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Superfluid turbulence in a nonuniform circular channel

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The Ohio State University, 1993

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SUPERFLUID TURBULENCE IN A NONUNIFORM CIRCULAR CHANNEL

Dissertation

**Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University**

by

Paul Joseph Murphy

The Ohio State University

1993

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Chapter I

Introduction

1.1 Introduction

The near simultaneous discovery by Kapitza¹ and Allen and Misener² of the superfluid phase of helium (called He-II) has led to a rich experimental and theoretical description of the unique properties of this hydrodynamic system. Over five decades later many of these properties have been understood, but not all. Perhaps the most studied is the excess dissipation that is caused by the presence of superfluid turbulence in flowing He-II. This property has been studied and is theoretically well understood for homogeneous turbulence in spatially uniform thermal counterflow.

As a logical extension of the earlier work done for spatially uniform counterflow a recent experiment has been done³ where the velocity field of the flowing helium had a small spatial nonuniformity. The measured excess dissipation varied dramatically from the prediction of the theory used in understanding spatially uniform homogeneous turbulence³.

The theoretical model disagreed with the measured magnitude of the excess dissipation by about 50%. The theoretical model also predicts the spatial and heat current dependence of the excess dissipation incorrectly.

The failure of the model to adequately account for the effects of weak nonuniformity is surprising. For the experiment described above the divergence is in one dimension and it can be argued that on the scale of the superfluid turbulence the velocity field did not vary by more than 1 part in 10,000. The apparent sensitivity of superfluid turbulence to the presence of nonuniformity was unexpected and interesting.

The experiment described in this thesis is similar to the above research but has two important differences. The first difference is that the flow field in my experiment is allowed to diverge in two dimensions. The second difference is that the cross sectional geometry of my flow channel is circular while the experiment done by Kafkalidis and Tough used a channel that is rectangular in cross section (hereafter called the nonuniform rectangular experiment). These differences will be described briefly here and more thoroughly later in the thesis.

The channel used in the nonuniform rectangular experiment was wedge-shaped and its cross section diverged from an aspect ratio (width to height) of 8:1 to 40:1. The channel used in my experiment is shaped like a cone and the diameter of the cross section varies by a factor of 2 over the length of the channel. Thus the velocity in the nonuniform rectangular channel varies inversely with the distance along the length of the channel while the flow field in my channel varies inversely with the length of channel squared.

The difference in the cross sectional geometry is very important. As will be described later in this chapter, uniform channels with rectangular cross sections experimentally enter only one state of turbulence, the T-II state. Experimentally uniform channels with circular cross sections enter two different states of turbulence, the T-I and T-II state. The nonuniform rectangular channel experiment observes only one state of superfluid turbulence which is consistent with the results found in the uniform rectangular channel case.

The experiment described in this thesis measures the effect that a nonuniform flow field has on the structure found in the uniform circular channel experiment. Reported in this thesis are the results of measuring the dissipation

for thermal counterflow in a conical flow channel. The questions I hope to answer are: Do the 2 turbulent states (T-I and T-II) survive in the presence of a nonuniform flow field? If they do survive how do the states compare to theory? Does the unexpected sensitivity to the nonuniform flow field found in the nonuniform rectangular experiment also occur in this experiment? If so does it affect the T-I and T-II state in the same way? The effect of diverging and converging flow will be studied. Do the results of the experiment depend on whether the flow diverges or converges? This is of interest since the nonuniform rectangular experiment has only been performed in a diverging flow configuration. Also of interest is the idea that the flow can be thought of as locally uniform. The theory applied to analyze the results of the nonuniform rectangular experiment made this assumption and its failure is hard to explain. The results of my experiment will be analyzed to examine the idea that the turbulence in the flow channel can be described as locally uniform. The theoretical analysis developed in this thesis depends on this idea of local uniformity and comparison of the theory with experiment will determine the validity of this assumption.

1.2 Basic Theoretical Description

In 1938 liquid Helium cooled below the λ point (2.1K) was discovered to flow through small channels and spaces with a viscosity, η , that was at least 1500 times less than the viscosity of liquid helium above the λ point^{1,2}. Kapitza called this phase of liquid helium 'superfluid helium'¹ (also called He-II). These results put only an upper limit on the viscosity of He-II and suggested that the actual viscosity of the fluid could in fact be 0. In a later experiment it was discovered that He-II did cause a viscous drag on an object and in fact had a viscosity about that of helium gas^{4,5}. These contradictory results led Tizsa⁶ to model liquid helium as two interpenetrating, inseparable fluids. One of these fluids, the normal fluid, behaves just like a regular fluid and has a density ρ_N and viscosity η . The other, called the superfluid, flows frictionlessly, carries no entropy and has a density ρ_S . The total density of the fluid is just the sum of the densities of the two fluids

$$\rho = \rho_S + \rho_N \quad (1)$$

In a landmark paper Landau^{7,8} used the description of Tizsa to derive thermodynamic properties of the two fluids comprising He-II. In that paper Landau proposed that He-II is made up of two noninteracting interspersed fluids. The

Figure 1 The dispersion relationship for elementary excitations in He-II. The shape of this curve was first suggested by Landau. This curve is the result of neutron scattering measurements.

superfluid, essentially the result of a Bose-Einstein condensation, has no viscosity and is considered to be an ideal fluid, i.e. irrotational and incompressible. The 'normal' fluid is made up of quasiparticle excitations and has a measurable viscosity. Landau proposed the dispersion relation for the quasiparticles making up the normal fluid. This dispersion relation, seen in Figure 1⁹, asserts that the normal fluid is made up of two different types of thermal excitations. The linear region of this curve gives rise to phonons. The quasiparticles associated with the minimum of the dispersion relation are called rotons. These quasiparticles have no classical counterpart.

In his paper Landau parameterizes the two fluids by their densities, ρ_N and ρ_S , as well as their velocity fields, $\mathbf{V}_N$ and $\mathbf{V}_S$. He then derives the equations of motion for both the normal and superfluid components of He-II

$$\rho_N \frac{\partial \mathbf{V}_N}{\partial t} + \rho_N (\mathbf{V}_N \cdot \nabla) \mathbf{V}_N = \eta \nabla^2 \mathbf{V}_N - \frac{\rho_N}{\rho} \nabla P - \rho_S S \nabla T \quad (2)$$

$$\rho_S \frac{\partial \mathbf{V}_S}{\partial t} + \rho_S (\mathbf{V}_S \cdot \nabla) \mathbf{V}_S = - \frac{\rho_S}{\rho} \nabla P + \rho_S S \nabla T \quad (3)$$

where S is the specific entropy of helium and η is the viscosity of the normal fluid. P and T are the pressure and

temperature at a point in space and may vary in time. It is also understood that for these equations $\mathbf{V}_N$ and $\mathbf{V}_S$ are functions of space and time. These equations are known as the Two Fluid Model.

Experiments have supported the Two Fluid Model as a correct description of the thermodynamic and hydrodynamic properties of He-II for small values of the velocity fields $\mathbf{V}_N$ and $\mathbf{V}_S$. At large values the Two Fluid Model breaks down and excess dissipation, larger than the dissipation due to just the normal fluid, is found in the flow of He-II. This excess dissipation, first discovered by Gorter and Mellink¹⁰ in 1949, was modeled by them as a mutual friction force between the normal fluid and the superfluid. They proposed modifying the Two Fluid Model by including a mutual friction force $\mathbf{F}_{SN}$ term (called the mutual friction approximation)

$$\rho_N \frac{\partial \mathbf{V}_N}{\partial t} + \rho_N (\mathbf{V}_N \cdot \nabla) \mathbf{V}_N = \eta \nabla^2 \mathbf{V}_N - \frac{\rho_N}{\rho} \nabla P - \rho_S S \nabla T + \mathbf{F}_{SN} \quad (4)$$

$$\rho_S \frac{\partial \mathbf{V}_S}{\partial t} + \rho_S (\mathbf{V}_S \cdot \nabla) \mathbf{V}_S = - \frac{\rho_S}{\rho} \nabla P + \rho_S S \nabla T - \mathbf{F}_{SN} \quad (5)$$

Experimentally Gorter and Mellink found that the excess dissipation depended cubically on the average

Figure 2 Typical Counterflow Apparatus (A) and Data (B).

relative velocity $\mathbf{V}_{NS} = \mathbf{V}_N - \mathbf{V}_S$. From this they concluded that the mutual friction force has the form

$$F_{SN} = A(T) \rho_S \rho_N V_{NS}^3 \quad (6)$$

where $A(T)$, the Gorter-Mellink coefficient, has a value of $\sim 50 \text{ cm g / s}$ and is weakly temperature dependent.

The precise cause of this excess dissipation was first suggested by Feynman¹¹ in 1955. Working from the idea, proposed by Onsager¹², that the superfluid can support vorticity whose circulation is quantized in units of $\kappa = h/m_4$, where m_4 is the mass of the ^4He atom, Feynman suggested that the superfluid has associated with it a tangle of these quantized vortex lines. The quasiparticles that make up the normal fluid can scatter off these quantized vortex lines and thus momentum can be transferred between the normal fluid and superfluid. This scattering, Feynman suggested, gives rise to the mutual friction force F_{SN} .

1.3 Simple Flow Experiments

At this point I am going to digress and describe the basic physics behind thermal counterflow. This is important since it is thermal counterflow that Vinen studied (his experiment and results are described in the next section)

and because the experiment described in this thesis utilizes thermal counterflow.

Figure 2a schematically shows the basic setup for thermal counterflow. A small chamber containing a heater and thermometer is connected to a large helium reservoir by a small uniform channel of length l and cross sectional area A . The helium reservoir is regulated at temperature T . When power is input into the small chamber the normal fluid in the chamber rushes through the channel to remove the excess entropy. Simple energy arguments give the average velocity of the normal fluid through the channel:

$$V_N = \frac{\dot{Q}}{\rho S T A} \quad (7)$$

The superfluid reacts so that there is no net mass flow. Thus for thermal counterflow

$$j = \rho_S V_S + \rho_N V_N = 0 \quad (8)$$

which implies that

$$V_S = -\frac{\rho_N}{\rho_S} V_N \quad (9)$$

The relative velocity $\mathbf{V}_{NS} = \mathbf{V}_N - \mathbf{V}_S$ can immediately be derived

$$V_{NS} = \frac{\dot{Q}}{\rho_S S T A} \quad (10)$$

Figure 3 also shows a typical result for a thermal counterflow experiment. As soon as thermal counterflow is set up in the channel the small chamber increases in temperature by an amount ΔT_L due to the viscosity of the normal fluid. As the power is raised above some critical input power $\dot{Q}_c$ an excess temperature difference $\Delta T'$ is observed. This excess dissipation is associated with the mutual friction force. In order to connect theory and experiment it is necessary to understand how the mutual friction force gives rise to this temperature difference.

In order to simplify the extreme complications of the Two Fluid Model it is necessary to make certain assumptions¹³. First we will look only at steady state behavior. Also we will note that we do not measure any thermodynamic variables locally but instead measure their spatial average. So we will average the Two Fluid Model incorporating the mutual friction approximation over space and time. Equations (4) and (5) become

$$0 = \eta \langle \nabla^2 V_N \rangle - \frac{\rho_n}{\rho} \langle \nabla P \rangle - \rho_S S \langle \nabla T \rangle + F_{SN} \quad (11)$$

$$0 = -\frac{\rho_s}{\rho} \langle \nabla P \rangle + \rho_s S \langle \nabla T \rangle - F_{SN} \quad (12)$$

where the $\langle \rangle$ indicate the spatial average over the axial components of the enclosed vector quantities. Adding equations (11) and (12) yields

$$\langle \nabla P \rangle = \eta \langle \nabla^2 V_N \rangle \quad (13)$$

Equation (13) is an interesting result because it states that the mutual friction does not affect the pressure difference if the velocity field V_n is the same as before the dissipation. In fact the pressure difference is just that due to the normal fluid viscosity. This result is known as the 'Allen and Reekie Rule' and has been verified experimentally to be a good approximation for thermal counterflow¹³. The discrepancy between the Allen and Reekie Rule and actual measurement is small and for the rest of this paper it will be assumed to rigidly apply.

Subtracting equations (11) and (12) yields an expression for the temperature gradient ∇T

$$\langle \nabla T \rangle = \frac{\eta}{\rho_s S} \langle \nabla^2 V_N \rangle + \frac{F_{SN}}{\rho_s S} \quad (14)$$

As can be seen from equation (14) the temperature gradient

is made up of 2 pieces: a part due to the normal fluid viscosity and a part due to the mutual friction. Equation (13) is the Poiseuille equation for $\langle \mathbf{v}_N \rangle$ and for a channel which is circular in cross section this equation can be solved for the laminar normal fluid profile. This solution yields:

$$\langle v_N \rangle = 2 V_N \left(1 - \frac{r^2}{a^2} \right) \hat{z} \quad (15)$$

where a is the radius of the circular channel and $\hat{z}$ is the axial direction. Using the above result we can then solve for the viscous contribution to the temperature gradient

$$\langle \nabla T \rangle = \frac{8\eta}{a^2 \rho_{ST}} V_N + \frac{F_{SN}}{\rho S} \quad (16)$$

The total temperature difference across the uniform circular channel is obtained by integrating the temperature gradient over the length of the channel.

$$\Delta T = \int_0^l \langle \nabla T \rangle dz = \langle \nabla T \rangle l \quad (17)$$

We can then write the temperature difference as

$$\Delta T = \Delta T_L + \Delta T' \quad (18)$$

where

$$\Delta T_L = \frac{8\eta l}{\rho S T a^2} V_N \quad (19)$$

and

$$\Delta T' = \frac{F_{SN} l}{\rho S} \quad (20)$$

As can be seen from equation (18) in order to investigate the excess dissipation due to F_{SN} it is necessary to measure and subtract off the temperature difference due to the normal fluid viscosity. It should be noted that the chemical potential difference across the flow channel is a direct measure of the mutual friction force. This is easily seen by using the Gibbs-Duhem relation

$$\nabla \mu = -S \nabla T + \frac{1}{\rho} \nabla P \quad (21)$$

substituting equations (13) and (14) into (21) we see immediately that

$$\langle \nabla \mu \rangle = \frac{F_{SN}}{\rho_S} \quad (22)$$

While it is not entirely obvious how one would go about directly measuring the chemical potential difference,

Figure 3 Experimental measurement of the excess dissipation for a High (A) and Low (B) aspect ratio channels. Please note that the line defining the excess temperature difference has a non-zero intercept.

