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Independent Stage Control of a Cascade Injector

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Abstract

A comprehensive investigation of independent stage control as applied to a cascade injector was completed. It was experimentally demonstrated that altering the number of active stages in the cascade could control the plume structure of this injector. Influence coefficient analysis showed that the first stage governed penetration height. Power law correlation comparison found the cascade injector penetrated higher than a normal, circular injector by a factor of 3.5.

Corresponding computational data showed agreement with the experimental data in terms of penetration distance, but less accuracy in predicting plume width. A turbulence model study indicated the Menter model to have the closest agreement with experimental data. However, the fine grid solution produced using Wilcox $k-\omega$ was later found to have better agreement. The computational data was also used to produce a synthetic signal similar to the experimental planar laser induced fluorescence signal that improved agreement with the experimental data.

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1.0 Introduction

One of the main factors governing combustion efficiency in a supersonic flowpath is the design of the fuel injection system. High flow velocities in the combustor create short residence times, which means that mixing and combustion processes must occur rapidly. Fuel injector designs for supersonic combustion ramjet (scramjet) engines balance penetration into the high-speed flow with dispersion and mixing. To achieve the approximate balance, a variety of fuel injection techniques have been studied.

In-stream injectors, such as struts and physical ramps, physically intrude into the combustor flowpath. This class of injectors is capable of providing good fueling coverage of the flowpath. However, the obstruction in the flowfield is a source of drag, total pressure loss, and friction; all of which may be disadvantageous in terms of thrust. Additionally, in-stream injectors must be designed to structurally and thermally withstand the extreme environment of the combustor. These design considerations typically necessitate some form of cooling, which can significantly increase the complexity of the system.¹

Wall injectors do not physically intrude into the combustor flowpath, but it has long been known that material injected into a supersonic cross-flow creates a strong shock pattern and produces a complex region of flow around the injector.² The simplest wall injection technique, the single-hole injector, has been extensively used to investigate the flow phenomena associated with injection. A circular hole injecting material normal to the cross-flow at an underexpanded injection condition will have the flow structure displayed in Figure 1. In this figure, the injectant is an obstruction to the supersonic cross-flow and a three-dimensional bow shock develops. The interaction between the bow shock and the boundary layer upstream of the injector causes a separation zone. This zone is a source of high pressure gradients which cause an increase in the local heat flux.¹⁻¹²

The injection plume has been characterized using both experimental and computational methods. In 1964, Zukoski and Spaid noted that the injectant was captured as a streamline in schlieren images and defined the maximum distance between that line and the wall as the penetration height.² This height was found to increase or decrease depending on the shock structure, which is dependant on the structure of the Mach disk.^{3,4} The Mach disk was also characterized using schlieren imaging by its vertical and horizontal displacement.^{5,6} Using smoke-wire and soap film visualization techniques, vortical structures following transverse injection were determined by Fric and Roshko to originate from the boundary layer. These vortices were indicative of the coupling that exists between the injection and its wake.⁷ The pressure field surrounding a single hole injector was investigated using pressure sensitive paint.^{8,9} The distance between the bow shock and the separation shock, the static pressure on the wall upstream of the injector, and the magnitude of a low pressure separation region following the injector were all observed to increase with increasing jet-to-freestream momentum flux ratio. The magnitude of the expansion region surrounding the injection port was not affected by changes in momentum flux ratio. Gruber et al.¹⁰ employed planar Rayleigh/Mie scattering to examine the vortical structures that govern jet structure and large-scale mixing. Although penetration height was independent of the molecular weight of the injectant, molecular weight altered the compressibility of the flow and thus had an effect on mixing characteristics. Laser Doppler velocimetry was used to explore the velocity flowfield.¹¹ The upstream and downstream separation zones, bow shock structure, Mach disk, and pairs of counter-rotating vortices were some of the flow features measured in this work. These data showed that downstream of the injector, counter-rotating kidney-shaped vortices dominated the flowfield. This study also provided data for the validation of analytical and numerical models. Most recently, Ben-Yakar et al.¹² used a variety of techniques to note that in addition to the momentum flux ratio, the jet shear layer appears to have an effect on the penetration characteristics.

Penetration data, however obtained, is typically nondimensionalized by a variety of parameters including the exit diameter of the injector, effective diameter, and momentum flux ratio and has been shown to follow a number of power law correlations.^{2, 6, 13, 14, 15}

At the present time, overexpanded injection from a circular hole has not been characterized to the same extent as the underexpanded injection condition. However, this case is expected to have features similar to overexpanded nozzle flow. As in overexpanded nozzle flow, the injectant pressure at the exit of the circular hole would be lower than the pressure outside of the hole and an oblique shockwave would form, as shown in Figure 2.¹⁶ The overexpanded injectant plume can be measured in terms of the penetration height and width in the same manner as an underexpanded jet.

To minimize the upstream separation region shown in Figure 1, reduce heat flux, and improve mixing, a variety of injection hole shapes and injection angles have been studied. Gruber et al.¹³ compared normal, circular injection to elliptical, normal injection and angled, circular injection. These researchers determined that both the shape of the injection port and the angle of injection have a notable effect on penetration. Penetration height was observed to increase as the momentum flux ratio between the jet and the freestream increased. Penetration profile data followed the power law correlation:

$$\frac{y}{d_{eff} J} = 1.23 \left(\frac{x}{d_{eff} J} \right)^{0.344} \quad (1)$$

where d_{eff} was the effective diameter:

$$d_{eff} = \sqrt{\frac{4 A_{xs}}{\pi}} \quad (2)$$

with A_{xs} representing the cross-sectional area, and J was the jet-to-freestream momentum flux ratio:

$$J = \frac{(\rho u^2)_{jet}}{(\rho u^2)_{freestream}} = \frac{(\gamma P M^2)_{jet}}{(\gamma P M^2)_{freestream}} \quad (3)$$

Gruber et al. also found that injector geometry affected large scale eddy structure in the near-field, which impacted the strength and location of the bow shock, and had an impact on the separation zone upstream of the injector.^{17, 18} When appropriately positioned with respect to the freestream flow, the elliptical, normal injector was found to have a weaker bow shock, resulting in a smaller separation zone.

Barber et al.¹⁹ determined that wedge-shaped injection ports have no upstream separation zone; a result attributed to the properties of the boundary layer upstream of the injector. Although there were no noticeable differences in shock structure, the injectant plume was observed to exhibit faster penetration and improved mixing characteristics compared to a normal, circular injector. Tamioka et al.²⁰ examined diamond shaped injectors. At low momentum flux ratios, the diamond injector had a higher penetration distance than a comparable normal, circular injector. However, this difference disappeared as the momentum flux ratio was increased. Angled injection from the diamond-shaped orifice offered a similar increase in penetration height as angled injection from a circular port. Adding a yaw angle to the angled injection from the diamond shaped orifice was found to increase the penetration height. This increase appeared to be the effect of increased vortical plume structure caused by changes in the pressure field. A similar increase was not observed when a yaw angle was added to an angled, circular jet.

Staged injection techniques use jet-to-jet interactions to improve the injection plume characteristics. In the early 1990's, Hollo et al.²¹ used laser-induced iodine fluorescence to nonintrusively investigate staged injection into a supersonic cross-flow. These researchers determined that the velocity field created by staging generates vortices that enables rapid mixing. Additionally, data collected using this diagnostic provided information that could be used for comparison to CFD solutions.^{22, 23}

An aerodynamic ramp is an array of wall jets that produces vortices similar to those produced by a physical ramp. This design is advantageous because fuel-vortex interactions can be used to

enhance fuel-air mixing without having a physical intrusion into the supersonic flowfield.²⁴ Compared to the physical ramp, the aerodynamic ramp had slightly reduced mixing. Due to the shock system, a total pressure loss was incurred, but this was also a smaller loss compared to the physical ramp injector. Donbar et al.²⁵ found that an aerodynamic ramp configuration had a slightly lower combustion efficiency than low angled injection.

In 1986, Briedenthal et al.²⁶ suggested that a series of injection ports increasing exponentially in width and injection speed would increase jet penetration and enhance mixing. This theory was supported by Clement and Rodriguez²⁷, who observed the oblique shock structure and linear increase in penetration height predicted by Briedenthal et al. However, these experiments only examined the effect of exponentially increasing injection port width, not injection speed. In 1997, Eroglu and Breidenthal²⁸ used flow visualization techniques, including laser-induced fluorescence (LIF), to study a fully exponential transverse jet. In this injection concept, an exponential increase in injection width and velocity with downstream position were used to control penetration and mixing characteristics.

A cascade injector is a staged injection technique designed to produce high levels of penetration across a high momentum air stream. The concept may enable fueling large engines from flowpath walls, minimizing the need for more intrusive fueling schemes. The first, most upstream, stage of the injector penetrates the flow and causes a shock structure to develop. The downstream stages are then able to penetrate farther into the flow due to the lower pressure region behind the shock structure. One such injector has been developed by Aerojet General Corporation. The injector block, shown in Figure 3, is a lateral array of three injectors. Each injector is a flush-mounted array of fourteen injection ports that are aligned with the engine flow axis. These ports are divided into four axial stages, each of which is supplied by a separate fuel manifold, as shown in Figure 4.

Mathur et al.²⁹ conducted a series of experiments to evaluate the effectiveness of this cascade injector. Wall static pressure measurements and conventional probing were used in conjunction with

shadowgraph, acetone planar laser induced fluorescence (PLIF), and pressure sensitive paint to investigate the injector at overexpanded (144 psia), pressure-matched (555 psia), and underexpanded (788 psia) injection conditions. These experiments were conducted in the AFRL/PRAS Research Cell 19 facility (RC19) at a total pressure of 38 psia, ambient total temperature, and a nominal facility Mach number of $M = 3.0$. Due to the presence of strong leading shock waves, shock reflections, and boundary layer separation at the wall opposite the injector, the authors concluded that the amount of penetration observed in these experiments was disproportionate to the size of the test section. Jet penetration at the underexpanded injection condition continued to increase until it spanned the height of the experimental test section. Furthermore, the pressure sensitive paint indicated that the lateral spacing between cascades inhibited effective mixing.

Corresponding numerical results obtained by Cox-Stouffer et al.³⁰ showed good agreement with the experimental data. Computations of the injectant flow used the Viscous Upwind aLgorithm for Complex Flow ANalysis (VULCAN) Navier-Stokes code.³¹ By assuming symmetry along the tunnel centerline, a half-tunnel computational domain was used. A 1.1 million node grid provided sufficient resolution. The computational domain extended from the facility plenum, through the Mach 3 nozzle, and through the 2.0 inch high by 6.0 inch wide constant-area test section of the experimental facility. Freestream Mach number, total pressure, and total temperature were specified to be 3, 265.5 kPa, and 87.5°F (303 K), respectively. Overexpanded, pressure-matched, and underexpanded fuel injection cases were studied. The pressure-matched and underexpanded cases were found to provide an excess level of penetration, where the jet penetration was observed to reach the opposite wall of the test section. As in the experimental work, this result suggested incorrect scaling of the injector with respect to duct height.

Both works suggested that staging strategies could be investigated to improve the effectiveness of the cascade injector; a recommendation that provided the motivation for this work. This thesis presents a comprehensive investigation of independent stage control as applied to the

Aerojet cascade injector. First, the results of an experimental investigation are presented. To avoid overpenetrating the test section, these experiments were conducted at overexpanded injection conditions. The effects of penetration and mixing were observed by examining combinations of active stages while maintaining a constant injection pressure condition. Although the overall mass flow rate changed, the mass flow rate through each active stage remained constant from case to case. Shadowgraph and schlieren imaging, Mie scattering, and Nitric Oxide Planar Laser Induced Fluorescence (NO-PLIF) were the diagnostic techniques used to reveal various features of the injectant plume and its interaction with a Mach 3 supersonic crossflow. Computational fluid dynamics (CFD) was then used to explore the mechanics governing the penetration and mixing characteristics of the cascade injector.

2.0 Experimental Analysis

This section describes the collection and analysis of experimental data. The data presented include shadowgraph and schlieren imaging, Mie scattering, and images from the NO-PLIF diagnostic technique. These data provided information regarding the injection plume characteristics.

2.1 Experimental Facility

A complete description of RC19 and its capabilities can be found in Reference 32. This facility is a modular test facility used for studies of fuel injection, inlet-combustor interactions, and flameholding in supersonic flows. A picture and schematic of RC19 appear as Figure 5 and Figure 6, respectively. The facility is supplied with air by the Research Air Facility, which is capable of supplying air at a maximum temperature of 1200°F and a maximum pressure of 750 psig. This air enters RC19 and is expanded into a plenum where the total temperature and total pressure are measured. These quantities are used to control the test conditions of the facility. A modular nozzle block assembly conditions the flow to a nominal Mach number of 2, 3, or 4.5. The air then passes through the test section, which can be configured into a full or half-tunnel arrangement with an aspect ratio of 1.16, 2, or 3. Finally, the flow is re-expanded and passed back to the Air Facility, where it is exhausted.

For these experiments, the nozzle block assembly provided a nominal facility Mach number of $M = 3$. The test section was 2 inches high by 6 inches wide. The bottom wall, where the cascade injector was mounted, had a 2.5 degree divergence. Fused silica windows were mounted in the other three walls to provide optical access to the test section.

Data were collected using a state-of-the-art pressure scanning system. Pressure taps, located in the facility nozzle and test section, had an uncertainty of ± 0.025 psi.³³ Instantaneous and time-averaged flow visualization techniques as well as laser-based diagnostics were also used to collect data.

2.2 Fuel Injection Hardware

The cascade injector block was developed by Aerojet General Corporation for use in scramjet engines. The injector block, shown in Figure 3, is a flush-mounted lateral array of three cascade injectors. Of the three injectors shown in this figure, only the center injector was used in the present experiments. The other two cascades were sealed with a 2-component epoxy. The center injector is composed of fourteen injection ports. These converging-diverging nozzles are grouped into four axial stages. Each stage was independently supplied by a fuel manifold, as shown in Figure 4. The fuel supplied to each stage could be turned on and off with a ball valve. Stagnation temperature and pressure were measured in the fuel manifold prior to the injection block.

