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entitled
Design of Hinge-Line Geometry to Facilitate Non-Plastic Folding In Thin Metallic Origami-
Inspired Devices
by
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Doctor of Philosophy Degree in Engineering

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Origami is the traditional art of paper folding, which yields objects that can be considered, in engineering terms, as mechanisms with relative motion between panels (paper) constrained by hinges (folds). Non-paper materials are often studied for origami-inspired applications in engineering. The proposed hinge material in this work is bulk metallic glass (BMG), chosen for its low stiffness, wear and corrosion resistance, biocompatibility, and extreme capacity for elastic deformation. Panel-spacing and geometry were examined to provide insight for the design of thin BMG folding membrane hinges to connect larger regions of thicker material (panels). Finite element analysis was performed to study the stress variation, distribution, and displacement along the hinge for several design variations, and several loading profiles are discussed to determine the necessity of modified rounded-edge panels. The results will directly aid in creating origami-inspired designs with membrane hinges, and applicable to the design of devices such as foldable electronics, optical systems, and deployable solar arrays.

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Chapter 1

Introduction

An earlier version of this study was presented at the ASME IDETC/CIE conference in Aug, 2018*, and permission has been granted by publisher and co-author for its use in this dissertation. The abstract and section 1.1 originally appeared in the conference paper. Compared to the earlier version, the dissertation provides thorough explanation for motivation and challenges in chapter 2, adds improvements to the geometry and boundary condition sets of the models in chapter 3, improves accuracy of the results in 4, and provides new results in chapter 4.

1.1 Overview

Bulk metallic glasses (BMG) are a wide variety of metallic alloys that have an amorphous micro structure. BMG is obtained by rapidly quenching molten metal with near-eutectic liquids such that crystal nucleation and growth is avoided (Suryanarayana & Inoue, 2011). It has extremely attractive mechanical properties such as high hardness, high yield strength, high fracture toughness, high elastic limit and low stiffness compared to conventional metallic materials such as steel. It also has good corrosive resistance,

biocompatibility, and excellent soft-magnetic properties. Researchers have demonstrated that BMG can be an ideal material for compliant mechanisms, which can be used as bearings and hinges to eliminate friction, reduce wear, and reduce manufacturing complexity (Homer et al., 2014). Origami has recently been described as a compliant mechanisms, because its motion is tied to deflection of the material (Greenberg, Gong, Magleby, & Howell, 2011). In this study, BMG produced in the form of thin ribbons is identified as a candidate material for origami hinges since it is thin, compliant, and has extreme capacity for elastic deformation compared to other metallic materials. Thin film BMG is also much stronger than natural fibers and polymers. Currently, BMG applications are in sporting goods, precision gearing, and valve springs due to its high cost in production and processing-dependent behaviors (Suryanarayana & Inoue, 2011).

Origami is the art of paper folding, and it can be extended to transform 2D materials to 3D objects that are able to bear loads or transfer motions. This transformation is achieved by the movements between the faces and creases on a sheet of material. Origami is often modeled as zero-thickness rigid panels connected by rotational hinges (Ishida Sachiko & Ichiro, 2014) , but many thickness accommodation methods has been developed to incorporate stronger materials with finite thickness (Zirbel et al., 2013). Origami designs are not only aesthetically pleasing, but also have promising applications such as emergency shelters (Thrall & Quaglia, 2014; Quaglia, Yu, Thrall, & Paolucci, 2014), robotics, deployable solar arrays, and space telescopes (Lang, 2004; Morris, McAdams, & Malak, 2016).

Membrane hinges combined with modified crease patterns are a common way to apply origami principles in field of engineering (Natori, Katsumata, Yamakawa,

Sakamoto, & Kishimoto, 2013). Compared with conventional, rotational hinges and other mechanical hinges, membrane hinges are easily manufactured and assembled (Crampton, Magleby, & Howell, 2017). Some work has been done to study the geometry of the crease (Nguyen, Terada, Tokura, & Hagiwara, 2015; Hou, Ma, Chen, & You, 2017) and how the membrane material folds under a prescribed type of loading (displacement controlled or force controlled) (Nguyen et al., 2015; Hou et al., 2017), however, there has not been a study to compare hinge stress and deformation across different loading conditions. Even though elastic sheet bending theories and various compliant beam bending theories (Greenberg et al., 2011; Howell, Magleby, & Olsen, 2013) provides a basic idea as to how crease geometry should be designed, there are many questions to be answered to facilitate using any particular thin sheet as simple membrane hinges. The material of interest, BMG has been previously featured in origami design with rolling contact (Nelson, Lang, Magleby, & Howell, 2016), but little research has been done to explore its ability to serve as membrane hinges.

The objective of this study is to explore the guidelines to follow when designing membrane hinges with materials such as BMG. Finite element analysis is used to study the stress variation, distribution and deformation patterns along the hinge for several models consisting of panels connected with thin flat membranes. The parameters include loading conditions, hinge width, and geometry modifiers such as supports and fillets. All results are compared and guidelines for using material with certain elastic limit as membrane hinges are given in the end.

1.2 Motivation

The following table summarized the benefits of using BMG in origami mechanisms compared to existing materials. As discussed in previous sections, the combination of high strength, high elastic limit, and low stiffness of BMG is very appealing in origami applications. Due to the high cost and limited availability of BMG, using it as membrane hinges is a good way to harvest the benefits of BMG. Combined with the excellent soft magnetic properties, high corrosion resistance, and possibly high fracture toughness, the BMG based origami mechanisms can find extended application in engineering field, especially as space arrays, custom RF devices and foldable electronics. By studying the loading conditions and stresses of BMG hinges, a guideline can be proposed to help with incorporating materials such as BMG as membrane hinges.

Table 1.1: Benefits of using BMG as origami hinges

Properties	Paper	metal	Polymer	BMG
High strength	no	yes	no	yes
High elastic limit	no	no	yes	yes
Ease of folding	yes	no	yes	unknown
Wear resistance	no	no	no	yes
Bio compatibility	no	no	no	yes
Corrosion resistance	no	no	yes	yes
Humidity resistance	no	no	yes	yes

1.3 Challenges

There are several challenges faced for this study:

- The material exhibits brittle-like behaviors, so the maximum stresses of the hinge material must be kept strictly under the yield strength at all times. It also means the BMG ribbon cannot be folded as is and its bending radius must be controlled.
- There is limited amount of research to characterize the hinge properties, and designing experiments to verify the validity of the study is difficult.
- Experience with FEA is very limited, and the software COMSOL must be learned from the beginning. The geometric nonlinearity caused by nonlinear large deformation and contact conditions must be solved. The loading conditions also need to be studied to include many different situations.

Chapter 2

Literature Review

2.1 Origami

Origami is the traditional art of paper folding. The word “origami” is from Japanese, where “ori-“means folding, and “-gami” means paper. To make an origami sculpture, a person needs to learn about the particular pattern, which used lines to indicate convex or concave (or “mountain” and “valley”) folds, transfer the pattern on to the paper, and fold the paper along the patterned lines. Origami is generally considered only for fun and art, however, in recent years, researchers have identified origami as compliant mechanisms, and it has great potential to be used in deployable structures, robotics and RF devices. Now, the concept of origami has been extended to transform 2D material to 3D objects that are able to bear loads or transfer motions. Figure 2-1 below shows a “flasher” in its deployed state with patterns in the left, and folded state in the right (Bhuiyan, Semer, & Trease, 2017). A flasher can be used in many devices and space structures such as deployable solar arrays.

Figure 2-1: A common origami mechanism: flasher in (left) deployed state with patterns displayed and (right) folded state (Bhuiyan et al., 2017).

To avoid future confusions, here are some terminologies used to describe origami mechanisms, as they slightly vary between research groups. Figure 2-2 is shown as an illustration.

- Deployed vs. Folded state: when the material is in flat sheet form, it is deployed, when it is at the end of the transformation from 2D to 3D, it is folded. The in-between stage is called folding process.
- Fold vs. creases: fold is the action, and crease is the structurally weakened part of the material which is created by the folding action.
- Creases vs. hinge: the function of a crease is equivalent to a hinge in a mechanism. There are many forms of hinges used in origami mechanism, including removal of material, membranes, pins and torsional springs.
- Creasing: the process to create a crease on the sheet material. It includes but is not limited to folding, material removal processes such as blanking,

indenting and etching, and assembly processes with membranes and rotational hinges.

- Faces vs. panels: faces are part of a sheet that remains unchanged between start and final stage. Usually, the face is considered zero-thickness in kinematic models. Panel specifically refer to the face when the face is considered with certain thickness.

Figure 2-2: Terminologies used with origami

2.1.1 Origami and compliant mechanisms

A mechanism is a mechanical device used to transform motion, force or energy (Howell, 2001). A traditional rigid body mechanism consists of rigid links that are connected at movable joints while a compliant mechanism uses the deflection of its flexible members to act like movable joint. The major benefits of compliant mechanisms include not only cost reduction from reduced part cost, assembly time, and complexity of manufacturing processing, but also increased performance from increased precision and

reliability (Howell, 2001). One example for compliant mechanism is the shampoo bottle lid, which uses the deflection of short plastic membrane to connect the lid and the cap, and hold the lid in open position. Some challenges for designing compliant mechanisms include the difficulty in analyzing the large geometric nonlinearities and coping with material limitations such as lack of strength and lack of study on fatigue properties (Howell et al., 2013). In order to reduce the level of complexity when designing compliant mechanism, Howell developed a method called Pseudo Rigid Body Model (PRBM). In PRBM, the flexible member of the mechanism is considered to be rigid links connected by a revolute joint or torsional spring, then the mechanism can be analyzed the same way with traditional mechanisms. Over the years, many new methods have been developed based on PRBM to improve the accuracy of the results (Ma & Chen, 2015), and finite element analysis has become a widely accepted way to analyze compliant members of the mechanisms (Lobontiu, 2002; Howell et al., 2013).

A research lead by Greenberg (2011) explained why origami mechanisms could be considered as compliant mechanisms: their motion is a result of the deflection of the material. As a result, PRBM is appropriate to be used to study origami mechanisms, when faces are modeled to be links and creases are modeled as joints (with or without stiffness). The motions between faces can be studied using kinematics, and computer programs such as freeform origami (Tachi, n.d.-a) can be used to solve complex folding problems.

Traditionally, origami mechanisms must be folded from one piece of paper, without cutting or gluing. When a crease is created by folding, the microscopic structure of paper is compromised, and it caused the stiffness to reduce at the crease (Rao,

Tawfick, Shlian, & Hart, 2013). As a result, the faces connected by the crease rotates first before they deforms during folding. In some designs, the faces undergo both rotational motion and deformation to achieve the final look, and in other designs, the faces remain a rigid link during the folding stage. The latter are called “rigid foldable” designs (Tachi, 2010), and they are of particular interests to researchers, especially when incorporating material thickness into modelling.

