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A NUMERICAL STUDY OF COUNTERROTATING DISKS IN SPIRAL GALAXIES

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By

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* * * * *

The Ohio State University

1997

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A handwritten signature in dark ink, reading "Barbara S. Ryden". The signature is fluid and cursive, with the first name "Barbara" and last name "Ryden" clearly legible. The middle initial "S." is written in a smaller, more compact script.

Adviser

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ABSTRACT

The serendipitous discovery in 1992 of a second spiral disk rotating in a sense opposite to the rest of the galaxy, followed by subsequent discoveries of other such counter-rotating disks, poses a formidable challenge to theorists. Current galaxy formation theories do not permit such a disk to be formed with the rest of the galaxy, and it appears to have been built up from external matter after the galaxy was formed. This dissertation aims to determine how a counterrotating disk can form in a spiral galaxy without destroying the preexisting disk. The most promising theory invokes the adiabatic infall, on a retrograde orbit, of intergalactic gas that dissipates energy and precesses to the plane of the primary disk. Using numerical simulations incorporating both stars and gas, I test this and competing theories for the formation of counterrotating disks. I show that under restricted conditions, dissipational infall of gas can indeed produce a counterrotating disk without excessive damage to the pre-existing disk. A cold, thin primary can survive secular gas infall and the production of a massive counterrotating disk if the infalling gas is well-dispersed in phase-space. A minor merger with a gas-rich dwarf galaxy can also yield a counterrotating disk if the primary galaxy's disk is massive and compact to begin with. Although the origin of counterrotating disks is an intriguing question in its own right, it has profound implications for the evolution and interaction histories of spiral galaxies.

To Carl Sagan, whose wonderfully evocative descriptions of the grandeur
of the universe inspired me to pursue a career in astronomy.

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Thakar, A.R., Ryden, B.S., Jore, K.P., Broeils, A.H. 1997, "NGC 4138: A Case Study in Counterrotating Disk Formation", *The Astrophysical Journal*, 479, 702.

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Thakar, A.R., and Ryden, B.S. 1994, "Sticky Particle Simulations of Counterrotating Disk Formation", *Bulletin of the American Astronomical Society*, 26, no. 4, 1441.

Thakar, A.R., Ryden, B.S., Jore, K.P., & Broeils, A.H. 1995, "NGC 4138 - A Smoking Gun for Counterrotating Disk Formation", *Bulletin of the American Astronomical Society*, 27, no. 4, 1442.

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Counterrotating Disks in Spirals	Prof. Barbara S. Ryden
N-body Simulations of Disk Galaxies	Prof. Jens V. Villumsen
Star Formation in the LMC	Prof. Darren Depoy
Narrow Band Photometry of MWC 560	Prof. Robert F. Wing

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CHAPTER 1

INTRODUCTION

1.1 Historical Perspective

A hundred years ago, nobody knew what a galaxy was. All our knowledge of galaxies has been obtained in the last hundred years, and it is fair to say that we still do not understand galaxies well. In fact, our view of galaxies has changed drastically in the last decade or so.

A galaxy is a collection of billions of stars, along with some gas and dust, all held together by gravity. There are three broad classes of galaxies, *elliptical*, *spiral*, and *irregular*. Our galaxy, the Milky Way, is a spiral galaxy. In the early days of the study of galaxies, as soon as their enormous sizes and distances became apparent, they were thought to be “island universes” that were self-contained and evolved mostly in isolation. But this concept has, in the last two decades, slowly given way to the idea that interactions between galaxies play a significant role in galaxy evolution. Observations of merger remnants and signatures of past interactions with increasingly sensitive instruments and advanced analysis techniques have prompted this view, but computer simulations have been largely responsible for its acceptance by the astronomical community.

Traditionally, astronomers have thought of elliptical galaxies as being smooth, featureless, non-rotating blobs supported against gravity by random motions, and ranging in shape from spherical to highly flattened oblate or prolate spheroids. Spiral galaxies, by contrast, have been described as prominently disk-like axisymmetric structures with ordered rotation in a single direction about the galactic center providing the main support against gravity. This picture may be too simple, however, to do justice to the real nature of most elliptical and spiral galaxies as they continue to be shaped by interactions and mergers.

1.2 Counterrotating Cores and Counterrotating Disks

Recent observations of line-of-sight velocity distributions (LOSVDs) in both elliptical and spiral galaxies are revealing their secret lives and stormy pasts. Unusual kinematics found in nearby galaxies are providing extremely valuable clues for divining the forces at work in galaxy formation. One of the more striking manifestations of unusual kinematics is the presence of counterrotating disks in both early and late type galaxies. In ellipticals, the disks are usually confined to the nuclear regions, forming a **counterrotating core**. In spiral galaxies, however, up to half of the stars and/or gas in the disk system can be in retrograde orbits, forming an extensive and kinematically cold ($v_{\text{rot}} \gg \sigma$) **counterrotating disk**.

The first kinematically peculiar core was discovered in the normal giant elliptical NGC 5813 as early as 1982 (Efsthathiou, Ellis & Carter 1982), and later efforts aimed at compiling rotation curves of ellipticals led to the discovery of more counterrotating cores (Franx & Illingworth 1988; Jedrzejewski & Schechter 1988; Bender 1988) in some ellipticals. These structures may be the result of mergers or gas infall. Since smaller

ellipticals have denser cores, the smaller secondary would survive the primary's tidal field provided that the two core radii differed by a factor of at least 6-10 (Kormendy 1984). Dynamical friction would then cause the secondary core to sink *en masse* to the center of the primary, virtually none the worse for wear. A counterrotating core would form in the case of a retrograde encounter, as further demonstrated by other simulations (Balcells & Quinn 1990; Barnes & Hernquist 1991; Hernquist & Barnes 1991). Mergers may not be the only explanation, however, and an interesting alternative to the merger theory has been suggested by Statler (1991), who shows that kinematically distinct cores can be the result of projection of major orbit families in a triaxial potential. The lower velocity dispersion of the core would provide unequivocal proof of the merger theory, since normal ellipticals have higher velocity dispersion at the center (Faber-Jackson Law).

1.3 Massive Counterrotating Disks in Spiral Galaxies

NGC 4550, a very ordinary-looking S0 galaxy near the center of the Virgo cluster of galaxies, jumped to instant fame in 1992 when astronomer Vera Rubin discovered, to her disbelief, that almost half the stars in the galaxy's disk were orbiting in the opposite direction (Rubin, Graham & Kenney 1992). In other words, there were *two* disks co-spatial with one another, and whereas one was spinning clockwise as seen from earth, the other was rotating counter-clockwise. The galactic dynamicist's bible (Binney & Tremaine 1987) dismisses such bi-directional or **counterrotating** disks with the words: "it is unlikely that this type of model is of practical importance."

Following the discovery of counterrotation in NGC 4550, several other instances of counterrotating stellar disks in spirals have been discovered (Braun, Walterbos &

Kennicutt 1992, Merrifield & Kuijken 1994). While counterrotating *gas* disks had been known for some time (Sage & Galletta 1994), such *extensive* and *stellar* counterrotating disks in spirals were a new phenomenon. Theorists are faced with the daunting task of explaining how, when standard galaxy formation scenarios allow angular momentum to be imparted in a single direction only, a galaxy can possess two disks with opposite angular momenta. Although under certain circumstances it is possible for stellar box orbits to scatter into tube orbits going in either direction (Evans & Collett 1994), thereby producing nearly equal populations of counterrotating stars, in general counterrotating disks must form from matter acquired externally *after* the single-disk galaxy has already formed. Furthermore, the lack of an obviously disturbed morphology rules out a disruptive merger between two spiral galaxies with opposite spins in which the second disk is acquired *en masse*. Simulations of such major mergers have shown that the sudden influx of stars in such a merger would all but destroy both disks (Barnes 1992, Barnes & Hernquist 1992a). It is much more likely that the matter destined for the new counterrotating disk is acquired gradually in the form of intergalactic gas and debris from previous mergers (Rix *et al.* 1992). If the orbital direction of the infalling material is opposite to that of the galaxy, the new disk formed would naturally rotate counter to the primordial disk.

1.4 Implications of Counterrotating Disks

The question of the origin of counterrotating disks is an important one for several reasons. Bi-directional disks have received scant attention from theorists primarily because they were not thought to occur in nature. Now that we know they exist, questions abound. Are all the varieties of counterrotating disks different evolutionary

stages of the same basic process? How do they affect spiral structure and stability? How would star formation be affected by the presence of the second disk? Is the scale length of a counterrotating disk determined by the angular momentum of the infalling gas or by some other processes? How stable or long-lived are these disks once they are formed? It has been noted (Merrifield & Kuijken 1994) that the process(es) most likely to form CR disks strongly affect the two properties that are used as the basis for the Hubble classification: *bulge size* and *strength of spiral structure*. If CR disks do indeed affect these properties, what are their implications for the Hubble Sequence? Are they a phase through which all or most spirals go through?

In a cosmological context, knowing how counterrotating disks form will provide crucial clues about how galaxies evolve and interact with their environment. While most astronomers would agree that galaxies interact, the importance of these interactions in shaping galaxies is a subject of considerable debate. Many important questions are still unanswered (Barnes & Hernquist 1992a). How much accretion can a disk galaxy stand? How much has already occurred? If counterrotating disks in spirals are indeed formed by an accretion or merger event, this would mean that disk galaxies can keep forming with time and do not have to grow from the inside out. If the gas infall process is the primary producer of counterrotating disks, this proves that spiral galaxies can absorb prodigious amounts of gas during their lives and can almost double their mass over time. This would significantly alter our current picture of spiral galaxies. The coldness of the disks seen in galaxies today can no longer restrict their interaction histories as much. If the counterrotating disks seen in spirals are mostly merger remnants, then this sheds light on other long-standing questions.

How important are mergers in disk galaxy formation? To what extent do they shape the Hubble sequence? How much do the Hubble types evolve with time?

The nature vs. nurture issue for galaxies is crucial to our understanding of galaxies as they appear today, but it also has profound cosmological implications. If interactions and mergers drive galaxy evolution, then the time-scale for primordial galaxy formation need not be large. The building blocks of galaxies may have formed within a relatively short time and subsequently evolved to their present states.

1.5 Numerical Simulations

N -body simulations, in which the astrophysical system is represented by a number of interacting bodies or particles, are the best investigative tools for studying such complex interactions between galaxies and their environments. These problems cannot be studied analytically without making simplifying assumptions that are hard to justify. Traditionally, N -body simulations have included only stars, since they are the major component by mass in most astrophysical objects. The only physical process that is necessary to model aggregates of stars is Newtonian gravity, and the N -body problem is essentially a problem of calculating N^2 gravitational interactions per simulation time-step, where N is the number of particles used to represent the astrophysical entity. When it is necessary to include gas in astrophysical simulations, gas dynamics must be incorporated in the calculations in addition to gravity. This is because the behavior of the gas differs from that of the stars in one important respect. The falling of gas into gravitational potential wells induces supersonic motions and leads to both thermalization of energy due to radiative shocks, and subsequent loss of energy by cooling. The net result of this is the loss of gravitational potential energy

(with a corresponding increase in the binding energy of the system), and is termed as **dissipation**.

Numerical simulations that include dissipational cooling of the infalling gas would show whether the proposed mechanisms are indeed feasible and able to produce the kind of disks that have been seen. There is a host of other questions that simulations can help answer. Although it is clear that counterrotating systems can be in the form of either cores or extended disks, do these necessarily represent two different evolutionary paths? It is possible that counterrotating cores in spirals represent a later stage in the evolution of counterrotating galaxies, especially if the existing disk has a fair amount of gas which causes the infalling gas to lose angular momentum due to viscous dissipation. The similarity of the profiles, masses, angular momenta and scale lengths of the counterrotating disks in NGC 4550 is remarkable, to say the least. Is the scale length of the second disk determined by the angular momentum of the infalling gas (Fall & Efstathiou 1980), or by dynamical and/or feedback processes between the gas disk and halo? How would star formation be affected by the presence of the second disk, given that spiral instabilities may be suppressed in counter-streaming disks (Kalnajs 1977; Araki 1987)? How stable are these disks once they are formed? Numerical studies recently carried out (Sellwood & Merritt 1994) seem to support theoretical predictions that counterrotating disks are less susceptible to spiral instabilities such as bars.

The origin and stability issues both beg a thorough numerical investigation involving a search of parameter space for both mergers and secondary infall along with a study of the evolution of the galaxy *after* the counterrotating disks are formed.

As mentioned above, the dissipationless (stars only, no gas) merger scenario has been modeled numerically by several workers. Hydrodynamical modelling of the secondary infall process and dissipational satellite mergers has not been attempted as yet. I have undertaken the task of developing such hydrodynamical simulations.

1.6 Project Overview

The objective of this dissertation is to use hydrodynamical numerical simulations to seek answers to the following questions:

- a) Is it possible to produce a massive counterrotating disk in a spiral galaxy through an accretion or merger event without destroying the preexisting primordial disk?
- b) If so, which process produces disks that provide the closest match to the observations?
- c) In either case, what effect does the formation of the counterrotating disk have on the primordial disk?
- d) What are the characteristics of the counterrotating disk formed?

The computationally intensive nature of the calculations involved in such simulations necessitates the use of supercomputers. All the simulations for this dissertation were carried out on the Cray Y-MP at the Ohio Supercomputer Center (OSC).

The numerical method is described in detail in Ch. 2. The description, results and analysis of the numerical simulations are presented in Ch. 3-5. Ch. 6 summarizes the main conclusions of this work and discusses future work. The modifications required to the simulation code to optimize it for execution on the Cray Y-MP are described in App. A.

CHAPTER 2

COMPUTATIONAL METHOD

2.1 Introduction

Traditionally, numerical simulations of self-gravitating systems have been done using N -body methods, in which the potential of a system of N particles is obtained by either the “action-at-a-distance” or the “field” approach. The simplest of the action-at-a-distance methods uses *direct summation* to calculate the gravitational potential at the location of a particle:

$$\Phi(\mathbf{r}_i) = -G \sum_{j \neq i} \frac{m_j}{[|\mathbf{r}_i - \mathbf{r}_j|^2 + \varepsilon^2]^{1/2}} \quad (2.1)$$

where ε is the “softening parameter”. While this technique is flexible, the CPU cost per step is very high, and it scales as $\sim O(N^2)$, so that it can only be used for small- N systems. The field approach assumes a global potential and propagates particles in this field to calculate the forces. The computational cost in this case scales only as $\sim O(N \log N)$, but the method is not as accurate and generally applicable as the action-at-a-distance approach, since it necessitates fitting the potential to some special model tuned to the geometry of the problem (e.g. the Fourier grid potential solver).

A considerable improvement in CPU cost can also be attained by a relatively new technique known as the “hierarchical tree” method (Barnes & Hut 1986). This technique has the additional advantage that it is effectively gridless and yet retains the essential ingredients of the action-at-a-distance approach. Particles are processed in a manner similar to direct summation, but *the potential from distant groups of particles is approximated using low-order multipole expansions*. If the required operations are performed using tree-structured data, then the CPU cost scales as $\sim O(N \log N)$. Besides being flexible and gridless, this method is also relatively easy to implement, which makes it very attractive. A generic version of a tree code written in the C Language (the TREE code, Barnes & Hut 1986, kindly provided by J. Barnes) was adapted for this problem. Significant changes were also necessary in order to optimize it for execution on the Cray Y-MP (see Appendix A).

The dissipational properties of the gas were modeled initially with a simple gas dynamics scheme consisting of **sticky particles**, in which the gas is represented by discrete particles that undergo inelastic collisions. This was a “quick and dirty” gas dissipation model that was easy to combine with the tree code and consumed a modest amount of CPU time. It allowed us to investigate the general characteristics of the problem and isolate the problems with our models of the primary galaxy and the source of the counterrotating disk. Although we were able to obtain many key insights into the origin of counterrotating disks with sticky particles, the microphysics of this model was not sophisticated enough to permit a study of the structure of the counterrotating disk formed. To study the newly formed disk in detail, we replaced the sticky particle method with a Monte Carlo-like approach known as **smoothed**

particle hydrodynamics or SPH (Lucy 1977, Gingold & Monaghan 1977). In SPH, the gas is assumed to be a continuous medium, and fluid elements are sampled and represented by particles. The dynamical equations are obtained from the Lagrangian form of the hydrodynamic conservation laws. SPH is most naturally implemented in three dimensions and there is no grid to constrain the spatial resolution or global geometry. Since SPH is also gridless and fully Lagrangian, combining a tree code with SPH presents no major problem (Hernquist & Katz 1989, hereafter HK).

Our implementations of the tree method and SPH, and the numerical difficulties encountered, are discussed in the following sections. The sticky particle method is described in detail in the next chapter. The simulation code was written in its entirety in the ANSI C Language.

2.2 The TREE Code

The TREE algorithm computes the acceleration on a given particle by walking through a tree consisting of *cells* and *particles*, where cells correspond to the internal nodes of the tree and particles correspond to the leaves. Each cell can contain pointers to up to 8 *descendants*, which may be either cells or particles. The force between particles is obtained as usual from the softened form of the Keplerian potential,

$$\phi(r) = -\frac{m_1 m_2}{(r^2 + \varepsilon^2)^{1/2}}, \quad (2.2)$$

where m_1 and m_2 are the particle masses and ε is the softening parameter, and $G = 1$. The force between a particle and a cell (i.e. a cluster of particles) is obtained from a multipole expansion of the cluster potential about its center of mass. If only the monopole approximation is used, then the force is given by (2) with m_2 being the

cluster mass, m_1 the particle mass, and r the distance between the particle and the cluster center of mass. The TREE code can also calculate the quadrupole moment (this is optional) recursively for more accuracy, since this is the next higher order term for expansion about the center of mass (the dipole term vanishes). The decision to consider a distant collection of particles as a cell or cluster is based on a user-defined fixed tolerance parameter θ . If $s/d < \theta$, where s is the cluster size and d is the separation between the particle and the cluster, then the cluster is not resolved into individual particles but its force contribution is approximated instead by the multipole expansion as described above.

2.2.1 Advantages of the TREE code

The gridless nature of the TREE code is its biggest advantage, and it provides the implementer the flexibility to include features that would introduce prohibitive complexity in other schemes. For example, HK have introduced individual particle time-steps such that each individual time-step is a power-of-two subdivision of the largest (system) time-step. This ensures that the system is synchronized at the end of the largest time-step even though the particles have their own time-steps. The lack of a constraining grid also makes the TREE code Lagrangian, so that spatial resolution is only limited by the number of particles possible in the simulation.

