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High-Fidelity Simulations of Transitional Flow Over Pitching Airfoils

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High-Fidelity Simulations of Transitional Flow Over Pitching Airfoils

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by

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Committee Chair: Dr. Paul Orkwis

Abstract

Presented is a high-fidelity, computational study of transitional flow over an airfoil as it is pitched up from an initial zero incidence to 40° at a nominally constant pitch rate, held, and then returned in a similar manner. The Reynolds numbers were chosen to bracket the regions of laminar and transitional flows applicable to prototypical micro air vehicle conditions, $5 \times 10^3 \leq Re_c \leq 4 \times 10^4$. A high-order, implicit large eddy simulation technique was employed to show the degree of fidelity required to capture these highly transitional flows. Two-dimensional analyses examining the effects of Reynolds number and pitch rate were conducted and a discussion is provided. Additionally, the impact of transition and spanwise extent on the flowfield and force histories were explored through three-dimensional, spanwise periodic simulations. These simulations were shown qualitatively to compare extremely well with available experimental observations.

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List of Symbols

C_l, C_d, C_m	lift, drag, and moment coefficients
c	chord
x, y, z	Cartesian coordinates
ξ, η, ζ	computational coordinates
u, v, w	Cartesian velocity components
M	Mach number
t	time
ρ	dimensionless density
T	dimensionless static temperature
p	dimensionless static pressure, $\frac{\rho T}{\gamma M_\infty^2}$
τ	dimensionless time, tU_∞/c
E_t	total energy per unit volume
e	internal energy
μ	molecular viscosity coefficient
λ	second molecular viscosity coefficient
Θ	heat flux vector
Re_c	chord Reynolds number, $\rho_\infty u_\infty c / \mu_\infty$
Pr	Prandtl number
α	geometric angle of attack
Ω	dimensionless pitch rate, $\Omega^+ c / U_\infty$

$\vec{\omega}$	dimensionless vorticity vector, $\vec{\omega}^+ c / U_\infty$
$\vec{U}$	conserved variable vector
$\hat{F}, \hat{G}, \hat{H}$	flux vectors
J	determinant of the inverse Jacobian, $\partial(\xi, \eta, \zeta, \tau) / \partial(x, y, z, t)$
$\tau_1, \tau_2, \tau_3, \tau_4$	dimensionless times of start and stop points in motion
a	motion smoothing parameter

Subscripts

v	denotes viscous quantity
∞	denotes free-stream value

Superscripts

$+$	denotes dimensional quantity
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Chapter 1

Introduction

Over the last few years, Micro Air Vehicle (MAV) development has sparked the interest of many within the research community, especially in the areas of intelligence, reconnaissance, and surveillance (ISR). Miniature, maneuverable, unmanned aircraft that can fly at low speeds or even hover have the potential to unlock a wide range of possibilities into exploring dangerous, inhospitable, or constricting environments. However, with this growing desire for MAV development, comes the need to understand and accurately model the flow physics associated with low Reynolds number, flapping flight.

Biological flyers use a great deal of natural aerodynamic mechanisms to their advantage while performing simple maneuvers in flight such as perching and landing. Fig. 1.1 shows a snapshot of a flare-like motion of a cardinal [1] at its maximum angle of attack as it attempts to perch. Through the high angle pitching motion, the cardinal is able to dynamically stall its wings, thereby temporarily generating higher lift beyond its normal static wing loading while simultaneously reducing its speed through the increased drag. This results in a precisely controlled landing.

The phenomenon of dynamic stall was first identified on rotor blades when conventional aerodynamics failed to predict the performance of high-speed helicopters [4]. Since then, extensive amounts of research, both experimentally and computationally, have been conducted

Figure 1.1: Biological Perching Maneuver [1]

to characterize and model the dynamic stall process for helicopter blades, turbine and compressor blades, and highly maneuverable aircraft. While much of the computational work focuses on laminar flow (Refs. [5, 6, 7, 8]), more recent work has been used to investigate the three-dimensional structures associated with dynamic stall (Ref. [9]). Figure 1.2 shows the dynamic stall process for a pitching airfoil. Initially and early in the motion, the flow around the airfoil remains attached (a-b). As the angle of attack increases higher, reversed flow is encountered as the separation point moves from the trailing edge towards the leading edge on the upper surface of the airfoil (c). The recirculation region builds into a leading edge vortex which becomes the stall vortex (d-e). Once this vortex is ejected and convects downstream, full stall is finally encountered (f).

The works of Carr [4], McCroskey [10], and Ekaterinaris and Platzer [11] provide comprehensive reviews of the progress in the prediction of dynamic stall. Much of the previous research done on dynamic stall vortex generation has been devoted to airfoils performing harmonic motions of only moderate amplitudes in a uniform stream. A few researchers have focused on non-periodic motions with constant pitch rate and high angle amplitudes.

The work of Visbal [7] focused on the simulation of the turbulent flow around a constant-rate pitching airfoil using the Favre-averaged Navier-Stokes equations to investigate the

Figure 1.2: Stages in pitching motion of an airfoil [2]

effects of compressibility on dynamic stall. Analyses of pitch rate and pitch axis location were included in this work, where it was concluded that increased pitch rate resulted in an angular delay of the dynamics stall process. The higher pitch rates also caused lower lift slope, $\frac{dC_L}{d\alpha}$, and moment coefficient values before separation reached the leading edge region. A similar delay in the dynamic stall process was experienced as the pitch axis location was moved downstream along the chord of the airfoil for a fairly high pitch rate of 0.2. It was also noted that aft-displacement of the pitch axis location caused a substantial reduction in the max lift coefficient. The compressibility effects were analyzed for free stream Mach numbers ranging from 0.2 to 0.6. For $M_\infty \geq 0.5$, fully developed, supercritical flow was observed over the airfoil. The dynamic stall process for these Mach numbers was found to be controlled by the shock/boundary layer interaction rather than by the formation of a leading-edge vortex. For the intermediate Mach numbers, $0.2 \leq M_\infty \leq 0.4$, the dynamic stall process was still characterized by the formation and convection of a leading edge vortex. With increased Mach number, the stalling of the airfoil would change from trailing to leading edge with an

earlier stall and decrease in maximum lift coefficient.

Gendrich, Koochesfahani, and Visbal [12] simulated the effects of initial acceleration on the laminar flow fields around rapidly pitching airfoils. In their study, a NACA 0012 airfoil was accelerated to a constant pitch rate from an initial zero degree incidence. Three angular accelerations were explored with a range of maximum pitch rates. Their findings had concluded that leading edge separation and the development of the dynamic stall vortex system was not influenced by the initial acceleration of the airfoil for the range of parameters analyzed. They also found that the lift, drag, and moment coefficients were only affected during the acceleration periods and slightly thereafter before reverting to their dependence only on instantaneous angle of attack and pitch rate. Additionally, the results compared quite favorably to experiments under the same conditions.

Three-dimensional effects were explored computationally for a pitching wing in Ref. [9]. In this work, second-order accurate three-dimensional simulations were conducted with up to sixth-order accuracy for the two-dimensional cases. While the results showed some three-dimensional variation of spanwise vorticity, the formations of the primary, secondary, and tertiary vortices were found to be very similar to their two-dimensional simulations both in angle of inception and chordwise location.

From only the few projects listed here, it is clear to see how complicated these kinds of maneuvers can be due to the complexity of the interrelated parameters including pitch rate, amplitude, and pitch axis location; not to mention the normal flowfield dependencies on Mach number, Reynolds numbers, and airfoil shape [5]. It becomes necessary to systematically formulate canonical cases to understand each of these dependencies and how they relate to each other.

Large flow unsteadiness, massive separation, motion-history effects, laminar to turbulent transition, three-dimensional flowfields dominated by vortex and acceleration effects all play a large role in the complexity of the MAV flight problem [13]. As a means of encompassing each of these areas, the AIAA Fluid Dynamics Technical Committee (FDTC) Low Reynolds

Number Discussion Group (LRDG) selected a set of transient, flare-like maneuvers as a canonical case study for MAV aerodynamics. The motivation for the transient, one-time motion as described by Ol *et al.* [13] was to avoid periodic motion conditions, where barring long-time relaxation of start-up transients, the response would be periodic as well. The problem was designed to be scalable for both wind tunnels and water tunnels, which placed bounds on Reynolds number and dimensionless pitch rate. The maximum angle of attack was chosen to go well into deep stall, and the angle of attack history relevant to applications such as perching, maneuvering, gust loading, and flapping. On the numerical side, the Reynolds number was again restricted by the desire to bracket laminar and transitional cases to allow for approaches ranging from immersed boundary methods to three-dimensional large eddy simulations (LES).

A variety of computational methods and measurement techniques were employed by a wide range of people in a collaborative effort to create a database of knowledge for MAV-type problems in unsteady aerodynamics. On the computational side, the work of Eldredge, Wang, and Ol [14] used a vortex particle method (VVP) [15], to analyze the effects of pitch rate, Reynolds number and wing cross section on vortex shedding and force generation. Their results showed that a flat plate and elliptical cross section produced similar results with only slightly more lift and drag for the flat plate case. Increasing pitch rate caused a steady increase in both lift and drag on the upstroke, while only the lower pitch rates were able to maintain positive lift and drag on the downstroke. The higher pitch rates did cause the airfoil to generate thrust on the downstroke. While their results show some favorable agreement with experimental data on the upstroke of the motion, their model lacks the capability to capture the motion-history effects of the downstroke of the motion.

Lian [16] used a pressure-Poisson immersed boundary method to investigate the effects of pitch rate, Reynolds number ($Re < 10^4$), and location of pitch axis. Laminar flow was assumed throughout these computations, so no turbulence effects were modeled. The most notable conclusions from his work indicated that the RANS method provided too much

numerical dissipation to capture the proper vortex shedding for lower pitch rate motions. Over prediction of the dissipation caused smearing of the discrete roll-ups of the trailing edge vortices which differed dramatically from available experimental data [13]. However, the RANS calculations matched fairly well with the experiments for higher pitch rates during the majority of the upstroke, where again, flow memory effects seemed to dominate. This was also the case with the lower-order method of Eldredge described above.

On the experimental side, Kilany, Judde and Soria [17] used three component - two dimensional (3C-2D) stereoscopic PIV (SPIV) in a free-surface water tunnel on a high pitch rate ($\Omega \approx 1.8$) motion at a Reynolds number of 7,500. Their work tracked the development of the leading and trailing edge vortices generated throughout the motion. With such a high pitch rate, the formation of the leading edge vortex was delayed until the downstroke of the motion with only trailing edge vortices being produced during the upstroke.

Williams *et al* [18] used particle image velocimetry in a wind tunnel to explore the effect of pitch rate and the formation of the dynamic stall vortex. Their measurements indicated a three-dimensional variation of the dynamic stall vortex towards the end of the upstroke. The variation continued on the the downstroke, where they noticed signs of a deterioration of the primary vortex into smaller, more discrete vortices. ate and Reynolds number.

For the current research, a high-order, implicit large eddy simulation (ILES) technique was employed to simulate the AIAA FDTC LRDG first canonical case for the pitch and return maneuver of a flat plate. The objectives were to study the effects of Reynolds number and pitch rate in two-dimensions, expand the two-dimensional simulations to spanwise periodic three-dimensional cases, explore the effect of spanwise extent, and validate the results against experimental water tunnel measurements [3]. This will then provide the high-fidelity computational component of the afore-mentioned collaborative effort for the canonical case.

1.1 Problem Description

Simulations of the pitch, hold, and return maneuver were performed for the flat plate depicted in Fig. 1.3. The plate has a nominal thickness of 2.5% with rounded leading and trailing edges. This promoted both the use of an O-grid topology and also, to some extent, forced laminar to turbulent transition as discussed by Ol [3]. Sharp edges could have fostered a more abrupt transition process acting like trips for the flow. The simplicity of the O-grid topology was also chosen for the easier implementation of boundary conditions on the rotating domain.

