Updating Bridge Deck Condition Transition Probabilities as New Inspection Data are
Collected: Methodology and Empirical Evaluation

THESIS

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Abstract

Deterioration is among the primary concerns regarding the structural performance and functionality of bridges and their components. In light of annual budget constraints, infrastructure agencies, such as state Departments of Transportation (DOTs) in the US, prioritize their bridge maintenance needs. To make trade-offs across bridges and over time, key inputs to the prioritization and decision making process are bridge condition assessments and predictions.

In this study, a Bayesian updating procedure is proposed to estimate a Markov Chain based concrete deck deterioration model in a manner that combines condition data collected over two inspection cycles and the deterioration information available prior to the collection of these condition data. Single period (one year) transition probabilities are estimated using Bayesian updating and maximum likelihood estimation, where in the case of the latter only the collected condition data over two inspection cycles are used.

A dataset of 357 bridge deck condition assessments based on AASHTO condition state definitions collected by a state infrastructure agency spanning two years is used to evaluate the performance of the two methods. Training and validation datasets are selected from the original dataset where the former is used for estimation and the latter for prediction and evaluation. An experimental design is conceived where five bridges with distinctively different deterioration are considered to belong to the two sets or neither in various combinations. The evaluation is based on measuring the degree of

similarity between reported condition states and those predicted based on the estimated transition probabilities using the two methods. While updating transition probabilities as new data are collected is found to be advantageous for many cases, this advantage is highly dependent on the deterioration nature of the bridge decks reflected in the training dataset. Finally, directions for future research are discussed.

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Chapter 1: Introduction and Motivation

Deterioration is among the primary concerns regarding the structural performance and functionality of bridges and their components. Such degradations can potentially lead to various forms of structural deficiencies and eventually failure if adequate maintenance, repair and rehabilitation actions are not performed at appropriate times. Moreover, the degraded functionality of bridge decks and the application of time-consuming repairs and rehabilitations lead to increased traffic delays and increased vehicle operating costs. In light of annual budget constraints, infrastructure agencies, such as state Departments of Transportation (DOTs) in the US, prioritize their bridge maintenance needs. Commonly, bridges with higher degradation levels receive higher priority for maintenance, repair and rehabilitation. To make trade-offs across bridges and over time, key inputs to the prioritization and decision-making process are bridge condition assessments and predictions.

To support such functionalities, the Federal Highway Administration (FHWA) developed the bridge management system AASHTOWare BrM (AASHTO BrM User Manual, 2015) – formerly known as Pontis (Pontis User Manual, 2005) – that has been successfully used in numerous practical decision-making settings. It is based on a discrete time Markov Chain deterioration model in which the type and extent of deterioration of bridge components are assessed using visual inspections. Critical parameters of discrete Markovian deterioration models are probabilities that describe the likelihoods of bridge

components transitioning from one discrete condition state to another. These transition probabilities are either determined based on expert judgement or, ideally, they are estimated based on field inspection data. Numerous studies focused on the estimation of transition probabilities for various infrastructure types including bridges, roadway pavements, and storm water and wastewater pipes. Examples include Carnahan et al. (1987), DeStefano and Grivas (1988), Jiang et al. (1988), Madanat et al. (1995), Madanat and Wan Ibrahim (1995), Madanat et al. (1997), Micevski et al. (2002), Mishalani and Madanat (2002), Baik et al. (2006), and Kobayashi et al. (2010).

The AASHTO Bridge Element Inspection Guide Manual (2010), subsequently referred to as the AASHTO Manual, establishes a national guideline for performance evaluation of bridge elements and collection of inspection data. Such data are used in both AASHTOWare BrM and this study for deterioration analysis of bridges. In the AASHTO Manual, the state of performance of a bridge element is expressed as a condition state vector. For example, for bridge decks, the focus of this study, four condition states are defined based on the extent of different types of defects, such as cracking, spalls and delamination. The definition of condition states of bridge decks is characterized in Table 1 (AASHTO, 2010). For a given bridge, each entry of the condition state vector is the proportion of the deck that is in the corresponding condition state.

Table 1. Definition of condition state for bridge deck based on the AASHTO Manual

Defect	Condition State 1	Condition State 2	Condition State 3	Condition State 4
Cracking	None to hairline	Narrow size and/or density	Medium size and/or density	The condition is beyond the limits established in condition state three (3) and/or warrants a structural review to determine the strength or serviceability of the element or bridge
Spalls/Delaminations/Patched Areas	None	Moderate spall or patch areas that are sound	Severe spall or patched area showing distress	
Efflorescence	None	Moderate without rust	Severe with rust staining	
Load Capacity	No reduction	No reduction	No reduction	

Source: AASHTO Bridge Element Inspection Guide Manual (2010)

The AASHTO Manual, which is adopted by all state transportation agencies in the US, requires regular observations and measurements of highway bridges to capture their physical and functional states. State DOTs usually carry out such bridge inspections once every one to two years, providing an opportunity to continually improve the estimates of bridge deterioration transition probabilities. However, typically and as reflected in Carnahan et al. (1987), DeStefano and Grivas (1988), Jiang et al. (1988), Madanat et al. (1995), Madanat and Wan Ibrahim (1995), Madanat et al. (1997), Micevski et al. (2002), Mishalani and Madanat (2002), Baik et al. (2006), and Kobayashi et al. (2010), resulting inspection data are used on a one-time basis to estimate deterioration models whereby subsequent observations are used only as inputs to forecast deterioration. While Micevski et al. (2002) employed an approach that updates prior

information, this information is assumed to be uninformative rather than based on previously collected data. Durango and Madanat (2002) addressed the notion of updating infrastructure deterioration estimates using an adaptive control framework. However, the results of the study are based on computation- and simulation-based analyses rather than an empirical ones.

The objective of this study is to develop a method that uses the bridge inspection data collected on an ongoing basis to update transition probabilities for concrete bridge decks. Doing so is expected to improve the representativeness of these probabilistic deterioration models and the accuracy of their predictions. Empirical data are used to demonstrate the transition probability updating method and to assess the value of such an approach.

The rest of this thesis is organized as follow. Chapter 2 summarizes the Markov Chain deterioration model and presents derivations of Markov Chain parameter estimation methods pertinent to this study. Chapter 3 describes the data used in this study as well as the procedures employed for preparing the dataset for analysis. Chapter 4 presents the set-up and results of an empirical analyses based on a preliminary experimental design and a refined experimental design. Chapter 5 summarizes the conclusions of this study and provides suggestions for future research.

Chapter 2: Deterioration Model and Parameter Estimation

2.1 Deterioration Model: Markov Chain

As noted previously, based on the condition state definitions, each entry in the condition state vector represents the proportion of the deck that is in that condition state. This definition could be interpreted as the probability of each unit of the deck being in each condition state, whereby for mathematical convenience a deck is divided into a contiguous set of small 1 ft $\times$ 1 ft units forming a grid. A widely-used approach to model such deterioration is the Markov Chain (Golabi et al., 1982, Mauch and Madanat, 2001). Markov Chain models are often used to represent the evolution of discrete and finite state variable at discrete periods (Ross, 1983). A mathematical representation of a Markov Chain is $Y^{(t+1)} = \mathbf{P} \cdot Y^{(t)}$, where $Y^{(t)}$ and $Y^{(t+1)}$ are the vectors that represent the probability mass functions of the discrete state at time t and time $t + 1$, respectively, and $\mathbf{P}$ is the transition probability matrix, which consists of transition probability elements p_{ij} . Each of these elements represents the probability of transitioning to state j at time $t + 1$ given that the condition state is i at time t . The matrix $\mathbf{P}$ could be invariant or may vary over time. In this study, the former is assumed.

