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SUPERSYMMETRIC GRAND UNIFIED THEORIES AND GLOBAL FITS TO LOW ENERGY DATA

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By

Tomáš Blažek, M.S., RNDr.

* * * * *

1996

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ABSTRACT

We present the details of a global χ^2 analysis of electroweak data, including fermion masses and mixing angles, in SO(10) SUSY GUTs. Just as precision electroweak data is used to test the Standard Model, the well determined Standard Model parameters are the precision electroweak data for testing theories beyond the Standard Model. In this work we use the latest experimentally measured values for these parameters. We study several models discussed in the literature. One of these models provides an excellent fit to the low energy data with $\chi^2 \sim 1$ for 3 degrees of freedom. We present graphs of constant χ^2 contours as functions of position in soft SUSY breaking parameter space, as well as our predictions for a few selected points in parameter space. We also study the sensitivity of our results to changes in various parameters. Finally, we discuss the consequences of our work in the context of a general MSSM analysis at the Z scale.

to Renata who has been passing her PhD in love, care and patience

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I thank my parents for all they have done for me.

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CHAPTER 1

UNIFICATION 101

1.1 Introduction

The idea that all known forces in Nature might be derived from a single unified force has been very attractive for a long time. After Einstein's early attempts to construct a theory unifying electromagnetism and gravity the idea has been revived in the 1970's with the advent of the so-called Standard Model of elementary particles[1]. According to the Standard Model, elementary particles can carry three different kinds of charges. The forces corresponding to these charges are completely uncorrelated with each other. In nature, they are easily distinguished by their intensity and so we speak about strong, electromagnetic and weak interactions. However, there are no differences in the origin of the three forces, since they are all imposed by the gauge symmetry. This symmetry became a cornerstone of modern elementary particle physics. It guarantees the freedom to rotate away unphysical (i.e. unmeasurable) phases independently at different space-time points. Naturally, with the underlying gauge principle common to all three different kinds of charges, the idea that under certain conditions they might be unified into a single charge has emerged even more appealing.

Over the next twenty-five years experimental laboratories from all around the world have been verifying the predictions of the Standard Model in all kinds of processes in a large number of experiments. One might expect that the wealth of the experimental data on various particle cross-sections and decays, and the unprecedented accuracy of some of the measurements provide enough hints as to what the extensions of the Standard Model could look like. Paradoxically, the Standard Model has turned out to be in perfect agreement with all the available data, while its various extensions based on numerous nice, attractive and fundamental theoretical ideas have been ruled out or were forced to be modified to much more complicated, and much less appealing versions. Still, the Standard Model remained in its original simple and elegant fashion, to the extent that today it would be more appropriate to call it the "Standard Theory".

If the Standard Model works so well, why is there a need for a more fundamental theory? The point is that one basic general principle - the gauge symmetry principle - is not sufficient to explain the structure of matter. Matter dynamics poses many questions which go beyond it: why exactly three forces, why can the electric charges of three quarks make up for the electric charge of the electron (why are solids around us electrically neutral), why is there a clear pattern in the masses of elementary particles establishing a hierarchy of families with identical quantum numbers, and what is the origin of the mass itself? All these questions are unexplained by the Standard Model which just parametrizes our ignorance of these and related issues. As an example, within the Standard Model, particle masses could be of any value and it is a plain mystery why the observed top quark is about 400,000 times heavier than the observed electron and they are still both equally elementary, structureless

and pointlike. Moreover, the gauge principle cannot be directly applied to the last of the observed forces, gravity, without introducing divergencies on the quantum level which, at best, render quantum gravity as a low energy effective theory of a more complete fundamental theory, possibly very different from the Standard Model.

Theories of Grand Unification have the potential to answer many of the open questions outlined above. In this introductory chapter we first go over the algebraic structure of the Standard Model, in order to set the conventions and address the Standard Model virtues and drawbacks in a more formal way. Then in the following section we introduce the ingredients of unification models: supersymmetry, minimal supersymmetric extension of the Standard Model, radiative electroweak symmetry breaking and Yukawa coupling unification, analogous to the gauge coupling unification. We pay special attention to the unification models based on an $SO(10)$ gauge group, since this class of models appears to be the minimal one. For the same reason, the global analysis of the $SO(10)$ unification models is, actually, the main topic of this project.

Finally, in the last subsection of this chapter we explain the main motivation for our work - to build a bridge between the low energy data, and a more complete theory (presumably string theory). We hope that such a theory would eventually emerge as the “theory of everything”, incorporating also gravitational effects into the unified theory of all observed interactions.

1.2 Standard Model

The Standard Model (SM) represents a theory of strong and electroweak interactions among quarks and leptons based on the gauge symmetry under the group

	$Q = \begin{pmatrix} u \\ d \end{pmatrix}$	u^c	d^c	$L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}$	e^c
$SU(3)_c$	3	$\bar{3}$	$\bar{3}$	1	1
$SU(2)_L$	2	1	1	2	1
$U(1)_Y$	1/3	-4/3	2/3	-1	2

Table 1.1: Assignment of quarks and leptons into the representations of the Standard Model gauge groups.

$G_0 = SU(3)_c \times SU(2)_L \times U(1)_Y$. Quarks and leptons of one generation fall into different multiplets of G_0 as specified in table 1.1.

The SM contains three copies of the fermionic generation displayed, with the same group structure but different masses. In addition, the SM assumes the existence of a Higgs scalar field, singlet under the $SU(3)_c$ and doublet under the $SU(2)_L$, and with hypercharge equal to -1/2. Hypercharge values (the last row) are assigned by hand to satisfy

$$Q = T_3 + Y/2 , \quad (1.1)$$

where Q is the electric charge and T_3 is the third component of the $SU(2)_L$ isospin of the respective field.

The gauged symmetry of the SM uniquely prescribes the form of the renormalizable interaction lagrangian

$$\mathcal{L}_{int} = -g_3 J_a^\mu G_{\mu a} - g_2 J_i^\mu W_{\mu i} - \frac{1}{2} g_1' J^\mu B_\mu \quad (1.2)$$

and at the same time introduces gauge couplings g_3 , g_2 and g_1' , and gauge fields G , W and B . The gauge couplings are dimensionless parameters of the theory characterizing

the strength of the corresponding interactions. They are unrelated to each other and undetermined by the theory. The gauge fields have to be in the adjoint representations of the corresponding gauge groups ($a=1,2,\dots,8, i=1,2,3$) and thus must remain massless from symmetry reasons. Using Weyl notation, the fermionic currents read

$$J_a^\mu = Q^* \bar{\sigma}^\mu \frac{\lambda_a}{2} Q + u^{c*} \bar{\sigma}^\mu \left(-\frac{\lambda_a}{2}\right)^* u^c + d^{c*} \bar{\sigma}^\mu \left(-\frac{\lambda_a}{2}\right)^* d^c \quad (1.3)$$

$$J_i^\mu = Q^* \bar{\sigma}^\mu \frac{\tau_i}{2} Q + L^* \bar{\sigma}^\mu \frac{\tau_i}{2} L \quad (1.4)$$

$$J^\mu = Q^* \bar{\sigma}^\mu \frac{1}{3} Q + u^{c*} \bar{\sigma}^\mu \left(-\frac{4}{3}\right) u^c + d^{c*} \bar{\sigma}^\mu \frac{2}{3} d^c + L^* \bar{\sigma}^\mu (-1) L + e^{c*} \bar{\sigma}^\mu (2) e^c \quad (1.5)$$

and one can easily check the group structure of quarks and leptons given in table 1.1.

As is well known, the electroweak gauge symmetry is not observed in nature. In the SM, it is assumed to be spontaneously broken in the Higgs sector, after the Higgs potential develops a nonzero minimum and the Higgs field h picks up an $SU(2)_L \times U(1)_Y$ symmetry breaking vacuum. However, one $U(1)$ generator still annihilates the vacuum. Denoted by Q , it satisfies the operator equation of the form (1.1), giving rise to electromagnetism. Consequently, the familiar electromagnetic current

$$J_{em}^\mu = J_{i=3}^\mu + J^\mu/2 \quad (1.6)$$

remains conserved.

The mass eigenstates of the gauge fields are determined by the mixings

$$\begin{pmatrix} Z^0 \\ A \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} W_3 \\ B \end{pmatrix} \quad (1.7)$$

and

$$W_\pm = \frac{1}{\sqrt{2}}(W_1 \mp i W_2). \quad (1.8)$$

Then the electroweak part of (1.2) becomes

$$\mathcal{L}_{int}^{ew} = -\frac{e}{2\sqrt{2}\sin\theta_W} [W_+^\mu J_{-\mu} + W_-^\mu J_{+\mu}] - \frac{e}{2\cos\theta_W \sin\theta_W} J_{NC}^\mu Z_\mu^0 - e J_{em}^\mu A_\mu \quad (1.9)$$

with

$$\sin\theta_W = \frac{g'_1}{\sqrt{g_1'^2 + g_2^2}}, \quad (1.10)$$

$$e = g'_1 \cos\theta_W = g_2 \sin\theta_W, \quad (1.11)$$

$$J_{\pm} = 2(J_1 \mp iJ_2) \quad (1.12)$$

and

$$J_{NC} = 2[J_3 - \sin^2\theta_W J_{em}]. \quad (1.13)$$

In the effective theory with heavy W and Z^0 states integrated out, $\mathcal{L}_{int}^{ew}$ of eq.(1.9) leads to the effective lagrangian of the weak interactions

$$\mathcal{L}_{Fermi} = \frac{G_{\mu}}{\sqrt{2}} [J_+^{\mu} J_{-\mu} + \rho J_{NC}^{\mu} J_{NC\mu}], \quad (1.14)$$

where

$$\frac{G_{\mu}}{\sqrt{2}} = \frac{e^2}{8 \sin^2\theta_W M_W^2} \quad (1.15)$$

and

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2\theta_W}. \quad (1.16)$$

The mechanism of the $SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$ symmetry breaking has to guarantee that $\rho \cong 1$ since the analysis of the neutral current experiments is very precise and places a strong constraint [2]

$$\rho_{exp} = 1.0012 \pm 0.0031. \quad (1.17)$$

The SM naturally satisfies the above limit by placing the Higgs field into a doublet of $SU(2)_L$. In fact, with the vacuum expectation value

$$\langle h \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}, \quad (1.18)$$

the gauge boson masses at tree level are

$$M_W^2 = \frac{1}{4}g_2^2 v^2 \quad (1.19)$$

and

$$M_Z^2 = \frac{1}{4}(g_1'^2 + g_2^2)v^2, \quad (1.20)$$

which, together with the condition (1.10), automatically yields $\rho^{tree} = 1$. Loop corrections both in the SM, and in any of its extensions, modify the value of the ρ parameter and provide an important test of the theory (see next chapter).

In addition to the interactions already described, the gauge symmetry allows for non-trivial Yukawa interactions of the form

$$\mathcal{L}_{Yuk}^{(SM)} = Q_i \tilde{h} \lambda_{ij}^u u_j^c + Q_i h \lambda_{ij}^d d_j^c + L_i h \lambda_{ij}^e e_j^c + h.c. , \quad (1.21)$$

where λ 's are the 3×3 Yukawa matrices operating in the family space, $\tilde{h} = i\sigma_2 h^*$ is an $SU(2)_L$ doublet, conjugated to h , and the contraction of the $SU(2)$ indices is assumed but not explicitly shown¹. In the broken phase these terms generate mass matrices for the fermions,

$$m_{ij}^f = \frac{v}{\sqrt{2}} \lambda_{ij}^f, \quad (1.22)$$

($f = e, d, u$), and hence, in the SM, the Yukawa matrices are solely responsible for the observed hierarchy in the quark and lepton masses. The mass matrices are diagonalized by a bi-unitary transformation of the form

$$V_f^{L*} m^f (V_f^{R*})^\dagger = m_{diag}^f. \quad (1.23)$$

Matrices $V_f^{L,R}$ transform the electroweak interaction eigenstates into the corresponding mass eigenstates, as shown in equations (B.19 - B.20). This transformation leaves

¹Our convention for $SU(2)$ contraction is defined in eq.(1.25) in this section.

the fermionic kinetic energy terms and the currents J_{em}^μ , J_{NC}^μ and J_a^μ , unaffected, but it modifies the weak charged currents J_+ and J_- adding the inter-generational Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix

$$V_{CKM} = V_u^L (V_d^L)^\dagger \quad (1.24)$$

into the definition of these currents. The CKM matrix is unitary and in the case of three generations it is completely determined by the magnitude of three mixing angles and by one phase.

In summary, Yukawa interaction terms, eq.(1.21), contain a large number of new, independent constants, undetermined by the theory. However, not all of them are observables. After the diagonalization the matrices V_u^R , V_d^R and V_e^R drop completely out of the theory and their left counterparts enter only through the CKM matrix. Thus, utilizing unitarity, we are left with 13 independent parameters in this sector. These include nine fermion masses and four independent entries of the CKM matrix.

Let us now summarize the remarkable achievements of the SM :

- It agrees with all the data of elementary particle physics. Among the especially remarkable, and non-trivial successes are e.g. the agreement with the precise measurements sensitive to one loop effects (like e.g. the detailed measurements at the Z peak); the perturbative QCD agreement with the observed jet cross-sections ranging over more than ten orders in magnitude; the absence of the flavor changing neutral currents (FCNC) at tree level, naturally suppressing FCNC to the level required by experiment; the potential for the explanation

of the observed CP violation in the K system through a non-removable complex phase in the CKM matrix; separate (perturbative) conservation of baryon number (B) and lepton number (L) in agreement with the non-observation of B and L violating processes; etc.

- It is a gauge theory. As such, once the particle content is specified, all the interactions of the spin $\frac{1}{2}$ and 1 fields are rigidly determined in a unique way prescribed by the gauge symmetry.
- It is a renormalizable theory. The $SU(2)_L \times U(1)_Y$ gauge symmetry is broken spontaneously, and that is the only way known how to reconcile renormalizability with the requirement that the gauge bosons, mediating the weak interactions, be massive. In addition, each family is anomaly free.
- The Higgs sector of the SM can be limited to an extremely economical choice. Just one Higgs doublet can simultaneously solve many problems: it spontaneously breaks the gauge symmetry, it gives mass to the W and Z bosons in a way which naturally explains why the ρ parameter is very close to one, and it gives masses to the fermions.

The unsatisfactory features of the SM, together with the reasons why one should try to look beyond it, can be summarized as follows:

- It does not represent a unified description of the fundamental interactions. Even the weak and electromagnetic interactions are not unified but from the symmetry point of view just mixed to another pair of independent interactions. Three gauge couplings are completely unrelated with unexplained different values.
- In addition to the three gauge couplings there are 13 unexplained, completely free parameters in the fermionic sector (9 masses and 4 entries in the CKM mixing matrix), and two parameters determining the Higgs sector - too many free parameters for the final fundamental theory. The hierarchy in the fermionic sector shows a clear pattern, yet the SM lacks any explanation of its origin.
- The presence of the scalar Higgs particle at the scale of $M_W \approx M_Z \approx 100\text{GeV}$ poses probably the most pressing theoretical problem: if the Higgs state is a bound state, why cannot we observe more signs of the binding (strong) force in the experimental data obtained at the comparable energies? And if the Higgs state is an elementary scalar, how can one avoid an extreme fine tuning to separate the Higgs mass (and consequently the electroweak symmetry breaking scale) from the Planck scale? The two scales are, in reality, over 16 orders in magnitude apart, while the quadratic mass corrections to the scalar mass would naturally make them very close to each other !
- The SM provides no explanation for the electric charge quantization. In fact, the electric charge of the quarks could be of any value, leaving the neutrality of the atoms, and the macroscopic matter around us completely unexplained.

- The fermionic assignment into the group multiplets is without any deeper principle. For instance, why do the lepton and quark representations choose fundamental representations only? Why does triangle anomaly cancel out as a miracle?
- Massless neutrino is in the theory with the use of a brute force: without any symmetry principle we rejected the presence of a singlet state ν^c .
- In the same way there is no deeper (symmetry) principle which would explain the baryon and lepton number conservations.
- There's no explanation why we have more families.
- Gravity is not incorporated into the SM.

In the next section we describe how grand unified theories shed some light on all but the last two of these shortcomings.

1.3 Building up Unification Picture

1.3.1 Gauge Coupling Unification at Very High Scale

The idea of the unification of the $SU(3)_c \times SU(2)_L \times U(1)_Y$ was born even before the Standard Model became well established. Already in 1974, it was noticed that the three running gauge couplings, when evolved to higher scales, seem to meet at a single point, and $SU(5)$ was identified as a minimal group describing the single unified gauge interaction [3].

Historically, this work represents a first successful prediction of an independent and completely undetermined SM parameter. (Taking α_{em} and α_s as inputs, $\sin\theta_W$ was

predicted within a few per cent.) Moreover, the $SU(5)$ group provides an explanation for the observed charge quantization. Electric charge is quantized since it is a traceless generator of the group which places quarks and leptons into common multiplets. A factor $1/3$ for quarks appears naturally from the number of colors.

From the very beginning, however, it was realized that the unification takes place at an unprecedented high energy of about 10^{16}GeV , leaving a Grand Desert down to the electroweak scale 10^2GeV . It is interesting to note that this picture assumes that there are no new substructures of the known matter particles in this broad energy range. With string theory at 10^{18}GeV in mind, it is plausible that the known quarks and leptons are actually the field-theoretical concepts derived directly as the massless modes of the string. This gives a new twist to our understanding of matter: the structure of matter may not resemble onion-like layers with always new substructure underneath the objects declared “elementary” lately. The chain “molecules - atoms - atomic nuclei - protons and neutrons - quarks and leptons” may have an end. Much work has to be done though, before the details of such an end become clear, because the energy scale is so high that any direct experimental investigation, probing this region, is not even thinkable.

Moreover, the idea of the Grand Desert implies that the potential for the discovery of new elementary particles in this energy interval is very limited: if there indeed are any new particles there has to be a whole set of them, fitting into a complete $SU(5)$ multiplet, in order not to change the slopes of the gauge couplings in the “desert”.

Figure 1.1: One loop evolution of the gauge couplings within the Standard Model. Here $\alpha_1 \equiv (5/3)\alpha'_1$ where α'_1 is constructed from the square of the g'_1 coupling constant introduced in the previous section on the Standard Model. The relative width of each line reflects current experimental uncertainties.

1.3.2 Why Supersymmetry as Ingredient ?

More recently, the precise measurements at the Z peak showed that, within the Standard Model, the gauge couplings do not merge exactly [5] (see figure 1.1). The departure from the unification point is significant, since when the exact unification is assumed and α_{em} and $\sin\theta_W$ are taken as the inputs at the Z scale, the predicted value for $\alpha_s(M_Z) \approx 0.7$ falls short of the measured value by about eight standard deviations. However, it was found that within the supersymmetric (SUSY) extension

Figure 1.2: One loop evolution of the gauge couplings within the Minimal Supersymmetric Standard Model (MSSM), assuming supersymmetric extension right from the electroweak scale 90GeV. Supersymmetric thresholds at this scale are included. As in Fig.1.1, $\alpha_1 \equiv (5/3)\alpha'_1$ where α'_1 is constructed from the square of the g'_1 coupling constant introduced in the previous section on the Standard Model. The relative width of each line reflects current experimental uncertainties.

of the SM, when the β -functions are modified because of the presence of the SUSY partners [4], the unification of the three gauge couplings is restored (see figure 1.2).²

At the same time, incorporating supersymmetry into the theory solves the problem of the stabilization of the gauge hierarchy across the Grand Desert. In supersymmetric

²SUSY changes the slopes in agreement with the note at the end of the previous subsection, since the SUSY partners to gauge bosons, and the additional Higgs particles (see the next subsection) do not come in complete SU(5) multiplets. Obviously, focusing the running couplings into the unification point after adding so many new particles, which fall into incomplete SU(5) representations, can still be a plain accident, but, in general, defocusing would be more likely to expect.

extensions of the Standard Model the Higgs particle is identified as an elementary scalar (and not *e.g.*, as a condensate). Radiative corrections to its mass are controlled by supersymmetry and amount to zero in the strict supersymmetric limit[6]. In this case, bosonic loops cancel exactly with fermionic loops, leaving the scalar mass at the value where it was once set. We know that, in real life, supersymmetry is broken and this property will not hold anymore. Yet, when SUSY is broken explicitly by the soft terms (mass terms of the SUSY particles, and bi- and tri-linear scalar couplings - see next subsection), the quadratic divergencies of bosonic and fermionic loops still cancel out [7] and only much milder logarithmically divergent terms are present. As a result, the Higgs mass, and the W and Z masses, are all protected and the scale of the electroweak symmetry breaking is secured, even if it is far away from the unification scale. In such scenario, the scale of the electroweak symmetry breaking becomes related to the scale of the SUSY breaking terms. Although elegant, such a solution to the hierarchy problem raises, in turn, new questions about the mechanism of supersymmetry breaking and the origin of the soft SUSY breaking terms. In any case, the vast difference between the Z scale and the unification scale becomes stabilized, but its origin remains unexplained.

In summary, supersymmetry removes the fine tuning, otherwise needed to set the gauge hierarchy, improves the ultraviolet behaviour of the theory, and keeps the gauge coupling unification alive. In addition to that, it couples perturbatively to the SM sector of the theory. This means that at the electroweak scale, if still below the SUSY threshold, the superpartners will change the SM predictions only via small loop effects, thus providing an order parameter (soft mass terms) which can couple the SUSY sector to the Standard Model as weakly as desirable by the experimental

limits. All these appealing features make SUSY a number one candidate for physics beyond the Standard Model.

1.3.3 Minimal Supersymmetric Standard Model with soft SUSY breaking terms

In section 1.2 we described the basic features of the Standard Model. For the sake of completeness we sketch here briefly its supersymmetric extension [8, 9], the Minimal Supersymmetric Standard Model (MSSM).

In MSSM, each SM particle acquires its super-partner. In addition to the SM gauge bosons, spin-1/2 gauginos are introduced, and the SM fermions are paired with their scalar partners - sfermions. The gauge structure in the SUSY sector follows exactly the gauge structure of the SM. Supersymmetry requires each matter multiplet appearing in the superpotential to fall into a left-handed chiral representation. Consequently, in order to give mass to all the quarks there have to be two different Higgs doublets.³ In our notation $\hat{H}_1 = (\hat{H}_1^0 \hat{H}_1^-)$ gives mass to the *down*-type quarks and charged leptons, and $\hat{H}_2 = (\hat{H}_2^+ \hat{H}_2^0)$ gives mass to the *up*-type quarks. The minimal superpotential is then, basically, determined by the Yukawa terms, eq.(1.21), since the gauge symmetry (together with the particle assignment into the gauge multiplets) fixes the rest. The superpotential reads

$$W = \epsilon_{ij} \hat{Q}^j \lambda^u \hat{u}^c \hat{H}_2^i + \epsilon_{ij} \hat{Q}^i \lambda^d \hat{d}^c \hat{H}_1^j + \epsilon_{ij} \hat{L}^i \lambda^e \hat{e}^c \hat{H}_1^j + \mu \epsilon_{ij} \hat{H}_1^i \hat{H}_2^j, \quad (1.25)$$

where the last term is necessary to give mass to the higgsinos. In our conventions, the 2×2 antisymmetric matrix ϵ_{ij} is defined by $\epsilon_{12} = -1$, and the hats distinguish

³Two independent Higgs doublets are also required for the triangle anomaly cancellation.

superfields from their component fields. λ 's are the Yukawa matrices introduced in (1.21).

In this work we will assume that SUSY is broken in the hidden sector and that it is communicated to the visible sector at the messenger scale, which is close to or above the unification scale. Hence in the visible sector below the messenger scale SUSY is broken explicitly by the soft terms ⁴

$$\begin{aligned}
-\mathcal{L}_{soft} = & \tilde{Q}^\dagger \mathbf{m}_Q^2 \tilde{Q} + \tilde{u}^c \mathbf{m}_u^2 \tilde{u}^{c\dagger} + \tilde{d}^c \mathbf{m}_d^2 \tilde{d}^{c\dagger} + \tilde{L}^\dagger \mathbf{m}_L^2 \tilde{L} + \tilde{e}^c \mathbf{m}_e^2 \tilde{e}^{c\dagger} \\
& + m_{H_1}^2 H_1^\dagger H_1 + m_{H_2}^2 H_2^\dagger H_2 \\
& + \epsilon_{ij} \tilde{Q}^j \mathbf{A}^u \tilde{u}^c H_2^i + \epsilon_{ij} \tilde{Q}^i \mathbf{A}^d \tilde{d}^c H_1^j + \epsilon_{ij} \tilde{L}^i \mathbf{A}^e \tilde{e}^c H_1^j + B\mu \epsilon_{ij} H_1^i H_2^j \\
& - \frac{1}{2} M_3 \lambda^a \lambda^a - \frac{1}{2} M_2 \lambda^i \lambda^i - \frac{1}{2} M_1 \lambda \lambda.
\end{aligned} \tag{1.26}$$

λ fields denote Majorana gauginos (as before $a=1,2,\dots,8$ represents the $SU(3)_c$ index, and $i=1,2,3$ stands for the $SU(2)_L$ index). Squarks and sleptons are denoted as their SM counterparts, with the additional *tilde* mark above. Their corresponding mass terms are 3×3 matrices in the family space. $\mathbf{A}$'s are the trilinear soft breaking matrices of dimension 1 and $B\mu$ is the dimension 2 coefficient of the bilinear soft breaking term⁵.

Throughout this work the following boundary conditions at the unification scale M_G will be applied:

⁴Note the sign at the gaugino terms: it guarantees that we get the same signs for gluinos, charginos and neutralinos as Haber and Kane [9] when using their conventions on the way to obtain the physical fermionic states. The defining equation (1.26) is, however, missing in [9]

⁵Alternatively, B can be understood as a dimension 1 coefficient multiplying the superpotential parameter μ . Since unconstrained at the GUT scale, and multiplicatively renormalized down, the value of B or $B\mu$ can be conveniently picked up *e.g.*, at the Z scale to fix the electroweak symmetry breaking. In this work we will use $B\mu$ as a dimension 2 parameter.

- universal squark and slepton masses

$$m_Q^2|_{M_G} = m_u^2|_{M_G} = m_d^2|_{M_G} = m_L^2|_{M_G} = m_e^2|_{M_G} = m_0^2 \cdot 1_{3 \times 3}; \quad (1.27)$$

- universal gaugino mass parameters

$$M_3|_{M_G} = M_2|_{M_G} = M_1|_{M_G} = M_{1/2}; \quad (1.28)$$

- and the A-matrices proportional to the corresponding Yukawa matrices

$$\begin{aligned} A^u|_{M_G} &= A_0 \lambda^u, \\ A^d|_{M_G} &= A_0 \lambda^d, \\ A^e|_{M_G} &= A_0 \lambda^e. \end{aligned} \quad (1.29)$$

- Finally, we introduce non-universal Higgs masses

$$\begin{aligned} m_{H_1}^2|_{M_G} &\neq m_0^2, \\ m_{H_2}^2|_{M_G} &\neq m_0^2, \end{aligned} \quad (1.30)$$

which are, in general, different from each other. Note that if the messenger scale of SUSY breaking is not M_G then our analysis is not completely self-consistent. In any complete SUSY grand unified theory (GUT) defined up to an effective cut-off scale $M > M_G$, the interactions above M_G will renormalize the soft breaking parameters. This will, in general, split the degeneracy of squark and slepton masses at M_G even if they are degenerate at M . However, bounds on flavor changing neutral current processes severely constrain the magnitude of possible splitting. Thus these corrections must be small. In addition, in theories where SUSY breaking is mediated by gauge exchanges with a messenger scale below (but near) M_G , the present analysis is

expected to apply unchanged, since in this case squarks and sleptons will be nearly degenerate at the messenger scale. The Higgs masses, on the other hand, are probably dominated by new interactions which also generate the μ -term in the superpotential, eq.(1.25). It is thus plausible to expect the Higgs masses to be split and independent of squark and slepton masses. The parameter A_0 could also be universal at the messenger scale, instead of the unification scale M_G , and the same comments apply to it as to the squark and slepton mass parameters.

1.3.4 Radiative Electroweak Symmetry Breaking

The simple relations between the soft SUSY breaking terms are not valid below the scale M_G . The renormalization group equations (RGEs), which run these terms down, break the universal pattern in eq's (1.27)-(1.29).⁶ An amazing property of the RGEs emerges if the top quark is heavy (as observed in experiment). In such a case, the large top Yukawa coupling drives the running Higgs mass parameter $m_{H_2}^2$ negative with the potential for triggering the radiatively induced electroweak symmetry breaking at a lower scale[10]. This dependence on scale is logarithmic, and therefore, crossing the Grand Desert and breaking the electroweak symmetry many orders of magnitude below the GUT scale turns into a possible scenario. Thus the simple pattern of eq's (1.27)-(1.30), expected to be initiated by a SUSY breakdown in some $SU(3)_c \times SU(2)_L \times U(1)_Y$ singlet states, is powerful enough to tune the vacuum expectation values (vev's)

$$\langle H_1 \rangle = \begin{pmatrix} v_d \\ 0 \end{pmatrix}, \quad (1.31)$$

$$\langle H_2 \rangle = \begin{pmatrix} 0 \\ v_u \end{pmatrix} \quad (1.32)$$

⁶The 2-loop RGEs, used in this study, are presented in appendix A.

of the scalar Higgs fields to guarantee that at the scale $\mathcal{O}(100\text{GeV})$

$$v \equiv \sqrt{2(v_u^2 + v_d^2)} \approx 246\text{GeV}, \quad (1.33)$$

as deduced from the gauge boson masses M_W and M_Z determined from experiment. Note a nice property of the MSSM Higgs sector that, with v given by the eq.(1.33), the SM relations (1.19)-(1.20) are left unchanged.

In the broken phase, there are three Goldstone modes G^+ , G^- and G^0 , eaten by the gauge bosons, leaving five physical degrees of freedom in the scalar Higgs sector. These include a charged Higgs state H^+ , a neutral, CP -odd state (i.e., a pseudoscalar) A^0 and two neutral, CP -even states H^0 and h^0 , distinguished by different masses. It is customary to define the angle β as the mixing angle in the charged Higgs sector:

$$\begin{pmatrix} (H_1^-)^* \\ H_2^+ \end{pmatrix} = \begin{pmatrix} \sin\beta & -\cos\beta \\ \cos\beta & \sin\beta \end{pmatrix} \begin{pmatrix} H^+ \\ G^+ \end{pmatrix}, \quad (1.34)$$

and the angle α as the mixing angle in the CP -even sector :

$$H_1^0 = \frac{1}{\sqrt{2}} (v \cos\beta + H^0 \cos\alpha - h^0 \sin\alpha + i A^0 \sin\beta - i G^0 \cos\beta), \quad (1.35)$$

$$H_2^0 = \frac{1}{\sqrt{2}} (v \sin\beta + H^0 \sin\alpha + h^0 \cos\alpha + i A^0 \cos\beta + i G^0 \sin\beta). \quad (1.36)$$

The quark and charged lepton mass matrices become

$$\mathbf{m}_{ij}^u = v_u \lambda_{ij}^u, \quad (1.37)$$

$$\mathbf{m}_{ij}^d = v_d \lambda_{ij}^d, \quad (1.38)$$

$$\mathbf{m}_{ij}^e = v_d \lambda_{ij}^e. \quad (1.39)$$

It is interesting to note that the observed hierarchy in the third generation masses $M_{top}/M_{bottom} \approx 50$ may not have its origin in the Yukawa coupling hierarchy $\lambda_{33}^u \gg$

λ_{33}^d , as is required in the SM (compare with eq.(1.22)). In the MSSM, these Yukawa entries may be about the same, with the mass hierarchy coming from the Higgs vevs⁷:

$$\tan\beta = \frac{v_u}{v_d} \approx 50. \quad (1.40)$$

It is a simple exercise to derive the conditions for the electroweak symmetry breaking from the (tree level) Higgs potential at the SUSY breaking scale (see appendix B). In the minimum of the potential, the H^0 and h^0 tadpoles have to drop out, leading to the conditions

$$-\frac{1}{8} (g_1'^2 + g_2'^2) v^2 (\sin^2\beta - \cos^2\beta) + m_{H_1}^2 + \mu^2 = B\mu \tan\beta, \quad (1.41)$$

$$+\frac{1}{8} (g_1'^2 + g_2'^2) v^2 (\sin^2\beta - \cos^2\beta) + m_{H_2}^2 + \mu^2 = B\mu \frac{1}{\tan\beta}, \quad (1.42)$$

where all quantities are evaluated at the SUSY breaking scale $\mathcal{O}(100\text{GeV})$. In the top-down approach, adopted in our study, these two equations determine v and $\tan\beta$.

The tree level pseudoscalar mass

$$m_A^2 = m_{H_1}^2 + \mu^2 + m_{H_2}^2 + \mu^2 = B\mu \left(\tan\beta + \frac{1}{\tan\beta} \right) \quad (1.43)$$

and the tree level Z boson mass, eq.(1.20), can be substituted into these equations to eliminate the less intuitive parameters. Note, that after subtracting the two equations one obtains a simple tree level relation between the Higgs mass parameters and the masses M_Z and m_A :

$$m_{H_1}^2 - m_{H_2}^2 = (m_A^2 + M_Z^2) \frac{\tan^2\beta - 1}{\tan^2\beta + 1}. \quad (1.44)$$

⁷In general, the top-bottom hierarchy in the MSSM could have its origin either in hierarchy in the corresponding Yukawa couplings, or in the Higgs vacuum expectation values, or partly in both of these effects, like *e.g.*, for $\lambda_{33}^u/\lambda_{33}^d \approx 7$ and at the same time $v_u/v_d \approx 7$. However, we show in the next subsection that there is a theoretical prejudice against the third possibility.

1.3.5 Yukawa Unification

Next in the row of the phenomenological successes of supersymmetric unification is the correct $m_b - m_\tau$ relation. A simple $SU(5)$ picture implies that these two masses (or the corresponding Yukawa couplings) are the same at the unification scale. When renormalized down, they differ significantly because m_b picks up large corrections due to, primarily, strong interactions. Amazingly, it was found that the renormalization effects can make up for the entire difference between $m_b(m_b)$ and M_τ ⁸.

At first, this effect was discovered in the framework of simple non-supersymmetric gauge unification assuming that the top quark is light[11]. Later [12], it was found that in the supersymmetric extension, where the unification occurs at a higher scale than expected in [11], the color interactions of the b quark make the predicted difference between m_b and M_τ even larger than found in experiment. But, remarkably, when the top quark was assumed heavy, with the Yukawa coupling of about one, and with the two MSSM Higgs doublets, the correct $m_b - M_\tau$ difference was re-established again [13].

This observation suggested that the third generation is special and that its Yukawa couplings may be important in driving the GUT scale couplings to the low scale. Hence, it is of great interest what can be said about the top quark mass from the simple $SU(5)$ relation $m_b|_{M_G} = m_\tau|_{M_G}$. This issue was studied in the works by many authors [14],[15] before the top quark was discovered. The resulting predictions are smeared out by the inaccuracy in the experimental values of $\alpha_s(M_Z)$ and $m_b(m_b)$ but, in general, they can be sketched as in figure 1.3. Here we mark the points

⁸Throughout this work we will always denote pole masses with a capital letter and running masses with a lower case letter.

Figure 1.3: An example of a constant m_b contour in the $\tan\beta$ - m_t plane assuming $m_b = m_\tau$ at the GUT scale.

in the $\tan\beta$ - m_t plane which agree with the low-energy values of b and τ masses, and which allow for the exact $m_b - m_\tau$ unification (to one loop) at the GUT scale (the latter being self-consistently determined by the gauge coupling unification). The experimentally allowed range for the top quark mass $m_t(m_t) = 167 \pm 6\text{GeV}$ is also shown in the figure⁹. Before the top quark was discovered, this figure suggested that it is naturally heavy. Once detected, and found, indeed, heavy, an important piece of the assumed SUSY GUT phenomenology got into the right place in the mosaic.

⁹More details on the experimental value of the top quark mass and other observables are in the next chapter. Here we used the common wisdom, that the difference between the measured pole mass M_t and the undressed, current mass $m_t(m_t)$ can be approximated by 8GeV to a sufficient precision.

At present, when the mass of the top quark is known, the significance of figure 1.3 can be re-interpreted in terms of the unknown parameter $\tan\beta$. The figure suggests that the unification scenario explains the origin of the *top – bottom* hierarchy either by the hierarchy in the Higgs vev's or by the hierarchy in the corresponding Yukawa couplings. Thus one gets a “large $\tan\beta$ scenario” or a “low $\tan\beta$ scenario”. In the first case, $\tan\beta \simeq 50$ and necessarily $\lambda_t|_{M_Z} \simeq \lambda_b|_{M_Z}$, which then implies that the unified Yukawa coupling for the b and τ has also to be large and about the same as the top quark Yukawa coupling at the GUT scale, paving the way for the $\lambda_{33}^u = \lambda_{33}^d = \lambda_{33}^e \approx 1$ unification. In the second case, $\tan\beta \simeq 1$ and the top quark Yukawa coupling is the only large coupling in the Yukawa sector. These features are not sensitive to the input values of $m_b(m_b)$ and $\alpha_s(M_Z)$.

1.3.6 SO(10) SUSY GUTs

Within $SU(5)$, there is no symmetry reason why the top quark Yukawa coupling should be unified with the bottom and tau Yukawa couplings. Yet, the fact that they are equal at the unification scale if $\tan\beta$ is large, may be viewed as a hint for a higher symmetry. The simplest at hand is $SO(10)$.

$SO(10)$ is a simple group of rank 5, hence it possesses one more diagonal generator than $SU(5)$. The $SU(5)$ group is actually its subgroup and, therefore, $SO(10)$ may successfully incorporate the simple picture of the gauge coupling unification described before. The reason, why $SO(10)$ is theoretically very appealing, is that the quarks and leptons of one family fit nicely into a sixteen-dimensional representation of $SO(10)$. (The last spot in the **16** multiplet is assumed to be taken by the singlet neutrino ν^c .) Such an assignment

- naturally explains the family structure of the SM,
- solves the mystery why the triangle anomaly is avoided within the SM, and
- provides a framework for the Yukawa coupling unification which includes all fermions of one family.

Remarkably, this kind of economy in $SO(10)$ extends beyond the fermionic multiplets. It is also the two Higgs doublets which can be placed (within their corresponding $SU(5)$ 5-plets) into a single 10-dimensional representation.

Thus $SO(10)$ is not just another higher gauge symmetry, but it is, in a sort of way, the minimal of many possible unification groups. Any higher group than $SO(10)$ contains right from the beginning many new, unobserved states mixed with the quarks and leptons in its complete multiplets, and (if not having $SO(10)$ as its subgroup) it becomes quite complicated to give the new particles mass at, or above, the unification scale. On the other hand, $SU(5)$, or any other group of rank 4 or 5, seems to be not high enough symmetry to provide predictions for the fermion masses and mixings; and the real models constructed so far require the introduction of so many independent initial parameters at the GUT scale, that, eventually, they lose any predictive power as far as the fermionic sector goes. Finally, the attempts to describe the GUT physics by a direct product of simple gauge symmetry groups seem to disregard the initial hint which had motivated our effort at the beginning - namely that the three MSSM gauge couplings appear to meet at the same point.

First attempts to utilize the minimality of $SO(10)$ to address the problem of fermion masses and mixings were made in [16]. The empirical relation between the

masses of the two lighter generations

$$\frac{m_s}{m_d} \approx \frac{1}{9} \frac{m_\mu}{m_e} \quad (1.45)$$

was assumed to result from an additional (Clebsch) factor of 3 at the 22 entry in the λ^c Yukawa matrix, as had been suggested - for $SU(5)$ - by Georgi and Jarlskog[17]. The family hierarchy was proposed to be borne out of the higher-dimensional operators after the breakdown of the GUT symmetry; the idea going back to Froggatt and Nielsen[18].¹⁰ Systematic search over the possible sets of effective $SO(10)$ operators was performed in [20]. This analysis concentrated on the minimal number of the operators needed, and came to the conclusion that one has to have a set of at least four effective operators to describe the data correctly. The remaining entries in the Yukawa matrices were filled with zeros, unexplained within the analysis, but nevertheless motivated by the assumption of the additional (unspecified) family symmetries.¹¹

Another virtue of $SO(10)$ appears to be a natural solution to the doublet-triplet splitting problem in the Higgs multiplet, without adding new large representations (larger than the adjoint representation) just to get over this problem. Any SUSY GUT incorporating the gauge coupling unification potentially runs into problems with too fast proton decay caused by the higgsino states carrying color. These states mediate proton decay via a dimension 5 operator. Since suppressed only by one power of the effective colored Higgs mass, the presence of a dimension 5 operator makes this issue more sensitive than in the case of dimension 6 operators, induced by, *e.g.*, the heavy gauge bosons. Apparently, colored Higgs states have to get mass at the scale of the $SU(5)$ symmetry breaking, while their colorless partners should remain massless

¹⁰A more introductory review can be found in the series of lectures by S. Raby[19].