2.3 Test Matrix

The test matrix, shown in Table 1, consisted of 11 stage combinations and two nominal injection pressure conditions. The nominal injection pressures used for these experiments were 144 psia and 250 psia, which corresponded to overexpanded injection conditions. The lower pressure condition was selected so that the baseline condition, where all four stages were active, could be compared with previous experimental results. The higher injection pressure condition was selected based on the upper limit of the NO supply pressure limitation in RC19 at the time of the experiments.

Cases with three injection stage combinations were tested at both the low and high pressure condition. Cases with two injection stage combinations were tested only at the low pressure condition because, for these cases, a stable high pressure condition could not be maintained long enough for data acquisition.

2.4 Instrumentation, Data Collection, and Analysis

This section details the diagnostic techniques and data processing used to examine the injection plume and its interaction with the supersonic crossflow.

2.4.1 Shadowgraph Imaging

Shadowgraph imaging is a spatial averaging technique used to directly observe flow phenomena. Parallel light rays are passed through the facility test section, producing an image that is captured by a high-speed camera. If the light rays do not encounter a density gradient, the light will pass straight through the test section with no change in intensity. If the light passes through a density gradient, it is deflected forming lighter and darker regions according to the relative amount of deflection. These regions reveal second order density gradients, which allow turbulent flow structure and shock wave locations to be observed.

In these experiments, light from a Mercury-arc lamp was directed through the tunnel test section using a 48-inch parabolic mirror. A Photron Fastcam Ultima APX CMOS camera captured 500 frames per second. The camera was equipped with two neutral density filters: optical density 3 and optical density 0.2.

Images were obtained for all low pressure cases listed in Table 1. Additionally, a backpressure study was completed to observe the effects of the pressure rise that would be associated with combustion. This study was composed of three cases. The first two cases had no injection at facility plenum total pressures of 38 psia and 50 psia, respectively. The third case was the low injection pressure condition of Case 1 (1,2,3,4) at a total pressure of 50 psia. Measurements of the bow shock angles for all cases were made using a protractor.

2.4.2 Schlieren Imaging

Schlieren technique is based on imaging the density gradients in the flow field. In this technique, light from a slit source is collimated then passed through the test section and into a lens. An inverted image is formed at the focal plane of the lens, where a knife edge is placed. If the light has passed through a density gradient, the knife edge will intercept more light at certain points than at others causing lighter and darker areas to be captured on the screen. This technique is useful in the location of shocks as well as for observing complex boundary-layer phenomena in supersonic flows.

In these experiments, light from a Mercury-arc lamp passed through an iris and was directed to a 48-inch parabolic mirror. This mirror collimated the light and directed it through the test section into another parabolic mirror. The light was reflected to the knife edge, which was a razor blade placed at the focal point of the mirror. Images projected onto a screen were captured using the previously described Photron CMOS camera.

Images were obtained for all low pressure cases listed in Table 1. Because the backpressure did not have a significant effect on the shock structure, corresponding schlieren images for the backpressure study were not taken. Measurements of the bow shock angles for all low pressure cases listed in Table 1 were made using a protractor.

2.4.3 NO-PLIF Diagnostic

NO-PLIF is a laser diagnostic technique that can be used to measure species concentration. A mixture of Nitric Oxide (NO) and air is injected into the test section through the injector. Laser energy causes the NO molecules to jump into their excited $A^2\Sigma^+$ electronic state. The molecules either collide with other molecules and are quenched, or fall back to the ground state while fluorescing. Molecule fluorescence is captured using a digital camera forming an image of the regions of flow containing NO.

In these experiments, NO-PLIF was used to examine penetration and plume width. Data were taken in both the transverse and cross-flow planes, which will be referred to as the side-view and end-view, respectively. For both orientations, a Lumonics Hyperdye (HD-300) dye laser, pumped by the second harmonic of an injection-seeded Q-switched Nd:YAG laser, generated radiation at 574 nm. This beam was frequency was doubled using an Inrad Autotracker and then mixed with the residual infra-red beam from the Nd:YAG laser using a second Autotracker. This combination, shown in Figure 7, produced a 3-mJ/pulse 226 nm beam which prompted the transition of NO between the ground state, $X^2\Pi_i$ ($v'' = 0$) and the excited state, $A^2\Sigma^+$ ($v' = 0$). This system was

tuned to capture the $R_1(8.5)$ transition and probe the $J = 8.5$ rotational state for the end-view and the overlapped $Q_1(12.5) + Q_2(19.5)$ transitions for the side-view.

The beam was directed to an optical table located under the test section. A laser sheet was formed by a negative cylindrical lens with a focal length of -50 mm followed by a positive spherical lens with a focal length of 1 m. This sheet had a constant height of approximately 75 mm (matching the mirror diameter) and a minimum width that was located inside the test section. The optical table was mounted on a three-dimensional traversing system that positioned the laser sheet in the appropriate location.

Both the beam energy and wavelength were continuously monitored to ensure that both were constant signal throughout data acquisition. To monitor the system, a portion of the beam was directed over a small CH_4 -air burner and then to a fast photodiode. The resulting NO fluorescence signal along with that from the photodiode were displayed continuously on an oscilloscope; constant wavelength was then ensured by periodically tuning the dye laser wavelength to maintain the fluorescent signal (from the flame).

Images were captured using a Princeton Instruments “Superblue” PIMAX intensified charge coupled device camera. The camera was equipped with a Cerco 45-mm, f/1.8 lens and a Schott glass UG-5 filter which was used to block scattering at 226 nm and fluorescence from the (0,0) band, and pass fluorescence from the A-X (0,1), (0,2), (0,3), ... bands. Because the camera orientation in the end-view was not normal to the laser sheet, a Scheimpflug mount was used to minimize perspective distortion. These components are shown in Figure 8 and Figure 9. To increase signal strength and enable the camera to frame at the 10 Hz laser repetition rate, camera pixels were binned 2 by 2 and cropped such that the images for both the end-view and the side-view were 256 pixels wide by 256 pixels tall.

An NO concentration in the injectant plume of approximately 200 parts per million (PPM) provided an adequate fluorescence signal. This concentration adding a small quantity of gas from a bottle containing 1% NO (by volume, equaling 10,000 PPM) and 99% N₂ to the air injectant stream until the mole fraction of NO within the injectant was approximately 200 PPM.

To collect end-view data at each test condition, the laser sheet was transmitted perpendicular to the injection plate at seven axial locations: $x = -0.5, 0.0, 0.5, 1.0, 1.5, 2.0,$ and 2.5 inches relative to the middle of the 14th injection port of the cascade. An ensemble of 200 end-view NO-PLIF images was collected at each location and injection condition. Background images with no injection were also taken at each location. The field of view for these images was 1.75 inches high by 3.5 inches wide. To capture side-view data, the laser sheet was transmitted through the top window at the spanwise centerline of the injector. The field of view for these images was 2.5 inches high by 3.0 inches long. Again, 200-image ensembles were obtained along with background images.

The raw end-view images were processed in several steps. First, each image in the ensemble was corrected for perspective distortion using an image of a dot card with known spacing between the dots. Plume images were normalized by recording NO PLIF images with the tunnel uniformly seeded with NO (with no flow in the tunnel). Each image was then divided by this reference image, removing the effects of laser sheet intensity distribution, collection efficiency, and camera fixed pattern noise. The background image was subtracted from each plume image, then the images were averaged and the standard deviation was determined. Each average image was normalized by its maximum NO signal. Penetration distance and the maximum injection plume width were identified for each case using the Matlab codes shown in Appendix A. For this study, the penetration distance was defined to be the maximum vertical position having at least 10% of the maximum signal. Maximum injection width was the largest difference between the first and last occurrence of 10% of the maximum signal at a given transverse position in the jet plume.

The raw side-view images were processed in a manner similar to the end-view images. Because the camera was positioned in an orientation normal to the tunnel test section, a perspective distortion correction was not necessary. Unfortunately, the reference images required to correct for the laser sheet intensity distribution, collection efficiency, and camera fixed pattern noise were not acquired during experimental testing. Although it was not possible to completely account for these variations, the effects were mitigated by using a running average of the local maximum NO signal to normalize the image. The upper penetration boundary of the plume was defined using 10% of this average local maximum. A higher percentage of the average local maximum signal was used to define the lower penetration boundary of the plume to mitigate the effect of laser scattering from the bottom wall. These boundaries and the maximum injection plume width were identified for each case using the Matlab codes shown in Appendix A.

A per unit mass comparison was completed to determine the effects of each stage on penetration distance for both the end and side view. An influence coefficient (IC) representing the sensitivity of penetration distance to mass flow for each case was defined in the following manner:

$$IC = \frac{\Delta y / y}{\Delta \dot{m} / \dot{m}} \quad (4)$$

where y was the penetration distance of the baseline case, Δy was the difference in penetration distances between the baseline case and the particular case of interest, $\dot{m}$ was the mass flow rate of the baseline case, and $\Delta \dot{m}$ was the difference in mass flow between the baseline case and the case of interest. Note that mass flow was defined:

$$\dot{m} = \rho u \sum_i (A^* Cd)_i \quad (5)$$

The quantity (ρu) was the same for each case and cancelled out in the division of $\Delta \dot{m}$ and $\dot{m}$.

Because each injection port in the cascade is a converging-diverging nozzle with a discharge coefficient, the geometric definition of effective diameter shown in Equation 1 would be less

accurate if applied to the cascade injector. Instead, an effective injector equivalent throat diameter based on measured flow properties, D_j^* , was used to nondimensionalize the experimental data³⁴:

$$D_j^* = \sqrt{\frac{4}{\pi} \frac{\dot{m}_f C^*}{P_t}} \quad (6)$$

In this equation, P_t was the total pressure of the injectant at the throat of the injection port. The mass flow rate of the injectant, $\dot{m}_f$, and the characteristic velocity, C^* , were determined using the mass flow parameter equation. Note that in the following equations the subscript i denotes the stage number.

$$\dot{m}_f = \frac{P_t}{RT_t} MA_i Cd_i \sqrt{\gamma RT_t} \left(1 + \frac{\gamma - 1}{2} M^2 \right)^{\frac{-(\gamma + 1)}{2(\gamma - 1)}} \quad (7)$$

$$C^* = \frac{\sqrt{\gamma RT_t}}{\gamma \sqrt{\left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma + 1}{\gamma - 1}}}} \quad (8)$$

In both equations, the ratio of specific heats (γ) was the constant value for air at standard conditions,

1.4. The specific gas constant (R) was $288.317 \frac{J}{kg K}$, calculated for the mixture of NO and air used

in these experiments. P_t and T_t were the throat values for the total pressure and total temperature, respectively. A_i was the nominal total throat area for each stage as defined in the design information provided by Aerojet General Corporation. M represented the Mach number, which was equal to 1 at the throat of each injection port. The discharge coefficient for each stage, Cd_i , was calculated using values obtained by bench testing each stage of the injector:

$$Cd_i = \frac{\dot{m}_{measured}}{\rho u A_i} \quad (9)$$

During this test, air flowed from the RC19 air supply through a 1000 SLMP (standard liters per minute, with standard temperature equal to 0 °F) mass flow meter ($\dot{m}_{measured}$) and into the cascade injector block. The throat values for injection pressure and temperature were calculated using the isentropic relations with values for stagnation pressure and temperature measured prior to the injection block:

$$P = \frac{P_{t_bench_test}}{\left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}}} \quad (10)$$

$$T = \frac{T_{t_bench_test}}{\left(1 + \frac{\gamma - 1}{2} M^2\right)} \quad (11)$$

These values were then used to calculate the density and velocity at the throat of the injector using the perfect gas law and the definition of the Mach number, respectively:

$$\rho = \frac{P}{RT} \quad (12)$$

$$u = M \sqrt{\gamma RT} \quad (13)$$

2.4.4 Mie Scattering

The set-up for this laser diagnostic technique was identical to NO-PLIF, with the exception of a camera filter. Removing the camera filter allowed scattering off of small ice crystals in the flow to be detected. The image formed was the inverse of a PLIF image.

Mie scattering images were taken only for Case 1 (1,2,3,4) at the low and high injection pressure conditions. Scattering off the bottom wall of the test section required significant reductions in laser power and gain, which resulted in poor quality Mie scattering images. These images were

used for qualitative visualization of the bow shock and plume structure. No measurements were made from these images.

2.5 Results and Discussion

This section presents and discusses the results of the experimental data analysis in two sections. First, the injectant plume structure is described. Next, the penetration and plume width are discussed. The section concludes with a discussion of the influence coefficients and a presentation of the nondimensional data.

2.5.1 Injectant Plume Structure

The injectant plume structure was described using shadowgraph and schlieren visualization, NO-PLIF images, and Mie scattering. Case 1 (1,2,3,4) at the low pressure condition is referred to as the baseline case and is used to describe a representative injection plume structure in the side view and the end view.

Shadowgraph and schlieren images of the baseline case, shown in Figure 10 and Figure 11, revealed the shock structure surrounding the cascade injector. The bow shock, which appears in both figures, had an angle of 37° . In Figure 10, the oblique shock wave reflects off the top wall of the test section approximately 2 inches downstream of the injector. Although neither image adequately captured the feature, both figures indicate the shock structure associated with overexpanded injection. The overall structure of the cascade was consistent with similar data from previous work.²⁹ Note that shadowgraph and schlieren images for all other cases appear in Appendix B.

A representative instantaneous side-view NO PLIF image is shown in Figure 12. In this image, dark regions correspond to freestream fluid containing no nitric oxide. Bright regions, on the other hand, are indicative of fluid with nitric oxide at some detectable concentration. The figure shows a structure of vortices following the bow shock that develop into a turbulent shear layer at a constant height. This region has a high level of standard deviation, as indicated by the light colored area in Figure 13. It is likely that large-scale mixing occurs along this interface between the jet and

crossflow fluids. The ensemble-averaged side-view NO-PLIF image is shown in Figure 14. In this image, the injectant boundary follows a diagonal, linear path until it reaches a constant penetration height at approximately $x = 1.0$ inches.

The Mie scattering image shown in Figure 15 displays the instantaneous end-view injection plume structure with the bow shock. The dark area in this image indicates the location of the injection plume. The lighter halo around this plume illustrates the location of the bow shock.

