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Probabilistic analysis of groundwater levels in hillside slopes

Reddi, Lakshmi Narayana, Ph.D.

The Ohio State University, 1988

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PROBABILISTIC ANALYSIS OF GROUNDWATER LEVELS
IN HILLSIDE SLOPES

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By

Lakshmi Narayana Reddi, B.tech., M.S.

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1988

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DEDICATION

To my wife Usha

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No one has been affected more by, has sacrificed more toward, or hoped more for this work than the author's wife, Usha. She has been and continues to be his support in life and no thanks can possibly be enough. The author feels the same way about the inspiring nature of his parents. His parents have always guided him through their perspicacious personalities. Though it may seem inappropriate in the present context, the author wishes to adorn this work with the quotation:

"There is no religion higher than truth" (Mme. Blavatsky, H.P.)

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CHAPTER I

INTRODUCTION

1.1 GENERAL

Landslides on steep hillside slopes are common in mountainous parts of the world. They are often associated with heavy rainfall. Rainfall has the effect of raising the porewater pressure and reducing the effective stress in the soil. Stability analyses of slopes are generally based on the effective stress principle, in which the effective normal stress on the slip surface is

$$\bar{\sigma} = \sigma - u \quad (1.1)$$

where σ = normal stress and u = porewater pressure. Thus, prediction of porewater pressure is an essential step in evaluation of landslide hazard. On hillside slopes, the fluctuation of groundwater surface is dependent on many related processes such as rainfall, evapotranspiration, runoff, infiltration, and drainage. Traditionally, the physics of the groundwater flow problem is mathematically represented by a governing differential equation. As observed by Johnson(1977), the mere statement of a governing differential equation for groundwater flow leaves one far from a practical prediction of actual groundwater levels in a well. Mathematical modeling usually necessitates several simplifications of the physical processes involved. In the face of such adversity of modeling, solutions that have been developed so far to the problem of predicting

groundwater movement either totally neglected or highly simplified such physical processes as evapotranspiration and unsaturated flow. For instance, recent work by Sangrey, D.A.(1984) on the problem faced a severe criticism by Campbell, A.M.(1986) for neglecting the importance of water movement in the unsaturated zone.

Solutions to the groundwater flow problem need parameters such as permeability and moisture capacity which are highly uncertain. Most of these parameters are known to have spatial as well as temporal variations. Any model however elaborate it may be, that ignores the variability of the parameters might lead to conclusions far from reality. But it is difficult, at least from economics viewpoint, to measure and describe these parameters for the whole domain. The estimation of groundwater parameters is thus, as aptly called by Freeze(1972), the "Achilles' Heel" of groundwater models.

The aim of the present research is to develop a groundwater model that accounts for all the important physical processes involved and yet simple to use. The presence of variability in groundwater parameters essentially leads to a probabilistic solution of the problem. The uncertainty in the groundwater fluctuations is therefore studied in relation to the uncertainty in the parameters. Since numerous groundwater models are available at present, the need for present study cannot be underscored without a knowledge of the state-of-the-art. For this reason, literature relevant to the present study is briefly reviewed in sections 1.2 and 1.3 of this chapter. Section 1.4 includes the scope of the present study.

1.2 DETERMINISTIC MODELS

The relationship between rainfall and groundwater fluctuations has been observed by many investigators in the past. This fact is very clear from the works of Veatch(1906) who believed

"that the rise and fall of the groundwater stands in no relation whatever to the amount of rain, since with the same quantity of precipitation it at times rises, and again falls, and even with considerably increasing quantities of rain it often falls constantly".

Rainfall alters the soil water regime first before percolating down to the water table. Recognizing this fact, earlier investigations concentrated on how soil properties affect the process of rainfall infiltration into soil. These studies are broadly termed as infiltration studies. Numerous formulations, some entirely empirical and others theoretically based, have been proposed over the years in repeated attempts to represent the physics of infiltration phenomenon (Green and Ampt(1911), Kostiaikov(1932), Horton(1940), Philip(1957c) and Holton(1961). Out of these, the studies of Green and Ampt(1911) and Philip(1957c) deserve brief description in the present context since their theoretical basis paved the way for recent developments.