¹¹In this thesis (see chapter 4), we go back to the ADHRS models and evaluate quantitatively their performance.

to be able to provide for the electroweak symmetry breaking at the scale fourteen orders of magnitude lower. The mechanism suggested in [22] shows that the adjoint 45 representation of $SO(10)$ coupled to the two 10's containing Higgs fields does naturally the right job if it acquires vev in the $B - L$ direction. This trick cannot be applied to the case of the $SU(5)$ gauge group since the $B - L$ operator is no longer a group generator and the doublet-triplet splitting can be arranged only at the expense of fine tuning among two different vevs. A realization of the Dimopoulos-Wilczek mechanism in specific $SO(10)$ models [23] shows that satisfying the proton decay limit does not require an excessively heavy Higgs triplet. In fact, the mass suppressing the dimension 5 operator turns out to be the effective triplet mass which gets large by see-saw mechanism induced by a Higgs doublet (contained in a new 10) with a mass from *below* the GUT scale.

For completeness, let's mention also the objections against $SO(10)$ SUSY GUTs. The first one originates from the clue provided by string theory. From group theory we know that in order to break $SO(10)$ into a lower-rank SM gauge group, any $SO(10)$ SUSY GUT has to contain a state in the adjoint representation. However, when generating GUTs from string theory, adjoint representations can be obtained only in the Kac-Moody algebras of higher levels (see the review article [24] and the references therein). It is proved to be impossible with the level 1 algebra [25], and it looks that it is very difficult with the level 2 algebra [26], demanding three chiral 16 dimensional representations. However, realistic models have been constructed recently with the level 3 algebra [27]. Hence this disadvantage is of formal, technical character, and not a real shortfall.

The second objection against the *minimal* $SO(10)$ SUSY GUTs (*i.e.*, those with $\tan\beta$ large) is that, in order to obtain radiative electroweak symmetry breaking, a large amount of fine tuning in the soft SUSY breaking sector of the theory is necessary[28]. At present, it is difficult to judge how significant such an objection is. Soft masses originate in the hidden sector which we know almost nothing about. Moreover, the fine tuning issue can be looked at from various perspectives. If one accepts the naturalness criterion which states that “observable properties of a system should not be *unusually unstable* with respect to minute variations in the fundamental parameters”[29], the difference is made by the term “unusually”. The MSSM with large $\tan\beta$ is more sensitive to the minute variations in the GUT values of soft SUSY breaking parameters than the MSSM with low $\tan\beta$. Therefore, the MSSM in the large $\tan\beta$ regime is *unusually unstable*, when compared to the sensitivity averaged over the whole parameter space, which is available to the MSSM - the prescription suggested in [29]. However, when we adopt a point of view that models under investigation belong to the class of minimal $SO(10)$ SUSY GUT models, there is always the same (rather high) sensitivity in each of these models, and for any region in the parameter space. There are no less sensitive models, and consequently there are no *unusually unstable* models. All models are sensitive, and so this instability is normal, usual in this class of models. One finds the same situation as with the sensitivity of Λ_{QCD} to the GUT value of α_s . Small variations of $\alpha_s(M_{GUT})$, or equivalently α_{GUT} , lead to substantially different values of Λ_{QCD} but it does not make GUT models unnatural, since the same kind of sensitivity occurs in all GUT models.

1.3.7 Testing Different GUTs with Global Analysis to Find Bridges to String Theory

There are many SUSY GUT models based on the $SO(10)$ gauge group. When focused on the fermionic sector, some models also include $U(1)$ family symmetries, higher non-abelian horizontal symmetries, or a set of discrete symmetries in order to restrict Yukawa matrices. Even the minimal search by ADHRS has found nine different 23 operators and six different 22 operators leading to 54 different, viable GUTs.

Then there are theories based on different gauge groups. Some assume a simple unified gauge group like $SU(5)$, $SU(6)$, $E(6)$, *etc.*, some prefer semi-simple groups like *e.g.*, $SU(5) \times U(1)$, and some others do not require a grand unified theory at all and use just the SM gauge group $SU(3) \times SU(2) \times U(1)$ to try to explain the pattern in the low energy data. Many contain low energy supersymmetry, while there are still some, which do not.

One thing is certain, all of these theories cannot describe Nature. Nevertheless, many are consistent with the low energy data to within an order of magnitude. One may believe that, with our present knowledge of field theory, this is the best we can achieve today. This work assumes we can do better. We propose to evaluate the performance of different models based on the results of a global fit to all independent low energy data. With our analysis, models will be assigned a corresponding χ^2 value of the best fit. That can quantify how well the model works, and sometimes, maybe, it can lead to significant modifications or rejections of some models. As a separate advantage, such global top-down analysis can easily accomodate cross-correlations

between different observables which is often hard to implement in a bottom-up approach. Successful models will then completely determine low energy phenomena, and provide for the additional predictions of the observables to be measured.

In general, this kind of analysis is much more meaningful for the models where the number of initial parameters is not exceeding the number of low energy observables. In this case, the obtained χ^2 value can determine the confidence level of a model, dependent on the number of degrees of freedom. We would like to stress, that we believe that all the initial parameters have to be included in such counting, not excluding those which the low energy data are not *very* sensitive to. As explained in the previous subsection, the $SO(10)$ SUSY GUTs represent excellent starting grounds for such an analysis.¹² Thus minimality is one of the guiding principles in this thesis. Minimality, after all, seems to be a good guidance for the description of Nature. Hence, the main hypothesis of our work is

- that *a few* effective operators derived from a SUSY grand unified theory dominate the quantitative behaviour of fermion masses and mixing angles at the unification scale, and
- that, eventually, a more fundamental theory, incorporating Planck or stringy dynamics, will generate this grand unified theory as its effective field theory. The dominant effective operators in the Yukawa sector will thus help to identify the unifying field theory and serve as a bridge to the correct compactification in string theory.

¹²We show in this work that even with a number of soft SUSY breaking parameters, see eq's(1.27)-(1.30), these models may still contain a positive number of degrees of freedom.

As a result of a long-term work in this direction, a set of different models could be pinpointed as potential effective field theories to which the low-energy limits of string theory could match. In this thesis, we start this investigation with the most predictive $\text{SO}(10)$ SUSY GUTs.

CHAPTER 2

OBSERVABLES

2.1 List of Independent Observables at Low Energies

In this chapter we describe the low energy observables which enter the χ^2 analysis, their measured values and the associated experimental and/or theoretical errors, and finally the way how we compute them from the initial parameters at the GUT scale. These are the precision electroweak data for testing grand unified theories.

From the historical perspective, the gauge boson and fermion masses, and their mixing angles are now very well measured. In the fermionic sector it is clear that fermions come in well defined families with an amazingly simple regularity in the hierarchy of masses and mixing angles. There is a hope that by understanding especially this sector we can find the correct matching conditions at the string scale. Now seems an appropriate time to begin such an endeavor. It was only ten years ago that the top quark mass and the CKM angle V_{ub} were unknown; while V_{cb} and the weak mixing angle $\sin^2\theta_W$ were measured, but with large error bars. Today, as a result of both experimental and theoretical progress, all but one of the parameters of the Standard Model are known to much better accuracy, while the Higgs mass has yet to be measured.

In the global analysis, our χ^2 function is computed out of twenty low energy observables.¹³ We have

- 6 observables associated with the Standard Model gauge and electroweak symmetry breaking sectors — M_Z , M_W , G_μ , α_{EM} , $\alpha_s(M_Z)$, and ρ_{new} ;
- 13 observables associated with fermion masses and mixing angles — M_t , $m_b(M_b)$, $(M_b - M_c)$, m_s , m_d/m_s , Q^{-2} , M_τ , M_μ , M_e , V_{us} , V_{cb} , V_{ub}/V_{cb} , and J ¹⁴; and
- the branching ratio for $b \rightarrow s\gamma$,

where ρ_{new} includes the added contribution to the electroweak ρ parameter, eq.(1.14) from new physics beyond the SM, Q is the Kaplan-Manohar-Leutwyler ellipse parameter[30] relating u , d , and s quark masses and J is the Jarlskog CP violating invariant[31] (see more on these parameters below). For fermion masses and mixing angles we use those combinations of parameters which are known to have the least theoretical and/or experimental uncertainties. For example, while the bottom and top quark masses are known reasonably well, the charm quark mass is not known as accurately. On the other hand, heavy quark effective theory relates the mass difference $M_b - M_c$ between the bottom and charm quark pole masses to about 5% accuracy, as described in more detail below. We thus use this relation, instead of the charm quark mass itself, to test the theory. Note also, that mass parameters denoted with a capital letter M are defined as pole masses, while m_b , m_s , m_d , m_u are defined as the $\overline{MS}$ running masses. The three light quark running masses are evaluated at the scale 1 GeV.

¹³In addition, there is a special penalty incorporated into the χ^2 ; added if one of the experimental bounds on sparticle masses is violated. This gaurantees that we remain in the experimentally allowed regions of parameter space.

¹⁴In the actual χ^2 analysis J is replaced by the hadronic matrix element $\hat{B}_K$ as will be explained later in this chapter.

The traditional Standard Model parameter $\sin^2 \theta_W(M_Z)|_{\overline{MS}}$ is not listed among the observables. It can be understood as being replaced by the Fermi constant G_μ . We made this change since G_μ is extracted from the muon lifetime formula [32] with very little sensitivity to the particle content at the scale M_Z and the underlying theory at this scale. In fact, it could be defined completely in the context of the effective four-fermi contact interaction plus electromagnetism, and the remaining uncertainty of the order M_μ^2/M_W^2 would then appear in the quantities derived from G_μ . On the contrary, the experimental value of $\sin^2 \theta_W(M_Z)$ in the $\overline{MS}$ scheme is derived from the precision measurements at the Z peak. Loop effects play an important role in its precise determination and hence one has to specify the theory at M_Z and its particle content, and then calculate all relevant radiative corrections. Thus the underlying theory is an explicit input for the extraction of the experimental value of $\sin^2 \theta_W(M_Z)$ and, naturally, it is commonly taken to be the Standard Model (SM). However, we adopt an approach where the minimal supersymmetric extension of the Standard Model (MSSM) is matched directly to the low energy $SU(3)_c \times U(1)_{em}$ effective theory by integrating out the superpartners at the same time as the top quark and W/Z bosons (at a single scale M_Z); hence there is no room for the SM. This approach is particularly sensible if there are light SUSY particles (lighter than Z and W gauge bosons), such as charginos, neutralinos, pseudoscalar Higgs, etc., for then there is no energy regime where the SM dominates[33]. Although not among the observables contributing to the χ^2 function, the SM value of $\sin^2 \theta_W(M_Z)$ is displayed among the model predictions in chapter 5.

2.2 Low Energy Observables – Experimental Values

Let us now start with the experimental observables and their errors.

- $M_Z = (91.1863 \pm 0.0020)\text{GeV}$

has been determined by the LEP experiments[34]. The excellent precision, leading to the error of just 2.0MeV, is obtained by tracking down many delicate effects including the effects of tides on the rock where the accelerator is placed, water level in Lake Geneva, and return currents from the electric express trains passing close by the CERN laboratory[35].

- $M_W = (80.356 \pm 0.125)\text{GeV}$

has been determined as the weighted average of the recent[34] and older[36] measurements at Fermilab given in table 2.1. It is interesting to note that LEP2

Experiment	CDF 1996	D0 1996	CDF 1990	LEP2 1996
$M_W/1\text{GeV}$	80.41 ± 0.18	80.37 ± 0.15	79.91 ± 0.39	80.3 ± 0.5

Table 2.1: Experimental values of M_W and their corresponding errors.

will eventually determine M_W with the accuracy 40MeV, but as of summer'96 their data sample was too small to get their value with the accuracy below 400MeV[37] and their value is not included in the average.

- $G_\mu = (1.16639 \pm 0.00002) \times 10^{-5} \text{GeV}^{-2}$,

derived from the muon lifetime formula with the electromagnetic radiative corrections included[38].

- $\alpha_{em} = 1/137.036$,

as found in the quantum Hall effect[39].

- $\alpha_s(M_Z) = 0.118 \pm 0.005$.

The most precise determinations of $\alpha_s(M_Z)$, as reported at the summer '96 conferences[40, 41], are displayed in table 2.2. We use the central value quoted by Schmelling in Warsaw[40]. However, for the error we choose a conservative

Experiment	$\alpha_s(M_Z)$
DIS CCFR	$0.119 \pm 0.004 \pm 0.004$
$\tau \rightarrow \text{hadrons}$ CLEO	$0.117 \pm 0.003 \pm 0.003 \pm 0.006$
lattice	0.118 ± 0.003 0.116 ± 0.003
$Z^0 \rightarrow \text{hadrons}$ LEP1	0.120 ± 0.003

Table 2.2: Most precise determinations of $\alpha_s(M_Z)$ and their corresponding errors as quoted at the summer '96 conferences.

value as suggested by Webber[42] or Burrows[41]. This accounts for the fact

that the systematic errors of individual α_s measurements are significant and not completely under control, and thus caution is required when adding them together.

- $\rho_{\text{new}} = (-0.6 \pm 2.6) \times 10^{-3}$.

This value is obtained from Langacker's combined fits to the precision electroweak data, presented at recent workshops[43].

- $M_t = (175 \pm 6)\text{GeV}$,

as the weighted average over the CDF result $(176.8 \pm 6.5)\text{GeV}$ and D0 result $(169 \pm 11)\text{GeV}$ [44]. We do not include the value $(155^{+18}_{-13})\text{GeV}$ derived from the precision measurements at LEP1[37]. Although this value is remarkably in agreement with the direct measurements, it is derived within the framework of the Standard Model, which is replaced by the MSSM framework in our analysis. If accepted, this value would reduce the central value by 4GeV to 172GeV [35].

- $m_b(M_b) = (4.26 \pm 0.11)\text{GeV}$.

This is our estimate from a number of results quoted in [45].

- $M_b - M_c = (3.4 \pm 0.2)\text{GeV}$,

as determined from the masses of the B and D mesons,

$$M(B) = M_b + \Lambda_{QCD} + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{m_b}\right), \quad (2.1)$$

$$M(D) = M_c + \Lambda_{QCD} + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{m_c}\right), \quad (2.2)$$

in the framework of heavy quark effective theory[46]. Note that the uncertainty in the difference of these two masses is of the order $\Lambda_{QCD}^2/m_b - \Lambda_{QCD}^2/m_c$, much

lower than the uncertainty of the c quark mass itself $m_c(M_c) = (1.3 \pm 0.3)\text{GeV}$ [47].

- $m_s(1\text{GeV}) = (180 \pm 50)\text{MeV}$,

as in [47]. In the next chapters we also include a specific discussion about the changes in our results for much lower value of m_s ,

$$m_s(1\text{GeV}) = (120 \pm 25)\text{MeV} , \quad (2.3)$$

suggested by recent lattice calculations[48].

- $(m_d/m_s)|_{1\text{GeV}} = 0.05 \pm 0.015$;

from [49]. See also the following comments to Q^{-2} . Light quark mass ratios are known to a better accuracy than their masses themselves.

- $Q^{-2} = (2.03 \pm 0.20) \times 10^{-3}$.

Among the three light quarks there is one good relation, (reparametrization invariant and free of $O(m_q)$ corrections) which severely constrains any theory of fermion masses. This is the Kaplan-Manohar-Leutwyler ellipse given by

$$1 = \frac{1}{Q^2} \frac{m_s^2}{m_d^2} + \frac{m_u^2}{m_d^2}, \quad (2.4)$$

or

$$Q = \frac{\frac{m_s}{m_d}}{\sqrt{1 - \frac{m_u^2}{m_d^2}}}, \quad (2.5)$$

where Q is the ellipse parameter. The experimental value for Q is obtained from a weighted average of lattice results and a chiral Lagrangian analysis, with significant contributions from the violation of Dashen's theorem[50]. We follow Donoghue[49] and assign a conservative 10% error to the experimental

value of Q^{-2} . Note that m_d/m_s derived from chiral Lagrangian analysis is not free of first order quark mass corrections, and hence σ is much smaller for Q^{-2} than for m_d/m_s [49].¹⁵

- Masses of charged leptons and Cabibbo angle V_{us} .

These quantities have been very well measured for a long time. Their values are quoted in table 2.3, which summarizes our experimental input.

- $V_{cb} = 0.0392 \pm 0.003$,

as in [52]. Note that the central value went down and the error bars have been steadily decreasing in the last five years, making this mixing angle a very significant constraint.¹⁶

- $(V_{ub}/V_{cb}) = 0.08 \pm 0.02$,

from the inclusive b decays [55]. The value for V_{ub}/V_{cb} changed dramatically in 1992 from approximately 0.15 ± 0.05 to its present value. The errors were and continue to be dominated by theoretical model dependence.

- This spot is reserved for a CP -violating observable. From the theoretical point of view it would be a CP -violating phase ξ in the CKM matrix, or a Jarlskog parameter J . The latter is defined by the expression –

$$J = \text{Im}(V_{ud}V_{ub}^*V_{cb}V_{cd}^*) \approx |V_{cd}||V_{ub}/V_{cb}||V_{cb}|^2 \sin \xi. \quad (2.6)$$

¹⁵In addition, we do not constrain m_u/m_d independently, as suggested by Leutwyler[51], which requires additional input from an expansion in $1/N_{color}$. Instead we quote values for m_u/m_d among the predictions in chapter 5.

¹⁶Note that the actual value of V_{cb} is correlated with the values of m_b and m_c [46, 54]. We have not included this correlation in our analysis.

What is really measured in experiment is rather the $K^0 - \overline{K}^0$ mixing observable

$$\epsilon_K = (2.26 \pm 0.02) \times 10^{-3}. \quad (2.7)$$

The largest uncertainty in such a comparison, however, comes in the value of the QCD bag constant $\hat{B}_K$. We thus exchange the Jarlskog parameter J for $\hat{B}_K$ in the list of low-energy data we are fitting. Our theoretical value of $\hat{B}_K$ is defined as that value needed to agree with ϵ_K for a set of fermion masses and mixing angles derived from the GUT-scale. We test this theoretical value against the “experimental” value of $\hat{B}_K$, which is given in table 2.3. That value, together with its error estimate,

$$\hat{B}_K = 0.8 \pm 0.1 \quad (2.8)$$

is picked up from recent lattice calculations[56]. The same mean value and standard deviation for $\hat{B}_K$ were also used in a recent review article [57].

- Branching ratio $BR(b \rightarrow s\gamma) = (2.32 \pm 0.92) \times 10^{-4}$, as measured at CLEO[58]. In the absence of direct observations of superpartners, this process places a significant constraint on the soft SUSY breaking parameters.

The experimental values for all the observables (X_i 's) are given in table 2.3, together with the associated standard deviations σ_i entering into the well-known formula for the χ^2 function

$$\chi^2 = \sum_{i=1}^{20} \frac{(X_i^{th} - X_i^{exp})^2}{\sigma_i^2}. \quad (2.9)$$

Observable	Central value X_i^{exp}	σ_i
M_Z	91.186	0.46
M_W	80.356	0.40
G_μ	1.166×10^{-5}	1.2×10^{-7}
α_{em}^{-1}	137.04	0.69
$\alpha_s(M_Z)$	0.118	0.005
ρ_{new}	-0.6×10^{-3}	2.6×10^{-3}
M_t	175.0	6.0
$m_b(M_b)$	4.26	0.11
$M_b - M_c$	3.4	0.2
m_s	180	50
m_d/m_s	0.05	0.015
Q^{-2}	0.00203	0.00020
M_τ	1.777	0.0089
M_μ	105.66	0.53
M_e	0.5110	0.0026
V_{us}	0.2205	0.0026
V_{cb}	0.0392	0.003
V_{ub}/V_{cb}	0.08	0.02
$\hat{B}_K$	0.8	0.1
$B(b \rightarrow s\gamma)$	2.32×10^{-4}	0.92×10^{-4}

Table 2.3: Experimental observables and their associated standard deviations as they enter the χ^2 function.

Note, that some observables are known so well (see the experimental errors listed above), that we have to assign the theoretical instead of experimental error as their standard deviation in the χ^2 evaluation. This is true for M_Z , M_W , G_μ , α_{em} and the charged lepton masses. As shown in table 2.3, we chose a simple prescription: σ is taken to be either the experimental error or 0.5%, whichever is larger.¹⁷ This is because our theoretical calculation also introduces an error; the numerical solution

¹⁷In chapter 5 we investigate the sensitivity of our results to the magnitude of this theoretical uncertainty. We show, for a comparison, how the quality of the fit changes if the theoretical uncertainty were on the level of 0.1%.

to the renormalization group equations is computed with some limited precision and the values of all observables are calculated within the first orders of the perturbative expansion in couplings. Thus 0.5% represents a conservative simple estimate of the combined theoretical error assigned to each low energy theoretical output ¹⁸.

2.3 Low energy observables – computed values

In our analysis we consider the minimal supersymmetric standard model defined at a GUT scale M_G with, in all cases (but one), tree level GUT boundary conditions on gauge couplings and Yukawa matrices. In this one case, we include an arbitrary ϵ_3 parameter, given by

$$\epsilon_3 \equiv \frac{\alpha_3(M_G) - \tilde{\alpha}_G}{\tilde{\alpha}_G} \quad \text{with} \quad \tilde{\alpha}_G \equiv \alpha_1(M_G) = \alpha_2(M_G) \quad (2.10)$$

which parametrizes the one loop threshold correction to gauge coupling unification.¹⁹ Following the introductory remarks in the previous chapter, we also include 7 soft SUSY breaking parameters — an overall scalar mass m_0 for squarks and sleptons, a common gaugino mass $M_{1/2}$, the parameters A_0 , B and μ , and in addition we have allowed for non-universal Higgs masses, m_{H_u} and m_{H_d} ²⁰. Thus the number of arbitrary parameters in the effective GUT includes the 3 gauge parameters, 7 soft SUSY breaking parameters and n_y Yukawa parameters. The number n_y and the form of the Yukawa matrices are model dependent. Various models with different n_y are discussed in chapters 4 and 5.

¹⁸For G_μ we take σ to be 1%. This is to account for the additional error resulting from our neglecting one loop SUSY box and vertex corrections.

¹⁹ ϵ_3 is calculable in any complete SUSY GUT. It is also constrained somewhat by the bounds on the nucleon lifetime[59].

²⁰compare with the eq's (1.27)-(1.30)

The effective theory between M_G and M_Z is the MSSM. We use the two loop SUSY renormalization group equations (RGEs) for dimensionless parameters and one loop RGEs for dimensionful parameters to renormalize the GUT scale couplings down to M_Z . However we have checked that the corrections to our results obtained by using two loop RGE for dimensionful parameters[60] are insignificant. Full two loop RGEs used are stated in appendix A. When in the framework of the MSSM we use Dimensional Reduction regularization in the Modified Minimal Subtraction Scheme. This renormalization procedure will be abbreviated as $\overline{DR}$ in the following text.

When crossing the Z scale we match the MSSM to the non-supersymmetric $SU(3)_c \times U(1)_{em}$ gauge theory, i.e. our effective theory below the Z threshold is standard QCD and electromagnetism. In this effective theory we perform the calculations using Dimensional Regularization in the Modified Minimal Subtraction scheme. Quantities renormalized according to this prescription will appear with a standard subscript $\overline{MS}$ in the following formulas. Below the Z scale, three loop QCD renormalization group equations run quark masses and α_s down to lower scales. When this running crosses the b and c mass thresholds the number of flavors is reduced by one each time. Finally we end up with the three-flavor QCD RGEs as we arrive at the scale of 1 GeV, where the three light quark masses are evaluated. In addition, QED running is included to one loop precision. (For further details see [61].)

At M_Z we calculate complete one loop MSSM corrections to the tree level W and Z masses, eq's(1.19) and (1.20). Thus the W and Z pole masses are given by the formulae:

$$M_W^2 = \frac{1}{4} g_2^2 v^2 + \delta M_W^2, \quad (2.11)$$

$$M_Z^2 = \frac{1}{4} \left(\frac{3}{5} g_1^2 + g_2^2 \right) v^2 + \delta M_Z^2, \quad (2.12)$$

where δM_W^2 and δM_Z^2 are the complete one loop self-energies in the $\overline{DR}$ scheme, given in appendix B (see also ref's [62] and [33]). $g_1 = \sqrt{\frac{5}{3}}g'_1$ and g_2 are the $\overline{DR}$ gauge couplings evaluated at M_Z in the MSSM.

The Higgs vacuum expectation value v is an implicit function of the soft SUSY breaking parameters and gauge and Yukawa couplings. It is determined from the relations (1.41) and (1.42) by self-consistently demanding minimization of the tree level Higgs potential. The actual value for v is found by minimizing χ^2 .

The theoretical value of G_μ is calculated according to the formula (compare with the tree level eq.(1.15))

$$G_\mu = \frac{\pi \alpha(M_Z)}{\sqrt{2} M_W^2 \sin^2 \theta_W(M_Z)} \frac{1}{1 - \Delta \hat{r}_W^{MSSM}}. \quad (2.13)$$

The right-hand side of this equation is consistently evaluated in the $\overline{DR}$ scheme within the context of the MSSM. The electromagnetic structure constant α and $\sin^2 \theta_W$ are determined by the relations among the gauge coupling constants α_1 and α_2 :

$$\sin^2 \theta_W(M_Z) = \frac{\frac{3}{5}\alpha_1(M_Z)}{\frac{3}{5}\alpha_1(M_Z) + \alpha_2(M_Z)}, \quad (2.14)$$

$$\alpha(M_Z) = \alpha_2(M_Z) \sin^2 \theta_W(M_Z). \quad (2.15)$$

$\Delta \hat{r}_W^{MSSM}$ in (2.13) contains the corrections from the W self-energy, and vertex and box diagrams. We follow refs. ([33],[63]) closely, but not to all details: we don't include the vertex and box diagrams containing SUSY particles. These have a minor effect (see results in [33]) and we compensate for them with an additional 1/2% error on G_μ , as already mentioned in a previous footnote. In fact, we use a reverse procedure to the one introduced in [33], where the equation (2.13) is utilized to derive the MSSM value of $\sin^2 \theta_W$ in the bottom-up approach starting from the precise value of G_μ .

When crossing the Z threshold, gauge couplings are subject to two different kinds of corrections. That is to account for the states which are integrated out at this scale, and for the transition from the $\overline{DR}$ scheme to the $\overline{MS}$ scheme. First, when the SUSY particles, the top quark and the Z and W gauge bosons are integrated out , the threshold corrections to $\alpha_s|_{\overline{DR}}$ and $\alpha|_{\overline{DR}}$ read

$$\frac{\delta\alpha_s}{\alpha_s} = \frac{\alpha_s}{\pi} \left(\frac{1}{12} \sum_{i=1}^{12} \log \frac{m_{\tilde{q}_i}}{M_Z} + \log \frac{M_{\tilde{g}}}{M_Z} + \frac{1}{3} \log \frac{M_t}{M_Z} \right), \quad (2.16)$$

$$\begin{aligned} \frac{\delta\alpha}{\alpha} = \frac{\alpha}{2\pi} \left(\frac{4}{9} \sum_{i=1}^6 \log \frac{m_{\tilde{q}_i}}{M_Z} + \frac{1}{9} \sum_{i=1}^6 \log \frac{m_{\tilde{q}_i}}{M_Z} + \frac{1}{3} \sum_{i=1}^6 \log \frac{m_{\tilde{l}_i}}{M_Z} \right. \\ \left. + \frac{4}{3} \sum_{i=1}^2 \log \frac{M_{C_i}}{M_Z} + \frac{1}{3} \log \frac{M_{H^+}}{M_Z} + \frac{16}{9} \log \frac{M_t}{M_Z} - 7 \log \frac{M_W}{M_Z} \right). \end{aligned} \quad (2.17)$$

The $\overline{DR}$ values of α_i , ($i = 1, 2, 3$) are converted to the corresponding $\overline{MS}$ values using the relations

$$\frac{1}{\alpha_i(M_Z)|_{\overline{MS}}} = \frac{1}{\alpha_i(M_Z)|_{\overline{DR}}} + C_i; \quad \text{with } C_i = \frac{C_2(G_i)}{12\pi}, \quad (2.18)$$

where the quadratic Casimir $C_2(G_i) = N$ for $SU(N)$ or equals zero for $U(1)$.

The strong coupling constant is now ready for comparison with the experimental value quoted in table 2.3. The electromagnetic fine structure constant may stay within the $\overline{DR}$ scheme and it is corrected by an additional factor 0.0684 ([33] and the references therein) to account for the running down to the zero momentum transfer value as extracted from Thomson scattering.

Note that $\sin^2 \theta_W$ in eq.(2.14) represents the MSSM value, in the $\overline{DR}$ scheme, which can differ by a few *per cent* from the SM value - as found in [33]. For completeness, we also calculate the SM value, in the $\overline{MS}$ scheme, and quote both quantities among the model predictions in chapter 5. Our SM value of $\sin^2 \theta_W(M_Z)$ is obtained

from the SM values of α_1 and α_2 (by a relation analogous to eq.(2.14)). The latter are derived from the corresponding MSSM values by applying the threshold corrections similar to (2.16) and (2.17), with the W , Z and top quark terms absent, since these particles are kept in the SM. These are the leading logarithmic thresholds (LLT) which make up for the exclusion in the SM of the SUSY partners and non-minimal Higgs sector.²¹ A constant due to the change from the $\overline{DR}$ scheme to the $\overline{MS}$ scheme is added to $\delta\alpha_2$, according to eq.(2.18).

In the absence of direct observations, the SUSY sector of the MSSM is tested by all observables calculated at the loop level. Obviously, the strongest constraints come from the phenomena which originate directly in loop effects. Because of this reason ρ_{new} , as well as the process $b \rightarrow s\gamma$, were added to the list of observables. Breaking of custodial $SU(2)$ by physics beyond the SM is tested using

$$\rho_{new} = \frac{\Pi_{new}^{ZZ}(0)}{M_Z^2} - \frac{\Pi_{new}^{WW}(0)}{M_W^2}, \quad (2.19)$$

where $\Pi_{new}^{ZZ}(0)$ and $\Pi_{new}^{WW}(0)$ stand for the contributions of physics beyond the SM to the vector boson self-energies at zero momentum and are given in appendix B.

For the fermionic sector, we have the Z -scale tree level relations (1.37)-(1.39) between the fermion mass matrices and Yukawa matrices. In addition, at M_Z we include the leading ($O(\tan\beta)$) one loop threshold corrections to these relations [64] (see the next chapter). The one-loop corrected fermion mass matrices are then diagonalized.

²¹Explicit forms of the LLT corrections to α_1 and α_2 are given e.g. by the equations (17) and (18) in ref.[33]. We agree with the authors of [33] that the LLT approximation cannot proceed in a consistent way at this point since the $SU(2) \times U(1)$ gauge symmetry is broken at a scale comparable to the masses of sparticles being integrated out. The difference compared to the full calculation, however, is not very significant, and for our purposes will be neglected. Thus our SM value of $\sin^2 \theta_W$ contains a theoretical uncertainty of about 1%, (which is larger than the experimental uncertainty) similar to the theoretical uncertainty assigned to G_μ .

The pole mass of the top quark is calculated from the diagonal running mass with the leading two loop QCD corrections included:

$$M_t = m_t(M_Z)(1 + \frac{4}{3} \frac{\alpha_s}{\pi} + K_t(\frac{\alpha_s}{\pi})^2), \quad (2.20)$$

where $K_t = 11.4$ [53]. We utilize the infrared fixed point behaviour of the top Yukawa coupling and the fact that α_s also changes only slightly between M_{top} and M_Z , and evaluate the top pole mass from the relevant couplings at the Z scale.²²

The other quark and lepton masses are run down to their corresponding mass scales as explained in the beginning of this subsection. M_b and M_c are calculated from the $\overline{MS}$ running masses, $m_b(M_b)$ and $m_c(M_c)$ using the two loop QCD threshold correction similar to (2.20). In this case, the exact two-loop calculation leads to $K_b = 12.4$ and $K_c = 13.3$ [53]. Note that there are infra-red renormalons which make this perturbative conversions ambiguous. However, these renormalon contributions cancel in the mass difference. Moreover, it has been argued that this ambiguity cancels when extracting V_{cb} via semileptonic B decays[46],[54].²³

The lepton pole masses are obtained with the help of the one loop QED corrections.

Upon diagonalizing the one loop corrected mass matrices at M_Z , we calculate the complete form of the CKM mixing matrix (note that the CKM matrix elements do not run below the Z scale), and then compute the theoretical value for $\hat{B}_K$ needed to fit ϵ_K [65]. The effects of SUSY box diagrams are neglected in this last calculation,

²²A more precise evaluation would use $\alpha_s(M_t)$ and $m_t(M_t)$ to obtain the pole mass M_t . The error obtained with our approximation, using values at M_Z instead, partially cancels the one due to the non inclusion of the logarithmic corrections to the top mass, leading to a value which differs by less than 2 GeV from the actual value of M_t .

²³The correction to the bottom mass $(M_b - m_b(M_b))/m_b(M_b)$ at two loops is $\approx 25\%$ less than the results found recently by Ball, Beneke and Braun using an infinite order resummation within a naive non-abelianization of the mass correction[54]. The theoretical error for this result is also estimated to be about 25%. By using two loop pole masses, combined with the mass relation for $M_b - M_c$, we obtain a small value for $m_c(M_c)$. This value increases when using the scheme of BBB[54].

while a potentially significant ($O(\tan\beta)$) one loop threshold correction to J [64] is included (automatically, since we start from the one loop corrected mass matrix $\mathbf{m}_d$) as a leading higher order effect to the SM box diagram. The SUSY box diagram corrections can be as large as 15% [67] and may affect the predictions for CP violating parameters, such as η or $\sin 2\alpha$ and $\sin 2\beta$. We discuss the sensitivity of our results to these corrections in chapter 5.

Finally, special attention is paid to the branching ratio for $b \rightarrow s\gamma$. We calculate the partial amplitudes for this process, as well as the amplitude for $b \rightarrow s \text{ gluon}$ (required to resum leading logarithmic corrections), using the complete quark and squark mixing matrices at low energies [68]. Large QCD corrections are resummed to leading order following [69]. The low scale in this evaluation is set equal to the pole mass M_b of the b quark, and the dependence of the branching ratio on this choice is neglected compared to the experimental error.

We then form a χ^2 function including the 20 low energy observables — 6 in the gauge sector (M_W , M_Z , α_{EM} , G_μ , α_s , ρ), 13 in the fermion mass sector (9 charged fermion masses, 3 quark mixing angles and $\hat{B}_K$) and the branching ratio for $b \rightarrow s\gamma$. This χ^2 function is then minimized self-consistently using Minuit by iteratively varying the GUT parameters with m_0 , $M_{1/2}$, and μ fixed. The procedure involves two nested minimizations. The GUT scale parameters m_{H_d} and m_{H_u} are varied separately in the second nested minimization, for each step in the main minimization process where the rest of the free parameters are varied. Such a nested minimization is time consuming, but on the other hand it guarantees a high precision in the optimization process which is very sensitive to radiative electroweak symmetry breaking. Analytic tree level conditions for the potential minimum are used to determine v and $\tan\beta$.

With these technical details we conclude the discussion on the theoretical calculation of the twenty observables given in table 2.3.

In addition to the 20 observables discussed above, we have to deal with the experimental lower bounds on sparticle masses. We set lower limits of 30 GeV on squark and slepton masses, expecting significant positive higher order corrections to the squared mass which would raise it above the experimental lower bound of 45 GeV. We require that the lightest neutralino (if below the LEP1 threshold) contribute to the invisible Z partial width as measured at LEP1 $-\Gamma(Z \rightarrow \text{invisibles}) = (-1.5 \pm 2.7) \text{ MeV}$ [70]. We also demand that the lightest chargino be heavier than 65 GeV. Finally we require the pseudo-scalar Higgs, A^0 , to have mass greater than 80 GeV. Note that these bounds are on the tree level running masses. However, in tables 5.6 and 5.7, where we show predictions of the GUT models discussed later, Higgs masses are displayed with the leading one loop corrections included.²⁴ As can be seen there, the 80 GeV lower limit on the tree masses provides enough room for the one loop corrected Higgs masses to stay above the experimental limits. In order to take all these bounds into account we have added a large penalty to the χ^2 function whenever one of these bounds is exceeded, thus forcing the optimization procedure to stay within the experimentally allowed region in parameter space.

²⁴The CP odd state A^0 , if light, (along with the other light Higgs states) receives significant corrections to its tree level mass in the regime of large $\tan\beta$ in which we work. If the pseudoscalar mass is less than M_Z , it receives negative mass squared corrections; the same is true for the lightest CP even Higgs h^0 [71].

CHAPTER 3

SUPERSYMMETRIC THRESHOLD CORRECTIONS TO FERMION MASS MATRICES IN THE LARGE $\tan\beta$ REGIME

3.1 Introduction

Minimal supersymmetric grand unified theories (SUSY GUTs) based on the gauge group $SO(10)$ require $\tan\beta$ (the ratio of the vacuum expectation values of the two Higgs scalar doublets present in the low energy theory) to be of order $M_{top}/M_{bottom} \approx 50$, see eq.(1.40). As explained in the first chapter, this follows from the unification of the top, bottom and tau Yukawa couplings at the GUT scale, M_{GUT} , and the necessity to fit the large top to bottom mass ratio at the weak scale[72]. Recent results using a general $SO(10)$ operator analysis for fermion masses and mixing angles seem to be in significant agreement with experiment [73]. It was shown in [74, 75, 76], however, that there are potentially large finite 1-loop corrections (proportional to $\tan\beta$) to the masses of the down-type quarks at the supersymmetric threshold. Note, these corrections were not included in the analysis of ref. [73]. They may be as large as several tens of per cent dependent on the sparticle spectrum[75]. Thus they must be included when analyzing any SUSY theory with large $\tan\beta$.

In this chapter we evaluate the finite 1-loop threshold corrections, proportional to $\tan\beta$, to the down quark mass matrix. These result in corrections to down quark masses. Charged lepton masses get such corrections too, and they are similar to the d -quark mass corrections from the electroweak SUSY sector. We emphasize that the non-diagonal elements of the down quark mass matrix also get potentially large corrections; thus leading to significant corrections to the CKM matrix.

As a consequence, for grand unified theories with large $\tan\beta$ these corrections may significantly alter the low energy predictions for the quark and lepton masses, for four of the CKM matrix elements and for the Jarlskog parameter J , eq.(2.6) (or, alternatively, for $\hat{B}_K$ in agreement with the approach adopted in the previous chapter). The angles α , β and γ of the unitarity triangle and the ratio $|\frac{V_{ub}}{V_{cb}}|$, however, are not corrected to this order. In the light of recent models for fermion masses these corrections may be useful in selecting among the various models. Moreover, if one model fits the data, it will only do so for a particular range of SUSY parameters.

3.2 1-Loop Corrections to the Down Quark Mass Matrix

When one integrates superpartners out of the minimal supersymmetric standard model (MSSM), there are significant $\mathcal{O}(\tan\beta)$ 1-loop corrections to the mass matrix of the down-type quarks originating in the diagrams with gluino – d -type squark and chargino – u -type squark loops yielding (see figures 3.1a,b,c ; for the notation and conventions used, and for a short derivation see chapter 1 and appendix B)²⁵

$$\mathbf{m}_d = (V_d^{L0})^\dagger (1 + \epsilon \Gamma_d + \epsilon V_{CKM}^{0\dagger} \Gamma_u V_{CKM}^0) \mathbf{m}_d^{0Diag} V_d^{R0}, \quad (3.1)$$

²⁵We would like to thank Uri Sarid for bringing our attention to the second chargino diagram, fig.3.1c.