In Figure 16, the axial development of the instantaneous injection plume for the baseline case is shown using NO-PLIF. The plume emerges as a thin initial jet that spreads and lifts away from the wall as it moves downstream from the injector. The penetration of the plume increases slowly until reaching a relatively constant penetration distance near the $x = 1.0$ inch position. The width of the injection plume remains relatively constant as the downstream axial position increases. Standard deviation images are shown in Figure 17. These images indicate relatively high levels of unsteadiness at the interface between the injectant and air in the crossflow. Ensemble-averaged images are shown in Figure 18. In these images, the plume begins as a 2-dimensional conical structure that spreads into an oval as it lifts away from the wall and moves downstream; behavior that is consistent with that shown in Figure 16.

The effects of increased injection pressure on the baseline case are shown in Figure 19 through Figure 22. Figure 19 and Figure 20 show the instantaneous and ensemble-averaged side-view NO-PLIF images, respectively. In these images the injectant boundary follows a much steeper slope before reaching a constant penetration height at approximately $x = 1.0$ inches. A significant increase in penetration distance, attributed to the increase in injection pressure, was observed. Figure 21 and Figure 22, respectively, display the instantaneous injection plume and ensemble-averaged images for the end-view. The plume structure and development in both figures is similar to those in Figure 16 and Figure 18. However, for the higher injection pressure condition, the plume shape

begins as a cylinder and develops into a 2-dimensional conical structure as axial position increases and the plume appears to lift farther from the wall. As observed in Figure 19 and Figure 20, penetration distance was observed to increase notably as a result of the increased injection pressure. However, the maximum injection plume width was not as significantly affected by this increase.

2.5.2 Plume Penetration and Spreading

Instantaneous and ensemble-averaged images for Case 1 (1,2,3,4) for two injection pressure conditions were discussed in the previous section. In all figures, the penetration distance increased to a relatively constant value near the $x = 1.0$ inch position and, in the end-view images, maximum plume width was observed to continually increase with increasing downstream position. Note that penetration and plume spread increased in response to higher injection pressure. Figure 23 and Figure 24, respectively, show the penetration profiles and maximum plume width resulting from the analysis of the ensemble-averaged images at the low and high injection pressure conditions. In Figure 23a and Figure 24a, an initially rapid growth rate slowed as downstream position increased for both injection conditions. Penetration distance was observed to be approximately 1.0 inch for the lower injection pressure case, and 1.4 inches for the case with higher injection pressure. This behavior is consistent with previous experimental results.²⁹ Figure 23b-e and Figure 24b-e show plume width to continually increase with increasing downstream position, ending at 0.79 inches for the lower injection pressure case and 0.86 inches for the case with higher injection pressure.

Figure 25 presents a comparison of the cases with three active stages with an injection pressure of 144 psia. The maximum penetration distance of $y = 0.96$ inches occurred for Case 4 (1,3,4). The minimum penetration of $y = 0.86$ inches occurred in Case 2 (1,2,3). Case 3 (1,2,4) had a final penetration distance of $y = 0.93$ inches, which was slightly greater than the final penetration distance of $y = 0.89$ inches for Case 5 (2,3,4). The penetration distance in Figure 25 appears to be most significantly affected by stages 1 and 4. The first stage of the cascade is important because its

interaction with the freestream flow forms the bow shock, which enables downstream stages to penetrate farther into the crossflow. The importance of stage 4 is demonstrated by the observed increase in penetration distance between Case 2 (1,3,4) and Case 5 (2,3,4). Lateral spread was not significantly affected by stage selection.

The penetration distance hierarchy observed in Figure 25 is more easily seen in Figure 26 where the cases with three active injection stages at the higher nominal injection pressure are plotted. The cases, ranked in order of maximum to minimum penetration distance are: Case 4 (1,3,4), Case 3 (1,2,4), Case 2 (1,2,3), and Case 5 (2,3,4). This ranking appears to indicate that stage 1 has a slightly higher impact on penetration than stage 4; Case 2 (1,2,3) penetrates slightly farther than Case 5 (2,3,4) at the higher injection pressure condition. The significantly higher penetration distance of Case 4 (1,3,4) compared with Case 5 (2,3,4) in this figure effectively shows the importance of stage 1.

In Figure 27, the cases having two active stages and an injection pressure of 144 psia are compared. The cases, ranked in order from maximum to minimum penetration distance, were: Case 8 (1,4), Case 10 (2,4), Case 11 (3,4), Case 7 (1,3), Case 6 (1,2), and Case 9 (2,3). The maximum final penetration distance for these cases was $y = 0.80$ inches, observed in Case 8 (1,4). The minimum final penetration distance of $y = 0.68$ inches occurred in Case 9 (2,3). Again, the penetration distances illustrate the importance of stages 1 and 4.

The penetration distances determined for all cases using the side-view and end-view data are recorded in Table 2 and Table 3, respectively. The flow in the near field of the injector is transient, so the level of penetration measured in the side-view and the end-view are not expected to be in good agreement in this region. Downstream of the injector after the injection plume has reached a constant height, the side-view and the end-view are expected to yield consistent results. Indeed, as shown in Table 4, the side-view and end-view NO-PLIF data indicated penetration heights that were typically less than 6% apart. Note that percent difference was calculated using the following formula:

$$\% \text{ difference} = \left[\frac{(\text{Side-view penetration} - \text{End-view penetration})}{\text{Side-view penetration}} \right] * 100 \quad (14)$$

The low percent difference between the side-view and end-view indicates that the method of correcting the side-view images was acceptable. However, because reference images were not acquired for the side-view images, the data have higher uncertainty. Although data from both the end-view and the side-view are presented in the figures and tables, the values quoted in the text are those obtained using the end-view data.

The maximum injectant plume widths as a function of axial location are shown in Figure 25 through Figure 27 and listed in Table 5. In all cases, the maximum injectant plume width increases as axial location progresses downstream; a trend which is attributed to mixing. For this reason, the maximum injection plume widths were compared at $x = 3.5$ inches, the most downstream location where data were acquired. At this location, the cases with three active injection stages at the low injection pressure condition ranked in order from maximum to minimum plume width were: Case 3 (1,2,4), Case 2 (1,2,3), Case 4 (1,3,4), and Case 5 (2,3,4). The order from greatest to least maximum plume width for cases with three active stages at the high injection pressure was much different. Case 3 (1,2,4) and Case 5 (2,3,4) had the widest maximum plume width, 0.79 inches. Case 4 (1,3,4) had a maximum plume width of 0.78 inches. Case 2 (1,2,3) had the smallest maximum injection plume width: 0.76 inches. The cases with two active injection stages, ranked in order from widest to smallest maximum plume width, were: Case 7 (1,3), Case 8 (1,4), Case 6 (1,2), Case 11 (3,4), Case 10 (2,4), and Case 9 (2,3).

The location of the maximum plume width with respect to the maximum plume height is shown in Table 6 for each case. For all cases, the location of the maximum width moves towards the top of the injection plume as downstream location increases. Cases with two active stages, which remained relatively close to the bottom of the test section, had the most significant increase. At the $x = 3.5$ inch location, the maximum plume widths for Cases 7 (1,3) and 6 (1,2) were 97.5% and 94.8%

of the injectant plume height, respectively. The maximum plume widths for cases with three active stages at an injection pressure of 250 psia were located at a much lower percentage of penetration height. Both observations are indicative of the typical plume structure for the injection pressure condition and number of active stages; cases with three active stages at the high injection pressure condition produced long, slender injection plumes while plumes produced in cases with two active stages were shorter and wider.

2.5.3 Influence Coefficients

Although Figure 25 through Figure 27 show that stage 4 has a notable impact on penetration distance, Table 7 indicates that this stage does not have a significant effect on the penetration distance on a per mass basis. For the cases with three active stages at an injection pressure of 144 psia, the stages that had an effect on penetration distance per unit mass in order from maximum to minimum effect were: stage 1, stage 2, stage 4, and stage 3. The stages that had an effect on penetration distance per unit mass at an injection pressure of 250 psia in order from greatest to least effect were: stage 1, stage 2, stage 4, and stage 3. Note that the influence coefficients of stages 3 and 4 in Table 7 are very close in value. This similarity is expected because stages 3 and 4 have mass flow rates that are close in magnitude. The cases having 2 active stages and an injection pressure of 144 psia, ranked from maximum to minimum effect on penetration distance per mass as indicated by the influence coefficients in Table 7 are: Case 11 (3,4), Case 9 (2,3), Case 10 (2,4), Case 8 (1,4), Case 7 (1,3), and Case 6 (1,2). This ranking indicates that the combination of stages having the most effect on penetration distance per unit mass was stages 1 and 2. However, it was noted that cases where stage 1 was active had larger influence coefficients than other cases. The combination of stages having the least effect on penetration distance per unit mass was stages 3 and 4. The influence coefficients shown in Table 7 suggest that the aerodynamic characteristics of stages 1 and 2 are very important when considering the penetration of this injector design. That is, despite having the

smallest mass flow rates, stages 1 and 2 appear to have the most pronounced impact on penetration distance.

2.5.4 Nondimensional Data

The effective diameters for each experimental test condition are listed in Table 8. Figure 28, Figure 29, and Figure 30 show the penetration height versus the axial location nondimensionalized by these effective diameters. In Figure 28a, cases with three active injection stages listed in order from maximum to minimum nondimensional penetration distance are: Case 3 (1,2,4), Case 4 (1,3,4), Case 1 (1,2,3,4), Case 2 (1,2,3), and Case 5 (2,3,4). At $x = 3.5$ inches, the penetration height for Case 2 (1,2,3) is slightly higher than that of Case 1 (1,2,3,4), but the rest of the order remains unchanged. For the side-view data at the higher injection pressure condition the penetration hierarchy changes. Case 1 (1,2,3,4) has the highest nondimensional penetration distance, followed closely by Case 4 (1,3,4) and Case 3 (1,2,4). Case 2 (1,2,3) has a higher nondimensional penetration than Case 5 (2,3,4) (Figure 29a). However, at $x = 3.5$ inches, the case order from maximum to minimum nondimensional penetration is identical to that of Figure 28a. The cases with two active injection stages, in order of maximum to minimum nondimensional penetration, were: Case 8 (1,4), Case 7 (1,3), Case 6 (1,2), Case 10 (2,4), Case 9 (2,3), and Case 11 (3,4).

Once again, the first injection stage was shown to have the greatest effect. In Figure 28 and Figure 29, the maximum nondimensional penetration height for Case 5 (2,3,4) was notably lower than other cases. In Figure 30, Case 6 (1,2), Case 7 (1,3), and Case 8 (1,4) had nondimensional penetration heights that were significantly taller than other cases. The order of nondimensional penetration heights indicates that the penetration height, which is dependent on the shock structure, is a function of the injection pressure and the shape of the injector.

Data presented in the literature are typically nondimensionalized by the effective diameter and the jet momentum ratio and determined to follow a power law correlation. These correlations enable injectors of different designs to be compared. Because effective diameters used to

nondimensionalize data presented in the literature do not include the effect of the discharge coefficient, the cascade data was nondimensionalized using the effective diameter as defined in Equation 2. These values are displayed with the effective injector throat diameters for each case in Table 8. Jet momentum ratio values for the cascade data were 0.65 and 1.1 at the low and high pressure conditions, respectively. Nondimensional cascade data are compared to the correlation proposed by Gruber et al. (Equation 1 at $J = 3$)¹³ in Figure 31. This figure shows the cascade injector to have significantly higher penetration with respect to a sonic, normal injector; a comparison which is also presented in Table 9. The cascade injector was observed to penetrate higher than a sonic normal injector by a factor of 3.5 or greater.

3.0 Computational Fluid Dynamics

This section describes the numerical analysis used to explore the mechanics governing the penetration and mixing characteristics of the cascade injector.

3.1 Computer Code and Modeling

The computational results were obtained using the VULCAN Navier-Stokes code, version 5.0.2.³¹ This code, developed jointly by AFRL and NASA's Langley Research Center, was introduced in 1997. It is derived from the NASA Langley Algorithm for Research in Chemical Kinetics (LARCK), a scramjet application code that combines the Wilcox $k-\omega$ turbulence model with a chemical kinetics model.³⁷ VULCAN uses a cell-centered finite volume scheme to solve the Reynolds-averaged conservation equations appropriate for calorically or thermally perfect gases in two-dimensional, axisymmetric, or three-dimensional form. These equations can be integrated in a fully elliptic manner or space-marched in parabolic and hyperbolic regions of flow. Full multi-grid capabilities allow these schemes to share information and enable an accelerated convergence rate for steady-state solutions. The inviscid fluxes can be evaluated using central differences, Roe's flux difference method, or a low-diffusion flux vector split scheme. A variety of one-equation and two-equation turbulence models are available and turbulence-chemistry interactions can be modeled based on the Magnussen and Hjertager eddy break-up model, temperature fluctuations, or changes in average species concentrations as determined by probability density functions. Chemical reactions are accounted for with finite rate kinetics models based on the law of mass action. Although VULCAN is a structured solver, it can solve grids that are not continuous across block boundaries (non-C(0) continuous connectivity). The code also contains a number of other features, including block level parallelization using MPI. Further description of this code and its capabilities can be found in the literature.^{31, 36}

The calculations in this study were performed using the low dissipation flux-split scheme of Edwards, derived for a mixture of thermally perfect gases. To minimize truncation error, the MUSCL parameter (κ) was set to $\frac{1}{3}$. The Van Leer flux limiter was applied to ensure Total Variation Diminishing (TVD). With the exception of the fine grid case, all solutions were advanced in time with a CFL number that started at 0.1 and was increased to a constant value of 4.0 as the initial transient behavior subsided. For the fine grid case, the CFL number was only increased to 2.0 to control numerical instabilities.

For the majority of the cases in this study, turbulence was modeled using the 1998 Wilcox $k-\omega$ model without the Pope correction term.³⁸ Previous studies have employed this model to examine injection into a supersonic cross-flow.³⁹ However, in order to complete a turbulence model study, two cases were run using other models. In the first case, turbulence was modeled using the Menter model.^{40,41} This two-equation model applies the Wilcox $k-\omega$ model²³ near the tunnel wall, then transitions to the Jones and Launder $k-\epsilon$ model⁴² in the outer edge of the boundary layer. In the second case, the explicit algebraic Reynolds-stress model of Gatski and Speziale⁴³ was applied with the Wilcox $k-\omega$ model.³⁸ For all cases, the turbulent Prandtl and Schmidt numbers were set to constant values of 0.89 and 0.5, respectively.

