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I, Sucharitha Rajendran , hereby submit this original work as part of the requirements for the degree of Master of Science in Mechanical Engineering.

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Experimental Investigation of Jet Breakup at Low Weber Number

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EXPERIMENTAL INVESTIGATION OF JET BREAKUP AT LOW WEBER NUMBER

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ABSTRACT

An experimental investigation on the disintegration of circular liquid jets, ejected into a stagnant ambient atmosphere at low Weber number, is presented in this thesis. The process of breakup of the liquid jet was captured using a real-time image processing high-speed digital camera system. In order to understand the influence of inertial, surface tension, and viscous forces on the process of breakup, a range of Weber numbers from 5 to 110 was experimentally tested. A syringe pump was used to provide a constant flow rate and produce a jet at a given Weber number. The effects of surface tension and viscosity were investigated by using two viscous liquids (ethylene glycol and propylene glycol) apart from water. Nozzle diameters from 0.279 mm to 1.753 mm were used to study its influence on the liquid jet breakup.

Results show that the jet breakup patterns for water at lower Weber numbers follow a different behavior than that for higher Weber numbers. In the former case, the breakup length depends not only on Weber number, but also quite significantly, on nozzle exit diameter. Moreover the functional dependence of jet breakup length in this range ($We < 100$), besides inertia and surface tension, is also governed by viscous and gravitational forces. The influence of liquid properties and nozzle diameter on jet breakup is discussed along with a parametric scaling of the different forces. A universal correlation to depict the breakup in any Newtonian liquid is established. The influence of elongational viscosity on the breakup of low Weber number large diameter jets is discussed along with the experimental findings.

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NOMENCLATURE

Parameters

d	Diameter of the nozzle exit [m]
d_j	Diameter of the jet [m]
R	Radius of the jet [m]
F_S	Surface Tension Force on the liquid jet [N]
F_D	Viscous Drag force on the liquid jet [N]
F_G	Gravitational force on the liquid jet [N]
F_I	Inertia force on the liquid jet [N]
g	Gravitational acceleration [m/s^2]
$\dot{m}$	Mass Flow rate of liquid pushed by the syringe pump [kg/s]
p_a	Pressure exerted by ambient air on the liquid-air interface [N/m^2]
P_l	Pressure exerted by the liquid on the liquid-air interface [N/m^2]
u	Velocity of the issuing jet [m/s]
L	Breakup Length [m]
l_c	Capillary Length [m] $\left(= \sqrt{\sigma / \rho g} \right)$

Greek Symbols

μ	Viscosity of the liquid [kg/m-s]
ρ_l	Density of the liquid [kg/m^3]
ρ_a	Density of the ambient air [kg/m^3]
σ	Surface Tension coefficient [N/m]
τ	Shear stress [N/m^2]

Σ_e Elongational stress [N/m²]

η Extensional viscosity

Non-Dimensional Numbers

Re Reynolds Number $\left(= \rho u d / \mu \right)$

We Weber Number $\left(= \rho u^2 d / \sigma \right)$

Fr Froude Number $\left(= u^2 / g d \right)$

Bo Bond Number $\left(= \rho g d^2 / \sigma \right)$

Oh Ohnesorge Number $\left(= \mu / \sqrt{\rho \sigma l_c} \right)$

Mo Morton Number $\left(= g \mu^4 / \rho \sigma^3 \right)$

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Chapter 1

INTRODUCTION

1.1 Overview

The formation of dispersed droplets from a bulk liquid into a gaseous medium finds significant usage in diverse fields of application; such as, agriculture, fuel combustion, ink-jet printing, firefighting, medicine and cosmetics, to name a few. Liquid sheet formed from a narrow slit or liquid jets from a small hole are often used for liquid atomization. Both, the liquid sheet and liquid jet would eventually breakup into drops due to their inherent instability. As soon as the liquid leaves the opening (slit/hole), disturbances develop on the surface and propagate to eventually breakup the liquid sheet or jet into drops.

The scales, on which liquid jets could occur, range from astronomical scales (Hughes, 1991) to nanoscales (Eggers, 2002). The basic process can be viewed as the result of the action of both internal and external forces. For a liquid stream emerging from a nozzle under the effect of gravity and no other external force, the surface tension of the liquid would pull the liquid and try form a sphere, as this would have the minimum surface area. The aerodynamic forces, gravitational forces and viscous forces present would induce instability in the liquid that ultimately cause the liquid jet to breakup. The incoming velocity of the liquid stream, and thus the aerodynamic force involved, can vary from very small to very large values. This would in turn, alter the nature in which the forces act and thus, would lead to different regimes of breakup. There are discussed later in Chapter 2.

1.2 Motivation

The instability and disintegration of liquid jets at low Weber numbers is of importance in many atomization applications. The breakup of the liquid stream arising from an orifice into an ambient atmosphere, basically leads to a discontinuity in the flow. This could be beneficial in a few applications, like spray coating, drug delivery etc., where it is intended to ensure a quick and effective breakup of the emerging liquid jet into drops. In some other applications, like impinging jet cooling, the disintegration of the liquid jet will result in discontinuous cooling effects that cause unsuitable thermal profile development on the object to be cooled. Thus, it is important to be able to predict the breakup pattern of liquid jet and understand the factors that it depends on.

1.3 Scope and Limitations of the Work

Though most engineering applications could involve complex geometries and liquids with varying rheological properties, quantitative analysis of a simple system will help provide a better understanding of the fundamental process of jet breakup. Hence, in this work, liquid streams, ejected from circular nozzle orifice into a quiescent ambient atmosphere, are studied.

To better understand the effect of viscosity on the phenomena of breakup of the liquid stream, three Newtonian liquids: water, ethylene glycol, and propylene glycol are used. The diameter of the nozzle through which liquid jet emerges, is varied from 0.279 mm to 1.753 mm to test the influence of initial jet diameter, i.e. the nozzle inner diameter. This investigation involves a study of breakup of low weber number Newtonian liquids as they are ejected into a stagnant atmosphere by a circular nozzle.

Chapter 2

PHENOMENA OF JET BREAKUP

2.1 Regimes of Jet Breakup

As the fluid is issued from the circular nozzle orifice into quiescent ambient air, at lower supplying flow rates, the fluid would drip from the orifice. When the supplying flow rate is increased beyond a certain limit, a continuous stream of fluid begins to develop at the orifice. This limit is determined by a dripping-jetting transition limit that many investigators have been studying (the dripping-jetting transition is discussed later in Section 2.1.2 of this chapter). The continuous stream formed, is seen to disintegrate at some distance downstream of the nozzle exit. As the liquid jet moves downstream from the orifice, the fluid stream that is initially smooth is found to develop distinct disturbances on its surface. When the amplitude of this disturbance is equal to the radius of the liquid jet, drops begin to be pinched off from the continuous stream. The distance from the nozzle orifice to the point where the first drop is pinched-off from the main jet stream is termed as the breakup length of the liquid jet (L).

Though a few researchers tried to analyze the breakup process, Sarvat (1833) was the first to provide quantitative data for his experimental examination of the jet breakup process. From his tests, he concluded that the breakup length was directly proportional to the velocity of the jet for a given jet diameter and, for a given jet velocity, breakup length was proportional to the jet diameter. His experimental results were used by many researchers who followed, to verify and explain their findings. Plateau (1843) observed that the liquid column would disintegrate only when its length was greater than its perimeter. In other words, the liquid stream was unstable to any disturbance that reduced the surface area. Plateau is accredited for recognizing the role of

surface energy or surface tension in the process of the disintegration. This later formed the basis of Lord Rayleigh's instability analysis.

Lord Rayleigh (1879) introduced the linear stability theory to jet breakup dynamics. He compared the surface energy of the undisturbed column, to that of the disturbed column. This established that a liquid column under the influence of no other force apart from surface tension would be unstable to axisymmetric disturbances with a wavelength greater than the circumference of the developing jet. The dominant wavelength was mathematically found to be $4.51 d$. This led to Rayleigh's prediction that the resulting drops would have a diameter equivalent to $1.89 d$. Though Rayleigh's linear stability was built on the assumption of an inviscid condition in stagnant surroundings, it has been found to be in general acceptance with later studies and theories of viscous turbulent jets that have varying surrounding air conditions.

2.1.1 Stability Curve

To gain a better knowledge of the breakup phenomenon, a breakup curve showing the relation between the breakup length and the issuing velocity of the jet was developed by Lin and Reitz, 1998. Other experimental and numerical studies have been seen to agree with the general structure of this breakup curve.

The unbroken jet length, i.e. breakup length, is found to initially increase, approximately linearly, with increasing issuing liquid velocity. Upon reaching a maximum, the breakup length begins to decrease with an increase in velocity (region A to B). The velocity at which the maximum breakup length is attained is termed as the *critical velocity*.

This region (A to B) comprises of the *Rayleigh breakup regime* and the *first-wind breakup regime*. Weber (1931) showed that in this linear portion of the stability curve, a small axisymmetric disturbance will grow exponentially till the disturbance is of the order of the jet radius. As the issuing velocity is increased further, in region B to C, *second-wind breakup regime* is found. Beyond this lies the *atomization regime*. Here, the breakup length continues to decrease with increasing issuing velocity till it is at the tip of the nozzle exit.

Haenlein (Lefebvre, 1989) was the first to identify these four distinct regimes of breakup in the disintegration of a liquid column. Ohnesorge (Lefebvre, 1989) was the first to establish criteria for determining the breakup mode of liquid jet. He used the relative importance of gravitational, inertial, surface tension, and viscous forces to classify the breakup. He divided the breakup phenomena into 3 regions based on two non-dimensional numbers: Reynolds number and the Ohnesorge number. Reitz (1998) tried to resolve the uncertainties in the Ohnesorge chart. He distinguished the following four regimes of breakup that are found when the issuing velocity is progressively increased.

Figure 1: Stability curve: Typical variation of L vs. U (Lin and Reitz, 1998)

Figure 2: Ohnesorge chart (Lefebvre, 1989)

Rayleigh breakup: This mode of breakup is formed due to interplay of primary disturbance in the liquid stream and the surface tension forces. Radially symmetric disturbances are a characteristic of this mode. Drops, that are approximately twice the diameter of the issuing jet, are pinched-off the main liquid stream when the amplitude of disturbance in the stream is equal to its radius. Here, the breakup length increases linearly with issuing velocity.

First-wind induced breakup: In this mode of breakup, the aerodynamic forces i.e. the drag due to the ambient air, play a role in breaking up the liquid jet unlike in the Rayleigh breakup mode. The diameters of the drops that are pinched off are of the order of the jet diameter. The jet breaks up many nozzle diameters downstream of the nozzle.