This study uses the same Markov Chain deterioration model that is adopted by AASHTOWare BrM Bridge Management System whereby the following assumptions are made: (a) the condition state drops at most by one (i.e., no multi-step transitions are assumed to occur), (b) no rehabilitation action is applied (i.e., no condition state

improvement is assumed to occur), and (c) each unit (1 ft × 1 ft of bridge deck) deteriorates independently. Although AASHTOWare BrM allows for age-variant transition probabilities for condition state 1 to 2 (considering the penetration and accumulation of chloride ions in the early deterioration phase), as noted previously the transition probability matrix P is assumed to be independent of age, as the available age data are not reliable (e.g., a 37-year-old bridge in the dataset is entirely in condition state 1, while a 7-year-old bridge is entirely in condition state 2). Based on the adopted Markov Chain model assumptions, Figure 1 shows the possible deterioration paths for each condition state in one period (one year in the case of the data considered subsequently).

Figure 1. Possible deterioration paths for each condition state

As a result, the Markov Chain model used in this study takes the following form:

$$Y_k^{(t+1)} = P \cdot Y_k^{(t)} \quad (1a)$$

$$\begin{bmatrix} y_{1k}^{(t+1)} \\ y_{2k}^{(t+1)} \\ y_{3k}^{(t+1)} \\ y_{4k}^{(t+1)} \end{bmatrix} = \begin{bmatrix} p_{11} & 0 & 0 & 0 \\ 1 - p_{11} & p_{22} & 0 & 0 \\ 0 & 1 - p_{22} & p_{33} & 0 \\ 0 & 0 & 1 - p_{33} & p_{44} \end{bmatrix} \cdot \begin{bmatrix} y_{1k}^{(t)} \\ y_{2k}^{(t)} \\ y_{3k}^{(t)} \\ y_{4k}^{(t)} \end{bmatrix} \quad (1b)$$

where, $y_{ik}^{(t)}$ = the probability that a $1\text{ft} \times 1\text{ft}$ unit of a bridge deck k is in condition state i at time t .

The only parameters to be determined are transition probabilities p_{11} , p_{22} , p_{33} and p_{44} . Note that the condition state vector is specific to a bridge and, therefore, the use of the index k is necessary. Also, note that the transition probabilities represent the general deterioration and, therefore, are independent of k . In the case of estimation, $Y_k^{(t)}$ and $Y_k^{(t+1)}$ across all bridge records used for estimation are obtained from data, and in the case of prediction, $Y_k^{(t)}$ is obtained from data and $Y_k^{(t+1)}$ is predicted based on Equation 1.

2.2 Parameter Estimation

Different statistical estimation methods could be used to estimate the transition probabilities as illustrated in Carnahan et al. (1987), DeStefano and Grivas (1988), Jiang et al. (1988), Madanat et al. (1995), Madanat and Wan Ibrahim (1995), Madanat et al. (1997), Micevski et al. (2002), Mishalani and Madanat (2002), Baik et al. (2006), and Kobayashi et al. (2010) whereby, as discussed previously, the methods rely on a one-time estimation for a given condition dataset and, therefore, do not take advantage of condition data that stream in on a regular basis to update the estimates of the transition probabilities. In this study, Bayesian estimation is used to determine transition probabilities for concrete bridge decks using inspection data over one period (i.e., two

consecutive inspections) while taking into account the available estimates directly determined from previous data (or expert judgement inspired by previous data), thus, allowing for the incorporation of new inspection data to update the estimates of deterioration models as these data become available. As a reference, maximum likelihood estimation (MLE) is also used whereby only the inspection data over one period are used.

2.2.1 Notation

For the convenience of developing and implementing the above estimation methods, a new set of variables are defined. The transition probabilities p_{ii} and $p_{i \ i+1} = 1 - p_{ii}$ indicate whether a unit on a deck remains in the same state i or transitions to the next lower state $i + 1$, respectively given that the current state is i . The variables $N_{ik}^{(t)}$ and $N_{ik}^{(t+1)}$ represent the area of a bridge deck k that is in condition state i at time t and the area that remains in condition state i at time $t + 1$, respectively. $N_{ik}^{(t)}$ and $N_{ik}^{(t+1)}$ can be derived from inspection data $Y_k^{(t)}$ and $Y_k^{(t+1)}$ as follows:

$$N_{1k}^{(t)} = y_{1k}^{(t)} \times A_k \quad (2a)$$

$$N_{2k}^{(t)} = y_{2k}^{(t)} \times A_k \quad (2b)$$

$$N_{3k}^{(t)} = y_{3k}^{(t)} \times A_k \quad (2c)$$

$$N_{4k}^{(t)} = (1 - y_{1k}^{(t)} - y_{2k}^{(t)} - y_{3k}^{(t)}) \times A_k \quad (2d)$$

$$N_{1k}^{(t+1)} = y_{1k}^{(t+1)} \times A_k \quad (2e)$$

$$N_{2k}^{(t+1)} = (y_{1k}^{(t+1)} + y_{2k}^{(t+1)} - y_{1k}^{(t)}) \times A_k \quad (2f)$$

$$N_{3k}^{(t+1)} = (y_{1k}^{(t+1)} + y_{2k}^{(t+1)} + y_{3k}^{(t+1)} - y_{1k}^{(t)} - y_{2k}^{(t)}) \times A_k \quad (2g)$$

$$N_{4k}^{(t+1)} = (1 - y_{1k}^{(t)} - y_{2k}^{(t)} - y_{3k}^{(t)}) \times A_k \quad (2h)$$

where $A_k = \sum_{i=1}^4 N_{ik}^{(t)} = \sum_{i=4}^4 N_{ik}^{(t+1)}$ is the deck area of bridge k . Based on the definition of $N_{ik}^{(t)}$, Equations 2a-2d are used to compute the deck area of bridge k originally in condition state i at time t from inspection records. $N_{2k}^{(t+1)}$ is calculated by excluding the part of area that was in condition state 1 at time t and deteriorates to condition state 2 at time $t + 1$ from the entire area in condition state 2 at time $t + 1$. Following a similar logic, $N_{3k}^{(t+1)}$ is computed by excluding the part of area that was in condition state 2 at time t and deteriorates to condition state 3 at time $t + 1$ from the entire area in condition state 3 at time $t + 1$.