3.2 Computational Domain and Grids

The computational domain extended from the entrance plane of the isolator through the 2.0 inch high by 6.0 inch wide experimental facility test section. A cascade injector, located in the diverging bottom wall of the test section, was modeled. For the medium grid cases, the injectant flow field from the discharge plane of the injector to the top wall of the test section was computed on a half-tunnel computational domain with 1.4 million grid points. This grid had 287 points in the I direction, 41 points in the J direction, and 120 points in the K direction. The fine grid contained approximately 11 million grid points with 573 points in the I direction, 81 points in the J direction,

and 239 points in the K direction. All grids were created with Gridgen, a commercial meshing toolkit used to create three-dimensional grids for complex geometries.⁴⁴ A two-dimensional image of the injector region of the medium grid is shown in Figure 32a. Note that in this figure and all other side-view images, the grid coordinates have been transformed such that $y = 0$ corresponds to the floor of the test section and the divergence is shown at the top of the domain. The medium resolution transverse grid at $x = 3.5$ inches is shown in Figure 32b. In the shear layer of the injection plume, the spacing between the y and z points were approximately $\Delta y = 0.09$ inches and $\Delta z = 0.011$ inches at $x = 2.0$ inches and $\Delta y = 0.1$ inches and $\Delta z = 0.014$ inches at $x = 3.5$ inches for the medium grid. The fine grid had spacing between the y and z points approximately equal to $\Delta y = 0.04$ inches and $\Delta z = 0.008$ inches at $x = 2.0$ inches and $\Delta y = 0.05$ inches and $\Delta z = 0.0083$ inches at $x = 3.5$ inches in the shear layer of the injection plume.

3.3 Boundary Conditions

An initial simulation of air through the $M = 3.0$ facility nozzle was completed using the average plenum conditions recorded during experimental testing as inflow boundary conditions to obtain a flow profile at the nozzle exit plane. This profile was provided as the inflow boundary condition of the isolator. Three chemical species were considered in all simulations: Nitrogen, Oxygen, and Air. The freestream was defined, by mass, as a mixture of 77% Nitrogen and 23% Oxygen. The injectant was air, defined, by mole, within VULCAN to be a mixture of 75% Nitrogen, 23% Oxygen, and trace amounts of Carbon Dioxide and Argon. Injectant properties at the exit planes of the injection ports were defined using experimental values for temperature and density. The computational cases corresponding to experimental Case 1 (1,2,3,4), Case 2 (1,2,3), and Case 5 (2,3,4) were directly compared. The global computational injectant mass flow rates had a less than 1% difference from the experimental data. Top, bottom, and side walls of the test section were

considered to be adiabatic. A symmetry condition was applied along the tunnel centerline for the entire domain. The variables at the exit of the domain were extrapolated.

3.4 Test Conditions and Solution Technique

The numerical study was composed of 12 cases which were subdivided into five areas of interest. First, a solution obtained using VULCAN version 5.0.2 was compared to the solution calculated with an older version of the code. This comparison ensured that any observed differences from previous work were not due to code updates. The second study was a comparison of three cases corresponding to Case 1 (1,2,3,4), Case 2 (1,2,3) and Case 5 (2,3,4) listed in Table 1. Case 1 (1,2,3,4) served as a baseline case while Cases 2 (1,2,3) and 5 (2,3,4) demonstrated the impact of stages 4 and 1, respectively. The third study explored the impact of the first stage in the cascade, which was determined experimentally to have the greatest effect on penetration distance per unit mass flow rate. In these simulations, the injection pressure of the first stage was incrementally increased from the baseline value of 144 psia to 250 psia while the other three stages were maintained at the baseline value. Next, a turbulence model study was conducted. This study compared the Wilcox $k-\omega$ model to the Menter model and the Gatski and Speziale explicit algebraic Reynolds-stress model. Finally, a grid sensitivity study compared medium and fine grid solutions obtained for Case 1 (1,2,3,4) at an injection pressure of 144 psia using the Wilcox $k-\omega$ turbulence model.

Convergence for all cases was determined by examining the residual as well as the mass-averaged one-dimensional flow properties. The residual, shown in Figure 33, was observed to drop approximately seven orders of magnitude before leveling off to a constant value. At this point, the mass-averaged one-dimensional flow properties showed little variation with respect to the number of iterations and the case was considered to be converged. Most cases reached convergence after 6,000

iterations. However, the fine grid case required 16,000 iterations before it was considered to be converged.

3.5 Results and Discussion

In this section, the CFD data is compared to previous computational results and the experimental data, the impact of the first stage on the injection plume characteristics is studied, the effects of turbulence modeling are discussed, the synthetic LIF signal is examined, and the results of a grid sensitivity study are presented.

3.5.1 Comparison to Previous Computational Results

The overexpanded injection case presented in Reference 30 was solved using VULCAN version 2.4.2. Using the input deck from this previous case, a numerical solution was generated using VULCAN version 5.0.2. Both cases produced similar solutions and it was concluded that updates to the code did not alter the fidelity of the CFD.

This exercise produced a numerical solution that could serve as a basis of comparison to previous numerical work. However, several notable differences between the computational set-up for the current experiments and previously published results³⁰ prevented direct comparison. The input deck from previous work, which originally modeled an array of three injectors, was modified to consider only a single injector along the tunnel centerline to be consistent with the present experiments. However, in the previous experiments, all stages of the injector were supplied from a single source such that the injection pressure through each stage was determined by its area. In the present experiments, the static pressure for each stage was controlled to maintain a constant injection pressure condition through each stage. Finally, the previously considered test section is a constant area duct while in the current experiments the bottom wall of the test section has a 2.5 degree divergence.

Despite these differences, the general trends of both solutions could be compared. Note that the previous numerical solution employed the Menter turbulence model so this comparison is based

on the current numerical simulation of Case 1 (1,2,3,4) at the low pressure condition using the Menter turbulence model. Although slight differences existed in the location and strength of the shock structure, the flow characteristics between the two cases were comparable.

3.5.2 Comparison to Experimental Data

The penetration boundaries for the CFD data were defined using the injectant mass fraction instead of the NO signal data. These boundaries were determined using Matlab codes that were consistent with the experimental data analysis (Appendix A). For the side-view, the upper boundary of the penetration distance was determined by 10% of the running average of the local maximum injectant mass fraction. The lower penetration boundary in the side-view was 25% of the local maximum injectant mass fraction; a higher value was used to mitigate the effect of scattering off of the floor of the tunnel test section. In the end-view, the dispersion boundary was defined as 10% of the local maximum injectant mass fraction.

Figure 34 presents a comparison of the side-view and end-view plume penetration boundaries of the experimental and computational data for Case 1 (1,2,3,4), respectively. In the side-view, the penetration boundaries correspond in the region around the injector, but become increasingly divergent with downstream position. In Case 1 (1,2,3,4), the computational data begins to over-predict the upper and lower penetration boundaries at approximately $x = 0.68$ inches and $x = 1.16$ inches, respectively. At the end of the experimental data domain, this over-prediction is 0.10 inches in magnitude for the upper boundary and 0.11 inches in magnitude at the lower boundary. In the end-view, the CFD data increasingly under-predicts the plume width with increasing downstream position. For Case 1 (1,2,3,4) at $x = 3.5$ inches, the magnitude of this under-prediction is approximately 0.19 inches. The trends of over-predicting penetration height in the side-view and under-predicting plume width in the end-view are present for all cases and both pressure conditions, as indicated in Table 10.

In both the experimental and computational data, the upper penetration boundary in the near field of the injector corresponds with the location of the shock wave caused by the injectant. This observation is illustrated using the CFD data in Figure 35 and Figure 36. In these figures, the penetration boundary defined by 10% of the global maximum injectant mass fraction is shown on a contour plot of the static pressure. In the region around the injector, the penetration boundary is aligned with the increase in static pressure that denotes the location of the shock wave.

Figure 35 and Figure 36 also show the increase in penetration that is obtained by increasing the injection pressure. These figures clearly show a greater increase for Case 2 (1,2,3) than for Case 5 (2,3,4). This occurrence, which is consistent with previous experimental data analysis²⁹, seems to indicate the importance of the first stage. Stage 4, which has the largest area and therefore supplies the most injectant mass flow, is not active in Case 2 (1,2,3), so a small increase in mass flow rate results in a significant increase in penetration distance.

3.5.3 First Stage Pressure Increase

Figure 37 shows a side-view and end-view comparison of cases with an incremental increase in the injection pressure of the first stage. These figures show an average incremental increase in penetration of approximately 0.025 inches per 53 psia of injection pressure. Because increasing the injection pressure of Stage 1 does not have a significant impact on penetration, this stage cannot be solely responsible for the penetration increase observed in Figure 35. Rather, this increase appears to be due to an increase in shock angle caused by the increase in mass flow. In Figure 38, there is no observable increase in shock angle because increasing the injection pressure of Stage 1 causes a minimal increase in overall mass flow rate. The shock angle increases 3 degrees between Figure 36a and b. Between Figure 35a and b, there is a 9-degree increase in shock angle. Both of these more substantial increases are caused by a significant increase in mass flow.

This conclusion is further supported by Figure 39, which displays streamlines overlaid on a plot of static pressure contours for Case 1 (1,2,3,4). Streamlines, which are tangential to the local

velocity vectors, indicate the location and amount of turning that occurs in the flow. As shown in the figure, these streamlines coalesce to form a definite boundary which governs the static pressure field. Both the core flow and the flow from the injector turn as they intersect each other. This turning creates the strong pressure waves that form shock waves in the flow. The location of the inflection point of the streamline defines the penetration boundary. At a higher pressure, the flow coming from the injector is able to travel a farther distance before it intersects the free stream flow and forms the shock, which results in increased penetration.

3.5.4 Turbulence Model Study

A comparison of the Wilcox $k-\omega$ model, Menter model, and Gatski and Speziale explicit algebraic Reynolds-stress model to the experimental data for Case 1 (1,2,3,4) is shown in Figure 40. The side-view penetration distance at the end of the experimental domain and the $x = 3.5$ inch end-view plume width at the location of the maximum experimental plume width are presented in Table 3. Although the Gatski and Speziale algebraic Reynolds-stress models over-predicts the penetration in the side-view by a greater distance than the Wilcox $k-\omega$ model, both schemes under-predict the plume width in the end-view by approximately the same amount. The Menter model is observed to under-predict both penetration and plume width. However, the Menter model under-predicts the plume width by a lesser amount than either of the other models.

The differences in the flowfields are best illustrated using the side-view and end-view contours of the eddy viscosity ratio which are displayed in Figure 41, Figure 42, and Figure 43. The eddy viscosity ratio, defined as the eddy viscosity divided by the molecular viscosity³⁶, can be considered a measure of the turbulent transport of momentum to the molecular, or laminar, transport of momentum. The lower penetration distance and larger plume width predicted by the Menter model are the result of the larger turbulent transport. As the turbulent viscosity increases, the mass diffusion due to turbulence, which is a function of the turbulent viscosity and the injectant species

gradient, also increases. In Figure 43b, the Menter model exhibits higher eddy viscosity ratio values indicative of larger turbulent viscosity within the plume penetration boundary.

The end-view mass fraction contours with the velocity vector field and the 10% penetration boundary of the Wilcox $k-\omega$ solution are shown in Figure 44. This figure illustrates the vortical structure of the injection plume. The turbulent transport of momentum reduces the strength of this vorticity. As seen in Figure 45, the Menter model, which has largest amount of turbulent transport, has the lowest one-dimensional, mass averaged streamwise vorticity.

The profile of the nozzle exit plane that was used as the inflow boundary condition for all computational cases was calculated using the Wilcox $k-\omega$ turbulence model. Figure 46 compares the boundary layer profiles of each turbulence model two inches upstream of the injector, which was approximately 12 inches downstream of the nozzle exit plane, on the centerline ($z = 0.0$ inches) and outside the computational plume width at $z = 0.5$ inches. No significant differences in boundary layer profile were observed at either location, indicating that the boundary layer thickness was not significantly affected by the turbulence model upstream of the injector.

3.5.5 Synthetic LIF Signal

The NO LIF signal depends on the local values of static temperature and pressure. Because this signal is not equivalent to the injectant mass fraction, the CFD solution was used to generate a synthetic LIF signal. Figure 47 and Figure 48, respectively, show the side-view and end-view static pressure, static temperature and injectant mass fraction fields from the CFD solution that were used to create the synthetic signal. The calculated temperature and pressure were used to derive ground-state Boltzmann fraction, net electronic quenching rate⁴⁵, and the laser-transition overlap integral relative to an arbitrary reference condition. These factors were then used to calculate the relative fluorescence signal from the computed injectant number density. The NO concentration in the injectant, 200 PPM, was sufficient for a strong fluorescence signal, but the injectant consisted of

mostly air. Among the species present in the injectant, only O_2 was a significant quencher of the excited NO. Other chemical species with large quenching cross sections (e.g., NO) were considered but existed only in small concentrations. Although the electronic quenching cross section for NO- O_2 collisions shows little variation at and above room temperature⁴⁵, no values have been reported at the low-temperature condition encountered in this work. It should also be noted that the laser-transition overlap integral did not include the effect of the collisional pressure shift or Doppler shift.

The penetration boundary defined by the synthetic LIF signal is displayed in Figure 47 and Figure 48. This boundary was determined using a Matlab code consistent with those used to determine the penetration boundaries for the experimental and CFD data. The upper and lower penetration boundaries, respectively, represent 10% and 25% of the local maximum synthetic LIF signal for each axial position. In the end-view, the plume width was defined as 10% of the local maximum synthetic LIF signal.