2.1.2 Origami and its applications, materials and processes

In recent years, origami has showed great potential to be used in the field of engineering. However, since it is a relatively new field of study, many origami designs are still in their prototype phase. Morris (2016) used datamining techniques and compiled a very thorough list of existing origami applications. Additionally, reviews and summary was done to cover synthesis and applications of active materials (Peraza-Hernandez, Hartl, Malak, & Lagoudas, 2014) and processing polymeric sheets for folding (Y. Liu, Genzer, & Dickey, 2016) in origami. Reynolds’ group compared various folding techniques and materials used for space payloads (Reynolds, Jeon, Banik, & Murphey, 2013). Crampton’s group compared a number of processes to fabricate origami mechanisms from stock sheet material, especially sheet metal (Crampton et al., 2017).

Here are a few examples of current origami application. An emergency shelter made of folded cardboard paper (Maanasa & Reddy, 2014) is able to be cheaply made and shipped in either creased sheet form or fully folded state, and deploy immediately whenever in need of sheltering. A sandwich panel with folded miura-ori core (Klett, Zeger, & Middendorf, 2017) can increase airflow in panel to reduce moisture build-up. A

space telescope prototype (Lang, 2004) folded in a flasher pattern can be deployed from a highly compact state.

Each material has their own strengths and weaknesses when applied in origami, and here is a comparison between common origami materials.

- Paper: paper is cheap and easily obtainable with a great range of thickness. Folding processes can be easily done by hand without using additional creasing procedures. Paper is weak against loading, impacts, and cannot stand environmental damages.
- Polymer and metallized polymer sheet: a wide range of polymer sheets can be used in origami. They are slightly stronger than paper and can withstand loading and some environmental damages. Some manipulations are needed to create creases on polymer sheets. Metallized polymer sheet (Mylar) belongs to this category here because of its strength, but in later section, it is considered a metal foil for its electrical conductivity.
- Traditional metal sheets and foils: metal sheets are strong, but creases created by folding or material removal processes offer very limited range of motion. Actual mechanical rotational hinges or polymer membranes are sometimes used in place of creases to allow a wider range of motion (Crampton et al., 2017). Metallic foils are sometimes used in combination with other materials for their electrical conductivity.
- Composite material: composite materials can offer the benefits of two of more different materials, and can have moderate amount of stiffness and range of motion. However, they are usually less common than other stock

materials.

Besides material, there are also various factors that affect the quality of any origami builds. Here are many commonly used creasing techniques to assist in folding.

- Hand folding following patterns
- Removing material at the designated crease location via blanking, indenting, chemical etching, laser cutting etc.
- Assembling panel material with the hinge material, which can be membranes or mechanical hinges.
- Using active materials as substrate to create self-folding action via heat or UV light, or magnetic field activation.

Membrane hinges are widely used in origami mechanisms because it can be easily assembled with panel materials. A wide range of materials, mainly polymeric materials can be used as membranes. Natori specifically compared different membrane space structures (Natori et al., 2013). In the scope of this dissertation, membrane hinges made of Bulk metallic glass are studied, aiming to provide some insights for creating strong membranes for deployable structures such as solar arrays and space telescopes.

2.1.3 Origami with metallic materials

This section focuses on origami mechanisms realized using metallic materials. A wide range of application can be achieved to work around the material limitations to utilize the strength and other properties such as electrical conductivity and shape memory effect. The applications are presented with a brief explanation of the material and creasing techniques used.

- Foldable resistors: heat shrinking PVC film is sandwiched between laser-cut Metallized Polymer Film (MPF) with designed pattern. PVC layer shrinks under uniform heat and self-folds as a results (Miyashita, Meeker, Tolley, Wood, & Rus, 2014).
- Inflatable cylinders: two card paper are folded into shape as guides, then MPF is sandwiched between the guides to enforce folding behaviors. Papers are then removed and MPF is taped to hold its cylindrical shape (Schenk, Kerr, Smyth, & Guest, 2013).
- Self-deployable tent graft: Shape memory alloy sheet is chemically etched and folded by hand at room temperature, and it is able to be deployed at body temperature (Kuribayashi et al., 2006).
- Morphing sandwich beam with miura-ori core: steel sheet cut using digital fabrication method with tab and slot design, then assembled by hand with simple tools (Cash, Warren, & Gattas, 2015).
- Frequency reconfigurable QHA: copper foil is sandwiched by Kapton substrates to reduce undesired creasing, then built with cardboard backing to gain some strength (X. Liu, Yao, Gibson, & Georgakopoulos, 2017).

2.1.4 Characterizing origami creases using experiments

Since deflection of the material at creases enabled origami mechanisms, several research groups designed experiments to characterize the crease with folding torques, hinge index, and folding angles. Table 2.1 is a summary of current publications to date, and these studies provided insights on how material folds with simple creasing. However,

they offer limited and inconsistent information about creases. It is important to have a widely accepted and consistent term to describe behavior of the hinges created by folding, but experimentally, it appears to be difficult. One of the future goals between this proposal and dissertation, is to design a simple experiment to verify the results of the proposed stress reducing modifications.

Table 2.1: Experimental setups of origami creases

Case	Material	Equipment	Result parameters	Reference
1	Paper	Tensile test machine, camera	Torque (linear load)	(Pradier, Cavoret, Dureisseix, Jean-Mistral, & Ville, 2016)
2	Cardboard	Custom circuit with load cell	Folding torque	(Qiu, Aminzadeh, & Jian S. Dai, 2013)
3	Varies natural and synthetic paper	Roller and sloped surface, SEM equipment	Fold and residual angles, microscopic images	(Rao et al., 2013)
4	Varies paper, polymer and metallic sheet	Custom fixture with angle control, digital microscope	Hinge index (original and residual angles)	(Francis, Blanch, Magleby, & Howell, 2013)
5	Varies paper and polymer sheet	Tensile test machine, optical microscope	Tensile test properties, radii and residual angles	(Abbott, Buskohl, Joo, Reich, & Vaia, 2014)

2.1.5 Origami modelling using Finite Element Analysis

The selection of studies in this section heavily influenced the methodology chosen for this dissertation. As mentioned before, origami can be modeled using PRBM. Commonly, origami patterns can be studied using kinematic models which assume the material to be zero thickness, to obtain the foldability or optimized geometric parameters (Greenberg et al., 2011). However, in real life, adjustments on patterns and folding processes must be made to accommodate material thickness. Thickness accommodating techniques are developed to help with fabrication from materials with finite thickness, and they usually involve adding crease length, applying material with high elastic limits as membranes hinges, using mechanical hinges, and trimming panel material (Morgan, Lang, Magleby, & Howell, 2016; Zirbel et al., 2013). Foldability and geometric constraints can be solved with modified kinematics models with computer programs (Tachi, n.d.-a, n.d.-b). However, the stresses and deformation of hinges cannot be studied in these models, so an assumption of these models is that the hinge material will not fail.

The stress and deformation analysis is not important for polymeric materials under low load, because they far exceed 50% elastic limit and have high ductility to resist sudden failure. In order to expand the application of origami into an engineering field, materials that have higher strength must be considered. There are less limitations on panel materials, however, there are many limitations associated with hinge material. For example, material with high strength, such as steel, have very small elastic limit around 0.2%, which greatly limited the range of motion for assembled origami mechanism. As a result, the stress and deformation at the crease need to be studied to make adjustments for existing crease design to accommodate for material limitations.

Finite element analysis (FEA) is a great tool to study the performance of hinges (Lobontiu, 2002), and is commonly used in origami analysis. In this section, several studies on crease geometry and folding process utilizing FEA is presented. The studies are arranged in the order of relevancy to this dissertation.

Nguyen (2015) compared the stress distributions at creases under a prescribed bending angle. Crease geometries are assumed to be obtained from grooving Aluminum sheets. He concluded that rectangular grooves are the best in terms of maximum bending angles before material self-intersects, and it creates the lowest bending force. Nguyen also concluded that how bending is applied at the crease has an effect on the force applied. The finding on crease geometry somewhat contradicts with what is considered common in flexure hinges, whose geometry is often corner-filled, circular or elliptical (Lobontiu, 2002). The reason why common flexure hinges are of certain shapes may be from manufacturing limitations with material removal techniques or tooling required. This raises the question: how can stress be reduced during bending from hand-like motions? New crease geometry can be designed and different ways of folding should be considered.

Hou (2017) studied how thickness of silicon rubber film affects the final geometry and stresses of cross-folded flat membranes. He also proposed two different methods to reduce the stress at the center of material. The membrane is folded using two steps, both of which are displacement controlled. As this study also uses displacement controlled folding, a question is raised: why is the membrane not folded with force or moment?

Figure 2-3: stress distribution of a cross folded membrane (2) effects of thickness on folding (Hou et al., 2017)

Sung, Erol, Frecker, & von Lockette (2016) studied how a certain active origami mechanism reacts to a magnetic field. The mechanism is fabricated with magnetically activated panels attached to compliant polymer substrate. This mechanism is activated with a magnetic field and experimental results matched with the FEA result. The final bended geometry, peak forces, and work created are studied to see if such origami mechanism can produce enough work during folding process. This is one of the few studies that are done using COMSOL Multiphysics among the origami research groups. Due to the availability of finite element analysis software, COMSOL is chosen to be used in this dissertation.

2.2 Bulk Metallic Glasses

The realization of origami in industry is often limited by the low strength material that have to be used. High strength materials usually come with low elastic limits which

greatly limit the motion of the creases. As material science advances, a new category of materials becomes available. Bulk Metallic Glasses (BMG) shows great potential to be used as flexible joints (Homer et al., 2014). In this section, BMG's material properties, synthesis and processing of BMG, and applications will be discussed in detail. The sources are from a combination of the book "Bulk Metallic Glasses", several credible reviews, and many journal articles retrieved from online databases.

2.2.1 Bulk metallic glasses: a brief introduction

As material science advances, many new types of metallic materials have been synthesized, such as metallic glasses, quasicrystals, nanocrystalline materials, and high temperature superconductors (Suryanarayana & Inoue, 2011). Metallic materials were traditionally considered crystalline before the discovery of metallic glasses. In 1960, Pol Duwez's research team at Caltech discovered that $\text{Au}_{75}\text{Si}_{25}$ alloy, which was obtained by rapidly quenching droplet of molten metal on highly conductive substrate, showed amorphous crystalline structure (Klement, Willens, & Duwez, 1960). Using this technique, which is known as the "gun" technique, the rate of quenching was estimated between 10^4 to 10^6 K/s, and thin foils and wires were able to be produced with maximum thickness of 50 microns. In the late 1980, Professor Inoue and Masumoto were able to produce a $\text{La}_{55}\text{Al}_{25}\text{Ni}_{20}$ glassy alloy rod with 1.2 mm diameter using water quenching method (Suryanarayana & Inoue, 2011). They achieved these results by systematically exploring the range of supercooled liquid region, and lowered the necessary rate of cooling down to 10^3 K/s. The supercooled liquid region is the temperature difference between crystallization temperature and glass transition temperature. Since then, metallic

glasses that were able to be produced with large section thickness were referred to as *bulk metallic glasses* (BMG). In 1992, the first commercial BMG, Vitreloy became available in many formulations, and a large number of research was done with this BMG. Currently, thousands of papers on BMG are published each year and it is an active field of research in material science, engineering and solid-state physics (Suryanarayana & Inoue, 2011).