Figure 1.3: Flat Plate Geometry

The maneuver of Eldredge *et al.* [14] was used in order to achieve a continuous motion that was sufficiently differentiable to avoid discontinuities in the angular acceleration. The pitch angle in dimensionless time is given by Eq. 1.1 and shown in Fig. 1.4.

$$\alpha(\tau) = \frac{\Omega_0}{2a} \ln \left[\frac{\cosh(a(\tau - \tau_1)) \cosh(a(\tau - \tau_4))}{\cosh(a(\tau - \tau_2)) \cosh(a(\tau - \tau_3))} \right] \quad (1.1)$$

Here, a controls the smoothing intensity, Ω_0 is the desired dimensionless pitch rate, and the dimensionless quantities, τ_1 through τ_4 , are given as

τ_1 = time from reference 0 until sharp start up of the unsmoothed ramp (seen in blue)

τ_2 = τ_1 + the unsmoothed pitch upstroke duration

τ_3 = τ_2 + the unsmoothed hold time at maximum angle

τ_4 = τ_3 + the unsmoothed pitch downstroke duration

Figure 1.4: Angle of Attack Time History for Pitch, Hold, and Return

Insight into how this motion was developed can be gained by instead looking at the pitch rate, which through a differentiation in time of Eq. 1.1 is

$$\dot{\alpha}(\tau) = \frac{1}{2}\Omega_0 [\tanh(a(\tau - \tau_1)) - \tanh(a(\tau - \tau_2)) - \tanh(a(\tau - \tau_3)) + \tanh(a(\tau - \tau_4))] \quad (1.2)$$

This shows that the pitch rate is simply composed of a summation of scaled hyperbolic tangent functions centered at the points where smoothing is desired as seen in Fig. 1.5.

Figure 1.5: Pitch Rate Time History for Pitch, Hold, and Return

As a is increased, the hyperbolic tangent functions approach their horizontal asymptotes much more rapidly, thereby decreasing their smoothing contribution. This causes the overall pitch rate to approach a step function with increasing a . The inverse is true when a is decreased, where values of $a \approx 2$ produce a nearly sinusoidal pitch rate. This produces a one-parameter family of curves that cover a wide range of motions from a near step function ($a = 100$) to a sinusoid ($a = 2$).

For the current study, the hold time, $\tau_3 - \tau_2$, was 0.05 convective times. With such a short hold time, the smoothing parameter, a , was chosen to be 100 to only allow a small amount of smoothing in order to avoid rounding off the hold. Three pitch rates, $\Omega_0 = 0.2, 0.4, 0.8$, were analyzed with the plate pitched about the quarter chord, and they will be discussed later. The angle of attack histories for the three pitch rates can be seen below.

Figure 1.6: Angle of Attack Time History for Various Pitch Rates

Chapter 2

Methodology

In this chapter, a discussion of the numerical scheme and procedure for the LES calculations used in this study is presented including details into the governing equations and their discretization along with a spatial filtering scheme.

2.1 Governing Equations

Due to the highly unsteady, viscous nature of the current study, the governing equations are the full compressible, three-dimensional Navier-Stokes equations. These equations can be transformed from Cartesian to curvilinear coordinates, (x, y, z, t) to (ξ, η, ζ, τ) and recast back in strong conservative form [19] to yield the following:

$$\frac{\partial}{\partial \tau} \left(\frac{\vec{U}}{J} \right) + \frac{\partial \hat{F}}{\partial \xi} + \frac{\partial \hat{G}}{\partial \eta} + \frac{\partial \hat{H}}{\partial \zeta} = \frac{1}{Re} \left[\frac{\partial \hat{F}_v}{\partial \xi} + \frac{\partial \hat{G}_v}{\partial \eta} + \frac{\partial \hat{H}_v}{\partial \zeta} \right] \quad (2.1)$$

The vector of conserved variables, $\vec{U}$, and the inviscid flux vectors, $\hat{F}$, $\hat{G}$, and $\hat{H}$, are given by

$$\vec{U} = [\rho, \rho u, \rho v, \rho w, \rho E]^T \quad (2.2)$$

$$\hat{F} = \frac{1}{J} \begin{bmatrix} \rho U \\ \rho u U + \xi_x p \\ \rho v U + \xi_y p \\ \rho w U + \xi_z p \\ (\rho E + p) U - \xi_t p \end{bmatrix}, \hat{G} = \frac{1}{J} \begin{bmatrix} \rho V \\ \rho u V + \eta_x p \\ \rho v V + \eta_y p \\ \rho w V + \eta_z p \\ (\rho E + p) V - \eta_t p \end{bmatrix}, \hat{H} = \frac{1}{J} \begin{bmatrix} \rho W \\ \rho u W + \zeta_x p \\ \rho v W + \zeta_y p \\ \rho w W + \zeta_z p \\ (\rho E + p) W - \zeta_t p \end{bmatrix} \quad (2.3)$$

where the directional contravariant velocities, U , V , and W , are

$$\begin{aligned} U &= \xi_t + \xi_x u + \xi_y v + \xi_z w \\ V &= \eta_t + \eta_x u + \eta_y v + \eta_z w \\ W &= \zeta_t + \zeta_x u + \zeta_y v + \zeta_z w \end{aligned} \quad (2.4)$$

and the volume specific energy, E , is

$$E = \frac{T}{\gamma(\gamma - 1)M_\infty^2} + \frac{1}{2} (u^2 + v^2 + w^2) \quad (2.5)$$

For simplicity, the terms with subscripts of the physical coordinates represent derivatives with respect to that given variable, i.e. $\xi_x = \partial \xi / \partial x$ and likewise for the other derivative terms. The viscous flux vectors, $\hat{F}_v$, $\hat{G}_v$, and $\hat{H}_v$, in indicial notation are

$$\hat{F}_v = \frac{1}{J} \begin{bmatrix} 0 \\ \xi_{x_i} \tau_{i1} \\ \xi_{x_i} \tau_{i2} \\ \xi_{x_i} \tau_{i3} \\ \xi_{x_i} (u_j \tau_{ij} - \Theta_i) \end{bmatrix}, \quad \hat{G}_v = \frac{1}{J} \begin{bmatrix} 0 \\ \eta_{x_i} \tau_{i1} \\ \eta_{x_i} \tau_{i2} \\ \eta_{x_i} \tau_{i3} \\ \eta_{x_i} (u_j \tau_{ij} - \Theta_i) \end{bmatrix}, \quad \hat{H}_v = \frac{1}{J} \begin{bmatrix} 0 \\ \zeta_{x_i} \tau_{i1} \\ \zeta_{x_i} \tau_{i2} \\ \zeta_{x_i} \tau_{i3} \\ \zeta_{x_i} (u_j \tau_{ij} - \Theta_i) \end{bmatrix} \quad (2.6)$$

Employing Stokes' hypothesis for the bulk viscosity coefficient ($\lambda = -2/3\mu$), components of the stress tensor and heat flux vector are given by

$$\tau_{ij} = \mu \left(\frac{\partial \xi_k}{\partial x_j} \frac{\partial u_i}{\partial \xi_k} + \frac{\partial \xi_k}{\partial x_i} \frac{\partial u_j}{\partial \xi_k} - \frac{2}{3} \delta_{ij} \frac{\partial \xi_l}{\partial x_k} \frac{\partial u_k}{\partial \xi_l} \right) \quad (2.7)$$

and

$$\Theta_i = - \left[\frac{1}{(\gamma - 1)M_\infty^2} \right] \left(\frac{\mu}{Pr} \right) \frac{\partial \xi_j}{\partial x_i} \frac{\partial T}{\partial \xi_j} \quad (2.8)$$

In indicial notation, the Cartesian coordinates and velocities correspond to $x_i (\equiv x, y, z)$, $i = 1...3$ and $u_i (\equiv u, v, w)$, $i = 1...3$, respectively, and the computational coordinates are given by $\xi_i (\equiv \xi, \eta, \zeta)$, $i = 1...3$. The repeated indices, i , j , k , or l , imply summations while δ_{ij} is simply the unit tensor. All variables have been nondimensionalized by their respective free-stream values, except for pressure which was normalized by twice the dynamic pressure, $\rho_\infty U_\infty^2$.

In order to close the previous set of equations, the perfect gas relation,

$$p = \frac{\rho T}{\gamma M_\infty^2} \quad (2.9)$$

along with Sutherland's law for viscosity are also used. The current set of equations is the full, *unfiltered* Navier-Stokes equations and is used without change in laminar, transitional or fully turbulent regions of the flow.

2.2 The Numerical Method

2.2.1 The Time Marching Scheme

Time-accurate solutions of Eq. 2.1 were computed numerically with the implicit, approximately factored, time-integration method of Beam and Warming [20]. This method has been simplified through the diagonalization of Pulliam and Chaussee [21] and supplemented with

the use of Newton-like subiterations to achieve second-order accuracy. In delta-form, the scheme can be written as

$$\begin{aligned}
& \left[\left(\frac{1}{J} \right)^{p+1} + \left(\frac{2\Delta\tau}{3} \right) \delta_\xi^{(2)} \left(\frac{\partial \hat{F}^p}{\partial \vec{U}} - \frac{1}{Re} \frac{\partial \hat{F}_v^p}{\partial \vec{U}} \right) \right] J^{p+1} \\
& \times \left[\left(\frac{1}{J} \right)^{p+1} + \left(\frac{2\Delta\tau}{3} \right) \delta_\eta^{(2)} \left(\frac{\partial \hat{G}^p}{\partial \vec{U}} - \frac{1}{Re} \frac{\partial \hat{G}_v^p}{\partial \vec{U}} \right) \right] J^{p+1} \\
& \times \left[\left(\frac{1}{J} \right)^{p+1} + \left(\frac{2\Delta\tau}{3} \right) \delta_\zeta^{(2)} \left(\frac{\partial \hat{H}^p}{\partial \vec{U}} - \frac{1}{Re} \frac{\partial \hat{H}_v^p}{\partial \vec{U}} \right) \right] \Delta \vec{U} \\
& = - \left(\frac{2\Delta\tau}{3} \right) \left[\left(\frac{1}{J} \right)^{p+1} \frac{3\vec{U}^p - 4\vec{U}^n + \vec{U}^{n-1}}{2\Delta\tau} + \vec{U}^p \left(\frac{1}{J} \right)_\tau^p \right. \\
& \left. + \delta_\xi^{(6)} \left(\hat{F}^p - \frac{1}{Re} \hat{F}_v^p \right) + \delta_\eta^{(6)} \left(\hat{G}^p - \frac{1}{Re} \hat{G}_v^p \right) + \delta_\zeta^{(6)} \left(\hat{H}^p - \frac{1}{Re} \hat{H}_v^p \right) \right]
\end{aligned} \tag{2.10}$$

In this expression, $\Delta \vec{U} = \vec{U}^{p+1} - \vec{U}^p$ where $\vec{U}^{p+1}$ corresponds to the $p+1$ approximation of $\vec{U}$ at the $n+1$ time level, while the δ -operator represents a spatial difference in the direction given by the subscript and the order given by the superscript. For the first subiteration, $p=1$ corresponds to $\vec{U}^n$. As $p \rightarrow \infty$, $\vec{U}^p \rightarrow \vec{U}^{n+1}$ [22].