Green and Ampt(1911) suggested a simple, approximate, yet useful approach to the infiltration process. The solution gives no information about soil moisture profile during infiltration, but does offer estimates of the infiltration rate and the cumulative infiltration functions of time, $i(t)$ and $I(t)$ respectively. The principal assumptions of the Green and Ampt approach are that there exists a distinct and precisely definable wetting front and that the matric suction at this wetting front remains effectively constant, regardless of time and position. Also, this approach assumes that, behind the wetting front, the soil

is uniformly wet and of constant conductivity. The wetting front is thus viewed as a plane separating a uniformly wetted zone from an as yet totally uninfiltreated zone. With these features, the Darcy's equation yields

$$Kt/\Delta\theta = L_f - (\Delta H_p) \ln \left[1 + \frac{L_f}{\Delta H_p} \right] \quad (1.2)$$

where ΔH_p is the pressure-head change from the surface to the wetting front, L_f is the depth of the wetting front, K is the hydraulic conductivity and $\Delta\theta$ is the wetness increment in the wetting front. The main drawback in using Green and Ampt approach is that, in reality, the wetting front may be so diffuse that its exact location cannot be approximated satisfactorily. For this reason, Green and Ampt approach found its applicability only in cases of coarse-textured soils which exhibit a sharp wetting front.

The first mathematically rigorous treatment of the infiltration process was given by Philip(1957). Philip(1957) considered infiltration into a semi-infinite, homogeneous, one-dimensional soil column of uniform initial moisture content to make the mathematical solution feasible. A combination of Darcy's equation and mass continuity equation yields the following general flow equation

$$\frac{\partial\theta}{\partial t} = \frac{\partial}{\partial z} \left(D \frac{\partial\theta}{\partial z} \right) - \frac{\partial K}{\partial z} \quad (1.3)$$

where D and K are diffusivity and permeability coefficients respectively. This is generally known as the Richards equation. Philip obtained the closed-form solution of Eq. 1.3 for the case where the top of soil column is maintained at a constant moisture content. The solution is sought in the form of power series

$$\begin{aligned}
 I(t) &= \sum_{n=1}^{\infty} j_n(\theta) t^{n/2} \\
 &= st^{1/2} + (A_2 + K_0)t + A_3 t^{3/2} + A_4 t^2 + \dots + A_n t^{n/2}
 \end{aligned} \tag{1.4}$$

where $I(t)$ is cumulative infiltration and the coefficients $j_n(\theta)$ are calculated from $K(\theta)$ and $D(\theta)$. The coefficient 's' is called sorptivity and is equal to

$$2(\theta_1 - \theta_0) \sqrt{D/\pi} \tag{1.5}$$

An approximate description of the infiltration solution is, therefore,

$$I(t) = st^{1/2} + At \tag{1.6}$$

For large times A is represented by the permeability, K . For very large values of time, Eq. 1.6 reduces to

$$i(t) = \frac{dI(t)}{dt} = K \tag{1.7}$$

In a classical treatment of the subject, Philip also gave solutions for soil moisture profile and higher-dimensional systems. Even though Philip's methodology is mathematically sound, the simplification that the soil column is semi-infinite is rarely satisfied in reality. This is because the fluctuations of water table at some depth alter the soil moisture profile in conjunction with the infiltration. Also, Philip's solution imposes restrictions on the choice of initial and boundary conditions. Flux boundary conditions as in the case of rainfall make analytical treatment almost impossible. In the face of such adversities and with the advent of digital computers, a new era of numerical solutions began.

