University of Cincinnati

Date: 11/6/2015

I, Ji Xu, hereby submit this original work as part of the requirements for the degree of Master of Science in Mechanical Engineering.

It is entitled:

Active Control of Vehicle Powertrain Noise using Adaptive Notch Filter with Inverse Model LMS Algorithm

Student's name: Ji Xu

This work and its defense approved by:

Committee chair: Teik Lim, Ph.D.

Committee member: Guohua Sun, Ph.D.

Committee member: Jay Kim, Ph.D.

Committee member: David Thompson, Ph.D.

19625

Active Control of Vehicle Powertrain Noise using Adaptive Notch Filter with Inverse Model LMS Algorithm

A thesis submitted to the
Graduate School
of the University of Cincinnati
in partial fulfillment of the
requirements for the degree of
MASTER OF SCIENCE
in the Department of Mechanical and Materials Engineering
of the College of Engineering and Applied Science

2015

By

Ji Xu

B.S., Mechanical Engineering, University of Cincinnati, USA, 2013

B.S., Mechanical Engineering, Shanghai Jiao Tong University, P.R. China, 2013

Committee Chair: Teik C. Lim, Ph.D.

Committee Members: Jay H. Kim, Ph.D.

David F. Thompson, Ph.D.

Guohua Sun, Ph.D.

ABSTRACT

Active noise control (ANC) systems have been gaining popularity in the last couple of decades, due to the deficiencies in passive noise abatement techniques. In the future, a novel combination of passive and active noise control techniques may be applied more widely, to better control the interior sound quality of vehicles. In order to maximize the effectiveness of this combined approach, smarter algorithms will be needed for ANC systems. These algorithms will have to be computationally efficient, with high stability and convergence rates. This will be necessary in order to accurately predict and control the interior noise response of a vehicle. Most of current ANC systems are configured with the filtered-x least mean square (FXLMS) algorithm or its modified versions. However, the traditional FXLMS algorithm often exhibits a frequency dependent convergence behavior, which leads to a poor tracking ability for time-varying frequencies, and unbalanced performance at individual harmonics. To improve the ANC system performance, a novel adaptive notch filter with inverse model least means square (ANF-IMLMS) algorithm is proposed in this study, as the basis for active control of vehicle powertrain noise. The proposed algorithm possesses the following two salient features as compared to the filtered-x LMS type algorithms: (1) rapid convergence speed, and (2) good computational efficiency. To verify the analysis, the proposed algorithm is evaluated through numerical simulation, which utilizes the measured powertrain responses. The data is taken under both steady state and transient conditions. Furthermore, a comparative study, between the proposed algorithm and several other newly developed algorithms, is conducted. The controlled results show obvious enhancement in terms of the convergence speed, and noticeable noise reductions for each engine harmonic over a broader frequency range.

ACKNOWLEDGEMENTS

I would like to express my gratitude towards Dr. Teik C. Lim, for giving me the opportunity to be introduced to the field of active noise control. I thank him for the guidance and support he has provided.

I am thankful to Dr. Mingfeng Li, Dr. Guohua Sun and Mr. Tao Feng for their technical assistance, suggestions and advice provided through the research work. I appreciate the help from all my colleagues at the Vibro-Acoustics and Sound Quality Research Laboratory.

TABLE OF CONTENT

ABSTRACT	II
ACKNOWLEDGEMENTS.....	IV
TABLE OF CONTENT	V
LIST OF TABLES	VII
LIST OF FIGURES	VII
1 INTRODUCTION	1
2 ACTIVE CONTROL OF VEHICLE POWERTRAIN NOISE USING ADAPTIVE NOTCH FILTER WITH INVERSE MODEL LMS ALGORITHM.....	6
2.1 INTRODUCTION	6
2.2 ALGORITHM DEVELOPMENT	7
2.2.1 Existing IMLMS algorithm	7
2.2.2 Proposed ANF-IMLMS algorithm.....	9
2.3 ANALYSIS.....	12
2.3.1 Convergence	12
2.3.2 Computational complexity	13
2.4 NUMERICAL SIMULATION.....	15
2.4.1 Steady State Case	16
2.4.2 Transient Case	20
2.5 CONCLUSIONS	22
3 COMPARATIVE STUDY OF SEVERAL ADAPTIVE ALGORITHMS FOR VEHICLE POWERTRAIN NOISE CONTROL	24
3.1 INTRODUCTION	24
3.2 BACKGROUND	25
3.2.1 Reference signal	25
3.2.2 FXLMS algorithm	26
3.2.3 Frequency dependent convergence behavior.....	28
3.3 MODIFIED ALGORITHMS	30

3.3.1 EE-FXLMS algorithm	30
3.3.2 NX-LMS algorithm	31
3.3.3 TF-FXLMS algorithm	33
3.3.4 ANF-IMLMS algorithm.....	36
3.4 RESULTS AND COMPARISON	38
3.4.1 Computational complexity	39
3.4.2 Convergence rate	41
3.4.3 Vehicle powertrain noise control.....	44
3.4.4 Vehicle powertrain sound tuning	48
3.5 CONCLUSIONS	51
4 CONCLUSIONS	53
REFERENCE.....	55

LIST OF TABLES

<i>Table 1. Computational complexities (operations per sample) of ANF-IMLMS algorithm, IMLMS algorithm and ANF-FXLMS algorithm.....</i>	<i>15</i>
<i>Table 2. Summary of algorithm characteristics.....</i>	<i>38</i>
<i>Table 3. Computational complexities (operations per sample) of various algorithms</i>	<i>40</i>
<i>Table 4. Summary of Controlled Result using the synthesized harmonic signal.....</i>	<i>44</i>

LIST OF FIGURES

<i>Figure 1. Block diagram of active noise control system using IMLMS algorithm.</i>	<i>7</i>
<i>Figure 2. Block diagram of active powertrain noise control system using proposed ANF-IMLMS algorithm.....</i>	<i>10</i>
<i>Figure 3. (a) Frequency response function of the secondary path (b) Impulse response function of the secondary path.</i>	<i>16</i>
<i>Figure 4. Estimated engine rotating speed for Case I and Case II.</i>	<i>18</i>
<i>Figure 5. Comparison of controlled responses for Case I: (a) the 2nd order (b) the 3rd order (c) the 4.5th order (d) spectra of time block data 1.5-2.0sec. (Keys: — baseline response; ANF-FXLMS; - - - ANF-IMLMS).....</i>	<i>19</i>
<i>Figure 6. Comparison of controlled responses for Case II: (a) the 1.5th order (b) the 2nd order (c) the 3rd order (d) spectra of time block data 1.5-2.0sec. (Keys: — baseline response; ANF-FXLMS; - - - ANF-IMLMS).....</i>	<i>20</i>
<i>Figure 7. Estimated engine rotating speed for Case III.</i>	<i>21</i>
<i>Figure 8. Comparison of controlled responses for Case III: (a) the 1.5th order (b) the 2nd order (c) the 3rd order (d) spectra of time block data 5.5-6.0sec. (Keys: — baseline response; ANF-FXLMS; - - - ANF-IMLMS).....</i>	<i>22</i>
<i>Figure 9. Illustration of the powertrain ANC system</i>	<i>25</i>
<i>Figure 10. Block diagram of the vehicle powertrain ANC system using the FXLMS algorithm..</i>	<i>27</i>

Figure 11. Block diagram of the vehicle powertrain ANC system using the EE-FXLMS algorithm	30
Figure 12. Block diagram of the vehicle powertrain ANC system using the NX-LMS algorithm	32
Figure 13. Block diagram of the vehicle powertrain ANC system using the TF-FXLMS algorithm	34
Figure 14. Block diagram of the vehicle powertrain ANC system using the ANF-IMLMS algorithm	36
Figure 15. (a) Frequency response function of the secondary path (b) Impulse response function of the secondary path.	41
Figure 16. Time history of the error signal after control using different algorithms.....	42
Figure 17. Estimated engine rotating speed from tachometer signal.....	45
Figure 18. Comparison of controlled response for engine speed at 4000 RPM: (a) 2nd order (b) 3rd order (c) spectrum of time block data 7.5-8s (Keys: — baseline response; — ANF-FXLMS; — TF-FXLMS; NX-LMS; - - - ANF-IMLMS).....	46
Figure 19. Comparison of controlled response for engine speed running up: (a) 2nd order (b) 3rd order (c) spectrum of time block data 7.5-8s (Keys: — baseline response; — ANF-FXLMS; — TF-FXLMS; NX-LMS; - - - ANF-IMLMS).....	47
Figure 20. Comparison of controlled response for engine speed at 4000 RPM: (a) 2nd order (b) 3rd order (c) spectrum of time block data 7.5-8s (Keys: — baseline response; — ANF-FXLMS; — TF-FXLMS; NX-LMS; - - - ANF-IMLMS; * Target).....	49
Figure 21. Comparison of controlled response for engine speed running up: (a) 2nd order (b) 3rd order (c) spectrum of time block data 7.5-8s (Keys: — baseline response; — ANF-FXLMS; — TF-FXLMS; NX-LMS; - - - ANF-IMLMS; * Target).....	50

1 INTRODUCTION

Vehicle noise, vibration, and harness (NVH) characteristic is one of the most important factors in the design of modern vehicles, as it contributes significantly to the perceived product quality, driving pleasure, and overall customer satisfaction [1-3]. Over the past decade, new challenges have been presented to vehicle NVH engineers – namely, the demand for more fuel efficient vehicles. This challenge has driven manufacturers to pursue lighter weight vehicle designs, which, unfortunately, often leads to more noise being transmitted into the vehicle cabin. In order to compensate for this undesirable effect, engineers have to enhance the application of noise control in these vehicles. Generally there are two categories of noise control techniques that are utilized in a vehicle cabin, namely passive and active control. Passive control techniques reduce interior noise, by modifying the vehicle structure, such as adding mass, tuning stiffness, and/or increasing damping. These methods of passive noise control can be detrimental to the goal of a lighter weight vehicle. Also, passive control is less effective in the lower frequency range. Overall, the pursuit of lighter weight vehicle design limits the ability to apply passive noise control effectively. Therefore, an alternative solution, namely active noise control (ANC), has been developed, to make up for the limitation of passive approaches [4-7]. The fundamental ANC concept is based on the superposition of destructive sound waves, which reduces the undesired noise, by superposing an out-of-phase secondary source on the undesired sound field. While the concept of ANC was proposed in the 1930s [8], it has only drawn the attention of the automotive industry in the last several decades [9-11]. This is mainly due to the evolution of digital signal processors, which have made ANC technology more feasible and affordable for automotive applications.