In order for a glassy alloy to be considered BMG, a few criteria have been established by Suryanarayana:

- The alloy systems have at least three components.
- The alloy can be produced at slow quenching rate of 10^3 K/s or less.
- The alloy can be produced in large section thickness of at least a few millimeters. However, the size requirement varies between studies and it can be as high as 10 millimeters. Also, in some studies, the term “bulk” refers to the multi-component alloy system.
- The alloy has a large supercool liquid region.

There are some other terms used for BMGs, such as noncrystalline material and amorphous metals. In the book “Bulk Metallic Glasses”, the use of “glass” in BMG usually indicated the material was obtained from continuously cooling of liquid metals, while amorphous metal indicated the material was obtained from other methods such as mechanical alloying. Noncrystalline material is just a general term for all materials that do not have crystalline structure. This dissertation follows this rule and “bulk metallic glasses” is the terms used throughout.

2.2.2 Material properties

BMG is an ideal candidate material for origami applications because of a combination of excellent material properties and ease of processing. BMG has high yield strength, high hardness, high fracture toughness, high elastic limit and low stiffness compared to conventional metallic materials such as steel. It also has good corrosive resistance, biocompatibility, and excellent soft-magnetic properties. However, the material properties are highly dependent both on the formulation and manufacturing processes. In this section, different properties of BMGs are listed and compared to common metallic alloys (table 2.2).

Table 2.2: Mechanical properties of BMG compared with other metal alloys

Properties	Vitreloy	Steel	Ti alloy
Alloying elements	Zr, Cu, Ni, Ti, Al, Be, Nb, I	Fe, C, Mn, P Si	Ti, Al, Sn, Zr
Hardness (Vickers)	>500	126	150-250
Yield strength (MPa)	1795-1850	200-2100	200-1400
Elastic modulus (GPa)	91-95	200	80-125
Elastic limit (%)	1.89-1.97	0.1-1	0.2%
Fracture toughness (MPa/m^{-1/2})	30-75	50-100	50-60
Density (Kg/m³)	6000-6900	7800-8000	4500-4800

Mechanical properties: mechanical properties of a material is very important to determine the possible applications for a material. Compared to common structural material such as steel and Titanium alloy, BMG has very desired properties due to its

unique microscopic structure. The most notable properties are its high hardness, high yield strength, high elastic limit, and low elastic modulus (Materion, 2017).

Chemical properties: There are limited information on the corrosive behaviors of BMG, compared to the amount of studies on the mechanical properties. In the 1970s, a Cr containing Fe based BMG was compared to its crystalline counterpart, and the BMG showed significantly higher corrosion resistance in different testing environments. Several factors may contribute to these anti-corrosive behaviors, including the high chemical homogeneity, lack of crystal structures and forming of passive films. In general, some groups of BMGs exhibit highly anti-corrosive behaviors, such as Zr- and Ti- based BMGs, and some others (Ni-, Cu-, and Fe-) showed vastly different corrosion rate under different testing environments (Suryanarayana & Inoue, 2011). The commercialized BMG, Vitreloy (Zr based) are highly corrosion resistant (Materion, 2017).

Magnetic properties: Since one of the most important application of melt spun BMG ribbons is on transformer core laminations, magnetic behaviors of BMG has been studied extensively, mostly in Fe based BMGs. Magnetic saturation, which indicates the maximum magnetic field a material can create, is used to describe the magnetic property of BMG. The highest magnetic saturation is 1.5T, and it can be further tailored with composition and annealing (Hasegawa, 2004; Suryanarayana & Inoue, 2011).

Ductility: BMG has an apparent lack of tensile ductility due to a shear localization instability. However, compared to conventional brittle material such as ceramics, BMG has high fracture toughness, and will not catastrophically fracture at internal or surface flaws at low stresses. The ductility of BMG can be improved with composition and processes such as cold rolling (Kruzic, 2016; Schroers, 2005; Schroers, Nguyen,

O’Keeffe, & Desai, 2007).

Fatigue properties: Historically, little research has been done on this topic, due to the lack of material to meet the testing requirements. The poor fatigue testing results on Vitreloy 1 gave BMGs a bad reputation when it became available. However, in more recent studies, researchers have found many composition of BMGs that exhibit great fatigue resistance, and as with other material properties of BMG, fatigue resistance is also highly dependent on composition and processes (Inoue & Takeuchi, 2011; Kruzic, 2016; Suryanarayana & Inoue, 2011).

2.2.3 Synthesis and processes

BMG is first obtained by the rapid quenching of molten metals to avoid passing the “nose” on a TTT graph. The early synthesis methods are called Rapid Solidification Processing (RSP), and the melt spinning technique is the most common way to produce BMG in ribbons, wires, and filaments (Suryanarayana & Inoue, 2011). Later on, as the required rate for cooling becomes lowered, many new techniques are developed to synthesize BMG parts such as water quenching and various casting techniques. BMG in other forms such as composites and foams are also able to be produced. In more recent studies, researchers are able to utilize the large supercooled liquid region of BMG and use thermoplastic forming methods to process BMG ingots into various geometries. (Schroers, 2005; Schroers, Nguyen, et al., 2007) In this section, melt spinning method, casting techniques and thermoplastic forming method will be briefly introduced.

As mentioned earlier, melt spinning method is the most common way of BMG synthesis. Because the thickness of the material produced using this method is very small,

the rate of cooling is very high, which causes the material to form a glassy alloy instead of crystalline alloy. In melt spinning, raw material is first melted in a small crucible, then molten metal is ejected under high pressure on to a spinning wheel which is usually made of copper. Parameters such as composition of raw material, melting temperature, nozzle diameter can all be adjusted to change the geometry of the final product, however, the thickness of the final product is limited to 50 microns because of the cooling rate requirements. (Suryanarayana & Inoue, 2011) The following figure shows a spin melting machine on the left and melt-spun ribbon on the right. (“Rapid Quench Machine System,” n.d.)

As the required cooling rate for BMG syntheses becomes lowered, direct casting of BMG also becomes possible. Many techniques have been developed by many research labs over the years. The techniques include water quenching method, high pressure die casting, copper mold casting, cap-cast technique, suction casting method, and the squeeze casting method, to name a few. Schroers (2010) summarized the advantages and disadvantages of direct casting as follows. The advantages include low melting temperatures due to carefully selected alloy composition, low shrinkage, one step process, homogeneous microstructure and consistent mechanical properties. The disadvantages include the coupled cooling and forming process, the high requirement for processing environment, the viscosity change in casting process, and high internal stresses. This method will not be discussed in this dissertation in detail, because the purposed topic so far only involves BMG in the ribbon form.

In more recent studies, researchers identified thermoplastic forming methods are the best way to process BMG, as it can be done with much lower cost compared to

traditional machining processes (Schroers, Nguyen, et al., 2007). The previous methods produce BMG part in finished form, which may be very complex. In thermoplastic forming, BMG ingots of very simple geometries are produced first, then, the ingots can be heated again to supercooled liquid region and process into desired geometry with much slower cooling rate using different techniques. When the ingots are heated to supercooled liquid regions, it transforms from glassy solid into a highly viscous metastable liquid before it eventually crystallizes. The material will have a much longer processing time, and as the metastable liquid is highly viscous, complex geometries can be easily achieved. Several examples of thermoplastic forming based methods are shown as follows:

- Injection molding: Vitreloy ingot is melted in a cold walled, water cooled crucible before it is molded using modified commercially available injection molding equipment. The product is a bi-stable mechanism to demonstrate BMG's potential to be used in compliant mechanisms (Homer et al., 2014).
- Miniature fabrication: due to the extremely high viscosity of BMG in the metastable liquid form, miniature parts and surface patterning can be achieved using heat and pressure. Patterned surface have applications such as micro-lens arrays, and 3D microparts have great potential to be used in MEMS, electric devices, and medical devices (Kumar, Desai, & Schroers, 2011).
- Blow molding: BMG disk is heated with the mold to process temperature, then, a pressure difference between the top and bottom of the disk is applied. The produced part is a highly accurate replica of the mold, which

demonstrated the net shaping ability of BMG for commercialization (Schroers, Pham, Peker, Paton, & Curtis, 2007).

2.2.4 Applications

The potential areas of applications for BMG have been explored systematically, and many successful applications are based on different advantages of BMG. Inoue summarized the advantages and disadvantages of BMG for potential applications in different fields (Inoue & Takeuchi, 2011), and the properties that are considered include mechanical, magnetic, chemical, processing, and aesthetic. The major limitation for its application is the high cost of material and highly process dependent behaviors. A few examples of BMG applications are presented in this section, some of which are successful commercial applications, while others are only possible applications. The general criteria for assessing if BMG can be used in certain application is summarized by Axinte (2012): (1) the cost of the material constitute a small part of the price, (2) performance is highly valued, (3) cost of failure is modest, (4) aesthetics are important, and (5) only limited quantities of materials are required.

BMG tools: compared to surgical knives made from steel, BMG knives have ultra-high hardness. The lack of grain boundaries of BMG makes the tool edge significantly smoother, and it also contributes to the wear/corrosion resistance of the edge (Axinte, 2012).

Compliant hinges: one of the most successful applications of BMG is on Digital Light Processor (DLP). (Tregilgas, 2004) AlTi amorphous alloy is used in place of Aluminum alloy as miniature hinges in DLP to serve as a robust mechanism to rotate the

mirror. Aluminum was preferred as a material before the presence of BMG for its manufacturing advantages and necessities, but it is not robust enough for the required fatigue life. These particular BMG hinges deform elastically and have almost infinite fatigue life. A single hinge is shown below on the left, and its location in an assembled DLP is shown on the right. Some generic BMG hinges are shown in the next figure (Kumar et al., 2011), and the geometry of one of the hinges resembles the geometry of the hinge that is studied in this dissertation.

Micro-lens arrays: For display field, micro-lens array is used to enhance the brightness of illumination and simplify the light guide module construction. The BMG micro-lens arrays that are obtained from hot embossing techniques are strong and hard, and it accurately replicated the surface patterns of the mold with parameter adjustments (Pan et al., 2008).

Anti-corrosive coating: An Fe-based BMG is commercialized under the name “AMO-beads”, and can be used as in peening shot to improve the surface quality of the treated parts. The surface hardness, strength, and corrosion resistance becomes much stronger (Inoue & Takeuchi, 2011).

