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A semantics for groups and events

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The Ohio State University, 1988

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"Meaning". In: *Language Files*, third edition, A. Bissantz and K. Johnson (eds.) Advocate Publishing Group, Reynoldsburg, Ohio, 1985. Reprinted as "What Is Meaning?" in *Language Files*, fourth edition, C. McManis, D. Stollenwerk and Z.-S. Zhang (eds.) Advocate Publishing Group, Reynoldsburg, Ohio, 1987.

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TABLE OF CONTENTS

ACKNOWLEDGMENTS	ii
VITA	iii
LIST OF FIGURES	vi
CHAPTER	
I. ADVERBIAL MODIFICATION AND THE INTERPRETATION OF DISTRIBUTIVE PREDICATES	1
0. A few preliminaries	1
1. The distributive-collective dichotomy	12
2. "Intrinsic" distributives and collectives	17
3. Distributive predication as predication on groups	19
4. Group-sensitive adverbials	26
5. Conjoining collective and distributive verb phrases	33
II. GROUP ACTION AND SPATIO-TEMPORAL PROXIMITY	50
0. Introduction	50
1. Vagueness and Ambiguity	51
2. The "proximity" uses of group-sensitive adverbials	54
3. Semantic effects of adverbial position	60
4. Quantifier scope	61
5. Event structure and group action	70
6. Fragment 2	79
7. Examples	85
8. Locative and temporal readings	100
9. Adjectival and object-oriented readings	107
10. Distributive predicates revisited	115
11. Conclusion	120
III. THE ALGEBRA OF GROUPS AND THE ALGEBRA OF EVENTS	123
0. Introduction: Tightening up the model	123
1. The algebra of groups	126
2. A note on <i>respectively</i> constructions	139

3. Committees, etc.	142
4. The algebra of events as a semilattice	151
5. Restricting the structure of events	154
6. Unaugmented and augmented frames	167
IV. QUANTIFIERS, GROUP-LEVEL PROPERTIES AND AGREEMENT	173
0. Are quantifiers sensitive to event structure?	173
1. Different types of group-level events	175
2. Event-sensitive quantification	177
3. Additivity, inclusion and exclusion	181
4. An argument-restriction account of agreement	192
5. Some problems	199
6. Agreement and the inclusion/exclusion distinction	205
7. Distributive determiners and plural agreement	209
8. <i>Fewer than two</i> and <i>more than one</i>	213
9. Remaining problems	215
LIST OF REFERENCES	224

LIST OF FIGURES

FIGURE

PAGE

1
2
3

78
78
116

CHAPTER I
ADVERBIAL MODIFICATION AND
THE INTERPRETATION OF DISTRIBUTIVE PREDICATES

0. A few preliminaries. Let us assume that the universe of discourse of a model for the semantic interpretation of English or similar languages contains, in addition to individuals in the usual sense, "groups" which have these individuals as members. These groups will be used in the interpretation of plural expressions. For example in a sentence like (1), we will assume that the verb phrase functions as a predicate on the group of students, a group which contains each of the individual students as a member:

(1) The students gathered in the hallway.

Note that groups are assumed to be elements of the universe of discourse, and not type-theoretic constructions built up on the basis of the universe of discourse¹. Perhaps this qualifies them in some sense as "individuals" themselves (cf. Link 1983), but at least for now I will reserve the term "individual" for non-groups, that is, for entities like John, Mary or Bill.

¹This does not exclude the possibility that the universe of discourse itself is defined inductively or type-theoretically on the basis of some more primitive set, however.

It may seem that letting groups into the universe of discourse must involve assigning them to the same logical type as individuals, and that if groups and individuals are of the same logical type, then groups must not be sets (or else individuals are sets). This is not the case, however; model structures have been developed in which the universe of discourse contains both sets and individuals (Hoeksema 1983, 1987). This chapter will mostly be concerned with issues that generalize across different formalizations of the notion of group, so I will take no stand for the present on whether groups are to be identified with sets or not.

In fact, let us assume just a bare minimum about the structure of the universe of discourse. Let a model frame F be a 4-tuple $\langle U, U_I, \sqsubseteq, \in \rangle$, where:

- (2)a. $U_I \subseteq U$
- b. $\sqsubseteq$ is a partial ordering in U .
- c. $\in$ is a binary relation in U such that
 - $(x \in g \text{ implies } x \in h) \text{ iff } g \sqsubseteq h$.

Intuitively, U is the class of groups and individuals (or more accurately it is the domain of discourse; in case there are any additional sorts of entity besides groups and individuals, we might also want to include these in U), U_I is the set of individuals, $\sqsubseteq$ is the subgroup relation, and $\in$ is the group-membership relation. By a model, we understand a 5-tuple $M = \langle U, U_I, \sqsubseteq, \in, I \rangle$ where $\langle U, U_I, \sqsubseteq, \in \rangle$ is a model frame and I is a function mapping expressions from a

fragment of some language to elements of U or type-theoretic constructs on U , so as to define truth conditions for sentences of the language in question.

Now a little thought should convince anyone that this definition will allow many models which are not plausible candidates for interpreting English.² However, I will defer the task of constraining the model structure until later, and simply point out now that this definition is general enough to allow most of the better-known proposals concerning model-structure for the interpretation of plurality.

Consider, for example, the sort of model suggested in Link (1987a). Here, the universe of discourse is simply required to be a complete join-semilattice E , partially ordered by a relation $\leq_1$. The atoms of E are understood as ordinary individuals, the non-atomic elements of E as groups. The partial order $\leq_1$ is understood simultaneously as the subgroup relation and the group membership relation.

To see that this sort of model is compatible with the definition above, let $U = E$, $U_I =$ the set A of atoms in E , and $\subseteq = \leq_1$. Condition (2)a., that $U_I \subseteq U$, follows immediately, since the set of atoms of a semilattice must be a subset of the semilattice itself. Condition (2)b., that $\subseteq$ be a partial ordering in U , is also immediate, since the

²For instance models whose frames form non-set-theoretic semilattices of the sort discussed in Landman (1987) pp. 17-21.

semilattice ordering $\leq_1$ is a partial order on E by definition.

Condition (2)c., that E be a binary relation in U such that $(x \in g \text{ implies } x \in h) \text{ iff } g \sqsubseteq h$, also follows very simply and straightforwardly: Since $\sqsubseteq = E = \leq_1$, all we need to show is that $(x \leq_1 g \text{ implies } x \leq_1 h) \text{ iff } g \leq_1 h$. Taking the left-to-right direction first, assume that $x \leq_1 g$ implies $x \leq_1 h$. For reductio, assume that $g \not\leq_1 h$. Set x equal to g . Since partial orders are reflexive, it follows that $x \leq_1 g$. By the reductio assumption, $x \leq_1 h$. But this contradicts our premise that $x \leq_1 g$ implies $x \leq_1 h$; therefore the reductio assumption was false, and $g \leq_1 h$. For the right-to-left direction, assume $g \leq_1 h$. For reductio, assume $x \leq_1 g$ and $x \not\leq_1 h$. Since $x \leq_1 g$ and $g \leq_1 h$, it follows by the transitivity of partial orderings that $x \leq_1 h$. But this contradicts the reductio assumption, which therefore must be false; $x \leq_1 g$ implies $x \leq_1 h$. So Link-style semilattices qualify as model frames, as defined above.

Consider now the sort of model proposed in Lønning (1987) or Link (1983, 1984a, 1984b). Here, the universe of discourse is a complete boolean algebra B with non-empty set of atoms A^0 . As in the join semilattice model outlined

³I follow Lønning in using B for the boolean algebra and E for its set of atoms; Link uses E for the algebra and A (or A^0) for the atoms. The requirement that B have a non-empty set of atoms is also Lønning's; Link imposes the stronger requirement that B be atomic.

above, atoms are understood as individuals and non-atoms as groups. To see that this sort of model is compatible with the requirements given in (2), let $U = B$, $U_I = E$, and $\subseteq = \leq$, where $\leq$ is the usual partial ordering imposed by the join operation in B . Reasoning proceeds exactly as in the case of the join semilattice, above. Note that since power set algebras are complete atomic boolean algebras, the requirements in (2) also allow models which take the universe of discourse to be a power set whose singletons are understood as individuals and non-singletons as groups. (This is the practical effect of proposals in Scha 1981, for example.)

A number of authors (e.g. Massey 1976, Wald 1977) have suggested that the domain of discourse should form a mereology, such as the Leonard and Goodman (1940) "Calculus of Individuals", with the mereological fusion operation giving, for any set of individuals, the group consisting of those individuals. The strong similarity between boolean algebra and mereology suggests that this sort of model too will meet our conditions, and in fact this is easy to show.

By a mereology we understand a pair $\langle A, l \rangle$ satisfying the axioms in (4), where A is a set, l is a primitive relation in A (understood as "is disjoint from") and $<$, $\circ$, and Fu are defined as in (3).

$$(3)a. \ x < y =_{def} x \ l \ y \rightarrow x \ l \ x$$

$$b. \ x \circ y =_{def} \exists z [x < z \wedge z < y]$$

$$c. x \text{ Fu } \alpha \text{ def } x \text{ l } \alpha \leftrightarrow [y \in \alpha \rightarrow x \text{ l } y]$$

$$(4)a. [\exists x x \in \alpha] \rightarrow [\exists y y \text{ Fu } \alpha]$$

$$b. [x < y \wedge y < x] \rightarrow x = y$$

$$c. x \circ y \leftrightarrow \neg x \text{ l } y$$

To show that mereologies qualify as frames under our definition, let $U = A$, let U_i be any subset of A , and let $\sqsubseteq = E = <$. We need only show that $<$ is a partial order (i.e. reflexive, transitive and antisymmetric); from there reasoning is identical to that for join semilattices and boolean algebras. To prove reflexivity, assume $\neg x < x$. This is equivalent to $\neg [x \text{ l } x \rightarrow x \text{ l } x]$ by definition (3)a. But this is a contradiction, so $<$ is reflexive. To prove transitivity, assume $w < x$ and $x < y$. By definition (3)a., $x \text{ l } w \rightarrow x \text{ l } w$ and $x \text{ l } y \rightarrow x \text{ l } x$. Therefore $x \text{ l } y \rightarrow x \text{ l } w$, which is equivalent to $w < y$. So $<$ is transitive. Antisymmetry is postulated directly as axiom (4)b.

The requirements in (2) also allow the sort of model suggested in Hoeksema (1983, 1987), where the universe of discourse U is generated by a set U_i as follows:

$$(5)a. U_0 = U_i$$

$$b. U_{n+1} = U_n \cup \text{POW}_{\leq 2}(U_n), \text{ where } \text{POW}_{\leq 2}(X) \text{ is the set of all subsets of } X \text{ with cardinality greater than or equal to 2.}$$

$$c. U = \bigcup_n U_n$$

Here, the elements of U_I are understood as individuals; groups are modeled as non-null non-singleton sets. The subgroup relation is modeled as the subset relation (restricted to non-null non-singletons), and the group membership relation is modeled as the set membership relation (also restricted to eliminate empty or singleton sets).

To confirm that this sort of model is admitted by (2), let $U = U$, $U_I = U_I$, $\sqsubseteq = \subseteq \cap (U \times U)$, and $\in = \in \cap (U \times U)$. Condition (2)a., that $U_I \subseteq U$, is immediate from (5), since any set is a subset of its union with other sets. Restricting the subset relation to U does nothing to alter the fact that it is a partial order, so (2)b. is met also. Condition (2)c., that $(x \in g \text{ implies } x \in h) \text{ iff } g \sqsubseteq h$, also continues to hold: It should be obvious that $(x \in g \text{ implies } x \in h) \text{ iff } g \subseteq h$ for all sets g, h ; therefore it also holds for all non-null non-singleton sets g, h . Since $\in$ and $\sqsubseteq$ differ from $\in$ and $\subseteq$ only in that they are undefined for null or singleton sets, it follows that $(x \in g \text{ implies } x \in h) \text{ iff } g \sqsubseteq h$.

Ladusaw (1982) also proposes a model in which groups are modeled by non-null non-singleton sets (and in which individuals are not modeled by sets); unlike Hoeksema's proposal, however, Ladusaw's does not allow for groups to contain other groups. In our terms, we take U_I as given, and let $U = U_I \cup \text{POW}_{\geq 2}(U_I)$, $\sqsubseteq = \subseteq \cap (U \times U)$, and $\in = \in \cap (U$

$\times U$). Ladusaw's system is thus like Hoeksema's after step 1 in the inductive definition of U . Reasoning similar to that outlined above for Hoeksema's system will show that Ladusaw's is also compatible with our notion of a model frame.

As a final example, consider the sort of model proposed in Landman (1987). Here, the domain of discourse A_ω is defined inductively on the basis of a set A as follows:

- (6)a. $A_0 = A$
- b. $A_{n+1} = \text{POW}(A_n) - \{\emptyset\}$
- c. $A_\omega = \text{POW}(\bigcup_{n < \omega} A_n) - \{\emptyset\}$

Formally, Landman's A_ω is fairly similar to Hoeksema's U except that A_ω does include singleton sets and does not include non-sets. There are more serious conceptual differences in how the elements of A_ω are to be construed, however. Though the elements of A_0 are termed "individuals", they are not elements of A_ω and play no role in the semantics of actual expressions, but serve only as the basis for defining A_ω . Ordinary singular predicates take singletons containing elements of A_0 (e.g. $\{j\}$) in their denotations; these singleton sets are the objects which model individuals in the intuitive sense. Groups are modeled as singletons containing sets; for example the group of John and Mary is modeled as $\{\{j, m\}\}$. Non-singletons are used as a technical device in the interpretation of certain predicates, but are not conceived as entities with

properties in their own right, and in some sense "are not really there"⁴.

To see that Landman's sort of model qualifies under the requirements set out in (2), let $U = A_\omega$; $U_I = \{X \in A_1 \mid \text{card}(X) = 1\}$; $\sqsubseteq = R \subseteq S(U) \times S(U)$ such that xRy iff $f(x) \subseteq f(y)$, where $S(U)$ is the set of singletons in U and f maps any singleton set onto its element; $\sqsubseteq = R \subseteq S(U) \times S(U)$ such that xRy iff $x \subseteq f(y)$.

Requirement (2)a., that $U_I \subseteq U$, follows from (6): Note first of all that $U_I = \{X \in A_1 \mid \text{card}(X) = 1\}$ and $A_1 = \text{POW}(A_0) - \{\emptyset\}$; therefore for all $x \in U_I$, it must be the case that $x \in \text{POW}(A_0)$, i.e. that $x \subseteq A_0$. The subsets of a set are also subsets of its union with other sets, so $x \subseteq \bigcup_{\alpha < \omega} A_\alpha$; in other words $x \in \text{POW}(\bigcup_{\alpha < \omega} A_\alpha)$. Since the cardinality of x is required to be 1, x is also an element of $\text{POW}(\bigcup_{\alpha < \omega} A_\alpha) - \{\emptyset\}$, that is, of A_ω . Since all elements of U_I are elements of A_ω and $U = A_\omega$, $U_I \subseteq U$.

Requirement (2)b., that $\sqsubseteq$ be a partial order, also holds. Partial orders, again, are just reflexive, transitive, antisymmetric relations. Recall that $x \subseteq y$ iff $f(x) \subseteq f(y)$, where f is a function mapping singleton sets onto their members. Since the subset relation is a partial order, it will be the case for all x, y, z that $f(x) \subseteq f(x)$, that $f(x) \subseteq f(y)$ and $f(y) \subseteq f(x)$ implies $f(x) \subseteq f(x)$, and

⁴Landman, p. 53. The use of non-singletons in Landman's semantics will be explained more fully below.

that $f(x) \subseteq f(y)$ and $f(y) \subseteq f(x)$ implies $f(x) = f(y)$. But then it must be the case that $x \subseteq x$ (reflexivity), that $x \subseteq y$ and $y \subseteq x$ implies $x = y$ (transitivity), and that $x \subseteq y$ and $y \subseteq x$ implies $x = y$ (antisymmetry). So $\subseteq$ is a partial order.

Requirement (2)c., that $(x \in g \text{ implies } x \in h) \text{ iff } g \subseteq h$, also goes through. What we need to show here is that $(x \subseteq f(g) \text{ implies } x \subseteq f(h)) \text{ iff } f(g) \subseteq f(h)$, where x , g and h are all singleton sets. For the left-to-right direction, assume $x \subseteq f(g)$ implies $x \subseteq f(h)$. This is equivalent to $f(x) \in f(g)$ implies $f(x) \in f(h)$. But for all sets or individuals y , $y = f(x)$ for some singleton x ; so we have $y \in f(g)$ implies $y \in f(h)$. But this is just the definition of "subset", so we have $f(g) \subseteq f(h)$. For the right-to-left direction, assume $f(g) \subseteq f(h)$. Then for all y , $y \in f(g)$ implies $y \in f(h)$, and in particular $f(x) \in f(g)$ implies $f(x) \in f(h)$ for any singleton x . But since $f(x)$ is the only element of x , it will be the case for any A that $f(x) \in A$ is equivalent to $x \subseteq A$, so it follows that $x \subseteq f(g)$ implies $x \subseteq f(h)$. Landman's proposed structure for the universe of discourse therefore is consistent with the requirements suggested above.

The fact that the requirements in (2) are so weak as to admit such structurally and conceptually different model frames as those proposed by Link, Lønning, Scha, Massey,

Wald, Hoeksema, Ladusaw and Landman makes these requirements "uninteresting" in some sense. In fact the requirements admit a number of structurally bizarre frames which probably no one would seriously suggest; there is no question at all but that they do not sufficiently constrain the class of models available for the interpretation of English, and I do not intend to suggest that they do.

There are advantages to making use of such an unconstrained class of model frames, however: it allows us to discuss general problems in the semantics of plurality in a relatively neutral format, where it is reasonably clear that the problems really are general, and not theory-dependent artifacts of the particular conception of groups we happen to be working in. In this way we can also better separate purely linguistic issues from ontological ones, and better determine exactly which aspects of linguistic structure have a bearing on how groups are structured.

One issue which generalizes across different theories of groups involves so-called "distributive" predicates (or perhaps more accurately, distributive readings or understandings⁵ of predicates). Early analyses of these

⁵I use the terms reading and understanding here as in Zwicky and Sadock (1975). An ambiguous expression (e.g. *bark*) is assigned more than one distinct reading by the grammar. However, even unambiguous expressions may be vague or "lack specification", allowing for a range of understandings which fail to qualify as distinct readings; e.g. *person* refers vaguely, rather than ambiguously, to both male and female people.

predicates treated them as applying to individuals; more recent analyses in general have treated them as applying to groups⁶. The remainder of this chapter will review the basic notion of distributivity and some of the evidence for treating distributives as group-level predicates, point out what appears to be a serious problem with such an approach, and outline an alternative account in which distributives are not necessarily group-level. This alternative account is offered mainly as a devil's-advocate solution; an examination of its defects will help justify refinements to the group-level analysis of distributivity to be offered in later chapters. Both the problem and the proposed solution will be presented from the perspective of the very general notion of a model frame as defined above, so it should be clear both that the problem is not specific to a particular theory of groups, and that the solution can be implemented in a very wide variety of structurally different models.

1. The distributive-collective dichotomy. It has long been noticed that sentences like (7) can be true in either of two distinct sorts of circumstance: in this example either John and Mary jointly lifted the piano, or else they each lifted the piano.

(7) John and Mary lifted the piano.

⁶See references on page 20.

The understanding in which John and Mary lifted the piano together is termed the collective understanding; the understanding in which they each lifted the piano is the distributive understanding.

I will take no position for the moment as to whether the collective/distributive dichotomy in this sort of example constitutes authentic ambiguity, or simple vagueness. Whether we adopt an ambiguity analysis or a vagueness analysis, the question arises of what parts of a sentence like (7) contribute the availability of both a collective and a distributive understanding. The three most immediate possibilities are that the subject NP is vague or ambiguous, that the main VP is, and that both are.

In the context of generalized quantifier theory, it seems particularly easy at first to treat this sort of example as ambiguous, with the subject noun phrase as the source of the ambiguity; in a sentence like (7), we let the noun phrase denote either the set of sets which contain the group of John and Mary as a member (for the collective reading), or else the set of sets which contain John and contain Mary (for the distributive reading).

We assume that the predicate denotation includes all the groups and individuals that lifted the piano, and let a sentence be true iff its predicate denotation is an element of its subject denotation. The ambiguity falls out automatically.

An alternative analysis in which noun phrases are unambiguous but verb phrases are ambiguous would assign only a "group" reading to the subject of (7) (so that it denoted either the group of John and Mary, or else the set of sets containing this group). On the collective reading, the VP denotes the set of all groups and individuals that lifted the piano, as before. On the distributive reading, the VP denotes the set containing all the individuals who lifted the piano, plus all groups of such individuals. If the group of John and Mary is in the distributive denotation of the VP, so is John and so is Mary (and vice versa); applying a distributive predicate to a group comes out logically equivalent to applying it to each of the group's members.

Note that ascribing ambiguity to the verb phrase does not require that verbs are lexically ambiguous. In fact, the systematicity with which the collective/distributive dichotomy occurs indicates that it is not a lexical matter. Another possibility, developed in detail by Roberts (1987a), is that the distributive reading is derived from the collective reading by an implicit adverb-like operator. For present purposes, however, it matters less what produces the ambiguity than that the ambiguity can be located in the verb phrase.

One can imagine a vagueness analysis like the ambiguity analysis just sketched, but in which each verb phrase has only a single denotation, equivalent to the union of its

collective and distributive denotations on the ambiguity analysis. On this view, a group could be in the denotation of *lifted the piano* either because it lifted the piano (as a group), or because each of its members lifted the piano.

The possibility also exists that both the noun phrase and the verb phrase are vague or ambiguous, though of course we would want to avoid positing multiple ambiguities unless necessary. Consideration of one particular type of double-ambiguity analysis will be helpful later in clarifying certain issues, however, and therefore seems worthwhile.

Given the preceding discussion, the most obvious sort of double-ambiguity analysis may be one in which a noun phrase like *John and Mary* denotes either the group of John and Mary or else the set of sets which contain both John and Mary, and in which a verb phrase like *lift the piano* denotes either the set of groups and individuals that lifted the piano, or else the set containing the individuals that lifted the piano, plus all groups of these individuals; in other words an analysis which essentially combine the two ambiguity analyses already discussed. There is at least one other relatively obvious possible double-ambiguity analysis, however, namely one in which distributively understood VPs denote sets of individuals, and do not have groups in their extensions at all. Collectively understood VPs continue to denote sets of groups and individuals, and noun phrases are ambiguous in the same way as before.

Probably the most fundamental contrast between this analysis and the others discussed so far is that it treats distributive predication as predication on individuals, rather than predication on groups. This is not an unintuitive claim; on its distributive reading, a sentence like *John and Mary lifted the piano* seems to predicate something of John and of Mary, rather than of the group of John and Mary as distinct from its members.

This view of distributive predication seems to force ambiguity in the noun phrase as well as the verb phrase: If *John and Mary* unambiguously denotes the group of John and Mary, then it will never be in the distributive denotation of *lifted the piano*, and *John and Mary lifted the piano* is incorrectly predicted to lack a (non-contradictory) distributive reading. Conversely, if *John and Mary* unambiguously denotes the set of sets that have both John and Mary as members, then it is hard to see where the collective reading comes from, since it involves predicating a property of the group of John and Mary, and not of John and Mary as individuals. It therefore appears that if verb phrases are ambiguous in the way suggested, then noun phrases must be as well.

As we shall see later, this impression is false, and comes from failing to consider other possible denotations for the noun phrase.

2. "Intrinsic" Distributives and Collectives. It will be noticed that certain predicates seem intrinsically distributive, and do not appear to allow collective understandings at all. For example the sentence *John and Mary were asleep* indicates that John and Mary were each asleep, and does not allow the possibility that the group of John and Mary somehow slept while John and Mary as individuals were awake.

It is actually somewhat questionable whether every member of a group must be asleep in order for the group as a whole to count as asleep. Sentence (8), e.g., seems to allow a small number of soldiers who were not asleep, especially if the total number of soldiers was large.

(8) The soldiers were asleep.

Allowing groups to sleep even if all their members do not does not amount to extending the collective/distributive dichotomy to sentences like (8). Even the distributive understandings of sentences with predicates like *lift the piano* or *make \$6000* tolerate a certain number of exceptions. For example (9) seems true as long as it is a "true generalization" that the T.A.'s make \$6000; if one or two exceptional T.A.'s make some other amount (9) can still be true:

(9) The T.A.'s make \$6000.

An interesting puzzle concerning distributivity and exceptionality arises if the subject is a conjoined series

of names (or definite descriptions, etc.), in which case no exceptions are allowed. Sentences of the form in (10) (where the ellipsis dots abbreviate a longer list of names) require that everyone listed in the subject NP was asleep, no matter how large the total group listed may be.

- (10) John and Mary and Bill and ... and Sue were
asleep.

It is somewhat mysterious why exceptions are tolerated in sentences like (8) or (9) when they are not tolerated in this sort of sentence; perhaps this effect is due to conversational implicature based on the Maxims of Relevance or Quantity, but perhaps it is due to a more fundamental contrast between conjoined noun phrases and definite descriptions.⁷

Conversely to the case of apparently intrinsic distributives there are predicates like *gather in the square* or *be large in number*, which seem to apply only to groups and not to individuals:

- (10)a. The protesters gathered in the square.
b. *John gathered in the square.

⁷David Dowty (p.c.), if I understand him correctly, suggests that a distributive predicate is true of a group if most of the group's members have the property in question. However, exceptions are not tolerated in sentences like (10), since in a conjunction structure exceptional individuals may simply be left off the list. Inclusion of such individuals would decrease brevity and convey irrelevant information, violating Grice's Maxims of Manner and Relation. Adherence to the Cooperative Principle thus requires that exceptional individuals must be left off the list.

(11)a. The protesters are large in number.

b. *John is large in number.

These predicates might be termed "intrinsically collective".

It is definitely not the case, however, that these predicates are immune to the distributive/collective alternation; on the distributive understanding the property simply distributes down to what seem intuitively like the member groups of a higher-order collection, rather than down to individuals:

(12)a. The committees gathered in their (respective) meeting-rooms.

b. Those two groups are (both) large in number.

Whether or not the objects in the extensions of words like *committee* or *group* are groups in the same sense as the group denoted by *John and Mary* is another question which will not be addressed until later. There are well-known arguments that suggest that they may be more like individuals, in which case the class of intrinsic collective predicates might turn out not to show any formal differences at all from the class that includes *lift the piano*. If so, it would be a real misnomer to continue calling them intrinsic collectives.

3. Distributive predication as predication on groups. So far we have distinguished a number of different issues, including the following: 1. Do sentences with both

collective and distributive understandings have vague, ambiguous or unambiguous subjects? 2. Do such sentences have vague, ambiguous or unambiguous verb phrases? 3. Do the denotations of distributively understood predicates include groups, or only individuals? 4. Do the denotations of predicates like *gather in the square* or *be large in number* include individuals, or only groups?⁸

All these issues are interrelated, of course, but I want to focus on one in particular, namely 3. In order not to prejudge the question of whether intrinsic distributivity is formally identical to the distributivity that arises with predicates which also allow collective understandings, perhaps this issue actually should be seen as composed of two more specific issues, one addressing intrinsic distributives and one addressing other predicates.

Early model-theoretic literature on plurality (Bennett 1972, 1975, Hausser 1974a, 1974b) generally assumed that all distributive predicates were simple predicates on individuals; the possibility of treating distributive predication as predication on groups was first raised by Bartsch (1972, 1973)⁹ and has since been assumed or argued for in some form or other by Scha (1981), Link (1983, 1984a,

⁸The possibility should also be considered here (and in issue 3) that the predicate denotations might include some third category besides individuals and groups. Cf. the comments on Landman (1987) below.

⁹Bartsch (1972) is an excerpt from the better known and more widely available Bartsch (1973).

1984b, 1987a), Hoeksema (1983, 1987), van Eijck (1983), Dowty and Brodie (1984), Dowty (1986), Root (1986), Lasersohn (1986, 1987), Roberts (1987a, 1987b), and Lønning (1987); in fact by virtually everyone who has taken a recent stand on the issue. (One notable exception is Landman 1987, who argues that distributive predication is neither predication on groups, nor simple predication on individuals, but involves a third category.)

In this section I will outline one well-known argument that distributive predication is predication on groups. In the following sections of this chapter I will illustrate what appears to be a serious defect in the standard treatment of distributives as group-level predicates, and formulate an alternative treatment which treats intrinsic distributives as predicates on individuals, and other distributively understood predicates as applying to the members of the groups to which they would apply on the standard group-level analysis. (In the more obvious cases, this means that these predicates will also be applying to individual-level arguments, rather than groups.) I will ultimately argue that this solution is incorrect and that distributive predicates are predicates of groups, but I think the analysis is worth considering, if only as an illustration of why the more complex formal machinery to be introduced in Chapter II is necessary.

Let me state more explicitly what I mean by the "standard treatment of distributives as group-level predicates". Essentially, this is just the VP-ambiguity analysis of distributivity sketched in Section 1 above, possibly extended to deal with intrinsic distributives as well as predicates like *lift the piano* or *earn \$6000*. (For our purposes, the vagueness analysis sketched immediately afterward will count as a version of the "standard treatment" as well.) To review, on this analysis, a distributive predicate applies not only to the individuals whose properties ultimately determine the satisfaction conditions of the predicate, but to all groups of such individuals as well. Thus the distributive denotation of, say, *earn \$6000* will include all individuals which earn \$6000, as well as all groups of such individuals. Note that these are groups whose members earn \$6000; these groups must be distinguished as a class from those which earn \$6000 as groups.

In the standard treatment, distributive VP denotations meet certain closure conditions. Most notably, they are closed under group-formation; if $x_1, \dots, x_n$ are in a distributive denotation α , so is the group of $x_1, \dots, x_n$. (Other closure conditions may be met as well; it should be obvious, for example, that if one closes a set of individuals under group formation, the resulting set will also be closed under group membership, subgroups, etc.) In

fact, the identifying feature of the standard treatment is precisely that some or all of the groups in a distributively understood VP denotation are there solely to satisfy these closure conditions, and not because of any properties they have as groups, independently of their members. Because of this, I will often refer to this sort of treatment of distributive predicates as the "closure-condition" theory of distributivity.

I know of four main lines of argument which have been proposed to support the closure-condition analysis of distributive predication. The first, due to Scha (1981), is that such an approach makes possible an adequate treatment of predicates like *contain* and *run parallel*. The second, implicit in Hoeksema (1983, 1987), is that closure-condition distributivity makes possible a semantic treatment of number agreement which correctly predicts the conditions under which conjoined singular noun phrases impose plural morphology on their verbs. The third, due to Roberts (1987a, 1987b), is that this approach correctly predicts the distribution of discourse anaphors relative to antecedents inside and outside distributive predicates. The fourth argument, from Dowty (1986), is based on the conjoinability of collective and distributive predicates. I want to outline this last argument in some detail, because the fragment to be given at the end of this chapter is designed

to cover without closure-condition distributivity the same sort of data as Dowty suggested requires it.

The argument goes essentially as follows: Suppose that distributive predicates denote sets of individuals, with no groups allowed, and that the denotation of a complex VP is the intersection of the denotations of each of the conjunct VPs from which it is formed. Then how do we account for the sentences in (13) (Dowty's (4) - (5))?

(13)a. John and Mary are a happy couple and are (each) well-adjusted individuals too.

b. The students closed their notebooks, left the room and then gathered in the hall after class.

The predicate *are a happy couple* presumably only has groups in its denotation, since couples are groups. But *are well-adjusted individuals* seems to be distributive, in which case it should only have individuals in its denotation. The intersection of the two predicates therefore is null, and (13)a. is predicted to be self-contradictory. But of course it is not; so (the argument goes) we must have been wrong in assuming that distributive predicates only have individuals in their denotations. Rather the denotation of *are well-adjusted individuals* must include all groups of well-adjusted individuals as well as the well-adjusted individuals themselves, and similarly for other distributive predicates.

I will eventually suggest that this argument is not valid, but let me first point out two spurious counterarguments and explain what's wrong with them. The first counterargument is that it is not really clear that *are well-adjusted individuals* is distributive; we do not want it to follow that John and Mary each have the property of being well-adjusted individuals, but only that they each have the property of being a well-adjusted individual. (I assume that the plural morphology on *individuals* in this sentence is not due to grammatical agreement.) This argument does not point out a serious problem, though, because it does not affect examples like (13)b. and because if we substitute a predicate like *be intelligent* for *be well-adjusted individuals*, example (13)a. does not suffer:

- (14) John and Mary are a happy couple, and are
intelligent too.

Here we do want it to follow that John is intelligent and Mary is intelligent.¹⁰

A second potential problem is that the distributive predicates in these examples are all "intrinsic" distributives. This makes it questionable whether the results extend to distributive understandings of other

¹⁰Note that there is still the difference between *is intelligent* and *are intelligent* to consider -- but as long as we maintain that *are intelligent* denotes a set of individuals, we probably also have to maintain that these two predicates express the same property, so this difference does not harm the argument.

predicates, which conceivably could be formally different from intrinsic distributives. It would be useful to have an example which involved a predicate which was not intrinsically distributive, but clearly understood as distributive in context, despite being conjoined to a collective. Examples of this sort are fairly hard to find, but I believe they do exist; for instance (15) has a reading in which John and Mary each had a beer:

(15) John and Mary met in the bar and had a beer.

4. Group-sensitive adverbials. Hoeksema (1983) presents a brief but interesting analysis of the English adverb *together*. I will argue that something at least more or less like Hoeksema's analysis is correct, not only for *together* but for a number of similar adverbials. If it is correct, however, then the closure-condition theory of distributivity would appear to be incorrect, despite arguments like Dowty's.

Hoeksema suggests that the denotation of *together* "is a function that yields, for any given plural predicate, its restriction to $U - U_1$, that is, to the set of all groups in the domain". In effect, *together* removes any individuals from a VP denotation, leaving all the groups.

As an example, suppose that John and Mary lifted a piano (as a group), and so did Bill and Sue, and Ed lifted one by himself. Then *lifted a piano* will have a denotation

containing the group of John and Mary, the group of Bill and Sue, and Ed; on a VP-ambiguity analysis it will also have an additional denotation for the distributive reading, which we will ignore for just a moment. Applying *together* simply removes any individuals from the VP denotation, which in this case means removing Ed. The resulting denotation for *lifted a piano together* will contain just the group of John and Mary and the group of Bill and Sue. This is result we want, since each of these two groups lifted the piano together, and nothing else did.

Hoeksema assumes a closure-condition treatment of distributivity, and it appears from his discussion that he intended for *together* to apply to distributives as well as collectives. It is clear, however, that this gives incorrect results.

Suppose that John, Mary and Bill each lifted the piano (alone), and that otherwise the piano was not lifted. Then on a closure-condition analysis, the distributive denotation for *lift the piano* will include John, Mary, Bill, and any groups that can be formed from just these individuals, viz. at least the group of John and Mary, the group of John and Bill, the group of Bill and Mary, and the group of John, Mary and Bill.¹¹ Applying *together* to this denotation will

¹¹Depending on algebraic properties of $\cup$ and other details of formalization, higher order groups might also be included, such as the group of the group of John and Mary and the group of John and Bill. In some models these higher order groups will be equivalent to lower order ones,

remove the individuals John, Mary and Bill, but leave all the groups intact. These groups will therefore count as having lifted the piano together, and a sentence like *John and Mary lifted the piano together* will come out true, even though it is clearly false in the situation described.

A Hoeksema-style analysis of *together* would appear to work quite well if we could restrict the domain of the adverb to collective predicates.¹² This would be easy if there were some systematic difference between collectives and distributives, for instance if collectives applied to groups but distributives did not. But if distributive predicates are predicates on groups, it is not so easy to find such a difference.

Note that it will not work simply to make *together* sensitive to the closure properties of distributive predicates, for instance by systematically excluding VP denotations which meet these closure conditions from its domain, since collective denotations can meet these conditions accidentally. Suppose that John and Mary lifted the piano together, and also each lifted it alone. Then the collective denotation will be closed under group formation,

however.

¹²There are some other adjustments which need to be made in addition, as we shall see.

subgroups and members¹³, but is still eligible for modification by *together*.

It would be possible to go intensional, of course, since distributive predicates will meet appropriate closure conditions in all possible worlds, while collective predicates will meet these conditions only accidentally in particular possible worlds. If *together* were sensitive to intensions of predicates and not just extensions, it could reliably select between collectives and distributives.

Unfortunately *together* does not give any other indication of creating an intensional context. It does allow substitution of referentially identical expressions:

(16)a. John and Mary lifted Raoul Duke together.

b. John and Mary lifted Hunter S. Thompson
together.

¹³See Hoeksema (1983) for arguments that the denotation of a plural predicate like *lift the piano* must contain the individuals that lifted the piano and not just the groups, even on its collective reading.

The claim here that the collective denotation will be closed under group formation is actually incorrect in models where group formation is not associative, but analogous examples can be constructed for such models as well.

Because closure of a set under a non-associative operation often yields an infinite result, such examples tend to be mathematical, but they are not hard to construct. Consider, for example, a generalized notion of sum, such that any individual number sums to itself, and any group of numbers sum in the standard way. Then the collective and distributive denotations of *sum to a non-negative number* will coincide in some models but not others (depending on the membership of *U*). Even so, *Five and seven together sum to a non-negative number* has only the collective reading.

It does not give rise to de dicto/de re or specific/non-specific ambiguities, does not block existential entailments (i.e. *John and Mary lifted a piano together* does entail that there was a piano that John and Mary lifted) and does not intuitively seem to require consideration of alternative situations. Given all this, it is very likely incorrect to treat *together* as an intensional operator.

It seems that we cannot both maintain a closure-condition treatment of distributivity and hold to a Hoeksema-style analysis of *together*. Of course to give up a general account of a systematic and widespread phenomenon like VP distributivity simply for the sake of single lexical item would be a high price to pay; but as it turns out *together* is just one of a number of adverbials for which the treatment of distributive predication as predication on groups is problematic. Some others, I would suggest, include *collectively*, *jointly*, *in common*, *as a group*, *as a team*, all comitative PPs, *in chorus*, *in unison*, *in harmony*, *by vote*, *by consensus*, and many other similar expressions. I will call such expressions "group-sensitive" adverbials.

Is it necessary to treat all these in the same way as Hoeksema treats *together*? If not, perhaps we could save the the idea that distributive predicate denotations contain groups because of closure conditions.