Rubin(1968) and Freeze(1971) pioneered the numerical solutions for the flow problem. Rubin(1968) obtained finite difference solution for two-dimensional, transient movement of water in a rectangular unsaturated soil slab. The two-dimensional flow equation used is an extension of Eq. 1.3 expressed in terms of suction pressure, i.e.

$$C(\Psi) \frac{\partial \Psi}{\partial t} = \frac{\partial}{\partial x} \left[K(\Psi) \frac{\partial \Psi}{\partial x} \right] + \frac{\partial}{\partial z} \left[K(\Psi) \frac{\partial \Psi}{\partial z} \right] + \frac{dK}{d\Psi} \frac{\partial \Psi}{\partial z} \quad (1.8)$$

This equation is made quasi-linear by Kirchoff's transformation

$$F = \int_{\Psi_0}^{\Psi} K(\xi) d\xi \quad (1.9)$$

The final equation is of the form

$$\frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial z^2} + \alpha \frac{\partial F}{\partial z} = \beta \frac{\partial F}{\partial t} \quad (1.10)$$

where

$$\alpha = \frac{d(\ln K)}{d\Psi} \quad \text{and} \quad \beta = \frac{C(\Psi)}{K(\Psi)}$$

Eq. 1.10 is solved using Alternating Direct Implicit Scheme (ADI). A typical solution obtained by Rubin is shown in Fig.1.1.

The most comprehensive flow model developed to this date is that of Freeze, R.A. (1971). This is a three-dimensional finite difference model of an entire groundwater basin, including both saturated and unsaturated flow regimes, allowing many kinds of

Figure 1.1: Moisture Content contours in a soil block (After Rubin, 1968)

heterogeneity and anisotropy, and even making provisions for hysteresis in permeability with fluctuations in pressure. The equation of flow similar to Eq. 1.8 is solved by the line successive overrelaxation technique. The model allows any generalized region shape and any configuration of time variant boundary conditions. When applied to natural flow systems, the model provides quantities of infiltration, groundwater recharge, water table depth, and stream base flow. Typical results obtained by applying Freeze's model to the flow problem in an embankment are shown in Fig. 1.2.

However, the difficulties in applying Freeze's model to the real world are as many as its ambitions. In Freeze's own words,

"complex models are open to the charge that their sophistication outruns the available data. To answer the charge, one can only reply that the deterministic simulation approach espoused here is potentially superior to any empirical technique that requires less data. If the technique can be shown to have practical value, it will encourage increased measurement of the necessary data".

Subsequent to Freeze's studies, several numerical solutions have been developed for the groundwater flow problem (Nieber, J.L. and Walter, M.F.(1981), for example). Most of these numerical methods overlooked the economics criterion. Even if the necessary data were collected the cost of employing a numerical model would exclude its application to many problems. The growth of numerical solutions has been mainly in the area of computational efficiency. Computational capabilities have grown much more than physical understanding of the flow problems. Sloan and Moore(1984) in their classical study, compared various groundwater flow models and concluded that

Figure 1.2: Numerical solution for the groundwater regime in an embankment (After Freeze, 1971)

the predictive accuracy of simpler physical models is as good as or even better than that of the complex numerical models. Sloan and Moore's comparisons are studied in a better perspective in chapter 2.

A deviation from such a realm of numerical studies is the work done by Sangrey, D.A. (1984). Sangrey developed a new methodology to predict the fluctuations of groundwater levels as a function of precipitation. His theoretical model comprises of two parts. The first part, called the Hydrometeorological model links the amount of precipitation, and other factors such as temperature, to the effective groundwater recharge. The second part called the Geohydrological model links the recharge to the height of groundwater in the aquifer. In the Hydrometeorological model, Sangrey used Goldschmidt's and Thornthwaite's formulae for evapotranspiration and derived recharge as a linear function of precipitation. In the Geohydrological model, Sangrey solved the following equation which is obtained by combining Darcy's law and the mass continuity equation.