Yarmchuck and Glaberson¹⁴ have developed a method for doing so which will be discussed in the next chapter.

Experimentally there are two different types of channel geometries that are generally studied, low aspect ratio and high aspect ratio channels. The reason that these two types are in separate categories is that they yield qualitatively different results. Figure 3 shows a representative set of data for each case. As we can see the turbulent state for the high aspect ratio channel is cubic (all data will be plotted as $(\Delta T/\dot{Q})^{1/2}$ vs $\dot{Q}$ to suppress the cubic dependence) and starts essentially from the critical heat current up to the highest powers. The low aspect ratio results show a strikingly different result. There exist two distinctly different turbulent states, identified by Tough as T-I and T-II¹³. There are associated with these two turbulent states two different critical heat currents $\dot{Q}_{c1}$ and $\dot{Q}_{c2}$. While these results seem at first to complicate matters it must be noted that the T-II state and the turbulent state found in high aspect ratio tubes have been identified as the same state. This state is associated with the formation of a fully developed random homogeneous tangle of vortex lines filling the flow channel. As yet the T-I state is an enigma.

1.4 The Vinen Equation

In a brilliant experiment Hall and Vinen^{15,16} measured the interaction between the normal fluid and the quantized vortex lines in rotating He-II. In 1956 in a series of 4 papers Vinen^{17,18,19,20} laid the basic groundwork for the theoretical and experimental study of superfluid turbulence. He studied the excess dissipation caused by mutual friction in thermal counterflow of helium through small (~1mm) rectangular tubes. Vinen argued from first principles that the interaction that led to the mutual friction force has the form

$$F_{SN} = \frac{B \rho_s \rho_n \kappa}{3 \rho} V_{NS} L \quad (23)$$

where B is the experimentally determined temperature dependent coefficient measured by Hall and Vinen in the rotating He-II experiment and L is the total length of vortex line per unit volume, called the line-length density. This connection between the line-length density and F_{SN} gives a framework for comparing models of vortex line density and experiment.

In an attempt to better understand the dynamics of large scale homogeneous superfluid turbulence, Vinen proposed a phenomenological theory for its growth and decay. The arguments he presents are based on a statistical

approach in which only the average effect of the vortex tangle on the superfluid is considered. He considers phenomenologically the effects of the experimental parameters, such as L , V_N , etc., on the vortex tangle and argues by dimensional analysis the form of each term.

Vinen theorizes that the steady state line density results from a dynamical balance between the generation of line length and its decay. For the generation term, $(dL/dt)_{\text{gen}}$, Vinen considers a plain vortex ring of radius b , core size a , and circulation κ in an undisturbed ideal fluid. The ring will have a self-induced velocity V_{SI} through the fluid given by

$$V_{SI} = \frac{\kappa}{4\pi b} \left(\ln \frac{8b}{a} - \frac{1}{4} \right) \quad (24)$$

If this ring is moving in He-II, then it will feel a force f due to interaction with the normal fluid

$$|f| = - \frac{\kappa \rho_s \rho_n B}{2\rho} V_{SI} \quad (25)$$

The effect of this force is to induce the vortex line to move at a velocity V_{nc} as described by the Magnus effect

$$V_{NC} = \frac{\rho_s f}{\rho_s \kappa} \quad (26)$$

For some flows the above Magnus effect will cause the vortex ring to expand. Vinen argues that the rate of this expansion depends on f , ρ_s and κ .

In the above argument, the rate of increase in the length of vortex line is independent of the amount of line present. Vinen argues that for the state of homogeneous turbulence this is not the case. He argues that the effect of other lines cannot be ignored; *''for the lines will be continually cutting across one another, so that parts of each line will be influenced by the large velocity fields in the immediate vicinity of neighbouring lines¹⁹.''* So Vinen argues that $(dL/dt)_{gen}$ also increases with L . Thus putting the pieces together Vinen states that $(dL/dt)_{gen}$ is proportional to f and depends also on L , κ and ρ_s . By a purely dimensional argument he determines

$$\left(\frac{dL}{dt}\right)_{gen} = \chi_1 \frac{f}{\rho_s \kappa} L^{\frac{3}{2}} \quad (27)$$

where χ_1 is a constant that depends on the precise geometrical arrangement of the vortex lines. So putting in the value of the force on the vortex line we get

$$\left(\frac{dL}{dt}\right)_{Gen} = \chi_1 \frac{B\rho_N}{2\rho} V_{NS} L^{\frac{3}{2}} \quad (28)$$

To understand the decay of the turbulence Vinen argued that the main effect will be analogous to the action of viscosity in high Reynolds number turbulence. In such a situation the decay of the turbulent energy is governed by the strength of the "eddies" or more exactly, in this case, the strength of the vortex tangle. From this argument Vinen states that the decay term, $(dL/dt)_{Decay}$, is

$$\left(\frac{dL}{dt}\right)_{Decay} = -\chi_2 \kappa L^2 \quad (29)$$

where χ_2 is an experimentally determined parameter.

Vinen then uses these two terms to formulate a dynamical equation for the density of vortex lines

$$\frac{dL}{dt} = \left(\frac{dL}{dt}\right)_{Gen} + \left(\frac{dL}{dt}\right)_{Decay} = \chi_1 \frac{B\rho_N}{2\rho} V_{NS} L^{\frac{3}{2}} - \chi_2 \kappa L^2 \quad (30)$$

This equation has a steady state form for the line density

$$L^{\frac{1}{2}} = \frac{\chi_1 B \rho_N}{2 \chi_2 \rho \kappa} V_{NS} \equiv \gamma V_{NS} \quad (31)$$

Using this result for the steady state line density in Vinen's equation for the mutual friction (equation (23)) we see immediately that the form of the mutual friction force proposed by Gorter and Mellink is recovered.

Vinen²⁰ later attempted to modify equation 18 to take into account the critical velocity below which $L=0$ and thus $F_{SN} = 0$. He proposed that the generation of vortex line does not occur close to the walls of the channel. He proposed that the vortex generation term is inactive within a characteristic distance from the wall on the order of $L^{-1/2}$. Vinen took this effect into account by multiplying $(dL/dt)_{Gen}$ by the factor $(1 - \alpha/L^{1/2}d)$, where d is the size of the flow chamber and α is an experimental parameter of order 1. Now taking into account this last term, the dynamical equation becomes

$$\frac{dL}{dt} = \chi_1 \frac{B \rho_N}{2 \rho} V_{NS} L^{\frac{3}{2}} \left(1 - \frac{\alpha}{L^{\frac{1}{2}} d} \right) - \chi_2 \kappa L^2 \quad (32)$$

The above form of the Vinen Equation is called the Modified

Vinen Equation (MVE). This equation can be solved for the steady state line density

$$L^{\frac{1}{2}} = \frac{\gamma V_{NS}}{2} \left(1 + \left(1 - \frac{4\alpha}{\gamma V_{NS} d} \right)^{\frac{1}{2}} \right) \quad (33)$$

It is interesting to note that the line density found in equation (33) is not defined for $V_{NS} < 4\alpha/\gamma d$. Thus if we define the critical velocity as $V_C = 4\alpha/\gamma d$ equation (33) becomes

$$L^{\frac{1}{2}} = \frac{\gamma V_{NS}}{2} \left(1 + \left(1 - \frac{V_C}{V_{NS}} \right)^{\frac{1}{2}} \right) \quad (34)$$

The model does phenomenologically fit some thermal counterflow data and will be useful later in the analysis of my data.

1.5 The Schwarz Theory

While the Vinen equation represented a great leap ahead in the basic understanding of superfluid turbulence it has serious flaws. It was phenomenological in nature, based on only average effects, and did not attempt to connect itself to the underlying microscopic physics. In its simplest form (eq. 31) Vinen's theory did not account for the critical

Figure 4 The motion of two crossed vortex lines. Note the destruction of areas of high radius of curvature.

velocities and could not account for the nonzero intercept of the line density (See Figure 3). While the MVE fixes these problems it does so by introducing another parameter. Curiously enough, this theory of superfluid turbulence presided as the only viable theory for over 20 years, until in 1978 Klaus W. Schwarz²¹ presented a theory based on microscopic considerations of a vortex tangle.

Schwarz derives the microscopic equation of motion of a point on a vortex by modeling the quantized vortex lines as classical vortices in an ideal fluid^{22,23}. The motion of the vortex line is a function of $\mathbf{v}_N$, $\mathbf{v}_S$ and the local self-velocity of the vortex line. Schwarz uses the local self-induction approximation which assumes that a point on a vortex line will propagate, in the absence of any other driving fields, with a velocity that depends only on the local radius of curvature. This approximation ignores the contribution to the self-induced motion due to the presence of other vortex lines. The contribution to the motion by these other vortex lines is small unless a vortex line comes close to another vortex line. At this point Schwarz 'reconnects' the vortex lines and continues the calculation(See Figure 4). By the physics involved in the reconnection the two neighboring vortex lines then move away from each other.

Interestingly the reconnection of the vortex lines solves an initially lethal problem with the Schwarz theory. If a calculation is initially started with a random tangle of vortex lines and it is assumed that the equations will obey the equations of motion for a classical vortex (which Schwarz's model for the motion of Superfluid turbulence does) the tangle will evolve locally in two different manners. regions of the vortex tangle that have a small radius of curvature will quickly disappear and the line length associated with these high curvature regions will vanish. However regions of the tangle with a large radius of curvature will begin to grow and line length will be added to the system. The problem is that the growth of vortex lines is then unbounded. This is where the reconnection ansatz comes in. As can be seen in Figure 4 the points where the reconnection takes place create regions of high radius of curvature. These regions of high curvature then decay away. As the vortex tangle grows the reconnections become more and more frequent until finally the system reaches a dynamical steady state.

Schwarz took the equation of motion for a point on a vortex and the reconnection ansatz and in detailed numerical studies predicted the dissipation of a vortex tangle under the influence of an externally induced velocity field. He started the numerical studies from a number of different

initial conditions and found that the final results were independent of initial condition. Schwarz calculated the final vortex configuration for both a space with periodic boundary conditions and for a space bounded by walls. The result of the calculations is that the line density is proportional to the square of the velocity field. His results agree well with Vinen's theory and fit the line density for pure superflow. While both theories fit the experiments, the Schwarz theory has no adjustable parameters.

Schwarz derives an analytical form for the line density for the case of homogenous turbulence by applying scaling to the equation of motion for a point on a vortex line²⁴. According to this scaling the steady state line density is given by

$$L^{\frac{1}{2}} = \frac{C_L V_{NS}}{\beta} \quad (35)$$

where

$$\beta = \frac{\kappa}{4\pi} \ln \left[\frac{C}{L^{\frac{1}{2}} a_0} \right] \quad (36)$$

This form of the steady state line density solved one of the unexplained problems with Vinen's form of the steady state line density. According to the simple form of Vinen's equation (see equation (31)) the functional form of $L^{1/2}$ vs V_{NS} is a straight line whose intercept is 0. However the experimental data fit a straight line with a nonzero intercept (Figure 3). In the scaled version the steady state line density has a small logarithmic correction that accounts for this apparent nonzero intercept by introducing a small nonlinearity in the $L^{1/2}$ vs V_{NS} .

1.6 The Experiment of Kafkalidis and Tough

In 1991 Kafkalidis and Tough³ (K&T) measured the effect of spatial inhomogeneity on superfluid turbulence in thermal counterflow. They constructed a high aspect ratio flow channel that diverged linearly from an aspect ratio of 8:1 to 40 :1 over the length of the channel (see Figure 5). The average velocity field in this apparatus varied as $1/r$ where r is measured from the point in space where the edges of the channel would meet if they extended that far. The opening angle of this channel is .08 radians and so the divergence is in some real sense small. K&T very accurately measured the temperature difference across the flow channel as well as at points along the flow channel length. These

Figure 5 Experimental setup of Kafkalidis and Tough.

measurements allowed K&T to deduce the $\dot{Q}$ and r dependence of the excess dissipation.

In analyzing their results K&T used the Schwarz form for the steady state line density (equation (35)). The problem is that this form of the line density is derived explicitly for the case of homogeneous superfluid turbulence. The experiment specifically violates this assumption. K&T assumed that the divergence in the experimental flow channel was weak enough that the local value of the line density is determined by the local average relative velocity $V_{NS}(r)$. This assumption is called the local uniformity approximation and seems valid since on the length scales associated with the vortex tangle, on the order of $L^{-1/2}$, the velocity field varies by no more than 1 part in 10^4 .

Some experimental results of K&T are shown in Figure 6. The solid lines are the predictions of the Schwarz theory using the local uniformity approximation. The theoretical prediction calculated the value of $\Delta T'$ along the channel as too high by about a factor of 2. More striking is the fact the functional form of the temperature difference is wrong for both $\dot{Q}$ and r !

Figure 6 Experimental results for diverging flow. The Q (A) and R (B) dependence is compared to the Schwarz theory using the local uniformity approximation.

Of further interest is the fact that K&T found that the transition between the laminar and turbulent states exists as a front in the channel and occurs at the point in the channel where the local velocity is the critical velocity. This result implies that the transition between the laminar and turbulent state depends only on the local velocity and is not effected by the nonuniformity of the channel.

1.7 Motivation for this Experiment

The surprising results of the K&T experiment showed that superfluid turbulence is much more sensitive to nonuniformity than had ever been expected. The K&T experiment kicked the scientific community out of the complacency that came as a result of the success of the Schwarz theory. The experiment described in this thesis is designed to extend the work of K&T into channels of small aspect ratio which diverge in two dimensions. The goal of this work is to see the effect of this divergence on the richer behavior found in small aspect ratio channels.

As mentioned in this chapter uniform channels of small aspect ratio have associated with them two distinct turbulent states as well as two critical heat currents (velocities). Of equal importance is the effect that spatial

nonuniformity has on the T-I state. As stated earlier in this chapter the character of the T-I state is enigmatic and any additional information about its behavior will help us to better understand its nature. Schwarz and Rozen in a recent experimental paper called the T-I state the patchy precursor to the T-II state. If this is so it is reasonable to expect that if the T-II state is affected by spatial nonuniformity then the T-I state, by nature of it's being the associated with the T-II state, should show some new behavior. This experiment will help us better understand the nature of this state.