The derivatives in the right-hand, explicit portion of Eq. 2.10 are evaluated using a high-order, compact finite difference, which is explained in more detail later. The implicit, left-hand side of the equation uses second-order central differences for all spatial derivatives while employing fourth-order, nonlinear dissipation terms [23, 24] in order to augment stability. This term in two-dimensions and in the ξ -direction can be seen in Eq. 2.11. Similar terms are also employed for the η - and ζ -directions.

$$\nabla_\xi (\sigma_{i+1,j} J_{i+1,j}^{-1} + \sigma_{i,j} J_{i,j}^{-1}) (\epsilon_{i,j}^{(2)} \Delta_\xi \vec{U}_{i,j} - \epsilon_{i,j}^{(4)} \Delta_\xi \nabla_\xi \Delta_\xi \vec{U}_{i,j}) \tag{2.11}$$

where Δ_ξ and ∇_ξ are forward and backward difference operators, respectively. The $\sigma_{i,j}$ term is the spectral radius scaling parameter is

$$\sigma_{i,j} = |U| + a\sqrt{\xi_x^2 + \xi_y^2} + |V| + a\sqrt{\eta_x^2 + \eta_y^2} \tag{2.12}$$

The second- and fourth-difference dissipation coefficients, $\epsilon^{(2)}$ and $\epsilon^{(4)}$, are given by

$$\epsilon_{i,j}^{(2)} = \kappa_2 \Delta t \max(\Gamma_{i+1,j}, \Gamma_i, \Gamma_{i-1,j}) \quad (2.13)$$

$$\epsilon_{i,j}^{(4)} = \max(0, \kappa_4 \Delta t - \epsilon_{i,j}^{(2)}) \quad (2.14)$$

with the coefficient to the second-difference dissipation term, Γ , at i, j as

$$\Gamma_{i,j} = \frac{|p_{i+1,j} - 2p_{i,j} + p_{i-1,j}|}{|p_{i+1,j} + 2p_{i,j} + p_{i-1,j}|} \quad (2.15)$$

Typical values of the constants are $\kappa_2 = 0.25$ and $\kappa_4 = 0.01$ [24], however, for the cases studied here, only fourth-order dissipation was used, so $\kappa_2 = 0$ while $\kappa_4 = 0.01$. The impact of the implicit damping coefficients is eliminated through the use of subiterations, so this allows their value to be determined for stability considerations exclusively [22]. The above dissipation model is appended to the diagonalized form of the approximately factored scheme results in a scalar pentadiagonal system.

As mentioned before, subiterations are utilized within a time step to maintain second-order temporal accuracy. This procedure is commonly used to reduce errors due to factorization, linearization, diagonalization, and explicit specification of boundary conditions [25]. In a variety of applications and simulations, including unsteady vortical flows, three subiterations per time step were found to be sufficient to preserve second-order accuracy in time [25, 26, 27]. A detailed analysis of the subiteration convergence for the current study is presented later. Additionally, the second-order, subiterative Beam-Warming method described was previously found to be well-suited for moving and deforming meshes [22].

2.2.2 Spatial Discretization

High-order spatial discretization of Eq. 2.1 is achieved using a compact finite difference formulation as described by Lele [28]. Given a discrete function, $f_i = f(x_i)$, Lele asserted

that the finite difference approximation of the derivative of the function at any point can be expressed as a linear combination of the surrounding functional values and derivatives. For simplicity, a uniform grid spacing of h is assumed. This results in the pentadiagonal system

$$\beta f'_{i-2} + \alpha f'_{i-1} + f'_i + \alpha f'_{i+1} + \beta f'_{i+2} = a \frac{f_{i+1} - f_{i-1}}{2h} + b \frac{f_{i+2} - f_{i-2}}{4h} + c \frac{f_{i+3} - f_{i-3}}{6h} \quad (2.16)$$

where the coefficients, a , b , c , α , and β , are found through relations determined by matching the Taylor series coefficients for the desired order of accuracy. The expansions for the left- and right-hand side terms of Eq. 2.16 around point i can be seen in Equations 2.17- 2.21. The Taylor expansions for these terms have been simplified and shown in a compact, summation notation to reveal the coefficients of the higher-order terms.

$$\frac{f_{i+1} - f_{i-1}}{2h} = \sum_{n=0}^{\infty} \frac{h^{2n}}{(2n+1)!} f_i^{(2n+1)} \quad (2.17)$$

$$\frac{f_{i+2} - f_{i-2}}{4h} = \sum_{n=0}^{\infty} \frac{2^{2n} h^{2n}}{(2n+1)!} f_i^{(2n+1)} \quad (2.18)$$

$$\frac{f_{i+3} - f_{i-3}}{6h} = \sum_{n=0}^{\infty} \frac{3^{2n} h^{2n}}{(2n+1)!} f_i^{(2n+1)} \quad (2.19)$$

$$(f'_{i-1} + f'_{i+1}) = 2 \sum_{n=0}^{\infty} \frac{h^{2n}}{(2n)!} f_i^{(2n+1)} \quad (2.20)$$

$$(f'_{i-2} + f'_{i+2}) = 2 \sum_{n=0}^{\infty} \frac{2^{2n} h^{2n}}{(2n)!} f_i^{(2n+1)} \quad (2.21)$$

Substituting these relations into the first derivative approximation of Eq. 2.16 yields

$$f_i^{(1)} + \sum_{n=0}^{\infty} 2(\alpha + 2^{2n}\beta) \frac{h^{2n}}{(2n)!} f_i^{(2n+1)} = \sum_{n=0}^{\infty} (a + 2^{2n}b + 3^{2n}c) \frac{h^{2n}}{(2n+1)!} f_i^{(2n+1)} \quad (2.22)$$

A family of schemes is then achieved by matching the Taylor coefficients on each side in order to eliminate the terms up to the desired order of accuracy. This leaves the recursive relation:

$$0^n + 2(2n + 1)(\alpha + 2^{2n}\beta) = (a + 2^{2n}b + 3^{3n}c) \quad n = 0, 1, 2, \dots \quad (2.23)$$

where the order of the term is given by $O(h^{2n})$. Thus, the relations for second- through tenth-order accurate schemes are given by

$$\begin{aligned} 1 + 2(\alpha + \beta) &= a + b + c && \text{(Second Order)} \\ 2 \cdot 3(\alpha + 2^2\beta) &= a + 2^2b + 3^2c && \text{(Fourth Order)} \\ 2 \cdot 5(\alpha + 2^4\beta) &= a + 2^4b + 3^4c && \text{(Sixth Order)} \\ 2 \cdot 7(\alpha + 2^6\beta) &= a + 2^6b + 3^6c && \text{(Eighth Order)} \\ 2 \cdot 9(\alpha + 2^8\beta) &= a + 2^8b + 3^8c && \text{(Tenth Order)} \end{aligned} \quad (2.24)$$

Resolution Analysis of Interior Discretization Scheme

As a means of comparison between the different families of schemes, the resolution of each can be determined through a Fourier wave number analysis [28]. Expanding the function, $f(x)$, which is assumed periodic over $[0, L]$, into its corresponding Fourier coefficients, results in

$$f(x) = \sum_{k=-N/2}^{N/2} \hat{f}_k \exp\left(\frac{2\pi j k x}{L}\right) \quad (2.25)$$

where $j = \sqrt{-1}$, k is the physical wave number, and $x \in (0, L)$. Next, a scaled wavenumber, $\omega = 2\pi k h / L = 2\pi k / N$, is introduced, along with a scaled coordinate, $s = x/h$, leaving

$$f(s) = \sum_{k=-N/2}^{N/2} \hat{f}_k \exp(j\omega s) \quad (2.26)$$

The derivative, $f'(s)$, is then readily attainable as

$$f'(s) = \sum_{k=-N/2}^{N/2} j\omega \hat{f}_k \exp(j\omega s) = j\omega f(s) \quad (2.27)$$

It is now assumed that the finite difference approximation to the derivative results in a modified wave number that relates to the original coefficients by

$$(\hat{f}'_k)_{fd} = j\omega' \hat{f}_k \quad (2.28)$$

Finally, taking a unit spacing centered around $s = 0$, the previous expressions can then be substituted into the difference approximation of Eq. 2.16.

$$\begin{aligned} \beta f'(-2) + \alpha f'(-1) + f'(0) + \alpha f'(1) + \beta f'(2) &= a \frac{f(1)-f(-1)}{2} + b \frac{f(2)-f(-2)}{4} + c \frac{f(3)-f(-3)}{6} \\ i\omega'(\beta(e^{-2i\omega} + e^{2i\omega}) + \alpha(e^{-i\omega} + e^{i\omega}) + 1) &= a \frac{e^{j\omega} - e^{-j\omega}}{2} + b \frac{e^{2j\omega} - e^{-2j\omega}}{4} + c \frac{e^{3j\omega} - e^{-3j\omega}}{6} \end{aligned} \quad (2.29)$$

which through trigonometric manipulation reveals the modified wave number as

$$\omega' = \frac{a \sin(\omega) + (b/2) \sin(2\omega) + (c/3) \sin(3\omega)}{2\beta \cos(2\omega) + 2\alpha \cos(\omega) + 1} \quad (2.30)$$

The wave number resolution of a particular scheme can then be plotted against the exact solution, $\omega' = \omega$, as seen in Fig. 2.1 with a summary of the coefficients for each of the plotted schemes shown in Table 2.1. Second-, fourth-, and sixth-order explicit central difference approximations ($\alpha = \beta = 0$) are included for comparison.

Figure 2.1: Resolution Analysis of Various Schemes

Table 2.1: Discretization Scheme Coefficients

OA	α	β	a	b	c	Stencils	
						LHS	RHS
2*	0	0	1	0	0	1	3
4*	0	0	4/3	-1/3	0	1	5
6*	0	0	3/2	-3/5	1/10	1	7
4	1/4	0	3/2	0	0	3	3
6	1/3	0	14/9	1/9	0	3	5
8	4/9	1/36	40/27	25/54	0	5	5
10	1/2	1/20	17/12	101/150	1/100	5	7

*** Denotes explicit scheme**

It is clear that each of the compact schemes surpasses the resolution capabilities of the explicit schemes. Even the fourth-order compact scheme provides better resolution than the

sixth-order explicit approximation. For the same order of accuracy, the compact approximations also require a smaller right-hand-side stencil size than their explicit counterpart. Another way to compare the resolution of each scheme is by defining the resolving efficiency [28] as $e = \omega_f/\pi$, where ω_f is the shortest well-resolved wave, which can be determined from the relation

$$\frac{|\omega'(\omega) - \omega|}{\omega} \leq \epsilon \quad (2.31)$$

where ϵ is a prescribed error tolerance. The largest wave number to meet this criterion corresponds to ω_f . For the schemes shown in Fig. 2.1, the resolving efficiency for a maximum error tolerance of 10% can be seen in Fig. 2.2.

Figure 2.2: Resolving Efficiency of Various Schemes

For this error tolerance, the advantage of compact over explicit schemes is evident with nearly 16% more resolution when comparing fourth- and sixth-order schemes. These efficiencies are compiled in Table 2.2.

Table 2.2: Resolving Efficiencies

	$O(h^2)$	$O(h^4)$	$O(h^6)$	$O(h^8)$	$O(h^{10})$
Explicit	0.2504	0.4439	0.5434		
Compact		0.5941	0.7016	0.7792	0.8178

Moving to higher order compact schemes does not result in as much of a benefit. Moving from fourth- to sixth-order compact gives around 11% more resolution, sixth- to eighth-order gives almost 8%, and eighth- to tenth-order yields about a 4% increase. Another aspect of the schemes to consider is the computational cost. The fourth- and sixth-order schemes only require tridiagonal systems to be solved, while the eighth- and tenth-order schemes require pentadiagonal systems to be solved. With only an 8% increase in resolution to go from sixth- to eighth-order, the trade-off computationally to go to a pentadiagonal system proves to be far too expensive. Additionally, simulations by Visbal and Gaitonde [22] showed the importance of the sixth-order accurate scheme over second-order when computing 2-D pitching airfoils with rigidly rotating meshes. The sixth-order scheme allowed for the development of significant shear-layer vortices that were not captured by the second-order scheme on much finer meshes. For this reason, the sixth-order, tridiagonal compact scheme was used for the current study.