Second-wind induced breakup: Here too aerodynamic forces play a significant role in the breakup process but the influence of surface tension forces is reduced. The drops formed here are

still of the order of the jet diameter but are much smaller than those formed in the first-wind induced breakup regime. Breakup occurs near the exit of the nozzle.

Atomization: Here complete disintegration of the liquid jet occurs at the nozzle tip. Only a small liquid core is left at the orifice. The droplet size usually decreases with increasing issuing velocity and are much smaller than the jet diameter.

Figure 3: (a) Rayleigh breakup (b) First wind-induced breakup (c) Second wind-induced breakup (d) Atomization regime (Lin and Reitz, 1998)

2.1.2 Transition from Dripping to Jetting

Many models and studies have tried to understand this transition from dripping to jetting. One of the more widely used transition correlation was proposed by Clanet and Lasheras (1999). They proposed a critical Weber number, given by Equation 2.1 at which this transition would occur.

$$We_c = 4 \frac{Bo_o}{Bo} \left[1 + KBo_oBo - ((1 + KBo_oBo)^2 - 1)^{1/2} \right]^2 \quad (2.1)$$

In the above Equation, Bo and Bo_o are the Bond numbers based on the inside and outside diameters of the nozzle and K is an empirical constant given a value 0.37 for the case of water injected into air. This model is seen to agree well with experimental result and provides a means to understand the influence of gravity, inertia and surface tension in the determination of transition to jetting flow. The authors note that viscous effects have not been taken into consideration. The model could be used for any Newtonian fluid injected vertically downwards into stagnant air and would provide the inviscid limit for the case.

Clanet and Lasheras (1999) defined the beginning of jetting flow to occur when the unbroken length of the liquid from the nozzle tip is about $20R$. When viscous liquids are used, this length could occur at very low flow rates and hence would not be an appropriate definition. Ambravaneswaran et al (2004) realized the inherent difficulty with this definition and defined the transition from dripping to jetting flow to occur when measurement of the dynamics of the system (e.g. L/R) undergo a sudden and large increase (or decrease). They use both computational and experimental techniques to verify their results. They define two phases of dripping: simple (where a single drop drips) and a complex phase (where multiple drops could drip). From the phase diagram (We vs. Oh) that was developed, it can be concluded that for Oh less than 0.5, there exists an intermediate phase that they define as complex dripping (where dripping is periodic). From their experiments, they note that at Oh greater than 0.5, the critical

Weber number can be computed considering the flow to be inviscid. For very viscous liquids, it was noted that the transition to jetting occurred from simple dripping phase and no complex dripping phase was observed. A quantitative means to predict the transition from dripping to jetting taking into account all the properties of the liquid is yet to be established.

2.2 Stability Theory

Lord Rayleigh (1879) suggested the linear stability theory can help explain the phenomena of breakup for a laminar inviscid flow in a stagnant condition. According to this, if the wavelength of the disturbance (λ) exceeds the circumference (πd) of the liquid stream (i.e. jet); it would breakup into droplets in such a manner that each bulge would form a droplet. Weber (1931) extended Rayleigh's analysis to viscous liquids and obtained the optimum wavelength (i.e. the most favorable one for drop formation) to be given by the following Equation.

$$\lambda_{opt} = \sqrt{2} \pi d \left(1 + \frac{3 \mu_l}{\sqrt{\rho_l \sigma d}} \right)^{0.5} \quad (2.8)$$

This optimum wavelength was found to be greater for viscous liquids than inviscid ones. The corresponding non-dimensional growth rate is obtained by linearized Navier-Stokes Equation. (Complete derivation in Eggers and Villermaux, 2008)

$$w = \frac{1}{2} \left(-3(kR)^2 Oh + \sqrt{9(kR)^4 Oh^2 + 4kR \left(1 - (kR)^2 \frac{I_1(kR)}{I_0(kR)} \right)} \right) \quad (2.9)$$

where, k is the wave number ($2\pi / \lambda$), R is the jet radius and I_1 and I_0 are the modified Bessel functions of the first kind of order 1 and 0, respectively. The above two equations are used to

obtain the growth rate vs. wave number plots that help understand the instability during the process.

The above analysis, models the liquid jet breakup as a linear instability problem, where, it is assumed that the jet breaks up into uniformly sized drops. However, this is not the case. Liquid jet is seen to breakup into fairly large sized primary droplets with a few satellite drops in-between them. In case of more viscous liquids, the primary drops formed, are sometimes seen to split up into further droplets, or are seen to merge to form larger sized drops. Since the contribution of the satellite drops are a small percent of the total volume of the system, linear instability analysis can still be used to approximately predict the primary drop diameter. However, to predict the exact breakup phenomena and the origin and dimensions of the satellite drops, non-linear analysis accounting for higher order terms, is required for modeling.

2.3 Velocity Relaxation

When the liquid jet emerges from the nozzle exit, the flow could either be laminar or turbulent. This is mainly governed by the Reynolds number of the flow within the nozzle. The critical Reynolds number in a pipe flow for the laminar to turbulent transition has been found to be around 2300 (Holman, 2002). This is based on the parabolic uniform velocity profile within the nozzle. Within the nozzle, the velocity of the jet is zero at the wall (i.e., no-slip i.e., the liquid sticks to the wall surface). But, as the liquid jet emerges from the nozzle, there is no wall to provide for the no-slip condition and the velocity profile relaxes and would ideally attain a uniform velocity distribution at some distance downstream. From linear stability analysis, if the Weber number of the liquid stream, as it exits the nozzle, was less than 3.3, the jet would become

unstable and disintegrate before the velocity profile relaxes. Whereas, if the Weber number were greater than 3.3, the velocity profile of the jet would relax to that of the Rayleigh regime jet before disintegration commenced. (Details of the same are well explained by Lin, 2003).

2.4 The Breakup Process

The action of jet breakup is determined by a number of parameters:

- Operating parameters: flow rate
- Thermophysical properties: surface tension, viscosity, density
- System geometry: nozzle shape, nozzle dimensions

These properties/ parameters manifest themselves into aiding and retarding forces that determine the process of breakup. The force balance in a control volume is shown in Figure 4.

The forces acting on the control volume of the liquid jet are:

1. Inertial Force: As the fluid arising from the nozzle tip is given a momentum, there exists an inertial force that can be expressed as the rate of change of momentum of the liquid flowing from the nozzle orifice.

$$\begin{aligned}
 F_I &= \frac{d(mu)}{dt} \\
 &= \rho_l \frac{\pi d^2}{4} \frac{d(uL)}{dt} \\
 &= \rho_l \frac{\pi d^2}{4} u^2
 \end{aligned} \tag{2.2}$$

where m the mass of the liquid and u is the velocity of the liquid at the nozzle exit.

Figure 4: Schematic of the forces acting in a control volume

2. Capillary force: Surface tension is a property of a liquid that has a tendency to resist external force, and drives the surface of the liquid towards a shape with the least surface energy. Capillary force is a manifestation of the surface tension and it acts to keep the liquid surface intact and hence aids the breakup process.

$$F_S = (\pi D_j) \sigma \quad (2.3)$$

3. Gravitational Force: This along with the inertial force retards the breakup process.

This force is mathematically expressed as:

$$\begin{aligned} F_G &= (\rho_l - \rho_a) g V \\ &= \rho_l g \frac{\pi d^2}{4} \cdot L \end{aligned} \quad (2.4)$$

where L is the length of liquid column at a given location. In the calculation of gravitational force, the density of air (1.225 kg/m^3) can be neglected as it is much smaller than that of water ($\sim 1000 \text{ kg/m}^3$).

4. Shear stress, τ : Shear stress originates from the forces that act parallel to the surface of the jet. The force due to the shear stress is given as

$$F = \tau A \quad (2.5)$$

where A is the cross-sectional area parallel to the force. Shear stress, τ , is given by Newton's law of Viscosity $\left(\tau = \mu \frac{\partial u}{\partial y}\right)$ for the Newtonian liquids used here.

Thus the shearing force in this case would become:

$$F = \mu (\pi d) u \quad (2.6)$$

Figure 5: Schematic of the forces that can be neglected

5. Viscous Drag: Drag force acts due to the relative motion between the moving liquid stream and the stagnant air around it. This force depends on the relative velocity between the two mediums and acts to decrease this velocity. If the

ambient air was in motion, there would be a significant drag as a result of this, and the resultant force would be expressed as follows:

$$F_D = C_d A \frac{1}{2} \rho_a u_r^2 \quad (2.7)$$

where C_d is the drag co-efficient, A is the reference cross-sectional, ρ_a is the density of the ambient air, and u_r is the relative velocity between the two media. In the current case under consideration, the ambience is stagnant. Hence u_r would be equivalent to the velocity of the jet u . In this study, the drag force is neglected, as the density of air is much lesser than that of the other liquids and hence the value of F_D would be much smaller than the other forces considered.

6. Pressure force due to p_a and p_L : The flow through the nozzle can be regarded as an isentropic flow, as there is no change of energy in the system. The nozzle used in this work, can be looked at as a converging nozzle with the flow going towards the throat. The flow rates considered for this study are low, and the Mach number of the flow is much less than unity ($M \ll 1$). For an isentropic subsonic flow, the exit pressure is the ambient pressure. Hence, in this case, the liquid exits the nozzle at atmospheric pressure and there is no pressure force acting on the jet.

The resultant forces that act on the jet stream are the inertial, gravitational, capillary (surface tension) and viscous forces as shown in Figure 6. While the inertial force (due to the momentum of the incoming jet), the gravitational force and the viscous force retard the breakup, the capillary force (due to surface tension) aids the process. Thus a larger capillary force would result in

shorter intact length (breakup length), while, a larger inertial, viscous or gravitational force would elongate the breakup length.

Figure 6: The forces acting on the surface of the liquid jet

Chapter 3

LITERATURE REVIEW

The earliest recorded study of liquid jet breakup was by Leonardo da Vinci in the *Codex Leicester* as shown in Figure 7. His thoughts on the topic, as has been quoted by Eggers and Villiermaux (2008) in their review, are: *‘How water has tenacity in itself and cohesion between its particles. This is seen in the process of a drop becoming detached from the remainder, this remainder being stretched out as far as it can through the weight of the drop which is extending it; and after the drop has been severed from this mass the mass returns upwards with a movement contrary to the nature of heavy things’.*

Figure 7: Sketch by Leonardo Da Vinci illustrating the impact of jets (Da Vinci, 1580)

Leonardo Da Vinci in the 15th century understood that the detachment of a drop from a stream was due to gravitational forces overcoming the capillary forces. Other researchers, like Hagen (1949), Plateau (1843, 1849) and Savart (1833), were able to establish that the breakup length depended on the critical wavelength of the initial amplitude of disturbance, though; they were unable to provide a method to predict the process.