Most of current ANC systems utilize the traditional filtered-x least mean square (FXLMS) algorithm, or its modified versions. Despite its recent successes, the FXLMS algorithm suffers from a low convergence rate and heavy computational burden [12]. The root cause of the performance degradation is the existence of a secondary path, which is also known as the electro-acoustic path, from the control speaker to the error microphone. To ensure the convergence of the system, the input reference signal is usually filtered by the estimated secondary path model, and the convergence rate is proportional to the power of the filtered input reference signal. Therefore, fast convergence can only be achieved, near the resonance of the secondary path dynamics. This frequency dependent convergence behavior is less of an issue when the system is controlling steady state signals. However, in real driving conditions, the powertrain noise is often transient in nature. For example, in a wide-open-throttle condition, the gas pedal is pressed down to the floor, and the slew rate of engine rotational speed can be greater than one thousand RPM per second. Other transient conditions may involve tip-in/tip-out of the gas pedal, and shifting gears when accelerating/decelerating. For this type of complicated conditions, the ANC systems need to track the variation in noise level quickly and accurately. However, the system with conventional FXLMS algorithm is often unable to perform such task due to the poor tracking ability.

To improve the performance of the ANC systems, a considerable amount of research work has been conducted recently. The FXLMS algorithm was independently derived by Widrow in the context of adaptive control, and Burgess, for ANC applications in 1980 [5, 13, 14]. The Modified-FXLMS (MFXLMS) algorithm was proposed by Bao in 1993 [15]. The MFXLMS algorithm takes advantage of a modification in the structure of the controller, such that, the gradient descent method behaves like the standard LMS algorithm. The MFXLMS algorithm was further improved

by Oliveira in 2010. This was done by adding a normalization filter before the controller. This new algorithm is known as the NX-LMS algorithm [16]. In the NX-LMS algorithm, the effect of the secondary path amplitude variation is nullified, by employing a normalization filter. In 2008 Thomas developed a relatively simpler structure, to improve the convergence behavior of the FXLMS algorithm. This algorithm is known as the eigenvalue equalization filtered-x least mean square (EE-FXLMS) algorithm [17, 18]. The overall convergence behavior is greatly improved by flattening the magnitude response of the estimated secondary path, while leaving the phase unchanged. In 2009, Duan proposed the time-frequency-domain FXLMS (TF-FXLMS) algorithm, which greatly reduces the computational cost, while still maintaining relatively good performance. He has successfully applied it to control the powertrain disturbance [12, 19]. Very recently, Li proposed the inverse model least mean square (IMLMS) algorithm, for harmonic noise control, in which, the inverse model of the secondary path is placed in series with the actual secondary path, to counteract its effect and further improve the convergence [20, 21]. The IMLMS algorithm has been implemented, to control the vehicle powertrain response successfully by Sun [22, 23].

Although the improved algorithms work well for certain application, there are drawbacks for each one of them. For example, the NX-LMS algorithm suffers from the complicated structure and high computational cost. The EE-FXLMS algorithm helps relieve the frequency dependent convergence behavior; but it could not completely eliminate such behavior. Meanwhile, the IMLMS algorithm requires a high order adaptive filter, which increases the computational burden on the processor. Moreover, it is found that, for the IMLMS algorithm, there is still certain discrepancies of convergence speeds at various frequencies. To address the above issues, an enhanced adaptive notch filter with IMLMS (ANF-IMLMS) algorithm, is proposed, as a basis for

active control of powertrain noise. The goal of the ANF-IMLMS algorithm is to incorporate the features of the adaptive notch filter into the IMLMS algorithm, to further reduce the computational cost and improve the system performance. Compared with both the original FXLMS algorithm and the IMLMS algorithm, the proposed ANF-IMLMS algorithm possesses advantages in two aspects: (1) capacity for rapid convergence due to unified convergence behavior across the entire frequency band, and (2) low computational cost since only two weights are required for each harmonic component, and there is no convolution process in the reference signal path. It is also noted that the tracking ability of the control algorithm is enhanced, due to the improved convergence behavior. Therefore, the proposed algorithm is more capable of controlling transient signals in real driving conditions.

In this study, the focus is on powertrain noise, which is one of the major components of vehicle interior noise. It is usually noticeable during vehicle idle condition or during rapid acceleration, so it has to be well treated to ensure a pleasant riding experience. Fortunately, the powertrain noise is dominated by harmonics in the low frequency range. Thus, the proposed ANF-IMLMS algorithm is suitable for the control of such noise.

In Chapter 2, the derivation of the proposed ANF-IMLMS algorithm is presented. The advantage of the new algorithm is analyzed theoretically. The convergence analysis is conducted by analyzing the input autocorrelation matrix and the eigenvalue spread of the input matrix. The computational complexity of the new algorithm is compared with that of the original IMLMS algorithm, and the adaptive notch filter with FXLMS (ANF-FXLMS) algorithm. To verify the effectiveness of the new algorithm, numerical simulations are performed, utilizing measured powertrain data. The data was recorded on a test vehicle for three cases. The engine speed stays

around 2000 rpm and 4000 rpm for the first two cases, and it increases from 1500 rpm to 5500 rpm for the third case. The control results of new algorithm are compared to that of the conventional ANF-FXLMS algorithm, to demonstrate the advantage of the new system. Part of this work is presented at the SAE Noise and Vibration Conference in 2015.

In Chapter 3, a comparative study of several selected algorithms is conducted. Although the ANF-IMLMS algorithm has been successfully applied to control the powertrain noise, the performance of the new algorithm is only compared to that of the traditional ANF-FXLMS algorithm. In this chapter, the control diagram of the EE-FXLMS algorithm, the NX-LMS algorithm, the TF-IMLMS algorithm, and the ANF-IMLMS algorithm are presented. The control results of all the algorithms are compared against each other, applying synthesized sinusoid signal. Several selected algorithms are then further compared, utilizing measured powertrain data. The salient features of each algorithm are analyzed along with the control results. Moreover, the concept of active noise control is extended to active sound quality control (ASQC) applying measured data, in which the individual engine orders are enhanced or attenuated to achieve the desired sound quality.

2 ACTIVE CONTROL OF VEHICLE POWERTRAIN NOISE USING ADAPTIVE NOTCH FILTER WITH INVERSE MODEL LMS ALGORITHM

2.1 Introduction

In this chapter, the ANF-IMLMS algorithm is developed based on the original IMLMS algorithm. The newly proposed algorithm is applied to control the powertrain noise under three different conditions: constant engine speed at low and high RPM, and varying engine speed condition. The control results are studied and compared with that of the conventional ANF-FXLMS algorithm. This chapter is organized as follows. Firstly, the original IMLMS is introduced in Section 2.2. Following is the basic configuration and the development of the proposed ANF-IMLMS algorithm. Theoretical equations are also derived. Section 2.3 presents a thorough analysis of the new algorithm, including the convergence analysis and computational complexity analysis. Compared with the conventional algorithm, the proposed algorithm possesses advantages in two aspects: (1) capacity for rapid convergence, and (2) low computational cost. The convergence speed is improved by enabling unified convergence behavior across different frequency components. The reduction of computational cost is achieved by reducing the filter length and eliminating the convolution process needed to generate the filtered reference signal. Finally, simulation results for measured powertrain data are presented in Section 2.4.

2.2 Algorithm Development

2.2.1 Existing IMLMS algorithm

Figure 1 shows the basic configuration of the ANC system using the IMLMS algorithm for general harmonic noise control, which is proposed by Li [20, 21]. In order to nullify the effect of the secondary path, an inverse model of the secondary path is placed in the reference signal path.

Figure 1. Block diagram of active noise control system using IMLMS algorithm.

Like many other tonal noises, powertrain noise contains a large number of harmonic components whose frequencies are proportional to the engine's rotational speed. Thus, the IMLMS algorithm is suitable for controlling such noise. In this case, a tachometer is used to measure the disturbance frequency, which is the speed of engine crankshaft. The estimated engine speed is then fed to a harmonic wave generator, to synthesize a series of sinusoidal signals. This signal serves as the reference signals. The reference signal at time n can be expressed as follows:

$$x_0(n) = a_i \cos(\omega_i n / f_s) = a_i \cos(\omega_i' n) \quad (1)$$

$$x_1(n) = a_i \sin(\omega_i n / f_s) = a_i \sin(\omega_i' n) \quad (2)$$

where a_i is the amplitude of the i^{th} order reference signal, ω_i is the i^{th} disturbance frequency, and f_s is the sampling frequency. ω_i is usually submultiples or multiples of the engine rotating speed (i.e., $\omega_i = 2\pi v/60$, v is the engine speed in rpm (revolution per minute)), and ω_i' is the generalized frequency. The objective of the ANC system is to minimize the square of residual noise at the target frequency.

