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Linear and nonlinear ultrasonic characterization of single-layered structures

Rypien, David Vincent, Ph.D.

The Ohio State University, 1990

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**300 N. Zeeb Rd.
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**LINEAR AND NONLINEAR ULTRASONIC
CHARACTERIZATION OF
SINGLE LAYERED STRUCTURES**

DISSERTATION

**Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University**

By

David Vincent Rypien, B.S., M.S., P.E.

The Ohio State University

1990

Dissertation Committee:

Laszlo Adler

Avraham Benatar

Stanislav Rokhlin

Approved by

**Adviser
Department of Welding Engineering**

**To my wife Ann
and my two black cats Max and Abby**

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To my wife Ann, for whom this dissertation is dedicated, for the times of your unending love and caring for me, I love you.

VITA

July 31, 1956	Born Youngstown, Ohio
1979	A.A.S., Stark Technical College, Canton, Ohio
1979-1989	Technical Analyst, The Timken Research Company, Canton, Ohio
1983	B.S., Walsh College, Canton, Ohio
1985	M.S., The Ohio State University, Columbus, Ohio
1989	P.E., State of Ohio, Columbus, Ohio
1984-Present	Research and Teaching Associate, The Ohio State University, Columbus, Ohio

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FIELDS OF STUDY

Major Field: Welding Engineering
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CHAPTER I

INTRODUCTION

1.1 Adhesive Joints

The advantages of using layered materials in structural design is that they can improve the load carrying capabilities of a structure and provide a means to transfer loads between two structure. The simplest layered structure is the adhesive joint which is used extensively in the automotive and aerospace industries. It is constructed by joining two plates (adherends) usually having the same or similar properties by a layer of glue or adhesive as shown in Fig. 1. The objective of this bonding process is to produce a bond that is as strong as the material it joins [1].

Strength assessment of adhesive bonds is currently conducted by destructively loading the joint, usually in uniaxial tension, until it fails [2]. A typical stress-strain curve for two adhesive joint samples are shown in Fig. 2, where S_u is the ultimate strength of the adhesive joint. The ultimate load at which the joint fails is a measure of the strength of the joint. This destructive testing technique is inadequate for a number of reasons. First, because of its destructive nature only a representative group of samples can be tested. Second, the destructive test provides information concerning the overall strength of the adhesive joint and little is known about its "weakest link," being the adherend-adhesive interface.

Figure 1. Diagram of typical single lap adhesive joint.

Figure 2. Stress-strain test results for two single lap adhesive joints with FM300K adhesive (Data Supplied by the American Cyanamide Corporation).
Sample 1, overlap length 9.1 mm, adhesive thickness 0.170 mm, and $S_u = 47.9$ MPa.
Sample 2, overlap length 9.1 mm, adhesive thickness 0.185 mm, and $S_u = 48.5$ MPa.

In recent years, nondestructive evaluation (NDE) methods have become useful tools for monitoring in quality control (QC) of adhesive joints. NDE can provide a number of advantages which far out weigh those of the already established destructive testing methods, e.g., NDE techniques can provide for total inspection of the samples without destroying their serviceability and are sensitive to the properties of the adhesive joint especially the bond layer. In particular, these techniques are able to assess the failure mechanisms in the adhesive layer or along the adherend-adhesive interface which will reduce either the cohesive or adhesive strength of the joint, respectively. It is not to say that there is no longer a need for destructive testing, instead the two techniques must complement each other. Recent reviews of nondestructive testing of adhesively bonded joints can be found in literature [3-6].

Ultrasonic NDE is an ideal candidate for a variety of reasons. First, ultrasonics can be used to detect gross defects like voids, lack of bonding, improper solidification and excessive thickness of the adhesive. Second, the linear and nonlinear elastic properties of the adhesive can be determined by nondestructively measuring the velocity of the propagating ultrasonic waves through the stress-free and prestressed layer, respectively. Third, using special ultrasonic guided wave modes, e.g., Lamb and interface waves which travel along the entire adhesive joint or just along the adhesive layer, respectively, can be used to determine the adhesive bond quality and the elastic properties of the adhesive layer.

1.2 Objectives

The purpose of this dissertation is to study the linear and nonlinear properties of the adhesive bonded joint by measuring the stress free and stress-dependent ultrasonic bulk

wave velocities for the adhesive layer and determine the dispersive properties of special guided wave modes which propagate through the adhesive joint and layer. There are four objectives in this study.

The first objective is to present a theoretical and experimental study on the acoustoelastic effects of a prestressed adhesive layer. Newly developed acoustoelastic equations will be presented which relate the ultrasonic bulk wave velocities of a prestressed adhesive layer to its second- and third-order elastic constants. In order to experimentally measure the acoustoelastic effects of the adhesive layer an ultrasonic spectroscopy technique for measuring the bulk wave velocities from the reflected frequency spectrum of the layer was further developed. This technique incorporates two ultrasonic transducers in which one will be used to measure the acoustoelastic effects, and the other to eliminate such effects as temperature and signal drift from other equipment components, analogous to using an active and dummy strain gauges in experimental stress analysis.

The second objective is to conduct a preliminary study to measure the linear and nonlinear properties of two solid materials, 6061-T6 aluminum and perspex, in order to obtain a better understanding of the materials which make up the adhesive joint. By measuring the stress free and stress-dependent ultrasonic bulk wave velocities of these solid materials the second- and third-order elastic constants can be calculated.

The third objective is to experimentally verify a technique to measure higher-order nonlinearities by eliminating the much stronger first-order acoustical birefringence effect of the shear wave in 6061-T6 aluminum. The intention of this portion of the study is to be

able to separate the stronger nonlinear effects of the aluminum adherend plates from the much weaker nonlinear effects of the adhesive layer.

Finally, the fourth objective will be to present both a theoretical and experimental study of guided waves in an adhesive joint. In particular, the frequency-dependent transmission spectrum will be determined as a function of angle of incidence for the Lamb and interface wave modes. Experimental verification of the theoretical predictions will be made using a double transmission ultrasonic immersion technique initially developed to measure the dispersive properties of Lamb waves in composite plates. In addition to comparing the theory and experiment for the frequency-dependent transmission coefficients for the Lamb waves in an adhesive joint, these results will also be compared to the theoretical calculations made for a single adherend plate in order to assess the acoustical coupling efficiency of the Lamb waves to the adhesive joint.

1.3 Summary of Chapters

This dissertation consist of six additional chapters. Chapter II is a background and literature review of the pertinent material used in this study.

Chapter III begins by presenting a technique used to measure the stress free and stress-dependent ultrasonic bulk waves velocities in 6061-T6 aluminum and perspex in which a LeCroy 9400 digital oscilloscope and its associated mathematical functions were used to rapidly measure the time delay between the front- and backwall echoes. The second- and third-order elastic constants for 6061-T6 aluminum and perspex were calculated and will be presented.

Chapter IV presents a proposed technique which was experimentally verified in which by choosing the proper polarization angle the effects of the first-order stress-dependent velocity terms are eliminated. The advantage of eliminating or reducing the strong effects of the first-order terms is that higher-order nonlinearities for a solid sample can be measured. Some preliminary results are presented in this chapter for 6061-T6 aluminum.

Chapter V begins by developing the displacement gradient equations for a prestressed adhesive layer. New acoustoelastic equations were developed which consider the effects of the stress-dependent bulk wave velocities on its second- and third-order elastic constants. In order to measure the acoustoelastic effects of the adhesive layer, an existing ultrasonic technique which uses the measured reflection spectrum from the layer was further developed. Measured results using this technique for FM300K adhesive material are presented.

Chapter VI presents the results of a theoretical and experimental study of the dispersion relations, specifically the frequency-dependent transmission coefficients for the Lamb and interface waves in an adhesive joint. Experimental measurements were made using a double transmission technique for Lamb and interface waves. A comparison will be made between the Lamb modes in the adhesive joint and those calculated for a single adherend plate to assess the acoustical coupling between the Lamb wave modes in a single plate and those in the adhesive joint.

Chapter VII summarizes the results obtained from this study for the linear and nonlinear properties of the solid and adhesively joined materials. The need for future work in these areas will be considered.

CHAPTER II

BACKGROUND AND LITERATURE REVIEW

2.1 Linear Elastic Wave Propagation

A linear elastic material behaves according to Hooke's law, which can be written in tensor notation as

$$T_{ij} = c_{ijlm}\gamma_{lm} \quad (2.1)$$

where $(i,j,l,m = 1,2,3)$ and repeated indices denote summation. It states that the applied stresses T_{ij} are linearly proportional to the produced strains γ_{lm} , and related by the materials elastic constant tensor c_{ijlm} .

In the most general case, a solid material has 81 elastic coefficients altogether. It follows from the symmetry of the stress-strain tensor that the number of coefficients reduce from 81 to 36. With the existence of an elastic potential which amounts to having the strain energy be a function of state and independent of path by which the state is reached further reduces the number of independent elastic constants from 36 to 21. For a linear elastic material the strain energy is a quadratic function of the strain with the elastic constant tensor as its leading coefficient, for this reason it will be referred to as the second-order elastic constant.

The materials in this study will be considered isotropic, in which case the physical properties of the material are independent of the material orientation, and only two independent elastic constants are needed, λ and μ , the Lam'e constants, to describe its behavior. The isotropic elastic constants are related to the components of the elastic constant tensor in the following manner:

$$\begin{aligned} c_{12} &= c_{13} = c_{23} = \mu \\ c_{44} &= c_{55} = c_{66} = \lambda \\ c_{11} &= c_{22} = c_{33} = \lambda + 2\mu \end{aligned} \quad (2.2)$$

the remaining coefficients are zero, and an abbreviated notation can be used for the elastic constant components [7].

For an isotropic solid, (2.1) can be written in terms of the second-order elastic constants as follows:

$$T_{ij} = \lambda \delta_{ij} \gamma_{kk} + 2\mu \gamma_{ij} \quad (2.3)$$

where

$$\gamma_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \quad (2.4)$$

is the infinitesimal strain tensor and u represents the displacement vector, and δ_{ij} is the Kronecker delta function.

2.1.1 Bulk Waves through an Isotropic Solid

The equation of motion for an isotropic solid (neglecting body forces) can be written as

$$\rho_0 \ddot{u}_i = (\lambda + 2\mu) \nabla \nabla \cdot u_i - \mu \nabla \times \nabla \times u_i \quad (2.5)$$

where ρ_0 is the material density in the unstrained state, and ∇ is the "del" operator. The particle displacement vector can be written as a combination of potential functions ϕ and ψ , in the following form

$$u = \nabla \phi + \nabla \times \psi \quad (2.6)$$

where ϕ and ψ represent the longitudinal and shear waves, respectively. Both bulk wave modes satisfy the following wave equations:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} + k_l^2 \phi = 0 \quad (2.7)$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial z^2} + k_t^2 \psi = 0 \quad (2.8)$$

Here,

$$k_l = \omega \sqrt{\frac{\rho_0}{\lambda + 2\mu}} \quad (2.9)$$

and

$$k_t = \omega \sqrt{\frac{\rho_0}{\mu}} \quad (2.10)$$

are the wave numbers ($k=2\pi/\lambda$, where λ in this case is the wavelength of the elastic wave) for the longitudinal and shear modes, respectively, where $\omega=2\pi f$. Substituting the above relationships into (2.9) and (2.10), we obtain two equations,

$$c_l = \sqrt{\frac{\lambda+2\mu}{\rho_0}} \quad (2.11)$$

and

$$c_t = \sqrt{\frac{\mu}{\rho_0}} \quad (2.12)$$

which relate the measured longitudinal and shear wave velocities and the second-order elastic constants.

It can be shown from (2.1) that for a propagating longitudinal wave the polarization is parallel to the direction of propagation. On the other hand, for a propagating shear wave the polarization is perpendicular to the direction of propagation. For example, if a plane shear wave propagates in the x-direction it can be polarized in either the xy or xz plane. For an ideal isotropic solid both polarizations will have the same velocity, determined by (2.12) [8].

There are a number of well established ultrasonic techniques used to measure the bulk wave velocities of a solid medium, e.g., pulse overlap and sing-around techniques, which are described in review articles [9-11]. Most commonly the bulk velocities of our

samples will be determined by dividing twice the sample thickness by the measured time interval between the front and back wall reflections of the sample.

2.1.2 Physical Considerations and Boundary Conditions

In this section we will consider the basic laws of physics that govern how sound waves propagate from one medium into another. Figure 3 illustrates the law of refraction for two interface conditions which are representative of actual testing conditions. An ultrasonic plane wave is incident from medium 1 into medium 2, at an angle θ_i , measured between the normal to the wave front and the z-axis. At the interface, a portion of the wave is reflected at an angle θ_r , and another portion is transmitted into medium 2 at an angle θ_t . The direction of the reflected and transmitted waves are determined by Snell's law [12],

$$\frac{\sin \theta_i}{c_{11}} = \frac{\sin \theta_r}{c_{11}} = \frac{\sin \theta_{tl}}{c_{12}} = \frac{\sin \theta_{tt}}{c_{12}} . \quad (2.13)$$

It can be seen in Fig. 3 that medium 2 contains two waves, one being a longitudinal and the other a shear wave. This occurs because of mode conversion at the interface between medium 1 and 2; the angles for the waves are related by (2.13).

When $\theta_i = 0$, i.e. at normal incidence, no mode conversion occurs and only pure longitudinal and shear wave modes are transmitted and reflected at the interface of medium 1 and 2. The reflected and transmitted longitudinal and shear amplitudes are related by the reflection and transmission coefficients, respectively, written as

Figure 3. Reflection and refraction of plane waves at the boundary of two interfaces; a) liquid/solid and b) solid/solid.

$$R_{l,t} = \frac{\phi_r}{\phi_i}, \frac{\psi_r}{\psi_i} = \frac{Z_2 - Z_1}{Z_1 + Z_2}, \quad (2.14)$$

$$T_{l,t} = \frac{\phi_t}{\phi_i}, \frac{\psi_t}{\psi_i} = \frac{2 Z_1}{Z_1 + Z_2} \quad (2.15)$$

where $Z_i = \rho_i c_{l,t}$ ($i=1,2$), is the material impedance for medium 1 and 2.

We must consider the boundary conditions at the interface ($z=0$) for the two cases shown in Fig. 3.

Figure 3a, is for a liquid/solid interface in which there is continuity of the normal displacements and stresses, u_z and T_{zz} , respectively, and $T_{xz} = 0$. At the liquid/solid interface only a longitudinal wave will be reflected, and by the law of reflection $\theta_r = \theta_i$.

Figure 3b, is for a solid/solid interface in which the interfaces are in perfect contact, i.e., there is continuity of normal and tangential displacements u_x and u_z , and stresses, T_{zz} and T_{xz} , respectively. For either an incident longitudinal or shear wave, mode conversion occurs at the solid/solid interface and reflected and transmitted longitudinal and shear waves are produced in mediums 1 and 2, with their directions of propagation determined by (2.13).

We also assume that the y-dimensions are large compared to the x- and z- and that the displacements along the y-direction are zero.

The components u_x and u_z of the particle displacements along the x and z axes and the stress components T_{zz} and T_{xz} may be represented in terms of the vector potentials ϕ and ψ according to the following equations [13]:

$$u_x = \frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial z} \quad (2.16)$$

$$u_z = \frac{\partial \phi}{\partial z} + \frac{\partial \psi}{\partial x} \quad (2.17)$$

$$T_{zz} = \lambda \left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} \right) + 2\mu \left(\frac{\partial^2 \phi}{\partial z^2} + \frac{\partial^2 \psi}{\partial z \partial x} \right) \quad (2.18)$$

$$T_{xz} = \mu \left(\frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi}{\partial z^2} + 2 \frac{\partial^2 \phi}{\partial x \partial z} \right) \quad (2.19)$$

2.1.3 Reflection From a Plane and Adhesive Layer

Consider an adhesive layer (thicknesses ranging from 50-300 μm) sandwiched between two thick adherend plates as a plane layer. Since this layer is so thin, normal time domain measurements used to determine the bulk wave velocities will not work. It has been shown by a number of authors [14-16], that by measuring the periodicity of the maxima and minima of the frequency-dependent reflection coefficient from the layer they could determine the properties of the layer. The theoretical formulation of this problem was presented by Brekhovskikh [17].