The penetration boundaries defined by the synthetic LIF signal, experimental data, and CFD solution in the side-view and end-view are compared in Figure 49. The CFD solution models Case 1 (1,2,3,4) with an injection pressure of 144 psia using the Wilcox $k-\omega$ turbulence model. In the side-view, the synthetic LIF signal reduces the amount of over-prediction for the upper penetration boundary with respect to the experimental data and improves the accuracy of the Wilcox $k-\omega$ model prediction. Despite this improvement, sources of uncertainty combined with the relatively small difference between the penetration boundaries defined by the synthetic LIF signal and the injectant mass fraction, indicate that the injectant mass fraction remains a valid parameter for comparison to the NO fluorescence. In the end-view, the penetration boundary determined by the synthetic LIF signal is indistinguishable from the penetration boundary defined by the CFD solution again indicating that a comparison between the NO-PLIF data and the CFD injectant mass fraction field is not unreasonable.

The shape and intensity of the injection plume captured in the experimental image is clearly reflected in the synthetic LIF signal, as illustrated by Figure 50 and Figure 51. The end-view image shows less agreement in shape with the experimental image because the CFD solution that it is based on underpredicts the plume width. However, the side-view image shows much better concurrence with the experimental data. Because the CFD solution does not contain the effects of scattering that are seen in the experimental data, the synthetic LIF signal structure is more distinct; an observation illustrated using Figure 50. In this figure, the synthetic LIF signal shows details of the injection from the individual ports that compose the cascade. These features are not well-defined in the static pressure or mass fraction contours from the CFD solution, but they are observed in the experimental data, and are pronounced in the static temperature field (Figure 47). These images highlight the strong dependence of the synthetic signal on the static temperature field.

3.5.6 Grid Sensitivity Study

Medium and fine resolution CFD solutions obtained for Case 1 (1,2,3,4) with an injection pressure of 144 psia using the Wilcox $k-\omega$ turbulence model were used to complete a grid resolution study. Increasing the grid resolution created a better model of the flow physics and allowed the features of the flowfield to be captured in greater detail. This statement is illustrated in Figure 52, which shows the side-view static pressure contours for the fine and medium grid resolutions. In both images, the same shock structure is indicated by a rise in static pressure, but the magnitude of the increase is larger for the fine grid. The fine grid also shows the local increase in static pressure created by the overexpanded injection condition in more detail. Figure 53 and Figure 54, respectively, compare the injectant mass fraction contours for the fine and medium resolution grids. Once again, the flow features appear to be similar, but the fine grid shows the injectant plume in more detail. However, as shown in Figure 55, the grid resolution actually has a noticeable impact on the penetration height. The penetration boundary defined by the fine grid is approximately 0.15

inches lower than the medium grid penetration boundary, which causes the fine grid to produce better agreement with the experimental data. In fact, as Figure 56 shows, the penetration boundary produced by the fine grid is in better agreement with the experimental data than the solution produced using the Menter turbulence model. Note that the plume width does not significantly change with grid resolution. While it is tempting to conclude that all simulations should be run using the fine grid resolution, the fine grid requires significantly more computational time on a greater number of processors, and is a more expensive investment.

4.0 Conclusions

It was experimentally demonstrated that the plume structure of this injector could be controlled by altering the number of active stages in the cascade. Cases with three active injection stages at a high injection pressure condition were observed to have plume structures that were tall and slender, while cases with two active injection stages were shorter and wider. All cases showed high standard deviation levels at the interface between the injectant and freestream, indicating a high level of unsteadiness at this location. Maximum plume width increased with increasing downstream position and was located in the upper half of the injection plume.

Good agreement was observed in penetration height determined using side-view and end-view data. Initially, analysis indicated that penetration height was strongly dependent on stages 1 and 4. When these stages were not active a noticeable decrease in penetration height occurred. However, analysis of the influence coefficients for each case showed that stage 1 had the greatest penetration distance per unit mass flow rate while stage 4 had the least.

Nondimensionalization by the effective injector equivalent throat diameter for each case showed that the penetration distance, which was dependant on shock structure, was a function of the injection pressure and the shape of the injector. Cascade injector data, nondimensionalized by the effective diameter and the jet momentum ratio was compared to the power law correlation of Gruber et al. This comparison showed the penetration height obtained by the cascade injector was greater than a normal, circular injector by at least a factor of 3.5.

Although not used during the experimental analysis, pitot pressure, cone static pressure, and total temperature profiles were obtained during testing. It is highly recommended that Raman scattering data be taken and combined with these profiles to gain quantitative mixing information.

All solutions obtained using CFD were found to be consistent with experimental data as well as previous computational results and similar trends were observed. The computational data showed

overall excellent agreement in terms of penetration distance, but was significantly less accurate in predicting plume width. As observed in both the experimental and computational data, the first stage produced the greatest effect on penetration distance per unit mass flow rate. However, an incremental increase in the injection pressure of the first stage did not produce a significant increase in penetration distance. By examining the static pressure and streamlines in the CFD solution, changes in penetration were attributed to the amount of turning that occurred in the flow field. Increasing the injection pressure of stage 1 produced a minimal increase in the overall mass flow rate and did not alter the location of the shock wave, so there was no corresponding increase in penetration distance. Further study on the effect of stage 1 by simulating injection through stages 2 and 3 only is recommended.

A turbulence model study showed that the Wilcox $k-\omega$ model and the Gatski and Speziale explicit algebraic Reynolds stress model over-predicted the amount of penetration height and the plume width. The Menter model, which showed the closest agreement with experimental data, under-predicted both penetration height and plume width. This model produced larger eddy viscosity values in the region surrounding the injection plume, resulting in a lower penetration distance and larger plume width. In addition to eddy viscosity ratio, the turbulent transport of mass and energy are governed by the turbulent Schmidt and Prandtl numbers, respectively. Variation of these numbers will alter the injection plume characteristics and may offer improved comparison with measured plume width. Because only the effects of the turbulence model on the baseline case were examined, a turbulence model study on Case 2 (1,2,3) and Case 5 (2,3,4) is recommended. Large eddy simulation of these cases would provide additional insight into the plume structure and penetration characteristics.

The CFD data was used to generate a synthetic LIF signal, which was similar to the experimental NO-PLIF signal. Although the synthetic signal offered better agreement with the

experimental data, this improvement was not great enough to invalidate the comparisons made using only the injectant mass fraction. Some uncertainty existed in the electronic quenching rate due to the low static temperatures of this case. Ideally, future measurements using NO-PLIF should be conducted at static temperatures at or above room temperature to mitigate the uncertainty in the calculated electronic quenching cross section.

A grid convergence study examined CFD solutions generated using medium and fine resolution grids. As expected, the fine resolution grid solution showed more detail in the static pressure and injectant mass fraction contours surrounding the injector. The plume width increased slightly as a result of the greater resolution. The fine grid resolution also reduced the penetration height by 0.15 inches, which was in better agreement with the experimental data. In fact, the penetration height predicted by the fine grid was shown to be in better agreement with the experimental data than the solution produced using the Menter turbulence model. Although the fine grid requires significantly more computational time on a greater number of processors, it is recommended that Case 2 (1,2,3) and Case 5 (2,3,4) be examined using this resolution, especially if a turbulence model study is conducted.

Though much remains to be learned about the Aerojet cascade injector, this study has examined independent stage control and provided insight into the mechanisms governing the penetration and mixing characteristics of the injector. Due to its complexity and plume characteristics, the Aerojet cascade injector is not well suited to be the primary injection technique in a scramjet engine. Although this injector demonstrates excellent penetration characteristics, the injection plume is narrow and the fuel would not cover a wide area. However, the injection plume characteristics described in this study make it an excellent choice for non-intrusively fueling specific areas of a supersonic combustor ranging from close to the injector wall to farther out in the flowfield.

6.0 References

1. Kutschenreuter, P. Scramjet Propulsion. Curran, E.T. and Murthy, S.N.B., ed., "Chapter 8: Supersonic Flow Combustors." American Institute of Aeronautics and Astronautics, Inc. Reston, VA. 2000.
2. Zukoski, E.E., and Spaid, F.W. "Secondary Injection of Gases into a Supersonic Flow." *AIAA Journal*, Vol. 2, No. 10, 1964, pp.1689-1696.
3. Schetz, J.A., and Billig, F.S., "Penetration of Gaseous Jets Injected into a Supersonic Stream." *Journal of Spacecraft*, Vol. 3, No. 11., 1966, pp. 1658-1665.
4. Schetz, J.A., Hawkins, P.F., and Lehman, H., "Structure of Highly Underexpanded Transverse Jets in a Supersonic Stream." *AIAA Journal*, Vol. 5, No. 3, 1967, pp. 882-884.
5. Schetz, J.A., Weinraub, R.A., and Mahaffety Jr., R.E., "Supersonic Transverse Injection into a Supersonic Stream." *AIAA Journal*, Vol. 6, No. 3, 1968, pp. 933-934.
6. Billig, F.S., Orth, R.C., and Lasky, M., "A Unified Analysis of Gaseous Jet Penetration." *AIAA Journal*, Vol. 9, No. 6, 1971, pp. 1048-1058.
7. Fric, T.F., and Roshko, A., "Vortical Structure in the wake of a Transverse Jet." *Journal of Fluid Mechanics*, Vol. 279, 1994, pp. 1-47.
8. Everett, D.E., and Dutton, J.C., "Pressure-Sensitive Paint Measurements of the Pressure Field about a Sonic Jet Injected Transversely into a Mach 1.6 Freestream." AIAA Paper 95-0524. 1995.
9. Everett, D.E., Woodmansee, M.A., Dutton, J.C., and Morris, M.J. "Wall Pressure Measurements for a Sonic Jet Injected into a Supersonic Crossflow." *Journal of Propulsion and Power*, Vol. 14, No. 6, 1998, pp. 861-868.
10. Gruber, M.R., Nejad, A.S., Chen, T.H., and Dutton, J.C., "Compressibility Effects in Supersonic Transverse Injection Flowfields." *Physics of Fluids*, Vol. 9, No. 5, 1997, pp. 1448-1461.
11. Santiago, J.G., and Dutton, J.C., "Velocity Measurements of a Jet Injected into a Supersonic Crossflow." *Journal of Propulsion and Power*, Vol. 13, No. 2, 1997, pp. 264-273.
12. Ben-Yakar, A., Mungal, M.G., and Hanson, R.K. "Time Evolution and Mixing Characteristics of Hydrogen and Ethylene Transverse Jets in Supersonic Crossflows." *Physics of Fluids*, Vol. 18, No. 2, 2006.
13. Gruber, M.R., Chen, T.H., and Nejad, A.S., "Mixing and Penetration Studies of Sonic Jets in a Mach 2 Freestream." *Journal of Propulsion and Power*, Vol. 11, No. 2, 1995, pp. 315-323.
14. McDaniel, J.C., and Graves, J. "Laser Induced Fluorescence Visualization of Transverse Gaseous Injection in a Nonreacting Supersonic Combustor." *Journal of Propulsion and Power*, Vol. 4, No. 6, 1988, pp. 591-597.
15. Rothstein, A.D., and Wantuck, P.D., "A Study of Normal Injection of Hydrogen into a Heated Supersonic Flow Using Planar Laser-Induced Fluorescence." AIAA Paper 1992-3423, 2003.
16. Anderson, J.D., Jr. Modern Compressible Flow with Historical Perspective, 2nd Edition. McGraw-Hill, Inc. Boston, MA. 1990.
17. Gruber, M.R., Nejad, A.S., Chen, T.H., and Dutton, J.C., "Bow Shock/Jet Interaction in Compressible Transverse Injection Flowfields." *AIAA Journal*, Vol. 34, No. 10, 1996, pp. 2191-2193.
18. Gruber, M.R., Nejad, A.S., Chen, T.H., and Dutton, J.C., "Large Structure Convection Velocity Measurements in Compressible Transverse Injection Flowfields." *Experiments in Fluids*, Vol. 22, 1997, pp. 397-407.
19. Barber, M.J., Schetz, J.A., and Roe, L.A., "Normal, Sonic, Helium Injection Through a Wedge-Shaped Orifice into Supersonic Flow." *Journal of Propulsion and Power*, Vol. 13, No. 2, 1997, 257-263.
20. Tomioka, S., Jacobsen, L.S., and Schetz, J.A., "Sonic Injection from Diamond-Shaped Orifices into a Supersonic Crossflow." *Journal of Propulsion and Power*, Vol. 19, No. 1, 2003, pp. 104-114.
21. Hollo, S.D., Hartfield Jr., R.J., and McDaniel, J.C., "Injectant Mole Fraction Measurements of Transverse Injection in Constant Area Supersonic Ducts." AIAA Paper 90-1632, 1990.
22. Hollo, S.D., McDaniel, J.C., and Hartfield, R.J., Jr. "Quantitative Investigation of Compressible Mixing: Staged Transverse Injection into Mach 2 Flow." *AIAA Journal*. Vol. 32, No. 3, 1994, pp. 528-534.

