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The ${}^7\text{Li}({}^3\text{He}, \text{p}){}^9\text{Be}$ reaction and primordial nucleosynthesis

Rath, Dipti Prakash, Ph.D.

The Ohio State University, 1989

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The ${}^7\text{Li}({}^3\text{He}, p){}^9\text{Be}$ Reaction and Primordial Nucleosynthesis

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree of Doctor of Philosophy in the Graduate
School of the Ohio State University

By

Dipti Prakash Rath, M.S.

* * * * *

The Ohio State University

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To my Family
whose love and encouragement
has inspired all my efforts.

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PUBLICATIONS

- The ${}^7\text{Li}({}^3\text{He},\text{p}){}^9\text{Be}$ Reaction and Primordial Nucleosynthesis; D.P. Rath, R.N. Boyd, H.J. Hausman, M.S. Islam and G.W. Kolnicki, Bull. Am. Phys. Soc. 34 (1989) 1191
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- The ${}^{63}\text{Cu}(\alpha,\text{p}){}^{66}\text{Zn}$ Cross Sections and the Corresponding Thermonuclear Reaction Rate; M.S. Islam, R.N. Boyd, P.B. Corn, D. Rath and X. Gu, Bull. Am. Phys. Soc. 34 (1989) 1193
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TABLE OF CONTENTS

Acknowledgements	iii
Vita	v
LIST OF TABLES	x
LIST OF FIGURES	xii
I Introduction	1
II Theory	10
2.1 Standard Big Bang Model (SM)	10
2.2 Nonuniform Density Model (NDM)	14
III Experimental details	25
3.1 Beamline	25
3.2 ^3He Beam Charge State	27

3.3	Detectors	30
3.4	Targets	36
3.5	Electronics and Data Aquisition	38
IV	Results and Analysis	43
4.1	Angular Distributions	43
4.2	Penetrability Factors	47
4.3	Total Cross Sections	55
4.4	Isospin Considerations	61
4.5	DWBA calculations	63
V	Astrophysical Discussions	67
5.1	Astrophysical S-factor	67
5.2	Thermonuclear Reaction Rate	74
VI	Conclusion	79
	Appendices	81
A	Effective beam energy in target	81

B Effective beam charge state	82
C Geometry of detector assembly	83
D Unicorn program parameters	84
E Algorithm to evaluate reaction rate	85
Bibliography	87

LIST OF TABLES

1.1	Estimate of LiBeB production (by number) by the galactic cosmic rays from the calculations by Meneguzzi et al. [Mene71], the experimental abundances and the uncertainty factors (e.g. $70(2)$ means 70^{+70}_{-35}) are from the Ref. [Aust81].	9
2.1	Summary of ^4He abundance determinations (including the uncertainties) in various locations [Schr77].	24
3.1	Data sheet of the various experimental quantities required to calculate the effective charge state of ^3He beam.	31
3.2	Bias sheet for the various detectors used in the experiment.	33
3.3	Possible reaction channels at $E(^3\text{He}) = 2.5$ MeV leading to particle products, the outgoing particle energies (computed from the programs KINMAT [Rinc81] and LOSTENERGY [Marm73]) are indicated at various locations within the detector telescopes.	37
4.1	Input parameters employed for DWBA calculations using the code CHUCK3 [Kunz84].	66
5.1	Parameters for fitting the S-factors according to Eqn. 5.3.	71

- 5.2 Summary sheet of the experimentally measured quantities, calculated penetrabilities (Eqn. 4.9) and the S-factors for ${}^7\text{Li}({}^3\text{He},p_0)$ reaction. . . 72
- 5.3 Summary sheet of the experimentally measured quantities, calculated penetrabilities (Eqn. 4.9) and the S-factors for ${}^7\text{Li}({}^3\text{H},n_0)$ reaction. . . 73

LIST OF FIGURES

1.1	Schematic curve of atomic abundances as a function of atomic weight based on the data of Suess and Urey [Sues56].	2
1.2	Isobar diagram for ${}^9\text{Be}$ and ${}^9\text{B}$ [Azje84].	8
2.1	Network of nuclear reactions included in the computer code of [Wago73]. The exothermic directions are indicated by arrows, while the dashed line indicates that individual nuclear abundances were only computed for mass number $A < 12$	15
2.2	Final abundances (by mass) produced in standard big bang models, whose only parameter is h , related to the present density and temperature [Schr77].	16
2.3	Sources of density fluctuation in the NDM.	17
2.4	Reaction network used in the primordial nucleosynthesis calculation of Boyd and Kajino [Boyd89]. The two new reaction pathways, ${}^7\text{Li}({}^3\text{H},n){}^9\text{Be}$ and ${}^9\text{Be}({}^3\text{H},n){}^{11}\text{B}$, have previously not been included in primordial nucleosynthesis calculations.	22
2.5	Growth of the (mass) abundances of the elements in primordial nucleosynthesis in the NDM with increasing time. This figure is for neutron-rich region. The dashed curves show the abundances predicted without the ${}^7\text{Li}({}^3\text{H},n){}^9\text{Be}$ and ${}^9\text{Be}({}^3\text{H},n){}^{11}\text{B}$ reactions included.	23

3.1	Schematic of Ohio State University Van De Graaff accelerator, beam-line 45° South.	26
3.2	Schematic of set up used to study the ^3He beam charge state.	29
3.3	Experimentally measured fractional charge state distribution of ^3He beam, the curve is from the fitting program CHARGE [Abdu88].	32
3.4	Schematic of the detector telescope components used in the experiment. "TA" refers to the Tantalum absorber foil in front of the detector, while " ΔE " and "E" refer, respectively, to the two detectors which comprise the detector telescopes.	34
3.5	Particle peaks arising from proton bombardment of the target at $E_{\text{beam}} = 1.975$ MeV and lab angle 140°, detected by a single surface barrier detector.	35
3.6	Schematic of the electronic circuitry employed for data acquisition.	40
3.7	2-dimensional E- ΔE plot at ^3He beam energy 1.3 MeV and lab angle 160°.	41
3.8	Energy spectra for protons and deuterons together (Fig. 3.8a), protons only (Fig. 3.8b), and deuterons only (Fig. 3.8c).	42
4.1	Nuclear energy level scheme for ^8Be [Ajze84].	44
4.2	Nuclear energy level scheme for ^9Be [Ajze84].	45
4.3	Differential cross sections for the proton groups from $^7\text{Li}(^3\text{He},p)^9\text{Be}$; p_0 (Fig. 4.3a), and p_1, p_2, p_3 (Fig. 4.3b). Energies are given as laboratory energies for the incident ^3He particles. The indicated errors on the data correspond only to the statistical uncertainties.	48

4.4	Differential cross sections for the deuteron groups (d_0, d_1) from ${}^7\text{Li}({}^3\text{He}, d){}^8\text{Be}$ reaction at 2.5 MeV beam energy. The indicated errors are only due to the statistical uncertainties.	49
4.5	Schematic of the potential governing the motion of one nucleus relative to another.	51
4.6	Sum of the nuclear and coulomb potentials specified by Eqn. 4.14. A typical incident energy and the corresponding limits of the numerical integration for penetrability are also indicated on the fig.	54
4.7	Results of WKB calculations for penetrability, the solid line uses a square well nuclear potential whereas the dotted line is with a more realistic Woods Saxon potential.	56
4.8	Total cross sections ($\sigma=4\pi a_0$, a_0 being the first legendre coefficient) for p_0 and p_2 groups.	58
4.9	Nuclear energy level scheme for ${}^{10}\text{B}$ [Ajze84].	59
4.10	Nuclear energy level scheme for ${}^{10}\text{Be}$ [Ajze84].	60
4.11	Total cross sections measured for ${}^7\text{Li}({}^3\text{He}, p_0){}^9\text{Be}$ and calculated for ${}^7\text{Li}({}^3\text{H}, n_0){}^9\text{Be}$. The indicated uncertainties are those for the measured ${}^7\text{Li}({}^3\text{He}, p)$ cross sections.	62
4.12	Results of the DWBA calculations for ${}^7\text{Li}({}^3\text{He}, p_0)$ and ${}^7\text{Li}({}^3\text{H}, n_0)$ differential cross sections at 1.0 MeV and 2.0 MeV bombarding energies. The ratios of the integrated cross sections at these energies are approximately equal to 4 and 2 respectively.	65

5.1	S-factors for ${}^7\text{Li}({}^3\text{He},p_0){}^9\text{Be}$ and ${}^7\text{Li}({}^3\text{H},n_0){}^9\text{Be}$, that for ${}^7\text{Li}({}^3\text{H},n_0)$ was derived from its calculated cross sections. The curves are resonance fits, together with flat backgrounds.	70
5.2	Thermonuclear reaction rates for ${}^7\text{Li}({}^3\text{H},n_0){}^9\text{Be}$	78

CHAPTER I

Introduction

The growth of human knowledge about the nucleosynthesis of the so-called light elements (D, ^3He , ^6Li , ^9Be , $^{10,11}\text{B}$) has been rather fascinating [Reev74, Aust81], and is quite distinct from those of the majority of heavier elements. It all began with the publication of the elemental abundance curve (Fig. 1.1; from [Fowl84]) by Suess and Urey in 1956 [Sues56] and the subsequent pioneering work by Burbidge, Burbidge, Fowler and Hoyle (B²FH) in 1957 [Burb57], where the synthesis of these elements was attributed to a collective process called X-process. The very name suggests there was little known about their nucleosynthesis at that point of time.

The difficulty in the nucleosynthesis of light elements arises from the fact that they are readily destroyed by hydrogen burning in the stellar interiors. In particular, ^9Be is destroyed by $^9\text{Be}(p,\alpha)^6\text{Li}$ and $^9\text{Be}(p,d)^8\text{Be}$ reactions. Therefore, stellar nucleosynthesis which produced almost all of the heavier elements, destroys instead of creating the light elements. In the astrophysical parlance, we say these elements are bypassed by the usual line of nucleosynthesis. This is evident on Fig. 1.1 where

Figure 1.1: Schematic curve of atomic abundances as a function of atomic weight based on the data of Suess and Urey [Sues56].

the abundances of Li, Be and B are seen to be several orders of magnitude lower than those of the neighboring elements indicating certain uniqueness in their nucleosynthetic history. Calculations show that during the typical age of a star ($\approx 10^9$ years), complete destruction of the light elements would occur if the stellar temperature is any higher than what is given below.

$$\text{D} \rightarrow 0.5 \text{ T}_6 \quad (\text{T}_6 = 10^6 \text{ }^\circ\text{K})$$

$$\text{Li} \rightarrow 2.0 \text{ T}_6$$

$$\text{Be} \rightarrow 3.5 \text{ T}_6$$

$$\text{B} \rightarrow 5.0 \text{ T}_6$$

Between 1955-1970, there was a series of papers [Fowl55, Bons60, Burn65, Bern67] which suggested that Li, Be and B could be produced by spallation reactions (of protons) on C, N and O in the stellar atmospheres of young stars in their so-called T-Tauri phase. This came to be known as the Autogenetic theory implying that each star (including our Sun) had produced its own supply of light elements during its T-Tauri phase. In 1970, a critical paper [Ryte70] was published which showed that accounting for light element abundance in T-Tauri stars would require the amount of energy pumped into particle acceleration to be comparable with the gravitational energy release of the stars implying an unlikely high efficiency for the acceleration mechanism. This put an end to the Autogenetic theory.