Figure 3.1: a) gluino, b),c) chargino exchange diagrams, in the interaction basis, which lead to $\tan\beta$ enhanced corrections to the d -quark mass matrix.

with $\epsilon = \frac{1}{16\pi^2} \tan\beta$ and

$$\Gamma_{dij} = \frac{8}{3} g_3^2 (\Gamma_{dL}^\dagger)_{i\alpha} \int dk \frac{M_{\tilde{g}}}{(k^2 + M_{\tilde{g}}^2)(k^2 + m_{\tilde{d}_\alpha}^2)} (\Gamma_{dR})_{\alpha l} \frac{(\mathbf{m}_d^{0Diag})_{lj}^{-1}}{\tan\beta}, \quad (3.2)$$

$$\begin{aligned} \Gamma_{u_{ij}} = & -\lambda_{u_i}^{0Diag} (\Gamma_{uR}^\dagger)_{i\alpha} \int dk \frac{U_{2A}^{*\dagger} m_{\chi_A} V_{A2}}{(k^2 + m_{\chi_A}^2)(k^2 + m_{\tilde{u}_\alpha}^2)} (\Gamma_{uL})_{\alpha j} (v_u)^{-1} \\ & + g_2 (\Gamma_{uL}^\dagger)_{i\alpha} \int dk \frac{U_{2A}^{*\dagger} m_{\chi_A} V_{A1}}{(k^2 + m_{\chi_A}^2)(k^2 + m_{\tilde{u}_\alpha}^2)} (\Gamma_{uL})_{\alpha j} (v_u)^{-1}. \end{aligned} \quad (3.3)$$

Uncorrected mass and mixing matrices are labeled by a “0” superscript. The 6×3 dimensional matrices Γ_{qL} and Γ_{qR} ($q = u, d$) correspond to the additional transformations necessary to diagonalize squark mass matrices in a SUSY basis where quark mass matrices are diagonalized. Expressions for the Γ ’s are rather complex since they involve the summation over the six-dimensional squark space. It has to be stressed though, that despite the explicit $\tan\beta$ term in the denominator of (3.2) and a similar $(v_u)^{-1} = (v_d \tan\beta)^{-1}$ term in (3.3), there will not be any actual $\tan\beta$ suppression in the elements of the Γ ’s. In the interaction basis (see the Feynman diagrams fig.3.1a,b,c) one can easily recognize the $R-L$ mixings among squarks in the loop and the mass insertions or mixings on the fermionic lines inside the diagram.

Since each of these mixings and mass insertions introduces a $\tan\beta$ unsuppressed quantity, the result represents a $\tan\beta$ unsuppressed correction. This correction is significant since it corrects a $\tan\beta$ suppressed mass matrix. To emphasize this fact the large Higgs vev ratio was pulled out into the ϵ ’s in (3.1). As a net effect, one can expect (at least some) terms in the Γ ’s to be of order $(0.1 - 1)$. These terms are then enhanced by a factor of $\tan\beta$ (multiplying a standard small loop factor $(16\pi^2)^{-1}$ in our definition of ϵ) and thus lead to significant mass matrix corrections.

Concentrating on these diagrams one has to mention that there are also neutralino diagrams which contribute by finite $\mathcal{O}(\tan\beta)$ terms to the d quark mass matrix. However, we have checked that (assuming degenerate gauginos at M_{GUT}) these contributions are less than the leading gluino corrections roughly by a factor of 16, as a result of smaller couplings, gaugino masses and group factors. Therefore these diagrams will not be discussed separately in this work although they are included in our numerical analysis in section 3.4 where we discuss their effects.

In order to gain some intuition for the Γ 's one can find an explicit form for them in the following approximation. First, neglect the second chargino diagram (fig.3.1c), since it is suppressed by a smaller coupling constant g_2 compared to the gluino and chargino diagrams, figure 3.1a and 3.1b, with g_3 and λ_t instead. Then in the evaluation of the remaining two diagrams assume that the squark mass matrices are diagonalized in generation space by the same rotations as the corresponding quark matrices. This approximation is valid assuming universal scalar masses and trilinear scalar interactions proportional to Yukawa interactions at the low energy SUSY scale.

That means that in this approximation the matrices Γ_{qL} and Γ_{qR} , ($q = d, u$) are diagonal in generation space and are not completely trivial only because of the mixing between squarks of the same generation. The integrals in (3.2) and (3.3) are then easy to do along with the summation over $\alpha = 1, \dots, 6$, i.e. over the squark mass eigenstates. The Γ 's are then proportional to the off-diagonal term of the down (up) squark mass matrix for each individual generation separately. We find

$$\Gamma_{d,ij} = \frac{8}{3} g_3^2 M_{\tilde{g}} \mu I_3(M_{\tilde{g}}^2, m_{\tilde{d}_{i1}}^2, m_{\tilde{d}_{i2}}^2) \delta_{ij} , \quad (3.4)$$

$$\Gamma_{u,ij} = U_{2A}^{*\dagger} m_{\chi_A} V_{A2} A_0 I_3(m_{\chi_A}^2, m_{\tilde{u}_{i1}}^2, m_{\tilde{u}_{i2}}^2) (\lambda_u^{0Diag})_{ij}^2 , \quad (3.5)$$

with the function I_3 given by

$$I_3(a, b, c) = \frac{ab \ln(\frac{a}{b}) + bc \ln(\frac{b}{c}) + ac \ln(\frac{c}{a})}{(a-b)(b-c)(a-c)}.$$

Terms suppressed by $\tan\beta$ have been neglected in these expressions. In this approximation both Γ matrices are diagonal which makes calculations of the corrections to the masses and mixing angles in terms of mass eigenstates simple. Besides that, note the large hierarchy in Γ_u , and a much milder hierarchy in Γ_d based just on the non-equality of the squark masses. Note, if Γ_d were completely proportional to the

identity matrix (i.e. the case of complete squark degeneracy), the gluino loop would not contribute to quark mixing corrections at all ²⁶.

One knows though, that the approximation used to derive (3.4) and (3.5) is not correct. The initial conditions at M_{GUT} need not be universal and, even if they were, squark masses and trilinear couplings run between the GUT (or string) and the low energy SUSY scales and violate our assumptions. As a result the explicit form of the potentially significant (i.e. $\tan\beta$ unsuppressed) elements in the Γ 's is clouded by the fact that they no longer remain diagonal in generation space. In order to evaluate these effects we have performed a numerical analysis. The results are found in section 3.4. We also show that our naive approximation, equations (3.4, 3.5), when suitably modified to take into account non-universal squark masses, gives results which agree to within 25% with the two-loop numerical analysis.

In order to figure out the explicit form of the 1-loop threshold corrections to the CKM matrix elements as well as to quark masses in terms of the Γ matrix elements one can define an unknown hermitian matrix $\mathbf{B}$ as

$$V_d^L = (1 + i\epsilon\mathbf{B}) V_d^{L0} \quad (3.6)$$

where, again, V_d^{L0} is the matrix diagonalizing down quarks in the absence of the SUSY corrections. Since there are no large (i.e. $\mathcal{O}(\tan\beta)$) corrections to the up quark mass matrix

$$V_{CKM} \equiv V_u^L V_d^{L\dagger} = V_u^{L0} (V_d^{L0})^\dagger (1 - i\epsilon\mathbf{B}) = V_{CKM}^0 (1 - i\epsilon\mathbf{B}) . \quad (3.7)$$

²⁶Also note that the same analysis could be done for the corrections to the up quark mass matrix and the above mentioned approximation would show that the relevant Γ matrices (analogous to (3.4) and (3.5)) become suppressed by $\tan\beta$ after the up (instead of down) quark mass matrix is pulled out of the expression analogous to (3.1). Thus in this case there are no corrections proportional to $\tan\beta$. There are however corrections to charged lepton masses proportional to $\tan\beta$. These are smaller than those for down quarks but are still significant and must be included in any fermion mass analysis.

$\mathbf{B}$ is determined through the diagonalization condition

$$(\mathbf{m}_d^{\text{Diag}})^2 \equiv \text{Diag}(m_{d_1}^2, m_{d_2}^2, m_{d_3}^2) = V_d^L \mathbf{m}_d \mathbf{m}_d^\dagger V_d^{L\dagger}, \quad (3.8)$$

where both V_d^L and $\mathbf{m}_d$ on the r.h.s. are to be expanded to first order in ϵ according to (3.6) and (3.1).

3.2.1 Corrections to Down Quark Masses

Diagonal elements of this matrix equation (3.8) specify the corrections to the masses of the d , s and b (d_1 , d_2 and d_3) quarks. Note that the terms containing unknown $\mathbf{B}$ elements drop out of these equations :

$$\frac{\delta m_{d_i}}{m_{d_i}} = \epsilon \text{Re}(\Gamma_d)_{ii} + \epsilon [V_{CKM}^{0\dagger} \text{Re}(\Gamma_u) V_{CKM}^0]_{ii}. \quad (3.9)$$

This is an exact formula where the effects of squark rotations are fully included in the Γ 's. Since the Γ_u matrix has some generation hierarchy (for more discussion on this see section 3.4) due to the Yukawa couplings in the charging loop the dominant correction from the chargino diagram goes to the b quark mass correction:

$$\left(\frac{\delta m_b}{m_b} \right)_{\chi^+} = \epsilon \text{Re}(\Gamma_u)_{33} + \epsilon \mathcal{O}(10^{-3}). \quad (3.10)$$

The suppression in the second term above is caused by the hierarchies present in (3.9). The largest next-to-leading correction indicated above results, for example, from the term $(V_{CKM}^{0\dagger})_{32} \text{Re}(\Gamma_u)_{23} (V_{CKM}^0)_{33}$ where two orders come from V_{cb}^* and at least one order from Γ_{u32} .

Note that the corrections to the masses of the s and d quarks can easily be as significant, or even larger than the correction to the b quark mass. While the gluino correction (which is the largest correction to each quark mass) to the b quark mass

is larger due to the smaller b squark masses (in a universal-like scenario where one starts with all soft squark masses equal at the GUT scale), the chargino correction may invert the net effect since it is always of opposite sign to the gluino correction and its contribution to the two lighter quarks is small.

3.2.2 Corrections to CKM Matrix Elements

The non-diagonal equations, i.e. those with zeros on the l.h.s. of the matrix equation (3.8), lead to

$$-i\epsilon\mathbf{B}_{ij} = \epsilon\Gamma_{d\,ij} + \epsilon(V_{CKM}^{0\dagger}\Gamma_u V_{CKM}^0)_{ij} (1 + \mathcal{O}(\frac{m_{d_i}^2}{m_{d_j}^2})), \quad (3.11)$$

where ij indices correspond to the 12, 13 or 23 combinations and the transposed elements (for $i > j$) are obtained by the hermiticity of $\mathbf{B}$. The diagonal elements of $\mathbf{B}$ remain undetermined by this procedure but to the first order in the ϵ -expansion they can be removed by phase redefinitions of the b , s and d fields. We thus set the diagonal elements of $\mathbf{B}$ to zero.

Then from (3.7) we can easily derive ²⁷

$$\begin{aligned} \delta V_{cb} = & \epsilon [V_{cd}\Gamma_{d\,13} + V_{cs}\Gamma_{d\,23}] \\ & + \epsilon [\{\delta_{2j} - (V_{CKM})_{23}(V_{CKM}^\dagger)_{3j}\}\Gamma_{u\,jk}(V_{CKM})_{k3}] \end{aligned} \quad (3.12)$$

$$\begin{aligned} \delta V_{ub} = & \epsilon [V_{ud}\Gamma_{d\,13} + V_{us}\Gamma_{d\,23}] \\ & + \epsilon [\{\delta_{1j} - V_{CKM})_{13}(V_{CKM}^\dagger)_{3j}\}\Gamma_{u\,jk}(V_{CKM})_{k3}] \end{aligned} \quad (3.13)$$

$$\begin{aligned} \delta V_{td} = & -\epsilon [V_{ts}\Gamma_{d\,21}^\dagger + V_{tb}\Gamma_{d\,31}^\dagger] \\ & - \epsilon [\{\delta_{3j} - (V_{CKM})_{31}(V_{CKM}^\dagger)_{1j}\}\Gamma_{u\,jk}^\dagger(V_{CKM})_{k1}] \end{aligned} \quad (3.14)$$

²⁷Zero superscripts are dropped from now on since they make no difference in the following expressions.

$$\begin{aligned} \delta V_{ts} = & \epsilon [V_{td} \Gamma_{d12} - V_{tb} \Gamma_{d32}^\dagger] + \epsilon [(V_{CKM})_{31} (V_{CKM}^\dagger)_{1j} \Gamma_{u,jk} \\ & - (V_{CKM})_{33} (V_{CKM}^\dagger)_{3j} \Gamma_{u,jk}^\dagger] (V_{CKM})_{k2} . \end{aligned} \quad (3.15)$$

As already mentioned, the numerical analysis shows that both Γ_u and Γ_d matrices keep track of the generation hierarchy from the Yukawa sector with the 33 element of the order of 0.1-1 and the relevant 12, 13 and 23 elements of small magnitude. Together with the hierarchy in the CKM matrix this implies that in each of the previous equations the dominant correction is the one containing the Γ_{u33} term. However the corrections due to the other terms are non-negligible resulting in a 25% effect. Thus a good approximation to the exact results of equations (3.12) - (3.15) is given by (for more detail, see section 3.4)

$$\frac{\delta V_{cb}}{V_{cb}} \approx -\epsilon [\Gamma_{u33} - \Gamma_{u22} - \frac{V_{cs}}{V_{cb}} \Gamma_{d23}] \equiv -\epsilon \Delta \quad (3.16)$$

$$\frac{\delta V_{cb}}{V_{cb}} \approx \frac{\delta V_{ub}}{V_{ub}} \approx -\epsilon \Delta \quad (3.17)$$

$$\frac{\delta V_{ts}}{V_{ts}} \approx \frac{\delta V_{td}}{V_{td}} \approx -\epsilon \Delta^* . \quad (3.18)$$

The results of equations (3.17) and (3.18) follow directly from the unitarity of the CKM matrix and the fact that these are the only terms which receive significant corrections.

Note that as a consequence the ratio V_{ub}/V_{cb} remains unchanged. In addition the numerical analysis shows that $\text{Re}\Gamma_{u33} \gg \text{Im}\Gamma_{u33}$ thus these dominant corrections to the CKM elements are equal in magnitude, but opposite in sign, to the chargino corrections to the b quark mass, eq.(3.10).

The other five CKM elements get the corrections of the form similar to (3.12) - (3.15). However, large Γ elements are always in the product with a small CKM matrix

element, and the terms containing large diagonal CKM matrix elements are in the same way pushed down by small Γ elements in these corrections. Hence the actual numerical values of the corrections to V_{ud} , V_{us} , V_{cd} , V_{cs} and V_{tb} are not significant, at least not at the present level of experimental accuracy. As an example, the dominant correction to, let's say V_{us} goes like

$$\delta V_{us} \sim \epsilon V_{ub} V_{ts} V_{tb}^* \Gamma_{u33}^* < 0.001 . \quad (3.19)$$

3.3 CP Violating Parameters

The Jarlskog parameter which measures CP violation can be obtained from the four CKM-matrix elements left after crossing out any row and any column of this matrix[77]:

$$J \sum_{\gamma, l} \epsilon_{\alpha\beta\gamma} \epsilon_{jkl} = \text{Im} [V_{\alpha j} V_{\beta k} V_{\alpha k}^* V_{\beta j}^*] . \quad (3.20)$$

Consider the product

$$J = \text{Im} [V_{cs} V_{tb} V_{cb}^* V_{ts}^*] \quad (3.21)$$

Using the formula (3.16) and (3.18) from the previous section it is easy to obtain the leading correction

$$\delta J \approx -2\epsilon \text{Re}[\Delta] J \quad (3.22)$$

This threshold correction to J may significantly alter the prediction for ϵ_K in SUSY GUT models with large $\tan\beta$.

Note, it is not obvious how this result is obtained for other equivalent definitions of J . For example, at first glance one might guess that $\delta J \approx 0$ for J defined by $J = \text{Im} [V_{ud} V_{cs} V_{us}^* V_{cd}^*]$. However, such a guess does not take into account that we have the imaginary part of the product in (3.20) and imaginary parts are small for

every CKM matrix element, even if its absolute value is close to one. In this case the small corrections to the large CKM matrix elements become important and, in fact, it is corrections to V_{cs} and V_{us} that lead to the result (3.22).

Finally, we note that although J changes, *the angles of the unitarity triangle remain uncorrected to this order*. This is easily understood from a geometrical point of view. For the “standard” choice of its sides – $|V_{ud}V_{ub}^*|$, $|V_{cd}V_{cb}^*|$ and $|V_{td}V_{tb}^*|$ – each side contains one element which gets a significant correction and (as a consequence of the unitarity of the CKM matrix discussed earlier) these corrections are identical in magnitude (see (3.16) and (3.18)). Hence, the sides are contracted (or stretched) by the same multiplicative factor and the angles stay the same. The area of the triangle gets corrected, of course, twice as much as the sides, and that is the reason for the factor of two in (3.22) (recall that J measures the area of the triangle).

3.4 Numerical Analysis and Conclusions

In order to get a feeling for these SUSY threshold corrections we took the initial conditions (values at the GUT scale) for the dimensionless couplings from the SO(10) models [73] which give predictions for the low energy data in good agreement with experiment. The initial values for the dimensionful soft SUSY breaking parameters were taken from ref.[75, 78] in order to guarantee the radiative electroweak symmetry breaking at the weak scale. We focused mainly on simple non-universal cases. The numerical results presented below were obtained for $m_{H_1}^2 = 2.0m_0^2$, $m_{H_2}^2 = 1.5m_0^2$ and all other scalar masses equal m_0^2 . Next, we used 2-loop renormalization group equations[79] to run all the couplings and mass parameters to the low energy scale.

Leading corrections to the CKM matrix elements have appeared practically independent of the exact value of the low energy SUSY scale between M_Z and 500 GeV (changes were within 1% of the mass or the CKM element in question). In the actual numerical analysis the Γ -matrices have been evaluated according to the following formulae (note that there are no divergent pieces from the integrals in (3.2) and (3.3) and that the chargino summation is easy to do) :

$$\Gamma_{d,ij} = \frac{8}{3}g_3^2 (\Gamma_{dL}^\dagger)_{i\alpha} \frac{-m_{d\alpha}^2}{m_{d\alpha}^2 - M_{\tilde{g}}^2} \ln \frac{m_{d\alpha}^2}{M_{\tilde{g}}^2} (\Gamma_{dR})_{\alpha j} \frac{M_{\tilde{g}}}{m_{d\alpha} \tan\beta} \quad (3.23)$$

$$\begin{aligned} \Gamma_{u,ij} = & \lambda_{u_i} (\Gamma_{uR}^\dagger)_{i\alpha} (m_{\tilde{u}\alpha}^2 - |M_2|^2) I_3(m_{\chi_1}^2, m_{\chi_2}^2, m_{\tilde{u}\alpha}^2) (\Gamma_{uL})_{\alpha j} \frac{\mu}{v_u} \\ & - g_2^2 (\Gamma_{dL}^\dagger)_{i\alpha} I_3(m_{\chi_1}^2, m_{\chi_2}^2, m_{\tilde{u}\alpha}^2) (\Gamma_{uL})_{\alpha j} M_2 \mu. \end{aligned} \quad (3.24)$$

M_2 is the wino mass parameter and terms suppressed by $\tan\beta$ were dropped. The summation is only over $\alpha = 1, \dots, 6$ (there's no summation over i, j on the r.h.s. of these equations). This summation could be done analytically in terms of the mass eigenvalues, however the expressions are long and don't provide much insight, so we keep rather the compact forms above.

Typical values for these matrices at the weak scale follow -

$$\begin{aligned} \epsilon\Gamma_d \approx & \begin{pmatrix} 0.180 + i3 \cdot 10^{-8} & -3 \cdot 10^{-6} - i9 \cdot 10^{-7} & 8 \cdot 10^{-5} + i2 \cdot 10^{-5} \\ -3 \cdot 10^{-6} + i7 \cdot 10^{-7} & 0.180 - i4 \cdot 10^{-8} & -4 \cdot 10^{-4} + i6 \cdot 10^{-6} \\ -9 \cdot 10^{-5} + i8 \cdot 10^{-5} & 3 \cdot 10^{-5} + i1 \cdot 10^{-5} & 0.218 - i3 \cdot 10^{-9} \end{pmatrix} \\ \epsilon\Gamma_u \approx & \begin{pmatrix} -0.022 + i5 \cdot 10^{-16} & -9 \cdot 10^{-7} + i3 \cdot 10^{-7} & -3 \cdot 10^{-6} + i6 \cdot 10^{-6} \\ -9 \cdot 10^{-7} - i3 \cdot 10^{-7} & -0.022 - i3 \cdot 10^{-11} & -1 \cdot 10^{-4} + i5 \cdot 10^{-9} \\ 1 \cdot 10^{-5} + i3 \cdot 10^{-5} & 5 \cdot 10^{-4} - i1 \cdot 10^{-8} & -0.116 + i7 \cdot 10^{-10} \end{pmatrix} \end{aligned}$$

where in this case we used the GUT scale values, $M_{1/2} = 400 \text{ GeV}$, $m_0 = 250 \text{ GeV}$, and $A_0 = -1100 \text{ GeV}$, the weak scale value, $\mu = 270 \text{ GeV}$, and Model 4 of [73] for the Yukawa matrices (with the weak scale values $\lambda_t = 1.01$, $\tan\beta = 53$ and $V_{cb}^0 = 0.038$ as output). With these inputs we find $M_{\tilde{g}} = 1029 \text{ GeV}$, $A_t = (A_u)_{33} = -(736 + i6 \cdot 10^{-7}) \text{ GeV}$, up-squark mass eigenvalues (in GeV) (976, 976, 951, 951, 869, 695)

and down-squark mass eigenvalues (in GeV) (980, 979, 949, 948, 820, 757). To gain some intuition for the size of the corrections, these particular values lead to $\delta m_b/m_b = 10.2\%$, $\delta m_s/m_s = 15.7\%$, $\delta m_d/m_d = 15.7\%$, $\delta V_{cb}/V_{cb} = \delta V_{ub}/V_{ub} = \delta V_{ts}/V_{ts} = \delta V_{td}/V_{td} = 8.0\%$ and $\delta J/J = 16.6\%$. As we discussed earlier the approximation of retaining only the $\Gamma_{u_{33}}$ term in equation(3.16) does not work extremely well since it predicts an 11.6% correction. However, this leading correction is then lowered by about 1.5% coming from the Γ_d term in (3.12)-(3.15) and by additional 2% from the subleading Γ_u terms. V_{ud} , V_{us} , V_{cd} , V_{cs} and V_{tb} get a relative correction less than 1%, e.g. $\delta V_{us}/V_{us} = 0.01\%$. Similarly, the corrections to the angles α , β and γ of the unitarity triangle are much below 1%. Neutralino corrections have been included in the above numerical analysis. Their effects are as follows: the b mass is reduced by 1.6% and the masses of s and d are reduced by 1.3%. Integrating out neutralinos has less than a 1% impact on the CKM elements and CP violating parameter, J .

We would like to emphasize that such corrections are generic for a large subspace of the allowed parameter space. The actual size of the corrections in our global analysis results of the next chapters can be found in table 4.1 and tables 5.2-5.5.

3.4.1 Approximate Formulae for Mass and Mixing Angle Corrections

In eqns. (3.4) and (3.5), we presented the results of a naive approximation which assumes that squark mass matrices are diagonalized in generation space by the same rotations as the corresponding quark matrices. This approximation is valid in the case of universal scalar masses and trilinear scalar interactions proportional to Yukawa interactions when, in addition, one also neglects the renormalization group running from M_G to the low energy SUSY scale. If one now includes the effect of RG running,

quark and squark mass matrices can no longer be diagonalized in generation space by the same unitary transformations and the A parameters are no longer universal. We have checked that a simple approximation for the corrections to down quark masses and the CKM matrix elements (valid to 25%) can be obtained by using the results of eqns. (3.4) and (3.5) with the values of squark and gluino masses obtained by RG running as input and by replacing A_0 with A_t (for the third generation) and the chargino mass with the low energy value of μ . This approximation has been widely used in the previous papers[74, 76, 75, 78, 80] where large bottom mass corrections have been recognized. In particular, in this improved approximation

$$\begin{aligned}\delta m_d &\approx (\tilde{\epsilon}_{d1} + \tilde{\epsilon}_{d2} \mathcal{O}(10^{-6})) m_d \\ \delta m_s &\approx (\tilde{\epsilon}_{s1} + \tilde{\epsilon}_{s2} \mathcal{O}(10^{-4})) m_s \\ \delta m_b &\approx (\tilde{\epsilon}_{b1} + \tilde{\epsilon}_{b2} (|V_{tb}|^2 + \mathcal{O}(10^{-6}))) m_b ,\end{aligned}\tag{3.25}$$

where

$$\tilde{\epsilon}_{di1} = \frac{2\alpha_s}{3\pi} \mu M_{\tilde{g}} I_3(M_{\tilde{g}}^2, m_{\tilde{d}_{i1}}^2, m_{\tilde{d}_{i2}}^2) \tan\beta \tag{3.26}$$

$$\tilde{\epsilon}_{di2} = \frac{1}{16\pi^2} \mu A_{u_i} \lambda_{u_i}^2 I_3(\mu^2, m_{\tilde{u}_{i1}}^2, m_{\tilde{u}_{i2}}^2) \tan\beta. \tag{3.27}$$

The analogous corrections for CKM matrix elements, also valid to about 25%, are given by

$$\frac{\delta V_{cb}}{V_{cb}} \approx \frac{\delta V_{ub}}{V_{ub}} \approx \frac{\delta V_{ts}}{V_{ts}} \approx \frac{\delta V_{td}}{V_{td}} \approx -\tilde{\epsilon}_{b2}, \tag{3.28}$$

where $\tilde{\epsilon}_{b2}$ is defined in the eq.(3.27).

An important feature of the b quark mass correction is that the gluino and chargino contributions are of the opposite signs and thus there is a partial cancellation between them. This effect with its consequences has been carefully studied in [75, 78, 80]. In

these papers it was shown that the magnitude of the gluino contribution is always two to three times larger than the chargino contribution and can be as large as 50% for universal scalar masses at M_G . For non-universal scalar masses the corrections can be smaller.

3.4.2 Consequences for Models of Fermion Masses

It is interesting to see what effect these corrections have for recent models of fermion masses and mixing angles. In the model of ref.[81] the value of $|V_{cb}|$ is of order .054. This is large compared to the latest experimental values. In this model, $\tan\beta$ can be either small or large. We would have to be in the large $\tan\beta$ regime for these corrections to be significant. In addition consider models 4,6 and 9 of ref.[73]. In these models $\tan\beta$ is expected to be large. Recall that the model independent experimental value of $|V_{cb}|$ is 0.039 ± 0.003 as stated in the previous chapter. For models 6 and 9, the predicted value of $V_{cb} \sim .048 - .052$ is at the upper end of the experimentally allowed range. For all these models we would choose $\mu M_{\tilde{g}} < 0$ so that the chargino correction to the b quark mass is positive and hence $\delta V_{cb}/V_{cb} < 0$. As a consequence the gluino correction is negative which gives $\delta m_b < 0$. This has the effect of decreasing the prediction for m_t , since a smaller top Yukawa coupling is now needed to fit the experimental ratio m_b/m_τ . These corrections apparently improve the predictions of the above models. However, the corrections to the strange and down quark masses, which are equal and negative, may be a problem since both ratios m_u/m_d and m_s/m_d were rather large and now the first one gets even bigger while the second one stays the same. This problem is exacerbated by the fact that the authors in [80] find no solutions for $|\frac{\delta m_b}{m_b}| < 10\%$ with $\mu M_{\tilde{g}} < 0$ consistent with

both the experimental rate for $b \rightarrow s\gamma$ and the cosmological constraint on the energy density of the Universe. There are solutions for larger values of $|\frac{\delta m_b}{m_b}|$ but this range of parameters may seriously be constrained by the ratios m_u/m_d and m_s/m_d .

For model 4 of ref.[73] however the situation may be better. In this model $|V_{cb}|$ is acceptably small ($V_{cb} \sim .038 - .044$). However J is too small and thus the bag constant, B_K , needed to fit ϵ_K is too large, i.e. greater than 1. In this case we need $\delta J/J > 0$. This would also increase $|V_{cb}|$ by half as much, which may be acceptable. In this case the chargino correction to the b quark mass is negative. Thus the gluino correction to m_b is positive and $\delta m_b > 0$. As a result the top quark mass prediction increases. This restricts the magnitude of the effect to values of $|\frac{\delta m_b}{m_b}| < 10\%$. In this case both m_s and m_d increase, which improves the agreement with experiment in the $m_s/m_d - m_u/m_d$ plane. Finally the $b \rightarrow s\gamma$ decay rate and the cosmological constraint can be satisfied[80].

Note that in either scenario the angles α , β and γ of the unitarity triangle and the ratio V_{ub}/V_{cb} remain unchanged. These correlations of quark mass and mixing angle predictions with the sign of $\mu M_{\tilde{g}}$ are very intriguing, especially since this sign may be determined independently once SUSY particles are observed. In a particular model the allowed maximal corrections to masses and mixing angles may represent new constraints on the magnitude and sign of the SUSY parameters.

In summary, finite SUSY corrections to the masses of the down-type quarks may be significant in the limit of large $\tan\beta$. In this chapter we have shown that the CKM matrix elements V_{cb} , V_{ub} , V_{ts} and V_{td} receive similar corrections, while the correction to the Jarlskog parameter is enhanced by a factor of two. The other elements of the CKM matrix and the angles of the unitarity triangle receive only small corrections,

down by a factor $\tan\beta$ or suppressed by the generation hierarchy present in Yukawa, CKM or Γ matrices.

CHAPTER 4

MODELS WITH FOUR EFFECTIVE OPERATORS

4.1 Introduction

In this chapter we discuss models with four effective mass operators at the GUT scale. These are the nine models found in the ADHRS study [82]. There are ($n_y =$) 5 independent Yukawa parameters in these models, bringing the total number of arbitrary parameters at the GUT scale to 15. With the twenty observables of chapter II, we are left with 5 degrees of freedom in these models.

We show the results of global analysis for each of these models. The best model appears to be model 4, and we study this model in much more detail. The best fits give $\chi^2 \sim 13 - 14$ for 5 degrees of freedom. All low energy observables are fit to within their 2σ bounds. The fits for 4 observables (V_{cb} , V_{ub}/V_{cb} , the Kaplan-Manohar-Leutwyler ellipse parameter Q and the Jarlskog CP violating invariant J) lie close to the 2σ border. We show that these results are true predictions of the theory, i.e. we see no other way to improve the results, except with the addition of a new operator contributing to the 13 and 31 elements of the Yukawa matrices. Thus we conclude that in order to get a more significant agreement with the low energy data one should either reexamine the assumptions of ADHRS, or start investigating models with at least five effective operators.

4.2 Summary of Earlier Systematic Search

Let us briefly recall the main points of the paper [82].²⁸

4.2.1 ADHRS Assumptions

- The Grand Unified Theory is supposed to be supersymmetric, and based on the $SO(10)$ gauge group.
- The effective theory below the GUT scale is the MSSM.
- The two Higgs doublets which are contained within the MSSM both belong to a single 10-dimensional representation of $SO(10)$. This is the minimal choice. As described in the introductory chapter, in this case the masses of the three fermions of the third generation can be explained by an operator

$$\mathcal{O}_{33} = A \mathbf{16}_3 \mathbf{10} \mathbf{16}_3, \quad (4.1)$$

at the GUT scale, where A is a single Yukawa coupling. At the same time this assumption leads to a large $\tan\beta$.

- All dimensionless couplings of the GUT should be of order unity. This implies that only the third family, $\mathbf{16}_3$, gets mass via a renormalizable operator. Given the previous assumption, this operator is necessarily that of eq.(4.1).
- Lighter families get masses via non-renormalizable operators. The structure of these higher-dimensional operators is partly responsible for the observed hierarchy in the fermionic sector.

²⁸We focus on the “22” texture only, avoiding the fine-tuning in the models with two 23 operators and no 22 operator.

- Each of the adjoint representations contained in the theory can break the symmetry only in one of the well defined dynamical directions. These are the $B - L$, Y (hypercharge), X ($SU(5)$ conserving) and T_{3R} (isospin of $SU(2)_R$) directions.
- The minimal number of effective operators is searched for, assuming that only a few operators are dominant.
- The threshold corrections could be safely neglected.

4.2.2 ADHRS Results

The minimal set of the effective operators was found to contain four operators : $\mathcal{O}_{33}$ of the type (4.1), $\mathcal{O}_{23}$, $\mathcal{O}_{22}$ and $\mathcal{O}_{12}$. Both $\mathcal{O}_{33}$ and $\mathcal{O}_{12}$ were found to be unique. There were six choices for the 22 operator,

$$\mathcal{O}_{22} =$$

$$\begin{aligned}
(a) \quad & 16_2 \frac{\bar{A}}{S_M} 10_1 \frac{A_1}{\bar{A}} 16_2 \\
(b) \quad & 16_2 \frac{S_G}{\bar{A}} 10_1 \frac{A_1}{S_M} 16_2 \\
(c) \quad & 16_2 \frac{\bar{A}}{S_M} 10_1 \frac{A_1}{S_M} 16_2 \\
(d) \quad & 16_2 10_1 \frac{A_1}{\bar{A}} 16_2 \\
(e) \quad & 16_2 10_1 \frac{\bar{A} A_1}{S_M^2} 16_2 \\
(f) \quad & 16_2 10_1 \frac{A_1 S_G}{\bar{A}^2} 16_2.
\end{aligned} \tag{4.2}$$

All give the same 0 : 1 : 3 Clebsch relation between up quarks, down quarks and charged leptons responsible for the Georgi-Jarlskog relation (1.45) mentioned in chapter 1 [83]. Therefore, these six choices are undistinguishable, as far as only the quark and charged lepton mass matrices are tested. However, they may be distinguished

by different predictions for the neutrino sector (if neutrinos are found massive), or by a pattern of proton decay branching ratios (if discovered experimentally). A_1 , A_2 and $\bar{A}$ are adjoint states with vacuum expectation values (vev) in the B-L, Y and X (SU(5) invariant) directions of SO(10) and S 's are various singlet fields with vevs at or above the unification scale.

In addition, there were nine choices for the 23 operator,

$$\mathcal{O}_{23} =$$

$$\begin{aligned} (1) \quad & 16_2 \frac{A_2}{S} 10_1 \frac{S}{\bar{A}} 16_3 \\ (2) \quad & 16_2 \frac{A_2}{S} 10_1 \frac{A_1}{\bar{A}} 16_3 \\ (3) \quad & 16_2 \frac{A_2}{\bar{A}} 10_1 \frac{S}{\bar{A}} 16_3 \\ (4) \quad & 16_2 \frac{A_2}{\bar{A}} 10_1 \frac{A_1}{\bar{A}} 16_3 \\ (5) \quad & 16_2 \frac{A_2}{S} 10_1 \frac{A_2}{\bar{A}} 16_3 \\ (6) \quad & 16_2 \frac{A_2}{\bar{A}} 10_1 \frac{A_2}{\bar{A}} 16_3 \\ (7) \quad & 16_2 10_1 \frac{S}{\bar{A}} \frac{S'}{\bar{A}} 16_3 \\ (8) \quad & 16_2 10_1 \frac{S}{\bar{A}} \frac{A_1}{\bar{A}} 16_3 \\ (9) \quad & 16_2 10_1 \frac{A_1}{\bar{A}} \frac{A_1}{\bar{A}} 16_3 \end{aligned} \tag{4.3}$$

with the numbers 1-9 giving also names to the models. For the sake of completeness, the unique 12 operator reads

$$\mathcal{O}_{12} = 16_1 \left(\frac{\bar{A}}{S_M} \right)^3 10_1 \left(\frac{\bar{A}}{S_M} \right)^3 16_2 . \tag{4.4}$$

A priori, there is no reason why supersymmetric thresholds at the electroweak scale should be negligible, as stated in one of the assumptions of ADHRS. In fact, as showed in chapter 3, threshold corrections to $m_b(M_b)$, $M_b - M_c$, V_{cb} and J (or $\hat{B}_K$)

can be of the same size as the experimental errors are. Therefore, they can become significant and lead to the decision to accept a model which would be otherwise rejected, if these corrections were not included. ²⁹

It should also be pointed out, that in the ADHRS analyses, the low-energy observables were separated into two distinct groups. Observables from one group were fixed to their central values, and served to find the GUT scale parameters. Observables from the second group were then derived as theoretical predictions of the models, and compared to the experimental values.

Applying this procedure, ADHRS deduced that models 4, 6 and 9 were the most promising candidates for the correct description of the data, available at that time.

4.3 Global Analysis Results

Compared to the original ADHRS analysis, our approach makes a difference in two respects. The first one is the inclusion of the supersymmetric threshold corrections at the weak scale, removing one of the technical ADHRS assumptions. Obviously, this makes the analysis much more self-consistent. A second improvement is the evaluation of model performances in a complete top-down approach using global analysis. There is no separation into two groups of observables any more, since within the global analysis, each observable is treated equally.

Our results, in a limited SUSY parameter space, can be found in the figures 4.1-4.9. In these nine figures, the solid line shows the total χ^2 value. The three dominant contributions to χ^2 are shown for every model. Note, that as the m_0 parameter

²⁹Note however, that if a model, which works well, is found with the SUSY corrections neglected, then it can always remain in agreement with the low energy data by pushing squark and slepton masses into a TeV region. In such a case the MSSM is tuned to decouple from the SM at the Z scale.

Figure 4.1: Best fit for model 1 (solid curve) in a limited SUSY parameter space, with three dominant contributions to χ^2 : from $\hat{B}_K$ (open squares), from V_{us} (solid triangles) and from the ellipse parameter Q^{-2} (open circles).

Figure 4.2: Best fit for model 2 (solid curve) in a limited SUSY parameter space, with three dominant contributions to χ^2 : from V_{us} (solid triangles), from V_{cb} (solid circles) and from $\hat{B}_K$ (open squares).

Figure 4.3: Best fit for model 3 (solid curve) in a limited SUSY parameter space, with three dominant contributions to χ^2 : from $\hat{B}_K$ (open squares), from V_{cb} (solid circles) and from the ellipse parameter Q^{-2} (open circles).

Figure 4.4: Best fit for model 4 (solid curve) in a limited SUSY parameter space, with three dominant contributions to χ^2 : from $\hat{B}_K$ (open squares), from V_{ub}/V_{cb} (open triangles) and from V_{cb} (solid circles).

Figure 4.5: Best fit for model 5 (solid curve) in a limited SUSY parameter space, with three dominant contributions to χ^2 : from $\hat{B}_K$ (open squares), from V_{us} (solid triangles) and from V_{cb} (solid circles).

Figure 4.6: Best fit for model 6 (solid curve) in a limited SUSY parameter space, with three dominant contributions to χ^2 : from α_s (solid dots), from V_{cb} (solid circles) and from the ellipse parameter Q^{-2} (open circles).

Figure 4.7: Best fit for model 7 (solid curve) in a limited SUSY parameter space, with three dominant contributions to χ^2 : from $M_b - M_c$ (solid squares), from V_{cb} (solid circles) and from the ellipse parameter Q^{-2} (open circles).

Figure 4.8: Best fit for model 8 (solid curve) in a limited SUSY parameter space, with three dominant contributions to χ^2 : from $M_b - M_c$ (solid squares), from V_{cb} (solid circles) and from the ellipse parameter Q^{-2} (open circles).

Figure 4.9: Best fit for model 9 (solid curve) in a limited SUSY parameter space, with three dominant contributions to χ^2 : from $\hat{B}_K$ (open squares), from V_{cb} (solid circles) and from the ellipse parameter Q^{-2} (open circles).

increases, squarks become heavier, and the SUSY loop corrections gradually vanish. Hence, from the figures we can conclude, that the best fits, within these models, are not very sensitive to the size of the SUSY corrections. (That means that no more study with different values of μ and $M_{1/2}$ is needed for models other than model 4.)