2.2.2 Maximum Likelihood Estimation

The event that a unit of deck remains in the same condition state or not at the beginning of the next time period (i.e., in the next year in the case of the data used in this study) can be described as a Bernoulli process, which has two outcomes, “success” (i.e., remains in the same state) and “failure” (i.e., transitions to the next state). Assuming that each 1 ft $\times$ 1 ft unit of bridge deck remains in the same condition state independently with the same probability p_{ii} and that all bridge decks deteriorate independently of age, the probability that the total area in condition state i at time $t + 1$ across all bridges whose records are used for estimation, $N_i^{(t+1)} = \sum_{\forall k} N_{ik}^{(t+1)}$, out of total deck area in condition state i at time t across all corresponding bridge records, $N_i^{(t)} = \sum_{\forall k} N_{ik}^{(t)}$, remain in the same condition state is given by:

$$p(N_i^{(t+1)}|p_{ii}) = \binom{N_i^{(t)}}{N_i^{(t+1)}} p_{ii}^{N_i^{(t+1)}} (1 - p_{ii})^{(N_i^{(t)} - N_i^{(t+1)})} \quad (3)$$

That is, the random variable $N_i^{(t+1)}$ follows a binomial distribution with parameter $N_i^{(t)}$ and probability of success p_{ii} . In the context of observing the realization $N_i^{(t+1)}$ and considering p_{ii} as an unknown variable, Equation 3 is the likelihood of observing the outcome $N_i^{(t+1)}$ as a function of p_{ii} . That is:

$$L(p_{ii}; N_i^{(t+1)}) = p(N_i^{(t+1)}|p_{ii}) = \binom{N_i^{(t)}}{N_i^{(t+1)}} p_{ii}^{N_i^{(t+1)}} (1 - p_{ii})^{(N_i^{(t)} - N_i^{(t+1)})} \quad (4)$$

By setting the first derivative of the likelihood function depicted by Equation 4 with respect to p_{ii} to zero and verifying that the second derivative is negative, the estimator of p_{ii} that maximizes the likelihood is determined to be the following:

$$\widehat{p_{u,MLE}} = \frac{N_i^{(t+1)}}{N_i^{(t)}} \quad (5)$$

That is, the maximum likelihood estimate of p_{ii} is the ratio of the deck area that remains in state i at time $t + 1$ to the deck area that was in state i at time t . In this approach, only the inspection data available from two consecutive inspections are used to estimate the transition probabilities and, as a result, the Markov Chain deterioration model.

2.2.3 Bayesian Estimation

Another approach to estimate transition probabilities is that of Bayesian estimation, whereby the transition probabilities are assumed to follow certain distributions, the parameters of which are updated when new condition observations are made. In this approach, the estimates of the transition probabilities available before the

two most recent sets of inspection data are collected are taken into account in the estimation process.

Based on the same set-up as maximum likelihood estimation, the event that a $1 \text{ ft} \times 1 \text{ ft}$ unit of the deck remains in the same condition state or not in the next year is also described as a Bernoulli process. In Bayesian theory, the probability that the event “succeeds” to remain in the same condition state is considered to be a random variable, rather than a fixed value as in the case of maximum likelihood estimation and, therefore, it follows a distribution. This defined distribution is referred to as a prior distribution with known prior parameter values that completely determine the probability density function. Bayes’ law is then used to combine the prior distribution and the recent sets of inspection data to update the prior distribution to what is referred to as a posterior distribution. Subsequently, the estimate of the probability that the event “succeeds” to remain in the same condition state could then be taken to be the mean or the mode of the posterior distribution.

For events that follow the Bernoulli process, the Beta distribution is commonly selected as the prior distribution for mathematical convenience whereby the posterior distribution is derived to also be a Beta distribution through the application of Bayes’ law (Gelman et al., 2014). More specifically, the posterior distribution of the probability p_{ii} of remaining in the same state i at time $t + 1$ given a state i at time t and given a bridge deck area $N_i^{(t+1)}$ that remains in state i at time $t + 1$ is given by the following:

$$p(p_{ii} | N_i^{(t+1)}) = \frac{p(N_i^{(t+1)} | p_{ii}) \times p(p_{ii})}{p(N_i^{(t+1)})}$$

$$\propto p(N_i^{(t+1)} | p_{ii}) \times p(p_{ii}) \quad (6)$$

where $p(p_{ii})$ is the prior Beta distribution of p_{ii} given by the following:

$$p(p_{ii}) = p_{ii}^{a_{prior}-1} \times (1 - p_{ii})^{b_{prior}-1} \quad (7)$$

where a_{prior} and b_{prior} are the parameters for the prior distribution. Equation 6 and 7

imply the following:

$$\begin{aligned} p(p_{ii} | N_i^{(t+1)}) &\propto p_{ii}^{N_i^{(t+1)}} (1 - p_{ii})^{N_i^{(t)} - N_i^{(t+1)}} \times p_{ii}^{a_{prior}-1} (1 - p_{ii})^{b_{prior}-1} \\ &\propto p_{ii}^{(a_{prior} + N_i^{(t+1)})-1} \times (1 - p_{ii})^{(b_{prior} + N_i^{(t)} - N_i^{(t+1)})-1} \end{aligned} \quad (8)$$

Therefore, the posterior distribution is given by the following:

$$p(p_{ii} | N_i^{(t+1)}) = \text{Beta}(a_{prior} + N_i^{(t+1)}, b_{prior} + N_i^{(t)} - N_i^{(t+1)}) \quad (9)$$

To illustrate the nature of the Beta distribution, this probability density function (pdf) for different sets of parameter values are presented in Figures 2 and 3. Figure 2 shows examples where the mean and mode of p_{ii} is set at 0.5, which takes place when the two parameter a and b are equal. As the values of parameters a and b increase, the distribution becomes tighter, implying a more reliable assessment of p_{ii} . As discussed in Chapter 4, the sum of the parameters of a and b represents the sample size. As expected, larger sample sizes lead to more reliable assessments of p_{ii} . As discussed in Chapter 4, the sum of the parameters of a and b represents the sample size. As expected, larger sample sizes lead to more reliable assessments of p_{ii} . Note that at parameter values $a = b = 1$, the Beta distribution reduces to a uniform distribution where p_{ii} is equally likely to take a value within an infinitesimal interval anywhere between 0 and 1, implying uninformative

information about p_{ii} . Figure 3 shows examples where the sum of a and b (i.e., the sample size) is fixed 100 while the mean of p_{ii} varies across the range from zero to one.

Figure 2. Pdf of Beta distributions with different parameters ($a = b = \text{constant}$)

Figure 3. Pdf of Beta distributions with different parameters ($a + b = 100$)

In this study, the mode of the posterior distribution is taken to be the point estimate of p_{ii} , which is given by the following:

$$\widehat{p_{u,BYE}} = \frac{a_{posterior}-1}{a_{posterior}+b_{posterior}-2} = \frac{a_{prior}+N_i^{(t+1)}-1}{a_{prior}+b_{prior}+N_i^{(t)}-2} \quad (10)$$

Note that when the parameters of the prior distribution a_{prior} and b_{prior} take the value of 1, corresponding to a uniform prior distribution as illustrated Figure 2, the Bayesian point estimate of p_{ii} is identical to the Maximum Likelihood estimate. That is, when the prior information is uninformative, Bayesian updating where prior information is captured does not lead to an estimate any different from the one where prior information is not captured, because the information is uninformative.