There is little doubt that other analyses are possible, of course. However, I know of only one other existing model-theoretic analysis of any of these adverbials: Bennett's (1975) analysis of *together*. Bennett takes *together* to map individual-level verb phrase denotations onto group-level verb phrase denotations. Unfortunately, nothing in Bennett's semantics for *together* determines which group-level VP denotation a particular individual-level VP denotation will get mapped onto. In fact the mapping is so arbitrary that it does not even preserve existential entailments, as Bennett himself pointed out (1975 p. 142). Thus (17) -- Bennett's example -- incorrectly receives a reading which does not entail the existence of a house:

(17) The gods build a house together.

It would probably not be hard to fix this problem through the use of meaning postulates restricting the interpretation of *together*, but I think to do so would miss the point. The attractive thing about Hoeksema's analysis, to my mind at least, is that it portrays *together* as operating in a precisely specifiable, deterministic way on information provided by the verb phrase it modifies; information, moreover, that this verb phrase must carry for independent reasons, and not just to guarantee the proper functioning of *together*. Bennett's analysis, even if patched up to assure

existential entailments in the right places, must portray *together* as non-logical and substantially arbitrary.¹⁴

Now, not all the adverbials listed above can be treated as logical operators, though I think that at least the collectivizing uses of *together*, *collectively*, *as a group*, and comitative *with* probably can be, and that logical operations play a substantial role in the semantics of *jointly* and *in common*. But even for clearly non-logical adverbials like *in harmony* or *by vote*, I think that any analysis that does not portray these as operating sensibly only on collective VPs is missing something essential. To tell if John and Mary performed a particular piece of music in harmony, one must know if they performed it at all, and to do this it may not suffice simply to check if John performed it and if Mary performed it, since some pieces require more than one person for their performance. Instead we must check to see if John and Mary performed it as a group -- and this amounts to checking if the group of John and Mary is in the collective denotation of the appropriate

¹⁴Bennett does argue briefly against an analysis which treats VPs modified by *together* as equivalent to their corresponding unmodified collective VPs (pp. 142-143), noting that (i) and (ii) are not synonymous:

(i) The mob applauds Mary.

(ii) The mob applauds Mary together.

I agree that these sentences are not synonymous, though I am not sure I would agree with Bennett as to the precise nature of the semantic difference. This sort of example will be more easily explained after a consideration of adverb position and the "proximity" readings of *together*, which will be treated in detail in Chapter II.

verb phrase. The most natural and easy way to do this is to let *in harmony* take collective predicates as its arguments, to the exclusion of distributives¹⁵; any other treatment would apparently have to let the mapping provided by *in harmony* somehow duplicate on its own the information which collective predicates must provide anyway. Besides producing this unnecessary duplication, such an account would also portray the semantic relation between the predicate modified by *in harmony* and that produced by the modification as substantially less direct.

But again, if distributive predicates are predicates of groups, it is not clear that selection for collectives is possible in principle. The problem is not specific to one particular analysis of one particular lexical item but to any analysis of a whole range of adverbials which is to make explicit the relation between modified predicates and those formed by modification.

5. Conjoining collective and distributive verb phrases. In order to account for the selection properties of group-sensitive adverbials, we need some analysis of how it is that they can properly distinguish collective and

¹⁵This does not exclude the possibility of a distributive understanding for *performed this piece in harmony*, since the distributivity may still be induced by an operator which takes the whole verb phrase inside its scope, or by ambiguity or vagueness in the subject noun phrase, etc. All that is excluded is for the predicate modified by *in harmony* to be distributive.

distributive verb phrases in the face of evidence that distributive verb phrases, like collectives, are predicates of groups. For example, it appears that we need an account of how distributive predicates can conjoin with collectives which does not require a closure-condition treatment of distributivity. This section will give one possible solution to the conjunction problem.

Of those authors that do not treat distributive predicates as predicates of groups, I know of two who give analyses of conjunctions of distributive and collective verb phrases: Hausser (1974b) and Landman (1987). Both Hausser and Landman use some sort of type-raising mechanism to give the effect of converting distributive predicates to predicates of groups just in case they conjoin with collective predicates.

Landman's type-raising rules are relatively complicated but similar in relevant effects¹⁶ to Hausser's; the following translation rule from Hausser (1974b) should serve to illustrate the general idea behind this approach. IV and

¹⁶For details of Landman's analysis see his Section 2.4. Landman differs from Hausser in formulating his type-raising operation to generalize across an infinite hierarchy of types, and in taking distributive plural predicates to apply to "sums" (i.e. non-singleton sets; cf. the outline of Landman's model structure in Section 0 above) rather than individuals. Because Landman's raising operation generalizes across types and can apply trivially when not needed, Landman is able to give a single schema for conjunction while Hausser requires four separate VP conjunction rules.

IV are the categories of individual-level and group-level verb phrases, respectively, and $Fa(\gamma, \delta)$ is ' γ and δ '¹⁷:

- (18) If $\gamma \in \text{PIV}$ and $\delta \in \text{PIV}$, and γ, δ translate into γ', δ' , respectively, then $Fa(\gamma, \delta)$ translates into $P[\gamma'(P) \wedge \wedge x[P\{x\} \rightarrow \delta'(x)]]$

Here P is functioning as a variable over groups. $\wedge x[P\{x\} \rightarrow \delta'(x)]$ is an open proposition in this group-level variable -- equivalent, for practical purposes, to a group-level predicate. The conjunction of a collective and a distributive predicate thus gives the set of groups which satisfy the collective predicate and all of whose members satisfy the distributive predicate.¹⁸

Building the type-raising mechanism directly into the conjunction rule complicates the notion of conjunction significantly, but is necessary if we are to avoid the problem of group-sensitive adverbials. If distributive predicates were lifted to the group level and conjoined to collectives in two separate operations, there would be no apparent obstacle to other operations applying between these

¹⁷This rule only covers the case where the collective predicate precedes the distributive. Hausser gives a separate rule for the opposite order, but it appears to contain a misprint.

¹⁸Hausser assumes, as we have not, that a given predicate is either uniformly group-level or uniformly individual-level; hence the complex predicate formed by conjunction contains only groups in its extension. Nothing in the general idea of raising distributive predicates to conjoin them with collectives actually requires this, however.

two, and in particular to adverbials like *together* applying to the distributive predicate before it is conjoined. But of course this sort of intervening operation should be ruled out; it would produce the same sort of incorrect readings as would be produced if distributive predicates had been group-level to start with.

Building the type-raising operation into the conjunction rule prevents group-sensitive adverbials from applying to distributive conjuncts, but it does not eliminate all incorrect modification patterns. Because the complex predicate formed by the conjunction rule is group-level, it is predicted to be eligible for modification by group-sensitive adverbials. Unfortunately, such modification does not appear to be possible.

To see this, note first that many group-sensitive adverbials, including *together*, have uses which indicate spatial proximity rather than collective action. In (19), for example, *together* indicates that John and Mary sat in the same area, not that the group of John and Mary somehow sat, independently of its members:

(19) John and Mary sat together.

The locative use of *together* and other group-sensitive adverbials will be covered in Chapter II; for present purposes it is simply worth noting that the collectivizing use of *together* does not require spatial proximity; (20) does not require that John and Mary were near each other.

(20) John and Mary made \$10,000 together.

Of course certain types of group action by their very nature require the members of the groups to be near each other in space. Lifting a piano is such an action, for example, so (21) does suggest that John and Mary were near each other.

(21) John and Mary lifted a piano together.

I think it is safe to say that the proximity requirement imposed by this sentence on John and Mary follows from the physical nature of piano-lifting, and not from the semantics of *together*.

Now, if distributive predicates raise to group-level in order to conjoin to collectives as Hausser and Landman suggest, the resulting complex predicate should be available for modification by *together* in its collectivizing use. A complex predicate formed by the conjunction of a collective and distributive and modified by *together* therefore should have a reading which is not subject to proximity restrictions. For example if we conjoin *walk to the store* with *bought a steak* (understood collectively) and then modify the whole thing with *together*, the resulting predicate should denote the set of groups which bought a steak and whose members walked to the store. This set includes groups whose members were not near each other when they walked to the store. But (22) does not allow an understanding where John and Mary walked to the store from

different directions, met each other there, and then pooled their money to buy a steak:

(22) John and Mary together walked to the store and
bought a steak.

I want to present a somewhat more complex type-raising solution that avoids both this problem and the more general problem of selection for collectives by group-sensitive adverbials by positing a level at which verb phrases denote sets of generalized quantifiers rather than sets of groups and/or individuals. Ordinary extensional predicates of the sort we have been considering so far have basic denotations at the group/individual level, and undergo type-raising to denote sets of generalized quantifiers before combining with their subjects. The lower-level denotations of distributively understood predicates (which are recoverable from the higher-level ones) are not required to be closed under group formation, so adverbial selection will not be a problem. Intrinsically distributive predicates will simply denote sets of individuals (at the lower type), and predicates like *lift the piano* be unambiguous in each of their types.

It will be easiest to explain how this analysis works if we develop a small fragment to illustrate it, which I will call Fragment 1. Rather than presenting the entire fragment at once, it will be convenient to present it a bit

at a time and give comments as we go. The fragment will include transitive and intransitive extensional verbs, collectivizing *together*, proper names, singular and plural common nouns, definite descriptions, simplex and conjoined verb phrases, and simplex and conjoined noun phrases. It will ignore matters of tense, agreement and other details not immediately relevant to the matters at hand. The reader is cautioned that this analysis will be rejected in later chapters. Interpretation is relative to a model frame as defined in Section 0.

Let us begin by defining denotation domains for basic expressions:

- (23)a. $D_{IV} = POW(U)$
- b. $D_{TV} = POW(U)^U$
- c. $D_{CN} = POW(U_I)$
- d. $D_{PN} = U_I$

Only basic expressions are required to denote in these domains; domains for derived expressions will be defined indirectly via the rules which interpret them.

Let's also have some basic expressions to denote in these domains. I assume that syntactic category labels are just names of sets of expressions:

- (24)a. $\langle walk \rangle, \langle sit \rangle, \langle matter \rangle^{19} \in IV$

¹⁹*Matter* in the sense of *Those things are unimportant; they don't matter at all.* *Matter* may seem like an odd verb to include in a simple little fragment like this one; I have included it because it is one of what turns out to be a surprisingly small number of lexical IVs which are neither

- b. *<lift>*, *<build>*, *<like>* ∈ TV
c. *<piano>*, *<rock>*, *<T.A.>* ∈ CN_{sg}
d. *<John>*, *<Mary>*, *<Bill>* ∈ PN

The angle brackets here are the usual angle brackets of sequence notation (in this case indicating one-place sequences). Expressions will combine by sequencing rather than concatenation; this gives them a nested tree-structure which concatenation does not produce.

I haven't bothered to list determiners or adverbs, which will be introduced syncategorematically. The syncategorematic treatment is adopted because only one determiner and one adverb will be included in the fragment, and is not intended to express any sort of theoretical claim.

Plural nouns may be derived by rule, as follows:

(25) If $\alpha \in \text{CN}_{2g}$ then $\beta = \text{PLUR}(\alpha) \in \text{CN}_{01}$.

where PLUR(x) is the result of adding plural morphology to x; $\|g\| = \|a\| \cup \{g \in U \mid \forall x[x \in g \rightarrow x \in \|a\|]\}$.

I haven't bothered to formalize the morphological function PLUR, though of course this would be necessary in a fully detailed fragment. The semantic portion of the rule stipulates that the denotation of a plural noun includes all the individuals from the denotation of the noun's singular counterpart, plus any groups all of whose members are in the

intrinsically distributive nor intrinsically collective.

denotation of the singular counterpart. For example the denotation of $\langle students \rangle$ will include all students and groups of students.²⁰

A noun phrase may consist of just a proper name; both the name and the NP formed from it will denote an individual:

(26) If $\alpha \in PN$ then $\beta = \langle \alpha \rangle \in NP$; $\|\beta\| = \|\alpha\|$.

A noun phrase can also be formed by adding *the* before a common noun. The resulting NP will denote an individual or group (or else its semantics will be undefined):

(27) If $\alpha \in CN$ ($CN = CN_{sg} \cup CN_{pl}$) then $\beta = \langle \langle the \rangle, \alpha \rangle \in NP$; $\|\beta\| = x$ if $x \in \|\alpha\|$ and $\forall y[(y \in \|\alpha\| \wedge y \neq x) \rightarrow y \in x \vee y \in x]$, undefined otherwise.

Note that if α is singular (i.e. if it denotes a set of individuals), $\langle \langle the \rangle, \alpha \rangle$ will receive an interpretation only if $\|\alpha\|$ is singleton. If α is plural, $\langle \langle the \rangle, \alpha \rangle$ will denote the maximal group in α .

Intrinsically distributive verbs are subject to meaning postulates assuring that only individuals are in their denotations:

(28)a. If $\alpha = \langle walk \rangle$ or $\alpha = \langle sit \rangle$ then $\|\alpha\| \in POW(U_I)$.

b. If $\alpha = \langle like \rangle$ then $\forall x[\|\alpha\|(x) \in POW(U_I)]$.

A verb phrase may consist of just an intransitive verb:

²⁰For evidence that plural noun denotations contain individuals, see Lasnik (1987). The pluralization operation here differs somewhat from the one given there, however.

(29) If $\alpha \in IV$ then $\beta = \langle \alpha \rangle \in VP$; $\|\beta\| = \|\alpha\|$.

A verb phrase may also consist of a transitive verb plus a noun phrase:

(30) If $\alpha \in TV$ and $\beta \in NP$ then $\gamma = \langle \alpha, \beta \rangle \in VP$;

$$\|\gamma\| = \|\alpha\|(\|\beta\|).$$

A verb phrase may be modified by *together*, which restricts the VP denotation to groups. The locative reading of *together* is ignored:

(31) If $\alpha \in VP$ then $\beta = \langle \alpha, \langle \text{together} \rangle \rangle \in VP$;

$$\|\beta\| = \|\alpha\| \cap (U - U_I).$$

Note that modification by *together* of an intrinsically distributive predicate yields the null set.

Verb phrases may be raised to the sets-of-generalized-quantifiers level:

(32) If $\alpha \in VP$ then $\beta = \langle \alpha \rangle \in VP$;

$$\|\beta\| = \{P \in \text{POW}(\text{POW}(U)) \mid \|\alpha\| \in P\}.$$

Verb phrases may be conjoined; this corresponds in the semantics to set intersection:

(33) If $\alpha \in VP$ and $\beta \in VP$ then $\gamma = \langle \alpha, \langle \text{and} \rangle, \beta \rangle \in VP$;

$$\|\gamma\| = \|\alpha\| \cap \|\beta\|.$$

It seems harmless to let VPs conjoin at either their higher or their lower types. Note though that if a distributive VP and a VP modified by *together* conjoin, the result will necessarily be empty at the lower type, but generally will be nonempty at the higher type.

Noun phrases may conjoin with a corresponding group-formation operation in the semantics:

- (34) If $\alpha \in NP$ and $\beta \in NP$ then $\gamma = \langle \alpha, \langle and \rangle, \beta \rangle \in NP$;
 $\|\gamma\| = g(\|\alpha\|, \|\beta\|)$, where g is a (possibly partial) function from $U \times U$ to $U - U_I$ such that for all y, z for which g is defined, $x \in g(y, z)$ iff $x = y$ or $x = z$.

The g operation (g for "group") maps a pair of individuals or groups onto the group containing just those two groups or individuals.²¹

Noun phrases may be raised to the generalized quantifier level. The raising operation used here is somewhat novel:

²¹I allow g to be a partial function in order to leave open the possibility that there are groups or individuals that are not contained in any groups, which will certainly be the case in at least some models admitted under our definition of a frame. (In giving a more restrictive definition of a frame we might consider ruling out this possibility, however.)

Aside from its potential partiality, we are secure in defining g as a function; $g(y, z)$ will always be unique if it exists at all, as a quick glance at (2) should make clear. The definition of a model frame does not allow distinct groups to have exactly the same members.

This requirement may actually be too strong, if we decide to treat membership in committees and similar objects as formally identical with ordinary group membership. Even so I will let the requirement stand without argument for now.

It may be somewhat surprising to realize that if committee membership cannot be treated as ordinary group membership in the sort of model frame defined in (2), it cannot be treated that way in any of the models shown above to instantiate this sort of frame; but cf. the treatments of the committee problem in Landman (1987) and in Chapter III, below.

(35) If $\alpha \in NP$ then $\beta = \langle \alpha \rangle \in NP$;

$$\|\beta\| = \{P \in POW(U) \mid \|\alpha\| \in P \vee s(\|\alpha\|) \subseteq P\},$$

where s is a function from $U - U_I$ to $POW(U)$ such that $s(x) = \{y \mid y \in x\}$.

The s operation (s for "set") maps any group onto the set of its members. In models where group formation is just set formation, s is the identity mapping on groups. Applied to an individual-denoting noun phrase, rule (35) gives the set of sets containing that individual, just as in the standard sort of type-raising on names due to Montague. Applied to a group-denoting noun phrase, rule (35) gives the set of sets which either contain the group or the group's members. For example, a set will be in the raised denotation of $\langle\langle the \rangle, \langle T.A.'s \rangle\rangle$ if it contains the group of T.A.'s or if it contains the individual T.A.'s.

Finally, a sentence may consist of a noun phrase and a verb phrase. I'll let the function-argument structure go in either direction and assume that phrases combine at the lowest type for which a pragmatically plausible interpretation is available:

(36) If $\alpha \in NP$ and $\beta \in VP$ then $\gamma = \langle \alpha, \beta \rangle \in S$;

$$\|\gamma\| = 1 \text{ if } \|\alpha\| \in \|\beta\| \text{ or } \|\beta\| \in \|\alpha\|, 0 \text{ otherwise.}$$

To illustrate how the sort of system outlined here allows conjunction of collective and distributive predicates, an example may be in order. A derivation of

John and Mary like Bill and lift the piano together follows. Assume that the sentence is true, i.e. that John likes Bill, Mary likes Bill, and John and Mary lift the piano as a group.

Start with *the piano*. $\llbracket \langle piano \rangle \rrbracket$ is the set of pianos. $\llbracket \langle \langle the \rangle, \langle piano \rangle \rangle \rrbracket = x$ if $x \in \llbracket \langle piano \rangle \rrbracket$ and $\forall y[(y \in \llbracket \langle piano \rangle \rrbracket \wedge y \neq x) \rightarrow y \sqsubseteq x \vee y \in x]$ and is undefined otherwise, by rule (27). Suppose that there is more than one piano. We may assume that pianos do not have members or subgroups, so for any piano p it will be false that $\forall y[(y \in \llbracket \langle piano \rangle \rrbracket \wedge y \neq p) \rightarrow y \sqsubseteq p \vee y \in p]$, and $\llbracket \langle \langle the \rangle, \langle piano \rangle \rangle \rrbracket$ will be undefined. On the other hand if there is only one piano, then $\forall y[(y \in \llbracket \langle piano \rangle \rrbracket \wedge y \neq p) \rightarrow y \sqsubseteq p \vee y \in p]$ is true and $\llbracket \langle \langle the \rangle, \langle piano \rangle \rangle \rrbracket =$ the unique piano p .

Now $\llbracket \langle lift \rangle \rrbracket$ is a function from groups and individuals (the liftees) to sets of groups and individuals (for each liftee the set of its lifters). Since we are assuming that John and Mary lift the piano, it must be the case that $g(\text{John}, \text{Mary}) \in \llbracket \langle lift \rangle \rrbracket(p)$, that is, that the group of John and Mary is in $\llbracket \langle \langle lift \rangle, \langle \langle the \rangle, \langle piano \rangle \rangle \rangle \rrbracket$. Since $g(\text{John}, \text{Mary})$ is in $U - U_I$ by the definition of g , it will also be in $\llbracket \langle \langle \langle lift \rangle, \langle \langle the \rangle, \langle piano \rangle \rangle \rangle, \langle together \rangle \rangle \rrbracket$, which is just the restriction of $\llbracket \langle \langle lift \rangle, \langle \langle the \rangle, \langle piano \rangle \rangle \rangle \rrbracket$ to $U - U_I$.

By rule (32), the higher-type denotation $\llbracket \langle \langle \langle \langle lift \rangle, \langle \langle the \rangle, \langle piano \rangle \rangle \rangle, \langle together \rangle \rangle \rangle \rrbracket = \{P \in \text{POW}(\text{POW}(U)) \mid \llbracket \langle \langle \langle lift \rangle, \langle \langle the \rangle, \langle piano \rangle \rangle \rangle, \langle together \rangle \rangle \rrbracket \in P\}$, the set of

generalized quantifiers which have the set of groups that lifted the piano as a member.

By the same token, $\{\langle\langle\text{like}\rangle, \langle\text{Bill}\rangle\rangle\}$ is a set of individuals (groups are excluded by meaning postulate (28)b.) and $\{\langle\langle\langle\text{like}\rangle, \langle\text{Bill}\rangle\rangle\rangle\} = \{P \in \text{POW}(\text{POW}(U)) \mid \{\langle\langle\text{like}\rangle, \langle\text{Bill}\rangle\rangle\} \in P\}$. Note that the intersection of this set with $\{\langle\langle\langle\text{lift}\rangle, \langle\langle\text{the}\rangle, \langle\text{piano}\rangle\rangle\rangle, \langle\text{together}\rangle\rangle\rangle\}$ will be non-empty -- one of its elements, e.g., will be $\{\langle\langle\langle\text{lift}\rangle, \langle\langle\text{the}\rangle, \langle\text{piano}\rangle\rangle\rangle, \langle\text{together}\rangle\rangle\rangle, \{\langle\langle\text{like}\rangle, \langle\text{Bill}\rangle\rangle\}\}$. This intersection will be the denotation of $\langle\langle\langle\langle\text{like}\rangle, \langle\text{Bill}\rangle\rangle\rangle, \langle\text{and}\rangle, \langle\langle\langle\langle\text{lift}\rangle, \langle\langle\text{the}\rangle, \langle\text{piano}\rangle\rangle\rangle\rangle, \langle\text{together}\rangle\rangle\rangle\rangle$.

It may be worth digressing just a moment at this point to take a look at higher-type VP denotations from the point of view of generalized quantifier theory. These VP denotations are sets of generalized quantifiers, of course, but they may also be considered as generalized quantifiers themselves. The type-lifting rule (32), for example, is easily seen to map any set in $\text{POW}(U)$ onto the ultrafilter it generates in $\text{POW}(\text{POW}(\text{POW}(U)))$ just as Montague's famous treatment of proper names can be recast in terms of a function mapping any individual in U onto the ultrafilter it generates in $\text{POW}(\text{POW}(U))$. In fact, lower-type VP denotations relate to the algebra of higher-type VP denotations in precisely the same way as individuals (and groups) relate to the more familiar algebra of higher-type

NP denotations. A conjoined higher-type VP denotation like $\llbracket \langle \langle \langle \langle \text{like} \rangle, \langle \text{Bill} \rangle \rangle \rangle, \langle \text{and} \rangle, \langle \langle \langle \langle \text{lift} \rangle, \langle \text{the} \rangle, \langle \text{piano} \rangle \rangle \rangle, \langle \text{together} \rangle \rangle \rangle \rrbracket$ forms a principle filter.²²

Turning now to the subject noun phrase of our example, *John and Mary*, we let $\llbracket \langle \text{John} \rangle \rrbracket = \text{John}$ and $\llbracket \langle \text{Mary} \rangle \rrbracket = \text{Mary}$. By rule (34), $\llbracket \langle \langle \text{John} \rangle, \langle \text{and} \rangle, \langle \text{Mary} \rangle \rangle \rrbracket = g(\text{John}, \text{Mary})$, the group of John and Mary (provided such a group exists). By the NP type-raising rule (35), $\llbracket \langle \langle \langle \text{John} \rangle, \langle \text{and} \rangle, \langle \text{Mary} \rangle \rangle \rangle \rrbracket = \{P \in \text{POW}(U) \mid g(\text{John}, \text{Mary}) \in P \vee s(g(\text{John}, \text{Mary})) \subseteq P\}$. Recall that s is a function from $U - U_I$ to $\text{POW}(U)$ such that $s(x) = \{y \mid y \in x\}$, so $s(g(\text{John}, \text{Mary})) = \{\text{John}, \text{Mary}\}$. So we have $\llbracket \langle \langle \langle \text{John} \rangle, \langle \text{and} \rangle, \langle \text{Mary} \rangle \rangle \rangle \rrbracket = \{P \in \text{POW}(U) \mid g(\text{John}, \text{Mary}) \in P \vee \{\text{John}, \text{Mary}\} \subseteq P\}$.

By the subject-predicate rule (36), then, $\llbracket \langle \langle \langle \langle \text{John} \rangle, \langle \text{and} \rangle, \langle \text{Mary} \rangle \rangle \rangle, \langle \langle \langle \langle \text{like} \rangle, \langle \text{Bill} \rangle \rangle \rangle, \langle \text{and} \rangle, \langle \langle \langle \langle \text{lift} \rangle, \langle \text{the} \rangle, \langle \text{piano} \rangle \rangle \rangle, \langle \text{together} \rangle \rangle \rangle \rangle \rrbracket = 1$, because $\{P \in \text{POW}(U) \mid g(\text{John}, \text{Mary}) \in P \vee \{\text{John}, \text{Mary}\} \subseteq P\} \in \{P \in \text{POW}(\text{POW}(U)) \mid \llbracket \langle \langle \text{like} \rangle, \langle \text{Bill} \rangle \rangle \rrbracket \in P\} \cap \{P \in \text{POW}(\text{POW}(U)) \mid \llbracket \langle \langle \langle \text{lift} \rangle, \langle \text{the} \rangle, \langle \text{piano} \rangle \rangle \rangle, \langle \text{together} \rangle \rangle \rrbracket \in P\}$. To see this, consider that John and Mary each like Bill and are therefore in $\llbracket \langle \langle \text{like} \rangle, \langle \text{Bill} \rangle \rangle \rrbracket$. Since $\{\text{John}, \text{Mary}\} \subseteq \llbracket \langle \langle \text{like} \rangle, \langle \text{Bill} \rangle \rangle \rrbracket$, then, $\llbracket \langle \langle \text{like} \rangle, \langle \text{Bill} \rangle \rangle \rrbracket \in \{P \in \text{POW}(U) \mid g(\text{John},$

²²It would be interesting to explore the algebraic relation between NP denotations and VP denotations in more detail and search for linguistic reflexes of this relation, but to do so would take us too far afield. See Keenan and Faltz (1985) for one detailed study of this relation. For an extension of the generalized quantifier framework to various categories other than NPs, see van Benthem (1986).

$Mary) \in P \vee \{John, Mary\} \subseteq P\}$, so $\{P \in POW(U) \mid g(John, Mary) \in P \vee \{John, Mary\} \subseteq P\} \in \{P \in POW(POW(U)) \mid \llbracket \langle \langle like \rangle, \langle Bill \rangle \rangle \rrbracket \in P\}$. Likewise, we know that $g(John, Mary) \in \llbracket \langle \langle lift \rangle, \langle the \rangle, \langle piano \rangle \rangle, \langle together \rangle \rrbracket$. Therefore $\llbracket \langle \langle lift \rangle, \langle the \rangle, \langle piano \rangle \rangle, \langle together \rangle \rrbracket \in \{P \in POW(U) \mid g(John, Mary) \in P \vee \{John, Mary\} \subseteq P\}$. So $\{P \in POW(U) \mid g(John, Mary) \in P \vee \{John, Mary\} \subseteq P\} \in \{P \in POW(POW(U)) \mid \llbracket \langle \langle lift \rangle, \langle the \rangle, \langle piano \rangle \rangle, \langle together \rangle \rrbracket \in P\}$. Since both $\{P \in POW(U) \mid g(John, Mary) \in P \vee \{John, Mary\} \subseteq P\} \in \{P \in POW(POW(U)) \mid \llbracket \langle \langle like \rangle, \langle Bill \rangle \rangle \rrbracket \in P\}$ and $\{P \in POW(U) \mid g(John, Mary) \in P \vee \{John, Mary\} \subseteq P\} \in \{P \in POW(POW(U)) \mid \llbracket \langle \langle lift \rangle, \langle the \rangle, \langle piano \rangle \rangle, \langle together \rangle \rrbracket \in P\}$, we know that $\{P \in POW(U) \mid g(John, Mary) \in P \vee \{John, Mary\} \subseteq P\} \in \{P \in POW(POW(U)) \mid \llbracket \langle \langle like \rangle, \langle Bill \rangle \rangle \rrbracket \in P\} \cap \{P \in POW(POW(U)) \mid \llbracket \langle \langle together \rangle, \langle lift \rangle, \langle the \rangle, \langle piano \rangle \rangle \rrbracket \in P\}$, i.e. that the sentence is true.

We have seen in this chapter that the facts of adverbial modification require collective and distributive predicates to be distinguishable in principle, contrary to an apparent claim of the popular "closure-condition" analysis of distributivity. One possible way of drawing this distinction was outlined, which abandoned the closure-condition treatment and made crucial use of a type-raising operation on verb phrases. The next chapter will show that the full range of facts cannot be handled so easily,

however, and that a more complicated sort of model structure is called for, which makes a closure-condition analysis of distributivity possible after all.

CHAPTER II

GROUP ACTION AND SPATIO-TEMPORAL PROXIMITY

0. Introduction. The fragment given at the end of the last chapter is unsatisfactory in a number of respects. Although it does show how collective and distributive predicates can be conjoined without requiring a closure-condition theory of distributivity, it does not address at all the other sorts of evidence which have been adduced in favor of the closure-condition approach. For example, any account of distributivity which pretends to completeness would have to address the complex anaphora facts which Roberts (1987a, 1987b) has covered in her detailed closure-condition account of distributivity; Hoeksema's (1983, 1987) argument from subject-verb agreement and Scha's (1981) argument from predicates like *contain* and *run parallel* would also have to be addressed.

Perhaps the fragment could be extended to meet Roberts', Hoeksema's and Scha's arguments. However, the fragment has some intrinsic defects which mere extensions cannot correct; more fundamental revisions are necessary. In this chapter, I will look at the semantics of *together* and other group-sensitive adverbials in more detail, and

will suggest that in order to account for the full range of readings associated with them, some additional formal apparatus is needed which will make a closure-condition treatment of distributivity possible after all.

1. Vagueness and ambiguity. Before looking at these adverbials, let me also point out what I take to be a serious problem with Fragment 1 which has nothing in particular to do with adverbial modification. The fragment treats sentences with both collective and distributive understandings as vague rather than ambiguous. If this is not clear, consider that the higher-type denotation of a noun phrase like *John and Mary* is a generalized quantifier which includes all the subsets of U which have the group of John and Mary as a member, as well as all the subsets of U which have John as a member and Mary as member. *John and Mary lift the piano* comes out true no matter whether John and Mary collectively lift the piano or each lift the piano, since in either case the denotation of *lift the piano* will be in this generalized quantifier. The sentence comes out true in either situation because the denotation of the noun phrase simply allows that much leeway, and not because of any ambiguity in a strict sense.

It may seem that this sort of sentence really is just vague between the two understandings, rather than ambiguous, so perhaps it is not clear what the problem is. However,

nearly all model-theoretic semantic analyses of the collective/distributive dichotomy treat it as a case of authentic ambiguity rather than vagueness, and any attempt at a vagueness analysis must face serious difficulties.¹

To see some of these difficulties, consider the sentences in (1):

(1)a. John and Mary made less than \$10,000 last year.

b. John and Mary made more than \$10,000 last year.

Now suppose John and Mary each made \$6,000 last year. In this case, both (1)a. and (1)b. will come out true; that is, it will simultaneously be true that John and Mary made less than \$10,000 and that they made more than \$10,000.

Of course, in the described situation, both these sentences are true, in a certain sense. But if these sentences are vague, rather than ambiguous, they each have only a single reading, and it is unclear whether appealing to "a certain sense" makes any sense. Note that we cannot say, under current assumptions, that in a certain sense it is true that John and Mary made more than \$10,000, but in another sense it is false; sentence (1)b. simply comes out true (as does (1)a.) and that is the end of it.

This result seems unsatisfactory. Suppose John and Mary are two T.A.'s in the local linguistics department, and

¹Dowty and Brodie (1984) appear to assume a vagueness treatment, but Dowty (1986) has since given an ambiguity analysis. The tendency to favor a vagueness analysis is fairly strong outside the model-theoretic literature; see especially Harnish (1976), Katz (1977:127-128).

one knows that T.A.'s get paid only \$6,000 a year. It would be natural (and true) in this sort of context to say that John and Mary do not get paid more than \$10,000 a year; but the vagueness hypothesis predicts that such a statement would be unambiguously false.

Note also that if John and Mary each made \$6,000 last year, both (2)a. and (2)b. are true:

(2)a. John and Mary made exactly \$6,000 last year.

b. John and Mary made exactly \$12,000 last year.

Now if there is a real ambiguity between distributive and collective readings, then once we choose one reading or the other, we can find for any group G at most one X such that G made exactly $\$X$ last year. But if sentences are simply vague between collective and distributive understandings, then for any group G all of whose members made the same amount (other than zero), there will be two distinct values X such that G made exactly $\$X$ last year; for example in the described situation these values are 6,000 and 12,000. It therefore makes no sense to ask what the exact amount is that John and Mary made last year, since there is more than one such amount; any reference to the exact amount that they made is predicted to be anomalous.

But in fact it is perfectly natural to ask what the exact amount is that two people made, even if (indeed, especially if) it is known that they each made the same amount. The vagueness hypothesis predicts just the

opposite, since only if the members of a group made different amounts (or nothing at all) will there be a unique amount which the group made exactly.

Given problems such as these, it seems reasonable to conclude that the collective and distributive understandings of sentences like those under discussion represent distinct readings, and that these sentences are ambiguous. Yet the technique proposed in Fragment 1 for conjoining collective and distributive predicates relies crucially on such sentences being vague, and therefore is highly suspect.²

2. The "proximity" uses of group-sensitive adverbials. As mentioned briefly in Chapter I, *together* may be used to indicate spatial proximity rather than collective action.

²For additional arguments in favor of an ambiguity analysis, see Gillon (1987). However, data like those discussed above are as problematic for Gillon's analysis as for a vagueness one. Gillon argues that a sentence with a plural NP subject is true iff the predicate of the sentence holds of each element of a "minimal cover" of the group denoted by the subject. A minimal cover K of a group G is a set of subgroups of G such that each member of G is contained in some member of K , and no member of K is a proper subgroup of the union of all other elements of K . (Gillon identifies groups with sets, so group union is just set union.)

Suppose that John, Mary and Bill are the T.A.'s, that John and Mary make \$4000 each, and Bill makes \$3000. Then Gillon's analysis predicts that *The T.A.'s make exactly \$7000* should be true, since each element of the minimal cover $\{g(\text{John, Bill}), g(\text{Mary, Bill})\}$ makes exactly \$7000. I doubt if anyone would judge the sentence as true in this situation, however. A more detailed critique of the minimal cover analysis is given in Lasersohn (to appear).

Thus (3) indicates that John and Mary were near each other as they stood:

(3) John and Mary stood together.

Together can also indicate temporal proximity; (4) may be used to assert that John and Mary stood up at the same time, but not necessarily at the same place:

(4) John and Mary stood up together.

On the reading of *stand* which is equivalent to *stand up*, and perhaps on its stative reading too, this holds of (3) as well.

"Proximity" *together* does not pose the same sort of problem for the closure-condition approach to distributivity as the collectivizing use of *together*, and it may seem that proximity *together* is simply irrelevant to the question of how distributive predicates should be formalized. I want to suggest, however, that it is no accident that both collective action and spatial or temporal proximity are signaled by the same lexical item, and that any adequate analysis of collectivizing *together* must be straightforwardly extendable to cover the proximity uses as well.

The collectivizing/proximity alternation displayed by *together* is paralleled by a similar alternation in the semantics of several other group-sensitive adverbials, e.g. comitative prepositional phrases, and *as a group*:

(5)a. John lifted a piano with Mary.

b. John sat with Mary.

(6)a. The students lifted the piano as a group.

b. The students sat as a group.

A similar sort of alternation shows up in expressions indicating isolation or individual action:

(7)a. John lifted the piano by himself.

b. John sat by himself.

(8)a. John lifted the piano alone.

b. John sat alone.

I have not been able to do a thorough cross-linguistic study; however, a quick survey of twelve languages showed that identical marking of collective action and spatio-temporal proximity, though apparently not universal, is extremely common and is not limited to a particular genetic or areal grouping.³ The examples in (9) and (10), from Tamil and Korean respectively, are typical:

(9)a. jānum meriyum sērṇṭu piyānovait tūkkīnārkaḷ

John-and Mary-and joined piano-acc lifted

'John and Mary lifted the piano together.'

b. jānum meriyum sērṇṭu uṭkārṭārkaḷ

³Languages surveyed include Arabic, Basque, Chinese, Finnish, French, German, Hebrew, Japanese, Korean, Spanish, Swedish and Tamil. Of these, all but Basque and Hebrew had an expression essentially equivalent to English *together* in both its collectivizing and spatial proximity uses, and all but Hebrew had an item equivalent to English *with* in both its collectivizing and spatial uses. Thanks to Kutz Arrieta, Sandy Feinstein, Lutfi Hussein, Hakan Kuh, Yongkyoon No, Jane Rauschenberg, Ying-Yu Sheu, Uma Subramaniam, Hideo Tomita and Zheng-Sheng Zhang for information and speaker judgments.

John-and Mary-and joined sat

'John and Mary sat together.'

(10)a. Johnkwa Maryka pianolul kaci tulessta

John-and Mary-nom piano-acc together lifted

'John and Mary lifted the piano together.'

b. Johnkwa Maryka kaci sofae ancessta

John-and Mary-nom together sofa-on sat

'John and Mary sat on the sofa together.'

Given these facts, I think it is well worth considering the proximity uses of *together* and other adverbials in more detail, with the goal of unifying their analysis with that of the collectivizing use.

The spatial understanding of *together* is available whenever it modifies distributive predicates (i.e. predicates which are either intrinsically distributive or understood distributively in context), except those which express states or actions which are not able to be spatio-temporally located. It is also available when *together* modifies collective predicates, though it is less prominent than the collectivizing reading when both are available. Because *gather* and *disperse* are "intrinsic" collectives, modification of them by collectivizing *together* would be superfluous, but if *together* takes its spatial proximity reading, we obtain an intuitively correct interpretation for (11)a. and a plausible explanation for the oddity of (11)b.

(11)a. The students gathered together.

b. ??The students dispersed together.

Especially in sentences describing social interaction, the spatial use of *together* sometimes carries implications of deliberate accompaniment. One would not ordinarily say (12), for instance, if John and Mary were strangers who just happened to have adjacent seats in the theater:

(12) John and Mary sat together.

Likewise the temporal understanding of *together* often carries implications of deliberately coordinated action. Sentence (13) seems less appropriate if John and Mary stood up simultaneously by coincidence than if they did so by plan:

(13) John and Mary stood up together.