2.2.5 Existing BMG application in origami

Currently, there is only one research (Nelson et al., 2016) that involves using BMG as hinges. A compliant mechanism called “D-core” is designed and assembled with various materials. Metallic glass film is one of the choices, however, the reason for the incorporation of BMG ribbons is not clearly explained. Besides this research, there are little known applications of BMG in origami.

Figure 2-4: Metallic hinges applied as D-core hinges (top left) unassembled (bottom left) assembled (right) deployed mechanism (Nelson et al., 2016)

Chapter 3

Methodology

The purpose of this chapter is to: (1) to study how to create a “fold” action in FEA analysis and (2) to study the deformation and stress variation during the displacement or load input to determine if modifications are needed. Designing suitable boundary conditions is the largest challenge and accommodating geometric nonlinearities requires extensive setups for FEA analysis. The approach here is to build a simple geometry to represent a single crease that is connect by panels on both sides, and by simulating the reaction of the crease under various folding conditions, it is able to provide some guidelines on how to incorporate elastic material such as BMG as membrane hinges to design origami-inspired mechanisms. Afterwards, physical models following the proposed design principles can be built to test the validity of the aforementioned principles.

3.1 Design considerations

Since the purpose of a hinge is to connect the panels and rotate without permanent deformation, the hinge material must be under the elastic limit during the whole folding

process (Hu, Marciniak, & Duncan, 2002). A basic model is constructed under the elastic sheet bending theory. BMG is a category of metallic alloys, so it has a wide range of mechanical properties. A specific formulation of BMG, $\text{Ti}_{41.5}\text{Zr}_{2.5}\text{Hf}_5\text{Cu}_{42.5}\text{Ni}_{7.5}\text{Si}_1$, that exhibits desired properties of Elastic modulus (E) of 103 GPa and Yield strength (S) of 2080 MPa is selected as the hinge material for this particular study (Suryanarayana & Inoue, 2011).

From elastic sheet bending theory (Hu et al., 2002), a sheet of material reaches its maximum deformation when the true strain of its outer surface matches its elastic limit. As the bending radius decreases, the stress difference between outer surface and inner surfaces increases until the material fails.

The minimum bending radius can be obtained by equation 1, where R/t is the ratio between the bending radius of the sheet and the thickness of the sheet, and ε is the elastic strain of the sheet material.

$$\left(\frac{R}{t}\right)_{\min} = \frac{1}{2\varepsilon_{\max}} - 1 \approx \frac{1}{2\varepsilon_{\max}} \quad \text{Equation (1)}$$

For this BMG material, its elastic limit is 2%, and $(R/t)_{\min}$ is approximately 25. Since the stress and deformation in a compliant material is coupled (Howell, 2001), when deformation is distributed evenly across the hinge, the shape of the hinge should remain circular and the maximum stress should remain under its yield strength. The width of the hinge must form a semi-circle when the material is fully folded. Several parameters are used with the following consideration:

- BMG ribbon has thickness from 20-50 microns (Suryanarayana & Inoue, 2011)
Hinge thickness $t = 40$ microns is chosen for this study.
- Minimum panel thickness $R_{\min}$ must be 1 mm or above from equation (1), and $R = 1.04$ mm is chosen here so the inner surface has a minimum bending radius of 1 mm.
- The minimum panel spacing which is equal to hinge width $w_1 = \pi R = 3.14$ mm.
Actual panel spacing is represented by w . Parametric studies are set up to study how the ratio between w and w_1 changes the stress. This ratio is named Hinge Width Index (HWI) for easier reference.
- Panel width 5 mm and in-plane thickness of 2 mm are relatively large compared to w and is chosen to reduce simulation time.

Figure 3-1: Basic model for finite element study

3.1.1 Modifications for the basic model

To control the bending radius and maximum stress during bending, four different modifications are proposed on the panels. They are split support, whole support, oval support and fillets, as shown in fig. 3-2. Their design parameters are kept comparable to

the base model. The center of all three supports rest at the top corner of the panel with radius of $R' = 1\text{mm}$, which is the minimum bending radius. To help with simulation, fillets of 0.25 mm are added with where needed. Panel fillets of different sizes r_F are add on to basic models to study the effects of fillets, since it is commonly used on compliant beams to add strength to them (Lobontiu, 2002).

Figure 3-2: Different modifications of basic model

3.1.2 Summary of parameters

Table 3.1 Summary for parameters used

t	Hinge thickness	0.04 mm
R_{min}	Minimum bending radius	1 mm
R	Panel thickness	1.04 mm
w₁	Nominal width of hinge	3.14 mm
w	Actual width of hinge	varied
HWI	Ratio between w and w₁	varied
L	Panel width	5 mm
R'	Circular supports radius	1 mm
a	Oval support major axis	varied
b	Oval support minor axis	1 mm
r_F	Radius of panel fillets	varied

3.1.3 Design questions

To test the validity of the design considerations from previous section, finite element analysis is set up and several design questions are brought into the study.

- Is elastic bending theory enough to describe the motion of the hinge section?
- Where is stress concentrated along the hinge?
- How does the hinge deform under different loading conditions?
- How does stress level increase during the loading?
- Are selected hinge widths best at reducing maximum stress? Can maximum stresses be reduced by adjusting the nominally calculated hinge width? If so, what is the optimal adjustment?
- Is it necessary to have supports and fillets?

3.2 Model setup for finite element analysis

The finite element analysis is performed using COMSOL Multiphysics following the standard simulation workflow.

Stationary analysis is run under the solid mechanics “node” under 2D environment. Geometries of each model are plotted with parameters incorporated, for enabling parametric studies later. Contact pairs are created at this point to avoid self-intersection of model and define a natural folding endpoint. In order to easily extract the stress data, 14 domains of equal sizes are added to the hinge domain, which creates 13 data points on the outer diameter (bottom) of the hinge for stress evaluation.

Material properties are set as follows: density $\rho = 6800 \text{ Kg/m}^3$, elastic modulus $E = 100 \text{ GPa}$, and Poisson’s ratio $\nu = 0.3$. These properties are chosen based on a special formulation of BMG that has an elastic limit of 2% as shown in the previous section.

Two types of meshes, triangular and quadrilateral are considered and a series of mesh convergence test are carried out to obtain appropriate accuracy of the results.

Folding displacements and loads are prescribed to be in small steps following a defined load profile. The load profiles are specific to each case and some are developed by calculation while some are obtained through trial and error. This step and the segregated solver are able to combat geometric nonlinearities of the model.

Lastly, the deformation and stress distributions are collected from predefined points across the hinge and plotted against the folding process.

3.3 Boundary conditions

One of the goals of this dissertation is to study how to simulate the “fold” action in FEA analysis. Admittedly, the boundary conditions are not based on any biomechanics study, but a general and simplified approximation of human hand movement. However, the proposed boundary conditions are meaningful to reveal some hidden information that cannot be done with other analytical studies. Three categories of boundary conditions are proposed here, and they are Pure Moment Folding (PCF), Displacement Controlled Folding (DCF), and Force Controlled Folding (FCF). The design concept of these folding conditions can be used by origami folding machines and robotic fingers. In real life scenario, a “fold” action initiated by human is likely a combination of all three types of motion, and usually with additional feedback from the eyes to make adjustments during the process. The following section describes all the boundary conditions and why the specific values are chosen. The order of its arrangements is based on the complexity of the setup to achieve a fully folded state.

3.3.1 Pure moment folding (PCF)

Pure moment folding is one of the more common boundary conditions used when analyzing compliant beams. Since COMSOL Multiphysics does not directly allow moments to be boundary conditions, some adjustments need to be made. First, a “symmetry” BC is added where the line of mirroring is to reduce simulation time. Then, a contact pair is set between panel and the side block to avoid self-interaction. On the leftmost boundary, two small segments are added to the top and bottom along the boundary. Each of the segment has a boundary load in the form of pressure applied to it

with the same magnitude but opposite direction. The force equivalent of the pressure is 1.5N and their directions rotates with the panel. Even though a moment cannot be directly applied to the model, this setup is the least complex and requires the least amount of experiments before achieving the folded state. It is worth of note that for hinges that are significantly longer than the base value, it cannot be fully folded, no matter the amount of the applied moment.

Figure 3-3: Pure moment folding setup

3.3.2 Displacement controlled folding (DCF)

Displacement control is achieved using “rigid connector” boundary conditions with the help of two circles ($r = 0.3 \text{ mm}$) added to the center of the panels. Both connectors can rotate freely but rigid connector 1 is fixed in place while the rigid connector 2 travels in x direction for a preset distance. The w chosen in this case is 1.6 mm ($\text{HWI} = 1.02$). This set up is slightly more complex than the previous case since geometries of the basic model need to be modified, and some manual calculation on

displacements is required to fully fold the design. In actual simulation, the simulation time is significantly lower than other load based folding, and several studies on filets and oval shaped supports are studied using this case. Additionally, since the panels can be fully folded even with extra-wide panel spacing, a parametric study is set up to study how the hinge deforms.

Figure 3-4: Displacement controlled folding setup

3.3.3 Force controlled symmetrical folding (FCSF)

The geometry and BCs are set up as shown in the following figure and it is very similar to pure moment folding case. First, a “symmetry” BC is added where the line of mirroring is to reduce simulation time. Then, a contact pair is set between panel and the side block to avoid self-interaction. A single boundary load F1 is added to the left end to mimic human hand motion, which “push” the panel up and “flip” the panel to the right until it is going to touch the opposite side. The increased complexity for this case lies in the fact that the amount and pattern of the boundary loads needs to be experimented before reaching the desired final folded state.

Figure 3-5: Symmetrical force loading setup

3.3.4 Force controlled unsymmetrical folding (FCUF)

One unsymmetrical loading profile is used in this case. A fixed constraint BC is added to the left end of panel and a boundary load F_3 is added to the right end of the panel. F_3 is designed to simulate hand motion. It goes up and toward the left end to “flip” the panel at first, then down and right to “match” and “press” the two panels together. A contact pair is set up differently compared to previous cases and they are shown in the following figure. This loading case require the most amount of setup because in order to have the desired folded state, huge amount of experimenting with the boundary loads is

required. Since the stress results in this case indicate that supports are needed to protect the hinge material, models with supports are studied using this setup. For supported cases, additional contacts pairs are set between curved edge and hinge, otherwise all BCs are kept the same.

Figure 3-6: Unsymmetrical force loading setup

3.3.5 Force controlled follower force folding (FCFF)

While the previous force controlled folding uses force based on global coordinate, the following set of force is set up based on local coordinate system. A Pressure of 500 MPa is applied at the end of the panel, and always towards the surface of the panel. It can be considered as the unsymmetrical force input but with the position feedback. With the addition of the feedback, the panels can be folded easily without much trial and error.

Figure 3-7: Follower force folding setup

3.4 Mesh setup and convergence testing

As the hinge section undergoes large deformation during loading, and it is the main area of interest, a more detailed mesh is required at the hinge while the panel section can have coarser mesh to reduce computational cost. Two different types of meshes with various resolution are studied and compared to determine the optimal choice. Additional tests are also performed to study the effects of mesh resolution at other areas.