Nonetheless, the strong realization that the spallation reactions could be significant sources for light element nucleosynthesis led to an alternative site for production

of these elements, namely the Interstellar Medium (ISM). It was hypothesized that the Galactic Cosmic Ray (GCR) flux could effectively spall ($\equiv$ break up) the constituents of the ISM to produce light elements in sufficient quantities [Mene71, Mene75]. This came to be known as the Galactogenetic theory.

The results of this theory [Mene75] for the light elements are shown in Table 1.1. It is clear that the GCR mechanism is capable of explaining the observed abundances of ${}^6\text{Li}$, ${}^9\text{Be}$ and ${}^{10,11}\text{B}$ within their uncertainties but it does not produce enough ${}^7\text{Li}$ (underproduced by a factor of 10) or D, ${}^3\text{He}$ and ${}^4\text{He}$ (underproduced by much larger factors).

The other inevitable mechanism that comes into the picture now is the primeval Big Bang (BB), discussed in some detail in Chapter 2. The standard BB theory [Wago67, Wago69, Wago73] has accurately predicted the observed mass fraction of ${}^4\text{He}$ in the universe and is consistent with the 3° microwave background radiation [Penz65]. BB results complement those of the GCR theory in a remarkable way. BB seems to produce most of those elements that are not produced in sufficient quantities by the GCR mechanism.

In conclusion, as far as the synthesis of ${}^9\text{Be}$ is concerned, it is principally attributed to the GCR mechanism. Production of ${}^9\text{Be}$ in the standard BB picture has, so far, been assumed to be negligible.

Recently, however, the Nonuniform Density BB Model (NDM) [Sale86, Alco87, Full88, Mala88a, Mala88b] in which the density of the universe at the time of big bang

nucleosynthesis is not assumed to be uniform, as it is in the SM, has gained considerable attention. Both of these models have been shown to produce the elements from ^2H to ^4He with the abundances thought to be the primordial values. However, the abundance of ^7Li predicted by the two models differs considerably; that traditionally taken [Boes85] to be the primordial value is well reproduced by the SM, provided the baryonic density of the universe is no more than a few percent of the critical density. However, the simplest version of the NDM fails to reproduce that value. Recent work by Malaney and Fowler [Mala88] has suggested that back diffusion of neutrons, if handled correctly, will alleviate the overproduction of ^7Li by the NDM, but other work by Kurki-Suonio [Kurk88] suggests that more problems are created than are solved by the inclusion of those effects.

Observationally the question of the primordial ^7Li abundance is nearly as confused as it is theoretically. Boesgaard and Steigman [Boes85] have argued that Population II disk stars give the best indication of the primordial ^7Li abundance, and the value they give is in good agreement with the results of the SM. However, Hobbs and Pilachowski [Hobb88] have recently observed some metal poor Population I halo stars with considerably higher ^7Li abundances. They argue that one possible interpretation of the discrepancy between these two groups of stars is that the halo stars give the correct value of the primordial ^7Li abundance, and that the ^7Li abundance is lowered to that observed in the other group of stars by stellar processing. The situation regarding the Li abundance was further called into question by Vauclair [Vauc88], who suggests that stellar destruction may indeed be responsible for the very low abundance value usually assumed to be the primordial abundance of ^7Li , and that the actual primordial abundance may well be considerably larger. The summary of

all this work is that the situation regarding the primordial ${}^7\text{Li}$ abundance is far from clear, and that an independent check of the SM and the NDM predicted primordial abundances would be most desirable.

Such a test, using ${}^9\text{Be}$, was recently suggested by Boyd and Kajino [Boyd89]. ${}^9\text{Be}$ has long been known to be produced in very low quantities in big bang nucleosynthesis. However, inclusion of a reaction previously omitted from network calculations of big bang nucleosynthesis yields, the ${}^7\text{Li}({}^3\text{H},n){}^9\text{Be}$ reaction, enhances the ${}^9\text{Be}$ abundance by many orders of magnitude from that previously found, and raises it, at least in the NDM, to an abundance level close to that recently observed by Rebolo et al. [Rebe88] in highly metal deficient dwarf stars. Previous measurements of the cross section [Val'm61, Sero62] for that reaction were done many years ago, and were presented as the yield to all ${}^9\text{Be}$ states. Since only the yield to the ${}^9\text{Be}$ ground state will ultimately produce ${}^9\text{Be}$ (the excited ${}^9\text{Be}$ states are above the n emission threshold and decay to ${}^8\text{Be}$ which subsequently decays to two α particles [Ajze84]), that yield was estimated from the data given; it produced a value which was inconsistent with cross sections [Ajze84] inferred from similar data, those from the ${}^9\text{Be}({}^3\text{H},n){}^{11}\text{B}$ reaction, by correcting for the difference in nuclear charge between the two target nuclei.

Another way of estimating the cross section for ${}^7\text{Li}({}^3\text{H},n){}^9\text{Be}$ would be to infer it from data for a similar reaction, the ${}^7\text{Li}({}^3\text{He},p){}^9\text{Be}$ reaction. In the past, large amount of experimental efforts has gone into the study of ${}^7\text{Li}({}^3\text{He},p){}^9\text{Be}$ reaction. These efforts were principally guided by the desire to establish the isobaric similarity between the energy-levels of ${}^9\text{Be}$ and ${}^9\text{B}$ [Kroe67]. The correspondence between the low-lying analog states [Ajze84] of these two nuclei are shown in Fig. 1.2. The spins and parities of these states are seen to be similarly ordered and the energy levels are

only slightly shifted because of the difference in coulomb energy. In addition, the lowest-lying $T = \frac{3}{2}$ states of these two mirror nuclei have received a great deal of attention for the study of isospin forbidden nucleon decays [Adlo70].

However, the astrophysical importance of this reaction has been generally overlooked. In the literature, angular distributions and differential cross sections are available at some energies [Woli61, Lync64, Cock67, Sana69] but total cross-sections over the desired range are non-existent. This provides the motivation of the present work to measure the total cross section of this reaction in a continuous manner over as much of the desired energy range as possible.

Table 1.1

Estimate of LiBeB production by the galactic cosmic rays from the calculations by Meneguzzi et al. [Mene71], the experimental abundances (by number) are from the Ref. [Aust81].

Nuclide	$(N_i/H)_{theory}$	$(N_i/H)_{expt}^*$
${}^6\text{Li}$	45×10^{-12}	$70(2) \times 10^{-12}$
${}^7\text{Li}$	66×10^{-12}	$900(2) \times 10^{-12}$
${}^9\text{Be}$	16×10^{-12}	$14(1.6) \times 10^{-12}$
${}^{10}\text{B}$	51×10^{-12}	$30(2) \times 10^{-12}$
${}^{11}\text{B}$	118×10^{-12}	$120(2) \times 10^{-12}$

* $70(2)$ means 70^{+70}_{-35}

CHAPTER II

Theory

2.1 Standard Big Bang Model (SM)

The realization that the universe could have originated according to a hot big bang (BB) scenario from a state of extreme density and temperature dates back to the work of Alpher, Bethe and Gamow ($\alpha\beta\gamma$) in the 1940s [Alph48, Alph50]. The accompanying nucleosynthesis in this so-called Gamow model, began with only neutrons; but this assumption was later shown to be invalid [Haya50, Alph53]. The current viewpoint concerning the origin of elements in BB nucleosynthesis emerged in the 1960s with the calculations by Wagoner, Fowler and Hoyle [Wago67].

In the 1920s, Hubble and his associates [Hubb29] made the significant discovery that the faint spiral nebulae (galaxies much like our own Milky Way galaxy but very distant) were moving away from us with velocities of recession proportional to their distances. This is generally referred to as the Hubble's law

$$v = H_0 r \quad (2.1)$$

where H_0 is the Hubble's constant. Hubble's law is consistent with a universe expanding in a uniform manner. There emerged two opposing viewpoints regarding the implications of this expansion.

1. Steady State Model [Bond48, Hoyl48] in which it was postulated that the universe retained a constant density in spite of the expansion by the creation of new matter.
2. BB Model [Frie22, Lema27] in which the universe originated from a state of much higher density in the past with gradually decreasing density as it expands.

The demise of the Steady State Model came with the Penzias and Wilson's discovery of the microwave background radiation [Penz65] and the subsequent proof that it has a blackbody spectrum. The density of a steady-state model was always what it is today, far too low to provide the photon interactions necessary for thermalization whereas in the BB model the background radiation can be explained as the remnant of the ever expanding and ever cooling universe.

Probably the most important of all evidences in support of the BB model is the accuracy with which it predicts the abundance ^4He . The abundance of ^4He has been extensively measured. The results for various nearby locations in our universe is given in Table 2.1 [Schr77]. The best value for the observed abundance is thought to be [Boes85]

$$0.242 \leq X_{obs.} \leq 0.248$$

where X is the mass fraction of ${}^4\text{He}$. This value is in extremely good agreement with the BB predicted primordial ${}^4\text{He}$ abundance [Ste86]

$$0.239 \leq X_{BB} \leq 0.251.$$

In fact, so far we do not know of any other method of production of ${}^4\text{He}$ which does as well as the BB nucleosynthesis. We are thus led to the belief that we are indeed living in a big bang universe. A point worth mentioning here is the result of calculations by Steigman et al. [Ste77, Yang79, Ste86], who have used the ${}^4\text{He}$ abundance to predict that the number N_L of light ν flavors (lepton types) in nature must be ≤ 4 . Interestingly, three lepton types (e^{-1} , μ^{-1} and τ^{-1}) have been already discovered implying that at most one additional light ν (extra lepton type) would be just barely allowed within the constraints of BB nucleosynthesis.

The assumptions inherent in the Standard Big Bang model are the following.

1. general relativity is the correct theory of gravity
2. the universe is homogeneous and isotropic
3. only the presently known particles were present during nucleosynthesis
4. the baryon number of the universe is positive
5. the lepton number of the universe is less than the photon number

In addition, it is commonly assumed that the so-called cosmological constant Λ

is 0 and the present photon temperature is 2.7° K. Then the final abundance of the elements produced within a volume element emerging from very high temperatures (BB conditions) depends on

1. The expansion rate $\theta = V^{-1} (dV/dt)$,
2. The difference between the proton and neutron mass fractions ($X_p - X_n$) during the onset of BB nucleosynthesis,
3. The baryon density (ρ_b) at the same point of time.