For our particular case where the two adherend plates of the adhesive joint are identical, the frequency-dependent reflection coefficient can be written in the following form:

$$R = \frac{Z_2^2 - Z_1^2}{Z_1^2 + Z_2^2 + 2i Z_1 Z_2 \cot \alpha d} \quad (2.20)$$

Experimental measurement of the frequency-dependent reflection coefficient from (2.20) for a plane layer can be done by using ultrasonic spectroscopy. Figures 4 and 5 show how this technique works. First, the reflected signal from the layer is obtained by normal incidence longitudinal or shear wave contact techniques as shown in Fig. 4. Figure 5 shows a typical A-scan of the reflected signals from the layer. Notice that the reflection from the upper interface of the layer cannot be time resolved from the bottom interface signal which makes velocity measurements difficult. Finally, the reflection signal from the layer is time gated and frequency analyzed. The dominant feature of the reflection spectrum is the position of the minima, as shown in Fig. 6, which occur due to destructive interference between the two reflected signals from each interface, and at frequencies where the thickness d of the layer is an integral number of half wavelengths, i.e.,

$$d = n \frac{\lambda}{2}, \quad n = (1, 2, 3, \dots) \quad (2.21)$$

From the location of these minima on the frequency axis and the depth and shape of the minima, values for the velocity and attenuation of the layer can be deduced if the impedance of the upper and lower, super- and substrates, respectively, are known. An

Figure 4. Schematic diagram for measuring the velocity in a layer using conventional normal incidence ultrasonic contact technique.

Figure 5. Typical A-scan of an adhesive joint at normal incidence.

Figure 6. Calculated reflection coefficient of an adhesive layer with various amounts of attenuation.

expression for the acoustic velocity in the layer was determined by substituting ($\lambda = c/f$) into (2.21) as follows

$$c_{l,t} = 2 \frac{f_{\min} d}{n}, \quad n = (1, 2, 3, \dots) \quad (2.22)$$

The use of this technique to correlate the cohesive strength of an adhesive joint to the measured longitudinal wave velocity and attenuation was described by Alers, et al., [15]. The results are shown in Figs. 7a and 7b, for the shear strength as a function of measured velocity and attenuation, respectively. In Fig. 7a, it would appear that the relationship between the measured velocity and the shear strength are linear [4]. In Fig. 7b it is difficult to determine the relationship between the longitudinal wave attenuation and the joint strength, e.g., for a measured attenuation coefficient of 20 Np/cm the bond strength ranges from 2800 to 3000 lbf (12.460 to 13.350 kN). However, in both cases the scatter of the data is less than desirable. Even though they were not able to show a conclusive relationship between cohesive bond strength and the ultrasonic velocity and attenuation from the measured frequency spectrum they were able to show that the material properties could be determined from this technique.

2.2 Nonlinear Wave Propagation

Nonlinear elastic wave propagation may occur in a solid as a result of one of the following factors. First, the amplitude of the propagating elastic wave may be large enough to cause finite strains. Second, the velocity of a propagating elastic wave through a stressed solid may be dependent upon the amount of externally applied

Figure 7. a) Correlation between the measured longitudinal sound velocity and the strength of the adhesive bond. b) Correlation between the measured longitudinal wave attenuation and the strength of the adhesive bond [15].

stress. Finally, the material itself may contain various absorption mechanisms, such that it is locally nonlinear [18].

The measurement of the material acoustoelastic effect, or the way in which its ultrasonic velocity changes with an applied external stress, is commonly used to measure the third-order elastic constants for a solid medium and will be defined in the next section.

2.2.1 Acoustoelastic Effect of an Isotropic Solid

The acoustoelastic effect for an isotropic solid was studied as early as 1953 by Hughes and Kelly [19]. They developed expressions, which will be reviewed later in this section, for the velocities of ultrasonic bulk waves in a prestressed isotropic material under hydrostatic pressure and uniaxial compression using the finite deformation theory outlined by Murnaghan [20]. Experimentally measured third-order elastic constants using this theory and expressions formulated by Thurston and Brugger [21] for cubic and isotropic solids were obtained for various polycrystalline metals by Smith, et al. [22]. Additional experimental results were obtained by a number of other authors [23, 24] for the stress dependent bulk wave velocities in aluminum and other materials.

The finite deformation theory used to describe the stress-dependent ultrasonic bulk wave velocities and the third-order elastic constants for an isotropic solid involves two departures from the linear elastic theory described in Section 2.1. First, since the strains in the solid are sufficiently large, the final coordinates of a point in the stressed

material are not interchangeable with the original coordinates, as in the linear elastic theory. Second, the expression for the strain energy function must be expanded in order to account for these large strains. These two elements will be described in more detail below.

2.2.1.1 Coordinate System

Consider a fixed Cartesian coordinate system shown in Fig. 8. A material point is considered to be in the undeformed or natural state when no external stresses are applied to the material. To measure the acoustoelastic effect of a material, the sample is usually prestressed in either uniaxial static tension or compression which will displace the material point to a new location. This new location is called the initial or reference state and is described by the coordinates $a_i(a,b,c)$. A superimposed ultrasonic bulk wave, either longitudinal or shear will propagate through the material which will again displace the material particle. The new particle location is called the deformed or final state and is denoted by $x_i(x,y,z)$. The x_i and a_i are associated by the rule that x_i is the new position at time t , of a particle whose reference position was a_i [25].

By choosing the reference state as an independent variable the final position x can be described as a function of time t , i.e.,

$$x_i = m(a_i, t) \quad (2.23)$$

where $m(a_i, t)$ describes the motion of the particle. The components of the displacements $u_i(u,v,w)$ are defined as follows

Figure 8. Coordinate system for a material point in the initial and deformed state.

$$u_i = x_i - a_i. \quad (2.24)$$

This coordinate system is referred to as the material description or Lagrangian coordinate system. The choice of the Lagrangian coordinate system has the following advantage. The equation of motion is written as

$$\rho_0 \ddot{u}_i = \frac{\partial T_{ij}}{\partial a_j} \quad (2.25)$$

and is linear, where T_{ij} is the engineering stress tensor, and the time derivatives are material derivatives which are equivalent to $\partial^2/\partial t^2$, and are actually related to experimentally measured quantities.

2.2.1.2 Strain Energy Function

The next step is to expand the elastic strain energy in terms of the strains. The elastic strain energy F for an isotropic solid including the third-order terms as a function of strains γ , is given by Pollard [8] as

$$F = F_0 + F_1 + \frac{1}{2!} c_{iklm} \gamma_{ik} \gamma_{lm} + \frac{1}{3!} c_{iklmno} \gamma_{ik} \gamma_{lm} \gamma_{no} \quad (2.26)$$

From (2.26), F_0 is the initial strain energy which in most cases is assumed to be zero, and F_1 is the potential energy due to static loading. The coefficient c_{iklmno} , of the terms in γ^3 is called the third-order elastic constants tensor. This sixth-order tensor has 729 possible components and is reduced significantly to only three, l , m , and n , using

Murnaghan's notation for an isotropic solid. The three independent third-order elastic constants l , m , and n are related in the following manner

$$\begin{aligned} l &= \frac{1}{2} c_{112} \\ m &= c_{155} \\ n &= 4 c_{456}. \end{aligned} \tag{2.27}$$

A summary of other notation for the third-order elastic constants can be found in [18].

2.2.2 Uniaxially Stressed Isotropic Solid

When an isotropic solid is prestressed under uniaxial tension and compression a slight anisotropy is introduced in the material. As a consequence of this anisotropy two phenomena may occur. First, the velocity of the ultrasonic waves propagating through the solid will depend on the direction and magnitude of the applied stress. Second, depending on the polarization direction of the propagating shear waves in relation to the direction of the applied stress, it is possible to have two shear waves propagating in one direction, one having a slightly different velocity than the other. This second phenomenon is called "acoustic birefringence," analogous to light waves in photoelasticity. It was proposed [26] that by eliminating the first-order effects of the shear birefringence effect, one could possibly measure higher-order nonlinearities in the material.

Figure 9. Sample Geometry and coordinate system for uniaxially loaded isotropic solid [22].

Figure 9 shows the associated geometry and coordinate system for three possible cases. Case I is a longitudinal wave, in which the particle motion or polarization direction is normal to the loading direction and parallel to the propagation direction. Case II is a shear wave, in which the polarization is parallel to the loading direction and normal to its propagation direction. Case III is also a shear wave which is produced by the same transducer as in Case II; however it is rotated by 90 degrees on the sample so that its polarization is normal both to the loading and propagation directions. The derivation of the acoustoelastic equations relating the stress-dependent ultrasonic velocities for the above cases will be reviewed [18].

2.2.2.1 Case I: Longitudinal Waves

For a longitudinal wave propagating in the z-direction, the displacements may be written as

$$\begin{aligned} u &= \xi a \\ v &= \chi b \\ w &= \chi c + Ae^{i[\omega t - (1+\chi)k_1 c]}. \end{aligned} \tag{2.28}$$

The following sign convention will be adapted, where the minus (-) indicates compressive stress and the plus sign (+) indicates tension.

First, let us consider the prestressed isotropic solid under uniaxial compression. Referring to (2.28), ξ is the compression along the a-direction and χ is the sample expansion along the b- and c-direction due to the Poisson effect. The displacements in

the w-direction are due to the superimposed ultrasonic wave having wave number k_1 and propagating in the c-direction. The term $(1+\chi)$ has been included in propagation distance to correct for the thickness changes of the material.

The expression for ξ and χ in terms of the applied stress $-\sigma_{11}$, may be determined in the following fashion:

$$\begin{aligned}
 T_{11} &= -\sigma_{11}, \\
 T_{22} &= T_{33} = T_{12} = T_{21} = T_{13} = T_{31} = 0, \\
 u &= \xi a & u_a &= \xi \\
 v &= \chi b & v_b &= \chi \\
 w &= \chi c & w_c &= \chi.
 \end{aligned} \tag{2.29}$$

The equations on the right-hand side of (2.29) are the displacement gradients. Substituting (2.29) into (A.7) from Appendix A, the following stress-displacement relations were determined

$$\begin{aligned}
 T_{11} &= (\lambda+2\mu)u_a + \lambda(v_b+w_c) \\
 T_{22} &= (\lambda+2\mu)v_b + \lambda(w_c+u_a) \\
 T_{33} &= (\lambda+2\mu)w_c + \lambda(u_a+v_b) \\
 T_{23} &= T_{32} = \mu(v_c+w_a) \\
 T_{31} &= T_{13} = \mu(w_a+u_c) \\
 T_{21} &= T_{12} = \mu(u_b+v_a).
 \end{aligned} \tag{2.30}$$

Substituting (2.29) into (2.30) the expressions for ξ and χ were obtained as,

$$\xi = -\frac{\lambda + \mu}{\mu} \sigma_{11}, \quad (2.31)$$

and

$$\chi = -\frac{\lambda}{2(\lambda + \mu)} \xi = \frac{\lambda}{2\mu(3\lambda + 2\mu)} \sigma_{11}, \quad (2.32)$$

respectively.

In the case of uniaxial tension the sign in the above expressions are reversed. Substituting (2.31) and (2.32) back into (2.28) and differentiating each term, the nonvanishing terms are u_a , v_b , w_c , and w_{cc} . For Case I, the third ($i = 3$) component of the equation of motion, listed in Appendix A, will be used to obtain its acoustoelastic equation.

Substituting the nonvanishing displacement gradients listed above into (A.14), one obtains the following

$$\rho_0 V_1^2 = \lambda + 2\mu \pm \frac{\sigma_{11}}{3K_0} \left[2 \frac{\lambda}{\mu} (m + \lambda + 2\mu) - 2l \right] \quad (2.33)$$

which relates the measured longitudinal wave velocity V_1 to the applied stress and the third-order elastic constants l and m . K_0 is the bulk modulus for an unstrained isotropic solid defined as

$$K_0 = \lambda + \frac{2}{3} \mu. \quad (2.34)$$

2.2.2.2 Case II: Shear Wave Polarized Parallel to Load

The displacement fields for a shear wave polarized in the loading direction and propagating in the z-direction are written as follows

$$\begin{aligned} u &= \xi a + A e^{i[\omega t - (1+\chi)k_t c]} \\ v &= \chi b \\ w &= \chi c \end{aligned} \quad (2.35)$$

where k_t is the shear wave number. The values for ξ and χ are the same as those calculated in the previous section. For Case II, the first component of the equation of motion, (A.10) will be used. The nonvanishing displacement gradients are u_a , u_c , u_{cc} , v_b , and w_c , by substituting these values into, (A.10), the following expression was obtained

$$\rho_0 V_{t1}^2 = \mu \pm \frac{\sigma_{11}}{3K_0} \left[-m - \frac{\lambda}{4\mu} n - \lambda - 2\mu \right] \quad (2.36)$$

where (2.36) relates V_{t1} , the stress-dependent shear wave velocity to the third-order elastic constants, m and n , for this particular polarization.

2.2.2.3 Case III: Shear Wave Polarized Normal to Load

By rotating the shear wave transducer by 90 degrees the same propagation direction is maintained as in Case I and II, however, the polarization direction is now normal to the loading direction. A technique described by Procter, Hsu and Sachse

[27, 28], can be used to determine the polarization of the transducer prior to conducting any measurements. The displacement fields for this case are written as

$$\begin{aligned} u &= \xi a \\ v &= \chi b + A e^{i[\omega t - (1+\chi)k_t c]} \\ w &= \chi c. \end{aligned} \quad (2.37)$$

The nonvanishing displacement gradients are u_a , v_b , v_c , v_{cc} , and w_c . Substituting these values into (A.12), the second component of the equation of motion was obtained:

$$\rho_0 V_{t2}^2 = \mu \pm \frac{\sigma_{11}}{3K_0} \left[\frac{\lambda + \mu}{2\mu} n + 2\lambda - m \right] \quad (2.38)$$

where (2.38) relates the stress-dependent shear wave velocity V_{t2} to the third-order elastic constants, m and n .

Examining (2.33), (2.36), and (2.38), we find three unknowns, l , m , and n , and three equations identical to those formulated by Hughes and Kelly [19]. By measuring the ultrasonic bulk wave velocities at no stress and then with an externally applied uniaxial stress the two, second- and three, third-order elastic constants, respectively, needed to describe an isotropic solid can be calculated.

2.2.3 Acoustoelastic Effect for a Plane Layer

Ultrasonic determination of the third-order elastic constants for an adhesive layer is a difficult task for a number of reasons. First, the adhesive joint can be considered as a three layered solid and it is difficult to model the nonlinear elastic wave propagation for such a geometry. Second, when an external stress is applied to the adhesive joint, the stress distribution along the layer may not be even. Third, existing ultrasonic techniques used to measure the stress-dependent velocity of a prestressed solid medium will not be adequate to measure the velocity of a prestressed layer.

The stress distribution along the adhesive layer of a single lap adhesive joint is well known. The sample geometry for a single lap adhesive joint was shown in Fig. 1. Goland and Reissner (G&R) [29] formulated the closed formed solutions for the normal and shear stress distribution along the adhesive layer. Their expressions were programmed in a computer listed in Appendix B, and the results for a particular joint are shown in Figs. 10 and 11.

Some more recent analysis of the stress distribution along the adhesive layer were considered. Sneddon [30], for instance, found a sign error in the normal stress expression of G&R. This correction was made and incorporated in the results shown in Fig. 10. Sneddon's formulation for the shear stress distribution was unchanged, and the normal stress distribution is only slightly reduced.

Renton and Vinson [31, 32] used the G&R formulation and included the effects of the edges of the adhesive layer on the shear stress distribution. Since the edges of the adhesive layer are free surfaces the shear stresses must vanish. An approximation

Figure 10. Normal stress distribution along the adhesive layer for 4.45 kN load.