Generally, the inverse model does not exist for a broadband noise. This is because the secondary path is normally a non-minimum phase system. However, the inverse model can be easily obtained for a harmonic noise. The following is to recap the derivation of the IMLMS algorithm. Assuming a single harmonic frequency, the response of the secondary path $S(z)$ and control filter $W(z)$ at a specific frequency (ω) can be represented by a gain and a phase shift:

$$S(z) = g e^{j\theta} \quad (3)$$

$$W(z) = g_0 e^{j\theta_0} \quad (4)$$

where g and θ are the gain and phase of the secondary path model at frequency ω , g_0 and θ_0 are the gain and phase of the controller at the same frequency, and $z = e^{j\omega}$. It is noted that the output filtered by the control filter and the inverse model of the secondary path, is still a sinusoidal signal with the same frequency as the reference, but with certain phase shifted and magnitude modified. Assuming the input signal is $x_1(n) = \sin(\omega n)$, and then the output should be $y'(n) = g_0 \sin(\omega n + \theta_0 - \theta)/g$. With the prior knowledge of estimated secondary path, one can easily obtain the output $y'(n)$, by manipulating input reference signal, through multiplying constants A and B obtained from the estimated secondary path:

$$A = \cos(\theta)/g \quad (5)$$

$$B = \sin(\theta)/g \quad (6)$$

$$\begin{aligned} y'(n) &= g_0 \sin(\omega n + \theta_0)A - g_0 \cos(\omega n + \theta_0)B \\ &= g_0 \sin(\omega n + \theta_0 - \theta)/g \end{aligned} \quad (7)$$

The filter weight update equation can be summarized as follows:

$$e(n) = d(n) - y(n) = d(n) - \mathbf{W}^T(n)\mathbf{X}_1(n) \quad (8)$$

$$\begin{aligned} \mathbf{W}(n+1) &= \mathbf{W}(n) - \mu \frac{\partial e^2(n)}{\partial \mathbf{W}(n)} \\ &= \mathbf{W}(n) + 2\mu e(n)\mathbf{X}_1(n) \end{aligned} \quad (9)$$

where $y(n)$ is the secondary canceling wave, μ is the convergence parameter, $e(n)$ is the residual noise, the filter weight vector $\mathbf{W}(n)$ is denoted as: $\mathbf{W}(n) = [w_0(n) \ w_1(n) \ \dots \ w_{L-1}(n)]^T$ and reference signal vector is $\mathbf{X}_1(n) = [x_0(n) \ x_1(n-1) \ \dots \ x_1(n-L+1)]^T$. L is the order of the control filter. From above equations, it can be seen that the secondary path effect is completely removed from the output. Furthermore, only the basic LMS algorithm is used, instead of using a filtered reference.

2.2.2 Proposed ANF-IMLMS algorithm

Previous section shows the derivation of the IMLMS algorithm, and the IMLMS algorithm has demonstrated its effectiveness for canceling powertrain noise [22, 23]. However, the implementation of this algorithm (as shown in Figure 1) requires a high order (L) adaptive filter. This is because it only utilizes the in-phase component of the sinusoidal reference signal. The

higher order filter also adds more computational burden on the processor. This section describes a new powertrain noise ANC system using the proposed ANF-IMLMS algorithm. Figure 2 shows the configuration of the control system with the new ANF-IMLMS algorithm. Note this control diagram is only for a single harmonic noise control. Multiple harmonic control case can be easily realized, by stacking the single harmonic control configuration in parallel form. This is because that the harmonic components at different frequencies are uncorrelated, and can be processed separately.

Figure 2. Block diagram of active powertrain noise control system using proposed ANF-IMLMS algorithm.

In this control diagram, the secondary path model $S(z)$, constants A and B , and reference signals $x_0(n)$ and $x_1(n)$ are the same as those in the configuration with the IMLMS algorithm. To achieve a faster convergence, the quadrature counterpart of the original output, is added to the system. The constants determined from the inverse model secondary path, are directly employed to manipulate the reference signals before the control filter. Instead of using an adaptive filter vector of length L , the new algorithm only requires two weights for the adaptive filter: $w_0(n)$ and

$w_1(n)$. Therefore, the proposed algorithm requires much less computational time as compared to the IMLMS algorithm.

Referring to the block diagram shown in Figure 2, $x_0'(n)$ and $x_1'(n)$ are the reference signals filtered by the inverse model of the secondary path. $y(n)$ is the secondary canceling wave. They can be expressed as follows:

$$x_0'(n) = Ax_0(n) + Bx_1(n) = \cos(\omega n - \theta)/g \quad (10)$$

$$x_1'(n) = -Bx_0(n) + Ax_1(n) = \sin(\omega n - \theta)/g \quad (11)$$

$$\begin{aligned} y(n) &= [w_0(n)x_0'(n) + w_1(n)x_1'(n)] * S(z) \\ &= w_0(n)x_0(n) + w_1(n)x_1(n) \end{aligned} \quad (12)$$

where $*$ is the notation for convolution in the time domain. It is noted that $y(n)$ contains both the in-phase and quadrature components of the target frequency, and the secondary path effect is completely removed. Similarly, the residual noise and weight update equation can be simplified as follows:

$$\begin{aligned} e(n) &= d(n) - y(n) \\ &= d(n) - w_0(n)x_0(n) - w_1(n)x_1(n) \end{aligned} \quad (13)$$

$$\begin{aligned} w_0(n+1) &= w_0(n) - \mu \frac{\partial e^2(n)}{\partial w_0(n)} \\ &= w_0(n) + 2\mu e(n)x_0(n) \end{aligned} \quad (14)$$

$$\begin{aligned} w_1(n+1) &= w_1(n) - \mu \frac{\partial e^2(n)}{\partial w_1(n)} \\ &= w_1(n) + 2\mu e(n)x_1(n) \end{aligned} \quad (15)$$

2.3 Analysis

2.3.1 Convergence

In this section, the convergence behavior of the proposed ANF-IMLS algorithm is evaluated. It is well known that convergence speed of the algorithm is determined, by the eigenvalue spread of the input autocorrelation matrix. Therefore, to investigate the convergence behavior, the input autocorrelation matrix needs to be introduced first:

$$\mathbf{R} = E(\mathbf{X}(n)\mathbf{X}^T(n)) \quad (16)$$

where $E(\)$ is the mathematical expectation. The $\mathbf{R}$ matrix represents the expectation of the product of input vectors. The eigenvalue spread, denoted by ρ , is defined as the ratio between the largest and the smallest eigenvalues of the input autocorrelation matrix.

$$\rho = \frac{\lambda_{max}}{\lambda_{min}} \quad (17)$$

If we assume unity amplitude ($a_i = 1$) for the reference signal, the input vector for each frequency component can be described as follows:

$$\mathbf{X}(n) = [x_0(n) \ x_1(n)]^T = [\cos(\omega_i'n) \ \sin(\omega_i'n)]^T \quad (18)$$

Substitute Eq. (18) into Eq. (16), one can obtain:

$$\begin{aligned}
\mathbf{R} &= E \begin{bmatrix} \cos(\omega_i' n)^2 & \cos(\omega_i' n) \sin(\omega_i' n) \\ \cos(\omega_i' n) \sin(\omega_i' n) & \sin(\omega_i' n)^2 \end{bmatrix} \\
&= \frac{1}{2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
\end{aligned} \tag{19}$$

It is noted from Eq. (19) that the off diagonal terms of the $\mathbf{R}$ matrix are all zeros, which is due to the orthogonality of the in-phase and quadrature components of the reference signal. The eigenvalues: λ_1 and λ_2 of the $\mathbf{R}$ matrix are identical. Therefore, the eigenvalue spread (ρ) equals 1 for individual sinusoidal frequency, which leads to a fast convergence behavior of the proposed algorithm. The time constant of the adaption is approximated as shown in Eq. (20), detailed derivation can be found in Ref. [5].

$$\tau_{mse} \leq \frac{\lambda_{max}}{\lambda_{min}} T = T \tag{20}$$

where the upper bond of time constant τ_{mse} is only related to the sampling period T , and there is no specific frequency information or secondary path information presented in Eq. (20). Thus, unified convergence speed is expected for all the harmonic components, and one optimum step size is made possible.

2.3.2 Computational complexity

This section evaluates the computational complexities of the proposed ANF-IMLMS algorithm, the IMLMS algorithm and the ANF-FXLMS algorithm. The detailed derivation of the conventional ANF-FXLMS can be found in Ref. [5]. This study is based on comparing the number of real multiplications and real additions required to generate one sample output for each harmonic

component. Two parameters are very important in this analysis: the order of adaptive filter (L), and the order of estimated secondary path model (M).

For the proposed ANF-IMLMS algorithm, the order of adaptive weight (L) is always 2. One is for in-phase reference signal and the other is for the quadrature counterpart. The computational requirement to calculate one sample output is 5 real additions and 8 real multiplications, which can be broken down as follows. According to Eqs. (10) and (11), 2 additions and 4 multiplications are required to generate the filtered reference signal. Another 2 additions and 2 multiplications are needed for updating the filter weights as shown in Eqs. (14) and (15). Furthermore, 1 addition and 2 multiplications are required to calculate the final output per Eq. (12).

To generate one sample output, the computation required by the IMLMS algorithm includes $(2L)$ real additions and $(2L+2)$ real multiplications. The requirement can be broken down into follows: 1 addition and 2 multiplications for generating the filtered reference per Eq. (7), L additions and L multiplications are needed to update the filter weight according to Eq. (9) and finally, $(L-1)$ additions and L multiplications for calculating the final output as shown in Eq. (8).

The computational requirement for the conventional ANF-FXLMS is analyzed similarly. It is noted that the reference signal needs to be filtered by the secondary path model for the conventional FXLMS algorithm, which is realized by a convolution process in the controller. To generate the filtered reference, $2(M-1)$ additions and $2M$ multiplications are needed. The rest is similar to the proposed ANF-IMLMS algorithm. Another 2 additions and 2 multiplications are needed for updating the filter weight; 1 addition and 2 multiplications are required to calculate the

final output. Thus, the conventional ANF-FXLMS requires a total of $(2M+1)$ additions and $(2M+4)$ multiplications.

Table 1. Computational complexities (operations per sample) of ANF-IMLMS algorithm, IMLMS algorithm and ANF-FXLMS algorithm.