The model used in the convergence study is the basic model with standard dimensions. To simplify the data extraction process, the hinge section is separated into six domains of equal size. The boundary conditions are chosen to be the follower force controlled folding, because it requires minimum setup, and the external load/reaction forces can be easily solved. Below is an image of the area of interest.

Figure 3-8: Hinge section of a basic model with domains added for data extraction; the black dot is the point of interest for mesh convergence study

The output for this mesh convergence study is the von Mises stress at the mid-point of the outer surface of the hinge. The stresses at other points are compared during the setup and they follow the exact same trend. The numerical results with the final mesh choice will be shown in Chapter 4.

3.4.1 Triangular meshes at hinge section

To create triangular meshes of particular size, the option of building custom sized meshes is toggled and the maximum size of each element needs to be specified. In this case, a parametric study is run with an extensive range of chosen sizes, namely 0.05 mm, 0.04 mm, 0.03 mm, 0.02 mm, 0.015 mm, 0.01 mm, 0.008 mm, and 0.006 mm. To conceptualize this range, table 3.2 is a comparison between three different meshes resolutions. Each section shown is one sixth of the hinge.

Table 3.2: Triangular mesh resolution comparison

Max element size	Resultant mesh	No. of layers
0.05mm		1
0.02mm		3
0.004mm		13

3.4.2 Quadrilateral meshes at hinge section

The quadrilateral meshes created in this case are in the form of “mapped” meshes. It means all the mesh elements are rectangular and aligned in a matrix form. After some experimentation, meshes of different resolutions are created with a wide range of element size and the aspect ratio (length of edge/width of edge) of the elements are close to 1. The following table shows all the meshes settings and the following figure put the settings into perspective.

Table 3.3: Distribution combinations for mapped meshes

# of column	15	20	30	40	50	60	80	100	120	140
# of rows	4	4	4	4	4	5	6	7	9	11
Max size	0.03	0.02	0.01	0.01	0.01	.008	.006	.005	.004	.003
	5	6	8	3	1		7	3	4	8

Table 3.4: Mapped mesh resolution comparison

No. of Columns	No. of layers	Resultant mesh
15	4
30	4
140	11

3.4.3 Triangular meshes at curved surfaces

An additional study is performed to study how the mesh resolution at curved surfaces of the proposed supports, meshes of three levels of resolution are created. The option that affected the number of nodes on curved surface is “curvature factor”. The smaller the curvature factor, the more nodes are place on the curved edge. The values are chosen to be 0.075, 0.05, 0.04, and 0.025 for the main curved surface, and twice of that for the rounded corner. Table 3.5 is an illustration of the meshes of the lowest and the highest of the resolution.

Table 3.5: Support mesh resolution comparison

Curvature factor	No. of node	mesh
0.075	19 (main curve) 15 (fillet curve)
0.025	53 (main curve) 17 (fillet curve)

3.5 Physical model and validations

The knowledge learned from FEA needs to be validated with physical models. Due to the limited availability of the BMG material studied, the validation needs to be carried out with material of better availability. Also, the stresses and loads on the hinge is difficult to measure directly due to its scale.

As a result, the validation of this study will be carried out in an indirect way.

- First, several physical models complying with all the finding of this study will be fabricated to demonstrate the usefulness of the study.
 - The different types of folding action affects the stress distribution of the hinge membrane.
 - Adding certain modifications can help with relieving the stresses
- Then, some models that did not meet the requirement will be made to observe the fail pattern.
- Additionally, panels with other modifications will be made to show off the flexibility of the designs

Chapter 4

Results and Discussion

Chapter 4 presented all the discoveries made through finite element analysis, combined with physical model validation. It was organized into four sections and they were arranged following their order of appearance in Chapter 3. Section 1 explained the final mesh selection and other setups on FEA geometry for data extraction; Section 2 presented the preliminary results obtained directly from folding the basic model with proposed boundary conditions; Section 3 proposed several methods to offset the undesired stress concentration from in the previous section, and the final section used 3D printed physical models to demonstrate the effectiveness of the proposed hinge-line geometry in controlling final folded geometry and protecting the hinge.

4.1 Selection of mesh and data collection

In Chapter 3, a mesh convergence test was designed to determine the geometry and resolution of an appropriate mesh, especially at the hinge section. The combination of evenly distributed quadrilateral mesh at the hinge section with triangular mesh for panels was chosen. Various points on the basic geometry were selected to study the corresponding values.

4.1.1 Finalized mesh selection

The focus of this study was the stress distribution at the hinge section of the basic geometry, which could undergo large, non-linear displacement and had a long and slender shape. Therefore, the mesh convergence test only focused on the hinge. Both evenly distributed quadrilateral “mapped” elements and triangular elements were considered and compared. In the following figures, von Mises stress of a pre-selected point at a randomly selected instance were plotted against both their maximum element size and the number of elements at the hinge section. For both types of meshes, convergence was reached, and error percentage was calculated based on converged stress value.

Figure 4-1: von Mises stress comparison from mesh convergence test, plotted against maximum elements size of each types of mesh

Figure 4-2: Von Mises stress comparison from mesh convergence test, plotted against number of elements of each type of mesh

Based on the following considerations, the finalized mesh was presented in figure 4.3.

- To balance high mesh resolution with reasonable simulation time, the maximum element size should be between 0.018 mm and 0.009 mm, which produced results that were 99.9% to 99.99% accurate.
- Mapped quadrilateral mesh was preferred because it had higher-order elements whose displacement could be interpolated to a higher degree. Compared to triangular mesh, it converged slightly slower, however, both meshes produced results at similar level of accuracy.
- Having three rows of mapped elements was determined at this point, since one of its edge was 0.013 mm. Having two rows of elements, in which case the max

element size was 0.02 mm, could not reach the desired accuracy and having four rows was not necessary.

- The number of column was determined to be 224, to keep the aspect ratio of each element close to 1. This number was kept the same even when the hinge became wider during parametric studies, however, the maximum size of each element would not exceed 0.018 mm in all cases.

Figure 4-3: (Bottom) Finalized mesh at the hinge section with (top) details enlarged

4.1.2 Data collection from finalized geometry

After the mesh was finalized, the basic model geometry was plotted with smaller domains at the hinge section to enable the data extraction. Several types of data were collected from COMSOL and analyzed using Excel.

- Image data: snapshot of each frame and animation of folding process to

visualize the effect of each set of boundary condition.

- Stress data: von Mises stress at the 13 evenly distributed points at the outer edge of the hinge for stress analysis.
- Other data: coordinates of the four tips of the panels to determine contact status.

4 panel tip points for coordinate data extraction

Hinge section

13 evenly space points for stress data extraction

Figure 4-4: Finalized geometry with black dots showing the points for data extraction.

In order to clearly and efficiently describe the folding process, some terms were used to describe the results as the following,

- Folding process: percentages were used to describe the folding process, with 0% meaning the model was at its original state and 100% meaning the model was fully folded. The stress and deformations at 34%, 66% and 100% were

plotted and compared unless otherwise noted.

- Position of hinge: Stress data from 13 evenly distributed points on the outer edge of the hinge was used in stress distribution graphs. The position of each point was presented in decimals between 0 and 1 of the total hinge width.
- Contact: The forms of contact were separated into three categories: close tip contact, full contact and far tip contact. Close tip contact was usually caused by short hinges, while far tip contact was usually caused by long hinges. In this chapter, white-filled dots represented non-fully contacted panels in graphs comparing stress change and distribution with increasing HWI.

Figure 4-5: All three forms of contact

4.2 Folding a basic model

The following section presented the stress analysis results for folding the basic model with three types of boundary conditions, namely pure moment controlled folding

(PCF), displacement controlled folding (DCF) and force controlled folding (FCSE, FCUF, FCFF). Pure moment controlled folding (PCF) was considered a basis for analysis as it could be explained with existing elastic sheet folding theory. Displacement controlled folding would always achieve the fully folded final state. Force controlled folding was designed to follow human folding actions, and it provided additional information compared to the previous folding controls.

After examination of all results presented in this section, both the dimensional design and folding methods affected the stress distribution for a basic origami crease, and some additional designs was needed to reduce the undesired stress concentration.

4.2.1 Pure moment controlled folding

Figure 4-6: Stress distribution for PCF

Figure 4-7: Max stress growth for PCF

The stress distribution and stress growth of pure moment controlled folding showed:

- The stress distributed evenly across the whole hinge with its maximum stress reaching the yield stress of the material, as intended by following the basic linear elastic bending theory presented in the previous chapter.
- The maximum stress increased *linearly* with the folding process.

During the folding process, the prescribed input moment *linearly* increased. As the displacement of a compliant beam was coupled with the stress, the stress level of elastic membranes was proportional to the inverse of the bending radius. In this case, increasing panel thickness was effective for reducing the final stress level of a folded membrane or adding safety factor to the base hinge-line design.

In order to study if increasing the hinge width would reduce the maximum stress, a parametric sweep was performed under the same set of boundary conditions, except that the hinge width was increased. Hinge Width Index (HWI) was used instead of the hinge width so the results would apply to crease designs with various bending radii.

Figure 4-8: Max stress increase vs increasingly wide hinges

Figure 4-9: Final stress distribution for PCF for various HWI

Table 4.1: Form of contact for PCF

Contact Form	Corresponding HWI
Close tip contact	$HWI < 1$
Full contact	1
Far tip contact	$HWI > 1$

Judging by the results obtained so far, it was inconclusive whether increasing hinge width was beneficial, since no panels came into full contact except the critical case (HWI=1). For shorter hinges that had close tip contact, significantly increasing the input moment would fully close the panels, as the contacted close tip became a point of rotation, which aided in the direction of folding. For longer hinges that had far tip contact, the contacted tip did not become a new point of rotation, so in principle, it could not be fully folded.

4.2.2 Displacement controlled folding (DCF)

Figure 4-10: Stress distribution for DCF

Figure 4-11: Max stress growth for DCF

The stress distribution and stress growth of displacement controlled folding showed• The final stress level was very close to yield stress and only slightly varied between different hinge positions. However, during the folding process, the stress significantly concentrated towards to center of the hinge.

- The stress level increased with the folding process, and it exceeded the yield stress before the panels came into full contact.

During the folding process, only the displacement of the panels was controlled and panels were free to rotate. As a result, the bending of the membrane was not controlled, therefore, the stress was not evenly distributed across the hinge. If additional moment was applied in conjunction with the placement, the absolute maximum stress could be reduced. For this study, increasing panel thickness was also effective since the bending radius could be increased all across the hinge.

In order to study if increasing the hinge width would reduce the maximum stress, a parametric sweep was performed under the same set of boundary conditions, except that the hinge width was increased. Hinge Width Index (HWI) was used instead of the hinge width so the results would apply to crease designs with various bending radii.