One immediately sees that the quality of the fits is quite poor. Note that model 4 apparently sticks out as the best. Therefore, this is the only model out of the ADHRS models which we check in more detail in the next section. With the ADHRS assumptions, other models get barely to the $\chi^2 = 30$ limit.

4.4 Model 4 of ADHRS

Let us analyze model 4 in detail, in order to see how it performs in a much larger portion of the SUSY parameter space, and whether its performance can be improved by subtle modifications either in the data or in the assumptions.

Based on the four operators, eq's (4.1 - 4.4), the resulting Yukawa matrices at M_G for model 4 are given by –

$$\lambda_u = \begin{pmatrix} 0 & C & 0 \\ C & 0 & -\frac{1}{3}B \\ 0 & -\frac{4}{3}B & A \end{pmatrix}$$

$$\lambda_d = \begin{pmatrix} 0 & -27C & 0 \\ -27C & Ee^{i\phi} & \frac{1}{9}B \\ 0 & -\frac{2}{9}B & A \end{pmatrix}$$

$$\lambda_e = \begin{pmatrix} 0 & -27C & 0 \\ -27C & 3Ee^{i\phi} & B \\ 0 & 2B & A \end{pmatrix}.$$

Note that Yukawa matrices are defined by the couplings — $H_u Q \lambda_u u^c + H_d Q \lambda_d d^c + H_d L \lambda_e e^c$, in agreement with the eq.(1.25), and that our notation follows the notation of chapter 1.

The best fits give $\chi^2 \sim 13 - 14$ for 5 degrees of freedom. See figures 4.10-4.12 for contour lines of constant χ^2 in the $m_0 - M_{1/2}$ plane for different values of μ .³⁰ The regions with lowest χ^2 contain significant (of order (4 - 9)%) one loop SUSY threshold corrections to fermion masses and mixing angles. These one loop SUSY threshold corrections scale roughly as $\frac{\mu M_{1/2}}{m_0^2}$ or $\frac{\mu A_{33}}{m_0^2}$. Hence, from figures 4.10-4.12, we see that these corrections are necessary to improve the agreement of the model with the data.

In table 4.1 we give the results for the point labelled “II” in figure 4.10. All low energy observables are fit to within their 2σ bounds.

4.4.1 Why Model 4 Cannot Work Better

The fits for 4 observables, (V_{cb} , V_{ub}/V_{cb} , $\hat{B}_K$ and Q) lie close to the 2σ border. Note that Minuit typically tries to equalize the contribution of all the observables to χ^2 . We now argue that these results are true predictions of the theory.

Consider the first three parameters. It was shown by Hall and Rasin[84] that the relation

$$\frac{V_{ub}}{V_{cb}} = \sqrt{\frac{\lambda_u}{\lambda_c}} \quad (4.5)$$

holds for any fermion mass texture in which the 11, 13 and 31 elements of the mass matrices are zero and perturbative diagonalization is permitted. Note λ_u , λ_c are the up and charm quark Yukawa couplings evaluated at a common renormalization scale. A typical value for the right-hand side of the equation is 0.05 which is too small for the left-hand side by more than 20%.

³⁰ $M_{1/2}$ and m_0 are GUT scale values, while μ is given at M_Z .

Figure 4.10: Model 4 global analysis results in the $m_0 - M_{1/2}$ plane, for a fixed value of the Higgs parameter $\mu(M_Z) = 80$ GeV. Solid (dashed) curves represent contour lines of constant $\chi^2 = 15$ (14) / 5dof. The star marks the point ($m_0=700\text{GeV}$, $M_{1/2}=240\text{GeV}$). In the text it is referred to as point II and the corresponding results obtained at this point are listed in table 4.1.

Figure 4.11: Model 4 global analysis results in the $m_0 - M_{1/2}$ plane, for a fixed value of the Higgs parameter $\mu(M_Z) = 160$ GeV. Solid (dashed) curves represent contour lines of constant $\chi^2 = 15$ (14) / 5dof.

Observable	Computed value	Contribution to χ^2	SUSY corrections [in %]
M_Z	91.12	<0.5	1.44 12.93
M_W	80.37	<0.5	
G_μ	1.166×10^{-5}	<0.5	
α_{em}^{-1}	137.0	<0.5	
$\alpha_s(M_Z)$	0.1162	<0.5	
ρ_{new}	$+1.74 \times 10^{-4}$	<0.5	
M_t	173.9	<0.5	0.75
$m_b(M_b)$	4.360	0.82	5.14
$M_b - M_c$	3.146	1.61	7.90
m_s	162.6	<0.5	3.68
m_d/m_s	0.0461	<0.5	0.00
Q^{-2}	0.00173	2.19	1.66
M_τ	1.777	<0.5	-1.94
M_μ	105.6	<0.5	-1.46
M_e	0.5113	<0.5	-1.46
V_{us}	0.2215	<0.5	0.00
V_{cb}	0.0450	3.79	1.53
V_{ub}/V_{cb}	0.0463	2.84	0.00
$\hat{B}_K$	0.9450	2.10	-3.09
$B(b \rightarrow s\gamma)$	2.388×10^{-4}	<0.5	
TOTAL χ^2		14.012	

Table 4.1: Model 4 - Results at point II. (See point II on figure 4.10.) Initial parameters: $1/\alpha_G = 24.33$, $M_G = 3.29 \cdot 10^{16} \text{ GeV}$, $\epsilon_3 = -4.72\%$, $A = 0.734$, $B = 5.67 \cdot 10^{-2}$, $C = 0.865 \cdot 10^{-4}$, $E = 1.04 \cdot 10^{-2}$, $\Phi = 1.76$, $\mu = 80.00 \text{ GeV}$, $m_0 = 700.00 \text{ GeV}$, $M_{1/2} = 240.00 \text{ GeV}$, $m_{H_u}/m_0 = 1.39$, $m_{H_d}/m_0 = 1.22$, $A_0 = 387.79 \text{ GeV}$, $B\mu = 120.01 \text{ GeV}^2$

Figure 4.12: Model 4 global analysis results in the $m_0 - M_{1/2}$ plane, for a fixed value of the Higgs parameter $\mu(M_Z) = 240$ GeV. Solid (dashed, double-dash-dotted) lines represent contour lines of constant $\chi^2 = 15$ (14,13) / 5dof.

We now show that the fits for V_{ub}/V_{cb} , V_{cb} and $\hat{B}_K$ are correlated. Consider the formula for ϵ_K given by

$$\epsilon_K \approx \left(\frac{V_{ub}}{V_{cb}} V_{cb}^2 \sin(\xi) \right) \hat{B}_K \times (\text{one loop factors}) \quad (4.6)$$

where the first factor is just the Jarlskog parameter, J - see eqn.(2.6). We see that if V_{ub}/V_{cb} is small, then V_{cb} and $\hat{B}_K$ must be increased to compensate. As a consequence, V_{cb} and $\hat{B}_K$ are both too large. The addition of 13 and 31 mass terms modifies relation (4.5) and can, in principle, accomodate larger values of V_{ub}/V_{cb} and thus lower values for V_{cb} and $\hat{B}_K$.

Now consider the ellipse parameter Q . This parameter is strongly controlled by the Georgi-Jarlskog relation[83]

$$\frac{m_s}{m_d} \approx \frac{1}{9} \frac{m_\mu}{m_e} \quad (4.7)$$

which is satisfied by model 4. This is an important zeroth order relation to try to satisfy. However unless there are small calculable corrections to this relation, it leads to values of $m_s/m_d \sim 25$ and thus values of Q which are too large. Note, that introducing 13 and 31 terms in the down quark and charged lepton mass matrices can also, in principle, perturb the zeroth order Georgi-Jarlskog relation.

Thus the disagreement between model 4 and the data seems significant. It is unlikely that it can be fixed with the inclusion of small threshold corrections to the Yukawa relations at M_G .³¹

4.4.2 What If One Experimental Value Were Wrong?

Before one adds new operators, however, it is worthwhile to consider the possibility that perhaps one of the experimental measurements is wrong. It is interesting to ask whether the agreement with the data can be significantly improved by removing one contribution to the χ^2 function; essentially discarding one piece of data³². In order to check this possibility we have artificially inflated the value of σ in the analysis for several observables (one at a time), to see if this significantly improves the fit. Our results are given in figure 4.13, where we also state, in the figure caption, the

³¹Note, there are 2 types of threshold corrections. First, there are the higher dimension operator corrections to the 4 operators; obtained when integrating out the Froggatt-Nielsen massive intermediates. These are expected to be of order 10%. Second, there are the usual higher loop corrections.

³²We thank M. Barnett for bringing this idea to our attention.

preferred value of the observable with the inflated error. As one can see, we find no significant improvements with this procedure.

4.4.3 What If Strange Quark Is Much Lighter?

To continue in this thread let's discuss how the performance of model 4 changes if recent lattice results on the low value of the strange quark mass prove to be correct [85]. We repeated the minimization of χ^2 at point II with the value $m_s(1\text{GeV}) = (120 \pm 25)\text{MeV}$ replacing the old data $(180 \pm 50)\text{MeV}$ of table 2.3. We found that the performance of model 4 gets slightly worse in this case: total χ^2 changes from 14.01 to 16.16 as m_s goes from 163MeV down to 156MeV in the best fits at point II.

4.4.4 What If $\tan\beta$ Is Low?

Models with low values of $\tan\beta$ were studied in a separate work [86]. Such models involve an extra free parameter describing the mixing between different Higgs doublets giving mass to fermions. The authors in [86] found that this additional parameter cannot improve the agreement of model 4 with the low energy data. The main problem appeared in fitting the masses of the lighter generations.

4.5 Conclusions

We re-analyzed the nine models of ADHRS with four effective operators. Although the original study [82] put forward models 4,6 and 9, we found that models 6 and 9 are not better than most of the other models. Only model 4 works exceptionally better. Yet, $\chi^2 \approx 14$ with five degrees of freedom means a confidence level of only 3% in this model. We showed that this low confidence level is quite robust and cannot

Figure 4.13: Results for Model 4 with one observable removed from the χ^2 function (the corresponding standard deviation of that observable is inflated by a large factor), at fixed values of the Higgs parameter $\mu(M_Z)=80\text{GeV}$ and $M_{1/2}=240\text{GeV}$. Eight observables tried, (and their values obtained at the minimum of χ^2) are as follows: α_s (0.1144) - dots, $BR(b \rightarrow s\gamma)$ ($2.29 \cdot 10^{-2}$) - diamonds, $M_b - M_c$ (2.84GeV) - solid squares, $1/Q^2$ ($1.50 \cdot 10^{-3}$) - open circles, V_{ub}/V_{cb} (0.045) - triangles, $m_b(M_b)$ (4.68GeV) - stars, B_K (1.12) - open squares, V_{cb} (0.0465) - solid circles. For comparison, the thick solid line represents the best χ^2 values of Model 4 with all observables present with their regular standard deviations as quoted in table 2.3. No significant improvement of Model 4 is observed. Recall that leaving out an observable from the χ^2 function means reducing the number of degrees of freedom by one.

be improved by a single change in the data or by a simple modification of the model at the GUT scale, resulting in low $\tan\beta$.

On the other hand, we have found that we can indeed improve the results of model 4 by adding one operator contributing to the 13 and 31 elements of the Yukawa matrices. We discuss this possibility in the next chapter. The additional terms correspond to one new effective mass operator. Of course there are many possible 13 operators. In this work we have not performed a search over all possible 13 operators. Instead we study two 13 operators which are motivated by two complete $SO(10)$ extensions of model 4 [87].

CHAPTER 5

MODELS WITH THE ADDITIONAL 13 OPERATOR MOTIVATED BY MODEL 4

5.1 Introduction

In this chapter we analyze models derived from complete SUSY GUTs discussed recently by Lucas and Raby[88]. These models were motivated by model 4 of [89]. They represent the simplest $SO(10)$ extensions of this model. An important assumption in these models is naturalness with respect to the $U(1)$ symmetries present. One of the three models studied is actually identical to model 4 of ADHRS when states with mass greater than M_G are integrated out; this is model 4b (see eq.(4.2)). The other two, model 4a and model 4c, include one new effective operator at the GUT scale. We find that model 4a gives a best fit with $\chi^2 \sim 3$ for 3 dof in a small corner of parameter space, while model 4c gives $\chi^2 \leq 1$ for 3 dof over a large subspace of the allowed parameter space.

Our main results are found in figures 5.1-5.6 and tables 5.2-5.5. These include graphs of constant χ^2 contours in the two-dimensional soft SUSY parameter space labelled by m_0 , a universal squark and slepton mass parameter, and $M_{1/2}$, a universal gaugino mass parameter both with values given at M_G . We have several such plots for different values of the Higgs mass parameter μ , given at M_Z . In the tables we

give the computed values for the low energy observables for a few selected points in parameter space. We also include values for the Higgs and SUSY spectra, the CP violating angles α , β and γ , measurable in neutral B decays, the Wolfenstein parameters η , ρ , and the ratio m_u/m_d for these points. In figure 5.7, we present a scatter plot for values of $(\sin 2\alpha, \sin 2\beta)$ predicted by models 4a and 4c for points in figures 5.1-5.3 and 5.4-5.6, respectively, with χ^2 values ≤ 3 . Finally we test the sensitivity of χ^2 to changes in various parameters and to an intermediate neutrino scale.

5.2 Effective Operators of Models 4a, 4b and 4c

Models 4a, 4b and 4c were constructed as simple extensions of model 4 of ADHRS. The label (a,b,c) refers to different possible 22 operators which give identical Clebsch relations for the 22 element of the Yukawa matrices, eq.(4.2).³³ In the extension to a complete SUSY GUT these different 22 operators lead to inequivalent theories. The different theories are defined by the inequivalent U(1) quantum numbers of the states. When one demands “naturalness”, i.e. includes all terms in the superspace potential consistent with the symmetries of the theory one finds an additional 13 operator for models 4a and 4c given by –

$$\mathcal{O}_{13} =$$

$$(a) \quad 16_1 \left(\frac{\tilde{A}}{S_M} \right)^3 10_1 \left(\frac{\tilde{A}A_2}{S_M^2} \right) 16_3 \quad (5.1)$$

³³Note, models d, e and f have the second family 16_2 coupled directly to 10_1 and a heavy 16. If this coupling is as large as the third generation Yukawa coupling, then we would obtain excessively large flavor changing neutral current processes, such as $\mu \rightarrow e\gamma$. Thus these models were not considered in [88].

model	u_u	u'_u	u_d	u'_d	u_e	u'_e
a	$-4/3$	$1/3$	-2	-9	-54	3
c	$-4/3$	$1/3$	$2/3$	-9	-54	-1

Table 5.1: u Clebsches for models 4a and 4c

$$(c) \quad 16_1 \left(\frac{\tilde{A}}{S_M} \right)^3 10_1 \left(\frac{A_2}{S_M} \right) 16_3 \quad (5.2)$$

Model 4b, on the other hand, is identical to model 4 of ADHRS when states with mass greater than M_G are integrated out, since the $U(1)$ symmetries present in this model do not allow any new operator. The results for this model are identical to the model 4 results presented in the previous chapter; thus we will not discuss it further. In models 4a and 4c, the addition of the 13 operator introduces two new real parameters in the Yukawa matrices at M_G ; thus we have $n_y = 7$ and three degrees of freedom. Models 4a and 4c differ only by the 13 operator. The resulting Yukawa matrices are given by –

$$\lambda_u = \begin{pmatrix} 0 & C & u_u D e^{i\delta} \\ C & 0 & -\frac{1}{3}B \\ u'_u D e^{i\delta} & -\frac{4}{3}B & A \end{pmatrix}$$

$$\lambda_d = \begin{pmatrix} 0 & -27C & u_d D e^{i\delta} \\ -27C & E e^{i\phi} & \frac{1}{9}B \\ u'_d D e^{i\delta} & -\frac{2}{9}B & A \end{pmatrix}$$

$$\lambda_e = \begin{pmatrix} 0 & -27C & u_e D e^{i\delta} \\ -27C & 3E e^{i\phi} & B \\ u'_e D e^{i\delta} & 2B & A \end{pmatrix}$$

where the new Clebsches for models 4(a,c) are given in table 5.1.

5.3 Results of Global Analysis

We find that model 4a gives a best fit $\chi^2 \sim 4$ for 3 dof, while model 4c gives $\chi^2 \leq 1$ for 3 dof. For model 4c, the recorded best fit comes with χ^2 as low as 0.168. It is found for $\mu = 160\text{GeV}$, $M_{1/2} = 400\text{GeV}$ and $m_0 = 2900\text{GeV}$. Our results are presented in figures 5.1-5.3, for model 4a, and in figures 5.4-5.6, for model 4c.

5.3.1 Why 13 Operator Helps

In order to understand why the 13 operator resolves the problems of Model 4 discussed in the previous chapter, consider the approximate formulas (valid to 10% and particular to model 4c) for V_{ub}/V_{cb} , V_{cb} , λ_s , λ_d , λ_μ , λ_e where the latter are the diagonalized Yukawa eigenvalues and all are evaluated at the GUT scale. We have

$$\begin{aligned}
 V_{ub}/V_{cb} &= \frac{9 C A}{4 B^2} \\
 V_{cb} &= \frac{4 B}{9 A} \\
 \lambda_s &= E \\
 \lambda_d &= \frac{729 C^2}{E} \left(1 - \frac{31 B D}{243 C A} e^{-i\delta} \right) \\
 \lambda_\mu &= 3 E \\
 \lambda_e &= \frac{243 C^2}{E} \left(1 - \frac{109 B D}{27 C A} e^{-i\delta} \right)
 \end{aligned} \tag{5.3}$$

$$\tag{5.4}$$

Recall, the contribution of the 13 operator is proportional to D . Consider first the electron and down quark Yukawa couplings. The electron gets a significant correction from the 13 operator whereas the correction to the down quark is much smaller. When one adds the 13 operator to a model which is already fit to the lepton masses, one must readjust the parameters. In order to keep the electron mass fixed, one needs to

Figure 5.1: Model 4a global analysis results in the $m_0 - M_{1/2}$ plane, for a fixed value of the Higgs parameter $\mu(M_Z) = 80$ GeV. Solid (dashed) curves represent contour lines of constant $\chi^2 = 6$ (4) / 3dof.

Figure 5.2: Model 4a global analysis results in the $m_0 - M_{1/2}$ plane, for a fixed value of the Higgs parameter $\mu(M_Z) = 160$ GeV. Solid (dashed, double-dash-dotted) curves represent contour lines of constant $\chi^2 = 6$ (4,3) / 3dof.

Figure 5.3: Model 4a global analysis results in the $m_0 - M_{1/2}$ plane, for a fixed value of the Higgs parameter $\mu(M_Z) = 240$ GeV. Solid (dashed, double-dash-dotted) curves represent contour lines of constant $\chi^2 = 6$ (4,3) / 3dof.

increase the parameter C [note, in the fits $\delta \sim 2\pi$]; the parameter E is kept fixed in order not to change the muon mass. As a consequence the ratio V_{ub}/V_{cb} increases, which allows for smaller values of $\hat{B}_K$. In addition we see that the Georgi - Jarlskog relation is now given by

$$\frac{m_s}{m_d} \approx \frac{1}{9} \left(\frac{1 - \frac{109}{27} \frac{B D}{C A} e^{-i\delta}}{1 - \frac{31}{243} \frac{B D}{C A} e^{-i\delta}} \right) \frac{m_\mu}{m_e}$$

As a consequence, the ratio $1/9$ is further decreased; hence Q is reduced. Thus one new operator is able to resolve four problems.

Figure 5.4: Model 4c global analysis results in the $m_0 - M_{1/2}$ plane, for a fixed value of the Higgs parameter $\mu(M_Z) = 80$ GeV. Solid (double-dash-dotted, dotted) lines represent contour lines of constant $\chi^2 = 6$ (3,1) / 3dof. The stars mark the points (400,280) and (700,240). In the text, these are referred to as points I and II. The results and predictions obtained at these points are listed in tables 5.2, 5.3 and 5.6.

Figure 5.5: Model 4c global analysis results in the $m_0 - M_{1/2}$ plane, for a fixed value of the Higgs parameter $\mu(M_Z) = 160$ GeV. Solid (double-dash-dotted, dotted) lines represent contour lines of constant $\chi^2 = 6$ (3,1) / 3dof. The star marks the point (1400,170). In the text, it is referred to as point III1 (point III at $\mu=160\text{GeV}$). The results and predictions obtained at this point are listed in tables 5.4 and 5.7.

Observable	Computed value	Contribution to χ^2	SUSY corrections [in%]
M_Z	91.12	< 0.1	
M_W	80.34	< 0.1	
G_μ	$1.164 \cdot 10^{-5}$	< 0.1	
α_{EM}^{-1}	136.9	< 0.1	1.25
$\alpha_s(M_Z)$	0.1132	0.93	12.66
ρ_{new}	$+9.87 \cdot 10^{-5}$	< 0.1	
M_t	173.5	< 0.1	0.93
$m_b(M_b)$	4.311	0.22	4.96
$M_b - M_c$	3.499	0.24	6.64
m_s	184.6	< 0.1	4.95
m_d/m_s	0.0496	< 0.1	0.00
Q^{-2}	0.00205	< 0.1	2.01
M_τ	1.776	< 0.1	-4.00
M_μ	105.7	< 0.1	-2.76
M_e	0.5110	< 0.1	-2.76
V_{us}	0.2205	< 0.1	0.00
V_{cb}	0.0406	0.21	2.49
V_{ub}/V_{cb}	0.0744	< 0.1	0.00
$\hat{B}_K$	0.8196	< 0.1	-5.05
$B(b \rightarrow s\gamma)$	$2.575 \cdot 10^{-4}$	< 0.1	
TOTAL χ^2		2.039	

Table 5.2: **Model 4c - Results at point I.** (see point I on fig. 5.4)
Initial parameters: $1/\alpha_G = 24.43$, $M_G = 2.50 \cdot 10^{16} \text{GeV}$, $\epsilon_3 = -4.76\%$, $A = 0.764$,
 $B = 5.26 \cdot 10^{-2}$, $C = 1.10 \cdot 10^{-4}$, $D = 4.63 \cdot 10^{-4}$, $\delta = 5.70$, $E = 1.25 \cdot 10^{-2}$, $\Phi = 1.07$,
 $\mu = 80.00 \text{GeV}$, $m_0 = 400.00 \text{GeV}$, $M_{1/2} = 280.00 \text{GeV}$, $m_{H_d}/m_0 = 1.77$, $m_{H_u}/m_0 = 1.59$,
 $A_0 = 322.21 \text{GeV}$, $B\mu = 120.00 \text{GeV}^2$

Observable	Computed value	Contribution to χ^2	SUSY corrections [in%]
M_Z	91.12	0.02	
M_W	80.38	< 0.02	
G_μ	$1.166 \cdot 10^{-5}$	< 0.02	
α_{EM}^{-1}	137.0	< 0.02	1.43
$\alpha_s(M_Z)$	0.1151	0.34	12.78
ρ_{new}	$+1.87 \cdot 10^{-4}$	0.09	
M_t	175.7	< 0.02	0.74
$m_b(M_b)$	4.287	0.06	5.43
$M_b - M_c$	3.440	0.04	7.56
m_s	189.0	0.03	3.68
m_d/m_s	0.0502	< 0.02	0.00
Q^{-2}	0.00204	< 0.02	1.78
M_τ	1.776	< 0.02	-2.08
M_μ	105.7	< 0.02	-1.50
M_e	0.5110	< 0.02	-1.50
V_{us}	0.2205	< 0.02	0.00
V_{cb}	0.0400	0.07	1.58
V_{ub}/V_{cb}	0.0772	< 0.02	0.00
$\hat{B}_K$	0.8140	< 0.02	-3.18
$B(b \rightarrow s\gamma)$	$2.382 \cdot 10^{-4}$	< 0.02	
TOTAL χ^2		0.7306	

Table 5.3: Model 4c - Results at point II. (see point II on fig. 5.4)
Initial parameters: $1/\alpha_G = 24.36$, $M_G = 3.17 \cdot 10^{16} \text{GeV}$, $\epsilon_3 = -4.89\%$, $A = 0.807$,
 $B = 5.44 \cdot 10^{-2}$, $C = 1.15 \cdot 10^{-4}$, $D = 4.94 \cdot 10^{-4}$, $\delta = 5.71$, $E = 1.31 \cdot 10^{-2}$, $\Phi = 1.04$,
 $\mu = 80.00 \text{GeV}$, $m_0 = 700.00 \text{GeV}$, $M_{1/2} = 240.00 \text{GeV}$, $m_{H_d}/m_0 = 1.42$, $m_{H_u}/m_0 = 1.24$,
 $A_0 = 458.35 \text{GeV}$, $B\mu = 120.66 \text{GeV}^2$

Observable	Computed value	Contribution to χ^2	SUSY corrections [in%]
M_Z	91.12	0.02	
M_W	80.38	< 0.02	
G_μ	$1.166 \cdot 10^{-5}$	< 0.02	
α_{EM}^{-1}	137.0	< 0.02	1.79
$\alpha_s(M_Z)$	0.1154	0.26	12.89
ρ_{new}	$+1.50 \cdot 10^{-4}$	0.08	
M_t	176.9	0.10	0.62
$m_b(M_b)$	4.275	< 0.02	6.27
$M_b - M_c$	3.429	0.02	8.76
m_s	188.3	0.03	2.85
m_d/m_s	0.0512	< 0.02	0.00
Q^{-2}	0.00203	< 0.02	1.68
M_τ	1.777	< 0.02	-1.41
M_μ	105.7	< 0.02	-0.82
M_e	0.5110	< 0.02	-0.81
V_{us}	0.2205	< 0.02	0.01
V_{cb}	0.0397	0.03	1.62
V_{ub}/V_{cb}	0.0797	< 0.02	0.00
$\hat{B}_K$	0.8000	< 0.02	-3.26
$B(b \rightarrow s\gamma)$	$2.311 \cdot 10^{-4}$	< 0.02	
TOTAL χ^2		0.5868	

Table 5.4: Model 4c - Results at point III1. (see point III1 on fig. 5.5)
Initial parameters: $1/\alpha_G = 24.51$, $M_G = 3.33 \cdot 10^{16} \text{GeV}$, $\epsilon_3 = -4.34\%$, $A = 0.852$,
 $B = 5.63 \cdot 10^{-2}$, $C = 1.21 \cdot 10^{-4}$, $D = 5.06 \cdot 10^{-4}$, $\delta = 5.70$, $E = 1.36 \cdot 10^{-2}$, $\Phi = 1.02$, μ
 $= 160.00 \text{GeV}$, $m_0 = 1400.00 \text{GeV}$, $M_{1/2} = 170.00 \text{GeV}$, $m_{H_d}/m_0 = 1.33$, $m_{H_u}/m_0 =$
 1.14 , $A_0 = -982.43 \text{GeV}$, $B\mu = 123.72 \text{GeV}^2$

Observable	Computed value	Contribution to χ^2	SUSY corrections [in%]
M_Z	91.12	< 0.10	1.85 12.53
M_W	80.35	< 0.10	
G_μ	$1.165 \cdot 10^{-5}$	< 0.10	
α_{EM}^{-1}	136.9	< 0.10	
$\alpha_s(M_Z)$	0.1124	1.25	
ρ_{new}	$+4.60 \cdot 10^{-5}$	< 0.10	
M_t	176.5	< 0.10	0.61
$m_b(M_b)$	4.306	0.18	9.73
$M_b - M_c$	3.486	0.19	13.45
m_s	178.5	< 0.10	4.09
m_d/m_s	0.0500	< 0.10	0.00
Q^{-2}	0.00203	< 0.10	1.91
M_τ	1.776	< 0.10	-1.93
M_μ	105.7	< 0.10	-1.06
M_e	0.5110	< 0.10	-1.06
V_{us}	0.2205	< 0.10	0.01
V_{cb}	0.0402	0.11	1.73
V_{ub}/V_{cb}	0.0777	< 0.10	0.00
$\hat{B}_K$	0.7949	< 0.10	-3.49
$B(b \rightarrow s\gamma)$	$2.269 \cdot 10^{-4}$	< 0.10	
TOTAL χ^2		1.9477	

Table 5.5: **Model 4c - Results at point III2.** (see point III2 on fig. 5.6)
Initial parameters: $1/\alpha_G = 24.65$, $M_G = 2.88 \cdot 10^{16} \text{GeV}$, $\epsilon_3 = -4.45\%$, $A = 0.888$,
 $B = 5.89 \cdot 10^{-2}$, $C = 1.23 \cdot 10^{-4}$, $D = 5.69 \cdot 10^{-4}$, $\delta = 5.74$, $E = 1.40 \cdot 10^{-2}$, $\Phi = 1.02$, μ
 $= 240.00 \text{GeV}$, $m_0 = 1400.00 \text{GeV}$, $M_{1/2} = 170.00 \text{GeV}$, $m_{H_d}/m_0 = 1.33$, $m_{H_u}/m_0 =$
 1.14 , $A_0 = -1079.39 \text{GeV}$, $B\mu = 128.50 \text{GeV}^2$

Observable	Predictions at point I				Predictions at point II			
$m_u/m_d _{1\text{GeV}}$	0.409				0.440			
$\sin^2\theta_W^{(MSSM)}$	0.2342				0.2334			
$\sin^2\theta_W^{(SM)}$	0.2323				0.2320			
$\sin 2\alpha$	0.953				0.966			
$\sin 2\beta$	0.513				0.522			
$\sin \gamma$	0.935				0.928			
ρ	-0.120				-0.130			
η	0.316				0.325			
$\tan\beta$	52.77				54.38			
CP even Higgses	73.43	110.81			75.24	110.35		
CP odd Higgs	73.45				75.25			
charged Higgs	115.65				116.92			
gluino	725				630			
charginos	70	260			67	232		
neutralinos	52	95	129	260	48	97	113	232
up squarks	474	614	747	771	487	603	887	905
down squarks	510	552	749	775	513	550	893	909
charged sleptons	66	338	423	448	294	550	716	718
sneutrinos	327		440	440	544		713	714

Table 5.6: Model 4c - More Results at points I and II. (see points I and II on fig. 5.4) In Higgs sector, we quote all masses with one loop corrections included. For superpartners, masses are at tree level. For squark and slepton masses, the first two columns are for the third family which are significantly split, while the third and fourth columns are mean values for the nearly degenerate states of the second and first families, respectively.

Observable	Predictions at point III1				Predictions at point III2			
$m_u/m_d _{1\text{GeV}}$	0.475				0.432			
$\sin^2\theta_W^{(MSSM)}$	0.2337				0.2344			
$\sin^2\theta_W^{(SM)}$	0.2325				0.2331			
$\sin 2\alpha$	0.973				0.977			
$\sin 2\beta$	0.531				0.516			
$\sin \gamma$	0.925				0.918			
ρ	−0.138				−0.140			
η	0.334				0.323			
$\tan\beta$	55.39				55.86			
CP even Higgses	71.70	112.36			66.70	112.05		
CP odd Higgs	71.70				66.70			
charged Higgs	115.43				112.22			
gluino	447				440			
charginos	101	218			122	274		
neutralinos	62	105	174	217	67	122	251	272
up squarks	558	677	1443	1460	546	666	1440	1458
down squarks	597	641	1459	1462	574	639	1457	1460
charged sleptons	600	1062	1397	1418	564	1051	1397	1418
sneutrinos	1052	1394 1395			1047	1394 1395		

Table 5.7: Model 4c - More Results at points III1 and III2. (see points III1 and III2 on figures 5.5 and 5.6) Higgs, squark and slepton masses are treated in the same way as for the results quoted on table 5.6.

Figure 5.6: Model 4c global analysis results in the $m_0 - M_{1/2}$ plane, for a fixed value of the Higgs parameter $\mu(M_Z) = 240$ GeV. Solid (double-dash-dotted, dotted) lines represent contour lines of constant $\chi^2 = 6$ (3,1) / 3dof. The star marks the point (1400,170). In the text, it is referred to as point III2 (point III at $\mu=240\text{GeV}$). The corresponding results and predictions obtained at this point are listed in tables 5.5 and 5.7.

5.4 Discussion of Model 4c Results

Let us now discuss our results for model 4c. We find that the preferred region of SUSY parameter space with a fixed value of $\mu = 80\text{GeV}$ corresponds to $M_{1/2} > 220\text{GeV}$ and $m_0 > 300\text{GeV}$. The lower bound on m_0 depends slowly on $M_{1/2}$ while the lower bound on $M_{1/2}$ appears to be independent of m_0 . These bounds are rather distinct (the χ^2 value rises steeply when getting closer to these values) since they result from the lower experimental limits on sparticle masses. No observable from our list in table 2.3 places a decisive constraint which would exclude a portion of

the $(m_0, M_{1/2})$ parameter space at fixed $\mu = 80\text{GeV}$. In other words, for this value of μ everything in the $(m_0, M_{1/2})$ plane that has not been excluded by direct experimental searches, is allowed (provided the other parameters are subject to the optimization procedure).

As μ increases (see figures 5.5 and 5.6 for $\mu = 160, 240\text{ GeV}$, respectively) the lower bound on $M_{1/2}$ goes down since it is determined by the LEP limits on the masses of the lightest chargino and neutralino. The latter become proportional to, but less than $M_{1/2}$ in the limit of large μ . In addition, as μ increases the χ^2 profile in the m_0 direction starts to change more smoothly, and the contour lines of constant χ^2 shift more towards the higher values of m_0 . This is explained by the fact that the constraints placed by the observables which receive one loop corrections enhanced by $\tan\beta$ start to be significant. (Especially the b quark mass correction and subsequently the correction to $M_b - M_c$.) Since these one-loop SUSY threshold corrections scale roughly as $\mu M_{1/2}/m_0^2$ or $\mu A_{33}/m_0^2$, we see that with increasing μ a larger suppression by m_0 is required to keep the potentially large corrections under control. The effect clearly shows up in figures 5.4, 5.5 and 5.6 combined.

5.4.1 Role of SUSY Threshold Corrections

Our results show that the SUSY sector can make its presence visible in the analysis of the SM fermion mass parameters, especially those suppressed by $\tan\beta$ at tree level, through the low energy threshold corrections. For $\chi^2 \sim 1$ the latter are up to 9%. Two notes are in order at this point: First, note that these corrections have the potential to be as large as 30-70% ³⁴[90]. It would be interesting if a window in

³⁴Clearly, such large corrections would not violate perturbativity, since the main reason for them being large would be that the corresponding tree level values happen to be $\tan\beta$ suppressed. That

parameter space with such large SUSY contributions was favored. However, our numerical optimization never disclosed such a region. Second, it is interesting to note that the best fit is found in the region of very large m_0 , where the effect of the SUSY threshold corrections to fermion masses and mixings is, in fact, minimized. As a result, in this region, with $\chi^2 < 1$ and the SUSY corrections to fermion masses negligible, the effective number of degrees of freedom is actually larger than 3, since there are 7 parameters in the Yukawa matrices determining the 13 low energy observables in the fermion mass sector. *This means that the Yukawa sector of the selected model does actually a much better job than appears at first glance.*

For the opposite sign of μ there is a significant restriction from the process $b \rightarrow s\gamma$. As has been noticed by many authors, the SM amplitude with the W in the loop is sufficiently big to explain the measured branching ratio. In the large $\tan\beta$ regime of the MSSM however, there are substantial partial amplitudes due to the charged Higgs and chargino loops. The charged Higgs amplitude is always of the same sign as the SM amplitude, while the sign of the chargino amplitude depends on the sign of μA_t . In order to get an agreement with the measured rate, the chargino amplitude, especially with large $\tan\beta$, has to enter with the opposite sign, and as a consequence a specific sign of μA_t is preferred. Due to the boundary conditions at the GUT scale which are used throughout this paper we always run into the region at the Z scale where $A_t < 0$, and as a result the sign of μ is fixed. ($\mu > 0$, in our convention.) With the bad sign of μ , the best χ^2 values are about 30.³⁵ For this reason we do not quote means that the higher order corrections would be well under control as for any other quantity not suppressed at tree level.

³⁵Obviously, if superpartners are *very* heavy, the SUSY spectrum decouples and the above statement is incorrect. However, for large $\tan\beta$, “*very heavy*” in this case means deep in the TeV region which we don’t consider in our analysis.

results or figures for $\mu < 0$. Interestingly enough, even if we exclude the branching ratio for $b \rightarrow s\gamma$ from the list of observables and repeat the optimization procedure for $\mu < 0$, the conspiracy between the one loop corrections proportional to μ (see the two previous paragraphs) does not work very well and significantly worse χ^2 values are obtained than in the case $\mu > 0$.

Finally note that $|\mu|$ less than about 65GeV is excluded since there would inevitably be a light higgsino-like neutralino and chargino; already ruled out by experiment. Moreover, the large values of m_0 selected by our fit, imply a suppression of the supersymmetric corrections to the anomalous magnetic moment of the muon, which could otherwise lead to very strong constraints on the allowed parameter space in the large $\tan\beta$ region [91].

In tables 5.2 - 5.5 we give the values of the initial parameters which minimize χ^2 for fixed particular values of μ , m_0 , $M_{1/2}$ labelled as points I-III marked in figures 5.4-5.6. For each low energy observable we quote its computed value, the partial contribution to χ^2 and the relative magnitude of the SUSY threshold corrections [in %]. We also include in tables 5.6 and 5.7 values for the SUSY and Higgs spectra, the unitarity triangle parameters ρ , η together with the corresponding angles α , β and γ , measurable in neutral B decays, and the predicted ratio for m_u/m_d .

5.4.2 Predictions for $\sin 2\alpha$ and $\sin 2\beta$

Note that the values for α , β and γ do not significantly change across different points in the SUSY parameter space. This fact is explicitly evident from figure 5.7.

Each “x”-symbol in this figure stands for a point anywhere in the SUSY parameter space with the minimum of χ^2 less than 3, with 3 degrees of freedom present both

(a)

(b)

Figure 5.7: Stability of the predictions for $\sin 2\alpha$ and $\sin 2\beta$, the parameters of the unitarity triangle. “x” symbols denote the points predicted by a) model 4c, b) model 4a, at those regions in the SUSY parameter space where $\chi^2 < 3 / 3\text{dof}$. Dots in the figures represent the boundary of the allowed region resulting from a combined general Standard Model fit ali for $f_{B_d} = 200 \pm 40 \text{ MeV}$, $B_{B_d} = 1.0$, $\hat{B}_K = 0.75 \pm 0.10$.

in model 4c and model 4a. Note that, under such a restriction in model 4a, only the corner in the SUSY parameter space with very large m_0 and $M_{1/2}$ contributes to figure 5.7b. Nevertheless, these two models give similar, narrowly spread predictions for $\sin 2\alpha$ and $\sin 2\beta$, grouped around the values 0.95 and 0.52, respectively, while a much larger region is allowed for these two observables as a result of a general Standard Model analysis [92].

5.4.3 Sensitivity to SUSY box diagrams

We have also checked the sensitivity of our results, for point I in figure 5.4, to a potential 10% positive enhancement of ϵ_K . We have found that the total χ^2 stays the same to within a few per cent and that our predictions for $\sin 2\alpha$ ($\sin 2\beta$) change from 0.953 (0.513) to 0.957 (0.527). Indeed, over a large range of points, we checked that the predictions for $\sin 2\alpha$ ($\sin 2\beta$) change by less than 1% (3%). Thus our results, for model 4c, are not very sensitive to SUSY box corrections to ϵ_K . On the other hand, for point II (figure 5.1) in model 4a, we find that a 10% positive enhancement of ϵ_K significantly improves the agreement with experiment; χ^2 changes from 6.99 to 4.57. This is because in this case, $\hat{B}_K$ was a significant constraint on the model.

5.4.4 Sensitivity to ϵ_3

It is an interesting feature of most of the points in the SUSY parameter space that the best fits favor a rather large negative GUT threshold correction ϵ_3 eq.(2.10). In figure 5.8 the sensitivity of the best fits to different lower bounds on ϵ_3 is displayed for values of μ and $M_{1/2}$ fixed and m_0 varied. In addition table 5.8 shows how the selected GUT scale parameters α_G , M_G and A (the 33 element in the Yukawa matrices) vary in the best fits at $m_0=700\text{GeV}$ (point II), in order to compensate for changes in

Figure 5.8: The dependence of the quality of the fit on the GUT threshold correction ϵ_3 , in model 4c. The thick solid line represents the results of model 4c with ϵ_3 unrestricted. The dotted line constrains ϵ_3 to be positive (and the optimization procedure always settles ϵ_3 very close to zero). Curves with solid squares, open circles and stars correspond to the results of the optimization performed with negative lower limit on ϵ_3 -3%, -2% and -1%, respectively. Curves with open triangles, solid circles, open squares and solid triangles correspond to the cases with lower bound on ϵ_3 greater than zero, namely 1%, 2%, 3% and 4%, respectively. In all cases the optimization procedure tends to yield the values of ϵ_3 very close to the lower bound. Table 5.8 displays the effect of different ϵ_3 lower bounds on various GUT parameters and low energy observables at point II (i.e. at $m_0=700\text{GeV}$).