23. Eklund, D.R., Northam, G.B., Dancey, C.L., and Wang, J.A., "Computational/Experimental Investigation of Staged Injection into a Mach 2 Flow." *AIAA Journal*, Vol. 32, No. 5, 1994, pp. 907-916.
24. Eklund, D.R., and Gruber, M.R., "Study of a Supersonic Combustor Employing and Aerodynamic Ramp Pilot Injection." AIAA Paper 99-2249, 1999.
25. Donbar, J., Powell, O., Gruber, M., Jackson, T., Eklund, D., and Mathur, T. "Post-test Analysis of Flush-Wall Fuel Injection Experiment in a Scramjet Combustor." AIAA-2001-3197, 2001.
26. Briedenthal, R.E., Kato, S.M., and Groenewegen, B.C. "On Turbulent Mixing in Nonsteady Jets." AIAA Paper 1986-42, 1986.
27. Clement, P., and Rodriguez, C. "Shock Wave Structure and Penetration Height in Transverse Jets." *AIAA Student Journal*. Vol. 27, No. 2, 1989, pp. 7-16.
28. Eroglu, A., and Briedenthal, R.E. "Exponentially Accelerating Jet in Crossflow." *AIAA Journal*. Vol. 36, No. 6, 1998, pp. 1002-1009.
29. Mathur, T., Cox-Stouffer, S., Hsu, K.-Y., Crafton, J., Donbar, J., and Gruber, M., "Experimental Assessment of a Fuel Injector for Scramjet Applications," AIAA Paper 2000-3703, 2000.
30. Cox-Stouffer, S.K., Gruber, M.R., and Bulman, M.J., "A Streamlined, Pressure-Matched Fuel Injector for Scramjet Applications," AIAA Paper 2000-3707, 2000.
31. White, J.A., and Morrison, J.H. "A Pseudo-Temporal Multi-Grid Relaxation Scheme for Solving the Parabolized Navier-Stokes Equations." AIAA Paper 99-3360, 1999.
32. Gruber, M.R., and Nejad, A.S., "New Supersonic Combustion Research Facility," *Journal of Propulsion and Power*, Vol. 11, No. 5, 1995, pp. 1080-1083.
33. Pressure Systems Incorporated, "System 8400 Users Manual, Version 3.0," May, 1994.
34. M. Bulman, Aerojet General Corporation, Private communication.
35. Rogers, R.C. "Mixing of Hydrogen Injected from Multiple Injectors Normal to a Supersonic Airstream." NASA TND-6476, Sept. 1971.
36. Hypersonic Air-Breathing Propulsion Branch, NASA Langley Research Center. VULCAN homepage. <http://vulcan-cfd.larc.nasa.gov/>
37. S.K. Cox-Stouffer, Taitech Inc., and R.A. Baurle, NASA, Private communication.
38. Wilcox, D.C. Turbulence Modeling for CFD, Second Edition. DCW Industries, Inc., 2004.
39. Baurle, R.A., Tam, C.-J., and Dasgupta, S. "Analysis of Unsteady Cavity Flows for Scramjet Applications." AIAA Paper 2000-3617, 2000.
40. Menter, F.R. "Zonal Two Equation k-w Models for Aerodynamic Flows." AIAA Paper 93-2906, 1993.
41. Menter, F.R. "Improved Two-Equation k-w Turbulence Models for Aerodynamic Flows." NASA TM-103975, Oct. 1992.
42. Jones, W.P., and Launder, B.E. "The Calculation of Low-Reynolds Number Phenomena with a Two-Equation Model of Turbulence." *International Journal of Heat and Mass Transfer*, Vol. 16, 1973, pp.1119-1130.
43. Gatski, T.B., and Speziale, C.G. "On Algebraic Stress Models for Complex Turbulent Flows." *Journal of Fluid Mechanics*, Vol. 254, 1993, pp. 59-78.
44. Pointwise, Inc. Gridgen Overview. <http://www.pointwise.com/gridgen/>
45. Settersten, T.B., Patterson, B.D., and Gray, J.A.. "Temperature- and species-dependent quenching of NO $A^2\Sigma^+$ ($v' = 0$) probed by two-photon laser-induced fluorescence using a picosecond laser", *Journal of Chemical Physics*, Vol. 124, 2006, pp. 234308-1–234308-14.

7.0 Tables

Table 1: Experimental Test Matrix

Case #	Active Stages	Nominal Injection Pressure (psia)
1	1,2,3,4	144, 250
2	1,2,3	144, 250
3	1,2,4	144, 250
4	1,3,4	144, 250
5	2,3,4	144, 250
6	1,2	144
7	1,3	144
8	1,4	144
9	2,3	144
10	2,4	144
11	3,4	144

Table 2: Side-view penetration boundaries.

Case #	Side-View Penetration (Upper)			Side-View Penetration (Lower)		
	x=1.0	x=1.5	x=2.0	x=1.0	x=1.5	x=2.0
1	0.86	1.0	1.0	0.010	0.15	0.19
2	0.77	0.83	0.85	0.087	0.14	0.17
3	0.74	0.91	0.95	0.0	0.096	0.14
4	0.79	0.94	0.98	0.0	0.13	0.16
5	0.73	0.87	0.89	0.008	0.14	0.16
1 hp	1.1	1.3	1.4	0.010	0.20	0.34
2 hp	0.96	1.0	1.1	0.13	0.17	0.25
3 hp	0.94	1.1	1.2	0.0	0.15	0.29
4 hp	0.98	1.2	1.3	0.0	0.16	0.24
5 hp	0.84	0.99	1.0	0.010	0.24	0.29
6	0.62	0.64	0.66	0.057	0.078	0.095
7	0.66	0.73	0.74	0.058	0.11	0.14
8	0.60	0.77	0.78	0.0	0.077	0.13
9	0.61	0.64	0.66	0.078	0.12	0.14
10	0.61	0.75	0.79	0.007	0.095	0.096
11	0.57	0.67	0.67	0.036	0.087	0.12

Table 3: End-view penetration boundaries.

Case #	End-View Penetration (on centerline)					
	x=1.0	x=1.5	x=2.0	x=2.5	x=3.0	x=3.5
1	0.81	1.0	1.0	1.0	1.0	1.0
2	0.75	0.82	0.82	0.85	0.84	0.86
3	0.67	0.86	0.90	0.91	0.92	0.93
4	0.72	0.90	0.92	0.94	0.94	0.96
5	0.83	0.83	0.85	0.85	0.88	0.89
1 hp	1.0	1.3	1.4	1.4	1.4	1.4
2 hp	0.92	1.1	1.1	1.1	1.1	1.1
3 hp	0.87	1.1	1.2	1.2	1.2	1.2
4 hp	0.92	1.2	1.3	1.3	1.3	1.3
5 hp	0.79	1.0	1.1	1.1	1.1	1.1
6	0.61	0.64	0.64	0.66	0.67	0.69
7	0.64	0.70	0.71	0.70	0.73	0.73
8	0.54	0.74	0.77	0.78	0.79	0.80
9	0.58	0.63	0.62	0.64	0.66	0.68
10	0.55	0.71	0.75	0.74	0.74	0.78
11	0.53	0.67	0.67	0.68	0.71	0.75

Table 4: Percent difference between side-view and end-view.

Case #	% Difference at x=2.0
1	1.1
2	2.9
3	4.9
4	5.8
5	4.6
1 hp	1.9
2 hp	-0.050
3 hp	-3.6
4 hp	0.62
5 hp	-3.6
6	3.5
7	4.2
8	0.88
9	6.1
10	4.7
11	1.1

Table 5: Maximum plume width.

Case #	x=1.0	x=1.5	x=2.0	x=2.5	x=3.0	x=3.5
1	0.43	0.61	0.61	0.68	0.73	0.79
2	0.40	0.50	0.58	0.63	0.73	0.74
3	0.38	0.54	0.61	0.64	0.73	0.76
4	0.41	0.55	0.59	0.63	0.69	0.70
5	0.40	0.50	0.51	0.55	0.59	0.64
1 hp	0.43	0.63	0.71	0.78	0.79	0.86
2 hp	0.38	0.55	0.59	0.64	0.69	0.76
3 hp	0.40	0.55	0.61	0.69	0.73	0.79
4 hp	0.41	0.59	0.65	0.68	0.73	0.78
5 hp	0.46	0.66	0.71	0.71	0.75	0.79
6	0.36	0.45	0.53	0.59	0.59	0.65
7	0.39	0.50	0.55	0.63	0.66	0.71
8	0.35	0.50	0.54	0.60	0.65	0.69
9	0.36	0.44	0.49	0.53	0.54	0.59
10	0.36	0.46	0.51	0.56	0.59	0.63
11	0.41	0.56	0.58	0.59	0.63	0.64

Table 6: Location of Max Plume Width in % of Max Plume Height.

Case #	x=1.0	x=1.5	x=2.0	x=2.5	x=3.0	x=3.5
1	52	61	60	65	70	75
2	53	61	70	73	87	85
3	56	62	68	70	79	82
4	57	61	64	66	73	73
5	48	60	60	64	67	72
1 hp	41	48	51	55	55	60
2 hp	41	51	54	57	61	69
3 hp	46	48	50	56	59	64
4 hp	45	50	52	53	57	61
5 hp	58	66	68	66	71	74
6	59	70	82	89	88	95
7	61	71	78	89	90	98
8	65	67	70	77	83	86
9	62	69	78	82	82	87
10	66	65	68	76	79	81
11	78	84	87	86	89	85

Table 7: Influence Coefficients.

Influence Coefficient	Active Stages	Effect	Side-View Penetration (Upper) at x=2.0 in	End-View Penetration at x=2.0 in	Maximum Plume Width at x = 2.0 in
1:2	1,2,3	stage 4	0.40	0.43	0.46
1:3	1,2,4	stage 3	0.30	0.42	0.47
1:4	1,3,4	stage 2	0.42	0.75	1.3
1:5	2,3,4	stage 1	1.1	1.4	2.1
1:2 hp	1,2,3	stage 4	0.50	0.47	0.48
1:3 hp	1,2,4	stage 3	0.60	0.44	0.31
1:4 hp	1,3,4	stage 2	0.80	0.71	0.81
1:5 hp	2,3,4	stage 1	2.3	2.0	2.1
1:6	1,2	stage 3,4	0.49	0.51	0.47
1:7	1,3	stage 2,4	0.48	0.52	0.61
1:8	1,4	stage 2,3	0.60	0.60	0.87
1:9	2,3	stage 1,4	0.62	0.67	0.73
1:10	2,4	stage 1,3	0.60	0.67	0.85
1:11	3,4	stage 1,2	1.4	1.4	2.0

Table 8: Effective throat diameter values.

Case #	D_j* (in.)	d_{eff} (in.)
1	0.047	0.076
2	0.038	0.055
3	0.040	0.066
4	0.041	0.071
5	0.043	0.071
6	0.029	0.039
7	0.031	0.047
8	0.033	0.060
9	0.033	0.047
10	0.035	0.060
11	0.037	0.065

Table 9: Percent increase of cascade injector from Gruber et al. correlation.

$x/(d_{eff} \cdot J)$	0.33	0.66	1	1.33	1.66	2	2.33	2.66
Case 1	4.1	4.2	4.6	4.8	5.0	5.2	5.3	5.6
Case 2	5.4	5.4	5.2	5.6	5.7	5.9	6.3	6.4
Case 3	5.1	4.7	5.0	5.3	5.4	5.8	6.0	6.1
Case 4	4.8	4.4	4.8	5.1	5.2	5.4	5.5	5.8
Case 5	5.0	5.4	5.4	5.6	6.0	6.3	6.4	6.8
Case 1 hp	3.6	3.9	4.1	4.3	4.7	4.7	4.9	5.1
Case 2 hp	3.9	4.6	4.9	4.9	5.0	5.1	5.3	5.4
Case 3 hp	3.6	4.0	4.3	4.4	4.5	4.7	4.9	5.1
Case 4 hp	3.5	3.9	4.1	4.2	4.4	4.6	4.7	4.9
Case 5 hp	4.0	4.0	4.2	4.6	5.0	5.0	5.2	5.4
Case 6	6.2	5.6	5.7	6.0	6.0	6.1	6.6	6.7
Case 7	5.3	5.4	5.3	5.8	5.8	5.9	6.0	6.3
Case 8	5.0	4.6	4.9	5.1	5.4	5.5	5.7	6.0
Case 9	6.5	5.1	5.9	6.0	6.3	6.5	6.9	7.1
Case 10	4.7	4.4	4.7	5.1	5.6	5.9	6.1	6.3
Case 11	4.4	4.1	4.7	4.9	5.3	5.7	6.1	6.2

Table 10: Experimental and computational data comparison (in inches).

Low Injection Pressure								
	Side-view upper penetration at $x = 2.2$ in.		Side-view lower penetration at $x = 2.2$ in.		End-view penetration at $x = 3.5$ in.		Plume Width at $x = 3.5$ in.	
Case #	Experimental	CFD	Experimental	CFD	Experimental	CFD	Experimental	CFD
1	1.0	1.1	0.19	0.30	1.1	1.2	0.79	0.43
2	0.85	1.0	0.19	0.31	0.86	1.1	0.74	0.38
5	0.89	0.95	0.15	0.28	0.89	1.0	0.64	0.38
High Injection Pressure								
	Side-view upper penetration at $x = 2.2$ in.		Side-view lower penetration at $x = 2.2$ in.		End-view penetration at $x = 3.5$ in.		Plume Width at $x = 3.5$ in.	
Case #	Experimental	CFD	Experimental	CFD	Experimental	CFD	Experimental	CFD
1	1.4	1.4	0.25	0.29	1.5	1.6	0.65	0.46
2	1.1	1.3	0.27	0.39	1.1	1.4	0.71	0.40
5	1.0	1.1	0.28	0.35	1.0	1.2	0.64	0.44

Table 11: Experimental data and turbulence model comparison (in inches).

	Side-view upper penetration at x = 2.2 in.	Side-view lower penetration at x = 2.2 in.	End-view Penetration at x = 3.5 in.	Plume Width at x = 3.5 in.
Experimental Data	1.0	0.19	1.1	0.79
Wilcox k-w Model	1.1	0.30	1.3	0.42
G.S. Alg. Reynolds Stress Model	1.2	0.29	1.2	0.44
Menter Model	0.97	0.23	1.1	0.50

8.0 Figures

Figure 1: Side-view structure of underexpanded, normal injection into a supersonic crossflow from a single-hole injector.¹³

Figure 2: Side-view structure of overexpanded, normal injection from a single-hole injector.

Figure 3: Cascade injector block.

Figure 4: Diagram of fuel manifold with stagnation pressure and temperature measurement locations.

Figure 5: Picture of RC19 facility.

Figure 6: RC19 facility schematic.

Figure 7: NO-PLIF laser components.

Figure 8: End-view NO-PLIF experimental setup.

Figure 9: RC19 optics layout.

Figure 10: Shadowgraph image of the baseline case (Case 1, $P_{inj} = 144$ psia).

Figure 11: Schlieren image of baseline case (Case 1, $P_{inj} = 144$ psia).

Figure 12: Instantaneous side-view NO-PLIF image (Case 1, $P_{inj} = 144$ psia).

Figure 13: Side-view standard deviation image (Case 1, $P_{inj} = 144$ psia).

Figure 14: Ensemble-averaged side-view NO-PLIF image (Case 1, $P_{inj} = 144$ psia).

Figure 15: Mie scattering image (Case 1, $P_{inj} = 144$ psia).

Figure 16: Axial development of instantaneous end-view NO-PLIF images (Case 1, $P_{inj} = 144$ psia).

Figure 17: Axial development of end-view standard deviation images (Case 1, $P_{inj} = 144$ psia).

Figure 18: Axial development of ensemble-averaged end-view NO-PLIF images (Case 1, $P_{inj} = 144$ psia).

Figure 19: Instantaneous side-view NO-PLIF image (Case 1, $P_{inj} = 250$ psia).