I doubt whether these effects are truth conditional, rather than due to conversational implicature. Suppose John and Mary are two characters in a novel, at this point in the story still strangers, but each quite well known to the reader. By coincidence they sit next to each other in the theater. In this situation, (14) seems acceptable:

(14) John and Mary sat together in the third row, each oblivious to the other and unaware of the effect they would have on each other's lives.

In contexts where deliberate accompaniment is unlikely or impossible, the suggestion of deliberate accompaniment simply drops, and no feeling of contradiction results.

Example (15) is a little grisly, but provides an illustration:

(15) John and Mary's corpses lay together on the
coroner's dissection table.

However, even these examples do not seem to be perfectly synonymous to similar examples where *together* is replaced by *next to each other* or *side by side*; *together* seems to indicate not just that John and Mary (or their bodies) were spatially near each other, but that this fact was especially significant in some way. Likewise in similar examples involving temporal *together*, more significance seems to be attached to the simultaneity of action than if *together* is replaced by *simultaneously* or *at the same time*. Perhaps there is some kind of conventional (as opposed to conversational) effect here after all.

Whether or not the spatial and temporal readings of *together* truth-conditionally imply something more than spatial or temporal proximity, it is clear that they do entail at least this much, and it is primarily with these spatial and temporal proximity entailments that we will be concerned in the discussion which follows.⁴

⁴There are truth-conditional semantic differences between *together* and *next to each other* or *side by side*, as illustrated in (i) and (ii) below, so an implicature analysis is by no means out of the question. Sentence (i) can be used to describe a situation where the milk and the water were mixed into a homogenous whole; (ii) requires that the milk and the water were mixed independently, but were next to each other when it happened.

(i) The milk and the water were mixed together.

3. Semantic effects of adverbial position. Even in its collectivizing use, *together* shows some semantic effects which the analysis of Fragment 1 cannot account for without some adjustment.⁵ Compare the a., b. and c. sentences in (16) through (18)⁶:

(16)a. John and Mary made \$20,000 together.

b. John and Mary together made \$20,000.

c. Together, John and Mary made \$20,000.

(17)a. John and Mary lifted five pianos together.

b. John and Mary together lifted five pianos.

c. Together, John and Mary lifted five pianos.

(18)a. John and Mary lifted every piano together.

b. John and Mary together lifted every piano.

c. Together, John and Mary lifted every piano.

Sentence (16)a. indicates that John and Mary participated in some joint enterprise which earned them \$20,000. In contrast (16)b. and c. simply indicate that John and Mary's combined income was \$20,000; these sentences are more natural than (16)a. to describe a situation, say, in which John and Mary are a married couple who worked at different

(ii) The milk and the water were mixed next to each other.

⁵Bennett's analysis of *together* does not account for these effects either. (Nor, of course, does Hoeksema's.)

⁶These sentences should be read with no special accent on *together*.

jobs paying \$10,000 each. Likewise (17)a. indicates that for each of five pianos, John and Mary lifted it together, while (17)b. and c. simply indicate that the total number of pianos lifted by John and/or Mary is at least five. Sentences (17)b. and c. can be used to describe, while (17)a. cannot, a situation in which John lifted two pianos by himself and Mary lifted three (and otherwise no pianos were lifted). The sentences in (18) show the same pattern; (18)a. means that John and Mary lifted each of the pianos together, while (18)b. and c. simply mean that between the two of them, John and Mary lifted all the pianos.

Placement of *as a group* produces identical effects to placement of *together*:

(19)a. The T.A.'s earned \$20,000 as a group.

b. The T.A.'s, as a group, earned \$20,000.

c. As a group, the T.A.'s earned \$20,000.

Placement of comitative PPs also shows a similar effect:

(20)a. John lifted five pianos with Mary.

b. John, with Mary, lifted five pianos.

c. With Mary, John lifted five pianos.

4. Quantifier scope. In each of (16) - (20), the a. sentence, with the adverbial in postverbal position, is truth-conditionally distinct from the b. and c. sentences, with the adverbial in sentence-initial or medial position.

The semantic alternation here is reminiscent of that first pointed out by Scha (1981) between what he called collective and cumulative quantification.

I don't believe anything as complex as Scha's treatment of cumulative quantification is necessary to account for the contrasts illustrated above, however. I will propose that the difference among the a., b. and c. sentences in these examples is due primarily to differences in the relative scope of the group-sensitive adverbial and the object noun phrase, and also partly to a simple distributivity entailment imposed by the operation which derives TV-modifying adverbials from VP-modifying ones. In the a. sentences, the object is necessarily outside the scope of the adverbial, while the b. and c. sentences allow the object to be inside its scope. This is, in fact, exactly the pattern we expect, since there is plenty of independent evidence that postverbal adverbs modify their preceding verbs (in the sense that they denote functions which take the denotations of these verbs as arguments), while preverbal adverbs modify (in this same sense) either the entire verb phrase or the sentence as a whole, and sentence-initial adverbs modify sentences (see, e.g., McConnell-Ginet 1982). In addition, the operation deriving TV/TVs from VP/VPs is semantically slightly more complex than the simple Geach-style type-raising one might expect, forcing a

distributive entailment with respect to the output TV's object argument.

Fragment 1 generated *together* only in postverbal position, made no provision for scope ambiguities, and did not even allow quantified NPs in object position. However, even if the fragment is extended to allow these, it will not produce the correct readings so long as the only contribution of *together* is a restriction to groups. I'll demonstrate this by making the necessary extensions.

The rule in (21) introduces *every*, which will be the only quantificational determiner. (Although *every* shows a number of differences from the cardinal number determiners illustrated in (16) - (17) and (19) - (20), its failure to show scope effects relative to a Hoeksema-style *together* operator should be sufficient to demonstrate the relevant defects in the analysis.) As before, syncategorematicity is not intended to express any sort of theoretical claim.

(21) If $\alpha \in \text{CN}_{\text{sg}}$ then $\beta = \langle \langle \text{every} \rangle, \alpha \rangle \in \text{NP}$;

$\|\beta\| = \{X \in \text{POW}(U) \mid \|\alpha\| \subseteq X\}$.

Rather than let transitive verbs take generalized quantifiers directly as their object arguments, I'll assume that such objects are quantified in.⁷ To keep things

⁷I want to avoid letting extensional TVs take generalized quantifiers directly as their object arguments because of the conjunction facts discussed in Dowty (1981) and Partee and Rooth (1983). Partee and Rooth also give arguments against letting extensional verb phrases denote sets of generalized quantifiers, which might appear to pose a problem for the sort of system outlined in Fragment 1. I

simple, I'll formalize quantifying in as an operation of pronoun replacement, Montague-style; the analysis can be easily be adapted to more sophisticated treatments of wide scope quantification using Cooper Storage, function composition, scope indexing⁸, GB-style Quantifier Raising, or whatever. First we need a rule for pronouns:

- (22) For all $i \in N$, $\langle it_i \rangle \in NP$; $\llbracket \langle it_i \rangle \rrbracket$ is undefined;
for each $s \in U^N$, $\llbracket \langle it_i \rangle \rrbracket^s = s(i)$.

Here N is the set of natural numbers. U^N is the set of functions from N into U (i.e. sequences of elements of U); these serve as assignments of values to variables. For each such sequence s , we have an interpretation function $\llbracket \cdot \rrbracket^s$ which maps the pronoun $\langle it_i \rangle$ onto the i^{th} element of s .

I depart slightly from standard practice by maintaining an invariant interpretation function $\llbracket \cdot \rrbracket$ independent of the various functions $\llbracket \cdot \rrbracket^s$. However, for expressions not containing free pronouns, $\llbracket \cdot \rrbracket$ gives the same value as $\llbracket \cdot \rrbracket^s$ for all s :

- (23) If $\llbracket \alpha \rrbracket$ is defined, then $\llbracket \alpha \rrbracket^s = \llbracket \alpha \rrbracket$ for all $s \in U^N$.

$\llbracket \cdot \rrbracket$ is not defined for pronouns, and hence not for any expression containing a free pronoun, so the following

think this fragment can be extended to include the full range of English quantifiers without running into the problems Partee and Rooth discuss, but because I will abandon this treatment of verb phrase semantics later in the chapter, I will not attempt to give a demonstration here.

⁸As in Cooper and Parsons (1976), Williams (1986, 1988), and Roberts (1987a).

general principle is needed to define the semantics of such expressions:

- (24) For any expression α defined by some rule R as $F(\beta_0, \dots, \beta_n)$, if R gives $\|\alpha\|$ as $G(\|\beta_0\|, \dots, \|\beta_n\|)$, then for all $s \in U^N$, $\|\alpha\|^s = G(\|\beta_0\|^s, \dots, \|\beta_n\|^s)$.

I will allow quantifying in to take place at the sentence level, or the verb phrase level:

- (25) If $\alpha \in S$ dominates $\langle it_i \rangle$ and $\beta \in NP$, then $\gamma = \text{PROSUB}_i(\alpha, \beta) \in S$, where PROSUB_i is the result of replacing the leftmost occurrence of $\langle it_i \rangle$ in α by β^0 ; for all $s \in U^N$, $\|\gamma\|^s = 1$ if $\{x \in U \mid \|\alpha\|^{s[x/i]} = 1\} \in \|\beta\|$, where $s[x/i]$ is a sequence like s except that $s[x/i](i) = x$.

- (26) If $\alpha \in VP$ dominates $\langle it_i \rangle$ and $\beta \in NP$, then $\gamma = \text{PROSUB}_i(\alpha, \beta) \in VP$; for all $s \in U^N$, $\|\gamma\|^s = \{x \in U \mid \{y \in U \mid x \in \|\alpha\|^{s[y/i]}\} \in \|\beta\|\}$.

If all an expression's pronouns are bound, the expression will receive the same value for each $\|\cdot\|^s$. In this case, it is safe to define $\|\cdot\|$ for the expression in question:

- (27) If $\|\alpha\|^s = \|\alpha\|^t$ for all $s, t \in U^N$, then $\|\alpha\| = \|\alpha\|^s$.

⁹ α is leftmost in a set of subexpressions $\{\alpha, \alpha_0, \dots, \alpha_n\}$ of an expression β if α is left of α_i for all i , $0 \leq i \leq n$. α is left of α_i if $\beta = \langle \beta_0, \dots, \beta_m \rangle$, $\beta_j = \alpha$ or β_j dominates α , $\beta_k = \alpha_i$ or β_k dominates α_i , and $j \leq k$. α dominates β if $\alpha = \langle \dots \beta \dots \rangle$ or α dominates γ and γ dominates β . In a more complete fragment, quantifying in would have to be sensitive to structural relations like c-command, and not just linear order, of course. Rules (22) and (23) also ignore the morphological effects of binding, such as the reflexive/non-reflexive alternation, and agreement of pronouns with their antecedents.

Hoeksema's original analysis of *together* was apparently intended to cover its postverbal use. If postverbal adverbs modify verbs rather than verb phrases, however, some adjustment will have to be made. The semantic type of *together* as defined by Hoeksema (and in Fragment 1) is inappropriate for a transitive verb modifier, or indeed for a modifier of any verb but an ordinary intransitive.

The fragment had transitive verbs denote in $\text{POW}(U)^v$. The sets in a TV denotation's range are the sets of groups and/or individuals which the subject noun phrase may denote or quantify over. As a first stab at formulating a TV-modifying version of *together*, we might simply let it eliminate the individuals from these sets, leaving only the groups, much as the VP-modifying version of *together* eliminates the individuals from the VP denotation it modifies:

(28) If $\alpha \in \text{TV}$ then $\beta = \langle \alpha, \langle \text{together} \rangle \rangle \in \text{TV}$;

$\|\beta\| = f : U \rightarrow \text{POW}(U - U_I)$ such that for all $x \in U$,
 $f(x) = \|\alpha\|(x) - U_I$.

In order to assure that the direct object of a complex TV appears between its lexical head and the adverb, we need to modify our original verb-object rule:

(29) If $\alpha \in \text{TV}$ and $\beta \in \text{NP}$ then $\gamma = \text{WRAP}(\alpha, \beta) \in \text{VP}$,

where WRAP is as given in (27), below; $\|\gamma\| = \|\alpha\|(\|\beta\|)$.

(30) Where $\alpha = \langle \alpha_0, \dots, \alpha_n \rangle$:

If $n > 0$, then $\text{WRAP}(\alpha, \beta) = \langle \alpha_0, \beta, \alpha_1, \dots, \alpha_n \rangle$;

if $n = 0$, then $\text{WRAP}(\alpha, \beta) = \langle \alpha, \beta \rangle$.

We also need to change out original *together* rule so that postverbal *together* only modifies verbs and not whole verb phrases:

(31) If $\alpha \in \text{IV}$ then $\beta = \langle \alpha, \langle \text{together} \rangle \rangle \in \text{IV}$;

$\|\beta\| = \|\alpha\| \cap (U - U_I)$.

Preverbal *together* does modify whole verb phrases:

(32) If $\alpha \in \text{VP}$ then $\beta = \langle \langle \text{together} \rangle, \alpha \rangle \in \text{VP}$;

$\|\beta\| = \|\alpha\| \cap (U - U_I)$.

I won't bother to give rules for sentence-initial *together*, since the relevant aspects of its semantics are the same as for medial *together*, and sentence-initial adverbials pose special problems for compositionality.

To see how these modifications fail to predict the differing semantic effects of preverbal and postverbal *together*, a couple of examples may be in order. I'll first work through *John and Mary lift every piano together*, and then through *John and Mary together lift every piano*.

By (28), $\|\langle \langle \text{lift} \rangle, \langle \text{together} \rangle \rangle\| = f : U \rightarrow \text{POW}(U - U_I)$ such that for all $x \in U$, $f(x) = \|\langle \text{lift} \rangle\|(x) - U_I$. Since $\|\langle \langle \text{lift} \rangle, \langle \text{together} \rangle \rangle\|$ is a function with domain U , it cannot take $\|\langle \langle \text{every} \rangle, \langle \text{piano} \rangle \rangle\|$ as an argument, which is an element of $\text{POW}(\text{POW}(U))$ by rule (20), above. However, for each s , $\|\langle \text{it}_i \rangle\|^s = s(i)$ by rule (22), and $s(i) \in U$.

The verb-object rule (29) does not define $\llbracket \langle \langle \text{lift} \rangle, \langle \text{it}_i \rangle, \langle \text{together} \rangle \rrbracket$ since $\llbracket \langle \text{it}_i \rangle \rrbracket$ is undefined, but (24), in conjunction with (29) and (22) gives $\llbracket \langle \langle \text{lift} \rangle, \langle \text{it}_i \rangle, \langle \text{together} \rangle \rrbracket^s$ as $\llbracket \langle \langle \text{lift} \rangle, \langle \text{together} \rangle \rrbracket(s(i))$ for each s . Note that $\llbracket \langle \langle \text{lift} \rangle, \langle \text{together} \rangle \rrbracket(s(i))$ is $\llbracket \langle \text{lift} \rangle \rrbracket(s(i)) - U_I$.

We can quantify in at this point, by rule (26); this gives $\llbracket \langle \langle \text{lift} \rangle, \langle \langle \text{every} \rangle, \langle \text{piano} \rangle \rangle, \langle \text{together} \rangle \rrbracket^s$ as $\{x \in U \mid \{y \in U \mid x \in \llbracket \langle \langle \text{lift} \rangle, \langle \text{it}_i \rangle, \langle \text{together} \rangle \rrbracket^{s[y/i]}\} \in \llbracket \langle \langle \text{every} \rangle, \langle \text{piano} \rangle \rrbracket\}$. Now, $s[y/i]$ is exactly like s except that $s[y/i](i) = y$. Since $\llbracket \langle \langle \text{lift} \rangle, \langle \text{it}_i \rangle, \langle \text{together} \rangle \rrbracket^s = \llbracket \langle \langle \text{lift} \rangle, \langle \text{together} \rangle \rrbracket(s(i)) = \llbracket \langle \text{lift} \rangle \rrbracket(s(i)) - U_I$, it follows that $\llbracket \langle \langle \text{lift} \rangle, \langle \text{it}_i \rangle, \langle \text{together} \rangle \rrbracket^{s[y/i]} = \llbracket \langle \text{lift} \rangle \rrbracket(y) - U_I$ for every s , hence that $\llbracket \langle \langle \text{lift} \rangle, \langle \langle \text{every} \rangle, \langle \text{piano} \rangle \rangle, \langle \text{together} \rangle \rrbracket^s = \{x \in U \mid \{y \in U \mid x \in \llbracket \langle \text{lift} \rangle \rrbracket(y) - U_I\} \in \llbracket \langle \langle \text{every} \rangle, \langle \text{piano} \rangle \rrbracket\}$, for every s .

Since for all $s, t \in U^M$, $\{x \in U \mid \{y \in U \mid x \in \llbracket \langle \text{lift} \rangle \rrbracket(y) - U_I\} \in \llbracket \langle \langle \text{every} \rangle, \langle \text{piano} \rangle \rrbracket\} = \{x \in U \mid \{y \in U \mid x \in \llbracket \langle \text{lift} \rangle \rrbracket(y) - U_I\} \in \llbracket \langle \langle \text{every} \rangle, \langle \text{piano} \rangle \rrbracket\}$, we have $\llbracket \langle \langle \text{lift} \rangle, \langle \langle \text{every} \rangle, \langle \text{piano} \rangle \rangle, \langle \text{together} \rangle \rrbracket = \{x \in U \mid \{y \in U \mid x \in \llbracket \langle \text{lift} \rangle \rrbracket(y) - U_I\} \in \llbracket \langle \langle \text{every} \rangle, \langle \text{piano} \rangle \rrbracket\}$, by (27).

By rule (34) of Fragment 1, $\llbracket \langle \langle \text{John} \rangle, \langle \text{and} \rangle, \langle \text{Mary} \rangle \rangle \rrbracket = g(\text{John}, \text{Mary})$, the group of John and Mary. By the subject-predicate rule (36), $\llbracket \langle \langle \langle \text{John} \rangle, \langle \text{and} \rangle, \langle \text{Mary} \rangle \rangle, \langle \langle \text{lift} \rangle, \langle \langle \text{every} \rangle, \langle \text{piano} \rangle \rangle, \langle \text{together} \rangle \rangle \rrbracket = 1$ if $g(\text{John}, \text{Mary}) \in \{x \in U \mid \{y \in U \mid x \in \llbracket \langle \text{lift} \rangle \rrbracket(y) - U_I\} \in \llbracket \langle \langle \text{every} \rangle, \langle \text{piano} \rangle \rrbracket\}$,

$\langle \text{piano} \rangle \rangle \mathbb{I}$, that is to say, if $\{y \in U \mid g(\text{John}, \text{Mary}) \in \mathbb{I}\langle \text{lift} \rangle \mathbb{I}(y) - U_I\} \in \mathbb{I}\langle \langle \text{every} \rangle, \langle \text{piano} \rangle \rangle \mathbb{I}$. Since $\mathbb{I}\langle \langle \text{every} \rangle, \langle \text{piano} \rangle \rangle \mathbb{I} = \{X \in \text{POW}(U) \mid \mathbb{I}\langle \text{piano} \rangle \mathbb{I} \subseteq X\}$, the sentence is true if $\{y \in U \mid g(\text{John}, \text{Mary}) \in \mathbb{I}\langle \text{lift} \rangle \mathbb{I}(y) - U_I\} \in \{X \in \text{POW}(U) \mid \mathbb{I}\langle \text{piano} \rangle \mathbb{I} \subseteq X\}$, that is, if $\mathbb{I}\langle \text{piano} \rangle \mathbb{I} \subseteq \{y \in U \mid g(\text{John}, \text{Mary}) \in \mathbb{I}\langle \text{lift} \rangle \mathbb{I}(y) - U_I\}$. By definition, $g(\text{John}, \text{Mary}) \notin U_I$, so this can be simplified to $\mathbb{I}\langle \text{piano} \rangle \mathbb{I} \subseteq \{y \in U \mid g(\text{John}, \text{Mary}) \in \mathbb{I}\langle \text{lift} \rangle \mathbb{I}(y)\}$. In plain English, *John and Mary lift every piano together* is true if each individual piano is in the set of things which the group of John and Mary lifted.

Compare this case with that described by *John and Mary together lifted every piano*. We start again with $\mathbb{I}\langle \text{lift} \rangle \mathbb{I}$, a function from U into $\text{POW}(U)$. This cannot take $\mathbb{I}\langle \langle \text{every} \rangle, \langle \text{piano} \rangle \rangle \mathbb{I}$ as an argument, so again we use $\langle \text{it}_i \rangle$ instead. For all s , $\mathbb{I}\langle \langle \text{lift} \rangle, \langle \text{it}_i \rangle \rangle \mathbb{I}^s = \mathbb{I}\langle \text{lift} \rangle \mathbb{I}(s(i))$.

Now we quantify in. By (26), $\mathbb{I}\langle \langle \text{lift} \rangle, \langle \langle \text{every} \rangle, \langle \text{piano} \rangle \rangle \mathbb{I}^s = \{x \in U \mid \{y \in U \mid x \in \mathbb{I}\langle \langle \text{lift} \rangle, \langle \text{it}_i \rangle \rangle \mathbb{I}^s(y/i)\} \in \mathbb{I}\langle \langle \text{every} \rangle, \langle \text{piano} \rangle \rangle \mathbb{I}\}$, for all s . This equals $\{x \in U \mid \{y \in U \mid x \in \mathbb{I}\langle \text{lift} \rangle \mathbb{I}(y)\} \in \mathbb{I}\langle \langle \text{every} \rangle, \langle \text{piano} \rangle \rangle \mathbb{I}\}$. Since this is the same at every s , $\mathbb{I}\langle \langle \text{lift} \rangle, \langle \langle \text{every} \rangle, \langle \text{piano} \rangle \rangle \mathbb{I}$ is defined, and equals $\{x \in U \mid \{y \in U \mid x \in \mathbb{I}\langle \text{lift} \rangle \mathbb{I}(y)\} \in \mathbb{I}\langle \langle \text{every} \rangle, \langle \text{piano} \rangle \rangle \mathbb{I}\}$.

$\mathbb{I}\langle \langle \text{every} \rangle, \langle \text{piano} \rangle \rangle \mathbb{I} = \{X \in \text{POW}(U) \mid \mathbb{I}\langle \text{piano} \rangle \mathbb{I} \subseteq X\}$, so $\mathbb{I}\langle \langle \text{lift} \rangle, \langle \langle \text{every} \rangle, \langle \text{piano} \rangle \rangle \mathbb{I} = \{x \in U \mid \{y \in U \mid x \in \mathbb{I}\langle \text{lift} \rangle \mathbb{I}(y)\} \in \{X \in \text{POW}(U) \mid \mathbb{I}\langle \text{piano} \rangle \mathbb{I} \subseteq X\}\}$. This reduces

to $\{x \in U \mid \llbracket \langle piano \rangle \rrbracket \subseteq \{y \in U \mid x \in \llbracket \langle lift \rangle \rrbracket(y)\}\}$. By (32), $\llbracket \langle \langle together \rangle, \langle \langle lift \rangle, \langle \langle every \rangle, \langle piano \rangle \rangle \rangle \rrbracket = \{x \in U \mid \llbracket \langle piano \rangle \rrbracket \subseteq \{y \in U \mid x \in \llbracket \langle lift \rangle \rrbracket(y)\}\} \cap (U - U_I)$, which is equivalent to $\{x \in U - U_I \mid \llbracket \langle piano \rangle \rrbracket \subseteq \{y \in U \mid x \in \llbracket \langle lift \rangle \rrbracket(y)\}\}$.

$\llbracket \langle \langle John \rangle, \langle and \rangle, \langle Mary \rangle \rangle \rrbracket = g(\text{John}, \text{Mary})$, as before, so $\llbracket \langle \langle \langle John \rangle, \langle and \rangle, \langle Mary \rangle \rangle, \langle \langle together \rangle, \langle \langle lift \rangle, \langle \langle every \rangle, \langle piano \rangle \rangle \rangle \rangle \rrbracket = 1$ if $g(\text{John}, \text{Mary}) \in \{x \in U - U_I \mid \llbracket \langle piano \rangle \rrbracket \subseteq \{y \in U \mid x \in \llbracket \langle lift \rangle \rrbracket(y)\}\}$, that is, if $\llbracket \langle piano \rangle \rrbracket \subseteq \{y \in U \mid g(\text{John}, \text{Mary}) \in \llbracket \langle lift \rangle \rrbracket(y)\}$.

But this is exactly the same truth condition as we derived for *John and Mary lift every piano together*. Since the two sentences are not synonymous, there is some error in the analysis.

5. Event structure and group action. The central feature of the analysis just sketched is that *together*, whether it modifies a transitive verb, intransitive verb or verb phrase, operates to restrict the subject argument of the verb or VP to groups. Given this kind of semantics for *together*, scope differences are not sufficient to produce the range of readings we want.

I don't know if some other device besides scope could be devised to produce these readings without altering the semantics of *together*. Rather than exploring this possibility, however, I want to suggest that the semantic

difference between *John and Mary together lifted every piano* and *John and Mary lifted every piano together* really is due to scope, and that *together* contributes something more to the semantics of the constructions in which it appears than just a restriction to groups.

The semantics I want to suggest for *together* involves not just a restriction to groups, but also conditions on "event structure". In order to implement it, we need to expand the notions of a model and model frame from Chapter I. Let a model frame be a 6-tuple $\langle U, U_I, \sqsubseteq_U, E, E, \sqsubseteq_E \rangle$, where $\langle U, U_I, \sqsubseteq_U, E \rangle$ is a model frame as defined in Chapter I and $\langle E, \sqsubseteq_E \rangle$ is a join semilattice. A model is a 7-tuple $\langle U, U_I, \sqsubseteq, E, E, \sqsubseteq_E, I.I \rangle$, where $\langle U, U_I, \sqsubseteq_U, E, E, \sqsubseteq_E \rangle$ is a model frame as just defined and $I.I$ is a function mapping expressions of a language to elements of U or E or type-theoretic constructs based on U and E , so as to define truth conditions for sentences of the language.

The elements of E are to be understood as "events", though the notion I have in mind here is more general, and perhaps also more abstract, than events in the ordinary, pretheoretic sense. Events are the sort of thing a sentence may be a true or false description of. As understood here they include states, processes, etc., and not just the more dynamic objects the term ordinarily suggests; I also want to allow such things as negative events (such as the event of John's not being with Mary), disjunctive events (such as the

event of John's either being with Mary or with Sue), and distinct events which take place at the same place and time, involving the same participants (such as the event of John crossing the finish line and the event of John winning the race).¹⁰ Events thus are rather abstract and "propositional".¹¹ Readers who are uncomfortable with the term "event" being used in this sense should feel free to substitute an alternative: "situation", "fact", "information state", "state of affairs" and "partial possible world" all suggest themselves -- with this last probably being closest to what I have in mind -- though these terms all have unfortunate connotations of their own as well.

The requirement that $\langle E, \sqsubseteq_s \rangle$ be a join semilattice ensures that any two events e, e' combine to form a third event $e \sqcup e'$, the join of e and e' . For example, if there are two pianos, and for each of them an event in which John lifts it, then the join of these two events will be a complex event in which John lifts both pianos. The join operation imposes the "subevent" relation $\sqsubseteq_s$ on E . I will

¹⁰There is of course a large philosophical literature on the question of negative and disjunctive events, and on the question of whether events are individuated by their spatio-temporal locations and participants. Unfortunately, this literature is too extensive to be reviewed here; for some idea of the issues involved see, e.g., Barwise and Perry (1983), Barwise and Etchemendy (1987), Davidson (1980) and the references cited there.

¹¹The relation between events and propositions will be discussed in Chapter III.

assume, following Situation Semantics¹², that the truth of propositions is generally "persistent" relative to $\sqsubseteq_s$; if some sentence S provides a true description of what happens in an event e , and e is a part of another event e' , then S provides a true (partial) description of what happens in e' . In this fragment, persistence will be assured through the use of lexical meaning postulates; a more general approach is adopted in Chapter III.

It is easy to make a case that the set of events should have more structure than that imposed by the semilattice requirement, but I think that it must have the structure of a semilattice at the very least, and it is all I will assume in this chapter. Again, a more restrictive account of event structure will be proposed in Chapter III.

Consider now what kinds of events will count as involving group action. Although piano-lifting sentences are surely becoming tiresome by now, there are advantages in seeing how familiar examples come out in the new context of an event-based model, so let us examine the conditions under which an event may be accurately described by the sentence *John and Mary lift the piano*.

There is of course the distributive reading in which John and Mary each lift the piano, but let us ignore this interpretation for the moment and focus instead on the collective reading. If the sentence is collectively

¹²See Barwise and Perry (1983).

understood, the image which it most readily conveys is one in which John and Mary stand at opposite ends of the piano and lift it with their arms. Probably no one would claim that this is the only sort of situation in which the sentence is true, however; we would probably want to weaken the requirement at least to the point where the group of John and Mary is required only to play a (more or less) immediate causal role in bringing about the lifting of the piano.

How does this contrast with the sort of event described by *John lifts the piano*? One is tempted to say that in events described by the first sentence, John and Mary each contribute to the lifting of the piano, while in events described by the second sentence John causes the piano to be lifted without any contribution from Mary.

Taking this stand produces immediate problems, however, if we consider sentences like *John and Mary lift every piano*. This sentence (like *John and Mary together lift every piano*, but unlike *John and Mary lift every piano together*) allows for the case where John lifted some of the pianos and Mary lifted the rest. But unless we adopt some radically non-standard semantics for *every* or for *and*, *John and Mary lifted every piano* means that for every piano, the group of John and Mary lifted it. Thus events in which only John or only Mary actually contributed to the lifting of a

piano have to count as liftings of that piano by the group of John and Mary.

I don't think this is as implausible as it may at first appear. It is actually common practice to attribute an action to a group even if only some of its members actually performed it. Imagine a competition in which teams are required to attempt various stunts, including lifting a piano. John and Mary form one team, Bill and Susan form another. During the competition, John lifts the piano; meanwhile Mary performs one of the other stunts, say shooting herself out of a cannon. When Bill and Sue's turn arrives, they succeed in doing almost all the stunts that John and Mary did, but fail at lifting the piano, and therefore lose the competition. In this sort of situation, it seems fair to say that John and Mary won the competition because they lifted the piano, while Bill and Sue didn't. This is despite the fact that Mary played no role in the actual lifting.

Now I don't think it's the case that if some individual lifts a piano, then any group containing that individual will also count as lifting the piano; in the situation just described we would not count John and Bill as lifting the piano, for instance. Nor am I sure it is the job of model-theoretic semantics to determine under what conditions action by an individual may be counted as action by a group containing that individual; surely the prime consideration

in the example just given is the pragmatic fact that the overall performance of the team is more relevant than the performances of its individual members. One might even want to claim for such cases that the domain of discourse is pragmatically restricted only to teams.

Note, however, that adopting such a view does not in itself explain the fact that the group as a whole can receive "credit" for the properties of its individual members. A simple restriction on the domain will not automatically transfer the properties of the excluded individuals up onto the groups to which they belong. Moreover, I hope to show below that a model-theoretic semantics can and should be sensitive to the distinction between events where an individual performs an action which for some reason is counted as a group action, and events where the group itself performs the action.

Such a distinction is easy to capture formally, given the semilattice structure of the set of events E . An event of John and Mary lifting the piano in which only John actually brings about the piano-lifting will have a subevent in which John lifts the piano. An event in which John and Mary lift the piano through "authentic" group action need not have such a subevent, but it will have a subevent of John and Mary lifting the piano which in turn does not have a subevent of John lifting the piano.

If this last point is not clear, consider an event e in which John and Mary, acting together, lift a piano. Any number of other things may also happen in this event; for example the event may contain a whole sequence of actions, of which John and Mary's lifting is just one. In particular it might have a subevent in which John lifts the piano by himself. Consider, however, the minimal subevent of e containing John and Mary's collective lifting. This minimal subevent, call it e' , consists just of the lifting itself, including whatever smaller events it comprises. Since e' involves "authentic" group action by John and Mary, and consists just of this group lifting, it will not contain a subevent of John lifting the piano.

In contrast, if John and Mary lift the piano solely through action on the part of John, as in the competition example above, then the minimal event containing John and Mary's lifting of the piano will have a subevent in which John lifts the piano.¹³ Figure 1 informally diagrams this sort of event; Figure 2 diagrams an event in which John and Mary lift the piano through "authentic" group action:

With this view of group-action event structure, it is straightforward to define a semantics for *together*,

¹³I will assume in what follows that this subevent is in fact a proper subevent. There may be some way to adjust formal analysis given below if we choose instead to actually identify the event of John and Mary lifting of the piano with the event of John lifting it, but I am not sure what these adjustments would be.

Figure 1

Figure 2

comitative PPs, and similar expressions which not only captures the distinction between preverbal and postverbal position, but also may be extended to cover spatial and temporal readings, and even to allow a closure-condition treatment of distributivity.

Consider the type of event which can be described by *John and Mary together lift a piano* or *John and Mary lift a piano together*. These, in contrast to *John and Mary lift a piano*, require what I have been calling "real" group action: action on the part of both John and Mary. We can capture this by letting *together* be sensitive to events with the sort of structure just argued to distinguish real group action from the sort of "pseudo" group action which is really not performed by the group as a whole.

6. Fragment 2. In order to capture this idea formally, we need to redefine our semantic types and rules in terms of events. Since this is a major change, it is probably better to construct a new fragment, Fragment 2, than to try to revise Fragment 1. This section gives rules for simple sentences formed from nondistributive verbs and verb phrases, quantificational NPs, proper names, definite descriptions, and conjoined terms, plus preverbal and postverbal *together* and comitative PPs in their collectivizing use. Extensions to cover the spatial and temporal readings of these adverbials will be given in later

sections of this chapter, as will extensions to cover what I will call their "object-oriented" and predicative readings, and distributive predicates.

I will follow Austin (1950) and recent work in Situation Semantics in taking an utterance to refer deictically, or, in Austin's words, demonstratively, to an event. In an Austinian truth theory, the semantics of a sentence provides an event "type"; the statement made in uttering the sentence is true iff the event deictically referred to by the utterance is of the type provided by the sentence semantics. Rather than defining event types as sequences of properties, individuals, "indeterminates", etc., as in Situation Semantics, or taking them as formal primitives, as in Link (1987b), I will simply identify event types with sets of events. Sentences thus denote sets of events, and may be thought of as predicates of events in a very conventional sense. Probably this means that event types will not be good for all the things Situation Semanticists and others would like, but it is sufficient for our purposes in this chapter.¹⁴

It will be convenient typographically to define denotation domains via the mediation of type indices. I'll assume type indexing as in (33). As before, U is the set of

¹⁴In Chapter III, event structure will be constrained in terms of a set of primitive "propositions", which play a role much like one might expect of abstract event types.

groups and individuals, U_i is the set of individuals, and E is the set of events.

- (33)a. Elements of U (and nothing else) are of type u .
- b. Elements of E (and nothing else) are of type e .
- c. For any type t , sets of items of type t (and nothing else) are of type $\langle t \rangle$.
- d. For any types t, t' , functions mapping items of type t to items of type t' (and nothing else) are of type $\langle t, t' \rangle$.

A function *TYPE* will be defined to set up correspondences between syntactic categories and semantic types. The following principle establishes the connection between the semantic type associated with a category and the denotations of expressions of that category:

- (34) For all types t and categories C , if $TYPE(C) = t$, then for every expression $\alpha \in C$, $\|\alpha\|$ is of type t .

TYPE is defined as follows:

- (35)a. $TYPE(S) = \langle e \rangle$.
- b. $TYPE(T) = u$.
- c. $TYPE(Q) = \langle \langle u \rangle \rangle$.
- d. $TYPE(CN) = \langle u \rangle$.
- e. $TYPE(VP) = TYPE(IV) = \langle e, \langle u \rangle \rangle$.
- f. $TYPE(TV) = \langle e, \langle u, \langle u \rangle \rangle \rangle$.
- g. For all categories C, C' , $TYPE(C/C') = \langle TYPE(C'), TYPE(C) \rangle$.

T (for "term") and Q (for "quantifier") are the categories of non-quantificational and quantificational noun phrases, respectively. Readers who prefer not to draw a major category distinction between these are invited to think of T and Q as abbreviating feature matrices which differ only with respect to some minor feature.

Fragment 2 has the following basic expressions and lexical meaning postulates:

- (36)a. $\langle John \rangle, \langle Mary \rangle, \langle Bill \rangle \in T$.
- b. For all natural numbers i , $\langle it_i \rangle \in T$;
 $\| \langle it_i \rangle \|$ is undefined;
 for all $s \in U^n$, $\| \langle it_i \rangle \| ^s = s(i)$.
- c. $\langle T.A. \rangle, \langle piano \rangle \in CN_{ns}$;
 $\| \langle T.A. \rangle \| \in U_I$.
 (Likewise for $\langle piano \rangle$.)
- d. $\langle lift \rangle, \langle build \rangle \in TV$;
 for all $e, e' \in E$ such that $e \sqsubseteq_s e'$, and all $x, y \in U$, $x \in \| \langle lift \rangle \| (e)(y)$ implies $x \in \| \langle lift \rangle \| (e')(y)$. (Likewise for $\langle build \rangle$.)
- e. $\langle the \rangle \in T/CN$;
 for all X of type $\langle u \rangle$, $\| \langle the \rangle \| (X) = x$ if $x \in X$ and $\forall y [y \in X \wedge y \neq x \rightarrow y \sqsubseteq_U x \vee y \in x]$,
 undefined otherwise.
- f. $\langle five \rangle \in Q/CN_{p1}$;
 for all X of type $\langle u \rangle$, $\| \langle five \rangle \| (X) = \{ Y \in POW(U) \mid \exists x \in Y [x \in X \wedge CARD(\{ y \in U \mid y \in x \}) = 5] \}$.

g. $\langle \text{every} \rangle \in Q/CN_{sg}$;

for all X of type $\langle u \rangle$, $\|\langle \text{every} \rangle\|(X) = \{Y \in \text{POW}(U) \mid X \subseteq Y\}$.

h. $\langle \text{together} \rangle \in VP/VP$;

for all f of type $\langle e, \langle u \rangle \rangle$ and all e of type e ,
 $\|\langle \text{together} \rangle\|(f)(e) = \{w \in U - U_I \mid \exists e' \sqsubseteq_E e [w \in f(e') \wedge \forall e'' \sqsubseteq_E e' [[\exists z z \in f(e'')] \rightarrow f(e'')] = f(e')]]\}$.

i. $\langle \text{with} \rangle \in (VP/VP)/T$;

for all x of type u , all f of type $\langle e, \langle u \rangle \rangle$, and
 all e of type e , $\|\langle \text{with} \rangle\|(x)(f)(e) = \{y \in U \mid \exists e' \sqsubseteq_E e [g(x, y) \in f(e') \wedge \forall e'' \sqsubseteq_E e' [[\exists z z \in f(e'')] \rightarrow f(e'')] = f(e')]]\}$.