This experiment will also investigate the front behavior seen in the K&T experiment. Of interest is the question of how robust is the front behavior seen by K&T. If fronts are a natural part of spatially nonuniform superfluid turbulence then the two transitions found in low aspect ratio tubes should also show front behavior. It should be possible to compare and contrast the data from this experiment and the experiment of K&T to determine the physical laws by which superfluid fronts form.

This experiment will also try to investigate the suppression of the T-II state that K&T described. As has been mentioned before the only type of turbulence that K&T saw was T-II. Since the channel used in this work has a

small aspect ratio states other than the T-II state may be observed and data on the T-II state may be limited.

Finally the experiment will investigate whether there is a difference between diverging and converging flow. This is critically important since the results of K&T indicate that the steady state superfluid turbulent state is very sensitive to spatial nonuniformity. If the theory were to depend on nonlocal parameters it seems reasonable to believe the final steady state configuration could depend on whether the flow is converging or diverging.

Chapter II

Experimental Design

2.1 Introduction

In this chapter I will discuss the overall design of the experimental probe and apparatus constructed for this work. I will present first a brief overview of the design criterion of the overall probe and then go into detail about the construction of the apparatus. Throughout this chapter I will describe the positive and negative aspects of the designs used and wherever possible make suggestions for improvement.

2.2 Probe considerations

Figure 7 shows a schematic diagram of the probe. It is designed to be used in a bath of liquid helium that is pumped to superfluid temperatures. The probe consists of a vacuum can in which the experiment is performed connected to a top flange by an indium O-ring seal. The top flange is suspended from an upper brass plate by three thin wall (.020") .5" diameter stainless steel tubes . One of these tubes serves as a pumping line for the vacuum can. Another

Figure 7 Schematic diagram of the experimental probe.

provides an access to the experiment for cabling. The probe is constructed using standard cryogenic techniques, including radiation baffles to attempt to minimize the heat leak into the liquid helium bath and pumping line offsets at the end of the stainless steel tubing, so radiation does not flood into the vacuum can.

2.1.1 The Needle Valve

A superleak tight valve is needed to protect the delicate CPG. This valve consists of a needle valve connected to a long stem reaching to the top flange of the probe so that the valve can be operated from the room. Turning the needle lifts it from its seat and connects the back of the flexible membrane capacitor to the helium reservoir. Closing the needle valve seals off the two volumes. Figure 8 shows the design for the needle valve²⁵.

2.1.2 The Vacuum Can

The vacuum can is designed to give the largest possible interior vacuum space and top opening for maximum flexibility in assembling the apparatus. The vacuum can is assembled from three pieces of machined brass (see Figure 9). The body of the can, seen in Figure 10, is 8.1" long and has an outer diameter of 3.5" and inner diameter of 3.375". It is desirable to make the can as light as possible thus

Figure 8 Needle Valve.

Figure 9 Vacuum can assembly.

Figure 10 Vacuum can body .

Figure 11 Vacuum can bottom.

Figure 12 Vacuum can collar.

reducing the thermal mass of the can and allowing for faster cooling of the apparatus. This also saves helium. However it is not recommended that the can wall be less than 1/16" for strength reasons. The can is the main source of protection for the experimental apparatus and thus must be able to take some punishment.

The bottom of the can is a disk of brass 3.5" in diameter with a 1/16" by 1/8" groove cut around its edge (see Figure 11). The disk is made to snugly fit into the vacuum can body. The interior of the can is machined away to reduce thermal mass. The disk is then hard soldered (silver alloy solder) into the vacuum can body.

The top collar of the vacuum can is a cylinder .6" long and machined to an outer diameter of 3.5" (see Figure 12). The bottom of the collar has a groove .0625" x .1" machined around its outer edge and fits snugly into the vacuum can body. The top of the collar is machined as a matching O-ring surface for the top flange. Eight 4-40 tapped holes lie outside the O-ring surface and match the bolt circle on the top flange. It should be noted that great care went into the design of this piece so that the top opening is as wide as possible. The tapped bolt circle was calculated to allow the heads of the bolts to be pushed to the very edge of the

probe. The large top opening allows for easy removal and reconnection of the entire vacuum can.

The indium O-ring surface accepts a ring made from 1/16" thick indium wire. The O-ring is constructed by measuring the wire on the O-ring surface and marking where it should be cut. The indium wire is then removed from the surface and cut at a 45° with a fresh razor blade that had been cleaned with methanol to remove any grease. The two fresh cuts are then overlapped and if done right weld weakly to each other. The O-ring is then soaked in methanol and allowed to air dry. At no time is it touched with bare hands and any handling is done wearing rubber gloves that have been washed to remove any grease. The indium O-ring surfaces are then cleaned with an orange wood scraper, methanol and plastic scrub pad (metal or steel wool must not be used as it would scratch the O-ring surface). When attaching the vacuum can it is critical that the O-ring seats itself properly. Usually the O-ring is seated using just four screws evenly spaced around the vacuum can. Just as the O-ring starts to compress the other 4 screws are attached and the O ring is compressed evenly until the screws are hand tight. Since indium is soft the O-ring will flow a small amount after the screws are tightened so after an hour the screws are retightened.

2.1.3 The Vacuum Tight Amphenol Connectors

A major problem associated with any probe is passing electrical cabling from a pressurized area to another under a different pressure. In the design of this probe I had to get wires from the room to the bath space and the vacuum can. In order to deal with this problem a new type of feed-through was developed. This feed-through modified standard 6 and 17 pin Amphenol connectors to be leak tight to high vacuum.

I do not take credit in any way for the design described in this section. The design is entirely Bob Kindler's. I think it is critical to document this design so it is not lost. Figures 13 and 14 show the step by step process for constructing the leak tight Amphenol connector.

Instructions:

1. Use male side of the connector.
2. Remove retaining ring from housing and push out pin assembly.
3. Check all pins, if not the solid type, to see if they are sealed with solder. If not, fill the wire end with Staybrite solder (464°F).

Figure 13 Amphenol assembly drawing.

STEP 8.
REASSEMBLE THE CONNECTOR
WITH ALL PARTS AND SCREW
ON A MATING CONNECTOR TO
KEEP ALL THE PINS LINED
UP WHILE POTTING THE STYCAST.

STEP 9.
REMOVE RETAINING RING AND TOP
PLASTIC PIN HOLDER, AND DISCARD.

STEP 10.
BECAUSE OF THE SMALL UNDERCUT
ON THE TOP OF THE HOUSING,
WRAP BLACK PLASTIC TAPE
AROUND THE CONNECTOR.
TRIM THE TAPE OFF
EVEN WITH THE TOP
OF THE HOUSING.

STEP 11.
MIX STYCAST 2860 WITH
CATALYST #9, AND SLOWLY
FILL TO THE TOP OF THE
HOUSING, MAKING SURE
THAT IT FLOWS AROUND
EACH PIN AND INTO THE
GROOVES.

STEP 12.
ALLOW STYCAST TO SET-UP AND
CURE BEFORE REMOVING THE MATING
CONNECTOR.

STEP 13.
MACHINE A JUNCTION BOX (SEE ATTACHED
DRAWING) AND LEAK CHECK.

STEP 14.
SINCE STYCAST DOESN'T LIKE VERY
HIGH TEMPERATURE, USE MULTICORE
ELECTRONIC SOLDER (63/37 361 DEGREES
F) TO SOLDER THE LEADS ONTO THE PINS.

PAGE
2 OF 2

LOW TEMPERATURE FACILITY
FOR:
DESIGN BY: BOB KINDLER
DRAWN BY: RICH KINDLER
DATE 10 MARCH 93
SCALE: OUT
MAT'L:
NEED:
ACAD NAME: AMPSEAL

Figure 14 Amphenol assembly drawing, (continued)

Figure 15 Junction box for leak tight Amphenol connector.

4. Sand blast the inside of the housing where the Stycast will go. This will allow the Stycast stick better.
5. Face off the flange and take a face cut off of the O.D. of the housing. Should be smooth to accept an O-ring seal.
6. Machine a 1/16" by .030" deep groove between the step and the retaining ring groove. This will help seal and hold the Stycast in the housing.
7. Leak check the housing, we have found them to be porous. On average 1 in 4 will have small holes and leak.
8. Re-assemble the connector with all parts and screw on the mating connector to keep all the pins lined up with the potted Stycast.
9. Remove the retaining ring and plastic pin holder. Discard the plastic holder.
10. Because of the small undercut on the top side of the connector, it is necessary to wrap black plastic tape around the connector to keep the Stycast from running out. Trim the tape off even with the top.
11. Mix Stycast #2850 with catalyst #9 and slowly fill to the top of the housing making sure it flows around each pin and into the grooves.

12. Allow the Stycast to setup and cure before removing the mating connector.
13. Machine a junction box (see Figure 15) and leak check
14. Since Stycast does not like very high temperatures, use Multicore Electronic Solder (63/37 361 °F) to solder on the leads and pins.

2.1.4 The Heat Sinks

Figure 16 shows the design for the main heat sink in the probe. This heat sink is hard soldered to the top flange and is thus in direct thermal contact with the helium bath. The bottom of the heat sink has an indium O-ring surface. A matching piece was machined out of brass and had three tapped holes machined in it for attaching vespel support rods. The primary purpose of this matching piece was to lend a certain amount of modularity to the experiment by allowing the flow channel to be removed easily.

The primary purpose of the heat sink is to remove all the heat that flows down the wires that come from room temperature through the stainless steel tubing. The heat sink is made of brass and has eleven 1/16" x 1/16" notches cut into its surface evenly spread out over its length. Into these notches 2.2 meters of 36 gauge copper wire coated with

Formvar were wrapped. Before wrapping the wire onto the sink the wire was folded over itself and twisted tightly so as to minimize inductance. After each layer of the wire was wrapped on a very thin (viscosity about that of water) layer of G.E. varnish was applied and allowed to dry. This process was continued until all but 3" of the wire was wrapped into the notch. At this point a last layer of G.E. varnish was applied. The 3" of wire was then unwrapped and one end of the wire was connected to a wire coming out of the tubing. The other end was connected to maganin wire and connected to the experiment.

The thermal impedance of the heat sink has been estimated to be, in the worst case, 35 K/W. The maganin wire used to connect the experimental apparatus to the heat sink has a thermal resistance on the order of 10^6 K/W per inch. Thus the heat sink anchors more than 99.999% of the heat flux coming down the wires from the exterior through the stainless steel tube. Since the heat leak through one of these wires is less than 100 μ W the heat leak to the apparatus is less than .01 μ W per wire which for this experiment is acceptable.

The other heat sink used for this experiment is an 8" long 1/4" diameter OFHC copper rod soldered to the top flange. The primary purpose of this rod is to provide a mounting surface for the supporting platform for the

Figure 16 Drawing of the main heat sink,

Figure 17 Pin Block,

capacitor. The rod also acts as a thermal anchor for the capacitor (the main thermal anchor for the capacitor however is through direct contact with the bath) and as a convenient place to connect and thermally anchor adjustable pin blocks (see Figure 17). The cables for the capacitor are connected to such a pin block to give them some thermal anchoring before connecting to the capacitor.

2.2.5 The Top Flange

The top flange has two purposes. It is the main connection assembly for both the experimental apparatus and the vacuum can. The top flange is made of brass and has an O-ring surface that matches the O-ring surface on the top collar on the vacuum can (see Figure 18). The main thermal anchor is hard soldered to the inside center of the top flange and a 3/8" hole has been drilled through the top flange to allow the experimental apparatus to be connected to the helium bath. A bolt circle that matches the bolt circle on the vacuum can is drilled around the edge of the plate. Drilled through and countersunk holes are placed to accept the pumping line offsets.

Figure 18 Top Flange,

2.2.6 The Pumping Line Offsets

The main purpose of the pumping line offsets is to stop radiation from flowing from the top of the probe (at 300K) to the bottom (at $<4.2\text{K}$). The offsets make the line coming down from room temperature turn before it enters the vacuum can. Of primary importance to the design of these offsets is that you should not be able to see through them, thereby insuring that there is no direct path between room temperature and the vacuum can. From the design used on this probe, Figure 19, we can see that this is not the case. Space considerations made making a complete offset difficult and at the time of the probe design it was not considered important. This was a mistake. The amount of radiation absorbed by my experiment was at least $20\mu\text{W}$, a factor of 200 times the acceptable heat leak. In order to fix this problem 2 graphite cylinders $3/8" \times 3/8"$ with an offset hole drilled into it (see Figure 20) were affixed into the drilled through holes that accept the pumping line offsets on the top flange. The other drilled pumping line offset through hole in the top flange had a thick bundle of cables coming through it and could not accept a graphite plug. The cables effectively blocked the hole and thus shielded the vacuum can from the radiation flux. The design of future pumping line offsets should not be made in the same manner as this one.

Figure 19 Pumping line offset used in this experiment. Note the offset does not cover the entire pumping line. This caused a substantial radiation leak.

Figure 20 Graphite plug used to correct the pumping line offset problem.

2.2.7 Wires

The wire used in the construction of the probe, except where noted, is 36 gauge maganin wire coated with HML. HML is extremely tough coating and durable which allows its use in a harsh environment with confidence. The wires coming down the stainless steel tubing are bundled in groups of four and these groups are placed inside teflon tubing as an added precaution against shorting to the stainless steel tubing walls.

The capacitor required that the leads connecting it to the leak tight Amphenol connectors be individually shielded by ground. I choose stainless steel coax to be the connection between the Amphenol connector and the heat sink. The connection between the heat sink and the pin block is maganin wire inside of a fiberglass sheath shielded by the outer shield of the stainless steel coax. The stainless steel coax is type "s 25" and was purchased from LakeShore Cryotronics²⁶.

2.3 The Experimental Apparatus

A block diagram of the apparatus is given in Figure 21. The apparatus consists of a heater, thermometer, flow channel and chemical potential gradiometer¹⁴ (CPG). It is designed to produce thermal counterflow in the flow channel and to measure the temperature and chemical potential differences, ΔT and $\Delta \mu$, across the channel. The CPG consists of two parts, a superleak and a flexible membrane capacitor which are connected to both the bath and the flow channel as shown in Figure 21. The apparatus also contains a bypass valve, which is a safety feature for the flexible membrane capacitor. The entire apparatus is thermally isolated by a vacuum system from an external bath that is regulated at a constant temperature, T ($\pm 10 \mu K$).