Adherend material: 2024-T3 aluminum.

Adherend thickness: 12 mm.

Adhesive material: FM300K.

Adhesive thickness: 0.300 mm.

Overlap length: 25.4 mm.

Figure 11. Shear stress distribution along the adhesive layer for 4.45kN load.

Adherend material: 2024-T3 aluminum.

Adherend thickness: 12 mm.

Adhesive material: FM300K.

Adhesive thickness: 0.300 mm.

Overlap length: 25.4 mm.

to this effect is shown in Fig. 10. Others [33, 34] have considered these effects and have shown fairly good agreement between their analysis and the G&R solutions.

Referring to the normal and shear stress distributions shown in Figs. 10 and 11, respectively, the best area to measure the bulk wave velocities of the adhesive layer would be at the center of the joint. This is apparent for two reasons: first, the stress distributions are fairly constant, and second, the measurements will not be affected by the edges of the sample.

Ultrasonic measurement of the nonlinear behavior of the adhesive joint has recently been studied. A theoretical approach presented by Achenbach, et al. [35], suggested a nonlinear model based on the stress-strain behavior of the adhesive joint, shown in Fig. 1. They proposed that by measuring the reflection from the deformed adhesive layer one could determine the strength of the adhesive joint directly.

An experimental study conducted by Nagy et al. [36], in which a dynamic acoustoelastic technique was introduced to measure the apparent velocity change of the adhesive layer as a function of applied load. They were able to predict the failure stress of the joint with fairly good agreement compared with destructive test results.

2.3 Guided Waves

Guided waves are bound and guided by the geometric configuration of the material in which they propagate [37]. The properties of a particular waveguide can be determined by studying the dispersive nature of the propagating guided wave modes.

A common type of guided wave is the Lamb wave, which in its simplest form can propagate along a thin plate, with displacements occurring both in the direction of wave propagation and normal to the plane of the plate [38]. In particular, the problem of modelling and generating Lamb waves in a submerged plate has been of interest for a number of years [39-41].

Lamb wave propagation in a single plate can be studied by considering its free or forced vibrations. In the first case the incident longitudinal and shear waves are set to zero, and the real or complex wave numbers corresponding to the true and leaky Lamb waves, respectively, are obtained from the solution of the resulting characteristic equations. Such calculations for the free vibrations of the plate are not expected to give good correlation with the experimental results whenever the guided mode is strongly leaky, but that does not happen to often. The second approach uses either an incident longitudinal or shear wave to determine the frequency dependent reflection and transmission coefficients of the plate material at different angles of incidence. The Lamb wave velocities C_{Lamb} for the various modes in a thin plate can be related to the angle of incidence by the following equation

$$C_{\text{Lamb}} = \frac{C_w}{\sin \theta_i} \quad (2.39)$$

where C_w is the phase velocity of the surrounding fluid. The forced solution approach is more realistic and will be used in the next section as an example to determine the dispersive characteristics of leaky Lamb waves in a single fluid loaded plate.

2.3.1 Lamb Wave Propagation through a Fluid Loaded Plate

Consider a thin plate of thickness d , with a Lamb wave propagating along the x -direction as shown in Fig.12. For the region occupied by the plate we introduce the following potential functions

$$\phi = \phi^+ \cosh \alpha z + \phi^- \sinh \alpha z \quad (2.40)$$

$$\psi = \psi^+ \cosh \beta z + \psi^- \sinh \beta z \quad (2.41)$$

where k is the Lamb wave number, and

$$\alpha = \sqrt{k^2 - k_t^2} \quad (2.42)$$

$$\beta = \sqrt{k^2 - k_l^2} \quad (2.43)$$

are the longitudinal and shear wave number components in the z -direction. According to Snell's law each of the above potential functions have the same k_x wave number component in the x -direction and is not shown. Calculation of the displacements by substituting equations (2.40) and (2.41) into (2.16) and (2.17) results in two types of Lamb waves which can propagate in the plate independent of one another. The first type of waves describes wave motion which is symmetric with respect to the plane $z = 0$. The second type of waves describe motion which is antisymmetric with respect to the plane $z = 0$. Both types of waves are shown in Figs. 12b and 12c, respectively.

Figure 12. a) Schematic diagram of Lamb wave problem for a single plate.
 b) Symmetric Lamb wave propagation in single plate.
 c) Antisymmetric Lamb wave propagation in a single plate.

The following potential functions are introduced for the surrounding fluid [39]:

$$\phi_i = \phi_0 e^{-\eta z} + \phi_r e^{\eta z}, \quad (2.44)$$

$$\phi_t = \phi_t e^{-\eta z} \quad (2.45)$$

where ϕ_0 and ϕ_t are the incident and transmitted longitudinal wave potential functions with

$$\eta = \sqrt{k^2 - k_w^2} \quad (2.46)$$

and where k_w is the wave number of the fluid. Along with the potential functions, the boundary conditions at $z = \pm d/2$ must be considered, i.e., the normal components of displacement and stress, u_z and T_{zz} , are continuous, and the tangential component of stress T_{xz} , vanishes at the fluid/solid boundary. Solving these boundary conditions, we get,

$$\mathbf{A} \begin{bmatrix} \phi_i \\ \phi_r \\ \phi_t \\ \phi^+ \\ \phi^- \\ \psi^+ \\ \psi^- \end{bmatrix} = \mathbf{0}, \quad (2.47)$$

where $\mathbf{A}$ is a 7x6 matrix and $\mathbf{0}$ is a zero vector.

Figure 13 shows numerical results for the longitudinal-to-longitudinal frequency-dependent transmission maxima for the leaky Lamb modes ($C_{\text{Lamb}} > C_{\text{t(aluminum)}}$) in a single aluminum plate having longitudinal and shear wave velocities and density of 6420 m/s, 3130 m/s, and 2700 kg/m³, respectively.

Referring to Fig. 13, the values of d and ω at which new roots appear are called "critical" thicknesses and frequencies, respectively. At critical frequencies the wave number k approaches zero, in other words the phase velocity approaches infinity, and the symmetric and antisymmetric modes represent standing longitudinal or shear waves in the plate. The relations between the critical thicknesses and longitudinal and shear wavelengths are given by [41]:

for symmetric modes:

$$2d = \lambda_l/2, 3\lambda_l/2, 5\lambda_l/2, \dots$$

$$2d = \lambda_t, 2\lambda_t, 3\lambda_t, \dots$$

for antisymmetric modes:

$$2d = \lambda_l, 2\lambda_l, 3\lambda_l, \dots$$

$$2d = \lambda_l/2, 3\lambda_l/2, 5\lambda_l/2, \dots$$

These values were determined and the results are shown in Fig. 13.

Figure 13. Calculated longitudinal-to-longitudinal transmission maxima for leaky Lamb waves in a single aluminum plate submerged in water.

2.3.2 Guided Waves in Adhesive Joints

There are two types of guided waves which are of particular interest in the study of adhesive joints. The first types are termed Lamb waves and propagate through the entire adhesive joint. The second types are termed interface waves, which propagate through the adhesive layer alone, and is evanescent to the bonded adherend materials [42]. Figure 14 shows the case for symmetric propagation of guided waves in the adhesive joint for both cases.

Lamb wave modes have been used as an ultrasonic inspection tool for the past fifteen years [43-46] to measure the attenuation and phase or group velocity of these modes through the sandwiched adhesive joint. The complex mode structure of these Lamb modes are rather difficult to evaluate and are dominated by the resonances of the adherend plates, while the adhesive layer and interface properties cause only rather weak effects.

On the other hand, the interface waves have been shown to offer superior sensitivity to both the cohesive and adhesive properties of the adhesive bond [47-49]. One difference between the interface and Lamb waves as was already pointed out, is that the interface waves are less sensitive to the elastic properties of the surrounding adherend materials and not sensitive to the thickness of the adherend plates at all. Which makes it difficult to produce and detect these wave modes in the adhesive joint.

The most common way to produce interface waves in an adhesive joint is by wedge transducers [41]. Figure 15 shows a schematic diagram of this technique.

Figure 14. Guided acoustic waves in an adhesive joint, a) Lamb type waves, and b) Interface type waves [42].

Figure 15. Schematic diagram of guided interface wave inspection by Rayleigh wave coupling [50].

Rayleigh waves are produced on one of the free surfaces of the adherend plates, which are coupled to one or more vertically polarized interface modes over the bonded area, and received on the opposite free surface of the bonded area. This technique works not only for true guided modes but also for slightly leaky ones which are attenuated by radiating into one or both of the surrounding media [50,51]. Unfortunately, the awkward geometry required for this type of guided wave technique cannot be easily adapted for laboratory and in-service inspection.

Recently, a direct excitation technique was developed [48,49] in which a longitudinal or shear wave transducer was coupled directly to the adhesive joint in such a way that the ultrasonic wave propagation direction was parallel to the adherend/adhesive interface. This technique was shown to be very effective in generating both symmetric and antisymmetric modes of Rayleigh (SV) and Love (SH) type wave polarizations in the adhesive layer. By measuring the destructive interference between the interface waves and the bulk waves generated in the adhesive layer and adherend plates, respectively, they found good agreement between measured and theoretically calculated dispersion properties of the interface waves.

A more practical approach is to produce interface waves having a higher phase velocity than the adherend materials. A double transmission immersion technique developed by Nagy et al., at The Ohio State University [54], was used to measure leaky Lamb waves in composite materials; because of its simple equipment and minimal alignment and was adapted to measuring guided waves in the adhesive joint.

CHAPTER III

MEASURING TECHNIQUE TO DETERMINE THIRD-ORDER ELASTIC CONSTANTS FOR 6061-T6 ALUMINUM AND PERSPEX

3.1 Introduction

In Chapter II the construction of the adhesive joint was described. To better understand the linear and nonlinear properties of the adherend plates and adhesive layer, a preliminary study was conducted to measure the second- and third-order elastic constants for 6061-T6 aluminum and perspex. The stress free and stress-dependent bulk wave velocities were measured for each solid material and the results are presented in this chapter.

3.2 Experimental Procedure

3.2.1 Sample Preparation

Two samples were considered in this study, 6061-T6 aluminum, and perspex. The 6061-T6 aluminum was rolled into a rod 8.87 mm x 12.45 mm x 152.40 mm, solution treated, and artificially aged. No prior history for the perspex was available. The dimensions of the perspex was 6.35 mm x 6.35 mm x 152.40 mm. The weight-volume densities for each sample were calculated by sectioning a small cube from each sample,

cleaning, accurately measuring the dimensions of each section using calipers to within $\pm 25.4 \mu\text{m}$, and weighing the samples to within $\pm 0.01 \text{ g}$. These results for the aluminum and perspex are shown in Tables 1 and 2, respectively.

Each sample was initially inspected using a standard normal incidence ultrasonic pulse/echo technique to determine, if any flaws existed in the sample, and to measure the samples attenuation. A single longitudinal transducer was used to scan the area of the sample. Any back reflected signal occurring in time between the main bang and backwall reflection were considered a flaw and the sample was discarded. The attenuation of the sample was measured by comparing the amplitudes of the multiple reflections from each of the successive backwall echoes. Highly attenuating samples were also discarded and a new ones made.

A strain gauge was attached to each sample at mid-length and parallel to the loading direction. The purpose of the strain gauge was to, measure any sample deflection caused by misalignment of the sample when mounting it in the testing fixture, and also to measure the strain of the sample as it was being loaded.

3.2.2 Measurement of Second-Order Elastic Constants

Prior to applying any external stress to the sample, ultrasonic bulk wave velocity measurements were made using the pulse/echo/overlap technique [55]. A schematic representation of the experimental system is shown in Fig. 16.

Figure 16. Schematic diagram of experimental system to measure the acoustoelastic effects in solids.

Table 1. Measured and calculated results for 6061-T6 aluminum.

	Results
Density (kg/m ³)	2650 ± 4.5
Longitudinal velocity (m/s)	6420 ± 10.8
Shear velocity parallel to load (m/s)	3197 ± 6.0
Shear velocity normal to load (m/s)	3161 ± 5.9
λ (MPa)	55053 ± 220
μ (MPa)	27085 ± 143
K_0 (MPa)	73110 ± 315

Table 2. Measured and calculated results for perspex.

	Results
Density (kg/m ³)	1148 $\pm$ 4.3
Longitudinal velocity (m/s)	2772 $\pm$ 5.2
Shear velocity parallel to load (m/s)	1396 $\pm$ 5.9
Shear velocity normal to load (m/s)	1393 $\pm$ 5.9
λ (MPa)	4347 $\pm$ 11.2
μ (MPa)	2237 $\pm$ 27.6
K_0 (MPa)	5838 $\pm$ 29.6

Ultrasonic longitudinal and shear wave contact transducers, (5 MHz, 6.35 mm in diameter), were coupled to the sample using a thin oil and a shear wave couplant manufactured by the Panametrics Corporation. Prior to coupling the shear wave transducer to the sample, the polarization of the shear transducer was determined by a technique described in [27,28]. The transducers were configured in the pulse/echo mode. A Panametrics 5052PR pulser/receiver unit was used to excite the transducers and receive, filter, and amplifier the returned echoes from the main bang and backwall echoes of the sample.

The time domain signal was then displayed and time averaged on a LeCroy 9400 digital oscilloscope. Since the first echo or "main bang" on the oscilloscope is a combination of the initial pulse of the system and the coupling effects of the transducer/sample interface, it was neglected and the time delay between the second and successive reflections were used instead. The time delay between each multiple reflected signal was measured ten times by overlapping the previous signal with the signal to be measured with an accuracy of ± 0.1 ns and averaged. The bulk wave velocities were calculated by dividing twice the sample thickness by the averaged time delay.

The calculated material densities and velocities were then substituted into equations (2.11) and (2.12), in order to determine the second-order elastic constants and are listed in Tables 1 and 2 for each sample.

3.2.3 Measurement of Third-Order Elastic Constants

The experimental set up is the same as shown in Fig. 16. The same transducers and couplant were used as described in the previous section. The transducers were mounted in a fixture which was built to minimize any tilting of the transducer during the loading of the sample.

Two load frames were used, one to produce a tensile load ranging from 0-17.8 kN with fixed grips, and the other was a compression press producing loads ranging from 0-44.5 kN. The samples were mounted and aligned in the grips of the tensile machine and in the compression press in such a way that no external moments were present at zero stress.

Since the ultrasonic bulk wave velocities must be measured accurately as a function of applied stress the following zero crossing technique was introduced which utilizes the arithmetic capabilities of the LeCroy 9400 oscilloscope. The first backwall reflection of the sample was expanded on the oscilloscope and an interleaved option used to increase the time resolution to ± 0.02 ns. A reference signal of 0.7 volts dc, was produced and divided by the back reflection signal, using the arithmetic ratio mode. The resulting wave form was asymptotic where the rf time domain signal of the sample crossed the zero, i.e., division by zero from the previous step. When an external stress is applied to the sample a slight change in the velocity will occur and as a consequence the position in time between the zero stress and loaded position of the asymptote will be displaced an amount proportional to the applied stress. This time displacement was used to calculate the stress-dependent velocities for all three cases.

Figure 17. Signal processing technique to measure stress-dependent velocity for solid samples.

The aluminum sample was loaded in compression and tension ranging from -19.33 to 11.04 kN, in increments of 2 kN. The perspex was loaded in compression and tension ranging from -0.81 to 0.40 kN, in increments of 0.1 kN. Each sample was loaded and unloaded ten times and its time delay measured and averaged in order to reduce the scatter in the data. These values were below the 0.2% yield strength of the materials and within the loading capabilities of the load frames for the tensile and compression machines. During the unloading process, each sample was given time to relax in order to reduce any viscoelastic effects in the material. This relaxation time was measured by observing when the time difference between the stressed and reference position of the asymptote returned to zero.

3.3 Results and Discussion

3.3.1 Acoustoelastic Effect for Aluminum and Perspex

Figures 18-21 show the results of the longitudinal and shear wave stress-dependent velocities for 6061-T6 aluminum and perspex, as a function of applied tensile and compressive stresses. Each set of results represents the measured data and "best fit" linear regression line which are listed below for each case.