Figure 20: Ensemble-averaged side-view NO-PLIF image (Case 1, $P_{inj} = 250$ psia).

Figure 21: Axial development of end-view NO-PLIF images (Case 1, $P_{inj} = 250$ psia).

Figure 22: Axial development of end-view ensemble averaged images (Case 1, $P_{inj} = 250$ psia).

(a) Side-view

(b) End-view, $x = 1.0$ in.

(c) End-view, $x = 2.0$ in.

(d) End-view, $x = 3.0$ in.

(e) End-view, $x = 3.5$ in.

Figure 23: Penetration Profiles (Case 1, $P_{inj} = 144$ psia).

(a) Side-view

(b) End-view, $x = 1.0$ in.

(c) End-view, $x = 2.0$ in.

(d) End-view, $x = 3.0$ in.

(e) End-view, $x = 3.5$ in.

Figure 24: Penetration Profiles (Case 1, $P_{inj} = 250$ psia).

Figure 25: Comparison of cases with 3 active injection stages ($P_{inj} = 144$ psia).

(a) Side-view

(b) End-view, $x = 1.0$ in.

(c) End-view, $x = 2.0$ in.

(d) End-view, $x = 3.0$ in.

(e) End-view, $x = 3.5$ in.

Figure 26: Comparison of cases with 3 active injection stages ($P_{inj} = 250$ psia).

(a) Side-view

(b) End-view, $x = 1.0$ in.

(c) End-view, $x = 2.0$ in.

(d) End-view, $x = 3.0$ in.

(e) End-view, $x = 3.5$ in.

Figure 27: Comparison of cases with 2 active injection stages ($P_{inj} = 144$ psia).

(a) Side-view

(b) End-view, $x = 1.0$ in.

(c) End-view, $x = 2.0$ in.

(d) End-view, $x = 3.0$ in.

(e) End-view, $x = 3.5$ in.

Figure 28: Nondimensionalized data for cases with 3 active injection stages ($P_{inj} = 144$ psia).

(a) Side-view

(b) End-view, $x = 1.0$ in.

(c) End-view, $x = 2.0$ in.

(d) End-view, $x = 3.0$ in.

(e) End-view, $x = 3.5$ in.

Figure 29: Nondimensionalized data for cases with 3 active injection stages ($P_{inj} = 250$ psia).

(a) Side-view

(b) End-view, $x = 1.0$ in.

(c) End-view, $x = 2.0$ in.

(d) End-view, $x = 3.0$ in.

(e) End-view, $x = 3.5$ in.

Figure 30: Nondimensionalized data for cases with 2 active injection stages ($P_{inj} = 144$ psia)

(a) Cases with 3 active injection stages ($P_{inj} = 144$ psia).

(b) Cases with 3 active injection stages ($P_{inj} = 250$ psia).

(c) Cases with 2 active injection stages ($P_{inj} = 144$ psia).

Figure 31: Power law correlation comparison between cascade injector and Gruber et al.¹³

(a) Side-view.

(b) End-view, $x = 3.5$ in.

Figure 32: Grid system for region surrounding cascade injector.

Figure 33: Residual for Case 1, $P_{inj} = 144$ psia.

(a) Side-view.

(b) End-view.

Figure 34: Comparison of CFD and experimental data for Case 1, $P_{inj} = 144$ psia.

(a) $P_{inj} = 144$ psia

(b) $P_{inj} = 250$ psia

Figure 35: Static pressure (Pa) contours with penetration boundary for CFD (Case 2).

(a) $P_{inj} = 144$ psia

(b) $P_{inj} = 250$ psia

Figure 36: Static pressure (Pa) contours with penetration boundary for CFD (Case 5).

(a) Side-view.

(b) End-view. $x = 3.5$ inches.

Figure 37: First stage pressure increase.

(a) $P_{inj} = 144$ psia.

(b) $P_{inj} = 197$ psia.

(c) $P_{inj} = 250$ psia.

Figure 38: Static pressure contours (Pa) with penetration boundary for first stage pressure increase.

Figure 39: Static pressure contours (Pa) with streamlines and penetration boundary for Case 1, $P_{inj} = 144$ psia.

(a) Side-view.

(b) End-view, $x = 3.5$ inches.

Figure 40: Turbulence model comparison.

(a) Side-view.

(b) End-view. $x = 3.5 \text{ in.}$

Figure 41: Gatski-Speziale algebraic Reynolds stress model ratio of eddy to molecular viscosity.

(a) Side-view.

(b) End-view. $x = 3.5$ inches.

Figure 42: Wilcox $k-\omega$ model ratio of eddy to molecular viscosity.

(a) Side-view.

(b) End-view, $x = 3.5$ inches.

Figure 43: Menter model ratio of eddy to molecular viscosity.

Figure 44: End-view mass fraction contours with velocity vector field and 10% penetration boundary.

Figure 45: One-dimensional, mass-averaged streamwise vorticity.

(a) On centerline ($z = 0.0$ in.)

(b) At $z = 0.5$ in.

Figure 46: Boundary layer profiles at $x = -2.0$ inches.

(a) Static Pressure Contours from CFD (Pa).

(b) Static Temperature contours from CFD (K).

(c) Injectant Mass Fraction contours from CFD.

Figure 47: Side-view contours with synthetic LIF signal penetration boundaries.

(a) Static Pressure Contours from CFD (Pa) at $x = 3.5$ inches.

(b) Static Temperature Contours from CFD (K) at $x = 3.5$ inches.

(c) Injectant Mass Fraction Contours from CFD at $x = 3.5$ inches.

Figure 48: End-view contours with synthetic LIF signal penetration boundaries.

(a) Side-view.

(b) End-view, $x = 3.5$ inches.

Figure 49: Comparison of penetration boundaries.

(a) Experimental NO-PLIF signal.

(b) Synthetic LIF signal.

Figure 50: Side-view experimental and synthetic LIF images.

(a) Experimental NO-PLIF signal at $x = 3.5$ inches.

(b) Synthetic LIF signal at $x = 3.5$ inches.

Figure 51: End-view experimental and synthetic LIF images.

(a) Fine grid resolution.

(d) Medium grid resolution.

Figure 52: Grid resolution comparison of static pressure contours (Pa) with penetration boundaries.

(a) Side-view, fine grid resolution.

(e) Side-view, medium grid resolution.

Figure 53: Side-view mass fraction contours for fine and medium grid resolution with penetration boundaries.

(a) End-view, fine grid resolution.

(b) End-view, medium grid resolution.

Figure 54: End-view mass fraction contours for fine and medium grid resolution with penetration boundaries.

(a) Side-view.

(b) End view.

Figure 55: Fine and medium grid resolution penetration boundaries with experimental data.

(a) Side-view.

(b) End-view.

Figure 56: Turbulence model study with fine grid penetration boundary.

Appendix A: Analysis Codes

```
% Program: runavg4med2.m
% Created: 6 April, 2006 (Heidi Meicenheimer)
% Modified: 9 April, 2006 (Heidi Meicenheimer)
% 1st Part of Script:
% Description: Reads in sections of data and user sets limits to define
% a region of interest in which the desired max value occurs. Plots are
% used to help set and verify limits. Identify local maximum value.
%-----
clear;
close all;
clc;

%Specify file name:
filename='c1.dat'
%Load data file:
data = dlmread(filename, ',');
%Set number of rows and columns to the size of the data file:
[numrow_data numcoln_data]=size(data);

%Specify number of data columns (x axis) to read at a time:
numcoln_readin=25;
j=1;
numcoln_left=numcoln_data;

%Output file for user input points:
mfid=fopen(['points_',filename, '.dat'], 'w+');
fprintf(mfid, '%s\n', 'LBD1 UBD1 ');
fprintf(mfid, '%s\n', '-----');

%Loop to define region of interest and plot data for set:
while numcoln_left>0;
clear start_index stop_index go_part1 small_data LBD1 UB1 axis_store;

start_index=(j-1)*numcoln_readin+1;
stop_index=numcoln_readin*j;
if stop_index>=numcoln_data;stop_index=numcoln_data;end;

go_part1=0;
figure(1);
while go_part1~=1;
close(1);
figure(1);
disp(['I am plotting the intensity columns from
',num2str(start_index), ' to ',num2str(stop_index), '.']);
small_data=data(:, [start_index:1:stop_index]);
plot(small_data, '.'); % Plots all Data
title(['Intensity columns from ',num2str(start_index), 'to ',num2str(stop_index), '.']);
LBD1=input('Lower Boundary (>0) = ');
UBD1=input('Upper Boundary = ');
hold on;line([LBD1 LBD1], [0 max(max(data))]);
hold on;line([UBD1 UBD1], [0 max(max(data))]);
go_part1=input('(Enter 1 to continue) or (Enter 0 to re-enter limits) ');
disp(' ');
axis_store=axis;
```

```

if go_part1==1;
%Write LBD1 and UBD1 data:
fprintf(mfid, '%7.1f\t%7.1f\n', LBD1, UBD1);
end;
end;

%Loop to determine lcoal maximum value:
if j==1;max_data_reduced=max(small_data);
elseif j~=1;max_data_reduced=[max_data_reduced max(small_data)];
end

j=j+1;
numcoln_left=(numcoln_data-stop_index);
end;

% 2nd Part of script:
% Description: Determines the location of 10% of the average local
% maximum for the upper boundary and the location of 25% of the average
% local maximum for the lower boundary. Data is written to a file in
% format that can be read into Tecplot. Contour is plotted to make sure
% data file isn't junk.
%-----
clear avg_max_data_reduced mf_data
%Set increment:
increment=1;
%Set number of points to average:
num2avgpts=4;

numleft=numcoln_data;

%Determine average local maximum:
i=1;
while numleft > num2avgpts;
avg_max_data_reduced(i)=mean(max_data_reduced([i:1:(i+num2avgpts)]));
numleft=(numcoln_data-i);
i=i+1;
end;
avg_max_data_reduced=[avg_max_data_reduced
ones(1,numleft)*avg_max_data_reduced(i-1)];

%Determine location of 10% and 25% of average local maximum:
figure(10);
for k=1:1:numcoln_data;
hold on;
mf_data(:,k)=data(:,k)./avg_max_data_reduced(k);
end;

for k=1:1:numcoln_data;
plot([1:1:numrow_data],mf_data(:,k),'.')
hold
on;plot(find(mf_data(:,k)>=.1),mf_data(find(mf_data(:,k)>=.1),k),'.r');
hold on;plot(find(mf_data(:,k)>=.25),mf_data(find(mf_data(:,k)>=.25),k),'.k');
% To plot individual points change k to correct index
if
isempty(min(find(mf_data(:,k)>=.1)))==1;min_row_index(k)=numrow_data;
else;min_row_index(k)=min(find(mf_data(:,k)>=.1))-1;end;

```

```

if
isempty(max(find(mf_data(:,k)>=.25)))==1;max_row_index(k)=numrow_data;
else;max_row_index(k)=max(find(mf_data(:,k)>=.25))+1;end;
axis([axis_store([1,2]) [0 1]]);
end;

%Output file for 10% boundary:
bfid=fopen(['bound10_',filename, '.dat'], 'w+');
for ii=1:1:numcoln_data;
    fprintf(bfid, '%7.12f\t%7.12f\n', ii, min_row_index(ii));
end

%Output file for 25% boundary:
dfid=fopen(['bound25_',filename, '.dat'], 'w+');
for ii=1:1:numcoln_data;
    fprintf(dfid, '%7.12f\t%7.12f\n', ii, max_row_index(ii));
end

%Plot of 10% and 25% boundaries:
figure(11);
plot([1:1:numcoln_data], [min_row_index], [1:1:numcoln_data], [max_row_index]);

%Output file for contour:
fid=fopen(['out_',filename, '.dat'], 'w+');
%Tecplot compatible header:
fprintf(fid, '%s\n', 'TITLE="exper_data"');
fprintf(fid, '%s\n', 'VARIABLES="X"');
fprintf(fid, '%s\n', 'Y');
fprintf(fid, '%s\n', 'INTENSITY');
fprintf(fid, '%s\n', 'ZONE I=210, J=250');
fprintf(fid, '%s\n', 'ZONETYPE=ORDERED');
fprintf(fid, '%s\n', 'DATAPACKING=POINT');
fprintf(fid, '%s\n', 'DT=(SINGLE SINGLE SINGLE )');
for ii=1:1:numcoln_data;
for jj=1:1:numrow_data

%Stores plot data
x(jj,ii)=ii;
y(jj,ii)=jj;
value(jj,ii)=mf_data(jj,ii);

%Writes data
fprintf(fid, '%7.12f\t%7.12f\t%7.12f\n', ii, jj, mf_data(jj,ii));
end
end
fclose(fid);
fclose(mfid);
fclose(bfid);
fclose(dfid);

%Plots data
figure(100);
surf(x,y,value);
view(2);
shading interp;
caxis([0 0.5]);
colorbar;

```

```

% Program:  runavg4med2_CFD.m
% Created:  9 April, 2006 (Heidi Meicenheimer)
% Modified: 12 April, 2006 (Heidi Meicenheimer)
% Modified: 18 April, 2006 (Heidi Meicenheimer)
% 1st Part of Script:
% Description: CFD data files are read in and combined into one data
% file using function files. User sets limits to define a region of
% interest in which the desired max value occurs. A plot is used to
% help set and verify limits. Local maximum values are identified.
% NOTE: The following function files must be in the same directory as
% the code:
%       data_to_rows_X.m
%       data_to_rows_Y.m
%       data_to_rows_data.m
%-----
clear;
close all;
clc;

%Set name of output file identifier:
filename='c1p144a'
%Name of data file for zone 3:
filename1='c1b1_zone3.dat'
%Name of data file for zone 4:
filename2='c1b1_zone4.dat'
%Name of data file for zone 5:
filename3='c1b1_zone5.dat'

%Call functions and combine CFD data into 1 matrix:
xdata=[data_to_rows_x(filename1) data_to_rows_x(filename2)
data_to_rows_x(filename3)];
ydata=[data_to_rows_y(filename1) data_to_rows_y(filename2)
data_to_rows_y(filename3)];
data=[data_to_rows_data(filename1) data_to_rows_data(filename2)
data_to_rows_data(filename3)];

%Define region of interest, identify local max:
%-----
clear totszi j i delta k coord_matrix

%Set number of rows and columns to the size of the data file:
[numrow_data numcoln_data]=size(data);

%Read in all data columns (x axis) at one time:
numcoln_readin=numcoln_data;
j=1;
numcoln_left=numcoln_data;

%Loop to define region of interest and plot data for set:
while numcoln_left>0;
clear start_index stop_index go_part1 small_data LBD1 UB1 axis_store;

start_index=(j-1)*numcoln_readin+1;
stop_index=numcoln_readin*j;
if stop_index>=numcoln_data;stop_index=numcoln_data;end;