In order to produce thermal counterflow in the flow channel heat must be input to the helium. Power, $\dot{Q}$, is applied by adjusting the power supply causing a current to flow to a 1000 Ohm heater connected to an OFHC copper heater block soldered to a CU-Ni tube filled with helium. Since the superleak has high thermal impedance all the heat input to the system passes through the flow channel producing thermal counterflow. The dissipation in the flow channel causes the helium in the "hot reservoir" (see Figure 21) to rise to a temperature $T + \Delta T$. Since the helium in the thermal

Figure 21 Block diagram of the experimental apparatus.

thermal reservoir is all at the same temperature and the tube to which the thermometer block is connected is isolated from the heater by a thermal standoff, the tube is in thermal equilibrium with the helium.

2.3.1 The Superleak

The construction of the superleak used in this experiment is the same as the one used by Griswold²⁷ and Lorenson²⁸. The superleak consists of a 1.5 cm long piece of Vycor glass rod. Vycor glass has small voids on the order of 60Å. Since the normal fluid has viscosity it is trapped and stopped by the Vycor. Since the superfluid has no viscosity it can flow through the Vycor. The glass also has a high thermal impedance so heat does not conduct across the superleak.

The superleak is constructed by machining two brass endcaps and soldering 32 mil i.d. Cu-Ni tubes into the endcaps (see Figure 22). The endcaps are then epoxied onto the Vycor rod being careful not to block off the ends of the rods with the Stycast 2850 (see Figure 22). To keep the epoxy from running while it is curing the entire assembly is grabbed by one brass end piece and rotated at ~1Hz. After the epoxy has cured the entire assembly is covered in a thin layer (~1mm) of Stycast 2850. The Stycast is applied to the

Figure 22 Construction of the Superleak.

rotating assembly with a #2 artists brush. The entire assembly is then heated with a heat gun until the Stycast cures. To avoid curing the Stycast too quickly the air hitting the superleak should not be too hot. I set the temperature by placing my hand in the air flow where the superleak was and making sure that it wasn't so hot that I couldn't keep it there.

After the entire apparatus is finished the superleak is leak checked. The superleak is first checked for leaks from the outside to the Vycor glass and then is checked to make sure that helium can flow through the Vycor glass. If it passes all these tests the superleak is stored in a desiccator until it is connected to the experiment.

After the experiment has been built and is in the cryostat the system is leak tested. At this point it is necessary to note that black stycast at room temperature is permeable to helium gas. If the needle valve is opened and the bath is filled with 1 atm of helium the vacuum can will show a small steadily increasing leak rate. The character of this leak is that its time constant is long, on the order of 10 sec. On one experimental run the leak rate in the vacuum can was 4.5×10^{-10} with no helium in. For the first minute after the helium was in the bath there was no increase in the leak rate. at 1:30 sec after 1 atm of helium was placed

in the bath the leak rate was at 8×10^{-10} . After 2 min the leak rate was 22×10^{-10} , after 2:30 it was 70×10^{-10} and after 3 min it was 90×10^{-10} . If I then pumped out the bath the leak rate then went away on a longer time scale. Do not let this leak fool you. The stycast around the joints are fine and will seal at low temperatures.

2.3.2 The Heater Apparatus

The heater apparatus consists of two pieces (see Figure 23 and Figure 24) of machined OFHC copper. The first piece is an adaptor that is soldered onto a .015" I.D. Cu-Ni tube that is soldered closed at one end. The adapter is tapped for a 4-40 screw that has been cut so that only a few threads are left. The second piece, the heater block, screws onto the first and has attached to it two heaters. The heaters are placed into two channels machined into the heater block and secured there by Stycast 2850.

The heaters are made from EvenOhm wire and have values of 980 Ohms and 1061 Ohms. EvenOhm wire has a resistance of 199.7 ohms per foot and its resistance doesn't vary by more than .1% between 300K and 4.2K. EvenOhm wire is quite brittle and thus does not withstand a great deal of stress. Frequent bending at a point in the wire will deform it and cause it to break. Therefore it was essential for the

*HEATER
BLOCK*

Figure 23 Heater block,

Figure 24 Heater block adaptor,

heaters used in this probe that the EvenOhm wire had no kinks in it, as constant thermal cycling would cause the wire to become weak and break at the kink point.

EvenOhm wire does not bond well to lead/tin solders. In general the electrical contact between the EvenOhm and the other wire is mechanical and the effect of the solder is to mechanically hold the joint together. This can have rather annoying consequences such as cold joints. To alleviate this problem the ends of the EvenOhm heaters are dipped in a pool of molten silver alloy solder for about two seconds. The silver alloy solder bonds well to the EvenOhm and the lead-tin solder bonds well to the silver alloy.

To make the heater five feet of EvenOhm wire was cut from a roll and checked for kinks. The ends of the wire were then stripped and dipped into molten silver alloy solder. 1" long 24 gauge copper wires were then soldered onto the ends of the EvenOhm and shrink tubing was put over the joints. The EvenOhm was then noninductively wrapped and checked again for kinks and then wrapped onto wood toothpicks that had been shaved down to fit into the channels machined in the heater block. The individual heaters were then potted into the channels with Stycast 2850. To avoid mechanical stress on the EvenOhm only the

copper wires soldered to the heaters penetrated outside the Stycast.

2.3.3 The Thermal Standoff

As mentioned at the beginning of this section, the heater must be isolated from the thermometer by a thermal standoff. As can be seen from Figure 21 the heater apparatus inputs heat to the helium through a Cu-Ni tube. Since there is heat flowing continually from the tube into the helium the tube is at a higher temperature than the helium due to the Kapitza resistance. The thermometer block measures the temperature of the Cu-Ni tube in thermal equilibrium with the "hot reservoir" and must be isolated from this extraneous temperature shift.

The thermal standoff consists of Stycast 1266 machined into a rod .1"-.2" long and 1/16" in diameter. A .015" hole is drilled lengthwise through the rod and then a .032" hole is counter-bored halfway through the rod. The Cu-Ni tubes connecting the heater and thermal standoff are then connected to the thermal standoff with a .005" tungsten wire which was passed through the standoff. The tubes were then epoxied to the thermal standoff using Stycast 1266. After

the Stycast cured the tungsten wire was pulled out to ensure that helium could flow through the standoff. The standoff was then checked for leaks from the outside.

2.3.4 The Thermometer Block

The thermometer block is a piece of machined OFHC copper (see Figure 25). The block has a .032" Cu-Ni tube soldered into a hole that has been drilled through the apparatus. Another hole has been drilled so that an encapsulated germanium-500 and a CG-1000 thermometer could be inserted. Although the thermometer block was designed to make thermal contact with the thermometers by screwing the assembly together the actual removal and reattachment of the screws was too violent for the delicate apparatus. Better thermal contact was made by thermally anchoring the thermometer leads to the thermometer block itself.

While both the Germanium-500 and the carbon glass-1000 thermometers were present in the experiment all of the data was taken with the germanium resistor. Early in the experiment the carbon glass resistor became noisy and thus its sensitivity was impaired.

Figure 25 Thermometer Block.

2.3.6 The Flow Channel

The building of the flow channel was much less complicated than the rest of the apparatus but involved a great deal of work. From preliminary calculations it was decided that the flow channel would be roughly conical in shape, have a small end diameter of approximately $100\mu\text{m}$ and diverge by about a factor of 2. The length of the flow channel would have to be between 2.5 cm and 12.5 cm to fit into the apparatus.

The flow channel used in this experiment was made by heating and drawing larger (1 mm id) cylindrical capillary tubing down to the approximate size. The drawn tubes are then cut and the inner diameter of the tubes at each of their ends was measured by imaging the ends of the tubes directly under a microscope. The tubes would then be recut and reobserved until a likely candidate of the proper length and end sizes were found. It was then necessary to measure the inner diameter of the tube as a function of its length. This was done by looking through the side of the tube and imaging the inner diameter with a microscope. The size of the images were then recorded at every .5cm along the length of the tube. The image size at each end of the tube was then compared to the actual size of the ends of the tube and this gave the conversion factor necessary to convert the images

Table 1 Values of the inner diameter of my flow channel as a function of length along the flow channel.

Length along the flow channel (cm)	Actual Diameter (μm)
0.0	200
0.5	186
1.0	172
1.5	152
2.0	143
2.5	138
3.0	130
3.5	125
4.0	123
4.5	117
5.0	112
5.11	111

Figure 26 Profile of the flow channel used in this experiment. The values for this plot are from Table 1

Figure 26 Profile of the flow channel used in this experiment. The values for this plot are from Table 1.

Figure 27 Flow channel apparatus.

then assumed that the conversion factor varied linearly over the length of the tube (see Table Table 1). The characterization of the tube used in this experiment is shown in Figure 26.

2.3.7 The Flow Channel Apparatus

The flow channel used in this experiment was constructed by placing the glass flow channel described above into an assembly to protect it and allow it to be easily connected to and removed from the experiment. The schematic for the flow channel assembly is shown in Figure 27. The glass rod had a .003" diameter 12" long tungsten wire run down the channel. The two ends of the glass tube were inserted into specially machined brass endcaps into which .032" Cu-Ni tubes had been soldered. One of the brass endcaps had a groove machined into it and this endcap was used to identify the fat end of the channel. The glass channel was then glued in place with Stycast 2850FT epoxy. The tungsten wire was then pulled out to ensure that the flow channel was not blocked. Finally a stainless steel sheath was slipped over the apparatus and glued onto one endcap with GE varnish to allow for the different thermal expansion coefficients of the stainless steel and the glass. The stainless steel sheath protected the flow channel from damage and gave the

construction mechanical rigidity. The sheath had a section cut out so that the flow channel could be accessed and leak checked.

The flow channel could possibly be broken if both ends of the channel assembly were grabbed and then twisted. In order to insure the survival of the flow channel an additional reinforcement is added to the flow channel apparatus. Whenever the channel is going to be soldered into or taken out of the apparatus a 1mm thin wall glass tube is epoxied between the stainless steel sheath and the end of the channel that the sheath is not G.E. varnished to. Just before the vacuum can is placed on the experiment for an experimental run the tube is cut. The tube must be cut before the experiment is cooled or the differential thermal contractions induced could and probably would cause the flow channel to break!

2.3.8 The Flexible Membrane Capacitor

The flexible membrane capacitor(FMC) is an integral part of the Chemical Potential Gradiometer(CPG). The FMC consists of two small volumes of helium separated by a .001" thick Mylar separator that has 2000Å of gold evaporated on one side (see Figure 28). A capacitor plate is positioned .001" from the metal side of the mylar. As the pressure

between the two small chambers changes the Mylar flexes and the capacitance between the Mylar and the capacitor plate changes in response.

The capacitor plate is housed in a capacitor housing and is electrically isolated from the housing by a capacitor spacer (see Figures 29,30 and 31). The capacitor housing is made of brass ,the capacitor plate is made of OFHC copper and the capacitor spacer is made of Macor. The capacitor housing has been machined to accept the indium O-ring seal used to attach Mylar mounting rings. These rings, described below, have the gold Mylar attached to them. The capacitor housing also has soldered to it two copper tubes to allow an electrical lead to access the capacitor plates and helium to access the interior of the capacitor. The macor capacitor spacer is epoxied into the capacitor housing using a coat of Stycast 1266 thinly spread onto the macor by a #2 artists brush. A 28 gauge copper wire is then soldered to the cutout in the capacitor plate making sure that the excess solder does not flow out of the cutout. If excess solder does flow out of the cutout then it is filed away to ensure a snug fit in the capacitor spacing. A drop of Stycast 1266 is then placed into the interior cutout in the macor separator and

Figure 28 Assembly diagram of the flexible membrane capacitor.

Figure 29 Capacitor housing.

Figure 30 Macor spacer,

Figure 31 Capacitor plate,

Figure 32 Mylar mounting ring.

MYLAR MOUNTING RINGS

NOTE: "A"

ALL DIMENSIONS MUST BE HELD AS CLOSE AS POSSIBLE. The tolerances given is the maximum allowable slop. Unless otherwise noted the tolerance is $\pm .005$.

NOTE "B"

Make 2 pieces with 3/32 groove on bottom and make 2 pieces without the groove.

LOW TEMP LAB SHOP

FOR: MURPHY/TOUGH

DESIGN BY: BOB KINDLER

DRAWN BY: RICH KINDLER

DATE: 07 MAR. 90

SCALE: 2:1

MAT'L: BRASS

NEED: 4 SEE NOTE "B"

Figure 33 Mylar holder and assembly geometry.

Figure 34 schematic diagram showing the steps involved in making the mylar assembly.

Figure 35 Pressure calibration of the flexible membrane capacitor.

the capacitor plate is dropped into place while pulling the copper wire lead attached to the capacitor plate through one of the two copper tubes that are attached to the housing. After the epoxy has cured the apparatus is turned around and a drop of Stycast 1266 is put on the top of the copper tube that the capacitor lead is coming through. Capillary action will pull some of the Stycast into the tube and effectively seal it. The apparatus is then blanked off and checked for leaks from the outside.

The gold mylar is attached to two mounting rings as seen in Figure 32. First a piece of gold mylar is inspected under a microscope for pock marks and imperfections. A 2"x2" piece is cut out of the bigger piece of gold mylar centered on the acceptable area and is glued to the mylar holder with 5 minute epoxy. A mylar mounting ring, without the groove (see Figure 32), is cleaned with methanol and mounted on a jig, smooth side up, allowing the ring to be held away from the tabletop (see Figure 33). A thin coat of Stycast 1266 is applied to the smooth side of the mounting ring and the mylar in the mylar holder is laid over the apparatus with the mylar side down and the acceptable area of the mylar centered on the ring. A petri dish is placed over the apparatus and a 200 g weight is added. After the Stycast has cured the mylar mounting ring is cut away from the rest of the mylar and any excess mylar is trimmed off using a razor blade.

In order to make the final assembly a wire lead must be attached to the gold mylar and a second mylar mounting ring, this one the one with the groove, must be glued to the gold side of the mylar. In order not to cause an electrical short all the gold where the ring touches the mylar must be removed with a cotton bud soaked in methanol except for a thin strip that goes to the edge (see Figure 34). This thin strip is where the lead is attached and will sit under the groove in the mounting ring. A 40 gauge copper wire is then soldered with indium onto the gold strip. This is a delicate procedure and was not attempted until at least five practice joints were successfully completed on a practice piece of mylar glued to a brass plate. If the solder joint was too tall it could be rolled down gently with a small metal tube. A thin coat of Stycast 1266 is then placed on the smooth side of the second mylar mounting ring and it is glued to the mylar keeping the groove over the solder joint. The mounting ring is checked for electrical isolation from the gold, readjusting it as necessary. After the Stycast has cured a bead of Stycast 2850 is put into the outer groove formed by the undercut on the outer diameter of the mounting rings. The mylar assembly is then blanked off and checked for leaks from the outside.