Together and *with* are entered lexically at their VP-modifying type. The TV-modifying adverbial denotations will be derived by the following rule:

(37) If $\alpha \in VP/VP$ then $\beta = \langle \alpha \rangle \in TV/TV$;

for all f of type $\langle e, \langle u, \langle u \rangle \rangle \rangle$, all e of type e ,
 and all x of type u , $\|\beta\|(f)(e)(x) = \{y \in U \mid \forall z [[z \in x \vee z = x] \rightarrow y \in \|\alpha\|(f'(z))(e)]\}$, where for
 all e of type e and all x of type u , $f'(x)(e) = f(e)(x)$.

Plural nouns are derived as in Fragment 1:

(38) If $\alpha \in CN_{sg}$ then $\beta = \text{PLUR}(\alpha) \in CN_{pl}$,

where $\text{PLUR}(x)$ is the result of adding plural
 morphology to x ; $\|\beta\| = \|\alpha\| \cup \{g \in U \mid \forall x [x \in g \rightarrow$

$$x \in \|\alpha\|\}.$$

Term phrases may be derived by applying a T/CN to a CN (CN = CN_{sg} $\cup$ CN_{pl}):

(39) If $\alpha \in \text{T/CN}$ and $\beta \in \text{CN}$ then $\gamma = \langle \alpha, \beta \rangle \in \text{T}$;

$$\|\gamma\| = \|\alpha\|(\|\beta\|).$$

Terms may be conjoined to give a new, group-denoting term:

(40) If $\alpha \in \text{T}$ and $\beta \in \text{T}$ then $\gamma = \langle \alpha, \langle \text{and} \rangle, \beta \rangle \in \text{T}$;

$$\|\gamma\| = g(\|\alpha\|, \|\beta\|).$$

A (VP/VP)/T may take a T argument to give a VP/VP:

(41) If $\alpha \in (\text{VP/VP})/\text{T}$ and $\beta \in \text{T}$ then $\gamma = \langle \alpha, \beta \rangle \in$

$$\text{VP/VP}; \|\gamma\| = \|\alpha\|(\|\beta\|).$$

A TV/TV may modify a TV:

(42) If $\alpha \in \text{TV/TV}$ and $\beta \in \text{TV}$ then $\gamma = \langle \beta, \alpha \rangle \in \text{TV}$;

$$\|\gamma\| = \|\alpha\|(\|\beta\|).$$

A TV may take a T argument to give a VP:

(43) If $\alpha \in \text{TV}$ and $\beta \in \text{T}$ then $\gamma = \text{WRAP}(\alpha, \beta) \in \text{VP}$;

$$\text{for all } e \text{ of type } e: \|\gamma\|(e) = \|\alpha\|(e)(\|\beta\|).$$

A VP/VP may modify a VP:

(44) If $\alpha \in \text{VP/VP}$ and $\beta \in \text{VP}$ then $\gamma = \langle \alpha, \beta \rangle \in \text{VP}$;

$$\|\gamma\| = \|\alpha\|(\|\beta\|).$$

A sentence may consist of a term and a VP:

(45) If $\alpha \in \text{T}$ and $\beta \in \text{VP}$ then $\gamma = \langle \alpha, \beta \rangle \in \text{S}$;

$$\|\gamma\| = \{e \in E \mid \|\alpha\| \in \|\beta\|(e)\}.$$

Since $\|\cdot\|$ is undefined for pronouns, we need a general principle defining the various functions $\|\cdot\|^s$ based on the rules for $\|\cdot\|$. (This is just (24) over again.)

- (46) For any expression α defined by some rule R as $F(\beta_0, \dots, \beta_n)$, if R gives $\|\alpha\|$ as $G(\|\beta_0\|, \dots, \|\beta_n\|)$, then for all $s \in U^N$, $\|\alpha\|^s = G(\|\beta_0\|^s, \dots, \|\beta_n\|^s)$.

If $\|\cdot\|$ is defined for some expression α , then $\|\alpha\|^s$ is constant across values for s :

- (47) If $\|\alpha\|$ is defined, then $\|\alpha\|^s = \|\alpha\|$ for all $s \in U^N$.

A Q/CN_{sg} and a CN_{sg} combine to form a Q :

- (48) If $\alpha \in Q/CN_{sg}$ and $\beta \in CN_{sg}$ then $\gamma = \langle \alpha, \beta \rangle \in Q$;
 $\|\gamma\| = \|\alpha\|(\|\beta\|)$.

Q s will only be introduced through quantifying in, which, as before, may take place at either the S or the VP level:

- (49) If $\alpha \in S$ dominates $\langle it_i \rangle$ and $\beta \in NP$, then $\gamma = \text{PROSUB}_i(\alpha, \beta) \in S$, where PROSUB_i is the result of replacing the leftmost occurrence of $\langle it_i \rangle$ in α by β ; for all $s \in U^N$, $\|\gamma\|^s = \{e \in E \mid \{x \in U \mid e \in \|\alpha\|^s(x/i)\} \in \|\beta\|\}$.

- (50) If $\alpha \in VP$ dominates $\langle it_i \rangle$ and $\beta \in NP$, then $\gamma = \text{PROSUB}_i(\alpha, \beta) \in VP$; for all $s \in U^N$ and all e of type e , $\|\gamma\|^s(e) = \{x \in U \mid \{y \in U \mid x \in \|\alpha\|^s(y/i)(e)\} \in \|\beta\|\}$.

If all an expression's pronouns are bound, the expression will receive the same value for each $\|\cdot\|^s$, in which case it is safe to define $\|\cdot\|$ for the expression in question:

- (51) If $\|\alpha\|^s = \|\alpha\|^t$ for all $s, t \in U^N$, then $\|\alpha\| = \|\alpha\|^s$.

7. Examples. Let's work through the sentences *John and Mary together lift every piano* and *John and Mary lift every piano together* again, this time using the rules of Fragment 2.

As a transitive verb, $\langle \text{lift} \rangle$ denotes a function which maps any event onto a function from individuals/groups to sets of individuals/groups. Intuitively, $\|\langle \text{lift} \rangle\|$ should map any event e onto a function which assigns each liftee in e the set of its lifters in e .

$\|\langle \text{lift} \rangle, \langle it_i \rangle\|$ is undefined (for all i), but $\|\langle \text{lift} \rangle, \langle it_i \rangle\|^s$ for each s is a function mapping any event e onto the set of lifters in e of the i^{th} element of s (where i is any natural number), as indicated in (52):

$$(52) \text{ For all } s, e: \|\langle \text{lift} \rangle, \langle it_i \rangle\|^s(e) = \|\langle \text{lift} \rangle\|(e)(s(i)).$$

$\|\langle \text{every} \rangle, \langle \text{piano} \rangle\|$ is the set of sets containing all the pianos, as indicated in (53):

$$(53) \|\langle \text{every} \rangle, \langle \text{piano} \rangle\| = \{X \in \text{POW}(U) \mid \|\langle \text{piano} \rangle\| \subseteq X\}.$$

Applying the VP-level quantifying-in rule (50) defines $\|\langle \text{lift} \rangle, \langle \langle \text{every} \rangle, \langle \text{piano} \rangle \rangle\|^s$ for each s as in (54):

$$(54) \text{ For all } s, e: \|\langle \text{lift} \rangle, \langle \langle \text{every} \rangle, \langle \text{piano} \rangle \rangle\|^s(e) = \{x \in U \mid \{y \in U \mid x \in \|\langle \text{lift} \rangle, \langle it_i \rangle\|^s(y/1)(e)\} \in \|\langle \text{every} \rangle, \langle \text{piano} \rangle\|\}.$$

This reduces as shown in (55):

$$(55) \text{ For all } s, e: \|\langle \text{lift} \rangle, \langle \langle \text{every} \rangle, \langle \text{piano} \rangle \rangle\|^s(e) =$$

$$\{x \in U \mid \{y \in U \mid x \in \llbracket \langle \text{lift} \rangle \rrbracket(e)(s\{y/i\}(i))\} \in \llbracket \langle \text{every} \rangle, \langle \text{piano} \rangle \rrbracket\}.$$

And reduces further as shown in (56):

$$(56) \text{ For all } s, e: \llbracket \langle \langle \text{lift} \rangle, \langle \text{every} \rangle, \langle \text{piano} \rangle \rangle \rrbracket^s(e) = \{x \in U \mid \{y \in U \mid x \in \llbracket \langle \text{lift} \rangle \rrbracket(e)(y)\} \in \llbracket \langle \text{every} \rangle, \langle \text{piano} \rangle \rrbracket\}.$$

Since this is the same for all s , we have (57):

$$(57) \text{ For all } e: \llbracket \langle \langle \text{lift} \rangle, \langle \text{every} \rangle, \langle \text{piano} \rangle \rangle \rrbracket(e) = \{x \in U \mid \{y \in U \mid x \in \llbracket \langle \text{lift} \rangle \rrbracket(e)(y)\} \in \llbracket \langle \text{every} \rangle, \langle \text{piano} \rangle \rrbracket\}.$$

Replacing $\llbracket \langle \text{every} \rangle, \langle \text{piano} \rangle \rrbracket$ with $\{X \in \text{POW}(U) \mid \llbracket \langle \text{piano} \rangle \rrbracket \subseteq X\}$ gives the reduction in (58):

$$(58) \text{ For all } e: \llbracket \langle \langle \text{lift} \rangle, \langle \text{every} \rangle, \langle \text{piano} \rangle \rangle \rrbracket(e) = \{x \in U \mid \{y \in U \mid x \in \llbracket \langle \text{lift} \rangle \rrbracket(e)(y)\} \in \{X \in \text{POW}(U) \mid \llbracket \langle \text{piano} \rangle \rrbracket \subseteq X\}\}.$$

This reduces as shown in (59):

$$(59) \text{ For all } e: \llbracket \langle \langle \text{lift} \rangle, \langle \text{every} \rangle, \langle \text{piano} \rangle \rangle \rrbracket(e) = \{x \in U \mid \llbracket \langle \text{piano} \rangle \rrbracket \subseteq \{y \in U \mid x \in \llbracket \langle \text{lift} \rangle \rrbracket(e)(y)\}\}.$$

Now, by the lexical meaning postulate for $\langle \text{together} \rangle$, $\llbracket \langle \text{together} \rangle, \langle \langle \text{lift} \rangle, \langle \text{every} \rangle, \langle \text{piano} \rangle \rangle \rrbracket$ is a function as shown in (60):

$$(60) \text{ For all } e: \llbracket \langle \text{together} \rangle, \langle \langle \text{lift} \rangle, \langle \text{every} \rangle, \langle \text{piano} \rangle \rangle \rrbracket(e) = \{w \in U - U_I \mid \exists e' \sqsubseteq_s e [w \in \{x \in U \mid \llbracket \langle \text{piano} \rangle \rrbracket \subseteq \{y \in U \mid x \in \llbracket \langle \text{lift} \rangle \rrbracket(e')(y)\}\} \wedge \forall e'' \sqsubseteq_s e' [\exists x x$$

$$\in \{x \in U \mid \llbracket \langle piano \rangle \rrbracket \subseteq \{y \in U \mid x \in \llbracket \langle lift \rangle \rrbracket(e'')(y)\}\} \rightarrow \{x \in U \mid \llbracket \langle piano \rangle \rrbracket \subseteq \{y \in U \mid x \in \llbracket \langle lift \rangle \rrbracket(e'')(y)\}\} = \{x \in U \mid \llbracket \langle piano \rangle \rrbracket \subseteq \{y \in U \mid x \in \llbracket \langle lift \rangle \rrbracket(e')(y)\}\}\}.$$

Now, replacing redundancies of the form $x \in \{x \in U \mid F(x)\}$ simply by $F(x)$, we obtain (61):

(61) For all e :

$$\begin{aligned} & \llbracket \langle \langle together \rangle, \langle \langle lift \rangle, \langle \langle every \rangle, \langle piano \rangle \rangle \rangle \rrbracket(e) = \\ & \{w \in U - U_I \mid \exists e' \sqsubseteq_E e [\llbracket \langle piano \rangle \rrbracket \subseteq \{y \in U \mid w \in \\ & \llbracket \langle lift \rangle \rrbracket(e')(y)\} \wedge \forall e'' \sqsubseteq_E e' [\exists z \llbracket \langle piano \rangle \rrbracket \subseteq \{y \\ & \in U \mid z \in \llbracket \langle lift \rangle \rrbracket(e'')(y)\} \rightarrow \{x \in U \mid \llbracket \langle piano \rangle \rrbracket \\ & \subseteq \{y \in U \mid x \in \llbracket \langle lift \rangle \rrbracket(e'')(y)\}\} = \{x \in U \mid \\ & \llbracket \langle piano \rangle \rrbracket \subseteq \{y \in U \mid x \in \llbracket \langle lift \rangle \rrbracket(e')(y)\}\}]\}. \end{aligned}$$

By the subject-predicate rule (45), we obtain (62):

$$\begin{aligned} (62) \quad & \llbracket \langle \langle John \rangle, \langle and \rangle, \langle Mary \rangle \rangle, \langle \langle together \rangle, \langle \langle lift \rangle, \\ & \langle \langle every \rangle, \langle piano \rangle \rangle \rangle \rrbracket = \{e \in E \mid \llbracket \langle \langle John \rangle, \langle and \rangle, \\ & \langle Mary \rangle \rangle \rrbracket \in \llbracket \langle \langle together \rangle, \langle \langle lift \rangle, \langle \langle every \rangle, \\ & \langle piano \rangle \rangle \rrbracket(e)\}. \end{aligned}$$

Since the subject of the sentence, $\langle \langle John \rangle, \langle and \rangle, \langle Mary \rangle \rangle$, denotes $g(\text{John}, \text{Mary})$, the group of John and Mary, this is equivalent to (63):

$$\begin{aligned} (63) \quad & \llbracket \langle \langle John \rangle, \langle and \rangle, \langle Mary \rangle \rangle, \langle \langle together \rangle, \langle \langle lift \rangle, \\ & \langle \langle every \rangle, \langle piano \rangle \rangle \rangle \rrbracket = \{e \in E \mid g(\text{John}, \text{Mary}) \in \\ & \{w \in U - U_I \mid \exists e' \sqsubseteq_E e [\llbracket \langle piano \rangle \rrbracket \subseteq \{y \in U \mid w \in \\ & \llbracket \langle lift \rangle \rrbracket(e')(y)\} \wedge \forall e'' \sqsubseteq_E e' [\exists z \llbracket \langle piano \rangle \rrbracket \subseteq \{y \\ & \in U \mid z \in \llbracket \langle lift \rangle \rrbracket(e'')(y)\} \rightarrow \{x \in U \mid \llbracket \langle piano \rangle \rrbracket \\ & \subseteq \{y \in U \mid x \in \llbracket \langle lift \rangle \rrbracket(e'')(y)\}\}]\}. \end{aligned}$$

$$\subseteq \{y \in U \mid x \in \llbracket \langle \text{lift} \rangle \rrbracket(e'')(y)\} = \{x \in U \mid \llbracket \langle \text{piano} \rangle \rrbracket \subseteq \{y \in U \mid x \in \llbracket \langle \text{lift} \rangle \rrbracket(e')(y)\}\} \}}.$$

Since $g(\text{John}, \text{Mary}) \in U - U_I$ by definition, this reduces to (64):

$$\begin{aligned} (64) \quad \llbracket \langle \langle \text{John} \rangle, \langle \text{and} \rangle, \langle \text{Mary} \rangle \rangle, \langle \langle \text{together} \rangle, \langle \langle \text{lift} \rangle, \\ \langle \langle \text{every} \rangle, \langle \text{piano} \rangle \rangle \rangle \rangle \rrbracket = \{e \in E \mid \exists e' \subseteq_E e \\ [\llbracket \langle \text{piano} \rangle \rrbracket \subseteq \{y \in U \mid g(\text{John}, \text{Mary}) \in \\ \llbracket \langle \text{lift} \rangle \rrbracket(e')(y) \wedge \forall e'' \subseteq_E e' [\exists z \llbracket \langle \text{piano} \rangle \rrbracket \subseteq \{y \\ \in U \mid z \in \llbracket \langle \text{lift} \rangle \rrbracket(e'')(y)\}] \rightarrow \{x \in U \mid \\ \llbracket \langle \text{piano} \rangle \rrbracket \subseteq \{y \in U \mid x \in \llbracket \langle \text{lift} \rangle \rrbracket(e'')(y)\} = \{x \in \\ U \mid \llbracket \langle \text{piano} \rangle \rrbracket \subseteq \{y \in U \mid x \in \\ \llbracket \langle \text{lift} \rangle \rrbracket(e')(y)\}\}]\} \}. \end{aligned}$$

So, *John and Mary together lift every piano* is true of an event e if e contains a subevent e' in which the group of John and Mary lifts every piano, and such that for every subevent e'' of e' , if anyone lifts every piano in e'' , the set of lifters of every piano in e'' is the same as the set of lifters of every piano in e' .

Note that this does not rule out the possibility that some of the pianos were lifted by John or Mary as an individual, rather than through "authentic" group action. It does, however, rule out events in which, e.g., John lifts all the pianos, Mary lifts none of them, and none of them are lifted through authentic group action. Such an event e , say one in which John lifts all the pianos by himself but the group also counts as lifting them, will have a proper

subevent e'' in which John lifts all the pianos. Since this will be a proper subevent of any piano-lifting e' ($e' \sqsubseteq_E e$) by the group, even a minimally small one, the set of lifters of every piano in e'' will not include the group, and hence will not be equal to the set of lifters of every piano in e' .

Now compare *John and Mary lift every piano together*. As before, $\llbracket \langle lift \rangle \rrbracket$ maps any e onto a function mapping each liftee in e onto the set of its lifters in e , and $\llbracket \langle together \rangle \rrbracket$ is a function mapping any possible VP denotation f and event e as shown in (65):

$$(65) \llbracket \langle together \rangle \rrbracket(f)(e) = \{w \in U - U_I \mid \exists e' \sqsubseteq_E e [w \in f(e') \wedge \forall e'' \sqsubseteq_E e' [[\exists z \ z \in f(e'')] \rightarrow f(e'') = f(e')]]\}.$$

Since $\langle together \rangle$ is of category VP/VP, $\langle together \rangle$ will not combine with $\langle lift \rangle$ unless it first undergoes rule (37), which generates $\langle\langle together \rangle\rangle$ as a TV/TV. This rule gives the semantics of $\langle\langle together \rangle\rangle$ as in (66), where f is now a possible TV denotation and $x \in U$:

$$(66) \text{ For all } f, e, x: \llbracket \langle\langle together \rangle\rangle \rrbracket(f)(e)(x) = \{y \in U \mid \forall z [[z \in x \vee z = x] \rightarrow y \in \llbracket \langle together \rangle \rrbracket(f'(z))(e)]\}, \text{ where for all } e \text{ of type } e \text{ and all } x \text{ of type } u, f'(x)(e) = f(e)(x).$$

Applying $\llbracket \langle\langle together \rangle\rangle \rrbracket$ to $\llbracket \langle lift \rangle \rrbracket$, then, we get (67).

$$(67) \text{ For all } e, x: \llbracket \langle\langle lift \rangle, \langle\langle together \rangle\rangle \rrbracket(e)(x) =$$

$$\{y \in U \mid \forall z [[z \in x \vee z = x] \rightarrow y \in \llbracket \langle together \rangle \rrbracket(\llbracket \langle lift \rangle \rrbracket'(z))(e)]\}, \quad \text{where}$$

$$\llbracket \langle lift \rangle \rrbracket'(z)(e) = \llbracket \langle lift \rangle \rrbracket(e)(z).$$

By the lexical meaning postulate for $\langle together \rangle$, (68) follows:

$$(68) \text{ For all } e, x: \llbracket \langle \langle lift \rangle, \langle together \rangle \rangle \rrbracket(e)(x) =$$

$$\{y \in U \mid \forall z [[z \in x \vee z = x] \rightarrow y \in \{w \in U - U_I \mid$$

$$\exists e' \sqsubseteq_E e [w \in \llbracket \langle lift \rangle \rrbracket'(z)(e') \wedge \forall e'' \sqsubseteq_E e' [[\exists v v$$

$$\in \llbracket \langle lift \rangle \rrbracket'(z)(e'') \rightarrow \llbracket \langle lift \rangle \rrbracket'(z)(e'') =$$

$$\llbracket \langle lift \rangle \rrbracket'(z)(e')]]]]\}.$$

By the equivalence of $\llbracket \langle lift \rangle \rrbracket'(z)(e)$ and $\llbracket \langle lift \rangle \rrbracket(e)(z)$, we get (69):

$$(69) \text{ For all } e, x: \llbracket \langle \langle lift \rangle, \langle together \rangle \rangle \rrbracket(e)(x) =$$

$$\{y \in U \mid \forall z [[z \in x \vee z = x] \rightarrow y \in \{w \in U - U_I \mid$$

$$\exists e' \sqsubseteq_E e [w \in \llbracket \langle lift \rangle \rrbracket(e')(z) \wedge \forall e'' \sqsubseteq_E e' [[\exists v v$$

$$\in \llbracket \langle lift \rangle \rrbracket(e'')(z) \rightarrow \llbracket \langle lift \rangle \rrbracket(e'')(z) =$$

$$\llbracket \langle lift \rangle \rrbracket(e')(z)]]]]\}.$$

Adding the object $\langle it_i \rangle$ now (for any odd i), $\llbracket \langle \langle lift \rangle, \langle it_i \rangle, \langle together \rangle \rangle \rrbracket$ is undefined, but for each sequence s we have (70):

$$(70) \llbracket \langle \langle lift \rangle, \langle it_i \rangle, \langle together \rangle \rangle \rrbracket^s =$$

$$\llbracket \langle \langle lift \rangle, \langle together \rangle \rangle \rrbracket(s(i)).$$

So $\llbracket \langle \langle lift \rangle, \langle it_i \rangle, \langle together \rangle \rangle \rrbracket^s$ maps any event e as in (71):

$$(71) \llbracket \langle \langle lift \rangle, \langle it_i \rangle, \langle together \rangle \rangle \rrbracket^s(e) = \{y \in U \mid \forall z$$

$$[[z \in s(i) \vee z = s(i)] \rightarrow y \in \{w \in U - U_I \mid \exists e' \sqsubseteq_E$$

$$\begin{aligned}
& e \ [w \in \llbracket \langle lift \rangle \rrbracket(e')(z) \ \wedge \ \forall e'' \sqsubseteq_S e' \ [(\exists v \ v \in \\
& \llbracket \langle lift \rangle \rrbracket(e'')(z)] \rightarrow \llbracket \langle lift \rangle \rrbracket(e'')(z) \\
& = \\
& \llbracket \langle lift \rangle \rrbracket(e')(z)]]]]].
\end{aligned}$$

Quantifying in now, we get (72):

(72) For all s , e :

$$\begin{aligned}
& \llbracket \langle \langle lift \rangle, \langle \langle every \rangle, \langle piano \rangle, \langle together \rangle \rangle \rrbracket^s(e) = \\
& \{x \in U \mid \{u \in U \mid x \in \llbracket \langle \langle lift \rangle, \langle it_i \rangle, \\
& \langle together \rangle \rrbracket^{s[u/i]}(e) \} \in \llbracket \langle \langle every \rangle, \langle piano \rangle \rangle \rrbracket\}.
\end{aligned}$$

This is equivalent to (73):

(73) For all s , e :

$$\begin{aligned}
& \llbracket \langle \langle lift \rangle, \langle \langle every \rangle, \langle piano \rangle, \langle together \rangle \rangle \rrbracket^s(e) = \\
& \{x \in U \mid \{u \in U \mid x \in \{y \in U \mid \forall z \ [[z \in s[u/i](i) \\
& \vee z = s[u/i](i)] \rightarrow y \in \{w \in U - U_I \mid \exists e' \sqsubseteq_S e \ [w \in \\
& \llbracket \langle lift \rangle \rrbracket(e')(z) \ \wedge \ \forall e'' \sqsubseteq_S e' \ [(\exists v \ v \in \\
& \llbracket \langle lift \rangle \rrbracket(e'')(z)] \rightarrow \llbracket \langle lift \rangle \rrbracket(e'')(z) = \\
& \llbracket \langle lift \rangle \rrbracket(e')(z)]]]]] \} \in \llbracket \langle \langle every \rangle, \langle piano \rangle \rangle \rrbracket\}.
\end{aligned}$$

This reduces to (74):

(74) For all e :

$$\begin{aligned}
& \llbracket \langle \langle lift \rangle, \langle \langle every \rangle, \langle piano \rangle, \langle together \rangle \rangle \rrbracket(e) = \\
& \{x \in U \mid \{u \in U \mid \forall z \ [[z \in u \vee z = u] \rightarrow x \in \{w \in U \\
& - U_I \mid \exists e' \sqsubseteq_S e \ [w \in \llbracket \langle lift \rangle \rrbracket(e')(z) \ \wedge \ \forall e'' \sqsubseteq_S e' \\
& [(\exists v \ v \in \llbracket \langle lift \rangle \rrbracket(e'')(z)] \rightarrow \llbracket \langle lift \rangle \rrbracket(e'')(z) = \\
& \llbracket \langle lift \rangle \rrbracket(e')(z)]]]]] \} \in \llbracket \langle \langle every \rangle, \langle piano \rangle \rangle \rrbracket\}.
\end{aligned}$$

Since $\llbracket \langle \langle every \rangle, \langle piano \rangle \rangle \rrbracket$ is $\{X \in \text{POW}(U) \mid \llbracket \langle piano \rangle \rrbracket \subseteq X\}$,

this is equivalent to (75):

(75) For all e :

$$\begin{aligned} \llbracket \langle \text{lift} \rangle, \langle \text{every} \rangle, \langle \text{piano} \rangle, \langle \text{together} \rangle \rrbracket (e) = \\ \{x \in U \mid \{u \in U \mid \forall z [\{z \in u \vee z = u\} \rightarrow x \in \{w \in U \\ - U_I \mid \exists e' \sqsubseteq_E e [w \in \llbracket \langle \text{lift} \rangle \rrbracket (e')(z) \wedge \forall e'' \sqsubseteq_E e' \\ [\{\exists v v \in \llbracket \langle \text{lift} \rangle \rrbracket (e'')(z)\} \rightarrow \llbracket \langle \text{lift} \rangle \rrbracket (e'')(z) = \\ \llbracket \langle \text{lift} \rangle \rrbracket (e')(z)]]]]\} \in \{X \in \text{POW}(U) \mid \llbracket \langle \text{piano} \rangle \rrbracket \subseteq \\ X\} \}. \end{aligned}$$

This in turn is equivalent to (76):

(76) For all e :

$$\begin{aligned} \llbracket \langle \text{lift} \rangle, \langle \text{every} \rangle, \langle \text{piano} \rangle, \langle \text{together} \rangle \rrbracket (e) = \\ \{x \in U \mid \llbracket \langle \text{piano} \rangle \rrbracket \subseteq \{u \in U \mid \forall z [\{z \in u \vee z = u\} \\ \rightarrow x \in \{w \in U - U_I \mid \exists e' \sqsubseteq_E e [w \in \llbracket \langle \text{lift} \rangle \rrbracket (e')(z) \\ \wedge \forall e'' \sqsubseteq_E e' [\{\exists v v \in \llbracket \langle \text{lift} \rangle \rrbracket (e'')(z)\} \rightarrow \\ \llbracket \langle \text{lift} \rangle \rrbracket (e'')(z) = \llbracket \langle \text{lift} \rangle \rrbracket (e')(z)]]]]\} \}. \end{aligned}$$

If we assume that individual pianos do not have members (i.e. that they do not appear in the range of $\mathbb{E}$)¹⁵, (77) follows:

(77) For all e :

$$\begin{aligned} \llbracket \langle \text{lift} \rangle, \langle \text{every} \rangle, \langle \text{piano} \rangle, \langle \text{together} \rangle \rrbracket (e) = \\ \{x \in U \mid \llbracket \langle \text{piano} \rangle \rrbracket \subseteq \{u \in U \mid x \in \{w \in U - U_I \mid \\ \exists e' \sqsubseteq_E e [w \in \llbracket \langle \text{lift} \rangle \rrbracket (e')(u) \wedge \forall e'' \sqsubseteq_E e' [\{\exists v v \\ \in \llbracket \langle \text{lift} \rangle \rrbracket (e'')(u)\} \rightarrow \llbracket \langle \text{lift} \rangle \rrbracket (e'')(u) = \\ \llbracket \langle \text{lift} \rangle \rrbracket (e')(u)]]]]\} \}. \end{aligned}$$

The semantics for the sentence as a whole is as in (78):

¹⁵Nothing as yet guarantees this formally. The more restrictive theory of group structure offered in Chapter III will do so, however.

$$(78) \llbracket \langle \langle \langle \text{John} \rangle, \langle \text{and} \rangle, \langle \text{Mary} \rangle \rangle, \langle \langle \text{lift} \rangle, \langle \langle \text{every} \rangle, \langle \text{piano} \rangle \rangle, \langle \text{together} \rangle \rangle \rrbracket = \{e \in E \mid \llbracket \langle \langle \text{John} \rangle, \langle \text{and} \rangle, \langle \text{Mary} \rangle \rangle \rrbracket \in \llbracket \langle \langle \text{lift} \rangle, \langle \langle \text{every} \rangle, \langle \text{piano} \rangle \rangle, \langle \text{together} \rangle \rangle \rrbracket(e)\}.$$

Since $\llbracket \langle \langle \text{John} \rangle, \langle \text{and} \rangle, \langle \text{Mary} \rangle \rangle \rrbracket = g(\text{John}, \text{Mary})$, this reduces to (79):

$$(79) \llbracket \langle \langle \langle \text{John} \rangle, \langle \text{and} \rangle, \langle \text{Mary} \rangle \rangle, \langle \langle \text{lift} \rangle, \langle \langle \text{every} \rangle, \langle \text{piano} \rangle \rangle, \langle \text{together} \rangle \rangle \rrbracket = \{e \in E \mid g(\text{John}, \text{Mary}) \in \{x \in U \mid \llbracket \langle \text{piano} \rangle \rrbracket \subseteq \{u \in U \mid x \in \{w \in U - U_I \mid \exists e' \sqsubseteq_E e [w \in \llbracket \langle \text{lift} \rangle \rrbracket(e')(u) \wedge \forall e'' \sqsubseteq_E e' [\exists v \ v \in \llbracket \langle \text{lift} \rangle \rrbracket(e'')(u)] \rightarrow \llbracket \langle \text{lift} \rangle \rrbracket(e'')(u) = \llbracket \langle \text{lift} \rangle \rrbracket(e')(u)]\}}\}}\}.$$

This reduces directly to (80):

$$(80) \llbracket \langle \langle \langle \text{John} \rangle, \langle \text{and} \rangle, \langle \text{Mary} \rangle \rangle, \langle \langle \text{lift} \rangle, \langle \langle \text{every} \rangle, \langle \text{piano} \rangle \rangle, \langle \text{together} \rangle \rangle \rrbracket = \{e \in E \mid \llbracket \langle \text{piano} \rangle \rrbracket \subseteq \{u \in U \mid g(\text{John}, \text{Mary}) \in \{w \in U - U_I \mid \exists e' \sqsubseteq_E e [w \in \llbracket \langle \text{lift} \rangle \rrbracket(e')(u) \wedge \forall e'' \sqsubseteq_E e' [\exists v \ v \in \llbracket \langle \text{lift} \rangle \rrbracket(e'')(u)] \rightarrow \llbracket \langle \text{lift} \rangle \rrbracket(e'')(u) = \llbracket \langle \text{lift} \rangle \rrbracket(e')(u)]\}}\}.$$

Since $g(\text{John}, \text{Mary}) \in U - U_I$ by definition, this reduces further to (81):

$$(81) \llbracket \langle \langle \langle \text{John} \rangle, \langle \text{and} \rangle, \langle \text{Mary} \rangle \rangle, \langle \langle \text{lift} \rangle, \langle \langle \text{every} \rangle, \langle \text{piano} \rangle \rangle, \langle \text{together} \rangle \rangle \rrbracket = \{e \in E \mid \llbracket \langle \text{piano} \rangle \rrbracket \subseteq \{u \in U \mid \exists e' \sqsubseteq_E e [g(\text{John}, \text{Mary}) \in \llbracket \langle \text{lift} \rangle \rrbracket(e')(u) \wedge \forall e'' \sqsubseteq_E e' [\exists v \ v \in \llbracket \langle \text{lift} \rangle \rrbracket(e'')(u)] \rightarrow \llbracket \langle \text{lift} \rangle \rrbracket(e'')(u) = \llbracket \langle \text{lift} \rangle \rrbracket(e')(u)]\}}\}.$$

In other words, *John and Mary lift every piano together* is true of an event e if for every piano u , e has a subevent e' in which the group of John and Mary lift u , and such that for all subevents e'' of e' , if anyone lifted u in e'' , the set of things that lifted u in e'' is the same as the set of things that lifted u in e' .

Note that this requires "authentic" group action on John and Mary's part in lifting each piano. Suppose there is a piano p which John lifts without Mary's help (and p is not otherwise lifted). It might still be the case that $g(\text{John, Mary}) \in \llbracket \langle \text{lift} \rangle \rrbracket(p)(e)$ -- that is, that the group of John and Mary lift p in e -- but it will not be the case that for some subevent e' of e such that $g(\text{John, Mary}) \in \llbracket \langle \text{lift} \rangle \rrbracket(p)(e')$, all subevents e'' of e' such that $\exists v v \in \llbracket \langle \text{piano} \rangle \rrbracket(p)(e'')$ are also such that $g(\text{John, Mary}) \in \llbracket \langle \text{piano} \rangle \rrbracket(p)(e'')$. This will not be the case because any e' in which the group lifts the piano will have John's solo lifting as a proper subevent. Since even a minimally small lifting by the group has this proper subevent in which John lifts the piano, we can always find a subevent e'' of e' in which somebody (John) lifts the piano, but the group of John and Mary does not.

The fragment produces a similar semantic difference between *John and Mary together lift five pianos* and *John and Mary lift five pianos together*, though through somewhat

different formal means. It would be tedious, I think, to work through the derivations of these sentences step by step as we have just done for *John and Mary together lift every piano* and *John and Mary lift every piano together*, but those who wish to do so may confirm that the following readings are assigned:

$$\begin{aligned}
 (82) \quad \llbracket \langle \langle \langle \text{John} \rangle, \langle \text{and} \rangle, \langle \text{Mary} \rangle \rangle, \langle \langle \text{together} \rangle, \langle \langle \text{lift} \rangle, \\
 \langle \langle \text{five} \rangle, \langle \text{pianos} \rangle \rangle \rangle \rrbracket = \{ e \in E \mid \exists e' \sqsubseteq_e e \exists x \in U \\
 [g(\text{John}, \text{Mary}) \in \llbracket \langle \text{lift} \rangle \rrbracket(e')(x) \wedge x \in \llbracket \langle \text{pianos} \rangle \rrbracket \\
 \wedge \text{CARD}(\{y \in U \mid y \in x\}) = 5 \wedge \forall e'' \sqsubseteq_e e' [\exists z \exists x \in \\
 U \mid z \in \llbracket \langle \text{lift} \rangle \rrbracket(e'')(x) \wedge x \in \llbracket \langle \text{pianos} \rangle \rrbracket \wedge \\
 \text{CARD}(\{y \in U \mid y \in x\}) = 5] \rightarrow \{w \in U \mid \exists x \in U \quad w \in \\
 \llbracket \langle \text{lift} \rangle \rrbracket(x)(e'') \wedge x \in \llbracket \langle \text{pianos} \rangle \rrbracket \wedge \text{CARD}(\{y \in U \mid y \in \\
 x\}) = 5] = \{w \in U \mid \exists x \in U \quad w \in \llbracket \langle \text{lift} \rangle \rrbracket(x)(e') \wedge x \\
 \in \llbracket \langle \text{pianos} \rangle \rrbracket \wedge \text{CARD}(\{y \in U \mid y \in x\}) = 5\} \} \}.
 \end{aligned}$$

$$\begin{aligned}
 (83) \quad \llbracket \langle \langle \langle \text{John} \rangle, \langle \text{and} \rangle, \langle \text{Mary} \rangle \rangle, \langle \langle \text{lift} \rangle, \langle \langle \text{five} \rangle, \\
 \langle \langle \text{pianos} \rangle \rangle, \langle \langle \text{together} \rangle \rangle \rangle \rrbracket = \{ e \in E \mid \exists x \in U \quad x \in \\
 \llbracket \langle \text{pianos} \rangle \rrbracket \wedge \text{CARD}(\{y \in U \mid y \in x\}) = 5 \wedge \forall z [\{z \in x \\
 \vee z = x\} \rightarrow \exists e' \sqsubseteq_e e \quad [g(\text{John}, \text{Mary}) \in \\
 \llbracket \langle \text{lift} \rangle \rrbracket(e')(z) \wedge \forall e'' \sqsubseteq_e e' \quad [\exists v \quad v \in \\
 \llbracket \langle \text{lift} \rangle \rrbracket(e'')(z) \rightarrow \llbracket \langle \text{lift} \rangle \rrbracket(e'')(z) = \\
 \llbracket \langle \text{lift} \rangle \rrbracket(e')(z)]] \} \}.
 \end{aligned}$$

The noun phrase *five pianos* is taken as denoting the set of sets containing a group of five pianos. *John and Mary together lift five pianos* is true of an event e if e has a subevent e' in which the group of John and Mary lift a

group of five pianos, and such that for any subevent e'' of e' , if anyone lifts a group of five pianos in e'' , then the set of lifters of five pianos in e'' is the same as the set of lifters of five pianos in e' .

In contrast, *John and Mary lift five pianos together* is true of e if there is a group of five pianos such that for each piano z in the group (and for the group itself), e has a subevent e' in which the group of John and Mary lifts z , and such that for each of its subevents e'' , if anyone lifts z in e'' , the set of lifters of z in e'' is the same as for e' .

The primary motivation, at least in the present context, for treating noun phrases introduced by cardinal number determiners in this way is that they show the same sort of patterns with respect to collectivity and distributivity as do non-quantificational terms, such as definite descriptions or conjoined names. *Five students lifted the piano* can mean either that each of five students did, or that a group of five students did so collectively. The collective reading in particular would be very difficult to obtain if *five students* were interpreted as a quantifier ranging over individual students, such as *every student*. In the present, group-level treatment, however, the collective reading falls out automatically, and the distributive reading can be derived via a VP-modifying operator much like Link and Roberts', to be introduced in section 10, below.