2.2.2 Limitations of the TREE code

The original version of the TREE code that we used as our starting point was about 10 times slower than the fastest Fourier grid codes using Fast Fourier Transforms (Hernquist 1987). This is because it was not vectorized so that it could run fast on

the Cray Y-MP. Vectorization refers to optimization for a “pipeline vector machine” that handles an entire array of numbers as a single “vector” and operates on it in parallel, in assembly-line fashion, instead of sequentially looping through it one element at a time. Even the speed payoff due to $N \log N$ scaling does not kick in until $N \geq 5000$. The primary bottlenecks in the tree code were the tree-walks (to compute the acceleration of each particle) and tree construction. The modifications necessary for vectorization of the tree code are described in Appendix A.

To maintain $N \log N$ scaling, the value of θ is practically limited to $0.3 \leq \theta \leq 1.0$. The error in the force calculation due to the multipole approximation is determined by the value of θ , hence there is an obvious tradeoff between the large θ required for speed and the small θ required for accuracy. The error due to the discrete Monte-Carlo sampling that is inherent in all N -body methods dominates (often by an order of magnitude) the error due to the tree force calculation (Barnes & Hut 1989) for judicious choices of N ($\gtrsim 10^4$) and θ ($0.8 \leq \theta \leq 1.0$).

Another problem with the TREE code is the conservation of linear momentum. Although all N -body methods violate conservation of energy and linear/angular momentum due to numerical errors, the momentum conservation law is most strongly violated by the TREE method. This is because of the lack of symmetry in the force calculation between a particle and a cell (or cluster of particles).

2.3 Numerical Difficulties

The development of a code to simulate thin disks in itself is not an easy task. This is because thin galactic disks (scale length/height $\gtrsim 10$), are fraught with instabilities

that make the construction of a stable numerical model quite tricky. In addition to the axisymmetric Jeans instability which causes local clumping of disk particles, rotating disks are susceptible to bar instabilities and bending (firehose) instabilities (Merritt & Sellwood 1994). Local stability is usually bought by bumping up the value of Q (Toomre's stability criterion, see Binney & Tremaine 1987), whereas the presence of a fixed dark halo and/or higher velocity dispersion fortify the disk against the tendency to form bars (Binney & Tremaine 1987). Bending modes seem to chronically affect thin disks, however, and it is not clear what stabilizes against them.

Apart from instabilities, an additional difficulty is that a sufficiently large number of disk particles N is required in the simulation. There are several reasons for this:

a) The smaller the number of particles, the more grainy the potential and hence greater the chance of two-body encounters in the disk (Hockney & Eastwood 1989). Furthermore, relaxation effects seem to depend only on the graininess or resolution of the potential, and are relatively independent of the N -body algorithm used (Hernquist & Barnes 1990). To make the disk free of relaxation effects for more than a few rotations, a sufficiently large number of particles is necessary.

b) The effects of the Poisson noise due to the discrete sampling of the mass distribution become more pronounced with smaller N , because the signal-to-noise ratio is proportional to $\sqrt{N}$. It is especially important to minimize the noise in spiral disks, because unwanted modes introduced due to the noise get swing-amplified as the disk rotates.

c) For the TREE code to provide significant savings over a direct N -body code, N should be $\gtrsim 5000$ (Hernquist 1987).

d) Finally, better resolution demands a higher value of N . In three dimensions, good resolution for the disk may be hard to achieve without a prohibitively large value of N .

The speed limitation of the TREE code discussed earlier limits the number of particles possible.

2.4 Smoothed Particle Hydrodynamics (SPH)

In SPH, the fluid is modeled as a collection of fluid elements. Since a real fluid has a very large number of such elements (each of infinitesimal size), for a numerical representation the fluid must somehow be “sampled” by a finite number of fluid “particles”, in such a way that the particle mass density follows the fluid mass density. Instead of adopting discrete particles to represent the fluid, however, the physical properties of the particles are “smoothed” or averaged instead, and the average value of any physical attribute of the fluid (e.g. density) at any point is estimated using a smoothing or averaging kernel. Thus if $f(r)$ is any property of the fluid at location r , then its average value is given by

$$\langle f(\mathbf{r}) \rangle = \int W(\mathbf{r} - \mathbf{r}', h) f(\mathbf{r}') d\mathbf{r}', \quad (2.3)$$

where $W(\mathbf{r} - \mathbf{r}', h)$ is the **smoothing/averaging/interpolating kernel** and h is the **smoothing length**. For numerical integration, this becomes

$$\langle f(\mathbf{r}_a) \rangle = \sum_b m_b \frac{f(\mathbf{r}_b)}{\rho_b} W(\mathbf{r}_a - \mathbf{r}_b, h) = \sum_b m_b \frac{f_b}{\rho_b} W_{ab}. \quad (2.4)$$

For example, the average density at point a is given by

$$\langle \rho_a \rangle = \sum_b m_b W_{ab}. \quad (2.5)$$

The ingenuity of SPH lies in the fact that *the kernel can be chosen so that it is differentiable analytically, so there is no need for differencing on a grid*. This makes SPH gridless and fully Lagrangian. The equations of motion in SPH are

$$\text{Continuity:} \quad \frac{\mathcal{D}\rho_a}{\mathcal{D}t} = - \sum_b m_b \mathbf{v}_{ab} \cdot \nabla_a W_{ab}, \quad (2.6)$$

$$\text{Momentum:} \quad \frac{\mathcal{D}\mathbf{v}_a}{\mathcal{D}t} = - \sum_b m_b \left(\frac{P_a}{\rho_a^2} + \frac{P_b}{\rho_b^2} + \Pi_{ab} \right) \nabla_a W_{ab}, \quad (2.7)$$

$$\text{Thermal Energy:} \quad \frac{\mathcal{D}u_a}{\mathcal{D}t} = \frac{1}{2} \sum_b m_b \left(\frac{P_a}{\rho_a^2} + \frac{P_b}{\rho_b^2} \right) \mathbf{v}_{ab} \cdot \nabla_a W_{ab}, \quad (2.8)$$

where $\mathcal{D}/\mathcal{D}t$ indicates the *Lagrangian derivative* (i.e. derivative following the motion), and Π_{ab} is the *artificial viscosity*. The particles are moved using

$$\frac{\mathcal{D}\mathbf{r}_a}{\mathcal{D}t} = \mathbf{v}_a,$$

and the time-step is subject to the Courant condition ($\delta t \leq h/a_s$, where a_s is the sound speed in the medium) as well as limits imposed by the force (forces must not get too large) and artificial viscosity (shear viscosity must not get too large - see limitations below).

2.4.1 The Kernel

The choice of the kernel is crucial to the success of the code. Whereas a Gaussian kernel is very useful if we want to get a physical interpretation of an SPH quantity, other kernels are much more suitable for real implementations. The basic requirements of a kernel are

$$\int W(\mathbf{r} - \mathbf{r}', h) d\mathbf{r}' = 1 \quad \text{and} \quad \lim_{h \rightarrow 0} W(\mathbf{r} - \mathbf{r}', h) = \delta(\mathbf{r} - \mathbf{r}').$$

A good kernel has compact support (i.e. is strongly peaked and approaches a delta function as $h \rightarrow 0$) and has a continuous second derivative. The compact support minimizes the errors due to interpolation, and the continuous second derivative minimizes the impact of particle disorder on the errors due to the use of summation interpolants in place of integral interpolants. A kernel which meets these requirements is the spline kernel (Monaghan & Lattanzio 1985).

2.4.2 Advantages of SPH

The main advantages of SPH are its flexibility due to its gridless implementation, its ease of implementation in three dimensions, and the ability to use a smoothing length (h) that is variable in space and time (the SPH analog of an adaptive grid) to include any physical effect (e.g. radiative cooling, star formation) so long as it can be expressed in terms of a global conservation equation. An attractive feature of the kernel scheme used is that the kernel can be computed in a separate subroutine, so that a different kernel can be used without having to change the whole program. The ability to interchange different kernels with ease provides a tremendous flexibility to the SPH user. Finally, the use of the symmetric pressure gradient term in the momentum and energy equations ensures the conservation of linear and angular momentum as well as energy. If the correct time-step is chosen, the total energy should be conserved to within 0.5% over 400 time steps (Monaghan 1992).

2.4.3 Limitations of SPH

SPH cannot handle very large density gradients since it only has a finite number of particles. The treatment of shocks is only as good as the artificial viscosity, and

a satisfactory general artificial viscosity is not currently available. All the artificial viscosity schemes used to date are rather *ad hoc* in nature, and one of the principal problems in SPH implementations of rotating disks is too much shear viscosity.

Errors due to the averaging process can only be calculated analytically for integral interpolants and it is difficult to estimate them for the summation interpolants unless all the particle positions are known. For the spherically symmetric spline kernel, the errors are $\sim O(h^2)$ (HK), where h is the smoothing length. In general, errors increase with the particle disorder. For moderate disorder (this applies to most astrophysical simulations),

$$\text{errors} \propto \frac{1}{n} \log n^2, \quad (2.9)$$

where n is the number of particles (Monaghan 1992). The ultimate yardstick for measuring the success of SPH is how well it solves problems with known solutions, and it has in fact been used successfully in a variety of applications (Monaghan 1992).

2.5 TREE+SPH

The combination of a TREE code and SPH implemented for this dissertation follows closely that of TREESPH (HK). The main differences are that a spline kernel is not used to soften gravity, and radiative cooling is omitted. Standard softening (eq. 2.2) is used for gravity, and an isothermal gas is assumed. This significantly reduces the computational resources required for each simulation, and this benefit is achieved at the expense of very little accuracy in the gas dynamics. Previous SPH simulations (Barnes & Hernquist 1991, Mihos & Hernquist 1994a) have established that for galaxy interaction simulations, an isothermal gas is indeed a good assumption.

CHAPTER 3

STICKY PARTICLE SIMULATIONS FOR COLD PRIMARY

Published Work: This chapter has been previously published as Thakar, A. R. & Ryden, B. S. 1996, “Formation of Massive Counterrotating Disks in Spiral Galaxies”, *Astrophysical Journal*, 461, 55. Figures and tables have been renumbered to ensure continuity, and references have been moved to the general bibliography at the end.

3.1 Abstract

We present results of numerical simulations of the formation of a massive counterrotating gas disk in a spiral galaxy. Using a hierarchical tree gravity solver combined with a sticky-particle gas dissipation scheme for our simulations, we have investigated three mechanisms: episodic and continuous gas infall, and a merger with a gas-rich dwarf galaxy. We find that both episodic and continuous gas infall work reasonably well and are able to produce a substantial gas counterrotating disk without upsetting the stability of the existing disk drastically, but it is very important for the gas to be well-dispersed in phase-space and not form concentrated clumps prior to its absorption by the disk galaxy. The initial angular momentum of the gas also plays a crucial role in determining the scale length of the counterrotating disk formed and the time

it takes to form. The rate of infall, i.e. the mass of gas falling in per unit time, has to be small enough to preclude excessive heating of the preexisting disk. It is much easier in general to produce a smaller counterrotating disk than it is to produce an extensive disk whose scale length is similar to that of the original prograde disk.

A gas-rich dwarf merger does not appear to be a viable mechanism to produce a massive counterrotating disk, because only a very small dwarf galaxy can produce a counterrotating disk without increasing the thickness of the existing disk by an order of magnitude, and the time-scale for this process is prohibitively long because it makes it very unlikely that several such mergers can accumulate a massive counterrotating disk over a Hubble time.

3.2 Introduction

In recent years, high spectral resolution combined with sophisticated computer spectral analysis techniques has yielded line-of-sight velocity distributions of galaxies which reveal the vast kinematic complexity shown by the stars and gas in some elliptical, lenticular and spiral galaxies. One of the most interesting phenomena is the presence of counterrotating disks, i.e. disks rotating in a sense opposite to that of the galaxy as a whole. In ellipticals, the disks are usually confined to the nuclear regions, forming counterrotating *cores*. In some S0/spiral galaxies, however, up to half of the stars and/or gas in the disk system can be in retrograde orbits, forming counterrotating *disks* that extend far beyond the nuclear regions, e.g. NGC 4550 (Rubin, Graham & Kenney 1992), NGC 4826 (Braun, Walterbos & Kennicutt 1992), NGC 4546 (Sage & Galletta 1994), NGC 7217 (Merrifield & Kuijken 1994), and most recently NGC 3626 (Ciri, Bettoni & Galletta 1995).

The counterrotating stellar cores seen in ellipticals seem to be the result of dissipationless, retrograde mergers between progenitors of different sizes (Kormendy 1984). Simulations of such mergers have shown that the core of the secondary galaxy sinks to the center of the primary as a result of dynamical friction (Balcells & Quinn 1990). Counterrotating disks in spiral galaxies, however, are intriguing because both disks can be cold and have similar scale lengths, as in NGC 4550 (Rix *et al.* 1992). It is very hard to imagine such disks being formed together, although it has been suggested that two *identical* counterrotating disks could conceivably arise due to the conversion of box orbits to tube orbits when a triaxial halo potential becomes axisymmetric (Evans & Collett 1994). Box orbits are equally likely to scatter in either direction, giving rise to equal populations of stars orbiting prograde and retrograde. In the general case, however, where the counterrotating disk is of a different size, has a significantly different velocity dispersion, or is inclined to the plane of the primary disk, one of the disks is almost certainly of external and secondary origin.

However, a merger between two disk galaxies with comparable masses but opposite spins would not be able to produce a counterrotating disk. Simulations of gas-rich major mergers (e.g. Barnes & Hernquist 1991, Hernquist & Barnes 1991, Barnes 1992) demonstrate that the associated tidal trauma plays havoc with the velocity fields of both disks and drives the gas to the center of the remnant, which resembles an elliptical galaxy with a kinematically distinct (sometimes counterrotating) gas core. A counterrotating disk in a spiral is much more likely to be formed by secondary infall of gas which was initially moving in a retrograde orbit and which was forced to settle in a flat disk aligned with the potential symmetry plane due to energy dissipation and

subsequent loss of angular momentum (Rix *et al.* 1992). The preexisting disk can remain unperturbed if it has little or no prograde gas to begin with, and if the gas build-up is adiabatic. This is especially relevant in the light of cosmological models which predict that galaxies continue to accrete matter throughout their lives (Ryden & Gunn 1987, Ryden 1988, White 1990), and that the spin angular momentum of a galaxy with respect to this infalling material changes on time-scales much smaller than the galaxy's lifetime (Ryden 1988, Ostriker & Binney 1989). Spirals must also have an ongoing supply of outside gas if they are to maintain their observed star-formation rates over times comparable to the age of the universe (White 1991). Thus it is quite possible that, over time, infalling gas may assume a retrograde orbit around a galaxy and settle gradually into a counterrotating disk as large as the existing disk.

Although major mergers may be safely ruled out by the observed properties of counterrotating disks in spirals, it is not so easy to dismiss retrograde mergers with gas-rich dwarf galaxies in which dissipation does play a significant role. The ubiquity of dwarf galaxies (especially in clusters) suggests that this may be a potentially frequent phenomenon and worth investigating. Dwarf galaxies generally contain a fraction (typically a tenth) of the mass of normal galaxies. In a gas-rich dwarf, most of the luminous mass is in the form of gas (Roberts & Haynes 1994). The stellar mass is thus only a few percent of the mass of the original galaxy, and therefore likely to be much less disruptive to the spiral galaxy's disk. Dwarf galaxies also have dark halos, and the question is whether the stellar and gas components of the dwarf can form a counterrotating disk in spite of the dark halo and its potentially disruptive influence on the primary disk.

We have undertaken the task of developing hydrodynamical N-body simulations to investigate the dissipational infall and dwarf-merger hypotheses for forming counterrotating disks. Our numerical code consists of two components: a hierarchical tree gravity solver and a simple gas dissipation scheme. The gravity solver is a specialization of the generic TREE code (Barnes & Hut 1986) which has emerged as the method of choice among N-body codes in the last few years due to the gridless flexibility it affords and the excellent blend of speed and accuracy it provides for most applications. The preferred method of incorporating gas dynamics into N-body simulations currently is Smoothed Particle Hydrodynamics (Lucy 1977, Gingold & Monaghan 1977, Monaghan 1992), which allows realistic treatment of the physics involved and also treats the gas as a continuous medium rather than discrete particles. However, crude gas dynamics which represent gas clouds as particles and dissipate energy via inelastic collisions between these clouds can be used as a short-cut to gain some quick insights. Such “sticky particle” schemes have proven to be quite useful in the past (Brahic 1977, Schwarz 1981, Negroponte & White 1983, Roberts & Hausman 1984, Shlosman & Noguchi 1993), and in fact they may represent the clumpy nature of the gas better than SPH. We have therefore adopted this approach as a first approximation in order to test our galaxy model and study the macroscopic behavior of the gas prior to implementing “real” gas dynamics with SPH.

Our physical model and numerical method are described in detail in the next section, followed by a discussion of the simulation results and their implications in section 3. Section 4 summarizes our conclusions.

3.3 Computational Method

3.3.1 Spiral Galaxy Model

We adopt a two-component spiral galaxy consisting of a disk and a halo. The disk mass distribution is exponential in both the radial and vertical directions. The initial velocities are assigned using the epicycle approximation. The shape and orientation of the Schwarzschild velocity ellipsoid in the plane of the disk is assumed to be constant so that the radial, azimuthal and vertical dispersions bear a constant ratio to each other everywhere in the plane. The radial dispersion σ_r is assumed to be proportional to the local disk surface density and is set by specifying the value of the stability parameter Q . The azimuthal dispersion σ_ϕ is obtained from σ_r and standard epicycle theory (Binney & Tremaine 1987, §3.2). For an exponential vertical mass distribution, the vertical dispersion $\sigma_z(z)$ can be obtained by integrating the Boltzmann equation for the plane-parallel approximation (van der Kruit 1988):

$$\begin{aligned}\sigma_z^2(z) &= \sigma_z^2(0)\{2 - \exp(-z/z_e)\}, \\ \sigma_z^2(0) &= \pi G \Sigma z_e\end{aligned}$$

where Σ is the disk surface density and z_e is the disk scale height. The rotation velocity is then calculated from the circular speed and the velocity dispersions by solving Jeans equation (Binney & Tremaine 1987, §4.2).

The halo mass distribution is that of a truncated isothermal sphere supported by a Maxwellian random velocity distribution (truncated at 2σ to avoid excessive evaporation). The halo is assumed to be four times as massive as the disk. The disk and halo are dissipationless and assumed to contain no gas initially.

Our units are $[t] = 1$ Myr, $[m] = 2.2 \times 10^{11} M_{\odot}$, and $[r] = 1$ kpc, which makes the velocity unit $[v] = 977.8$ km/s. In these units, the disk parameters are: mass $M_d = 0.25$, total radius $R_d = 21$, scale length $R_e = 3.5$, scale height $z_e = 0.3$, and the value of Q at $R = 8.5$ is 1.20. The halo parameters are: mass $M_h = 1.0$, total radius $R_h = 50$, core radius $R_c = 1.0$, and velocity dispersion $\sigma_h = 0.13$.