	ANF-IMLMS	IMLMS	ANF-FXLMS
Additions	5	$2L$	$2M+1$
Multiplications	8	$2L+2$	$2M+4$

Clearly, the amount of computation saved by the proposed ANF-IMLMS algorithm depends on the order of adaptive filter (L) when compared to the IMLMS algorithm, and the order of secondary path model (M) when compared to the ANF-FXLMS algorithms. Table 1 summarizes the complexity (operations per sample) for each of the algorithms mentioned above. The order of estimated secondary path (M) is set to be 256 in this study for numerical simulation. Even though the order of adaptive filter (L) and the order of secondary path model (M) are user-selective, L and M need to be above certain level to ensure the convergence of algorithms.

2.4 Numerical Simulation

In order to evaluate the performance of the proposed algorithm, numerical simulations are performed, applying measured powertrain data. Both the proposed ANF-IMLMS algorithm and the conventional ANF-FXLMS algorithm are tested and compared. The primary powertrain disturbance and tachometer signal were measured on a test vehicle at sampling frequency of 4096 Hz. Three different cases are studied. The first two cases are run under the steady state condition, when the engine speed is kept constant. The last case is run under a transient condition when the

engine speed increases gradually. The simulations are run in Matlab/Simulink environment. The error microphone is placed at driver's head position, and the control speaker is placed in the driver side door panel. The transfer function from the control speaker to the error microphone measured in the test vehicle, and this function is used as the secondary path model. In the simulation, it is assumed that the estimated secondary path model is the same as the real one. The estimated secondary path is modeled as a 256-tap finite impulse response filter. This step is usually implemented through off-line system identification, before the active noise control system is turned on. Figure 3 (a) and (b) show the frequency response function (FRF) and impulse response function (IRF) of this estimated secondary path, respectively. For demonstration purposes, only single input single output (SISO) ANC system is shown here. However, it can be extended to a multiple input and multiple output (MIMO) ANC system.

Figure 3. (a) Frequency response function of the secondary path (b) Impulse response function of the secondary path.

2.4.1 Steady State Case

The first two cases involve active control of powertrain noise under the steady state condition. Figure 4 shows the estimated engine speed, which is kept at around 2000 rpm for Case

I, and around 4000 rpm for Case II. The effective range of ANC system is limited in the low frequency range. For demonstration purposes, the goal for Case I is to reduce the 2nd order, the 3rd order and the 4.5th order response as much as possible. For each algorithm, the step size is set to be the largest value while still remains stability. Figure 5 (a)-(c) show the time histories of controlled results and baseline responses, for the 2nd, the 3rd and the 4.5th order, respectively. Solid line represents the baseline response when the ANC system is inactivated, dashed line represents the controlled result using proposed ANF-IMLMS algorithm, and dotted line represents that of the conventional ANF-FXLMS algorithm. Both algorithms can reduce the noise at target orders effectively. Difference is that the conventional system needs more time to converge and yield the best performance for lower orders. However, the proposed algorithm has unified performance for all the orders. Figure 5 (d) shows the frequency spectra of baseline response and controlled responses, using time block data from 1.5 to 2.0 second. It can be observed that the proposed algorithm can suppress the noise to background level, at all target order within the first second. Meanwhile, the conventional algorithm can only do that for the 3rd order and the 4.5th order; it requires more time to converge for lower orders at which the magnitude responses of secondary path is smaller. Figure 6 shows the simulation results for the Case II. Similarly, Figure 6 (a)–(c) show the time histories of individual orders, and Figure 6 (d) shows the frequency spectra using time block data from 1.5 to 2.0 second. The goal for this case is to reduce the 1.5th, the 2nd and the 3rd order responses. Very similar result can be observed. The conventional ANF-FXLMS algorithm is less effective at lower orders where the magnitude response of secondary path is smaller. The results, again, verified that the proposed algorithm can eliminate the frequency dependent behavior and achieve better overall performance.

Figure 4. Estimated engine rotating speed for Case I and Case II.

Figure 5. Comparison of controlled responses for Case I: (a) the 2nd order (b) the 3rd order (c) the 4.5th order (d) spectra of time block data 1.5-2.0sec. (Keys: — baseline response; ANF-FXLMS; - - - ANF-IMLMS)

Figure 6. Comparison of controlled responses for Case II: (a) the 1.5th order (b) the 2nd order (c) the 3rd order (d) spectra of time block data 1.5-2.0sec. (Keys: — baseline response; ANF-FXLMS; --- ANF-IMLMS)

2.4.2 Transient Case

Figure 7 shows the estimated engine speed for Case III, in which the engine speed ramps up from 1500 rpm to 5500 rpm in ten seconds. Figure 8 (a)-(d) show the simulation results before and after control using the various algorithms. In this case, the powertrain disturbance and the secondary path for each order are time-varying due to the increasing speed. The results also indicate the tracking ability of each algorithm. One can expect the algorithm with better tracking

ability to achieve more reduction of powertrain noise. Similarly, Figure 8 (a)-(c) show the time histories of responses for the 1.5th, the 2nd and the 3rd order, respectively. Figure 8 (d) represents the frequency spectra, using time block data from 5.5 to 6.0 second. As can be seen from Figure 8 (a)-(c), the proposed algorithm and conventional algorithm have relative competitive performance for the 3rd order. However, for the 1.5th order and the 2nd order, the conventional ANF-FXLMS algorithm can't achieve any reduction until certain speed, which is unacceptable and indicates the poor tracking ability of the conventional algorithm. The corresponding result is shown in Figure 8 (d), the proposed algorithm can achieve about 10 dB more reduction at the 1.5th order and 5 dB more reduction at the 2nd order, than the conventional algorithm.

Figure 7. Estimated engine rotating speed for Case III.

Figure 8. Comparison of controlled responses for Case III: (a) the 1.5th order (b) the 2nd order (c) the 3rd order (d) spectra of time block data 5.5-6.0sec. (Keys: — baseline response; ANF-FXLMS; --- ANF-IMLMS)

2.5 Conclusions

The study develops and presents a novel adaptive notch filter with inverse model least mean square algorithm for active control of powertrain noise. Based on the analysis, the new algorithm provides a faster convergence speed. This is because the secondary path effect is eliminated completely, and the eigenvalue spread of the input correlation matrix is always unity for each frequency component. Another advantage of the new algorithm is the reduced computational cost since only an adaptive notch filter is used, and the convolution process for the

generation of the filtered reference signal is eliminated. To verify the effectiveness of the proposed algorithm, numerical simulations are performed, applying measured powertrain data. Both constant and run-up engine speed cases are analyzed. Results demonstrate, that the ANC system configured with the proposed algorithm, has faster overall convergence behavior and better tracking ability than the one with conventional algorithm.

3 COMPARATIVE STUDY OF SEVERAL ADAPTIVE ALGORITHMS FOR VEHICLE POWERTRAIN NOISE CONTROL

3.1 Introduction

In Chapter 2, the adaptive notch filter with inverse model least mean square (ANF-IMLMS) algorithm was successfully applied to control the powertrain noise. Compared to the conventional adaptive notch filter with filtered-x least mean square (ANF-FXLMS) algorithm, the new algorithm can effectively reduce the noise level in a broader frequency range. However, the comparison between the ANF-IMLMS algorithm and other newly proposed algorithms remains unknown. In this chapter, a comparative study is conducted. Several advanced algorithms are introduced, and the performance of each algorithm is studied by comparing the controlled results. This chapter is organized as follows. First, background information about active noise control and the basic configuration of the conventional FXLMS algorithm are presented in Section 3.2. Then, several recently developed adaptive algorithms, such as the EE-FXLMS algorithm, the NX-LMS algorithm, the TF-FXLMS algorithm, and the ANF-IMLMS algorithm, are introduced in Section 3.3. In Section 3.4, the advantages and disadvantages of aforementioned algorithms, in terms of the convergence speed and computational complexity, are analyzed. Several numerical simulations are conducted to verify the analysis. The algorithms are applied numerically to control harmonic responses first. Then, the ANC systems configured with various algorithms are implemented to control the powertrain noise. Moreover, the concept of ANC is extended to the active sound quality control (ASQC), in which one engine order is targeted to be enhanced and one order to be reduced. Finally, conclusions of the study and suitable recommendations are made in Section 3.5.

3.2 Background

3.2.1 Reference signal

In general, the noise inside a vehicle cabin can be divided into two categories. The first category is powertrain noise. This noise is synchronized with the engine's rotation and is the noise that is usually noticeable during vehicle's idle condition and during rapid acceleration. Good control of this noise is necessary in order to ensure an enjoyable operating experience. The other noise category is the noise caused by random sources, such as road surface irregularities and turbulent airflow.

Figure 9. Illustration of the powertrain ANC system

Figure 9 shows the flow diagram of the ANC system for powertrain noise. Powertrain noise is dominated by the harmonics of the engine's rotational speed. The dominant harmonic components depend on the number of cylinders and the firing order of the cylinders. Coherent reference signals can be synthesized using the estimated engine rotational speed, which is

measured by a tachometer. Reference signals can then be created by a harmonic wave generator, which can be expressed as:

$$x_0(n) = a_i \cos(\omega_i n / f_s) = a_i \cos(\omega_i' n) \quad (21)$$

$$x_1(n) = a_i \sin(\omega_i n / f_s) = a_i \sin(\omega_i' n) \quad (22)$$

where a_i is the amplitude of the i^{th} order reference signal, ω_i is the i^{th} disturbance frequency, and f_s is the sampling frequency. ω_i is usually submultiples or multiples of the engine rotating speed (i.e., $\omega_i = 2\pi i v / 60$, v is the engine speed in rpm (revolution per minute)), and ω_i' is the generalized frequency. The objective of the ANC system is to minimize the square of residual noise at the target frequency.

3.2.2 FXLMS algorithm

Figure 10 shows the configuration of basic FXLMS algorithm applied to control the powertrain noise. Detailed analysis of the FXLMS algorithm can be found in reference [5].