Lord Rayleigh (1879) proposed a linear stability analysis and with that, marked the beginning of the study in the field of stability and breakup of liquid columns. Lord Rayleigh analyzed an infinite column of inviscid liquid undergoing breakup. He showed that the instability in the liquid column was only due to surface tension forces. His solution for the growth rate and wavelength of a destructive symmetric disturbance also showed that at low velocities, the size of droplets formed as a result of the liquid column breakup were approximately twice the diameter of the jet. Weber (1931) modified Rayleigh's theory by considering the viscosity of the liquid and the aerodynamic force due to the ambient atmosphere. His analysis showed that the breakup length of the jet would decrease with increase in the jet velocity as there would be an increase in the growth rate of the prevailing disturbance. This meant that the occurrence of a critical velocity in the breakup curve in Figure 1 was a result of the action of aerodynamic forces. Weber's theory predicted a maximum in the jet breakup curve by relating the critical point in the curve with the gaseous Weber number. However, the location of this maximum was not in complete agreement with the experimental measurements of other researchers (Grant and Middleman (1966), Sterling and Sleicher (1975), Lefebvre (1989), Fenn and Middleman (1969), Kalaaji et al. (2003)).

Grant and Middleman (1966) tested several liquids using a range of nozzles with diameters from 0.3 mm to 1.4 mm, and nozzle aspect ratios ranging from 7 to 150. They used a high speed electronic flash unit (0.5 μ s) to capture shadowgraph images of the liquid jets. They found that

Weber's prediction overestimated the critical velocity for low Ohnesorge number jets and underestimated it for high Ohnesorge number jets. They provide a correlation to predict the breakup in both laminar and turbulent jets. But, they have observed, that the predicted correlation was in poor agreement with the experimental results at sub-atmospheric conditions.

Following the work done by Grant and Middleman (1966), Fenn and Middleman (1969), were able to establish that the critical velocity observed in the breakup curve is a function of both viscous stresses and aerodynamic pressure forces. Their analysis yielded a critical gaseous Weber number of 5.3 that was found to be greater than Weber's prediction. For Weber number lower than the established critical gaseous Weber number, the critical velocity for breakup was observed to be independent of ambient pressure forces and depended only on the shear stress caused due to motion of the liquid jet. At larger Weber numbers, the effect of aerodynamic pressure forces became significant and increased the instability in the jet. They suggested a correlation to predict the critical velocity when gaseous Weber number was lower than 5.3. This correlation implied that the origin of a critical maximum velocity was due to shear stresses that develop on the surface of the liquid jet due to its motion.

Sterling and Sleicher (1975) performed experiments with three different liquids and considered nozzles with both small and large aspect. Shadowgraph images taken at 24 frames per s were used to capture the breakup process. With their experiment they were able to establish that the results reported by Fenn and Middleman (1969) were due to velocity-profile relaxation effects, and that the aerodynamic forces affected the jet breakup. Their results too agreed that Weber's theory overestimated the effect of aerodynamic forces and thus, they provided modification to Weber's theory. Their modification was capable of predicting breakup for shorter nozzles but overestimated the breakup length for extended nozzles. With this, they came to a conclusion, that

for extended nozzles, velocity relaxation effects could be neglected and the transition from Rayleigh-breakup to first-wind induced breakup regime was purely due to the increased significance of the aerodynamic forces.

The role of velocity-relaxation and its cause has not been well-established as yet. Mansour and Chigier (1994) studied the stability of laminar and turbulent liquid jets. Though the growth rates for laminar and turbulent liquid jets were found to be the same, the initial disturbance amplitude value was small for laminar jets and was large for turbulent jets. This finding was in agreement with that of Phinney (1972) in reasoning that, the cause for a maximum in the stability curve was due to the increase of the initial amplitude of disturbance and not due to the aerodynamic forces.

An experimental investigation for low velocity liquid jets was conducted by Leroux et.al (1997) to understand the physics behind the origin of the maximum in the stability curve. They defined a critical gas density ρ^* as a function of both viscosity of the fluid and the nozzle length. Aerodynamic forces were found to be of significance only when the gas density was greater than ρ^* . Their experimental data was in agreement with the predictions by Sterling and Sleicher (1975). They provided a modification to Weber's prediction of the critical velocity and accommodated the critical gas density ρ^* in the aerodynamic term in Weber's dispersion Equation.

Earlier studies available in the literature [Fenn and Middleman (1966); Grant and Middleman (1966); and Leroux et.al. (1997)] show that the transition from Rayleigh to first-wind induced regime for high Ohnesorge jets is due the aerodynamic forces that act on the liquid jet. For low Ohnesorge jets however, there has been no conclusive result. Also, a criterion, to establish the transition from low to high Ohnesorge jet, is yet to be clearly defined. While Grant and

Middleman (1966) and Phinney (1972) suggest that the increase in the initial amplitude of the disturbance to be the transition, Sterling and Sleicher (1975) suggest that this transition is a manifestation of the velocity profile relaxation effect.

To study the growth rates in the Rayleigh and first wind-induced regime Kalaaji et al. (2003) experimentally subjected the liquid jet to sinusoidal perturbations. They found that Sterling and Sleicher's prediction underestimated the rate of amplification of the disturbance, as the spatial nature of the disturbance was not taken into account.

Sallam et al. (1999) found that the correlation suggested by Grant and Middleman for turbulent jets, covered more than two modes of jet breakup as it was inclusive of the transition from laminar to turbulent and from turbulent to aerodynamic bag/shear breakup regimes. Sallam et al. (2002) identified three modes of liquid column breakup based in Weber and Reynolds number: weakly turbulent Rayleigh- like breakup, turbulent breakup, and aerodynamic bag/shear breakup. Water and ethanol were the jet liquids considered. In their analysis they have also incorporated the results of some previous work [Chen and Davis (1964), Grant and Middleman (1966) and Wu and Faeth (1995)]. They propose correlations to predict the breakup pattern in each of the three breakup modes. Li et al. (2007) tried to establish a relation between breakup length and the velocity for a circular impinging jet both experimentally and numerically by using two nozzle diameters.

A few researchers analyzed the effect of geometry of the nozzle to the liquid jet breakup phenomena. The effect of nozzle length on the breakup was studied by Takahashi and Kitamura (1969). They used water and aqueous glycerin as fluids for the liquid jet. The ambient conditions were also varied. They tested the breakup in a stagnant atmosphere of air and in kerosene. They

found a critical nozzle length to diameter ratio, until which the laminar breakup length increased and beyond which, the breakup length reached stagnation. They found this value to be 15 for water in ambient air and 10 for glycerin in ambient air. The critical limit was 10 for both liquids in ambient kerosene. They explained this by stating that, initial amplitude of disturbance decreased as nozzle length increased.

The effect of cavitation on flow as it emerges from the nozzle exit was studied by Vahedi et al. (2003). Taking inspiration from Chigier and Reitz (1996), they experimentally investigated two geometries of nozzle design: cone-up nozzle and a cone-down (constricted) nozzle. It was found that, while the jets issuing from a cone-up nozzle followed the Ohnesorge classification, the jet issuing from a constricted nozzle did not. This was ascribed to separation of flow caused by the abrupt inlet that the liquid faces in a cone-down nozzle. This separation of flow within the nozzle eliminated the effect of wall-friction that initiated the disturbances in the jet, which ultimately would cause the jet to breakup. Hence, as they experimentally observed, the breakup lengths for the cone-down nozzles were higher than those for cone-up nozzles.

The effect of the length of the nozzle on the value of breakup length was tested by Umemura et al. (2011). Water was issued from stainless steel nozzles under gravity in a quiescent ambient atmosphere. They conducted the experiment with two sets of nozzles: one short and the other long. They found the breakup length of the liquid jet to be two valued: the breakup length was longer for the long nozzle. The factors (nozzle exit reflection, velocity profile relaxation, vortex shedding from nozzle entrance) that lead to jet breakup are based on the location of the initial disturbance amplitude. In the case of long nozzle lengths, the velocity at the exit prohibited creation of the disturbance at the exit; whereas, for short nozzles, this disturbance was seen to be

created at the nozzle exit. This, they concluded was the reason for the occurrence of two valued breakup.

3.1 Research Objective

From the literature survey, it is seen that past studies have not been able to conclusively predict the breakup pattern for low Weber number (low velocity) liquid jet. Literature on the effect of geometrical parameters, have primarily concentrated on the nozzle length and the shape, but, not on the diameter of the nozzle itself. The present investigation was undertaken with the following objective: to resolve the breakup pattern at low jet Weber number and to see the influence of jet exit diameter on the breakup length. The ambient pressure (p_a) is not varied. Weber number (We), nozzle diameter (d) and jet inlet velocity (u) are the operating parameters. The jet breakup patterns that arise in these conditions are investigated. To study the effect of viscosity and surface tension on the breakup process, ethylene glycol and propylene glycol, apart from water, are used as working fluids. An empirical correlation to predict breakup length in this low velocity regime is proposed.

Chapter 4

EXPERIMENTAL METHOD

4.1 Overall Setup

Fluid, pumped from a reservoir (collecting tank) by a NEXUS 3000 syringe pump, was used to produce a uniform jet emerging from a simple stainless steel needle. The setup of the experiment is shown in Figure 8. To overcome the flow rate limitation of the syringe pump, a multi-syringe plate attachment is used. *Thermo* Syringes, each having volume of 60 ml, is used. Three to four such syringes are used in parallel to attain the required flow rate. Each syringe is attached to a dual check valve that allows liquid to be drawn from the collecting tank (reservoir) as the syringe plunger is being withdrawn, and allows injection into the needle (nozzle) when the syringe plunger is pushed. The fluid, pumped at a flow rate controlled by the syringe pump, is injected into a stagnant ambient atmosphere through simple stainless steel needles. With increment of flow rate beyond a certain limit, liquid stream (or jet) is formed from the stainless steel needle. This liquid stream breaks up at some distance from the nozzle exit and is collected back in the collecting tank (reservoir) from where it can be withdrawn again by the syringe pump.