But given a group-level treatment of such noun phrases, the semantic differences associated with adverbial position cannot be produced merely by scope. If, for example, we derived the TV-modifying version β of a VP-modifying adverbial α via a simple Geach style type raising rule¹⁶, the relative position of *together* and *five pianos* should have no semantic effect whatsoever. An optional distributivity operator will do us little good in this case, because (barring special intonation not normally associated with this operator) objects of verbs modified by postverbal adverbs obligatorily take a distributive reading.

In the current fragment, the appropriate semantic effect is produced by building a kind of distributive quantification into the rule for deriving TV/TVs from VP/VPs, which is repeated here as (84):

(84) If $\alpha \in \text{VP/VP}$ then $\beta = \langle \alpha \rangle \in \text{TV/TV}$;

for all f of type $\langle e, \langle u, \langle u \rangle \rangle \rangle$, all e of type e ,
and all x of type u , $\|\beta\|(f)(e)(x) = \{y \in U \mid \forall z$
[[$z \in x \vee z = x$] $\rightarrow y \in \|\alpha\|(f'(z))(e)\}$, where for
all e of type e and all x of type u , $f'(x)(e) =$
 $f(e)(x)$.

If a complex TV formed by adverbial modification takes a group level argument, the adverbial quantifies over the

¹⁶E.g. as follows (cf. (37)): If $\alpha \in \text{VP/VP}$ then $\beta = \langle \alpha \rangle \in \text{TV/TV}$; for all f of type $\langle e, \langle u, \langle u \rangle \rangle \rangle$, all e of type e , and all x of type u , $\|\beta\|(f)(e)(x) = \|\alpha\|(f'(x))(e)$, where for all e of type e and all x of type u , $f'(x)(e) = f(e)(x)$.

members of the group. Thus, as indicated in (83), if John and Mary lift five pianos together, then there is a group of five pianos, such that for each of that group's members, the event e' of John and Mary lifting it has no subevents e'' in which that member of the group is lifted, but in which the set of its lifters is different from that for e' . In contrast, as indicated in (82), *John and Mary together lifted five pianos* only requires that the whole event of John and Mary lifting the group not have subevents where a different set of lifters lifts the group.

The pattern here is one general to TV-modifying adverbials, not specific to *together*; for example *John built five boxes slowly* (pronounced without special accent on *slowly*) requires that each of the five boxes was built slowly, while *John slowly built five boxes* only requires that the whole event of the five boxes being built was slow. Similarly, *John moved all the books slowly* requires that each book was moved slowly, while *John slowly moved all the books* only requires that the whole event of moving all the books was slow; this is despite the fact that *all the books* allows for collective reference to the group of all books, and not just universal quantification over the set of individual books. For this reason it seems reasonable to build the appropriate semantic mechanisms into the general rule deriving TV/TVs from VP/VPs, as I have done in this fragment.

The reader may confirm that the fragment also predicts similar semantic alternations according to the position of comitative PPs.

8. Locative and temporal readings. The analysis just outlined for the collectivizing use of *together* (and comitative *with*) can be extended straightforwardly to the locative and temporal uses of these expressions as well.

Consider what it means for e to be an event in which John and Mary stand up together (i.e. at the same time). There may be much more going on in e than just John and Mary standing up; e could be a large event which lasts several years, for instance. However, e presumably will contain a subevent e' which consists just of John and Mary's standing up (plus whatever smaller events it may comprise, such as John's standing up and Mary's standing up). Let t be the smallest interval¹⁷ of time during which e' occurs, or, as I will call it, the "running time" of e' . Now, for any t' , if t' is the running time of a subevent e'' of e' in which someone stands up, then it is the running time of e' . That is to say, every standing-up which is a subevent of John and Mary's standing up must have the same running time as John and Mary's standing-up. In particular, John's standing up

¹⁷I assume that instantaneous moments count as a variety of interval.

takes the same stretch of time as Mary's standing up (and they both take the same stretch of time as the group's standing up).

Now consider what it means for e to be an event in which John and Mary sit together (i.e. near each other). Here, John and Mary must sit not only at the same time, but also in the same location, at least if we consider the division of space into locations as fairly coarse-grained. (It is of course impossible for John and Mary to occupy literally the same space.) Again, e may be a large event that plays over a sizable area. But it will contain a subevent e' which consists of just John and Mary's sitting (plus any subevents). Let I be the smallest location in which e' takes place, or, as I will call it, the "running space" of e' , and let t be the running time of e' ¹⁸. Now for any location I' and interval t' , if I' is the running space of a subevent e'' of e' in which someone sits, then it is the running space of e' , and if t' is the running time of e'' then it is the running time of e' . That is, any sitting which is a subevent of John and Mary's sitting must have the same running space and time as John and Mary's sitting.

In order to implement these analyses formally, we need to expand our notions of a model and model frame again, to include times and locations. Let a model frame be a 12-

¹⁸Choose e' such that t has no proper subintervals at which John and Mary sit (e.g. by letting t be a "moment").

tuple $\langle U, U_I, \sqsubseteq_U, E, F, \sqsubseteq_F, T, \sqsubseteq_T, \tau, L, \sqsubseteq_L, \sigma \rangle$ and a model be a 13-tuple $\langle U, U_I, \sqsubseteq_U, E, F, \sqsubseteq_F, T, \sqsubseteq_T, \tau, L, \sqsubseteq_L, \sigma, \|\cdot\| \rangle$, where $U, U_I, \sqsubseteq_U, E, F, \sqsubseteq_F$ and $\|\cdot\|$ are as before, $\langle T, \sqsubseteq_T \rangle$ and $\langle L, \sqsubseteq_L \rangle$ are complete atomic boolean algebras, τ is a $\sqsubseteq$ -homomorphism from F into T , and σ is a $\sqsubseteq$ -homomorphism from F into L . Intuitively, T and L are sets of times and locations respectively and $\sqsubseteq_T$ and $\sqsubseteq_L$ are the inclusion relations on T and L .¹⁹ The function τ maps events onto their running times; it is the "temporal trace" function, in the terminology of Link (1987b). Likewise σ is the "spatial trace" function; it maps events onto their running spaces.

Let us also define a function K which maps any event e onto the ordered pair $\langle \tau(e), \sigma(e) \rangle$. This function will be of use in defining the semantics of the locative uses of *together* and similar adverbials.

Recall now the definition given above (in (36)g.) for the collectivizing use of *together*:

(85) $\langle \text{together} \rangle \in \text{VP/VP}$;

For all f of type $\langle e, \langle u \rangle \rangle$ and all e of type e ,
 $\|\langle \text{together} \rangle\|(f)(e) = \{w \in U - U_I \mid \exists e' \sqsubseteq_F e [w \in f(e') \wedge \forall e'' \sqsubseteq_F e' [[\exists z \ z \in f(e'')] \rightarrow f(e'') = f(e')]]]\}$.

¹⁹ T should also be ordered according to some precedence relation, of course, but since temporal precedence will not play a role in what follows, I do not include it in the model.

The temporal and locative understandings can be defined using an identical format, except that either τ (for the temporal use) or K (for the locative use) is substituted for f in the final clause²⁰:

(86) $\langle together_{temp} \rangle \in VP/VP$;

For all f of type $\langle e, \langle u \rangle \rangle$ and all e of type e ,
 $\llbracket \langle together_{temp} \rangle \rrbracket(f)(e) = \{w \in U - U_I \mid \exists e' \sqsubseteq_E e$
 $[w \in f(e') \wedge \forall e'' \sqsubseteq_E e' [[\exists z z \in f(e'')] \rightarrow \tau(e'')] = \tau(e')]]\}$.

(87) $\langle together_{loc} \rangle \in VP/VP$;

For all f of type $\langle e, \langle u \rangle \rangle$ and all e of type e ,
 $\llbracket \langle together_{loc} \rangle \rrbracket(f)(e) = \{w \in U - U_I \mid \exists e' \sqsubseteq_E e [w$
 $\in f(e') \wedge \forall e'' \sqsubseteq_E e' [[\exists z z \in f(e'')] \rightarrow K(e'')] = K(e')]]\}$.

The same substitutions may be made in the definition of comitative *with*, to give temporal and locative understandings. The collectivizing use is defined in (88) (repeating (36)h.); definitions for the temporal and locative uses follow in (89) and (90):

(88) $\langle with \rangle \in (VP/VP)/T$;

for all x of type u , all f of type $\langle e, \langle u \rangle \rangle$, and
all e of type e , $\llbracket \langle with \rangle \rrbracket(x)(f)(e) = \{y \in U \mid \exists e'$

²⁰The subscripts *temp* and *loc* in the following definitions are added only to maintain the notational convenience of defining $\llbracket \cdot \rrbracket$ as a function. They are not to be considered syntactic features in any theoretical sense, and may be eliminated if we take $\llbracket \cdot \rrbracket$ to be a relation rather than a function (as I believe any serious treatment of ambiguity ought to do).

$$\sqsubseteq_E e [g(x, y) \in f(e') \wedge \forall e'' \sqsubseteq_E e' [[\exists z z \in f(e'')] \rightarrow f(e'') = f(e')]]].$$

$$(89) \langle with_{temp} \rangle \in (VP/VP)/T;$$

for all x of type u , all f of type $\langle e, \langle u \rangle \rangle$, and all e of type e , $\llbracket \langle with_{temp} \rangle \rrbracket(x)(f)(e) = \{y \in U \mid \exists e' \sqsubseteq_E e [g(x, y) \in f(e') \wedge \forall e'' \sqsubseteq_E e' [[\exists z z \in f(e'')] \rightarrow \tau(e'') = \tau(e')]]]\}$.

$$(90) \langle with_{loc} \rangle \in (VP/VP)/T;$$

for all x of type u , all f of type $\langle e, \langle u \rangle \rangle$, and all e of type e , $\llbracket \langle with_{loc} \rangle \rrbracket(x)(f)(e) = \{y \in U \mid \exists e' \sqsubseteq_E e [g(x, y) \in f(e') \wedge \forall e'' \sqsubseteq_E e' [[\exists z z \in f(e'')] \rightarrow K(e'') = K(e')]]]\}$.

Giving three distinct semantic definitions for *together* (and three for *with*), as I have done here, amounts to a claim that these words are ambiguous, rather than vague, between the collectivizing, locative and temporal uses. These three uses seem intuitively distinct enough to me to justify such a treatment; however, the analysis is easy to adapt so that *together* and *with* each receives only a single semantic definition, which is neutral, or in some sense vague, between the three uses in question. Rather than stipulating in the final clause that $f(e'') = f(e')$, that $\tau(e'') = \tau(e')$, or that $K(e'') = K(e')$, we introduce a function variable, say f' , ranging over $\{f, K, \tau\}$, and give a single definition stipulating that $f'(e'') = f'(e')$. (I'll skip the details of formalization.) Note that such an approach

leaves a certain amount of work to the pragmatics, which must somehow decide which of the three possible values for f' is appropriate in context; however, this sort of decision is necessary in an ambiguity analysis as well, since there the pragmatics must decide which of the three distinct semantic definitions is the appropriate one.

It is probably worthwhile at this point to introduce some intransitive verbs and postverbal IV-modifying adverbials:

(91)a. $\langle sit \rangle, \langle stand \rangle \in IV$;

for all $e, e' \in E$ such that $e \sqsubseteq_E e'$, and all $x \in U$, $x \in \models \langle sit \rangle \models (e)$ implies $x \in \models \langle sit \rangle \models (e')$.

(Likewise for $\langle stand \rangle$.)

b. If $\alpha \in VP/VP$ then $\alpha \in IV/IV$.

c. If $\alpha \in IV$ and $\beta \in IV/IV$ then $\gamma = \langle \alpha, \beta \rangle \in IV$;

$\models \gamma \models = \models \beta \models (\models \alpha \models)$.

d. If $\alpha \in IV$ then $\beta = \langle \alpha \rangle \in VP$;

$\models \beta \models = \models \alpha \models$.

As mentioned briefly above, this approach to the semantics of locative *together* requires that the division of space into locations be fairly coarse-grained, so that, say, John and Mary can count as sitting in the same location even though they do not literally occupy the same space. Of course, this point holds for the temporal reading as well;

John and Mary may count as standing up together even if their actions do not temporally coincide to the exact millisecond. I don't think this is a serious problem; we may either assume that σ and τ simply do not provide a very fine-grained mapping, or else assume that T and L themselves are not very fine-grained structures.

This latter option may seem odd at first, and I am not sure if it can really be maintained in the long run, but I think it is worth considering. It is now quite standard practice to assume that the universe of discourse U contains only those individuals which are relevant to the discourse being interpreted. It seems fair to suppose that T and L will also contain only those times and locations which are relevant, and not necessarily the "whole" structure of time and space. For everyday purposes, it is irrelevant whether John stood up at 2:47:31.67 P.M. or rather 2:47:31.68 P.M., and perhaps the set of intervals T should simply not contain any intervals small enough to capture this difference.²¹ Likewise it may matter more that John sat on the loveseat (just where Mary happened to be sitting too!) than that he sat on the left side of the loveseat, so perhaps L should not distinguish sublocations within the loveseat itself. As smaller and smaller locations become relevant to our

²¹Note that since T is required to be atomic, it will still contain "moments" in the formal sense of intervals with no non-zero subintervals.

purposes, however, two individuals will have to be closer and closer to count as together.

9. Adjectival and object-oriented readings. Both *together* and comitative *with* show readings we still have not accounted for. For example, either *together* or a comitative PP may be used predicatively:

(92)a. John and Mary are together.

b. John is with Mary.

Either *together* or a comitative PP may take what I will call an "object-oriented" reading:

(93)a. John lifted the piano and the rock together.

b. John lifted the piano with the rock.

In these sentences the piano and the rock are understood as being lifted simultaneously. These examples contrast with all our previous ones in that the object noun phrase is understood as denoting a group (or part of a group) whose members participate collectively or coterporally in some event. Compare (94), where, as in our previous examples, it is the subject noun phrase which denotes such a group (or part of a group)²²:

(94)a. John and Mary lifted the piano together.

b. John lifted the piano with Mary.

²²Actually, both sentences in (85) are ambiguous, but the object-oriented readings are considerably less prominent than the subject-oriented readings.

The pattern exhibited in (92) - (94) is one familiar from the behavior of ordinary adjectives. Most of these can occur predicatively, of course:

(95) John is drunk.

Most of them can also occur postverbally, with either a subject-oriented or an object-oriented reading:

(96)a. John walked home drunk.

b. John saw Mary drunk.

Under certain circumstances, a postverbal adjective can take a "resultative" reading. Thus in (97), the door is understood as becoming green as a result of John's painting it:

(97) John painted the door green.

Arguably at least, resultative readings are also available for *together* and for comitative PPs:

(98)a. The workers gathered together.

b. The workers gathered with their supporters.

All this seems to indicate that *together* may always be an adjective, and not, as we have been assuming, an adverb. Likewise, comitative PPs may only have an adjectival use, and not, strictly speaking, any adverbial use at all.

I don't think this argument is as strong as it might appear, though the case that comitative PPs should always be treated as adjectival in some sense is probably stronger than the case that *together* should. An entirely adjectival

treatment of *together* faces difficulties in accounting for the its preverbal use. Adjectives may appear preverbally, but if so, they are normally set off by pauses or intonational breaks:

(99)a. John, drunk, staggered around wildly.

b. *John drunk staggered around wildly.

Comitative PPs also are most natural in this position with pauses or breaks:

(100)a. John, with Mary, lifted every piano.

b. ?John with Mary lifted every piano.

Together does not require pauses or breaks, and in this sense patterns like an ordinary adverb rather than an adjective:²³

(101)a. John and Mary slowly lifted every piano.

b. John and Mary together lifted every piano.

It's also worth noting that the alternation between subject-oriented and object-oriented readings also occurs with

²³At first glance, *together* seems to occur without pauses or breaks much more naturally in its collectivizing use than in its proximity uses: *John and Mary together sat* sounds fairly odd. However, *John and Mary, together again after all this time, sat down and told each other what had happened* sounds fine, and if one has heard this sort of sentence, *John and Mary, together, sat* also sounds OK. In certain circumstances no breaks are required, however: *John and Mary are miserable when they're apart, but together are quite happy* sounds fine to me, as does *John and Mary together waited out the worst storm they had ever been through*. I suspect this pattern really has nothing to do with the collectivizing/proximity relation, but rather with the "weight" of the VP following *together*; pauses are necessary with light VPs, unnecessary with heavy ones. Pauses are also necessary if *together* heads a larger phrase.

certain expressions which are almost surely not adjectival, e.g. *as* phrases:

(102)a. John is competent as a researcher (but incompetent as a teacher).

b. We hired John as a researcher (not as a teacher).

Note that *as* phrases may not be used predicatively:

(103) *John is as a researcher.

Note incidentally that *as* phrases prefer pauses or breaks if they occur preverbally:

(104)a. John, as a researcher, is quite competent.

b. ?John as a researcher is quite competent.

Whether or not *together* and comitative PPs are always adjectival, it is clear that they do have predicative and object-oriented uses, and that these uses should be related to the subject-oriented uses we have dealt with so far.

Let us assume a function LOC mapping any event *e* onto the set of individuals and groups²⁴ located in the running space of *e* during the running time of *e*. We can now derive the predicative use of locative *together* by applying $\llcorner \langle together_{loc} \rangle \llcorner$ to LOC as part of a lexical rule. I repeat the definition of $\llcorner \langle together_{loc} \rangle \llcorner$ as (105):

²⁴Since groups cannot be located independently of their members, we require $x \in LOC(e) \leftrightarrow \forall y \in x [y \in LOC(e)]$.

- (105) For all f of type $\langle e, \langle u \rangle \rangle$ and all e of type e ,
 $\| \langle together_{loc} \rangle \| (f)(e) = \{ w \in U - U_I \mid \exists e' \sqsubseteq_E e [w \in f(e') \wedge \forall e'' \sqsubseteq_E e' [[\exists z z \in f(e'')] \rightarrow K(e'') = K(e')]]] \}$.

Applying this function to LOC gives, for each e , the set indicated in (106):

- (106) $\| \langle together_{loc} \rangle \| (LOC)(e) = \{ w \in U - U_I \mid \exists e' \sqsubseteq_E e [w \in LOC(e') \wedge \forall e'' \sqsubseteq_E e' [[\exists z z \in LOC(e'')] \rightarrow K(e'') = K(e')]]] \}$.

If John and Mary are together in e , then we can identify an $e' \sqsubseteq_E e$ such that John and Mary are present in $\sigma(e')$ at $\tau(e')$ and such that for all subevents e'' of e' in which something is located, the running spaces and times of e'' are the same as those for e' . Specifically, let e' be the event of John and Mary being at the location $\sigma(e')$ at time $\tau(e')$.²⁵ The subevents of e' will presumably be things like the event of John being at $\sigma(e')$ at $\tau(e')$ and the event of Mary being at $\sigma(e')$ at $\tau(e')$. The requirement that $K(e'') = K(e')$ assures that John and Mary will be in the same place, at the same time.

Together also has a predicative temporal use, though this is somewhat less common. One can say *John and Mary are together* in order to convey the fact that their actions are well synchronized (e.g. if they are playing a piece of music together). To derive this reading, I assume a function ACT

²⁵Again, choose e' such that $\tau(e')$ is minimally small.

which maps any event e onto the set of individuals and groups which perform some sort of action in e . The adverbial temporal $\llcorner \langle together_{temp} \rangle \llcorner$ is then applied to this function as part of a lexical rule. The definition of $\llcorner \langle together_{temp} \rangle \llcorner$ is repeated as (107):

(107) $\langle together_{temp} \rangle \in VP/VP$;

For all f of type $\langle e, \langle u \rangle \rangle$ and all e of type e ,
 $\llcorner \langle together_{temp} \rangle \llcorner (f)(e) = \{w \in U - U_I \mid \exists e' \sqsubseteq_E e$
 $[w \in f(e') \wedge \forall e'' \sqsubseteq_E e' [[\exists z z \in f(e'')] \rightarrow \tau(e'')$
 $= \tau(e')]]\}.$

Applying this function to ACT gives (108):

(108) $\llcorner \langle together_{temp} \rangle \llcorner (ACT)(e) =$

$\{w \in U - U_I \mid \exists e' \sqsubseteq_E e [w \in ACT(e') \wedge \forall e'' \sqsubseteq_E e'$
 $[[\exists z z \in ACT(e'')] \rightarrow \tau(e'') = \tau(e')]]\}.$

As far as I can tell, *together* has no collectivizing predicative use. The predicative uses of comitative *with* may be derived in a manner analogous to that just outlined for *together*.

Object-oriented uses can be derived via a second rule creating TV/TVs from VP/VPs. Our earlier, subject-oriented rule is repeated as (109):

(109) If $\alpha \in VP/VP$ then $\beta = \langle \alpha \rangle \in TV/TV$;

for all f of type $\langle e, \langle u, \langle u \rangle \rangle \rangle$, all e of type e ,
 and all x of type u , $\llcorner \beta \llcorner (f)(e)(x) = \{y \in U \mid \forall z$
 $[[z \in x \vee z = x] \rightarrow y \in \llcorner \alpha \llcorner (f'(z))(e)], \text{ where for}$

all e of type e and all x of type u , $f'(x)(e) = f(e)(x)$.

The rule for the object-oriented postverbal use is given in (110):

- (110) If $\alpha \in VP/VP$ then $\beta = \langle\langle\alpha\rangle\rangle \in TV/TV$;
 for all f of type $\langle e, \langle u, \langle u \rangle \rangle \rangle$, all e of type e ,
 and all x of type u , $\|\beta\|(f)(e)(x) = \{v \in U \mid x \in \|\alpha\|(\lambda e^*\{u \in U \mid v \in f(e^*)(u)\})(e)\}$.

Here e^* is an ordinary variable over events. (I avoid using a prime for clarity in the derivation below.) $\lambda e^*\{u \in U \mid v \in f(e^*)(u)\}$ is a function mapping any e^* onto the set $\{u \in U \mid v \in f(e^*)(u)\}$.

To illustrate how this works, a derivation of *John lifts Mary and Bill together* follows.

Applying (110) to $\langle togethertemp \rangle$ gives (111):

- (111) For all f of type $\langle e, \langle u, \langle u \rangle \rangle \rangle$, all e of type e ,
 and all x of type u ,
 $\|\langle\langle\langle togethertemp \rangle\rangle\rangle\|(f)(e)(x) = \{v \in U \mid x \in \|\langle togethertemp \rangle\|(\lambda e^*\{u \in U \mid v \in f(e^*)(u)\})(e)\}$.

Modifying $\langle lift \rangle$ with $\langle\langle\langle togethertemp \rangle\rangle\rangle$ gives (112):

- (112) For all e of type e , and all x of type u ,
 $\|\langle\langle lift \rangle, \langle\langle\langle togethertemp \rangle\rangle\rangle\rangle\|(e)(x) = \{v \in U \mid x \in \|\langle togethertemp \rangle\|(\lambda e^*\{u \in U \mid v \in \|\langle lift \rangle\|(e^*)(u)\})(e)\}$.

This is equivalent to (113) by the meaning postulate for $\langle together_{temp} \rangle$:

$$\begin{aligned}
 (113) \text{ For all } e \text{ of type } e, \text{ and all } x \text{ of type } u, \\
 \llbracket \langle lift \rangle, \langle \langle together_{temp} \rangle \rangle \rrbracket(e)(x) = \\
 \{v \in U \mid x \in \{w \in U - U_I \mid \exists e' \sqsubseteq_E e [w \in \lambda e^* \{u \in U \mid \\
 v \in \llbracket \langle lift \rangle \rrbracket(e^*)(u)\}(e') \wedge \forall e'' \sqsubseteq_E e' [\exists z z \in \lambda e^* \{u \in U \mid \\
 v \in \llbracket \langle lift \rangle \rrbracket(e^*)(u)\}(e'') \rightarrow \tau(e'') = \tau(e')]\}}\} \}.
 \end{aligned}$$

This reduces to (114) by lambda conversion:

$$\begin{aligned}
 (114) \text{ For all } e \text{ of type } e, \text{ and all } x \text{ of type } u, \\
 \llbracket \langle lift \rangle, \langle \langle together_{temp} \rangle \rangle \rrbracket(e)(x) = \\
 \{v \in U \mid x \in \{w \in U - U_I \mid \exists e' \sqsubseteq_E e [w \in \{u \in U \mid \\
 v \in \llbracket \langle lift \rangle \rrbracket(e')(u)\} \wedge \forall e'' \sqsubseteq_E e' [\exists z z \in \{u \in U \mid \\
 v \in \llbracket \langle lift \rangle \rrbracket(e'')(u)\} \rightarrow \tau(e'') = \tau(e')]\}}\} \}.
 \end{aligned}$$

Adding in the object NP $\langle Mary \rangle, \langle and \rangle, \langle Bill \rangle$, we get (115):

$$\begin{aligned}
 (115) \text{ For all } e \text{ of type } e, \llbracket \langle lift \rangle, \langle Mary \rangle, \langle and \rangle, \\
 \langle Bill \rangle \rrbracket, \langle \langle together_{temp} \rangle \rangle \rrbracket(e) = \\
 \{v \in U \mid g(Mary, Bill) \in \{w \in U - U_I \mid \exists e' \sqsubseteq_E e \\
 [w \in \{u \in U \mid v \in \llbracket \langle lift \rangle \rrbracket(e')(u)\} \wedge \forall e'' \sqsubseteq_E e' \\
 [\exists z z \in \{u \in U \mid v \in \llbracket \langle lift \rangle \rrbracket(e'')(u)\} \rightarrow \tau(e'') = \tau(e')]\}}\} \}.
 \end{aligned}$$

Since $g(Mary, Bill)$ is in $U - U_I$ by definition, this reduces to (116):

$$\begin{aligned}
 (116) \text{ For all } e \text{ of type } e, \llbracket \langle lift \rangle, \langle Mary \rangle, \langle and \rangle, \\
 \langle Bill \rangle \rrbracket, \langle \langle together_{temp} \rangle \rangle \rrbracket(e) =
 \end{aligned}$$

$$\{v \in U \mid \exists e' \sqsubseteq_E e [g(\text{Mary}, \text{Bill}) \in \{u \in U \mid v \in \llbracket \langle \text{lift} \rangle \rrbracket(e')(u) \} \wedge \forall e'' \sqsubseteq_E e' [\exists z z \in \{u \in U \mid v \in \llbracket \langle \text{lift} \rangle \rrbracket(e'')(u) \} \rightarrow \tau(e'') = \tau(e')]]]\}.$$

This reduces further to (117):

$$(117) \text{ For all } e \text{ of type } e, \llbracket \langle \langle \text{lift} \rangle, \langle \langle \text{Mary} \rangle, \langle \text{and} \rangle, \langle \text{Bill} \rangle \rangle, \langle \langle \langle \text{together}_{temp} \rangle \rangle \rangle \rrbracket(e) = \\ \{v \in U \mid \exists e' \sqsubseteq_E e [v \in \llbracket \langle \text{lift} \rangle \rrbracket(e')(g(\text{Mary}, \text{Bill})) \wedge \forall e'' \sqsubseteq_E e' [\exists z v \in \llbracket \langle \text{lift} \rangle \rrbracket(e'')(z) \rightarrow \tau(e'') = \tau(e')]]]\}.$$

Adding in the subject, *John*, we get (118):

$$(118) \llbracket \langle \langle \text{John} \rangle, \langle \langle \text{lift} \rangle, \langle \langle \text{Mary} \rangle, \langle \text{and} \rangle, \langle \text{Bill} \rangle \rangle, \langle \langle \langle \text{together} \rangle \rangle \rangle \rrbracket = \\ \{e \in E \mid \exists e' \sqsubseteq_E e [\text{John} \in \llbracket \langle \text{lift} \rangle \rrbracket(e')(g(\text{Mary}, \text{Bill})) \wedge \forall e'' \sqsubseteq_E e' [\exists z \text{John} \in \llbracket \langle \text{lift} \rangle \rrbracket(e'')(z) \rightarrow \tau(e'') = \tau(e')]]]\}.$$

10. Distributive predicates revisited. Building reference to events into VP semantics allows us to return to a closure-condition theory of distributivity, where the extension of a distributive predicate (in a given event) is closed under group formation. In fact it may have been noticed that in the treatment of locative and temporal readings for *together* and *with* given above, such an account of distributivity was already presupposed.

Suppose that John and Mary collectively lift the piano, and also each lift it alone. This, recall, was the kind of

situation that posed such problems for the closure-condition theory argued against in Chapter I, since it apparently did not allow the collective and distributive denotations of the VP *lift the piano* to be distinguished as the selection facts required. Under the present approach to VP semantics, however, the collective and distributive denotations of a predicate are systematically different, even in situations like the one just described.

If *lift the piano* is understood distributively, then any event e in which John and Mary lift the piano will have a subevent e' in which John lifts the piano, and a subevent e'' in which Mary lifts the piano. This is just the opposite from the kind of event I am claiming may be described by *John and Mary together lift the piano*; the

Figure 3

semantics of *together* assures that any event describable by this sentence will have a subevent of piano-lifting by John and Mary which in turn will not have a subevent in which John lifts the piano or Mary lifts the piano. It may be the case that John and Mary also lift the piano individually, but these events will not be subevents of the collective lifting event itself.

The sort of event in which John and Mary distributively lift the piano is diagrammed informally in Figure 3.

We may derive optional distributive readings by an operation on VPs²⁶:

(119) If $\alpha \in VP$, then $\beta = \langle \alpha \rangle \in VP$;

for all $e \in E$, $\|\beta\|(e) = \bigcup_n \|\beta\|_n(e)$, where:

(i) $\|\beta\|_0(e) = \|\alpha\|(e)$;

(ii) For all i (i a positive integer), $\|\beta\|_i(e) = \|\beta\|_{i-1}(e) \cup \{x \in U - U_i \mid \forall y y \in x \rightarrow y \in \|\beta\|_{i-1}(e)\}$.

Intrinsic distributivity can be assured through the use of meaning postulates:

²⁶This operation just closes the denotation of a VP relative to some e under recursive group formation. It is essentially the same operation proposed by Jack Hoeksema (1983) for the derivation of U from U_i ; cf. Ch. I (3). (Hoeksema attributes the formalization to van Benthem.) In models where group formation is associative, the operation can be considerably simplified so that it can apply in a single step, without requiring the relatively complex induction given here.

(120)a. For all $e \in E$ and all $x \in U$, $x \in \llbracket \langle sit \rangle \rrbracket(e)$
implies $\forall y \ y \in x \rightarrow y \in \llbracket \langle sit \rangle \rrbracket(e)$.

(Likewise for $\langle stand \rangle$.)

b. For all $e \in E$ and all $x, y \in U$, $x \in \llbracket \langle lift \rangle \rrbracket(e)(y)$ implies $\forall z \ z \in y \rightarrow x \in \llbracket \langle lift \rangle \rrbracket(e)(z)$.

(Likewise for $\langle build \rangle$.)

As an example of how this approach to distributivity works, consider a small model containing just three events: e_1 , e_2 and $e_1 \sqcup e_2$. Suppose that p is the piano, that $\llbracket \langle lift \rangle \rrbracket(e_1)(p) = \{\text{John}\}$, and that $\llbracket \langle lift \rangle \rrbracket(e_2)(p) = \{\text{Mary}\}$ (i.e. that $\llbracket \langle \langle lift \rangle, \langle the \rangle, \langle piano \rangle \rangle \rrbracket(e_1) = \{\text{John}\}$ and $\llbracket \langle \langle lift \rangle, \langle the \rangle, \langle piano \rangle \rangle \rrbracket(e_2) = \{\text{Mary}\}$).

By the lexical meaning postulate given in (36)d. for $\langle lift \rangle$, $\text{John}, \text{Mary} \in \llbracket \langle lift \rangle \rrbracket(e_1 \sqcup e_2)(p)$. Since $\llbracket \langle lift \rangle \rrbracket(e_1 \sqcup e_2)(p) = \llbracket \langle \langle lift \rangle, \langle the \rangle, \langle piano \rangle \rangle \rrbracket(e_1 \sqcup e_2)$, we have that $\text{John}, \text{Mary} \in \llbracket \langle \langle lift \rangle, \langle the \rangle, \langle piano \rangle \rangle \rrbracket(e_1 \sqcup e_2)$.

Applying the distributivity operation (119) to $\langle \langle lift \rangle, \langle the \rangle, \langle piano \rangle \rangle$ gives (121):

(121) For all $e \in E$, $\llbracket \langle \langle \langle lift \rangle, \langle the \rangle, \langle piano \rangle \rangle \rangle \rrbracket(e)$
= $\cup_n \llbracket \langle \langle \langle lift \rangle, \langle the \rangle, \langle piano \rangle \rangle \rangle \rrbracket_n(e)$, where:
(i) $\llbracket \langle \langle \langle lift \rangle, \langle the \rangle, \langle piano \rangle \rangle \rangle \rrbracket_0(e) =$
 $\llbracket \langle \langle lift \rangle, \langle the \rangle, \langle piano \rangle \rangle \rrbracket(e);$

- (ii) For all i (i a positive integer), $\lll\langle\langle\langle\text{lift}\rangle, \langle\langle\text{the}\rangle, \langle\text{piano}\rangle\rangle\rangle\rangle\lll_i(e) = \lll\langle\langle\langle\text{lift}\rangle, \langle\langle\text{the}\rangle, \langle\text{piano}\rangle\rangle\rangle\rangle\lll_{i-1}(e) \cup \{x \in U - U_i \mid \forall y y \in x \rightarrow y \in \lll\langle\langle\langle\text{lift}\rangle, \langle\langle\text{the}\rangle, \langle\text{piano}\rangle\rangle\rangle\rangle\lll_{i-1}(e)\}$.

By (i), $\text{John}, \text{Mary} \in \lll\langle\langle\langle\text{lift}\rangle, \langle\langle\text{the}\rangle, \langle\text{piano}\rangle\rangle\rangle\rangle\lll_0(e_1 \sqcup e_2)$. Therefore by (ii), $g(\text{John}, \text{Mary}) \in \lll\langle\langle\langle\text{lift}\rangle, \langle\langle\text{the}\rangle, \langle\text{piano}\rangle\rangle\rangle\rangle\lll_1(e_1 \sqcup e_2)$. Hence $g(\text{John}, \text{Mary}) \in \lll\langle\langle\langle\text{lift}\rangle, \langle\langle\text{the}\rangle, \langle\text{piano}\rangle\rangle\rangle\rangle\lll(e_1 \sqcup e_2)$.

By the subject-predicate rule (45), then, $\lll\langle\langle\langle\text{John}\rangle, \langle\text{and}\rangle, \langle\text{Mary}\rangle\rangle\rangle, \langle\langle\langle\text{lift}\rangle, \langle\langle\text{the}\rangle, \langle\text{piano}\rangle\rangle\rangle\rangle\lll = \{e_1 \sqcup e_2\}$.

Note that if the distributive reading of *lift the piano* is modified by *together* (in its collectivizing use), the sentence must come out false. The denotation of *together lift the piano* will be as in (122):

(122) For all e of type e ,

$$\begin{aligned} & \lll\langle\langle\langle\text{together}\rangle\rangle\rangle\lll(\lll\langle\langle\langle\text{lift}\rangle, \langle\langle\text{the}\rangle, \langle\text{piano}\rangle\rangle\rangle\rangle\lll)(e) = \\ & \{w \in U - U_i \mid w \in \lll\langle\langle\langle\text{lift}\rangle, \langle\langle\text{the}\rangle, \langle\text{piano}\rangle\rangle\rangle\rangle\lll(e) \wedge \exists e' \sqsubseteq_E e [w \in \lll\langle\langle\langle\text{lift}\rangle, \langle\langle\text{the}\rangle, \langle\text{piano}\rangle\rangle\rangle\rangle\lll(e') \wedge \forall e'' \sqsubseteq_E e' [\exists z z \in \lll\langle\langle\langle\text{lift}\rangle, \langle\langle\text{the}\rangle, \langle\text{piano}\rangle\rangle\rangle\rangle\lll(e'') \rightarrow \lll\langle\langle\langle\text{lift}\rangle, \langle\langle\text{the}\rangle, \langle\text{piano}\rangle\rangle\rangle\rangle\lll(e'') = \lll\langle\langle\langle\text{lift}\rangle, \langle\langle\text{the}\rangle, \langle\text{piano}\rangle\rangle\rangle\rangle\lll(e')]\}\}. \end{aligned}$$

The only event e in our model such that $g(\text{John}, \text{Mary}) \in \lll\langle\langle\langle\text{lift}\rangle, \langle\langle\text{the}\rangle, \langle\text{piano}\rangle\rangle\rangle\rangle\lll(e)$ is $e_1 \sqcup e_2$. But there is a subevent e' of $e_1 \sqcup e_2$ such that $\exists z z \in \lll\langle\langle\langle\text{lift}\rangle,$

$\langle\langle the \rangle, \langle piano \rangle \rangle \rangle \Vdash (e'')$ and $\Vdash \langle\langle lift \rangle, \langle\langle the \rangle, \langle piano \rangle \rangle \rangle \Vdash (e_1 \sqcup e_2)$.

Specifically, either e_1 or e_2 is such a subevent. Consider e_1 ; the case with e_2 is analogous. Since $\text{John} \in \Vdash \langle\langle lift \rangle, \langle\langle the \rangle, \langle piano \rangle \rangle \rangle \Vdash (e_1)$, it follows that $\text{John} \in \Vdash \langle\langle lift \rangle, \langle\langle the \rangle, \langle piano \rangle \rangle \rangle \Vdash_0 (e_1)$ and hence that $\text{John} \in \Vdash \langle\langle lift \rangle, \langle\langle the \rangle, \langle piano \rangle \rangle \rangle \Vdash (e_1)$ by (118). So we have that $\exists x \ x \in \Vdash \langle\langle lift \rangle, \langle\langle the \rangle, \langle piano \rangle \rangle \rangle \Vdash (e_1)$. But $\Vdash \langle\langle lift \rangle, \langle\langle the \rangle, \langle piano \rangle \rangle \rangle \Vdash (e_1) \neq \Vdash \langle\langle lift \rangle, \langle\langle the \rangle, \langle piano \rangle \rangle \rangle \Vdash (e_1 \sqcup e_2)$; we already know that $g(\text{John}, \text{Mary}) \in \Vdash \langle\langle lift \rangle, \langle\langle the \rangle, \langle piano \rangle \rangle \rangle \Vdash (e_1 \sqcup e_2)$, but it is easy to see that $g(\text{John}, \text{Mary}) \notin \Vdash \langle\langle lift \rangle, \langle\langle the \rangle, \langle piano \rangle \rangle \rangle \Vdash (e_1)$. So $\Vdash \langle\langle John \rangle, \langle and \rangle, \langle Mary \rangle \rangle, \langle\langle together \rangle, \langle\langle lift \rangle, \langle\langle the \rangle, \langle piano \rangle \rangle \rangle \rangle \Vdash = \emptyset$.