Figure 4-12: Max stress vs increasing HWI for DCF

Figure 4-13: Final Stress distribution for DCF

Table 4.2: Form of contact for DCF.

Contact form	Corresponding HWI
Full contact	All range

DCF was designed to fully close the panels, and it was important because

whatever the folding control scheme was, when the panels were fully folded, the hinges would have the same final stress distribution. As a result, DCF was the best in studying the stress distribution change with increasing hinge width. An extended range of hinge width was also studied and the results were in figure 4-14:

Figure 4-14: Bending stress distribution for various HWI for DCF with longer hinges

Figure 4-15 Hinge geometry for various HWI for DCF

Figure 4-16: Max stress vs increasing HWI for DCF

For models with significantly longer hinges, when they were fully folded by DCF, the stress concentrated at the middle of the hinge. For $HWI > 2.2$, at the side of the hinge, the direction of the stress was the inverted. Significantly increasing the hinge width (more than double) could reduce the maximum stress to the yield stress, which was much less effective than slightly increasing the panel thickness to accommodate for the higher absolute stress.

It was also to note that since von Mises stress was not able to reflect the direction of the bending stress, in DCF analysis with long hinges, second Piola-Kirchhoff stress, which measured the bending stress, was used to demonstrate the change in the stress direction.

4.2.3 Force controlled symmetrical folding

Figure 4-17: Stress distribution for FCSF

Figure 4-18: Max stress growth for FCSF

Force controlled folding was used to represent human hand folding, and three sets of different boundary conditions were investigated. The stress distribution was similar to that of the DCF, but it consistently grew during the folding process.

In order to study if increasing the hinge width would reduce the maximum stress, a parametric sweep was performed under the same set of boundary conditions, except that

the hinge width was increased. Hinge Width Index (HWI) was used instead of the hinge width so the results would apply to crease designs with various bending radii.

Figure 4-19: Max stress vs increasing HWI for FCSF

Figure 4-20: Final stress for various HWI for FCSF

Table 4.3: Forms of contact for FCSF

Contact form	Corresponding HWI
Full contact	$\text{HWI} \leq 1.02$
Far tip contact	$\text{HWI} > 1.05$

Similar to the case of PCF, increasing the hinge width would reduce the stress concentration, however, the panels would not come into full contact.

4.2.4 Results from unsymmetrical force folding

Figure 4-21: Stress distribution for FCUF

Figure 4-22: Max stress growth for FCUF

FCUF was significantly different from the previously introduced folding control because it was unsymmetrical. In order to move one panel into place, a high boundary load was used and it directly resulted in high stress concentration at the fixed end of the hinge that could lead to material failure. Increasing the panel thickness could not reduce the absolute maximum stress in this case (it related to loading), and it remained to be seen whether increasing hinge width could alleviate this issue.

Figure 4-23: Max stress vs increasing HWI for FCUF

Figure 4-24: Stress distribution for various HWI for FCUF

Table 4.4 Forms of contact for FCUF

Contact form	Corresponding HWI
Close tip contact	$\text{HWI} < 0.95$
Full contact*	$0.97 \sim 1.02$
Far tip contact	$\text{HWI} > 1.05$
* Panels are slightly miss-matched	

As it turned out, increasing the hinge width would not bring down the maximum stress concentrated at the fix end of the hinge. Additional modifications were needed. In real life, membranes would be under various uncontrolled and unpredictable folding and loading conditions like FCUF, and there was definitely a need to implement the basic hinge-line geometry. Supports and fillets that could control the bending radius during folding process was studied in section 4.3 as a proposed solution.

4.2.5 Results from follower force folding

Figure 4-25: Stress distribution for FCFF

Figure 4-26 Max stress growth for FCFF

For FCFF, the boundary load was normal to the rotated panel. As a result, the effective stress was resulted from a pure moment combined with a force at the hinge. It was a very effective way to fold the panels, but additional position feedback was needed in the folding process. Since this loading case was a combination of a pure moment and a force, increasing panel thickness could be helpful in adding some safety factors.

Figure 4-27 Max stress vs increasing HWI for FCFF

Figure 4-28: Final stress distribution for various HWI for FCFF

Table 4.5: Forms of Contact for FCFF

Contact form	Corresponding HWI
Close tip contact	HWI < 0.95
Full contact	0.97~1.02
Far tip contact	HWI > 1.05

The HWI study produced results that had the same pattern compared to FCSF. It was not unexpected, and as explained before, a follower force could be viewed as a combination of a moment and a force that always pointed towards the opposing panel. Stress were always evenly distributed along the hinge for moment controlled folding, so the FCFF stress distribution took on the pattern of a force controlled folding stress distribution.

4.3 Different methods to offset the high stress concentration

After thorough investigation of the stress distribution associated with each type of folding conditions, increasing panel thickness, thus increasing bending radius would effectively reduce stress concentration in membrane hinges. However, in some situations, unexpected high stress could occur during folding, and it could not be controlled by simply using thicker panels or increasing hinge width. In this section, the effect of proposed hinge-line designs in the form of supports was studied. Effect of adding fillets, a commonly seen feature in compliant mechanisms, was compared.

4.3.1 Adding circular supports

During elastic folding of membrane hinges, it was found that hinges that bend with constant radius had the lowest maximum stress. Supports, which could help with restricting the radius of bending membrane were proposed. FEA was used to study how two types of circular supports performed on controlling the unexpected high stress. The loading case was selected to be force controlled unsymmetrical folding. Compared to a basic, unsupported model, both proposed supports greatly reduced the absolute maximum

stress introduced by the loading conditions.

Figure 4-29: Side profile of a split support

Figure 4-30: High stress caused by loading conditions was reduced by adding split support.

Figure 4-31: Side profile of a whole support

Figure 4-32: High stress caused by loading conditions was reduced by adding whole support.

Figure 4-33 Whole support (WS) vs split support (SS)

Both split support and whole support was able to control the bending of the membrane. There were some slight differences in the stress distribution results, making one type of support better than the other in some applications.

- Loading conditions: split support design was symmetrical, so panels would be folded from any side; whole support design must have the supported side fixed.
- Stress control level: split support design could have higher stress concentration when the tips contact with each other and reduce the bending radius; whole support design could strictly control the bending radius.
- Overall Dimension: split support design had the same thickness with that of panels; whole support design added extra height to unfolded geometry.
- Adaptability: Split design could be easily adapted into various shapes for additional functionality, such as controlling the folded angle.

In short, whole support fully controlled the bending radius of the membrane, while split support had more potential applications.

Figure 4-34: Split supports that took on various shapes (left) for thick panels (middle) control folded panel angle for movement (right) strictly control folded panel angle for stowage.

Three types of adapted split support design provided additional functionalities besides assist in folding:

- For panels whose thickness (T) greatly exceed the minimum bending radius (R') of the membrane (so $T > R'$), a support in figure 4.34 (left) could be used. Without supports, the membrane must still fold in the shape of semi-circle, so the hinge width should be calculated with $w = \pi T$. When using the support, the hinge width $w' = \pi R' + 2(T - R')$, which was shorter than w , and it should be the shortest hinge width possible.
- For panels that could be in movement, using support shown in figure 4.34 (middle) could control the final folded panel angle and reduce the impact between panels during folding.

- The support design in figure 4.34 (right) could lock panels from further movement and protect the membranes from impact external loading on the membrane itself.

4.3.2. Adding oval supports

Compared to circular supports, supports with inconstant radius were studied to see if they could have potential applications, and the short answer was negative. This study was carried out using the displacement controlled method so the panels always fully fold. Also, the hinge width was adjusted to be 2% larger than the perimeter of the supports, to ensure proper contact between panels and membrane.

Figure 4-35: Side profile of an oval support. The displayed support had $a/b=0.8$

Figure 4-36: Stress distribution of oval supported hinges

Figure 4-37: Max stress variation with different oval shapes

From the stress distribution between oval supports of different aspect ratios (a/b), It was clear that the membrane fully followed the contour of the support, and the stress was concentrated where the supports had smaller radius. Interestingly, having small support radius close to the sides of the membrane ($a/b = 0.8$) was much worse than having small radius at the center of the membrane ($a/b = 1.2$) for maximum stress.

4.3.3 Adding fillets

Fillets was a feature commonly implemented in compliant mechanism, because they were: (1) often unavoidable from manufacturing processes and (2) help with adhesion between panels and membranes. The adverse effects of fillets was not often discussed. In this section, fillets were added to the basic model and analyzed under displacement controlled folding. Two situations were considered: (1) hinge width was kept the same and (2) double fillets radius was added to the hinge.

Figure 4-38: basic design with fillets with unspecified change in hinge width

Figure 4-39: Stress distribution with increasing fillet radius (fixed hinge width)

Figure 4-40: Maximum stress increased significantly with fillets (fixed hinge width).

Figure 4-41: Stress distribution with increasing fillet radius (extended hinge width)

Figure 4-42: Maximum stress increased slightly with fillets (extended hinge width)

Combining the results from both situations, the following conclusions could be made:

- Adding fillets to the hinge-line was equivalent to shortening the hinge width, which was undesired as explained before.
- When fillets were unavoidable or desired, adjustments need to be made to basic hinge-line design elements, like increasing panel thickness and increasing panel width.

4.4 Physical model validation

FEA results could only be meaningful if they were verified against real life scenarios. Unfortunately, directly measuring stresses on a folded membrane was not an easy task. The limited availability of proposed material, lack of material processing equipment, and the difficulty in measuring large strain/flexure at such small scale were all the challenges faced in this task. A compromised plan was proposed to qualitatively verify the stress results obtained from FEA using 3D printed panels and plastic membrane.

4.4.1 Qualitative testing of various hinge-line geometries

Measuring the bending stress of a material that was under large strain was not easy. Bending stress, which was the stress of interest in this FEA, could be usually measured by attaching strain gauges to the top and bottom surface. There were two challenges associated with testing stresses in this way:

- Commercially available strain gauges was not able correctly measure the large range of strain for this application.
- The strain gauges themselves were too thick compared to the thickness of the membrane.

Besides the lack of stress testing methods, BMG ribbon, which was the material of interest, was not easy to manipulate with.

- BMG was a high cost material and only a small quantity was available for this study.
- BMG ribbon's mechanical properties was not provided, and BMG ingots made for tensile test often had different properties. Since the differences between BMG ingots and BMG ribbons were largely resulted from processing, little could be done to obtain the exact mechanical property of BMG ribbon. Measuring the mechanical property of a thin elastic sheet was a still a challenge in general.
- One of the major contribution of this dissertation was the supported panel design. Since BMG ribbon's minimum bending radius was around only 1mm, it was difficult to attach it to rigid panels of 1 mm thick and supports to them.

In the end, a compromised solution was proposed. Since all the study was carried out focusing on thin membrane folding in the elastic region, alternative thin elastic sheet of a reasonable elastic limit was used as the hinge. Instead of quantitatively measuring the bending stress across the hinge, qualitatively checking the failure pattern of folded test assembly was used.