Figure 8: Experimental set-up

4.2 Image Capture System

The breakup of the emerging liquid jet from the nozzle exit is captured using a high speed digital camera system: Hi-D cam – II version 3.0 – (NAC Image technology). This had an $8 \times$ zoom lens that enables clear imaging of the breakup process. The system has an adjustable stand and hence can be set in line with the nozzle exit in such a way, that the full breakup length of the liquid jet from the nozzle exit is captured. The frame rate of the camera was varied from 2000 fps to 6000 fps. For a higher incoming velocity of the liquid jet, more frames per second were required to clearly observe the process of breakup. To provide illumination, a single bulb focusing light system (ARRI) with glossy reflectors was used. The light incident from this light system was reflected off a white screen placed behind the setup to ensure better quality of images. The images are captured using a computerized data acquisition system connected to the high-speed camera system. The captured images were analyzed to determine the breakup length of the immerging jets in each case using Image-Pro Plus 4.0 (Media Cybernetics), an image processing software. In order to make accurate measurements of the breakup length; the average of many images captured over a specific time, is taken. To ensure correct measurement in the software (Image Pro), each image needs to be calibrated with respect to some known measurement. The outer diameter of the nozzle (stainless steel needle) is used for calibrating each image.

4.3 Materials and Liquids Used

Twelve stainless steel needles with different diameters ranging from 0.279mm to 1.753mm were used as nozzles to produce the liquid jet. Three pure liquids were used: water, ethylene glycol and propylene glycol. The properties of these three liquids are tabulated in Table 1.

Table 1: Properties of pure liquids used in the experiment

Properties	Water	Ethylene Glycol	Propylene Glycol
Density, ρ (kg/m³)	998	1113.2	1036
Surface Tension, σ (mN/m)	72.8	48.4	47.1
Viscosity, μ (mPa.s)	1	16.1	40.4
Capillary Length, l_c (mm)	2.728	2.096	2.153
Morton number, Mo	2.55×10^{-11}	5.22×10^{-06}	2.41×10^{-04}

4.4 Experimental Test Conditions

In this experiment, liquid flow rate (Q) and the diameter of inlet nozzle (d) are the main operating parameters. Table 2 gives a summary of the range of test parameters that are present in the experiment. The objective was to maximize the Weber number range in the experiments. For a given liquid, as properties do not change, increase in velocity of incoming liquid and increase in diameter of the liquid jet are the only two factors that could maximize the Weber number. The diameter range used for the experiment is given in Table 2. Increment in velocity, depends on the flow rate increment in the syringe pump.

The maximum flow rate in the NEXUS 3000 syringe pump for one 60 ml *Thermo* Syringe is 88.8 ml/min. The maximum flow rate required for obtaining the required range of Weber numbers for the three liquids used exceeds the maximum attainable flow rate for one syringe in the syringe pump as can be seen in Table 2. Thus, three or four syringes are placed in parallel: four syringes in the case with water as working fluid and three syringes when ethylene glycol and propylene glycol were used as the working fluids.

This enabled a wide range of Weber numbers to be tested: 5 to 110 for water; 5 to 80 for ethylene glycol and 5 to 60 for propylene glycol. This also determined the range of Reynolds number obtained for the experiment. In the case with water as the working fluid, Reynolds number (Re) range was from 1375 – 3736, which includes laminar, transitional, and turbulent flow modes. For ethylene glycol and propylene glycol however, the maximum attainable Reynolds Number was 118 and 47 respectively. Thus for these two liquids, the flow was always laminar. The ambient atmospheric conditions were not varied.

The high speed camera system is used to take a series of images which are then studied to understand the process of breakup and the factors governing it.

Table 2: Range of experimental test conditions

Properties	Water	Ethylene Glycol	Propylene Glycol
Nozzle Diameter, d (mm)	0.381 – 1.753	0.508 – 1.499	0.599 – 1.753
Velocity, u (m/s)	0.455 – 5.352	0.352 – 2.495	0.360 – 1.496
Flow rates, Q (ml/min)	7.95 – 114.05	5.92 – 309.2	9.39 – 110.95
Weber number, We	5 – 110	5 – 70	5 – 50
Reynolds number, Re	317.7 – 3735.5	23.0 – 147.68	9.2 – 51.19
Bond Number, Bo	0.011 – 0.413	0.058 – 0.693	0.067 – 0.663
Ohnesorge Number, Oh	0.052 – 0.097	0.0028 – 0.007	0.14 – 0.24

Chapter 5

RESULTS AND DISCUSSION

In a typical case, the liquid is ejected, from the bottom surface of the stainless steel needle (nozzle), with a round cylindrical cross section. The forces of inertia, surface tension, viscous shear and gravity act upon the incipient liquid jet. Since the ambient is stagnant, effects of the dynamic force of an air stream would be absent. Interplay of the aforementioned forces will lead to the breaking up of the liquid column. While inertial, gravitational, and viscous forces retard the breakup process, the capillary force (due to surface tension) assists the breakup.

Three Newtonian liquids are used as working fluid: water, ethylene glycol and propylene glycol. These liquids were chosen in order to interpret the accurate significance of each of the forces in the actual breakup process.

5.1 Water

When water is used as the working fluid, two modes of breakup phenomena are encountered. At low Weber number, for a given diameter, drops are seen to be pinched off the main stream at some distance from the nozzle exit. At higher Weber numbers for the same nozzle diameter, instead of drops, ligaments are pinched off from the main water stream at some distance from the nozzle exit. These ligaments then further breakup, to form drops. As the diameter of the nozzle is increased, this ligament formation is noticed to occur at progressively smaller Weber Numbers.

This can be better understood by examining the breakup for the two sample diameters shown in Figure 9. For the water jet emerging from the smaller diameter nozzle (0.406 mm), ligamented breakup is seen to occur for Weber numbers of 80 and beyond. For the jet emerging from larger diameter nozzle (1.499 mm), ligamented breakup is observed to commence at a smaller Weber number of 30.

While examining the Reynolds number for each of the case, it is noted that ligamented breakup mode for the smaller nozzle (0.406 mm diameter) starts at Reynolds number of 1533 ($We = 80$) and for the larger nozzle (1.499 mm diameter), at Reynolds number of 1804 ($We = 30$). For the other nozzle diameters too, the ligamented form of breakup is observed to occur at around Reynolds number of 1500 - 1850. This may be attributed to the change in flow behavior from laminar to turbulent. This mode, where ligaments are pinched off from the main jet stream, may thus be associated with transitional flow behavior.

For the cases in which ethylene glycol and propylene glycol are used as the working fluid, the maximum Reynolds numbers tested is 103 and 47 respectively. Thus all the cases presented for these two liquids are only laminar and hence breakup of liquid jets into ligaments is not observed.

Figure 9: Breakup process showing ligamented and non-ligamented breakup for water

5.2 Viscous Fluids: Ethylene Glycol & Propylene Glycol

As has been stated in the previous section, the maximum Reynolds number tested for these two liquids are seen to be well in the laminar flow region. Hence, no ligamented breakup mode is observed. It is observed that the breakup length i.e. the undisturbed liquid column length for propylene glycol is longer than that of ethylene glycol which in turn, is longer than that of water.

When ethylene glycol is used as the working fluid, for smaller Weber number and smaller diameters, the breakup is similar to that of the non-ligamented breakup mode of water. But as Weber number and diameter increases, a series of bulges connected by liquid segments are noticed. Each bulge is then pinched off from the main stream to form droplets. Sample images in cases depicting the same, are shown in Figure 10a.

When propylene glycol is used as working fluid, a slightly different trend is noticed. For lower Weber numbers and smaller diameter nozzles, breakup pattern is similar to that of the non-ligamented breakup mode of water. But, for larger diameters at low Weber numbers, bulges connected by long thin threads of liquid are seen. These then pinch off to form droplets. As Weber number is further increased, the length of these connecting threads is observed to decrease. Images showing this trend are shown in Figure 10b.

Figure 10: Breakup process in viscous liquid jets

5.3 Stability Curve

As discussed in Chapter 2, the stability curve i.e. breakup length versus velocity curve, is used to see the general behavior of the breakup length with the issuing velocity of the jet. The stability curve for the cases involving different nozzle diameters for each of the three working fluids: water, ethylene glycol and propylene glycol, has been shown in Figures 11, 12 and 13 respectively. From the Figure 11, it can be seen that, the experimental results obtained for water, are in agreement with the general shape suggested for the stability curve. The peak in the stability curve depicts the maximum breakup length for a given diameter. The plot for ethylene glycol and propylene glycol (Figures 12 and 13) show no distinct maxima. As the Reynolds number range for the experiments with these two liquids are very low, the flow would be purely laminar and hence the resulting stability curve would be exclusively in the linear portion of the general stability curve shown in Figure 1.

Many correlations for predicting the maxima of the stability curve have been suggested. Amongst those, the correlation suggested by Grant and Middleman (1966) is the most commonly used. A comparison of the critical velocity (maxima of the stability curve) found experimentally for water, and that predicted by Grant and Middleman (1966) is shown in Figure 14. Grant and Middleman suggested a correlation relating Reynolds number (Re) at the critical velocity with Ohnesorge number (Oh) for each jet diameter as shown below.

$$Re^* = 325 Oh^{-0.28} \quad (5.1)$$

It is seen from Figure 14 that their prediction agrees well with the current experimental data.

Figure 11: Stability curve for water

Figure 12: Stability curve for ethylene glycol

Figure 13: Stability curve for propylene glycol

Figure 14: Critical velocity comparison with Grant and Middleman (1966)

5.4 Comparison with Past Studies

Many past studies have tried to predict the process of breakup of a liquid column falling under gravity in a stagnant ambient atmosphere. Different factors and geometries have been considered, as has been discussed in Chapter 2. Grant and Middleman (1966) suggested a correlation that they derived from the linear stability theory. Miesse (1955) used experimental predictions to formulate a correlation to describe the jet disintegration. In his paper, water and liquid nitrogen were used as the working fluids. The experiments conducted included a wide range of Weber numbers from ~ 45 to ~ 750 . Sallam et al. (2002) too, used experimental results of various other researchers, apart from their own, to define various regimes or modes of breakup and provided a means of predicting them.

The experimental values of the breakup lengths for the three liquids used, has been compared with the correlation predicted of Grant and Middleman (1966), Meisse (1955) and Sallam et al. (2002) in Figure 15,16 and 17. As can be seen in the figures, while Miesse's correlation over-predicts the breakup length, Sallam et al.'s correlation under-predicts it. Grant and Middleman's correlation is the best fitting correlation and is able to approximately predict the breakup for the viscous liquids too. The effect of nozzle diameter on the breakup is however not predicted by their correlation.