11. Conclusion. I claimed above that it was no accident that group action and spatial and temporal proximity were expressed in the same way. I have tried to account for this fact formally by constructing a model-theoretic semantics in which restrictions to group action and to spatially and temporally proximate action are parallel in logical structure.

In some sense, however, the question of parallelism here is a cognitive one, and not purely a matter of logic. Other logical systems are, after all, imaginable.

We have here an analogous problem to more familiar problems from the linguistic theory of syntactic universals. We can explain certain syntactic patterns by constructing, for example, our theory of syntactic features in a certain way. But even after we have done so, the question still remains of why languages use the particular feature system they do. It would be incorrect to suggest that the feature system does not explain anything; but even so, there is another level of explanation beyond it which still has to be given.

In the same way, I have tried to explain why particular meanings tend to be expressed by the same lexical items, by developing a particular sort of model structure and interpretation function, in such a way that the appropriate sorts of events show systematic similarities to one another. But the question remains why languages use the particular sort of model structure and interpretation function they do. Somehow or another it appears to be cognitively "natural" to regard participation in the same event as fundamentally similar to location in the same area, or action at the same time.

Perhaps this is simply because the most salient sorts of event play out in a particular area, at a particular time. Perhaps it is also due partly to the fact that many types of collective action require their participants to be spatially near each other and to act at the same time.

Whether the parallelism is something that can be explained simply as due to generalization from these sorts of salient or prototypical events, or if instead it is due to more fundamental constraints on the structure of human cognition, is perhaps the most interesting question one can ask with regard to the data covered in this chapter. It is not, however, a question model-theoretic semantics can be expected to answer.

CHAPTER III
THE ALGEBRA OF GROUPS AND
THE ALGEBRA OF EVENTS

0. Introduction: Tightening up the model. There are still some readings for *together* left unaccounted for by the rules of Fragment 2. For example what I like to call the "assembly" reading, illustrated in (1)a., is unaccounted for, as are a variety of more or less idiomatic readings, such as those illustrated in (1)b. and c.

(1)a. John put the bicycle together.

b. John and Mary are sleeping together.

c. John really has his head together.

Each of these readings is interesting in its own right, and for its relation to the collectivizing and proximity readings already discussed. But it is certainly possible to spend too much time on the meaning of a single word, and rather than try to account for every reading of *together* here, I'll leave off discussion it entirely and turn to a different problem, that of the algebraic structure of U and E .

In Chapters I and II, only a minimal amount of structure was imposed on these two sets. The set U of groups and individuals was constrained in terms of a frame $\langle U, U_I, \sqsubseteq_U, E \rangle$ as follows:

(2)a. $U_I \subseteq U$

b. $\sqsubseteq_U$ is a partial ordering in U .

c. E is a binary relation in U such that

$(x E g \text{ implies } x E h) \text{ iff } g \sqsubseteq_U h$.

Note here that the relation E is definable entirely in terms of $\sqsubseteq_U$, and therefore does not impose any restrictions of its own. In fact, all that (2) really requires is that U have a distinguished subset U_I , and a partially ordered subset (the field of $\sqsubseteq_U$).

These restrictions are extremely lax. For instance, there is no guarantee that for any two individuals, there is a group containing those individuals (let alone a group containing just those individuals). Nor is there anything to require groups to have members, or to prevent ordinary individuals like John or Mary from having members.

Although it was useful in Chapters I and II to maintain a maximally general (i.e. permissive) notion of what sorts of sets could qualify as U , a more restrictive definition is necessary if all meaningful sentences are to receive a well-formed semantics, and if all legitimate inferences are to be licensed by the formal system. As things stand now, for

example, a simple sentence like (3) will be uninterpretable relative to some models:

(3) John and Mary sit.

Because there is no guarantee of a group containing John and Mary (even assuming that John and Mary are both elements of U), the subject noun phrase of this sentence will fail to refer in some models, and the sentence as a whole will not receive a truth value. But this is clearly wrong; if John and Mary are both in U , the group of John and Mary should be as well. Perhaps we should require U to be closed under group formation.

Stipulating that U must be closed under the group formation operation would constrain the structure of U somewhat, but still leaves tremendous leeway unless the formal properties of this operation are also restricted. As things stand, we have allowed the group formation operation to be either associative or non-associative; idempotent or non-idempotent; unary, binary or n -ary for n of any cardinality whatsoever. We have allowed for models in which groups are identified with sets, or with lattice-theoretical joins, or with mereological sums, or with any of a variety of other sorts of formal object. Section 1 of this chapter will be concerned with this sort of issue.

Our definition of the set of events E simply as a join semilattice is also overly permissive. In order to capture

formally the intuitive idea of a piano-lifting by John counting as a piano-lifting by John and Mary, we allowed an event of the group of John and Mary lifting the piano to have a subevent in which only John lifts the piano. In a certain sense the former, group-level event seems derivative of the latter, individual-level event; the ontologically parsimonious might object to its being considered a separate event at all. While I happen not to agree with this view, I do think there is some value in viewing this sort of group-level event as being dependent on or licensed by the individual-level event. Once this licensing relation is acknowledged, the question arises of what possible positions in the lattice structure of E a derivative group-level event may occupy relative to the individual-level event which licenses it. I will address this issue in Sections 4 - 6 of this chapter, and propose an analysis which constrains the relative position of the licensor and licensee, as well as the class of structures which may serve as E .

1. The algebra of groups. Disputes about the nature of groups, as represented model-theoretically, have tended to center around two sorts of issue, which have sometimes not been sufficiently well kept apart. The question of whether groups are to be considered some kind of individual, or if instead they are some kind of abstract object such as sets, is in principle independent of the question of whether

groups relate formally to their members in the same way as a set $\{x, y\}$ relates to x and y , or rather in the way that an element $x*y$ of some other algebra $A = \langle A, * \rangle$ (a semilattice or mereology, for example) relates to x and y .

Ulrich Blau and Godehard Link have given arguments like the following against taking groups to be sets:

The *set* of cards is abstract, outside of space and time, ... and it cannot be shuffled, cut, or marked. The *cards* are concrete, in space and time, ... and they can be shuffled, cut and marked. (Blau 1981)

If my kids turn the living room into a mess, I find it hard to believe that a set has been at work ... sums or collections of concrete objects remain concrete, i.e., more generally, sums are of the same kind as the objects they are made of (Link 1984a)

In contrast, arguments in favor of a set-theoretic approach have often been based on purely algebraic concerns. Hoeksema (1983), for example, cites the non-associativity of set formation (in contrast to the associativity of the mereological sum and lattice-theoretic join operations) as an advantage for a set-theoretic treatment. A non-associative operation allows for the non-equivalence of the different possible bracketings of a conjoined noun phrase, as in (4):

- (4) [[Blücher and Wellington] and Napoleon] fought against each other near Waterloo. $\neq$ [Blücher and [Wellington and Napoleon]] fought against each other near Waterloo.

Landman (1987) gives similar arguments, plus many others.

It is not hard to see that this sort of algebraic issue is independent of the concrete/abstract issue. Suppose, for example, we wanted a model like Hoeksema's, but in which all the elements of U were non-sets. Recall Hoeksema's original definition for U (given in (3) of Chapter I, above):

$$(5)a. U_0 = U_I$$

$$b. U_{n+1} = U_n \cup \text{POW}_{\leq 2}(U_n), \text{ where } \text{POW}_{\leq 2}(X) \text{ is the set of all subsets of } X \text{ with cardinality greater than or equal to 2.}$$

$$c. U = \bigcup_n U_n$$

It is easy to construct non-set-theoretic frames which mirror all the essential structure of U as Hoeksema defines it. Rather than defining U inductively, we assume the pair $\langle U, U_I \rangle$ is given, with $U_I \subseteq U$, as always. Retain (5)a. and b., but replace c. with (6), so that the induction does not define U , but a separate set U_ω .

$$(6) U_\omega = \bigcup_n U_n$$

Now require an isomorphism i between U_ω and U :

$$(7) i : U_\omega \rightarrow U \text{ such that for all } x, y \in U_\omega, \\ x \in y \text{ iff } i(x) \in i(y).$$

U will have all the same structure as U_ω , but nothing requires it to have any sets as members at all. U "looks" just like U_ω at the formal level, but gives us the conceptual freedom not to think of groups as abstract sets.

The availability of this sort of reifying isomorphism makes it appear likely that set-theoretic models can in general be translated into non-set-theoretic models with all relevant aspects of their formal structure intact. This renders the difference between them ignorable in practice, no matter what one's views on the abstractness of groups.

Some arguments against taking groups to be sets have been of a more formal nature. For example, Link has claimed that identifying groups with sets incorrectly predicts that (8) should be true:

(8) Peter, Paul and Mary have three elements.

I am not sure that identifying groups with sets actually does make this prediction, but as Landman has pointed out, if it does, then Link's own approach must predict that (9) is true, and this seems just as strange:

(9) Peter, Paul and Mary are the join of three atoms.

The oddity of both (8) and (9) is probably best described as due to the fact that the mathematical structure of a term's referent (whether set-theoretic, lattice-theoretic or whatever) is ordinarily not of explicit concern in everyday language use.

Link also notes that many predicates apply both to groups and to singular objects, suggesting that groups and singular objects therefore should be treated "on a par". This is not a very forceful argument. Link is somewhat vague as to what he means by "on a par", but the

satisfaction of a predicate by both sets and individuals is not a difficult problem. Although not allowed by familiar type theories such as Russell's, (or Montague's adaptation of it), mixed-level predicates are expected if we make use of a "cumulative" type system, such as those suggested by Landman (1987) and implicit in Hoeksema (1983).¹

To many people the most attractive feature of Link's algebraic approach may be that it portrays the denotation domains of plural and mass nouns as fundamentally similar; both form complete join semilattices. Moreover, a systematic correspondence exists between elements of the two domains, so that the "consists of" function mapping groups or individuals onto their material substance forms a join homomorphism.

Nothing in this picture precludes the idea that groups are sets, however. Because the algebraic properties of sets can be mimicked to greater or lesser degrees by systems whose elements are not actually sets at all, there is no reason to expect that treating groups as sets will prevent the plural and mass domains from sharing a significant amount of formal structure. Even if we decide to retain a lattice-theoretic treatment of mass expressions, the idea of

¹Cumulative types appear to have been introduced first in Chwistek (1924-1925); see also Quine (1969) Chapter XII. Of course ZF set theory (and similar set theories) are cumulative as well; a semantics which defined denotation domains directly, without the mediation of type indices, would allow mixed-level predicates quite naturally.

a structure-preserving function mapping groups and individuals onto their material substance is easy to implement. It is simple to define a homomorphism between, say, a Hoeksema-style domain and an arbitrary join semilattice: simply require that $h(X) = \sqcup\{h(x) \mid x \in X\}$ for all $X \in U - U_I$.²

To my mind there have been no convincing arguments given against the formal modeling of groups as sets. Of course this by no means means that groups should be modeled as sets, and certainly not that the notion of a group as expressed in natural language is equivalent to that of a set. But the closest competitors to a set-theoretic approach (join semilattices, boolean algebra, mereology) all share defects which make the idea of groups as sets attractive.

Not least of these by any means is the non-associativity of set formation, already mentioned above. There has been little or no dispute about the necessity for non-associative group formation. Even Link, whose lattice-theoretic approach would appear at first to predict that group formation is associative, has given examples like the following to show that it is not (1984a):

²I am not suggesting that a join semilattice actually is the best sort of formal object to serve as the mass domain; see Landman (1987) for evidence that elements of this domain, like elements of the count domain, combine by a non-associative operation.

(10) The Leitches and the Latches like each other.

As Link puts it, "the liking is part of external family relations; nothing is said about internal family affairs".

Landman gives examples like the following:

(11) The cards below seven and the cards from seven up are separated.

If group formation is associative, then the group of the cards below seven and the cards from seven up will be the same as the group of the cards below ten and the cards from ten up, but (11) is not equivalent to (12):

(12) The cards below ten and the cards from ten up are separated.

Non-associativity is also necessary to account for the distribution of *both*. Normally, of course, *both* requires that its complement denote a two-member group:

(13)a. Both John and Mary are asleep.

b. *Both John, Mary and Bill are asleep.

But the total number of members in a group may not be the same as the total number of individuals from which the group is ultimately formed. The following sentence appeared in the text of a recent paper on Welsh syntax (Willis 1986):

(14) In contrast, both Awbery and Jones and Thomas need extra statements in their grammars to make the distinction.

This sentence may sound non-sensical until one is told that

Jones and Thomas collaborated on one grammar of Welsh, and Awbery wrote a separate one.

The non-associativity of group formation can be dealt with in a lattice-theoretical (or boolean or mereological) model, but only at some cost. Link (1984a) develops a system in which there are, in effect, two distinct types of groups: "sums", i.e. non-atomic joins of elements in the algebra which serves as the domain of discourse; and "impure atoms", which are atoms in this algebra.³ Impure atoms thus have a formal status much like ordinary individuals such as John or Mary, but are group-like in their intuitive properties.

Formally, Link takes the domain of discourse to be a complete atomic boolean algebra whose set of atoms A consists of two disjoint sets B (the pure atoms, corresponding to individuals in the pretheoretic sense) and G (the impure atoms). He further requires a one-one function γ from proper joins of elements of B into G . This assures, for example, that if John and Mary are elements of B , then there will be two distinct groups containing just John and Mary: their join, $\text{John} \sqcup \text{Mary}$; and their impure atom, $\gamma(\text{John} \sqcup \text{Mary})$.

³Link uses the term "group" to refer to impure atoms but not to sums, a terminology shared by Landman (1987). Because I have been using the term "group" in a somewhat different way (as in fact Link himself does in 1987a), I will use "impure atom" throughout.

Sum formation will associate: by the standard axioms of boolean algebra, $(\text{John} \sqcup \text{Mary}) \sqcup \text{Bill} = \text{John} \sqcup (\text{Mary} \sqcup \text{Bill})$. Impure atom formation will not associate: $\gamma(\gamma(\text{John} \sqcup \text{Mary}) \sqcup \text{Bill}) \neq \gamma(\text{John} \sqcup \gamma(\text{Mary} \sqcup \text{Bill}))$.

Although this approach is adequate in allowing non-associative groups, the duplication it produces between sums and impure atoms is questionable. The sums in particular now seem entirely superfluous. Landman suggests that sums should be used only in the denotations of distributive predicates, so that their properties are entirely parasitic on the properties of the atoms which generate them; any time a property is ascribed to a group as a group, it is actually the impure atom that satisfies the predicate. But even this, it seems to me, is overly generous to the sums; there is no reason whatsoever to believe that the groups which satisfy a distributive predicate are formed by separate, associative operation while other groups are formed by a non-associative operation.

It is true, of course, that the subjects of intrinsically distributive predicates do associate, so that (15)a. and b., for example, are equivalent:

(15)a. $[[\text{John and Mary}] \text{ and Bill}]$ are asleep.

b. $[\text{John and } [\text{Mary and Bill}]]$ are asleep.

But the reader may easily confirm that Fragment 2 predicts

this, even for models which do not have an associative group formation operation.⁴

If group formation is modeled as set formation, non-associativity is expected, since, e.g., $\{\{\text{John, Mary}\}, \text{Bill}\} \neq \{\text{John}, \{\text{Mary, Bill}\}\}$. No intermediate level of associative "sums" is necessary.

Another advantage of a set-theoretic treatment is that the set membership relation is distinct from the subset relation. This distinction is not available in models where groups are simply the boolean or lattice-theoretical joins or mereological sums of the individuals which make them up; in such models the relation of John to the group of John and Mary is precisely the same as the relation of the group of John and Mary to the group of John, Mary and Bill. (Here again the device of impure atoms can be made to produce the desired effect, however.)

The member/subgroup distinction is useful in giving the semantics of predicates like *be similar*, whose denotations are closed under subgroups but not members. If John, Mary and Bill are similar, it follows that John and Mary are

⁴Note, incidentally, that subjects of derived distributives do not necessarily associate. Sentence (i) has a reading in which John and Mary lifted the piano collectively, and Bill lifted it by himself:

(i) John and Mary, and Bill, lifted the piano.
This is not a counterexample to Landman's claim, however, since he (and Link) would take the subject as denoting the sum of Bill and the impure atom of John and Mary.

similar, that John and Bill are similar, and that Mary and Bill are similar, but it does not follow that John is similar, that Mary is similar or that Bill is similar. Massey (1976), whose mereological model does not distinguish subgroups from members, is forced to account for these inferences through the use of highly questionable abstract syntactic deep structures, but Hoeksema, who treats groups as sets, is able to account for them with a very simple meaning postulate.

Maintaining a non-associative group formation operation and a distinction between subgroups and members allows for higher-order groups, a possibility that lattice-theoretic models do not allow for, barring appeals to impure atoms. Examples like (10) and (11) above show quite clearly that at least second order groups are necessary; examples calling for third order groups or higher are somewhat harder to construct and process, but seem to have well-defined readings: E.g. *The Leitches and Latches, and the Reitches and Ratches, are similar in that they hate each other* has a reading in which the Leitches and Latches hate each other and the Reitches and Ratches hate each other. The subject of the sentence denotes a two-membered third order group containing the second order group of the Leitches and the Latches as one member and the second order group of the Reitches and the Ratches as the other member.

It seems unlikely that the grammar of a language sets a maximum order on the groups to which it allows reference. This suggests that the class of groups and individuals U in a model should be closed under recursive group formation. Since recursive closure of a set under set formation (or similar non-associative operations) results in a set with transfinite cardinality, we obtain a somewhat disturbing consequence: U must always be transfinite.⁵ In contrast, a system like Link's allows for finite models.

Note that the cardinality question here is not intrinsically tied to the issue of set-theoretic versus lattice-theoretic models. If we extend Link's device of impure atoms to account for groups of arbitrary order, similar cardinality results will obtain. The real issue is whether reference to groups past a certain order is grammatically disallowed, and if so, how we determine the cut-off point. This seems to be an analogous issue to the old syntactic question of whether there is a grammatically imposed limit on sentence length, and an analogous answer seems in order. I will assume there is no such limit.

Despite the advantages of regarding groups as sets, there are reasons to think that the notion of a group is not

⁵In fact, assuming a set-theoretic model like Hoekesma's or Landman's, if the set of individuals U_i is itself transfinite, U will have a minimum cardinality of aleph-omega, an unimaginably huge number.

actually equivalent to that of a set. A number of sets seem to have no counterparts among the groups expressed in natural language. In particular there seems to be no reason to admit an empty group, and singleton groups are also somewhat questionable; no one, to my knowledge, has argued explicitly for their existence, and I can see no good reason for assuming them. In fact, Hoeksema (1987) has pointed out that data like (16) - (17) seem to indicate that an individual cannot form a group with itself:

(16)a. John is similar to Mary.

b. John and Mary are similar.

(17)a. John is similar to himself.

b. *John and John are similar.

Of course other factors, such as the repetition of a proper name referring both times to the same individual, may play a role in the unacceptability of (17)b.; *?John and himself are similar* is perhaps not quite so bad, though still not a perfectly acceptable sentence.

An obvious problem for the view that an individual cannot form a group with itself comes up with sentences like *Cicero and Tully were the same person*. Perhaps all these examples show, however, is that group formation needs to be defined at a level of discourse markers or "pegs" (in the sense of Landman (1986)) in addition to the final level of model-theoretic interpretation. I will not give an analysis of such sentences here, or show how to adapt the pegs theory

to the study of plurality, but will simply assume that some account along these lines will work and that an individual may not form a group with itself. Note that if an individual cannot form a group with itself, the question of whether the group formation operation is idempotent simply does not arise.

It seems fair, then to adopt a model in which groups are formalized as non-null, non-singleton sets. As it happens, this is virtually identical to the sort of model suggested by Hoeksema.⁶ For the remainder of this dissertation, then, I will adopt a Hoeksema-style domain of discourse. U is defined on the basis of U_1 as in (5), above; ε is identified with ε (restricted to U , of course), and $\subseteq_U$ is identified with $\subseteq$ (also restricted to U).

2. A note on *respectively* constructions. In general, arguments against treating groups as sets have been accompanied by alternative suggestions which portray the algebra of groups as significantly poorer than might be expected in a set-theoretic treatment, in the sense that fewer distinct groups are generated by the same set of

⁶There is a difference in that Hoeksema does not admit all non-null non-singleton sets as groups, but only finite ones. I don't think this is a serious problem; talk of infinite groups is pragmatically very restricted, and the formal changes in the definition of U which would be necessary to admit infinite sets are not difficult.

individuals. (In large part this relative poverty of structure is due to the assumption of associative group formation.) Before committing totally to a set-theoretic treatment, however, it may be worth considering whether it too may fail to provide a fine-grained enough distinction among groups, whether in fact a richer algebra may be called for.

We have assumed from the outset that group formation is commutative. Our initial definition of a frame, general as it was, did not allow distinct groups to have precisely the same membership; thus $g(x, y) = g(y, x)$ for all $x, y \in U$. In general this appears to be correct; (18)a. and b. are logically equivalent:

(18)a. John and Mary ate a pizza.

b. Mary and John ate a pizza.

However, this sort of equivalence breaks down in sentences containing the adverb *respectively*:

(19)a. John and Mary ate a pizza and drank a beer,
respectively.

b. Mary and John ate a pizza and drank a beer,
respectively.

This sort of data suggests that we may have to distinguish between the group of John and Mary and the group of Mary and John, for instance by modeling groups as tuples or sequences rather than sets.

In fact I doubt strongly that there can be any compositional treatment of this sort of sentence unless conjunction can be interpreted as sequence formation. If *ate a pizza and drank a beer respectively* denotes the sequence $\langle \llbracket \text{ate a pizza} \rrbracket, \llbracket \text{drank a beer} \rrbracket \rangle$ and *John and Mary* denotes $\langle \text{John}, \text{Mary} \rangle$, we can obtain correct truth conditions for sentences like (19)a. through the use of a subject-predicate rule as in (20):

(20) If $\alpha \in T$ and $\beta \in VP$ then $\gamma = \langle \alpha, \beta \rangle \in S$;

$\llbracket \gamma \rrbracket = 1$ if $\llbracket \alpha \rrbracket_0 \in \llbracket \beta \rrbracket_0, \dots, \llbracket \alpha \rrbracket_n \in \llbracket \beta \rrbracket_n$,

where $\llbracket \alpha \rrbracket = \langle \llbracket \alpha \rrbracket_0, \dots, \llbracket \alpha \rrbracket_n \rangle$ and $\llbracket \beta \rrbracket =$

$\langle \llbracket \beta \rrbracket_0, \dots, \llbracket \beta \rrbracket_n \rangle$.

If no ordering is maintained among the denotations of the conjuncts forming the subject of a *respectively* sentence, there seems little hope of setting up the appropriate correspondence with the VP conjuncts.

However, this sort of rule is probably better understood as simultaneously predicating separate properties of each of the elements of the sequence, rather than as predicating some sort of complex property of a group. I would strongly resist any move to treat plural terms in general as denoting sequences. The intuition is strong that *John and Mary* and *Mary and John* normally refer to the same object. Treating the subjects of ordinary sentences like (18)a. and b. as denoting sequences would greatly complicate the task of explaining their equivalence. Presumably, we

would have to employ meaning postulates specifying that if a particular sequence satisfies a given predicate, so will any permutation of the sequence -- unless the predicate is formed by *respectively* conjunction.

Treating groups as sequences also poses a problem in defining the semantics of terms formed from plural nouns, rather than by conjunction. What is the denotation of *the students*, if groups are sequences? Most likely we do not want to treat such terms as denoting sequences at all, since sentences like (21) are anomalous:

- (21) *The students ate a pizza and drank a beer,
respectively.

Because arguments like (22) seem valid, it is questionable whether conjoined terms should be analyzed as denoting sequences either:

- (22) The students ate a pizza. The students were John,
Mary and Bill. Therefore, John, Mary and Bill ate
a pizza.

I think it is safe to conclude, then, that group formation is commutative, and that sequences are not the appropriate formal device for modeling groups.

3. Committees, etc. The non-commutativity of conjunction in *respectively* sentences is not the only, or even the most obvious, reason for suspecting that a richer, more fine-grained algebra of groups may be called for. The classic

problem of committees, families, communities and other "social" collective objects calls into question our assumption that no two distinct groups may have exactly the same membership. It is well-known that two different committees may have the same members -- even in all possible worlds -- yet have different properties. (For example they may be chaired by different people, discuss different sorts of business, etc.) If we consider committees to be a kind of group, then the identity of a group must not be determined by its membership.

Whether we call committees groups is largely a matter of choice, since in this context "group" is a technical term which we may define in any way we want. However, there are a number of differences between social collectives like committees and the purely logical sort of groups we have been concerned with so far. These differences require that some sort of distinction be made, and, to my mind, justify a terminological split.

As already mentioned, ordinary groups, such as those to which we refer using conjoined names, appear to be determined by their membership, while committees and similar objects are not. In addition, committees are somewhat freer than ordinary groups in how many members they can have. There is nothing non-sensical about a committee with only one member, or even, as Bennett (1975) has pointed out, about a committee which temporarily has no members at all.

On the other hand, committees may be subject to restrictions on membership which do not hold for ordinary groups. The judgments here are perhaps not as clear as one would like, but it seems that the members of a committee or similar organization must be individuals, or perhaps other committee-like objects. They cannot, I think, be ordinary groups -- a committee whose members were John, Mary, and the group of Bill and Sue would be a strange committee indeed, unless Bill and Sue formed some sort of committee-like entity themselves, or counted as though they were a single individual for the purposes of the committee rules.⁷ In contrast, an ordinary "logical" group consisting of John, Mary, and the group of Bill and Sue is something we have already seen to be necessary.

The relative independence of committees from their membership, the possibility of singleton and empty committees, and the ineligibility of ordinary groups for membership in committees are all predicted if: (i) committees are members of U_I -- "individuals" rather than groups; (ii) committees are related to their members by a partial function $m : U_I \times T \rightarrow \text{POW}(U_I)$. Since the range of m is $\text{POW}(U_I)$, the set of sets of individuals, rather than $U - U_I$, the set of groups, we allow singleton and empty committees even though we do not allow singleton and empty

⁷For instance if Bill and Sue are a married couple jointly appointed to a position which is normally held by a single individual.

groups, and we also exclude committees whose membership is anything other than a first-order set.

It may seem odd at first to treat committees as individuals, but formally all this means is that they must be present in the first step of the group-formation induction (5), rather than being properly generated on the basis of the set of individuals.

The treatment of committees suggested here is reminiscent of Link's device of impure atoms, in that committees are group-like in their intuitive properties, but are modeled as individuals. The primary difference is that Link defines a one-one function from sums into the set of impure atoms, so that for each sum there is an impure atom which is "the" impure atom corresponding to it; for example, corresponding to the sum $\text{John} \sqcup \text{Mary}$ is the impure atom $\gamma(\text{John} \sqcup \text{Mary})$. Now $\gamma(\text{John} \sqcup \text{Mary})$ is not a committee with John and Mary as members, though this might also be modeled as an impure atom⁸; instead, $\gamma(\text{John} \sqcup \text{Mary})$ is what we might call the "plain vanilla" impure atom of John and Mary -- it has no properties other than those we attribute intuitively to the group of John and Mary.

⁸Nothing rules out the possibility of "extra" impure atoms which do not serve as the value of γ for any sum. Presumably, if we were to model committees as impure atoms it would be as these "extra" impure atoms.

In the current system, unlike Link's, "plain vanilla" impure atoms are unnecessary, since non-associative group formation is assumed from the start. Committees, which have properties independent of the groups which make them up, may still be admitted as individuals, but because there is no γ function or plain vanilla impure atoms, we do not reproduce the whole structure of groups at the individual level, and avoid the sort of massive duplication which characterizes Link's system.

This approach to committees has real advantages over the treatment suggested in Landman (1987). Landman does not admit committees at all as independent entities in the universe of discourse, instead treating all their properties as properties of the groups which make them up. This would initially appear to disallow different committees with the same membership, but Landman is able to reconstruct the effect of like-membered committees with different properties by relativizing the property sets of groups and individuals to the "roles" or "aspects" under which they are conceived. Thus a particular group may have certain properties as Committee A, but a different set of properties as Committee B. The group itself remains the same in both cases, however.

Landman's case rests primarily on the fact that even ordinary groups and individuals appear to have different

properties in different roles. For example, (23)a., b. and c. can all be true, even if John is both the judge and the janitor:

(23)a. The judge makes exactly \$50,000.

b. The janitor makes exactly \$10,000.

c. John makes exactly \$60,000.

Here, John has different properties relative to his role as a judge than he does relative to his role as a janitor, or relative to his role just as John.

A similar example involves a situation where the judges happen to moonlight as hangmen. Unsatisfied with courtroom conditions, the judges go on strike. As Landman points out, prisoners who had already been condemned to death would be in for an unpleasant surprise if they concluded from this that the hangmen were on strike.

According to Landman, the problem of different committees with the same members is essentially the same one as that of John's two incomes, or the judges and the hangmen. Just as John can have different properties as a judge than he does as a janitor, or the judges can have different properties as judges than they do as hangmen, so also the group of (say) John and Mary can have different properties as Committee A than they do as Committee B. We feel little temptation to introduce John-as-a-judge and John-as-a-janitor as two distinct entities in the universe of discourse; if the committee problem really is the same,

we are no more justified in introducing Committee A and Committee B as distinct entities.

Formally, Landman assumes a generalized quantifier treatment of terms, framed in a property theory which allows multiple ultrafilters for each group or individual. Each of the ultrafilters associated with a group or individual corresponds to one of its "guises". Thus John has one property set in his guise as a judge, and a different one in his guise as a janitor. A group or individual can be globally inconsistent in its properties, but is consistent relative to each guise.

This idea is explanatory only if the notion of a "guise" is clarified somewhat. Landman identifies guises as properties; the property set of a given group or individual varies according to a property against which it is evaluated -- say the judge property or the janitor property.

Relativization to a guise/property is expressed naturally in English using an *as*-phrase. In the judges-and-hangmen example, the prisoners get hanged because the judges are on strike as judges, but not on strike as hangmen. In the case of John's two jobs, it is possible for the judge to earn exactly \$50,000 and the janitor to earn exactly \$10,000 even though they are one and the same person because we ordinarily understand *The judge earns exactly \$50,000 to*

mean more specifically "the judge earns exactly \$50,000 as a judge", and likewise for the janitor sentence.

By the same token, Committee A can pay a visit to South Africa without Committee B doing so even if both committees have the same members, because when these members actually make the trip, they do so as Committee A. The property of going to South Africa is a property not of the committee, which does not exist as an object in its own right, but of the group of the committee's members, relative to their guise as Committee A.⁹

The problem is that not all properties of a committee can be reduced to properties of the committee's members, even relative to their guise as the committee. For example, (24)a. and b. are both acceptable, but in comparison to (25)a., (25)b. sounds extremely odd:

(24)a. Committee A paid an official visit to South Africa.

b. Committee A was founded in 1925.

(25)a. John and Mary paid an official visit to South Africa as Committee A.

b. ??John and Mary were founded in 1925 as Committee A.

⁹Note that since guises are identified with properties, this approach requires us to distinguish the "Committee A property" from the "Committee B property". Although committees no longer exist as groups or individuals, they do exist as properties that groups or individuals can "have".

There is an interesting grammatical correlation to these facts: It is well known that certain dialects of English allow plural verb agreement with nouns like *committee*. Even these dialects require singular agreement for predicates like *be founded in 1925*, however:

(26)a. *Committee A are paying an official visit to
South Africa.

b. *Committee A were founded in 1925.

Perhaps it would be possible, in some roundabout way, to reduce properties like having been founded in 1925 to properties of the members of a committee, rather than the committee itself; such a reduction might even be desirable, if our project were to develop a restrictive theory of metaphysics¹⁰. But in giving a semantics for natural language, where primary concerns in developing a model-theoretic ontology include a special regard for observed grammatical structure and straightforwardness and simplicity in the statement of truth conditions, it seems better to admit committees as objects in their own right, with properties independent of those of their members.

¹⁰One real challenge to any such reductionist account is Bennett's (1975) observation that a committee can be temporarily without members, already mentioned above. Note that since Landman disallows empty groups, his analysis predicts that memberless committees should not occur.

4. The algebra of events as a semilattice. A detailed elaboration and defense of an algebraic theory of event structure would be a massive undertaking, deserving a book of its own rather than three short sections in this dissertation. Nonetheless a few worthwhile comments can be made even in the brief format to be adopted here.

Many of the issues which arise with regard to the algebra of groups and individuals are actually more easily settled as they pertain to the algebra of events. There seems little reason, for example, to suspect that whatever operation forms complex events out of simpler ones might be non-commutative. The complex event of John eating a pizza and Mary drinking a beer is intuitively the same as the event of Mary drinking a beer and John eating a pizza. Of course the order in which we describe the subevents of this complex event may convey an impression that these subevents occurred in that same order, but by now it should be uncontroversial to attribute this effect to conversational implicature rather than model-theoretic interpretation.

Likewise the issue of associativity does not seem likely to provoke much dispute. The event of John eating a pizza and Mary drinking a beer, and Bill smoking a cigarette seems to be no different than the event of John eating a pizza, and Mary drinking a beer and Bill smoking a cigarette. I can find no convincing examples which would

require complex events to be formed by a non-associative operation.

Perhaps an advocate of a non-associative operation might argue that gerunds and other deverbal nominals refer to events, and that conjunction of such nominals is non-associative, as shown by the nonequivalence of (27)a. and b.

(27)a. [[John's eating a pizza and Mary's drinking a beer] and Bill's smoking a cigarette] were quite unexpected.

b. [John's eating a pizza and [Mary's drinking a beer and Bill's smoking a cigarette]] were quite unexpected.

Sentence (27)a. can be used to state that one did not expect that both John would eat a pizza and Mary would drink a beer -- though either one of these events by itself might not have been surprising at all¹¹ -- and that one did not expect that Bill would smoke a cigarette. In contrast, (27)b. can be used to mean that one did not expect that John would eat a pizza, and also did not expect that both Mary would drink a beer and Bill would smoke a cigarette.

It is highly questionable, however, whether the connective *and* in this sort of example really corresponds to the operation forming complex events out of simpler ones, or rather to the ordinary group formation operation. If events

¹¹Suppose, for example, that one had been under the impression that John and Mary together had enough money for either a pizza or a beer, but not both.

can be construed as a sort of individual, as the possibility of nominal reference to them indeed suggests, then there is no reason not suppose that there can be groups of events, or even groups of groups of events, etc.

Nor is it advisable to identify the group of two events with the complex event which has just those two as parts. In the examples from (27), for instance, it is not the whole event of John eating a pizza, Mary drinking a beer, and Bill smoking a cigarette that is described as having been unexpected. Note also that the verb phrases in (27)a. and b. take plural agreement. In contrast, when sentences describing complex events serve as antecedents for event anaphora, the anaphors are always singular:

(28) John made silly faces and Mary stood on her head.

It was really funny.

I will assume without discussion that the operation which forms complex events from simpler ones is idempotent. This leaves us with a commutative, associative, idempotent operation. One of the standard definitions of a semilattice is as a set closed under a commutative, associative, idempotent operation, so our original assumption that the set of events forms a semilattice appears to be justified.¹²

¹²Of course our original assumption was that the set of events forms a join semilattice, and not just that it forms a semilattice. However, any semilattice may be viewed as a join semilattice: Assume a set A closed under a commutative, associative, idempotent operation $*$. Now

5. Restricting the structure of events. To require that the set of events form a semilattice is not to impose a very severe restriction at all. We have already seen fit to impose additional restrictions in the form of meaning postulates, assuring, for example, that if a group sits in some event e , then e has subevents in which each member of the group sits. Other meaning postulates assure "persistence" of truth relative to $\sqsubseteq_E$.

It is not hard to see that more restrictions are necessary. For example, if John eats a pizza in e and Mary drinks a beer in e , then e should have a (proper) subevent in which John eats a pizza, and separate one in which Mary drinks a beer. Nothing as yet assures this.

It clearly will not do simply to require that if any two distinct sentences are true of an event e , then for each of these sentences, e has a distinct subevent in which that sentence is true. Such an approach would require distinct subevents even when the sentences in question were logically equivalent. But by "logically equivalent sentences" we presumably mean sentences which are true in precisely the same events in all models, so this approach leads to a contradiction.

define $\sqsubseteq_A$ as in (i):

(i) $x \sqsubseteq_A y \iff x * y = y$

Every pair of elements in A will have a least upper bound relative to $\sqsubseteq_A$.

What we want is to require that when two logically independent sentences -- or, even better, two logically independent propositions -- are true in some event, then the event will have distinct subevents for each of these two propositions. A principle like (29) might do the trick (though obviously it would need to be clarified and made more precise):

- (29) Independence principle (informal version): For all propositions p , q , if there is an event e in which p is true, and p is logically independent from q , then there is an event e' in which p is true and q is not.

We understand p to be independent from q if p can be true while q is not, that is, if p does not entail q . Since *John eats a pizza* and *Mary drinks a beer* express logically independent propositions, this principle will require that if each of these sentences is true of some event, then there are separate events in which *John eats a pizza* is true (but *Mary drinks a beer* is not), and in which *Mary drinks a beer* is true (but *John eats a pizza* is not).

However, this "independence principle" is incompatible with several aspects of our earlier analysis of group action sentences, including the treatment of sentences in which a group gets "credit" for action by only some of its members. A sentence like *John and Mary lifted the piano* can be judged true, even if only John did the actual lifting. It was

argued in Chapter II that in this sort of situation, the event e of the group lifting the piano has a proper subevent e' in which John lifts the piano but in which it is not true that the group lifts the piano. The independence principle will require in addition an event e'' in which the group lifts the piano, but in which it is not true that John lifts the piano. It cannot be the case that e'' has a subevent in which John lifts the piano, since then the persistence meaning postulate for *lift* would require it also to be true in e'' that John lifts the piano -- precisely what the independence principle disallows. So it will never be the case that the "smallest" event of the group lifting the piano will include the event of John lifting the piano, and we lose our account of "team credit" events entirely.

The independence principle also poses problems for sentences which attribute a property to a group only by virtue of some totaling function on the properties of the group's members. It was implicit in the analysis of Chapter II that if John lifts two pianos and Mary lifts another three pianos, then it follows that the group of John and Mary lifts the group of all five pianos. The smallest event in which the group lifts these pianos should have the event of John lifting his two pianos as a part, and the event of Mary lifting her three pianos as another part. But the proposition expressed by *John and Mary lift five pianos* is independent from that expressed by *John lifts two pianos and*

Mary lifts three pianos, and the independence principle therefore requires that in any model where the former proposition is true of some event, there must be some event where this proposition is true but the latter one is not. But again, this is incompatible with our assumption that the group-level event has the individual-level events as parts.