Figure 10. Block diagram of the vehicle powertrain ANC system using the FXLMS algorithm

From Figure 10, it can be derived that the residual noise $e(n)$ can be expressed as,

$$e(n) = d(n) - y(n) \quad (23)$$

$$y(n) = \mathbf{S}(n) * [\mathbf{W}^T(n)\mathbf{X}(n)] \quad (24)$$

where n is the time index, $d(n)$ is the primary undesired noise, which is the powertrain noise in this case. $S(n)$ is the impulse response of the secondary path $S(z)$, and $*$ denotes linear convolution. The adaptive filter weight and the reference signal are

$$\mathbf{W}(n) = [w_0(n) \ w_0(n) \ \cdots \ w_{L-1}(n)]^T \quad (25)$$

$$\mathbf{X}(n) = [x(n) \ x(n-1) \ \cdots \ x(n-L+1)]^T \quad (26)$$

The objective is to minimize the mean square error (MSE), which can be estimated as follows:

$$\xi(n) = E[e^2(n)] \quad (27)$$

where $E[\cdot]$ represents the mathematical expectation. The MSE $\xi(n)$ can be minimized using steepest descend method, which updates the adaptive weight vector in the negative gradient direction:

$$\mathbf{W}(n+1) = \mathbf{W}(n) + \mu \mathbf{X}'(n)e(n) \quad (28)$$

where $\mathbf{X}'(n) = \mathbf{S}(n) * \mathbf{X}(n)$, it can be seen that the adaptive filter weight is updated by filtered reference signal $\mathbf{X}'(n)$, therefore it's named filtered-x LMS algorithm.

3.2.3 Frequency dependent convergence behavior

Feintuch developed a frequency-domain FXLMS algorithm in 1993, for which the convergence behavior can be analyzed more intuitively [24]. It is reported that the step size of adaption (μ), needs to meet the following stability condition:

$$\mu P_x(\omega) < \frac{2}{|S(\omega)|^2} \quad (29)$$

where $P_x(\omega)$ is the power of the original reference signal, at frequency ω . $|S(\omega)|$ is the amplitude of the secondary path, at frequency ω . In general, the upper bound on μ is inversely proportional to the power of the filtered reference signal. Therefore, even if the power of the original reference signal is made uniform across different frequencies, the secondary path model will still affect the upper bond of the step size, causing certain discrepancies at various frequencies. If a fixed step size is used, components at different frequencies will exhibit various convergence behavior.

$$\tau_{mse} \leq \frac{\lambda_{max}}{\lambda_{min}} T \leq \frac{\max_{\omega} |X(e^{j\omega})|^2}{\min_{\omega} |X(e^{j\omega})|^2} T \quad (30)$$

Similar results can be obtained from making a time domain analysis. It is reported that the overall convergence rate is determined by the dynamic range of the autocorrelation matrix of the filtered reference signal [25]. Equation 30 shows the relationship between the convergence time and the dynamic range of the eigenvalue. λ_{max} and λ_{min} are the maximum and the minimum eigenvalues of the autocorrelation matrix. T is the sampling period, and τ is the time constant, which is the time that is required for any signal to decay to $1/e$ ($\approx 37\%$) of its initial value. Due to the fact that MSE is a squared value, τ_{mse} is the time required for the MSE to be reduced by a factor of e^2 . Therefore, the larger the dynamic range, the longer it takes for the ANC system to converge.

Both the time domain and the frequency domain analysis come to the same conclusion, that the overall convergence is limited by the frequency component, which possesses a smaller secondary path amplitude. If the FXLMS algorithm is directly applied to control the powertrain response, the engine orders with larger secondary path amplitudes can be reduced in a more effective manner. However, the system will have minimal effect on the orders with smaller secondary path amplitudes. As a result, unbalance performance is expected. Recently, many researchers and engineers have developed modified algorithms, to improve ANC system performance. Several advanced algorithms are selected and introduced in the following section.

3.3 Modified Algorithms

3.3.1 EE-FXLMS algorithm

Thomas et al. developed the EE-FXLMS algorithm in 2008 [17, 18]. The basic configuration of the EE-FXLMS algorithm for vehicle powertrain response is shown in Figure 11. Compared to other modified algorithms, the major advantage of the EE-FXLMS algorithm is the simplicity of the structure. This algorithm is essentially a FXLMS algorithm, with minor modifications. The effectiveness of the EE-FXLMS algorithm comes from reducing the dynamic range of the eigenvalue of the auto-correlation matrix. The convergence time is proportional to the dynamic range of the eigenvalue spread, as indicated in Equation 30. Therefore, reducing the dynamic range can effectively reduce the convergence time. The eigenvalue-equalization process, which is highlighted in Figure 11, can be used to reduce the dynamic range. It is implemented in the following steps[17]:

Figure 11. Block diagram of the vehicle powertrain ANC system using the EE-FXLMS algorithm

1. Get the time-domain response $\mathbf{S}(n)$ of the secondary path through system identification.
2. Perform fast Fourier transform (FFT) on $\mathbf{S}(n)$ to obtain the frequency response.
3. Divide each frequency response value by its magnitude.
4. Perform inverse fast Fourier transform (IFFT) to get a new $\mathbf{S}(n)$ and use it as normal.

This process successfully flattens the magnitude response of the estimated secondary path model $\hat{S}(z)$, which is used in the system. However, the magnitude response of the actual secondary path $S(z)$ still exists, which leads to discrepancies in the convergence rates at various frequencies. Essentially, the EE-FXLMS algorithm reduces the upper bound of the time constant τ_{mse} , from the second order of the ratio $(\frac{\max_{\omega} |X(e^{j\omega})|}{\min_{\omega} |X(e^{j\omega})|})$, multiplied by the sampling period to the first order, which effectively reduces the convergence time. However, the process does not completely remove the frequency dependent convergence behavior.

3.3.2 NX-LMS algorithm

The NX-LMS algorithm is another alternative used to overcome the secondary path dynamics. It was proposed by Oliveira [16] in 2010, based on the modified FXLMS (MFXLMS) algorithm. The concept of MFXLMS algorithm is to modify the structure of the standard FXLMS algorithm, using a modified error. Thus, the gradient descent method behaves like a standard LMS schemes. Moreover, it is reported that the MFXLMS algorithm is as tolerable as the FXLMS algorithm, in terms of the secondary path modeling error. For example, the phase angle of the estimated secondary path $\hat{S}(z)$ should be within $\pm 90^\circ$ of the actual one. Thus, the robustness of the FXLMS algorithm is retained in the MFXLMS algorithm. Based on the MFXLMS algorithm, Oliveira added another normalization filter, in the form of the scheduled gain N , to eliminate the

effect of the secondary path amplitude variations, which forms the NX-LMS algorithm. The basic configuration of the NX-LMS algorithm for vehicle powertrain response is shown in Figure 12. When applying the NX-LMS algorithm, additional computational cost is needed, which is one of the disadvantages of the NX-LMS algorithm. Compared to the conventional FXLMS algorithm, the NX-LMS algorithm requires additional steps, to generate the modified error, which includes a convolution process and two extra additions. The convolution process of the controller output $u(n)$, and the estimated secondary path $\hat{S}(z)$, is used to estimate the actual secondary source, which is received at the error microphone location.

Figure 12. Block diagram of the vehicle powertrain ANC system using the NX-LMS algorithm

3.3.3 *TF-FXLMS algorithm*

The capability of ANC system is often limited by the computational power of digital signal processor, especially for complex multi-input multi-output (MIMO) system. Therefore, to improve the system performance, some computationally-efficient algorithms have been proposed. For example, a frequency domain implementation of the FXLMS algorithm has been developed [26]. One critical shortcoming of frequency domain approach is the block delay between the input reference signal and secondary canceling wave. Therefore, this approach cannot be used to control broadband random noise such as road noise. To address this drawback, a delayless subband adaptive filtering technique was proposed by Morgan and Thi in 1995 [27, 28]. In the delayless implementation, the gradient estimate used to update the filter coefficients is calculated in the frequency domain, however, the actual filtering process to calculate the secondary canceling wave is implemented in the time domain.

Figure 13. Block diagram of the vehicle powertrain ANC system using the TF-FXLMS algorithm

Similar idea was described by Duan to develop the time-frequency-domain FXLMS (TF-FXLMS) algorithm, and it was successfully implemented to control the powertrain response [12]. Figure 13 shows the block diagram of the TF-FXLMS algorithm proposed by Duan with overlap-save implementation. The overlap-save implementation is a method to convert circular convolution to linear convolution [29]. In this study, a 50% overlap is used. Therefore, for one update, N samples of new reference signal and error signal are accumulated in the buffer to form the $2N$ data vectors.

$$\mathbf{x}(k) = [x(kN - N + 1) \ x(kN - N + 2) \ \cdots \ x(kN)]^T \quad (31)$$

$$\mathbf{e}(k) = [e(kN - N + 1) \ e(kN - N + 2) \ \cdots \ e(kN)]^T \quad (32)$$

For input reference signal, the overlap-save implementation keeps the previous block of N samples and pads them with the new data to form the $2N$ vector, expressed as $[\mathbf{x}(\mathbf{k} - \mathbf{1}) \ \mathbf{x}(\mathbf{k})]^T$. However, for the error signal, it pads N -point zero data to form the $2N$ vector, which can be expressed as $[\mathbf{0} \ \mathbf{e}(\mathbf{k})]^T$. Then, the reference signal and error signal are transformed into frequency domain by $2N$ point FFT.

$$\mathbf{X}(k) = [X_0(k) \ X_1(k) \ \cdots \ X_{2N-1}(k)]^T = FFT([\mathbf{x}(k - 1) \ \mathbf{x}(k)]^T) \quad (33)$$

$$\mathbf{E}(k) = [E_0(k) \ E_1(k) \ \cdots \ E_{2N-1}(k)]^T = FFT([\mathbf{0} \ \mathbf{e}(k)]^T) \quad (34)$$

Another advantage of TF-FXLMS algorithm and other frequency domain algorithms is the ability to decouple and isolate the reference signal spectral components into separate frequency bins. Convergence speed can be improved by setting an individual step size for each bin to be inversely proportional to the signal power at that bin.

$$\mathbf{W}(k + 1) = \mathbf{W}(k) + \hat{\mu} \overline{\mathbf{S}(k) \mathbf{X}(k)} \mathbf{E}(k) \quad (35)$$

$$\hat{\mu}_m = \frac{\mu}{|X_m \hat{S}_m|^2} \quad (36)$$

where $\overline{(\cdot)}$ denotes the complex conjugate operation, $\mathbf{S}(k)$ is transformed from the secondary path transfer function $\mathbf{S}(n)$ by the FFT process, $\hat{\mu}_m$ is the step size for the m^{th} frequency bin, $|\cdot|$ represents the norm operator, and $X_m \hat{S}_m$ represents the filtered reference signal in the corresponding m^{th} frequency bin.