Case I: longitudinal waves,

$$\text{6061-T6:} \quad V_1 = 0.116\sigma + 6420, \quad (3.1)$$

$$\text{perspex:} \quad V_1 = 0.035\sigma + 2772. \quad (3.2)$$

Figure 18. Longitudinal wave velocity as a function of stress for 6061-T6 aluminum.

Figure 19. Shear wave velocity as a function of stress for 6061-T6 aluminum at two polarization angles.

Figure 20. Longitudinal wave velocity as a function of stress for perspex.

Figure 21. Shear wave velocity as a function of stress for perspex at two polarization angles.

Table 3. Calculated third-order elastic constants for 6061-T6 aluminum.

	Calculated Results
l ($\times 10^{10}$ N/m ²)	- 29.8 ± 0.3 - $20.2 < l < - 38.8^a$
m ($\times 10^{10}$ N/m ²)	- 26.4 ± 1.1 - $23.7 < m < - 40.1^a$
n ($\times 10^{10}$ N/m ²)	- 44.7 ± 0.2 - $27.6 < n < - 43.6^a$
A ($\times 10^4$ MPa)	8.65 7.90 ^b

^a Range of values calculated from [22-24].^b From [55].

Table 4. Calculated third-order elastic constants for perspex.

	Calculated Results
$l \text{ (} \times 10^{10} \text{ N/m}^2\text{)}$	$- 2.5 \pm 0.6$
$m \text{ (} \times 10^{10} \text{ N/m}^2\text{)}$	$- 1.1 \pm 0.4$
$n \text{ (} \times 10^{10} \text{ N/m}^2\text{)}$	$- 0.8 \pm 0.4$

Case II: shear wave polarized parallel to load,

$$6061\text{-T6: } V_{t1} = -0.111\sigma + 3197, \quad (3.3)$$

$$\text{perspex: } V_{t1} = -0.466\sigma + 1396. \quad (3.4)$$

Case III: shear wave polarized normal to load,

$$6061\text{-T6: } V_{t2} = 0.059\sigma + 3161, \quad (3.5)$$

$$\text{perspex: } V_{t2} = 0.0079\sigma + 1393. \quad (3.6)$$

It can be seen that in all cases the velocities are linearly proportional to the applied stresses.

The third-order elastic constants were calculated by choosing an average value from the stress and velocity from Figs.18-21 and substituted along with the material densities and second-order elastic constants determined earlier, into (2.33), (2.36), and (2.38) for each material. These calculated results are listed in Tables 3 and 4, for 6061-T6 aluminum and perspex, respectively.

A range of values for the third-order elastic constants was calculated by taking the high and low values from [22-24] and are listed in Table 3. Smith et al., found that the reason for such a large scatter of values, especially for aluminum alloys, was that the third-order elastic constants were sensitive to alloy types and previous fabrication history. The initial test results for the calculated third-order elastic constants fall within the range of values. However, this is not the case for the final test results which will be discussed in the next section.

The trend of the stress-dependent bulk wave velocities as a function of applied stress corresponds well to previously published results [24]. The direction of the slope in each case above can be explained in the following manner in which an increase in the effective modulus occurs when the atoms of the material are forced closer together by compressive stresses, a more complex change occurs when shear stresses act on adjacent rows of atoms this changing the effective shear modulus of the material [11]. Consider the solid prestressed in uniaxial compression as shown in Fig. 9. For Case I: in which the longitudinal wave is polarized normal to the direction of the applied stress, the reduction in the stress-dependent velocity is caused by a slight reduction in the effective modulus and an increase in the material density. For Case II: in which the shear wave is polarized parallel to the direction of the applied stress, the reduction in velocity is caused by a reduction in the shear modulus caused by the shear strain under the action of the applied stress. Finally, for Case III: in which the shear wave is polarized normal to direction of the applied stress, the smallest changes in the stress-dependent velocity occur due to a slight increase in the shear modulus for this particular polarization direction.

In conjunction with the slope of the stress-dependent velocities, the acoustoelastic constant A can be calculated and compared to previous results for the case of the longitudinal wave. The acoustoelastic constant was determined by dividing the change in stress by the change in the normalized longitudinal velocity [56], calculated, and compared to previous results for 6061-T6 listed in Table 3. The agreement between these results is good (within 10 %). This is a good indication because the sensitivity of the longitudinal wave velocity to the stress dependence is the lowest for this particular case compared to the shear waves in the other two cases.

3.3.2 Effects of Texture and Loading

It was assumed that the materials in this study are isotropic. This implies that for a polycrystalline bulk sample which consists of randomly oriented crystalline grains, that the effective elastic constants describing the material will result from averages of the anisotropic single crystal values. One then would assume that in the case where the shear wave is polarized in two directions the shear wave velocity should be invariant of the polarization at zero stress and both shear wave velocities will have the same velocity. If we neglect some slight anisotropy in the shear wave velocities, probably due to some texture caused by a previous extrusion process, this seems to be true for perspex.

However, in the case of the aluminum sample, two different shear wave velocities were measured at zero stress. Two possible explanations for this difference will be presented here. First, the sample was cut from a large plate which was rolled causing a slight alignment of the crystalline structure or texture in the sample. Second, as a consequence of the rolling some residual stress may have been induced. From Fig. 19, the amount of residual stress in the sample can be estimated from the position where the two shear velocity slopes intersect. From the location of this point in our initial test results we estimate approximately 210 MPa (tension) which we attribute to the effects of rolling, and where the value is below the yield strength for 6061-T6 aluminum ($S_y = 270$ MPa).

CHAPTER IV

APPLICATION OF ACOUSTICAL BIREFRINGENCE EFFECTS TO MEASURE HIGHER-ORDER NONLINEARITIES

4.1 Introduction

In the previous chapter the stress-dependent shear wave velocities were measured in the two principal polarization directions of stress for a uniaxially loaded 6061-T6 aluminum sample. As a consequence of the applied stress two different shear velocities were measured and found to be dependent on the direction of polarization with respect to the direction of applied stress. This first-order effect is very strong for shear waves polarized parallel to the principal axis of stress. In order to separate these much stronger effects from the weak effects produced by higher-order nonlinearities and the nonlinear effects of the adhesive layer in the case of the adhesive joint, the first-order effects of the aluminum must be reduced or even eliminated. A technique was proposed by Nagy [60] and experimentally verified in this chapter in which by selecting the proper polarization angle the effects of the first-order terms can be eliminated. Theoretical calculations were made to determine the optimal polarization angle, the stress-dependent shear wave velocities were measured to verify these calculations, and are presented in the following sections.

4.2 Theory

The acoustical birefringence effect of shear ultrasonic waves has been studied by a number of authors to measure the effects of applied stress on a solid material [61-64]. The higher-order acoustoelastic relation for two different polarizations of a shear wave can be written in the following form:

$$V_1 = V_t + \Delta V_1, \quad (4.1)$$

where

$$\Delta V_1 = V_{10} + V_{11}\sigma + V_{12}\sigma^2 + \dots,$$

and

$$V_2 = V_t + \Delta V_2, \quad (4.2)$$

where

$$\Delta V_2 = V_{20} + V_{21}\sigma + V_{22}\sigma^2 + \dots$$

In equations (4.1) and (4.2), V_1 and V_2 are the shear wave velocity coefficients, and V_t is the shear wave velocity.

Consider an ultrasonic shear wave polarized at an arbitrary angle θ , with respect to the axis of the principal stress, as shown in Fig. 22. The received signal S from the two shear wave components can be written as

$$S = \cos^2\theta \exp\left[i \omega \frac{2d}{V_1}\right] + \sin^2\theta \exp\left[i \omega \frac{2d}{V_2}\right], \quad (4.3)$$

where $2d$ represents the two way travel of the shear wave through the sample of thickness d .

Figure 22. Schematic diagram of shear wave polarized at an arbitrary angle.

In order to measure the weak higher-order nonlinearities of a material the optimal shear wave polarization angle where the stronger first-order effects can be reduced or eliminated must be determined. At this particular angle the velocity coefficient can be written as

$$V_{\text{eff}} = V_t + \Delta V_{\text{eff}}, \quad (4.4)$$

and the received ultrasonic signal as

$$S_{\text{eff}} \cong \exp\left[i\omega \frac{2d}{V_{\text{eff}}}\right]. \quad (4.5)$$

In general, the right hand side of (4.5) can be written as

$$\exp\left[i\omega \frac{2d}{V_i}\right] \cong \exp\left[i\omega \frac{2d}{V_t}\right] \left[1 + i\omega \frac{2d}{V_t^2} \Delta V_i\right] \quad (4.6)$$

where $i = 1, 2$, or eff .

Substituting (4.3) into (4.6) the received transducer response can be written as

$$S = \exp\left[i\omega \frac{2d}{V_t}\right] \left\{ \cos^2\theta + \sin^2\theta + i\omega (\Delta V_1 \cos^2\theta + \Delta V_2 \sin^2\theta) \frac{2d}{V_t^2} \right\}. \quad (4.7)$$

Similarly, by substituting (4.4) into (4.6) an expression for the received transducer at the optimal polarization can be written as

$$S_{\text{eff}} = \exp\left[i \omega \frac{2d}{V_t}\right] \left(1 + i\omega (\Delta V_{\text{eff}}) \frac{2d}{V_t^2}\right) \quad (4.8)$$

At the optimal polarization angle (4.7) and (4.8) are identical and

$$\Delta V_{\text{eff}} = \Delta V_1 \cos^2\theta + \Delta V_2 \sin^2\theta. \quad (4.9)$$

Substituting the values for ΔV_1 and ΔV_2 from (4.1) and (4.2) into (4.9) we get

$$\begin{aligned} \Delta V_{\text{eff}} = & V_{10} \cos^2\theta + V_{20} \sin^2\theta + \sigma(V_{11} \cos^2\theta + V_{21} \sin^2\theta) \\ & + \sigma^2(V_{12} \cos^2\theta + V_{22} \sin^2\theta). \end{aligned} \quad (4.10)$$

In order to eliminate the first-order effects from (4.10) and find an expression to determine the optimal polarization angle we set the following σ terms to zero as follows

$$0 = V_{11} \cos^2\theta + V_{21} \sin^2\theta \quad (4.11)$$

where V_{11} and V_{21} are the slopes determined from the shear waves polarized parallel and perpendicular to the principal stress, respectively. Solving (4.11) the needed relationship to determine the angle of polarization is as follows

$$\tan^2 \theta = -\frac{V_{11}}{V_{21}}. \quad (4.12)$$

At this particular polarization the stress-dependent velocity can be written as

$$V_{t\theta} = V_{t0} + V_{t\theta} \sigma^2 \quad (4.13)$$

where

$$V_{0\theta} = (V_{01} - V_t) \cos^2\theta + (V_{02} - V_t) \sin^2\theta. \quad (4.14)$$

Measuring the stress-dependent shear wave velocities at this polarization angle calculated from (4.12) will result in the elimination of the first-order effects in (4.1) and (4.2). It should be noted that the first-order effects still exists at this polarization angle however the sign of one component is opposite the other and in essence their effects cancel each other out .

4.3 Results and Discussion

The polarization angle for 6061-T6 aluminum was calculated to be 28 degrees by substituting the values of the slopes from the stress-dependent shear wave velocities from (3.3) and (3.5) into (4.12).

The stress-dependent shear wave velocity as a function of applied stress at this angle, for the 6061-T6 aluminum sample is shown in Fig. 23. It is evident from the results that the birefringence effects of the shear waves have been eliminated as compared to the results shown in Fig. 19. A polynomial "best fit" routine was used to determine the relationship between the stress-dependent shear wave velocities and the applied stresses resulting in the following equation

$$V_t = 3174 - (32.6 \times 10^{-6}) \sigma^2 \quad (4.15)$$

Figure 23. Shear velocity as a function of stress for a polarization angle of 28 degrees.

where $V_{0\theta} = 3174$ m/s which is the shear velocity at zero stress for a 28 degree polarization angle. The first-order terms have also been eliminated as is evident from (4.15). The $V_{0\theta}$ term on the right hand side of (4.15) is a combination of the second-order elastic constants and the coefficient of the σ^2 term is a combination of the third- and fourth-order elastic constants, and is related to higher-order nonlinearities of the material. The magnitude of this second-order coefficient is 0.1% of the first-order terms in (3.3) and (3.5).

CHAPTER V

MEASUREMENT OF THE ACOUSTOELASTIC EFFECTS FOR AN ADHESIVE LAYER

5.1 Introduction

The measurement of the acoustoelastic effects of an adhesive layer is not an easy task for a number of reasons. First, when an external stress is applied to the adhesive layer sandwiched between two adherend plates its stress distribution is no longer uniform or may not even be known. Second, normal ultrasonic velocity measurements will not work because of the thin layer.

In this chapter theoretical calculations are made to determine the displacement gradients for a prestressed adhesive layer. The acoustoelastic equations will be derived for a single element having both normal and shear stress components. An experimental technique which uses the frequency-dependent reflection spectrum obtained from the adhesive layer will be further developed in order to measure the acoustoelastic effect of the layer.

5.2 Theory

It was shown by Goland and Reissner [29] that when a single lap adhesive joint is pulled in tension the stress distribution along the adhesive layer is no longer even and is composed of a normal and shear stress components. If the mid-section of the adhesive layer is chosen and a sufficient overlap length and layer thickness is used, the stress distribution is fairly even.

The displacement gradients in this area can be determined by examining how the stresses effect a small element removed from the adhesive layer at this point. When the adhesive joint is loaded in uniaxial tension the small element is compressed and rotated by the normal σ_0 and shear τ_0 stress components.

The contributions of the stress components can be written as a function of the strain-displacement equations in the following manner [57],

$$\begin{aligned}
 \epsilon_{xx} &= u_a \\
 \epsilon_{yy} &= v_b \\
 \epsilon_{zz} &= w_c \\
 \gamma_{xy} &= \gamma_{yz} = 0 \\
 \gamma_{zx} &= u_c + w_a.
 \end{aligned}
 \tag{5.1}$$

The normal displacement gradients have the same form as (2.31) and (2.32) determined for the case of a solid prestressed in uniaxial compression. No displacements occur in the y-direction due to the shear strain. On the other hand, the shear strain in the zx-direction can be written in the following form,

$$\gamma_{zx} = \sin\left(\frac{\tau_0}{\mu}\right) + \left[1 - \cos\left(\frac{\tau_0}{\mu}\right)\right] \quad (5.2)$$

where the two terms on the right-hand side of (5.2) relate the lateral and thickness changes of the layer due to the shear stress component τ_0 . For very small values of shear strain, i.e., the following approximation can be made

$$\sin\left(\frac{\tau_0}{\mu}\right) \cong \frac{\tau_0}{\mu}, \text{ and } \cos\left(\frac{\tau_0}{\mu}\right) \cong 1, \quad (5.3)$$

which simplifies (5.2) to the very familiar shear strain equation,

$$\gamma_{zx} = \frac{\tau_0}{\mu} \quad (5.4)$$

for the two dimensional state of pure shear.

By superimposing the results obtained from (2.31) and (2.32), and with (5.2) the displacement gradients for the prestressed adhesive layer can be written directly as

$$\begin{aligned} u &= u_a + u_c = \pm \frac{\lambda}{6\mu K_0} \sigma_0 + \sin\left(\frac{\tau_0}{\mu}\right) \\ v &= v_b = \pm \frac{\lambda}{6\mu K_0} \sigma_0 \\ w &= w_a + w_c = \left[1 - \cos\left(\frac{\tau_0}{\mu}\right)\right] \pm \frac{(-\lambda - \mu)}{-3\mu K_0} \sigma_0, \end{aligned} \quad (5.5)$$

where the (+) and (-) signs are used when an externally applied tensile or compressive stresses are applied to the adhesive joint, respectively. K_0 is the bulk modulus defined in Chapter II, for the adhesive material.