The nuclear reactions are incorporated into the framework by representing them in the form

$$N_i({}^{A_i}Z_i) + N_j({}^{A_j}Z_j) \Leftrightarrow N_k({}^{A_k}Z_k) + N_l({}^{A_l}Z_l) \quad (2.2)$$

where N_m is the number of nuclei with mass number A_m and atomic number Z_m . The rate equation that determines the evolution of the abundance of any nucleus i then has the form

$$\frac{dY_i}{dt} = \sum_{j,k,l} N_i \left(\frac{Y_l^{N_l} Y_k^{N_k}}{N_l! N_k!} [lk]_j - \frac{Y_i^{N_i} Y_j^{N_j}}{N_i! N_j!} [ij]_k \right) \quad (2.3)$$

where

$$Y_i = \frac{X_i}{A_i},$$

$$\sum_i X_i = 1,$$

$$[ij]_k = \rho_b N_A \langle \sigma v \rangle .$$

and the sum includes all reactions involving nucleus i . The reaction rates $\langle \sigma v \rangle$ are determined from the experimental cross sections. This procedure is outlined in more detail in chapter 5.

A typical reaction network employed in the standard BB calculations and the main results of the theory are shown in Figs. 2.1 and 2.2 respectively [Schr77].

2.2 Nonuniform Density Model (NDM)

Recently there has been a lot of renewed interest in the so-called Nonuniform density BB model (NDM) [Sale86, Alco87, Full88, Appl88] as opposed to the uniform density standard BB model (SM). Various theoretical arguments and computer simulations raise the possibility that the first-order phase transition from quark-gluon plasma to hadronic matter [Fig. 2.3], which must have occurred at some point during the early evolution of the universe may have introduced nonuniform fluctuations in the baryon density. In this scenario isothermal density fluctuations could arise because, when macroscopic regions of different phases coexist together in equilibrium, the quark-gluon plasma phase has a higher baryon concentration than does the hadron gas phase.

In addition, differential diffusivity of neutrons and protons in such environments

Figure 2.1: Network of nuclear reactions included in the computer code of [Wago73]. The exothermic directions are indicated by arrows, while the dashed line indicates that individual nuclear abundances were only computed for mass number $A < 12$.

Figure 2.2: Final abundances (by mass) produced in standard big bang models, whose only parameter is h , related to the present density and temperature [Schr77].

Quark-gluon $\rightarrow$ Hadronic matter phase transition 17 transition

Differential diffusivity of n and p

Figure 2.3: Sources of density fluctuation in the NDM.

could give rise to low density neutron rich regions and high density proton rich regions. This happens because protons, which interact via coulomb scattering, have small mean free paths and could not diffuse across great lengths whereas neutrons, which scatter more weakly via their magnetic dipole moment and nucleon-nucleon interaction, can.

The resulting nucleosynthesis can then be very different from that of the SM in which it is implicitly assumed that during the epoch of nucleosynthesis the baryonic density and chemical composition are homogenous. We now define the baryonic density parameter

$$\Omega = (\rho_b/\rho_c) \tag{2.4}$$

where ρ_b is the baryonic density and ρ_c is the critical density required to close the universe. In terms of the Hubble constant,

$$\rho_c = (3H_0^2/8G) = 5.7 \times 10^{-30} (H_0/55)^2 \text{ gm/cm}^3 \tag{2.5}$$

The values for H_0 are somewhat uncertain and the two limiting values, 55 and 110 km sec⁻¹ Mpc⁻¹ [Peeb78], yield 5.7×10^{-30} and 22.8×10^{-30} gm/cm³ as the limits for ρ_c .

The SM finds that ρ_b is required by the primordial abundances to be no more than a few percent of ρ_c ($0.01 \leq \Omega \leq 0.20$). Observationally, however, the study of the dynamics of galactic clusters suggests [Rees84] that the value of Ω could be greater

than the upper limit derived from primordial nucleosynthesis. In addition, theoretically there are reasons to believe that Ω could be as large as 1. As a consequence of these discrepancies, attempts have been made [Schr81, Zee82, Turn84] to account for the dark matter component by non-baryonic mass in the form of axions, massive neutrinos, or other relic particles from the big bang. It is often stated that primordial nucleosynthesis is the major constraint against the dark matter being baryonic.

In contrast, in the NDM, it is possible to reconcile the light isotope abundances with an $\Omega=1$ universe [Sale86]. Such a scenario would be naturally very attractive because there is no more need for any exotic forms of missing mass (the required additional dark matter could be now entirely baryonic), and the Friedman model solution for the state of the universe would be one of expanding for ever with constant curvature *i.e.* a flat universe.

One serious drawback of all NDM calculations is the inevitable over-production of ${}^7\text{Li}$ associated with them [Full88]. This is principally due to the large contribution from the ${}^3\text{He}(\alpha,\gamma){}^7\text{Be}$ reaction in the high baryon density regions, and ${}^3\text{H}(\alpha,\gamma){}^7\text{Li}$ reaction in the low baryon density regions. However, very recently, it has been argued [Mala88b] that the late time diffusion of neutrons into proton-rich regions of an inhomogeneous $\Omega=1$ early universe can significantly reduce the high ${}^7\text{Li}$ abundance to levels consistent with stellar observations. There are also counter-arguments [Kurk88] which show that this effect apparently does not solve the ${}^7\text{Li}$ anomaly, and its inclusion may create disagreements between predictions of the NDM with the abundances of some of the other lighter nuclides. In summary, the difficulty in the interpretation of the primordial ${}^7\text{Li}$ abundance data poses problems in the attempts to distinguish between the two BB models.

In the light of the above state of affairs, an independent check of the NDM predictions, using ${}^9\text{Be}$, was recommended by Boyd and Kajino [Boyd89]. ${}^9\text{Be}$ is somewhat more robust to stellar burning than ${}^7\text{Li}$ and therefore would be a better candidate. The abundance of this nuclide is less sensitive to stellar processing than that of ${}^7\text{Li}$. ${}^9\text{Be}$ has long been known to be produced in very low quantities in big bang nucleosynthesis. However, unfortunately, all BB calculations have so far yielded vanishingly small abundances for this nuclide. The SM and NDM predict the number ratio (${}^9\text{Be}/{}^1\text{H}$) to be 2.0×10^{-23} and 1.6×10^{-16} respectively.

Two reactions that could potentially contribute to the synthesis of ${}^9\text{Be}$ in the NDM are ${}^7\text{Li}({}^3\text{H},n){}^9\text{Be}$ and ${}^7\text{Li}({}^3\text{He},p){}^9\text{Be}$, previously not included in primordial nucleosynthesis calculations. Contribution of the reaction ${}^7\text{Li}({}^3\text{He},p){}^9\text{Be}$ turns out to be not important because ${}^7\text{Li}$ is produced in the n -rich region whereas ${}^3\text{He}$ is produced in the p -rich region thus limiting their interaction probability. That of ${}^7\text{Li}({}^3\text{H},n){}^9\text{Be}$, however, is believed to be rather significant, since both ${}^7\text{Li}$ and ${}^3\text{H}$ are produced in the n -rich regions.

Boyd and Kajino [Boyd89] have performed a calculation using a modified reaction network shown in Fig. 2.4; it is the same for nuclides up to C as that used in other recent calculations [Kaji88] except for the inclusion of the ${}^7\text{Li}({}^3\text{H},n){}^9\text{Be}$ and ${}^9\text{Be}({}^3\text{H},n){}^{11}\text{B}$ reactions. The results of the calculations are shown for the neutron-rich region in Fig. 2.5; the abundances produced in that region are nearly identical to those of Kajino et al. [Kaji88], with the exception of those of ${}^9\text{Be}$ and ${}^{10}\text{B}$, whether or not the ${}^7\text{Li}({}^3\text{H},n)$ reaction is included. However, that for ${}^9\text{Be}$ is increased by three orders of magnitude in the NDM by inclusion of that reaction; when the neutron- and proton-rich regions are combined, the abundance ratio is $({}^9\text{Be}/{}^1\text{H}) = 1 \times 10^{-13}$. Furthermore,

(${}^9\text{Be}/{}^1\text{H}$) is increased in the SM by seven orders of magnitude to 1.5×10^{-16} . In all these calculations, the parameter which fixes the fraction of the volume occupied by higher density regions was taken to be 0.5. Other parameters of the calculation are the same as those used by Kajino et al. [Kaji88].

The direct experimental measurement of the ${}^7\text{Li}({}^3\text{H},n){}^9\text{Be}$ cross section, although highly desirable, is presently complicated by the lack of availability of usable ${}^3\text{H}$ beam as well as the difficulty associated with the detection of the outgoing reaction products which requires a TOF facility. Under the circumstances, one way of estimating the cross section for ${}^7\text{Li}({}^3\text{H},n){}^9\text{Be}$ would be to infer it from data for a similar reaction, the ${}^7\text{Li}({}^3\text{He},p){}^9\text{Be}$ reaction. ${}^3\text{H}$ and ${}^3\text{He}$ being mirror nuclei, it is hoped that realistic estimates of the desired cross section could be made by suitably correcting for the difference in coulomb penetrability of the two nuclei.

Unfortunately there is also a paucity of existing data for the ${}^7\text{Li}({}^3\text{He},p){}^9\text{Be}$ reaction. However, since the cross section for that reaction is fairly simple to measure we have done so, and have used these data to infer the cross section for ${}^7\text{Li}({}^3\text{H},n){}^9\text{Be}$.

Figure 2.4: The reaction network used in the primordial nucleosynthesis calculation of Boyd and Kajino [Boyd89]. The two new reaction pathways, $^7\text{Li}(^3\text{H},n)^9\text{Be}$ and $^9\text{Be}(^3\text{H},n)^{11}\text{B}$, have previously not been included in primordial nucleosynthesis calculations.

Figure 2.5: Growth of the (mass) abundances of the elements in primordial nucleosynthesis in the NDM with increasing time. This figure is for neutron-rich region. The dashed curves show the abundances predicted without the ${}^7\text{Li}({}^3\text{H},n){}^9\text{Be}$ and ${}^9\text{Be}({}^3\text{H},n){}^{11}\text{B}$ reactions included.

Table 2.1

Summary of ^4He abundance determinations (including the uncertainties) in various locations [Schr77].

Location	Mass Fraction of ^4He
Interstellar medium and and young stars	0.26-0.32
Nearby normal galaxies	0.22-0.34
Large Magellanic cloud	0.24-0.27
Small Magellanic cloud	0.21-0.28
Galaxy I Zw 18	0.21-0.29
Galaxy II Zw 40	0.13-0.23

CHAPTER III

Experimental details

3.1 Beamline

The experiment was carried out in the beamline 45° south of the Ohio State University CN Van de Graaff accelerator. A schematic diagram of the beamline is shown in Fig.

3.1. The basic components on the beamline are

1. Ortec Scattering chamber,
2. Quadrupole Focussing Magnets,
3. Steering Magnets,
4. Beam Scanner,
5. Two Pumping Stations,
6. Two LN₂ Cold Traps,
7. Two Slit Boxes,
8. Faraday Cup.

OSU VAN DE GRAAFF ACCELERATOR BEAMLINE 45° S

Figure 3.1: The schematic of Ohio State University Van De Graaff accelerator, beamline 45° South.

The scattering chamber was equipped with a movable target ladder capable of accomodating four targets at the same time, any one of which could be centered with respect to the beam. The base plane of the scattering chamber could be rotated to achieve scattering angles between 15° and 160° . Two pairs of detectors were simultaneously deployed, one each in the forward and backward angles.