Figure 15: Comparison with other correlations for water

Figure 16: Comparison with other correlations for ethylene glycol

Figure 17: Comparisons with other correlations for propylene glycol

For all the three liquids, the correlation suggested by Grant and Middleman does not agree completely. Also, by observing the experimental data, one can conclude that the non-dimensional breakup length i.e. L/d is dependent on the diameter of the nozzle.

Correlations suggested by Miesse (1955) and Sallam et al (2002) do not agree with the experimental results found. The deviation of their correlations for ethylene glycol and propylene glycol is greater than that for water. Their correlations did not include any property of the liquid

itself and hence, this suggests that, the breakup length could depend on the properties of the pure liquid such as viscosity, surface tension etc.

5.5 Parameters Involved

Breakup length of a liquid column plays a crucial role in many applications. For some applications the breakup process assists its function, whereas, for others, breakup is undesirable. Hence, a means to effectively predict the factors influencing breakup of a liquid column has been an area of constant interest.

In order to understand the breakup, one would need to look at the forces involved in the process. These forces are divided into the aiding (surface tension) and retarding (inertia, viscous forces and gravity) forces, as has already been discussed in Chapter 1. Parameters governing these forces are the following eight:

- Breakup length, L
- Nozzle diameter or exit jet diameter, d
- Velocity of jet at nozzle exit, u
- Density of the liquid, ρ
- Surface Tension of the liquid, σ
- Viscosity of the liquid, μ
- Acceleration due to gravity, g
- Capillary length, l_c

The capillary length defined here, is the breakup length of the liquid column, when the only forces acting on it are the capillary force (resulting due to surface tension) and the gravitational force. The column is considered to be freely falling under gravity and no initial inertia is provided. The resultant force balance on the liquid column is schematically depicted in Figure 18.

The liquid column is approximated to be a cylinder of diameter equal to nozzle exit diameter, d . Hence, the gravitational force F_G , is given by:

$$F_G = \rho g \left(\frac{\pi d^2}{4} \right) l_c \quad (5.2)$$

The capillary force i.e. the force due to surface tension, F_S , can be defined using the capillary pressure (p_s) as:

$$F_S = p_s \left(\frac{\pi d^2}{4} \right) \quad (5.3)$$

The capillary pressure is defined by the Young–Laplace Equation as:

$$p_s = \frac{\sigma}{l_c} \quad (5.4)$$

Hence, by equating the gravitational and capillary forces, $F_G = F_S$, the capillary length l_c , is derived as:

$$l_c = \sqrt{\frac{\sigma}{\rho g}} \quad (5.5)$$

Dimensional analysis of the breakup length in terms of the eight parameters stated above

is performed in order to arrive at a reasonable hypothesis about the breakup process. The Buckingham π theorem is used for performing the dimensional analysis.

Figure 18: Forces acting on freely falling liquid column

5.5.1 Buckingham π Analysis

The main variables that would be involved in the forces that lead to breakup, as discussed before, are: L , d , u , ρ , σ , μ , g and l_c .

The dimensions of the above eight parameters are:

L	$[L]$
d	$[L]$
u	$[L/T]$
ρ	$[ML^{-3}]$
σ	$[MT^{-2}]$
μ	$[ML^{-1}T^{-1}]$
g	$[LT^{-2}]$
l_c	$[L]$

The primary or fundamental variables involved in the above eight are Mass [**M**], Length [**L**], and time [**T**]. Since there are three basic or fundamental dimensions involved, in accordance with the Buckingham π theorem, these parameters need to be grouped into **five** ($8 - 3 = 5$) **non-dimensional π groups**.

Three parameters (d, u, ρ) are chosen in such a manner that they can collectively represent the three fundamental dimensions involved in the problem. The fundamental dimensions; $[M]$, $[L]$ and $[T]$ are expressed in terms of the representative three dimensional parameters: d, u, ρ .

$$[M] = \rho d^3$$

$$[L] = d$$

$$[T] = d/u$$

To obtain the non-dimensional π groups, each of the remaining five dimensions (L, σ, μ, g and l_c) are divided by their dimensional equivalent expressed in terms of d, u and ρ . Thus, the π groups are:

$$\pi_1 = \frac{L}{[L]} = \frac{L}{d} \quad (5.6)$$

$$\pi_2 = \frac{g}{[LT^{-2}]} = \frac{g}{d \frac{d^{-2}}{u^{-2}}} = \frac{g d}{u^2} = \frac{1}{Fr} \quad (5.7)$$

$$\pi_3 = \frac{\sigma}{[MT^{-2}]} = \frac{\sigma}{\rho d^3 \frac{d^{-2}}{u^{-2}}} = \frac{\sigma}{\rho u^2 d} = \frac{1}{We} \quad (5.8)$$

$$\pi_4 = \frac{\mu}{[ML^{-1}T^{-1}]} = \frac{\mu_l}{\rho d^3 d^{-1} \frac{d^{-1}}{u^{-1}}} = \frac{\mu}{\rho u d} = \frac{1}{Re} \quad (5.9)$$

$$\pi_5 = \frac{l_c}{[L]} = \frac{l_c}{d} \quad (5.10)$$

The above five Equations are the non-dimensional π groups. The functional relationship between these dimensionless groups can be expressed as:

$$\pi_l = f(\pi_2, \pi_3, \pi_4, \pi_5) \quad (5.11)$$

Hence, non-dimensional breakup length can be expressed as:

$$\frac{L}{d} = f(Fr, We, Re, \frac{l_c}{d}) \quad (5.12 \text{ a})$$

Bond number (Bo) is the ratio of Weber number (We) to Froude number (Fr). It represents the ratio of gravitational force to the capillary force. Hence, the above expression can also be represented as a function of Bo as shown below.

$$\frac{L}{d} = f(Bo, We, Re, \frac{l_c}{d}) \quad (5.12 \text{ b})$$

5.6 Breakup Length

Understanding the force balance, as shown in Figure 6, shows that the retarding forces (inertia, viscous and gravity) should result in an increment of the breakup length and the aiding forces (surface tension) would lead to decrease in the breakup length. Thus it can be said that any general prediction for breakup length of a liquid column must be in accordance with the following.

$$\frac{L}{d} \sim \frac{\text{Capillary}}{\text{Viscous} \times \text{Inertia} \times \text{Gravitational}}$$

The effects of each of the dimensionless quantities obtained by the Buckingham π analysis on the breakup length are examined. It is seen that breakup length depends on Weber number and Reynolds number directly and inversely with Froude number. Also, as nozzle diameter increases, breakup length is seen to broadly increase.

From the relationship of the dimensionless quantities with the breakup length, the factor $\left(\frac{L}{d}\right)\left(\frac{1}{Re^2 Bo}\right)$ is obtained. A graph of this factor with the dimensionless diameter, d/l_c , is shown for the three liquids used in Figures 19, 20, 22 and 24. The graphs interestingly show that breakup length depends on the nozzle diameter. The slope of the graph in each case (for the different Weber numbers and the different liquids tested) is around -3. Thus a general Equation to represent the plots in these figures would be:

$$\frac{L}{d} \frac{1}{Re^2 Bo} = C \left(\frac{d}{l_c}\right)^{-3} \quad (5.13)$$

The coefficient of each line (C) is then plotted with Weber number to obtain the correlation predicting the breakup length. These graphs for the three working liquids are shown in Figure 21, 23 and 25. The resultant Equation for water, ethylene glycol and propylene glycol thus becomes:

Table 3: Correlation for each liquid used

$$\text{Water} \quad \frac{L}{d} \frac{We^{2/3}}{Re^2 Bo} = 0.000132 \left(\frac{d}{l_c} \right)^{-3} \quad (5.14)$$

$$\text{Ethylene Glycol} \quad \frac{L}{d} \frac{We^{2/3}}{Re^2 Bo} = 0.058 \left(\frac{d}{l_c} \right)^{-3} \quad (5.15)$$

$$\text{Propylene Glycol} \quad \frac{L}{d} \frac{We^{2/3}}{Re^2 Bo} = 0.5 \left(\frac{d}{l_c} \right)^{-3} \quad (5.16)$$

For the case with water, the ligamented breakup mode does not agree with the correlation found. Figure 19 shows only the non-ligamented mode of breakup. This along with the case of ligamented breakup mode is shown in Figure 20. The transition from non-ligamented to ligamented breakup mode is discussed in Section 5.8 of this chapter.

When ethylene glycol and propylene glycol are used as working fluids, the Reynolds number for the cases involved is less than 200 and hence the flow is always laminar. It is seen that the predicted correlation is in good agreement with the experimental results (Figures 22 – 25). However, at low Weber numbers and large nozzle diameter, anomalies exist. These are discussed in the following Section 5.7.

Figure 19: Non-ligamented data for water

Figure 20: Correlation for water

Figure 21: Coefficients vs. Weber number for water

Figure 22: Correlation for ethylene glycol

Figure 23: Coefficients vs. Weber number for ethylene glycol

Figure 24: Correlations for propylene glycol

Figure 25: Coefficients vs. Weber number for propylene glycol

From the experimental correlation derived for the three liquids, a general Equation for any pure liquid could be written as follows.

$$\frac{L}{d} \frac{We^{2/3}}{Re^2 Bo} = C_2 \left(\frac{d}{l_c} \right)^{-3} \quad (5.17)$$

Observing the values of C_2 for the three liquids used, it is concluded that the value of C_2 depends directly on viscosity i.e. for the liquid with larger viscosity, the value is higher. But as the left-hand side of the general expression is dimensionless, the determining factor of C_2 too must be dimensionless. Hence, Morton number, a number based purely on the properties of the liquid, is used. The value of C_2 is thus seen to vary as $34 Mo^{0.51}$. Thus the empirical correlation can be written as follows.

$$\frac{L}{d} \frac{We^{2/3}}{Re^2 Bo} = 34 Mo^{0.51} \left(\frac{d}{l_c} \right)^{-3} \quad (5.18)$$

In the above Equation, Weber number (We), Reynolds number (Re) and Bond number (Bo) are based on the diameter of the nozzle as the characteristic length here, is the nozzle diameter. Since the aim is to express the breakup length as a function of nozzle diameter, the dimensionless parameters, We, Re and Bo are expressed in terms of l_c , the capillary length. Thus the correlation based on l_c is:

$$\frac{L}{d} \frac{We_{l_c}^{2/3}}{Re_{l_c}^2} = 34 Mo^{0.51} \left(\frac{d}{l_c} \right)^{1/3} \quad (5.19)$$

Bo_{l_c} is unity and hence does not appear in the above Equation. The graph depicting the experimentally derived correlation above is shown in Figure 26 for all the three liquids.