3.3.2 Infall Model

We allow the halo to virialize initially by holding the disk frozen for a few crossing times. The disk and halo are then allowed to evolve together for about 10-12 crossing times before the gas is brought into the picture. The gas is configured as a rectangular slab of uniform density and constant velocity dispersion. In accordance with the typical random velocities of molecular clouds, the velocity dispersion of the gas is assumed to be ~ 10 km/s. The density of the slab determines how much gas is added to the galaxy per unit time, and hence is kept low enough to ensure that the gas does not fall in faster than the disk can handle. The maximum infall rate which the disk can withstand without significant heating was experimentally determined to be a few percent (2-5 %) of its mass per crossing time. The length of the initial rectangle of gas is therefore proportional to the total mass of the gas. With the origin at the center of the galaxy and the galactic disk in the xy plane, the gas slab is placed slightly above the xy -plane and parallel to the y -axis. It is then given a velocity (i.e. each gas particle is given the same orbital velocity) in the negative y direction which is an appropriate fraction of the Keplerian velocity $\sqrt{GM/R}$, where M is the mass of the galaxy, and R is the distance of the gas slab from the y -axis. This initial velocity (and hence the initial angular momentum) is set to be small enough that the gas falls into

the galaxy’s potential well, and large enough that it does so within a Hubble time. The direction of the velocity is also set to ensure that the gas orbit is retrograde with respect to the primary disk’s rotation.

We follow the motion of the gas and disk particles until all the gas settles into the plane of the disk. Although it is possible to comment on the size, orientation and general appearance of the counterrotating disk formed, the simple sticky particle scheme cannot be used to study the more complex physics that must come into play after this point, such as shocks, star formation, heating by supernovae, etc. There is also the implicit assumption that no gas particles are lost to star formation prior to their forming a counterrotating disk. As a result, we can only study the gross properties of the infall process, such as the time-scale of counterrotating disk formation, the overall size and shape of the counterrotating disk formed, and the thickening of the primary disk.

3.3.3 Gas-Rich Dwarf Merger Model

Dwarf galaxies range from dwarf ellipticals with ellipsoidal mass distributions to dwarf irregulars with unknown mass distributions. There is no “garden-variety” dwarf galaxy model as such, and even properties such as mass, radius and gas content vary widely. Hence we decided to use a spherical Plummer model to represent our dwarf galaxy. In this model, the mass density function is assumed to be a polytrope of index 5:

$$\rho(r) = \left(\frac{3}{4\pi}\right) \frac{M}{R^3[1 + (r/R)^2]^{5/2}}$$

where M is the total mass, and R determines the size (radius) of the sphere. Actually, the three mass components - gas, stars and dark matter - form concentric Plummer

spheres with the gas sphere being the smallest and the dark matter being the largest. Initial positions and velocities of the equal-mass particles in each Plummer model are determined according to a scheme described in the literature (Aarseth *et al.* 1974).

The dwarf galaxy (secondary) is placed in the plane of the primary's disk at a large enough distance from the primary that its gravitational influence on the disk orbits is negligible; the initial separation is a little more than 3 halo radii (150 kpc). The secondary is also given a small non-zero velocity which places it in a retrograde orbit around the primary's disk so that if a second disk is formed it will rotate counter to the original disk.

We carried out three simulations of mergers between our spiral galaxy and a gas-rich dwarf galaxy. The primary difference among these is the mass of the dwarf galaxy. This is the quantity which has the most significant impact on the outcome of the simulation. The important parameter values are summarized in Table 1 for the three runs dubbed S1, S2 and S3.

3.3.4 Numerical Code

For the gravity solver, we have adapted a C-language version of the hierarchical TREE code (Barnes & Hut 1986) for the infall problem and implemented it on the Cray Y-MP at the Ohio Supercomputer Center. Fundamental changes to the TREE code were necessary to optimize and in particular vectorize it so that it would run efficiently on the Cray (Hernquist 1990, Makino 1990). Performance characteristics of the TREE code have been discussed elsewhere (Hernquist 1987), and an error analysis (Barnes & Hut 1989) has shown that errors can be made quite small with judicious choice of parameters, especially for large particle numbers ($> 10^4$). To ensure good accuracy,

Run No.	Dwarf Mass	Dwarf GMF ^a	Dwarf MLR ^b	Dwf/Sp TMR ^c	Dwf/Sp LDR ^d
S1	0.30	0.6	3.0	24%	40%
S2	0.10	0.6	2.0	8%	20%
S3	0.05	0.6	2.0	4%	10%

^a Gas Mass Fraction - fraction of luminous mass in gas particles.

^b Mass to Light Ratio - ratio of total mass to luminous (stars + gas) mass.

^c Total Mass Ratio - ratio of total mass of dwarf to total mass of spiral.

^d Luminous to Disk mass Ratio - ratio of luminous mass in dwarf to disk mass in spiral.

Table 1: Parameter values for gas-rich dwarf merger simulations.

we use quadrupole moments for all simulations, and an opening angle (clumping parameter) $\theta = 0.8$.

In all the simulations presented here, we use 32k particles to represent the disk, 16k particles for the halo, and up to 20k particles for the gas (depending on its mass). Each gas particle is about 1.3 times as massive as a disk particle. The disk crossing time is ~ 100 Myr, so the disk is held frozen for $t = 0 - 500$ Myr to allow the halo to virialize. Following this, the disk and halo are allowed to evolve together for $t = 500 - 1500$ Myr. The gas is then introduced at $t = 1500$ Myr.

One concern in simulations of disks is the minimization of two-body encounters so as to keep the disk as relaxation-free as possible for the duration of the simulations. Although relaxation rates in tree codes are comparable to those in PM codes with similar force resolution (Hernquist & Barnes 1990), and the number of disk particles

we chose is enough to keep the disk relatively relaxation-free for the longest of our simulations in the absence of any other effects (Hockney & Eastwood 1989), the discreteness of the halo heats up the disk significantly as time progresses. Since our halo is 4 times as massive as the disk, each halo particle is 8 times as massive as a disk particle. Hence halo particles passing through the disk can disrupt the disk and alter its vertical structure much like the $\sim 10^6 M_\odot$ black holes postulated by Lacey & Ostriker (1985). A test simulation showed that doubling the number of halo particles (to 32k) reduces the increase in disk thickness by 40-50% on average. The increased heating of the disk with radius for our more discrete halo is consistent with the predicted swing amplification of the halo discreteness noise by the disk (Sellwood 1989).

The effect of the discrete halo on the disk also necessitates the choice of a rather high softening length, $\epsilon = 1.0$ kpc. This limits our vertical resolution to 1 kpc, and changes in thickness which amount to less than this cannot be considered significant. This limited resolution is still enough to enable us to identify broad trends in the behavior of the disk, however.

We have chosen a time-step of 5 Myr. Since the resolution is set by ϵ , the time-step is small enough that the average particle (whose velocity ~ 0.2) does not move more than the spatial resolution length in one time-step. Furthermore, since the minimum collision radius for sticky-particle collisions is $r_{\min} = \epsilon/2$ (see below), this also ensures that very few if any collisions are missed between successive time-steps, since the relative velocity between colliding particles is usually much less than 0.2.

3.3.5 Sticky Particle Method

The sticky particle method we adopted includes features from several previous implementations (Brahic 1977, Schwarz 1981, Negroponte & White 1983). We use the standard scheme for binary inelastic collisions, which conserves linear and angular momentum and dissipates kinetic energy by reducing and reversing the relative radial velocity of the colliding particles. The tangential component of the relative velocity is left intact. The final velocities of colliding particles p and q are given by (Brahic 1977):

$$\begin{aligned} \mathbf{v}'_p &= \frac{1}{2} \{ \mathbf{v}_p + \mathbf{v}_q + [\mathbf{v}_{pq} \cdot \mathbf{i}] \alpha_c \mathbf{i} + (\mathbf{v}_{pq} \cdot \mathbf{j}) \mathbf{j} \}, \\ \mathbf{v}'_q &= \frac{1}{2} \{ \mathbf{v}_p + \mathbf{v}_q - [\mathbf{v}_{pq} \cdot \mathbf{i}] \alpha_c \mathbf{i} - (\mathbf{v}_{pq} \cdot \mathbf{j}) \mathbf{j} \} \end{aligned}$$

where $\mathbf{v}_p$ and $\mathbf{v}_q$ are the initial velocities, $\mathbf{v}_{pq} = \mathbf{v}_p - \mathbf{v}_q$, $\mathbf{i}$ is a unit vector in the radial direction from p to q , $\mathbf{j}$ is a unit vector along the tangential component of $\mathbf{v}_p - \mathbf{v}_q$, and α_c is the coefficient of restitution ($-1 < \alpha_c < 0$ for an inelastic collision). The energy lost after one such collision is $(1 - \alpha_c^2)[\mathbf{i} \cdot (\mathbf{v}_p - \mathbf{v}_q)]^2$. The stickiness of the gas particles is adjusted by varying the coefficient of restitution α_c ; the number of collisions can also be controlled by the minimum and maximum collision radii.

We allowed each particle to be involved in at most one collision per time-step in order to make the collisions at any given time-step independent of each other. The second particle chosen for the collision is the one nearest to the current particle which meets the other criteria necessary for a collision to occur and which has not already been paired with another particle. Hence multiple collisions are not possible and only binary collisions are handled. This was necessary for the collisions to be computed in parallel. For this reason our gas is probably not excessively dissipative even for

rather low values of α_c (~ -0.25). We chose $\alpha_c = -0.5$ for the simulations reported here.

A collision between two particles occurs when the distance between them is less than a pre-defined “collision radius” (r_c), which varies with the density to make the number of collisions roughly independent of density. This avoids the overlapping of clouds in dense regions (Negroponte & White 1983). To avoid collisions in very dense regions (where r_c is small and can cause a runaway increase in density), we define a minimum collision radius $r_{\min}$ whose value is set to half the gravitational softening length (or resolution length) ϵ ; to limit the search volume around each particle in which to look for potential collisions, a maximum collision radius $r_{\max}$ was used (we adopted a value of 3.0 for $r_{\max}$). It was also necessary to have a minimum relative K.E. cutoff $E_{\min}$ so that particles bound to each other do not engage in too many successive low-energy collisions. This limit was specified by setting the minimum relative collision velocity to be 0.010. If the relative K.E. between two colliding particles is less than $E_{\min}$, the particles undergo an elastic collision instead of an inelastic one. Thus particles do not interpenetrate.

Finally, to the standard collision model we added the refinement necessary to distinguish between particles moving toward each other and those moving away from each other. Thus collisions are avoided if the particles are separating (as implied by the angle between the relative radial velocity and the relative radial position vector), even if they are within the collision radius.

3.4 Results and Discussion

The list of input parameters and initial conditions that can have a tangible effect on the outcome of the simulations is quite long. For dissipational infall, the parameters which we decided to test include the infall rate of the gas, the mass of the galactic halo, the “stickiness” of the gas (as specified by the coefficient of restitution α_c), and the magnitude and direction of the initial angular momentum of the gas (the former being specified by the initial velocity of the gas, the latter by the inclination of the infall trajectory to the plane of the disk). For the gas-rich dwarf merger, the primary input parameter tested is the mass of the dwarf galaxy.

For all simulations, results are shown starting at $t = 1.5$ Gyr, which is the point at which the gas/dwarf-galaxy is introduced.

3.4.1 Episodic Infall

Our *episodic* infall is really *periodic* infall, since the episodes are equally spaced in time. The formation of a counterrotating gas disk is illustrated in Figs. 1 and 2. We introduce a new slab of gas after every 1.5 Gyr. The slab is placed slightly (5 kpc) out of the plane of the primary disk. The mass of each slab is 8% of the mass of the disk. The face-on (top) view (Fig. 1) shows the gas forming a disk considerably smaller than the primary stellar disk, and the edge-on view (Fig. 2) indicates that the thickness of the gas disk is comparable to that of the stellar disk.

The thickness of the primary disk is studied more quantitatively by plotting the mean half-thickness as a function of radius at different epochs during the simulation (Fig. 3). This is done by calculating the mean z -coordinate at each (x, y) location

Figure 1: Face-on view of counterrotating disk formation by periodic infall. The galactic (primary) disk is shown only in the first panel for clarity. The galactic halo is not shown. Each slab of gas is 8% as massive as the disk. The square panels are 200 kpc on each side. Note that all panels are not equally spaced in time. Time is in Gyr.

by binning the z values in bins of size 1 kpc^2 . The average of the mean z -coordinates is then computed for each value of R . This ensures that any tilt or warp in the disk plane is accounted for, although there is some error introduced due to the fact that the thickness is measured vertically (parallel to z -axis) even though the disk may be tilted. Since we are more concerned with the trend in the thickness rather than its precise value, we have neglected this error.

Figure 2: Edge-on view of periodic gas infall. The galactic disk is shown only in the first panel. The gas is initially 5 kpc above the plane of the disk, so the infall is nearly coplanar.

There is a significant increase in the thickness of the disk, especially in the outer regions, as the simulation progresses. By $t = 9.0$, or 7.5 Gyr after the infall begins, the disk thickness has increased by a factor of 5-6 at some radii. The mass of the gas accumulated in the counterrotating disk by this time is about 40% of the primary disk's mass. As more gas is added, the thickness increases gradually and the heating of the disk progresses inwards towards the center, as more and more gas finds its way there.

In the meantime, the primary disk also develops a bar which gets more pronounced as more gas falls in, since the colder gas is more susceptible to bar formation. The final velocity fields of the primary disk and the gas (Fig. 4) confirm that the gas is counterrotating and also indicate that it has lost most of its velocity dispersion due to collisions and hence is significantly colder than the primary disk.

Once there is some counterrotating gas in the disk, successive episodes of infall can build up the counterrotating disk faster because the gas gets rapidly captured by the disk. The incoming gas particles collide with the ones already in the disk and hence get decelerated enough to be instantly inducted into the counterrotating flow. The heightened collision rate inhibits the “flinging” normally caused by tidal stripping, thereby facilitating the quick capture of the gas.

3.4.2 Continuous Infall

The discrete infall events in the periodic scheme have a somewhat *ad-hoc* nature to them. The gas is more or less “spoon-fed” to the spiral galaxy in a “palatable” form every so often. Although we do not know enough about the nature of real gas infall, a more realistic situation perhaps is one in which all of the gas is initially placed in a configuration which will allow it to make its own way to the galactic disk over an extended period of time (to keep the infall rate sufficiently low), the only stipulation being that it should do so on a retrograde orbit. To simulate such *continuous* gas infall, we make the initial rectangular slab of gas sufficiently long that its infall time is close to a Hubble time (~ 12 Gyr). The mass of the slab is set to 80% of the mass of the primary disk, so that the counterrotating disk formed will be almost as massive as the primary disk.

Figure 3: (a) Top (left panels) and side (right panels) views of the primary (galactic) disk for different epochs during periodic infall. The width of each square panel is 40 kpc. (b) The mean half-thickness of the disk as a function of radius for each epoch. Each panel shows the effect of adding more gas in steps of 8% of the primary disk's mass, so that by $t = 9.0$, the total mass of gas in the counterrotating disk is about 40% of the primary disk mass.

Figure 4: Velocity fields of the disk and counterrotating gas at $t = 8.5$. Only a small subset ($\sim 6\%$) of the total number of particles is shown for clarity. The left panel shows the primary disk, rotating counterclockwise, and the right panel shows the gas, rotating clockwise.

In our first test of continuous gas infall, we made the cross-section of the rectangular slab the same as it was for periodic infall. The results in this case are shown in Fig. 5. It is clear that most of the length of the slab breaks up into individual clumps well before it falls into the galaxy. The clumps formed are large enough to have an appreciable effect on the disk, with the largest of them approaching the sizes of dwarf galaxies, with masses of the order of $10^9 M_\odot$. Even after tidal stripping, the cores of these are dense enough to remain intact and spiral into the center of the galaxy due to dynamical friction, which is proportional to the mass of the clumps to first order (Binney & Tremaine 1987, eq. 7-26):

$$t_{\text{fric}} \sim \frac{r_i^2 v_c}{GM} \sim \frac{1}{M}$$

Figure 5: (a) Top and (b) side views of counterrotating disk formation by continuous gas infall. The initial configuration of the gas is a long, thin rectangular slab of uniform density. Self-gravity causes the slab to collapse into a thin filament and then further disintegrate into clumps before falling into the galaxy.

where t_{fric} is the time it takes for an object with mass M and initial distance r_i to spiral in under dynamical friction to the center of an isothermal mass distribution whose circular velocity is v_c . A simple calculation for $M = 10^9 M_\odot$ and a maximum initial impact parameter of 50 kpc yields $t_{\text{fric}} \simeq 6$ Gyr for $r_i = 20$ kpc (the edge of the primary disk, where $v_c \simeq 180$ km/s).

After about 4.5 Gyr (by $t = 6.0$), we see a counterrotating disk starting to form. Most of the gas has fallen in by about $t = 10.0$, and the final size of the counterrotating

Figure 6: The thickness of the disk measured for different epochs during continuous infall. By $t = 9.0$, the thickness increases by a factor of 6-8 in some parts of the outer disk.

disk formed is quite small ($\lesssim$ half the size of the primary disk), as would be expected with most of the gas particles being clumped together very closely in phase-space and subjected to dynamical friction by the time they come close to the primary disk. The effect on the primary disk of “digesting” these clumps of gas is measured in terms of the mean half-thickness as a function of radius for different epochs during the infall (Fig. 6).

Figure 7: (a) Top and (b) side views of continuous infall with a thicker (less dense) initial slab of gas than in Fig. 5. The counterrotating disk formed is slightly larger than the primary disk.

Clearly, the clumping of the gas particles poses a serious problem for the formation of a counterrotating disk whose size is comparable to that of the primary disk, and it also has a detrimental effect on the tranquillity of the primary disk. To minimize

the clumping, we tried substantially reducing the density of the slab by increasing its cross-section but keeping its length the same to preserve the infall rate. The increased dynamical time in this case makes it much more difficult for the gas particles to coalesce into clumps before they fall into the galaxy. The results are shown in Fig. 7. It is evident that the clumping is much less pronounced than in Fig. 5, and the counterrotating disk formed in this case is much larger than before. It is almost the same size as the primary disk. Thus if the gas initially occupies a larger volume in phase-space, it will tend to form a larger counterrotating disk.

Another factor which can affect the size of the counterrotating disk formed is the magnitude of the initial angular momentum of the gas. Fig. 8 shows the result of increasing the initial velocity of the gas in the positive x direction by a small amount (20% of y -velocity) so that the initial pericenter distance (and hence the angular momentum) is larger than before. The effect on the size of the counterrotating disk is remarkable, and we now get a counterrotating disk which is actually larger than the primary disk. Thus small changes in the initial angular momentum of the gas can have a significant impact on the scale length of the resulting counterrotating disk. Although both the initial phase-space density and the angular momentum of the gas affect the size of the counterrotating disk, the former is more important because it keeps the gas clump-free and therefore less susceptible to dynamical drag and also less likely to disrupt the primary disk.