Singly ionized ^3He particles with bombarding energies ranging from 0.8 to 2.5 MeV were obtained from the accelerator. The bombarding energies were corrected for the energy loss in the target. The computed effective beam energies corresponding to the specific bombarding energies of the experiment are given in Appendix A along with an outline of the procedure. Typical beam currents on the target ranged between 50 to 100 nA. The low energy runs (≤ 1 MeV) were taken with a shorting rod installed in the mid plane of the accelerating column, also a reduced VdG tank pressure (100 psi; interlock at 75 psi) was used. This arrangement significantly improved both the quantity and stability of the beam.

3.2 ^3He Beam Charge State

Since the effective charge state of the beam after passing through the target and reaching the Faraday cup is of crucial importance in determining the amount of charge received by the Faraday cup, hence the absolute differential cross sections, an independent measurement of the fractional charge states (1^+ and 2^+) of the ^3He beam was attempted at a few energies in the energy range of interest. The availability of a variable magnet, located at the end of beamline 0° , capable of scanning the various

charge states made this measurement possible. A schematic of the experimental set-up is shown in Fig. 3.2. The ^3He beam after passing through a Li+C target, was analysed by the variable magnetic field to identify the two possible charge states. For each charge state, the elastically scattered ^3He particles from Carbon were detected by a solid state surface barrier detector for the purpose of normalization since the elastic scattering yield does not depend on the charge state if the energies and scattering angles are the same. The effective charge state of the beam can then be determined from the yields of the elastic peaks. The details are given below.

Let X_i = Fraction of charge state i^+

Y_i = Yield of the elastic peak for charge state i^+

Q_i = Collected charge scaler for yield Y_i

Then,

$$X_1 + X_2 = 1 \quad (3.1)$$

$$\text{and } \frac{X_1}{X_2} = \frac{(Q_1/Y_1)}{(Q_2/Y_2)} = \frac{Q_1 Y_2}{Q_2 Y_1} \quad (3.2)$$

Simultaneous solution of these equations gives

$$X_2 = \left(1 + \frac{Q_1 Y_2}{Q_2 Y_1}\right)^{-1} \quad (3.3)$$

Finally,

$$\begin{aligned} X &= ^3\text{He effective charge state} \\ &= X_1.1 + X_2.2 = (1 - X_2).1 + X_2.2 = 1 + X_2 \end{aligned} \quad (3.4)$$

Figure 3.2: The schematic of set up used to study the ^3He beam charge state.

A sample data sheet is shown in Table. 3.1 and the effective charge states for ^3He calculated in the above manner are shown in Fig. 3.3. The curve is a fit to the data using the fitting program CHARGE [Abdu88]. For comparison, the effective charge states for the specific beam energies of the present experiment, calculated from ^4He data found in literature [Mari68] are given in Appendix B. The difference in the region of overlap is seen to be less than 3%. Therefore, henceforth we will adopt the effective charge states from this reference [Mari68] in extracting the absolute values for the differential cross sections.

3.3 Detectors

The reaction products were detected by a pair of detector telescopes, each consisting of a 1000μ silicon surface barrier E detector preceded by a 100μ silicon surface barrier ΔE detector. A monitor detector was used alongside the detector telescopes to observe any possible depletion of the targets. The solid angle subtended by each of the detector telescopes was 2.763 msr. The monitor detector subtended 0.340 msr. The associated geometry including the location and dimension of the collimators is given in Appendix C. To reduce noise pick-up, the detector telescopes were electrically isolated from their holders by annular housings made of Teflon. A schematic of the detector set-up is shown on Fig. 3.4. A bias sheet for the detectors is given in Table 3.2.

The above arrangement clearly discriminated the proton groups from the competing deuteron groups originating from the $^7\text{Li}(^3\text{He},d)^8\text{Be}$ reaction. An appropriate

Table 3.1

A data sheet of the various experimental quantities required to calculate the effective charge state of ^3He beam.

E_{beam}	Q_1	Q_2	Y_1	Y_2	X_1	X_2	X
1.50	10288	82763	5419	6358	0.1273	0.8727	1.873
1.75	15257	105517	6282	5687	0.1158	0.8842	1.884
2.00	10822	121068	3833	3300	0.0715	0.9285	1.928
2.25	15932	390647	3676	3744	0.0399	0.9601	1.960

Figure 3.3: Experimentally measured fractional charge state distribution of ^3He beam, the curve is from the fitting program CHARGE [Abdu88].

Table 3.2

The bias sheet for the various detectors used in the experiment.

	ΔE_1	E_1	ΔE_2	E_2	Elastic
Detector	23-709H	11-030D	23-709I	14-105E	18-094C
Bias(V)	+30	+250	+30	+200	+150
Reverse Current(μa)	0.15	2.0	0.20	0.7	0.3

Figure 3.4: The schematic of the detector telescope components used in the experiment. "TA" refers to the Tantalum absorber foil in front of the detector, while " ΔE " and "E" refer, respectively, to the two detectors which comprise the detector telescopes.

Figure 3.5: The particle peaks arising from proton bombardment of the target at $E_{\text{beam}} = 1.975 \text{ MeV}$ and lab angle 140° , detected by a single surface barrier detector.

thickness of Tantalum was used in front of the telescopes to absorb other emitted particles, especially the α s from the ${}^7\text{Li}({}^3\text{He},\alpha){}^6\text{Li}$ reaction and the elastically scattered ${}^3\text{He}$ particles, while ensuring at the same time that the lowest as well as the highest energy protons stopped completely within the depletion layer of the E detector. The proton energies typically ranged from 8 to 12 MeV. A typical list of the simultaneously occurring nuclear reactions leading to particle products is given in Table 3.3. The energy loss of the particles as they pass through the various materials (e.g. Tantalum absorber and Silicon detectors) were calculated from the program LOSTENERGY [Marm73].

3.4 Targets

Targets were prepared by evaporating natural LiF powder from Tungsten boats onto $10\mu\text{gm}/\text{cm}^2$ carbon foils. To avoid accretion of contaminants from air onto the targets, the prepared targets were allowed to cool inside the evaporation chamber before venting the chamber with inert gas. The ${}^7\text{Li}$ thickness of the targets were determined by measuring the yield of the ${}^7\text{Li}(p,p_0){}^7\text{Li}$ reaction [Bard66] at 140° with a proton bombarding energy of 1.975 MeV. At this energy and angle, it was possible to kinematically resolve the elastic peaks corresponding to the isotopes ${}^6,{}^7\text{Li}$. Isotopic abundance of ${}^7\text{Li}$ in natural Li is about 93%. A typical energy spectrum is shown in Fig. 3.5. The ${}^7\text{Li}$ content of the targets typically ranged from 10 to $20\mu\text{gm}/\text{cm}^2$. The target thickness was monitored at the start and the end of the experimental runs by the monitor detector to ensure that there was no appreciable depletion of the Li content on the targets. The thicknesses were found to remain constant within

Table 3.3

Possible Reaction Channels at $E(^3\text{He})=2.5$ MeV leading to particle products, the outgoing particle energies (computed from the programs KINMAT [Rinc81] and LOSTENERGY [Marm73]) are indicated at various locations within the detector telescopes.

Reaction	E_x	E	TA+ ΔE	TA+ ΔE +E
		45°, 130°	45°, 130°	45°, 130°
$^7\text{Li}(^3\text{He},p)^9\text{Be}$	0.00	12.97,10.42	11.93, 9.18	-ve,-ve
$^7\text{Li}(^3\text{He},p)^9\text{Be}$	1.69	11.36,8.99	10.20, 7.58	-ve,-ve
$^7\text{Li}(^3\text{He},p)^9\text{Be}$	2.43	10.65,8.36	9.43,6.86	-ve,-ve
$^7\text{Li}(^3\text{He},p)^9\text{Be}$	3.05	10.06,7.83	8.78,6.24	-ve,-ve
$^7\text{Li}(^3\text{He},d)^8\text{Be}$	0.00	12.59,9.13	10.78,6.75	-ve,-ve
$^7\text{Li}(^3\text{He},d)^8\text{Be}$	3.04	9.95,6.90	7.74,3.78	-ve,-ve
$^{12}\text{C}(^3\text{He},p)^{14}\text{N}$	0.00	6.97,5.72	5.21,3.58	-ve,-ve
$^{12}\text{C}(^3\text{He},p)^{14}\text{N}$	2.31	4.70,3.68	2.00,-ve	-ve,-ve
$^{12}\text{C}(^3\text{He},p)^{14}\text{N}$	3.95	3.06,2.25	-ve,-ve	-ve,-ve
$^{19}\text{F}(^3\text{He},p)^{21}\text{Ne}$	0.00	14.05,12.80	13.08,11.75	-ve,-ve
$^{19}\text{F}(^3\text{He},p)^{21}\text{Ne}$	0.35	13.71,12.48	12.72,11.41	-ve,-ve
$^{19}\text{F}(^3\text{He},p)^{21}\text{Ne}$	1.75	12.34,11.17	11.26,10.00	-ve,-ve
$^7\text{Li}(^3\text{He},\alpha)^6\text{Li}$	0.00	11.36,6.90	-ve,-ve	-ve,-ve

10% during the runs. LN_2 traps were used on the beamline in the proximity of the scattering chamber to reduce carbon build-up on the targets.

3.5 Electronics and Data Acquisition

A schematic of the electronic circuitry used in the experiment for CAMAC based data acquisition with Lecroy 3500 system is shown in Fig. 3.6. The program UNICORN [Rinc85] performed the data manipulation and analysis. The parameter sheet for UNICORN is given in Appendix D. The four basic signals to be processed were

$$\begin{aligned} \text{Forward Telescope} & \begin{cases} 1. E_1 \\ 2. \Delta E_1 \end{cases} \\ \text{Backward Telescope} & \begin{cases} 3. E_2 \\ 4. \Delta E_2. \end{cases} \end{aligned}$$

The linear signal from each of the E detectors was amplified by ORTEC 571 module. One of the two outputs was directly sent to a Lecroy 3511 ADC while the second one was used to generate a logic pulse by sending it through a Single Channel Analyzer SCA . The threshold levels of the SCA were adjusted to eliminate low energy events. The output of the SCA was sent through the gate generator, *fanned out*, and finally sent to the E and ΔE ADC s used in the coincident mode. This allowed the sorting algorithm of the data acquisition system to select only the events of interest. A third signal from the logic *fan out* was sent to the $SCALER$ to be compared with the integrated area of the energy spectrum in the determination of the dead time

factor. The count rate of events was typically 10 Hz, and the deadtime was less than 1%. The other *SCALER* inputs were the integrated charge of the beam, and the real run time of the event files.

Appropriate software windows were set to define the following histograms

$$\text{Forward Telescope} \left\{ \begin{array}{l} 1. 2D (E_1, \Delta E_1) \\ 2. 1D (E_1) \\ 3. 1D (E_1), \text{ protons only} \\ 4. 1D (E_1), \text{ deuterons only} \end{array} \right.$$

$$\text{Backward Telescope} \left\{ \begin{array}{l} 5. 2D (E_2, \Delta E_2) \\ 6. 1D (E_2) \\ 7. 1D (E_2), \text{ protons only} \\ 8. 1D (E_2), \text{ deuterons only} \end{array} \right.$$

The E- ΔE plot at 1.3 MeV is shown Fig. 3.7. The corresponding energy spectrum as well as the constituent spectra for protons and deuterons are shown in Figs. 3.8a, 3.8b and 3.8c respectively. The various particle peaks of interest are seen to be clearly resolved in these spectra. The high energy background in the proton spectrum is due to the ${}^6\text{Li}({}^3\text{He}, p){}^8\text{Be}$ reaction, which has a higher Q-value than that for ${}^7\text{Li}({}^3\text{He}, p){}^9\text{Be}$.