Non-Periodic Boundary Conditions

Special care must be taken with the boundary points of a computational domain. In order to maintain the tridiagonal system defined from the sixth-order compact scheme of the interior nodes and the five-point, right-hand-side stencil, the boundary point derivatives, f'_1 and f'_2 , can be approximated by the one-side biased relations

$$f'_1 + \alpha_1 f'_2 = \frac{1}{h}(a f_1 + b f_2 + c f_3 + d f_4 + e f_5) \quad (2.32)$$

and

$$\alpha_2 f'_1 + f'_2 + \alpha_2 f'_3 = \frac{1}{h}(af_1 + bf_2 + cf_3 + df_4 + ef_5) \quad (2.33)$$

The right-hand-side, five-point stencil is also assumed for these relations. As with the interior scheme, the Taylor series expansion can be substituted into the previous relations in order to find the corresponding relations for the desired order-of-accuracy. Thus, matching the Taylor coefficients for point 1, yields

$$\begin{aligned} 1 + \alpha_1 &= b_1 + 2c_1 + 3d_1 + 4e_1 && \text{(First Order)} \\ 2\alpha_1 &= b_1 + 2^2c_1 + 3^2d_1 + 4^2e_1 && \text{(Second Order)} \\ 3\alpha_1 &= b_1 + 2^3c_1 + 3^3d_1 + 4^3e_1 && \text{(Third Order)} \\ 4\alpha_1 &= b_1 + 2^4c_1 + 3^4d_1 + 4^4e_1 && \text{(Fourth Order)} \\ 5\alpha_1 &= b_1 + 2^5c_1 + 3^5d_1 + 4^5e_1 && \text{(Fifth Order)} \end{aligned} \quad (2.34)$$

with the leading term coefficient, a_1 , determined through the identity,

$$a_1 + b_1 + c_1 + d_1 + e_1 = 0 \quad (2.35)$$

Similarly, the relations for point 2 are

$$\begin{aligned} 1 + 2\alpha_2 &= -a_2 + c_2 + 2d_2 + 3e_2 && \text{(First Order)} \\ 0 &= a_2 + c_2 + 2^2d_2 + 3^2e_2 && \text{(Second Order)} \\ 6\alpha_2 &= -a_2 + c_2 + 2^3d_2 + 3^3e_2 && \text{(Third Order)} \\ 0 &= a_2 + c_2 + 2^4d_2 + 3^4e_2 && \text{(Fourth Order)} \\ 10\alpha_2 &= -a_2 + c_2 + 2^5d_2 + 3^5e_2 && \text{(Fifth Order)} \end{aligned} \quad (2.36)$$

with b_2 determined through the identity,

$$a_2 + b_2 + c_2 + d_2 + e_2 = 0 \quad (2.37)$$

The maximum achievable order for each boundary point system is fifth using these formulations. The coefficients for the one-side biased stencils of boundary point 1 and 2 can be seen in Tables 2.3 and 2.4, respectively.

Table 2.3: Boundary Point 1 Coefficients

Scheme	α_1	a_1	b_1	c_1	d_1	e_1	OA
C2	1	-2	2	0	0	0	2
C3	2	-5/2	2	1/2	0	0	3
C4	3	-17/2	3/2	3/2	-1/6	0	4
C5	4	-37/12	2/3	3	-2/3	1/12	5

Table 2.4: Boundary Point 2 Coefficients

Scheme	α_2	a_2	b_2	c_2	d_2	e_2	OA
C4	1/4	-3/4	0	3/4	0	0	4
C5	3/14	-19/28	-5/42	6/7	-1/14	1/84	5

Similar expressions can be derived for the N and $N - 1$ boundary points with only differences in the signs of the coefficients as the one-sided biased stencils switch directions. In order to maintain stability, the fifth- and fourth-order one-side biased schemes have been chosen for the second and first boundary points, respectively.

2.2.3 Spatial Filtering

Unlike the standard LES approach, no additional sub-grid stress (SGS) and heat flux terms are appended to the governing equations found in Section 2.1. Instead, a high-order, low-pass filter operator is applied to the conserved dependent variables during the solution of the standard Navier-Stokes equations in order to provide dissipation. This highly-discriminating, Padé-type filter selectively damps only the evolving, poorly resolved high-frequency content

of the solution [26, 27, 29, 30]. This filtering regularization procedure provides an attractive alternative to the use of standard sub-grid-scale (SGS) models, and has been found to yield suitable results for several turbulent flows on LES level grids. A reinterpretation of this implicit LES (ILES) approach in the context of an Approximate Deconvolution Model [31] has been provided by Matthew *et al* [32]. The filter is applied to the conserved variables along each transformed coordinate direction once after each time step or sub-iteration. For transitional and turbulent flows, the high-fidelity spatial filtering provides an effective implicit LES approach in lieu of traditional SGS models, as demonstrated in References [26] and [27].

The filter is derived in a similar manner as the compact differencing scheme discussed previously. If a typical component of the solution vector is denoted by ϕ , filtered values, $\hat{\phi}$, at interior points are given by

$$\alpha_f \hat{\phi}_{i-1} + \hat{\phi}_i + \alpha_f \hat{\phi}_{i+1} = \sum_{m=0}^M \frac{a_m}{2} (\phi_{i+m} + \phi_{i-m}) \quad (2.38)$$

where the explicit stencil on the right-hand-side is determined by the desired order-of-accuracy. Eq. 2.38 is based on templates proposed in References [28] and [33] and with proper choice of coefficients, provides a $2N^{th}$ -order formula on a $2N+1$ point stencil. As with the compact scheme for the spatial discretization, these coefficients can be found through matching the a Taylor coefficients. Substituting the expansions about point, i , into Eq 2.38 yields,

$$\begin{aligned} \hat{\phi}_i + \alpha_f \sum_{n=0}^{\infty} \frac{(1+(-1)^n) \hat{\phi}_i^{(n)}}{n!} &= \sum_{m=0}^N \frac{a_m}{2} \left(\sum_{n=0}^{\infty} \frac{m^n (1+(-1)^n) \phi_i^{(n)}}{n!} \right) \\ \hat{\phi}_i + 2\alpha_f \sum_{n=0}^{\infty} \frac{\hat{\phi}_i^{(2n)}}{(2n)!} &= \sum_{m=0}^N a_m \left(\sum_{n=0}^{\infty} m^{2n} \frac{\phi_i^{(2n)}}{(2n)!} \right) \end{aligned} \quad (2.39)$$

It should be noted that this formulation has eliminated the odd derivatives leaving a non-dispersive filter. This ensures that no waves will be amplified. Matching each coefficient

from the above expression leaves

$$0^{2n} + 2\alpha_f = \sum_{m=0}^N m^{2N} a_m \text{ for } N = 0, 1, 2, \dots \quad (2.40)$$

From this relation, if α_f is left as a free variable, there are then $N+1$ unknowns ($\alpha_0, \alpha_1, \dots, \alpha_N$) but only N equations. To close the set of equations, another desirable constraint can be imposed by looking at the frequency response of the filter. This is found through another Fourier analysis. Substituting the respective Fourier expansions into the filter equation (Eq. 2.38), yields

$$\sum_k \hat{b}_k (1 + \alpha_f (e^{-i\omega} + e^{i\omega})) = \sum_k b_k \left(\sum_{m=0}^N \frac{a_m}{2} (e^{-im\omega} + e^{im\omega}) \right) \quad (2.41)$$

where b_k and $\hat{b}_k$ are the Fourier coefficients for the original and filtered functions, respectively. Now, through trigonometric substitution and rearranging, the frequency response of the filter is given as

$$\frac{\hat{b}_k}{b_k} = SF(\omega) = \frac{\sum_{m=0}^N a_m \cos(m\omega)}{1 + 2\alpha_f \cos(\omega)} \quad (2.42)$$

where this relation is simply a function of the wave number, ω . To completely eliminate the highest frequency mode, a constraint of $SF(\pi) = 0$ is employed. This yields the final equation,

$$\sum_{m=0}^N (-1)^m a_m = 0 \quad (2.43)$$

The coefficients for up to a tenth-order filter are tabulated in Table 2.5 [30, 34].

Table 2.5: Interior Point Filter Coefficients

OA	a_0	a_1	a_2	a_3	a_4	a_5
2	$\frac{1+2\alpha_f}{2}$	$\frac{1+2\alpha_f}{2}$	0	0	0	0
4	$\frac{5+6\alpha_f}{8}$	$\frac{1+2\alpha_f}{2}$	$\frac{-1+2\alpha_f}{8}$	0	0	0
6	$\frac{11+10\alpha_f}{16}$	$\frac{15+34\alpha_f}{32}$	$\frac{-1+2\alpha_f}{16}$	$\frac{1-2\alpha_f}{32}$	0	0
8	$\frac{93+70\alpha_f}{128}$	$\frac{7+18\alpha_f}{16}$	$\frac{-1+2\alpha_f}{32}$	$\frac{1-2\alpha_f}{16}$	$\frac{-1+2\alpha_f}{128}$	0
10	$\frac{193+126\alpha_f}{256}$	$\frac{105+302\alpha_f}{256}$	$\frac{-1+2\alpha_f}{64}$	$\frac{45(1-2\alpha_f)}{512}$	$\frac{5(-1+2\alpha_f)}{256}$	$\frac{1-2\alpha_f}{512}$

It is also seen from the frequency response that $|\alpha_f| < 0.5$ in order to ensure that $0 < SF(\omega) < 1$. With higher values of α_f , the frequency response remains close to 1 for more of the wave number spectrum, thereby providing less dissipation. A special case of $\alpha_f = 0.5$ corresponds to an identity where the frequency response goes to 1, so there is no filtering [29, 30]. The resolutions of various ordered filters can be seen in Fig. 2.3. In each of these cases $\alpha_f = 0.3$. It is seen that at the higher wave numbers where the compact schemes lack resolution, the implicit filters selectively dampen out only the poorly resolved waves, while avoiding the lower wave numbers. Higher-order filtering provides less dissipation to the lower wave numbers while still suppressing the higher, under-resolved wave numbers.

Figure 2.3: Interior Filtering Schemes for $\alpha_f = 0.3$

The filtering scheme selected for this study, was the eighth-order accurate formulation with $\alpha_f = 0.3$ to provide two-orders of magnitude more resolution than the spatial discretization scheme.

Near-Boundary Filter Formulations

Because of the large stencil size of the higher-order schemes, modifications must be made as one moves towards a boundary to not extend outside the domain. For example, as the *F8* scheme with a nine-point centered stencil moves towards a boundary, it cannot properly span the last four points near the boundary. For this reason, one-side biased schemes must be formulated to shift the explicit stencil or the order of the filter must be reduced to shorten the required stencil. High-order, one-side biased filters can be derived in the same way as the compact scheme at the boundary. At a near-boundary point, i , a filter formula is given by

$$\alpha_f \hat{\phi}_{i-1} + \hat{\phi}_i + \alpha_f \hat{\phi}_{i+1} = \sum_{n=1}^{11} a_{n,i} \phi_n \quad (2.44)$$

where $i \in \{2, \dots, 5\}$. Again, this choice of the filter retains the tridiagonal form of the filter.

The resolution of the one-sided-biased filters can be determined again through a Fourier analysis of Eq. 2.44. From Ref. [30], the spectral function, SF , at $i \in \{2, \dots, 5\}$ is

$$SF(\omega) = \frac{\sum_{n=1}^{11} a_{n,i} \{\cos((n-i)\omega) + j \sin((n-i)\omega)\}}{1 + 2\alpha_f \cos(\omega)} \quad (2.45)$$

with similar definitions for j and ω as before. It should be noted that the spectral function for the boundary filter formulations does have an imaginary part due to the asymmetry of Eq. 2.44.

Enforcing the conditions $Real(SF(\pi)) = 0$ as before to remove the highest frequency content and matching Taylor series coefficients yields the coefficients, $a_{n,i}$. The coefficients for the first five points off the boundary are shown in Tables 2.6 through 2.9 [29, 30]. Similar expressions can be derived for the $N - 4$ through $N - 1$ boundary points with merely a sign difference.

For the current study, the fourth-, fourth-, sixth-, and eighth-order accurate, one-side biased schemes were selected for the second through fifth boundary points, respectively, to be coupled with the eighth-order accurate interior formulation.