Small differences in the inclination of the infall trajectory (in other words the initial direction of the angular momentum vector), appear to be less important to the final outcome, although highly inclined trajectories have been shown to have significant

Figure 8: (a) Top and (b) side views of continuous infall with a thick initial slab of gas and a higher initial gas velocity than in Fig. 7. This gives the gas more angular momentum and the result is a larger counterrotating disk. For $t > 9.0$ Gyr, the image scale is doubled so that each panel width is 100 kpc (instead of 200 kpc).

dynamical consequences (Sofue 1994). We do not see an appreciable difference in the outcome or the time-scale when the initial z -position of the gas is altered by 5 kpc in either direction (corresponding to an inclination of 1 in 10). Also, as the edge-on views in Fig. 8 indicate, the plane of the galactic disk itself changes as the infall progresses. This means that the gas which initially precesses to the plane of the primary disk must again realign itself as that plane itself shifts. The inner gas is aligned with the primary disk plane since it has had more time to do so, whereas the outer gas is still tilted with respect to it. This differential precession of the counterrotating disk is also clearly apparent in the lower half of Fig. 8, and a classic integral-sign warp develops as a result.

Beyond $t = 12.0$ Gyr, the counterrotating disk becomes more and more centrally concentrated. In our simple model which does not include shocks, star formation or re-heating by supernovae, it is not too surprising that the cool gas eventually makes its way to the center as it loses energy in collisions. This “overcooling” is a problem that has been observed also in simulations of dissipative galaxy formation (Navarro & White 1994), in which the primary disk formed is more centrally concentrated than real spiral disks due to cool gas making its way to the center.

The counterrotating gas disk is also more susceptible than the primary disk to bending instabilities such as warps. This is due to the coldness of its velocity field, as shown in Fig. 9. The kinetic energy dissipated by the gas in collisions causes it to be significantly colder than the primary disk (stellar) particles.

In order to study the role played by the collisional energy dissipation, we ran a simulation with the collisions turned off. The behavior of this collisionless fluid was

Figure 9: Velocity fields of the disk (left) and counterrotating gas (right) at $t = 9.0$ Gyr for the continuous infall simulation in Fig. 7. Only a small subset ($\sim 12\%$) of the total number of particles is shown for clarity.

dramatically different; in the time it takes for a collisional fluid to form a counterrotating disk ($\sim 7-9$ Gyr), the collisionless fluid particles had not even begun to form a disk-like configuration. Most of them were on wildly eccentric orbits which took them far beyond the neighborhood of the primary disk rather than spiraling in towards the center of the disk. Thus tidal stripping and dynamical friction by themselves are not enough to produce a counterrotating disk within a Hubble time, and a mechanism for redistributing angular momentum, provided by inelastic collisions in this case, is necessary.

The initial angular momentum of the gas is the primary determinant of the time required to form the counterrotating disk. Besides tidal torques, net gas angular momentum can only be lost to the disk and halo particles through dynamical friction,

and hence is a liability when the gas tries to form a counterrotating disk. When the initial y -velocity imparted to the gas is doubled, with everything else staying the same, the time required for the gas to fall in and circle the disk is at least doubled. Other factors, such as the stickiness of the gas or the inclination of the infall trajectory, are less important in determining the time-scale. A less sticky gas does not necessarily take longer to form a counterrotating disk. If $\alpha_c = -0.5$ instead of -0.25 , the gas is less clumpy and more dispersed, but it takes about the same time to form a counterrotating disk, although the disk has a larger scale length and scale height. The initial height of the gas has a slightly more pronounced effect on the time-scale of the counterrotating disk formation, since the gas must precess to the plane of the disk first. Collisional dissipation facilitates this task, however, and it does not take very long for the gas to reorient its angular momentum vector.

A more massive (and extended) halo is also instrumental in setting the time-scale of formation for the counterrotating disk. It provides a deeper potential well for the gas to fall into, so the gas can have a higher-energy orbit and the infall can proceed faster without making the angular momentum too high to form a counterrotating disk. Our initial attempts at continuous infall were unsuccessful because the halo was only twice as massive as the disk ($M_h = 0.5$), and the maximum relative velocity of the gas was too low to allow the infall to complete within a Hubble time.

One of the necessary constraints on a successful model for counterrotating disk formation is that the original prograde disk must remain relatively unperturbed by the addition of retrograde gas whose mass is comparable to its own mass. This condition can only be met if the gas is added a little at a time. Simulations of dissipationless

mergers have shown that a disk can be substantially heated by the acquisition of a satellite whose mass is only a few percent of the disk mass (Quinn 1987). In the case of dissipational infall, more mass can be added without disrupting the existing disk, although there is still a limit to how much gas mass can be added per dynamical time. We find that the heating of the disk increases sharply when the gas mass exceeds a few percent of the disk mass per crossing time; this is largely because the gas tends to form large clumps regardless of the value of α_c . The large clumps disrupt the disk as discussed above. The disk also forms a bar in response to the infalling gas, and the bar gets more prominent as the infall rate increases. The sensitivity of the disk to the amount of gas falling in constrains the infall rate and prevents the rapid formation of a counterrotating disk if the primary disk is to remain undisturbed.

3.4.3 Dwarf Merger

Although some of the clumps of gas in the continuous infall case above had masses comparable to dwarf galaxies, an actual merger with a gas-rich dwarf is still worth investigating separately, because a real dwarf galaxy contains stars and dark matter in addition to gas. Dwarf galaxies also come in a range of sizes, and we wanted to find out if it was possible to obtain a counterrotating disk without severely disrupting the primary disk for any range of dwarf masses.

It is quite impossible to distinguish between the different mass constituents of both galaxies in a black and white image, so it is necessary to display the important components separately to study their behavior. The gas and stars in the dwarf galaxy, and the disk stars in the spiral galaxy, are the three constituents of interest. The evolution of the halos of both galaxies is not shown.

Figure 10: (a) Top and (b) side views of a merger with a massive gas-rich dwarf galaxy. The two galaxies merge rapidly, before there is a chance to form a counterrotating disk. Both the stellar and gas particles in the dwarf are shown. The primary disk is shown only in the first panel.

The evolution of the spiral disk and the gas in the dwarf galaxy for run S1 is quite rapid. The stronger mutual gravitational attraction, combined with stronger tidal effects and increased dynamical friction on the massive dwarf, causes the two galaxies to merge within a relatively short time (Fig. 10). The dwarf is massive enough to survive the primary's tidal field, and sinks to the center of the primary relatively unscathed. As a result, there is no chance for a counterrotating disk to form in this nearly head-on merger.

Figure 11: Formation of a counterrotating disk by a merger with an intermediate mass gas-rich dwarf galaxy (run S2). Only the gas particles in the dwarf are shown. The luminous mass (stars + gas) in the dwarf is one-fifth the mass of the primary disk. The counterrotating disk formed is less than half the size of the primary disk. The primary disk is shown only in the first two panels.

In run S2, the mass of the dwarf is reduced by a factor of 3 by halving the dark halo mass as well as the luminous mass. The dwarf gas evolution along with the primary disk is shown in Fig. 11 (top view). A counterrotating disk is formed in about 5-6 Gyr.

Finally, in run S3, the mass of the dwarf is halved again. Fig. 12 shows the evolution of the dwarf gas, and Fig. 13 shows the stars in the dwarf.

Figure 12: Merger with a gas-rich dwarf whose luminous mass is only about one-tenth of the mass of the primary disk (run S3). Only the gas particles are shown and the primary disk is shown only in the first two panels. The counterrotating disk formed in this case is almost as extended as the primary disk, but much less massive. It also takes much longer to form (compared to S2).

The formation of a tidal tail each time the secondary makes a close pass past the primary shows how mass is gradually stripped from the dwarf and forms a counter-rotating disk in the spiral. The gas forms a more flattened disk than the stars (not shown). Although a counterrotating disk is eventually formed in this case also, it takes significantly longer to form as the less massive dwarf is subject to gravitational “flinging” (due to tidal effects) several times before it totally merges with the primary.

Figure 13: The evolution of the stars in the dwarf galaxy with time for run S3. By the end of the simulation, the stars are still dispersed over a large region and have quite eccentric orbits (see velocity fields in Fig. 15).

The time taken to form the counterrotating disk is about 9 Gyr, which is prohibitively long if several such mergers are required to form a massive counterrotating disk.

The effect of the influx of gas and stars on the primary's disk is studied by comparing the thickness of the disk at various epochs as the simulation progresses (Fig. 14). From the disk thickness plots shown in Fig. 14, it is clear that even with such a small dwarf mass (S3), the disk still suffers considerable damage in the outer portions, although the thickening is less in the inner half of the disk for S3.

Figure 14: The thickness of the disk at different epochs for runs S2 and S3. In the outer parts of the disk, the heating of the disk is quite drastic, with an increase in thickness of more than 10 times.

The final velocity fields of the primary disk, the gas counterrotating disk, and the stellar counterrotating disk are compared for runs S2 and S3 in Fig. 15. The more massive dwarf results in a smaller, more centrally concentrated counterrotating disk, but the velocity field of the stars is dominated by random motions.

In general, the stars do not acquire an ordered retrograde motion because of their collisionless evolution. This is seen more clearly in the angular momentum plots (Fig. 16), which show the mean angular momentum of the dwarf's stars to be much lower

Figure 15: The velocity fields of the primary disk, secondary gas and secondary stars (from left to right) are compared for S2 (top panels) and S3 (bottom panels).

than that of the gas. The mass profiles plotted in Fig. 17 confirm that the stars in the dwarf do not have a disk-like (exponential) mass profile.

To summarize the situation with dwarf mergers, if the dwarf is too massive ($M_{Dw} \gtrsim 0.2M_{Sp}$), it merges rapidly with the primary and no counterrotating disk is formed. If the dwarf is too small ($M_{Dw} \lesssim 0.05M_{Sp}$), the counterrotating disk takes $\gtrsim 9$ Gyr to form, and since several such mergers will be required to produce a substantial counterrotating disk, there is not enough time to form one. For an intermediate dwarf mass, a small counterrotating disk forms within 5-6 Gyrs and two

Figure 16: The mean z -component of the angular momentum plotted as a function of radius for dwarf mergers S2 and S3. The angular momenta of both the gas (dotted line) and the stars (dashed line) belonging to the dwarf galaxy are directed opposite to that of the primary's disk (solid line). The mean angular momentum of the dwarf stars is significantly lower than that of the gas, however.

or three such events could conceivably produce a massive counterrotating disk within a Hubble time. The biggest problem with such a merger, however, is the thickening of the disk due to the effect of the dark matter and stars in the dwarf galaxy, most of which do not accompany the gas in forming a counterrotating disk. The behavior of the gas is consistent with simulations of gaseous mergers and spiral accretion (Sofue 1994) in which the stripping and accretion of gas proceeds at a much greater pace than for the stars, especially for retrograde mergers.

Figure 17: The mass profile of the stars (dashed line) and gas (dotted line) compared to the mass profile of the primary disk (solid line) for runs S2 and S3.

As noted in §2.4, the thickness of the primary disk increases with time, albeit much more slowly, even when the disk and halo are allowed to evolve together *in isolation* (in the absence of any gas infall or merger). The disk also shows traces of a bar and a tilt in this case. The thickening of the disk due to gas infall or a dwarf merger must therefore be compared with the thickening of an isolated disk, *not* with the initial pre-interaction disk. Fig. 18 compares the thickness of the disk at $t = 1.5$ (initial) with its thickness at $t = 9.0$ for an isolated galaxy as well as for dissipational infall and a dwarf merger. Whereas the thickness increases only slightly (and only

in the outer half of the disk) for dissipational infall, the entire disk shows a dramatic increase in thickness (an order of magnitude) for the dwarf merger. In fact, with our limited resolution, it is not possible to say whether dissipational infall does cause a non-zero increase in the disk thickness.

Figure 18: A comparison of the initial thickness (solid line) of the primary disk with its thickness at $t = 9.0$ Gyr for four different simulations: an isolated galaxy (no gas infall or merger), periodic gas infall, continuous gas infall, and a gas-rich dwarf merger (S3).

3.4.4 Comparison with Previous Work on Mergers

Results of equal mass mergers cannot generally be extended to unequal mass mergers (Villumsen 1982); hence we restrict ourselves to mergers between spirals and dwarfs.

There has been very little work to date on modelling such mergers, and practically none on retrograde mergers. Most studies of “gas-rich mergers” concentrate on mergers between a gas-rich disk galaxy with a gasless dwarf (e.g. Mihos & Hernquist 1994a, Hibbard & Mihos 1995). Simulations of dissipationless mergers between spiral galaxies and smaller satellites (Quinn 1987, Quinn, Hernquist & Fullagar 1993) have shown the dramatic impact that even a very small satellite, whose mass is only a few percent of the primary’s mass, can have on the disk of the primary. The disk thickness increases by a factor of $\sim 2-3$ for a satellite mass of 4% of the disk mass. The disk also spreads radially as a result of the spiral response to the perturbation due to the satellite. Our simulations provide confirmation of the radial and vertical spreading of the disk even in the case of a retrograde merger. As Figure 19 shows, there are fewer particles near the center and more particles at larger r and z for the post-merger disk.

To get a better idea of the role played by the gas, we made all the particles dissipationless in run S3 and compared the results with those obtained for S3. We found no appreciable difference in the thickness of the disk between the dissipationless and dissipational mergers. Of course, no counterrotating disk is formed in the totally dissipationless merger, since the collisionless gas particles now behave the same as the stars in S3. In the final analysis, although the presence of gas affects the behavior of the dissipationless particles to some extent (the stars tend to clump where the gas does), it does very little to alleviate the disruption of the disk caused by the stars and dark matter. The situation might be different if there was a lot more gas in the dwarf galaxy and little or no dark matter. This is consistent with an analytical study

Figure 19: The final ($t = 9.0$ Gyr) vertical density profile of the primary disk shown at various radii for the isolated disk, following periodic and continuous infall, and after dwarf merger S3. The density is averaged over azimuth for a given radius.

of spiral-dwarf mergers by Tóth & Ostriker (1992) which concludes that the gas in the dwarf will have a negligible impact on the heating of the disk unless the dwarf is almost entirely ($> 90\%$) gaseous.

Another observation from simulations of dissipationless mergers is that denser, massive cores in the secondary show up as dense cores in the primary after the merger (Balcells & Quinn 1990). This is also demonstrated in Figure 15, which shows that

the secondary's stars form a denser core in S2 (intermediate mass dwarf) than in S3 (low-mass dwarf).

3.5 Conclusions

Our experiments with dissipational infall indicate that the accretion of a counterrotating gas disk whose mass is comparable to that of the existing stellar disk is possible without excessive damage to the existing disk if the gas is acquired gradually in small batches. The severe restrictions on the mass that can be accreted in the form of a satellite galaxy through dissipationless mergers (Quinn 1987, Quinn, Hernquist & Fullagar 1993) are much less stringent in the case of gas infall, and the ability of the gas to dissipate energy and redistribute angular momentum allows it to form a counterrotating disk within a Hubble time. The gas infall can be in the form of successive episodes of infall, or a pseudo-continuous stream (since the gas breaks up into clumps anyway). The self-gravity of the gas, which has been taken into consideration in our simulations, is an important factor in determining both the clumpiness of the gas and its effect on the existing disk. A clumpy gas will tend to heat up the primary disk and form a small, centrally concentrated counterrotating disk. The initial angular momentum of the gas crucially affects both the size of the disk formed and the time required for its formation.

Collisional dissipation enables the gas to quickly redistribute and reorient its angular momentum, allowing it to settle into the plane of the disk to form a counterrotating disk within a Hubble time. A collisionless gas is unable to do this, and hence dissipationless infall may be ruled out as a probable mechanism for forming counterrotating disks.

A merger with a gas-rich dwarf galaxy can produce a small counterrotating disk whose mass is a small fraction of the primary disk mass, but it would be very difficult to accumulate an extensive counterrotating disk whose mass is comparable to that of the primary disk with this process. Although we tried only three representative cases of a merger with a gas-rich dwarf galaxy, it is clear that obtaining a massive counterrotating disk by this method is a difficult proposition. The need to form such a disk within a Hubble time with minimal disruption of the primary disk is virtually impossible to satisfy with one or more mergers of this kind. The disruptive influence of the dwarf's dark halo and to some extent the stellar matter in the dwarf is strong enough to heat the primary disk considerably, causing a sharp increase in its thickness. Extremely gas-rich dwarf galaxies which have little or no dark matter could succeed in producing counterrotating disks, but unless these represent the majority of dwarf galaxies, the probability of forming massive counterrotating disks is still significantly diminished.

The bleak outlook for dwarf mergers means that a potentially prolific mechanism for producing counterrotating disks is lost. Since dwarf galaxies may form naturally in tidal tails of encounters between giant galaxies (Hernquist & Barnes 1992), dwarf mergers would have increased significantly the probability of obtaining counterrotating disks in spirals, especially in clusters of galaxies. Without them, we are left with dissipational gas infall as the primary producer of massive counterrotating disks in spirals. The conditions for such infall to succeed, namely a relatively clump-free gas maintaining a retrograde orbit and a low infall rate, must persist over nearly a Hubble time, and this leads us to believe that they must not be a frequent phenomenon even

though most spiral galaxies accrete gas throughout their lives. Although we would expect to see a few more because our vision has improved considerably, it is not likely that many more will be seen.

The sticky particle scheme only allows us to qualitatively address basic questions, such as whether gas infall or a gas-rich dwarf merger are likely routes to counterrotating disks. It is not competent enough to permit inquiry into the detailed characteristics of the gas disk (scale length, height) or the intricacies of its effect on the primary disk. We expect to carry out such a detailed study after incorporating SPH gas dynamics in our code.

CHAPTER 4

NGC 4138 - A CASE STUDY

Published Work: This chapter has been previously published as Thakar, A. R., Ryden, B. S., Jore, K.P., Broeils, A.H. 1996, "NGC 4138 - A Case Study in Counterrotating Disk Formation", *Astrophysical Journal*, 479, 702. Figures and tables have been renumbered to ensure continuity, and references have been moved to the general bibliography at the end.

4.1 Abstract

The Sa(r) galaxy NGC 4138 has been recently found to contain an extensive counterrotating disk which appears to be still forming. Up to a third of the stars in the disk system may be on retrograde orbits. A counterrotating ring of H II regions, along with extended counterrotating H I gas, suggests that the retrograde material has been recently acquired in the gas phase and is still trickling in. Using numerical simulations, we have attempted to model the process by which the counterrotating mass has been accreted by this galaxy. We investigate two possibilities: continuous retrograde infall of gas, and a retrograde merger with a gas-rich dwarf galaxy. Both processes are successful in producing a counterrotating disk of the observed mass and

dimensions without heating up the primary significantly. Contrary to our experience with a fiducial cold, thin primary disk, the gas-rich merger works well for the massive, compact primary disk of NGC 4138 even though the mass of the dwarf galaxy is a significant fraction of the mass of the primary disk. Although we have restricted ourselves mainly to coplanar infall and mergers, we report on one inclined infall simulation as well. We also explore the possibility that the $H\alpha$ ring seen in the inner half of the disk is a consequence of counterrotating gas clouds colliding with corotating gas already present in the disk and forming stars in the process.