An extreme case is worth noting. When $a_{prior} = b_{prior} = 0$ and bridge deck area in a condition state i remains the same in years t and $t + 1$ (i.e., $N_i^{(t+1)} = N_i^{(t)}$), the estimator based on mode in Equation 10 produces a negative point estimate for transition probability p_{ii} . In such a case, an estimator based on the mean of the posterior distribution is used to estimate the transition probability as follows:

$$\widehat{p_{u,BYE}} = \frac{a_{posterior}}{a_{posterior} + b_{posterior}} = \frac{a_{prior} + N_i^{(t+1)}}{a_{prior} + b_{prior} + N_i^{(t)}} \quad (11)$$

It should be noted that when $N_i^{(t)}$ and $N_i^{(t+1)}$ are sufficiently large, which is true in the empirical analysis (more than 100,000 ft² in most cases), the difference between the mean and mode of the Beta distribution is negligible (less than 0.001% in the empirical study presented in Chapter 4). While, the substitution above is reasonable, in the analyses presented in Chapter 5, the case of $a_{prior} = b_{prior} = 0$ is intentionally avoided.

Chapter 3: Data

To evaluate the performance of the transition probability estimates determined by Bayesian updating method in predicting future condition states of concrete decks, an empirical investigation is conducted based on available data. Predictions based on Bayesian updating estimates are compared to prediction based on maximum likelihood estimates. In this analysis, the inspection records of a set of bridges for the year 2015 and 2016 are compiled. Once various discrepancies and inconsistencies are identified and subsequently addressed, a total of 357 bridge deck records are considered. This dataset is then divided into training and validation datasets, which are discussed in more detail in Chapter 4.

3.1 Data Collection and Definition

The data used in this study are gathered from the Ohio Department of Transportation bridge inspection database for the years 2015 and 2016. Figure 4 shows an example of a collected inspection record for a particular bridge. The variables in the inspection report database and their definitions are presented in Table 2.

	A	B	C	D	E	F	G	H	I
1	STATE	STRUCNUM	EN	EPN	TOTALQTY	CS1	CS2	CS3	CS4
2	39	100021	12		8132	4431	3501	200	0
3	39	100021	107		970	670	200	100	0
4	39	100021	205		6	6	0	0	0
5	39	100021	215		113	63	50	0	0
6	39	100021	234		113	88	25	0	0
7	39	100021	305		84	64	20	0	0
8	39	100021	311		20	12	8	0	0
9	39	100021	331		388	288	100	0	0

Figure 4. Example inspection report for a bridge in 2015

Table 2. Variables in the inspection report database and their definitions

Name	Definitions
STATE	STATE is the Federal Information Processing Standards (FIPS) code assigned to each state; the value is 39 for all bridges in Ohio.
STRUCNUM	STRUCNUM is the structure file number to identify bridges; this number is unique to each bridge.
EN	EN is the bridge element number certain bridge element. The reinforced concrete deck element, the focus of this study, is assigned EN = 12.
EPN	EPN is the parent element number for bridge elements. The wearing surface is the only other element that shares the same parent element number of the reinforcement bridge deck element. A blank EPN entry in a report (as is the case shown in Figure 4) indicates the absence of a protection system.
TOTALQTY	TOTALQTY is the total quantity of the bridge element. The unit of quantity for bridge decks is square foot.
CS1, CS2, CS3 and CS4	CS1, CS2, CS3 and CS4 are the quantity of a bridge element in condition states 1 to 4, respectively. In this project, they represent the deck area in condition states 1 to 4 based on the condition state definitions provided by AASHTO Bridge Element Inspection Guide Manual (2010).

3.2 Data Filtering

Some procedures are applied in this study to organize, compile, and select the original inspection records. This section provides a description of the preparation of the dataset used.

3.2.1 Bridge Element Filter

The focus of this study is reinforced concrete bridge decks. Thus, all inspection records with the element number (EN) of 12, indicating bridge decks, are selected.

3.2.2 Inspection Records Pair-Up

Since not all bridges are inspected every year and the analysis necessitates that condition inspection data have to be available for two consecutive years, inspection reports in 2015 and 2016 are paired based on the structure number (STRUCNUM). For all the records of reinforced concrete decks in 2015, 914 out of 3,183 decks are inspected in 2016. For these bridges, the inspection records in 2015 and 2016 are extracted. Figure 5 shows the example of a set of bridges that are inspected in 2015 but not in 2016.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1	2015								2016							
2	Structure	EN	EPN	TOTAL	CS1	CS2	CS3	CS4	Structure	EN	EPN	TOTAL	CS1	CS2	CS3	CS4
3	100021	12		8132	4431	3501	200	0	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A
4	100048	12		8132	5581	2501	50	0	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A
5	100137	12		9680	5780	3700	200	0	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A
6	100145	12		9680	6980	2600	100	0	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A
7	100226	12		12760	3035	9150	500	75	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A
8	100234	12		12760	3620	8600	520	20	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A
9	100331	12		5964	1464	4000	500	0	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A
10	100366	12		5964	464	5000	500	0	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A

Figure 5. Example of bridge decks that are inspected in 2015 but not in 2016

3.2.3 Bridge Deck Area Discrepancy

The area of bridge decks is found to vary from year to year. Small variation could be a result of measurement errors, while large differences could be indicative of record keeping errors. In this study records that exhibit large differences are discarded. Considering the empirical cumulative distribution function of the difference in deck area between 2015 and 2016, records with differences between -5 to $+5$ square feet are deemed acceptable. All other records are not considered further. Following the application of this filter, 456 bridge deck records remain.

3.2.4 Deterioration Assumption Consistency

The estimation methodologies presented in Chapter 3 are based on the assumption that there are no improvements in the condition state of bridge decks from year t to $t + 1$ (i.e., no transitions from 2 to 1, 3 to 2 or 1, and 4 to 3, 2 or 1) and no multi-step deterioration transition within this one unit period (i.e., no condition state transitions from 1 to 3 or 4, and from 2 to 4). The subset of bridges that comply with these criteria considering a tolerance of -5 to $+5$ square feet are, therefore, selected for analysis in this study. The filter associated with these criteria can be expressed as follows:

$$N_i^{(t+1)} - N_i^{(t)} \leq 5 \quad (12)$$

$$N_i^{(t+1)} \geq -5 \quad (13)$$

Equation 12 is intended to achieve consistency in the dataset with the assumption that no multi-step transitions in the condition state of bridge decks between two consecutive inspections take place. However, this equation does not guarantee that this assumption is strictly met for all bridge records in the dataset. Equation 13 guarantees that there is no

improvement in the condition state of bridge decks once the tolerance of 5 square feet is corrected for. In both equations, the tolerance of 5 square feet is considered to account for errors in observations and reporting by inspectors, among other factors. Thus, errors within this tolerance are corrected accordingly. Following the application of these two filters, a total of 357 bridge deck records remain in the dataset.

Chapter 4: Empirical Analysis

4.1 Analysis Set-Up

The dataset consisting of 357 bridge deck records described in Chapter 3 is divided into training and validation datasets. The training dataset contains 282 to 287 records (depending on the various cases considered as discussed subsequently) and is used to determine estimates of transition probabilities using the MLE and Bayesian updating methods. The estimated transition probabilities are then applied to the year 2015 inspection records of bridge decks in the validation dataset containing the remaining 75 to 70 records to predict their condition state in the year 2016. The similarity between the predicted and reported condition states is used as a measurement to assess the performance of the two methods.