The Mylar assembly is then placed into the capacitor housing and checked to see if the Mylar touches the capacitor plate. If it does a .001" facing cut is taken off the plate

and the process repeated until the mylar just doesn't touch the plate. Then the gold side of the mylar is placed towards the capacitor plate and the mylar assembly is connected to the capacitor housing on one side and a blank off on the other with indium O-rings (NOTE: the indium O-rings go in the outer groove. The inner groove is to catch excess indium and stop it from going into the capacitor assembly.) The capacitor is then checked for leaks from the outside.

The final check of the capacitor is to make sure it has a linear response with pressure. The capacitor was connected across the Datametrics Electronic Manometer and a small pressure difference was created between the chambers. The change in capacitance was then noted and the pressure was changed. The capacitor used in my experiment had the linear response shown in Figure 35.

In the apparatus the FMC sits on a plate of OFHC copper that is attached to the bottom of the copper rod heat sink described above. The FMC is held in place by a brass clamp and is in thermal equilibrium with the bath.

2.3.9 The Chemical Potential Gradiometer

The chemical potential gradiometer is made up of a superleak and the FMC connected to the experiment as shown in

Figure 21. This apparatus was first used by Yarmchuck and Glaberson⁷ to measure directly the chemical potential difference between the hot reservoir and the bath. A brief description of the basic idea of the device is given below.

Following the analysis from section 1.3 we can see that the chemical potential difference $\Delta\mu$ is related to the temperature difference $\Delta T'$ (see equations 20 and 22) by

$$\Delta\mu = -S\Delta T' \quad (37)$$

As we can see from the above relation the chemical potential difference is directly related to the temperature difference due to the presence of excess dissipation in the flow. The Superleak acts like a chemical potential short so the helium on the each side of the superleak is at the same chemical potential. Thus the two small chambers in the FMC are at different temperatures. Since the chambers are at the same temperature there must be a pressure difference between them

$$\Delta P = \rho\Delta\mu \quad (38)$$

Thus the pressure difference is related to $\Delta T'$

$$\Delta P = -\rho S\Delta T' \quad (39)$$

The pressure difference causes a displacement of the metalized mylar membrane in the CPG. This displacement is detected as a change in capacitance measured with a General Radio 1615a

capacitance bridge connected to an Ithaco Dynatrac 343 lock-in amplifier. The sensitivity of the capacitance measurement (1 part in 100,000) gives the CPG a sensitivity of less than $.1\mu\text{K}$ at $T = 1.8\text{K}$ to $.5\mu\text{K}$ at $T = 1.23\text{K}$. The high sensitivity of the gradiometer makes it possible to detect the presence of intrinsic noise in the dissipation around $\dot{Q}_{c2}$.

There are three reasons why a CPG is particularly appropriate for these experiments. The first reason is the high sensitivity obtained with a capacitance measurement. Since large driving voltages are used the signal to noise ratio can be made very large. Germanium resistors tend to self heat and thus lose sensitivity as larger drive voltages are used. The second reason is that the CPG is sensitive to just the dissipation due to the presence of vorticity in the channel. This sensitivity to the presence of turbulence gives us a way to very accurately measure the laminar to turbulent transition. The last reason is that the CPG is differential and is insensitive to bath temperature fluctuations. The bath fluctuation can plague sensitive resistance measurements of the temperature difference. For these experiments we anticipate much of the important data will involve excess temperature differences ΔT much less than 1 mK . Since the bath temperature fluctuates about $10\mu\text{K}$ the error associated with resistive temperature difference measurements can be quite

large. The differential nature of the CPG eliminates these temperature fluctuations as a source of error.

Chapter III

Experimental Procedure

3.1 Introduction

The data collected in this experiment are the temperature difference ΔT and excess temperature difference $\Delta T'$ as a function of the heat current $\dot{Q}$ through the nonuniform channel. As discussed in the previous chapter the total temperature difference is obtained from a change in the resistance of the germanium resistor attached to the thermometer block. The excess temperature difference is obtained from a change in the capacitance of the FMC. In order to discern these temperature changes it is necessary to measure the resistance and capacitance changes very accurately and to calibrate these changes against a known temperature scale. This chapter describes the procedures involved in making these measurements and calibrating the sensors used to detect the temperature changes.

Figure 36 Schematic Diagram of the Kelvin bridge.

Figure 37 Equivalent circuit for the Kelvin bridge.

3.2 The Resistance Measurements

The value of the germanium resistor is measured by a Kelvin bridge and a PAR-124A lock-in amplifier (see Figure 37). The bridge allows a standard four terminal measurement on an unknown resistance (R_X) by adjusting the decade resistance (R_D) until the voltage seen by the lock-in is zero. The germanium resistance is driven by an oscillator in the Par-124 at 205Hz and 3.2 mV rms excitation. The excitation voltage was chosen to maximize the sensitivity of the resistance measurements without causing the resistor to significantly self heat. The time constant on the lock-in amplifier is set at 300 ms, and the lock-in is set in band pass mode with a Q of 10.

Figure 37 shows the equivalent circuit for the Kelvin bridge. The two oscillators in the diagram represent the voltages across the germanium and decade resistors. The circuit is driven so that the voltage across the germanium resistor is 3.2 mV (V_X). This defines the voltage across the decade resistance as $V_X(R_D/R_X)$. The impedance seen by the lock-in is

$$Z = \frac{R_2 + R_1}{2} = 5.5 \text{ k}\Omega \quad (40)$$

The voltages V_A , V_B , and the current i_0 can then be

determined:

$$i_0 = \frac{V_X (R_D/R_X) + V_X}{2 (R_1 + R_2)} \quad (41)$$

$$V_A = V_X (R_D/R_X) - \frac{V_X R_1 (R_D/R_X + 1)}{2 (R_1 + R_2)} \quad (42)$$

$$V_B = i_0 R_2 = \frac{V_X R_1 (R_D/R_X + 1)}{2 (R_1 + R_2)} \quad (43)$$

Equations 42 and 43 define the voltage difference across the lock-in amplifier $\Delta V = V_A - V_B$:

$$\Delta V = \frac{V_X}{2} \left[\frac{R_D - R_X}{R_X} \right] \quad (44)$$

We note that

$$\frac{2\Delta V}{V_X} = \frac{\Delta R}{R_X} \quad (45)$$

Thus for a drive voltage $V_X = 3$ mV and a voltage sensitivity on the lock-in of 500 nV we see from the above equation that the resistance can be measured to an accuracy of about 0.008%.

3.3 Capacitance Measurements

The value of the FMC is measured using a General Radio 1615-A capacitance bridge and an Ithaco 393 lock-in amplifier. The FMC is driven with a 400Hz signal with a RMS value of 9.3 volts. The FMC had a nominal capacitance of 50 pF and is measured as a 3 terminal capacitance measurement to an accuracy of .1 fF. A detailed description of the circuit is found in the General Radio 1615-A manual. When the FMC is measured the variable capacitor in the 1615-A is tuned until the off balance voltage detected on the 393 lock-in amplifier is zero. The output of the Ithaco 393 is connected to a Hewlett-Packard 7045B X-Y chart recorder so the off balance signal from the capacitance bridge can be recorded in time. With this technique it was possible to detect the intrinsic fluctuations that arise at the T-I/T-II transition.

3.4 Resistance Thermometer Calibration

The temperature calibration of the germanium resistor is key to the success of our temperature measurements. The resistor is calibrated against the 1958 ^4He N.B.S. (now N.I.S.T.) temperature standard. The first step in the calibration is cooling the experiment to the desired temperature at which data is to be taken. The bath is

regulated at this temperature ($\pm 10\mu\text{K}$) using the same temperature regulation circuit used by Courts²⁹, Baehr³⁰, and Opatowsky³¹. This circuit is explained in detail in an appendix of Opatowsky's thesis. The temperature of the bath is determined by measuring the vapor pressure of the helium above the bath and converting the vapor pressure into a temperature using the 1958 ^4He temperature standard. The resistance and vapor pressure are recorded at the base temperature and the temperature is then raised about 1mK and the resistance and vapor pressure are recorded. The temperature is then increased again and the procedure repeated until the temperature of the bath is 50mk above the original temperature. The temperature is then lowered in about five equal steps repeating the vapor pressure and resistance measurements until finally the original vapor pressure is remeasured. This final point is a check of the calibration. If this value varies by more than 1 Ω the entire calibration is repeated.

The resistance (R) of the germanium resistor is then plotted against the vapor pressure (Vp). The vapor pressure is also plotted against temperature using the 1958 ^4He scale and a second order polynomial is fit to the Vp vs T graph. This fit is used to convert the vapor pressures in the R vs. Vp graph into temperature. The data are then plotted as R vs. T and fit to a second order polynomial. Experimentally

it is known that the value of the germanium resistance drifts slowly in time so the actual value of the resistance cannot be trusted as a direct measure of the temperature. However the change in the resistance ΔR is an excellent measure of the change in the temperature ΔT . In order to calibrate this the curve fitted to the R vs. T graph is solved for the value of the resistance at the original temperature. This resistance is then subtracted off the measured values and the resulting ΔR is plotted against ΔT . A best second order polynomial is fit through the data and this curve is used in the experimental runs to convert resistance differences into temperature differences. The calibrations for this experiment are shown in Figures 38 through 45.

The above resistance calibration was done for all temperatures except 1.8 K. At 1.8 K the system is working near the upper end of the range of pressures that the electric manometer can read. It is not possible to increase the temperature 50 mk and achieve regulation. Therefore at 1.8 K the calibration was done as far as possible and this truncated version was used. The error that the calibration causes in the measurement of ΔT is at worst only a few percent and should not affect the overall conclusions.

Figure 38 Calibration of the germanium resistance difference ΔR vs. temperature difference ΔT for 1.23K,

Figure 39 Calibration of the germanium resistance difference ΔR vs. temperature difference ΔT for 1.3K.

Figure 40 Calibration of the germanium resistance difference ΔR vs. temperature difference ΔT for 1.4K,

Figure 41 Calibration of the germanium resistance difference ΔR vs. temperature difference ΔT for 1.5K.

Figure 42 Calibration of the germanium resistance difference ΔR vs. temperature difference ΔT for 1.6K.

Figure 43 Calibration of the germanium resistance difference ΔR vs. temperature difference ΔT for 1.7K,

Figure 44 Calibration of the germanium resistance difference ΔR vs. temperature difference ΔT for 1.75K.

Figure 45 Calibration of the germanium resistance difference ΔR vs. temperature difference ΔT for 1.8K,

3.5 Input Power

The circuit used for setting the input power $\dot{Q}$ is shown in Figure 46 . The power supply is a Hewlett-Packard 6111A constant voltage power supply and is set at 10 V. The decade resistor (R_D) varies from 0 Ω to 1,111,110 Ω and is used to set the current in the heater. The safety resistance R_S has a resistance of 18 k Ω and insures that the power in the heater is never above 300 μ W. After R_D is set the current in the circuit is measured to four significant figures using a Hewlett-Packard 3465A Digital Multimeter. A table of approximate R_D values vs. input power was tabulated to help in setting the appropriate input power values.

3.6 Data Acquisition

The day to day operation of the experiment is described in this section. It should be noted that the individual experimental data runs took between 5-10 hours to complete. Runs longer than 10 hours are not possible since the bath runs out of helium. Being systematic and a bit compulsive about the day to day running of the experiment (especially where the capacitor is concerned) is healthy since a mistake here can be catastrophic.

Figure 46 Schematic diagram for the power regulation circuit.

Upon starting an experimental run the first thing that is done is to check that the needle valve to the FMC is open. Helium can then be transferred into the system and the system cooled and regulated to the temperature at which data is to be taken. The helium in the flow channel provides a very good link between the bath and the germanium resistor. Thus the germanium resistor should track any variations in the bath temperature without much lag. If it does not then the flow channel could be plugged. In this case the small amount of power that the germanium resistor is supplying to the hot reservoir can vaporize the helium in the reservoir and thus thermally isolate the germanium resistor. If the channel is plugged allowing the experiment to rise above nitrogen temperatures and pumping out any residual air or pumping and flushing the system at room temperature usually fixes the problem.

Once regulation has been achieved the base values of the capacitance and resistance are recorded. The needle valve is then closed and the values of the FMC and the germanium resistor begin to change. These values should settle down in 30 sec. The resistor should come back to its initial value before closing the needle valve. A substantial change (1Ω) indicates that there could be a heat leak into the system causing thermal counterflow in the flow channel. The subsequent temperature rise along the channel is seen by

the germanium resistor and is reflected as a resistance change from its base reading with the needle valve open. In this experiment heat leak due to radiation down the pumping line (see chapter 2) caused such a shift of the resistance. Tracking down the leak and correcting it caused the germanium resistor to have the same value before and after I closed the needle valve.

The FMC does not settle down to its original value but instead shifts approximately $+0.2$ pF. The reason for this remains a mystery but is probably due to imperfections in the mylar that do not relax after closing the needle valve. The shift in the FMC is seen again after data acquisition has begun. At a ΔC of $.2$ pF the capacitor will undergo another positive shift that is obvious when the zero point of the capacitor is taken again. After the second shift has occurred the mylar will relax to its original (after closing the needle valve) value after a few minutes. To eliminate the effect of this spurious shift the data is taken from high $\dot{Q}$ (thus high ΔC) to low $\dot{Q}$. Taking data this way incorporates the shift of the capacitor in the data and uses the fact that the shift is still there for a while at $\dot{Q} = 0$. Thus the potential systematic error in the data is eliminated.

After the system has reached equilibrium the values of

the capacitor and germanium resistor, called the base values are recorded. Since the helium has just been pumped through the λ point a laminar data run is done. After the system has gone turbulent a "laminar line" is almost experimentally impossible to get. This is probably due to the presence of vorticity left over from the turbulent state. This turbulence, not present in the fresh helium-II acts as a seed for the turbulent state and causes the turbulent state to appear at a lower heat current than in fresh Helium-II.