4.2 Introduction

NGC 4138, an early-type spiral galaxy within the Virgo supercluster, has no distinguishing feature other than a nuclear ring of star-forming H II regions similar to rings seen in many other early-type spirals (see, for example, Pogge & Eskridge 1993). Nevertheless, it has recently joined the growing list of spiral galaxies that are now known to possess *two* disks which rotate opposite to each other. The spiral/S0 galaxies currently known to have such gaseous and/or stellar counterrotating disks are NGC 4550 (Rubin, Graham & Kenney 1992), NGC 4826 (Braun, Walterbos & Kennicutt 1992), NGC 7217 (Merrifield & Kuijken 1994), NGC 7332 (Fisher, Illingworth & Franx 1994), NGC 4546 (Galletta 1987), NGC 3626 (Ciri, Bettoni & Galletta 1995), NGC 3593 (Bertola et al. 1996) and most recently, NGC 4138 (Jore, Broeils & Haynes 1996, hereafter JBH). This latest instance of a counterrotating disk was discovered as part of a survey of “boring” spiral (Sa) galaxies which had no unusual morphological features. The counterrotating disk consists of both stars and gas which add up to about a third of the mass of the primary disk, and it appears to be still forming

or has formed recently, as evinced by the extended counterrotating neutral gas surrounding the primary disk and the nuclear ring of star-forming H II regions which is also counterrotating (assuming that the counterrotating material does not remain in the gas phase for long). An H α emission-line imaging survey of S0 galaxies (Pogge & Eskridge 1987) previously detected the inner ring of H II regions in NGC 4138, which is classified as S0⁺ by the *Third Reference Catalogue of Bright Galaxies* (de Vaucouleurs et al. 1991) although the *Revised Shapley-Ames Catalogue* (Sandage & Tammann 1981) classifies it as Sa(r). A follow-up study (Pogge & Eskridge 1993) confirmed the existence of the ring, resolving many “blobs” into discrete H II regions.

The examples of counterrotating disks currently known run the gamut from counterstreaming stars (e.g., NGC 4550) to counterstreaming gas (e.g., NGC 7332), and this variety among the specimens, along with the observed differences in the velocity dispersions of the two components, strongly implies an external (merger or slow accretion) origin for the retrograde material. The presence of ionized gas along with stellar counterrotation suggests that at least some of this material was in the gas phase when accreted and formed stars later. However, in the cases where the retrograde disk mass is comparable to the primary prograde disk mass, the merger or accretion theory invoked must explain how the primary disk can remain unperturbed in spite of the influx of a large amount of stars and gas. Simulations of major mergers show clearly that the acquisition of a large number of stars is quite a disruptive process which leaves a remnant that resembles an elliptical galaxy more than a disk galaxy (e.g. Barnes 1992; Barnes & Hernquist 1992a). This is true even for gas-rich mergers between disk galaxies that yield a counterrotating disk (Hernquist & Barnes 1991).

We can therefore rule out major mergers in which a large amount of dissipationless matter is dumped into the galaxy within a relatively short time. This leaves minor gas-rich mergers and secular gas infall as the processes most likely to produce massive counterrotating disks without heating up the primary disk inordinately.

Whereas examples of counterrotating gas in S0s and spirals have been known for some time (see Sage & Galletta 1994 and references therein), the unexpected discoveries of a few massive, stellar counterrotating disks in galaxies previously thought to be “normal” naturally raise the troubling question: how many more have been missed? Systematic searches for counterrotation in spiral galaxies are just getting under way, and so far the results are mixed. A recent survey of 28 S0’s with high signal-to-noise spectra (Kuijken, Fisher & Merrifield 1996) revealed no new cases of stellar counterrotation but uncovered counterrotating gas in almost a quarter of the sample. This implies that most if not all counterrotating disks must start out as gas disks, and suggests that star formation may somehow be inhibited in the retrograde gas. Furthermore, most of the counterrotating gas disks are substantially smaller than the corresponding primary disks. These findings argue against a high frequency of massive, stellar counterrotating disks in S0 galaxies. Theoretical studies of the origin of massive counterrotating disks are necessary to complement the observations and make some predictions which can be checked against future observations.

We have been investigating the origin of such massive counterrotating disks in spiral galaxies with numerical simulations in which we try to produce a massive counterrotating disk in a fiducial spiral galaxy (Thakar & Ryden 1996, hereafter TR). However, given the large number of input parameters that can have an appreciable

effect on the outcome, it is of great practical value to adopt a phenomenological approach and study individual galaxies so that we can narrow down the range of parameters that are relevant to existing counterrotating disks.

As in TR, we investigate two mechanisms here for producing a counterrotating disk in NGC 4138. The first process is gradual or secular gas infall, and the second is a gas-rich dwarf satellite merger. Minor mergers and accretion have been receiving increased attention in the last few years, due partly to the fact that major mergers have been shown, through numerical simulations, to be unable to produce some of the observed features in spiral galaxies, and certainly can be ruled out in cases where cold, thin disks still dominate the galaxy morphology. The picture that is emerging is that the importance of accretion and minor mergers as a means for spiral galaxy evolution may have been underestimated. The secular evolution of galaxies most likely is driven by such events from later types towards S0 (e.g. Walker, Mihos & Hernquist 1996). There is also some evidence from numerical experiments to support the idea that spiral disks are more resistant to minor mergers if the angular momentum of the merger orbit is high enough for the primary disk to respond in a coherent manner by tilting rather than warping or heating (Huang & Carlberg 1996).

The prevalence of counterrotating gas disks is clearly indicative of the importance of gas in the process(es) responsible for forming them. Indeed, TR found it quite impossible to produce a counterrotating stellar disk within a Hubble time from a dissipationless merger without wrecking the primary disk. However, it is not clear whether it is necessary for all the retrograde matter accreted to be in the form of gas. A gas-rich dwarf merger may work nearly as well as pure gas infall in some cases,

especially if the dwarf has a low M/L ratio. There are some difficulties associated with secular gas infall, particularly the source of the infalling gas and the need for the gas to maintain retrograde angular momentum with respect to the primary disk over a good fraction of a Hubble time. Alternatives to gas infall, of which gas-rich dwarf mergers is the most promising, must be thoroughly investigated to determine their ability to produce massive counterrotating disks.

Based on numerical experiments with a cold and thin primary, TR found that gas infall was able to produce massive counterrotating disks without excessive heating of the primary disk, but the same could not be said of gas-rich dwarf mergers, which were found to be problematic. For the significantly hotter and more compact primary of NGC 4138, gas-rich dwarf mergers do a much better job, and in fact produce a counterrotating disk in a significantly shorter time than continuous infall. We briefly describe our numerical method in section 2. Our simulation results are presented in section 3. Analysis of these results and comparison with previous work in galaxy interactions are in section 4. Section 5 lists our conclusions.

4.3 Computational Method

The numerical method used is described in detail in TR. We use a tree code to compute the gravitational forces for all particles and a sticky particle dissipation scheme for the gas dynamics. We started with a basic version of the tree gravity solver (Barnes & Hut 1986) written in the C language and optimized it for execution on the Cray Y-MP at the Ohio Supercomputer Center. To this we added a sticky particle code which implements a simple form of gas dynamics via inelastic collisions which dissipate kinetic energy and conserve linear and angular momentum. Our

Disk				Halo		
Mass	Radius	Scale Len.	Scale Ht.	Mass	Radius	Core Rad.
M_d	R_D	R_d	h_d	M_h	R_h	R_c
2.0×10^{10}	6.0	1.25	0.2	1.5×10^{11}	20	5

Note. — Masses are in $M_\odot$, lengths in kpc.

Table 2: Parameters for NGC 4138 Model.

sticky particle code is a combination of previous schemes (Brahic 1977; Schwarz 1981; Negroponte & White 1983), and is also optimized for the Cray Y-MP.

For the simulations described here, we use twice the number of particles to represent the halo (32k instead of 16k as in TR). The softening length is smaller (0.5 kpc instead of 1 kpc used by TR) to give a better resolution. The primary stellar disk is represented as before with 32k particles. It is dissipationless initially and contains no gas (except for infall simulation I6, see below). The number of gas particles is also the same, being 20k for gas infall models and 15k for dwarf merger models. The dissipationless matter in the dwarf galaxy is represented by a varying number of particles depending on its mass. The mass of each dissipationless particle in the dwarf is comparable to the mass of each particle in the primary disk ($\sim 6 \times 10^5 M_\odot$). The total number of particles is 84k for the infall simulations and 86k for the dwarf mergers (except one large N dwarf merger which contains 162k particles).

Figure 20: The initial rotation curves for the primary disk and halo in our numerical model compared with the corresponding rotation curves derived from observations of NGC 4138 (JBH).

Although the two-component model for the spiral galaxy is the same as before, the model parameters are different since we are modelling NGC 4138 instead of a fiducial spiral galaxy. The parameter values for our model of NGC 4138, based on the observations and mass modelling described by JBH, are summarized in Table 2. The disk in this case is radially much hotter than the disk used by TR, and the value of Q_o (the value of the stability parameter Q at $R_o = 8.5$ kpc) we use is 4.0. This was necessary for the model rotation curve and primary disk velocity dispersion values to be consistent with the “observed” rotation curve and velocity dispersions

of NGC 4138 obtained by JBH (Fig. 20). Although the initial galaxy prior to the accretion of the counterrotating material was in all probability different from the one we see today, we had no choice but to start with a model as closely resembling the observed post-accretion galaxy as possible. To investigate what happens if we start with a radially cold primary instead, we ran one dwarf merger simulation with a $Q_0 = 1.5$ primary disk.

Model	M_g^a	V_0^b	X_0^c	J_z^d	r_{nn}^e	Z_0^f
I1	1.0×10^{10}	0.3	25	0.45	3.0	0
I2	1.0×10^{10}	0.3	10	0.18	3.0	0
I3	1.0×10^{10}	0.2	10	0.12	3.0	0
I4	1.0×10^{10}	0.3	20	0.36	3.0	15
I5	1.0×10^{10}	0.3	25	0.45	1.0	0
I6	$1.1 \times 10^{10}\dagger$	0.3	25	0.45	1.0	0

^atotal gas mass, in $M_\odot$

^bthe initial velocity of the gas in the negative y direction, specified in units of the centripetal velocity $\sqrt{G(M_h + M_d)/X_0}$, where $M_h + M_d$ is the total mass of the primary (halo+disk)

^cthe initial distance of the gas column from the center of the primary disk, in kpc

^dthe magnitude of the angular momentum (quantified crudely by taking the product $m_g \times V_0 \times R_0$, where m_g is the total mass of the infalling gas in our simulation units)

^ethe search radius for nearest neighbors used by the sticky particle code, in kpc

^fthe initial height of the gas column above the $x - y$ plane, in kpc

[†]I6 has additional prograde gas in the primary disk with one-tenth the mass of the infalling gas.

Table 3: Parameters for Gas Infall Simulations.

The initial conditions and some physical parameters of the infalling gas are different in the present simulations from those used by TR. The velocity dispersion of the gas is higher than before (~ 30 km/s instead of ~ 10 km/s used in TR), the mass and

initial angular momentum of the infalling gas are different to suit the current problem, and we have experimented with the “stickiness” of the gas to assess its impact on the final outcome. We have also studied the effect of an inclined infall orbit and the presence of a small amount of preexisting prograde gas in the disk. The parameter values for the different infall simulations (dubbed I1-I6) are listed in Table 3. The magnitude of the initial angular momentum, J_z , of the gas can be altered by changing the initial velocity of the gas V_0 or the initial distance R_0 of the gas slab from the center of the primary disk, or both. The total mass of the gas falling in is taken to be 50% of the mass of the primary disk, even though the counterrotating mass observed is at most 30% of the mass of the disk. The extra gas is included to allow for some gas which is dispersed over large distances and does not fall into the primary disk by the end of the simulation.

Similarly, for the dwarf mergers, the total amount of luminous mass (gas+stars) in the dwarf galaxy is chosen to be 50% of the primary disk’s mass. The dwarf galaxies we have chosen have higher gas contents than do typical dwarfs. Our values for M_g/M_T , the ratio of the gas mass to the total mass, range between 0.27 and 0.53, whereas the typical range for M_{HI}/M_T , the ratio of neutral gas mass to total mass, is 0.09 – 0.24 for dwarf galaxies (Roberts & Haynes 1994). However, the latter does not take into account the molecular gas content in dwarf galaxies, which is not very well quantified so far. The mass model for the dwarf galaxy is a spherical Plummer model with the density profile being a polytrope of index 5 (as described in TR). The parameters for the dwarf galaxy in each of the dwarf merger simulations (M1-M3) are listed in Table 4. The total mass of the dwarf galaxy therefore ranges between $\sim 9\%$

Model	M_g^a	M_*^b	M_D^c	M_g/M_T^d	R_*^e	R_g^f	R_h^g
M1	8×10^9	2×10^9	5×10^9	0.53	4.0	4.0	4.0
M2	8×10^9	2×10^9	2×10^{10}	0.27	4.0	4.0	8.0
M3 ^h	8×10^9	2×10^9	2×10^{10}	0.27	4.0	4.0	8.0

^amass of gas, in $M_\odot$

^bmass of stars, in $M_\odot$

^cmass of dark matter, in $M_\odot$

^dratio of gas mass to total mass

^eradius of stellar sphere, in kpc

^fradius of gas sphere, in kpc

^gradius of halo, in kpc

^hM3 uses twice the number of particles as M2

Table 4: Dwarf Galaxy Parameters for Gas-Rich Dwarf Mergers.

and $\sim 18\%$ of the mass of the primary galaxy. Although the physical parameters for the dwarf galaxy are identical for M2 and M3, the latter uses a radially colder primary disk and a larger number of particles in the simulation (64k each for the disk and halo, and 34k for the dwarf galaxy) to minimize numerical heating even further.

In all the simulations presented here, the primary disk is held frozen for $t = 0 - 0.2$ Gyr, which amounts to ~ 5 dynamical times. This is to allow the halo to virialize. The disk and halo are then evolved in isolation for $t = 0.2 - 0.5$ Gyr so that they may achieve equilibrium. The gas column (for infall) or the dwarf galaxy is introduced at $t = 0.5$ Gyr. In most cases, the simulation results are shown starting at $t = 0.9$ Gyr, so that the simulation is already in progress. The initial time-step is not shown.

With the simple gas dynamics scheme we are using, we do not attempt to study the internal structure of the gas disks formed in our simulations, and hence we follow each simulation only up to the point where a flat disk-like structure with retrograde orbits is formed. The sticky particle scheme does not give an accurate description of the detailed gas dynamics, so we restrict ourselves to studying the overall characteristics of the objects such as the size and shape of the counterrotating disk formed (if any) and the impact of the accretion or merger on the primary disk. For the same reason, we also do not study the effect of including large quantities of preexisting gas in the primary disk with this method. We hope to report on these aspects in the near future using SPH simulations which will provide a more realistic description of the gas dynamics with the inclusion of shocks, viscosity, and eventually even star formation.

4.4 Results

4.4.1 Continuous Infall

The models for continuous infall and the gas-rich dwarf merger are basically the same as those used by TR. For gas infall, a long (length = 150 kpc, width = 10 kpc), rectangular slab of gas is allowed to fall into the primary spiral galaxy on a retrograde orbit. We chose this model because it allows us to impart the appropriate angular momentum to the gas in a simple manner, and because we do not really know what the characteristics of real gas infall are.

Our long column of gas (which breaks up into clumps before it falls into the primary galaxy) can be thought of as tidal debris left over from a previous interaction. Figs. 21-28 depict the behavior of the infalling gas for the different infall simulations

Figure 21: Gas infall with large initial angular momentum for the gas (I1). The top half shows the “top” view and the bottom half shows the “side” view. The primary disk is shown only in the first panel for each view for clarity. The galactic halo is not shown. The mass of the gas is 50% of the mass of the disk. The square panels are 80 kpc on each side, except for the last panel, which is 40 kpc on each side. Time is in Gyr.

(I1-I6). I1 and I2 compare large initial angular momentum (J_z) infall with small initial angular momentum infall respectively for a moderately dissipative gas (coefficient of restitution $\alpha = -0.5$). I3 shows the effect of very low initial gas angular momentum on the production of a counterrotating disk. I4 illustrates infall along an orbit which is considerably tilted with respect to the plane of the primary disk. I5 is the same as I1 but with a different parameterization of the sticky gas which produces a smaller counterrotating disk. Finally, in I6 we show the effect of placing a prograde ring of

Figure 22: Top (upper half) and side (lower half) view of gas infall with just enough initial angular momentum to still yield a counterrotating disk (I2). Panel widths are 80 kpc except for the last panel in each view, which is 40 kpc wide.

gas in the disk in addition to the infalling gas. The mass and initial density of the infalling gas are the same in all simulations reported here.

For I1, the large J_z case (Fig. 21), the resulting counterrotating disk is more than twice as large as the primary disk. The effect of lowering the initial angular momentum of the gas can be seen in I2 (Fig. 22). This approaches the minimum retrograde J_z that the gas can have and still yield a counterrotating disk. Although the counterrotating disk formed is slightly smaller in I2 than in I1, it is still significantly larger than the primary disk. If J_z for the gas is reduced further, a counterrotating disk does not form (Fig. 23). The evolution of the gas is now very different from the previous two simulations. A shell forms at $t = 1.7$ Gyr, and thereafter the gas falls rapidly back into the primary disk. The infall proceeds at a significantly faster rate, and the side view shows that the end result at $t = 3.7$ Gyr resembles a spheroid rather than a disk (only the $x - z$ plane view is shown, but the $y - z$ plane view is almost identical). Fig. 24 provides a comparison of the velocity fields of the primary disk

Figure 23: Top (upper half) and side (lower half) view of gas infall with very low initial angular momentum (I3). Panel widths are 80 kpc except for the last panel in each view, which is 40 kpc wide.

and the gas for the I2 and I3. The velocity field of the gas is not ordered for I3, and it is hard to discern a well-defined orbital direction. The difference also shows up in the comparison of the specific angular momenta of the primary and secondary disks for the two simulations (Fig. 25). Even though the net angular is retrograde for I3, we are clearly getting out of the regime that yields a counterrotating disk.