Figure 5.9: The same as in figure 5.8, with a tighter theoretical uncertainty on precisely measured electroweak observables. In this figure the uncertainty for M_Z , G_μ and α_{em} is set to 0.1% and the actual experimental error 130MeV is assigned to $\sigma(M_W)$; as opposed to the uncertainty of 0.5% for M_Z , M_W and α_{em} , and $\sigma(G_\mu)=1.0\%$, assumed in figure 5.8. As before, in all restricted cases the optimization procedure tends to yield the values of ϵ_3 very close to the lower bound. Table 5.9 displays the effect of different ϵ_3 lower bounds on various GUT parameters and low energy observables at point II (i.e. at $m_0=700\text{GeV}$).

ϵ_3	$1/\alpha_G$	$M_G \times 10^{-16}$	A	$\alpha_s(M_Z)$	$G_\mu \times 10^5$	M_{top}	$m_b(M_b)$	χ^2_{total}
-4.89%	24.362	3.17	0.807	0.1151	1.166	175.7	4.287	0.731
-3.00%	24.628	2.10	0.808	0.1154	1.161	176.1	4.291	0.959
-2.00%	24.767	1.69	0.802	0.1156	1.159	176.1	4.295	1.279
-1.00%	24.907	1.37	0.813	0.1158	1.157	176.6	4.295	1.694
0.00%	25.044	1.11	0.818	0.1160	1.155	177.0	4.296	2.244
+1.00%	25.177	0.89	0.794	0.1162	1.153	176.4	4.304	2.944
+2.00%	25.314	0.73	0.810	0.1164	1.151	177.1	4.302	3.675
+3.00%	25.450	0.59	0.827	0.1166	1.149	177.9	4.303	4.583
+4.00%	25.578	0.48	0.797	0.1167	1.147	177.1	4.309	5.534

Table 5.8: **Model 4c - Varying Lower Bound on ϵ_3 at point II.** Best fit values of selected GUT parameters and low energy observables are displayed for different lower bounds on ϵ_3 . Note that the optimization procedure pushes ϵ_3 to its lower limit. The first line corresponds to the unrestricted case and the corresponding values are taken over from table 5.3 for the sake of completeness.

ϵ_3	$1/\alpha_G$	$M_G \times 10^{-16}$	A	$\alpha_s(M_Z)$	$G_\mu \times 10^5$	M_{top}	$m_b(M_b)$	χ^2_{total}
-5.36%	24.295	3.53	0.807	0.1150	1.166	175.6	4.286	1.20
-3.00%	24.494	2.55	0.808	0.1186	1.165	177.4	4.418	7.49
-2.00%	24.579	2.21	0.803	0.1201	1.165	177.9	4.471	13.97
0.00%	24.759	1.64	0.818	0.1229	1.164	179.6	4.560	34.02

Table 5.9: **Model 4c - Sensitivity of the results of table 5.8 to the magnitude of the theoretical uncertainties.** In comparison to table 5.8, the best fit values of the same GUT parameters and low energy observables are displayed for different lower bounds on ϵ_3 , but this time for the uncertainty $\sigma=0.1\%$ for M_Z , G_μ and α_{em} , and the experimental value $\sigma=130\text{MeV}$ for M_W . ($\sigma=0.5\%$ for M_Z , M_W and α_{em} , and $\sigma=1.0\%$ for G_μ was assumed in table 5.8, as well as throughout this paper.) The first line corresponds to the case when ϵ_3 is unrestricted.

the lower bound on ϵ_3 . Our study shows that even small positive GUT threshold corrections 1-2% to α_s are plausible in model 4c. As one can see in table 5.8, this would require a rather low value of the GUT scale, below 1×10^{16} GeV, and lower α_G . As a result, the best fit value of $\alpha_s(M_Z) \sim 0.116$ is getting closer to its central value 0.118, while G_μ is getting worse. On the other hand, the A parameter of the Yukawa matrices is quite insensitive to the change in ϵ_3 and to the induced changes in α_G and M_G , and so are the masses of the heaviest generation.

We note that our results in this regard differ from those of a recent paper by Pierce et al.[93] in which a general MSSM analysis is performed. We believe that a significant part of the difference results from our assumed theoretical uncertainties in the electroweak observables. Recall, in our analysis we have assumed an overall 0.5% theoretical uncertainty (σ) for M_Z , M_W , and α_{EM} and a 1% uncertainty for G_μ . As commented previously, we believe these uncertainties correctly account for the theoretical errors introduced by neglecting higher orders in perturbation theory and by the numerical analysis. However for comparison, we have changed the theoretical uncertainties for M_Z , α_{EM} , and G_μ to 0.1% and for M_W to the experimental value of 130 MeV; the new results, for model 4c, are displayed in figure 5.9 and table 5.9. In table 5.9 we see that in this case α_s varies significantly with changes in ϵ_3 . Moreover, as ϵ_3 increases the quality of the fit quickly deteriorates. This is seen graphically in figure 5.9. We find, for point II in model 4c with $\epsilon_3 = -3\%$ (-2%), $\chi^2 = 7.49$ (13.97) with the dominant contributions to χ^2 coming from $M_Z - 2.11$ (3.67), $G_\mu - 0.89$ (1.81), $\alpha_{EM} - 0.74$ (1.46) and $m_b(M_b) - 2.05$ (3.69). Thus the theoretical uncertainties for the precisely measured electroweak parameters allow for quite a bit of flexibility in the values of ϵ_3 and $\alpha_s(M_Z)$.

5.4.5 Notes on Higgs Sector

Since we have focused on fermion masses, we have not discussed the Higgs sector of particular models in great detail in this work. However, in figure 5.10 we show the sensitivity of the best fits to the assumed lower bound of the pseudoscalar mass, at fixed values of $\mu=80\text{GeV}$ and $M_{1/2}=240\text{GeV}$ in model 4c. As one can see, requiring a heavier pseudoscalar (A) forces the model into the region of larger m_0 and thus heavier squarks and sleptons. Or in other words, in order to have light squarks and sleptons, one necessarily also has a light A.

To understand these properties, it is instructive to analyze the renormalization group evolution of the mass parameters. For a top quark mass $M_t \simeq 175 \text{ GeV}$, the low energy soft supersymmetry breaking mass parameters are approximately given by [94],

$$\begin{aligned}
m_{H_u}^2 &\simeq \frac{2}{3} \left(m_{H_u}^2(0) - m_0^2 \right) - 2.5 M_{1/2}^2, \\
m_{H_d}^2 &\simeq \frac{2}{3} \left(m_{H_d}^2(0) - m_0^2 \right) - 2.3 M_{1/2}^2 - \frac{0.2}{3} \left(2 m_0^2 + m_{H_d}^2(0) \right), \\
m_Q^2 &\simeq \frac{5 m_0^2}{9} - \frac{m_{H_u}^2(0) + m_{H_d}^2(0)}{9} + 5 M_{1/2}^2, \\
m_D^2 &\simeq \frac{5 m_0^2}{9} - \frac{2 m_{H_d}^2(0)}{9} + 5 M_{1/2}^2,
\end{aligned} \tag{5.5}$$

where m_Q and m_D are the left and right handed sbottom mass parameters, and we have ignored the A_0 contribution, as well as the small $m_{H_u}(m_{H_d})$ effect on the running of $m_{H_u}(m_{H_d})$. Moreover, in the large $\tan \beta$ regime there is a simple relation, eq.(1.44) between the Higgs mass parameters and the pseudoscalar Higgs mass derived in chapter 1 :

$$m_A^2 \simeq m_{H_d}^2 - m_{H_u}^2 - M_Z^2. \tag{5.6}$$

Figure 5.10: The variation of the quality of the fit for different lower bounds on tree level pseudoscalar mass, in model 4c. The thick solid line represents the results of model 4c with $m_A^{tree} > 80\text{GeV}$. When very close to this limit, the pseudoscalar mass receives negative corrections, and its one loop corrected value turns out to be at the experimental limit (see tables 5.6 and 5.7 for the results at point $m_0=700\text{GeV}$). Curves with solid and open circles, solid squares, and stars correspond to the lower limit on m_A^{tree} of 130GeV, 200GeV, 300GeV and 500GeV, respectively.

Using the fact that $m_{H_u}^2 \simeq -M_Z^2/2 - \mu^2$ (compare with equations (1.42) and (1.20)), we obtain

$$m_{H_u}^2(0) - m_0^2 \simeq 4M_{1/2}^2 - \frac{3}{2} \left(\mu^2 + \frac{M_Z^2}{2} \right). \quad (5.7)$$

Since the best fit is obtained for moderate values of the one-loop down quark mass corrections, low values of the μ parameter $\mu^2 \ll m_0^2$ are preferred. For $M_{1/2} > \mu \simeq \mathcal{O}(M_Z)$, an approximate relation between $m_{H_u}(0)$ and m_0 is obtained as a function of the gaugino mass parameter $M_{1/2}$. Moreover,

$$0.9 m_{H_d}^2(0) - m_{H_u}^2(0) \simeq 0.1 \left(2 m_0^2 - 3 M_{1/2}^2 \right) + \frac{3}{2} (m_A^2 + M_Z^2). \quad (5.8)$$

From the above equations, the qualitative behaviour of our solutions may be understood. For instance, for $m_0 \simeq 3M_{1/2}$, as is the case at point II on figure 5.4 (see table 5.3), and $M_{1/2} > \mu, M_Z$, from Eq. (5.7) we get that $m_{H_u}(0) \simeq 1.2 m_0$. Moreover from Eq. (5.8), we obtain that $m_{H_d}(0)$ may vary from values of order $1.4 m_0$, for low values of m_A , up to values of order $2 m_0$ for larger values of m_A , without inducing a problem in the sbottom sector. Larger values of $m_{H_d}(0)$, however, induce lower values of the sbottom mass parameters, implying unwanted large values for the gluino-induced bottom mass corrections. Therefore, the best fit is obtained for values of the CP-odd Higgs mass close to its experimental bound, or, equivalently, for values of $m_{H_d}(0)$ of the order of $1.4 m_0$. Larger (smaller) values of $m_0/M_{1/2}$ imply smaller (larger) values of $m_{H_{u,d}}(0)/m_0$, as is clearly seen from Eqs. (5.7), (5.8) (see tables 5.2-5.5).

The relation between the CP-odd mass and m_0 may also be understood from the above equations. For large values of the CP-odd mass, and $m_0 > M_{1/2}$, $m_{H_d}^2(0)$ must be significantly larger than $m_{H_u}^2(0)$. For these large values of $m_{H_d}^2(0)$, large values of

m_0^2 are required in order to avoid a very low value of the sbottom mass. The results displayed in figure 5.10 are just a reflection of this fact.

5.4.6 What If Strange Quark Is Much Lighter?

As in the previous chapter, let us also discuss how the performance of models 4a and 4c changes if recent lattice results on the low value of the strange quark mass prove to be correct [95]. With $m_s(1\text{GeV}) = (120 \pm 25)\text{MeV}$ replacing the value $(180 \pm 50)\text{MeV}$ of table 2.3, the χ^2 value of model 4c at point II changes from 0.731/3dof to a significantly worse value of 5.44/3dof. More than a half of it comes from the m_s value itself, 163.4MeV at the best fit, indicating that model 4c cannot get to such low values of the strange mass as reported by the lattice groups, even with the help of the fifth effective operator. The rest of the χ^2 value is being shared by a low value for $\alpha_s(M_Z)$, a high V_{cb} and $\hat{B}_K$, and low V_{ub}/V_{cb} , contributing by less than 0.8 to the total χ^2 each. We checked that this behaviour of model 4c is typical for different points as well, and that the best fit values of the strange quark mass $m_s < 160\text{MeV}$ never occurs. Model 4a, on the contrary, favors values of $m_s(1\text{GeV})$ less than 180MeV. However, decreasing m_s cannot cure the problems of this model: high values of $\hat{B}_K$ and V_{cb} , and a low value for V_{ub}/V_{cb} . At the same point II, these observables contribute 2.85, 0.75 and 1.76, respectively, to χ^2 which totals 6.82 despite values of m_s as low as 136.5MeV. This represents only a marginal improvement to the χ^2 value of 6.99 at the same point with the original larger experimental value of the s -quark mass.

5.4.7 Intermediate Neutrino Scale

Finally, let us study the changes in the predictions of model 4c caused by the presence of a right-handed neutrino singlet at an intermediate neutrino scale $M_\nu = 10^{12}\text{GeV}$. Such a right-handed neutrino scale is motivated by a see-saw mechanism [96] giving rise to light neutrino masses. In fact, assuming a simple scenario, where the hierarchy in neutrino Yukawa couplings follows the pattern of charged fermions, Majorana mass of about 10^{12}GeV implies that the tau neutrino would have mass in the range $1 - 10\text{eV}$, which would make it a suitable hot dark matter candidate [97]. In our case, $SO(10)$ symmetry relates the Yukawa coupling of the τ -neutrino to the Yukawa couplings of other third generation particles. The minimal $SO(10)$ (and that is, what we assume for model 4c) prescribes

$$\lambda_{33}^u = \lambda_{33}^d = \lambda_{33}^e = \lambda_{33}^\nu = A, \quad (5.9)$$

where A is the coefficient of the 33 operator, eq.(4.1), and it is of order one. Since there is new, non-negligible Yukawa coupling in the theory, the RGEs between the GUT scale M_G and the neutrino scale M_ν are modified. At the same time, one has to introduce also the corresponding new tri-linear scalar coupling and sneutrino mass matrix (see Appendix A).³⁶

These modifications may have an impact on the low energy values of some couplings and change the performance of models. Recall that *e.g.*, the correct values for b and τ masses are obtained after a rather delicate balance between the gauge couplings pulling in one way and the Yukawa couplings pulling the other way is established.

³⁶Below the scale M_ν , the right-handed neutrinos are integrated out, and the theory is the MSSM as discussed before in this thesis.

Figure 5.11: Effects of the right-handed neutrino at the scale 10^{12}GeV : model 4c global analysis results in the $m_0 - M_{1/2}$ plane, for a fixed value of the Higgs parameter $\mu(M_Z) = 80 \text{ GeV}$. Solid (double-dash-dotted, dotted) lines represent contour lines of constant $\chi^2 = 6 (3,1) / 3\text{dof}$. The fainter curves are the previous results for M_v above the GUT scale.

Figure 5.12: Effects of the right-handed neutrino at the scale 10^{12}GeV : model 4c global analysis results in the $m_0 - M_{1/2}$ plane, for a fixed value of the Higgs parameter $\mu(M_Z) = 160 \text{ GeV}$. Solid (double-dash-dotted, dotted) lines represent contour lines of constant $\chi^2 = 6 (3,1) / 3\text{dof}$. The fainter curves are the previous results for M_v above the GUT scale.

Figure 5.13: Effects of the right-handed neutrino at the scale 10^{12}GeV : model 4c global analysis results in the $m_0 - M_{1/2}$ plane, for a fixed value of the Higgs parameter $\mu(M_Z) = 240 \text{ GeV}$. Solid (double-dash-dotted, dotted) lines represent contour lines of constant $\chi^2 = 6 (3,1) / 3\text{dof}$. The fainter curves are the previous results for M_ν above the GUT scale.

In our particular case, the performance of model 4c does not get better after the changes in the β -functions above M_ν are introduced. The results are shown in figures 5.11, 5.12 and 5.13, where they are directly compared to the previous results where ν^c was assumed as being integrated out right from the beginning at the GUT scale (figures 5.4-5.6) As we can see, the quality of the fits gets slightly worse, and the best results are obtained for a heavier SUSY spectrum than in the case $M_\nu > M_G$. Nevertheless, model 4c remains a very good candidate model even with an additional intermediate right-handed neutrino scale.

5.5 Conclusions

The results of our analysis, as well as the whole project presented here, may be understood from two different perspectives.

The emphasis of this paper is to reanalyze the fermion mass sector in the context of the minimal $SO(10)$ SUSY GUT in order to gain insight into the underlying flavor physics. From this perspective, we analyzed the best working model of ADHRS with four effective operators in the Yukawa sector at the GUT scale. The best fit allows us to assign a confidence level of about 3-4% to this model. Next, we found that the addition of a new operator, giving rise to the 13 (and 31) entries in the Yukawa matrices, may improve the performance of the model. Substantial improvement however, is not automatic, as has been evidenced with model 4a that has a confidence level $< 50\%$ for the best fits in the corner of SUSY parameter space with very large m_0 and $M_{1/2}$, and large μ . On the other hand, we showed that model 4c provides an excellent fit to all 20 low energy observables, with confidence level better than 68% in a large region of the allowed SUSY parameter space. We have not performed a complete search over all possible 13 operators. Model 4c was picked as a candidate model suggested by a recent formulation of a complete $SO(10)$ SUSY GUT[88]. Whether or not this particular model is close to the path nature has chosen remains to be seen. One important test will be via the CP violating decays of the B. We predict a value for $\sin 2\alpha$ which is insensitive to the SUSY breaking parameters (see figure 5.7) and tables 5.6 and 5.7), whereas in the SM the value of $\sin 2\alpha$ is unrestricted[92]. Another important test will come from nucleon decay rates which is discussed in a recent paper[98].

As a separate matter, we considered the sensitivity of our results to different lower bounds for ϵ_3 . For the assumed theoretical uncertainties, as given in table 2.3, our analysis of model 4c indicates that the best fits favor a negative GUT threshold correction to α_s of the order $-(4-5)\%$. We have checked for the whole $m_0 - M_{1/2}$ plane at fixed $\mu = 80\text{GeV}$ that a correction of -3% instead of $-(4-5)\%$ can easily be accommodated and gives basically the same best values of χ^2 as obtained in the unrestricted analysis. In this case, for $\epsilon_3 = -3\%$, the contour lines of figure 5.4 move almost uniformly towards higher values of m_0 by about 50 GeV. Note that the study of the complete SO(10) model[88] giving rise to the effective operators of model 4c shows that such negative corrections to $\alpha_s(M_G)$ can easily be obtained without any fine tuning. To understand how sensitive our results might be to this threshold correction we studied a narrower region in the parameter space, determined by fixed $M_{1/2}=240\text{GeV}$ at the previously fixed $\mu=80\text{GeV}$ (see figure 5.8 and table 5.8). It turns out that $\epsilon_3 > 0$ remains a possibility, at the expense of lower GUT scale and α_G , resulting in a worse agreement with the low energy value of G_μ . Fermion masses are not very sensitive to these variations. In addition, from table 5.8 it is clear that as ϵ_3 increases, so does $\alpha_s(M_Z)$. These results are however strongly dependent on the magnitude of our assumed theoretical uncertainties as discussed in the previous section and shown in figure 5.9 and table 5.9.

Note that model 4c is also flexible enough to accommodate recent analyses of the R_b anomaly and low values for $\alpha_s(M_Z)$ [99, 100]. For the Higgs sector of the MSSM, the best fits at a number of points (e.g. also at the sample points I - III; see tables 5.6 and 5.7) end up with low values of the pseudoscalar Higgs mass m_A . This preference is not strong though, as can be inferred from figure 5.10 where we show how the quality

of the fits change with the increasing lower bound on m_A . However, the low value of m_A , such as at point III2 for instance, leaves the door open for a natural explanation of the $1\text{-}2\sigma$ increase in the partial width $Z \rightarrow b\bar{b}$, without ever asking for it in the course of our analysis. At the same time, as can be argued on general grounds, one of the CP even Higgs states is always almost degenerate with the pseudoscalar state. For $m_A < M_Z$ it is the lighter h^0 state, while for $m_A > M_Z$ it is the heavy Higgs state H^0 . In the latter case, the h^0 mass equals $M_Z \cos^2 2\beta + \text{rad. corrections}$ where the radiative corrections are rather substantial in our case. Thus our study indicates that there are no additional constraints on a light CP even Higgs state between the experimental limit of about 55 GeV (for $\tan\beta \simeq 60$) and the upper MSSM limit of about 130 GeV.

Another interesting feature of the best fits is the tendency towards lower values of $\alpha_s(M_Z) \sim 0.115$. As with low m_A , it just happens to be in agreement with a potentially significant positive pseudoscalar contribution to R_b . The origin of rather low α_s in the best fits can be traced to the optimization efforts to suppress the gluino correction to an already large enough value for $m_b(M_Z)$. This correction has to be positive, since its sign is correlated with the sign of the chargino-stop partial amplitude to the process $b \rightarrow s\gamma$ [101], and the latter is fixed in the large $\tan\beta$ regime, as described in the previous section. At the same time lower values of $\alpha_s(M_Z)$ are welcome in order to suppress the QCD logarithmic corrections to $b \rightarrow s\gamma$. Here the point is that with $\tan\beta$ large the chargino-stop loop tends to be too large, most of the times outweighing by too much the W and H^+ loops which contribute with the opposite sign. Thus the optimization favors lower $\alpha_s(M_Z)$ as the way to make up

for a rather large net $b \rightarrow s\gamma$ amplitude by minimizing the enhancement from the renormalization to the M_b scale.

Finally, we have showed that the right-handed neutrino at the scale 10^{12}GeV can be incorporated into model 4c, although the quality of the fits gets worse in this case.

Our analysis can also be viewed from a different perspective. Let us neglect for a moment the underlying GUT physics and the origin of the Yukawa matrices at the GUT scale and view this analysis simply as an MSSM global fit in the large $\tan\beta$ regime. (See also [100].) From this new perspective a special feature of our approach is that we run complete sets of fermion and sfermion mass matrices down to the Z scale, instead of just the leading 33 elements. We now try to draw some general conclusions coming from such an analysis.

Our results suggest that there is no narrow, strongly preferred region in the SUSY $(m_0, M_{1/2})$ parameter plane for low values of the μ parameter $\mu(M_Z) \sim 80\text{GeV}$. Hence, at present one cannot make strong conclusions in the large $\tan\beta$ regime about the masses of squarks, sleptons and gluinos, which leaves open various channels for Tevatron and LEP II experiments.

In addition, a definite statement can be made about the sign of μ following from the structure of the partial amplitudes to the process $b \rightarrow s\gamma$, as already explained in more detail in the previous section. In the conventions we use μ has to be positive. Note, this conclusion, however, depends on the assumption of universal squark and slepton masses and universal A parameter at M_G which constrains the sign of A_t at M_Z .

Finally, by keeping the complete 3×3 mass matrices for both fermions and sfermions one can study flavor dependent processes in a theory which fits the low

energy data; for example, in rare B and K decays, $B - \bar{B}$ mixing, lepton flavor violating processes or even $Z \rightarrow b\bar{b}$.

In summary, we performed a detailed global analysis of independent SM parameters (plus the extra low energy data constraining the SUSY sector) in several models based on SO(10) SUSY GUTs. At the present time, when direct evidence for physics beyond the Standard Model evades experimental observations, this kind of analysis serves as the best test of new physics and actually starts to compete with SM electroweak precision tests. Our ultimate goal is to identify a set of effective theories, defined at a scale M ($M = M_{Planck}$, M_{string} or M_{GUT}) which accurately fit the low energy data and thus determine the boundary conditions at the scale M for some more fundamental theory valid above M . We have shown that this set is non-trivial; model 4c is one element of the set. More elements of the set need to be found in order to determine (a) whether they can be distinguished by purely low energy measurements, or (b) whether one or more elements can be constructed as the low energy limit of a string.

Finally, some theoretical uncertainties in the present analysis can and should be removed in the future. By including the SUSY box contributions to G_μ we can remove the additional 1/2% uncertainty included in the evaluation of χ^2 for this quantity. In addition, the SUSY box corrections to ϵ_K can be as large as 15%[102]. These should be included in order to remove the uncertainties in the predictions for CP violating parameters; although these uncertainties do not appear to be significant at the representative points studied in model 4c.

APPENDIX A

RENORMALIZATION GROUP EQUATIONS

In this appendix we show the two loop renormalization group equations (RGEs) for the couplings of the MSSM with an additional right-handed singlet neutrino (MSSM+ ν^c). As explained in chapter 2, in order to speed up our numerical analysis we use the two loop equations for dimensionless couplings and only one loop equations for dimensionful couplings. However, we incorporated the full two loop RGEs into our codes, and we have checked at a number of points that the full two loop results differ insignificantly from those which we obtain in a simplified analysis.

Two loop equations for the MSSM couplings, including the corresponding soft SUSY breaking terms, were originally derived in the work of Martin and Vaughn [103]. To study the effect of a massive right-handed neutrino present in the theory above an intermediate scale, between the GUT scale and the Z scale, we expanded their work by adding the terms coming from the singlet neutrino Yukawa coupling and related neutrino soft terms.

If below the intermediate neutrino scale, the right-handed neutrino couplings Y_ν , h_ν , m_ν^2 , M and B_ν (see below) should be put to zero in the equations of this appendix. Then one is left with the MSSM RGEs of Martin and Vaughn. We have checked, that

this is, indeed, the case. As a separate check, on a one-loop level our additional neutrino terms agree with the RGEs known in the literature [104].

In the MSSM+ ν^c , the gauge group is $SU(3)_c \times SU(2)_L \times U(1)_Y$, with chiral superfields (denoted with a hat) $\hat{Q}$ and $\hat{L}$ for the $SU(2)_L$ -doublet quarks and leptons; $\hat{u}^c$, $\hat{d}^c$, $\hat{e}^c$ and $\hat{\nu}^c$ for the $SU(2)_L$ -singlet quarks and leptons, and two Higgs doublet chiral superfields $\hat{H}_u$ and $\hat{H}_d$. The superpotential is

$$W = \hat{u}^c Y_u \hat{Q} \hat{H}_u + \hat{d}^c Y_d \hat{Q} \hat{H}_d + \hat{e}^c Y_e \hat{L} \hat{H}_d + \hat{\nu}^c Y_\nu \hat{L} \hat{H}_u + \mu \hat{H}_u \hat{H}_d + \frac{1}{2} \hat{\nu}^c M \hat{\nu}^c \quad (\text{A.1})$$

where Y_u , Y_d , Y_e , Y_ν and M are each 3×3 Yukawa matrices.³⁷ M is a symmetric Majorana mass matrix for right-handed neutrinos.

The soft SUSY-breaking Lagrangian contains tri- and bi-linear scalar couplings

$$-\mathcal{L} = \tilde{u}^c \mathbf{h}_u \tilde{Q} H_u + \tilde{d}^c \mathbf{h}_d \tilde{Q} H_d + \tilde{e}^c \mathbf{h}_e \tilde{L} H_d + \tilde{\nu}^c \mathbf{h}_\nu \tilde{L} H_u + B H_u H_d + \frac{1}{2} \tilde{\nu}^c B_\nu \tilde{\nu}^c + \text{h.c.} \quad (\text{A.2})$$

where $\mathbf{h}_u$, $\mathbf{h}_d$, $\mathbf{h}_e$, $\mathbf{h}_\nu$ and B_ν are again 3×3 matrices in family space, and a tilde is used to denote the scalar component of each SM fermion. There are also scalar masses

$$\begin{aligned} -\mathcal{L} = & m_{H_u}^2 H_u^\dagger H_u + m_{H_d}^2 H_d^\dagger H_d \\ & + \tilde{Q}^\dagger \mathbf{m}_Q^2 \tilde{Q} + \tilde{L}^\dagger \mathbf{m}_L^2 \tilde{L} + \tilde{u}^c \mathbf{m}_u^2 \tilde{u}^{c\dagger} + \tilde{d}^c \mathbf{m}_d^2 \tilde{d}^{c\dagger} + \tilde{e}^c \mathbf{m}_e^2 \tilde{e}^{c\dagger} + \tilde{\nu}^c \mathbf{m}_\nu^2 \tilde{\nu}^{c\dagger} . \end{aligned} \quad (\text{A.3})$$

Here again $\mathbf{m}_Q^2$, $\mathbf{m}_L^2$, $\mathbf{m}_u^2$, $\mathbf{m}_d^2$, $\mathbf{m}_e^2$ and $\mathbf{m}_\nu^2$ are 3×3 matrices in family space. Finally, the gauginos for the subgroups $SU(3)_c$, $SU(2)_L$ and $U(1)_Y$ have SUSY breaking

³⁷In this appendix, we use the Yukawa matrices $\mathbf{Y}_i$, defined by having doublets in the superpotential on the right. In the other parts of this thesis we use λ_i 's, defined by having doublets on the left (compare with eq.(1.25)). Obviously, $\mathbf{Y}_i = (\lambda_i)^T$, for $i = u, d, e, \nu$. The same relation is also induced upon the trilinear matrices $\mathbf{h}_i$, used in this appendix, and $\mathbf{A}_i$ elsewhere in this thesis.

masses M_3 , M_2 , and M_1 , respectively, and enter the lagrangian with an unconventional sign :

$$\mathcal{L} = +\frac{1}{2}M_3\lambda^a\lambda^a + \frac{1}{2}M_2\lambda^i\lambda^i + \frac{1}{2}M_1\lambda\lambda, \quad (\text{A.4})$$

where $a = 1, 2, \dots, 8$ and $i = 1, 2, 3$ enumerate the adjoint states of the subgroups.

Note, that we compromise our notation (see eq's(1.25) and (1.26)) of Yukawa and trilinear matrices, as well as the bilinear term, so that we be able to compare our re-derived RGEs directly with the original results of Martin and Vaughn. Note, that an overall sign in their equations (4.2) and (4.3) (both in the published version and in the second revision in LANL archives), which are the analogies to our eq's (A.2) and (A.3), should be reversed, in order to be consistent with the rest of their work. In addition, we differ by an overall sign for the gaugino mass terms, eq.(A.4) above. That sign is all right though, since it is consistent with Martin and Vaughn's convention for gaugino states λ , which includes an extra i .³⁸ Thus we conclude that all the RGEs of Martin and Vaughn ought to be identical to our results, when the right-handed neutrino terms are disregarded, which is, indeed, the case.

At this point we begin with the results for two-loop β -functions for the three gauge couplings :

$$\frac{d}{dt}g_a = \frac{g_a^3}{16\pi^2}B_a^{(1)} + \frac{g_a^3}{(16\pi^2)^2} \left[\sum_{b=1}^3 B_{ab}^{(2)} g_b^2 - \sum_{x=u,d,e,\nu} C_a^x \text{Tr}(Y_x^\dagger Y_x) \right]. \quad (\text{A.5})$$

Here $B_a^{(1)} = (33/5, 1, -3)$ for $U(1)_Y$ (in a GUT normalization), $SU(2)_L$, and $SU(3)_c$ respectively, and

$$B_{ab}^{(2)} = \begin{pmatrix} 199/25 & 27/5 & 88/5 \\ 9/5 & 25 & 24 \\ 11/5 & 9 & 14 \end{pmatrix} \quad (\text{A.6})$$

³⁸We thank Steve Martin for clarifications on this subject.

and

$$C_a^{u,d,e,\nu} = \begin{pmatrix} 26/5 & 14/5 & 18/5 & 6/5 \\ 6 & 6 & 2 & 2 \\ 4 & 4 & 0 & 0 \end{pmatrix}. \quad (\text{A.7})$$

Note that we define $t = \ln \mu$.

The two-loop renormalization group equations for the three gaugino mass parameters can then be written easily in terms of the same coefficients:

$$\begin{aligned} \frac{d}{dt} M_a = & \frac{2g_a^2}{16\pi^2} B_a^{(1)} M_a + \frac{2g_a^2}{(16\pi^2)^2} \left[\sum_{b=1}^3 B_{ab}^{(2)} g_b^2 (M_a + M_b) \right. \\ & \left. + \sum_{x=u,d,e,\nu} C_a^x \left(\text{Tr}[Y_x^\dagger h_x] - M_a \text{Tr}[Y_x^\dagger Y_x] \right) \right]. \end{aligned} \quad (\text{A.8})$$

The two-loop beta functions for the superpotential parameters are:

$$\frac{d}{dt} \mu = \frac{1}{16\pi^2} \beta_\mu^{(1)} + \frac{1}{(16\pi^2)^2} \beta_\mu^{(2)}, \quad (\text{A.9})$$

$$\frac{d}{dt} Y_{u,d,e,\nu} = \frac{1}{16\pi^2} \beta_{Y_{u,d,e,\nu}}^{(1)} + \frac{1}{(16\pi^2)^2} \beta_{Y_{u,d,e,\nu}}^{(2)}, \quad (\text{A.10})$$

with:

$$\beta_\mu^{(1)} = \mu \left\{ \text{Tr}(3Y_u Y_u^\dagger + 3Y_d Y_d^\dagger + Y_e Y_e^\dagger + Y_\nu Y_\nu^\dagger) - 3g_2^2 - \frac{3}{5}g_1^2 \right\} \quad (\text{A.11})$$

$$\begin{aligned} \beta_\mu^{(2)} = & \mu \left\{ -\text{Tr}(9Y_u Y_u^\dagger Y_u Y_u^\dagger + 9Y_d Y_d^\dagger Y_d Y_d^\dagger + 6Y_u Y_d^\dagger Y_d Y_u^\dagger \right. \\ & + 3Y_e Y_e^\dagger Y_e Y_e^\dagger + 3Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger + 2Y_\nu Y_e^\dagger Y_e Y_\nu^\dagger) \\ & + [16g_3^2 + \frac{4}{5}g_1^2] \text{Tr}(Y_u Y_u^\dagger) + [16g_3^2 - \frac{2}{5}g_1^2] \text{Tr}(Y_d Y_d^\dagger) + \frac{6}{5}g_1^2 \text{Tr}(Y_e Y_e^\dagger) \\ & \left. + \frac{15}{2}g_2^4 + \frac{9}{5}g_1^2 g_2^2 + \frac{207}{50}g_1^4 \right\} \end{aligned} \quad (\text{A.12})$$

$$\beta_{Y_u}^{(1)} = Y_u \left\{ \text{Tr}(3Y_u Y_u^\dagger + Y_\nu Y_\nu^\dagger) + 3Y_u^\dagger Y_u + Y_d^\dagger Y_d - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2 \right\} \quad (\text{A.13})$$

$$\begin{aligned}
\beta_{Y_u}^{(2)} = & Y_u \left\{ -\text{Tr}(9Y_u Y_u^\dagger Y_u Y_u^\dagger + 3Y_u Y_d^\dagger Y_d Y_u^\dagger + 3Y_u Y_\nu^\dagger Y_\nu Y_u^\dagger + Y_\nu Y_e^\dagger Y_e Y_\nu^\dagger) \right. \\
& - 3Y_u^\dagger Y_u \text{Tr}(3Y_u Y_u^\dagger + Y_\nu Y_\nu^\dagger) - Y_d^\dagger Y_d \text{Tr}(3Y_d Y_d^\dagger + Y_e Y_e^\dagger) \\
& - 4Y_u^\dagger Y_u Y_u^\dagger Y_u - 2Y_d^\dagger Y_d Y_d^\dagger Y_d - 2Y_\nu^\dagger Y_\nu Y_\nu^\dagger Y_\nu \\
& + [16g_3^2 + \frac{4}{5}g_1^2] \text{Tr}(Y_u Y_u^\dagger) + [6g_2^2 + \frac{2}{5}g_1^2] Y_u^\dagger Y_u + \frac{2}{5}g_1^2 Y_d^\dagger Y_d \\
& \left. - \frac{16}{9}g_3^4 + 8g_3^2 g_2^2 + \frac{136}{45}g_3^2 g_1^2 + \frac{15}{2}g_2^4 + g_2^2 g_1^2 + \frac{2743}{450}g_1^4 \right\} \quad (\text{A.14})
\end{aligned}$$

$$\beta_{Y_d}^{(1)} = Y_d \left\{ \text{Tr}(3Y_d Y_d^\dagger + Y_e Y_e^\dagger) + 3Y_d^\dagger Y_d + Y_u^\dagger Y_u - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{15}g_1^2 \right\} \quad (\text{A.15})$$

$$\begin{aligned}
\beta_{Y_d}^{(2)} = & Y_d \left\{ -\text{Tr}(9Y_d Y_d^\dagger Y_d Y_d^\dagger + 3Y_u Y_d^\dagger Y_d Y_u^\dagger + 3Y_e Y_e^\dagger Y_e Y_e^\dagger + Y_\nu Y_e^\dagger Y_e Y_\nu^\dagger) \right. \\
& - Y_u^\dagger Y_u \text{Tr}(3Y_u Y_u^\dagger + Y_\nu Y_\nu^\dagger) - 3Y_d^\dagger Y_d \text{Tr}(3Y_d Y_d^\dagger + Y_e Y_e^\dagger) \\
& - 4Y_d^\dagger Y_d Y_d^\dagger Y_d - 2Y_u^\dagger Y_u Y_u^\dagger Y_u - 2Y_\nu^\dagger Y_\nu Y_\nu^\dagger Y_\nu \\
& + [16g_3^2 - \frac{2}{5}g_1^2] \text{Tr}(Y_d Y_d^\dagger) + \frac{6}{5}g_1^2 \text{Tr}(Y_e Y_e^\dagger) \\
& + \frac{4}{5}g_1^2 Y_u^\dagger Y_u + [6g_2^2 + \frac{4}{5}g_1^2] Y_d^\dagger Y_d \\
& \left. - \frac{16}{9}g_3^4 + 8g_3^2 g_2^2 + \frac{8}{9}g_3^2 g_1^2 + \frac{15}{2}g_2^4 + g_2^2 g_1^2 + \frac{287}{90}g_1^4 \right\} \quad (\text{A.16})
\end{aligned}$$

$$\beta_{Y_e}^{(1)} = Y_e \left\{ \text{Tr}(3Y_d Y_d^\dagger + Y_e Y_e^\dagger) + 3Y_e^\dagger Y_e + Y_\nu^\dagger Y_\nu - 3g_2^2 - \frac{9}{5}g_1^2 \right\} \quad (\text{A.17})$$

$$\begin{aligned}
\beta_{Y_e}^{(2)} = & Y_e \left\{ -\text{Tr}(9Y_d Y_d^\dagger Y_d Y_d^\dagger + 3Y_u Y_d^\dagger Y_d Y_u^\dagger + 3Y_e Y_e^\dagger Y_e Y_e^\dagger + Y_\nu Y_e^\dagger Y_e Y_\nu^\dagger) \right. \\
& - 3Y_e^\dagger Y_e \text{Tr}(3Y_d Y_d^\dagger + Y_e Y_e^\dagger) - Y_\nu^\dagger Y_\nu \text{Tr}(3Y_u Y_u^\dagger + Y_\nu Y_\nu^\dagger) \\
& \left. - 4Y_e^\dagger Y_e Y_e^\dagger Y_e - 2Y_\nu^\dagger Y_\nu Y_\nu^\dagger Y_\nu - 2Y_\nu^\dagger Y_\nu Y_e^\dagger Y_e \right\}
\end{aligned}$$

$$\begin{aligned}
& + \left[16g_3^2 - \frac{2}{5}g_1^2 \right] \text{Tr}(\mathbf{Y}_d \mathbf{Y}_d^\dagger) + \frac{6}{5}g_1^2 \text{Tr}(\mathbf{Y}_e \mathbf{Y}_e^\dagger) + 6g_2^2 \mathbf{Y}_e^\dagger \mathbf{Y}_e \\
& + \frac{15}{2}g_2^4 + \frac{9}{5}g_2^2 g_1^2 + \frac{27}{2}g_1^4 \}
\end{aligned} \tag{A.18}$$

$$\beta_{\mathbf{Y}_\nu}^{(1)} = \mathbf{Y}_\nu \left\{ \text{Tr}(3\mathbf{Y}_u \mathbf{Y}_u^\dagger + \mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger) + 3\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu + \mathbf{Y}_e^\dagger \mathbf{Y}_e - 3g_2^2 - \frac{3}{5}g_1^2 \right\} \tag{A.19}$$

$$\begin{aligned}
\beta_{\mathbf{Y}_\nu}^{(2)} = \mathbf{Y}_\nu \bigg\{ & -\text{Tr}(9\mathbf{Y}_u \mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{Y}_u^\dagger + 3\mathbf{Y}_u \mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{Y}_u^\dagger + 3\mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger + \mathbf{Y}_\nu \mathbf{Y}_e^\dagger \mathbf{Y}_e \mathbf{Y}_\nu^\dagger) \\
& - 3\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu \text{Tr}(3\mathbf{Y}_u \mathbf{Y}_u^\dagger + \mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger) - \mathbf{Y}_e^\dagger \mathbf{Y}_e \text{Tr}(3\mathbf{Y}_d \mathbf{Y}_d^\dagger + \mathbf{Y}_e \mathbf{Y}_e^\dagger) \\
& - 2\mathbf{Y}_e^\dagger \mathbf{Y}_e \mathbf{Y}_e^\dagger \mathbf{Y}_e - 4\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu - 2\mathbf{Y}_e^\dagger \mathbf{Y}_e \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu \\
& + \left[16g_3^2 + \frac{4}{5}g_1^2 \right] \text{Tr}(\mathbf{Y}_u \mathbf{Y}_u^\dagger) + \left[6g_2^2 + \frac{6}{5}g_1^2 \right] \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu + \frac{6}{5}g_1^2 \mathbf{Y}_e^\dagger \mathbf{Y}_e \\
& + \frac{15}{2}g_2^4 + \frac{9}{5}g_2^2 g_1^2 + \frac{207}{50}g_1^4 \}
\end{aligned} \tag{A.20}$$

Obviously, the β -functions for $\mathbf{Y}_{u,d,e,\nu}$ are 3×3 matrices in family space.