$$S \frac{\partial H}{\partial t} = R + K \nabla(H \nabla H) \quad (1.11)$$

where R is recharge given by hydrometeorological model, S is specific yield, and K is permeability coefficient. The decay solution of one-dimensional form of Eq. 1.11 and for the boundary conditions of a sloping aquifer [$H=0$ at $x=L$ and $\frac{\partial H}{\partial x} = 0$ (Fig.1.3)] is obtained as

$$H_d(x,t) = \frac{RL^2}{Kh_a} \frac{16}{\pi^3} \sum_{n=0}^{\infty} (-1)^n \exp[-(2n+1)^2 \frac{t}{t_0}] (2n+1)^{-3} \cos(2n+1) \frac{\pi x}{2L} \quad (1.12)$$

$$\text{where, } t_0 = \frac{4SL^2}{\pi^2 Kh_a}$$

Figure 1.3: Characteristics of a sloping aquifer in Sangrey's study

The contribution of total recharge to the groundwater is approximated by the superposition of recharges of each time interval. In this way the contribution of one pulse of recharge between times separated by time length Δt is obtained as the difference of the decay function before and after Δt . Even though the mathematical solution is accepted to be elegant, Sangrey's hydrometeorological model faced severe criticism by Yen, B.C. and Campbell, A.M. The hydrometeorological model primarily neglected the effect of unsaturated zone on groundwater level fluctuations. Soil moisture regime in the unsaturated zone, antecedent precipitation and nature of soil cover can greatly affect the linear relationship assumed between recharge and precipitation.

Beven(1982) took yet another approach to model the subsurface stormflow. His methodology is based on kinematic approximations in both unsaturated and saturated zones. For the unsaturated zone, the following assumptions are made:

1. Capillary potential gradient can be neglected
2. Saturated conductivities are high compared to rainfall rates, i
3. At the start of infiltration, water content at the soil surface immediately adjusts so that $K(\theta) = i$

Under these conditions, the movement of the wetting front is given by

$$\frac{dz}{dt} = \frac{i}{\theta_w(z) - \theta(z, t=0)} \quad (1.13)$$

where $\theta_w(z)$ is the water content at which $K(\theta)=i$. Beven suggested the kinematic wave equation

$$\nu \frac{\delta h}{\delta t} = -K_s \sin \alpha \frac{\delta h}{\delta x} + i \quad (1.14)$$

in the case of saturated flow in a sloping soil mantle of constant hydraulic conductivity K_s and effective porosity ν overlying a relatively impermeable bedrock surface of slope α . 'h', in Eq. 1.14 denotes the height of groundwater surface on the slope. 'x' and 't' are the indices for downslope direction and time respectively. This equation was shown to be a good approximation when a parameter $\lambda = 4i \cos \alpha / (K_s \sin^2 \alpha)$ is less than about 0.75. Beven used Eqs. 1.13 and 1.14 together to solve for the saturated-unsaturated groundwater regime on the slope. The simplifications behind Beven's formulation restricted its application only to steep slopes, high saturated hydraulic conductivities, and low rainfall rates.

Significant contribution was made by Eagleson(1978) to the understanding of physics of unsaturated flow. Eagleson subdivided the vertical movement of soil moisture between the surface and the water table into three categories according to the predominant forces involved.

1. Infiltration and Exfiltration: These processes occur in the soil moisture zone under the complementary effects of capillarity and gravity in the wet season and capillarity alone in the dry season.
2. Percolation: This process occurs in the intermediate zone under the domination of gravity forces.
3. Capillary rise: This process occurs between the water table and the intermediate zone under the complementary effects of gravity and capillarity.

These processes are analyzed separately, and their effects are linearly superimposed. Infiltration and Exfiltration are described by the Philip equation which assumes the medium to be effectively semi-infinite. The internal soil moisture at the beginning of the storm period is assumed to be uniform at its long-term space-time average. The exfiltration equation is modified for the presence of natural vegetation through the

introduction of a distributed sink representing the moisture extraction by plant roots. Gravitational percolation to groundwater is assumed to be steady throughout the rainy season and capillary rise from water table is assumed to be steady throughout the year and to take place to the dry surface.

1.3 STOCHASTIC MODELS

The deterministic models reviewed in the previous section include many hydrogeological parameters. However, most of these parameters are highly variable and can only be estimated approximately. Much of the difficulty stems from spatial and temporal variations of the parameters. The variabilities of the parameters greatly reduce predictive capabilities of the deterministic models. These facts gave rise to a relatively new branch of hydrology known as "Stochastic Hydrology". According to Clarke(1973), a model is termed stochastic or deterministic depending on whether or not it contains random variables. That is, if any of the independent variables is thought of as having a distribution in probability, then the model is stochastic. The general objective of stochastic models is to evaluate how the uncertainties in groundwater systems influence the flow observations and modeling.