I want to claim even so that the independence principle is correct at some level of description. It is vital to the notion of an event as intended here that if two independent things happen in some event, then that event has two independent subevents corresponding to them. The problem is that even though the sentence *John and Mary lift five pianos* does express a logically independent proposition from *John lifts two pianos and Mary lifts three pianos*, the event of the group lifting five pianos is in some other sense completely dependent on the events of John lifting two pianos and Mary lifting three; and even though the sentence *John and Mary lift the piano* does express a logically independent proposition from *John lifts the piano*, in the case of "team credit" predication the event of the group lifting the piano is again completely dependent on the event of John lifting the piano.

Indeed, in such cases the group-level event exists only by virtue of people's perceptions of the individual-level event -- John's lifting of the piano will count as a lifting

by the group of John and Mary only if John and Mary are perceived as forming some sort of "team". Despite the fact that piano-lifting is a purely physical activity, the group-level event has no independent physical realization from the individual-level event. It exists as a separate event by convention only.

Because of the derivative nature of this sort of event, as well as events in which a group has a property only by virtue of totaling on the properties of its members, some sort of special treatment is warranted. I want to sketch a three-step definition of the set of admissible event structures: We first define the class of step 1 event frames or preframes, which respect the independence principle. Step 1 frames provide the input to an operation assigning properties to groups through totaling on the properties of their members. The result is a step 2 frame or unaugmented frame. A step 2 frame may then be augmented with team credit events to obtain a step 3 frame or augmented frame. Actual sentences from English are to be evaluated relative to the class of augmented frames. Although an augmented frame as a whole need not respect the independence principle, the only exceptions will be those licensed by the process deriving them from step 1 frames.

Consider now what step 1 frames should be like formally; step 2 and 3 frames will be covered in the next

section. A step 1 frame $F = \langle E_F, \sqsubseteq_F \rangle$ should form a join semilattice, of course; let us also make the simplifying assumption that this semilattice is complete.

In order to assure that the independence principle will hold for F , we need to make reference to propositions. Up to now we have avoided this except in informal discussion; let us now assume a set of propositions P , where P forms a complete boolean algebra relative to the standard connectives $\wedge$, $\vee$ and $\neg$.

I take propositions as primitive for convenience and to avoid getting sidetracked on philosophical issues. Ultimately, though, it would be highly undesirable to maintain both events and propositions as independent formal primitives, especially under the relatively abstract conception of events adopted here. The two notions are similar enough that one should almost certainly be modeled on the basis of the other; events from preframes may even be identified as propositions, as I will suggest below.

For heuristic purposes, however, it will be useful to assume two distinct structures, one of events and one of propositions. Starting from the relatively uncontroversial assumption that the structure of propositions forms a boolean algebra, we can then show how a few simple stipulations about the relation between an event and the propositions which are true in it will constrain the class

of admissible event structures much more fully than our original definition, which allowed any join semilattice.

Let us assume a function P mapping any event e onto a non-empty set of propositions, intuitively the propositions which hold true in e .

We should also require (30), in order to avoid contradictory events, to assure that the (boolean¹³) entailments of a proposition which is true in an event e will also be true in e , and to assure that the conjunction of any propositions true in e will also be true in e .¹⁴ (' $\Rightarrow$ ' indicates entailment; formally it may be taken as denoting $\sqsubseteq_P$, the boolean partial order of P .)

- (30) For all step 1 frames F , events $e \in E_F$, propositions $p, q \in P$, and sets of propositions $X \subseteq P$: (i) If $p \in P(e)$ then $\neg p \notin P(e)$; (ii) If $p \in P(e)$ and $p \Rightarrow q$, then $q \in P(e)$; and (iii) If $X \subseteq P(e)$ then $\bigwedge X \in P(e)$.

I will also assume that no two events support exactly the same propositions; events are distinguished by what is true in them:

¹³The inference from *John lifted two pianos and Mary lifted another three pianos* to *John and Mary lifted five pianos* will be accounted for in the mapping between step 1 and step 2 frames, rather than through the boolean partial order on P .

¹⁴The conditions in (30) jointly guarantee that for any e , $P(e)$ will be a proper principle filter in the set of propositions.

- (31) For all step 1 frames F and events $e, e' \in E_F$,
 $P(e) = P(e')$ iff $e = e'$.

We can now state the independence principle more formally as the stipulation in (32):

- (32) Independence principle: For every step 1 frame F ,
 and proposition $p \in P$ such that there exists an e
 $\in E_F$ such that $p \in P(e)$, the following holds: $\exists e$
 $\in E_F$ $p \in P(e) \wedge \forall q \in P[p \neq q \rightarrow q \notin P(e)]$.

That is, if there is an event in E_F in which p is true, then there is one in which only p (plus whatever other propositions p entails) is true.

Let us now define the notion of a "simple" event, with respect to some proposition. Intuitively, an event is simple with respect to p (in F) iff p is true in e and no propositions from which p is logically independent are true in e . Formally:

- (33) $\text{Simple}(e, p)$ iff $p \in P(e) \wedge \forall q \in P[p \neq q \rightarrow q \notin P(e)]$.

If the independence principle is satisfied, then if a proposition p holds in some event e in frame F , then F contains an event e' which is simple with respect to p .

For any event e , there is at most one proposition p such that e is simple with respect to p . Proof: Assume p, p' , such that $\text{Simple}(e, p)$ and $\text{Simple}(e, p')$. By the

definition of Simplicity, $p \in P(e)$ and $p' \in P(e)$. Now distinguish three cases. Case 1: $p \neq p'$. Then $p' \notin P(e)$, by the definition of simplicity. Of course this contradicts our earlier result that $p' \in P(e)$, so Case 1 is excluded. Case 2: $p' \neq p$. Then $p \notin P(e)$, again by the definition of simplicity. This contradicts our earlier result that $p \in P(e)$, so Case 2 is excluded. Case 3: $p \Rightarrow p'$ and $p' \Rightarrow p$. Therefore $p = p'$ by the antisymmetry of partial orders. (Q.E.D.)

For any event e , there is at least one proposition p such that e is simple with respect to p . Proof: Let $p = \bigwedge \{q \in P \mid q \in P(e)\}$. Then $p \in P(e)$ by clause (iii) of (30). Since $p \Rightarrow q$ for all $q \in P(e)$, it will also be true that $\forall q \in P[p \neq q \rightarrow q \notin P(e)]$. So e will be simple with respect to p . (Q.E.D.)

The last two results together assure that we are always safe in talking about the proposition with respect to which a given event e is simple.

The events in E are actually in one-one correspondence with those propositions which are true of some event or other in E , and thus are somewhat abstract and proposition-like themselves. In fact, there is no reason not to identify the set of propositions with the set of possible step 1 events. However, the one-one correspondence will be destroyed by the processes mapping from step 1 frames to

step 2 and 3 frames; for example in cases where John lifts the piano but the group of John and Mary gets credit for it, we want to "add in" a new, dependent event of the group lifting the piano. At least at the level of augmented frames, therefore, the distinction between events and propositions will have to be maintained.

Are there any other requirements which ought to be imposed on step 1 frames? Most likely there are several, but one in particular will be of interest in what follows: we should require that propositions should be "persistent" relative to $\subseteq_F$. This was guaranteed for particular verbs by lexical meaning postulates in Fragment 2; a more general stipulation like that given in (34) now seems desirable:

- (34) For all step 1 frames F , events $e, e' \in E_F$, and propositions $p \in P$, if $p \in P(e)$ and $e \subseteq_F e'$, then $p \in P(e')$.

In fact I will assume not only that if p is true in e then it is also true in all superevents of e , but also that if p is true in e then e is a superevent of e_p , the event which is simple with respect to p . Accordingly, let us revise (34) as (35):¹⁵

- (35) For all step 1 frames F and propositions p , where

¹⁵As it turns out, (35) is actually no stronger than (34) for finite frames. I do not know if it is stronger for infinite frames or not.

e_p is the event which is simple with respect to p ,
 $\{e \in E_F \mid p \in P(e)\} = \{e \in E_F \mid e_p \sqsubseteq_F e\}.$

A few simple proof sketches will now show that the notion of a step 1 frame is considerably more restrictive than that of a join semilattice, so that the event structures of some possible frames from Fragment 2 will not qualify.

Any step 1 frame contains a zero element. Proof: Let $p = v\{q \in P \mid \exists e \in E_F q \in P(e)\}$, and let e_p be the event which is simple with respect to p . Now for any event e , where q is the proposition with respect to which e is simple, $q \Rightarrow p$. Therefore $p \in P(e)$ by (30)ii. So $e_p \sqsubseteq_F e$ by (35). (Q.E.D.)

The existence of a zero element guarantees that any step 1 frame F forms a (complete) lattice, and not just a semilattice. Proof: Every subset X of E_F has at least one lower bound, namely the zero element. Let Y be the set of lower bounds for X . Then $\sqcup Y$ will be the greatest lower bound (meet) for X . (Q.E.D.)¹⁶

¹⁶I will use ' $\sqcap X$ ' to indicate the meet of X , ' $e \sqcap e'$ ' to indicate the meet $\{e, e'\}$, ' 0_F ' to indicate the zero element of E_F and ' 1_F ' to indicate the unit element of E_F . (The existence of a unit element is guaranteed by our earlier requirement that E_F be complete as a join semilattice.)

For any $X \subseteq E_F$, $\sqcup X$ is simple for $\wedge\{p \in P \mid \exists e \in X \text{ Simple}(e, p)\}$. Proof: For the sake of notational brevity, let us write p_X for $\{p \in P \mid \exists e \in X \text{ Simple}(e, p)\}$. Now, for any event in which $\wedge p_X$ is true, each element of p_X is also true, by (30)ii. That is, $\{e^* \in E_F \mid \wedge p_X \in P(e^*)\} \subseteq \{e^* \in E_F \mid \forall e \in X e \sqsubseteq_F e^*\}$. Likewise, if each element of p_X is true in e , then $\wedge p_X$ is true in e by (30)iii. That is, $\{e^* \in E_F \mid \forall e \in X e \sqsubseteq_F e^*\} \subseteq \{e^* \in E_F \mid \wedge p_X \in P(e^*)\}$. Therefore $\{e^* \in E_F \mid \wedge p_X \in P(e^*)\} = \{e^* \in E_F \mid \forall e \in X e \sqsubseteq_F e^*\}$. But this is just the set of upper bounds for X . Since $\sqcup X$ is by definition the least upper bound for X , it follows that $\{e^* \in E_F \mid \wedge p_X \in P(e^*)\} = \{e^* \in E_F \mid \sqcup X \sqsubseteq_F e^*\}$. From this it follows from (35) that $\sqcup X$ is simple with respect to $\wedge p_X$. (Q.E.D.)

For any $X \subseteq E_F$, $\sqcap X$ is simple for $\vee\{p \in P \mid \exists e \in X \text{ Simple}(e, p)\}$ (that is, for $\vee p_X$). Proof: $\vee p_X$ is true in each element of X , by (30)ii. So for all $e \in X$, where e^* is the event which is simple with respect to $\vee p_X$, $e^* \sqsubseteq_F e$, by (35); that is, e^* is a lower bound for X . Assume a lower bound e^{**} for X such that $e^* \sqsubseteq_F e^{**}$. Let p^{**} be the proposition with respect to which e^{**} is simple. By (35), $\vee p_X \in P(e^{**})$; therefore $p^{**} \Rightarrow \vee p_X$ by the definition of simplicity (33). Also by (35), $p^{**} \in P(e)$ for each $e \in X$; therefore $p \Rightarrow p^{**}$ for each $p \in p_X$, again by (33). But for all q such that $p \Rightarrow q$ for each $p \in p_X$, it follows that $\vee p_X \Rightarrow q$, since $\vee p_X$ is the least upper bound for p_X in the boolean

algebra P . Therefore $\forall p_X \Rightarrow p^{**}$. So $\forall p_X = p^{**}$, by the antisymmetry of the partial order $\Rightarrow$. Therefore $e^* = e^{**}$ by (31); e^* is the greatest lower bound for X . (Q.E.D.)

We can now show that step 1 frames are distributive.

Proof: Let p, p', p'' be the propositions with respect to which e, e', e'' are simple (respectively). Then $e \sqcap (e' \sqcup e'')$ is simple with respect to $p \vee (p' \wedge p'')$ and $(e \sqcap e') \sqcup (e \sqcap e'')$ is simple for $(p \vee p') \wedge (p \vee p'')$, by the results of the last two paragraphs. But $p \vee (p' \wedge p'') = (p \vee p') \wedge (p \vee p'')$, by the distributivity of P . Therefore $e \sqcap (e' \sqcup e'') = (e \sqcap e') \sqcup (e \sqcap e'')$, by (31). Similar reasoning will show that $e \sqcup (e' \sqcap e'') = (e \sqcup e') \sqcap (e \sqcup e'')$.

Step 1 frames are also complemented. Proof: Let e be simple for p , 1_F be simple for p_1 , and 0_F be simple for p_0 . Now $p_1 \Rightarrow p_1 \vee \neg p$. Therefore $p_1 \vee \neg p \in P(1_F)$, by (30)ii. So, there exists some event in E_F which is simple for $p_1 \vee \neg p$, by the independence principle (32). Call this event $\neg e$. To show that $\neg e$ is the complement of e , we need to demonstrate that $e \sqcup \neg e = 1_F$, and that $e \sqcap \neg e = 0_F$. By our conjunction result, $e \sqcup \neg e$ is simple for $p \wedge (p_1 \vee \neg p)$. By distributivity, this equals $(p \wedge p_1) \vee (p \wedge \neg p)$. Since $p_1 \Rightarrow p$ and $p \wedge \neg p = 0_F$, this equals p_1 . Since 1_F is simple for p_1 , it follows that $e \sqcup \neg e = 1_F$. Now, by our disjunction result, $e \sqcap \neg e$ is simple for $p \vee p_1 \vee \neg p$. But this equals

1_F . Let e' be any event in E_F and p' be the proposition it is simple for. $1_F \vee p' = 1_F$, so by the disjunction result, $(e \sqcap -e) \sqcap e' = e \sqcap -e$. That is, $e \sqcap -e \sqsubseteq_F e'$ for any $e' \in E_F$, so $e \sqcap -e = 0_F$. (Q.E.D.)

We have now shown that a step 1 frame F forms a complemented distributive lattice, that is, a boolean algebra.

In fact, it is easy to see that, if F is "flipped upside-down", it is isomorphic to a proper ultrafilter in the algebra of propositions, namely the ultrafilter $Q = \{q \in P \mid p \Rightarrow q\}$, where p is the proposition for which 1_F is simple. (I forego a proof.) Again, there seems to be no reason not to actually identify F with Q , and treat propositions as step 1 events. Of course Q forms a boolean algebra: For all $x, y \in Q$, let $x \sqcup_Q y = x \vee y$, let $x \sqcap_Q y = x \wedge y$, and let $-_Q x = p \vee \neg x$. Let $0_Q = p$, and let $1_Q = 1_F$.

6. Unaugmented and Augmented frames. There is quite a bit more that could be said about step 1 frames, but I take it that the essential point, that they form a considerably more restrictive class than the class of semilattices, is now sufficiently clear. Let us therefore turn to the question of how step 1 frames may be adjusted to allow for the analysis of group action suggested in Chapter II.

In order to do this, we have to be able to relate propositions indicating that a particular member or subgroup of some group has a property to propositions indicating that the group itself has that property, or some related property. This will be easiest if we work in terms of English sentences, so let us briefly outline how Fragment 2 could be revised to accomodate the richer notion of a frame introduced in this section.

Fragment 2 made no provision for propositions; sentences simply denoted sets of events, verb phrases denoted functions from events to sets of groups and/or individuals, etc. Yet clearly we want the set of events denoted by a sentence to correspond precisely to the set of events in which the proposition expressed by the sentence holds true.

No actual changes need be made in our compositional rules to accomplish this; we simply revise the meaning postulates governing verbal denotations to assure that the set of events in a given sentence denotation will correspond exactly to the set of events in which some particular proposition holds true. Let Ext be a function mapping any proposition and event frame onto the set of events in that frame in which the proposition holds true, as given in (36):

$$(36) \text{ Ext}(p, F) =_{\text{def}} \{e \in E_F \mid p \in P(e)\}.$$

Now we require:

- (37) For any n -place verb α , there is a proposition $p \in P$, such that for any model M containing an event frame F , and groups or individuals $x_1, \dots, x_n \in U_M$,

$$\{e \in E_F \mid x_1 \in \|\alpha\|(e) \dots (x_n)\} = \text{Ext}(p, F).$$

This general meaning postulate replaces the "persistence" meaning postulates given in the lexical entries of individual verbs in Fragment 2; it assures persistence and more.

Now, let us say that a proposition p is a group-level totaling of two propositions q, q' if q asserts that some group or individual a stands in a relation R to some other group or individual b , q' asserts that some group or individual c stands in R to another group or individual d , and p asserts that the group of a and c stands in R to the group of b and d . Formally, let f be a function from events to functions from groups and/or individuals sets of groups and/or individuals, such that for any model M , the restriction f_M of f to $E_M \times U_M \times \text{POW}(U_M)$ is a possible 2-place predicate denotation. Now, where for all models M containing an event frame F , $\text{Ext}(q, F) = \{e \in E_F \mid a \in f_M(e)(b)\}$ and $\text{Ext}(q', F) = \{e \in E_F \mid c \in f_M(e)(d)\}$:

- (38) p is a group-level totaling of q, q' iff for all models M containing an event frame E_F , $\text{Ext}(p, F) = \{e \in E_F \mid \{a, c\} \in f_M(e)(\{b, d\})\}$.

To derive a step 2 frame F^* from a step 1 frame F , we first set up an event structure isomorphic to that of F .

Assignments of of propositions to events in F are preserved for their isomorphic images in F^* , but we also add to $P(e)$, for all $e \in E_{F^*}$, any group-level totalings of any propositions $q, q' \in P(e)$:

- (39) $F = \langle E_{F^*}, \sqsubseteq_{F^*} \rangle$ is a step 2 frame if F is a step 1 frame and: (i) There exists an isomorphism $i : E_{F^*} \rightarrow E_F$; (ii) for all $e \in E_{F^*}$, $P(e) = \{p \in P \mid p \in P(i(e))\} \cup \{p \in P \mid \exists q, q' \in P(i(e)) [p \text{ is a group-level totaling of } q, q']\}$.

Deriving a step 3 frame from a step 2 frame involves augmenting it with "team credit" events. Let us say that a proposition q is a team credit extension of another proposition p iff p ascribes to some member or subgroup of a group the same property as q ascribes to the group itself. More specifically: Let a be a group or individual and f be a function from events to sets of groups and/or individuals, such that for any model M , the restriction f_M of f to $E_M \times \text{POW}(U_M)$ is a possible 1-place predicate denotation. Now, where for all models M containing an event frame F , $\text{Ext}(p, F) = \{e \in E_F \mid a \in f_M(e)\}$:

- (40) q is a team credit extension of p iff for all models M containing an event frame F , $\text{Ext}(q, F) = \{e \in E_F \mid b \in f_M(e)\}$ and either $a \in b$ or $a \sqsubseteq_U b$.

The actual augmentation process involves adding, immediately "above" the event e which is simple with respect to a given proposition p , an event e^* in which some proposition q , a team credit extension of p , is also true. The new event should inherit all the propositions from the original event e ; $P(e^*)$ should be closed under conjunction and entailment; and all propositions in $P(e^*)$ should persist upward through the lattice. Formally, let us define a relation Aug which two event lattices stand in if one is just like the other except for the addition of an event as just described:

(41) Aug(F , F^*) iff:

- a. $E_{F^*} = E_F \cup \{e^*\}$, for some $e^* \notin E_F$.
- b. For any $e', e'' \in E_F$, if $e' \sqsubseteq_F e''$ then $e' \sqsubseteq_{F^*} e''$.
- c. There exists an $e \in E_F$ such that $e \sqsubseteq_{F^*} e^*$; for any $e' \in E_F$, $e^* \sqsubseteq_{F^*} e'$ iff $e \sqsubseteq_F e'$ and $e \neq e'$; and for any $e' \in E_F$, $e' \sqsubseteq_{F^*} e^*$ iff $e' \sqsubseteq_F e$.
- d. For all $p \in P$, if $p \in P(e)$, then $p \in P(e^*)$; if Simple(e, p) then $q \in P(e^*)$ for some q , a group-level extension of p .
- e. For all $e', e'' \in E_{F^*}$ and all $p, q \in P$, if $p \in P(e')$ and $p \Rightarrow q$ then $q \in P(e')$; if $X \subseteq P(e')$ then $\bigwedge X \in P(e')$; if $p \in P(e')$ and e'

$\sqsubseteq_{F*} e''$ then $p \in P(e'')$; $p \notin P(e)$ except as required here or in d., above.

This definition relates two frames only if one is like the other except for the addition of exactly one event of the appropriate type. However, we obviously want to allow for frames which contain any number of team credit events. We therefore count as a step 3 frame any event lattice which stand in the transitive closure of the Aug relation to a step 2 frame, as well as all the step 2 frames themselves:

(42) $F* = \langle E_{F*}, \sqsubseteq_{F*} \rangle$ is a step 3 frame iff:

- a. $F*$ is a step 2 frame, or
- b. F is a step 2 frame and $\text{Aug}^+(F, F*)$.

CHAPTER IV
QUANTIFIERS, GROUP-LEVEL PROPERTIES AND AGREEMENT

0. Are quantifiers sensitive to event structure? Fragment 2 included two quantificational determiners: *every* and *five*. (It also included *the*, which presupposed uniqueness in the singular, and which therefore was also quantificational in some sense or other.) *Every* mapped the denotation of its CN argument onto the set of sets including that CN denotation as a subset. *Five* mapped a CN denotation onto the set of sets containing a five-membered group from the CN denotation. In either case the result was a set of sets--more or less the standard treatment since Montague, at least if one considers denotation domains purely from the standpoint of what sorts of objects in the model-theoretic ontology they include.

From a more "functional" perspective, however, this analysis was not quite so standard. Quantificational NPs are normally taken as denoting sets of possible VP denotations. Given the event-based treatment of verbal semantics in Fragment 2, however, the relation between possible NP denotations and possible VP denotations was portrayed as somewhat less direct, since NPs did not take

event structure into account. To calculate the truth value of a sentence, it was no longer possible simply to check if the denotation of the predicate was in the denotation of the subject. Instead, the verb phrase provided a possibly distinct set of groups and/or individuals for each event. A sentence was designated as true of a given event (i.e. the event was contained in the sentence denotation) iff the set provided by the verb phrase for that event was in the denotation of the subject NP.¹

This treatment of quantificational noun phrases implicitly made a very strong claim which was not motivated, or even mentioned, during the original presentation and discussion of Fragment 2, namely that quantifiers are insensitive to event structure. A sentence like *Every student ate a pizza* is true in an event e if the set of things that ate a pizza in e includes the set of all students as a subset; the internal structure of e itself does not matter.

This claim appears to be correct for noun phrases introduced by *every*, and for many other quantifiers. As it turns out, however, there is a certain class of quantifiers for which it is not correct.

¹I simplify somewhat here; recall that in Fragment 2 quantificational NPs were always quantified in, and were never handled by the subject-predicate rule itself. Both the subject-predicate rule and the quantifying-in rules involved abstracting a subset of U from a VP or open formula denotation.

1. Different types of group-level events. Before turning explicitly to the interaction of quantifiers and events, let us review the kinds of structures an event of group action can have. Consider for example what it means for a group to earn \$10,000, or what it means for a group to lift every piano. In Chapters II and III we saw that this could mean any of at least four different things: In one case, each of the members of the group earns \$10,000 or lifts every piano. This situation corresponds to the distributive reading of the verb phrase. In another case -- what we might call the "pure" collective case² -- the group enters into some joint enterprise which earns them \$10,000, or goes around to each of the pianos and lifts it as a group. In a third case, involving "additive" properties, some of the members of the group earn an amount possibly less than \$10,000, and the other members do too, but the total amount earned by the group as a whole is at least \$10,000; or, some of the members of the group lift some of the pianos, and other members lift the rest. In the fourth case, involving "team credit", some of the members of the group earn \$10,000 or lift all the pianos, but the group as a whole gets credit for their action because it is seen to function as some sort of team. The collective reading of the verb phrase

²Not to be confused with the "purely collective" predicates of Dowty (1986).

corresponds to any of these last three cases; predicates are vague, rather than ambiguous, among the pure collective, additive, and team credit understandings.

It was argued in Chapter II that the distributive, pure collective, additive and team credit understandings corresponded to distinct event structures. In the distributive case, the event of the group earning \$10,000 has a subevent for each member in which that member earns \$10,000, and the event of the group lifting every piano has a subevent for each member in which that member lifts every piano. In the pure collective case, the (smallest) event of the group earning \$10,000 does not have any subevents in which any members of the group earn money at all, and the smallest event of the group lifting every piano does not have subevents in which any members of the group lift any pianos. In the additive case, the event of the group earning \$10,000 does have subevents in which its members or subgroups earn money, and the event of the group lifting every piano has subevents in which its members or subgroups lift pianos. (None of these members or subgroups is required to earn the whole \$10,000 or lift all the pianos, however.) In the team credit case, the event of the group earning \$10,000 has a subevent in which one of its members or subgroups earns \$10,000, and the event of the group

lifting every piano has a subevent in which some member or subgroup lifts all the pianos.³

2. Event-sensitive quantification. Consider some different types of noun phrases, and whether they are sensitive to the distinctions among distributive, pure collective, additive and team credit events. First consider conjoined terms. Sentence (1) is true in all four kinds of event:

(1) John and Mary made \$10,000.

This sentence is true if John made \$10,000 and Mary made \$10,000 (the distributive case). It is true if John and Mary, working on some joint project, made \$10,000 (the pure collective case). It is also true if John and Mary's combined income was at least \$10,000 (the additive case). Note that this holds true regardless of whether John and Mary work together or are associated with one another in any way; although the sentence does normally suggest some sort of close association between John and Mary, the effect is pragmatic. The argument in (2) seems to me to be undeniably valid:

(2) John made \$5000. Mary made \$5000. Therefore John and Mary made (a total of) \$10,000.

³Note that the team credit event structure may be considered a special subcase of the additive event structure. It does have special properties which set it off from other sorts of additive events, however, as will become clear in the discussion below.

Finally, sentence (1) is true if John made \$10,000 but one views John's income as belonging to the group (the team credit case).

Now consider noun phrases introduced by *the*. Sentence (3) is true in all four types of event:

(3) The inventors made \$10,000.

This sentence is true if each of the inventors made \$10,000; or if the group of inventors, acting together on a single project, made \$10,000; or if the combined total income of the inventors was at least \$10,000; or if one of the inventors made \$10,000 for the whole team.

Similar results hold for noun phrases introduced by cardinal numbers. For me at least, these produce a slight preference for the distributive reading, but the other three understandings are very clearly available as well.

(4) Six inventors made \$10,000.

Sentence (4) is true if each member of a group of six inventors earned \$10,000, or if the group as a whole earned this amount on some joint project, or if their combined income was at least \$10,000, or if one of the members of the group earned this amount for the team as a whole.

Similar results hold for unstressed *some*⁴, which I will write here as *sm*:

⁴Numerous syntactic and semantic differences have been pointed out between stressed and unstressed *some*. See Postal (1966), Perlmutter (1970), Stockwell et al. (1973), Carlson (1977), Wald (1977), Lasnik (1987).

(5) *Sm* inventors made \$10,000.

The difference between (3) and (5) is simply one of definiteness; otherwise the truth conditions are the same. Sentence (5) is also much like (4), differing only in that the group is not required to have six members. The additive understanding is perhaps somewhat harder to get here than in previous examples -- my intuitions about the argument in (6) are not as strong as for the argument in (2):

(6) John and Mary are inventors. John made \$5000.
 Mary made \$5000. Therefore *sm* inventors made (a
 total of) \$10,000.

However, the argument in (7) seems more clearly correct, and if we accept both this argument and that in (2), it follows that an additive understanding must be available for sentences whose subjects are introduced by *sm*:

(7) John and Mary made \$10,000. John and Mary are
 inventors. Therefore *sm* inventors made \$10,000.

A pragmatic explanation for the difficulty of obtaining an additive interpretation for (5) will be offered below.

Now consider sentences whose subjects are introduced by the determiner *no*, such as (8):

(8) No inventors made \$10,000.

Here again the most prominent understanding may at first seem to be the distributive one; the sentence is falsified by the existence of an individual inventor who made at least

\$10,000.⁵ Under certain circumstances a group may falsify the sentence as well, however: Suppose that John invented the electric doorknob, Mary invented the pocket spud-bar, and Bill and Sue together invented the velcro tennis racket. Now if Bill and Sue sell their patent for \$10,000, sentence (8) seems to be false. Thus, an evaluation of the sentence requires us to check for pure collective events. Team credit events will reduce to one of these two cases, since some member or subgroup of the group in question will have made \$10,000 on its own, and this will be enough already to falsify the sentence.

Note, however, that if John sold his patent for \$5000 and Mary also sold hers for \$5000, this is not enough to falsify the sentence, despite the fact that John and Mary are inventors and *John and Mary made \$10,000* is true. Additive event structures are irrelevant to the truth of (8), or any sentence whose subject is introduced by *no*. This is true even for sentences containing expressions which normally bring out additive understandings, such as *a total of*:

(9) No inventors made a total of \$10,000.

(Cf. *The inventors made a total of \$10,000.*)

⁵Since we have been allowing non-distributive predicate denotations to contain individuals as well as groups, there is actually no reason to believe that falsification of the sentence by an individual shows that the distributive reading is necessarily involved.

Now consider noun phrases introduced by *only*. Sentence (10) requires two things: that John earned \$10,000, and that no one else did.

(10) Only John made \$10,000.

What does it mean that no one else earned \$10,000? Consider again the situation with the inventors, sketched above. If it is clear in context that the universe of discourse include the inventors of various products, and Bill and Sue earned \$10,000 on their invention of the velcro tennis racket, then (10) is falsified, just as (8) was. It is also falsified if some individual earned \$10,000, and as before, the team credit situation reduces to one of these two cases.

But again, an additive situation is not enough to falsify the sentence. Suppose that Mary got \$5000 for the pocket spud bar and Bill and Sue got \$5000 for the velcro tennis racket. This does not falsify (10), even though it is true that Mary and Bill and Sue together made \$10,000.

3. Additivity, inclusion and exclusion. Checking to see if a sentence is true or not can involve either (or both) of two different sorts of operation. One either must check to see if some particular group or individual, or quantity of groups or individuals, has the property expressed by the verb phrase, or else one must check to see if some group or individual, or quantity of groups or individuals, does not have the property expressed by the verb phrase. For

example, to see if *Five inventors made \$10,000*, one checks to see if at least one group of five inventors made \$10,000. To see if *No inventors made \$10,000* is true, one checks to make sure that there are not any groups of inventors that made \$10,000, except those that made this amount only by virtue of their individual incomes totaling at least this much.

Let us refer to noun phrases like *five inventors*, which require checking that something does have a property, as "inclusive" noun phrases, and to noun phrases like *no inventors*, which require checking to see that things do not have a property, as "exclusive" noun phrases. Note that some NPs, e.g. those introduced by *only*, are both inclusive and exclusive.

The two classes of exclusive noun phrase we have examined so far, those introduced by *no* and those introduced by *only*, both systematically discount additive events. If a group has a property only by virtue of some totaling function on the properties of its members, then that group will not be enough to falsify a sentence whose subject is introduced by *no* or *only*. There is a good functional/pragmatic reason why this should be so: Given any property for which an appropriate totaling function can be defined, it will almost always be possible to find a group which has that property; just keep adding more and more members. For example unless one sets an astronomically

high value for X , one can always find a group of inventors whose combined income is at least $\$X$, simply by adding more and more inventors to the group until the appropriate value is reached. If additive event structures were taken into account, a sentence like *No inventors made $\$10,000$* would be almost trivially false.

A similar explanation can be offered why sentences with *sm* resist an additive understanding. By adding in more and more inventors, it is possible to find a group of inventors which makes *Sm inventors made $\$X$* true for any reasonable value for X . If one takes additive event structures into consideration, the sentence becomes almost trivially true.

It was argued above that the difficulty of obtaining an additive understanding with *sm* is pragmatic, rather than semantic. One might ask, therefore, whether similar considerations hold for *no* and *only*. In the case of *no*, it seems to me that it is simply impossible to get an additive reading, and that therefore the explanation should be semantic. Exactly how to set up the semantics to derive the impossibility of an additive reading is a more difficult question, however.

One possibility that may come to mind is to redefine the semantic type of noun phrases so that they denote sets of possible VP denotations -- or, to keep things more in keeping with the idea of relativizing truth to events as

adopted above, we might redefine the type of NPs so that they denote functions from events to sets of possible VP denotations. This event argument would then function as the event relative to which the truth of the sentence as a whole is to be evaluated.

For example, on this approach *no inventors* would map any event e onto a set containing *made \$10,000* iff no inventors made \$10,000 in e . Because *made \$10,000* itself takes an event argument, perhaps there is some way to make the denotation of *no inventors* sensitive to the structure of this argument, and in particular to allow *made \$10,000* to be an element of *no inventors*(e) even if *made \$10,000*(e) does contain groups of inventors, provided that the subevents of e are such things as the event of one inventor making \$5,000 and the event of another inventor also making \$5,000.

Unfortunately, as sketched here, this approach appears to require non-compositional rules. The denotation of *made \$10,000* is a function from events to sets of groups and individuals -- for each e the set of groups and individuals that made \$10,000 in e . To see if any such group is in the set only by virtue of adding together the incomes of its members, we have to know what the income of these members is in the subevents of e . That is, for any such member, we have to know for what values of x that member is in *made*(e')(x), where $e' \sqsubseteq_e e$. Now at the level where we

combine *no inventors* with *made \$10,000*, *made* has already combined with *\$10,000*, so that *made* is no longer accessible without violating compositionality.

The fact that *only*, which is also exclusive, also systematically discounts additive event structures suggests a way this general approach might be made more compositional. Rather than letting VP denotations map events onto sets of groups and individuals, suppose we let them map events onto pairs of such sets. The first set in the pair will be terms the inclusion set, and the second set will be termed the exclusion set.⁶

The intuitive idea behind the pairing is this: when an inclusive noun phrase "checks" whether a group or individual has the property denoted by the verb phrase, it checks whether that group or individual is contained in the inclusion set provided by the VP (for a given event argument). When an exclusive noun phrase checks that a group or individual does not have the property in question, it checks whether that group or individual is in the exclusion set provided by the VP. For example if the VP in question is *made \$10,000*, the inclusion set will contain all those groups and individuals that made \$10,000, and the

⁶As will be seen, the inclusion and exclusion sets are similar in function to "positive" and "negative" denotations, as suggested, e.g., by Cooper (1983). I prefer the present terminology so that we can reserve the term denotation for the larger function of which the inclusion and exclusion sets serve as values at a particular argument.

exclusion set will contain all those groups and individuals that do not count as having made \$10,000 for the purposes of exclusive quantification. Certain groups may be in both sets; for example if John and Mary made \$5,000 each (in some event e), then the group of John and Mary will be in the inclusion set of $\llbracket \text{made } \$10,000 \rrbracket(e)$, but will be in its exclusion set too.

By extending the pairing of inclusive and exclusive denotations down to the level of transitive verbs, we can give a compositional semantics which allows for the simultaneous truth of *John and Mary made \$10,000* and *No inventors made \$10,000*, even if John and Mary are inventors. Let transitive verbs denote functions mapping events onto pairs of functions from groups and individuals to sets of groups and individuals. The first function gives inclusive values; for instance if the verb is *make* (in its monetary sense) and Bill makes \$10,000 in e , then the first element of $\llbracket \text{make} \rrbracket(e)$ should map some group of 10,000 dollars⁷ onto a set containing Bill.

The second function in the pair provides exclusive values. Obviously, it should be related to the inclusion function in a systematic way. The easy part is given in (11); anything which is not in the set provided by the

⁷Dollars as abstract units of monetary value, of course, not individual dollar bills. I treat sums of money as groups for the sake of convenience; it may ultimately be more advisable to treat them as single objects for which units of currency provide measure functions.

inclusion function for a given value should be in the set provided by the exclusion function. (Subscripts indicate ordering in the pair, so that $\|\alpha\|(e) = \langle \|\alpha\|(e)_1, \|\alpha\|(e)_2 \rangle$.)

- (11) For all $\alpha \in TV$, $e \in E$, $x, y \in U$, if $x \notin \|\alpha\|(e)_1(y)$ then $x \in \|\alpha\|(e)_2(y)$.

For instance if $\text{John} \notin \|\text{see}\|(e)_1(\text{Mary})$, i.e. if John doesn't see Mary in e , then John should be in the exclusion set for the VP *see Mary* (relative to e).

But we want to allow groups which are in the inclusion set of a VP to be in the exclusion set too, if they are in the inclusion set because of an additive event structure:

- (12) For all $\alpha \in TV$, $e \in E$, $x, y \in U$, if $x \in \|\alpha\|(e)_1(y)$ and if for all $e' \sqsubseteq_E e$ such that $x \in \|\alpha\|(e')_1(y)$ there exists some w , a member or proper subgroup of x , and some z , a member or proper subgroup of y , such that $w \in \|\alpha\|(e')_1(z)$, then $x \in \|\alpha\|(e)_2(y)$.

For instance suppose that John and Mary made \$5,000 each in e . Then the group made \$10,000 in e . But (assuming that the group did not otherwise make any money) for any subevent e' of e in which the group makes \$10,000, John made \$5,000 in e' , and so did Mary. Therefore (12) lets the group of John and Mary into the exclusion set of *make* for the event e and sum of money in question. Note that if the group of John and Mary earned \$10,000 in e through pure collective action in addition to the \$10,000 it earned though the

summing of their separate incomes of \$5,000, then the group will not be a member of this exclusion set.

We need to revise the verb-object rule to handle paired denotations. This may be done in the obvious way:⁸

- (13) If $\alpha \in TV$ and $\beta \in T$, then $\gamma = \langle \alpha, \beta \rangle \in VP$; for all $e \in E$, $\|\gamma\|(e) = \langle \|\alpha\|(e)_1(\|\beta\|), \|\alpha\|(e)_2(\|\beta\|) \rangle$.

Finally, we need to give the semantics for quantifiers in such a way that they may be sensitive to the separation of inclusion and exclusion sets in the VP denotations. As above, we take noun phrases to denote functions from events to sets of possible VP denotations. Inclusive subject noun phrases will check, at each e , the inclusion set of their VP; exclusive NPs will check exclusion sets. For example, we may define *five* and *no* as follows:

- (14)a. For all $e \in E$, $X \in POW(U)$ and possible VP denotations f , $f \in five(X)(e)$ iff there exists an $x \in f(e)_1 \cap X$ such that $CARD(\{y \in U \mid y \in x\}) = 5$.

