Car A, B and D on Road Surface NVH 3

Figure 0-4 : ODF for Car A, B and C on Road Surface NVH 4

B.2 Virtual References for ODF

Figure 0-5 : VR for Car B, C and D on Road Surface NVH 1

Figure 0-6 : VR for Car A, C and D on Road Surface NVH 2

Figure 0-7 : VR for Car A, B and D on Road Surface NVH 3

Figure 0-8 : VR for Car A, B and C on Road Surface NVH 4

B.3 Fixed Time Shifted SDF

Figure 0-9 : Fixed Time Shifted SDF and VR for Car B, C and D on surface NVH 1

Figure 0-10 : Fixed Time Shifted SDF and VR for Car A, C and D on surface NVH 2

Figure 0-11 : Fixed Time Shifted SDF and VR for Car A, B and D on surface NVH 3

Figure 0-12 : Fixed Time Shifted SDF and VR for Car A, B and C on surface NVH 4

B.4 Random Time Shifted SDF

Figure 0-13 : Random Time Shifted SDF and VR for Car B, C and D on surface NVH 1

Figure 0-14 : Random Time Shifted SDF and VR for Car A, C and D on surface NVH 2

Figure 0-15 : Random Time Shifted SDF and VR for Car A, B and D on surface NVH 3

Figure 0-16 : Random Time Shifted SDF and VR for Car A, B and C on surface NVH 4

B.5 Flipped Time SDF

Figure 0-17 : Flipped Time SDF and VR for Car B, C and D on surface NVH 1

Figure 0-18 : Flipped Time SDF and VR for Car A, C and D on surface NVH 2

Figure 0-19 : Flipped Time SDF and VR for Car A, B and D on surface NVH 3

Figure 0-20 : Flipped Time SDF and VR for Car A, B and C on surface NVH 4

B.6 Half Flipped Time SDF

Figure 0-21 : Half Flipped Time SDF and VR for Car B, C and D on surface NVH 1

Figure 0-22 : Half Flipped Time SDF and VR for Car A, C and D on surface NVH 2

Figure 0-23 : Half Flipped Time SDF and VR for Car A, B and D on surface NVH 3

Figure 0-24 : Half Flipped Time SDF and VR for Car A, B and C on surface NVH 4

B.7 Extended ODF

Figure 0-25 : Extended ODF and VR for Car B, C and D on surface NVH 1

Figure 0-26 : Extended ODF and VR for Car A, C and D on surface NVH 2

Figure 0-27 : Extended ODF and VR for Car A, B and D on surface NVH 3

Figure 0-28 : Extended ODF and VR for Car A, B and C on surface NVH 4

B.8 Extended SDF

Figure 0-29 : Extended SDF and VR for Car B, C and D on surface NVH 1

Figure 0-30 : Extended SDF and VR for Car A, C and D on surface NVH 2

Figure 0-31 : Extended SDF and VR for Car A, B and D on surface NVH 3

Figure 0-32 : Extended SDF and VR for Car A, B and C on surface NVH 4

B.9 Auto Power Comparison of Analytical Data

Figure 0-33 : Auto Power Comparison of Analytical DF for Car B, C and D on surface NVH 1

Figure 0-34 : Auto Power Comparison of Analytical DF for Car A, C and D on surface NVH 2

Figure 0-35 : Auto Power Comparison of Analytical DF for Car A, B and D on surface NVH 3

Figure 0-36 : Auto Power Comparison of Analytical DF for Car A, B and C on surface NVH 4

B.10 Virtual References of Experimental Data

Figure 0-37 : VR of Measured Data using Extended ODF and SDF for Car B, C and D on surface NVH 1

Figure 0-38 : VR of Measured Data using Extended ODF and SDF for Car A, C and D on surface NVH 2

Figure 0-39 : VR of Measured Data using Extended ODF and SDF for Car A, B and D on surface NVH 3

Figure 0-40 : VR of Measured Data using Extended ODF and SDF for Car A, B and C on surface NVH 4

B.11 Auto Power Comparison of Experimental Data

Figure 0-41 : Auto Power Comparison of Measured DF for Car B, C and D on surface NVH 1

Figure 0-42 : Auto Power Comparison of Measured DF for Car A, B and D on surface NVH 2

Figure 0-43 : Auto Power Comparison of Measured DF for Car A, B and D on surface NVH 3

Figure 0-44 : Auto Power Comparison of Measured DF for Car A, B and C on surface NVH 4