The squared Hellinger distance (HD^2) metric is commonly used to measure the degree of similarity between two probability mass functions (Yang et al., 2000). This metric is used in this study to assess the similarity between two condition states, whether measured or predicted. The squared Hellinger distance is expressed as following:

$$HD^2 = \frac{1}{2} \sum_{i=1}^k (\sqrt{p_i} - \sqrt{q_i})^2 \quad (14)$$

where p_i and q_i are the elements of two probability mass functions for which the degree of similarity is to be assessed. The upper and lower limits of HD^2 are 1 and 0,

respectively. A smaller value indicates a higher degree of similarity between the two probability mass functions.

To use measured condition states in calculating HD^2 , the reported condition state vectors of bridge decks must be converted to probability mass functions. This conversion is achieved by normalizing the state vector depicting the bridge deck area in each state by dividing the area in each state by the total area. In the case of predicted condition states, the predictions based on Equation 1a directly take the form of a probability mass function. Based on an exploratory analysis, a few bridge decks are identified as having unique characteristics. As a result, the records for these bridge decks are given special treatment in the analysis. As noted previously, there are 357 records of bridge decks in the dataset. Among them, 70 reflect observed deterioration between 2015 and 2016. Figure 6 shows the extent of this deterioration as measured by HD^2 capturing the degree of similarity between the condition state vectors in 2015 and 2016. In this figure, the values are ordered from largest to smallest along the x-axis. Clearly, four bridge decks exhibit substantial deterioration as reflected in the large HD^2 values with respect to those of the other bridge decks.

Figure 6. Bar chart of HD^2 between 2015 and 2016 for all bridges

Note that bridge decks represented in the training dataset contribute to the estimates in accordance to their bridge deck area whereby decks with larger deck areas have larger contributions. The proportion of deck area for these four bridge decks with respect to the total area of all bridge decks in the training dataset (if these four bridges were to be included in the training dataset) are 0.18%, 2.96%, 0.05%, and 0.08%, respectively.

In addition, one bridge deck is found to exhibit a different deterioration pattern with respect to all other bridge decks. More specifically, this bridge deck is the only one

that has some deck area in condition state 3 in 2015 where part of this deck area deteriorates to condition state 4 in 2016. The inspection record for this bridge deck is shown in Table 3. All the other bridge decks either have no deck area in condition state 3 in 2015 or have some deck area in condition state 3 in 2015 but none of this area deteriorates to state 4 in 2016. Note that the proportion of deck area for this bridge with respect to the total area of all bridge decks in the training set is 0.26%.

Table 3. Inspection records for unique deterioration pattern bridge (Structure number: 7904983, unit: square feet)

Year	Total Deck Area	Area in Condition State 1	Area in Condition State 2	Area in Condition State 3	Area in Condition State 4
2015	14433	12267	1877	289	0
2016	14433	12234	1876	289	34

Given that these five bridge decks appear to be distinctly different from all other decks in the dataset, the impact their corresponding records could have on the results may be substantial. Therefore, two scenarios are considered. In Scenario 1 the records of these five bridge decks are included in the validation dataset, resulting in a total of 282 records in the training dataset. In Scenario 2 these five records are included in the training dataset resulting in a total of 287 records in this dataset. The records of all other bridges are randomly assigned to each of the two datasets.

To evaluate the performance of the maximum likelihood and Bayesian estimation methods, transition probabilities are determined using the training dataset. Applying the

estimated transition probabilities to the condition state vector of bridge decks in the validation dataset for the year 2015, the condition states of these bridges are predicted for the year 2016. The similarity between the predicted and reported condition states for 2016 are then compared across the two methods.

As discussed in Section 2.2.3, in the Bayesian estimation method, the transition probabilities available before the collection of condition data for 2015 and 2016 are assumed to follow Beta distributions defining the priors in the Bayesian framework. In Section 3.3 the Beta distribution is described through its parameters a_{prior} and b_{prior} . For the purpose of this analysis, the Beta distribution is conveniently parameterized through the mean and sample size, referred to as the prior mean and confidence “parameters”. The relationship between this parameterization and the one in Section 2.2.3 is as follows:

$$prior\ mean = \frac{a_{prior}}{a_{prior} + b_{prior}} \quad (15)$$

$$confidence = a_{prior} + b_{prior} \quad (16)$$

where prior mean represents the prior belief about the mean value of p_{ii} and confidence represents the level of belief on prior mean.

In this analysis the confidence is set to vary between 2 and 400,000 square feet. When the confidence is set to 2, the Bayesian point estimate (the mode of the posterior distribution) is equal to the maximum likelihood estimate. The upper limit sample size of 400,000 is selected based on recommendations in Pontis Technical Manual (1993). This manual suggests that it is reasonable to assume that the prior distribution of transition probabilities is developed based on observations for 10 to 20 bridges. Considering that

the average deck area for the considered bridge dataset is about 20,000 square feet, 20 bridge observations yield the upper limit of 400,000 square feet for the assumed confidence.

As for the prior mean, 10,000 mean values of the prior beta distribution (p_{11} , p_{22} and p_{33}) are generated independently from a uniform distribution that ranges between zero and one. Recall, since single-state transitions are assumed, $p_{12} = 1 - p_{11}$, $p_{23} = 1 - p_{22}$ and $p_{34} = 1 - p_{33}$. Given that in practice, some informative prior mean values are likely to be available (based on historical data or expert judgement), basing this analysis on prior means generated uniformly between zero and one is expected to bias the results against the advantage Bayesian updating may offer. Each set of the prior transition probabilities is updated using the training dataset through the application of the Bayesian updating method presented in Section 2.2.3.

Combined with confidence values ranging from 2 to 400,000 with 400 steps of increment, 4,000,000 sets of transition matrices are determined based on Bayesian estimation. Note that the range for confidence starts at 2 instead of 0 to avoid the negative mode point estimate as discussed in Section 2.2.3. Applying these transition matrices to 2015 condition state vectors of bridge decks in the validation dataset, the condition state vectors of these bridges are predicted for 2016. The similarity between the predicted and reported condition states for 2016 are then measured using the HD^2 metric.

The HD^2 metric is computed for each bridge deck in the validation dataset for all generated prior mean values of p_{11} , p_{22} and p_{33} and the levels of confidence in these mean

values. Thus, for a given bridge deck and one set of generated mean values, a graph of HD^2 versus confidence can be plotted. Figures 7 and 8 show two examples of such plots.

Since, as discussed in Section 2.2.3, the Bayesian estimates are approximately equal to the maximum likelihood estimates at confidence equals 2 for large values of $N_i^{(t)}$ and $N_i^{(t+1)}$, which are the case for the training datasets, Figure 7 indicates a case where some of the greater-than-zero confidence in the prior mean values lead to Bayesian estimates that produce a superior prediction for 2016 (lower HD^2) than that of the maximum likelihood estimates. In this analysis, the confidence level associated with the minimum HD^2 for a given prior mean and bridge is selected to represent the confidence level for the Bayesian updating method. Naturally, in practice the selected confidence level may not be associated with the minimum HD^2 or a bridge, and, therefore, this selection is expected to bias the results in favor of the Bayesian updating. Figure 8 indicates a case where a confidence greater than zero leads to an inferior prediction to that based on the maximum likelihood estimates. That is, the Bayesian and maximum likelihood estimates would reflect the same prediction performance when the confidence in the prior mean values is assumed zero for the Bayesian estimation method.