3.3.4 ANF-IMLMS algorithm

The inverse model least mean square (IMLMS) algorithm was first developed by Li et al. [20, 21] for harmonic noise control. Generally, there are two approaches that are used to compensate for the effect of the secondary path [30]. The first is to place an identical filter in the reference signal path to the weight update of the LMS algorithm, which realizes the FXLMS algorithm. Another approach is to place an inverse filter, $1/S(z)$, in series with $S(z)$, to remove its effect. For the broadband noise, the inverse model does not necessarily exist. Fortunately, it can be easily realized for narrowband noise. In Chapter 2, the adaptive notch filter with inverse model least mean square (ANF-IMLMS) algorithm was proposed to control the powertrain disturbance. The goal of ANF-IMLMS algorithm is to incorporate the features of the adaptive notch filter into the IMLMS algorithm, to further reduce the computational cost and improve the system performance.

Figure 14. Block diagram of the vehicle powertrain ANC system using the ANF-IMLMS algorithm

Figure 14 shows the block diagram of the ANF-IMLMS algorithm for powertrain noise.

To realize the inverse model, two constants A and B are used. The constants are expressed as:

$$A = \cos(\theta)/g \quad (37)$$

$$B = \sin(\theta)/g \quad (38)$$

where g and θ represent the gain and phase of the secondary path $S(z)$ at target frequency. As show in Figure 14, $x_0'(n)$ and $x_1'(n)$ are the reference signals filtered by the inverse model of the secondary path. $y(n)$ is the secondary canceling wave. They can be expressed as follows:

$$x_0'(n) = Ax_0(n) + Bx_1(n) = \cos(\omega n - \theta)/g \quad (39)$$

$$x_1'(n) = -Bx_0(n) + Ax_1(n) = \sin(\omega n - \theta)/g \quad (40)$$

$$\begin{aligned} y(n) &= [w_0(n)x_0'(n) + w_1(n)x_1'(n)] * S(z) \\ &= w_0(n)x_0(n) + w_1(n)x_1(n) \end{aligned} \quad (41)$$

where $*$ is the notation for linear convolution. It's noted that $y(n)$ contains both the in-phase and quadrature components of the target frequency and the secondary path effect is completely removed. The residual noise and weight update equations can be expressed as follows:

$$e(n) = d(n) - w_0(n)x_0(n) - w_1(n)x_1(n) \quad (42)$$

$$w_0(n+1) = w_0(n) + 2\mu e(n)x_0(n) \quad (43)$$

$$w_1(n+1) = w_1(n) + 2\mu e(n)x_1(n) \quad (44)$$

3.4 Results and Comparison

This section compares the aforementioned algorithms from two aspects: computational complexity and convergence rate. The computational complexity of the various algorithms is studied by comparing the number of operations that are needed to generate one sample output for one frequency component, while the convergence rate is studied by comparing how fast the system can reduce the undesired noise, and the level of the mean square error (MSE), at the end of the simulation period. Synthesized harmonic signal is used, to compare the convergence rate. Numerical simulation applying measured powertrain data is then conducted, to verify the ANC system's performance in more realistic conditions. It is noted that, for most powertrain active noise control system, the FXLMS algorithm is not directly used due to the computational cost. Instead, it is implemented, together with an adaptive notch filter, namely ANF-FXLMS algorithm. Therefore, the ANF-FXLMS algorithm is compared, together with the other four modified algorithms. The characteristics of the five algorithms are summarized in Table 2. Detailed analysis is shown in the following sections.

Table 2. Summary of algorithm characteristics

	<i>Convergence Speed</i>	<i>Computational Cost</i>
<i>ANF-FXLMS</i>	<i>Very slow</i>	<i>High</i>
<i>EE-FXLMS</i>	<i>Slow</i>	<i>High</i>
<i>NX-LMS</i>	<i>Fast</i>	<i>High</i>
<i>TF-FXLMS</i>	<i>Fast</i>	<i>Low</i>
<i>ANF-IMLMS</i>	<i>Fast</i>	<i>Very low</i>

3.4.1 Computational complexity

One of the limiting factors of the ANC system is its computational capacity. The digital signal processor can only handle a certain load in real-time application. Therefore, the algorithm needs to be computationally-efficient, to maximize the benefits of the ANC system. This section studies the computational complexity of the aforementioned algorithms, by comparing the number of real multiplications, and real additions, that are needed to generate one sample output, per frequency component.

The computational complexity for the feed-forward ANC system can be divided into three parts: (1) generate reference signal; (2) update controller weights; and (3) generate controller output. For the NX-LMS algorithm and the TF-FXLMS algorithm, there is one additional procedure involved, to generate the modified error signal. This is because the two algorithms do not use the raw error signal directly.

The computational complexity of a specific algorithm can also vary, according to two parameters: the order of adaptive filter (L) and the order of estimated secondary path model (M). Table 3 summarizes the computational complexity, for each of the algorithms. It can be observed that the computational cost of the ANF-IMLMS algorithm in our simulation is much lower than the other algorithms. For the ANF-FXLMS algorithm and the ANF-IMLMS algorithm, the order of adaptive filter L is always 2, due to the nature of the notch filter. The order of adaptive filter L is set to be 128 in the simulations, for the EE-FXLMS algorithm, the NX-LMS algorithm and the TF-FXLMS algorithm. On the other hand, the order of the estimated secondary path model M is set to be 256 for all the algorithms. It is noted that the block size (N) is the same as the order of

adaptive filter (L), for the TF-FXLMS algorithm. The computational complexity analysis of the TF-FXLMS algorithm is conducted by calculating the operations that are needed to generate $2N$ samples of output, and then, dividing the total number by $2N$. Detailed analysis can be found in reference [12].

Table 3. Computational complexities (operations per sample) of various algorithms

Algorithm	Oper.	Generate Reference	Update Weights	Generate Output	Generate Error	Total	Total in Simulation
ANF-FXLMS $L=2, M=256$	+	$2(M-1)$	L	$L-1$	N/A	$2*M+2*L-3$	513
	x	$2M$	L	L	N/A	$2*M+2*L$	516
EE-FXLMS $L=128, M=256$	+	$M-1$	L	$L-1$	N/A	$M+2*L-2$	510
	x	M	L	L	N/A	$M+2*L$	512
NX-LMS $L=128, M=256$	+	$M-1$	L	$L-1$	$M+1$	$2M+2L-1$	767
	x	$M+1$	L	L	M	$2M+2L+1$	769
TF-FXLMS $L=N=128; M=2N=256$	+	$4*\text{Log}_2(2N)$	$4*\text{Log}_2(2N)+9$	$N-1$	$4*\text{Log}_2(2N)$	$12*\text{Log}_2(2N)+N+8$	232
	x	$4*\text{Log}_2(2N)$	$4*\text{Log}_2(2N)+21$	N	$4*\text{Log}_2(2N)$	$12*\text{Log}_2(2N)+N+21$	245
ANF-IMLMS $L=2, M=256$	+	2	L	$L-1$	N/A	$2*L+1$	5
	x	4	L	L	N/A	$2*L+4$	8

This study is based on several assumptions:

1. For all the algorithms, the cost of multiplying the constant step size in the weight update process is neglected.
2. For the TF-FXLMS algorithm, the cost of transforming the secondary path model, from the time domain to the frequency domain, is neglected since it is implemented off-line.
3. For the EE-FXLMS algorithm, the cost of generating the new secondary path model is neglected

4. For the ANF-IMLMS algorithm, the cost of generating the table of constant A and B, is neglected, so is the cost of looking up the constant A and B, according to the speed.

3.4.2 Convergence rate

In order to compare the convergence rates of the modified algorithms, numerical simulations were conducted, by applying certain synthesized signal. The simulations were conducted in the Matlab/Simulink environment. The secondary path model was obtained in a test vehicle, as the transfer function from the driver side door speaker to the error microphone, which was placed above the driver's head position. The frequency response function, and the impulse response function of the secondary path model are shown in Figure 15.

Figure 15. (a) Frequency response function of the secondary path (b) Impulse response function of the secondary path.

For this simulation, the undesired noise was synthesized using several pure harmonic signal at different frequencies. In this case, 100 Hz, 300 Hz, and 500 Hz, were selected as the test frequencies. The initial amplitude of all the frequency components is set to be one, which corresponds to 0 dB. A background white noise at -40 dB is added, to make the problem more realistic. Sampling frequency of 4092 Hz is used. Please note, for each algorithm, different

frequency components are attenuated separately, and then, summed together, to obtain the overall results. This is expected to be similar to all the frequency components being attenuated simultaneously. This can be done since the components at different frequencies are uncorrelated, and can be processed separately. The step size for one algorithm, across the different frequencies, are set to be the same. In order to get a fair comparison, the step sizes are also set to be the optimal values, meaning that, the largest value for the step size of a specific algorithm is used. This is done to obtain the most reduction while still having a stable system.