Examining (5.5), the u-displacement components are composed of the Poisson effect and the lateral rotation due to the normal and shear stresses, respectively. The only displacement occurring in the v-component is due to the Poisson effect. Finally, the w-displacement component is composed of the normal stress component and a small thickness change due to the shear stress component. These displacement gradients can be substituted directly into the equation of motion found in Appendix A.

5.3 Acoustoelastic Equations for the Adhesive Layer

The acoustoelastic equations relate the stress-dependent ultrasonic bulk wave velocities to the third-order elastic constants of the adhesive layer. The next step is to superimpose an elastic wave propagating along the c-direction (z axis) in the prestressed layer which has the following form

$$u_i = A e^{i(\omega t - \zeta k c)} \quad (5.6)$$

where

$$\zeta = \frac{\Delta \eta}{\eta} = 1 \pm \frac{(-\lambda - \mu)}{-3\mu K_0} \sigma_0 - \cos\left(\frac{\tau_0}{\mu}\right), \quad (5.7)$$

and k is the wave number for the elastic wave. The relative change in path length ζ , is due to the normal and shear stress components acting on the thickness of the adhesive element.

In order to calculate the third-order elastic constants for the adhesive layer three possible cases were considered. Case I, is a longitudinal wave which propagates normal and is polarized parallel to the externally applied load. Case II, is a shear wave which propagates normal and is polarized parallel to the externally applied load. Case III, is a shear wave which propagates and is polarized normal to the externally applied load. The coordinate system and sample geometry for each case is shown in Fig. 24.

5.3.1 Case I: Longitudinal Waves

For a longitudinal wave propagating in the w -direction, we can write the displacement fields by superimposing (5.4) and (5.5) in order to obtain the following expressions

$$\begin{aligned} u &= \pm \frac{\lambda}{6\mu K_0} \sigma_0 + \sin\left(\frac{\tau_0}{\mu}\right) \\ v &= \pm \frac{\lambda}{6\mu K_0} \sigma_0 \\ w &= \left[1 - \cos\left(\frac{\tau_0}{\mu}\right)\right] \pm \frac{(-\lambda - \mu)}{-3\mu K_0} \sigma_0 + A e^{i(\omega t - \zeta k c)} \end{aligned} \quad (5.8)$$

The nonvanishing displacement gradients are u_a , u_c , v_b , w_a , w_c and w_{cc} . Substituting these displacement gradients into the third-component of the equation of motion (A.14), the following expression was obtained

$$\rho_0 V_1^2 = \lambda + 2\mu \pm \frac{\sigma_0}{3 K_0} \left[-\frac{\lambda}{\mu} (2\lambda + 9\mu + 4m) - 2(3\mu + 1 + 2m) \right], \quad (5.9)$$

Figure 24. Schematic diagram of acoustoelastic measurements of adhesive layer.

which relates the measured stress-dependent longitudinal wave velocity V_l , and the third-order elastic constants l and m for the adhesive layer.

5.3.2 Case II: Shear Wave Polarized Parallel to External Load

For this particular case, the displacement fields for a shear wave polarized in the u -direction can be written as

$$\begin{aligned} u &= \pm \frac{\lambda}{6\mu K_0} \sigma_0 + \sin\left(\frac{\tau_0}{\mu}\right) + A e^{i(\omega t - \zeta k_c)} \\ v &= \pm \frac{\lambda}{6\mu K_0} \sigma_0 \\ w &= \left[1 - \cos\left(\frac{\tau_0}{\mu}\right)\right] \pm \frac{(-\lambda - \mu)}{-3\mu K_0} \sigma_0 \end{aligned} \quad (5.10)$$

where k_t is the shear wave number in the adhesive layer. The nonvanishing displacement gradients are u_a , u_c , u_{cc} , v_b , w_a , and w_c . Substituting these displacements gradients into the first component of the equation of motion (A.10), the following expression was obtained

$$\rho_0 V_{t1}^2 = \mu \pm \frac{\sigma_0}{3 K_0} \left[-2(\lambda + \mu) - m - \frac{\lambda}{4\mu} n \right], \quad (5.11)$$

where V_{t1} is the stress-dependent shear wave velocity as a function of the third-order elastic constants m and n for this particular polarization.

5.3.3 Case III: Shear Wave Polarized Normal to External Load

For this particular case, the displacement fields for a shear wave polarized in the v -direction can be written as

$$\begin{aligned} u &= \pm \frac{\lambda}{6\mu K_0} \sigma_0 + \sin\left(\frac{\tau_0}{\mu}\right) \\ v &= \pm \frac{\lambda}{6\mu K_0} \sigma_0 + A e^{i(\omega t - \zeta k c)} \\ w &= \left[1 - \cos\left(\frac{\tau_0}{\mu}\right)\right] \pm \frac{(-\lambda - \mu)}{-3\mu K_0} \sigma_0 \end{aligned} \quad (5.12)$$

The nonvanishing displacement gradients are u_a , u_c , v_b , v_c , v_{cc} , w_a , and w_c . Substituting these displacements into the second-component of the equation of motion (A.12), we obtain the following expression

$$\rho_0 V_{t2}^2 = \mu \pm \frac{\sigma_0}{3 K_0} \left[-2(\lambda + \mu) - m - \frac{\lambda}{4\mu} n \right], \quad (5.13)$$

where V_{t2} is the stress-dependent shear wave velocity. For this particular sample geometry there are three unknowns, i.e., l , m , and n , and only two equations.

5.4 Experimental Procedure

5.4.1 Sample Preparation

The adhesive bonded samples used in this portion of the study were furnished by the Northrop Corporation. These samples were constructed by glueing together two

adherend panels made from 2024-T3 aluminum, measuring 22.9 cm x 24.1 cm x 0.95 cm, with American Cyanamide FM300K adhesive and BR-127 primer. Prior to glueing the panels together the surfaces of each adherend plate were prepared according to ASTM D3933-80 [58]. The assembled panels were cured in an autoclave for 1 hr at 350 degrees and 0.17 MPa atmosphere to provide a superior bond quality. Destructive tensile shear tests were performed on two samples in accordance with ASTM D1002-79 [2], and are shown in Fig. 2. The density of the FM300K adhesive material in its unstrained state was provided by the manufacturer and is listed in Table 5.

The panels were then sectioned and machined into single lap shear specimens according to D1002 at The Ohio State University. A 2.54 cm overlap length was used to provide a more even stress distribution at the center of the joint (see Figs. 10 and 11). An average adhesive layer thickness for each sample was determined by optically measuring both sides and at four points along each side of the adhesive joint with a resolution of ± 2 μm . Each sample was inspected using normal incidence pulse/echo ultrasonic techniques to detect any gross defects in the adhesive layer. Samples containing defects were discarded.

5.4.2 Determination of Second-Order Elastic Constants

The second-order elastic constants for the FM300K adhesive material were calculated from the measured bulk wave velocities of the adhesive layer with no externally applied stress. The longitudinal and shear waves were produced by either a 15 MHz, or 5 MHz, 0.64 cm diameter broadband contact transducers, respectively.

Each transducer was coupled to the adhesive joint, as shown in Fig. 5. The time domain signal from the adhesive layer was gated and displayed on a LeCroy 9400 oscilloscope (see Fig. 6). The frequency spectrum of the reflected signal was obtained by applying a Fast Fourier Transform (FFT) to the signal and then was stored. A reference spectrum of each transducer was obtained from the back reflection of a single 9.5 mm, 2024-T3 aluminum panel and stored. The adhesive layer spectrum was deconvolved by the reference spectrum to eliminate the effects of the transducer over the working frequency range. The longitudinal and shear wave velocities of the adhesive layer were calculated using (2.22). The values for the second-order elastic constants were then calculated by substituting the previously calculated velocities into (2.11) and (2.12). These calculated values are listed in Table 5 and will be used in the next chapter for numerical calculation of guided waves in the adhesive layer.

5.3.3 Measuring the Third-Order Elastic Constants

Figure 25 shows a schematic diagram of the experimental system used to measure the stress-dependent frequency response of the adhesive layer. Two identical transducers, either longitudinal or shear wave contact types described in the previous section were used. The active transducer was used to transmit and receive the ultrasonic signals from the statically prestressed adhesive joint. The reference transducer was used to transmit and receive ultrasonic signals from the stress-free sample. The purpose of using two transducers was to reduce the drift of the analog spectrum analyzer (manufacturers specifications for drift are 20 kHz/10 mins after a 2 hr warm up following initial start up), and to compensate for temperature effects, analogous to using an active and dummy strain gauge for stress measurements.

Table 5. Measured results for FM300K adhesive material from Samples 1E and 8.

	Measured Results
Density (kg/m ³)	1180 ^d
Longitudinal velocity (m/s)	2250 ± 2.1
Shear velocity parallel to external load (m/s)	980 ± 1.3
Shear velocity normal to external load (m/s)	980 ± 1.3
λ (MPa)	3707 ± 57.9
μ (MPa)	1133 ± 29.3
K_0 (MPa)	4462 ± 77.4
Thickness (μm)	296 ± 2.0

^d Values supplied by the America Cyanamide Company.

Prior to making any measurements a reference sample having the same adhesive layer properties and frequency response as the sample to be tested was found. These characteristics were ultrasonically measured using the system in Fig. 25, e.g., Fig. 26 shows the frequency response for shear measurements of a 296 μm thick adhesive layer at a monitoring frequency of 3.41 MHz. Usually, the reference sample was obtained from a section cut from either adjacent sides of the same panel from which the active sample was obtained.

Initially, the active transducer was coupled to the adhesive sample and clamped by a fixture used to eliminate any rotation of the transducer during the loading process. Two types of couplant were used, for longitudinal transducers a thick grease, and for the shear transducers a special shear wave couplant manufactured by the Panametrics Company.

Strain gauges were attached to the active sample along the load line at position 0 shown in Fig. 1. The strain gauges were used to indicate if any external moments were placed on the sample during alignment of the sample in the tensile and compressive loading frames. The reference transducer was coupled to the unstrained sample in the same manner as above.

Referring to Figs. 25 and the signal outputs shown in Fig. 27, the primary signal generator on the left-hand side produces a 3 kHz, 2 volts peak-to-peak square wave signal (see Fig. 27b) needed to synchronize and trigger the various components in the system. Since two different pulser/receiver units were used, a Panametrics 5052UAX50 for the reference transducer, and a Panametrics 5052R with stepless gate for the active transducer, each requiring a different trigger voltage, it was necessary to have a second signal generator

Figure 25. Schematic diagram of equipment used to measure acoustoelastic effects in adhesive layer.

Figure 26. Frequency response of active and reference adhesive joint samples.
Monitoring frequency is 3.41 MHz.

to trigger the active transducer. The second signal generator and active transducer were triggered on the upward leg of the 3 kHz square wave trigger signal produced by the primary signal generator. The reference transducer was then triggered on the downward leg of the same trigger signal. The time difference between the signal bursts from each transducer was long enough so that no interference from successive signal bursts occurred. These signals were displayed on a HP4337 analog oscilloscope which was used as an aid in adjusting the proper trigger levels of the system (see Figs. 27c and 27d).

The reflection from the adhesive layer of both samples, which occurs shortly after the first pulse of each burst, was time gated using a stepless gate of $\approx 2 \mu\text{s}$ in duration, mixed and sent to the HP8557 analog spectrum analyzer. A sample spectrum is shown in Fig. 26, notice that the time sweep selected on the spectrum analyzer provides enough resolution to see how the active and reference spectrum are interweaved. A particular frequency was selected along the slope of one of the minima from the spectrum of the adhesive layer. This point was centered on the spectrum analyzer and the frequency span adjusted to zero. At this setting the frequency is not swept and the signal appears at the same level across the entire display. As each transducer was triggered the amplitude of the center frequency signal from the spectrum analyzer was fed into a Model SR-510 lock-in amplifier (see Figs. 27e and 27f).

When a static stress was applied to the active sample a slight shift in the frequency minimum will occur, either to the right or left which corresponds to a change in the bulk wave velocity, as a consequence of this shift the amplitude of the active signal will increase or decrease, respectively. On the other hand, the reference amplitude remains constant throughout the loading process. The lock-in amplifier, was used in dynamic measurements

Figure 27. Signal outputs from the signal generator, transducers, spectrum analyzer, and lock-in amplifier.

by Nagy [36] and incorporated in this study to measure the difference between two successive signals, e.g., in our case the amplitudes of the frequency response at 3.41 MHz for the active and reference transducers with a carrier frequency of 3 kHz. The difference between the two signals was displayed on the LeCroy 9400 digital oscilloscope as a function of time. Initially, when both the active and reference samples are at rest the resultant amplitude difference is zero. A calibration constant was used to convert the amplitude difference expressed in millivolts on the LeCroy 9400 to frequency shift in MHz (the resolution of the frequency shift for this system was ± 10 kHz).

The stress-dependent longitudinal and shear wave velocities were measured for two samples. Each sample was loaded in both tension and compression ranging from 0 to 4.45 kN at 0.89 kN increments.

5.5 Results and Discussion

The stress-dependent bulk wave velocities as a function of normal stress for samples 1E and 8 are shown in Figs. 28-31. The bulk wave velocities were corrected using (5.7) for the thickness changes as a consequence of applied stress to the adhesive layer. From (5.9), (5.11), and (5.13) it can be seen that the normal stress is the only component of stress which contributes to the stress-dependent bulk wave velocities. The average normal stress was calculated using the corrected Goland and Reissner model over the area of the adhesive joint covered by the transducer and this value was plotted in Figs. 28-31, instead of the applied average shear stress.

A "best fit" regression line (solid line) was fit for the entire range of data points, shown in Figs. 28-31, it appears that the results have a different slope as shown by the dashed lines with the arrows indicating the external loading direction. We attribute this difference to some buckling in the sample when it was loaded in compression, i.e., a higher normal stress component (tension) occurred in the sample than was predicted by our model. The "best fit" regression line results for samples 1E and 8 are listed below:

Case I: longitudinal waves,

$$1E: \quad V_1 = -0.80\sigma + 2250, \quad (5.14)$$

$$8: \quad V_1 = -0.81\sigma + 2250. \quad (5.15)$$

Case II: shear wave polarized parallel to external load,

$$1E: \quad V_{t1} = -0.28\sigma + 980, \quad (5.16)$$

$$8: \quad V_{t1} = -0.27\sigma + 980. \quad (5.17)$$

Case III: shear wave polarized normal to external load,

$$1E: \quad V_{t2} = -0.26\sigma + 980, \quad (5.18)$$

$$8: \quad V_{t2} = -0.28\sigma + 980. \quad (5.19)$$

The acoustoelastic equations for the case of the shear waves are identical and that the major contribution to the stress-dependent bulk wave velocities is the normal stress component. Judging from the measured stress-dependent shear wave results in Figs. 31 and 33 and the slopes in (5.16) through (5.19) our assumptions is justifiable. This is

Figure 28. Longitudinal wave velocity as a function of the normal stress for Sample 1E.

Figure 29. Shear wave velocity as a function of normal stress for Sample 1E.

Figure 30. Longitudinal wave velocity as a function of normal stress for Sample 8.

Figure 31. Shear wave velocity as a function of normal stress for Sample 8.

because no matter what direction the shear wave is polarized it will always be polarized perpendicular to the normal stress component.

Two equations relating the third-order elastic constants were determined by substituting the appropriate stress and bulk wave velocities from either Figs. 28 and 29 or 30 and 31 into Eqs. (5.9) and (5.11).

For example, using Figs. 28 and 29, and a normal stress component of 5 MPa (compression), the respective longitudinal and shear wave velocities are 2254 m/s and 981 m/s, respectively. Substituting these and the second-order elastic constants from Table 5 into (5.9) and (5.11) the following two equations were obtained,

for longitudinal waves,

$$-6.44 \times 10^{10} = 15.09 m + 2 l \quad (5.20)$$

and for shear waves,

$$-0.97 \times 10^{10} = m + 0.82 n. \quad (5.21)$$

With our particular choice of sample geometry we were unable to calculate all three third-order elastic constants for the adhesive layer.