To quantify the improvement in prediction by using Bayesian estimation at the optimal confidence value for a set of prior mean values, the following measure is defined:

$$Reduction = \frac{HD_{MLE}^2 - HD_{BYE}^2}{HD_{MLE}^2} \quad (17)$$

where HD_{MLE}^2 and HD_{BYE}^2 are the HD^2 values for the predictions based on the maximum likelihood and Bayesian estimates, respectively. These inputs to this measure of

Reduction are depicted in Figure 7. Naturally, for a case like the one shown in Figure 8, the measure Reduction takes a value of zero.

Figure 7. Example HD^2 versus confidence plot where Bayesian estimates at the optimal confidence level are superior to those of maximum likelihood

Figure 8. Example HD^2 versus confidence plot where Bayesian estimates at the optimal confidence level are equivalent to those of maximum likelihood

There are also possible cases where the HD^2 of the Bayesian updating based predictions takes a minimum value at the upper bound of the considered range of confidence level (400,000) as depicted in Figures 9 and 10. That is, for such cases, HD_{BYE}^2 takes a value greater than 400,000. Figure 9 indicates a case where the gradient of the relationship between HD^2 of the Bayesian updating based predictions and the confidence level is ascending, which means that with increasing confidence level, the improvement in prediction of Bayesian updating produces diminishes for that bridge. Figure 10 indicates a case where the gradient of the relationship between HD^2 of the Bayesian updating based predictions and the confidence level is descending, which

means that with increasing confidence level, the improvement in prediction of Bayesian updating produces an enhancement for that bridge.

In either case, it is desirable to consider confidence levels greater than 400,000 in determining HD^2_{BYE} . Doing so is applied in the analysis based on the refined experimental design described in Section 4.3.

Figure 9. Example HD^2 versus confidence plot where Bayesian estimates optimize at maximum confidence level with ascending gradient

Figure 10. Example HD^2 versus confidence plot where Bayesian estimates optimize at maximum confidence level with descending gradient

4.2 Preliminary Experiment

All 4,000,000 sets of the Bayesian updated transition probabilities based on the combinations of prior mean values and confidence levels and the one set of maximum likelihood estimated transition probabilities are applied to the bridge decks of the Scenario 1 and Scenario 2 validation datasets as defined in Section 4.1. Figure 11 shows the number of cases for each bridge where the Bayesian estimates are superior to the maximum likelihood estimates (i.e., Reduction > 0%) under Scenario 1 as shown in Figure 11(a) and under Scenario 2 as shown in Figure 11(b). Note that in Scenario 1 there are 75 bridge deck records in the validation set, while in Scenario 2 there are 70 records.

In Figure 11, Bridge ID is defined to be the rank among the 75 bridge deck records in the validation dataset of Scenario 1 based on the descending order of the values of HD^2 measuring the similarities of the reported condition states in 2015 and 2016. The counts reported in Figure 11 for each bridge represent the number of sets of prior transition probabilities out of the 10,000 generated ones that lead to Bayesian estimates that produce predictions that are superior to the prediction produced by the maximum likelihood estimates.

Based on Figure 11, the performance of Bayesian updating is clearly different for the two Scenarios. Under Scenario 1, where the records of the five distinctly different bridge decks belong to the validation dataset, Figure 11(a) indicates that Bayesian updating is superior to maximum likelihood for a substantially large number of cases associated with a fairly small number of bridge decks, specifically those that exhibit more substantial deterioration between 2015 and 2016.

The bridge decks that are associated with the superiority of Bayesian updating over maximum likelihood include the four decks found to exhibit the most deterioration between 2015 and 2016 – Bridge ID values of 1 through 4 in Figure 11. They also include the one and only deck that exhibits deterioration that involves a transition to state 4 in 2016 – Bridge ID value of 5 in Figure 11 (the deck with the eighth largest change in deterioration between 2015 and 2016 when considering all bridge decks as depicted in Figure 6). These results are consistent with the expectation that for decks that experience deterioration that is not well represented in the training dataset, incorporating prior

information with the two most recent condition inspections via Bayesian updating is advantageous.

Figure 11. Bar chart by Bridge ID when Reduction > 0%; (a) Scenario 1; (b) Scenario 2

Under Scenario 2, where the records of the five distinctly different bridge decks identified in Section 4.1 belong to the training dataset, Figure 11(b) indicates that Bayesian updating is superior to maximum likelihood only for a small number of cases spanning most bridge decks in the validation dataset. This result suggests that due to the unrepresentative impact the five distinctly different decks have on the estimates, only a small set of prior mean values and confidence levels lead to more accurate Bayesian estimation based predictions with respect to the maximum likelihood estimation based predictions. That is, only a few combinations of prior mean values and confidence levels

lead to Bayesian estimates that counter the maximum likelihood estimates that are corrupted by the presence of the five distinctly different bridges in the training dataset.

Since the performance of Bayesian updating is highly dependent on the distribution of the five distinctively different bridges, a more refined experiment is designed to explore the impact of these bridges on Bayesian updating.

4.3 Refined Experimental Design

This section focuses on the refined experiment involving the five distinctively different bridges and presents the analysis results. The five bridges are divided into two categories based on their unique characteristics: the four bridges that exhibit substantial deterioration are categorized as “severe deterioration bridges” and the bridge with some deck area transitioning from condition state 3 to 4 is categorized as an “extreme deterioration bridge”. The refined experimental design consists of scenarios where the combinations of bridges in the two categories are in the training set, in the validation set, or neither in the training nor in the validation set. That is a total of nine scenarios are considered as shown in Table 4. Note that the scenarios are expressed as “S_XE_X”, where “S” represents “severe deterioration bridges” and “E” represents “extreme deterioration bridge”. “X” takes the letters “N”, “T” and “V” denoting that the bridge category is “neither in training nor in validation set”, “in training set”, and “in validation set”, respectively. Also, note that the two scenarios in the preliminary experiment are correspond to Scenarios S_VE_V and S_TE_T, respectively.

Table 4. Combination of all possible Scenarios

		Extreme deterioration bridge		
		Neither	In Training	In Validation
Severe deterioration bridges	Neither	$S_N E_N$	$S_N E_T$	$S_N E_V$
	In Training	$S_T E_N$	$S_T E_T$	$S_T E_V$
	In Validation	$S_V E_N$	$S_V E_T$	$S_V E_V$

Recall the discussion regarding the confidence level associated with the minimum HD^2 of the Bayesian updating based predictions in Section 4.1, where in some cases this confidence level falls beyond the upper bound of the confidence range considered, 400,000. To address this situation while considering the same number of confidence values to maintain the computation cost at a reasonable level, an exponential scale ranging from 2 to 10^9 square feet with an initial step size of 400 is applied. For the bridges where HD_{BYE}^2 is associated with confidence levels greater than 400,000, the values determined from the results based on the exponential scale are used instead.

To thoroughly examine the performance of condition state predictions based on Bayesian updating estimates against those based on Maximum Likelihood, four more refined metrics are defined.