- b. For all $e \in E$, $X \in POW(U)$ and possible VP denotations f , $f \in no(X)(e)$ iff $X \subseteq f(e)_2$.

A sentence is true of an event e if the denotation of its predicate is in the denotation of its subject applied to e . (I assume here for simplicity that subjects combine directly

⁸Because adverbial modification will not be of special importance in this chapter, I combine the verb and its object by simple sequencing rather than wrapping as in Chapter II.

with their VPs, rather than through quantifying in. Note though that the quantifying in rule, if formalized, would have to mimic the pairing of inclusion and exclusion sets posited for VPs.)

Note that ordinary sentence negation should not be viewed as a VP operator switching the inclusion and exclusion sets of a predicate. Suppose that John made \$5,000 and Mary made \$6,000 in *e* (and otherwise the group did not make money). Then the group of John and Mary is in the exclusion set of *made \$10,000* at *e*, and hence, under the theory that negation performs this sort of switch, would also be in the inclusion set of *didn't make \$10,000*. But *John and Mary did not make \$10,000* is false in the situation described. A more adequate account of negation would appeal to the propositional complement operation introduced in Chapter III, but will not be worked out in detail here.

The splitting of VP denotations into inclusion and exclusion sets as just suggested only makes sense if it really holds true that exclusive noun phrases consistently discount additive event structures. This claim is a little hard to judge, because apart from noun phrases introduced by *no* and *only* there are few if any clear examples of exclusive NPs, and many of the plausible candidates seem to ignore groups altogether. For instance it is not really clear that *few*, despite its many similarities with *no*, is exclusive.

To see if *Few inventors made \$10,000* is true, does one check how many inventors did make \$10,000, or how many did not? I will eventually suggest that *few* is exclusive, but in either case the question of additive event structures just does not come up, because the sentence seems to be about the incomes of individual inventors, and not about groups of inventors at all. Likewise *No inventor made \$10,000* (in contrast to our previous example *No inventors made \$10,000*) is falsified only by an individual who made \$10,000 and not by groups.

One possible set of counterexamples may come from sentences with *only*. We already saw above that these ordinarily do discount additive event structures, but there is a possibility that this effect is pragmatic rather than semantic. Consider the budget of a small city. The payroll for the police department totals \$1,000,000, the payroll for the fire department also totals \$1,000,000, and the payroll for the sanitation department totals \$500,000. In this situation, sentence (15) seems false:

(15) Only the police officers get paid \$1,000,000.

The sentence is false because the firefighters also get paid \$1,000,000. Since it is the combined income the firefighters that is in question, this appears to be a case where *only* excludes a group from having a property even if it has that property only by virtue of a totaling operation on the properties of the group's members.

Compare another example, however. Suppose now that the fire department payroll is only \$500,000. In this case, (15) is true -- despite the fact that the combined income of the firefighters and the sanitation workers is \$1,000,000. What is different about this case? It seems clear that the reason why (15) was false in the original situation but true in this one is that in the original situation there were two distinct payrolls which (each) totaled \$1,000,000, while in the new situation there is only one. To calculate the truth value of the sentence, one compares the lump sums allocated to single entries in the overall budget.

Because we are really concerned here with the allocation of lump sums to individual budget entries, rather than free totaling of the incomes of individual city employees, perhaps this sort of example is best seen as not involving an additive event structure after all. If not, then the semantic division of VP denotations into inclusion and exclusion sets seems warranted.

While it is far from firmly established at this point that such the division of VP denotations into inclusion and exclusion sets is necessary -- the facts about *only* could probably also be explained in a theory without the division, but with a strong pragmatic mechanism to restrict the availability of additive understandings -- I take it that the division is at least plausible and worth exploring. As

it turns out, dividing VP denotations into inclusion and exclusion sets leads to a number of interesting (though tentative) results: among other things it appears to make possible the solution of certain problems in the semantics of subject-verb agreement. The remainder of the chapter will be concerned with this topic.

4. An argument-restriction account of agreement. The agreement of subject and verb for number in English has often been claimed to be (at least partly) semantically conditioned. Particularly strong claims to this effect have been made in informal feature-based semantic theories (see, e.g., Juul (1975), Reid (1984)); however, the lack of precision with which such theories are generally formulated makes such claims hard to evaluate in any rigorous way. Much more successful in this respect, and therefore also better known and more influential, has been the model-theoretic analysis proposed in Hoeksema (1983).

Hoeksema's analysis is intended primarily to account for the conditions under which conjoined singular noun phrases in subject position impose plural agreement on their verb phrase, and the conditions under which they impose singular agreement. Consider data like the following:

(16)a. John and Mary are/*is paving their driveway.

b. A student and a professor were/*was talking in the hallway.

- c. This plate and that glass need/*needs to be washed.
- d. Every student and every professor keeps/*keep a spare piece of chalk nearby.
- e. No piano and no tuba is/*are beyond my ability to lift.
- f. Many a day and many a night has/*have passed.

Assuming a generalized quantifier treatment of noun phrases, Hoeksema proposes the following as a descriptive generalization to cover this sort of data: Conjoined atomic quantifiers impose plural agreement, but conjoined non-atomic quantifiers impose singular agreement. A generalized quantifier $Q \in \{0, 1\}^{\text{Pow}(U)}$ is atomic iff its minimal elements (i.e. those sets X such that $Q(X) = 1$ with no proper subsets Y such that $Q(Y) = 1$) are singleton in every model.

The generalized quantifier denoted by a proper name will be atomic; the minimal element of, e.g., *John* will always be $\{\text{John}\}$, which is singleton. Definite descriptions and demonstrative NPs likewise denote atomic quantifiers; if there is a unique professor in U , *the professor* will have the singleton set containing this professor as its sole minimal element, and otherwise will have no minimal elements at all, in which case the quantifier will be trivially atomic. Indefinite descriptions will also denote atomic

quantifiers; the minimal elements of *a student* will be the singleton sets containing each of the individual students.

In contrast, noun phrases introduced by *every* are not atomic; the minimal element of, e.g., *every student* will be the set containing all the individual students, which will not be singleton in models containing more than one student. *No piano* also denotes a non-atomic quantifier; its minimal element will always be the empty set, which is not singleton.

Readers familiar with the Irene Heim's (1982) dissertation or extensions of Kamp's (1981) Discourse Representation Theory will notice that the noun phrases which denote atomic quantifiers in a generalized quantifier analysis of noun phrase semantics are precisely those which can be treated as being constants or variables of type *e* (using now the conventional Montagovian type indices, not those of Fragment 2), and in fact Hoeksema himself suggests that the generalization should be stated in this way in his more recent (1987) paper. The distinction between atomic and non-atomic quantifiers thus corresponds essentially to the distinction between the syntactic categories *T* and *Q* as used in Fragment 2, and there seems little doubt that Hoeksema's generalization could be captured through the use of a syntactic rule which was sensitive to this category distinction.

To adopt such a rule would miss the point of Hoeksema's analysis entirely, however, and would do little to explain why number agreement shows the pattern it does. If we ignore the semantic differences between T and Q, there seems little reason to expect that conjoined singular Ts should impose plural agreement and conjoined singular Qs should impose singular agreement rather than vice versa, or any other imaginable pattern. Indeed, since the motivation for distinguishing T from Q is largely (if not entirely) semantic, and not syntactic, it is questionable whether we should purely syntactic rules to be sensitive to the distinction at all.

A much more illuminating approach is to note some semantic differences between T-conjunction and Q-conjunction. The conjunction of singular Ts denotes the group whose members are the individuals denoted by the conjuncts; *John and Mary are paving their driveway* predicates something of the group of John and Mary. Of course the situation is complicated somewhat by the occurrence of distributive predicates, but the advantages of a group-level treatment of distributives are already well documented. In contrast, the conjunction of singular Qs denotes the intersection of the denotations of the conjuncts. Thus *Every student and every professor keeps a spare piece of chalk nearby* means that the set of things

which keep a spare piece of chalk nearby is among the sets containing all the students, and all the professors.

Given the fact that singular Ts denote individuals, for which the set intersection operation is not defined, but which are in the domain of the group formation operation, it is hardly surprising that T-conjunction should give correspond semantically to group formation. If we assume that plural agreement is an indicator of group-level predication, it follows that conjoined singular Ts should impose plural agreement.

Conversely, if two Qs quantify over individuals, so will their intersection. No group-level predication is introduced, so intersective conjunction of singular Qs should impose singular agreement.

There is much left vague in this account, most importantly the formal mechanisms which would actually assure the appearance of plural or singular morphology on predicates. Hoeksema's solution⁹ is to impose systematic type-theoretic differences between singular and plural verb phrases and noun phrases. The group-forming conjunction defined for atomic quantifiers does not preserve types, but

⁹I will follow the 1983 version of Hoeksema's analysis except when indicated otherwise. Although Hoeksema (1987) does suggest a number of revisions, he does not make clear how to implement all these revisions formally. In some cases it is not at all evident that the suggested revisions will produce the desired effects without substantial (but unspecified) modifications elsewhere in the grammar.

maps singular-typed quantifiers onto plural-typed ones. The intersective conjunction defined for non-atomic quantifiers does preserve types, however, mapping singular-typed quantifiers onto other singulars, and plurals onto other plurals.

More specifically, singular verb phrases are held to denote in $\text{POW}(U_I)$. That is, they denote sets of individuals only, and have no groups in their denotations at all. Plural verb phrases, however, denote in $(\text{POW}(U) - \text{POW}(U_I)) \cup \{\emptyset\}$. They denote sets which do not contain just individuals -- either there has to be some group or other in the VP denotation, or else the VP denotation is empty. Note that plural VP denotations may contain individuals; in fact Hoeksema requires that a plural VP denotation contain all the individuals from the corresponding singular. What is ruled out are plural VP denotations which contain individuals but no groups.

Singular noun phrases denote functions from $\text{POW}(\text{POW}(U_I))$ into $\{0, 1\}$; i.e. they denote (characteristic functions of) sets of possible singular verb phrase denotations. Plural noun phrases denote functions from $\text{POW}((\text{POW}(U) - \text{POW}(U_I)) \cup \{\emptyset\})$ into $\{0, 1\}$; they denote characteristic functions of sets of possible plural VP denotations.

A sentence is true if the denotation of its subject maps the denotation of its predicate onto 1, false if it

maps this denotation onto 0. Since the set of possible singular VP denotations is disjoint from the set of possible plural VP denotations, no plural VP denotation will ever be in the domain of a singular NP denotation, and no singular VP denotation will be in the domain of a plural NP denotation. Thus, sentences in which the subject and predicate are of unmatched number will not receive a well-defined semantics.

The conjunction of two non-atomic quantifiers Q , Q' gives the characteristic function of the intersection of the sets characterized by Q and Q' . Formally, non-atomic conjunction is taken as the standard boolean meet operation. Non-atomic quantifiers are functions from either the boolean algebra $\text{POW}(U_I)$ or the boolean algebra $\text{POW}((\text{POW}(U) - \text{POW}(U_I)) \cup \{\emptyset\})$ into the minimal boolean algebra $2 (= \{0, 1\})$. Let B be either of these former two boolean algebras. Then for any set $X \in B$, $[Q \wedge Q'](X) = Q(X) \wedge_2 Q'(X)$. Thus, for example, *every professor and every student* will denote that function which, for any set $X \in \text{POW}(U_I)$, maps X onto 1 if X contains all the professors and all the students, maps X onto 0 if X does not contain all the professors or all the students, and is undefined otherwise. Note that this function, like the functions denoted by the conjunct noun phrases, is defined only for elements of $\text{POW}(U_I)$ and so will sensibly combine (as subject) only with singular predicates.

The conjunction of atomic quantifiers Q , Q' forms groups on the basis of the minimal elements of the two quantifiers, yielding a plural quantifier. Specifically, let $[Q \ \& \ Q']$ be that function from $(\text{POW}(U) - \text{POW}(U_I)) \cup \{\emptyset\}$ into 2 such that $[Q \ \& \ Q'](X) = 1$ iff for some m , m' , minimal elements of Q , Q' respectively, $m \cup m' \in X$. Recall here that Hoeksema models groups as sets, so the union of (distinct) minimal elements of Q and Q' will be a group. For example, since the minimal element of $\llcorner John \llcorner$ is $\{John\}$, and the minimal elements of $\llcorner a \ student \llcorner$ are the singleton sets containing the individual students, the union of the minimal element of $\llcorner John \llcorner$ and any minimal element of $\llcorner a \ student \llcorner$ will be group containing John and some student or other. Thus $\llcorner John \ and \ a \ student \llcorner$ will map any X onto 1 of X contains a group containing John and a student. Any such X will contain at least one group, and therefore is a possible plural VP denotation but not a possible singular VP denotation; *John and a student* will sensibly serve as subject only of plural predicates.

5. Some problems. The basic strategy of the analysis of number agreement just sketched is to restrict the range of possible arguments a subject noun phrase can take. The function denoted by a singular noun phrase is defined only for those sets which can serve as denotations of singular verb phrases, and the functions denoted by plural noun

phrases are defined only for those sets which can serve as denotations for plural verb phrases. Syntactically, nothing prevents a subject and predicate of unmatched number from combining; but semantically, such a combination does not receive a coherent interpretation, since the denotation of the verb phrase will be outside the domain of the subject denotation.

A number of objections can be (and have been) made to this analysis, of varying degrees of legitimacy and force. These objections can be separated into two categories: those that purport to argue against semantically conditioned agreement generally, and those that do not address this general issue, but only the specific formal semantic mechanisms Hoeksema suggests for producing agreement. In this section I want to present some arguments from this latter category. (More general arguments will be discussed briefly at the end of the chapter.) In the next section I will suggest an alternative analysis which, like Hoeksema's, portrays number agreement as driven essentially by the semantics, and which, like Hoeksema's, uses the device of argument restrictions to produce the desired effects. By dividing VP denotations into inclusion and exclusion sets, however, this new analysis will assign correct truth conditions in a number of cases where Hoeksema's analysis gives incorrect or inconsistent results.

One argument against a Hoeksema-style agreement system is offered in Roberts (1987a, p. 171), where it is attributed to Barbara Partee. A singular verb phrase can serve quite easily and naturally as the antecedent for a plural VP anaphor, and vice versa:

(17)a. John bought a house, and Bill and Mary did too.

b. Bill and Mary bought a house, and John did too.

Since Hoeksema assumes a type-theoretic difference between singular and plural predicates, this sort of anaphora is unexpected.

This argument only has force if it can be established that the antecedent and anaphor have to be of the same type, of course. Note though that the binding of an anaphor by an antecedent of a different type is not a formal impossibility, by any means; as Partee and Rooth (1983, p. 367) point out, generalized quantifiers (Montagovian type $\langle\langle e, t \rangle, t \rangle$) can be made to bind variables of type e with no change to Montague's quantifying-in rule at all. While it is not entirely clear that this possibility has direct application to the VP-anaphora facts, it does remove some of the force of the argument. Even if antecedents and anaphors must be of the same type, however, it does not necessarily follow that singular and plural VPs cannot stand in an antecedent/anaphor relation to one another, even in a system very much like Hoeksema's. If we assume a cumulative type system, where each type includes all types lower than it on

the hierarchy, there is nothing to prevent a set of individuals from belong to at least some of the same types as a set of sets, hence for a singular VP denotation to belong to at least some of the same types as a plural one.

Another, more substantial, criticism of Hoeksema's agreement system was made by van Eijk (1983). Since Hoeksema disallows plural VP denotations which only have individuals as members, in a world where no one except just a single man walks the denotation of the plural VP *walk* (according to van Eijk) comes out empty. Since the empty set will always be mapped onto 1 by monotone decreasing quantifiers, a sentence like *No men walk* comes out true. But obviously, this is not an accurate characterization of the sentence's truth conditions.

Van Eijk is actually incorrect in supposing that the denotation of *walk* comes out empty in the situation described. Hoeksema (p. 70) sets up a "linkage requirement" stipulating that a plural predicate denotation must contain all the individuals from the denotation of the predicate's singular counterpart. Thus the plural VP *walk* is required to contain the single walking man. This is no real defense for Hoeksema, however, since, as van Eijk rightly points out, the denotation domain Hoeksema defines for plural VPs excludes sets of individuals, thus prohibiting the denotation of *walk* from containing the single walking man

unless it also contains at least one group. The fact is, the domain specifications and the linkage condition impose contradictory requirements on the denotation of *walk*. The system as a whole is simply inconsistent.

Another argument, in some respects converse to van Eijk's, can be given by adapting one of Hoeksema's own arguments about the domain of plural VPs. Hoeksema motivates the inclusion of individuals in plural predicate denotations as follows: Suppose that plural predicate denotations contain only groups, and no individuals, so that, e.g., *were dancing* denotes the set of all groups that were dancing. Now, how do we confirm or disconfirm a sentence like *Only girls were dancing*? According to Hoeksema, examination of the VP denotation will not be sufficient, since one individual boy or other non-girl might have been dancing. If, on the other hand, the denotation of *were dancing* contains all the dancing entities, whether groups or individuals, the presence of this one individual non-girl in the set will be enough to determine that the denotation of the subject *only girls* must map this set onto 0.

Hoeksema's choice of examples here is somewhat unfortunate: since *were dancing* is a distributive predicate, if some non-girl was dancing, then (provided at least one other individual was dancing) this non-girl will be

contained in at least one group of dancing individuals. Since this group contains a non-girl, it will not be in the denotation of *girls*, and hence will suffice to disconfirm the sentence. If we substitute a non-distributive predicate, say, *built a table*, for *were dancing*, however, the argument seems quite forceful.

The problem is that this argument can be used not only to motivate the inclusion of individuals in plural predicate denotations, but also to motivate the inclusion of groups in singular predicate denotations. Suppose that John built one table, and that Mary and Bill built another one. Then, in Hoeksema's system, the denotation of the singular predicate *has built a table* is just {John}, the singleton set containing John. But now how do we confirm or disconfirm the sentence *Only John has built a table*? Presumably this sentence is false in the situation described, since Mary and Bill also built a table. But the denotation of *has built a table* comes out the same in this situation as in a situation where no one but John built a table, in which case the sentence is true. The denotation of the predicate *has built a table* simply does not provide enough information to determine whether or not it is true that only John built a table.

If, however, we let *has built a table* denote the set containing not just the individuals that have built a table, but also the groups, then we can let *only John* denote a

function mapping the singleton set of John onto 1 and all other sets onto 0. If John built a table and Mary and Bill also built one then *has built a table* will be {John, {Mary, Bill}}, and we can determine on this basis that *Only John has built a table* is false.

Van Bijk's argument seems to show that we must allow for plural predicate denotations that only contain individuals, i.e. for plural predicates of the type which Hoeksema reserves for singulars. The argument from *only*, just outlined, seems to show that we must allow for singular predicate denotations that contain groups, i.e. for singular predicates of the type which Hoeksema reserves for plurals.

The problems raised by these arguments would all be solved if we abandoned the idea of a systematic type difference between singular and plural predicates, and let both singulars and plurals denote sets containing either individuals or groups or both. But if singular and plural predicates have the same denotation domain, rather than disjoint domains as Hoeksema suggests, an argument-restriction account of number agreement cannot work.

6. Agreement and the inclusion/exclusion distinction. It seems a high price to pay, to give up the idea that agreement -- especially the agreement imposed by conjoined subjects -- is semantically conditioned. It is interesting

and useful to note that the problematic examples in this respect, those involving *no* and *only*, coincide neatly with those which motivated the division of VP denotations into inclusion and exclusion sets; more specifically it is exclusive group-level quantification which appears to pose a problem. This suggests that by letting VPs denote functions which provide both an inclusion and an exclusion set, perhaps we could find a way to assign singular and plural predicates disjoint denotation domains, without producing the incorrect semantic results associated with Hoeksema's system.

Suppose that marking a predicate with singular morphology restricts its inclusion set to individuals, and marking it with plural morphology restricts its inclusion set to groups. In either case the exclusion set is unaffected. For example, the denotation of the uninflected VP *build a table* maps any event *e* onto the pair whose first element is the set of all groups and individual that build a table in *e* and whose second element is the set of groups and individuals that do not build a table in *e*; the denotation of the singular form *builds a table* maps *e* onto the pair whose first element is the set of individuals that build a table in *e* and whose second element is the set of groups and individuals that do not build a table in *e*; and the denotation of the plural form *build a table* maps *e* onto the pair whose first element is the set of all groups that build

a table in e and whose second element is the set of all groups and individuals which do not build a table in e .

In this sort of system the denotations of singular and plural predicates are drawn from disjoint domains, and a semantic account of agreement becomes possible after all. We let singular noun phrases denote functions which map each e onto (the characteristic function with domain $D_{VP[-p]}$ of) a set of possible singular VP denotations, and we let plural noun phrases denote functions which map each e onto (the characteristic function with domain $D_{VP[+p]}$ of) a set of possible plural VP denotations. A sentence α is true of an event e if the function provided by its subject for argument e , when applied to the denotation of its predicate, yields the value 1; it is false of e if it yields the value 0. Note that in either case the subject and predicate must match in number; if not, the predicate denotation will be outside the domain of the subject function and no truth value will be assigned.

Consider a couple of examples. We let $\llbracket \text{every inventor} \rrbracket$ map each e onto that function which maps a possible singular VP denotation f onto 1 if $\llbracket \text{inventor} \rrbracket \subseteq f(e)_1$, which maps all other singular VP denotations onto 0, and which is undefined for plural VP denotations. We let $\llbracket \text{sm inventors} \rrbracket$ map each e onto that function which maps a possible plural VP denotation f onto 1 if $\llbracket \text{inventors} \rrbracket \cap f(e)_1 \neq \emptyset$, which maps

all other plural VP denotations onto 0, and which is undefined for singular VP denotations.

Now consider the examples which were problematic under Hoeksema's analysis. Let $\llbracket no\ men \rrbracket$ map each e onto that function which maps a possible plural VP denotation f onto 1 if $\llbracket men \rrbracket \subseteq f(e)_2$, which maps all other plural VP denotations onto 0, and which is undefined for singular VP denotations. Now suppose that no one walks but just a single man m . Hoeksema's analysis is inconsistent on what the VP denotation should be in this case. In the current analysis, we let $\llbracket walk \rrbracket(e)_1 = \emptyset$, since no groups walk in e , but let $\llbracket walk \rrbracket(e)_2 = U - \{m\}$, since everything but m does not walk in e . Since m is a man and $\llbracket men \rrbracket$ is the set of all men and groups of men, it follows that $\llbracket men \rrbracket \not\subseteq \llbracket walk \rrbracket(e)_2$. Therefore the sentence is false, which is the correct result.

The problems with *only* are also eliminated. We let $\llbracket only\ John \rrbracket$ map each e onto that function which maps a possible singular VP denotation f onto 1 if $John \in f(e)_1$ and $f(e)_2 = U - \{John\}$, which maps all other singular VP denotations onto 0, and which is undefined for plural VP denotations. Now suppose that John built a table, and that Mary and Bill also built one (collectively). Then it will not be the case that $\llbracket has\ built\ a\ table \rrbracket(e)_2 = U - \{John\}$, since the group of Mary and Bill will fail to be in $\llbracket has$

built a table $\| (e)_2$. The sentence is therefore false, which is the correct result.

7. Distributive determiners and plural agreement. Certain determiners, including *many*, *most*, *several*, *few*, stressed *some*, and a number of others, produce a strong dispreference for collective readings. Thus *Many inventors made \$10,000* means that each of many individual inventors made \$10,000, not that some group of many inventors collectively made \$10,000. Likewise *Many inventors are numerous* has only a non-sensical reading meaning that each of a large number of inventors is numerous.

In previous work (Lasnik 1986, 1987) I suggested that such determiners are insensitive to the groups in the denotations of their noun and VP argument denotations, despite requiring these arguments to be plural. For example (assuming Hoeksema-style denotation domains), *many* was assigned the following semantics:

- (18) For all $X \in D_{N[+p]}$, $\| \text{many} \| (X) = \{ Y \in D_{VP[+p]} \mid \text{CARD}(X \cap Y \cap U_I) \text{ is large} \}$.

(It is assumed that the pragmatics will determine just how great a set's cardinality has to be in order to count as "large" in context.) Thus *Many inventors built a table* was

predicted to be true iff the set of individuals in both *||inventors||* and *||built a table||* was sufficiently large.¹⁰

In the analysis proposed here, however, the inclusion sets associated with a plural VP are prohibited from containing any individuals, so this sort of solution will not work. What I would like to suggest instead is that determiners like *many* check the inclusion sets of their VP arguments not for individuals, but for groups whose membership consists of some other group, plus all of that group's members. Such groups systematically occur in the denotations of distributive predicates.

To see how this proposal works, consider what distributive predicates must be like in the new system for verbal semantics proposed above. I would like to suggest that the operation which derives optional distributive readings for predicates from their non-distributive counterparts may apply only to VPs which are unmarked for

¹⁰Examples like *Many students gathered in the hall*, if acceptable, are problematic for such an account. I find this sort of sentence less than perfectly natural, but they are certainly far better than *Many students are numerous*. In Lasnik (1987) I suggested, following Dowty's (1986) analysis of *all*, that the contrast in acceptability was due to the fact that predicates like *gather in the hall* impose "subentailments" on the members of any group in their extension; for example, the members of any group in the extension of *gather in the hall* must be in the hall at the same time as the other members of the group. In the case of *be numerous*, however, there are no such subentailments; the properties of a group's individual members are irrelevant to the question of whether the group satisfies the predicate. I will assume here that something like this explanation is correct.

number. Such VPs may have both groups and individuals in their inclusion sets. An uninflected VP to which this operation has already applied may then be marked for number, however, restricting the inclusion sets of the resulting finite VP either to groups or to individuals. The effect of the distributivity operation on the inclusion set of a VP denotation (at a given event) is simply to close it under group formation; a straightforward adaptation of rule (119) from Chapter II will do the trick:

(19) If $\alpha \in VP$ then $\beta = \langle \alpha \rangle \in VP$;

for all $e \in E$, $\|\beta\|(e)_1 = \bigcup_n \|\beta\|_n(e)_1$, where:

(i) $\|\beta\|_0(e)_1 = \|\alpha\|(e)_1$;

(ii) for all i (i a positive integer),

$$\|\beta\|_i(e)_1 = \|\beta\|_{i-1}(e)_1 \cup \{x \in U - U_I \mid \forall y \ y \in x \rightarrow y \in \|\beta\|_{i-1}(e)_1\}.$$

Though we will not be particularly concerned with it, the operation should also have an effect on the exclusion set; it recursively removes all groups whose members are not in the exclusion set at the previous step in the induction:

(20) Where α, β are as in (19), for all $e \in E$, $\|\beta\|(e)_2 = \bigcap_n \|\beta\|_n(e)_2$, where:

(i) $\|\beta\|_0(e)_2 = \|\alpha\|(e)_2$;

(ii) for all i (i a positive integer),

$$\|\beta\|_i(e)_2 = \|\beta\|_{i-1}(e)_2 - \{x \in U - U_I \mid \forall y \ y \in x \rightarrow \{y \notin \|\beta\|_{i-1}(e)_2\}\}.$$

Assume an event e in which John builds a table, so does Mary, and so (collectively) does the group of Bill and Sue. Then the inclusion set of the distributive denotation of *build a table* (unmarked for number) at e will contain John, Mary, the group of Bill and Sue, the group of John and Mary, the group of John and the group of Bill and Sue, the group of John, Mary, and the group of Bill, Sue and Ed, etc. Most importantly it will contain the group of John, Mary, and the group of John and Mary. It will not, however, contain the group of Bill, Sue, and the group of Bill and Sue; this distinction corresponds to the fact that the group of John and Mary is in the set only by virtue of the fact that John built a table and Mary built a table, while the group of Bill and Sue is in the set because they built a table collectively.

Marking the VP with plural morphology restricts the inclusion set to groups, eliminating John and Mary, but leaving the group of John, Mary, and the group of John and Mary, and all the other groups as well. Determiners may be defined so as to be sensitive to groups with this sort of structure, thereby producing the effects of distributive quantification

We may define *many* as in (21):

- (21) For all $X \in \text{POW}(U)$, all $e \in E$ and all $f \in \text{DVP}[+p1]$, $\llbracket \text{many} \rrbracket(X)(e)(f) = 1$ if $\exists x \in X \cap f(e)$

$$[\{x\} \cup \{y \in U \mid y \in x\} \in f(e)_1] \wedge \text{CARD}(\{y \in U \mid y \in x\}) \text{ is large}] ; 0 \text{ otherwise.}$$

In English: A sentence like *Many inventors build a table* is true of an event *e* if there is some *x* such that *x* is in *||inventors||* and in the inclusion set of *||build a table||*(*e*), and such that the group of *x* and all *x*'s members is in the inclusion set of *||build a table||*(*e*), and such that *x* has a large set of members.

8. Fewer than two and more than one. The inclusion/exclusion distinction allows us to treat a class of examples which have previously appeared to be problematic for any semantic account of number agreement, namely those sentences whose subjects are introduced by the strings *fewer than two* and *more than one*¹¹. Subjects introduced by *fewer than two* regularly impose plural agreement, despite the fact that at most one individual may have the property in question in the event described; conversely *more than one* requires singular agreement, even though more than one individual is involved:

(22)a. Fewer than two inventors are/*is hiding in this closet.

¹¹ *Fewer than two* has actually been much more problematic for previous theories than *more than one*, which can be treated as semantically singular in precisely the same way as *every*.

- b. More than one inventor is/*are hiding in this closet.

An explicit account of the internal compositional semantics of phrases like *fewer than two inventors* and *more than one inventor* would require an analysis of the comparative construction and is outside the scope of this study. However, the following semantics suggests itself for the phrases as wholes:

(23)a. For all $e \in E$, $f \in D_{VP}[+p1]$, $\llbracket \text{fewer than two inventors} \rrbracket(e) = 1$ if $CARD((\llbracket \text{inventors} \rrbracket - f(e)_2) \cap U_I) < 2$; 0 otherwise.

b. For all $e \in E$, $f \in D_{VP}[-p1]$, $\llbracket \text{more than one inventor} \rrbracket(e) = 1$ if $CARD(\llbracket \text{inventor} \rrbracket \cap f(e)_1) > 1$; 0 otherwise.

A plural VP denotation f is mapped onto 1 by the denotation of *fewer than two inventors* at e iff all individual inventors, with fewer than two possible exceptions, are in the exclusion set of f at e . Because the quantifier is defined only for plural VP arguments, plural agreement is predicted, despite the fact that a sentence like (22) actually disallows the case where a group of inventors is hiding in the closet. Likewise a singular VP denotation f is mapped onto 1 by the denotation of *more than one inventor* at e if more than one inventor is in the inclusion set of f at e .

9. Remaining problems. Besides the arguments from VP anaphora, *no, only, fewer than two* and *more than one*, a number of objections have been made to the claim that number agreement is semantically conditioned. Most of these are fairly well known and already much discussed; beyond the analysis given above I have little new to add to the debate. Even so a brief review seems in order. Where possible I will make informal suggestions as to how a semantic treatment might be developed, but in all cases the formal details are yet to be worked out. In some cases it seems unlikely that a semantic treatment is appropriate; although I am claiming that number agreement is in general semantically conditioned, in a small set of circumstances an "overlay" of syntactic principles appears to be involved.

Probably best known are arguments from committees and other "social" collective objects. In certain dialects, NPs headed by nouns like *committee* impose singular agreement on their verbs, and in others they impose plural agreement (at least in certain circumstances). Traditional grammar generally described this fact as due to a difference across dialects as to whether agreement was imposed semantically or syntactically; since committees are collective objects, in dialects with semantic agreement the noun *committee* imposes plural morphology on its verb, but since the noun is singular, in dialects with syntactic agreement, it imposes

singular morphology. Examples like *This committee was founded in 1925*, already mentioned in Chapter III, which require singular agreement even in dialects which otherwise allow plural agreement with *committee*, indicate the situation is not so simple, however.

I argued in Chapter III that committees are better considered to be individuals of a sort than groups. If nothing else is said, the present framework would predict singular agreement in all examples even though it is semantically conditioned. The question, then, is how to obtain plural agreement in those cases where it is acceptable.

It seems very unlikely that anyone would want to appeal to syntax here; it is precisely because committees are collective objects that they impose plural agreement, and the analysis should capture this fact. One possibility is to define a function F mapping any committee or other social collective onto the group of its members. A noun phrase like *the committee* could then be treated as ambiguously denoting either the committee itself (an abstract individual) or the value of F when applied to the committee. We could then use meaning postulates to prevent committees (abstract individuals) from appearing in the denotations of predicates like *meet in this room*, *decide to disband*, etc., for which *committee* imposes plural agreement. The only way an NP like *the committee* could logically combine with such

predicates, then, would be if its denotation were derived by application of the function F , so that it now denoted a group rather than an abstract individual.

One interesting problem which this sort of analysis would raise is how to treat cases where a committee has only one member, or none at all. I am not sure what sort of agreement is used in cases like this, or even if speakers of the dialects in question have consistent intuitions; a survey of speaker judgments is definitely in order.

Another well-known set of problematic examples for semantic number agreement involve plural nouns for which there is no corresponding singular, such as *scissors*, *pants*, *pliers*, etc. Certainly semantics seems to be involved in determining which nouns fall into this class; all of them seem to refer to bipartite objects having a particular shape. However, these bipartite objects cannot be identified with the groups of their parts; the separated halves of a pair of pliers do not constitute a pair of pliers.

This in itself does not eliminate the possibility of a semantic treatment, however. One could still stipulate that the objects in the extensions of such nouns must be in U rather than U_I , hence "groups" in the relevant sense rather than "individuals". To make this move would require us to abandon the idea that U is freely generated on the

basis of U_I by application of the group formation operation, however. Either we would need a subsidiary non-logical operation which added scissors, pliers, etc. to $U - U_I$, or else the inductive definition of U would have to start from some larger set than U_I .

A third set of problematic examples involves "proximity" agreement, in which a verb agrees not with the head noun of its subject, but with the nearest noun, as in the following example from Francis (1986), where it is attributed to Ronald Reagan:

- (24) The sheer weight of these figures make them harder to understand.

There seems little hope at all of giving a semantic account of this sort of agreement. On the other hand, a syntactic account would be almost as problematic, since agreement is normally with the head. Examples like (24) are surprisingly common, but even so it may be best to treat this sort of agreement as extragrammatical, resulting from processing/production considerations rather than the semantics or syntax of the competence grammar itself.

A similar class of examples for which a grammatical account may be more appropriate was pointed out by Morgan (1972): If a conjoined NP appears immediately following the copula in a *there*-insertion sentence, the copula agrees with the nearest conjunct:

(25)a. There was a man and two women in the room.

b. There were two women and a man in the room.

Some speakers show similar effects with disjoined subjects:

(26)a. Either Harry or his parents are/*is coming.

b. Either Harry's parents or his wife ?is/*are coming.

Perhaps these examples should also be explained as extragrammatical, but the *there*-insertion examples in particular sound, at least to my ears, much less like a production error than (24) does. Again, a semantic account seems hopeless, so a syntactic account seems the only reasonable alternative. Since only a highly specific subclass of *there*-insertion sentences is involved, however, this argument should not be taken as indicating that number agreement in general is syntactic, however.

There-insertion sentences present an additional problem: Singular agreement appears to be possible even if the postverbal NP is plural. For many speakers (myself included), such examples are grammatical only if the verb is cliticized onto *there*:

(27) There's two inventors hiding in this closet.

For such speakers it might be best to regard *there's* as a single lexical item rather than a syntactic combination of *there* and *is*, in which case the question of verbal agreement simply does not arise. Dialects which allow (unreduced)

there is with plurals may be more of a problem, however. It would be interesting to discover whether or not such dialects coincide closely with those that allow *is* with third person plural subjects generally; if so, we could take *is* as ambiguous or neutral for number.

Morgan also points out that when pseudo-cleft sentences take reflexive subjects, they impose singular agreement even when plural:

(28) Themselves was/*were all they could see.

Here again it may be that a syntactic principle is called for; Morgan suggests that a subject must be nominative to impose agreement, and that otherwise third person singular agreement is imposed as a default. Note, however, that aside from the default specification, this principle only specifies the conditions under which agreement takes place; it does not characterize the formal mechanisms by which agreement morphology is realized, which might still be treated as semantic.

Roberts (1987a : 170-171) seems to suggest that any mismatch in morphological number between subject and its verb is problematic for a semantic account of agreement, and gives examples from *there*-insertion, committees, and proximity agreement. In many cases, however, mismatched number appears to correspond to a special semantic

interpretation, and therefore is best treated as semantically conditioned.

One interesting agreement mismatch occurs in sentences evaluating a particular quantity of objects as being "a lot", "a little", "too much", "not enough", etc. Such sentences may have plural subjects with singular verbs:

(29)a. This many inventors is more than I really want to think about.

b. Three defeats in a row was too much for our hero.

c. Ten toes is as many as I would really care to have.

Such examples would appear to be extremely problematic for a purely syntactic theory of agreement which merely manipulated uninterpreted features. A semantic account seems much more promising, since what distinguishes these examples is that they are about quantities of inventors, defeats or toes, rather than being about the inventors, defeats or toes themselves.

Similar considerations hold for examples where conjoined atomic noun phrases impose singular rather than plural agreement, such as the well-known example (30):

(30) Ham and eggs is my favorite breakfast.

It is quite clear that singular agreement occurs here because *ham and eggs* is taken as referring to a particular dish, and not to a group in the normal sense. Likewise the

differing agreement in examples (31)a. and b., from van Eijck (1981), corresponds directly to the fact that *his aged servant and the subsequent editor of his papers* refers to a single individual in (31)a. but a group in (31)b.

(31)a. His aged servant and the subsequent editor of his papers was with him at his deathbed.

b. His aged servant and the subsequent editor of his papers were with him at his deathbed.

A final argument against semantic agreement comes from Krifka (1987). The numbers *zero* and *one point zero* impose plural morphology, but do not seem to imply semantic plurality:

(28)a. Zero tables have been built since we arrived.

b. One point zero liters of water flow through this valve per minute.

A natural treatment of plural *zero* is available in the agreement system developed above: let $\llbracket zero \rrbracket(X)(e)$ map a plural VP denotation f onto 1 if X is a subset of the exclusion set of f at e . Other plural VP denotations should be mapped onto 0; the function should be undefined for singular VP arguments.

One point zero is more problematic, however. A similar treatment to the case of *zero* or *fewer than two* is technically possible, but seems intuitively wrong; *one point zero* does not "feel" at all like an exclusive quantifier and

should not have access to the exclusion sets of its VP arguments. I have no suggestions at this point as to how a semantic treatment of *one point zero* should work. Any such treatment ought to part of a more general study of the semantics of fractional parts of objects, however, and perhaps such a study will uncover some parallel with other plural determiners.

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