CHAPTER VI

GUIDED WAVES IN AN ADHESIVE JOINT

6.1 Theory

There are two types of guided waves which propagate through an adhesive joint, i.e., Lamb type modes and interface waves. In particular, theoretical and experimental results will be presented for the frequency-dependent transmission properties of these wave modes above the shear velocity of the adherend material, where the waves are "leaky," and more practical to measure. For both types of guided waves we assume ideal boundary conditions at the interfaces, although other boundary conditions can be studied by introducing the proper boundary conditions.

6.1.1 Lamb Waves

Lamb waves propagate along the entire adhesive joint, i.e., in the adherend-adhesive-adherend sandwiched structure as a whole. The geometry of the layered structure is shown in Fig. 32. In this particular problem we are interested in determining the longitudinal-to-longitudinal transmission coefficient similar to the single plate problem presented in Chapter II.

Figure 32. Schematic diagram of Lamb wave problem for the adhesive joint.

The potential functions which correspond to the incident, reflected, and transmitted longitudinal waves in the fluid have the same form as (2.44) and (2.45), respectively. The Lamb waves which propagate along the three-layered solid are represented by a combination of longitudinal and shear wave potential functions each having the form of (2.40) and (2.41), respectively, and

$$\Gamma_j = [\phi_j^+, \phi_j^-, \psi_j^+, \psi_j^-] \quad (6.1)$$

where $j=1, 2, 3$ corresponding to the adherend-adhesive-adherend layers, respectively. Γ_j consists of 12 elements, four potential functions per each layer.

The boundary conditions at the fluid/adherend interfaces requires that both normal displacement and stress components be continuous and the tangential component of stress vanish at this interface. The boundary conditions at the adherend/adhesive interfaces require that both the normal and tangential components of displacement and stress be continuous. For the Lamb waves in the adhesive joint submerged in a fluid we must solve for a total of 14 boundary conditions, in which we get

$$\mathbf{B} \begin{bmatrix} \phi_i \\ \phi_r \\ \phi_t \\ \Gamma_1 \\ \Gamma_2 \\ \Gamma_3 \end{bmatrix} = 0, \quad (6.2)$$

where $\mathbf{B}$ is a 15x14 matrix. Solution of (6.2), for the frequency-dependent transmission coefficient will be described in Section 6.1.3.

6.1.2 Interface Waves

Interface waves propagate through the adhesive layer only. The geometry of the interface wave model is shown in Fig. 33. The adhesive layer is sandwiched between two semi-infinite half-spaces. The appearance of this problem is similar to the single plate submerged in a fluid, shown in Fig. 10, and there is only a superficial difference between these two cases [59]. The single plate submerged in a fluid vibrates pretty much like it would do between free boundaries, while the adhesive layer bound between the adherend half-spaces vibrates more like a rigidly held plate.

In addition to the incident, reflected, and transmitted longitudinal waves which propagate in the surrounding medium, we must also include the same shear wave components for the solid adherend half-spaces. For the interface wave problem (6.1) only consists of four elements since the interface wave travels along a single layer. Applying the eight boundary conditions for the adherend/adhesive interfaces, we get

$$C \begin{bmatrix} \phi_i \\ \phi_r \\ \phi_t \\ \psi_i \\ \psi_r \\ \psi_t \\ \Gamma_1 \end{bmatrix} = 0 \quad (6.3)$$

where C is a 10×8 matrix. In addition to the longitudinal-to-longitudinal transmission coefficient the shear-to-shear transmission coefficient must also be considered. To be more

Figure 33. Schematic diagram of interface wave problem for the adhesive joint.

specific, for angles of incidence between $\theta_i = 5$ to 13 degrees the longitudinal-to-longitudinal transmission coefficient will be calculated, and for θ_i between 15 and 27 degrees which is past the first critical of the aluminum adherends and the shear-to-shear transmission coefficient will be calculated. For θ_i ranging from 13 to 15 degrees is a mixed region where the first critical angle occurs and difficult to calculate the frequency-dependent transmission coefficients.

6.1.3 Numerical Analysis

Equations (2.46), (6.2) and (6.3) for Lamb waves in a single plate, three-layered adhesive joint, and interface waves in the adhesive layer, respectively, are not in a suitable form for calculating the frequency-dependent transmission properties in each case. These equations can be rewritten in a more suitable form as follows

$$\mathbf{D}_{1 \times m} \begin{bmatrix} \phi_r \\ \phi_t \\ \psi_r \\ \psi_t \\ \Gamma_j \end{bmatrix} = \mathbf{E}_{1 \times m} \phi_i + \mathbf{F}_{1 \times m} \psi_i \quad (6.4)$$

where the matrices $\mathbf{D}$, $\mathbf{E}$, and $\mathbf{F}$ contain the elements of the reflected, transmitted, and the incident longitudinal and shear waves, respectively. The dimensions l and m are determined by the number of variables and boundary conditions required for each problem. In order to determine the frequency-dependent transmission or reflection coefficients of the guided waves we assume only one incident wave which has an amplitude of one, i.e., either $\phi_i = 1$ and $\psi_i = 0$, or $\phi_i = 0$ and $\psi_i = 1$.

Equation (6.4) can be further simplified by multiplying both sides by the inverse matrix D^{-1} which results in

$$I \begin{bmatrix} \phi_r \\ \phi_t \\ \psi_r \\ \psi_t \\ \Gamma_j \end{bmatrix} = D^{-1} E_{1xm} \phi_i + D^{-1} F_{1xm} \psi_i \quad (6.5)$$

where I is the identity matrix. Furthermore, by multiplying out both sides of (6.5) results in

$$\begin{bmatrix} \phi_r \\ \phi_t \\ \psi_r \\ \psi_t \\ \Gamma_j \end{bmatrix} = G_{1xm} \phi_i + H_{1xm} \psi_i \quad (6.6)$$

where G and H are new matrices each containing elements of $D^{-1} E$ and $D^{-1} F$, respectively. Equation (6.6) is in the form used to determine the longitudinal-to-longitudinal or shear-to-shear transmission or reflection coefficients for each problem. For example, consider the longitudinal-to-longitudinal transmission coefficient for the interface waves. Rewriting (6.6) we get

$$\begin{bmatrix} \phi_r \\ \phi_t \\ \psi_r \\ \psi_t \\ \Gamma_1 \end{bmatrix} = \begin{bmatrix} g_{11} \\ g_{21} \\ \vdots \\ g_{81} \end{bmatrix} \phi_i \quad (6.7)$$

where the transmission coefficient for the longitudinal-to-longitudinal waves correspond to the numerical value of element g_{21} .

Equation (6.5) was programmed into a minicomputer to facilitate numerical calculations of the Lamb waves through a single plate, adhesive joint, and interface waves in the adhesive layer listed in Appendix C. The structure of the program is similar for all three cases and a flowchart is shown in Fig. 34.

Initially, the values for the bulk wave velocities of the fluid, adherend, and adhesive layers, along with their respective material densities and dimensions are stored in a data file. The bulk wave velocities for each layer were formatted as a complex number in order to account for the frequency-dependent material properties such as absorption and viscoelastic effects.

Depending on the problem the operator has a choice of calculating either the longitudinal-to-longitudinal or shear-to-shear reflection or transmission coefficients for each case. Once this choice has been made, two types of analysis can be made. The first type is a scan routine in which the angle of incidence or guided wave velocity (related by (2.39)) is held constant and the amplitude of the transmission or reflection coefficient is calculated as a function of frequency. This routine also has a smoothening subroutine ranging in sample sizes from 2 to 9 points per sample, similar to the smoothening routine of our digital oscilloscope. The second type of routine available is a search routine in which the program searches between specified minimum and maximum guided wave velocities and frequencies, the program determines either the transmission maxima or

Figure 34. Flowchart of program to calculate transmission or reflection properties of guided wave modes.

reflection minima in these specified regions within a preselected resolution. Both routines were used in this study.

6.2 Experimental Procedure

6.2.1 Sample Preparation

Two sample geometries were used to study Lamb and interface waves in the adhesive joint. The samples used for Lamb wave studies were made using 1.6 mm thick 2024-T3 aluminum adherend plates bonded together with 150 μm thick FM300K adhesive. Samples used for the interface wave study were made using 9.5 mm thick 2024-T3 aluminum adherend plates bonded together by FM300K adhesive material ranging in thicknesses from 250-300 μm .

Each sample was prepared by the Northrop Corporation as described in Section 5.3. Instead of using single lap joints the samples were sectioned into 51 mm x 241 mm panels. Optical and ultrasonic measurements were made to determine the adherend-adhesive-adherend dimensions and second-order elastic constants from each group of samples. These results were used in the numerical analysis program to predict the frequency-dependent transmission properties for the guided wave modes in the adhesive joints.

6.2.2 Double Transmission Technique

Figure 35 shows a schematic diagram of the double transmission ultrasonic technique originally used to measure the dispersive properties of Lamb waves in composite

plates [54] was adapted to measure both types of guided waves in an adhesive joint. A broadband immersion transducer (effective bandwidth ranging from 2-20 MHz) was aligned normal to a thick glass reflector (water path distance 17.5 cm). The adhesive joint samples was mounted on a goniometer device and suspended between the transducer and glass reflector. The advantage of using the goniometer is that the angle of incidence can be changed simply by rotating the sample. Another important advantage of this simple geometric arrangement over the conventional two transducer pitch/catch or through transmission techniques is that there is no need for further alignment. The lateral displacement of the through transmitted beam is compensated during the backward propagation.

Figure 36 shows the received double transmitted ultrasonic signal through an adhesive joint at $\theta_i = 20$ degrees. The signal contains a number of periodically repeated wavelets with exponentially decaying amplitudes. This reverberation is caused by multiple reflections within the layered structure of the adhesive joint. The whole signal, 8 μ s in length, corresponds to the Lamb modes generated in the joint at this particular angle of incidence. Similarly, the first wavelet of the transmitted signal includes the lagging reradiated components of the interface waves in the adhesive layer. Because of the one-to-ten adhesive-to-adherend thickness ratio, the reverberation of the interface layer is not resolved in the time domain as a pulse train, however, with accurate time gating and subsequent frequency analysis of this first arrival signal, reveals the presence of interface wave modes generated at this angle of incidence [52].

Lamb wave measurements were made on the samples having the 1.6 mm adherend plates. Prior to making any measurements on the adhesive samples the reflected signal

Figure 35. Schematic diagram of double transmission leaky Lamb wave inspection of adhesive joints [54].

from the glass reflector at normal incidence was frequency analyzed and stored. This resulting spectrum is the transducer response over the measuring range of the experiments. The Lamb wave modes in the adhesive joint were measured by time gating the entire length of the signal as shown in Fig. 36. Following frequency analysis of the signal it was normalized by the transducer spectrum to account for the frequency response of the transducer over the measuring frequency range of the Lamb modes. The location of the transmission maxima were recorded as a function of frequency and from the resulting normalized spectrum as a function of angle of incidence ranging from $\theta_i = 5$ to 27 degrees, 3250 to 17000 m/s range of guided wave velocities, respectively. At two particular angles of incidence, $\theta_i = 5$ and 20 degrees the transmission coefficient amplitude as a function of frequency was also recorded.

Measurement of the interface waves were made using the samples having the 9.5 mm adherend plates to insure that there was sufficient time separation between the first arrival signal and other components contributing to the Lamb modes of the joint. To account for the transducer and adherend plate response, an aluminum plate with the same thickness as double thick adherend plate was placed between the transducer and glass reflector. The double transmitted signal was frequency analyzed and normalized by the direct response of the transducer through the reference plate, and stored. The interface waves were measured by accurately time gating $\cong 800$ ns of the first arrival signal. The interface wave modes were measured from $\theta_i = 5$ to 27 degrees in a similar fashion as the Lamb waves above.

6.3 Results and Discussion

6.3.1 Lamb Waves

Figure 37 shows the theoretically predicted and experimentally measured transmission maxima as a function of Lamb wave velocity and frequency. The experimentally measured Lamb waves in the adhesive joint were also compared to the theoretical calculations for the single adherend plate and are shown in Fig. 38.

The sample parameters used in the numerical calculations were as follows: the longitudinal and shear velocities, densities, and dimensions used in these calculations were 6320 m/s, 3130 m/s, 1.58 mm, and 2700 kg/m³ for the 2024-T3 aluminum adherends and 2250 m/s, 980 m/s, 0.150 mm, and 1180 kg/m³ for the FM300K adhesive material, respectively. The longitudinal-to-longitudinal transmission maxima were calculated using the search routine for the Lamb wave velocities ranging from 3200 to 17000 m/s with a resolution of 200 m/s, and as a function of frequencies ranging from 0.1 to 20 MHz, with a resolution of 10 kHz.

The results of the numerical calculations shown in Fig. 37 for the Lamb wave maxima is very difficult to evaluate. The discontinuities in the calculated results occur for two reasons, first, the search routine has a resolution of 10 kHz and between two closely spaced modes the program may miss one mode, and second, the threshold amplitude value for this routine is 0.1 and any weak modes below this level will be ignored.

The superimposed experimental results in Fig. 37 seem to be in agreement with the calculated results. The question is are we measuring the properties of the adhesive layer? From Fig. 37 it is difficult to tell. For instance where two or more closely spaced modes

Figure 37. Theoretical and experimental transmission maxima for leaky Lamb waves in an adhesive joint.

Figure 38. Theoretical transmission maxima for leaky Lamb waves in a single adherend plate versus experimentally measured transmission maxima for the measured leaky Lamb waves in the adhesive joint (same as in Fig. 37).

were predicted we were only able to measure one which usually occur between the modes. Below the longitudinal wave velocity of the adherend it is even more difficult to evaluate. To answer the above question we can consider the acoustical coupling efficiency of the Lamb wave in the adhesive joint.

Without the adhesive layer, each adherend plate would vibrate at its characteristic frequencies defined by (2.21). If an adhesive material was used which was very thin, attenuation-free, with fairly high density and elastic constants comparable to those of the adherend plates strong acoustic coupling would be produced and the joint would vibrate somewhat like a double thick adherend plate. If instead, a somewhat thicker, more attenuating layer, with lower density and elastic constants were used weak coupling would be produced and the adherend plates would vibrate much like the way they do when both boundaries are free. Unfortunately, the second category best describes our samples for the acoustical coupling between the adherend plates as being weak [53]. To illustrate this point we compared our measured Lamb wave results for an adhesive joint to calculated results for a single adherend plate shown in Fig. 38. In our case the adherend plates of the adhesive joint are of the same thickness, and from the measured results the adhesive joint vibrates as two independent plates insensitive to the properties of the adhesive layer. The agreement between the two results is astonishingly good.

The amplitude of the longitudinal-to-longitudinal transmission coefficient was calculated as a function of frequency for two angles of incidences, i.e., $\theta_i = 5$ and 20 degrees with Lamb wave velocities corresponding to 17000 and 4336 m/s, respectively. These results along with the measured data are shown in Figs. 39 and 40. In Fig. 39 the agreement between the amplitude and location of the transmission maxima is good. This is

Figure 39. Longitudinal-to-longitudinal transmission coefficient for leaky Lamb modes in adhesive joint at $\theta_i = 5$ degrees, corresponding to a Lamb wave velocity of 17000 m/s.

Figure 40. Longitudinal-to-longitudinal transmission coefficient for leaky Lamb modes in adhesive joint at $\theta_i = 20$ degrees, corresponding to a Lamb wave velocity of 4336 m/s.

Figure 41. Theoretical transmission coefficient for leaky Lamb modes in a single adherend plate versus experimentally measured transmission coefficient for leaky Lamb waves in an adhesive joint at $\theta_i = 5$ degrees, corresponding to a Lamb wave velocity of 17000 m/s.