Metric a: The mean of the extent of reduction among all the 10,000 sets of prior probabilities is first determined for each bridge, then the weighted mean reduction across bridges is determined where the weight for a bridge is equal to the deck area of the bridge divided by the total area of all bridges in the validation set. The process is mathematically represented as follow:

$$\text{Metric a} = \sum_i (\sum_j \frac{R_{ij}}{10000} \times W_i) \quad (18)$$

where R_{ij} is the extent of HD^2 reduction, R , achieved using Bayesian updating estimates with respect to the HD^2 associated with the Maximum Likelihood estimate for bridge with index i and prior probability with index j , and W_i is the weight for bridge i .

Metric b: This metric is similar to Metric a except that only cases where the extent of reduction, R , is larger than zero are considered. Moreover, the weight for a bridge is equal to the deck area of the bridge divided by the total deck area of all the bridges in the validation set that have at least one case that exhibits non-zero extent of reduction. The formulation of the metric is represented as follows:

$$\text{Metric b} = \sum_{\forall i,s,t.I'_i=1} \sum_j (R_{ij} \times I_{ij} / \sum_j I_{ij}) \times W'_i \quad (19)$$

where R_{ij} is the same as in Metric a, and indicator functions I'_i and I_{ij} as well as the weighting parameter W'_i are defined as:

$$I'_i = \begin{cases} 1, & \sum_j \frac{R_{ij}}{10000} > 0 \\ 0, & \text{otherwise} \end{cases} \quad (20a)$$

$$I_{ij} = \begin{cases} 1, & R_{ij} > 0 \\ 0, & \text{otherwise} \end{cases} \quad (20b)$$

$$W'_i = W_i \times \frac{\sum_l W_l}{\sum_l W_l \times I'_l} \quad (20c)$$

Metric c: This metric is similar to Metric b except that only the top 10% of bridges with highest mean reductions, R , are considered. In this case, the reductions are weighted based on the deck areas of the selected bridges. This metric is mathematically represented as follows:

$$\text{Metric c} = \sum_{\forall i,s,t.I''_i=1} \sum_j (R_{ij} \times I_{ij} / \sum_j I_{ij}) \times W''_i \quad (21)$$

where R_{ij} , I_{ij} are the same as in Metric b, and I_i'' and W_i'' , which are an indicator function and a weighting parameter for bridge i , respectively, are defined as:

$$I_i'' = \begin{cases} 1, & i = 1 \text{ to } 7, \forall i: \sum_j \left(R_{ij} \times \frac{I_{ij}}{\sum_j I_{ij}} \right) < \sum_j \left(R_{i+1j} \times \frac{I_{i+1j}}{\sum_j I_{i+1j}} \right) \\ 0, & \text{otherwise} \end{cases} \quad (22a)$$

$$W_i'' = W_i \times \frac{\sum_l W_l}{\sum_l W_l \times I_l''} \quad (22b)$$

Metric d: To assess the performance of Bayesian updating regarding the distinctively different bridges, this metric is very similar to that of Metric c but applied only to the five bridges when they are included in the validation set. That is, Metric d is applicable to the five scenarios where either set of bridges in the two categories are in the validation set. This metric, for the applicable scenarios, is defined as follows:

$$\text{Metric d} = \sum_{\forall i, s, t, I_i'''=1} \sum_j (R_{ij} \times I_{ij} / \sum_j I_{ij}) \times W_i''' \quad (23)$$

where R_{ij} , I_{ij} are the same as in metric b. I_i''' and W_i''' are an indicator function and a weighting parameter for bridge i , respectively, are defined as:

$$I_i''' = \begin{cases} 1, & \text{if Bridge } i \text{ is among the five bridges} \\ 0, & \text{otherwise} \end{cases} \quad (24a)$$

$$W_i''' = W_i \times \frac{\sum_l W_l}{\sum_l W_l \times I_l'''} \quad (24b)$$

The results of the refined experiment based on the four metrics are presented in Tables 5 through 8. Considering results for Metrics a, b, and c for the S_{VEV} and S_{TET} scenarios, the comparison confirm the conclusions based on the preliminary experimental results of Section 4.2, which are based on the “counts” measure used for that analysis.

Table 5. Results for metric a

		Extreme deterioration bridge		
		Neither	In Training	In Validation
Severe deterioration bridges	Neither	1.21%	1.20%	2.26%
	In Training	0.37%	0.37%	0.93%
	In Validation	11.25%	11.24%	12.06%

Table 6. Results for metric b

		Extreme deterioration bridge		
		Neither	In Training	In Validation
Severe deterioration bridges	Neither	31.89%	25.58%	33.48%
	In Training	29.62%	27.66%	29.98%
	In Validation	44.21%	39.00%	45.15%

Table 7. Results for metric c

		Extreme deterioration bridge		
		Neither	In Training	In Validation
Severe deterioration bridges	Neither	48.53%	48.62%	53.52%
	In Training	49.06%	49.06%	49.06%
	In Validation	68.72%	67.86%	70.58%

Table 8. Results for metric d

		Extreme deterioration bridge		
		Neither	In Training	In Validation
Severe deterioration bridges	Neither	NA	NA	91.20%
	In Training	NA	NA	47.86%
	In Validation	80.00%	80.00%	83.83%

Considering all nine scenarios and all four metrics, for the most part the weighted mean reduction values are considerably smaller than using Metric a and progressively increase when using Metrics b, c, and d in this order. This pattern implies that a large

number of bridge decks do not or barely benefit from predictions based on Bayesian updating estimates. This result is consistent with the intuition that since the majority of bridges do not deteriorate extensively over one year, the MLE method provides reasonably good estimates of transition probabilities for the bridges, and thus, there is limited opportunity for Bayesian updating to produce estimates that lead to better predictions than those based on the MLE method.

When considering cases where Bayesian updating offers an improvement over MLE though, as captured in Metric b, substantial reductions are observed. This result suggests that when Bayesian updating does offer an advantage, the resulting improvements are worthwhile. To systematically and more thoroughly analyze the value of Bayesian updating, more targeted comparisons among the various scenarios are considered and presented subsequently.

Comparison 1

Comparison 1 focuses on the effect of the scenarios where the distinctively different bridges belong to the training set with respect to the scenario where they do not belong to either (considered as a reference). Tables 9-12 show the same Tables 5-8 with the pertinent scenarios not crossed out. For Scenarios S_{TEN} and S_{NET} based on Metrics a and b, the weighted mean reduction values are smaller than those of Scenario S_{NEN} , while for Metric c, the weighted mean reduction values for Scenarios S_{TEN} and S_{NET} are comparable to those of Scenario S_{NEN} .