```



```

go_part1=0;
figure(1);
while go_part1~=1;
close(1);
figure(1);
disp(['I am plotting the intensity columns from
',num2str(start_index), ' to ',num2str(stop_index),'.']);
small_data=data(:,[start_index:1:stop_index]);
plot(small_data,'.-'); % Plots all Data
title(['I am plotting the intensity columns from ',num2str(start_index) , ...
' to ',num2str(stop_index),'.']);
LBD1=input('Lower Boundary (>0) = ');
UBD1=input('Upper Boundary = ');
hold on;line([LBD1 LBD1],[0 max(max(data))]);
hold on;line([UBD1 UBD1],[0 max(max(data))]);
go_part1=input(' (Enter 1 to continue) or (Enter 0 to re-enter limits)
');
disp(' ');
axis_store=axis;
end;

%Loop to determine local maximum value:
if j==1;max_data_reduced=max(small_data);
elseif j~=1;max_data_reduced=[max_data_reduced max(small_data)];
end

j=j+1;
numcoln_left=(numcoln_data-stop_index);
end;

% 2nd Part of script:
% Description: Determines the location of 10% of the average local
% maximum for the upper boundary and the location of 25% of the average
% local maximum for the lower boundary. Data is written to a file in
% format that can be read into Tecplot. Contour is plotted to make sure
% data file isn't junk.
%-----
clear avg_max_data_reduced mf_data
%Set increment:
increment=1;
%Set number of points to average:
num2avgpts=4;

numleft=numcoln_data;

%Determine average local maximum:
i=1;
while numleft > num2avgpts;
avg_max_data_reduced(i)=mean(max_data_reduced([i:1:(i+num2avgpts)]));
numleft=(numcoln_data-i);
i=i+1;
end;
avg_max_data_reduced=[avg_max_data_reduced
ones(1,numleft)*avg_max_data_reduced(i-1)];

%Determine location of 10% and 25% of average local maximum:
figure(10);

```

```

for k=1:1:numcoln_data;
hold on;
mf_data(:,k)=data(:,k)./avg_max_data_reduced(k);
plot([1:1:numrow_data],mf_data(:,k),'.r')
hold
on;plot(find(mf_data(:,k)>=.1),mf_data(find(mf_data(:,k)>=.1),k),'.r');
hold on;plot(find(mf_data(:,k)>=.25),mf_data(find(mf_data(:,k)>=.25),k),'.k');
% To plot individual points change k to correct index

%INTERPOLATION:
%-----
%Loop for interpolation of 10% upper boundary:
% This if is for when the find command does find any values that are
>=0.1:
if isempty(max(find(mf_data(:,k)>=.1)))==1;
    ub_row_index(k)=numrow_data;
    ub_mf_values(k)=mf_data(ub_row_index(k),k);
    ub_values_x(k)=xdata(ub_row_index(k),k);
    ub_values_y(k)=ydata(ub_row_index(k),k);
else;
    ub_row_index(k)=max(find(mf_data(:,k)>=.1));
% This if statement is for when the index is at the end of the matrix:
    if ub_row_index(k)==numrow_data;
        ub_values_x(k)=xdata(ub_row_index(k),k);
        ub_values_y(k)=ydata(ub_row_index(k),k);
%This if statement is for when two values next to each other are the same:
    elseif mf_data(ub_row_index(k)+1,k)==mf_data(ub_row_index(k),k)
        ub_values_x(k)=xdata(ub_row_index(k),k);
        ub_values_y(k)=ydata(ub_row_index(k),k);
    % Interpolates values
    else
        ub_values_x(k)=interp1([mf_data(ub_row_index(k)+1,k) mf_data(ub_row_index(k),k)],...
            [xdata(ub_row_index(k)+1,k) xdata(ub_row_index(k),k)],.1);
        ub_values_y(k)=interp1([mf_data(ub_row_index(k)+1,k) mf_data(ub_row_index(k),k)],...
            [ydata(ub_row_index(k)+1,k) ydata(ub_row_index(k),k)],.1);
    end;
end;
% Used to command to get rid of NaN values:
if
isnan(ub_values_x(k))==1;ub_values_x(k)=xdata(ub_row_index(k),k);end;
if
isnan(ub_values_y(k))==1;ub_values_y(k)=ydata(ub_row_index(k),k);end;

%Loop for interpolation of 25% lower boundary:
% This if is for when the find command does find any values that are >=0.25:
if isempty(min(find(mf_data(:,k)>=.25)))==1;
    lb_row_index(k)=numrow_data;
    lb_mf_values(k)=mf_data(lb_row_index(k),k);
    lb_values_x(k)=xdata(lb_row_index(k),k);
    lb_values_y(k)=ydata(lb_row_index(k),k);
else;
    lb_row_index(k)=min(find(mf_data(:,k)>=.25));
%This if statement is when the index is at the beginning of the matrix:
    if lb_row_index(k)==1;
        lb_values_x(k)=xdata(lb_row_index(k),k);
        lb_values_y(k)=ydata(lb_row_index(k),k);
%This if statement is for when two values next to each other are the same:
    elseif mf_data(lb_row_index(k)-1,k)==mf_data(lb_row_index(k),k)

```

```

        lb_values_x(k)=xdata(lb_row_index(k),k);
        lb_values_y(k)=ydata(lb_row_index(k),k);
        %Interpolates values:
        else
            lb_values_x(k)=interp1([mf_data(lb_row_index(k),k)
mf_data(lb_row_index(k)-1,k)],...
            [xdata(lb_row_index(k),k) xdata(lb_row_index(k)-1,k)],.25);
            lb_values_y(k)=interp1([mf_data(lb_row_index(k),k)
mf_data(lb_row_index(k)-1,k)],...
            [ydata(lb_row_index(k),k) ydata(lb_row_index(k)-
1,k)],.25);
        end;
    end;
    % Used to command to get rid of NaN values
    if
        isnan(lb_values_x(k))==1;lb_values_x(k)=xdata(lb_row_index(k),k);end;
    if
        isnan(lb_values_y(k))==1;lb_values_y(k)=ydata(lb_row_index(k),k);end;
    %-----

axis([axis_store([1,2]) [0 1]]);

end;
figure(11);
plot(ub_values_x,ub_values_y,lb_values_x,lb_values_y);

%Output file:
fid=fopen(['out_',filename,'.dat'],'w+');
%Tecplot compatible header:
fprintf(fid,'%s\n','    X=    Y=    Mass Fraction= ');
fprintf(fid,'%s\n','-----');
for ii=1:1:numcoln_data;
for jj=1:1:numrow_data

%Stores plot data
x(jj,ii)=xdata(jj,ii);
y(jj,ii)=ydata(jj,ii);
value(jj,ii)=mf_data(jj,ii);

%Writes data
fprintf(fid,'%7.12f\t%7.12f\t%7.12f\n',xdata(jj,ii),ydata(jj,ii),mf_data(jj,ii));
end
end
fclose(fid);

%Contour plot data:
figure(100);
surf(x,y,value);
view(2);
shading interp;
caxis([0 1.0]);
colorbar;

% Output file for upper boundary
fid=fopen(['out_ub_',filename,'.dat'],'w+');

```

```

fprintf(fid,'%s\n','    X=    Y= ');
fprintf(fid,'%s\n','-----');
for iijj=1:1:numcoln_data;
    fprintf(fid,'%7.12f\t%7.12f\n',ub_values_x(iijj),ub_values_y(iijj));
end
fclose(fid);

% Output file for lower boundary
fid=fopen(['out_lb_',filename,'.dat'],'w+');
fprintf(fid,'%s\n','    X=    Y= ');
fprintf(fid,'%s\n','-----');
for iikk=1:1:numcoln_data;
    fprintf(fid,'%7.12f\t%7.12f\n',lb_values_x(iikk),lb_values_y(iikk));
end
fclose(fid);

```

```

% Program:  runavg4med2_CFD.m
% Created:  9 April, 2006 (Heidi Meicenheimer)
% Modified: 12 April, 2006 (Heidi Meicenheimer)
% Description:  Function to correctly order data.
%-----
function xdata = data_to_rows_x_slif(filename);

%Specify file name:
% do NOT put .dat in the filename for CFD data
%Load data file:
dataCFD = dlmread(filename, ' ');

%Read in CFD data:
%-----
%Determine size of specified matrix:
totszi=length(dataCFD);
j=1;
i=1;
%While loop to look at every value in specified matrix:
while i<=totszi;
    delta=0; %initialize delta
    clear coord_matrix; %clear variables from previous iteration
    k=1;
    %While loop to identify when x value stops increasing
    while delta<=0;
        %if-else loop calculates delta unless at last value of overall
        matrix, then program stops
        if i~=totszi;
            delta=dataCFD(i,1)-dataCFD(i+1,1); %calculated delta is
                                                %(current value-next value)
                                                %i.e. negative mean increasing

        else
            delta=1;
        end
        coord_matrix(k)=i; %defines coordinate matrix, to be used later
        i=i+1;
        k=k+1;
    end;
    %organized x matrix:
    xdata(j,:)=dataCFD(coord_matrix(1:(k-1)),1)';
    %organized y matrix:
    ydata(j,:)=dataCFD(coord_matrix(1:(k-1)),2)';
    %organized mass fraction matrix:
    data(j,:)=dataCFD(coord_matrix(1:(k-1)),5)';
    j=j+1;
end;

```

```

% Program:  runavg4med2_CFD.m
% Created:  9 April, 2006 (Heidi Meicenheimer)
% Modified: 12 April, 2006 (Heidi Meicenheimer)
% Description:  Function to correctly order data.
%-----
function ydata = data_to_rows_y_slif(filename);

%Specify file name:
% do NOT put .dat in the filename for CFD data
%Load data file:
dataCFD = dlmread(filename, ' ');

%Read in CFD data:
%-----
%Determine size of specified matrix:
totszi=length(dataCFD);
j=1;
i=1;
%While loop to look at every value in specified matrix:
while i<=totszi;
    delta=0; %initialize delta
    clear coord_matrix; %clear variables from previous iteration
    k=1;
    %While loop to identify when x value stops increasing
    while delta<=0;
        %if-else loop calculates delta unless at last value of overall
        %matrix, then program stops
        if i~=totszi;
            delta=dataCFD(i,1)-dataCFD(i+1,1); %calculated delta is
                                                %(current value-next value)
                                                %i.e. negative mean increasing

        else
            delta=1;
        end
        coord_matrix(k)=i; %defines coordinate matrix, to be used later
        i=i+1;
        k=k+1;
    end;
    %organized x matrix:
    xdata(j,:)=dataCFD(coord_matrix(1:(k-1)),1)';
    %organized y matrix:
    ydata(j,:)=dataCFD(coord_matrix(1:(k-1)),2)';
    %organized mass fraction matrix:
    data(j,:)=dataCFD(coord_matrix(1:(k-1)),5)';
    j=j+1;
end;

```

```

% Program:  runavg4med2_CFD.m
% Created:  9 April, 2006 (Heidi Meicenheimer)
% Modified: 12 April, 2006 (Heidi Meicenheimer)
% 1st Part of Script:
% Description:  Function to correctly order data.
%-----
function data = data_to_rows_data_s(filename);

%Specify file name:
% do NOT put .dat in the filename for CFD data
%Load data file:
dataCFD = dlmread(filename, ' ');

%Read in CFD data:
%-----
%Determine size of specified matrix:
totszi=length(dataCFD);
j=1;
i=1;
%While loop to look at every value in specified matrix:
while i<=totszi;
    delta=0; %initialize delta
    clear coord_matrix; %clear variables from previous iteration
    k=1;
    %While loop to identify when x value stops increasing
    while delta<=0;
        %if-else loop calculates delta unless at last value of overall
        matrix, then program stops
        if i~=totszi;
            delta=dataCFD(i,1)-dataCFD(i+1,1); %calculated delta is
                                                %(current value-next value)
                                                %i.e. negative mean increasing

        else
            delta=1;
        end
        coord_matrix(k)=i; %defines coordinate matrix, to be used later
        i=i+1;
        k=k+1;
    end;
    %organized x matrix:
    xdata(j,:)=dataCFD(coord_matrix(1:(k-1)),1)';
    %organized y matrix:
    ydata(j,:)=dataCFD(coord_matrix(1:(k-1)),2)';
    %organized mass fraction matrix:
    data(j,:)=dataCFD(coord_matrix(1:(k-1)),5)';
    j=j+1;
end;

```

Appendix B: Shadowgraph and Schlieren Images

Case 1, $P_{inj}=144\text{psia}$

Case 1, $P_{inj}=144\text{psia}$

Case 2, $P_{inj}=144\text{psia}$

Case 2, $P_{inj}=144\text{psia}$

Case 3, $P_{inj}=144\text{psia}$

Case 3, $P_{inj}=144\text{psia}$

Case 4, $P_{inj}=144\text{psia}$

Case 4, $P_{inj}=144\text{psia}$

Case 5, $P_{inj}=144\text{psia}$

Case 5, $P_{inj}=144\text{psia}$

Case 6, $P_{inj}=144\text{psia}$

Case 6, $P_{inj}=144\text{psia}$

Case 7, $P_{inj}=144\text{psia}$

Case 7, $P_{inj}=144\text{psia}$

Case 8, $P_{inj}=144\text{psia}$

Case 8, $P_{inj}=144\text{psia}$

Case 9, $P_{inj}=144\text{psia}$

Case 9, $P_{inj}=144\text{psia}$

Case 10, $P_{inj}=144\text{psia}$

Case 10, $P_{inj}=144\text{psia}$

Case 11, $P_{inj}=144\text{psia}$

Case 11, $P_{inj}=144\text{psia}$