Figure 42. Theoretical transmission coefficient for leaky Lamb modes in a single adherend plate versus experimentally measured transmission coefficient for leaky Lamb waves in an adhesive joint at $\theta_i = 20$ degrees, corresponding to a Lamb wave velocity of 4336 m/s.

because at $\theta_i = 5$ degrees the Lamb modes are dominantly thru transmitting longitudinal waves with their particle displacements perpendicular to the adherend-adhesive interface and are evanescent to the adhesive layer. On the other hand, the agreement between theory and experiment is rather poor especially at high frequencies which is a consequence of the rather weak coupling efficiency of the adhesive layer. This can be seen by comparing the experimental results of the transmission coefficient for the Lamb waves in the adhesive joint at these two angles of incidence to the theoretically calculated transmission coefficient for Lamb waves in a single adherend plate shown in Figs. 41 and 42. The agreement between theory and experiment in Fig. 42 is better than that shown in Fig. 40.

6.3.2 Interface Waves

Figure 43 shows the theoretical predictions and experimental results for the longitudinal-to-longitudinal transmission coefficient through an FM300K adhesive layer at 5 degrees angle of incidence. The longitudinal and shear wave velocities and densities used in these calculations were 6320 m/s, 3130 m/s, and 2700 kg/m³ for the aluminum adherend and $2250 + 45i$ and $980 + 20i$, and 1180 kg/m³ for the FM300K adhesive material. The imaginary components of the longitudinal and shear sound velocities account for absorption loss in the adhesive material which increases proportionally with frequency. Figure 44 shows the calculated and measured shear-to-shear transmission coefficient of the same adhesive layer at 20 degrees angle of incidence. Although the agreement is somewhat worse than in the longitudinal case, most of the modes are still readily identified. Because of the far more irregular spectrum of the shear transmitted signal, sometimes certain weak modes are not detected as distinct transmission maximum.

Figure 43. Longitudinal-to-longitudinal transmission coefficient for leaky interface waves modes in the adhesive layer at $\theta_i = 5$ degrees, corresponding to a trace velocity of 17000 m/s.

Figure 44. Shear-to-shear transmission coefficient for leaky interface waves modes in the adhesive layer at $\theta_i = 20$ degrees, corresponding to a trace velocity of 4336 m/s.

Figure 45. Theoretical and experimental transmission maxima for leaky interface waves in a 300 μm FM300K adhesive layer.

Figure 45 shows the calculated and measured transmission maxima as a function of guided wave velocity and frequency of the leaky interface waves in a 300 μm thick adhesive layer. Numerical calculations were made for frequencies ranging from 0.1 to 20 MHz, which for the lower frequency range is below the effective bandwidth of the measuring transducer. Figure 45 combines the longitudinal transmission data for the region of guided wave velocities ranging from 6400 to 17000 m/s and the shear transmission coefficient for guided wave velocities ranging from 3200 to 6200 m/s. The change of transmission modes was necessary partly because the first critical angle of the aluminum adherend was exceeded and partly because the shear modes below the first critical angle cannot be resolved from the much stronger longitudinal modes. The general agreement between the calculated and measured transmission maxima is good. Discrepancies in the interface wave results occur in a similar fashion as in the Lamb wave problem, i.e., the lack of resolution between two closely spaced modes and the insensitivity to very weak modes. The use of guided interface waves to nondestructively evaluate their improved sensitivity over Lamb wave modes to the properties of the adhesive layer was demonstrated by Nagy et al. [52, 53].

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

Ultrasonic characterization of the linear and nonlinear properties in solid and single layered materials can be determined from the measured bulk wave velocities and dispersive properties of special guided wave modes. In particular, a theoretical and experimental studies of the second- and third-order elastic constants for two solid materials and the adhesive layer were conducted. A second study was conducted to determine the frequency-dependent transmission properties of the Lamb and interface waves for an adhesive joint.

In the beginning of this study the second- and third-order elastic constants for 6061-T6 aluminum and perspex were measured. There was no problem in measuring the second-order elastic constants using well established ultrasonic pulse/echo techniques. However, determining the third-order elastic constants required that the stress-dependent ultrasonic bulk wave velocities be measured accurately and quickly. This task was accomplished by using a zero crossing technique and the mathematical capabilities of the LeCroy 9400 digital oscilloscope. In essence, by dividing a dc reference signal by the back reflection of the ultrasonic signal from the sample an asymptote is produced at the point where the rf back reflected signal crosses zero. When the sample was stressed in either uniaxial tension or compression the asymptote would shift corresponding to a change in velocity. The advantage of this technique is that it requires a minimal amount of equipment

and provides a time resolution of 0.02 ns, which is more than adequate for measuring the acoustoelastic effects in solid materials. The measured results for 6061-T6 aluminum showed good agreement with previously published results.

For some cases measuring the third-order elastic constants may not provide enough information concerning the nonlinear behavior of the material. Normally the strong first-order effects of the shear wave birefringence effects are measured, i.e., with the polarization direction either normal or parallel to the direction of the applied stress. At these particular polarizations it was found that the shear wave velocities are linearly proportional to the applied stress. It was shown that by eliminating or reducing the first-order effects of the shear wave birefringence the weaker higher-order nonlinearities could be measured. An optimal polarization angle for the shear wave transducer was calculated from the slopes of the first-order stress-dependent shear wave velocities. The measured results at this new polarization angle were no longer linear, instead, the shear wave velocity was dominantly a quadratic function of the applied stress. Further work in this area is needed in order to relate these higher-order nonlinearities to the properties of the material under study.

To measure the acoustoelastic effects of the adhesive layer the single lap adhesive joint was chosen. The reason for such a choice was quite clear, first, the stress distribution along the adhesive layer is well known and, second, it might be possible to relate our measured results to destructive test results carried out in industry using this particular sample design.

Measuring the second-order elastic constants for the adhesive layer was done by calculating the bulk wave velocities from the frequency-dependent reflection coefficient

from the bondline using a well established ultrasonic interference technique. However, this technique was inadequate to measure the stress-dependent bulk wave velocities. One of the major problems in measuring the stress-dependent velocities from the frequency spectrum using the digital oscilloscope was its lack of resolution. The analog spectrum analyzer provided the solution, but caused another problem, i.e. drift due to temperature changes. The solution to that problem was to use two transducers, analogous to an active and dummy strain gauges used in stress analysis. The active transducer measured the signal from the stressed sample and the reference transducer measured the signal from a stress-free sample, i.e. the frequency response of the active transducer changed as a function of applied stress while the reference response remained constant. Measuring the difference between the two amplitudes at a preselected frequency were related to the difference of the change in velocity as a function of stress.

It is well known that when a single lap adhesive joint is uniaxially stressed both normal and shear stresses are produced in the adhesive layer. From the known stress components and the finite deformation theory the acoustoelastic equations which relate the measured stress-dependent bulk wave velocities to the second- and third-order elastic constants in the layer were developed. The equations predicted that the normal component of stress was the only contribution to the stress-dependent bulk wave velocities for the layer. Considering the choice of sample geometry, the shear wave propagates parallel to the direction of the normal stress and the polarization direction is always perpendicular to the direction of stress no matter what direction the shear transducer was rotated. As a consequence of this result both shear wave acoustoelastic equations were identical and we were left with three unknowns and two equations.

The measured stress-dependent shear wave velocities at two different polarizations angles with respect to the externally applied stress on the adhesive joint were identical. We conclude that our experimental results agree with our theoretical predictions even though only two samples were measured. In order to calculate the third-order elastic constants for the adhesive layer, further work is needed in this area using different sample geometries which would provide at least three different acoustoelastic equations.

Ultrasonic measurements of the bulk wave modes in the adhesive layer up to this point were conducted at normal incidence using contact transducers. A double transmission immersion ultrasonic technique was developed to produce Lamb wave modes in the adhesive joint at various angles of incidence. A broadband transducer is aligned normal to a thick glass reflector and the adhesive joint mounted on a goniometer fixture is placed between the transducer and reflector. To change the angle of incidence all that is necessary is to rotate the sample. The main advantage of this simple experimental set up over the more commonly used two transducers pitch/catch and through transmission techniques is that no further mechanical alignment is necessary.

The leaky Lamb and interface waves which propagate along the adhesive joint and layer alone, respectively, were studied. The adhesive joint was modelled as a three-layered structure submerged in a fluid for leaky Lamb wave propagation through the entire joint. On the other hand, for leaky interface wave propagating through the adhesive layer alone the adhesive joint was modelled as a layer sandwiched between two solid semi-infinite half-spaces. Our primary interests were to determine the frequency-dependent transmission coefficient as a function of angle of incidence for each type of wave mode using a forced vibration solution and for guided wave phase velocities greater than the shear wave velocity

of the adherend plates. A computer program was developed to facilitate the numerical calculations for each problem. In order to account for the frequency-dependent material properties such as absorption and viscoelasticity a complex phase velocity was used for the adhesive layers. For example, if one were to consider the viscoelastic properties of the adhesive layer, the real component of the phase velocity can be used to determine the storage modulus and the imaginary component the loss modulus. In our calculations no imaginary component was used in the Lamb wave problem for either layer material. A small imaginary component, approximately 2 % of each respective bulk wave velocity, was added to the adhesive layer bulk wave velocities in order to account for absorption and to get a better agreement between our calculations and experiments for the interface waves.

The theoretical predictions and experimentally measured results for the Lamb and interface waves were compared. Because of the three-layered structure the large number of Lamb modes which propagate through the structure makes it very difficult to identify which modes are related to the adherend and adhesive layer. In the case of the Lamb waves at low angles of incidence the agreement between the numerical predictions and experiment was good only because the dominant feature of the transmission coefficient was from the through transmitted longitudinal waves. At higher angles of incidence the agreement was rather poor. We attribute this discrepancy to the poor acoustical coupling efficiency of the adhesive joint and the large number of competing modes especially at high frequencies. It can be shown that a weakly coupled adhesive joint vibrates much like two single plates with free boundaries. To verify this claim the measured Lamb results for the adhesive joint were compared to the theoretical predictions for a single adherend plate. The agreement is astonishingly good which indicates that the Lamb wave modes depend strongly on the adherend plates and to a lesser extent on the adhesive layer and interface between them.

In the second case the agreement between the theoretical prediction and experimental results for the interface waves is quite good. Still there are some discrepancies in the results especially when two closely spaced modes cannot be resolved, however, this is a minor discrepancy due to the technical limitations rather than to the more substantial differences between theoretical predications and experimental methods. It was shown that the interface waves are more sensitive to the adhesive layer properties and are much more easier to interpret and evaluate.

A number of new and existing theoretical and experimental techniques were presented in this dissertation study. To adequately characterize the linear and nonlinear materials properties of either a solid or layered structure using only one technique may not be enough. It is hoped that this work will provide a guide to choosing the proper ultrasonic techniques in order to characterize solid and layered materials and most importantly provide an impetus for continued research in the areas covered in this study.

APPENDICES

APPENDIX A
REVIEW OF FINITE DEFORMATION THEORY
AND EQUATION OF MOTION

A.1 Finite Deformation Theory and the Nonlinear Equation of Motion

We begin by expanding the elastic strain energy function in terms of the strains. The elastic strain energy for an isotropic solid including the third-order terms as a function of strains γ is given as

$$\begin{aligned}
 \phi = & [(\lambda+2\mu)/2](\gamma_{11} + \gamma_{22} + \gamma_{33})^2 \\
 & - 2\mu(\gamma_{22}\gamma_{33} - \gamma_{23}\gamma_{32} + \gamma_{33}\gamma_{11} - \gamma_{13}\gamma_{31} + \gamma_{11}\gamma_{22} - \gamma_{12}\gamma_{21}) \\
 & + [(\lambda + 2\mu)/3](\gamma_{11} + \gamma_{22} + \gamma_{33})^3 \\
 & - 2\mu(\gamma_{11} + \gamma_{22} + \gamma_{33})(\gamma_{22}\gamma_{33} - \gamma_{23}\gamma_{32} + \gamma_{12}\gamma_{11} \\
 & - \gamma_{13}\gamma_{31} + \gamma_{11}\gamma_{22} - \gamma_{12}\gamma_{21}) \\
 & + \nu(\gamma_{11}\gamma_{22}\gamma_{33} - \gamma_{11}\gamma_{23}\gamma_{32} - \gamma_{21}\gamma_{12}\gamma_{33} + \gamma_{21}\gamma_{13}\gamma_{32} \\
 & + \gamma_{31}\gamma_{12}\gamma_{23} - \gamma_{31}\gamma_{13}\gamma_{22}).
 \end{aligned} \tag{A.1}$$

where the Lagrangian strain measure γ used in (A.1) is defined as

$$\gamma = 1/2(JJ^T - \delta). \tag{A.2}$$

In (A.2), δ is the identity matrix

$$\delta = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad (\text{A.3})$$

and J is the Jacobian matrix

$$J = \begin{bmatrix} 1+u_a & u_b & u_c \\ v_a & 1+v_b & v_c \\ w_a & w_b & 1+w_c \end{bmatrix}, \quad (\text{A.4})$$

and where the subscripts in (A.4) denote the variable by which the argument is differentiated. The Jacobian matrix represents the limiting value of the ratio of the new volume at any time t to the old volume, and J^T in (A.2) is the transposed matrix of J . If we consider the principle of conservation of mass the determinant of J can be expressed as

$$|J| = \frac{\rho_0}{\rho} \quad (\text{A.5})$$

where ρ is the material density in the deformed state.

The Lagrangian strain components can be obtained as a function of displacement gradients by substituting the above quantities into (A.2), which are listed below:

$$\begin{aligned}
\gamma_{11} &= u_a + \frac{1}{2}(u_a^2 + v_a^2 + w_a^2) \\
\gamma_{22} &= v_b + \frac{1}{2}(u_b^2 + v_b^2 + w_b^2) \\
\gamma_{33} &= w_c + \frac{1}{2}(u_c^2 + v_c^2 + w_c^2) \\
\gamma_{23} &= \frac{1}{2}(u_b u_c + v_c(1+v_b) + w_b(1+w_c)) = \gamma_{32} \\
\gamma_{23} &= \frac{1}{2}(u_b u_c + v_c(1+v_b) + w_b(1+w_c)) = \gamma_{32} \\
\gamma_{21} &= \frac{1}{2}(w_a w_b + u_b(1+u_a) + v_a(1+v_b)) = \gamma_{12},
\end{aligned} \tag{A.6}$$

note that the components of the Lagrangian strain are symmetric.

The engineering stress tensor T can be expressed as a function of the Jacobian and the strain energy function as follows

$$T = J \left(\frac{\partial \phi}{\partial \gamma} \right) \tag{A.7}$$

By substituting (A.6) into (A.1) we then obtain the strain energy in terms of displacements. Finally, by differentiating (A.1) and substituting the result along with (A.4) into (A.7) we obtain the engineering stress tensor in terms of displacement gradients.