For the sake of completeness, let's also write down the RGEs for the last super-potential parameter - Majorana matrix M :

$$\begin{aligned}
\frac{d}{dt}M = & \frac{1}{16\pi^2} 2 \left\{ M \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu + (\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)^T M \right\} \\
& + \frac{1}{(16\pi^2)^2} 2 \left\{ M \left[-\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu - \mathbf{Y}_\nu^\dagger \mathbf{Y}_e \mathbf{Y}_e^\dagger \mathbf{Y}_\nu \right. \right. \\
& \quad \left. \left. - \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu \text{Tr}(3\mathbf{Y}_u \mathbf{Y}_u^\dagger + \mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger) + \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu (3g_2^2 + \frac{3}{5}g_1^2) \right] \right. \\
& \quad \left. + [the\ same\ bracket]^T M \right\} .
\end{aligned} \tag{A.21}$$

Next, the two loop β -functions for the soft-breaking trilinear scalar couplings read:

$$\frac{d}{dt}h_{u,d,e,\nu} = \frac{1}{16\pi^2} \beta_{h_{u,d,e,\nu}}^{(1)} + \frac{1}{(16\pi^2)^2} \beta_{h_{u,d,e,\nu}}^{(2)} . \tag{A.22}$$

$$\beta_{h_u}^{(1)} = h_u \left\{ \text{Tr}(3\mathbf{Y}_u \mathbf{Y}_u^\dagger + \mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger) + 5\mathbf{Y}_u^\dagger \mathbf{Y}_u + \mathbf{Y}_d^\dagger \mathbf{Y}_d - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2 \right\}$$

$$+Y_u \left\{ 2\text{Tr}(3h_u Y_u^\dagger + h_\nu Y_\nu^\dagger) + 4Y_u^\dagger h_u + 2Y_d^\dagger h_d + \frac{32}{3}g_3^2 M_3 + 6g_2^2 M_2 + \frac{26}{15}g_1^2 M_1 \right\} \quad (\text{A.23})$$

$$\begin{aligned} \beta_{h_u}^{(2)} = & h_u \left\{ -\text{Tr}(9Y_u Y_u^\dagger Y_u Y_u^\dagger + 3Y_u Y_d^\dagger Y_d Y_u^\dagger + 3Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger + Y_\nu Y_e^\dagger Y_e Y_\nu^\dagger) \right. \\ & - 5Y_u^\dagger Y_u \text{Tr}(3Y_u Y_u^\dagger + Y_\nu Y_\nu^\dagger) - Y_d^\dagger Y_d \text{Tr}(3Y_d Y_d^\dagger + Y_e Y_e^\dagger) \\ & - 6Y_u^\dagger Y_u Y_u^\dagger Y_u - 2Y_d^\dagger Y_d Y_d^\dagger Y_d - 4Y_d^\dagger Y_d Y_u^\dagger Y_u \\ & + [16g_3^2 + \frac{4}{5}g_1^2] \text{Tr}(Y_u Y_u^\dagger) + 12g_2^2 Y_u^\dagger Y_u + \frac{2}{5}g_1^2 Y_d^\dagger Y_d \\ & \left. - \frac{16}{9}g_3^4 + 8g_3^2 g_2^2 + \frac{136}{45}g_3^2 g_1^2 + \frac{15}{2}g_2^4 + g_2^2 g_1^2 + \frac{2743}{450}g_1^4 \right\} \\ & + Y_u \left\{ -6\text{Tr}(6h_u Y_u^\dagger Y_u Y_u^\dagger + h_u Y_d^\dagger Y_d Y_u^\dagger + h_d Y_u^\dagger Y_u Y_d^\dagger + 2h_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger) \right. \\ & - 2\text{Tr}(h_\nu Y_e^\dagger Y_e Y_\nu^\dagger + h_e Y_\nu^\dagger Y_\nu Y_e^\dagger) \\ & - 6Y_u^\dagger Y_u \text{Tr}(3h_u Y_u^\dagger + h_\nu Y_\nu^\dagger) - 2Y_d^\dagger Y_d \text{Tr}(3h_d Y_d^\dagger + h_e Y_e^\dagger) \\ & - 4Y_u^\dagger h_u \text{Tr}(3Y_u Y_u^\dagger + Y_\nu Y_\nu^\dagger) - 2Y_d^\dagger h_d \text{Tr}(3Y_d Y_d^\dagger + Y_e Y_e^\dagger) \\ & - 6Y_u^\dagger Y_u Y_u^\dagger h_u - 8Y_u^\dagger h_u Y_u^\dagger Y_u - 4Y_d^\dagger Y_d Y_d^\dagger h_d - 4Y_d^\dagger h_d Y_d^\dagger Y_d \\ & - 2Y_d^\dagger Y_d Y_u^\dagger h_u - 4Y_d^\dagger h_d Y_u^\dagger Y_u \\ & + [32g_3^2 + \frac{8}{5}g_1^2] \text{Tr}(h_u Y_u^\dagger) + [6g_2^2 + \frac{6}{5}g_1^2] Y_u^\dagger h_u + \frac{4}{5}g_1^2 Y_d^\dagger h_d \\ & - [32g_3^2 M_3 + \frac{8}{5}g_1^2 M_1] \text{Tr}(Y_u Y_u^\dagger) - [12g_2^2 M_2 + \frac{4}{5}g_1^2 M_1] Y_u^\dagger Y_u \\ & - \frac{4}{5}g_1^2 M_1 Y_d^\dagger Y_d + \frac{64}{9}g_3^4 M_3 - 16g_3^2 g_2^2 (M_3 + M_2) - \frac{272}{45}g_3^2 g_1^2 (M_3 + M_1) \\ & \left. - 30g_2^4 M_2 - 2g_2^2 g_1^2 (M_2 + M_1) - \frac{5486}{225}g_1^4 M_1 \right\} \quad (\text{A.24}) \end{aligned}$$

$$\begin{aligned} \beta_{h_d}^{(1)} = & h_d \left\{ \text{Tr}(3Y_d Y_d^\dagger + Y_e Y_e^\dagger) + 5Y_d^\dagger Y_d + Y_u^\dagger Y_u - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{15}g_1^2 \right\} \\ & + Y_d \left\{ 2\text{Tr}(3h_d Y_d^\dagger + h_e Y_e^\dagger) + 4Y_d^\dagger h_d + 2Y_u^\dagger h_u + \frac{32}{3}g_3^2 M_3 + 6g_2^2 M_2 + \frac{14}{15}g_1^2 M_1 \right\} \quad (\text{A.25}) \end{aligned}$$

$$\begin{aligned}
\beta_{\mathbf{h}_d}^{(2)} = & \mathbf{h}_d \left\{ -\text{Tr}(9\mathbf{Y}_d\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_d^\dagger + 3\mathbf{Y}_u\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_u^\dagger + 3\mathbf{Y}_e\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_e^\dagger + \mathbf{Y}_\nu\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_\nu^\dagger) \right. \\
& - \mathbf{Y}_u^\dagger\mathbf{Y}_u\text{Tr}(3\mathbf{Y}_u\mathbf{Y}_u^\dagger + \mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger) - 5\mathbf{Y}_d^\dagger\mathbf{Y}_d\text{Tr}(3\mathbf{Y}_d\mathbf{Y}_d^\dagger + \mathbf{Y}_e\mathbf{Y}_e^\dagger) \\
& - 6\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_d^\dagger\mathbf{Y}_d - 2\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_u^\dagger\mathbf{Y}_u - 4\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_d^\dagger\mathbf{Y}_d \\
& + \left[16g_3^2 - \frac{2}{5}g_1^2\right]\text{Tr}(\mathbf{Y}_d\mathbf{Y}_d^\dagger) + \frac{6}{5}g_1^2\text{Tr}(\mathbf{Y}_e\mathbf{Y}_e^\dagger) + \frac{4}{5}g_1^2\mathbf{Y}_u^\dagger\mathbf{Y}_u + \left[12g_2^2 + \frac{6}{5}g_1^2\right]\mathbf{Y}_d^\dagger\mathbf{Y}_d \\
& - \frac{16}{9}g_3^4 + 8g_3^2g_2^2 + \frac{8}{9}g_3^2g_1^2 + \frac{15}{2}g_2^4 + g_2^2g_1^2 + \frac{287}{90}g_1^4 \Big\} \\
& + \mathbf{Y}_d \left\{ -6\text{Tr}(6\mathbf{h}_d\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_d^\dagger + \mathbf{h}_u\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_u^\dagger + \mathbf{h}_d\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_d^\dagger + 2\mathbf{h}_e\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_e^\dagger) \right. \\
& - 2\text{Tr}(\mathbf{h}_\nu\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_\nu^\dagger + \mathbf{h}_e\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathbf{Y}_e^\dagger) \\
& - 2\mathbf{Y}_u^\dagger\mathbf{Y}_u\text{Tr}(3\mathbf{h}_u\mathbf{Y}_u^\dagger + \mathbf{h}_\nu\mathbf{Y}_\nu^\dagger) - 6\mathbf{Y}_d^\dagger\mathbf{Y}_d\text{Tr}(3\mathbf{h}_d\mathbf{Y}_d^\dagger + \mathbf{h}_e\mathbf{Y}_e^\dagger) \\
& - 2\mathbf{Y}_u^\dagger\mathbf{h}_u\text{Tr}(3\mathbf{Y}_u\mathbf{Y}_u^\dagger + \mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger) - 4\mathbf{Y}_d^\dagger\mathbf{h}_d\text{Tr}(3\mathbf{Y}_d\mathbf{Y}_d^\dagger + \mathbf{Y}_e\mathbf{Y}_e^\dagger) \\
& - 6\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_d^\dagger\mathbf{h}_d - 8\mathbf{Y}_d^\dagger\mathbf{h}_d\mathbf{Y}_d^\dagger\mathbf{Y}_d - 4\mathbf{Y}_u^\dagger\mathbf{h}_u\mathbf{Y}_u^\dagger\mathbf{Y}_u - 4\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_d^\dagger\mathbf{h}_u \\
& - 4\mathbf{Y}_u^\dagger\mathbf{h}_u\mathbf{Y}_d^\dagger\mathbf{Y}_d - 2\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_d^\dagger\mathbf{h}_d \\
& + \left[32g_3^2 - \frac{4}{5}g_1^2\right]\text{Tr}(\mathbf{h}_d\mathbf{Y}_d^\dagger) + \frac{12}{5}g_1^2\text{Tr}(\mathbf{h}_e\mathbf{Y}_e^\dagger) + \frac{8}{5}g_1^2\mathbf{Y}_u^\dagger\mathbf{h}_u \\
& + \left[6g_2^2 + \frac{6}{5}g_1^2\right]\mathbf{Y}_d^\dagger\mathbf{h}_d - \left[32g_3^2M_3 - \frac{4}{5}g_1^2M_1\right]\text{Tr}(\mathbf{Y}_d\mathbf{Y}_d^\dagger) - \frac{12}{5}g_1^2M_1\text{Tr}(\mathbf{Y}_e\mathbf{Y}_e^\dagger) \\
& - \left[12g_2^2M_2 + \frac{8}{5}g_1^2M_1\right]\mathbf{Y}_d^\dagger\mathbf{Y}_d - \frac{8}{5}g_1^2M_1\mathbf{Y}_u^\dagger\mathbf{Y}_u + \frac{64}{9}g_3^4M_3 - 16g_3^2g_2^2(M_3 + M_2) \\
& - \frac{16}{9}g_3^2g_1^2(M_3 + M_1) - 30g_2^4M_2 - 2g_2^2g_1^2(M_2 + M_1) - \frac{574}{45}g_1^4M_1 \Big\} \quad (\text{A.26})
\end{aligned}$$

$$\begin{aligned}
\beta_{\mathbf{h}_e}^{(1)} = & \mathbf{h}_e \left\{ \text{Tr}(3\mathbf{Y}_d\mathbf{Y}_d^\dagger + \mathbf{Y}_e\mathbf{Y}_e^\dagger) + 5\mathbf{Y}_e^\dagger\mathbf{Y}_e + \mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu - 3g_2^2 - \frac{9}{5}g_1^2 \right\} \\
& + \mathbf{Y}_e \left\{ \text{Tr}(6\mathbf{h}_d\mathbf{Y}_d^\dagger + 2\mathbf{h}_e\mathbf{Y}_e^\dagger) + 4\mathbf{Y}_e^\dagger\mathbf{h}_e + 2\mathbf{Y}_\nu^\dagger\mathbf{h}_\nu + 6g_2^2M_2 + \frac{18}{5}g_1^2M_1 \right\} \quad (\text{A.27})
\end{aligned}$$

$$\begin{aligned}
\beta_{\mathbf{h}_e}^{(2)} = & \mathbf{h}_e \left\{ -\text{Tr}(9\mathbf{Y}_d\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_d^\dagger + 3\mathbf{Y}_u\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_u^\dagger + 3\mathbf{Y}_e\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_e^\dagger + \mathbf{Y}_\nu\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_\nu^\dagger) \right. \\
& - 5\mathbf{Y}_e^\dagger\mathbf{Y}_e\text{Tr}(3\mathbf{Y}_d\mathbf{Y}_d^\dagger + \mathbf{Y}_e\mathbf{Y}_e^\dagger) - \mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\text{Tr}(3\mathbf{Y}_u\mathbf{Y}_u^\dagger + \mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger)
\end{aligned}$$

$$\begin{aligned}
& -6\mathbf{Y}_e^\dagger \mathbf{Y}_e \mathbf{Y}_e^\dagger \mathbf{Y}_e - 2\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu - 4\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu \mathbf{Y}_e^\dagger \mathbf{Y}_e \\
& + \left[16g_3^2 - \frac{2}{5}g_1^2\right] \text{Tr}(\mathbf{Y}_d \mathbf{Y}_d^\dagger) + \frac{6}{5}g_1^2 \text{Tr}(\mathbf{Y}_e \mathbf{Y}_e^\dagger) + \left[12g_2^2 - \frac{6}{5}g_1^2\right] \mathbf{Y}_e^\dagger \mathbf{Y}_e \\
& + \frac{15}{2}g_2^4 + \frac{9}{5}g_2^2 g_1^2 + \frac{27}{2}g_1^4 \Big\} \\
& + \mathbf{Y}_e \Big\{ -6\text{Tr}(6\mathbf{h}_d \mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{Y}_d^\dagger + \mathbf{h}_u \mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{Y}_u^\dagger + \mathbf{h}_d \mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{Y}_d^\dagger + 2\mathbf{h}_e \mathbf{Y}_e^\dagger \mathbf{Y}_e \mathbf{Y}_e^\dagger) \\
& - 2\text{Tr}(\mathbf{h}_\nu \mathbf{Y}_e^\dagger \mathbf{Y}_e \mathbf{Y}_\nu^\dagger + \mathbf{h}_e \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu \mathbf{Y}_e^\dagger) \\
& - 6\mathbf{Y}_e^\dagger \mathbf{Y}_e \text{Tr}(3\mathbf{h}_d \mathbf{Y}_d^\dagger + \mathbf{h}_e \mathbf{Y}_e^\dagger) - 2\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu \text{Tr}(3\mathbf{h}_u \mathbf{Y}_u^\dagger + \mathbf{h}_\nu \mathbf{Y}_\nu^\dagger) \\
& - 4\mathbf{Y}_e^\dagger \mathbf{h}_e \text{Tr}(3\mathbf{Y}_d \mathbf{Y}_d^\dagger + \mathbf{Y}_e \mathbf{Y}_e^\dagger) - 2\mathbf{Y}_\nu^\dagger \mathbf{h}_\nu \text{Tr}(3\mathbf{Y}_u \mathbf{Y}_u^\dagger + \mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger) \\
& - 6\mathbf{Y}_e^\dagger \mathbf{Y}_e \mathbf{Y}_e^\dagger \mathbf{h}_e - 8\mathbf{Y}_e^\dagger \mathbf{h}_e \mathbf{Y}_e^\dagger \mathbf{Y}_e - 4\mathbf{Y}_\nu^\dagger \mathbf{h}_\nu \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu - 4\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger \mathbf{h}_\nu \\
& - 4\mathbf{Y}_\nu^\dagger \mathbf{h}_\nu \mathbf{Y}_e^\dagger \mathbf{Y}_e - 2\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu \mathbf{Y}_e^\dagger \mathbf{h}_e \\
& + \left[32g_3^2 - \frac{4}{5}g_1^2\right] \text{Tr}(\mathbf{h}_d \mathbf{Y}_d^\dagger) + \frac{12}{5}g_1^2 \text{Tr}(\mathbf{h}_e \mathbf{Y}_e^\dagger) + \left[6g_2^2 + \frac{6}{5}g_1^2\right] \mathbf{Y}_e^\dagger \mathbf{h}_e \\
& - \left[32g_3^2 M_3 - \frac{4}{5}g_1^2 M_1\right] \text{Tr}(\mathbf{Y}_d \mathbf{Y}_d^\dagger) - \frac{12}{5}g_1^2 M_1 \text{Tr}(\mathbf{Y}_e \mathbf{Y}_e^\dagger) - 12g_2^2 M_2 \mathbf{Y}_e^\dagger \mathbf{Y}_e \\
& - 30g_2^4 M_2 - \frac{18}{5}g_2^2 g_1^2 (M_1 + M_2) - 54g_1^4 M_1 \Big\} \tag{A.28}
\end{aligned}$$

$$\begin{aligned}
\beta_{\mathbf{h}_\nu}^{(1)} &= \mathbf{h}_\nu \Big\{ \text{Tr}(3\mathbf{Y}_u \mathbf{Y}_u^\dagger + \mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger) + 5\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu + \mathbf{Y}_e^\dagger \mathbf{Y}_e - 3g_2^2 - \frac{3}{5}g_1^2 \Big\} \\
&+ \mathbf{Y}_\nu \Big\{ 2\text{Tr}(3\mathbf{h}_u \mathbf{Y}_u^\dagger + \mathbf{h}_\nu \mathbf{Y}_\nu^\dagger) + 2\mathbf{Y}_e^\dagger \mathbf{h}_e + 4\mathbf{Y}_\nu^\dagger \mathbf{h}_\nu + 6g_2^2 M_2 + \frac{6}{5}g_1^2 M_1 \Big\} \tag{A.29}
\end{aligned}$$

$$\begin{aligned}
\beta_{\mathbf{h}_\nu}^{(2)} &= \mathbf{h}_\nu \Big\{ -\text{Tr}(9\mathbf{Y}_u \mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{Y}_u^\dagger + 3\mathbf{Y}_u \mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{Y}_u^\dagger + 3\mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger + \mathbf{Y}_\nu \mathbf{Y}_e^\dagger \mathbf{Y}_e \mathbf{Y}_\nu^\dagger) \\
&- \mathbf{Y}_e^\dagger \mathbf{Y}_e \text{Tr}(3\mathbf{Y}_d \mathbf{Y}_d^\dagger + \mathbf{Y}_e \mathbf{Y}_e^\dagger) - 5\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu \text{Tr}(3\mathbf{Y}_u \mathbf{Y}_u^\dagger + \mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger) \\
&- 2\mathbf{Y}_e^\dagger \mathbf{Y}_e \mathbf{Y}_e^\dagger \mathbf{Y}_e - 6\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu - 4\mathbf{Y}_e^\dagger \mathbf{Y}_e \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu \\
&+ \left[16g_3^2 + \frac{4}{5}g_1^2\right] \text{Tr}(\mathbf{Y}_u \mathbf{Y}_u^\dagger) + \left[12g_2^2 + \frac{12}{5}g_1^2\right] \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu + \frac{6}{5}g_1^2 \mathbf{Y}_e^\dagger \mathbf{Y}_e \\
&+ \frac{15}{2}g_2^4 + \frac{9}{5}g_2^2 g_1^2 + \frac{207}{50}g_1^4 \Big\}
\end{aligned}$$

$$\begin{aligned}
& +Y_\nu \left\{ -6\text{Tr}(6h_u Y_u^\dagger Y_u Y_u^\dagger + h_u Y_d^\dagger Y_d Y_u^\dagger + h_d Y_u^\dagger Y_u Y_d^\dagger + 2h_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger) \right. \\
& \quad - 2\text{Tr}(h_\nu Y_e^\dagger Y_e Y_\nu^\dagger + h_e Y_\nu^\dagger Y_\nu Y_e^\dagger) \\
& \quad - 2Y_e^\dagger Y_e \text{Tr}(3h_d Y_d^\dagger + h_e Y_e^\dagger) - 6Y_\nu^\dagger Y_\nu \text{Tr}(3h_u Y_u^\dagger + h_\nu Y_\nu^\dagger) \\
& \quad - 2Y_e^\dagger h_e \text{Tr}(3Y_d Y_d^\dagger + Y_e Y_e^\dagger) - 4Y_\nu^\dagger h_\nu \text{Tr}(3Y_u Y_u^\dagger + Y_\nu Y_\nu^\dagger) \\
& \quad - 4Y_e^\dagger Y_e Y_e^\dagger h_e - 4Y_e^\dagger h_e Y_e^\dagger Y_e - 8Y_\nu^\dagger h_\nu Y_\nu^\dagger Y_\nu - 6Y_\nu^\dagger Y_\nu Y_\nu^\dagger h_\nu \\
& \quad - 4Y_e^\dagger h_e Y_\nu^\dagger Y_\nu - 2Y_e^\dagger Y_e Y_\nu^\dagger h_\nu \\
& \quad + \left[32g_3^2 + \frac{8}{5}g_1^2 \right] \text{Tr}(h_u Y_u^\dagger) + \left[6g_2^2 + \frac{6}{5}g_1^2 \right] Y_\nu^\dagger h_\nu + \frac{12}{5}g_1^2 Y_e^\dagger h_e \\
& \quad - \left[32g_3^2 M_3 + \frac{8}{5}g_1^2 M_1 \right] \text{Tr}(Y_u Y_u^\dagger) \\
& \quad - \left[12g_2^2 M_2 + \frac{12}{5}g_1^2 M_1 \right] Y_\nu^\dagger Y_\nu - \frac{12}{5}g_1^2 M_1 Y_e^\dagger Y_e \\
& \quad \left. - 30g_2^4 M_2 - \frac{18}{5}g_2^2 g_1^2 (M_1 + M_2) - \frac{414}{25}g_1^4 M_1 \right\} \tag{A.30}
\end{aligned}$$

These are again 3×3 matrices in family space.

The bi-linear soft SUSY breaking terms satisfy the two-loop renormalization group equation:

$$\frac{d}{dt} B_\xi = \frac{1}{16\pi^2} \beta_{B_\xi}^{(1)} + \frac{1}{(16\pi^2)^2} \beta_{B_\xi}^{(2)}, \tag{A.31}$$

with $B_\xi = B$ for the Higgs term and $B_\xi = B_\nu$ for the Majorana soft term, see eq.(A.2).

$$\begin{aligned}
\beta_B^{(1)} &= B \left\{ \text{Tr}(3Y_u Y_u^\dagger + 3Y_d Y_d^\dagger + Y_e Y_e^\dagger + Y_\nu Y_\nu^\dagger) - 3g_2^2 - \frac{3}{5}g_1^2 \right\} \\
&+ \mu \left\{ 2\text{Tr}(3h_u Y_u^\dagger + 3h_d Y_d^\dagger + h_e Y_e^\dagger + h_\nu Y_\nu^\dagger) + 6g_2^2 M_2 + \frac{6}{5}g_1^2 M_1 \right\} \tag{A.32}
\end{aligned}$$

$$\begin{aligned}
\beta_B^{(2)} &= B \left\{ -\text{Tr}(9Y_u Y_u^\dagger Y_u Y_u^\dagger + 9Y_d Y_d^\dagger Y_d Y_d^\dagger + 6Y_u Y_d^\dagger Y_d Y_u^\dagger \right. \\
&\quad + 3Y_e Y_e^\dagger Y_e Y_e^\dagger + 3Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger + 2Y_\nu Y_e^\dagger Y_e Y_\nu^\dagger) \\
&\quad \left. + \left[16g_3^2 + \frac{4}{5}g_1^2 \right] \text{Tr}(Y_u Y_u^\dagger) + \left[16g_3^2 - \frac{2}{5}g_1^2 \right] \text{Tr}(Y_d Y_d^\dagger) + \frac{6}{5}g_1^2 \text{Tr}(Y_e Y_e^\dagger) \right\}
\end{aligned}$$

$$\begin{aligned}
& + \frac{15}{2}g_2^4 + \frac{9}{5}g_1^2g_2^2 + \frac{207}{50}g_1^4 \Big\} \\
& + \mu \Big\{ -4\text{Tr}(9\mathbf{h}_u\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_u^\dagger + 9\mathbf{h}_d\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_d^\dagger + 3\mathbf{h}_u\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_u^\dagger + 3\mathbf{h}_d\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_d^\dagger \\
& \quad + 3\mathbf{h}_e\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_e^\dagger + 3\mathbf{h}_\nu\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger + \mathbf{h}_e\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathbf{Y}_e^\dagger + \mathbf{h}_\nu\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_\nu^\dagger) \\
& \quad + [32g_3^2 + \frac{8}{5}g_1^2]\text{Tr}(\mathbf{h}_u\mathbf{Y}_u^\dagger) + [32g_3^2 - \frac{4}{5}g_1^2]\text{Tr}(\mathbf{h}_d\mathbf{Y}_d^\dagger) + \frac{12}{5}g_1^2\text{Tr}(\mathbf{h}_e\mathbf{Y}_e^\dagger) \\
& \quad - [32g_3^2M_3 + \frac{8}{5}g_1^2M_1]\text{Tr}(\mathbf{Y}_u\mathbf{Y}_u^\dagger) - [32g_3^2M_3 - \frac{4}{5}g_1^2M_1]\text{Tr}(\mathbf{Y}_d\mathbf{Y}_d^\dagger) \\
& \quad - \frac{12}{5}g_1^2M_1\text{Tr}(\mathbf{Y}_e\mathbf{Y}_e^\dagger) - 30g_2^4M_2 - \frac{18}{5}g_1^2g_2^2(M_1 + M_2) - \frac{414}{25}g_1^4M_1 \Big\} .
\end{aligned} \tag{A.33}$$

$$\beta_{B_\nu}^{(1)} = 2[B_\nu\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu + (\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu)^T B_\nu] + 4[M\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu + (\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu)^T M] \tag{A.34}$$

$$\begin{aligned}
\beta_{B_\nu}^{(2)} = & 2\Big\{ B_\nu[-\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu - \mathbf{Y}_\nu^\dagger\mathbf{Y}_e\mathbf{Y}_e^\dagger\mathbf{Y}_\nu \\
& \quad - \mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\text{Tr}(3\mathbf{Y}_u^\dagger\mathbf{Y}_u + \mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu) + \mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu(3g_2^2 + \frac{3}{5}g_1^2)] \\
& \quad + [\textit{the same bracket}]^T B_\nu \Big\} \\
& + 4\Big\{ M[-\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger\mathbf{h}_\nu - \mathbf{Y}_\nu^\dagger\mathbf{h}_\nu\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu - \mathbf{Y}_\nu^\dagger\mathbf{Y}_e\mathbf{Y}_e^\dagger\mathbf{h}_\nu - \mathbf{Y}_\nu^\dagger\mathbf{h}_e\mathbf{Y}_e^\dagger\mathbf{Y}_\nu \\
& \quad - \mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\text{Tr}(3\mathbf{h}_u\mathbf{Y}_u^\dagger + \mathbf{h}_\nu\mathbf{Y}_\nu^\dagger) - \mathbf{Y}_\nu^\dagger\mathbf{h}_\nu\text{Tr}(3\mathbf{Y}_u\mathbf{Y}_u^\dagger + \mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger) \\
& \quad + \mathbf{Y}_\nu^\dagger\mathbf{h}_\nu(3g_2^2 + \frac{3}{5}g_1^2) - \mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu(3g_2^2M_2 + \frac{3}{5}g_1^2M_1)] \\
& \quad + [\textit{the same bracket}]^T M \Big\} .
\end{aligned} \tag{A.35}$$

Finally, we turn to the β -functions for the scalar (mass)² terms in the MSSM+ ν^c .

It is convenient to define the quantities

$$S = m_{H_u}^2 - m_{H_d}^2 + \text{Tr}[\mathbf{m}_Q^2 - \mathbf{m}_L^2 - 2\mathbf{m}_u^2 + \mathbf{m}_d^2 + \mathbf{m}_e^2] \tag{A.36}$$

and

$$S' = \text{Tr}[-(3m_{H_u}^2 + \mathbf{m}_Q^2)\mathbf{Y}_u^\dagger\mathbf{Y}_u + 4\mathbf{Y}_u^\dagger\mathbf{m}_u^2\mathbf{Y}_u + (3m_{H_d}^2 - \mathbf{m}_Q^2)\mathbf{Y}_d^\dagger\mathbf{Y}_d - 2\mathbf{Y}_d^\dagger\mathbf{m}_d^2\mathbf{Y}_d]$$

$$\begin{aligned}
& + (m_{H_s}^2 + m_L^2) Y_e^\dagger Y_e - 2 Y_e^\dagger m_e^2 Y_e + (-m_{H_s}^2 + m_L^2) Y_\nu^\dagger Y_\nu \\
& + \left[\frac{3}{2} g_2^2 + \frac{3}{10} g_1^2 \right] \{ m_{H_s}^2 - m_{H_d}^2 - \text{Tr}(m_L^2) \} + \left[\frac{8}{3} g_3^2 + \frac{3}{2} g_2^2 + \frac{1}{30} g_1^2 \right] \text{Tr}(m_Q^2) \\
& - \left[\frac{16}{3} g_3^2 + \frac{16}{15} g_1^2 \right] \text{Tr}(m_u^2) + \left[\frac{8}{3} g_3^2 + \frac{2}{15} g_1^2 \right] \text{Tr}(m_d^2) + \frac{6}{5} g_1^2 \text{Tr}(m_e^2) \quad (\text{A.37})
\end{aligned}$$

and

$$\begin{aligned}
\sigma_1 &= \frac{1}{5} g_1^2 \{ 3(m_{H_s}^2 + m_{H_d}^2) + \text{Tr}[m_Q^2 + 3m_L^2 + 8m_u^2 + 2m_d^2 + 6m_e^2] \} \\
\sigma_2 &= g_2^2 \{ m_{H_s}^2 + m_{H_d}^2 + \text{Tr}[3m_Q^2 + m_L^2] \} \\
\sigma_3 &= g_3^2 \text{Tr}[2m_Q^2 + m_u^2 + m_d^2] .
\end{aligned}$$

Then we obtain for each of the β -functions in the standard form

$$\frac{d}{dt} m^2 = \frac{1}{16\pi^2} \beta_{m^2}^{(1)} + \frac{1}{(16\pi^2)^2} \beta_{m^2}^{(2)} \quad (\text{A.38})$$

the following results:

$$\begin{aligned}
\beta_{m_{H_s}^2}^{(1)} &= 2\text{Tr}[3(m_{H_s}^2 + m_Q^2) Y_u^\dagger Y_u + 3 Y_u^\dagger m_u^2 Y_u + 3 h_u^\dagger h_u \\
& + (m_{H_s}^2 + m_L^2) Y_\nu^\dagger Y_\nu + Y_\nu^\dagger m_\nu^2 Y_\nu + h_\nu^\dagger h_\nu] \\
& - 6g_2^2 |M_2|^2 - \frac{6}{5} g_1^2 |M_1|^2 + \frac{3}{5} g_1^2 \mathcal{S} \quad (\text{A.39})
\end{aligned}$$

$$\begin{aligned}
\beta_{m_{H_s}^2}^{(2)} &= -6\text{Tr} \left[6(m_{H_s}^2 + m_Q^2) Y_u^\dagger Y_u Y_u^\dagger Y_u + 6 Y_u^\dagger m_u^2 Y_u Y_u^\dagger Y_u \right. \\
& + (m_{H_s}^2 + m_{H_d}^2 + m_Q^2) Y_u^\dagger Y_u Y_d^\dagger Y_d + Y_u^\dagger m_u^2 Y_u Y_d^\dagger Y_d \\
& + Y_u^\dagger Y_u m_Q^2 Y_d^\dagger Y_d + Y_u^\dagger Y_u Y_d^\dagger m_d^2 Y_d + 6 h_u^\dagger h_u Y_u^\dagger Y_u + 6 h_u^\dagger Y_u Y_u^\dagger h_u \\
& + h_d^\dagger h_d Y_u^\dagger Y_u + Y_d^\dagger Y_d h_u^\dagger h_u + h_d^\dagger Y_d Y_u^\dagger h_u + Y_d^\dagger h_d h_u^\dagger Y_u \\
& + 2(m_{H_s}^2 + m_L^2) Y_\nu^\dagger Y_\nu Y_\nu^\dagger Y_\nu + 2 Y_\nu^\dagger m_\nu^2 Y_\nu Y_\nu^\dagger Y_\nu \\
& \left. + \frac{1}{3} (m_{H_s}^2 + m_{H_d}^2 + m_L^2) Y_\nu^\dagger Y_\nu Y_e^\dagger Y_e + \frac{1}{3} Y_\nu^\dagger m_\nu^2 Y_\nu Y_e^\dagger Y_e \right]
\end{aligned}$$

$$\begin{aligned}
& + \frac{1}{3} Y_\nu^\dagger Y_\nu m_L^2 Y_e^\dagger Y_e + \frac{1}{3} Y_\nu^\dagger Y_\nu Y_e^\dagger m_e^2 Y_e + 2 h_\nu^\dagger h_\nu Y_\nu^\dagger Y_\nu + 2 h_\nu^\dagger Y_\nu Y_\nu^\dagger h_\nu \\
& + \frac{1}{3} h_e^\dagger h_e Y_\nu^\dagger Y_\nu + \frac{1}{3} Y_e^\dagger Y_e h_\nu^\dagger h_\nu + \frac{1}{3} h_e^\dagger Y_e Y_e^\dagger h_e + \frac{1}{3} Y_e^\dagger h_e h_e^\dagger Y_e \Big] \\
& + \left[32g_3^2 + \frac{8}{5}g_1^2 \right] \text{Tr} \left[(m_{H_e}^2 + m_Q^2) Y_u^\dagger Y_u + Y_u^\dagger m_u^2 Y_u + h_u^\dagger h_u \right] \\
& + 32g_3^2 \left\{ 2|M_3|^2 \text{Tr}[Y_u^\dagger Y_u] - M_3^* \text{Tr}[Y_u^\dagger h_u] - M_3 \text{Tr}[h_u^\dagger Y_u] \right\} \\
& + \frac{8}{5} g_1^2 \left\{ 2|M_1|^2 \text{Tr}[Y_u^\dagger Y_u] - M_1^* \text{Tr}[Y_u^\dagger h_u] - M_1 \text{Tr}[h_u^\dagger Y_u] \right\} + \frac{6}{5} g_1^2 S' \\
& + 33g_2^4 |M_2|^2 + \frac{18}{5} g_2^2 g_1^2 (|M_2|^2 + |M_1|^2 + \text{Re}[M_1 M_2^*]) + \frac{621}{25} g_1^4 |M_1|^2 \\
& + 3g_2^2 \sigma_2 + \frac{3}{5} g_1^2 \sigma_1
\end{aligned} \tag{A.40}$$

$$\begin{aligned}
\beta_{m_{H_e}^2}^{(1)} &= 2 \text{Tr} \left[3(m_{H_e}^2 + m_Q^2) Y_d^\dagger Y_d + 3Y_d^\dagger m_d^2 Y_d + 3h_d^\dagger h_d \right. \\
&\quad \left. + (m_{H_e}^2 + m_L^2) Y_e^\dagger Y_e + Y_e^\dagger m_e^2 Y_e + h_e^\dagger h_e \right] \\
&\quad - 6g_2^2 |M_2|^2 - \frac{6}{5} g_1^2 |M_1|^2 - \frac{3}{5} g_1^2 S
\end{aligned} \tag{A.41}$$

$$\begin{aligned}
\beta_{m_{H_e}^2}^{(2)} &= -6 \text{Tr} \left[6(m_{H_e}^2 + m_Q^2) Y_d^\dagger Y_d Y_d^\dagger Y_d + 6Y_d^\dagger m_d^2 Y_d Y_d^\dagger Y_d \right. \\
&\quad \left. + (m_{H_e}^2 + m_{H_e}^2 + m_Q^2) Y_u^\dagger Y_u Y_d^\dagger Y_d \right. \\
&\quad \left. + Y_u^\dagger m_u^2 Y_u Y_d^\dagger Y_d + Y_u^\dagger Y_u m_Q^2 Y_d^\dagger Y_d + Y_u^\dagger Y_u Y_d^\dagger m_d^2 Y_d \right. \\
&\quad \left. + 2(m_{H_e}^2 + m_L^2) Y_e^\dagger Y_e Y_e^\dagger Y_e + 2Y_e^\dagger m_e^2 Y_e Y_e^\dagger Y_e \right. \\
&\quad \left. + \frac{1}{3} (m_{H_e}^2 + m_{H_e}^2 + m_{H_e}^2) Y_\nu^\dagger Y_\nu Y_e^\dagger Y_e \right. \\
&\quad \left. + \frac{1}{3} Y_\nu^\dagger m_\nu^2 Y_\nu Y_e^\dagger Y_e + \frac{1}{3} Y_\nu^\dagger Y_\nu m_L^2 Y_e^\dagger Y_e + \frac{1}{3} Y_\nu^\dagger Y_\nu Y_e^\dagger m_e^2 Y_e \right. \\
&\quad \left. + 6h_d^\dagger h_d Y_d^\dagger Y_d + 6h_d^\dagger Y_d Y_d^\dagger h_d \right. \\
&\quad \left. + h_u^\dagger h_u Y_d^\dagger Y_d + Y_u^\dagger Y_u h_d^\dagger h_d + h_u^\dagger Y_u Y_d^\dagger h_d + Y_u^\dagger h_u h_d^\dagger Y_d \right. \\
&\quad \left. + 2h_e^\dagger h_e Y_e^\dagger Y_e + 2h_e^\dagger Y_e Y_e^\dagger h_e \right]
\end{aligned}$$

$$\begin{aligned}
& + \frac{1}{3} \mathbf{h}_\nu^\dagger \mathbf{h}_\nu \mathbf{Y}_e^\dagger \mathbf{Y}_e + \frac{1}{3} \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu \mathbf{h}_e^\dagger \mathbf{h}_e + \frac{1}{3} \mathbf{h}_\nu^\dagger \mathbf{Y}_\nu \mathbf{Y}_e^\dagger \mathbf{h}_e + \frac{1}{3} \mathbf{Y}_\nu^\dagger \mathbf{h}_\nu \mathbf{h}_e^\dagger \mathbf{Y}_e \Big] \\
& + \left[32g_3^2 - \frac{4}{5}g_1^2 \right] \text{Tr}[(\mathbf{m}_{H_d}^2 + \mathbf{m}_Q^2) \mathbf{Y}_d^\dagger \mathbf{Y}_d + \mathbf{Y}_d^\dagger \mathbf{m}_d^2 \mathbf{Y}_d + \mathbf{h}_d^\dagger \mathbf{h}_d] \\
& + 32g_3^2 \left\{ 2|M_3|^2 \text{Tr}[\mathbf{Y}_d^\dagger \mathbf{Y}_d] - M_3^* \text{Tr}[\mathbf{Y}_d^\dagger \mathbf{h}_d] - M_3 \text{Tr}[\mathbf{h}_d^\dagger \mathbf{Y}_d] \right\} \\
& - \frac{4}{5}g_1^2 \left\{ 2|M_1|^2 \text{Tr}[\mathbf{Y}_d^\dagger \mathbf{Y}_d] - M_1^* \text{Tr}[\mathbf{Y}_d^\dagger \mathbf{h}_d] - M_1 \text{Tr}[\mathbf{h}_d^\dagger \mathbf{Y}_d] \right\} \\
& + \frac{12}{5}g_1^2 \left\{ \text{Tr}[(\mathbf{m}_{H_d}^2 + \mathbf{m}_L^2) \mathbf{Y}_e^\dagger \mathbf{Y}_e + \mathbf{Y}_e^\dagger \mathbf{m}_e^2 \mathbf{Y}_e + \mathbf{h}_e^\dagger \mathbf{h}_e] + 2|M_1|^2 \text{Tr}[\mathbf{Y}_e^\dagger \mathbf{Y}_e] \right. \\
& \quad \left. - M_1 \text{Tr}[\mathbf{h}_e^\dagger \mathbf{Y}_e] - M_1^* \text{Tr}[\mathbf{Y}_e^\dagger \mathbf{h}_e] \right\} - \frac{6}{5}g_1^2 S' + 33g_2^4 |M_2|^2 \\
& + \frac{18}{5}g_2^2 g_1^2 (|M_2|^2 + |M_1|^2 + \text{Re}[M_1 M_2^*]) + \frac{621}{25}g_1^4 |M_1|^2 \\
& + 3g_2^2 \sigma_2 + \frac{3}{5}g_1^2 \sigma_1
\end{aligned} \tag{A.42}$$