In getting the laminar data speed is of the utmost importance. Once input power is greater than $\dot{Q}_{c1}$ with the system in the laminar state, it is metastable. The thermal relaxation time of the system is very fast ($\ll 1$ sec) so data can be taken quickly. The power on the system is increased in 5-10 μ W steps and the resistance values are read as fast as possible. The current and the resistance are recorded and the current is later converted into power. This procedure is continued until a jump in the CPG indicates the presence of turbulence. The input power is then disconnected and a baseline value of the germanium resistor is taken for confirmation. This method has achieved laminar data out to about 10 $\dot{Q}_{c1}$.

After the laminar data run is complete the input power is set at the maximum power for which data will be taken.

The first time this is done requires some guess work and simple calculations. After the system is in equilibrium the values of the current, the germanium resistance, the FMC and the vapor pressure are recorded. The power is then lowered and the process repeated. In general for this experiment the power was incremented in $1 \mu\text{W}$ intervals. After the run is over a note is taken of where the capacitor stopped changing. This point marks a power close to $\dot{Q}_{c1}$. The power is then reset to a value just above the point where the FMC stopped changing and lowered in steps of $.2\mu\text{W}$ until the capacitor stops changing. This process is done a few times to determine $\dot{Q}_{c1}$ accurately. $\dot{Q}_{c1}$ has the operational definition of being the lowest $\dot{Q}$ that can sustain turbulence. When all the data have been taken the needle valve is opened and liquid helium is transferred to the bath space.

After the experimental run is complete the values of the current, the germanium resistance and the capacitance are entered into the computer. All analysis for this work is done using the SigmaPlot³² software package. First the value of the laminar slope is determined by calculating the ΔT for the largest $\dot{Q}$ and fitting a straight line through that point and the origin. The resistance and capacitance data are then converted into ΔR and ΔC data by subtracting off the base resistance and capacitance measurement. The excess

temperature difference is then calculated. For the resistance data it is done by using the ΔR vs. ΔT calibration discussed above and calculating the total temperature change. The laminar temperature difference is then calculated and subtracted off. The conversion from ΔC to ΔT is discussed below.

In order to confirm the data, another data run on another day is taken. The same procedure is done and if the runs match each other another temperature is then chosen for the next run.

3.7 Capacitor Calibration

As has been discussed in the previous chapter the CPG is only sensitive to the excess dissipation caused by the presence of quantized vorticity in the flow channel. The extreme sensitivity that is obtained from the capacitance measurement insures that the excess dissipation can be measured quite accurately. What we are missing is the calibration of the capacitor, which allows the change in capacitance to be converted into $\Delta T'$.

If we know the pressure sensitivity of the capacitor then using the Gibbs-Duhem relation, as discussed in chapter 2, we get the conversion required. However as we can see in

Figure 35 the pressure dependence of the capacitor is temperature dependent. Thus we cannot predict it at the low temperatures at which these experiments were done. Therefore the CPG was calibrated against the germanium resistor. The procedure for doing so took advantage of the linearity of the capacitor vs. pressure at capacitive differences below 2.5 pF. The capacitance shift ΔC was plotted against the excess temperature difference $\Delta T'$ calculated from the germanium resistor. For $\Delta C \sim 2$ pF, $\Delta T'$ was always greater than 1 mK at all temperatures and thus the error associated with the resistive prediction of $\Delta T'$ was small. A straight line was fit, on the ΔC vs. $\Delta T'$ plot, through the origin and several points around $\Delta C = 2$ pF. The slope of the line was then used for the calibration of the CPG (see figure 47). All $\Delta T'$ data associated with a ΔC of 2 pF or less was calculated using the CPG instead of the resistor.

A final check was done on the capacitor after all the data runs were done and the system had warmed up to room temperature. Since the calibration of the capacitor is known the pressure sensitivity of the capacitor can then be calculated for each temperature. With the small range of temperatures involved, the capacitor should not undergo any major mechanical changes. Therefore we expect the pressure sensitivity of the capacitor to be the same at all temperatures. Figure 48 shows that this is in fact the case

Figure 47 ΔC vs. $\Delta T'$ calibration curves. The inconsistent data at 1.3K is discussed in the text.

Figure 48 The universal pressure vs. capacitance change curve for the experiment between the temperatures of 1.23K and 1.8K. The spurious data at 1.3 K is discussed in the text.

Figure 49 Close up of the pressure vs. capacitance change graph seen in the last figure. Note the scatter in the 1.3K data. The data is compared only to the data for 1.23K for clarity. 1.23K is representative of the other data.

with one exception. At 1.3 K the pressure data do not agree with all the other temperatures. Since data was taken below 1.3 K (at 1.23 K) and above 1.3 K the conclusion that there is a natural shift in the FMC is not supported. It is noted that just before the 1.3 K data were taken the system underwent catastrophic warm up due to the failure of the helium vacuum jacket. The problem was repaired and the system was then cooled down and the 1.3 K data were taken. This was the last temperature taken for converging flow and the system was then warmed up and the channel turned around. Figure 49 shows an enlargement of the ΔP vs. ΔC graph for $\Delta C < .1$ pF. As can be seen from this graph the other temperatures approach the origin linearly while the data for 1.3 K scatters. The obvious conclusion from this graph is that something anomalous happened to the FMC during this cool down and the CPG data was adversely effected. As such all CPG data at 1.3 K was discarded.

An interesting result of this experiment is that it is now possible to use the universal pressure sensitivity of the CPG as a calibration. The method for doing so is to cool the experiment and take data as described above. Then using the data for $\Delta T'$ derived from the germanium resistor verify that the pressure sensitivity of the FMC has not changed. If it has not then this pressure sensitivity can be used as the calibration of the CPG. This procedure is not necessary to

run the experiment and was not used for calculating the values of $\Delta T'$ discussed in this experiment. However it does provide a useful check and gives the calibration of the FMC for use in a future experiment.

Chapter IV

Analysis

4.1 Introduction

In analyzing this experiment I must take into account the different states the system can find itself in; the laminar state, the T-I state and the T-II state. The analysis of the laminar state data depends greatly on the assumption that the nonuniformity in the tube is not 'large'. Testing this assumption by comparing the laminar data to theory has a direct effect on how reasonable it is to go ahead with the same assumption in the turbulent case. Critical to the modeling of the turbulent case is the effect geometry has on the critical heat currents $\dot{Q}_{c1}$ and $\dot{Q}_{c2}$. I therefore attempt to model the different behaviors I might expect from the critical heat current data, $\dot{Q}_{c1}$ and $\dot{Q}_{c2}$.

4.2 Laminar

Detailed analysis of the laminar flow of helium through small uniform channels has been done several times in the past and so I will only summarize the major results.

Landau's two fluid model yields a set of equations for the flow of normal fluid through a uniform circular channel. The first equation is London's Equation:

$$\nabla P = \rho S \nabla T \quad (46)$$

The second equation is the Poiseuille equation, equation (13), for the axial component of the normal fluid velocity, $V_{nz}(r)$. These equations can be exactly solved for a channel of length l , and circular cross section of radius a . The geometry of the channel naturally suggests using cylindrical coordinates (r, θ, z) with z pointed along the axis of the cylinder. The solution yields a normal fluid profile of:

$$V_{nz}(r) = 2V_n \left(1 - \frac{r^2}{a^2}\right) \quad (47)$$

where V_n is the average velocity across the channel cross section:

$$\int V_{nz}(r) dA = \frac{\dot{Q}}{\rho S T} \Rightarrow V_n = \frac{\dot{Q}}{\rho S T A} \quad (48)$$

The temperature difference ΔT_L can then be calculated:

$$\Delta T_L = \frac{8\eta_n l \dot{Q}}{\pi a^4 (\rho S)^2 T} \quad (49)$$

Figure 50 The Geometries used in analyzing the conical flow problem.

The flow channel used in this experiment more closely resembles a cone than a uniform cylindrical channel. Therefore I am more interested in the laminar dissipation of a cone (see figure 50). In order to solve this problem I can take two different approaches. The first is to attempt a direct solution of the Navier-Stokes equation for flow in a cone. The second is to approximate the flow channel as an infinite series of infinitesimally short straight cylinders whose radii are changing as a function of the length of the flow channel. This assumption of local uniformity is called the Local Uniformity Approximation (LUA). Using the LUA I can then use the direct solution to the uniform circular tube problem and integrate the temperature difference directly. Both methods of solving this problem are outlined below.

In order to solve the Navier Stokes equation I first make a series of assumptions about the flow in a small conical channel. The geometry has spherical symmetry and thus spherical coordinates (r, θ, ϕ) are chosen. The line of symmetry along the cone is oriented in the r direction along the $\theta = \phi = 0$ direction. Since the flow in our system is modelled as coming from a point source I assume that the θ and ϕ components of the normal fluid velocities are zero. Since the flow is incompressible the radial velocity component is of the form:

Figure 51 Laminar data for converging flow at $T=1.23 \text{ K}$.

Figure 52 Laminar data for converging flow at $T=1.3$ K.

Figure 53 Laminar data for converging flow at $T=1.4 \text{ K}$.

Figure 54 Laminar data for converging flow at $T=1.5$ K.

Figure 55 Laminar data for converging flow at $T=1.6\text{ K}$.

Figure 56 Laminar thermal impedences as a function of temperature. The three lines are the three geometries discusses in the text.

$$V_r = \frac{\dot{Q}}{r^2} \Theta(\theta) \quad (50)$$

The velocity must disappear at the walls so I assume:

$$\Theta(\theta) = C(1 - \frac{\theta^2}{\alpha^2}) \quad (51)$$

This form of the θ velocity field is analogous to the parabolic profile found in the uniform channel. Using these assumptions and the fact that θ (~ 0.001 radian for my flow channel) is small the Navier-Stokes equation yields:

$$\frac{\partial P}{\partial r} = - \frac{4\eta_r C \dot{Q}}{\alpha^2 r^4} \quad (52)$$

$$\frac{\partial P}{\partial \phi} = 0 \quad (53)$$

$$\frac{\partial P}{\partial \theta} = 0 \quad (54)$$

C is determined by demanding that the average velocity be given by equation 3:

$$\int V_r \cdot dA = \frac{\dot{Q}}{\rho S T} \rightarrow C = \frac{2}{\pi \alpha^2} \frac{1}{\rho S T} \quad (55)$$

The Navier-Stokes equation simplifies to only one variable, r . Using London's equation it is possible to solve for ΔT_L

$$\frac{dT}{dr} = \frac{1}{\rho S} \frac{dP}{dr} = - \frac{4\eta_n C \dot{Q}}{\rho S \alpha^2 r^4} \quad (56)$$

$$\Delta T_L = \int_{r_o}^r \frac{dT}{dr} dr = \frac{8\eta_n \dot{Q}}{3\alpha^4 \pi (\rho S)^2 T} \left(\frac{1}{r^3} - \frac{1}{r_o^3} \right) \quad (57)$$

The second way of deriving the laminar temperature difference makes use of the fact that the radius of the cross sectional area of the cone is known as a function of the position z in the cone (see figure 50). The line defining the outer edge of the cone is given by:

$$R(z) = a \left(1 + \frac{\beta - 1}{l} z \right) \quad (58)$$

Using the LUA and Equation (19) the laminar temperature difference is given by:

$$\Delta T_L = \frac{8\eta_n \dot{Q}}{\pi a^4 (\rho S)^2 T_0} \int_0^l \frac{dz}{[1 + (\frac{\beta-1}{l})]^4} \quad (59)$$

This equation can be integrated to give the solution seen in equation 12.

Neither solution to ΔT_L is an exact solution but the approximations used in both schemes are reasonable. In fact it should be emphasized that the approximation used here directly tests the idea that the flow can be characterized by using a theory that only holds for the uniform case. The success or lack thereof of this description of the laminar thermal impedance impacts on our assumption made in the introduction that it is not unreasonable to expect that the homogeneous theory applied locally will describe the flow found in our weakly diverging case.

The fact that the two approximations for the thermal dissipation of a cone yield the same result give us a way to calculate the laminar thermal impedance of the flow channel used in this experiment. The value of the cross sectional radius of the flow channel is known as a function of the length of the channel and has been fit to the form:

$$R(z) = \frac{1}{2} [.01147 + .0009077z - .000119z^2 + .000056646z^3] \text{ (cm)} \quad (60)$$

Thus using Equations (59) and (60) the laminar temperature difference across the channel is given by:

$$\Delta T_L = \frac{8\eta_n \dot{Q}}{\pi (\rho S)^2 T} \int_0^{5.11 \text{ cm}} \frac{dz}{R(z)^4} \quad (61)$$

The integral in equation 16 is purely geometric and can be done numerically, yielding a value of $2.5 \times 10^9 \text{ cm}^{-3}$.

Figures 51 through 55 show the laminar data taken in this experiment. They show that the laminar temperature difference ΔT_L depends linearly on the input power $\dot{Q}$. The linear dependence of ΔT_L on $\dot{Q}$ is predicted from the Two Fluid Model for all the geometries and derivation methods discussed. While this functional dependence supports the analysis as done it cannot alone verify the validity of the approximations made in the analysis section. In order to verify these assumptions we must look at the laminar thermal impedances.

The laminar thermal impedance $\Delta T_L / \dot{Q}$ is plotted as a function of temperature in Figure 56 and is compared to the different geometric models developed in the analysis section. The

theoretical results plotted for a straight cylindrical channel are obtained from equation 19 using a channel diameter of $111\text{ }\mu\text{m}$ (the diameter of the small end of our channel) and length = 5.11 cm (the length of our channel). The theoretical results for a conical channel are obtained from Eq. 12 using the diameters of the small end and large end of our flow channel ($111\text{ }\mu\text{m}$ and $200\text{ }\mu\text{m}$ respectively) as well as the length of our channel. The theoretical laminar thermal impedances for our channel geometry are given by Eq. 16. As can be seen from figure 56 the theoretical analysis taking into account the nonlinear profile of our channel best explains our data. The fit to our data is particularly good at low temperatures where both the thermal impedance and the sensitivity of our experiment increase.

The accuracy of the fit allows us to conclude that the approximation of local uniformity (local uniformity approximation or LUA) is valid for the channel used in this experiment. The prediction of the laminar thermal impedance by modeling the dissipation in the flow as being locally uniform is supported by the experimental evidence. The validity of the LUA is critical to the analysis of the T-I and T-II state.