Table 2.6: Coefficients for boundary filter formulas at point 5

OA	$a_{1,5}$	$a_{2,5}$	$a_{3,5}$	$a_{4,5}$	$a_{5,5}$	$a_{6,5}$	$a_{7,5}$	$a_{8,5}$	$a_{9,5}$	$a_{10,5}$	$a_{11,5}$
10	$\frac{-1+2\alpha_f}{1024}$	$\frac{10-20\alpha_f}{512}$	$\frac{-45+90\alpha_f}{1024}$	$\frac{15+98\alpha_f}{128}$	$\frac{407+210\alpha_f}{512}$	$\frac{63+130\alpha_f}{256}$	$\frac{-105+210\alpha_f}{512}$	$\frac{15-30\alpha_f}{128}$	$\frac{-45+90\alpha_f}{1024}$	$\frac{5-10\alpha_f}{512}$	$\frac{-1+2\alpha_f}{1024}$

Table 2.7: Coefficients for boundary filter formulas at point 4

OA	$a_{1,4}$	$a_{2,4}$	$a_{3,4}$	$a_{4,4}$	$a_{5,4}$	$a_{6,4}$	$a_{7,4}$	$a_{8,4}$	$a_{9,4}$	$a_{10,4}$	$a_{11,4}$
8	$\frac{1-2\alpha_f}{256}$	$\frac{-1+2\alpha_f}{32}$	$\frac{7+50\alpha_f}{64}$	$\frac{25+14\alpha_f}{32}$	$\frac{35+58\alpha_f}{128}$	$\frac{-7+14\alpha_f}{32}$	$\frac{7-14\alpha_f}{64}$	$\frac{-1+2\alpha_f}{32}$	$\frac{1-2\alpha_f}{256}$	0	0
10	$\frac{1-2\alpha_f}{1024}$	$\frac{-5+10\alpha_f}{512}$	$\frac{45+934\alpha_f}{1024}$	$\frac{113+30\alpha_f}{128}$	$\frac{105+302\alpha_f}{512}$	$\frac{-63+126\alpha_f}{256}$	$\frac{105-210\alpha_f}{512}$	$\frac{-15+30\alpha_f}{128}$	$\frac{45-90\alpha_f}{1024}$	$\frac{-5+10\alpha_f}{512}$	$\frac{1-2\alpha_f}{1024}$

Table 2.8: Coefficients for boundary filter formulas at point 3

OA	$a_{1,3}$	$a_{2,3}$	$a_{3,3}$	$a_{4,3}$	$a_{5,3}$	$a_{6,3}$	$a_{7,3}$	$a_{8,3}$	$a_{9,3}$	$a_{10,3}$	$a_{11,3}$
6	$\frac{-1+2\alpha_f}{64}$	$\frac{3+26\alpha_f}{32}$	$\frac{49+30\alpha_f}{64}$	$\frac{5+6\alpha_f}{16}$	$\frac{-15+30\alpha_f}{64}$	$\frac{3-6\alpha_f}{32}$	$\frac{-1+2\alpha_f}{64}$	0	0	0	0
8	$\frac{-1+2\alpha_f}{256}$	$\frac{1+30\alpha_f}{32}$	$\frac{57+14\alpha_f}{64}$	$\frac{7+18\alpha_f}{32}$	$\frac{-35+70\alpha_f}{128}$	$\frac{7-14\alpha_f}{32}$	$\frac{-7+14\alpha_f}{64}$	$\frac{1-2\alpha_f}{32}$	$\frac{-1+2\alpha_f}{256}$	0	0
10	$\frac{-1+2\alpha_f}{1024}$	$\frac{5+502\alpha_f}{512}$	$\frac{979+90\alpha_f}{1024}$	$\frac{15+98\alpha_f}{128}$	$\frac{-105+210\alpha_f}{512}$	$\frac{63-126\alpha_f}{256}$	$\frac{-105+210\alpha_f}{512}$	$\frac{15-30\alpha_f}{128}$	$\frac{-45+90\alpha_f}{1024}$	$\frac{5-10\alpha_f}{512}$	$\frac{-1+2\alpha_f}{1024}$

Table 2.9: Coefficients for boundary filter formulas at point 2

OA	$a_{1,2}$	$a_{2,2}$	$a_{3,2}$	$a_{4,2}$	$a_{5,2}$	$a_{6,2}$	$a_{7,2}$	$a_{8,2}$	$a_{9,2}$	$a_{10,2}$	$a_{11,2}$
4	$\frac{1+14\alpha_f}{16}$	$\frac{3+2\alpha_f}{4}$	$\frac{3+2\alpha_f}{8}$	$\frac{-1+2\alpha_f}{4}$	$\frac{1-2\alpha_f}{16}$	0	0	0	0	0	0
6	$\frac{1+62\alpha_f}{64}$	$\frac{29+6\alpha_f}{32}$	$\frac{15+34\alpha_f}{64}$	$\frac{-5+10\alpha_f}{16}$	$\frac{15-30\alpha_f}{64}$	$\frac{-3+6\alpha_f}{32}$	$\frac{1-2\alpha_f}{64}$	0	0	0	0
8	$\frac{1+254\alpha_f}{256}$	$\frac{31+2\alpha_f}{32}$	$\frac{7+50\alpha_f}{64}$	$\frac{-7+14\alpha_f}{32}$	$\frac{35-70\alpha_f}{128}$	$\frac{-7+14\alpha_f}{32}$	$\frac{7-14\alpha_f}{64}$	$\frac{-1+2\alpha_f}{32}$	$\frac{1-2\alpha_f}{256}$	0	0
10	$\frac{1+1022\alpha_f}{1024}$	$\frac{507+10\alpha_f}{512}$	$\frac{45+934\alpha_f}{1024}$	$\frac{-15+30\alpha_f}{128}$	$\frac{105-210\alpha_f}{512}$	$\frac{-63+126\alpha_f}{256}$	$\frac{105-210\alpha_f}{512}$	$\frac{-15+30\alpha_f}{128}$	$\frac{45-90\alpha_f}{1024}$	$\frac{-5+10\alpha_f}{512}$	$\frac{1-2\alpha_f}{1024}$

2.3 Treatment of Metric Identities

A by-product of recasting the transformed governing equations into strong-conservation form are four metric identities for the surface and volume conservation of a given cell. The surface conservation identities are given by

$$\begin{aligned}(\xi_x/J)_\xi + (\eta_x/J)_\eta + (\zeta_x/J)_\zeta &= 0 \\(\xi_y/J)_\xi + (\eta_y/J)_\eta + (\zeta_y/J)_\zeta &= 0 \\(\xi_z/J)_\xi + (\eta_z/J)_\eta + (\zeta_z/J)_\zeta &= 0\end{aligned}\tag{2.46}$$

while the volume conservation identity, better known as the *geometric conservation law* (GCL), is given by

$$(1/J)_\tau + (\xi_t/J)_\xi + (\eta_t/J)_\eta + (\zeta_t/J)_\zeta = 0\tag{2.47}$$

As discussed in Ref. [35], for well behaved transformations where the mapping functions are sufficiently differentiable, the surface and volume conservation identities vanish completely. In the case of a finite-discretization, however, care must be taken to enforce these identities individually to reduce numerical errors. For the surface conservation identities of Eq. 2.46, this is achieved by evaluating the transformation metrics in a manner first described in Ref. [35] for lower-order methods in which the metric relations

$$\begin{aligned}(\xi_x/J) &= y_\eta z_\zeta - y_\zeta z_\eta \quad , \quad (\xi_y/J) = x_\zeta z_\eta - x_\eta z_\zeta \quad , \quad (\xi_z/J) = x_\eta y_\zeta - x_\zeta y_\eta \quad , \\(\eta_x/J) &= y_\zeta z_\xi - y_\xi z_\zeta \quad , \quad (\eta_y/J) = x_\xi z_\zeta - x_\zeta z_\xi \quad , \quad (\eta_z/J) = x_\zeta y_\xi - x_\xi y_\zeta \quad , \\(\zeta_x/J) &= y_\xi z_\eta - y_\eta z_\xi \quad , \quad (\zeta_y/J) = x_\eta z_\xi - x_\xi z_\eta \quad , \quad (\zeta_z/J) = x_\xi y_\eta - x_\eta y_\xi\end{aligned}\tag{2.48}$$

are evaluated by considering their analytically equivalent "conservative" form:

$$\begin{aligned}
(\xi_x/J) &= (y_\eta z)_\zeta - (y_\zeta z)_\eta \quad , \quad (\xi_y/J) = (x_\zeta z)_\eta - (x_\eta z)_\zeta \quad , \quad (\xi_z/J) = (x_\eta y)_\zeta - (x_\zeta y)_\eta \quad , \\
(\eta_x/J) &= (y_\zeta z)_\xi - (y_\xi z)_\zeta \quad , \quad (\eta_y/J) = (x_\xi z)_\zeta - (x_\zeta z)_\xi \quad , \quad (\eta_z/J) = (x_\zeta y)_\xi - (x_\xi y)_\zeta \quad , \\
(\zeta_x/J) &= (y_\xi z)_\eta - (y_\eta z)_\xi \quad , \quad (\zeta_y/J) = (x_\eta z)_\xi - (x_\xi z)_\eta \quad , \quad (\zeta_z/J) = (x_\xi y)_\eta - (x_\eta y)_\xi
\end{aligned} \tag{2.49}$$

Visbal and Gaitonde [36] expanded this notion to higher-order using the same high-order discretizations described earlier for the flux vectors of Equations 2.3 and 2.6 to evaluate the derivatives in Eq. 2.49. Handling of the derivatives in this manner ensures that the volume conservation is consistent. This was shown through freestream preservation and improved accuracy on general three-dimensional meshes in References [36] and [30].

In order to ensure the GCL identity of Eq. 2.47, again Visbal and Gaitonde showed a novel way to satisfy it numerically. The time-derivative of the conserved variable vector of the governing equations (Eq. 2.1) is split using chain-rule differentiation as follows:

$$(\vec{U}/J)_\tau = (1/J)\vec{U}_\tau + \vec{U}(1/J)_\tau \tag{2.50}$$

The first term involves the determinant of the inverse Jacobian, J^{-1} , which is evaluated in a standard fashion using the instantaneous values of the grid coordinates. The second term, which includes the time-derivative of the Jacobian, requires special treatment. Rather than attempting to compute the time-derivative of J^{-1} directly from the grid coordinates at various time levels (either analytically or numerically), the GCL identity of Eq. 2.47 is invoked to evaluate $(1/J)_\tau$:

$$(1/J)_\tau = -[(\xi_t/J)_\xi + (\eta_t/J)_\eta + (\zeta_t/J)_\zeta] \tag{2.51}$$

where

$$\begin{aligned}
\xi_t/J &= -[x_\tau(\xi_x/J) + y_\tau(\xi_y/J) + z_\tau(\xi_z/J)] \\
\eta_t/J &= -[x_\tau(\eta_x/J) + y_\tau(\eta_y/J) + z_\tau(\eta_z/J)] \\
\zeta_t/J &= -[x_\tau(\zeta_x/J) + y_\tau(\zeta_y/J) + z_\tau(\zeta_z/J)]
\end{aligned} \tag{2.52}$$

Here, x_τ , y_τ , and z_τ are the grid speed terms that can be determined analytically or numerically depending on the motion. This implementation of the GCL terms ensures that the identity is satisfied numerically by the differencing scheme to reduce grid induced errors in the solution.

Chapter 3

Preliminary Considerations

3.1 Grid Description

An O-grid topology was chosen for both the flat plate and SD7003 airfoil geometries described in Section 1.1. A nominal grid size for the flat plate was chosen as 650 points around the airfoil surface (ξ -direction) with 392 points projecting outward from the airfoil surface (η -direction). A circular farfield boundary was used nearly 100 chords around the airfoil. A description of the grid as well as the leading and trailing edge spacing and the initial outward spacing can be seen in Table 3.1. Also, in order to maintain sufficient resolution in the near-body wake region as the airfoil is rotated, 65% of the surface points are biased to the upper side of the airfoil. A depiction of the near- and farfield grid can be seen in Figures 3.2 and 3.3, respectively, where only every other grid point in the outward direction and every third point in the surface direction are shown for clarity.