Figure 3.6: The schematic of the electronic circuitry employed for data acquisition.

Figure 3.7: The 2-dimensional E- ΔE plot at ${}^3\text{He}$ beam energy 1.3 MeV and lab angle 160° .

Figure 3.8: The energy spectra for protons and deuterons together (Fig. 3.8a), protons only (Fig. 3.8b), and deuterons only (Fig. 3.8c).

CHAPTER IV

Results and Analysis

4.1 Angular Distributions

Angular distribution data were obtained in 50 KeV intervals in the ^3He bombarding energy range 0.8 to 2.5 MeV. For each energy, the yields of the 4 proton groups (p_0 , p_1 , p_2 , p_3) from $^7\text{Li}(^3\text{He},p)^9\text{Be}$ reaction (Fig. 3.8b) corresponding to excitations in ^9Be at $E_x = 0.0$, 1.69, 2.43 and 3.05 MeV were measured between lab angles 15° and 160° in 20° steps. The fitting procedure ERRFIT [Rinc86] was utilized. The yield of the deuteron groups (d_0 , d_1) from $^7\text{Li}(^3\text{He},d)^8\text{Be}$ (Fig. 3.8c) for transitions to the ground state and the broad first excited state of ^8Be at 3.04 MeV was also measured. The level schemes for $^8,9\text{Be}$ are exhibited in Figs. 4.1 and 4.2 respectively.

From the yields of the various particle groups the differential cross sections were determined using the program YIELD [Rath88] as follows.

Figure 4.1: The nuclear energy level scheme for ${}^8\text{Be}$ [Ajze84].

$$\frac{d\sigma}{d\Omega} = \frac{(\text{Yield})}{(\text{BeamFlux}) (\text{TargetDensity}) (\text{SolidAngle}) (\text{DetectorEfficiency})} \quad (4.1)$$

For computational purpose, it can be rewritten as

$$\frac{d\sigma}{d\Omega}_{(mb/sr)} = 26.602 \frac{Y}{B} \frac{T}{S} \frac{C}{D} \frac{1}{A} \frac{1}{E} \quad (4.2)$$

where

Y = Yield Counts

T = Target atomic mass in amu

C = Effective charge state of the beam

B = Beam current full scale in μa

S = Charge scaler #

D = Target density in $\mu g/m/cm^2$

A = Solid angle in sr

E = Detector efficiency in %

The differential cross sections for the p_0 group at a few representative bombarding energies are shown in Fig. 4.3a. The excitation function for this proton group has been previously measured [Woli61, Din66] at two lab angles, 20° and 165° . The present results are in fair agreement with those measurements. The differential cross sections for the other proton groups (p_1 , p_2 , p_3) at 2.4 MeV are shown in Fig. 4.3b. The curves represent legendre polynomial fits to the experimental data.

$$\frac{d\sigma}{d\Omega} = \sum a_l P_l(\cos\theta) \quad (4.3)$$

The characteristic differential patterns of these proton groups are consistent with the results at 3 MeV reported earlier [Sana69]. The differential cross sections for the deuteron groups (d_0 , d_1) at 2.5 MeV, displayed on Fig. 4.4, are in agreement with previous measurements [Bond77]. The indicated errors on the data correspond only to the statistical uncertainties.

4.2 Penetrability Factors

The wave function describing the relative motion of particles with relative angular momentum l is expressible as

$$\psi_l(r\theta\phi) = \frac{\chi_l(r)}{r} Y_{lm}(\theta\phi) \quad (4.4)$$

where the solution $\chi_l(r)$ to the radial part of the Schrodinger equation satisfies the differential equation

$$\frac{d^2\chi_l(r)}{dr^2} + \frac{2\mu}{\hbar^2} [E - V_l(r)] \chi_l(r) = 0 \quad (4.5)$$

with $V_l(r)$ being the effective radial potential for the l th partial wave:

Figure 4.3: Differential cross sections for the proton groups from ${}^7\text{Li}({}^3\text{He}, p){}^9\text{Be}$; p_0 (Fig. 4.3a), and p_1 , p_2 , p_3 (Fig. 4.3b). Energies are given as laboratory energies for the incident ${}^3\text{He}$ particles. The indicated errors on the data correspond only to the statistical uncertainties.

Figure 4.4: The differential cross sections for the deuteron groups (d_0, d_1) from ${}^7\text{Li}({}^3\text{He},d){}^8\text{Be}$ reaction at 2.5 MeV beam energy. The indicated errors are only due to the statistical uncertainties.

$$V_l(r) = \begin{cases} \frac{l(l+1)\hbar^2}{2\mu r^2} + \frac{Z_1 Z_2 e^2}{r} & r > R \\ \frac{l(l+1)\hbar^2}{2\mu r^2} + V_{coul} + V_{nucl} & r < R. \end{cases} \quad (4.6)$$

where R is the nuclear channel radius.

The potential is shown schematically in Fig. 4.5. The penetrability factor is then expressed in terms of the solutions of the radial wave equation as follows

$$P_l = \frac{\chi_l^*(\infty)\chi_l(\infty)}{\chi_l^*(R)\chi_l(R)}. \quad (4.7)$$

The exact solution of the radial wave equation is impossible because the nuclear potential is not well known. However in the WKB approximation, the following expression for the penetrability factor can be derived [Clay83].

$$P_l = \left(\frac{E_B - E}{E} \right)^{1/2} \exp \left[-\frac{2\sqrt{2\mu}}{\hbar} \int_R^{R_0} \left(\frac{E_{coul} R}{r} + \frac{E_l r^2}{r^2} - E \right)^{1/2} dr \right] \quad (4.8)$$

where E is the center of mass energy,

$$E_B = E_{coul} + E_l,$$

$$E_{coul} = \text{Coul Barrier} = 1.44 \left(\frac{Z_1 Z_2}{R} \right),$$

$$E_l = \text{Centrifugal Barrier} = \frac{l(l+1)\hbar^2}{2\mu r^2},$$

Z_1 and Z_2 being the atomic numbers of the reacting nuclei, and R the channel radius.

Figure 4.5: The schematic of the potential governing the motion of one nucleus relative to another. For $r < R$, the strongly attractive short-range nuclear force results in a deep negative potential. For $r > R$, the coulomb force dominates. The radial motion of the two nuclei adds an effective centrifugal potential.

Equation 4.5 can be integrated to give

$$P_l = \left(\frac{E_{coul}}{E} \right)^{1/2} \exp(-W_l) \quad (4.9)$$

where

$$W_{l=0} = \left(\frac{4Z_1Z_2e^2}{\hbar v} \right) \left[\frac{\pi}{2} - \sin^{-1} \left(\frac{E}{E_{coul}} \right)^{1/2} - \left(\frac{E}{E_{coul}} \right)^{1/2} \left(1 - \frac{E}{E_{coul}} \right)^{1/2} \right],$$

with the leading coefficient expressible as

$$\frac{4Z_1Z_2e^2}{\hbar v} = 0.525 \left(\frac{ARZ_1Z_2E_{coul}}{E} \right)^{1/2}, \quad (4.10)$$

and

$$W_{l \neq 0} = W_0 + 2 \left[\frac{l(l+1)E_l}{E_{coul}} \right]^{1/2} \left[1 - \left(\frac{E}{E_{coul}} \right)^{1/2} \right]. \quad (4.11)$$

The case of the zero angular momentum is of special interest because most nuclear reactions of astrophysical interest occur through $l=0$ (s-wave) interactions. The dominance of s-wave interactions comes about because the penetrability factor is smallest for $l=0$.

The penetrability factor is somewhat sensitive to the values assumed for the channel radius R . We have assumed the channel radii to be

$$R = 1.1(A_p^{1/3} + A_t^{1/3}) \text{ fm} \quad (4.12)$$

where A_p and A_t are the mass numbers of the projectile and target nuclei.

In the above calculation the fuzziness of the nuclear radius is not taken into consideration. This can be remedied to some extent by using a more realistic form of the nuclear potential (e.g. Woods-Saxon) and numerically evaluating the following expression

$$P = \frac{4}{\left(2\theta + \frac{1}{2\theta}\right)^2} \quad (4.13)$$

where

$$\theta = \exp \left(\int k \, dx \right) \quad \text{and} \quad \hbar k = \sqrt{2\mu(V - E)}$$

for the penetrability factor, also derived [Merz70] under WKB approximation. The limits of integration for θ are the points at which the incident particle's energy is equal to the potential; thus they depend on the energy of the incident particle and nature of the barrier height. For example, for the energy indicated on Fig. 4.6, the integration would be carried out between points a and b .

The results of such a calculation is shown in Fig. 4.7. The potential ($V_{nuc} + V_{coul}$) used in this calculation is given in Eqn. 4.12 and shown in Fig. 4.6 as a function of the radial distance. For the sake of comparison the penetrability factors computed from Eqn. 4.6 are also indicated on Fig. 4.7. The Woods-Saxon parameters for the nuclear part of the potential were adopted from literature [Pere76] with small adjustments.

Figure 4.6: The sum of the nuclear and coulomb potentials specified by Eqn. 4.14. A typical incident energy and the corresponding limits of the numerical integration for penetrability are also indicated on the fig.

$$V(r) = \begin{cases} \frac{-140.0}{1+e^{\left(\frac{r-2.29}{0.508}\right)}} + \frac{1.44Z_1Z_2e^2}{2R_{coul}} \left(3 - \left(\frac{r}{R_{coul}}\right)^2\right) & r < R_{coul} \\ \frac{-140.0}{1+e^{\left(\frac{r-2.29}{0.508}\right)}} + \frac{1.44Z_1Z_2e^2}{r} & r > R_{coul} \end{cases} \quad (4.14)$$

$$R_{coul} = 1.3 (7^{1/3}) = 2.49 \text{ fm}$$

This coulomb radius is different from the radius given in Eqn. 4.12. The results of the barrier calculation, however, are not sensitive to the coulomb radius, as it is a slowly varying function of the radius.