Appendix F explains the uncertainty analysis used. The experimental error bar for each data point is also depicted by error bars in Figure 26. The maximum and mean deviation from the correlation for each of the three liquids experimentally tested have been shown in Table 4.

Table 4: Percentage deviation

Liquid tested	Mean deviation (%)	Maximum deviation (%)
Water	10.5	23.2
Ethylene Glycol	11.1	26.3
Propylene Glycol	11.8	25.8

Figure 26: Correlations and error bards for all the three liquids

5.7 Anomalies for Ethylene glycol and Propylene glycol

The correlation for ethylene glycol and propylene glycol is observed to have anomalies as is depicted seen in Figures 22 and 24. This could be attributed to the high viscosities of these two liquids and hence, the involvement of extensional flows.

Extensional flows: There are basically two kinds of flows: shear and extensional. Shear flows occur when the liquid layers pass over each other, while, extensional flows occur when adjacent liquid particles are pulled towards or away from each other. This can be better understood by looking at the pictorial representation in Figure 27. The resistance to shear flow is termed as shear viscosity (μ) and the resistance to extensional flow is termed as extensional or elongation viscosity (η).

Figure 27: Liquid motion in shear and extensional flow (Barnes, 2000)

Trouton (1906) was the first to understand and use the extensional viscosity, though under a different term ‘coefficient of viscous traction’. In his paper he has analytically derived the value of extensional viscosity for Newtonian liquids $\eta = 3 \mu$.

Extensional viscosity is a result of the elongation or pulling force exerted on the liquid element, while shear viscosity arises due to the shearing force exerted. This is shown pictorially in Figure 28.

Figure 28: Pictorial representation for shear and extensional flow

For the current investigation, uniaxial elongation of the liquid element is assumed. The shear rate of deformation is defined as $\dot{\gamma} = \dot{\gamma}/t$ and the uniaxial extensional deformation rate is defined as $\dot{\epsilon} = \dot{\epsilon}/t$.

For Newtonian liquids, simple shear flow can be expressed as $\tau = \mu \dot{\gamma}$ and uniaxial stretching flows are expressed as $\Sigma_e = \eta \dot{\epsilon}$, where τ and Σ_e are the shear stress and extensional stress respectively.

Therefore, the extensional rate, $\dot{\epsilon}$ can be expressed as

$$\dot{\epsilon} = \Sigma_e / \eta_e \text{ where } \Sigma_e = F/A$$

and $A = \text{cross-sectional area} = \pi d^2/4$

$\dot{\epsilon}$ is non-dimensionalized by $\sqrt{g/d}$ and the diameter, d , is non-dimensionalized with the capillary length l_c . On plotting dimensionless elongation rate ($\dot{\epsilon}$) with d/l_c , it is seen that for both ethylene glycol and propylene glycol, the following Equation is satisfied.

$$\frac{\dot{\epsilon}}{\sqrt{g/d}} = Mo^{-0.25} \sqrt{\frac{We}{d}} \quad (5.20)$$

From the plots in Figure 29 and 31, it is observed that at very low Weber numbers and large diameters, the dimensionless elongation rate is almost a constant. Thus plotting the graphs in Figure 30 and 32, it is seen that these points that represent the anomalous behavior, actually lie outside the predicted elongation behavior.

Hence, it can be concluded that, extensional flow comes into play and its significance is realized when inertial forces are low.

In these conditions, surface tension would be the only force opposing the viscous force of the liquid. The ratio of surface tension force to that of the viscous force is noted in Appendix D. For ethylene glycol, it is seen that this ratio is of the order one and is greater than unity, whereas, for propylene glycol, this ratio is less than unity. Thus, while surface tension force is dominating for ethylene glycol, viscous forces dominate in the case of propylene glycol. As, surface tension forces tend to aid breakup and hence lower the breakup length, the breakup length for ethylene glycol is lower than that predicted by Equation 5.18 while for propylene glycol, it is higher than the prediction (Figure 22 and 24). For the case of water, the ratio of capillary to extensional

viscous force is much larger than one (in the order of 10) as is shown in Appendix D. This explains why the effect of extensional viscosity is not noticed for water.

Figure 29: Dimensionless $\dot{\epsilon}$ vs. dimensionless diameter for ethylene glycol

Figure 30: Elongation rate as a function of diameter for ethylene glycol

Figure 31: Dimensionless $\dot{\epsilon}$ vs. dimensionless diameter for propylene glycol

Figure 32: Elongation rate as a function of diameter for propylene glycol

5.8 Transition from Non-ligamented to Ligamented Mode of Breakup for Water

Deviation from the correlation 5.14 is seen while conducting the experiment with water as the working fluid. The non-ligamented breakup mode agrees with the derived correlation while the ligamented mode does not. There exists a transition from non-ligamented to ligamented mode of breakup. As discussed in Section 5.1, ligamentation is seen to occur as the Reynolds number of the flow increases and the fluid flow goes into transition/turbulent flow regime from the laminar flow regime. This transitional regime could not be attained for the experiments that use ethylene glycol and propylene glycol as working fluid, since, this would require higher values of Reynolds number, and hence, higher flow rates (more than the limit of the NEXUS 3000 Syringe Pump). For water, this transition from non-ligamented to ligamented mode is seen to be given by an experimentally derived transition line as shown in Figure 33. The Equation of this line is given as:

$$\frac{l_c}{d} = 1 + \left(\frac{We}{f - We} \right)^{1/3} \quad (5.21)$$

$$\text{where, } f = \frac{49}{Mo^{0.01}}$$

This Equation is arrived at with only the data for ligamented mode of breakup for water and hence depicts the transition for water alone. In order to arrive at a universal transition Equation, more pure liquids would have to be tested at higher Reynolds numbers (transition/turbulent flow regime).

Figure 33: Transition for water data

Chapter 6

SUMMARY AND CONCLUSIONS

6.1 Conclusions

The low Weber number liquid jet disintegration has been studied experimentally for three Newtonian liquids. The results of the work can be summarized into the following key findings.

1. At Weber number less than 100, there exist two modes of breakup for water: the non-ligamented and the ligamented mode. A transition Equation for the same has been derived and is given for water by.

$$\frac{l_c}{d} = 1 + \left(\frac{We}{f - We} \right)^{1/3}$$

$$\text{where, } f = \frac{49}{Mo^{0.01}}$$

2. Breakup length in the non-ligamented mode is found to be in agreement with the general stability curve established by Lin and Reitz (1998). The critical velocity for non-ligamented breakup mode for water is found to be in agreement with the conclusions of Grant and Middleman (1966).
3. A universal correlation to predict the breakup length at low Weber number Newtonian liquids has been established as:

$$\frac{L}{d} \frac{We_{l_c}^{2/3}}{Re_{l_c}^2} = 34 Mo^{0.51} \left(\frac{d}{l_c} \right)^{1/3}$$

It is seen, that the breakup length is not just a function of inertial and capillary forces, but also of gravitational and viscous forces. It is also seen that the breakup length depends on the initial diameter of the jet i.e. the nozzle orifice diameter.

4. For viscous liquids like ethylene glycol and propylene glycol, the correlation developed for the breakup length is found to deviate at low Weber numbers and large diameters. The cause for this anomalous behavior is found to be due to the increased significance of elongational viscosity. Its effect on the breakup length is seen to depend on the capillary force and its ability to overcome the extensional viscous force. For ethylene glycol, since the capillary force is greater than extensional viscous force, the breakup length is lower than that predicted by the above equation. While, for propylene glycol, the breakup length is higher than predicted as the capillary force is lesser than the extensional viscous force.

6.2 Recommendations for Future Work

Due to the limitation of the syringe pump used for this experimental investigation, only a limited range of cases could be tested. A wider range would open up a large scope for research.

1. The current work involves studying the breakup pattern for low Weber numbers for three Newtonian liquids. For water, the maximum Reynolds number attained in the current setup was 3735, whereas for ethylene glycol and propylene glycol it was only 147 and 51 respectively. An experimental set up that enables higher Reynolds number flow for the two viscous liquids considered would help visualize the laminar-turbulent transitional breakup mode. This could then assist in providing a universal transition Equation similar to Equation 5.20.

2. The dripping regime, defined by Chen and Davis (1964) is given by Equation 2.1. It does not explicitly include any viscous effects. It is observed through this work, that at low Weber numbers and large nozzle diameters, the effects of extensional viscosity become important. An experimental probe into this could help clarify the exact role of extensional force on dripping.
3. For the viscous liquids considered, the effect of extensional force is noted. However, a means to predict its significance and influence has not been developed. This would need more experimental data for cases with low inertial force.
4. The ligamented mode of breakup needs more clarification in order to understand its behavior. As is seen from Figure 15, this mode too is not well predicted by past literature. Studies similar to this work, but at higher Weber numbers and Reynolds numbers, for a wide range of diameters will help elucidate this phenomenon.