Figure 16. Time history of the error signal after control using different algorithms

The time history of the controlled results are shown in Figure 16. The first three columns are the controlled results, at different frequencies, for a single harmonic. The last column is the direct summation of the first three columns. The first row represents the controlled result, using

the conventional ANF-FXLMS algorithm. The step size is fixed for all frequency components. Therefore, it may be optimized for one frequency, but, not for the others. Hence, unbalanced control results are expected. As shown in Figure 16, the harmonic at 500 Hz converges very slowly, compared to the other harmonics. This is due to the fact that the secondary path amplitude is low, at 500 Hz, which leads to a low power filtered reference signal. For the ANF-FXLMS algorithm, the convergence speed is usually proportional to the power of the filtered reference signal. Hence, the secondary path dynamics leads to unbalanced control results. The second row in Figure 16 shows the controlled result from applying the EE-FXLMS algorithm. As expected, the frequency dependent convergence behavior is alleviated. The difference between the convergence speeds, at various frequencies, is reduced, when compared to that of the ANF-FXLMS algorithm. However, certain levels of discrepancy still exist. The third, the fourth, and the fifth row represent the controlled results from applying the NX-LMS algorithm, the TF-FXLMS algorithm, and the ANF-IMLMS algorithm, respectively. The reason for grouping these results together is because the three algorithms can remove the frequency dependent convergence behavior completely. However, the performance of these three algorithms are very different, especially, the result of the TF-FXLMS algorithm, as shown in the fourth row. It is found through simulation, that the TF-FXLMS algorithm is very sensitive to step size. A small tuning/change in the step size may cause very different results, and even cause the system to diverge. Therefore, the robustness of the TF-FXLMS is poor, when compared to the others. On the other hand, the controlled results of the NX-LMS algorithm, and the ANF-IMLMS algorithm, are similar.

Table 4 shows the summary of the controlled results, at the end of the simulation. It tabulates the mean square error (MSE), at various frequencies. If the undesired noise is completely

canceled, one can expect the MSE to be around the background level (-40 dB) for the individual frequency component, and the MSE of the summed result to be around -30 dB. In general, most of the algorithms can reduce the noise to the background level, except the ANF-FXLMS algorithm, which is due to the slow convergence rate at 500 Hz. For the NX-LMS algorithm, it is noted that there is a certain error state at 500 Hz, which might be caused by the modified structure. However, it still manages to reduce the overall level, down to around -29 dB.

Table 4. Summary of Controlled Result using the synthesized harmonic signal

<i>Algorithm</i>	<i>MSE(dB) 100 Hz</i>	<i>MSE(dB) 300 Hz</i>	<i>MSE(dB) 500 Hz</i>	<i>MSE (dB) Sum.</i>
<i>ANF-FXLMS</i>	-40	-39.8	-8.7	-8.7
<i>EE-FXLMS</i>	-40.1	-39.5	-39.6	-30.4
<i>NX-LMS</i>	-37.4	-39	-29.3	-28.9
<i>TF-FXLMS</i>	-40	-40.1	-40	-30.5
<i>ANF-IMLMS</i>	-39.9	-39.8	-39.3	-30.4

3.4.3 Vehicle powertrain noise control

The performance of the aforementioned algorithms can be further examined, by performing a series of numerical simulations, applying measured powertrain data. In this simulation, the primary powertrain disturbance was measured by an error microphone, which was placed at the driver's head position. The ANC system is designed to minimize the noise at the error microphone location. A tachometer signal was recorded, and served as the input reference signal. The secondary path model was measured by an offline system identification process. The electro-acoustic transfer function, from the driver side door speaker to the error microphone, was measured and used as the secondary path model. The frequency response function, and the impulse response

function of the secondary path model are shown in Figure 17. Data was acquired for two cases. In the first case, the engine rotational speed was kept around 4000 RPM, to evaluate the steady state performance. The second case was a partially open throttle test, which means the test vehicle was driven, with a constant acceleration of 0.2g, and the engine rotational speed increases from 1500 RPM to 5500 RPM in 10 seconds. For both cases, the time history of engine speed is shown in Figure 17.

Figure 17. Estimated engine rotating speed from tachometer signal.

For demonstration purposes, the goal of this simulation is to minimize the second and third order response, as much as possible. Figure 18 shows the simulation results for the first case. Figure 18 (a) and Figure 18 (b) represent the time histories of the controlled results, for the second order and third order respectively, while Figure 18 (c) shows the frequency spectra of the time data, from 7.5-8 seconds. The solid black line represents the baseline response, without the ANC system. The solid magenta line, solid green line, dotted blue line and the dashed red line represent the controlled results, from applying the ANF-FXLMS algorithm, the TF-FXLMS algorithm, the

NX-LMS algorithm, and the ANF-IMLMS algorithm, respectively. It is noted that, for the first case, all algorithms have a relatively close performance. The magenta line shows a certain amount of unbalance between the second and the third order. However, the difference is not very severe. This is also verified in the frequency spectra. All of the algorithms can reduce the second and the third order response, effectively.

Figure 18. Comparison of controlled response for engine speed at 4000 RPM: (a) 2nd order (b) 3rd order (c) spectrum of time block data 7.5-8s (Keys: — baseline response; — ANF-FXLMS; — TF-FXLMS; NX-LMS; - - - ANF-IMLMS)

Figure 19. Comparison of controlled response for engine speed running up: (a) 2nd order (b) 3rd order (c) spectrum of time block data 7.5-8s (Keys: — baseline response; — ANF-FXLMS; — TF-FXLMS; NX-LMS; - - - ANF-IMLMS)

Figure 19 shows the controlled result for the second case. Similarly, Figure 19 (a) and Figure 19 (b) represent the time histories of the controlled results for the second and the third order respectively, and Figure 19 (c) shows the frequency spectra of the time data, from 7.5-8 seconds. However, the algorithm performance can be distinctly noted in this test. As can be observed in both Figure 19 (a) and Figure 19 (b), the magenta line, which represents the application of the ANF-FXLMS algorithm, shows less reduction than the others, for the first four seconds. Its performance is only as good as the other algorithms in the last couple seconds, when the engine speed ramps up. It is also interesting to notice that the TF-FXLMS algorithm behaves very

differently from all the other algorithms. It shows less reduction for most of the simulation. For the TF-FXLMS algorithm, it is harder to track down the cause of the performance degradation. One reason is possibly due to the block processing of the filter weight update, where the system applying the TF-FXLMS algorithm is not able to track the changes in the primary noise, as fast as its time domain counterparts. The last two algorithms, which needs to be examined, are the NX-LMS algorithm and the ANF-IMLMS algorithm. It is noted that both algorithms show good performance throughout the entire simulation. The controlled results are relatively close to each other, with minor differences, due to the different algorithm structures. In general, both the NX-LMS algorithm and the ANF-IMLMS algorithm can reduce the noise level, and track the variation in the primary disturbance, effectively.

3.4.4 Vehicle powertrain sound tuning

Traditional ANC system tends to reduce the noise level as much as possible. However, the perceived sound quality is often the fundamental concern. A quieter vehicle does not necessarily mean it has better sound quality. In reality, to make the engine sound more natural, the active sound quality control (ASQC) is often used. The concept of ASQC is an extension of ANC. For the ASQC of powertrain disturbance, the engine order can be either enhanced, or attenuated selectively, according to a predetermined sound quality matrix. For example, the ASQC technology can be applied on the turbocharged vehicle, to make the engine sound more pleasing.

In order to further verify the performance of the aforementioned algorithms, more simulations are conducted, applying the ASQC technology. The same data is used, including the primary disturbance, tachometer signal, and the estimated secondary path model. Similarly, two

engine orders, namely, the second order, and the third order, are controlled. However, instead of trying to attenuate both engine orders, the goal of this simulation is to enhance the second order, to certain predetermined values, and to attenuate the third order simultaneously. The two specific test cases are run again. As mentioned before, the first case is a steady state test, and the second case is a transient test.

Figure 20. Comparison of controlled response for engine speed at 4000 RPM: (a) 2nd order (b) 3rd order (c) spectrum of time block data 7.5-8s (Keys: — baseline response; — ANF-FXLMS; — TF-FXLMS; NX-LMS; - - - ANF-IMLMS; * Target)

Figure 21. Comparison of controlled response for engine speed running up: (a) 2nd order (b) 3rd order (c) spectrum of time block data 7.5-8s (Keys: — baseline response; — ANF-FXLMS; — TF-FXLMS; NX-LMS; - - - ANF-IMLMS; * Target)

Figure 20 and Figure 21 show the control results for the steady state test, and the transient test, respectively. The figure arrangement is similar to the previous ANC test results, except for the extra magenta stars. The magenta stars in Figure 20 (a), 20 (c), 21 (a) and 21 (c) represent the target sound level for the second engine order. There is no target value for the third order, because the system tends to attenuate the third order, as much as possible. Similar observation can be made from the control results. For steady state test, all the tested algorithms have a comparable performance. However, the difference can be distinguished in the transient test, as shown in Figure 21. The system configured with the ANF-FXLMS algorithm has a poor performance in the first

four seconds, when the engine speed is relatively low. This is mainly due to the frequency dependent convergence behavior, which is inherent in the ANF-FXLMS algorithm. For the TF-FXLMS algorithm, different results can be observed between the second and third order. It may be noted that, for the second order, which is enhanced, the TF-FXLMS algorithm shows similar performance, when compared to the NX-LMS algorithm and the ANF-IMLMS algorithm. However, for the third order, the TF-FXLMS algorithm still shows a poor performance. Thus, it may be concluded that TF-FXLMS algorithm is as good as the other algorithms, for engine order enhancement, but, not for engine order cancellation. For the last two algorithms: the NX-LMS algorithm and the ANF-IMLMS algorithm, their performances are still among the best, and are relatively close to each other, either for the engine order enhancement, or cancellation.