The elastic sheet material of choice was a plastic sheet cut from a file holder. It

was the perfect material for testing failure patterns:

- The material was low cost and available in relatively large quantity.
- Plastic sheet could be cut simply with a knife, and the cutting edge was clean.
- The thickness of the plastic sheet was 0.4 mm and had a minimum elastic bending radius of around 5 mm. when it was bended beyond elastic range, plastic deformation could be easily spotted, but permanent creasing only appeared when the bending radius was less than 2 mm.
- The elastic modulus of the material was reasonable. Bending it with easy, and folded membrane could easily spring back with light load.

Since 3D printing machines were readily available, 3D printed panels with different hinge-line designs was used. The panels used in this testing had various thickness between 4 mm and 15 mm, and supports of different geometries were added to the base designs to test their effectiveness against high stress concentration.

As a summary, the proposed testing procedure was as follows:

Step 1: test the plastic sheet's minimum bending radius and design panels with various thicknesses and support geometry.

Step 2: Print the panels using 3D printing machine (Makerbot Replicater Plus) and bond the membrane to the panels using sheet adhesive using the specified distance as panel spacing.

Step 3: fold the assembly by hand for at least five time and observe the failure pattern.

The pattern was then compared to the expected failure pattern.

Figure 4-43: Three steps for testing bending failure pattern (top) step1 material testing and design panels (middle) step 2 print panels using 3D printer (bottom) step 3 assemble the components and fold the assembly

4.4.2 Test assembly

The previous sub-section had introduced the proposed testing method, and in this sub-section, detailed information on the fabrication process was presented.

Before the proper panel design could be finalized, one crucial piece of material property was needed: minimum bending radius. It was directly related to the yield strain of the material and not available. As a result, a separate tester was designed to measure the bending radius of *any* thin, elastic membrane.

The Minimum Bending Radius tester (MBR tester) was similar to a go/no-go wrench, which was used to test if a part's dimension was within the allowable range. The MBR tester had a high resolution, 3D printed, cone-shaped surface of known radius profile, and when an elastic sheet was pressed against the curved surface, it would follow the contour of the curved surface. If the radius of the tester curve was smaller than the elastic limit of the membrane, permanent deformation could be observed. Several iterations of the tester were fabricated, and the 3rd and 4th iteration were used to measure the minimum bending radius of the material (table 4.6).

After the minimum bending radius of 5 mm was obtained, the Test Assembly (TA) could be fabricated. Panels of several thickness T were printed and assembled with the clear membrane. The panel was spaced following the equation

$$w = HWI * w_1 = HWI * \pi T \quad (\text{eq. 4.1})$$

The selected HWI was 1. During the assembly process, the actual hinge width could be slightly higher than the assigned value.

Table 4.6: Comparison between different iterations of MBR testers.

Iteration	Geometry	range	comment
1		3-15 mm	Low print quality, radius of middle and right cone too large
2		5-20 mm	Low print quality, range was not low enough
3		2-5mm	Double cone was used to help the membrane conform to the surface
4		5-15mm	Built after previous iteration for testing other materials

Table 4.7: List of all test assemblies

TA	Panel thickness	Support	Geometry
1	4 mm	no
2	4 mm	whole
3	5 mm	whole
4	6 mm	whole
5	7 mm	whole

4.4.3 Test results

Each test assembly was folded in a specified way and the membrane section was examined and its failure pattern was compared to the expectations gained from the previous sections. All the test assembly behaved as expected, and qualitatively verified the results obtained from FEA.

Table 4.8: List of test results

TA	Expectation	Fold method	reason	Test result
1	Deformation at the middle of the hinge	DCF & FCSF	Stress was highest at the middle of the hinge	As expected
2	Deformation at the curved surface	FCUF	Radius was too small at the support	As expected
3	No or very slight level of deformation	FCUF	Radius was at critical value	As expected
4	No deformation	FCUF	Radius exceeded MBR by 20%	As expected
5	No deformation	FCUF	Radius exceeded MBR by 40%	As expected

4.5 Potential application

As the last section of the chapter, some potential applications of hinge-line designs were suggested in Table 4.9. All the models demonstrated in this part was fabricated following the procedure used for the test assembly in the previous section. It was to note that for the following hinge-line demonstration, only one basic test assembly was made with panel thickness of 15 mm and $HWI=1.02$. Three sets of supports were printed separately as inserts. The radius of inserts were 15 mm, 15 mm and 30 mm.

To demonstrate how the application guide could be used, a small loading-lifting mechanism was designed and fabricated (fig. 4-44) . The mechanism consisted of an outer shell and a 4 link linear fold-core. The fold-core was made from bonding the clear membrane to the 10mm thick, 20 mm wide panels and designed to be folded under the displacement controlled folding conditions. The outer shell was designed to control linear the movement of the panels, and bamboo sticks was used to allow the panel to rotate properly.

Depending on what type of supports were used, the mechanism could have different applications. When angle controlling supports were used, applying a horizontal input force from right would allow the panel to move, rotate 90 degrees and move some load up (top of the mechanism)/down (bottom of the mechanism) . When no support (as shown in figure) or split supports were used, apply a horizontal force from the right would allow the panel to travel a long distance and a higher amount of energy was stored in the membran. In short, this mechanism could be used as load lifting mechanism or a linear spring with high range of motion.

Table 4.9: Potential application guide

Hinge-line design	Applicable field
<p>General origami crease under symmetrical folding</p> <ul style="list-style-type: none"> • membrane not prone to sudden failure • membrane not protected against external loading or impact
<p>Origami crease under any folding conditions</p> <ul style="list-style-type: none"> • membranes protected throughout folding process • load lifting mechanisms • spring of high range of movement
<p>Origami crease with supported side fixed</p> <ul style="list-style-type: none"> • membrane <i>fully</i> protected throughout folding process • used in place of common hinges
<p>Origami crease that needed angle control</p> <ul style="list-style-type: none"> • membrane protected throughout folding process • fold angle and bending radius could be adjusted • membrane could be shortened

Figure 4-44: Proposed mechanism that could be used for load lifting (left) or energy storage (right)

Chapter 5

Conclusions

This conclusion section of the dissertation briefly summarized the FEA findings and a design guideline is proposed for designing hinge-line geometry for origami inspired devices. Original contributions are explained, and the direction of future work is also pointed.

5.1 Key FEA findings

- The stress distribution across the thin elastic membrane hinge depended on the loading conditions and contact pattern between the panels.
- Increasing panel thickness could effectively reduce stress level of the hinge.
- The hinge width, which was equivalent to panel spacing, should be calculated based on both the panel thickness, membrane thickness, and the elastic limit of the material.
- Supports of circular profile was effective in reducing stress concentration and could add functionality to the mechanism.

5.2 Guideline for design

A brief guideline for designing panel geometry when using elastic membrane as a hinge is created using the results.

Step 1: Obtain hinge material property elastic modulus and membrane thickness. Or prepare a minimum bending radius tester.

Step 2: Calculate or measure the minimum bending radius $R_{\min}$.

Step 3: Prepared 3D printed panels of thickness T or cut it from any stock material of thickness of T . Remember T should be greater than $R_{\min}$

Step 4: Select appropriate supports for the desired applications. Supports add functionality and protection for membranes so they are strongly recommend.

Step 5: Panel spacing w can be determined using T . For basic models, use $w = \pi T$, for supported models, w should be the perimeter of the curved surface.

Step 6: Install the membrane using preferred fastening methods.

5.3 Original contributions

This dissertation had three major contributions, and they were mainly based on Finite element analysis.

- Proposed five sets of boundary condition for FEA to fully fold an origami crease composed of panels and membrane hinges.
- Discovered that the stress concentration of an elastic membrane was depended on the loading conditions.
- Proposed various hinge-line designs in the form of supports to control the stress of the membrane.
- Provided qualitative validation of the proposed hinge-line design.

5.4 Future work

Several aspects of this study can be improved in the future.

- The scope of this is limited to 2D and a single crease, and it would be very beneficial to extend the work to 3D vertices and multi components.
- Developing low cost measurement system to measure the bending stress of an origami crease.
- Expanding the membrane material to an active material such as SMA and design load-lifting mechanisms that could be easily controlled.

5.5 Final Conclusion

This study used FEA to obtain stress analysis of various origami hinge-line designs under different loading conditions. A design guideline was presented to justify the choice of loading conditions, panel thickness, panel spacing and panel modifications. The results were qualitatively verified with new testing method and several potential applications were proposed. This analysis could also serve as a guide to set up compliant beam study with geometric non-linearity caused by large displacement and contact pairs using COMSOL Multiphysics

References

- Abbott, A. C., Buskohl, P. R., Joo, J. J., Reich, G. W., & Vaia, R. A. (2014). Characterization of creases in polymers for adaptive origami structures. In *ASME 2014 Conference on Smart Materials, Adaptive Structures and Intelligent Systems* (p. V001T01A009-V001T01A009). <https://doi.org/10.1115/SMASIS20147480>
- Axinte, E. (2012). Metallic glasses from “alchemy” to pure science: Present and future of design, processing and applications of glassy metals. *Materials and Design*, 35, 518–556. <https://doi.org/10.1016/j.matdes.2011.09.028>
- Bhuiyan, M. E. H., Semer, D., & Trease, B. P. (2017). Dynamic Modeling and Analysis of Strain Energy and Centrifugal Force Deployment of an Origami Flasher. In *ASME 2017 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference* (pp. 1–14). Cleveland, Ohio.
- Cash, T. N., Warren, H. S., & Gattas, J. M. (2015). Analysis of miura-type folded and morphing sandwich beams. In *ASME 2015 International Design Engineering Technical Conferences & Computers and Information in Engineering Conference* (pp. 1–9).
- Crampton, E. B., Magleby, S., & Howell, L. L. (2017). Realizing origami mechanisms from metal sheets. In *ASME 2017 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference* (pp. 1–10).

- Francis, K. C., Blanch, J. E., Magleby, S. P., & Howell, L. L. (2013). Origami-like creases in sheet materials for compliant mechanism design. *Mechanical Sciences*, 4(2), 371–380. <https://doi.org/10.5194/ms-4-371-2013>
- Greenberg, H. C., Gong, M. L., Magleby, S. P., & Howell, L. L. (2011). Identifying links between origami and compliant mechanisms. *Mechanical Sciences*, 2(2), 217–225. <https://doi.org/10.5194/ms-2-217-2011>
- Hasegawa, R. (2004). Applications of amorphous magnetic alloys. *Materials Science and Engineering A*, 375–377(1–2 SPEC. ISS.), 90–97. <https://doi.org/10.1016/j.msea.2003.10.258>
- Homer, E. R., Harris, M. B., Zirbel, S. A., Kolodziejaska, J. A., Kozachkov, H., Trease, B. P., ... Hofmann, D. C. (2014). New methods for developing and manufacturing compliant mechanisms utilizing bulk metallic glass. *Advanced Engineering Materials*, 16(7), 850–856. <https://doi.org/10.1002/adem.201300566>
- Hou, D., Ma, J., Chen, Y., & You, Z. (2017). Analysis of cross folding an elastic sheet. In *ASME 2017 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference* (pp. 1–8). <https://doi.org/10.1115/DETC2017-67388>
- Howell, L. L. (2001). *Compliant Mechanisms*. New York, NY: John Wiley & Sons.
- Howell, L. L., Magleby, S. P., & Olsen, B. M. (Eds.). (2013). *Handbook of Compliant Mechanisms*. Wiley.
- Hu, J., Marciniak, Z., & Duncan, H. (2002). *Mechanics of Sheet Metal Forming* (2 nd). Butterworth-Heinemann.