4.3 Total Cross Sections

From the first legendre coefficients of the fits to the differential cross sections of the reactions, the total cross sections were estimated at each energy using the relation

$$\sigma = 4\pi a_0 \quad (4.15)$$

The total cross sections for the two predominant proton groups (p_0 , p_2) from ${}^7\text{Li}({}^3\text{He},p){}^9\text{Be}$ are shown in Fig. 4.8. In ${}^7\text{Li}({}^3\text{He},p){}^9\text{Be}$ reaction, there is evidence for three resonances at bombarding energies 1.1, 1.5 and 2.2 MeV. They correspond to states in the compound nucleus ${}^{10}\text{B}$ at excitations 18.43, 18.80 and 19.29 MeV respectively (Q value for ${}^7\text{Li}({}^3\text{He},\gamma){}^{10}\text{B}$ reaction is 17.79 MeV). The energy level scheme for ${}^{10}\text{B}$ is shown in Fig. 4.9 [Ajze84]. These resonances have been previously

Figure 4.7: The results of WKB calculations for penetrability, the solid line uses a square well nuclear potential whereas the dotted line is with a more realistic Woods Saxon potential.

observed in both ${}^7\text{Li}({}^3\text{He},\gamma){}^{10}\text{B}$ and ${}^7\text{Li}({}^3\text{He},\alpha){}^6\text{Li}$ reactions [Paul65a, Paul65b]. The 1.1 MeV resonance has also been suggested by the 110° excitation functions for the p_0 and p_2 groups from ${}^7\text{Li}({}^3\text{He},p){}^9\text{Be}$ reaction [Stan71].

The ${}^7\text{Li}({}^3\text{He},p_0){}^9\text{Be}$ cross sections were next converted to those for ${}^7\text{Li}({}^3\text{H},n_0){}^9\text{Be}$ by using the penetrability factors given in Eqn. 4.6.

The cross-section for ${}^7\text{Li}({}^3\text{H},n){}^9\text{Be}$ reaction for transition to the ground state of ${}^9\text{Be}$, estimated in the present work, exhibits a broad resonance at a bombarding energy 0.8 MeV and a weak structure at 1.7 MeV. These would correspond to known states in the compound nucleus ${}^{10}\text{Be}$ at excitations 17.79 and 18.55 MeV respectively (Q value for ${}^7\text{Li}({}^3\text{H},\gamma){}^{10}\text{Be}$ reaction is 17.25 MeV). Similar results are suggested by the data of Val'mer et al. [Val'm61] on the neutron yield of the ${}^7\text{Li}({}^3\text{H},n)$ reaction, although the data are for all reactions resulting from ${}^7\text{Li}({}^3\text{H},n)$, so are difficult to compare directly to the present results. The energy levels of ${}^{10}\text{Be}$ are displayed in Fig. 4.10 [Ajze84].

The p_0 cross-sections measured for ${}^7\text{Li}({}^3\text{He},p)$ and calculated for ${}^7\text{Li}({}^3\text{H},n)$ are shown together in Fig. 4.11. The indicated errors are those for the measured ${}^7\text{Li}({}^3\text{He},p)$ cross section. It is to be noted, however, that the calculated ${}^7\text{Li}({}^3\text{H},n_0)$ cross sections are sensitive to the values assumed for the channel radius. To produce the cross sections indicated by the solid circles we have assumed the channel radius to be $R = 1.1 (A_{Li}^{1/3} + A_{He}^{1/3}) \text{ fm}$, where A_{Li} and A_{He} are the mass numbers of the target and projectile nuclei. This differs from the standard prescription [Clay83] for the radius, but gave essentially identical results for the penetrability as those obtained from the WKB approximation using actual nuclear potentials instead of the

Figure 4.8: Total cross sections ($\sigma=4\pi a_0$, a_0 being the first legendre coefficient) for p_0 and p_2 groups.

2

Figure 4.10: The nuclear energy level scheme for ^{10}Be [Ajze84].

usual hard sphere approximation. Such a modification appears to be important when the projectile and the target are of comparable size. Using a channel radius given by the standard prescription²⁰ of $R = 1.4 A_L^{1/3} \text{ fm}$ gave a cross section which was larger by about a factor of two.

4.4 Isospin Considerations

The virtual identity of the np and pp forces or equivalently near charge independence of the nuclear forces, manifests itself in the properties of the so-called mirror nuclei, which are obtained one from the other by transforming all neutrons into protons, and vice versa. More convincing are the arguments derived from even A nuclei. The pair ^{10}Be and ^{10}B is one such example.

To treat the charge independence in a more intuitive manner, isospin formalism is introduced and the nucleon is endowed with one additional degree of freedom namely its isospin. A nucleus with A nucleons, Z protons, and N neutrons, has a total charge Ze . The total charge can be written in terms of the third component of the total isospin as

$$Ze = e(I_3 + \frac{1}{2}A) \quad (4.16)$$

where

Figure 4.11: Total cross sections measured for $^7\text{Li}(^3\text{He},p_0)^9\text{Be}$ and calculated for $^7\text{Li}(^3\text{H},n_0)^9\text{Be}$. The indicated uncertainties are those for the measured $^7\text{Li}(^3\text{He},p)$ cross sections.

$$I_3 = \sum_i^A I_{3,i} .$$

For the two nuclei ^{10}Be ($Z=4$) and ^{10}B ($Z=5$), the above equation yields respectively the values -1 and 0 for I_3 implying that ^{10}Be states must all have total isospin of at least 1 whereas ^{10}B states have 0 and possibly 1 or higher isospins. In literature [Ajze84], the higher lying states in ^{10}B all have isospin equal to 1 but no information is yet available about the similar states in ^{10}Be , the compound nucleus for the $^7\text{Li}(^3\text{H},n)^9\text{Be}$ reaction. Since, however, the ^{10}Be states must all have isospin of at least 1, we have assumed that the resonances excited in the $^7\text{Li}(^3\text{He},p)$ reaction, which correspond to low-lying states in ^{10}Be , also have isospin of 1, and therefore that they would also be excited in the $^7\text{Li}(^3\text{H},n)$ reaction. The resonances observed in the neutron yield data of Val'mer et al. [Val'm61] appear to indicate the validity of this assumption.

4.5 DWBA calculations

An independent theoretical check of the relative magnitudes of the differential cross sections for $^7\text{Li}(^3\text{He},p)^9\text{Be}$ and $^7\text{Li}(^3\text{H},n)^9\text{Be}$ was made by using the DWBA code CHUCK3 [Kunz84]. The optical model parameters for these reactions at such low energies as investigated in the present work are not available in literature. Therefore, the parameters [Pere76] available at the lowest bombarding energies were adopted for the calculation. The input files employed for the 1 MeV calculation are given in Table 4.1. The results of this calculation at a two representative energies are shown in

Fig. 4.12. The difference between ${}^7\text{Li}({}^3\text{He},\text{p}){}^9\text{Be}$ and ${}^7\text{Li}({}^3\text{H},\text{n}){}^9\text{Be}$ differential cross sections as a function of the bombarding energy seems to reflect the pattern of the experimental results.

Since, however, DWBA represents only a part of the reaction amplitudes for the low energy reactions investigated in the present work, the above calculations are meant to be only a consistency check to see the qualitative features of the coulomb barrier corrections between the two reactions in the energy range of interest. The ratio of the integrated cross sections is approximately equal to 4 at 1.0 MeV, and 2 at 2.0 MeV.

Figure 4.12: The results of the DWBA calculations for ${}^7\text{Li}({}^3\text{He}, p){}_0\text{Be}$ and ${}^7\text{Li}({}^3\text{H}, n){}_0\text{Be}$ differential cross sections at 1.0 MeV and 2.0 MeV bombarding energies. The ratios of the integrated cross sections at these energies are approximately equal to 4 and 2 respectively.

```

1      4      LI7(H3,P)BE9 EX=0.0 L=0 ELAB=1.0
55.    3.0
10     2     -3.0
1      19.
1.0    3.      2.      7.      3.      1.40      0.61      1.
1      1
1.     -150.    1.20    .70      -7.      1.31      0.77
-4.     -10.    1.20    .70
11.202  1.      1.      9.      4.      1.4      1.
2      2
-1.     -45.    1.35    .65      -11.0    1.50    .33
-2      1      0      2      2      1      0      0 900.
-16.676 2.      1.      7.      3.
-1.     -1.      1.2    .70      0.
0.      0.      2.      2.      50.
9

```

```

1      4      LI7(H3,N)BE9 EX=0.0 L=0 ELAB=1.0
55.    3.0
10     2     -3.0
1      19.
1.0    3.      1.      7.      3.      1.40      0.61      1.
1      1
1.     -150.    1.20    .70      -7.      1.31      0.77
-4.     -10.    1.20    .70
10.439  1.      0.      9.      4.      1.4      1.
2      2
-1.     -45.    1.35    .65      -11.0    1.50    .33
-2      1      0      2      2      1      0      0 900.
-16.676 2.      1.      7.      3.
-1.     -1.      1.2    .70      0.
0.      0.      2.      2.      50.
9

```

Table 4.1: Input parameters employed for DWBA calculations using the code CHUCK3 [Kunz84].

CHAPTER V

Astrophysical Discussions

5.1 Astrophysical S-factor

In stellar environments, the nuclear reactions which occur between charged particles are greatly inhibited by their coulomb barrier. At low energies the cross sections are typically rapidly falling functions due to the exponential Gamow velocity factor which represents the probability of penetration through the coulomb repulsion. The astrophysical S-factor, defined below, on the other hand represents the intrinsically nuclear parts of the probability for the occurrence of a nuclear reaction, whereas the other factors represent energy-dependences that are of nonnuclear nature.

The S-factor for a nuclear reaction is given by

$$S(E) = \sigma(E)E \exp\left(\frac{2\pi Z_1 Z_2 e^2}{\hbar v}\right) \quad (5.1)$$

where $\sigma(E)$ is the interaction cross section, Z_1 and Z_2 are the atomic numbers of the reacting nuclei, and v is the relative velocity.

It is evident from the definition that the advantage of using the S-factor is that the strongly energy dependent factors in the nuclear cross section are now explicitly factored out and the function $S(E)$, in the absence of resonances, is typically slowly varying with energy. This feature is especially useful when extrapolations must be made into the low astrophysical energy regions of interest, where experimental data are not available as is frequently the case. The reliability of estimating the S-factor at the extrapolated energies is much higher than that of the cross section.

For numerical convenience, the S-factor is usually written in the form [Clay83]

$$S(E) = \sigma(E) E \exp(bE^{-1/2}) \quad (5.2)$$

where the value of the parameter b is

$$b = 31.28 Z_1 Z_2 A^{1/2} \text{ keV}^{1/2}$$

and A is the reduced atomic weight defined to be

$$A = \frac{A_1 A_2}{A_1 + A_2}.$$

If the center of mass energy E is expressed in keV and $\sigma(E)$ in barn, then $S(E)$ will be in its appropriate unit, namely keV.barn.

The S-factors for both ${}^7\text{Li}({}^3\text{He}, p){}^9\text{Be}$ and ${}^7\text{Li}({}^3\text{H}, n){}^9\text{Be}$ reactions are shown in

Fig. 5.1. The curves represent fits to the data points, assuming broad resonances at laboratory energies 0.8 and 1.7 MeV, together with a flat background [Rolf88].

$$S(E) = \frac{c_0}{(c_1 - E)^2 + c_2^2} + \frac{c_3}{(c_4 - E)^2 + c_5^2} + c_6 \quad (5.3)$$

The values of the parameters c_i thus obtained are given in Table 5.1. A tabulated data sheet of the S-factors along with the other relevant experimental quantities from which they were determined appears in Tables 5.2 and 5.3.