Table 3.1: Nominal Grid Dimensions

Dimensions	Points	$\Delta s_{LE}/c$	$\Delta s_{TE}/c$	$\Delta s_{init}/c$	N_{Upper}	N_{Lower}
650 x 392	254,800	0.0005	0.0003	0.000025	422	226

A nominal mesh was generated using the hyperbolic grid generator in Gridgen [37]. The

Figure 3.1: Surface Point Distribution for Flat Plate

Figure 3.2: Near-Body Computational Mesh for Flat Plate

Figure 3.3: Full Computational Mesh for Flat Plate

equations for the hyperbolic system are given by

$$x_\xi x_\eta + y_\xi y_\eta = 0 \quad (3.1)$$

$$x_\xi y_\eta - x_\eta y_\xi = V(\xi, \eta) \quad (3.2)$$

where $V(\xi, \eta)$ is the cell volume distribution and can be found simply from a specified growth rate. The system is solved through an iterative marching method from the airfoil surface. To begin, 281 points were generated outward using a growth rate of 1.025 from an initial spacing of 0.000025. At this point, in order to achieve a faster stretching of the mesh, the growth rate was adjusted to 1.04 and 131 more points were generated. The growth rate was modied once more to 1.07 and 80 more points were produced. This resulted in 392 points outward from the airfoil surface ending with a circular farfield boundary more than 100 chords away. For the spanwise periodic simulations, the surface grid above was extruded uniformly in the spanwise direction (ζ -direction) to achieve a spacing of $0.0033c$ between coinciding points. This spacing was chosen to correspond to the average spacing around the airfoil.

The spatial resolution of this grid was found to be sufficient for this study when compared to two coarser meshes, but the results are shown later.

3.2 Boundary Conditions

The boundary conditions were prescribed as follows. Along the airfoil surface, a no-slip adiabatic condition was employed in conjunction with a third order, extrapolated zero normal pressure gradient. The surface velocity components (u_s, v_s, w_s) were determined from the imposed pitching motion, namely

$$\begin{aligned} u_s(t) &= \dot{\alpha}(t)(y - y_0) \\ v_s(t) &= -\dot{\alpha}(t)(x - x_0) \\ w_s(t) &= 0.0 \end{aligned} \quad (3.3)$$

where (x_0, y_0) is the center of rotation in a z -constant plane. In the computational and experimental results presented later, the airfoils were pitched about the quarter chord. Along the far field boundary, located more than 100 chords away from the airfoil, freestream conditions were specified. It should be noted that near this boundary, the grids are stretched rapidly. This stretching in conjunction with the low-pass spatial filter provides a buffer-type treatment found previously [36] to be quite effective in reducing spurious reflections. Along the branch cut of the O-grid, spatial periodicity is imposed by means of a five-point grid overlap to allow for the high-order stencil required by the filter discussed earlier. For the 3-D cases, the spanwise boundaries were also prescribed as periodic with a similar overlap.

Chapter 4

Results

The numerical methodology described in Chapter 2 has been implemented in the extensively validated flow solver, FDL3DI [22, 30, 34]. This code has the ability to run in parallel and is scalable to a large number of processors. For the results presented here, simulations ran on anywhere from 12 to 2000 processors. The decomposition of the grids maintains a five-point overlap between adjacent blocks to allow for the higher-order stencils described earlier.

In this section, the computational results of the pitch, hold, and return motion of a flat plate are presented. Analyses of the spatial and temporal resolutions lead off the chapter in order to determine the proper length and time scales associated with this maneuver. Next, pitch rate and Reynolds number effects in two-dimensions are described. Transition during the motion is also explored through an extension of the two-dimensional simulations to spanwise periodic, three-dimensional cases. This also prompted an analysis of the spanwise extent in order to capture the necessary spanwise wave numbers and instabilities encountered during the motion. An experimental correlation is also made with the computational results to show favorable agreement throughout the motion.

All simulations were initialized from a static solution computed at zero degree incidence. Details into the startup of the airfoil motion in the periodic trailing edge shedding cycle were not considered in this investigation. In order to minimize compressibility effects, a small free

stream Mach number of 0.05 was used for all computations.

4.1 Spatial and Temporal Resolution Study

Two-dimensional simulations were performed in order to analyze the spatial and temporal resolutions for the current study. A lower Reynolds number of 5,000 was used in order to facilitate a more realistic solution where the flow may physically remain two-dimensional for the majority of the maneuver. At this Reynolds number, the flat plate also exhibits no bluff-body shedding at a zero incidence, which removes the uncertainty with the initial condition of unsteady startups. This gives a much clearer view of the deficiencies in the spatial and temporal resolutions that are being examined rather than sorting through the startup uncertainty. Because there is no time-mean method of comparison for this transient maneuver, the force and moment histories will be used as the metrics of comparison. These are spatially integrated quantities over the body, so they provide a means of quantifying the general effect of changes in the spatial and temporal resolutions.

4.1.1 Spatial Refinement

Grid convergence requires the use of several meshes that maintain similar distributions of points in each of the spatial directions. These distributions are easily retained by removing every other grid point or by adding a grid point proportionally between each adjacent cell. Although this simplistic approach maintains the proper distributions, halving or doubling the number of grid points in either or both the ξ - and η - directions changes the overall number of grid points by a factor of two or four. With such drastic differences in spatial resolution, the different meshes are bound to exhibit major differences and hinder the ability to show adequate refinement. To combat this, a method of bi-cubic spline interpolation is used in order to reduce or increase the number of grid points in either direction by any chosen factor. This is achieved by first mapping any dependent variable, $\phi(x, y)$, onto the computational

plane of for a given mesh, $\phi(x, y) \rightarrow \phi(\xi, \eta)$. This can be done by simply using the array indexing values, namely, $\xi = 1, 2, \dots, n_\xi$ and $\eta = 1, 2, \dots, n_\eta$. Next, the computational plane can be equally subdivided into the new number of cells in any given direction where the ξ and η interpolation points are given by

$$\xi_i = 1 + (i - 1) \frac{n_{\xi, old} - 1}{n_{\xi, new} - 1} \quad \eta_j = 1 + (j - 1) \frac{n_{\eta, old} - 1}{n_{\eta, new} - 1} \quad (4.1)$$

where $i = 1, 2, \dots, n_{\xi, new}$ and $j = 1, 2, \dots, n_{\eta, new}$. This will give new values for ξ and η that may not be integers. Interpolation of any dependent variable within a cell can then be performed from the four cell vertices. The advantage of the computational plane is that it is orthogonal with unit spacing in either direction. This makes it much easier to locate the appropriate surrounding nodes to use for interpolation.

Through a method of cubic spline interpolation as described in Ref. [38]. Given a discrete function $\phi_i = \phi(x_i)$, $i = 1, 2, \dots, N$, any new value within an interval ($x_i \leq x \leq x_{i+1}$) can be interpolated through the relation

$$\phi = A\phi_i + B\phi_{i+1} + C\phi_i'' + D\phi_{i+1}'' \quad (4.2)$$

where A and B weight the linear x -dependence of ϕ given by

$$A = \frac{x_{i+1} - x}{x_{i+1} - x_i} \quad B = \frac{x - x_i}{x_{i+1} - x_i} \quad (4.3)$$

and C and D weight the cubic x -dependence of ϕ given by

$$C = \frac{1}{6}(A^3 - A)(x_{i+1} - x_i)^2 \quad D = \frac{1}{6}(B^3 - B)(x_{i+1} - x_i)^2 \quad (4.4)$$

For spline interpolation, the second derivatives are determined globally through the tridiag-

onal system

$$\frac{x_i - x_{i-1}}{6}\phi''_{i-1} + \frac{x_{i+1} - x_{i-1}}{3}\phi''_i + \frac{x_{i+1} - x_i}{6}\phi''_{i+1} = \frac{\phi_{i+1} - \phi_i}{x_{i+1} - x_i} - \frac{\phi_i - \phi_{i-1}}{x_i - x_{i-1}} \quad (4.5)$$

The second derivatives at the boundaries, ϕ''_1 and ϕ''_N , are set to zero to close the set of N equations. These boundary conditions correspond to a *natural* cubic spline.

In the computational plane, these equations can be reduced further knowing that the plane exhibits unit spacing between nodes, i.e. $\xi_{i+1} - \xi_i = 1$. The linear weighting coefficients, A and B , reduce to the weighting of ϕ to the next highest and lowest integers, respectively,

$$A = CEIL(\xi) - \xi \quad B = \xi - FLOOR(\xi) \quad (4.6)$$

and the cubic weighting coefficients, C and D , reduce to

$$C = \frac{1}{6}(A^3 - A) \quad D = \frac{1}{6}(B^3 - B) \quad (4.7)$$

The second derivative relation in either direction becomes

$$\phi''_{i-1} + 4\phi''_i + \phi''_{i+1} = 6(\phi_{i+1} - 2\phi_i + \phi_{i-1}) \quad (4.8)$$

with the boundary conditions, $\phi''_1 = \phi''_N = 0$, remaining unchanged. The same system is solved in the η -direction by simply changing the weighting coefficients for that direction. One advantage of cubic over linear interpolation is the ability to maintain curvature of the geometry.

Two scale factors were chosen to coarsen the nominal flat plate grid discussed earlier. The overall grid scale-factors were chosen to approach 0.75 and 0.50. This required directional scale factors around 0.8660 and 0.7071, respectively. Table 4.1 shows the resulting grid dimensions for the desired scale factors.

Table 4.1: Grid Dimensions

Grid	Dimensions	Points	ξ -Scaling	η -Scaling	Coarsening Factor
Nominal	650 x 392	254,800	1.0000	1.0000	1.0000
C1	563 x 339	190,857	0.8662	0.8648	0.7490
C2	460 x 277	127,420	0.7077	0.7066	0.5001

Each coarsened grid was also initialized with the interpolated solution of the nominal grid. This allowed for only minor transients to be generated at the start of each simulation, minimizing the computational time to converge to a steady state solution. A depiction of each near-body grid colored by its corresponding outward-direction stretching ratio can be seen in Fig. 4.1. The images show that the coarsened grids have indeed maintained geometric similarity to the original grid.

Figure 4.1: Nominal, 75%, and 50% Coarsened Meshes

The force and moment coefficient histories for each grid can be seen in Fig. 4.2. A very nice agreement is seen between each of the cases for the majority of the maneuver. Slight deviations do appear on the return portion of the motion near 18° . Although there are differences, the small shift in the histories appears to be collapsing going from the coarsest resolution, C2, to finer resolutions, C1 and the Nominal case. The flowfields of each simulation are compared in Fig. 4.3 through the spanwise vorticity to provide a qualitative

difference in solutions. It is very difficult to discern even small differences in these images for the majority of the maneuver. At the return to zero-incidence, the train of vorticity connecting the ejected dynamic stall vortex system does exhibit more dissipation in the coarser grids, however, it does not appear to affect the convection of the system downstream or the shear layer vortices near the surface. This provides a good measure of confidence in the spatial resolution of the nominal grid, which is chosen for the remainder of the study.

Figure 4.2: Effect of Spatial Resolution on Force and Moment Coefficients

Figure 4.3: z -Vorticity Contours for Various Spatial Resolutions

4.1.2 Temporal Resolution

Proper temporal resolution is also a fundamental prerequisite for a valid solution to ensure that the significant transient flow effects are being captured. Figure 4.4 shows the force and moment histories for various time steps of $\Delta\tau = (8, 4, 2, 1) \times 10^{-4}$. As with the spatial resolution cases, the histories match very well for the pitch-up portion of the motion, but begin to deviate slightly on the return. Larger differences occur in the lift and moment

coefficients when compared to the drag. The general shape of each of the curves is similar, but there is again a shift to the right on the downstroke. The histories do collapse with higher temporal resolution resulting in a very little change moving from a time step of 2×10^{-4} to 1×10^{-4} . This makes 1×10^{-4} a reasonable time step for the remainder of the study.