The β -functions for $\mathbf{m}_Q^2, \mathbf{m}_L^2, \mathbf{m}_u^2, \mathbf{m}_d^2, \mathbf{m}_e^2, \mathbf{m}_\nu^2$ are 3×3 matrices:

$$\begin{aligned}
\beta_{\mathbf{m}_Q^2}^{(1)} = & (\mathbf{m}_Q^2 + 2\mathbf{m}_{H_u}^2) \mathbf{Y}_u^\dagger \mathbf{Y}_u + 2\mathbf{Y}_u^\dagger \mathbf{m}_u^2 \mathbf{Y}_u + \mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{m}_Q^2 + 2\mathbf{h}_u^\dagger \mathbf{h}_u \\
& + (\mathbf{m}_Q^2 + 2\mathbf{m}_{H_d}^2) \mathbf{Y}_d^\dagger \mathbf{Y}_d + 2\mathbf{Y}_d^\dagger \mathbf{m}_d^2 \mathbf{Y}_d + \mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{m}_Q^2 + 2\mathbf{h}_d^\dagger \mathbf{h}_d \\
& - \frac{32}{3}g_3^2 |M_3|^2 - 6g_2^2 |M_2|^2 - \frac{2}{15}g_1^2 |M_1|^2 + \frac{1}{5}g_1^2 S
\end{aligned} \tag{A.43}$$

$$\begin{aligned}
\beta_{\mathbf{m}_Q^2}^{(2)} = & -(2\mathbf{m}_Q^2 + 8\mathbf{m}_{H_u}^2) \mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{Y}_u^\dagger \mathbf{Y}_u - 4\mathbf{Y}_u^\dagger \mathbf{m}_u^2 \mathbf{Y}_u \mathbf{Y}_u^\dagger \mathbf{Y}_u - 4\mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{m}_Q^2 \mathbf{Y}_u^\dagger \mathbf{Y}_u \\
& - 4\mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{Y}_u^\dagger \mathbf{m}_u^2 \mathbf{Y}_u - 2\mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{m}_Q^2 - (2\mathbf{m}_Q^2 + 8\mathbf{m}_{H_d}^2) \mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{Y}_d^\dagger \mathbf{Y}_d \\
& - 4\mathbf{Y}_d^\dagger \mathbf{m}_d^2 \mathbf{Y}_d \mathbf{Y}_d^\dagger \mathbf{Y}_d - 4\mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{m}_Q^2 \mathbf{Y}_d^\dagger \mathbf{Y}_d - 4\mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{Y}_d^\dagger \mathbf{m}_d^2 \mathbf{Y}_d - 2\mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{m}_Q^2 \\
& - [(\mathbf{m}_Q^2 + 4\mathbf{m}_{H_u}^2) \mathbf{Y}_u^\dagger \mathbf{Y}_u + 2\mathbf{Y}_u^\dagger \mathbf{m}_u^2 \mathbf{Y}_u + \mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{m}_Q^2] \text{Tr}(3\mathbf{Y}_u^\dagger \mathbf{Y}_u + \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu) \\
& - [(\mathbf{m}_Q^2 + 4\mathbf{m}_{H_d}^2) \mathbf{Y}_d^\dagger \mathbf{Y}_d + 2\mathbf{Y}_d^\dagger \mathbf{m}_d^2 \mathbf{Y}_d + \mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{m}_Q^2] \text{Tr}(3\mathbf{Y}_d^\dagger \mathbf{Y}_d + \mathbf{Y}_e^\dagger \mathbf{Y}_e) \\
& - \mathbf{Y}_u^\dagger \mathbf{Y}_u \text{Tr}(6\mathbf{m}_Q^2 \mathbf{Y}_u^\dagger \mathbf{Y}_u + 6\mathbf{Y}_u^\dagger \mathbf{m}_u^2 \mathbf{Y}_u + 2\mathbf{m}_L^2 \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu + 2\mathbf{Y}_\nu^\dagger \mathbf{m}_\nu^2 \mathbf{Y}_\nu) \\
& - \mathbf{Y}_d^\dagger \mathbf{Y}_d \text{Tr}(6\mathbf{m}_Q^2 \mathbf{Y}_d^\dagger \mathbf{Y}_d + 6\mathbf{Y}_d^\dagger \mathbf{m}_d^2 \mathbf{Y}_d + 2\mathbf{m}_L^2 \mathbf{Y}_e^\dagger \mathbf{Y}_e + 2\mathbf{Y}_e^\dagger \mathbf{m}_e^2 \mathbf{Y}_e)
\end{aligned}$$

$$\begin{aligned}
& -4\{Y_u^\dagger Y_u h_u^\dagger h_u + h_u^\dagger h_u Y_u^\dagger Y_u + Y_u^\dagger h_u h_u^\dagger Y_u + h_u^\dagger Y_u Y_u^\dagger h_u\} \\
& -4\{Y_d^\dagger Y_d h_d^\dagger h_d + h_d^\dagger h_d Y_d^\dagger Y_d + Y_d^\dagger h_d h_d^\dagger Y_d + h_d^\dagger Y_d Y_d^\dagger h_d\} \\
& -h_u^\dagger h_u \text{Tr}[6Y_u^\dagger Y_u + 2Y_\nu^\dagger Y_\nu] - Y_u^\dagger Y_u \text{Tr}[6h_u^\dagger h_u + 2h_\nu^\dagger h_\nu] \\
& -h_u^\dagger Y_u \text{Tr}[6Y_u^\dagger h_u + 2Y_\nu^\dagger h_\nu] - Y_u^\dagger h_u \text{Tr}[6h_u^\dagger Y_u + 2h_\nu^\dagger Y_\nu] \\
& -h_d^\dagger h_d \text{Tr}[6Y_d^\dagger Y_d + 2Y_e^\dagger Y_e] - Y_d^\dagger Y_d \text{Tr}[6h_d^\dagger h_d + 2h_e^\dagger h_e] \\
& -h_d^\dagger Y_d \text{Tr}[6Y_d^\dagger h_d + 2Y_e^\dagger h_e] - Y_d^\dagger h_d \text{Tr}[6h_d^\dagger Y_d + 2h_e^\dagger Y_e] \\
& + \frac{2}{5}g_1^2 \left\{ (2m_Q^2 + 4m_{H_u}^2)Y_u^\dagger Y_u + 4Y_u^\dagger m_u^2 Y_u + 2Y_u^\dagger Y_u m_Q^2 + 4h_u^\dagger h_u - 4M_1 h_u^\dagger Y_u \right. \\
& \quad - 4M_1^* Y_u^\dagger h_u + 8|M_1|^2 Y_u^\dagger Y_u + (m_Q^2 + 2m_{H_d}^2)Y_d^\dagger Y_d + 2Y_d^\dagger m_d^2 Y_d \\
& \quad \left. + Y_d^\dagger Y_d m_Q^2 + 2h_d^\dagger h_d - 2M_1 h_d^\dagger Y_d - 2M_1^* Y_d^\dagger h_d + 4|M_1|^2 Y_d^\dagger Y_d \right\} \\
& + \frac{2}{5}g_1^2 S' - \frac{128}{3}g_3^4 |M_3|^2 + 32g_3^2 g_2^2 (|M_3|^2 + |M_2|^2 + \text{Re}[M_2 M_3^*]) \\
& + \frac{32}{45}g_3^2 g_1^2 (|M_3|^2 + |M_1|^2 + \text{Re}[M_1 M_3^*]) + 33g_2^4 |M_2|^2 \\
& + \frac{2}{5}g_2^2 g_1^2 (|M_2|^2 + |M_1|^2 + \text{Re}[M_1 M_2^*]) + \frac{199}{75}g_1^4 |M_1|^2 \\
& + \frac{16}{3}g_3^2 \sigma_3 + 3g_2^2 \sigma_2 + \frac{1}{15}g_1^2 \sigma_1
\end{aligned} \tag{A.44}$$

$$\begin{aligned}
\beta_{\mathbf{m}_L^2}^{(1)} = & (m_L^2 + 2m_{H_d}^2)Y_e^\dagger Y_e + 2Y_e^\dagger m_e^2 Y_e + Y_e^\dagger Y_e m_L^2 + 2h_e^\dagger h_e \\
& + (m_L^2 + 2m_{H_u}^2)Y_\nu^\dagger Y_\nu + 2Y_\nu^\dagger m_\nu^2 Y_\nu + Y_\nu^\dagger Y_\nu m_L^2 + 2h_\nu^\dagger h_\nu \\
& - 6g_2^2 |M_2|^2 - \frac{6}{5}g_1^2 |M_1|^2 - \frac{3}{5}g_1^2 S
\end{aligned} \tag{A.45}$$

$$\begin{aligned}
\beta_{\mathbf{m}_L^2}^{(2)} = & -(2m_L^2 + 8m_{H_d}^2)Y_e^\dagger Y_e Y_e^\dagger Y_e - 4Y_e^\dagger m_e^2 Y_e Y_e^\dagger Y_e - 4Y_e^\dagger Y_e m_L^2 Y_e^\dagger Y_e \\
& - 4Y_e^\dagger Y_e Y_e^\dagger m_e^2 Y_e - 2Y_e^\dagger Y_e Y_e^\dagger Y_e m_L^2 - 4Y_\nu^\dagger Y_\nu Y_\nu^\dagger m_\nu^2 Y_\nu - 2Y_\nu^\dagger Y_\nu Y_\nu^\dagger Y_\nu m_L^2 \\
& - (2m_L^2 + 8m_{H_u}^2)Y_\nu^\dagger Y_\nu Y_\nu^\dagger Y_\nu - 4Y_\nu^\dagger m_\nu^2 Y_\nu Y_\nu^\dagger Y_\nu - 4Y_\nu^\dagger Y_\nu m_L^2 Y_\nu^\dagger Y_\nu
\end{aligned}$$

$$\begin{aligned}
& - \left[(\mathbf{m}_L^2 + 4m_{H_d}^2) \mathbf{Y}_e^\dagger \mathbf{Y}_e + 2\mathbf{Y}_e^\dagger \mathbf{m}_e^2 \mathbf{Y}_e + \mathbf{Y}_e^\dagger \mathbf{Y}_e \mathbf{m}_L^2 \right] \text{Tr}(3\mathbf{Y}_d^\dagger \mathbf{Y}_d + \mathbf{Y}_e^\dagger \mathbf{Y}_e) \\
& - \left[(\mathbf{m}_L^2 + 4m_{H_u}^2) \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu + 2\mathbf{Y}_\nu^\dagger \mathbf{m}_\nu^2 \mathbf{Y}_\nu + \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu \mathbf{m}_L^2 \right] \text{Tr}(3\mathbf{Y}_u^\dagger \mathbf{Y}_u + \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu) \\
& - \mathbf{Y}_e^\dagger \mathbf{Y}_e \text{Tr}[6\mathbf{m}_Q^2 \mathbf{Y}_d^\dagger \mathbf{Y}_d + 6\mathbf{Y}_d^\dagger \mathbf{m}_d^2 \mathbf{Y}_d + 2\mathbf{m}_L^2 \mathbf{Y}_e^\dagger \mathbf{Y}_e + 2\mathbf{Y}_e^\dagger \mathbf{m}_e^2 \mathbf{Y}_e] \\
& - \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu \text{Tr}[6\mathbf{m}_Q^2 \mathbf{Y}_u^\dagger \mathbf{Y}_u + 6\mathbf{Y}_u^\dagger \mathbf{m}_u^2 \mathbf{Y}_u + 2\mathbf{m}_L^2 \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu + 2\mathbf{Y}_\nu^\dagger \mathbf{m}_\nu^2 \mathbf{Y}_\nu] \\
& - 4 \left\{ \mathbf{Y}_e^\dagger \mathbf{Y}_e \mathbf{h}_e^\dagger \mathbf{h}_e + \mathbf{h}_e^\dagger \mathbf{h}_e \mathbf{Y}_e^\dagger \mathbf{Y}_e + \mathbf{Y}_e^\dagger \mathbf{h}_e \mathbf{h}_e^\dagger \mathbf{Y}_e + \mathbf{h}_e^\dagger \mathbf{Y}_e \mathbf{Y}_e^\dagger \mathbf{h}_e \right\} \\
& - 4 \left\{ \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu \mathbf{h}_\nu^\dagger \mathbf{h}_\nu + \mathbf{h}_\nu^\dagger \mathbf{h}_\nu \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu + \mathbf{Y}_\nu^\dagger \mathbf{h}_\nu \mathbf{h}_\nu^\dagger \mathbf{Y}_\nu + \mathbf{h}_\nu^\dagger \mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger \mathbf{h}_\nu \right\} \\
& - \mathbf{h}_e^\dagger \mathbf{h}_e \text{Tr}[6\mathbf{Y}_d^\dagger \mathbf{Y}_d + 2\mathbf{Y}_e^\dagger \mathbf{Y}_e] - \mathbf{Y}_e^\dagger \mathbf{Y}_e \text{Tr}[6\mathbf{h}_d^\dagger \mathbf{h}_d + 2\mathbf{h}_e^\dagger \mathbf{h}_e] \\
& - \mathbf{h}_e^\dagger \mathbf{Y}_e \text{Tr}[6\mathbf{Y}_d^\dagger \mathbf{h}_d + 2\mathbf{Y}_e^\dagger \mathbf{h}_e] - \mathbf{Y}_e^\dagger \mathbf{h}_e \text{Tr}[6\mathbf{h}_d^\dagger \mathbf{Y}_d + 2\mathbf{h}_e^\dagger \mathbf{Y}_e] \\
& - \mathbf{h}_\nu^\dagger \mathbf{h}_\nu \text{Tr}[6\mathbf{Y}_u^\dagger \mathbf{Y}_u + 2\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu] - \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu \text{Tr}[6\mathbf{h}_u^\dagger \mathbf{h}_u + 2\mathbf{h}_\nu^\dagger \mathbf{h}_\nu] \\
& - \mathbf{h}_\nu^\dagger \mathbf{Y}_\nu \text{Tr}[6\mathbf{Y}_u^\dagger \mathbf{h}_u + 2\mathbf{Y}_\nu^\dagger \mathbf{h}_\nu] - \mathbf{Y}_\nu^\dagger \mathbf{h}_\nu \text{Tr}[6\mathbf{h}_u^\dagger \mathbf{Y}_u + 2\mathbf{h}_\nu^\dagger \mathbf{Y}_\nu] \\
& + \frac{6}{5} g_1^2 \left\{ (\mathbf{m}_L^2 + 2m_{H_d}^2) \mathbf{Y}_e^\dagger \mathbf{Y}_e + 2\mathbf{Y}_e^\dagger \mathbf{m}_e^2 \mathbf{Y}_e + \mathbf{Y}_e^\dagger \mathbf{Y}_e \mathbf{m}_L^2 + 2\mathbf{h}_e^\dagger \mathbf{h}_e \right. \\
& \quad \left. - 2M_1 \mathbf{h}_e^\dagger \mathbf{Y}_e - 2M_1^* \mathbf{Y}_e^\dagger \mathbf{h}_e + 4|M_1|^2 \mathbf{Y}_e^\dagger \mathbf{Y}_e \right\} - \frac{6}{5} g_1^2 S' \\
& + 33g_2^4 |M_2|^2 + \frac{18}{5} g_2^2 g_1^2 (|M_2|^2 + |M_1|^2 + \text{Re}[M_1 M_2^*]) + \frac{621}{25} g_4^4 |M_1|^2 \\
& + 3g_2^2 \sigma_2 + \frac{3}{5} g_1^2 \sigma_1
\end{aligned} \tag{A.46}$$

$$\begin{aligned}
\beta_{\mathbf{m}_u^2}^{(1)} &= (2\mathbf{m}_u^2 + 4m_{H_u}^2) \mathbf{Y}_u \mathbf{Y}_u^\dagger + 4\mathbf{Y}_u \mathbf{m}_Q^2 \mathbf{Y}_u^\dagger + 2\mathbf{Y}_u \mathbf{Y}_u^\dagger \mathbf{m}_u^2 + 4\mathbf{h}_u \mathbf{h}_u^\dagger \\
& - \frac{32}{3} g_3^2 |M_3|^2 - \frac{32}{15} g_1^2 |M_1|^2 - \frac{4}{5} g_1^2 S
\end{aligned} \tag{A.47}$$

$$\begin{aligned}
\beta_{\mathbf{m}_u^2}^{(2)} &= - (2\mathbf{m}_u^2 + 8m_{H_u}^2) \mathbf{Y}_u \mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{Y}_u^\dagger - 4\mathbf{Y}_u \mathbf{m}_Q^2 \mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{Y}_u^\dagger - 4\mathbf{Y}_u \mathbf{Y}_u^\dagger \mathbf{m}_u^2 \mathbf{Y}_u \mathbf{Y}_u^\dagger \\
& - 4\mathbf{Y}_u \mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{m}_Q^2 \mathbf{Y}_u^\dagger - 2\mathbf{Y}_u \mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{Y}_u^\dagger \mathbf{m}_u^2 - (2\mathbf{m}_u^2 + 4m_{H_u}^2 + 4m_{H_d}^2) \mathbf{Y}_u \mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{Y}_u^\dagger \\
& - 4\mathbf{Y}_u \mathbf{m}_Q^2 \mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{Y}_u^\dagger - 4\mathbf{Y}_u \mathbf{Y}_d^\dagger \mathbf{m}_d^2 \mathbf{Y}_d \mathbf{Y}_u^\dagger - 4\mathbf{Y}_u \mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{m}_Q^2 \mathbf{Y}_u^\dagger - 2\mathbf{Y}_u \mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{Y}_u^\dagger \mathbf{m}_u^2
\end{aligned}$$

$$\begin{aligned}
& - \left[(\mathbf{m}_u^2 + 4m_{H_u}^2) \mathbf{Y}_u \mathbf{Y}_u^\dagger + 2\mathbf{Y}_u \mathbf{m}_Q^2 \mathbf{Y}_u^\dagger + \mathbf{Y}_u \mathbf{Y}_u^\dagger \mathbf{m}_u^2 \right] \text{Tr}[6\mathbf{Y}_u^\dagger \mathbf{Y}_u + 2\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu] \\
& - 4\mathbf{Y}_u \mathbf{Y}_u^\dagger \text{Tr}[3\mathbf{m}_Q^2 \mathbf{Y}_u^\dagger \mathbf{Y}_u + 3\mathbf{Y}_u^\dagger \mathbf{m}_u^2 \mathbf{Y}_u + \mathbf{m}_L^2 \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu + \mathbf{Y}_\nu^\dagger \mathbf{m}_\nu^2 \mathbf{Y}_\nu] \\
& - 4 \left\{ \mathbf{h}_u \mathbf{h}_u^\dagger \mathbf{Y}_u \mathbf{Y}_u^\dagger + \mathbf{Y}_u \mathbf{Y}_u^\dagger \mathbf{h}_u \mathbf{h}_u^\dagger + \mathbf{h}_u \mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{h}_u^\dagger + \mathbf{Y}_u \mathbf{h}_u^\dagger \mathbf{h}_u \mathbf{Y}_u^\dagger \right\} \\
& - 4 \left\{ \mathbf{h}_u \mathbf{h}_d^\dagger \mathbf{Y}_d \mathbf{Y}_u^\dagger + \mathbf{Y}_u \mathbf{Y}_d^\dagger \mathbf{h}_d \mathbf{h}_u^\dagger + \mathbf{h}_u \mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{h}_u^\dagger + \mathbf{Y}_u \mathbf{h}_d^\dagger \mathbf{h}_d \mathbf{Y}_u^\dagger \right\} \\
& - 4\mathbf{h}_u \mathbf{h}_u^\dagger \text{Tr}[3\mathbf{Y}_u^\dagger \mathbf{Y}_u + \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu] - 4\mathbf{Y}_u \mathbf{Y}_u^\dagger \text{Tr}[3\mathbf{h}_u^\dagger \mathbf{h}_u + \mathbf{h}_\nu^\dagger \mathbf{h}_\nu] \\
& - 4\mathbf{h}_u \mathbf{Y}_u^\dagger \text{Tr}[3\mathbf{h}_u^\dagger \mathbf{Y}_u + \mathbf{h}_\nu^\dagger \mathbf{Y}_\nu] - 4\mathbf{Y}_u \mathbf{h}_u^\dagger \text{Tr}[3\mathbf{Y}_u^\dagger \mathbf{h}_u + \mathbf{Y}_\nu^\dagger \mathbf{h}_\nu] \\
& + \left[6g_2^2 - \frac{2}{5}g_1^2 \right] \left\{ (\mathbf{m}_u^2 + 2m_{H_u}^2) \mathbf{Y}_u \mathbf{Y}_u^\dagger + 2\mathbf{Y}_u \mathbf{m}_Q^2 \mathbf{Y}_u^\dagger + \mathbf{Y}_u \mathbf{Y}_u^\dagger \mathbf{m}_u^2 + 2\mathbf{h}_u \mathbf{h}_u^\dagger \right\} \\
& + 12g_2^2 \left\{ 2|M_2|^2 \mathbf{Y}_u \mathbf{Y}_u^\dagger - M_2^* \mathbf{h}_u \mathbf{Y}_u^\dagger - M_2 \mathbf{Y}_u \mathbf{h}_u^\dagger \right\} \\
& - \frac{4}{5}g_1^2 \left\{ 2|M_1|^2 \mathbf{Y}_u \mathbf{Y}_u^\dagger - M_1^* \mathbf{h}_u \mathbf{Y}_u^\dagger - M_1 \mathbf{Y}_u \mathbf{h}_u^\dagger \right\} - \frac{8}{5}g_1^2 S' \\
& - \frac{128}{3}g_3^4 |M_3|^2 + \frac{512}{45}g_3^2 g_1^2 (|M_3|^2 + |M_1|^2 + \text{Re}[M_1 M_3^*]) + \frac{3424}{75}g_1^4 |M_1|^2 \\
& + \frac{16}{3}g_3^2 \sigma_3 + \frac{16}{15}g_1^2 \sigma_1
\end{aligned} \tag{A.48}$$

$$\begin{aligned}
\beta_{\mathbf{m}_d^2}^{(1)} &= (2\mathbf{m}_d^2 + 4m_{H_d}^2) \mathbf{Y}_d \mathbf{Y}_d^\dagger + 4\mathbf{Y}_d \mathbf{m}_Q^2 \mathbf{Y}_d^\dagger + 2\mathbf{Y}_d \mathbf{Y}_d^\dagger \mathbf{m}_d^2 + 4\mathbf{h}_d \mathbf{h}_d^\dagger \\
& - \frac{32}{3}g_3^2 |M_3|^2 - \frac{8}{15}g_1^2 |M_1|^2 + \frac{2}{5}g_1^2 S
\end{aligned} \tag{A.49}$$

$$\begin{aligned}
\beta_{\mathbf{m}_d^2}^{(2)} &= - (2\mathbf{m}_d^2 + 8m_{H_d}^2) \mathbf{Y}_d \mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{Y}_d^\dagger - 4\mathbf{Y}_d \mathbf{m}_Q^2 \mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{Y}_d^\dagger - 4\mathbf{Y}_d \mathbf{Y}_d^\dagger \mathbf{m}_d^2 \mathbf{Y}_d \mathbf{Y}_d^\dagger \\
& - 4\mathbf{Y}_d \mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{m}_Q^2 \mathbf{Y}_d^\dagger - 2\mathbf{Y}_d \mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{Y}_d^\dagger \mathbf{m}_d^2 - (2\mathbf{m}_d^2 + 4m_{H_u}^2 + 4m_{H_d}^2) \mathbf{Y}_d \mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{Y}_d^\dagger \\
& - 4\mathbf{Y}_d \mathbf{m}_Q^2 \mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{Y}_d^\dagger - 4\mathbf{Y}_d \mathbf{Y}_u^\dagger \mathbf{m}_u^2 \mathbf{Y}_u \mathbf{Y}_d^\dagger - 4\mathbf{Y}_d \mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{m}_Q^2 \mathbf{Y}_d^\dagger - 2\mathbf{Y}_d \mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{Y}_d^\dagger \mathbf{m}_d^2 \\
& - \left[(\mathbf{m}_d^2 + 4m_{H_d}^2) \mathbf{Y}_d \mathbf{Y}_d^\dagger + 2\mathbf{Y}_d \mathbf{m}_Q^2 \mathbf{Y}_d^\dagger + \mathbf{Y}_d \mathbf{Y}_d^\dagger \mathbf{m}_d^2 \right] \text{Tr}(6\mathbf{Y}_d^\dagger \mathbf{Y}_d + 2\mathbf{Y}_e^\dagger \mathbf{Y}_e) \\
& - 4\mathbf{Y}_d \mathbf{Y}_d^\dagger \text{Tr}(3\mathbf{m}_Q^2 \mathbf{Y}_d^\dagger \mathbf{Y}_d + 3\mathbf{Y}_d^\dagger \mathbf{m}_d^2 \mathbf{Y}_d + \mathbf{m}_L^2 \mathbf{Y}_e^\dagger \mathbf{Y}_e + \mathbf{Y}_e^\dagger \mathbf{m}_e^2 \mathbf{Y}_e) \\
& - 4 \left\{ \mathbf{h}_d \mathbf{h}_d^\dagger \mathbf{Y}_d \mathbf{Y}_d^\dagger + \mathbf{Y}_d \mathbf{Y}_d^\dagger \mathbf{h}_d \mathbf{h}_d^\dagger + \mathbf{h}_d \mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{h}_d^\dagger + \mathbf{Y}_d \mathbf{h}_d^\dagger \mathbf{h}_d \mathbf{Y}_d^\dagger \right\}
\end{aligned}$$

$$\begin{aligned}
& -4\{h_d h_u^\dagger Y_u Y_d^\dagger + Y_d Y_u^\dagger h_u h_d^\dagger + h_d Y_u^\dagger Y_u h_d^\dagger + Y_d h_u^\dagger h_u Y_d^\dagger\} \\
& -4h_d h_d^\dagger \text{Tr}(3Y_d^\dagger Y_d + Y_e^\dagger Y_e) - 4Y_d Y_d^\dagger \text{Tr}(3h_d^\dagger h_d + h_e^\dagger h_e) \\
& -4h_d Y_d^\dagger \text{Tr}(3h_d^\dagger Y_d + h_e^\dagger Y_e) - 4Y_d h_d^\dagger \text{Tr}(3Y_d^\dagger h_d + Y_e^\dagger h_e) \\
& + [6g_2^2 + \frac{2}{5}g_1^2] \{(\mathbf{m}_d^2 + 2m_{H_d}^2)Y_d Y_d^\dagger + 2Y_d \mathbf{m}_Q^2 Y_d^\dagger + Y_d Y_d^\dagger \mathbf{m}_d^2 + 2h_d h_d^\dagger\} \\
& + 12g_2^2 \{2|M_2|^2 Y_d Y_d^\dagger - M_2^* h_d Y_d^\dagger - M_2 Y_d h_d^\dagger\} \\
& + \frac{4}{5}g_1^2 \{2|M_1|^2 Y_d Y_d^\dagger - M_1^* h_d Y_d^\dagger - M_1 Y_d h_d^\dagger\} + \frac{4}{5}g_1^2 \mathcal{S}' \\
& - \frac{128}{3}g_3^4 |M_3|^2 + \frac{128}{45}g_3^2 g_1^2 (|M_3|^2 + |M_1|^2 + \text{Re}[M_1 M_3^*]) + \frac{808}{75}g_1^4 |M_1|^2 \\
& + \frac{16}{3}g_3^2 \sigma_3 + \frac{4}{15}g_1^2 \sigma_1
\end{aligned} \tag{A.50}$$

$$\begin{aligned}
\beta_{\mathbf{m}_e^2}^{(1)} &= (2\mathbf{m}_e^2 + 4m_{H_d}^2)Y_e Y_e^\dagger + 4Y_e \mathbf{m}_L^2 Y_e^\dagger + 2Y_e Y_e^\dagger \mathbf{m}_e^2 + 4h_e h_e^\dagger \\
& - \frac{24}{5}g_1^2 |M_1|^2 + \frac{6}{5}g_1^2 \mathcal{S}
\end{aligned} \tag{A.51}$$

$$\begin{aligned}
\beta_{\mathbf{m}_e^2}^{(2)} &= -(2\mathbf{m}_e^2 + 8m_{H_d}^2)Y_e Y_e^\dagger Y_e Y_e^\dagger - 4Y_e \mathbf{m}_L^2 Y_e^\dagger Y_e Y_e^\dagger - 4Y_e Y_e^\dagger \mathbf{m}_e^2 Y_e Y_e^\dagger \\
& - 4Y_e Y_e^\dagger Y_e \mathbf{m}_L^2 Y_e^\dagger - 2Y_e Y_e^\dagger Y_e Y_e^\dagger \mathbf{m}_e^2 - (2\mathbf{m}_e^2 + 4m_{H_u}^2 + 4m_{H_d}^2)Y_e Y_\nu^\dagger Y_\nu Y_e^\dagger \\
& - 4Y_e \mathbf{m}_L^2 Y_\nu^\dagger Y_\nu Y_e^\dagger - 4Y_e Y_\nu^\dagger \mathbf{m}_\nu^2 Y_\nu Y_e^\dagger - 4Y_e Y_\nu^\dagger Y_\nu \mathbf{m}_L^2 Y_e^\dagger - 2Y_e Y_\nu^\dagger Y_\nu Y_e^\dagger \mathbf{m}_e^2 \\
& - [(m_e^2 + 4m_{H_d}^2)Y_e Y_e^\dagger + 2Y_e \mathbf{m}_L^2 Y_e^\dagger + Y_e Y_e^\dagger \mathbf{m}_e^2] \text{Tr}[6Y_d^\dagger Y_d + 2Y_e^\dagger Y_e] \\
& - 4Y_e Y_e^\dagger \text{Tr}[3\mathbf{m}_Q^2 Y_d^\dagger Y_d + 3Y_d^\dagger \mathbf{m}_d^2 Y_d + \mathbf{m}_L^2 Y_e^\dagger Y_e + Y_e^\dagger \mathbf{m}_e^2 Y_e] \\
& - 4\{h_e h_e^\dagger Y_e Y_e^\dagger + Y_e Y_e^\dagger h_e h_e^\dagger + h_e Y_e^\dagger Y_e h_e^\dagger + Y_e h_e^\dagger h_e Y_e^\dagger\} \\
& - 4\{h_e h_\nu^\dagger Y_\nu Y_e^\dagger + Y_e Y_\nu^\dagger h_\nu h_e^\dagger + h_e Y_\nu^\dagger Y_\nu h_e^\dagger + Y_e h_\nu^\dagger h_\nu Y_e^\dagger\} \\
& - 4h_e h_e^\dagger \text{Tr}[3Y_d^\dagger Y_d + Y_e^\dagger Y_e] - 4Y_e Y_e^\dagger \text{Tr}[3h_d^\dagger h_d + h_e^\dagger h_e] \\
& - 4h_e Y_e^\dagger \text{Tr}[3h_d^\dagger Y_d + h_e^\dagger Y_e] - 4Y_e h_e^\dagger \text{Tr}[3Y_d^\dagger h_d + Y_e^\dagger h_e]
\end{aligned}$$

$$\begin{aligned}
& + \left[6g_2^2 - \frac{6}{5}g_1^2 \right] \{ (\mathbf{m}_e^2 + 2m_{H_u}^2) \mathbf{Y}_e \mathbf{Y}_e^\dagger + 2\mathbf{Y}_e \mathbf{m}_L^2 \mathbf{Y}_e^\dagger + \mathbf{Y}_e \mathbf{Y}_e^\dagger \mathbf{m}_e^2 + 2\mathbf{h}_e \mathbf{h}_e^\dagger \} \\
& + 12g_2^2 \{ 2|M_2|^2 \mathbf{Y}_e \mathbf{Y}_e^\dagger - M_2^* \mathbf{h}_e \mathbf{Y}_e^\dagger - M_2 \mathbf{Y}_e \mathbf{h}_e^\dagger \} \\
& - \frac{12}{5}g_1^2 \{ 2|M_1|^2 \mathbf{Y}_e \mathbf{Y}_e^\dagger - M_1^* \mathbf{h}_e \mathbf{Y}_e^\dagger - M_1 \mathbf{Y}_e \mathbf{h}_e^\dagger \} \\
& + \frac{12}{5}g_1^2 \mathcal{S}' + \frac{2808}{25}g_1^4 |M_1|^2 + \frac{12}{5}g_1^2 \sigma_1 . \tag{A.52}
\end{aligned}$$

$$\beta_{\mathbf{m}_\nu^2}^{(1)} = (2\mathbf{m}_\nu^2 + 4m_{H_u}^2) \mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger + 4\mathbf{Y}_\nu \mathbf{m}_L^2 \mathbf{Y}_\nu^\dagger + 2\mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger \mathbf{m}_\nu^2 + 4\mathbf{h}_\nu \mathbf{h}_\nu^\dagger \tag{A.53}$$

$$\begin{aligned}
\beta_{\mathbf{m}_\nu^2}^{(2)} = & - (2\mathbf{m}_\nu^2 + 8m_{H_u}^2) \mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger - 4\mathbf{Y}_\nu \mathbf{m}_L^2 \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger - 4\mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger \mathbf{m}_\nu^2 \mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger \\
& - 4\mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu \mathbf{m}_L^2 \mathbf{Y}_\nu^\dagger - 2\mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger \mathbf{m}_\nu^2 - (2\mathbf{m}_\nu^2 + 4m_{H_u}^2 + 4m_{H_d}^2) \mathbf{Y}_\nu \mathbf{Y}_e^\dagger \mathbf{Y}_e \mathbf{Y}_\nu^\dagger \\
& - 4\mathbf{Y}_\nu \mathbf{m}_L^2 \mathbf{Y}_e^\dagger \mathbf{Y}_e \mathbf{Y}_\nu^\dagger - 4\mathbf{Y}_\nu \mathbf{Y}_e^\dagger \mathbf{m}_e^2 \mathbf{Y}_e \mathbf{Y}_\nu^\dagger - 4\mathbf{Y}_\nu \mathbf{Y}_e^\dagger \mathbf{Y}_e \mathbf{m}_L^2 \mathbf{Y}_\nu^\dagger - 2\mathbf{Y}_\nu \mathbf{Y}_e^\dagger \mathbf{Y}_e \mathbf{Y}_\nu^\dagger \mathbf{m}_\nu^2 \\
& - [(\mathbf{m}_\nu^2 + 4m_{H_u}^2) \mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger + 2\mathbf{Y}_\nu \mathbf{m}_L^2 \mathbf{Y}_\nu^\dagger + \mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger \mathbf{m}_\nu^2] \text{Tr}[6\mathbf{Y}_u^\dagger \mathbf{Y}_u + 2\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu] \\
& - 4\mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger \text{Tr}[3\mathbf{m}_Q^2 \mathbf{Y}_u^\dagger \mathbf{Y}_u + 3\mathbf{Y}_u^\dagger \mathbf{m}_u^2 \mathbf{Y}_u + \mathbf{m}_L^2 \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu + \mathbf{Y}_\nu^\dagger \mathbf{m}_\nu^2 \mathbf{Y}_\nu] \\
& - 4\{ \mathbf{h}_\nu \mathbf{h}_\nu^\dagger \mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger + \mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger \mathbf{h}_\nu \mathbf{h}_\nu^\dagger + \mathbf{h}_\nu \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu \mathbf{h}_\nu^\dagger + \mathbf{Y}_\nu \mathbf{h}_\nu^\dagger \mathbf{h}_\nu \mathbf{Y}_\nu^\dagger \} \\
& - 4\{ \mathbf{h}_\nu \mathbf{h}_e^\dagger \mathbf{Y}_e \mathbf{Y}_\nu^\dagger + \mathbf{Y}_\nu \mathbf{Y}_e^\dagger \mathbf{h}_e \mathbf{h}_\nu^\dagger + \mathbf{h}_\nu \mathbf{Y}_e^\dagger \mathbf{Y}_e \mathbf{h}_\nu^\dagger + \mathbf{Y}_\nu \mathbf{h}_e^\dagger \mathbf{h}_e \mathbf{Y}_\nu^\dagger \} \\
& - 4\mathbf{h}_\nu \mathbf{h}_\nu^\dagger \text{Tr}[3\mathbf{Y}_u^\dagger \mathbf{Y}_u + \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu] - 4\mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger \text{Tr}[3\mathbf{h}_u^\dagger \mathbf{h}_u + \mathbf{h}_\nu^\dagger \mathbf{h}_\nu] \\
& - 4\mathbf{h}_\nu \mathbf{Y}_\nu^\dagger \text{Tr}[3\mathbf{h}_u^\dagger \mathbf{Y}_u + \mathbf{h}_\nu^\dagger \mathbf{Y}_\nu] - 4\mathbf{Y}_\nu \mathbf{h}_\nu^\dagger \text{Tr}[3\mathbf{Y}_u^\dagger \mathbf{h}_u + \mathbf{Y}_\nu^\dagger \mathbf{h}_\nu] \\
& + \left[6g_2^2 + \frac{6}{5}g_1^2 \right] \{ (\mathbf{m}_\nu^2 + 2m_{H_u}^2) \mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger + 2\mathbf{Y}_\nu \mathbf{m}_L^2 \mathbf{Y}_\nu^\dagger + \mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger \mathbf{m}_\nu^2 + 2\mathbf{h}_\nu \mathbf{h}_\nu^\dagger \} \\
& + 12g_2^2 \{ 2|M_2|^2 \mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger - M_2^* \mathbf{h}_\nu \mathbf{Y}_\nu^\dagger - M_2 \mathbf{Y}_\nu \mathbf{h}_\nu^\dagger \} \\
& + \frac{12}{5}g_1^2 \{ 2|M_1|^2 \mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger - M_1^* \mathbf{h}_\nu \mathbf{Y}_\nu^\dagger - M_1 \mathbf{Y}_\nu \mathbf{h}_\nu^\dagger \} . \tag{A.54}
\end{aligned}$$

In this appendix, we have presented the two-loop renormalization group equations for all couplings in the MSSM+ ν^c . In the limit Y_ν , h_ν , m_ν^2 , M and $B_\nu = 0$ these results coincide with the MSSM equations obtained in [103].