To investigate the impact of a non-coplanar or inclined infall orbit, we performed one simulation in which the infall proceeds along an orbital inclination of $\sim 30^\circ$ with respect to the plane of the primary disk ($x - y$ plane). The results are shown in Fig. 26. A few shells can be seen forming early on in the gas, and the later panels show that although a counterrotating disk does appear to form, it is not aligned with the primary disk even by $t = 5.7$ Gyr. This is evident from the side views, in which the gas counterrotating disk and the primary disk appear to be inclined by almost 45° with respect to each other. Furthermore, the primary disk flips by almost 90° not once but *twice*, each flip taking ~ 1.5 Gyr to accomplish. It is clear that a near-

Figure 24: Velocity fields of the disk (bottom) and counterrotating gas (top) compared for I3 (left) vs. I1 (right). Only a fraction of the particles are sampled, and sampling is proportional to radius.

Figure 25: The mean specific angular momentum of the disk and counterrotating gas compared for I3 (open squares) and I1 (filled squares). In our sign convention, $J_z > 0$ for prograde orbits and $J_z < 0$ for retrograde orbits.

coplanar orbit is necessary for the counterrotating gas to settle quickly into the plane of the primary disk. TR experimented with a small orbital inclination (5°) and found it to have no appreciable impact on the outcome.

Although simulations I1-I4 illustrate the general aspects of infall simulations adequately, they use a sticky particle formulation in which the dissipative characteristics of the gas are less than ideal. The resulting counterrotating disk is considerably larger than the primary disk regardless of the initial angular momentum.

The radial distribution of the particles also shows a clear underdensity of particles at the inner radii (Fig. 32). Since we observed this behavior even when the gas was made very “sticky” (by choosing a smaller coefficient of restitution α), we previously concluded that this was a limitation of our infall model (Thakar 1996) rather than an artifact of the sticky particle model chosen for the gas. However, upon choosing a smaller maximum radius for selecting collisional neighbors (instead of a smaller α), the resulting counterrotating disks were appreciably smaller than before. This parameter change has effectively made the gas more dissipative by strongly favoring collisions between nearby particles (which on the average have higher relative velocities due to their mutual gravity) over those between remote particles, even though the total number of collisions is reduced because of the smaller number of nearest neighbors found for each gas particle. Since preferentially selecting collisions between nearby particles is physically more realistic, we adopt this parametrization for all the remaining simulations discussed below.

The results for the new sticky particle formulation are shown in Fig. 27 for I5, for which all the other inputs are the same as I1. A comparison of the primary disk shown in the first panel of the side view in Fig. 27 with the counterrotating disk at $t = 4.1$ Gyr indicates that the two are approximately the same size, with the counterrotating disk being slightly smaller. The enlarged views for the last time-step reveal some of the structure of the disk. The gas is distributed fairly evenly with radius, and most of it is within the radius of the primary disk. The counterrotating disk clearly does not have an exponential profile, and it is tilted by nearly 45° to the original plane of the primary disk. The primary disk also tilts by a comparable amount, as can be

Figure 26: Top (far left) and side (left) views of the gas, and top (right) and side (far right) views of the primary disk for gas infall with an orbital inclination of $\sim 30^\circ$ (I4). Panel widths are in kpc.

Figure 27: Top (left) and side (center) views of the gas, and side views of the primary disk (right), for continuous gas infall with a smaller search radius for sticky particle neighbors (I5). The primary disk is shown for reference also in the first panel of each of the gas views. For the gas, the square panels are 80 kpc wide, with the last one in each view being 20 kpc wide. For the primary disk, all panels are 20 kpc wide.

seen in the side views of the primary disk. It is not clear what causes the primary disk to tilt so much, especially when it does not tilt nearly as much when prograde gas is included in it (see below).

The nuclear ring of H II regions is a prominent feature of NGC 4138, but the discovery that it is also counterrotating was a surprise. Although nuclear star formation rings in early type spirals are not unusual, this is the first counterrotating star forming ring known. Could this ring be a result of collisions between corotating and

Figure 28: Top (left) and side (right) view of continuous gas infall with a prograde ring of gas in the primary disk initially at ~ 1.2 kpc from the center (I6). The prograde gas is also shown. The last two panels show magnified views of the counterrotating disk. The square panels are 80 kpc wide, except for the last one, which is 20 kpc wide.

counterrotating gas clouds? If so, why do the resulting star forming regions end up counterrotating? We placed a prograde ring of gas, with one-tenth the mass of the infalling gas, at the same location as the observed ring and ran the infall simulation I6 with everything else being the same as for I5 above. The results are shown in Fig. 28. The apparent thickness of the counterrotating disk in the side view is due to its projected inclination. The only discernible differences between the gas disk formed in this case and the one formed in the previous simulation (without prograde gas) are the size and the inclination of the counterrotating disk with respect to the original plane of the primary. Without prograde gas, the disk is a bit larger and tilts by a considerable amount, whereas with prograde gas, the disk is smaller and suffers much less tilting. It is reasonable for the prograde gas to rob the counterrotating gas of kinetic energy and thereby produce a smaller counterrotating disk, but what could the prograde gas have to do with the tilting? We are not quite sure, but some arguments on this subject are presented in §4.1.

4.4.2 Gas-Rich Dwarf Merger

The dwarf merger model we have used is described in detail in TR. We present the results of three gas-rich dwarf mergers here, the first with very little dark matter (M1) and the remaining two with a moderate amount of dark matter (M2 and M3). All mergers are coplanar and the dwarf galaxy models are identical except for the amount of dark matter. M3 uses the same dwarf galaxy model as M2, but has a colder primary disk and twice the number of collisionless particles as M2..

In the first dwarf merger (M1), shown in Fig. 29, we see a counterrotating disk forming by $t \sim 3.1$ Gyr. Although particles still appear to be falling in at $t = 4.5$

Gyr, these are mostly stellar particles from the dwarf, and almost all of the gas particles have fallen into retrograde orbits around the primary disk by this time.

Figure 29: Top (upper half) and side (lower half) views of a gas-rich dwarf merger with very little dark matter in the dwarf (M1). The primary disk is shown as light grey, and both the stars and the gas in the dwarf are shown as dark particles. The dark halos are not shown. The square panels are 80 kpc wide, except for the last panel, which is 40 kpc wide.

The formation of a tidal bridge and two tidal tails is evident from the top view in Fig. 29. The primary disk tilts in response to the merger, and the counterrotating disk which forms is consequently also tilted with respect to the original plane of the encounter. The sizes of the primary and counterrotating disks are comparable.

With the additional dark matter in the second dwarf merger (M2), the counterrotating disk is produced much faster (Fig. 30). There is less tidal stripping of the

Figure 30: Top (upper half) and side (lower half) views of a gas-rich dwarf merger (M2) with 4 times the amount of dark matter as in M1.

dwarf, although as in M1, two distinct tidal tails are formed. By $t \sim 2.5$ Gyr, most of the gas is within the radius of the primary disk, but it has not quite made it to the plane of the disk yet. The gas disk becomes steadily smaller after this, and a comparison of the panels for the last time-step in M1 and M2 shows that the gas disk in M2 is more compact and less inclined to the original plane of the primary. However, even by $t = 4.5$ Gyr, the counterrotating gas does not appear to be very flat, and its apparent thickness is not due to a projected inclination.

The two dwarf mergers are compared in Fig. 31. Although the counterrotating disk is tilted in M1, it is not as thick as it appears because it is also tilted in the $y - z$

plane. Such is not the case for the counterrotating disk in M2, which actually is thick. The second merger produces a puffer counterrotating disk whose velocity field is not completely circularized. A comparison of the angular momenta of the two disks shows this more clearly (middle row of panels). The angular momentum for the counterrotating disk in M2 is significantly lower than that for M1. The primary disk in M2 also gains a pronounced bar (not shown), but does not appear to tilt nearly as much as in M1.

The stellar component of the dwarf galaxy also attempts to participate in the counterrotating disk formation, but with much less success. The stellar particles are much more spatially dispersed in both M1 and M2, although there is some flattening discernible in both simulations. In M1, the stellar particles also follow the inclination of the primary disk. The velocity fields of the stars from the dwarf, however, are not at all regular or circularized. In general the angular momenta of the dwarf stars are much lower than those of the counterrotating gas particles, but they are definitely of the same polarity as the gas. So some of the stars do in fact form a counterrotating stellar disk, with more success in M1 than in M2.

The faster pace of M2 apparently leaves less time for the primary disk to tilt in response to the incoming material. The presence of the extra dark matter fosters a significantly thicker counterrotating disk than in M1. Surprisingly (see §4.2), the thickness of the primary disk is not affected much by the additional dark matter in M2.

For the third merger simulation (M3), the merger details are the same as for M2, except that the dwarf galaxy is introduced at $t = 1.0$ Gyr instead of $t = 0.5$ Gyr.

Figure 31: A comparison of the two dwarf mergers M1 and M2, showing three species: the primary disk (left), the gas in the dwarf (center) and the stars in the dwarf (right). The angular momentum comparison plots are in the middle (open squares for M1, filled squares for M2), with the velocity fields and the side views for M1 above the A.M. plots, and the velocity fields and side views for M2 below them.

This is because the colder primary in M3 requires more time to achieve equilibrium since it is more susceptible to local axisymmetric clumping. The final characteristics of the counterrotating disk are the same as in M2. The main difference between M3 and M2 is in the heating of the primary disk, and this is discussed in the following section.

4.5 Discussion

4.5.1 Size, Shape and Orientation of Counterrotating Disks

Both infall and mergers produce counterrotating disks with sizes which compare well with the observed size of the counterrotating disk in NGC 4138, in terms of mass as well as extent. In some cases, the counterrotating disk is slightly larger in extent than the primary disk, but this is not a big issue because we usually only follow our simulations up to the point where most of the gas falls into the primary disk. The size of the gas disk reduces slowly with time, but we do not wait until it matches that of the primary very closely (to conserve our limited CPU resources, and also because we do not have star formation in our simulations). The mass of the counterrotating disk is usually $\sim 30\%$ of the mass of the primary disk, since we start with a total secondary mass (in gas for infall and gas+stars for mergers) of $\sim 50\%$ of the primary disk mass and $\sim 40\%$ of it is lost in material dispersed over a large volume around the primary galaxy (see below). This is close to the upper limit of the mass of the counterrotating disk derived by JBH.

In the infall simulations, most of the dispersed gas accounts for a very small fraction ($\gtrsim 5 - 10\%$) of the total gas mass. However, we invariably find one or two

large clumps of gas, with masses in the range $\sim 10 - 30\%$ of the total gas mass, flung to large distances (100-200 kpc) from the primary disk. The reason for this is that the initial column of infalling gas breaks up into clumps, the last one or two of which are massive enough to survive the tidal field of the primary disk. This also explains why it does not happen in the dwarf merger simulations, although there still is a small amount of gas which is spread out over a large volume. The dispersed stars from the dwarf account for the remaining mass that is not in the counterrotating disk.

Figure 32: A comparison of the radial mass distribution of the counterrotating disk for infall and dwarf mergers. The y -axis shows the number density N of gas particles.

We do not see a consistent radial profile for the counterrotating disks formed in our simulations. The radial profiles shown in Fig. 32 are not exponential. The profile

for I6 shows a sharp peak at the location of the prograde gas, which has been included. It also shows a higher density for gas at radii above and below the range where the prograde gas is ($\sim 1.5 \pm 0.5$ kpc). In all the counterrotating disks formed, we see a drop in the density in the nuclear region of the disk. This is true even for I6, so the presence of a small amount of prograde gas does not help. The fact that the size of this “hole” is of the order of the gravitational softening length may be a clue as to its cause. This may also be an indication of the lowest angular momentum achievable by the gas as a result of insufficient dissipation in our models (see below). It should be noted, however, that the H I data for NGC 4138 also show a depression in the center (Fig. 3 in JBH).

The thickness of the counterrotating gas disks is also a matter of concern. We find that in the majority of our simulations, the counterrotating disk is somewhat thicker than the primary disk. This is especially so for M2 (and M3). Moreover, the thickness does not reduce appreciably with time on scales of a few dynamical times. One factor that may be causing this is that our gas starts out with a higher velocity dispersion than in TR, and it does not cool as much as it should. There is a cutoff for the collisions for relative velocities $\lesssim 10$ km/s, which is quite low and should allow the gas to cool down. However, if most of the cooling takes place before the gas disk takes shape, the gas will undergo much fewer collisions in the disk state. Alternatively, the thickness may be due to some other limitation of the gas dynamics used.

Except for the inclined infall simulation, in all other cases the counterrotating disk seems to faithfully follow the orientation of the primary disk, even in cases where the primary disk tilts by a considerable amount. This is reassuring but not too surprising,

since most of the change in the orientation of the primary seems to occur after the counterrotating disk is mostly formed. The largest disk inclination is seen in infall simulation I5. A calculation of the torques acting on the disk due to the halo and the gas reveals that the rate of change of the disk's angular momentum matches the total torque acting on it. Neither the halo nor the gas can provide enough torque individually to explain the change in the disk's angular momentum. The total torque acting on the disk fluctuates considerably on time-scales as short as a few Myr. We have also calculated the axial ratios and axial vectors of the halo at different times by diagonalizing the normalized inertia tensor,

$$M_{ij} = \sum m x_i x_j,$$

where the x_i are the components of the unit vector parallel to the radius vector of the particle. There is a slight flattening of the halo along the z -axis, and the axial vectors change with time in a manner consistent with the inclination experienced by the disk. The inclination of the disk therefore is intimately linked to the details of the dynamical evolution of the halo and the evolving gas, and different initial conditions are apt to result in very different behavior of the disk.

4.5.2 Impact of Accretion on Primary Disk

If a counterrotating disk forms which resembles the observed disk, then the next question is whether the primary disk suffers a lot of heating as a result of the accretion of the new disk via either infall or a merger. A measure of the heating of the primary disk is its mean thickness as a function of radius. We have plotted the mean half-thickness in Fig. 33 for the important infall and merger simulations. As

explained in TR, the primary disk is also heated in the absence of any accretion due to the discreteness of the particle representation and two-body encounters with halo particles. This effect must be taken into account when trying to measure the thickening due to the infall or merger. We have shown a thickness plot for the isolated primary disk evolved up to $t = 4.5$ Gyr in addition to the post-accretion plots in Fig. 33. Within ~ 4 kpc, where the thickness measurements are more reliable, the total increase in thickness is not much higher than that due to the numerical heating. When the numerical heating is reduced, as in the case of simulation M3, we see a sharp drop in the disk thickening. Since the two effects are not strictly separable, it is hard to make a quantitative statement about the thickening due to accretion only. The best we can say is that the average increase for $R \lesssim 4$ kpc is at most of the order of the numerical thickening.

Whereas the results for infall are consistent with the findings of TR, a comparison of Fig. 33 with Fig. 17 in TR reveals that the gas-rich dwarf merger performs much better for NGC 4138 than for the fiducial primary in TR. This is especially remarkable considering that the dwarf galaxy in TR was only $\sim 4\%$ as massive as the primary galaxy, with its luminous mass being $\sim 10\%$ of the mass of the primary disk, whereas the corresponding figures for the dwarf in M1 are $\sim 9\%$ and $\sim 50\%$. In M2, the total mass of the dwarf is $\sim 18\%$ of the mass of the primary galaxy. Thus the dwarf galaxies chosen for the NGC 4138 simulations are considerably more massive relative to the primary galaxy than in TR, and the merger time-scales are also much shorter.

The secret to the success of the dwarf mergers in NGC 4138 lies in the nature of its primary disk. The compact and massive primary disk, due to its higher surface

Figure 33: A comparison of the thickness of the disk after continuous infall and dwarf mergers. The solid line shows the initial disk thickness at $t = 0$. The dotted line is the disk thickness at $t = 4.5$ Gyr for an isolated disk evolved without any infall or merger. Thicknesses for $R \gtrsim 4$ kpc are not reliable due to small numbers of particles at larger disk radii.

density, has a higher vertical velocity dispersion and is therefore initially thicker and much more resistant to vertical heating than the primary used in the simulations by TR. The ratio of scale length to scale height for NGC 4138's disk is only ~ 5 , compared to ~ 10 for the disk used by TR. We chose to model a disk that was radially hot as well because it matched the observed parameters of NGC 4138's current disk well. Of course, the primary disk could have been quite different prior to the formation of the counterrotating disk, and may have been significantly colder radially. This case

is represented by M3, where we start with a radially colder disk, and use twice as many collisionless particles to minimize numerical heating. The post-merger disk is now even closer to the observed NGC 4138 disk since it suffers less vertical heating along the way.

4.5.3 Comparison with Observations

The lack of remnants of any tidal features in the vicinity of NGC 4138 may be used as an argument against a dwarf merger origin for the counterrotating disk, but it is not a serious problem. If the merger takes only a few Gyrs to complete, as we have shown, then there has been plenty of time for any tidal features to be absorbed by the primary by now. The extended neutral gas may represent the last of the tidal tails falling back into the primary. The higher mass of the dwarf speeds up the merger considerably. Low-mass satellites would not have been able to spiral in within a Hubble time, especially on a retrograde orbit (Binney & Tremaine 1987 §7.1 and eq. 7-27, and Walker, Mihos & Hernquist 1996 Fig. 13), nor would a single low-mass merger have been able to provide enough mass for the counterrotating disk.

There is nothing in the observations of NGC 4138 to date that precludes gas infall as a means of obtaining a counterrotating disk. The extended neutral gas observed could be the “smoking gun” from the infall process which produced the counterrotating disk. Although there is some uncertainty about the mass of this extended gas, it is not likely to be more than a few percent of the mass of the counterrotating disk, which is consistent with our infall simulations I5 and I6, where the mass of gas around the primary galaxy amounts to a few percent of the mass of the counterrotating disk. The large gas clumps we see at further distances in both

of these simulations are more problematic, since no counterpart is observed, at least in H I gas, around NGC 4138 (JBH). They owe their existence at least in part to our initial conditions, which allow the tail end of the infalling gas to form clumps dense enough to resist the tidal influence of the primary. Reducing the initial length of the infalling slab of gas by leaving out some of the gas would produce smaller or no leftover clumps of gas.

The presence of A stars along with the absence of O and B stars from the primary disk (JBH) suggests that star formation took place less than a Gyr ago in the primary but ceased thereafter. Does the influx of counterrotating gas suppress star formation in the primary? Our simulations indicate that large quantities of counterrotating gas make their way inside the primary disk only at later times, within the last 1 – 1.5 Gyr of the simulation. However, keeping in mind that this may be due to a limitation of our models, we cannot rule out the presence of counterrotating gas in the primary disk when the last burst of star formation took place. Although galaxies which harbor counterstreaming gas do not appear to have any ongoing star formation (e.g., Fisher, Illingworth & Franx 1994), there is no direct proof to date that counterstreaming gas inhibits star formation.

On the other hand, since a small amount of primordial prograde gas in the primary does not cause an appreciable inflow of gas to the center (simulation I6), we are more inclined to believe that collisions between counterstreaming gas clouds trigger star formation rather than inhibiting it. Incoming counterrotating gas would generally have the larger specific angular momentum, as demonstrated in Fig. 34. The direction of the orbits acquired by stars resulting from the interaction between corotating and

counterrotating gas would depend upon the relative masses of the two gas components. A more massive prograde component should produce significantly more stars with prograde orbits. Small amounts of prograde gas would produce stars with mostly retrograde orbits. In either case, some gas will flow inwards.