Chapter 8

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APPENDIX A

Water Jet Data

We	Inner Diameter (mm)	Flow rate (ml/min)	Jet Velocity (m/s)	Re	Breakup Length (mm)
5	0.28	4.19	1.14	317.72	N/A
	0.38	6.69	0.98	371.29	N/A
	0.41	7.36	0.95	383.28	N/A
	0.51	10.30	0.85	428.73	N/A
	0.56	11.89	0.81	449.73	N/A
	0.64	14.40	0.76	479.33	N/A
	0.84	21.83	0.66	550.64	34.20
	0.97	26.98	0.61	590.90	35.74
	1.22	38.30	0.55	664.13	52.66
	1.35	44.44	0.52	697.86	54.66
	1.50	52.23	0.49	736.46	49.02
	1.75	66.05	0.46	796.41	51.67
8	0.28	5.31	1.45	401.89	N/A
	0.38	8.47	1.24	469.65	N/A
	0.41	9.31	1.20	484.81	N/A
	0.51	13.03	1.07	542.30	N/A
	0.56	15.05	1.02	568.87	N/A
	0.64	18.22	0.96	606.31	N/A
	0.84	27.62	0.83	696.51	46.99
	0.97	34.13	0.78	747.43	51.88
	1.22	48.45	0.69	840.06	65.61
	1.35	56.22	0.66	882.74	64.72
	1.50	66.07	0.62	931.56	77.23
	1.75	83.55	0.58	1007.39	64.83
10	0.28	5.93	1.62	449.33	9.90
	0.38	9.47	1.38	525.08	18.99
	0.41	10.41	1.34	542.03	21.58
	0.51	14.57	1.20	606.31	30.41
	0.56	16.82	1.14	636.02	28.55
	0.64	20.37	1.07	677.88	33.42

	0.84	30.88	0.93	778.73	45.20
	0.97	38.15	0.87	835.65	56.81
	1.22	54.17	0.77	939.22	73.50
	1.35	62.85	0.74	986.93	75.38
	1.50	73.87	0.70	1041.51	78.85
	1.75	93.41	0.65	1126.30	84.35
20	0.28	8.39	2.29	635.45	14.78
	0.38	13.39	1.96	742.58	25.97
	0.41	14.72	1.90	766.55	33.84
	0.51	20.61	1.69	857.45	43.21
	0.56	23.79	1.62	899.46	41.85
	0.64	28.80	1.52	958.66	47.64
	0.84	43.66	1.32	1101.29	59.18
	0.97	53.96	1.23	1181.79	80.02
	1.22	76.61	1.09	1328.25	92.50
	1.35	88.88	1.04	1395.73	106.78
	1.50	104.46	0.99	1472.92	89.16
	1.75	132.11	0.91	1592.83	53.42
30	0.28	10.27	2.80	778.26	18.78
	0.38	16.39	2.40	909.47	26.63
	0.41	18.03	2.32	938.83	35.50
	0.51	25.24	2.08	1050.16	46.88
	0.56	29.14	1.98	1101.61	53.57
	0.64	35.27	1.86	1174.12	65.27
	0.84	53.48	1.62	1348.79	67.26
	0.97	66.08	1.51	1447.40	97.18
	1.22	93.82	1.34	1626.77	104.52
	1.35	108.86	1.28	1709.41	132.02
	1.50	127.94	1.21	1803.95	49.18
	1.75	161.80	1.12	1950.81	61.05
40	0.28	11.86	3.23	898.66	22.59
	0.38	18.93	2.77	1050.16	27.11
	0.41	20.82	2.68	1084.07	36.87
	0.51	29.15	2.40	1212.62	52.43
	0.56	33.64	2.28	1272.04	58.72
	0.64	40.73	2.14	1355.75	70.12

	0.84	61.75	1.87	1557.45	67.60
	0.97	76.31	1.74	1671.31	105.93
	1.22	108.34	1.55	1878.43	111.13
	1.35	125.70	1.47	1973.86	130.65
	1.50	147.73	1.40	2083.02	63.32
	1.75	186.83	1.29	2252.60	57.44
50	0.28	13.26	3.62	1004.73	25.50
	0.38	21.16	3.09	1174.12	28.02
	0.41	23.28	3.00	1212.02	42.03
	0.51	32.59	2.68	1355.75	45.96
	0.56	37.61	2.55	1422.18	53.39
	0.64	45.54	2.40	1515.78	74.85
	0.84	69.04	2.09	1741.29	77.97
	0.97	85.31	1.94	1868.58	79.67
	1.22	121.12	1.73	2100.15	133.35
	1.35	140.54	1.65	2206.84	65.37
	1.50	165.17	1.56	2328.89	69.90
	1.75	208.88	1.44	2518.48	62.75
60	0.28	14.53	3.96	1100.63	26.85
	0.38	23.18	3.39	1286.18	29.51
	0.41	25.50	3.28	1327.71	49.09
	0.51	35.70	2.94	1485.15	36.21
	0.56	41.20	2.80	1557.92	53.49
	0.64	49.89	2.63	1660.45	71.07
	0.84	75.63	2.29	1907.48	78.75
	0.97	93.46	2.13	2046.93	54.43
	1.22	132.69	1.89	2300.60	142.88
	1.35	153.95	1.80	2417.47	74.40
	1.50	180.93	1.71	2551.17	65.15
	1.75	228.82	1.58	2758.86	63.94
70	0.28	15.69	4.28	1188.82	29.28
	0.38	25.04	3.66	1389.23	29.25
	0.41	27.55	3.55	1434.09	46.41
	0.51	38.56	3.17	1604.15	30.37
	0.56	44.50	3.02	1682.74	41.57
	0.64	53.88	2.84	1793.49	83.46

	0.84	81.69	2.47	2060.32	67.43
	0.97	100.94	2.30	2210.93	46.95
	1.22	143.32	2.05	2484.93	54.76
	1.35	166.29	1.95	2611.17	79.43
	1.50	195.43	1.85	2755.58	65.83
	1.75	247.15	1.71	2979.91	66.50
80	0.28	16.78	4.57	1270.90	27.47
	0.38	26.77	3.91	1485.15	29.87
	0.41	29.45	3.79	1533.10	37.08
	0.51	41.22	3.39	1714.91	32.65
	0.56	47.58	3.23	1798.93	29.45
	0.64	57.60	3.03	1917.32	91.87
	0.84	87.33	2.64	2202.57	50.60
	0.97	107.91	2.46	2363.59	40.52
	1.22	153.21	2.19	2656.50	76.73
	1.35	177.77	2.08	2791.46	62.98
	1.50	208.92	1.97	2945.84	69.61
	1.75	264.22	1.82	3185.66	64.54
90	0.28	17.79	4.85	1347.99	30.31
	0.38	28.40	4.15	1575.24	21.47
	0.41	31.24	4.02	1626.10	49.29
	0.51	43.72	3.59	1818.93	29.89
	0.56	50.46	3.43	1908.05	33.71
	0.64	61.10	3.22	2033.63	89.16
	0.84	92.63	2.80	2336.18	33.56
	0.97	114.46	2.61	2506.96	46.11
	1.22	162.51	2.32	2817.65	68.01
	1.35	188.55	2.21	2960.79	67.45
	1.50	221.60	2.09	3124.53	70.54
	1.75	280.24	1.94	3378.90	75.83
100	0.28	18.76	5.11	1420.91	30.85
	0.38	29.93	4.38	1660.45	29.04
	0.41	32.93	4.24	1714.06	25.49
	0.51	46.08	3.79	1917.32	27.00
	0.56	53.19	3.61	2011.26	30.58
	0.64	64.40	3.39	2143.63	56.36

	0.84	97.64	2.95	2462.55	33.16
	0.97	120.65	2.75	2642.57	40.56
	1.22	171.30	2.45	2970.06	74.43
	1.35	198.75	2.33	3120.94	66.19
	1.50	233.58	2.21	3293.55	71.17
	1.75	295.40	2.04	3561.67	71.30
110	0.28	19.67	5.36	1490.26	30.29
	0.38	31.39	4.59	1741.49	26.72
	0.41	34.53	4.45	1797.72	26.95
	0.51	48.33	3.97	2010.90	26.93
	0.56	55.79	3.79	2109.43	31.20
	0.64	67.55	3.55	2248.26	47.93
	0.84	102.40	3.09	2582.75	36.15
	0.97	126.54	2.88	2771.55	42.85
	1.22	179.66	2.57	3115.02	63.44
	1.35	208.45	2.44	3273.27	74.69
	1.50	244.99	2.31	3454.30	74.50
	1.75	309.82	2.14	3735.52	83.94

APPENDIX B

Ethylene Glycol Jet Data

We	Inner Diameter (mm)	Flow rate (ml/min)	Jet Velocity (m/s)	Re	Breakup Length (mm)
5	0.51	7.96	0.65	22.98	15.64
	0.56	9.18	0.62	24.10	21.73
	0.64	11.12	0.59	25.69	25.33
	0.84	16.85	0.51	29.51	35.09
	0.97	20.83	0.47	31.67	23.64
	1.22	29.57	0.42	35.59	30.62
	1.35	34.31	0.40	37.40	34.88
	1.50	40.32	0.38	39.47	40.21
8	0.51	10.06	0.83	29.06	17.74
	0.56	11.62	0.79	30.49	27.10
	0.64	14.06	0.74	32.49	34.05
	0.84	21.32	0.64	37.33	45.03
	0.97	26.35	0.60	40.06	37.22
	1.22	37.40	0.53	45.02	51.83
	1.35	43.40	0.51	47.31	41.89
	1.50	51.01	0.48	49.93	47.27
10	0.51	11.25	0.93	32.49	16.74
	0.56	12.99	0.88	34.09	26.93
	0.64	15.72	0.83	36.33	33.27
	0.84	23.84	0.72	41.74	53.24
	0.97	29.46	0.67	44.79	46.48
	1.22	41.82	0.60	50.34	51.87
	1.35	48.52	0.57	52.89	56.41
	1.50	57.03	0.54	55.82	56.11
20	0.51	15.91	1.31	45.95	21.81
	0.56	18.37	1.25	48.21	27.00
	0.64	22.24	1.17	51.38	46.34
	0.84	33.71	1.02	59.02	62.09
	0.97	41.66	0.95	63.34	66.65

	1.22	59.14	0.84	71.19	99.81
	1.35	68.62	0.80	74.80	104.14
	1.50	80.65	0.76	78.94	87.63
30	0.51	19.49	1.60	56.28	26.79
	0.56	22.49	1.53	59.04	39.44
	0.64	27.23	1.43	62.93	53.37
	0.84	41.29	1.25	72.29	68.41
	0.97	51.02	1.16	77.57	84.38
	1.22	72.43	1.03	87.19	106.98
	1.35	84.04	0.98	91.61	124.88
	1.50	98.77	0.93	96.68	131.63
40	0.51	22.50	1.85	64.99	38.51
	0.56	25.97	1.76	68.17	38.44
	0.64	31.45	1.65	72.66	51.02
	0.84	47.67	1.44	83.47	82.01
	0.97	58.91	1.34	89.57	89.59
	1.22	83.64	1.19	100.67	140.52
	1.35	97.05	1.14	105.79	152.42
	1.50	114.05	1.08	111.64	N/A
50	0.51	25.16	2.07	72.66	38.74
	0.56	29.04	1.97	76.22	44.64
	0.64	35.16	1.85	81.24	67.16
	0.84	53.30	1.61	93.32	91.02
	0.97	65.86	1.50	100.15	100.45
	1.22	93.51	1.34	112.56	141.15
	1.35	108.50	1.27	118.27	N/A
	1.50	127.52	1.20	124.82	N/A
60	0.51	27.56	2.27	79.60	N/A
	0.56	31.81	2.16	83.50	59.13
	0.64	38.51	2.03	88.99	75.20
	0.84	58.39	1.76	102.23	97.13
	0.97	72.15	1.64	109.70	108.48
	1.22	102.44	1.46	123.30	132.48
	1.35	118.86	1.39	129.56	N/A
	1.50	139.69	1.32	136.73	N/A