APPENDIX B

WEYL NOTATION, MASS EIGENSTATES AND ONE-LOOP RESULTS IN MSSM

In this appendix we display the MSSM formulas incorporated into the numerical code used to evaluate global fits to low energy data. As a general remark, note that whenever possible, we tried to hold up to the same conventions as used in the Physics Report review article by Haber and Kane [105]. Before we begin, let us first review Weyl notation, which is a very formalism adopted in this work.

B.1 Weyl Notation

In this thesis, fermions are described either by Weyl spinors or by Dirac bi-spinors. In Weyl notation, there is a two-component spinor field for a fermion, and another two-component field for its antifermion. These spinors are completely unrelated and independent on each other. In fact, that is exactly the notation needed for the symmetries of the Standard Model (SM) which stipulates that these fields are in different gauge group multiplets. Thus within the SM, and within the MSSM as well, this notation is preferred over Dirac notation where bi-spinors necessarily require the presence of particular projectors in almost every term. However, in the loop calculations at the Z scale, where the chiral gauge symmetries are broken, it is again

useful to go back to Dirac notation and utilize the well-known trace formulas for the Dirac γ matrices.

The use of Weyl notation can be briefly illustrated on the example of the electron field e_α and the positron field e_α^c ($\alpha = 1, 2$). The more familiar Dirac bi-spinor field can be expressed as

$$\Psi_e = \begin{pmatrix} e_\alpha \\ (\epsilon^{\alpha\beta} e_\beta^c)^* \end{pmatrix}. \quad (\text{B.1})$$

The lower two components transform actually as a different representation of the Lorentz group than e and e^c , and form a spinor denoted as $(e^{c*})^{\dot{\alpha}}$, $\dot{\alpha} = 1, 2$. The $\text{SL}(2, \mathbb{C})$ contraction matrix ϵ is defined as

$$\epsilon_{\alpha\beta} = -\epsilon^{\alpha\beta} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad (\text{B.2})$$

and the same holds for the dotted indices. Raising and lowering of the indices on the spinors e and e^c , and on their complex conjugates, is always defined in an orderly way: the matrix ϵ goes first and then follows a spinor, as for instance, in eq.(B.1). There is one uncomfortable property in the Haber&Kane's convention that the complex conjugated spinors are contracted differently than the spinors with undotted indices:

$$\begin{aligned} e e^c &= e^\alpha e_\alpha^c, \\ e^* e^{c*} &= e_{\dot{\alpha}}^* (e^{c*})^{\dot{\alpha}}. \end{aligned}$$

On the other hand, a nice feature is that $e e^c = e^c e$ and $e^* e^{c*} = e^{c*} e^*$, which means that one does not need to pay attention to the ordering in mass terms (see eq.(B.6) below). This property goes somehow against intuition since the two elements of these spinors are anticommuting fields. Nevertheless, it is a simple exercise left for the reader that our set of conventions indeed leads to this conclusion.

In this, so-called, chiral representation, the Dirac γ matrices read

$$\gamma_\mu = \begin{pmatrix} 0 & \sigma_{\mu\alpha\dot{\beta}} \\ \bar{\sigma}_\mu^{\dot{\alpha}\beta} & 0 \end{pmatrix}, \quad (\text{B.3})$$

$$\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \quad (\text{B.4})$$

where $\sigma^\mu = (1, \vec{\sigma})$ and $\bar{\sigma}^\mu = (1, -\vec{\sigma})$, with $\vec{\sigma}$ being the standard Pauli matrices. This representation is chiral since the usual left and right projectors $P_{L,R} \equiv (1 \pm \gamma_5)/2$ clearly pick up one of the two Weyl spinors present in the Dirac bi-spinor of eq.(B.1).

In Weyl notation, the e and e^c kinetic terms in the lagrangian are explicitly decoupled —

$$\bar{\Psi}_e \gamma_\mu (i\partial^\mu) \Psi_e = e^* \bar{\sigma}_\mu (i\partial^\mu) e + e^{c*} \bar{\sigma}_\mu (i\partial^\mu) e^c, \quad (\text{B.5})$$

while the mass term is extremely simple and, contrary to the previous terms, it explicitly couples e to e^c :

$$\mathcal{L}_{mass} = m_e e e^c + h.c. \quad . \quad (\text{B.6})$$

Dirac equation

$$(\gamma_\mu p^\mu - m_e) \Psi = 0 \quad (\text{B.7})$$

translates into Weyl notation as

$$(\bar{\sigma}_\mu p^\mu)^{\dot{\alpha}\beta} e_\beta = m_e (e^{c*})^{\dot{\alpha}}, \quad (\text{B.8})$$

$$(\sigma_\mu p^\mu)_{\alpha\dot{\beta}} (e^{c*})^{\dot{\beta}} = m_e e_\alpha. \quad (\text{B.9})$$

Clearly, in the massless case these equations decouple and, after the division by $2E = 2|\vec{p}|$, they explicitly describe a left-handed field (with helicity -1/2)

$$\frac{\frac{1}{2} \vec{\sigma} \cdot \vec{p}}{|\vec{p}|} e = -\frac{1}{2} e, \quad (\text{B.10})$$

and a right-handed field (with helicity $+1/2$)

$$\frac{\frac{1}{2} \vec{\sigma} \cdot \vec{p}}{|\vec{p}|} e^{c*} = +\frac{1}{2} e^{c*}, \quad (\text{B.11})$$

as expected, since in the massless limit, helicity states are of specific chirality.

In Dirac notation, the charge conjugation operator is defined in this representation as $C = i\gamma^2\gamma^0$ and the charge conjugated bi-spinor can be derived as

$$\Psi^c = C \bar{\Psi}^T. \quad (\text{B.12})$$

Using Weyl notation

$$C = \begin{pmatrix} \epsilon_{\beta\alpha} & 0 \\ 0 & \epsilon^{\dot{\beta}\dot{\alpha}} \end{pmatrix} \quad (\text{B.13})$$

and

$$\Psi^c = \begin{pmatrix} e_{\alpha}^c \\ e^{*\dot{\alpha}} \end{pmatrix}. \quad (\text{B.14})$$

Compared with relation (B.1), electron and positron fields clearly exchange their positions in eq.(B.14) as one would expect from the operation of charge conjugation.

Let's finish this introductory part with the note that the gauge currents, mentioned in (1.2-1.5), can be derived by making the usual transition to covariant derivatives $\partial^\mu \rightarrow \partial^\mu + ig A^\mu{}^a T^a$ in the fermionic kinetic terms, analogous to (B.5). More technical details can be found in appendix A of Haber&Kane's Physics Report [105].

B.2 Mass eigenstates of MSSM

The MSSM lagrangian is determined by the superpotential (1.25) (and the extra soft SUSY breaking terms (1.26), which can, however, be neglected far above the SUSY breaking scale). These relations are written in terms of interaction eigenstates in a nice compact form due to high symmetry present in the theory. However, a large

part of this symmetry is broken at the Z scale, and many states mix under the less symmetry which is left over.

In particular, the fermionic sector of the theory is derived from the superpotential according to [106]

$$\mathcal{L}_{Yuk}^{(MSSM)} = -\frac{1}{2} \frac{\partial^2 W}{\partial A_i \partial A_j} \xi_i \xi_j + h.c. , \quad (B.15)$$

where A_i 's and ξ_i 's are the scalar and spinor components of chiral multiplets. Eq.(B.15) then generates Yukawa terms, and fermion mass matrices after the electroweak symmetry breaking. Fermion mass matrices, eq.(1.37-1.39), or equivalently the corresponding Yukawa matrices, are diagonalized by bi-unitary transformations

$$\lambda^{u \text{ diag}} = V_u^L{}^* \lambda^u (V_u^{R\dagger})^* , \quad (B.16)$$

$$\lambda^{d \text{ diag}} = V_d^L{}^* \lambda^d (V_d^{R\dagger})^* , \quad (B.17)$$

$$\lambda^{e \text{ diag}} = V_e^L{}^* \lambda^e (V_e^{R\dagger})^* , \quad (B.18)$$

where, by definition, the diagonal values are real and positive. Thus we define the mass eigenstates ξ and η

$$\xi_f = V_f^L f , \quad (B.19)$$

$$\eta_f^* = V_f^R f^{c*} , \quad (B.20)$$

as mixings of interaction eigenstates f and f^c , where $f = u, d, e$. Recall that if we find more convenient to work in Dirac notation, we can always change our fields to Dirac bi-spinors following the prescription

$$\Psi_f = \begin{pmatrix} \xi_{f\alpha} \\ \eta_f^{*\dot{\alpha}} \end{pmatrix} , \quad (B.21)$$

of eq.(B.1). Also recall, that the CKM matrix is defined according to eq.(1.24).

At the Z scale, scalar masses are generated by the following sources:

- *D*-terms from the kinetic terms:

$$V_D = (D^a D^a + (D')^2)/2, \quad (\text{B.22})$$

where

$$D^a = -g T_{ij}^a A_i^* A_j, \quad (\text{B.23})$$

$$D' = -\frac{1}{2} g' y_i A_i^* A_i, \quad (\text{B.24})$$

and the A_i fields are the scalar parts of chiral multiplets as in (B.15).

- *F*-terms from the superpotential:

$$V_F = \left| -\frac{\partial W}{\partial A_i} \right|^2 \quad (\text{B.25})$$

- Tri-linear and bi-linear soft SUSY breaking scalar couplings, eq.(1.26).
- SUSY breaking masses, eq.(1.26).

To clarify our notation let us define that the squark/slepton interaction eigenstates bear the subscript L or R . Thus $\tilde{u}_L^i$, $\tilde{d}_L^i$, $\tilde{e}_L^i$ and $\tilde{\nu}_L^i$ are the squarks and sleptons from the doublets $\tilde{Q}^i$ and $\tilde{L}^i$ in eq.(1.26), and

$$\tilde{u}_R^i = (\tilde{u}^{ci})^*, \quad (\text{B.26})$$

$$\tilde{d}_R^i = (\tilde{d}^{ci})^*, \quad (\text{B.27})$$

$$\tilde{e}_R^i = (\tilde{e}^{ci})^* \quad (\text{B.28})$$

are redefining singlet squarks and sleptons. The generation index $i = 1, 2, 3$. With this notation the squark and slepton mass matrices read

$$- \mathcal{L}_{\tilde{q}\tilde{l}mass} =$$

$$\begin{aligned}
& \begin{pmatrix} \tilde{u}_L^\dagger & \tilde{u}_R^\dagger \end{pmatrix} \begin{pmatrix} \mathbf{m}_Q^2 + v_u^2 \lambda^{u*} (\lambda^{u*})^\dagger + D_{uLL} & v_u \mathbf{A}^{u*} - \mu v_d \lambda^{u*} \\ v_u (\mathbf{A}^{u*})^\dagger - \mu v_d (\lambda^{u*})^\dagger & \mathbf{m}_u^2 + v_u^2 (\lambda^{u*})^\dagger \lambda^{u*} + D_{uRR} \end{pmatrix} \begin{pmatrix} \tilde{u}_L \\ \tilde{u}_R \end{pmatrix} \\
& + \begin{pmatrix} \tilde{d}_L^\dagger & \tilde{d}_R^\dagger \end{pmatrix} \begin{pmatrix} \mathbf{m}_Q^2 + v_d^2 \lambda^{d*} (\lambda^{d*})^\dagger + D_{dLL} & v_d \mathbf{A}^{d*} - \mu v_u \lambda^{d*} \\ v_d (\mathbf{A}^{d*})^\dagger - \mu v_u (\lambda^{d*})^\dagger & \mathbf{m}_d^2 + v_d^2 (\lambda^{d*})^\dagger \lambda^{d*} + D_{dRR} \end{pmatrix} \begin{pmatrix} \tilde{d}_L \\ \tilde{d}_R \end{pmatrix} \\
& + \begin{pmatrix} \tilde{e}_L^\dagger & \tilde{e}_R^\dagger \end{pmatrix} \begin{pmatrix} \mathbf{m}_L^2 + v_d^2 \lambda^{e*} (\lambda^{e*})^\dagger + D_{eLL} & v_d \mathbf{A}^{e*} - \mu v_u \lambda^{e*} \\ v_d (\mathbf{A}^{e*})^\dagger - \mu v_u (\lambda^{e*})^\dagger & \mathbf{m}_e^2 + v_d^2 (\lambda^{e*})^\dagger \lambda^{e*} + D_{eRR} \end{pmatrix} \begin{pmatrix} \tilde{e}_L \\ \tilde{e}_R \end{pmatrix} \\
& + \tilde{\nu}_L^\dagger (\mathbf{m}_L^2 + D_{\nu LL}) \tilde{\nu}_L, \tag{B.29}
\end{aligned}$$

where D_{fLL} and D_{fRR} are the contributions from the D -terms:

$$D_{uLL} = \frac{1}{4}(v_u^2 - v_d^2)(-g_2^2 + y_Q g_1^2) \tag{B.30}$$

$$D_{uRR} = \frac{1}{4}(v_u^2 - v_d^2)y_{uc}g_1^2 \tag{B.31}$$

$$D_{dLL} = \frac{1}{4}(v_u^2 - v_d^2)(+g_2^2 + y_Q g_1^2) \tag{B.32}$$

$$D_{dRR} = \frac{1}{4}(v_u^2 - v_d^2)y_{dc}g_1^2 \tag{B.33}$$

$$D_{eLL} = \frac{1}{4}(v_u^2 - v_d^2)(+g_2^2 + y_L g_1^2) \tag{B.34}$$

$$D_{eRR} = \frac{1}{4}(v_u^2 - v_d^2)y_{ec}g_1^2 \tag{B.35}$$

$$D_{\nu LL} = \frac{1}{4}(v_u^2 - v_d^2)(-g_2^2 + y_L g_1^2), \tag{B.36}$$

v_u and v_d are the vacuum expectation values of the Higgs fields (see eq's(1.32-1.31)) and the hypercharge values y_f are listed in table 1.1.

The mass eigenstates $\tilde{u}^\alpha$ (and similarly $\tilde{d}^\alpha$ and $\tilde{e}^\alpha$), with $\alpha = 1, 2, \dots, 6$, are defined in two steps. First, the $\tilde{u}_L$ and $\tilde{u}_R$ squarks are rotated by the same unitary matrices V_u^L and V_u^R as their respective fermionic partners (see eq's(B.19-B.20)). Only after

that, the additional 6×3 matrices Γ_{uL} and Γ_{uR} finish up the diagonalization:

$$\tilde{u}^\alpha = \Gamma_{uL}^{\alpha i} (V_u^L)_{ij} \tilde{u}_L^j, \quad (\text{B.37})$$

$$\tilde{u}^\alpha = \Gamma_{uR}^{\alpha i} (V_u^R)_{ij} \tilde{u}_R^j. \quad (\text{B.38})$$

At the same time, the diagonalized mass matrix is obtained according to

$$\mathcal{M}_u^{2\text{diag}} = \left(\Gamma_{uL} \mid \Gamma_{uR} \right) \begin{pmatrix} V_u^L & 0 \\ 0 & V_u^R \end{pmatrix} \mathcal{M}_u^2 \begin{pmatrix} V_u^{L\dagger} & 0 \\ 0 & V_u^{R\dagger} \end{pmatrix} \begin{pmatrix} \Gamma_{uL}^\dagger \\ \Gamma_{uR}^\dagger \end{pmatrix}, \quad (\text{B.39})$$

where $\mathcal{M}_u^2$ is the up-squark mass matrix in eq.(B.29).

For three sneutrinos the analogous formula to (B.37)

$$\tilde{\nu}^\alpha = \Gamma_{\nu L}^{\alpha i} (V_e^L)_{ij} \tilde{\nu}_L^j \quad (\text{B.40})$$

defines the three mass eigenstates (exceptionally $\alpha = 1, 2, 3$), while the formula

$$\mathcal{M}_{\tilde{\nu}}^{2\text{diag}} = \Gamma_{\nu L} V_e^L \mathcal{M}_{\tilde{\nu}}^2 V_e^{L\dagger} \Gamma_{\nu L}^\dagger \quad (\text{B.41})$$

defines the additional 3×3 rotation matrix $\Gamma_{\nu L}$, which is, in fact, the upper part of the 6×3 matrix Γ_{eL} . Note that we have used the freedom in the massless neutrino sector to rotate the neutrinos by the same unitary matrix as the electrons, $V_\nu^L \equiv V_e^L$, and that rotation is then reflected in the sneutrino rotations as well.

In the gaugino/higgsino sector we follow Haber&Kane almost in every detail. In analogy to the W bosons (compare with eq.(1.8)), we define

$$\lambda^\pm = (\lambda^1 \mp i\lambda^2)/\sqrt{2} \quad (\text{B.42})$$

for the $SU(2)_L$ gauginos. Because of an “i” in the gaugino-sfermion-fermion interaction (see eq.(B2) in [105])

$$\mathcal{L} = ig T_{ij}^a \lambda^a \xi_j A_i^* + h.c., \quad (\text{B.43})$$

we define mass eigenstates (gluinos, charginos, and neutralinos) in terms of $-i\lambda$ instead of just the initial spinor λ contained in vector multiplet. The projections of Weyl spinors from the Dirac chargino states $\tilde{\chi}_A$ ($A = 1, 2$) then given by

$$-i\lambda^+ = P_L V_{1A}^\dagger \tilde{\chi}_A, \quad (\text{B.44})$$

$$-i\lambda^- = P_L U_{1A}^\dagger \tilde{\chi}_A^c, \quad (\text{B.45})$$

and analogous projections to the charged higgsino states are

$$\tilde{h}_2^+ = P_L V_{2A}^\dagger \tilde{\chi}_A, \quad (\text{B.46})$$

$$\tilde{h}_1^- = P_L U_{2A}^\dagger \tilde{\chi}_A^c. \quad (\text{B.47})$$

The 2×2 matrices V and U are defined by the bi-unitary diagonalization of the chargino mass matrix

$$\mathcal{M}_{\tilde{\chi}}^{diag} = U^* \begin{pmatrix} M_2 & g_2 v_u \\ g_2 v_d & \mu \end{pmatrix} V^\dagger. \quad (\text{B.48})$$

Similar relations between the interaction and mass eigenstates in the neutralino sector read

$$-i\lambda = P_L N_{1i}^\dagger \tilde{\chi}_i^0, \quad (\text{B.49})$$

$$-i\lambda^3 = P_L N_{2i}^\dagger \tilde{\chi}_i^0, \quad (\text{B.50})$$

$$\tilde{h}_1^0 = P_L N_{3i}^\dagger \tilde{\chi}_i^0, \quad (\text{B.51})$$

$$\tilde{h}_2^0 = P_L N_{4i}^\dagger \tilde{\chi}_i^0, \quad (\text{B.52})$$

where the 4×4 unitary matrix N is determined from

$$\mathcal{M}_{\tilde{\chi}^0}^{diag} = N^* \begin{pmatrix} M_1 & 0 & -g_1 v_d/\sqrt{2} & g_1 v_u/\sqrt{2} \\ 0 & M_2 & g_2 v_d/\sqrt{2} & -g_2 v_u/\sqrt{2} \\ -g_1 v_d/\sqrt{2} & g_2 v_d/\sqrt{2} & 0 & -\mu \\ g_1 v_u/\sqrt{2} & -g_2 v_u/\sqrt{2} & -\mu & 0 \end{pmatrix} N^\dagger. \quad (\text{B.53})$$

B.3 Higgs Potential

As described at the beginning of the previous section, scalar potential for each particle contains contributions from a number of different sectors. Following the path outlined for the squarks and sleptons, the tree level Higgs potential eventually leads to a rather simple form

$$\begin{aligned}
V(H_1^0, H_2^0) = & (m_{H_1}^2 + \mu^2) |H_1^0|^2 + (m_{H_2}^2 + \mu^2) |H_2^0|^2 - B\mu (H_1^0 H_2^0 + h.c.) \\
& + \frac{1}{8} (g_1^2 + g_2^2) (|H_1^0|^2 - |H_2^0|^2)^2.
\end{aligned} \tag{B.54}$$

To derive eq's(1.41-1.42) one just needs to plug the expressions (1.35-1.36) into the formula above, and require that the tadpole terms be equal to zero.

B.4 One-loop Corrections to Vector Boson Masses

In the following, we present a complete one-loop calculation of the vector boson self-energies. Our calculations have been performed in the Feynman gauge and Dimensional Reduction scheme.

B.4.1 Corrections to M_Z^2

Gauge and Higgs sectors contribute to the polarization tensor Π^{ZZ} by the following terms

$$\begin{aligned}
\Pi_{g+H}^{ZZ}(p^2) = & \frac{g_1^2 + g_2^2}{16\pi^2} [c_{\beta-\alpha}^2 M_Z^2 B_0(M_{H^0}^2, M_Z^2, p^2) + s_{\beta-\alpha}^2 M_Z^2 B_0(M_{h^0}^2, M_Z^2, p^2) \\
& + 2s_w^4 M_W^2 B_0(M_W^2, M_W^2, p^2) \\
& + c_{\beta-\alpha}^2 \tilde{B}_{22}(M_{H^0}^2, M_Z^2, p^2) + s_{\beta-\alpha}^2 \tilde{B}_{22}(M_{h^0}^2, M_Z^2, p^2) \\
& + s_{\beta-\alpha}^2 \tilde{B}_{22}(M_{A^0}^2, M_{H^0}^2, p^2) + c_{\beta-\alpha}^2 \tilde{B}_{22}(M_{A^0}^2, M_{h^0}^2, p^2) \\
& + (c_W^2 - s_W^2)^2 (\tilde{B}_{22}(M_{H^+}^2, M_{H^+}^2, p^2) + \tilde{B}_{22}(M_{W^+}^2, M_{W^+}^2, p^2))]
\end{aligned}$$

$$\begin{aligned}
& + c_W^4 (-6A_0(M_W^2) + 8B_{22}(M_W^2, M_W^2, p^2) \\
& - (4p^2 + 2M_W^2) B_0(M_W^2, M_W^2, p^2))]. \tag{B.55}
\end{aligned}$$

The first two lines represent the contribution from the loops with one Higgs and one vector boson, next three lines are contributions from Higgs loops only, and the rest comes from the gauge bosons and gauge ghosts inside the loop. We use a shorthand notation like *e.g.*, $s_W^2 \equiv \sin^2 \theta_W$ and $c_{\beta-\alpha}^2 \equiv \cos^2(\beta - \alpha)$. The Passarino-Veltman functions are defined at the end of this chapter.

The result for quark and lepton contributions reads

$$\begin{aligned}
\Pi_{q+l}^{ZZ}(p^2) = & \frac{g_1^2 + g_2^2}{16\pi^2} \sum_{f=u,d,e,\nu} N_{c_f} \left[\frac{2}{3} X A_0(m_f^2) + \frac{1}{3} \left(-\frac{1}{3} p^2 + 2m_f^2 \right) X \right. \\
& \left. + \left(\frac{1}{3} p^2 X + \frac{2}{3} m_f^2 X - \frac{1}{2} m_f^2 \right) B_0(m_f^2, m_f^2, p^2) \right], \tag{B.56}
\end{aligned}$$

where

$$X = 4Q_f^2 s_W^4 - 4I_{3f} Q_f s_W^2 + \frac{1}{2} \tag{B.57}$$

and Q_f and I_{3f} stand, respectively, for the electric charge and isospin of the doublet spinor in the fermionic bi-spinor state. N_{c_f} is the color factor: 3 for quarks and 1 for leptons.

Squark and slepton contributions are

$$\begin{aligned}
\Pi_{\tilde{q}+\tilde{l}}^{ZZ}(p^2) = & \frac{g_1^2 + g_2^2}{16\pi^2} \left[\sum_{f=u,d,e} N_{c_f} \sum_{\alpha,\beta=1}^6 \tilde{B}_{22}(m_{\tilde{f}_\alpha}^2, m_{\tilde{f}_\beta}^2, p^2) \right. \\
& \times [2Q_f s_W^2 - \Gamma_{fL} \Gamma_{fL}^\dagger]_{\alpha\beta} [2Q_f s_W^2 - \Gamma_{fL} \Gamma_{fL}^\dagger]_{\beta\alpha} \\
& \left. + \sum_{\alpha=1}^3 \tilde{B}_{22}(m_{\tilde{\nu}_\alpha}^2, m_{\tilde{\nu}_\alpha}^2, p^2) \right]. \tag{B.58}
\end{aligned}$$

Neutralino loops result in

$$\begin{aligned}
\Pi_{\tilde{\chi}^0}^{ZZ}(p^2) = & \frac{g_1^2 + g_2^2}{16\pi^2} \frac{1}{2} \sum_{i,j=1}^4 (N_{i3} N_{j3} - N_{i4} N_{j4})^2 \left[\frac{1}{2} A_0(i) + \frac{1}{2} A_0(j) \right. \\
& \left. - 2B_{22}(i, j, p^2) + \frac{1}{2} (p^2 - m_i^2 - m_j^2 - 2m_i m_j) B_0(i, j, p^2) \right], \tag{B.59}
\end{aligned}$$

and, finally, chargino loops add

$$\begin{aligned}\Pi_{\tilde{\chi}^+}^{ZZ}(p^2) &= \frac{g_1^2 + g_2^2}{16\pi^2} \sum_{A,B=1}^2 [(a_{AB}b_{AB} + c_{AB}d_{AB}) (\frac{1}{2}A_0(A) + \frac{1}{2}A_0(B)) \\ &\quad - 2B_{22}(A, B, p^2) + \frac{1}{2}(p^2 - m_A^2 - m_B^2 - 2m_A m_B) B_0(A, B, p^2)) \\ &\quad + (a_{AB}b_{AB} - c_{AB}d_{AB}) m_A m_B B_0(A, B, p^2)],\end{aligned}\quad (\text{B.60})$$

where

$$a_{AB} = \frac{1}{2}(V_{A1}V_{1B}^\dagger + U_{A1}^*U_{1B}^\dagger) + (c_W^2 - s_W^2)\delta_{AB}, \quad (\text{B.61})$$

$$b_{AB} = a_{BA}, \quad (\text{B.62})$$

$$c_{AB} = \frac{1}{2}(-V_{A1}V_{1B}^\dagger + U_{A1}^*U_{1B}^\dagger), \quad (\text{B.63})$$

$$d_{AB} = c_{BA}. \quad (\text{B.64})$$

B.4.2 Corrections to M_W^2

Gauge and Higgs sectors contribute to the polarization tensor Π^{WW} by the following terms

$$\begin{aligned}\Pi_{g+H}^{WW}(p^2) &= \frac{g_2^2}{16\pi^2} [c_{\beta-\alpha}^2 M_W^2 B_0(M_{H^0}^2, M_W^2, p^2) + s_{\beta-\alpha}^2 M_W^2 B_0(M_{h^0}^2, M_W^2, p^2) \\ &\quad + s_w^4 M_Z^2 B_0(M_Z^2, M_W^2, p^2) + s_w^2 M_W^2 B_0(0, M_W^2, p^2) \\ &\quad + c_{\beta-\alpha}^2 \tilde{B}_{22}(M_{H^0}^2, M_W^2, p^2) + s_{\beta-\alpha}^2 \tilde{B}_{22}(M_{h^0}^2, M_W^2, p^2) \\ &\quad + s_{\beta-\alpha}^2 \tilde{B}_{22}(M_{H^+}^2, M_{H^0}^2, p^2) + c_{\beta-\alpha}^2 \tilde{B}_{22}(M_{H^+}^2, M_{h^0}^2, p^2) \\ &\quad + \tilde{B}_{22}(M_{A^0}^2, M_{H^+}^2, p^2) + \tilde{B}_{22}(M_Z^2, M_W^2, p^2) \\ &\quad + c_W^2 (-3A_0(M_Z^2)) + (-3A_0(M_W^2)) \\ &\quad + s_W^2 (A_0(M_W^2) - (4p^2 + M_W^2)B_0(0, M_W^2, p^2) + 8B_{22}(0, M_W^2, p^2)) \\ &\quad + c_W^2 (A_0(M_W^2) + A_0(M_Z^2) - (4p^2 + M_W^2 + M_Z^2)B_0(M_Z^2, M_W^2, p^2) \\ &\quad + 8B_{22}(M_Z^2, M_W^2, p^2))] \end{aligned}\quad (\text{B.65})$$

The first two lines represent the contribution from the loops with one Higgs and one vector boson, next three lines are contributions from Higgs loops only, and the rest comes from the gauge bosons and gauge ghosts inside the loop. We use a shorthand notation like *e.g.*, $s_W^2 \equiv \sin^2 \theta_W$ and $c_{\beta-\alpha}^2 \equiv \cos^2(\beta - \alpha)$.

The result for quark and lepton contributions reads

$$\begin{aligned} \Pi_{q+\ell}^{WW}(p^2) = & \frac{g_2^2}{16\pi^2} [3 \sum_{i,j=1}^3 |(V_{CKM})_{ij}|^2 [\frac{1}{2}A_0(m_{u_i}^2) + \frac{1}{2}A_0(m_{d_j}^2) \\ & - 2B_{22}(i, j, p^2) + \frac{1}{2}(p^2 - m_{u_i}^2 - m_{d_j}^2 - 2m_{u_i}m_{d_j})B_0(i, j, p^2)] \\ & + \sum_{j=1}^3 [\frac{1}{2}A_0(m_{d_j}^2) \\ & - 2B_{22}(0, m_{e_j}^2, p^2) + \frac{1}{2}(p^2 - m_{e_j}^2)B_0(0, m_{e_j}^2, p^2)]]. \end{aligned} \quad (\text{B.66})$$

Squark and slepton contributions are

$$\begin{aligned} \Pi_{\tilde{q}+\tilde{\ell}}^{WW}(p^2) = & \frac{g_2^2}{16\pi^2} [6 \sum_{\alpha,\beta=1}^6 \tilde{B}_{22}(m_{d_\alpha}^2, m_{\tilde{u}_\beta}^2, p^2) \\ & \times (\Gamma_{dL} V_{CKM}^\dagger \Gamma_{uL}^\dagger)_{\alpha\beta} (\Gamma_{uL} V_{CKM} \Gamma_{dL}^\dagger)_{\beta\alpha} \\ & + 2 \sum_{\beta=1}^3 \sum_{\alpha=1}^6 \tilde{B}_{22}(m_{\tilde{e}_\alpha}^2, m_{\tilde{\nu}_\beta}^2, p^2) \\ & \times (\Gamma_{eL} \Gamma_{\nu L}^\dagger)_{\alpha\beta} (\Gamma_{\nu L} \Gamma_{eL}^\dagger)_{\beta\alpha}]. \end{aligned} \quad (\text{B.67})$$

Finally, loops containing neutralino and chargino yield

$$\begin{aligned} \Pi_{\tilde{\chi}^0+\tilde{\chi}^\pm}^{WW}(p^2) = & \frac{g_2^2}{16\pi^2} 4 \sum_{i=1}^4 \sum_{A=1}^2 [(a_{Ai} b_{Ai} + c_{Ai} d_{Ai}) (\frac{1}{2}A_0(A) + \frac{1}{2}A_0(i) \\ & - 2B_{22}(A, i, p^2) + \frac{1}{2}(p^2 - m_A^2 - m_i^2 - 2m_A m_i)B_0(A, i, p^2)) \\ & + (a_{Ai} b_{Ai} - c_{Ai} d_{Ai}) m_A m_i B_0(A, i, p^2)], \end{aligned} \quad (\text{B.68})$$

where

$$a_{Ai} = \frac{1}{2}(V_{A1} N_{i2} + U_{A1}^* N_{i2} - \frac{1}{\sqrt{2}}(V_{A2} N_{i4} - U_{A2}^* N_{i3})), \quad (\text{B.69})$$

$$b_{Ai} = a_{Ai}^* , \quad (B.70)$$

$$c_{AB} = \frac{1}{2}(-V_{A1}N_{i2} + U_{A1}^*N_{i2} + \frac{1}{\sqrt{2}}(V_{A2}N_{i4} + U_{A2}^*N_{i3})) , \quad (B.71)$$

$$d_{Ai} = c_{Ai}^* . \quad (B.72)$$

B.5 Corrections to Fermion Mass Matrices Proportional to $\tan\beta$

At the SUSY threshold there are the following 1-loop diagrams contributing significantly to fermion mass matrices in large $\tan\beta$ limit.

i) *gluino diagram*

In Dirac notation, the quark-squark-gluino interaction, relevant for the results in chapter 3, reads

$$\mathcal{L}_{int} = -\sqrt{2}g_3 \left(\frac{\lambda^A}{2}\right)_{ab} \{+(\bar{d}_a P_R \tilde{g}^A) \tilde{d}_{Lb} - \tilde{d}_{Ra}^{\dagger} (\bar{\tilde{g}}^A P_R d_b)\} + h.c.. \quad (B.73)$$

According to eq's(B.37-B.38), the d -squark interaction eigenstates are turned into the mass eigenstates by

$$\tilde{d}_{Ri} = (V_d^{R\dagger})_{ij} (\Gamma_{dR}^{\dagger})_{j\alpha} \tilde{d}_{\alpha} \quad (B.74)$$

$$\tilde{d}_{Li} = (V_d^{L\dagger})_{ij} (\Gamma_{dL}^{\dagger})_{j\alpha} \tilde{d}_{\alpha} . \quad (B.75)$$

As before, the indices $i, j, \dots$ denote generation indices 1,2,3, the greek letters denote squark indices 1 to 6 and that the implicit summation over the repeating indices is assumed. The diagram with the gluino and d -type squarks in the loop contributes to the d -quark self-energy matrix (amputated two-point function) as :

$$\begin{aligned} -i\Sigma = & (-i\sqrt{2}g_3)^2 C_2(\mathbf{3}) \int \frac{d^d k}{(2\pi)^d} (-) P_R < \tilde{g} \bar{\tilde{g}} > P_R \\ & V_d^{L\dagger} \Gamma_{dL}^{\dagger} < \tilde{d} \bar{\tilde{d}} > \Gamma_{dR} V_d^R + P_L \dots P_L + \not{p} - terms . \end{aligned} \quad (B.76)$$

Only the term with the two right-handed projectors corrects the mass matrix. The term indicated as $P_L \dots P_L$ contains similar corrections to $\mathbf{m}^\dagger$. To get to the formula (3.1) in chapter 3 one performs the rotation to Euclidean space and integrates out angular variables. The integral measure dk in (3.2) stands for $k^2 d(k^2)$ and the integration limits are assumed to be zero and infinity. Note that in chapter 3 $\mathbf{m}_d^0$ was appended to these equations in not a very elegant way, but that is for later convenience.

ii) chargino diagram

Quark-squark-chargino interaction that is relevant for the diagrams in chapter 3 reads

$$\begin{aligned} \mathcal{L}_{int} = & (\bar{d} P_R (V^{*†})_{2A} \tilde{\chi}_A^c) \lambda_u \tilde{u}_R + \tilde{u}_L^\dagger \lambda_d ((U^{*†})_{2A} \tilde{\chi}_A^c P_R d) \\ & - g_2 (\bar{d} P_R (V^{*†})_{1A} \tilde{\chi}_A^c) \tilde{u}_L + h.c. . \end{aligned} \quad (\text{B.77})$$

u squarks are rotated to their mass eigenstates in exactly the same way as the d squarks above, defining the $\Gamma_{uR,L}$ matrices. Contribution to the d quark self-energy from this interaction reads

$$\begin{aligned} -i\Sigma = & (i)^2 \int \frac{d^d k}{(2\pi)^d} U_{A2} V_{B2} P_R \langle \tilde{\chi}_A^c \tilde{\chi}_B^c \rangle P_R \lambda_u V_u^{R0\dagger} \Gamma_{uR}^\dagger \langle \tilde{u} \tilde{u}^\dagger \rangle \Gamma_{uL} V_u^{L0} \lambda_d - \\ & - U_{A2} V_{B1} P_R \langle \tilde{\chi}_A^c \tilde{\chi}_B^c \rangle P_R g_2 V_u^{L0\dagger} \Gamma_{uL}^\dagger \langle \tilde{u} \tilde{u}^\dagger \rangle \Gamma_{uL} V_u^{L0} \lambda_d + P_L \dots P_L + \text{p terms} . \end{aligned}$$

Recall that the matrices U and V diagonalize the chargino mass matrix. The fact that one of their indices is 1 (2), traces back the wino (higgsino) interaction in the quark-squark-chargino vertex of the loop. Summation over $A, B=1,2$ is assumed. In order to derive the equation (3.1) one has to use the relations between the diagonalized and non-diagonalized Yukawa and squark mass matrices, which are derived at the

beginning of this appendix. The vev of the scalar Higgs H_d is added in order to pull out the mass matrix on the r.h.s. for future convenience and when combined with $\tan\beta$ (which is pulled out into the ϵ) it yields $(v_u)^{-1}$ in the final expression (3.3) given in chapter 3.

B.6 Passarino-Veltman Functions

Passarino-Veltman functions [107] are extremely useful for fast evaluation of loop integrals. In this section we discuss the most important scalar integrals and give the results for the more complicated Passarino-Veltman functions, used in this appendix, in terms of the simpler scalar integrals.

In our conventions, $A_0(m^2)$ is defined by the relation

$$\mu_0^\epsilon \int \frac{d^d k}{(2\pi)^d} \frac{1}{k^2 - m^2} \equiv -\frac{i}{16\pi^2} A_0(m^2), \quad (\text{B.78})$$

where we assume the usual shift in the pole: $m^2 \equiv m^2 - i\epsilon$. Obviously, $A_0(m^2)$ is divergent. However, its quadratic divergence cancels out in dimensional regularization. Logarithmically divergent piece has a simple structure and can be easily separated out. The finite part can be readily evaluated using dimensional regularization:

$$A_0(m^2) = -m^2(\text{Reg} + 1 - \ln m^2), \quad (\text{B.79})$$

where the regulator is

$$\text{Reg} = \frac{2}{\epsilon} - \gamma_E + \ln 4\pi + \ln \mu_0^2. \quad (\text{B.80})$$

In the modified minimal subtraction scheme the first three terms in eq.(B.80) are cancelled by counterterms, and only the finite scale dependent logarithm remains.

Scalar two-point function $B_0(m_1^2, m_2^2, p^2)$ is defined by

$$\mu_0^\epsilon \int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2 - m_1^2)((k+p)^2 - m_2^2)} \equiv +\frac{i}{16\pi^2} B_0(m_1^2, m_2^2, p^2). \quad (\text{B.81})$$

It is also logarithmically divergent, and the divergent piece can be expressed in terms of the same regulator and again easily extracted out. The finite piece can be transformed into a simple integral of one variable

$$B_0(m_1^2, m_2^2, p^2) = \text{Reg} - \int_0^1 dx \ln(-p^2 x(1-x) + m_1^2(1-x) + m_2^2 x - i\epsilon). \quad (\text{B.82})$$

This integral can be evaluated analytically, giving different results in the three regions:

a) $p^2 < (m_1 - m_2)^2$, b) $(m_1 - m_2)^2 < p^2 < (m_1 + m_2)^2$ and c) $(m_1 + m_2)^2 < p^2$.

The explicit formulas can be looked up in the review article by Bohm, Hollik and Spiesberger [108].

Finally, we give the formulas for the functions B_{22} and $\tilde{B}_{22}$, which appear in the results of this chapter, in terms of B_0 and A_0 :

$$\begin{aligned} B_{22}(m_1^2, m_2^2, p^2) = & \frac{1}{6} \left[\frac{A_0(m_1^2) + A_0(m_2^2)}{2} + \left(\frac{1}{3} p^2 - m_1^2 - m_2^2 \right) \right. \\ & + \left[\frac{1}{2} p^2 - m_1^2 - m_2^2 + \frac{(m_1^2 - m_2^2)^2}{2p^2} \right] B_0(m_1^2, m_2^2, p^2) \\ & \left. + \frac{1}{2p^2} (m_1^2 - m_2^2) (A_0(m_1^2) - A_0(m_2^2)) \right] \end{aligned} \quad (\text{B.83})$$

and

$$\tilde{B}_{22}(m_1^2, m_2^2, p^2) = B_{22}(m_1^2, m_2^2, p^2) - \frac{A_0(m_1^2) + A_0(m_2^2)}{4}. \quad (\text{B.84})$$

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