3.5 Conclusions

This chapter presents a comparative study of several newly developed adaptive algorithms, for the application of powertrain ANC system. Their salient features are briefly summarized and compared, in terms of the convergence rate and computational efficiency. In order to verify the performance of the ANC system employed with these adaptive algorithms, numerical simulations are conducted, utilizing both synthesized signal and measured powertrain data. Two specific cases of vehicle operating condition are tested. The engine's rotational speed is kept at a constant level in the first case, and increases gradually in the second case. Based on the simulation results, the conventional ANF-FXLMS algorithm shows a severe frequency dependent convergence behavior; the EE-FXLMS algorithm can alleviate such behavior, but cannot completely remove it. On the other hand, the TF-FXLMS algorithm is able to eliminate such frequency dependent behavior, and save computational cost. However, this algorithm shows poor robustness due to the frequency

domain implementation. Among the five tested algorithms, the NX-LMS algorithm and the ANF-IMLMS algorithm show the best performance. However, if computational cost is a concern, the ANF-IMLMS algorithm would be a better choice, due to its low computational complexity and good convergence behavior.

4 CONCLUSIONS

Active noise control technology has provided an alternative solution to tackle vehicle NVH problems. Normally, vehicle interior noise level is controlled by passive technique, which tends to add mass to the vehicle. Therefore, an increasing number of manufactures has adopted ANC technology. A smart combination of passive and active control could make a real difference. However, current ANC systems still has drawbacks. The key factor that limits the ANC system performance is the existence of the secondary path, which is the electro-acoustic path from the control speaker to the error microphone. Not only the computational cost is being increased, but also the convergence speed is being reduced by the secondary path. For steady state signal, the slow convergence speed is less an issue since the system will eventually suppress the undesired noise to background level. However, the system will have minimal effect or even no effect on fast-varying transient signal, such as one in a wide-open-throttle condition, where the slew rate of engine speed can be more than one thousand RPM per second. Therefore, the objective of this research is to tackle those two disadvantages simultaneously, aiming to develop a computationally-efficient algorithm with high convergence speed for active noise control system.

Previously, inverse model lease mean square algorithm was proposed for general harmonic noise control, which has been proven to be effective for powertrain response. However, our analysis shows that the IMLMS algorithm requires high computational capacity, which is one of the major disadvantages. Enlightened by the idea of adaptive notch filter, an enhanced IMLMS algorithm, namely adaptive notch filter with inverse model least mean square algorithm, is proposed in this study. As widely known, the adaptive notch filter utilize both the in-phase and quadrature component of a reference signal. Therefore it has a very narrow frequency band, which

is perfect for single frequency active noise control. Another good feature of the notch filter is the low computational cost, since it only requires two control filters for each controlled frequency. The proposed algorithm incorporates the virtues of both adaptive notch filter and the inverse model LMS algorithm. Therefore, it has two salient features compared with filtered-x LMS type algorithms: rapid convergence speed and low computational cost. The analysis is also validated by simulation results applying measured powertrain data. The basic configuration of the proposed algorithm is introduced in Chapter 2, and the controlled results of the new algorithm are compared to that of the conventional ANF-FXLMS algorithm. Chapter 3 further compares the proposed algorithm with several other newly developed algorithms. The computational cost and convergence speed of each algorithm is analyzed and compared. Overall, the proposed algorithm has better performance.

Future work with this algorithm involves multi-input multi-output (MIMO) system modelling and simulation. Instead of using one control speaker to control the noise at one target error microphone location, the MIMO system is more complicated but closer to real life application. The balance and tradeoff between different channels and locations needs to be taken into account. However, overall result is expected to be better than the current SISO system. Then, on-road testing is to be conducted to verify the simulation results. Another aspect of future work might involve studying and improving the controlled result at driver ear location. While the ANC system can reduce the noise level at target error microphone location, the controlled result at driver ear location might be a different scenario. The relationship between different control locations still needs to be studied. One possible approach is to investigate the transfer function between different locations, and apply the transfer function difference to predict the controlled result at driver ear location.

REFERENCE

- [1] M. Allman-Ward, J. Venor, R. Williams, M. Cockrill, H. Distler, A. Crewe, *et al.*, "The interactive NVH simulator as a practical engineering tool," SAE Technical Paper2003.
- [2] X. Wang, *Vehicle noise and vibration refinement*: Elsevier, 2010.
- [3] L. Yi, M. Tianfei, and Y. Weimin, "The summary of study on vehicle NVH performance," *Automotive Engineering*, vol. 24, pp. 177-186, 2002.
- [4] S. M. Kuo and D. R. Morgan, "Active noise control: a tutorial review," *Proceedings of the IEEE*, vol. 87, pp. 943-973, 1999.
- [5] S. M. Kuo and D. Morgan, *Active noise control systems: algorithms and DSP implementations*: John Wiley & Sons, Inc., 1995.
- [6] S. J. Elliott and P. A. Nelson, "Active noise control," *Signal Processing Magazine, IEEE*, vol. 10, pp. 12-35, 1993.
- [7] T. L. Lago, "Overview on ANVC Systems and the Future with Smart Embedded Solutions," *SAE International Journal of Passenger Cars-Mechanical Systems*, vol. 8, 2015.
- [8] L. Paul, "Process of silencing sound oscillations," ed: Google Patents, 1936.
- [9] R. Shoureshi and T. Knurek, "Automotive applications of a hybrid active noise and vibration control," *Control Systems, IEEE*, vol. 16, pp. 72-78, 1996.
- [10] A. Kinoshite and H. Aoki, "Active noise control system for automotive vehicle," ed: Google Patents, 1993.
- [11] S. Elliott, I. Stothers, P. Nelson, A. McDonald, D. Quinn, and T. Saunders, "The active control of engine noise inside cars," in *INTER-NOISE Congress and Conference Proceedings*, 1988, pp. 987-990.

- [12] J. Duan, M. Li, and T. C. Lim, "A computational-efficient active sound tuning system for steady-state and transient vehicle powertrain response," *International Journal of Vehicle Noise and Vibration*, vol. 10, pp. 77-92, 2014.
- [13] B. Widrow and E. Walach, *Adaptive Inverse Control, Reissue Edition: A Signal Processing Approach*: John Wiley & Sons, 2008.
- [14] J. C. Burgess, "Active adaptive sound control in a duct: A computer simulation," *The Journal of the Acoustical Society of America*, vol. 70, pp. 715-726, 1981.
- [15] C. Bao, P. Sas, and H. Van Brussel, "A novel filtered-x LMS algorithm for active noise control: Fast control processing," *Journal A*, vol. 34, pp. 89-94, 1993.
- [16] L. P. R. de Oliveira, B. Stallaert, K. Janssens, H. Van der Auweraer, P. Sas, and W. Desmet, "NEX-LMS: A novel adaptive control scheme for harmonic sound quality control," *Mechanical Systems and Signal Processing*, vol. 24, pp. 1727-1738, 8// 2010.
- [17] J. K. Thomas, S. Lovstedt, J. D. Blotter, S. D. Sommerfeldt, and B. Faber, "Eigenvalue equalization filtered-x (EE-FXLMS) algorithm applied to the active minimization of tractor noise in a mock cabin," *Noise Control Engineering Journal*, vol. 56, pp. 25-34, 2008.
- [18] J. K. Thomas, S. P. Lovstedt, J. D. Blotter, and S. D. Sommerfeldt, "Eigenvalue equalization filtered-x algorithm for the multichannel active noise control of stationary and nonstationary signals," *The Journal of the Acoustical Society of America*, vol. 123, pp. 4238-4249, 2008.
- [19] J. Duan, M. Li, T. C. Lim, M.-R. Lee, F. Vanhaaften, M.-T. Cheng, *et al.*, "Comparative study of frequency domain filtered-x LMS algorithms applied to vehicle powertrain noise control," *International Journal of Vehicle Noise and Vibration*, vol. 5, pp. 36-52, 2009.
- [20] M. Li, J. Duan, and T. C. Lim, "Enhanced inverse model LMS algorithm for active control of harmonic response clusters," in *INTER-NOISE Congress and Conference Proceedings*, 2012, pp. 9327-9335.

- [21] M. Li, T. C. Lim, and J. Duan, "Active control of sinusoidal disturbances using inverse model LMS algorithm," in *NOISE-CON Congress and Conference Proceedings*, 2013, pp. 876-882.
- [22] G. Sun, T. Feng, J. Xu, M. Li, and T. Lim, "Modified FxLMS Algorithm with Equalized Convergence Speed for Active Control of Powertrain Noise," *SAE International Journal of Passenger Cars-Mechanical Systems*, vol. 8, 2015.
- [23] G. Sun, T. Feng, M. Li, J. Xu, and T. C. Lim, "Active control of vehicle powertrain noise using inverse model LMS algorithm," in *Proceedings of the International Congress and Exposition on Noise Control Engineering*, 2014.
- [24] P. L. Feintuch, N. J. Bershad, and A. K. Lo, "A frequency domain model for filtered LMS algorithms-stability analysis, design, and elimination of the training mode," *Signal Processing, IEEE Transactions on*, vol. 41, pp. 1518-1531, 1993.
- [25] S. S. Narayan, A. M. Peterson, and M. J. Narasimha, "Transform domain LMS algorithm," *Acoustics, Speech and Signal Processing, IEEE Transactions on*, vol. 31, pp. 609-615, 1983.
- [26] Q. Shen and A. Spanias, "Time and frequency domain X block LMS algorithms for single channel active noise control," *Proc. 2nd Int. Congr. Recent Developments in Air-and Structure-Borne Sound Vibration*, pp. 353-360, 1992.
- [27] J. Thi and D. R. Morgan, "Delayless subband active noise control," in *Acoustics, Speech, and Signal Processing, 1993. ICASSP-93., 1993 IEEE International Conference on*, 1993, pp. 181-184.
- [28] D. R. Morgan and J. C. Thi, "A delayless subband adaptive filter architecture," *Signal Processing, IEEE Transactions on*, vol. 43, pp. 1819-1830, 1995.
- [29] J. J. Shynk, "Frequency-domain and multirate adaptive filtering," *IEEE Signal Processing Magazine*, vol. 9, pp. 14-37, 1992.

- [30] D. R. Morgan, "An analysis of multiple correlation cancellation loops with a filter in the auxiliary path," *Acoustics, Speech and Signal Processing, IEEE Transactions on*, vol. 28, pp. 454-467, 1980.