The final step is to obtain a general equation of motion in terms of the displacement gradients by differentiating the proper stress components in (A.7) and substituting the results in the following the equation of motion written as

$$\rho_0 \ddot{u}_i = \frac{\partial T_{ij}}{\partial a_j} \tag{A.8}$$

For example, the first component of (A.8), i.e., ($i=1$), can be written as

$$\rho_0 \ddot{u} = \frac{\partial T_{11}}{\partial a} + \frac{\partial T_{12}}{\partial b} + \frac{\partial T_{13}}{\partial c} \quad (\text{A.9})$$

with the resulting equation in terms of the displacement gradients as

$$\begin{aligned} \rho_0 \ddot{u} = & (\lambda + 2\mu)[u_{aa} + v_{ab} + w_{ac} + u_a(3u_{aa} + u_{bb} + u_{cc} + v_{ab} + w_{ac}) \\ & + v_a v_{aa} + w_a w_{aa} + u_b(v_{bb} + 2u_{ab} + w_{bc}) \\ & + v_b(u_{aa} + u_{bb} + u_{cc} + v_{ab}) + w_b w_{ab} + v_c v_{ac} \\ & + u_c(w_{cc} + 2u_{ac} + v_{bc}) + w_c(u_{aa} + u_{bb} + u_{cc} + w_{ac})] \\ & + \mu[u_{bb} + u_{cc} - v_{ab} - w_{ac} - u_a(v_{ab} + w_{ac}) + v_a(v_{bb} + v_{cc} + 2u_{ab}) \\ & + w_a(w_{bb} + w_{cc} + 2u_{ac}) + u_b(v_{aa} - w_{bc}) \\ & + v_b(-2u_{aa} - 2u_{cc} + v_{cc} - v_{ab}) + w_b(2u_{bc} - w_{ab}) \\ & + u_c(w_{aa} + w_{bb} - v_{bc}) + v_c(2u_{bc} - v_{ac}) - w_c(2u_{aa} + 2u_{bb} + w_{ac})] \\ & + 2(1 + 2m)[(u_b + v_b + w_c)(u_{aa} + v_{bb} + w_{cc})] \\ & + m[u_a(u_{bb} + u_{cc} - 3v_{ab} - 3w_{ac}) \\ & + (u_b + v_a)(v_{aa} + v_{bb} + 2u_{ab} + w_{bc}) \\ & + w_a(w_{aa} + w_{cc} + 2u_{ac} + v_{bc}) \\ & + v_b(-4u_{aa} + u_{bb} + u_{cc} - 5w_{ac} - 3v_{ab}) \\ & + w_b(v_{cc} + w_{ab}) + u_c(w_{aa} + w_{cc} + 2u_{ac} + v_{bc}) \\ & + v_c(v_{ac} + w_{ab}) + w_c(-4u_{aa} + u_{bb} + u_{cc} - 3w_{ac} - 5v_{ab})] \\ & + 1/4n[(u_b + v_a)(v_{cc} - w_{bc}) + (u_c + w_a)(w_{bb} - v_{bc}) \\ & + (v_c + w_b)(2u_{bc} - v_{ac} - w_{ab}) \\ & + 2v_b(w_{ac} - u_{cc}) + 2w_c(v_{ab} - u_{bb})]. \end{aligned} \quad (\text{A.10})$$

In order to obtain the second- and third-components of the equation of motion each variable in (A.10) was permuted and the results are shown below.

A.2 Second-component of the Equation of Motion

Substituting $i = 2$, into (A.8) the general form the equations of motion is obtained as

$$\rho_0 \ddot{v} = \frac{\partial T_{21}}{\partial a} + \frac{\partial T_{22}}{\partial b} + \frac{\partial T_{23}}{\partial c} \quad (A.11)$$

The second component following permutation from the first-component (A.10), can be written in terms of the displacement gradients as

$$\begin{aligned} \rho_0 \ddot{v} = & (\lambda + 2\mu)[v_{bb} + w_{bc} + u_{ba} + v_b(3v_{bb} + v_{cc} + v_{aa} + w_{bc} + u_{ba}) \\ & + w_b w_{bb} + u_b u_{bb} + v_c(w_{cc} + 2v_{bc} + u_{ca}) \\ & + w_c(v_{bb} + v_{cc} + v_{aa} + w_{bc}) + u_c u_{bc} + w_a w_{ba} \\ & + v_a(u_{aa} + 2v_{ba} + w_{ca}) + u_a(v_{bb} + v_{cc} + v_{aa} + u_{ba})] \\ & + \mu[v_{cc} + v_{aa} - w_{bc} - u_{ba} - v_b(w_{bc} + u_{ba}) + w_b(w_{cc} + w_{aa} + 2v_{bc}) \\ & + u_b(u_{cc} + u_{aa} + 2v_{ba}) + v_c(w_{bb} - u_{ca}) \\ & + w_c(-2v_{bb} - 2v_{aa} + w_{aa} - w_{bc}) + u_c(2v_{ca} - u_{bc}) \\ & + v_a(u_{bb} + u_{cc} - w_{ca}) + w_a(2v_{ca} - w_{ba}) - u_a(2v_{bb} + 2v_{cc} + u_{ba})] \\ & + 2(1 + 2m)[(v_b + w_c + u_a)(v_{bb} + w_{bc} + u_{ba})] \\ & + m[v_b(v_{cc} + v_{aa} - 3w_{bc} - 3u_{ba}) \\ & + (v_c + w_b)(w_{bb} + w_{cc} + 2v_{bc} + u_{ca}) \\ & + u_b(u_{bb} + u_{aa} + 2v_{ba} + w_{ca})] \end{aligned} \quad (A.12)$$

$$\begin{aligned}
& +w_c(-4v_{bb}+v_{cc}+v_{aa}-5u_{ba}-3w_{bc}) \\
& +u_c(w_{aa}+u_{bc})+v_a(u_{bb}+u_{aa}+2v_{ba}+w_{ca}) \\
& +w_a(w_{ba}+u_{bc})+u_a(-4v_{bb}+v_{cc}+v_{aa}-3u_{ba}-5w_{bc})] \\
& +1/4n[(v_c+w_b)(w_{aa}-u_{ca})+(v_a+u_b)(u_{cc}-w_{ca}) \\
& +(w_a+u_c)(2v_{ca}-w_{ba}-u_{bc}) \\
& +2w_c(u_{ba}-v_{aa})+2u_a(w_{bc}-v_{cc})].
\end{aligned}$$

A.3 Third Component of the Equation of Motion

In a similar fashion as in the previous section, the third component of the equation of motion, i.e., substituting $i = 3$ into (A.8), can be written in the general and displacement gradient forms as

$$\rho_0 \ddot{w} = \frac{\partial T_{31}}{\partial a} + \frac{\partial T_{32}}{\partial b} + \frac{\partial T_{33}}{\partial c}, \quad (A.13)$$

$$\begin{aligned}
\rho_0 \ddot{w} = & (\lambda+2\mu)[w_{cc}+u_{ca}+v_{cb}+w_c(3w_{cc}+w_{aa}+w_{bb}+u_{ca}+v_{cb}) \\
& +u_c u_{cc}+v_c v_c+w_a(u_{aa}+2w_{ca}+v_{ab}) \\
& +u_a(w_{cc}+w_{aa}+w_{bb}+u_{ca})+v_a v_{ca}+u_b u_{cb} \\
& +w_b(v_{bb}+2w_{cb}+u_{ab})+v_b(w_{cc}+w_{aa}+w_{bb}+v_{cb})] \\
& +\mu[w_{aa}+w_{bb}-u_{ca}-v_{cb}-w_c(u_{ca}+v_{cb})+u_c(u_{aa}+u_{bb}+2w_{ca}) \\
& +v_c(v_{aa}+v_{bb}+2w_{cb})+w_a(u_{cc}-v_{ab}) \\
& +u_a(-2w_{cc}-2w_{bb}+u_{bb}-u_{ca})+v_a(2w_{ab}-v_{ca}) \\
& +w_b(v_{cc}+v_{aa}-u_{ab})+u_b(2w_{ab}-u_{cb})-v_b(2w_{cc}+2w_{aa}+v_{cb})] \\
& +2(1+2m)[(w_c+u_a+v_b)(w_{cc}+u_{ca}+v_{cb})]
\end{aligned}$$

$$\begin{aligned}
& +m[w_c(w_{aa}+w_{bb}-3u_{ca}-3v_{cb}) \\
& +(w_a+u_c)(u_{cc}+u_{aa}+2w_{ca}+v_{ab}) \\
& +v_c(v_{cc}+v_{bb}+2w_{cb}+u_{ab}) \\
& +u_a(-4w_{cc}+w_{aa}+w_{bb}-5v_{cb}-3w_{ca}) \\
& +v_a(u_{bb}+v_{ca})+w_b(v_{cc}+v_{bb}+2w_{cb}+u_{ab}) \\
& +u_b(u_{cb}+w_{ca})+v_c(-4w_{cc}+w_{bb}+w_{aa}-3v_{cb}-5u_{ca})] \\
& +1/4n[(w_a+u_c)(u_{bb}-v_{ab})+(w_b+v_c)(v_{aa}-u_{ab}) \\
& +(u_b+v_a)(2w_{ab}-u_{cb}-v_{ca}) \\
& +2u_a(v_{cb}-w_{bb})+2v_b(u_{ca}-w_{aa})].
\end{aligned}$$

APPENDIX B
GOLAND AND REISSNER STRESS DISTRIBUTION
PROGRAM FOR SINGLE LAP ADHESIVE JOINT

The Basic program to calculate the shear and normal stresses along the adhesive layer for a single lap adhesive joint using the formulation of Goland and Reissner [29] is listed below.

```

10 REM 'stress profile'
12 DIM SST(25), NST(25)
15 REM sample dimensions use metric
20 M = .0095
21 H = .00017
22 C = (.0254*.363)/2
23 L = .0254*3.25
25 REM material properties, c=adhesive
26 E = 7E+10
27 RH = .34
28 EC = 4.4E+09
29 GC = 1.46E+09
35 REM loading information
36 T = (2490/1!)*175.1968
40 U1 = (2/M)*SQR((3*(1-RH^2)*T)/(E*M))
45 U2 = SQR(2)*U1/4
47 CHU2 = .5*(EXP(U2*C)+EXP(-U2*C))
48 CHU1 = .5*(EXP(U1*L)+EXP(-U1*L))
49 SHU1 = .5*(EXP(U1*L)-EXP(-U1*L))
50 SHU2 = .5*(EXP(U2*C)-EXP(-U2*C))
52 K = (CHU2*SHU1)/((SHU1*CHU2)+(2*SQR(2)*CHU1*SHU2))
54 KP = (K*C/M)*SQR(3*T*(1-RH^2)/E)
58 BTA = SQR((8*GC*M)/(E*H))
60 GMA = ((6*EC*M)/(E*H))^.25
65 LAM = (GMA*C)/M
68 CHLM = .5*(EXP(LAM)+EXP(-LAM))
69 SHLM = .5*(EXP(LAM)-EXP(-LAM))
72 R1 = CHLM*SIN(LAM)+SHLM*COS(LAM)
73 R2 = SHLM*COS(LAM)-CHLM*SIN(LAM)

```

```

75 DEL = .5*(.5*(EXP(2*LAM)-EXP(-2*LAM))+SIN(2*LAM))
80 X = C/20
85 PRINT "          position          normal          shear"
86 PRINT ""
90 FOR ANN=0 TO 20
100 CHBX = .5*(EXP(ANN*X*BTA/M)+EXP(-ANN*X*BTA/M))
103 LAMX = LAM*ANN*X/C
104 CHLX = .5*(EXP(LAMX)+EXP(-LAMX))
105 SHLX = .5*(EXP(LAMX)-EXP(-LAMX))
110 BCM = (BTA*C)/M
115 CL = COS(LAM*ANN*X/C)
117 SL = SIN(LAM*ANN*X/C)
120 SHBM = .5*(EXP(BCM)-EXP(-BCM))
130 R11 = (R1*(LAM)^2*K/2)+(LAM*KP*SHLM*SIN(LAM))
140 R22 = (R2*(LAM)^2*K/2)+(LAM*KP*CHLM*COS(LAM))
150 SST(ANN) = (T/(8*C))*((BCM*(1+3*K)*CHBX)/SHBM + 3*(1-K))
160 NST(ANN) = (T*M)/(C^2*DEL)*((R22*CHLX*CL)+(R11*SHLX*SL))
170 PRINT USING"          #####.##";(X*ANN)/C,NST(ANN)/6890
180 NEXT ANN

```

APPENDIX C FORTRAN PROGRAM TO CALCULATE THE DISPERSIVE PROPERTIES OF THE GUIDED WAVES

The Fortran program listed below was used to calculate the frequency-dependent transmission or reflection coefficients for the guided wave modes in a single layer and adhesive joint.

```

c * Evaluating maximum or minimum of transmission
c * or reflection coefficient from an adhesive
c * layer in a solid substrate.
c *
c *      c11, csl, r1      solid
c * -----
c *      c12, cs2, r2      layer
c * -----
c *      c11, csl, r1      solid
c *
c *****

      implicit real*8 (a-h,o-z)
      real*8 hu,hf,hb
      complex*16 gana(4)
      common/csl/cc(4),r1,r2,ff,gana,mode,p12,nsel
      common/csl2/sax1,ix,ixz,ll,h1,h2,dta
5      open(UNIT=10,FILE='gtr.dat')
      read(10,*) c11,csl,c12,cs2,r1,r2,flevel
      close (UNIT=10)
      write(*,60)
66      format(1x,'Mode Selection (1-3):')
1      /1x,'1=Shear incidence and shear reflection (minimum)'
1      /1x,'2=Long. incidence and long. reflection (minimum)'
1      /1x,'3=Long. incidence and long. transmission (maximum)'
1      /1x,'4=Shear incidence and Shear Transmission (maximum)'
      read(*,*) nsel
      write(*,*) "1"-Scanning "0"-Searching along fd axis"
      read(*,*) nsel
      if(nsel.eq.0) goto 4
      write(*,*) "Maximum value of C-?"
      read(*,*) cl
      write(*,*) "Minimum value of C-?"
      read(*,*) csim
      write(*,*) "Scanning step for C-?"
      read(*,*) chl
      goto 3
4      write(*,*) "Phase Velocity-?"
      read(*,*) cl
3      write(*,*) "Start-point of fd-?"
      read(*,*) fmin
      write(*,*) "End-point of fd-?"
      read(*,*) fmax
      write(*,*) "Step of fd-?"
      read(*,*) hl
      x0=0d0
      write(*,*) "Do you want change other parameters?"

1      Yea=1/No=0
      read(*,*) lya
      if (lya) 8,6,7
7      write(*,*) "Stop of fd-?"
      read(*,*) hl
      write(*,*) "Imaginary part of C-?"
      read(*,*) x0
6      cc(1)=c11*cl1
      cc(2)=csl*csl
      cc(3)=c12*cl2
      cc(4)=cs2*cs2
      open(UNIT=20,FILE='lanb.dat')
      write(*,20) c11,csl,r1,c12,cs2,r2
      write(20,20) c11,csl,r1,c12,cs2,r2
20      format(1x,'Longitudinal in solid=',f8.5,' Shear in
solid=',
1      f8.5,' r1=',f4.2/1x,'Longitudinal in layer=',f8.5,
1      ' shear in layer=',f8.5,' r2=',f4.2)
22      pi=4d0*datan(1d0)
      x=x0
24      fh=fmin
      yi=0d0
      y2=0d0
      if(nsel.lt.3) goto 21
      yi=ld5
      y2=ld5
21      xl=cl
23      ff=pi*fh
      call deta(xl,s,y,nsel,sm21)
      if(nsel.ne.0) goto 25
      write(20,50) fh,y,xl,sm21,sm2
      write(*,50) fh,y,xl,sm21,sm2
      fh=fh*hl
      if(fh.gt.fmax) goto 70
      goto 23
25      yy=y2
      y2=y1
      yi=y
      if((y2-yy)*(y2-y1)) 52,52,53
53      if(nsel.lt.3) goto 56
      if(y2.lt.y1) goto 52
      goto 57
58      if(y2.gt.y1) goto 52
57      write(20,50) fh,y1,xl,sm11,sm21
      write(*,50) fh,y1,xl,sm11,sm21
50      format(1x,'FD=',f8.5,' Ref=',d12.6,' V=',f8.5,
1      ' N=',d13.7,' D=',d13.7)
52      fh=fh
      fh=fh*hl
      if(fh.gt.fmax) goto 60
      y1=y
      sm11=sm21

```

[illegible][illegible]

```

aa=bb
l=j
42 continue
if (l.eq.n) goto 45
do 43 ii=k,nn
ci=a(b,ii)
a(k,ii)=a(l,ii)
43 a(l,ii)=ci
45 do 50 ji=k,nn
do 50 i=k,nn
ci=a(ji,k)*a(k,l)
50 a(ji,l)=a(ji,i)-ci/a(k,k)
sum=ld0
do 60 i=1,nn
sum=sum+cdabs(a(i,l))
60 if(key1.eq.1) goto 70
sum1=sum
key1=1
goto 15
70 goto (71,72,72,71) sum1
71 sum2=sum+cdabs(ec(2)*ec(2))
goto 74
72 sum2=sum+cdabs(ec(1)*ec(1))
74 sum=sum2/sum1
return
end

```


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