Under the above hypothesis, the A stars seen in the primary disk are an indication that counterrotating gas was present throughout the primary disk less than a Gyr ago. The star formation would have proceeded inwards as more and more counterrotating gas found its way there, ending earlier at radii where prograde gas was exhausted faster. The annulus of counterrotating G and K giants around the $H\alpha$ ring supports these ideas. The radial distribution of corotating G and K giants reflects the primordial radial distribution of prograde gas (decreasing smoothly from the center outwards).

If the counterrotating gas owes its origin to a merger instead of infall, at least some of the counterrotating G and K giants could have been inherited from the dwarf galaxy. In our dwarf merger simulations, we found that even though the stars from the dwarf have net counterrotation, they form an oblate structure which resembles an extended (radius $\gtrsim 3$ times radius of primary) and rather thick (scale height $\sim 2 - 3$ kpc) disk, rather than the compact and flat disk formed by the gas. This distinction is clearer in M1 where the gas disk is flatter. Observationally, such an extended counterrotating stellar population may be hard to separate from the halo stars and hence missed altogether.

Figs. 35 and 36 show the final rotation curves for an infall (I5) and dwarf merger (M1) simulation respectively. The counterrotating gas is colder than the primary

Figure 34: A comparison of the mean specific angular momentum of the counterrotating gas (open squares) with that of the corotating (prograde) gas for I6.

stars. The rotation curve of the counterrotating gas is slightly higher than that of the primary, and its velocity dispersions are systematically lower than those of the primary stars. This is qualitatively in agreement with the observed rotation curves, but our velocity dispersion values for the primary disk stars do not agree well with the observed values reported by JBH. Our values are significantly lower, in some cases by as much as a factor of 2. The velocity dispersion values for the counterrotating

Figure 35: The final rotation curves and velocity dispersion plots for the primary disk (filled points) and the counterrotating gas (open points) after gas infall (I5). The initial disk rotation curve is drawn in for comparison. The velocity dispersion values are somewhat smaller than the observed values. The disk inclination is taken into account in calculating the rotation speed.

gas agree with the observations much better. Since our gas never gets converted into stars, we do not have values for the velocity dispersions for counterrotating stars.

4.5.4 Comparison with Previous Work

The heating experienced by our primary disk after mergers with dwarfs that have half its own mass (not counting the dark matter) is modest compared with that due to a dissipationless merger between a large disk galaxy like the Milky Way, and a

Figure 36: The final rotation curves and velocity dispersion plots for the primary disk (filled points) and the counterrotating gas (open points) for the dwarf merger M1. The initial disk rotation curve is drawn in for comparison. Both the rotation curves and the velocity dispersion values are higher than those for the infall simulation, although part of the difference may be due to the error in the inclination correction. The dispersion values are closer to the observed values in this case.

dense dwarf galaxy with one-tenth the mass of the primary disk (Walker, Mihos & Hernquist 1996). The latter causes the disk to thicken by $\sim 60\%$ at the solar circle, which is not surprising considering that the merger is dissipationless, the primary is cold and thin, and a good portion of the satellite's mass ($\sim 45\%$) survives intact and sinks to the center of the primary. It is also a prograde merger which completes rapidly, in less than 3 Gyr. This is due to the interaction of the satellite with the disk

particles, caused by the small initial distance of the satellite from the disk (6 disk scale lengths). There is not enough time for the primary to completely tidally strip the satellite and thereby transfer orbital energy to the tidal remnants. The halo also does not have the opportunity to absorb some of this energy. A large initial orbital momentum merger (in which the satellite starts out at several disk radii) of a 10% mass satellite, by contrast, is much less damaging to the primary (Huang & Carlberg 1996). This is more in agreement with our simulations, in which the infalling gas as well as the dwarf galaxy are initially placed at a large average distance (~ 20 disk radii) from the primary disk. The relatively low density of the satellite in each of our mergers (central density of primary is ~ 40 times the central density of the dwarf) also contributes to more efficient stripping of the satellite.

The higher initial angular momentum and lower density satellites are only part of the story, however. As mentioned in §4.5.2, the nature of the primary has a lot to do with the amount of heating experienced. The fragility of cold and thin galactic disks has been demonstrated time and again in the context of minor mergers by numerical (TR; Walker, Mihos & Hernquist 1996; Quinn, Hernquist & Fullagar 1993; Quinn & Goodman 1986) and analytical (Tóth & Ostriker (1992); Quinn & Goodman 1986) studies, but the response of early-type disks to minor mergers has not received much attention. The resistance of these disks to minor mergers appears to be significantly higher. The production of counterrotating disks and bulges via retrograde minor mergers is much more likely for S0s/Sas than for later types. The counterrotating bulge recently reported in the Sb galaxy NGC 7331 (Prada et al. 1996) is most likely a result of very low angular momentum gas infall rather than a minor merger.

The origin of the gas for continuous infall is an open question, and for this reason the dwarf merger process is potentially a more frequent producer of counterrotating disks. The formation of dwarf galaxies in tidal tails of interactions between larger galaxies has numerical (Barnes & Hernquist 1992b; Elmegreen, Kaufmann & Thomasson 1993; Hibbard & Mihos 1995) as well as observational (Hibbard et al. 1994; Hunsberger, Charlton & Zaritsky 1996) support. It provides a possible mechanism for obtaining gas-rich dwarf galaxies which have less dark matter than the norm. The tidal features observed (Hibbard et al. 1994) in NGC 7252 (the “Atoms for Peace” galaxy), for instance, contain clumps with properties typical of dwarf irregular galaxies in terms of their high atomic gas content, luminosity, velocity width and M/L ratios ($\sim 1.5 - 7.5$ for $50 \leq H_0 \leq 75$). These tidally formed dwarfs, however, are substantially less massive than the dwarfs used in simulations M1-M3.

The S0 galaxy NGC 7332 (Fisher, Illingworth & Franx 1994) shows extended counterstreaming [O III] gas in which the counterrotating gas has higher velocity than the prograde gas and stars. This is cited as support for recent accretion and agrees with the behavior seen in our infall simulation with prograde gas (I6), where we find that the counterrotating gas has higher angular momentum than the corotating gas. We do not, however, see any clear gas motion towards the center as a result of the interaction of the counterstreaming components. An analytical study of two counterrotating gas disks lying one over the other predicts that the counterrotating gas should drag the corotating gas inward with equal masses of corotating and counterrotating gas accreting rapidly (Lovelace & Chou 1996). Although we see enhanced accretion for infall with prograde gas (I6), the amount of prograde gas in our test

is small and cannot be expected to dramatically affect the accretion and inflow rate of the counterrotating gas. We do however see some evidence of the pile-up of gas into rings as predicted by Lovelace, Jore & Haynes (1996) as a consequence of a two-stream instability in counterrotating systems. This is evident in Fig. 32, especially for M2 and I6, where local peaks and troughs dominate the radial gas profile.

The evolution of the tidal material seen in our simulations is in general agreement with the behavior described by Hibbard (1995) as a generic result of mergers. For the lowest orbital angular momentum case (I3), we see the tidal material falling back early in shells, and as the orbital angular momentum increases, so does the fallback time as the tidal material forms larger and larger loops. We also confirm that all the tidal material remains bound and eventually will fall back into the primary galaxy.

4.6 Conclusions

Based on numerical simulations using a fiducial spiral primary, TR predicted that counterrotating disks should be rare because they require special conditions to persist over several Gyrs. This was partly due to the fact that dwarf mergers were not able to produce counterrotating disks with the required properties within the required time frame. Our experiments with NGC 4138 lead us to be more optimistic about dwarf mergers. The combined effects of a massive, compact primary and a smaller dynamical time (due to the smaller primary) gives a dwarf merger a much better chance. For a dwarf merger model based on the observationally determined parameters of the galaxy, we have found a configuration that produces a counterrotating disk closely matching the one observed, although the dwarf galaxy must be predominantly gaseous with little to moderate amounts of dark matter.

The results of minor mergers involving large primaries with cold, thin disks cannot be extended to all types of spirals. Early type spirals, with hot, massive and compact disks of the type that NGC 4138 has, appear to be more resistant and less fragile. The initial orbital angular momentum of the satellite plays a role in determining the damage done to the primary disk as well in numerical simulations, and it must be taken into account when interpreting the results.

Continuous infall also succeeds in producing a counterrotating disk that matches the observed one in terms of mass and size. The time frame for infall (~ 5 Gyr) is somewhat longer than for dwarf mergers ($3 - 4$ Gyr). A common characteristic of the counterrotating disk resulting from either process is that it does not have an exponential profile. This is not a problem for NGC 4138, since there is no evidence of its counterrotating disk being exponential, but counterrotating disks in some other galaxies are exponential (e.g., NGC 4550, NGC 3593).

The nuclear star formation ring may owe its origin to an interaction between prograde and retrograde gas. The counterstreaming gas hypothesis is viable for both infall and mergers. The inferred star formation histories of both disks do not pose serious problems for this hypothesis currently, but knowledge of the colors and ages of the counterrotating stars is necessary to resolve this issue.

We have only tested a restricted set of infall and merger possibilities so far. This is partly because these processes are not well constrained observationally (especially infall), but also because of our CPU resource limitations. There is a need to explore alternative infall and merger models before a reliable prediction can be made regarding the overall ease of obtaining massive CR disks in all types of spiral galaxies.

CHAPTER 5

SPH SIMULATIONS

The Sticky Particle gas dynamics was replaced with Smoothed Particle Hydrodynamics (SPH) to investigate the structure of the counterrotating disk. The technical details of SPH are discussed in Ch. 2. We have selected the most promising candidates among the simulations presented in the previous chapter to rerun with SPH. We report here on three infall simulations and two gas-rich dwarf merger simulations, and discuss the differences between our previous results and the SPH results. As stated in Ch. 2, an isothermal gas is assumed for all the SPH simulations, and its temperature is assumed to be 10^4 K.

The questions that we hoped to answer with the SPH simulations were:

- a) What is the size (scale length) of the counterrotating disk and how does it depend on the initial conditions?
- b) What is the radial profile of the counterrotating disk and what are the physical processes controlling it?
- c) What is the morphology of the counterrotating disk? Are spiral arms visible?
- d) Is there any difference in the heating experienced by the primary disk in the SPH simulations?

5.1 Gas Infall

Infall simulations I1 and I2 from Ch. 4 were rerun with the SPH code. The simulation snapshots are shown in Figs. 37 and 38. Only the last few time steps are shown for I2. The column of gas does not collapse or form clumps as in the sticky particle case, and in fact the gas disperses more before it starts to shock as soon as the first batch of particles orbit around the primary and collide with the rest of the incoming gas (at $t \lesssim 1.0$ Gyr in Fig. 37).

The formation of the counterrotating disk commences significantly faster in the SPH simulations. As can be seen from Fig. 37, by $t = 1.6$ Gyr, a disk is already forming, and by $t = 2.4$ Gyr, more than two-thirds of the gas particles have assembled in the disk configuration (this is evident from the radial distribution plots which are not shown here, although radial density plots are included below). However, the evolution of the disk structure after it is in place proceeds very slowly. There is not much difference in the disk structure between $t = 2.4$ Gyr and $t = 3.0$ Gyr in Fig. 37.

The size of the counterrotating disk formed is about the same as the primary disk, but the appearance is quite different from the sticky particle results. Spiral structure is clearly discernible, and there is a pronounced underdensity of particles at intermediate radii that persists for at least a Gyr. Even though I2 has less angular momentum than I1, there is not much difference in the size or appearance of the counterrotating disk. The velocity fields for I1 are shown in Fig. 39. The counterrotating disk is kinematically much colder than the primary, which in fact does heat up during the simulation by an amount comparable to the sticky particle case (see below). There is also a density cusp in the center of the new disk, which indicates that viscous

Figure 37: Infall simulation I1 with SPH. The “top” view (top half) and the “side” view (bottom half) are shown. The primary disk is shown only in the first panel for each view for clarity. The square panels are 80 kpc wide, except for the last two in each view, which are 20 kpc wide.

Figure 38: Top (upper half) and side (lower half) view of gas infall simulation I2. Panel widths are 20 kpc.

dissipation does cause some particles ($\sim 5 - 10\%$) to spiral in to the very center. A large percentage of the particles, however, stay at large radii.

To investigate this issue further, we reduced the length of the initial column of gas by a third of its original length (100 kpc instead of 150 kpc), thereby making it denser and more susceptible to clumping. The results of this third infall simulation, which was not previously performed with sticky particles, are shown in Fig. 40. The shorter column of gas means that the disk formation is speeded up even further because the gas is available at a faster rate. Although there is more spiral structure now evident, especially at intermediate radii, the overall size of the disk has not changed. This suggests that the size is not controlled by the initial configuration of the gas, but rather depends on the physical processes involved in the gas dynamics. Fig. 41 shows

Figure 39: Velocity fields of the disk (left) and counterrotating gas (right) compared for I1. Only a fraction of the particles is sampled, and sampling is proportional to radius (hence the distribution of vectors does not reflect the density). Each panel is 17 kpc wide.

the evolution of the disk up to $t = 3.7$ Gyr, and although there is some change in the disk size and radial distribution, it is a very slow process. This is discussed further below.

5.2 Gas-Rich Dwarf Merger

Running a dwarf merger simulation with SPH is quite difficult if the gas is assumed to be isothermal. This is because the gas in our dwarf galaxy is configured as a Plummer sphere and it is unstable if the temperature is too low. Using the gas temperature that we used for the infall simulations, i.e., 10^4 K, does not work for the gas in the dwarf galaxy. The Jeans instability causes the gas to collapse within a few dynamical times. The Jeans length has to be increased by using a higher temperature (hence

Figure 40: Top (upper half) and side (lower half) view of gas infall with a shorter initial gas column. Panel widths are 80 kpc, except for the last three panels in each view, which have been magnified as indicated.

Figure 41: Evolution of the counterrotating disk for the third infall simulation. Top (upper half) and side (lower half) views are shown. Panel widths are 20 kpc.

isothermal sound speed) for the gas. However, if the temperature is increased too much, the gas pressure is too high and the gas expands indefinitely.

The simulation M2 described in Ch. 4 was rerun with SPH. This is the run with a substantial amount of dark matter in the dwarf galaxy. Initially a higher temperature was adopted for the gas, and the temperature was reduced to 10^4 K when the dwarf was tidally stripped and the counterrotating disk began to form ($t \sim 1.7$ Gyr). The simulation snapshots are shown in Fig. 42. The disk formed is somewhat smaller than the primary disk, and shows a pronounced bar. The side views indicate that the barred disk is quite thin. Both of these characteristics are departures from the sticky particle results, in which the counterrotating disk was circular and thick. The bar is quite long-lived since it persists at least for 0.3 Gyr (from $t = 2.1$ to $t = 2.4$ Gyr).

Figure 42: Top (upper half) and side (lower half) view of gas-rich dwarf merger M2 run with SPH. Panel widths are 80 kpc, except for the last three panels in each view, which have been magnified as indicated. The primary disk is shown only in the first three panels in each view.

Figure 43: Velocity fields of the disk (left) and counterrotating gas (right) compared for dwarf merger. Only a fraction of the particles is sampled, and sampling is proportional to radius. Each panel is 20 kpc wide.

The velocity fields for the primary and counterrotating disk are shown in Fig. 43, and as in the sticky particle case, the primary's field is much hotter than the counterrotating disk's field, more so than for gas infall. The dwarf merger does heat up the primary velocity field significantly more than gas infall, due to the presence of dark matter and stars in the dwarf.

To circumvent the difficulties associated with the isothermal gas during the approach of the dwarf galaxy, we repeated simulation M2 in another way. SPH was introduced to replace the sticky particle gas dynamics at the point where the dwarf galaxy has been tidally stripped and the disk has begun to form ($t \sim 2.9$ Gyr). The results of this simulation are displayed in Fig. 44. Once again, the evolution of the disk is very different from the sticky particle case. Now the counterrotating

Figure 44: Top (upper half) and side (lower half) view of the counterrotating disk formed after continuation of M2 beyond $t = 2.9$ Gyr with SPH instead of sticky particles. Panel widths are 20 kpc.

disk looks very similar to that formed by gas infall, but it is significantly smaller (by $\sim 30\%$) in radial extent. It is therefore considerably smaller than the primary disk. There is some evidence for a small bar, but it is not comparable in size to the one seen in Fig. 42.

5.3 Discussion

The general characteristics of the infall and merger processes are very similar for the sticky particle and SPH simulations, the only exception being the lack of severe clumping of the infalling gas in the SPH simulations. The inelastic collisions in the sticky particle simulations cause the gas to clump much more. In the case of SPH, the gas appears to be too warm for it to coalesce into clumps. However, once the disk

formation begins, the SPH gas undergoes more viscous dissipation than the sticky particle gas, and the end result is that the SPH disk is about the same size radially as before, and is also much thinner.

Figure 45: The radial density profiles for the counterrotating disks formed.

Although the sticky particle and SPH simulations evolve in a similar fashion up to the disk formation stage, the similarities end there. There are sharp differences between the counterrotating disks formed by SPH and sticky particles. The difference in the radial profiles of the primary and the counterrotating disks is quite dramatic. Fig. 45 shows plots of the radial density distribution of the counterrotating disks for the infall and merger simulations. All the profiles have been plotted for comparable time steps at which the disks reach a stable configuration ($\gtrsim 2.5$ Gyr). For the

infall simulations, there is a sharp increase in the density at the center, followed by a density trough between 1 and 3 kpc, and a local maximum at ~ 4 kpc. In contrast with the sticky particle simulations, where the gas piled up into rings at various radii (see Fig. 32), only a single ring at ~ 4 kpc is seen. The profile of the barred disk in the first SPH dwarf merger is very different from the infall profiles and from the sticky particle merger profiles. There is a hole in the center and then the profile approximates an exponential. The second merger, where SPH is only introduced after the dwarf has been merged, shows two main mass concentrations, one at the center and one at ~ 3 kpc.

The radial profiles change very slowly with time. This is illustrated in Fig. 46, where the profiles for $t = 2.9$ and $t = 3.7$ Gyr are plotted for the third infall simulation. There is only a very modest increase in the number density of particles at radii $\lesssim 3$ kpc over an interval of 0.8 Gyr.

Since the counterrotating disk profile is far from exponential, it is not possible to calculate a scale length for it that can be compared with the scale length of the primary. It does appear slightly larger than the primary in extent because more of its particles are concentrated at larger radii. The disk formed is also much thinner than in the sticky particle simulations. This is consistent with it being kinematically much colder than the primary disk.