70	0.51	29.77	2.45	85.97	N/A
	0.56	34.36	2.33	90.19	54.43
	0.64	41.60	2.19	96.12	90.53
	0.84	63.07	1.91	110.42	112.32
	0.97	77.93	1.78	118.49	126.96
	1.22	110.64	1.58	133.18	N/A
	1.35	128.38	1.50	139.94	N/A
	1.50	150.88	1.42	147.68	N/A

APPENDIX C

Propylene Glycol Jet Data

We	Inner Diameter (mm)	Flow rate (ml/min)	Jet Velocity (m/s)	Re	Breakup Length (mm)
5	0.56	9.39	0.64	9.14	24.81
	0.64	11.37	0.60	9.74	36.84
	0.84	17.24	0.52	11.19	52.58
	0.97	21.30	0.49	12.01	49.03
	1.22	30.24	0.43	13.50	68.05
	1.35	35.09	0.41	14.18	81.23
	1.50	41.23	0.39	14.97	136.54
	1.75	52.15	0.36	16.19	170.07
8	0.56	11.88	0.81	11.56	33.56
	0.64	14.38	0.76	12.32	42.07
	0.84	21.80	0.66	14.16	67.16
	0.97	26.94	0.61	15.19	59.49
	1.22	38.25	0.55	17.07	79.30
	1.35	44.38	0.52	17.94	90.67
	1.50	52.16	0.49	18.93	134.01
	1.75	65.96	0.46	20.48	188.50
10	0.56	13.28	0.90	12.93	N/A
	0.64	16.08	0.85	13.78	47.05
	0.84	24.37	0.74	15.83	70.99
	0.97	30.12	0.69	16.99	69.91
	1.22	42.76	0.61	19.09	91.01
	1.35	49.62	0.58	20.06	100.86
	1.50	58.31	0.55	21.17	150.11
	1.75	73.75	0.51	22.89	184.59
20	0.56	18.78	1.28	18.28	N/A
	0.64	22.74	1.20	19.49	42.57
	0.84	34.47	1.04	22.38	90.75
	0.97	42.60	0.97	24.02	85.82
	1.22	60.48	0.86	27.00	132.68

	1.35	70.17	0.82	28.37	116.34
	1.50	82.47	0.78	29.94	153.90
	1.75	104.29	0.72	32.38	N/A
30	0.56	23.00	1.56	22.39	N/A
	0.64	27.85	1.47	23.86	N/A
	0.84	42.22	1.28	27.42	82.18
	0.97	52.17	1.19	29.42	112.88
	1.22	74.07	1.06	33.07	148.52
	1.35	85.94	1.01	34.74	131.89
	1.50	101.00	0.95	36.67	173.63
	1.75	127.73	0.88	39.65	N/A
40	0.56	26.56	1.80	25.86	N/A
	0.64	32.16	1.69	27.56	N/A
	0.84	48.75	1.47	31.66	N/A
	0.97	60.24	1.37	33.97	106.12
	1.22	85.53	1.22	38.18	148.43
	1.35	99.24	1.16	40.12	178.43
	1.50	116.63	1.10	42.34	N/A
	1.75	147.49	1.02	45.79	N/A
50	0.56	29.69	2.02	28.91	N/A
	0.64	35.95	1.89	30.81	N/A
	0.84	54.50	1.65	35.39	N/A
	0.97	67.35	1.53	37.98	N/A
	1.22	95.62	1.37	42.69	170.69
	1.35	110.95	1.30	44.86	171.93
	1.50	130.39	1.23	47.34	N/A
	1.75	164.90	1.14	51.19	N/A

APPENDIX D

Capillary Force to Extensional Viscous Force Ratio

For Water:

	0.279	0.381	0.406	0.508	0.559	0.635	0.838	0.965	1.219	1.346	1.499	1.753
5	21.27	24.85	25.65	28.70	30.10	32.08	36.86	39.55	44.45	46.71	49.29	53.31
8	16.81	19.65	20.28	22.69	23.80	25.36	29.14	31.27	35.14	36.93	38.97	42.14
10	15.04	17.57	18.14	20.29	21.29	22.69	26.06	27.97	31.43	33.03	34.86	37.69
20	10.63	12.43	12.83	14.35	15.05	16.04	18.43	19.78	22.23	23.36	24.65	26.65
30	8.68	10.15	10.47	11.72	12.29	13.10	15.05	16.15	18.15	19.07	20.12	21.76
40	7.52	8.79	9.07	10.15	10.64	11.34	13.03	13.98	15.72	16.51	17.43	18.85
50	6.73	7.86	8.11	9.07	9.52	10.15	11.66	12.51	14.06	14.77	15.59	16.86
60	6.14	7.17	7.41	8.28	8.69	9.26	10.64	11.42	12.83	13.48	14.23	15.39
70	5.68	6.64	6.86	7.67	8.05	8.57	9.85	10.57	11.88	12.48	13.17	14.25
80	5.32	6.21	6.41	7.17	7.53	8.02	9.21	9.89	11.11	11.68	12.32	13.33
90	5.01	5.86	6.05	6.76	7.10	7.56	8.69	9.32	10.48	11.01	11.62	12.56
100	4.76	5.56	5.74	6.42	6.73	7.17	8.24	8.84	9.94	10.44	11.02	11.92
110	4.53	5.30	5.47	6.12	6.42	6.84	7.86	8.43	9.48	9.96	10.51	11.37

For Ethylene Glycol:

	0.508	0.559	0.635	0.838	0.965	1.219	1.346	1.499
5	1.53	1.61	1.71	1.97	2.11	2.37	2.49	2.63
8	1.21	1.27	1.35	1.56	1.67	1.88	1.97	2.08
10	1.08	1.14	1.21	1.39	1.49	1.68	1.76	1.86
20	0.77	0.80	0.86	0.98	1.06	1.19	1.25	1.32
30	0.63	0.66	0.70	0.80	0.86	0.97	1.02	1.07
40	0.54	0.57	0.61	0.70	0.75	0.84	0.88	0.93
50	0.48	0.51	0.54	0.62	0.67	0.75	0.79	0.83
60	0.44	0.46	0.49	0.57	0.61	0.68	0.72	0.76
70	0.41	0.43	0.46	0.53	0.56	0.63	0.67	0.70

For Propylene Glycol:

	0.508	0.559	0.635	0.838	0.965	1.219	1.346	1.499
5	0.61	0.65	0.75	0.80	0.90	0.95	1.00	1.08
8	0.48	0.51	0.59	0.63	0.71	0.75	0.79	0.85
10	0.43	0.46	0.53	0.57	0.64	0.67	0.71	0.76
20	0.30	0.32	0.37	0.40	0.45	0.47	0.50	0.54
30	0.25	0.27	0.30	0.33	0.37	0.39	0.41	0.44
40	0.22	0.23	0.26	0.28	0.32	0.33	0.35	0.38
50	0.19	0.21	0.24	0.25	0.28	0.30	0.32	0.34

APPENDIX E

Dimensionless Elongation Rate

For Ethylene Glycol:

	0.508	0.559	0.635	0.838	0.965	1.219	1.346	1.499
5	143.6	137.2	129.3	114.0	107.3	97.8	94.3	91.0
8	228.8	218.5	205.5	180.4	169.1	152.8	146.7	140.6
10	285.6	272.6	256.3	224.6	210.3	189.4	181.6	173.7
20	569.5	543.3	510.2	445.7	416.4	372.7	356.0	339.0
30	853.5	814.0	764.2	666.7	622.4	556.0	530.4	504.3
40	1137.4	1084.6	1018.2	887.8	828.4	739.3	704.9	669.6
50	1421.4	1355.3	1272.2	1108.9	1034.4	922.6	879.3	834.9
60	1705.3	1626.0	1526.1	1330.0	1240.4	1105.9	1053.8	1000.2
70	1989.3	1896.7	1780.1	1551.0	1446.4	1289.2	1228.2	1165.5
80	2273.2	2167.4	2034.1	1772.1	1652.5	1472.6	1402.6	1330.8

For Propylene Glycol:

	0.508	0.559	0.635	0.838	0.965	1.219	1.346	1.499
5	10.64	10.02	8.83	8.31	7.56	7.29	7.03	6.71
8	10.59	9.96	8.74	8.19	7.39	7.10	6.80	6.42
10	10.57	9.93	8.70	8.15	7.34	7.03	6.72	6.32
20	10.53	9.89	8.64	8.07	7.22	6.90	6.57	6.12
30	10.52	9.88	8.62	8.04	7.18	6.85	6.51	6.06
40	10.52	9.87	8.61	8.03	7.17	6.83	6.49	6.03
50	10.51	9.87	8.60	8.02	7.15	6.82	6.47	6.01
60	10.51	9.86	8.60	8.02	7.15	6.81	6.46	5.99

APPENDIX F

Uncertainty Analysis

Any experimental set-up will be affected by errors induced due to instruments, measurements, methods used etc. Calculating the uncertainty in each measurement will give an estimate of the correctness or accuracy of the obtained results.

While using the Image-Pro software to measure the breakup length of the liquid jet images, one needs to calibrate and scale the image to enable correct measurements. Calibration is done so as to be able to assign a length scale to each pixel length of the image. There are two ways in which this calibration can be done. One way is to assign a value of measurement for each pixel dimension. For example, if each pixel is assigned a value of 1 mm, then the measure length of 30 pixels would be 30 mm. This method could be done if the distance between the camera and the image to be captured is always maintained as a constant. This however, cannot be done for the current experiment. Hence, another method, where a reference physical object is measured for each image captured. Here, the nozzle outer diameter was taken as the reference. The nozzle outer diameter is measured with the help of vernier calipers whose uncertainty is ± 0.01 mm. Hence the uncertainty in the measurement (using Image-Pro) of breakup length is ± 0.01 mm.

The other parameter that is measured in the experiment is the flow rate. This is fixed using the NEXUS-3000 syringe pump whose uncertainty in measurement is ± 0.0001 ml/min.

To measure the uncertainties in subsequent calculations, the Single Sample Error Propagation Method is used. Here, the overall measurement uncertainty of a parameter, P is given by:

$$\delta P = \sqrt{\sum_{i=1}^n \left(\frac{\partial P}{\partial x_i} \delta x_i \right)^2}$$

where, $P = f(x_1, x_2, x_3 \dots)$

x_n represents the variable or measurements on which the parameter P depends.

Using the above relation and the uncertainty in measurements of d, L, and the flow rate, the errors for each value or data point can be calculated. The percentage error is calculated as $\delta P / P \times 100$.