Figure 4.4: Effect of Temporal Resolution on Force and Moment Coefficients

4.1.3 Convergence of Newton Subiterations

Of similar importance as the temporal resolution is the convergence of the Newton subiterations discussed earlier. Previous work has called three subiterations sufficient, but that work was mostly done on stationary meshes. With the addition of dynamic problems comes

the need to monitor the subiteration convergence to ensure that the error incurred at the end of the iterations is below that of the spatial or temporal discretization schemes. For this reason, a similar analysis to the temporal and spatial resolutions was done again comparing the force and moment histories of simulations with 3, 6, and 12 subiterations per time step. These results can be seen in Fig. 4.5. For almost the entire maneuver, there does not appear to be any change in the histories when more subiterations were used, indicating that three subiterations has provided sufficient convergence. As with the other resolution analyses, a small deviation does occur on the downstroke of the motion. However, this change is smaller than that incurred due to the time step in the previous section thereby providing closure that three subiterations is a viable choice for this work.

Figure 4.5: Effect of Newton Subiterations on Force and Moment Coefficients

4.2 Effect of Reynolds Number in 2-D

In this section, the impact of Reynolds number is examined in two-dimensions. A dimensionless pitch rate of 0.4 was used with four Reynolds numbers, $Re = (0.5, 1, 2, 4) \times 10^4$. Figure 4.6 shows the lift, drag, and moment coefficient histories throughout the motion. With increased Reynolds number, larger oscillations develop in each of the components towards the end of the upstroke, highlighting an earlier ejection and splitting of the dynamic stall vortex. This is reiterated in the z -vorticity contours of Fig. 4.7 at the maximum angle.

The dynamic stall vortex at this point is a single coherent structure at $Re = 5,000$, but for the higher Reynolds numbers, it has already deteriorated and split into a number of smaller vortices. This shows the significance that viscous effects have on the formation and destruction of the vortices associated with this motion.

The remainder of the motion exhibits similar trends with smaller and more numerous vortices intermittently dispersed throughout the flowfield. This is amplified with the higher Reynolds numbers where more and more vortices are being split and convected, culminating in a very disorganized flowfield. The large number of small, but strong vortices may be an indication that three-dimensionality at the higher Reynolds numbers becomes critical to alleviate the growing intensities through out-of-plane interactions and dissipation.

Figure 4.6: Effect of Reynolds Number on Force and Moment Coefficients

Figure 4.7: z -Vorticity Contours for Various Reynolds Numbers

4.3 Effect of Pitch Rate in 2-D

A range of pitch rates were simulated for the flat plate as a means to capture the general effect. Again, a Reynolds number of 5,000 was used with dimensionless pitch rates, Ω_0 , of 0.2, 0.4, and 0.8. The z -vorticity and streamwise velocity contours for halfway through the upstroke, at the max angle, halfway through the downstroke, and at the return position are

shown in Figures 4.8 and 4.9, respectively. It is interesting to examine the development of the leading edge vortex system on the upstroke of the motion for the different pitch rates in these images. For the first two images in the sequence, comparing them from left to right, it can be seen that doubling the pitch rate, nearly halves the size of the vortex system for the same angle of attack.

It is also interesting to note that relative to the body, the primary dynamic stall vortex is essentially in the same position when comparing the 0.1 pitch rate at $20^\circ \uparrow$ with the 0.2 pitch rate at 40° and the 0.4 pitch rate at $20^\circ \downarrow$ on the downstroke. Moving diagonally across the images in this manner actually compares nearly the same solution time of the simulations rather than the same angle of attack. This stays true throughout the maneuver. This is an indication that the size of the dynamic stall vortex is more a function of time rather than a function of pitch rate for this particular motion and fixed axis location. Increasing pitch rate forces more abrupt ejections early on, where the vortices split, which could be an indication that three-dimensionality becomes critical to correctly capture the interactions of the smaller, but more numerous vortices.

A similar trend can be seen from dye injection results of Ol [3] for various pitch rates at a Reynolds number of 10,000, shown in Fig. 4.10. Again, comparing images diagonally shows nearly the same size and position of the dynamic stall vortex relative to the body. The vorticity contours for these cases do compare favorably as well with the general flow structures seen in these dye injections.

Figure 4.8: z -Vorticity Contours for Various Pitch Rates at $Re = 5,000$

Figure 4.9: u -Velocity Contours for Various Pitch Rates at $Re = 5,000$

Figure 4.10: Water Tunnel Dye Injection Results for Various Pitch Rates (Ref. [3])

The force and moment coefficient histories are compared in Fig. 4.11. They have been plotted as a function of angle of attack for comparison purposes, although in time, the 0.4 and 0.8 pitch rate motions occur two and four times faster than the 0.2 case, respectively. During the upstroke of the motion, the lift, drag, and moment coefficients vary almost linearly with angle of attack with the pitch rate only effecting the initial offset of the curve and not the slope. Compared to static thin airfoil theory, the lift slope during the upstroke for each of these cases is about 30% lower than the thin airfoil theory prediction of $2\pi\alpha$. Additionally, the highest pitch rate produced a negative lift and drag (thrust) and positive moment for the latter half of the downstroke. This is not observed for the lower pitch rates, where the lift and drag were almost exclusively positive and moment mostly negative.

Eldredge, Wang, and Ol [14] noticed similar trends of the lift and drag for even higher pitch rates ($0.7 \leq \Omega_0 \leq 2.1$), where they also discovered a collapse of the lift and drag curves on the upstroke when they were normalized by the pitch rate. The computations here, however, show no such normalization. Instead, the additional lift, drag, and moment offset between each curve appears to be proportional to the pitch rate. That is, the additional lift,

drag, or moment between 0.2 and 0.8 is close to two times that between 0.2 and 0.4. The trend on the downstroke, however, becomes a little more convoluted where three-dimensional effects and transition effects become significant. The next section will show that three-dimensional effects do, indeed, dominate on the downstroke of the motion.

Figure 4.11: Effect of Pitch Rate on Force and Moment Coefficients

4.4 Impact of Transition

Two- and three-dimensional simulations were conducted for the pitch and return motion in order to investigate the role of transition. For many of the cases shown earlier, the two-dimensional simulations force only planar interactions between vortices. With more and stronger vortices, the assumption of only planar interactions becomes less and less physical. Admitting the third dimension and out-of-plane velocity allows the vortices to form and dissipate naturally, which can cause very different interactions. For these computations, a Reynolds number of 10,000 was used with a dimensionless pitch rate of 0.4. The three-dimensional, spanwise periodic simulation was conducted using a span-to-chord ratio of 0.8. The results were also span-averaged at each time step for the comparison to the 2-D computations.

Vorticity and streamwise velocity contours can be seen at various points throughout the motion in Figures 4.12 and 4.13, respectively. Halfway through the upstroke, it is seen that the development of the dynamic stall vortex system is nearly identical between both cases. At the peak angle, the shear layer vortices traversing up the aft portion of the airfoil can be seen to match very well indicating a laminar separation. However, at this same point in the cycle, the dynamic stall vortex system begins to deviate between the two cases. The secondary, counterrotating vortex (seen in red) has already begun to transition and break down in the three-dimensional case while remaining coherent in the laminar case. This causes a significant difference in the ejection of the primary dynamic stall vortex, which also has begun to transition. Instead of one coherent vortex ejection, the dynamic stall vortex ejects with a secondary, counter-rotating vortex in the two-dimensional case. This system forms a vortex dipole. From here, larger differences continue to arise on the downstroke of the motion. Without spanwise relief, the two-dimensional case produces many small, coherent vortices rather intermittently dispersed around the airfoil. The spanwise periodic case, on the other hand, relaxes these vortices producing fewer, but coherent structures. This produces a more simplified flow field.

A measure of the three-dimensionality of the spanwise periodic case is given in Fig. 4.14 by the instantaneous out-of-plane velocity component at the midplane of the simulation. Halfway through the upstroke, the flow is completely two-dimensional shown through the absence of the w -velocity component. At the peak angle, the vortical interactions from the primary, secondary, and tertiary recirculating regions present in the dynamic stall vortex system have begun to produce out-of-plane motions. Halfway through the downstroke, the slight redirections of velocity to the spanwise direction can be seen to be carried through the shear layer interactions of the different vortical structures as the dynamic stall vortex is convected downstream. Finally, at the return to zero-incidence, the largest intensities of out-of-plane velocity are encountered as the prominent vortices are ejected and dissipated.

Figure 4.12: Two-/Three-Dimensional z -Vorticity Comparison

Figure 4.13: Two-/Three-Dimensional u -Velocity Comparison

Figure 4.14: Instantaneous w -Velocity at the Midplane

The pressure distributions around the airfoil surface at the afore-mentioned positions in time can be seen in Fig. 4.15. The similarity between the two cases is again seen in the middle of the upstroke in Fig. 4.15(a) where the the leading edge vortex in both cases causes a nearly identical suction spike for almost 20% of the chord. Moving to the hold in Fig. 4.15(b), the two-dimensional case begins to under-predict the pressure due to the over-prediction of the secondary, counter-rotating vortex mentioned above. The aft portion of the pressure distribution still matches well. Halfway through the downstroke in Fig. 4.15(c), the pressure spike from the leading edge vortex system has traversed further downstream to

nearly 60% and 75% chord in the spanwise periodic and two-dimensional cases, respectively. The 15% difference can be attributed to the induced velocity of the secondary vortex in the two-dimensional simulation. Finally, the return to zero incidence in Fig. 4.15(d) shows a continued under-prediction of the suction spike for the two-dimensional case as the leading edge vortex system convects almost completely off past the airfoil's trailing edge. This is also the case for the pressure distribution on the lower surface of the airfoil, which had not differed until that point.

Figure 4.15: Two-/Three-Dimensional Pressure Coefficient Distributions

The force and moment coefficient histories are also compared for the two cases in Fig. 4.16. The force and moment histories reiterate the commonality between the two cases during the majority of the upstroke, where the lift, drag, and moment coefficients match very well.

Near the top of the upstroke, where the ejection of the dynamic stall vortex takes place, the force and moment coefficients all begin to deviate. For the spanwise periodic case, the lift coefficient plateaus through the majority of the downstroke while the ejected dynamic stall vortex moves downstream just above the upper surface of the plate. This keeps the circulation around the airfoil nearly constant. Once the vortex passes the trailing edge of the airfoil towards the end of the downstroke, however, there is a sharp drop in the lift coefficient. In the laminar case, the counter rotating vortex ejected with the dynamic stall vortex causes a small drop in lift on the beginning of the downstroke before leveling off. Towards the end of the downstroke, there is a sharp spike in the lift where the shear layer vortices near the trailing edge are swept down the airfoil surface by the still coherent, previously ejected vortices, which also results in a large drop in the moment coefficient.

To summarize, the two- and three-dimensional simulations compare very closely for the majority of the upstroke of the motion. Major differences begin to appear immediately preceding the hold of the motion, where the dynamics stall vortex system in the spanwise periodic case transitions and begins to break down as it convects. From that point onward, the two simulations develop dramatically differently. The remainder of the motion is dominated by three-dimensional effects that actually simplify the flowfield into fewer, coherent structures. This can explain why other researchers have had trouble matching experimental data on the downstroke of the motion when using two-dimensional methods.

Figure 4.16: Impact of Transition on Force and Moment Coefficients

4.5 Experimental Correlation

The spanwise periodic results for a span-to-chord ratio of 0.8 can be compared to experimental PIV data from water tunnel simulations conducted at the Air Force Research Laboratory [3]. It should be noted that the experimental measurements have been ensemble averaged over 50 cycles of the motion, while the computational results have been span-averaged.