- Inoue, A., & Takeuchi, A. (2011). Recent development and application products of bulk glassy alloys. *Acta Materialia*, 59(6), 2243–2267.
<https://doi.org/10.1016/j.actamat.2010.11.027>
- Ishida Sachiko, & Ichiro, H. (2014). Introduction to Mathematical Origami and Origami Engineering. In *Forum of Mathematics for Industry 2014*.
<https://doi.org/10.1007/978-4-431-55342-7>
- KLEMENT, W., WILLENS, R. H., & DUWEZ, P. (1960). Non-crystalline Structure in Solidified Gold-Silicon Alloys. *Nature*, 187(4740), 869–870.
- Klett, Y., Zeger, C., & Middendorf, P. (2017). Experimental characterization of pressure loss caused by flow through foldcore sandwich structures. In *International Design Engineering Technical Conferences and Computers and Information in Engineering Conference* (pp. 1–7). Cleveland, Ohio.
- Kruzic, J. J. (2016). Bulk Metallic Glasses as Structural Materials: A Review. *Advanced Engineering Materials*, 18(8), 1308–1331. <https://doi.org/10.1002/adem.201600066>
- Kumar, G., Desai, A., & Schroers, J. (2011). Bulk metallic glass: The smaller the better. *Advanced Materials*, 23(4), 461–476. <https://doi.org/10.1002/adma.201002148>
- Kuribayashi, K., Tsuchiya, K., You, Z., Tomus, D., Umemoto, M., Ito, T., & Sasaki, M. (2006). Self-deployable origami stent grafts as a biomedical application of Ni-rich TiNi shape memory alloy foil. In *Materials Science and Engineering A* (Vol. 419, pp. 131–137). <https://doi.org/10.1016/j.msea.2005.12.016>
- Lang, R. (2004). Origami : Complexity in Creases (Again). *Engineering & Science*, 8–19.

- Liu, X., Yao, S., Gibson, J., & Georgakopoulos, S. V. (2017). Frequency reconfigurable qha based on kapton origami helical tube for gps, radio and wimax applications. In *ASME 2017 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference* (pp. 1–6).
<https://doi.org/10.1115/DETC2017-68048>
- Liu, Y., Genzer, J., & Dickey, M. D. (2016). “2D or not 2D”: Shape-programming polymer sheets. *Progress in Polymer Science*, 52, 79–106.
<https://doi.org/10.1016/j.progpolymsci.2015.09.001>
- Lobontiu, N. (2002). *Compliant Mechanisms: Design of Flexure Hinges*. CRC Press.
- Ma, F., & Chen, G. (2015). Modeling large planar deflections of flexible beams in compliant mechanisms using chained beam-constraint-model . *Journal of Mechanisms and Robotics*, 8(2), 021018. <https://doi.org/10.1115/1.4031028>
- Maanasa, V. L., & Reddy, S. R. L. (2014). Origami-innovative structural forms & applications in disaster management. *International Journal of Current Engineering and Technology*, 4(5), 3431–3436.
- Materion. (2017). Data Sheet: Bulk Metallic Glass – The Next Metal.
- Miyashita, S., Meeker, L., Tolley, M. T., Wood, R. J., & Rus, D. (2014). Self-folding miniature elastic electric devices. *Smart Materials and Structures*, 23(9).
<https://doi.org/10.1088/0964-1726/23/9/094005>
- Morgan, M. R., Lang, R. J., Magleby, S. P., & Howell, L. L. (2016). Towards developing product applications of thick origami using the offset panel technique. *Mechanical Sciences*, 7(1), 69–77. <https://doi.org/10.5194/ms-7-69-2016>

- Morris, E., McAdams, D. A., & Malak, R. (2016). The state of the art of origami-inspired products: a review. In *ASME 2016 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference* (p. V05BT07A014). <https://doi.org/10.1115/DETC2016-59629>
- Natori, M. C., Katsumata, N., Yamakawa, H., Sakamoto, H., & Kishimoto, N. (2013). Conceptual model study using origami for membrane space structures. In *ASME 2013 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference* (pp. 1–12). <https://doi.org/10.1115/DETC2013-13490>
- Nelson, T. G., Lang, R. J., Magleby, S. P., & Howell, L. L. (2016). Curved-folding-inspired deployable compliant rolling-contact element (D-CORE). In *Mechanism and Machine Theory* (Vol. 96, pp. 225–238). <https://doi.org/10.1016/j.mechmachtheory.2015.05.017>
- Nguyen, H. T. T., Terada, K., Tokura, S., & Hagiwara, I. (2015). Development of grooving technique for origami-forming. In *ASME 2015 International Design Engineering Technical Conferences & Computers and Information in Engineering Conference* (pp. 1–10). <https://doi.org/10.1115/DETC2015-47079>
- Pan, C. T., Wu, T. T., Chen, M. F., Chang, Y. C., Lee, C. J., & Huang, J. C. (2008). Hot embossing of micro-lens array on bulk metallic glass. *Sensors and Actuators, A: Physical*, 141(2), 422–431. <https://doi.org/10.1016/j.sna.2007.10.040>
- Peraza-Hernandez, E. A., Hartl, D. J., Malak, R. J., & Lagoudas, D. C. (2014). Origami-inspired active structures: A synthesis and review. *Smart Materials and Structures*, 23(9). <https://doi.org/10.1088/0964-1726/23/9/094001>

- Pradier, C., Cavoret, J., Dureisseix, D., Jean-Mistral, C., & Ville, F. (2016). An Experimental Study and Model Determination of the Mechanical Stiffness of Paper Folds. *Journal of Mechanical Design*, 138(4), 041401. <https://doi.org/10.1115/1.4032629>
- Qiu, C., Aminzadeh, V., & Jian S. Dai. (2013). Kinematic and stiffness analysis of an origami-type carton. In *ASME 2013 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference* (Vol. Volume 6B:, pp. 1–9). Retrieved from <http://proceedings.asmedigitalcollection.asme.org/proceeding.aspx?articleid=1830747>
- Quaglia, C. P., Yu, N., Thrall, A. P., & Paolucci, S. (2014). Balancing energy efficiency and structural performance through multi-objective shape optimization: Case study of a rapidly deployable origami-inspired shelter. *Energy and Buildings*, 82, 733–745. <https://doi.org/10.1016/j.enbuild.2014.07.063>
- Rao, A., Tawfick, S., Shlian, M., & Hart, A. J. (2013). Fold mechanics of natural and synthetic origami papers. In *ASME 2013 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference* (p. V06BT07A049-V06BT07A049). <https://doi.org/10.1115/DETC2013-13553>
- Rapid Quench Machine System. (n.d.). Retrieved from <https://www.youtube.com/watch?v=76k8FJsluYA>
- Reynolds, W. D., Jeon, S. K., Banik, J. A., & Murphey, T. W. (2013). Advanced folding approaches for deployable spacecraft payloads. In *ASME 2013 International Design*

- Engineering Technical Conferences and Computers and Information in Engineering Conference* (pp. 1–10). Portland, Oregon. <https://doi.org/10.1115/DETC2013-13378>
- Schenk, M., Kerr, S. G., Smyth, A. M., & Guest, S. D. (2013). Inflatable cylinders for deployable space structures. In *First Conference Transformables 2013* (pp. 1–6). Retrieved from <http://www.markschenk.com/research/files/schenk2013-Transformables.pdf>
- Schroers, J. (2005). The superplastic forming of bulk metallic glasses. *The Journal of The Minerals, Metals & Materials Society (TMS)*, 57, 35–39. <https://doi.org/10.1007/s11837-005-0093-2>
- Schroers, J. (2010). Processing of bulk metallic glass. *Advanced Materials*, 22(14), 1566–1597. <https://doi.org/10.1002/adma.200902776>
- Schroers, J., Nguyen, T., O’Keeffe, S., & Desai, A. (2007). Thermoplastic forming of bulk metallic glass—Applications for MEMS and microstructure fabrication.pdf. *Materials Science and Engineering A*, 449–451(2), 898–902.
- Schroers, J., Pham, Q., Peker, A., Paton, N., & Curtis, R. V. (2007). Blow molding of bulk metallic glass. *Scripta Materialia*, 57(4), 341–344. <https://doi.org/10.1016/j.scriptamat.2007.04.033>
- Sung, E., Erol, A., Frecker, M., & von Lockette, P. (2016). Characterization of Self-Folding Origami Structures Using Magneto-Active Elastomers. In *ASME 2016 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference* (pp. 1–7). <https://doi.org/10.1115/DETC2016-59919>
- Suryanarayana, C., & Inoue, A. (2011). *Bulk Metallic Glasses* (1 st). Boca Taton, FL.

- Tachi, T. (n.d.-a). Freeform Origami. Retrieved from www.tsg.ne.jp/TT/software/
- Tachi, T. (n.d.-b). Rigid Origami simulator. Retrieved from www.tsg.ne.jp/TT/software/
- Tachi, T. (2010). Geometric considerations for the design of rigid origami structures. *International Association for Shell and Spatial Structures (IASS) Symposium 2010*, 12(Figure 2), 458–460. <https://doi.org/10.1016/j.mpaic.2011.07.005>
- Thrall, A. P., & Quaglia, C. P. (2014). Accordion shelters: A historical review of origami-like deployable shelters developed by the US military. *Engineering Structures*, 59, 686–692. <https://doi.org/10.1016/j.engstruct.2013.11.009>
- Tregilgas, J. H. (2004). Amorphous titanium aluminide hinge. *Advanced Materials and Processes*.
- Zhang M, Trease BP. Design of Hinge-Line Geometry to Facilitate Non-Plastic Folding in Thin Metallic Origami-Inspired Devices. ASME. IDETC/CIE, Volume 5B: 42nd Mechanisms and Robotics Conference (2018):V05BT07A066. doi:10.1115/DETC2018-86267.
- Zirbel, S. A., Magleby, S. P., Howell, L. L., Lang, R. J., Thomson, M. W., Sigel, D. A., ... Trease, B. P. (2013). Accommodating thickness in origami-based deployable arrays. In *ASME 2013 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference* (Vol. d, pp. 1–12). <https://doi.org/10.1115/DETC2013-12348>