4.3 Excess Dissipation

Figures 57 through 64 show the turbulent data for the channel for converging flow. The filled circles show the data points where excess intrinsic noise was measured. Comparison between these data sets and the uniform data found in Figure 3 shows that the general features found in the uniform data are preserved in the converging data. The data show two distinct turbulent states with two transitions. Of particular interest is the apparent broadening of the transition region around the second critical heat current. The region of noise that defines the transition region exists over a much larger domain of powers. In most cases it was not possible to take data above the transition region since the total temperature was becoming too large. In Figures 57 thru 64 the arrow shows the place in the data where the temperature difference is greater than 20 mK. Above this temperature difference the downward curvature in the data is spurious and is due to the variation in the thermodynamic parameters of the fluid across the channel.

In order for this analysis to continue it is necessary to understand how to connect the line density L to the mutual friction force F_{SN} . The mutual friction at any point in the channel due to the interaction of the normal fluid

Figure 57 Excess dissipation data for converging flow at $T=1.23 \text{ K}$.

Figure 58 Excess dissipation data for converging flow at $T=1.3\text{ K}$.

Figure 59 Excess dissipation data for converging flow at $T=1.4\text{ K}$.

Figure 60 Excess dissipation data for converging flow at $T=1.5 K$.

Figure 61 Excess dissipation data for converging flow at T=1.6 K.

Figure 62 Excess dissipation data for converging flow at $T=1.7$ K.

Figure 63 Excess dissipation data for converging flow at $T=1.75\text{ K}$.

Figure 64 Excess dissipation data for converging flow at $T=1.8 \text{ K}$.

with the vortex lines depends on how large L is at that point in the channel and how fast the relative velocity $\mathbf{V}_{NS}$

is at that point in the channel. In general $\mathbf{F}_{SN}$ can be written as

$$F_{ns} = \rho_s \kappa \alpha g L V_{ns} \quad (62)$$

where κ is the quantized circulation of a vortex line, $\alpha = B\rho_N/2\rho$ is a friction coefficient and g is a geometric factor. Thus $\nabla T'$ is defined in terms of L and $\mathbf{V}_{NS}$.

Unfortunately the experiment that was done was not able to measure the temperature gradient directly. The experimental quantity measured $\Delta T'$ is related to the gradient by:

$$\Delta T' = \int \nabla T' dr \quad (63)$$

Since in general L and $\mathbf{V}_{NS}$ are not uniform but depend on the spatial dimensions:

$$\Delta T' = \frac{\kappa \alpha g}{S} \int L(\vec{r}) V_{ns}(\vec{r}) d\vec{r} \quad (64)$$

where $L(\vec{r})$ and $V_{NS}(\vec{r})$ are the line density and relative velocity at a point in space, $\vec{r}$.

At present there exists no theoretical description of $L(\vec{r})$ in a nonuniform velocity field. However in our experiment the divergence is extremely small. Therefore we will assume that the divergence is small enough that the equations describing homogeneous SFT are valid throughout the channel. In place of the uniform velocity found in the homogeneous theory we will use the local average velocity as defined by the local crosssectional area of the flow channel. In this case the general forms of $L(\vec{r})$ and $\mathbf{v}_{NS}(\vec{r})$ become just $L(z)$ and $\mathbf{v}_{NS}(z)$. As in the laminar case z is zero at the entrance of the channel and traverses the length of the channel (see figure 50). Using the above assumptions the relative velocity $\mathbf{v}_{NS}(z)$ is given by

$$V_{ns}(z) = \frac{\dot{Q}}{\rho_s ST \pi R^2(z)} \quad (65)$$

and $R(z)$ is defined in equation 11. Now the equation for the temperature difference becomes

$$\Delta T' = \frac{\kappa \alpha g}{S} \int L(z) V_{ns}(z) dz \quad (66)$$

The set of assumptions used to derive equation 66 is the same set used in the laminar analysis and describes the LUA for the dissipative case. The only missing piece of the

model is a prescription for the line density $L(z)$. The rest of the dissipative analysis will concern itself with this problem.

4.4 T-II State

For states in our diverging flow that are analogous to the T-II state in uniform geometries the theory that best describes L is the Schwarz theory. According to Schwarz the geometric term in Eq 23 is:

$$g = (I_{II} - C_L I_I) \quad (67)$$

Where I_{II} , C_L , and I_I are parameters defined or calculated by Schwarz. As mentioned in Chapter 1 Schwarz derive the line density in the channel. Using the LUA the line density everywhere in the channel would be given by

$$L(z) = \frac{C_L^2}{\beta^2} V_{ns}^2(z) \quad (68)$$

where

$$\beta = \text{Log} \left(\frac{C}{L^{\frac{1}{2}}(z) a_0} \right) \quad (69)$$

a_0 is the core size of a vortex and c is a term calculated by Schwarz. Using the idea that the Schwarz theory can be applied locally uniquely defines the temperature gradient throughout the channel and thus the temperature difference across the channel can be calculated:

$$\Delta T' = \kappa \alpha (I_{II} - C_L I_I) \left(\frac{C_L}{\beta} \right)^2 \frac{\dot{Q}^3}{(\rho_s S T \pi)^3} \int_0^1 \frac{dz}{R^6(z)} \quad (70)$$

The form of the last equation implies that plotting $(\Delta T' / \dot{Q})^{1/2}$ vs. $\dot{Q}$ will compress the cubic dependence of the data into a straight line (to within the logarithmic factor β). The approximate cubic dependence can be seen in Figures 57 thru 64.

As is evident from the figures the Schwarz theory using the LUA does not agree well with the data. It should be noted that for most temperatures the system did not exit the transition region and get into the fully developed T-II state. The extreme width of the transition region did not allow, for most temperatures, data to be taken in the case where the channel was filled completely with the T-II state. For the measurements where the T-II state filled the channel the temperature difference across the channel was very high (> 20 mK). When the temperature difference across the channel becomes large the thermodynamic parameters of

the helium can vary greatly from one part of the channel to another. The effect of this variation is a spurious downward curve in the data. The theoretical calculations attempted to take into account this temperature dependence (thus the theoretical line curves down at high temperature) and as can be seen from the figures still overshoots the data. Unfortunately no data exist for this system that do not have this problem so a direct comparison with the nonuniform rectangular channel is difficult. The main result here is that the data are not inconsistent with the results of K&T. The turbulence in the T-II state is suppressed.

4.5 The Critical Heat Currents

In uniform flow channels the critical velocity depends on the size of the flow channel and from a simple scaling argument it can be shown that this dependence is approximately:

$$V_c d = \text{constant} \quad (71)$$

where d is the diameter of the flow channel. The above rule has experimentally been shown to approximately hold for both $\dot{Q}_{c1}$ and $\dot{Q}_{c2}$ in uniform tubes. More important equation 71 also holds for the critical velocity associated with the stationary fronts in the nonuniform rectangular channel.

Therefore using Eq 71 we determine the $V_c(z)$:

$$V_c(z) = \frac{V_{c1} d_c}{2R(z)} \quad (72)$$

where

$$V_{c1} = \frac{\dot{Q}_{c1}}{\rho S T \pi \left(\frac{d_c}{2} \right)^2} \quad (73)$$

and d_c is the diameter of the point in the flow tube where turbulence first appears.

As mentioned before d_c is the diameter of the flow at the point where turbulence first appears. However this experiment does not directly measure d_c or even V_c . What is measured is $\dot{Q}_{c1}$ and $\dot{Q}_{c2}$. So an important goal of our data analysis is to try to fix d_c . This can be done by using equation 71 at both transitions and comparing the results to critical velocity data in uniform tubes. The choice of d_c has a very large effect on the data analysis. If d_c is chosen at the small end of the channel the model allows for the existence of stationary fronts between the T-I and laminar states. If d_c is chosen at the fat end of the channel then, according to the model, the entire channel fills with turbulence as soon as the input power crossed

$\dot{Q}_{c1}$. So the choice of d_c can have a profound effect on our understanding of the properties of the T-I state.

The critical velocity data are plotted in Figures 65 and 66. The data are plotted as $V_c d$ vs T using the diameter of both the small and large end of the flow channel. Note that both the $\dot{Q}_{c1}$ and the $\dot{Q}_{c2}$ data from this experiment agree well with data from uniform channels for d_c equal to the small end of the channel. This agreement suggests that there is a front present for both the laminar/T-I and T-I/T-II transitions.

The T-I/T-II transition behaving as a front explains the broadening of the transition. As is noted in Figure 3 the T-I/T-II transition in uniform channels is wide. The effect of the front is to broaden this transition. The transition region first enters the channel at $\dot{Q}_{c2}$ and begins to exit the channel at approximately $2\dot{Q}_{c2}$. The transition region would completely exit the channel if the transition didn't have an intrinsic width in the uniform case. Since the transition region has an inherent width the power must be increased further to fully clear the channel of the transition region (see figure 67).

Figure 65 Comparison of the critical velocity in the converging channel to uniform data for $\dot{Q}_{c1}$.

Figure 66 Comparison of critical velocity data with data from uniform channels for $\dot{Q}_{c2}$.

Figure 67 Graphical representation of the T-I/T-II transition.

4.6 T-I State

At lower powers the system enters a region analogous to the T-I state in uniform channels. As noted in Chapter 1 the Schwarz theory correctly calculates the properties of the T-II state only so a direct application of the Schwarz theory in the T-I state would be invalid. Ladner³³ used a modified form of Vinen's equation successfully to fit the temperature differences associated with the T-I state in uniform tubes. While the modified Vinen equation (MVE) lacks the rigorous derivation found in the Schwarz theory it is the best theory available and will suffice for comparisons between the uniform and nonuniform T-I states. Vinen's equation defines the geometric factor found in Eq. 23 as

$$g = \frac{2}{3} \tag{74}$$

I extend the same assumptions for using the Schwarz theory on the T-II state to using this modified Vinen equation in the T-I state. I apply the equation locally to calculate the local line density using the local relative velocity as defined by Equation 28. The modified Vinen equation yields the following expression for calculating the line density everywhere in the channel

$$L_o^{\frac{1}{2}}(z) = \frac{1}{2} \gamma V_{ns}(z) \left[1 + \left[1 - \frac{V_c(z)}{V_{ns}(z)} \right]^{\frac{1}{2}} \right] \quad (75)$$

The equation has two adjustable parameters, γ and $V_c(z)$. This theory then allows us to write down the temperature difference

$$\Delta T' = \frac{\kappa \alpha \gamma^2}{6} \left(\frac{\dot{Q}}{\rho_s S T \pi} \right)^3 \int_0^1 \frac{dz}{R^6(z)} H(z) \quad (76)$$

$$H(z) = 2 - V_c(z) \frac{\rho_s S T \pi R^2(z)}{\dot{Q}} + 2 \sqrt{1 - V_c(z) \frac{\rho_s S T \pi R^2(z)}{\dot{Q}}} \quad (77)$$

The dependence of ΔT on $\dot{Q}$ is almost the same as for the T-II case (this time it doesn't have the weak logarithmic term β but the critical velocity does give the theory a non zero intercept). $V_c(z)$ is determined from the values of $\dot{Q}_{c1}$ and will be discussed in the next section. γ is allowed to vary to best fit $\Delta T'$.

With the determination of $V_c(z)$ from the critical heat current data it is possible to fit equations 76 and 77 to the converging flow data. The fits to the data are shown in Figures 68 through 74. Note that no capacitor data exist for

Figure 68 Modified Vinen equation fit to the T-I data for $T=1.23\text{ K}$.

Figure 69 Modified Vinen equation fit to the T-I data for $T=1.4\text{ K}$.

Figure 70 Modified Vinen equation fit to the T-I data for $T=1.5\text{ K}$.

Figure 71 Modified Vinen equation fit to the T-I data for $T=1.6\text{ K}$.

Figure 72 Modified Vinen equation fit to the T-I data for $T=1.7$ K.

Figure 73 Modified Vinen equation fit to the T-I data for T=1.75 K.

Figure 74 Modified Vinen equation fit to the T-I data for $T=1.8 \text{ K}$,

Figure 75 Modified Vinen Equation using $d_c = 200 \text{ } \mu\text{M}$.

Figure 76 Modified Vinen Fit to $V_c(z) = \text{constant}$.

Figure 77 Comparison of fit parameter γ for uniform and converging flow.

1.3 K and so no analysis can be done for that temperature. As can be seen from the figures the fit to the data is good. The downward curvature of the theory at low powers is a direct result of using equations 72 and 73 as the model for $V_C(z)$. This statement can be supported by choosing other forms for $V_C(z)$. Figure 75 shows the result choosing d_c equal to the large end of the channel. As is evident from the figure the downward curvature is not reproduced in the model. Figure 76 shows the results of choosing $V_C(z)$ equal to its value at the entrance of the channel and kept constant throughout the channel. As the figure shows the curvature of the data is far too large.

Figure 77 shows the fit parameter γ compared to the γ fit by Ladner. Interestingly the parameters are unchanged from the uniform case. This result coupled to the results about the critical velocity fronts leads to an interesting conclusion. The T-I state in the converging flow is unchanged from the T-I state in the uniform flow. The LUA seems valid for the T-I state, which is markedly different from the results of the T-II state. Also the critical condition is a local phenomena. This explanation seems accurate for both $\dot{Q}_{c1}$ through $\dot{Q}_{c2}$.

Figure 78 Comparison of excess dissipation for converging and diverging flows.

4.7 Converging and Diverging Flow

Figure 78 shows a comparison for diverging and converging flow. As is obvious from this graph the excess dissipation is the same for both cases. This leads to the obvious conclusion that the excess dissipation does not depend on the direction of the flow with respect to the channel. This conclusion is not inconsistent with the idea that the dissipation in the flow depends on the local parameters of the flow and as such does not undermine the validity of the LUA.

Chapter V

Conclusion

5.1 Introduction

The experimental results given in the previous chapter describe a system whose individual states (laminar, T-I, T-II) and transitions ($\dot{Q}_{C1}$, $\dot{Q}_{C2}$) respond differently to the nonuniformity in the flow field. These differing responses provide a deeper understanding of superfluid turbulence. This chapter reviews the results for each of the states discussed previously and attempts to incorporate these results into a coherent discussion of the effects of nonuniformity on superfluid turbulence as a whole.

5.2 Laminar

The laminar behavior of the counterflow is critical to understanding the effects of the nonuniform channel geometry on the laminar dissipation. The laminar dissipation measured in this experiment is consistent with the analysis using the idea that the dissipation in the channel is defined as if the properties of the flow were locally uniform, i.e. the

local uniformity approximation (LUA) discussed in chapter 4. This result leads directly to two conclusions. The first is that the characterization of the flow channel geometry is quite reasonable and thus the use of this characterization for the analysis of the turbulent states is justified. The second conclusion is that the assumption that was made about the 'smallness' of our nonuniformity is valid. This result supports the use of the LUA for the T-I and T-II state.