The comparisons of spanwise-vorticity and streamwise velocity can be seen in Figures 4.17 and 4.18, respectively. In both comparisons, a very nice agreement is seen throughout

the maneuver even on the downstroke. The main differences come in the strength of the computed vortices. The computational results produce more intense vortices and velocity concentrations. One explanation of this could be a product of “phase”-averaging versus span-averaging. As more and more cycles of the motion are averaged together, the general flow fields will be very similar, but there most likely a small difference in position of the large structures. This difference in position can cause a smearing effect for the overall structure in the averaged result. On the other hand, the position of the large scale structures will most likely not vary as much between different planes in the span-wise direction, thereby increasing their contributions in the span-averaged solution.

Another source of discrepancies could possibly arise from the end walls of the water tunnel in the experiment. Reflected flow off the walls through the core of the large dynamic stall vortex could cause slightly more deterioration of that structure in the experiment leading to the small differences.

Figure 4.17: Experimental/Computational z -Vorticity Correlation

Figure 4.18: Experimental/Computational u -Velocity Correlation

4.6 Analysis of Spanwise Extent

Span of the given wing section can play an important role in the development and the consequential breakdown of the vortical structures inherent to this maneuver. Periodic boundary conditions in the span-wise direction allow for a smaller computational domain, but can inadvertently constrain the resolved wave numbers across the span if enough of the span is not modeled. Conversely, in order to make a simulation tractable, it is desired to model enough of the span to capture the dominating spanwise content while avoiding over-modeling nearly periodic content. With this fine line between over-constraining and over-computing the domain, it becomes necessary to examine the changes in the solution by varying the span to ensure an optimal spanwise extent is chosen. This section highlights those changes for span-to-chord ratios of 0.1, 0.2, 0.4, and 0.8. Each of these cases maintains the same spanwise spacing corresponding to 30, 60, 120, and 240 cells for each spanwise extent, respectively. Again, a Reynolds number of 10,000 and dimensionless pitch rate of 0.4 are used.

The force and moment coefficient histories for the motion are shown in Fig. 4.19. While a smooth collapse of these curves is not apparent, the change from 0.4 to 0.8 is very small. The largest difference in the forces and moment appears between spans of $0.2c$ and $0.4c$. The majority of the upstroke matches very well until the ejection of the dynamic stall vortex system nearly 1.5 convective times into the maneuver. From that point on, the 0.1 and 0.2 span-to-chord ratio cases are very similar. Likewise, the 0.4 and 0.8 cases are also similar. The z -vorticity contours for the span-averaged solutions can be compared in Fig. 4.20. Qualitatively, these spans exhibit very similar flowfields until the ejection of the primary dynamic stall vortex after the second image of the motion sequence. From that point, the $0.1c$ and $0.2c$ cases exhibit a pairing of the primary dynamic stall vortex with a secondary, co-rotating vortex after the main ejection as seen in the third image of the motion sequence. This secondary vortex infuses the circulation in this region, which results in an ejection of the counter-rotating, shear layer vortex seen in the fourth image. The $0.4c$ and

0.8c cases, on the other hand, allow for more span-wise relief during the main ejection, which provides more dissipation, inhibiting the secondary, co-rotating vortex ejection. Without the added circulation, the shear layer vortex is not discharged as abruptly as with the shorter spans. Instead, it is traversed downstream along the upper surface of the airfoil towards the trailing edge without begin ejected. This results in a slightly higher lift and lower pitching moment as seen again in Fig. 4.19. The small differences between the $s/c = 0.4$ and $s/c = 0.8$ cases gives a measure of confidence that enough of the span is being modeled for the scope of this maneuver.

Figure 4.19: Effect of Span of Force and Moment Coefficients

Figure 4.20: Span-Averaged z -Vorticity Comparison for Various Spans

Figure 4.21: Span-Averaged u -Velocity Comparison for Various Spans

Additionally, the span-averaged and spanwise variation of the pressure distributions along the surface of the airfoil are examined in Figures 4.22 and 4.23, respectively. The commonality between the two smaller spans and likewise between the two larger spans is evident in these figures. Halfway through the downstroke, Figures 4.22(c) and 4.23(c) show an over-prediction of the pressure for the $0.1c$ and $0.2c$ cases due to the addition of the co-rotating vortex ejected with the DSV as described earlier. Continuing to the return position in Figures 4.22(d) and 4.23(d), the two lower span-to-chord ratios under-predict the suction spike

in the pressure distributions due to the ejection of the counter-rotating shear layer vortices from the aft portion of the airfoil surface. Also, at this point, the two higher span-to-chord ratios both show more pronounced spanwise pressure variations.

Figure 4.22: Span-Averaged Pressure Coefficient Distributions for Various Spans

(a) 20° (upstroke)

(b) 40°

(c) 20° (downstroke)

(d) 0°

Figure 4.23: Spanwise Variation of Surface Pressure Coefficient

Another interesting aspect to explore, is the transition process for the various spanwise

extents, however this involves determining a suitable metric to analyze. One such metric is the body-aligned vorticity component oriented from the trailing to leading edge of the airfoil. This value in the body's frame of reference is shown in Fig. 4.24 during the ejection of the dynamic stall vortex with the viewing plane set at the center of this vortex. The vorticity magnitude is shadowed off the back side of the plate for reference. While this sequence of images shows only the case with a span of $0.4c$, the other spans produce similar results. Integrating the square of this value over an x -constant plane in the moving reference frame gives a loose measure of the three-dimensionality of the flow at a given instance in time. Normalizing these values with their planar areas allows the transition process of each spanwise extent to be compared. The body-aligned vorticity development in time for each span can be seen in Fig. 4.25, where the plotted values correspond to the maximum integrated quantities at each time step.

Figure 4.24: Body-Aligned Vorticity Development ($\tau = 0.8 - 1.6$ in 0.1 increments)

Figure 4.25: Integrated Body-Aligned Vorticity Development

Nearly the same exponential growth of the body-aligned vorticity component can be seen for each span until a convective time around 1.5 where the vorticity plateaus. The growth is proportional to $\exp(18.35\tau)$. In order to visualize the transition process a little more clearly, Fig. 4.26 shows iso-surfaces of the vorticity magnitude ($|\vec{\omega}| = 36$) at $\tau = 0.5, 1.0, 1.5, 2.0$, and 2.5 convective times. The surfaces are colored by velocity magnitude. For the first three sequences, the motion is characterized by laminar sheets of vorticity that remain mostly two-dimensional. At $\tau = 1.5$, spanwise instabilities become evident in the variations in the sheet encompassing the dynamic stall vortex. The variation is consistent for each of the spanwise extents at this point and is the start of the detachment of the DSV. After this point, the body-aligned vorticity component has plateaued as indicated back in Fig. 4.25. At $\tau = 2.0$, the DSV has already detached and the iso-surfaces show many smaller scale, three-dimensional structures across the span in the area surrounding the DSV. Finally, at $\tau = 2.5$, a very similar turbulent leading edge vortex system is seen for each of the four cases. This turbulent region continues to convect downstream and dissipates in the process as the flow reattaches and returns to laminar. With such striking similarities between each of the spanwise extents throughout the transition process, it can be concluded that the spanwise

extent does not play a large role in the overall transition process on the upstroke of the motion. The span does influence how the secondary vortex system is dissipated and ejected, however, which has lasting effects on how the DSV is convected, as discussed above.

Figure 4.26: Iso-Surfaces of $|\vec{\omega}| = 36$ at $\tau = 0.5, 1.0, 1.5, 2.0, 2.5$

Around the same points in time as the previous figure, the dynamic stall process is highlighted in Fig. 4.27 through velocity profiles and z -vorticity contours near the surface of the plate.. The images depicted in this figure are for a span to chord ratio of 0.1, but are extremely similar for the other spans as well. Initially, the flow around the plate is attached when the plate is at zero incidence (Fig. 4.27(a)). As the plate pitches up, a recirculation region begins to form near the leading edge as the separation point moves from the trailing edge towards the leading edge (Fig. 4.27(b)). Next, the primary region continues to grow into a large, leading edge vortex, while a secondary recirculating region begins to form from the shear layer near the surface of the plate (Fig. 4.27(c)). In Fig. 4.27(d), the tertiary recirculating region has formed in front of the primary and secondary vortices. Finally, the primary dynamic stall vortex is ejected and the secondary and tertiary vortices break down (Fig. 4.27(e)-4.27(f)).

(a) $\tau = 0.0$

(b) $\tau = 0.6$

(c) $\tau = 0.8$

(d) $\tau = 1.2$

(e) $\tau = 1.4$

(f) $\tau = 1.6$

Figure 4.27: Dynamic Stall Process for $s/c = 0.1$

Chapter 5

Summary

An implicit large eddy simulation (ILES) technique was employed to simulate the AIAA FDTC LRDG first canonical case for the pitch and return maneuver of a flat plate. In two-dimensions, the effect of Reynolds number on development of the flow field and force and moment histories were examined. With $Re = (0.5, 1, 2, 4) \times 10^4$, it was found that the higher Reynolds number simulations experienced instabilities associated with the formation of the dynamic stall vortex that resulted in very abrupt and random detachments. This formed many small but coherent vortices throughout the flowfield that would have otherwise been dissipated through the spanwise relief of a three-dimensional simulation. The effect of Reynolds number on the force and moment coefficients became evident with the growing instability of the DSV as well, where large oscillations appeared towards the end of the upstroke, which were more pronounced with higher Reynolds number. The flow memory effects for the rest of the maneuver proved to be too overwhelming, and wildly varying forces were experienced for the remainder of the motion. The benign effects of Reynolds number experienced by Lian [16], Ol [3], and Eldredge *et al* [14] were not found in the two-dimensional analysis conducted here.

A series of dimensionless pitch rates ($\Omega_0 = 0.2, 0.4, 0.8$) were also simulated in two-dimensions. The size of the dynamic stall vortex seemed to be unaffected by the pitch rate,

where even the lower pitch rates still achieved a DSV roughly the same size as the higher pitch rates. Comparing images at the same solution time rather than angle-of-attack yielded very similar flow fields despite the angle. The size and position of the DSV was very closely tied to the solution time rather than the angle of attack. On the upstroke of the motion, the force and moment coefficients showed a nearly linear dependence on angle-of-attack. This was not the case on the downstroke of the motion, where it can be inferred that three-dimensional effects play a large role in the convection of the DSV and subsequent ejection of secondary and tertiary vortical structures.

Next, an examination into the impact of transition was conducted by analyzing the two-dimensional and spanwise periodic simulation with a spanwise extent of $0.8c$. As predicted, the DSV began a process of transition from the start of the motion until its ejection near the end of the downstroke when it experienced fully turbulent regions interspersed with regions of transitional and laminar flow. The mixed system convected downstream as a single, coherent structure in the span-averaged solution, as opposed to the convecting pair of co-rotating vortices in the two-dimensional case. The secondary and tertiary ejections of the shear layer vortices were substantially diminished in the spanwise periodic case due to the spanwise dissipation. This was indeed evident in the force and moment histories on the downstroke of the motion, where the large gradients experienced in the two-dimensional simulation especially in the lift and moment coefficients were subdued in the spanwise periodic case.

The spanwise extent was also analyzed in this study to determine the required domain size as to not over-constrain the spanwise wave numbers. For the majority of the upstroke of the motion, the lowest spanwise extent of $0.1c$ and even the two-dimensional simulation matched well with the higher spans. Also, the transition process for each span was compared by analyzing the development of the body-aligned vorticity component, where a common exponential growth rate was observed. It was not until the detachment of the DSV at the top of the upstroke that the flowfields and force histories began to deviate. It was found that a spanwise extent of at least $0.4c$ was required to capture the true ejection of the DSV

and subsequent convection downstream.

While many other computational methods have been able to show nice agreement to the experimental results on the upstroke of the motion, the downstroke has proven to be a continued challenge. While the other methods required much lower Reynolds numbers and higher pitch rates, which have both been shown to reduce the three-dimensionality of the flowfield, the method presented here has shown extremely favorable agreement with the experimental results throughout the maneuver proving that the spanwise periodic ILES computations do capture the true physics of the laminar, transitional, and turbulent regions of the flow for the desired Reynolds number and kinematics.

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