Table 9. Comparison 1: Results for metric a

		Extreme deterioration bridge		
		Neither	In Training	In Validation
Severe deterioration bridges	Neither	1.21%	1.20%	2.26%
	In Training	0.37%	0.37%	0.93%
	In Validation	11.25%	11.24%	12.06%

Table 10. Comparison 1: Results for metric b

		Extreme deterioration bridge		
		Neither	In Training	In Validation
Severe deterioration bridges	Neither	31.89%	25.58%	33.48%
	In Training	29.62%	27.66%	29.98%
	In Validation	44.21%	39.00%	45.15%

Table 11. Comparison 1: Results for metric c

		Extreme deterioration bridge		
		Neither	In Training	In Validation
Severe deterioration bridges	Neither	48.53%	48.62%	53.52%
	In Training	49.06%	49.06%	49.06%
	In Validation	68.72%	67.86%	70.58%

Table 12. Comparison 1: Results for metric d

		Extreme deterioration bridge		
		Neither	In Training	In Validation
Severe deterioration bridges	Neither	NA	NA	91.20%
	In Training	NA	NA	47.86%
	In Validation	80.00%	80.00%	83.83%

To interpret these results, the effects of the distinctively different bridges belonging to the training set is explored. When the distinctively different bridges are in the training set, they contaminate the training set and distort estimations of transition

probabilities using the MLE method. This effect becomes appreciable when the distinctively different bridges have large deck areas or when there are many such bridges in the training dataset. Thus, MLE will produce unrepresentative estimates of transition probabilities for the majority of bridges in the validation set, which experience no or limited deterioration. Bayesian updating has the potential to improve estimates of transition probabilities in such situations. The performance of Bayesian updating with respect to MLE depends on the relative effect of the corruption of the training set and the improvement Bayesian updating offers. In this comparison, for Metrics a and b, which consider all or many of the bridges in the validation set, the effect of the corruption of the training set is stronger than the improvement Bayesian updating offers. While for metric c, the two effects appear to counter each other almost equally. The reasons behind the differences observed based on Metrics a and b on the one hand and Metric c on the other are worth further exploration.

Comparison 2

Comparison 2 focuses on the effect of the scenarios where the distinctively different bridges belong to the validation set with respect to the scenario where they do not belong to either (considered as reference). Table 13-16 show the same Tables 5-8 with the pertinent scenarios not crossed out. For Scenarios S_{VE_N} , S_{NE_V} and S_{VE_V} , the weighted mean reduction values are larger than those of Scenario S_{NE_N} considering Metrics a, b, and c. This pattern implies that when either category of bridges is in the validation set, Bayesian updating is superior to MLE. Also, for Metric d, the applicable weighted mean reduction values are even larger than the corresponding values of Metrics

a, b, and c. This result implies that Bayesian updating is more beneficial for the distinctively different bridges.

Table 13. Comparison 2: Results for metric a

		Extreme deterioration bridge		
		Neither	In Training	In Validation
Severe deterioration bridges	Neither	1.21%	1.20%	2.26%
	In Training	0.37%	0.37%	0.93%
	In Validation	11.25%	11.24%	12.06%

Table 14. Comparison 2: Results for metric b

		Extreme deterioration bridge		
		Neither	In Training	In Validation
Severe deterioration bridges	Neither	31.89%	25.58%	33.48%
	In Training	29.62%	27.66%	29.98%
	In Validation	44.21%	39.00%	45.15%

Table 15. Comparison 2: Results for metric c

		Extreme deterioration bridge		
		Neither	In Training	In Validation
Severe deterioration bridges	Neither	48.53%	48.62%	53.52%
	In Training	49.06%	49.06%	49.06%
	In Validation	68.72%	67.86%	70.58%

Table 16. Comparison 2: Results for metric d

		Extreme deterioration bridge		
		Neither	In Training	In Validation
Severe deterioration bridges	Neither	NA	NA	91.20%
	In Training	NA	NA	47.86%
	In Validation	80.00%	80.00%	83.83%

These results can be attributed to the nature of the dataset where most bridge decks do not deteriorate or deteriorate by a small amount between the two inspection years and only a few decks exhibit severe or extreme deterioration as discussed in Section 4.1. When the distinctively different bridges are in the validation set, they are poorly represented in the training set and, therefore, the MLE estimates of the transition probabilities are not representative of the deterioration of these bridge decks. Consequently, Bayesian updating is capable of producing estimates that are likely to lead to better predictions for the distinctively different bridges. This conclusion is particularly evident from the results using Metric d where the performance of Bayesian updating for these bridges is distinctly superior to those of the other bridges. Note that the improvements associated with Scenario S_{VE_N} are larger than those associated with Scenario S_{NE_V} , which is likely due to the larger deck areas associated with the four severe deterioration bridges.

Chapter 5: Summary and Future Research

A Bayesian updating procedure is proposed to estimate a Markov Chain based concrete deck deterioration model in a manner that combines condition data collected over two inspection cycles and the deterioration information available prior to the collected condition data. Single period (one year) transition probabilities are estimated using Bayesian updating and maximum likelihood estimation where in the case of the latter only the collected condition data over two inspection cycles are used.

A dataset of bridge deck condition assessments based on AASHTO condition state definitions collected by a state infrastructure agency spanning two years is used to evaluate the performance of the two methods. A training and validation datasets are selected from the original dataset where the former is used for estimation and the latter for prediction and evaluation.

The evaluation is based on measuring the degree of similarity between reported condition states and those predicted based on the estimated transition probabilities using the two methods. While Bayesian updating is found to be superior to maximum likelihood estimation for many cases, this superiority is highly dependent on the deterioration nature of the bridge decks reflected in the training dataset. And, when

Bayesian updating offers an improvement over MLE, the resulting improvements are substantial and, therefore, worthwhile.

Regarding the nature of the dataset, when the distinctively different bridges belong to the training set, the ability of Bayesian updating to compensate for the corruption in the MLE-based transition probabilities varies across the bridge decks for which predictions are applied and at best compensates equally for the corrupted MLE results. However, when the distinctively different bridges are in the validation set and, therefore, the MLE-based transition probabilities are not representative of their deterioration, Bayesian updating is able to produce superior predictions for these bridge decks.

Several possible areas for future research are worth pursuing. In the analysis presented in Section 4.2 and 4.3, one set of randomly generated prior mean values of the prior distribution is generated. It is important to replicate the experiment based on multiple sets of randomly generated mean values of the prior distribution. Doing so will allow for assessing the confidence in the identified patterns.

Also, recall that it is assumed in the analysis that the confidence associated with the minimum prediction error produced by Bayesian updating is known, which is expected to lead to results that favor Bayesian updating. In addition, recall that using the uniform distribution in generating the mean value of the prior distribution in the analysis is expected to lead to results that unfavorably reduce the value of Bayesian updating.

Therefore, conducting additional evaluation analyses that address these limitations would be valuable.

In addition, use larger and more varied dataset and applying an evaluation experiment where more than two years of condition records are available would be worthwhile. For example, more deterioration patterns should be identified to distinguish the bridges. Also, different methods in combining the inspection records should be explored when inspection records span more than two time points.

Moreover, the sensitivities of the results to various assumptions are worthwhile to explore. Such assumptions could then be relaxed if the results are found to be sensitive to them, especially those with high sensitivity. For example, adjacent bridge deck units are not expected to deteriorate independently as is assumed in this study. Also, deterioration could lead to more than a one-level change in condition state, unlike the single-level change assumed in this study. In addition, the transition probabilities may not be age-independent as is assumed in this study. Moreover, confidence in the prior mean values are not likely to be known as is also assumed in considering the Bayesian estimates that correspond to the maximum improvement in the quality of the corresponding predictions with respect to the quality of the predictions based on the maximum likelihood estimates. Furthermore, observed condition values are not error-free as is assumed in the estimation and evaluation aspects of the study.

Furthermore, additional variables could be taken into account in the deterioration model such as the effects of age, environment, loading, and protection systems.

Additional variables could be collected to classify the bridges into different categories where bridges within each category are expected to have similar deterioration pattern. Thus, prediction errors for each category could be reduced compared to the case where all bridges are treated identically.

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