The differences seen in the structure and radial profiles of the counterrotating disks produced by sticky particles and SPH indicate that the viscosity (at least the amount we have assumed) by itself is not enough to produce a disk that is similar in size and shape to the primary disk. The thinness of the disks formed suggests that it

Figure 46: The radial density profiles for the counterrotating disk compared at $t = 2.9$ (solid line) and $t = 3.7$ (dotted line) Gyr, for the third gas infall simulation.

is not the lack of viscous dissipation that is to blame for this. Other processes such as radiative cooling, and more importantly the existence of primordial prograde gas in the primary, must also play a critical role. Repeating the infall simulation I6 from Ch. 4 (which has a prograde ring of gas in the primary) with SPH indicates that prograde gas is much more effective at driving the counterrotating gas to the center with SPH than was the case with sticky particles. All of the prograde gas, and most of the counterrotating gas that comes into contact with it, ends up in the center of the primary within a few dynamical times. The velocity fields of the prograde gas ring and the incoming retrograde gas are shown in Fig. 47 for several time steps.

Figure 47: Velocity fields of the prograde gas (top) and counterrotating gas (bottom) at equally spaced time steps between $t = 0.5$ and $t = 1.1$ Gyr for infall with a prograde gas ring in the primary. Each panel is 4 kpc wide. Only a fraction of the particles are shown for clarity.

Although the inclusion of star formation in our simulations is important so that we may study how and when the gas is converted into stars, it probably will not affect the radial profile of the disk too much. Previous SPH studies with star formation (Mihos, Richstone & Bothun 1992; Mihos, Bothun & Richstone 1993; Mihos & Hernquist 1994b) have shown that in galaxy interactions, the star formation rate is modest until most of the gas finds its way to the center and is confined to a small dense region. Thereafter the gas typically gets depleted very fast (few $\times 10^8$ yr) as a starburst ensues.

The heating of the primary disk in the SPH simulations is comparable to that seen in the sticky particle runs. It is of the order of the numerical heating that is present even in the absence of any interaction due to the discreteness of the particle

Figure 48: A comparison of the thickness of the disk after continuous infall and dwarf mergers. The solid line shows the initial disk thickness at $t = 0$. The dotted line is the disk thickness for an isolated disk evolved without any infall or merger. Thicknesses for $R \gtrsim 4$ kpc are not reliable due to small numbers of particles at larger disk radii.

representation. The mean half-thickness of the primary disk serves as a measure of the heating experienced by it, and it is plotted as a function of radius for various simulations in Fig. 48. A comparison of the plots with Fig. 33 shows general qualitative agreement. There is no reason for the vertical heating to be considerably more with SPH than with sticky particles, so this is not a surprise. As before, the dwarf mergers heat up the disk a little more than gas infall due to the presence of stars and dark matter in the dwarf. The SPH plots also show more heating in the inner regions of

the disk. This is consistent with the larger number of particles at smaller radii in the SPH counterrotating disk compared with its sticky particle counterpart.

Although the gas infall scenario has been reexamined with SPH to some extent, the problems encountered with the dwarf galaxy's gas stability have so far precluded a comparable investigation of dwarf mergers even with the restricted SPH code.

CHAPTER 6

CONCLUSIONS

The aim of this dissertation was to investigate the following questions:

- a)* Is it possible to produce a massive counterrotating disk in a spiral galaxy through an accretion or merger event without destroying the preexisting primordial disk?
- b)* If so, which process produces disks that provide the closest match to the observations?
- c)* In either case, what effect does the formation of the counterrotating disk have on the primordial disk?
- d)* What are the characteristics of the counterrotating disk formed?

The findings from the numerical simulations described in Chs. 3-5 are summarized in the following sections. The last section discusses future work in this field.

6.1 Formation of Massive Counterrotating Disks in Spirals

The constraints on the process that produces a massive counterrotating disk in a spiral galaxy depend on the type of the host or primary spiral galaxy to a large extent.

For a late-type spiral with a cold, thin disk, the adiabatic infall process works quite well, but a gas-rich dwarf merger produces an unacceptable amount of heating

in the primary disk even for a modest-sized dwarf galaxy with a small amount of dark matter. The dwarf merger process can therefore be ruled out as a producer of massive and extensive counterrotating disks, although it should be able to produce counterrotating cores or bulges.

By contrast, for an early-type spiral with a hot, massive and compact disk, the gas-rich dwarf merger process is not problematic as long as the dwarf galaxy mass is not dominated by dark matter (dark-to-luminous mass ratio $\lesssim 4$). Gas infall is also successful in producing massive counterrotating disks in such galaxies, although it takes longer than mergers to do so.

A restricted subset of initial conditions has been tested so far owing to CPU and time limitations, but the simulations completed have established the basic viability of the processes tested. The impact of the counterrotating disk formation on the primary disk has also been studied in some detail, although it has not been possible to quantify this effect due to the complications introduced by numerical heating. The important role played by gas and dissipation in these processes has also been established: dissipationless simulations fail to yield counterrotating disks within a Hubble time.

Based on the findings presented in this dissertation, it is not possible to make an authoritative prediction about the frequency of counterrotating disks in spiral galaxies, but it is clear that there are some real difficulties encountered in producing them by the known processes that I have tested. These difficulties include the large time-scales involved, the requirement that the gas be clump-free, the heating experienced by thin primaries in dwarf mergers, and restrictions on the initial angular momentum

of the gas. This suggests that large counterrotating disks are likely to be very rare in late-type spirals (Sb and later), and the likelihood of finding them in early-type spirals is much better. There are definite indications that the very process that produces them transforms the spiral galaxy into an earlier type, so the overwhelming majority of the ones that do exist are likely to be found in S0 or Sa galaxies.

6.2 Comparison of Accretion and Mergers

Gas infall, by definition, is a gradual process that takes longer to produce a counterrotating disk than does a dwarf merger. In general, the time-scale for accretion by gas infall is $\gtrsim 50t_d$, where t_d is the dynamical time of the primary galaxy. The success of the gas infall process is linked to the time-scale in some ways: the longer it takes the gas to accrete, the more likely it is that the gas will coalesce into concentrated clumps, some of which are massive enough to qualify as dwarf galaxies. In the gas infall model we have adopted, the infalling column of gas breaks up into clumps, and often the last one or two clumps are massive enough to survive the primary's tidal field. This only happens in the sticky particle simulations, however, and the SPH results suggest that clumping to this extent might not happen for a real gas if it is warm ($\sim 10^4$ K).

The initial angular momentum of the gas also plays a critical role in the infall models. It affects the time-scale of the infall as well as the size of the counterrotating disk formed. If the initial angular momentum of the gas is too large, a counterrotating disk will not be produced in a Hubble time. If it is too low, a counterrotating bulge or spheroid is produced instead. In between these extremes, the size of the counterrotating disk is proportional to the magnitude of the initial angular momen-

tum. Finally, the direction of the gas angular momentum is also crucial. Gas orbits inclined by more than a few degrees to the plane of the primary disk will not be able to produce a counterrotating disk in the same plane easily.

A gas-rich dwarf merger is by comparison a simpler process with fewer free parameters. Although the amount of dark matter and the fraction of mass in the form of gas are very important parameters, the process is less susceptible to sabotage by other factors such as small changes in the magnitude of the angular momentum, or the time-scale. The flip side of this is that the damage done to the primary disk can be extensive if the disk is cold and thin to begin with. This imposes sharp constraints on the mass of the dwarf galaxy, especially its dissipationless mass fraction. These constraints are severe enough to limit the viability of this process only to early-type spirals with hot or dense disks.

6.3 Effect on Primordial Disk

The impact of the counterrotating disk formation on the primary disk has been discussed in detail in Chs. 3-5. The main issue of concern is the vertical heating experienced by the primary disk (see Figs. 18 and 33). The amount of heating depends both on the process that yields the counterrotating disk as well as the nature of the primary disk itself. In general, the cold and thin disks in late-type spirals are significantly heated even by gradual gas infall, whereas the hot and dense disks in early-type spirals like NGC 4138 can withstand dwarf mergers where the mass of the dwarf is as much as 50% of the mass of the primary disk. The moral of the story is that one cannot make a general statement about the heating of all spiral disks due to minor mergers or accretion.

6.4 Nature and Stability of Counterrotating Disk

Viscous dissipation plays an important role in determining the characteristics of the counterrotating disk formed, as the results of the SPH simulations demonstrate. We obtain significantly thinner disks with SPH than with sticky particles, even though the sticky particle gas was more dissipative. Shocking in the gas gives rise to more spiral structure in the forming disk. The “hole” in the inner regions of the counterrotating disk appears to be a real effect in the gas infall process, at least in the absence of prograde gas and radiative cooling. However, the underdensity does not go all the way to the center in the case of the SPH disks, which show a density cusp at the very center. This piling up of mass at the center would enhance the bulge, and if it is large enough, provide fuel for a central source.

The scale length of the disk is crucially dependent on the initial angular momentum of the gas as well as its dissipative characteristics. Our inability to obtain an exponential profile even with SPH suggests that other physical processes, such as the presence of prograde gas, radiative cooling, and feedback by supernovae, may be important in determining the radial profile of the disk.

The disks produced by gas-rich dwarf mergers are significantly barred, and very thin compared with the primary disk. The severity of the bar instability is inversely related to the temperature assumed for the gas. If SPH is introduced after the dwarf galaxy has already merged, the bar is much smaller, and the disk morphology and radial profile are similar to the infall results. In this case the counterrotating disk is also smaller than the primary by $\sim 30\%$. The disks formed by SPH are stable and show spiral structure that evolves slowly with time (on order of few Gyr).

6.5 Future Work

Although this dissertation attempts to answer some fundamental questions about the formation of counterrotating disks, there is much more that we need to know about them before their origin, evolution and impact on spiral structure are fully understood. Some important directions for future research are outlined below.

To obtain answers to the detailed questions about counterrotating disks, the quality of the simulations will need to be further enhanced by the addition of physical processes like star formation, radiative cooling and re-heating by supernovae. This will require significant modifications to the current version of the SPH code. More realistic models for the astrophysical entities involved in the simulations, namely the spiral galaxy and the source of counterrotating gas, are also warranted. Self-consistent disk-bulge-halo models of spiral galaxies (*e.g.* Hernquist 1993) can be incorporated into the code. A very important step in the enhancement of the code will be its implementation on a massively parallel processor (MPP) architecture. This will allow it to exploit the new generation of MPP supercomputers. In addition, certain optimizations can be applied to the code to increase the degree of parallelism and make it execute faster. These include the speeding up of nearest neighbor searches by searching for groups of particles rather than individual particles, and choosing time-steps in such a way that neighbors are updated together as far as possible.

In addition to providing a higher degree of realism, the code improvements listed above will permit a significantly larger number of particles ($\sim 10^6$) to be used, which will serve to minimize the two-body effects in the disk due to numerical discreteness (Hockney & Eastwood 1989) and maximize the resolution. Such high resolution

simulations can then be used to probe deeper into the mystery of counterrotating disks as described below.

A detailed examination of the gas infall parameters is necessary to determine how they affect the final scale length and height of the counterrotating disk. An important question is whether the counterrotating gas triggers star formation in the primary disk if there is preexisting prograde gas in it. We have been able to shed very little light on this question in our case study of NGC 4138 (Thakar et al. 1997), and definite answers can only be obtained with an SPH code that includes star formation (and preferably feedback from supernovae). The source of the infalling gas is also a mystery, and exploring other gas infall models may yield some indication of the likely source(s). Observational evidence of accretion and infall in disk galaxies (*e.g.* Zaritsky 1995) can make a very important contribution to this work, since it would provide clues as to the configuration and rate of infall.

Further study of dwarf mergers is clearly warranted. Prograde mergers with smaller satellites have been shown to alter the state of the primary disk enough to change the galaxy's Hubble type (Walker, Mihos & Hernquist 1996), and I would like to compare the situation with retrograde mergers. Numerical (Hernquist & Barnes 1992) and observational (Hunsberger, Charlton & Zaritsky 1996) evidence for the formation of dwarf galaxies in tidal interactions increases the likelihood of mergers between dwarf and spiral galaxies.

The evolution of the host galaxy after counterrotating disk formation needs to be studied to answer the following questions regarding the long-term stability and impact of counterrotating disks on their hosts: Do counterrotating disks make the

galaxy more stable, or do they in fact introduce new instabilities (*e.g.* Sellwood & Merritt 1994)? What about the stellar populations and metallicities seen in galaxies with counterrotating stellar disks or star forming regions? Can the model we adopt for the formation of counterrotating disks also explain the level of enrichment seen in the integrated stellar spectra of these galaxies? If a metal-rich population is diluted by stars resulting from the influx of metal-poor gas, what would be the effect on the integrated spectrum? Numerical simulations that begin with the counterrotating disks already formed can help to answer these questions.

Appendix A

OPTIMIZATION FOR CRAY Y-MP

The changes required to the original tree code and the gas dynamics codes in order to optimize them for execution on the Cray Y-MP (at the Ohio Supercomputer Center) are described here. The Cray Y-MP is a pipelined vector machine, i.e. it can operate on entire arrays of data (vectors) in parallel by pipelining the operation in the same way that an assembly line can operate on many cars at a time. In order to benefit from this feature, the program has to be written in such a way that array operations are maximized. It is also necessary to ensure that as far as possible, operations on different array elements are independent of one another so that they can be executed in parallel. Maximizing array operations and eliminating “vector dependencies” is known as *vectorizing* the code.

The tree code as well as the gas dynamics codes had to be vectorized so that they would run efficiently on the Y-MP. For the tree code, the three main vectorization challenges were the tree-construction, tree-walk, and gravity calculation. Performance of the code on the Y-MP was monitored using the `perfview` and `flowview` commands in conjunction with the **Hardware Performance Monitor (HPM)**.

A.1 TREE Code

The vectorization of the TREE code required fundamental changes to the original version, which utilized recursion to accomplish tree operations. The recursive logic had to be replaced by iterative methods, and this was done according to prescriptions available in the literature (Hernquist 1990; Makino 1990). Two main functional units of the code needed re-working: the **tree construction** and the **gravitational force calculations** (which require tree-walks).

A.1.1 Tree Construction

The most natural and elegant way to build a tree is to start with the root and build each subnode one at a time recursively. This is a **depth-first search**, since each subnode is built completely before proceeding with the next one. There can be little parallelism in this method because each subnode depends upon its descendants. An alternative, however, is a **breadth-first search**, in which we subdivide all subnodes at a given level of the tree simultaneously. This requires considerably more book-keeping, but it affords a much higher degree of parallelism since all subnodes at a given level in the tree are independent and hence can be treated in parallel. The vectorized tree construction algorithm (Makino 1990) is:

```
create the root node and attach all particles to the root
flag all particles as not_done
 $l = 1$       (root level)
while there are still particles that are marked not_done
    for each remaining particle
```

```

        determine which subnode it belongs to
    for all subnodes at level  $l$ 
        count the number of particles in the subnode
    for all subnodes at level  $l$ 
        if at least one particle in subnode then
            create subnode and connect to parent
    for all particles marked not_done
        if particle is lone occupant of its cell then
            mark the particle as done
     $l = l + 1$  (go to next level)
end (while)

```

A.1.2 Tree Traversal

Once the tree is built, assigning mass, center-of-mass coordinates and quadrupole moments to the nodes of the tree can also be vectorized by traversing the tree bottom up, i.e. from the leaves to the root, one level at a time (Hernquist 1990). We start at the deepest level and again make use of the fact that the nodes at a given level can be processed in parallel. It is also necessary to maintain a list of all the cells in the tree when the tree is built. To expedite this process further, we keep an array of pointers to the first cell at each level of the tree, denoted by “fcell” in the algorithm described below, which is also used to calculate center-of-mass coordinates and quadrupole moments ($\text{NSUB} \equiv \text{max. number of subcells} = 8$ for 3-D tree code).

```

for  $l = \text{maxlevel}$  downto 1
    initialize mass of all cells at this level to 0

```

```

fcell = first cell at this level, lcell = last cell at this level
for j =1 to NSUB
    for i =fcell to lcell
        if subcell j is not empty then
            add mass of subcell j to mass of cell i
    end (for)

```

A.1.3 Force Calculation

There is inherent parallelism in the N-body force calculation since the forces on individual particles can be evaluated independently. For each particle, we build an interaction list containing the force terms for that particle. These interactions can then be processed in parallel in a vector loop. We still need to vectorize the code that builds the interaction lists, and that is accomplished in a manner similar to the mass assignment described above, except that the tree traversal is from the top (root) down (to the leaves) now. This algorithm is briefly described below (Hernquist 1990). Note that we create two interaction lists, one for leaf nodes (particles), and one for interior nodes (cells), because the latter may involve quadrupole moments.

```

nnodes = 1, node[1] = root
while nnodes > 0
    for i = 1 to nnodes
        if node[i] is a leaf then
            add node[i] to leaf interaction-list
        else
            add node[i] to interior-node list
    end (for)

```

```

        n_intnodes = n_intnodes + 1
    for i = 1 to n_intnodes
        if node[i] is near enough to be subdivided then
            add node[i] to near list
            n_near = n_near + 1
        else
            add node[i] to node interaction-list
    for j = 1 to NSUB
        for i = 1 to n_near
            if subnode[j] is not empty then
                add subnode[j] to list of nodes for next level
                nnodes = nnodes + 1
    end (while)

```

A.2 Sticky Particle (SP) Code

The SP code also had two functional units that needed to be vectorized: the **nearest-neighbor searching** to determine which particles should collide, and the **collision calculations** to determine the changes to the velocities of the particles involved in the collisions. The former required a tree-walk very similar to that required to build the force interaction lists, so the same prescription as above was used. The collisions presented a problem because multiple collisions per particle at a given time-step could not be vectorized - there is a vector dependency involved as the velocity change due to one collision must be applied before the next collision can occur. In order to make the collision calculations vectorizable, it was necessary to make every collision

independent of every other collision, and this could only happen if every particle was involved in at most one collision at each time-step. **Hence only binary collisions were implemented and no particle was allowed to collide with more than one particle at a time.** This is not a major problem, and in fact previous implementations of SP collisions have almost all allowed only binary collisions. The amount of dissipation “lost” by the absence of multiple collisions can be made up to some extent by increasing the “stickiness” of the gas particles. Restricting the collisions in this way allowed us to vectorize all the SP calculations.

A.3 SPH Code

The optimization of the nearest-neighbor searching was applicable to the SPH code as well. The remainder of the SPH code was optimized using standard techniques. The main challenge was the tradeoff between memory and CPU efficiency, because the SPH routines required considerably more memory to store the nearest neighbor lists so as to avoid searching repeatedly for them.

A.4 Other Optimization

To make the code even more efficient, we used the Cray intrinsic FORTRAN functions `WHENEQ`, `WHENNE` and `WHENIGT (3F)` to select array elements matching a given criterion. These were incorporated into the C code using the Interlanguage Communication facility. **Inlining** optimization was also applied to the entire code. After all these changes, we measured an **overall factor ~ 5 improvement** in the efficiency. A test run requiring 2 hours with the original code ran in under 25 minutes with the optimized code.

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