5.3 The Critical Heat Currents

There are 2 major conclusions from the analysis of the critical heat currents. Comparing the results for the critical heat currents from this work to the critical heat currents from uniform channels it is clear that the flow first goes turbulent at the small diameter end of the channel. This is the point where the highest counterflow velocity is found and thus should be the first place in the flow channel where the critical velocities are exceeded. This intuitive result also supports a local description of the turbulence.

The second conclusion is that the turbulent states exhibit stationary front behavior. The T-I/Laminar front behaved in a manner that was quantitatively consistent with a critical velocity depending inversely on the local

diameter d . The results of fitting the T-I state to the Modified Vinen Equation using a sharp front between the laminar and T-I state are excellent and the fit parameter agrees with values found for uniform channels. Other choices for the critical velocity did not lead to good fits. The T-I/T-II transition, being more complex, did not allow for the same degree of quantitative analysis. The dramatic broadening of this transition in the nonuniform channel is entirely consistent with a T-I/T-II front and a critical velocity proportional to $1/d$.

Both of these conclusions support the idea that superfluid turbulence depends only on the parameters of the local flow environment and not globally on the nonuniformity in our geometry. These results support the validity of using the LUA in the turbulent case since it is exactly this idea of locality on which the LUA is based.

5.4 The T-I State

There are two conclusions that come from analyzing the T-I data. The first conclusion is that the T-I state is the same as found in the uniform case. The fit parameter γ is essentially the same in the converging case as in the uniform case. The successful fit of the MVE to the

nonuniform data and the fact that the fit parameter γ is unchanged from the uniform case is direct support for the LUA. The dissipation associated with the T-I state depends only on the local flow environment.

The second conclusion about the T-I state comes by comparing the results of the T-I analysis to the T-II analysis. From this comparison we conclude that the T-I state differs in some fundamental way from the T-II state. As we see from Figures 57 through 64 the Schwarz theory using the LUA does not adequately describe the T-II state we observe. Thus the T-II state is not just an extension of the T-II state found in the uniform case. As stated in the first paragraph of this section the T-I state is equivalent to the T-I state in uniform channels. Thus we may conclude from this analysis that the T-I state is not just a scaled down version of the T-II state!. This is important since the T-I state has yet to be theoretically described and its structure is still largely enigmatic. What this experiment tells us is that the T-I state does not react the same way that the T-II state does to minor perturbations in its flow environment.

5.5 The T-II State

The overall conclusions associated with the T-II state are in agreement with the conclusions associated with the nonuniform rectangular channel experiment. In a nonuniform geometry the transition into the T-II state occurs in a front like manner. This means that for a range of heat currents above the critical value the channel is partially filled with the fully developed T-II state. The T-II state fills more and more of the channel as the heat current is increased until at some heat current the entire channel is filled with the T-II state. Also the dissipation associated with this T-II state is less than is predicted by an application of the Schwarz theory using the LUA. So far there has been no adequate theory proposed to explain this deficit, though a phenomenological fit to the nonuniform rectangular channel data has been proposed by Kafkalidis and Tough. Unfortunately the experiment reported in this thesis was unable to gather much useful T-II data and thus a more quantitative comparison between the nonuniform rectangular experiment and this research could not be done.

5.6 The Converging and Diverging Case

The quantitative agreement between the converging and diverging counterflow cases is excellent. The T-II, T-I,

transition and laminar states all quantitatively agree at all temperatures. This agreement allows us to conclude that the direction of the flow has no effect on the actual state of the system. This conclusion also states that if the local line density depends on the gradient of some parameter of the flow at a point cannot depend on the sign of that gradient! Thus the idea that the local line density depends on the local flow parameters and local environment is not invalidated by this result.

5.7 Conclusion

The conclusions associated with the T-I and T-II states seem to contradict one another on the point of whether the vortex tangle is defined by the local parameters of the flow. The effect of the nonuniformity on the laminar state, the T-I state, the two critical heat currents, and the converging/diverging flow agreement all support the conclusion that the nonuniformity in this experiment is small and thus the LUA is valid. This is the intuitive result since the opening angle of the cone is less than .001 radian and thus at any point in the flow the effects of the nonuniformity should be vanishingly small.

The T-II state however exhibits a large departure from the Schwarz theory using the LUA. This result is in direct

conflict with the conclusions stated above but is consistent with the conclusions associated with the nonuniform rectangular experiment. This conflict leads to three viable conclusions: the assumption of local uniformity for the T-II state is incorrect, the theory for the uniform homogeneous superfluid turbulent state is incorrect, or that the incorporation of even a small nonuniformity in the flow field gives rise to a different mechanism for vortex motion.

The first two conclusions are hard to believe. It can be argued that an individual vortex line does not interact with line far away from itself and that the intrinsic interaction length is the average line-line spacing. For the fully developed T-II state the line-line spacing is on the order of microns and thus the idea that vortex lines 'talk' to each other over large distances and react to the channel nonuniformity is not likely. Secondly the idea that the homogeneous state in the uniform case is not understood is counter to theoretical developments by Schwarz. The success of the Schwarz theory in describing the homogeneous state in the uniform case is extremely good.

The third conclusion allows us to keep the idea of the LUA and the validity of the Schwarz theory. The Schwarz theory is a microscopic theory for the motion of individual vortex lines and in that form it is entirely general.

However in its microscopic form the theory is computationally intensive and a numerical calculation of fully developed turbulence has never been attempted for nonuniform counterflow velocity fields. Almost all cases where the Schwarz theory has been numerically applied have been in a spatially uniform counterflow field. It is quite possible that the original equation of motion contains in it subtle physics that only occurs in a nonuniform case. The equation could then give rise to effects which are zeroed out in the uniform calculations but are present in my experiment. In a recent publication exactly such an effect has been reported by Schwarz. In this work Schwarz simulates the motion of a vortex line in the flow through a pinhole. The complex geometry of the pinhole induces a spatially diverging flow field in which the vortex line moves. The vortex line grows in length but the main cause of this growth is not mutual friction. The main cause of vortex growth is vortex line stretching due to the nonuniformity of the flow. This result shows that the Schwarz theory can yield new physics in the presence of nonuniform flows.

This research strongly suggests that the effect of nonuniformity in the flow field on the Schwarz model be studied. Unless the results of the nonuniform rectangular channel experiment and the research of this thesis can be

reconciled to the Schwarz theory, the Schwarz theory may have to be rethought.

APPENDIX A

THEORETICAL CALCULATION OF THE THERMAL IMPEDANCE FOR
LAMINAR FLOW IN A CONE WITH A SMALL OPENING ANGLE

The laminar thermal impedance of flow in a cone can be calculated using the Two Fluid Model

$$\rho_N \frac{\partial V_N}{\partial t} + \rho_N (V_N \cdot \nabla) V_N = \eta \nabla^2 V_N - \frac{\rho_N}{\rho} \nabla P - \rho_S S \nabla T \quad (\text{A-1})$$

$$\rho_S \frac{\partial V_S}{\partial t} + \rho_S (V_S \cdot \nabla) V_S = - \frac{\rho_S}{\rho} \nabla P + \rho_S S \nabla T \quad (\text{A-2})$$

I am interested in steady state behavior so all the time derivatives of Equations A-1 and A-2 are zero. Adding equation A-1 and A-2 yields

$$\rho_N (V_N \cdot \nabla) V_N + \rho_S (V_S \cdot \nabla) V_S = \eta \nabla^2 V_N - \nabla P \quad (\text{A-3})$$

Equation A-2 is rearranged to give

$$\rho_S S \nabla T = \frac{\rho_S}{\rho} \nabla P + \rho_S (V_S \cdot \nabla) V_S \quad (\text{A-4})$$

I now assume that the θ and ϕ components of V_n and V_s are zero. I also assume that the radial components of the

velocity fields can be written as $V_{\theta r}(r, \theta)$ and $V_{sr}(r)$. Using the geometry defined in Figure 50 and spherical coordinates Equations A-3 and A-4 become

$$\rho_s V_{sr} \frac{\partial V_{sr}}{\partial r} + \rho_N V_{Nr} \frac{\partial V_{Nr}}{\partial r} = -\frac{\partial P}{\partial r} + \eta \left[\frac{1}{r} \frac{\partial^2 (r V_{Nr})}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 V_{Nr}}{\partial \theta^2} + \frac{\cot \theta}{r^2} \frac{\partial V_{Nr}}{\partial \theta} - \frac{2 V_{Nr}}{r^2} \right] \quad (\text{A-5})$$

$$0 = -\frac{1}{r} \frac{\partial P}{\partial \theta} + \eta \left[\frac{2}{r^2} \frac{\partial V_{Nr}}{\partial \theta} \right] \quad (\text{A-6})$$

$$0 = \frac{\partial P}{\partial \phi} \quad (\text{A-7})$$

$$S \frac{\partial T}{\partial r} = \frac{1}{\rho} \frac{\partial P}{\partial r} + V_{sr} \frac{\partial V_{sr}}{\partial r} \quad (\text{A-8})$$

The incompressibility condition, $\nabla \cdot \mathbf{V}_s = \nabla \cdot \mathbf{V}_n = 0$ requires

$$V_{Nr}(r) = \frac{C_N}{r^2} \Theta(\theta) \quad (\text{A-9})$$

$$V_{sr}(r) = \frac{C_s}{r^2} \quad (\text{A-10})$$

I will choose $\Theta(\theta)$ to be the simplest symmetric function that goes to zero at $\theta = \alpha$ (at the walls). Therefore I assume

$$\Theta(\theta) = \left(1 - \frac{\theta^2}{\alpha^2}\right) \quad (\text{A-11})$$

The constant C_N can be found by demanding that the average normal fluid velocity be given by Equation 7 ($\theta \rightarrow 0$).

$$\begin{aligned}\int V_{Nr} \cdot dA &= \frac{C_N}{r^2} \int_0^{2\pi} \int_0^\alpha \left(1 - \frac{\theta^2}{\alpha^2}\right) r^2 \sin\theta d\theta d\phi = \frac{\dot{Q}}{\rho ST} \\ \Rightarrow C_N &= \frac{2}{\pi \alpha^2} \frac{\dot{Q}}{\rho ST}\end{aligned}\quad (\text{A-12})$$

C_s is derived from the counterflow condition $V_s = -(\rho_N/\rho_s)V_N$ (for $\theta \rightarrow 0$)

$$\begin{aligned}\int V_{Sr} \cdot dA &= \frac{C_s}{r^2} \int_0^{2\pi} \int_0^\alpha r^2 \sin\theta d\theta d\phi = -\frac{\rho_N}{\rho_s} \frac{\dot{Q}}{\rho ST} \\ \Rightarrow C_s &= -\frac{\rho_N}{\rho_s} \frac{\dot{Q}}{\rho ST \pi \alpha^2}\end{aligned}\quad (\text{A-13})$$

Using the above assumptions the Two Fluid Model simplifies to

$$\begin{aligned}-\frac{2}{r^5} \left[C_s^2 \rho_s + \rho_N C_N^2 \left(1 - \frac{\theta^2}{\alpha^2}\right) \right] &= \\ -\frac{\partial P}{\partial r} - 2\eta \frac{C_N}{r^4} \left[\frac{\theta \cot\theta + 1}{\alpha^2} \right]\end{aligned}\quad (\text{A-14})$$

$$\frac{\partial P}{\partial \theta} = -\frac{4\eta C_N \theta}{r^4}\quad (\text{A-15})$$

I am interested in the small angle limit for this solution so I let $\theta \rightarrow 0$. Equations A-14 and A-15 become

$$-\frac{2}{r^5}[C_S^2\rho_S+\rho_NC_N^2]=$$

$$-\frac{\partial P}{\partial r}-2\eta\frac{C_N}{r^4}\left[\frac{2}{\alpha^2}\right] \quad (\text{A-16})$$

$$\frac{\partial P}{\partial \theta}=0 \quad (\text{A-17})$$

and the temperature gradient (Eq. A-8) is:

$$S\frac{\partial T}{\partial r}+\frac{1}{\rho}\frac{\partial P}{\partial r}-2\frac{C_S^2}{r^5} \quad (\text{A-18})$$

As can be seen from the Equation A-17 the pressure is independent of θ . Using Equations A-18 and A-16 it is now possible to solve for the temperature gradient

$$S\frac{\partial T}{\partial r}=-\frac{2\eta C_N}{\rho r^4}\left[\frac{2}{\alpha^2}\right]+\frac{2}{\rho r^5}[C_S^2\rho_S+\rho_NC_N^2-\rho C_S^2] \quad (\text{A-19})$$

The temperature gradient consists of a viscous and an inertial term. It is of interest to determine the contribution each term contributes to the laminar temperature gradient for the flow described in this dissertation. The ratio of the inertial and viscous terms is

$$Ratio=\frac{1}{4\eta r}\left[\frac{\dot{Q}}{\rho ST\pi\alpha^2}\right]\frac{4\rho_N-\left[\frac{\rho_N}{\rho_S}\right]^2\rho_N}{\frac{2}{\alpha^2}} \quad (\text{A-20})$$

At $T=1.3\text{K}$ with $\alpha = .001$ and an input power of $100 \mu\text{W}$ this ratio is 0.18. This means that the inertial terms are a factor of five times smaller than the viscous terms at heat currents many times higher than the critical heat current.

These inertial terms can be ignored and Eq. A-19 becomes

$$S \frac{\partial T}{\partial r} = - \frac{4\eta C_N}{\rho r^4 \alpha^2} \quad (\text{A-21})$$

Using Eq. A-12 it is now possible to write the temperature gradient

$$\frac{\partial T}{\partial r} = - \frac{8\eta \dot{Q}}{(\rho S)^2 \pi T r^4 \alpha^4} \quad (\text{A-22})$$

Recognizing that αr is the local cross sectional radius of the cone $R(r)$ Equation A-22 can then be identified equivalent to the laminar gradient of a straight cylinder using the local cross sectional radius.

$$\frac{\partial T}{\partial r} = - \frac{8\eta \dot{Q}}{(\rho S)^2 \pi T (R(r))^4} \quad (\text{A-23})$$

The idea that the local temperature gradient in a cone is the same as that of a cylinder using the local cross sectional radius of curvature gives support to the Idea of the Local Uniformity Approximation (LUA).

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