Figure 5.1: S-factors for ${}^7\text{Li}({}^3\text{He}, p_0){}^9\text{Be}$ and ${}^7\text{Li}({}^3\text{H}, n_0){}^9\text{Be}$, that for ${}^7\text{Li}({}^3\text{H}, n_0)$ was derived from its calculated cross sections. The curves are resonance fits, together with flat backgrounds.

Table 5.1

The parameters for fitting the S-factors according to Eqn. 5.3.

${}^7\text{Li}({}^3\text{H},n_0)$	${}^7\text{Li}({}^3\text{He},p_0)$	
$c_0 = 128.06$	966.12	keV.b.MeV ²
$c_1 = 0.67$	0.67	MeV
$c_2 = 0.22$	0.17	MeV
$c_3 = 43.63$	6535.45	keV.b.MeV ²
$c_4 = 1.20$	1.07	MeV
$c_5 = 0.56$	0.84	MeV
$c_6 = 224.02$	1624.54	keV.b

Table 5.2

Summary sheet of the experimentally measured quantities, calculated penetrabilities (Eqn. 4.9) and the S-factors for ${}^7\text{Li}({}^3\text{He}, p_0)$ reaction.

E_{cm} MeV	$\sigma({}^3\text{He})$ mb	$\delta\sigma({}^3\text{He})$ mb	$S({}^3\text{He})$ MeV-b	$\delta S({}^3\text{He})$ MeV-b	$P_l({}^3\text{He})$
0.420	0.0665	0.0061	16.20	1.490	0.00418
0.490	0.1862	0.0171	19.78	1.820	0.00997
0.515	0.2799	0.0325	23.06	2.680	0.01300
0.522	0.3174	0.0365	24.45	2.810	0.01390
0.552	0.5054	0.0500	29.82	2.950	0.01840
0.594	1.0088	0.0948	41.97	3.940	0.02650
0.628	1.2703	0.0978	41.26	3.180	0.03420
0.677	1.6518	0.1520	38.76	3.570	0.04790
0.699	1.8880	0.1736	38.66	3.560	0.05510
0.747	2.2075	0.1633	34.60	2.560	0.07240
0.784	2.2941	0.1720	29.74	2.230	0.08780
0.854	3.0211	0.2326	28.41	2.190	0.12100
0.891	3.1787	0.2352	25.65	1.900	0.14100
0.950	3.2737	0.1571	21.14	1.010	0.17600
0.995	3.4407	0.2133	18.99	1.180	0.20500
1.029	3.6385	0.1746	18.01	0.864	0.22800
1.065	3.5658	0.2139	15.79	0.947	0.25400
1.092	3.7695	0.1772	15.45	0.726	0.27300
1.154	3.7979	0.2261	13.03	0.782	0.32100
1.206	4.3989	0.1980	13.36	0.601	0.36200
1.251	4.6739	0.2103	12.78	0.575	0.39800
1.312	5.3320	0.2239	12.76	0.536	0.44800
1.336	5.0053	0.2202	11.39	0.501	0.46800
1.371	5.3265	0.2503	11.30	0.531	0.49700
1.553	6.8503	0.2736	10.56	0.423	0.64500
1.666	7.0304	0.2742	9.17	0.358	0.73100
1.736	6.7584	0.2636	8.02	0.313	0.78100
1.814	6.9011	0.2898	7.43	0.312	0.83100

Table 5.3

Summary sheet of the experimentally measured quantities, calculated penetrabilities (Eqn. 4.9) and the S-factors for ${}^7\text{Li}({}^3\text{H},n_0)$ reaction.

E_{cm} MeV	$P_l({}^3\text{H})$	P_l^{RATIO}	$\sigma({}^3\text{H})$ mb	$\delta\sigma({}^3\text{H})$ mb	$S({}^3\text{H})$ MeV-b	$\delta S({}^3\text{H})$ MeV-b
0.420	0.250	59.81	3.9770	0.365	1.2720	0.1168
0.490	0.358	35.91	6.6860	0.614	1.5250	0.1401
0.515	0.398	30.61	8.5680	0.995	1.7650	0.2050
0.522	0.409	29.42	9.3380	1.074	1.8730	0.2154
0.552	0.456	24.78	12.524	1.239	2.2590	0.2235
0.594	0.523	19.73	19.904	1.870	3.1300	0.2941
0.628	0.575	16.81	21.354	1.644	3.0500	0.2348
0.677	0.647	13.51	22.316	2.053	2.8130	0.2588
0.699	0.679	12.32	23.260	2.139	2.7830	0.2560
0.747	0.742	10.25	22.627	1.674	2.4480	0.1811
0.784	0.788	8.97	20.578	1.543	2.0750	0.1556
0.854	0.864	7.14	21.571	1.661	1.9330	0.1489
0.891	0.898	6.37	20.248	1.498	1.7170	0.1270
0.950	0.943	5.36	17.547	0.842	1.3740	0.0659
0.995	0.970	4.73	16.274	1.009	1.2060	0.0748
1.029	0.986	4.32	15.718	0.754	1.1220	0.0538
1.065	0.998	3.93	14.013	0.841	0.9625	0.0578
1.092	1.000	3.66	13.796	0.648	0.9229	0.0433
1.154	1.000	3.11	11.811	0.703	0.7463	0.0444
1.206	1.000	2.76	12.141	0.546	0.7348	0.0330
1.251	1.000	2.51	11.731	0.528	0.6861	0.0309
1.312	1.000	2.23	11.890	0.499	0.6663	0.0280
1.336	1.000	2.14	10.711	0.471	0.5906	0.0260
1.371	1.000	2.01	10.706	0.503	0.5776	0.0271
1.553	1.000	1.55	10.602	0.424	0.5191	0.0208
1.666	1.000	1.37	9.6320	0.376	0.4491	0.0175
1.736	1.000	1.28	8.6510	0.337	0.3927	0.0153
1.814	1.000	1.20	8.2810	0.348	0.3659	0.0154

5.2 Thermonuclear Reaction Rate

The thermonuclear reaction rate per pair of particles depends on their velocity distribution $\phi(v)$ (assumed to be Maxwellian) and the magnitude of their nuclear interaction cross section.

$$\lambda = \langle \sigma v \rangle = \int_0^\infty v \sigma(v) \phi(v) dv \quad (5.4)$$

where

$$\phi(v)dv = \left(\frac{\mu}{2\pi kT} \right)^{3/2} \exp\left(-\frac{\mu v^2}{2kT} \right) 4\pi v^2 dv$$

Eqn. 5.4 may be rewritten in terms of energy as

$$\lambda = \langle \sigma v \rangle = \int_0^\infty v(E) \sigma(E) \psi(E) dE \quad (5.5)$$

where now

$$\psi(E)dE = \phi(v)dv = -\frac{2}{\sqrt{\pi}} \frac{E^{1/2}}{(kT)^{3/2}} \exp\left(-\frac{E}{kT} \right) dE$$

is the velocity distribution expressed in terms of energy.

We are thus led to the final expressions for the thermonuclear reaction rate per mole of particles

$$N_A < \sigma v > = \left(\frac{8}{\mu\pi} \right)^{1/2} \frac{N_A}{(kT)^{3/2}} \int_0^\infty E \sigma(E) \exp\left(-\frac{E}{kT}\right) dE \quad (5.6)$$

in terms of the cross section or

$$N_A < \sigma v > = \left(\frac{8}{\mu\pi} \right)^{1/2} \frac{N_A}{(kT)^{3/2}} \int_0^\infty S(E) \exp\left(-\frac{E}{kT} - bE^{-1/2}\right) dE \quad (5.7)$$

in terms of the S-factor. In these equations, N_A is the Avogadro's number.

In practical units Eqn. 5.7 assumes the form

$$N_A < \sigma v > = 1.20 \times 10^9 \left(\frac{8}{A\pi} \right)^{1/2} \frac{1}{(13.81T_9)^{3/2}} \int_0^\infty S(E) \exp\left(-\frac{E}{0.086T_9} - 0.032 bE^{-1/2}\right) dE \quad (5.8)$$

where the reaction rate is now in cm^3/sec , $S(E)$ in keV.b , E in MeV , the temperature in T_9 , and A and b are as defined in Eqn. 5.2.

In the absence of resonances, the effectiveness of a nuclear reaction for nucleosynthesis will be determined by the location of the so-called Gamow window. It is centered around the energy where the argument of the exponential function in the integral in Eqn. 5.5 becomes a maximum giving rise to the majority of contributions to the integral. This feature makes it feasible to carry out a numerical integration, if so desired. The energy and the width of the Gamow window are given by

$$E_0 = \left(\frac{bkT}{2} \right)^{2/3} = 1.220(Z_1^2 Z_2^2 AT_6^2)^{1/3} \text{ keV}, \quad (5.9)$$

$$\Delta = \frac{4}{\sqrt{3}}(E_0 kT)^{1/2} = 0.75(Z_1^2 Z_2^2 A T_6^5)^{1/6} \text{ keV.} \quad (5.10)$$

In the particular case when the S-factor is nearly constant, Eqn. 5.7 reduces to the simple expression

$$N_A \langle \sigma v \rangle = \frac{4.336 \times 10^5}{A Z_1 Z_2} S_0(\text{keV.b}) \tau^2 \exp(-\tau) \text{ cm}^3/\text{sec} \quad (5.11)$$

where

$$\tau = 42.48 \left(\frac{Z_1^2 Z_2^2 A}{T_6} \right)^{1/3}$$

The Gamow window for the ${}^7\text{Li}({}^3\text{H},n)$ reaction, at $T_9=1$, is centered around $E_{cm} = 0.3 \text{ MeV}$ with a width of 0.4 MeV . Evidently, the low energy tail of the 0.8 MeV resonance in the above reaction will be important for the primordial nucleosynthesis of ${}^9\text{Be}$. It is therefore essential that the effect of this resonance on the ${}^7\text{Li}({}^3\text{H},n)$ reaction be known.

The thermonuclear reaction rate for ${}^7\text{Li}({}^3\text{H},n)$ was calculated by numerical integration of Eqn. 5.8 in two ways

- i) using the S-factor determined from the ${}^7\text{Li}({}^3\text{H},n_0)$ cross sections which were calculated from the ${}^7\text{Li}({}^3\text{He},p_0)$ data. This approach utilizes the careful approach to calculation of the penetrability discussed in section 4.2.
- ii) assuming the ${}^7\text{Li}({}^3\text{H},n_0)$ S-factor to be same as that determined for ${}^7\text{Li}({}^3\text{He},p_0)$ from the measured cross sections. Since the S-factor should reflect just the nuclear

aspect of the reactions, this should also be a reasonable approach.

The results of the two approaches are shown together in Fig. 5.2. They are seen to differ by about an order of magnitude. The upper cut off energy of the numerical integration was taken to be 2.5 MeV which was also the uppermost energy of the present investigation. This cut off would suffice for ${}^7\text{Li}({}^3\text{H},n){}^9\text{Be}$ reaction because of its very low Gamow energy.

The algorithm used to numerically evaluate the integral in Eqn. 5.8 as a function of the temperature T_9 is given in Appendix E.

In the study of Boyd and Kajino [Boyd89] of the effect of the ${}^7\text{Li}({}^3\text{H},n)$ reaction on the primordial abundance of ${}^9\text{Be}$, it was assumed that the rate for this reaction was half of that for ${}^7\text{Li}(d,n)$ reaction [Caug85]. This reaction rate is also indicated on Fig 5.2.

Figure 5.2: The thermonuclear reaction rates for ${}^7\text{Li}({}^3\text{H}, n_0){}^9\text{Be}$.

CHAPTER VI

Conclusion

The present indirect investigation of the ${}^7\text{Li}({}^3\text{H},\text{n}){}^9\text{Be}$ reaction via the ${}^7\text{Li}({}^3\text{He},\text{p}){}^9\text{Be}$ reaction has reconfirmed the possibility that the contribution of BB to the nucleosynthesis of ${}^9\text{Be}$, especially in the nonuniform model, could be more significant than hitherto assumed.

It can be seen, however, that the rate assumed by Boyd and Kajino [Boyd89] is larger, perhaps by a factor of 5, than that inferred from the present results. If the present results turn out to be more representative of the correct reaction rate than that assumed by Boyd and Kajino, the predicted ${}^9\text{Be}$ abundance will be decreased from that predicted in [Boyd89]. It should be noted, however, that the decrease in the predicted reaction rates would not be as large as that indicated in Fig. 5.2, since processes other than ${}^7\text{Li}({}^3\text{H},\text{n})$ are also responsible for producing ${}^9\text{Be}$. However, given that the predicted abundances [Boyd89] are already an order of magnitude lower than the present observational limit of ${}^9\text{Be}$ in highly metal deficient stars, the prospects for detecting an abundance lower than that by another factor of three or so are bleak.

However, it should also be noted that the uncertainty on the present value of the thermonuclear reaction rate is rather sensitive to assumed parameters such as those of the coulomb barrier.

Precise predictions for the number ratio (${}^9\text{Be}/{}^1\text{H}$) arising out BB nucleosynthesis and a clear distinction between the two BB models will have to await future measurements of the ${}^7\text{Li}({}^3\text{H},n){}^9\text{Be}$ cross section over the entire energy-range of astrophysical interest, which, as pointed out before, goes down to fraction of a MeV in ${}^3\text{H}$ energy.

*We shall not cease from exploration.
And the end of all our exploring
Will be to arrive where we started
And know the place for the first time.*

T.S. Eliot (Four Quartets)

APPENDIX A

Effective beam energy in target

E_{beam} MeV	$\rho(^7\text{Li})$ $\mu\text{gm}/\text{cm}^2$	$\rho(^{19}\text{F})$ $\mu\text{gm}/\text{cm}^2$	$\rho(^7\text{Li})/2$ $\mu\text{gm}/\text{cm}^2$	$\rho(^{19}\text{F})/2$ $\mu\text{gm}/\text{cm}^2$	E_{eff} MeV
0.800	17.12	46.47	8.56	23.23	0.746
0.900	17.12	46.47	8.56	23.23	0.849
1.000	11.77	31.95	5.88	15.97	0.967
1.100	11.77	31.95	5.88	15.97	1.067
1.250	11.77	31.95	5.88	15.97	1.220
1.400	17.89	48.56	8.94	24.28	1.357
1.500	12.67	34.39	6.33	17.19	1.470
1.600	17.89	48.56	8.94	24.28	1.560
1.750	12.67	34.39	6.33	17.19	1.723
1.900	12.67	34.39	6.33	17.19	1.874
2.000	20.87	56.65	10.43	28.32	1.959
2.100	17.39	47.20	8.69	23.60	2.066
2.250	17.39	47.20	8.69	23.60	2.218
2.400	11.71	31.78	5.85	15.89	2.380
2.500	11.71	31.78	5.85	15.89	2.480
2.600	5.24	14.22	2.62	7.11	2.591

APPENDIX B

Effective beam charge state

E MeV	E/4 MeV	v 10^8 cm/sec	X_2	X
0.800	0.20000	6.20	0.68	1.68
0.900	0.22500	6.60	0.74	1.74
1.000	0.25000	6.95	0.78	1.78
1.100	0.27500	7.25	0.81	1.81
1.250	0.31250	7.75	0.85	1.85
1.400	0.35000	8.20	0.88	1.88
1.500	0.37500	8.50	0.90	1.90
1.600	0.40000	8.75	0.92	1.92
1.750	0.43750	9.15	0.93	1.93
1.900	0.47500	9.55	0.94	1.94
2.000	0.50000	9.80	0.94	1.94

APPENDIX C

Geometry of detector assembly

APPENDIX D

Unicorn program parameters

File: parsp.dat Printed Tue Jul 25 21:21:12 1989 Login: RATH Page: 1

```

1
2  ACORN Rev TR:1.00 25-JUL-89 21:20:36 Run ID: LI7 Seq#: 12
3  Sort program: DEV$LIB:[ACORN]UNISORT.EXE;238
4
5  User mode: OFF
6  Eventfile: DEV$SCRATCH:[RATH]G_97.EVT;1

7  Event buffer size: 256 words
8  --- Coincident Groups ---
9    CG# # of Sources
10   1    2
11   2    4
12  --- Histograms ---
13  # Type Sizes CG# SS# Offsets Gains Cond'n Label Pointer
14  1 2D      1
15      X: 64 1 0 128
16      Y: 64 2 0 128
17  2 1D 512 1 1 0 16 0 E1 4096
18  3 1D 512 1 2 0 16 0 dE1 4608
19  4 2D      2 0 E2-dE2 5120
20      X: 64 3 0 128
21      Y: 64 4 0 128
22  5 1D 512 2 3 0 16 0 E2 9216
23  6 1D 512 2 4 0 16 0 dE2 9728
24  7 1D 512 1 1 0 16 1 E1(d) 10240
25  8 1D 512 1 1 0 16 2 E1(p) 10752
26  9 1D 512 1 2 0 16 1 dE1(d) 11264
27 10 1D 512 1 2 0 16 2 dE1(p) 11776
28 11 1D 512 2 3 0 16 3 E2(d) 12288
29 12 1D 512 2 3 0 16 4 E2(p) 12800
30 13 1D 512 2 4 0 16 3 dE2(d) 13312
31 14 1D 512 2 4 0 16 4 dE2(p) 13824
32  --- Conditions ---
33  # Type
34  1 inside 2D Window: XSS: 1 XGain: 128 xlimits: 0 40 YSS: 2
35  2 inside 2D Window: XSS: 1 XGain: 128 xlimits: 0 50 YSS: 2
36  3 inside 2D Window: XSS: 3 XGain: 128 xlimits: 2 40 YSS: 4
37  4 inside 2D Window: XSS: 4 XGain: 128 xlimits: 5 39 YSS: 3

```

APPENDIX E

Algorithm to evaluate reaction rate

```

DIMENSION YY(200), XSEQ(200), YSEQ(200)
COMMON ZP, ZT, XMP, XMT, EL, EH, A0, A1, A2, A3, A4, A5, A6
OPEN (UNIT=2, FILE='RTDATA.DAT', STATUS='NEW', CARRIAGECONTROL='LIST')
WRITE (*, 60)
60  FORMAT (' Enter projectile charge # : ', $)
ACCEPT *, ZP
WRITE (*, 61)
61  FORMAT (' Enter target charge # : ', $)
ACCEPT *, ZT
WRITE (*, 62)
62  FORMAT (' Enter projectile mass # : ', $)
ACCEPT *, XMP
WRITE (*, 63)
63  FORMAT (' Enter target mass # : ', $)
ACCEPT *, XMT
WRITE (*, 87)
87  FORMAT (' Enter T9 min : ', $)
ACCEPT *, XLO
WRITE (*, 88)
88  FORMAT (' Enter T9 max : ', $)
ACCEPT *, XHI
204 17  WRITE (*, 17)
17  FORMAT (' Enter lower limit of integration in MeV (cm) : ', $)
ACCEPT *, EL
WRITE (*, 86)
86  FORMAT (' Enter upper limit : ', $)
ACCEPT *, EH
202 64  WRITE (*, 64)
64  FORMAT (' Enter A0 : ', $)
ACCEPT *, A0
WRITE (*, 65)
65  FORMAT (' Enter A1 : ', $)
ACCEPT *, A1
WRITE (*, 66)
66  FORMAT (' Enter A2 : ', $)
ACCEPT *, A2
WRITE (*, 68)
68  FORMAT (' Enter A3 : ', $)
ACCEPT *, A3
WRITE (*, 81)
81  FORMAT (' Enter A4 : ', $)
ACCEPT *, A4
WRITE (*, 82)
82  FORMAT (' Enter A5 : ', $)
ACCEPT *, A5
WRITE (*, 83)
83  FORMAT (' Enter A6 : ', $)
ACCEPT *, A6
100  DELTA=(XHI-XLO)/55.0
X=XLO
JJ=0
333  X=X+DELTA
IF (X.GT.XHI) GO TO 8887
CALL S6 (X,Y)
JJ=JJ+1
XSEQ(JJ)=X
YSEQ(JJ)=Y
YY(JJ)=Y
GO TO 333
8887  DO 808 I=1, JJ
DO 807 N=1, I
IF (YY(I)-YY(N)) 806, 807, 807
806  YYY=YY(I)
YY(I)=YY(N)
YY(N)=YYY
807  CONTINUE
808  CONTINUE
DO 600 J=1, JJ
WRITE (2, 55) XSEQ(J), YSEQ(J)
55  FORMAT (1X, E10.3, ', ', E10.3)
600  CONTINUE
9997  WRITE (*, +)
TYPE *, 'OUTPUT IS IN FILE RTDATA.DAT'
CALL EXIT
END

```



```

C      SUBROUTINE S6 (X,Y)
COMMON ZP,ZT,XMP,XMT,EL,EH,A0,A1,A2,A3,A4,A5,A6
REACTION RATE * Avon IN CM3/SEC
PI=3.1415926
AVO=6.022E23
XKEV=1.602E-6
XK=1.38E-16
AMU=1.66E-24
D=1.0/3.0
DD=1.0/6.0
ZPSQ=ZP*ZP
ZTSQ=ZT*ZT
XPT=XMP*XMT/(XMP+XMT)
XMU=XPT*AMU
ED=(EH-EL)/5000.0
FJ=0.0
DO 30 KJI=1,4999
XKJI=XKJI
YJ=EL+ED*XKJI
EO=0.001+1.220*(ZPSQ+ZTSQ*XPT*(1000.0*X)**2.0)**D
DELTA=0.001*0.750*(ZPSQ+ZTSQ*XPT*(1000.0*X)**3.0)**DD
TAU=42.48*(ZPSQ+ZTSQ*XPT/(1000.0*X))**D
BBB=31.23+ZP*ZT*SQR(XPT/1000.0)
EEJ=(YJ/(0.086*X))+(BBB/(SQR(YJ)))
CJ=EXP(-EEJ)
DJ=((A0)/((YJ-A1)**2.0+A2**2.0))
1  +((A3)/((YJ-A4)**2.0+A5**2.0))+A6
33  EJ=CJ*DJ
FJ=FJ+EJ*ED
30  CONTINUE
Y=1.545E-3*FJ*SQR(8.0/(XMU*PI))/((13.81*X)**1.5)
RETURN
END

```

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