Dettinger and Wilson (1981) divided the uncertainty in groundwater systems into two classes: intrinsic uncertainty and information uncertainty. The first class derives from the lack of knowledge of certain natural properties or processes and is an irreducible uncertainty inherent to the system. It denotes the physical variability in the system undiminished by any number of observations made on the system. The second class is the result of 'noisy' or incomplete information about the system and may be reduced by various strategies, notably further measurements. Information uncertainty represents

the lack, in quantity or quality, of information concerning the groundwater system. Intrinsic uncertainty and information uncertainty play different roles in groundwater predictions. Studying the effect of intrinsic uncertainty of the parameters gives the predictions as mean values about which there are small scale fluctuations described statistically by the covariance structure. On the other hand, studying the effect of information uncertainty allows the incorporation of sampled data which can reduce the uncertainty in the predictions. The development of stochastic models is to study these two effects.

The various stochastic approaches developed so far can be divided into two main groups:

1. Full distribution analyses
2. First and second moment analyses

Full distribution methods use a complete specification of the probabilistic properties of all inputs and parameters of the flow system and attempt to specify completely the probability distribution of the resulting flow. First and second moment analyses, on the other hand, assume that the first two moments of a random variable are sufficient to characterize it. That is, they consider only the mean and variance-covariance of the inputs, parameters, and the outputs.

Full distribution analyses can be further subdivided into two classes: method of derived distributions and Monte Carlo simulations. The derived distributions approach is an analytical method to derive the probability distribution of a random function given the distributions of its independent variables. Eagleson(1978) used this approach to evaluate water balance dynamics due to stochastic precipitation events. Uncertainty is introduced into water balance equations through probability density functions of inde-

pendent climatic variables. Eagleson then sought the derived probability distributions of the dependent water balance elements: surface runoff, evapotranspiration, and groundwater runoff. This approach is also adopted by Sagar and Kisiel(1972), Bakr, et al.(1978), and Dagan(1979). However, the derived distributions approach becomes extremely complicated unless applied to simple systems with relatively simple probabilistic properties.

A more popular full distribution analysis is the Monte Carlo simulation. Random generation technique is used in this method. It involves numerous replications of flow system simulations, with the parameters and inputs of each simulation generated at random from their respective probability distributions. The results of the simulations are compiled to form estimates of the probability distribution of the predicted groundwater levels. Unlike the derived distributions method, this method can be computerized and can be applied to complicated flow systems. Freeze(1975) presented the results of several sets of Monte Carlo simulations that show the effects of stochastic parameter distributions on two flow problems: steady state saturated flow and transient consolidation of a saturated clay layer. The stochastic variables considered are hydraulic conductivity, compressibility, and porosity. The primary output is the statistical distribution of hydraulic head through space and time. Monte Carlo method has been used for flow problems by several other investigators including Warren and Price(1961), Smith and Freeze(1979). A drawback of this method is that the results are not in the form of a closed form analytical solution. In this respect, derived distributions approach is advantageous. Also, the computational costs are very high in the method of Monte Carlo simulations.

The first and second moment methods have gained popularity because they are based on simple stochastic representations of random variables. The random variables are characterized only by the first two moments, that is, mean and variance. First and second moment methods can be used mainly in two ways: Perturbation and Taylor series expansions. In Perturbation analyses, the differential equation governing the flow system is perturbed slightly, yielding a new equation that governs the random components of the variables in the system. Using this approach, Bakr, et al.(1978) studied the effect of variation of hydraulic conductivity on piezometric levels. Hydraulic conductivity is characterized by a covariance function and the theory of spectral analysis is used to solve perturbed forms of the stochastic differential equation governing the flow. Anderson J. and Shapiro A.M. (1983), in another study, used perturbation technique to evaluate the effect of permeability variation on steady state, unsaturated infiltration. Anderson and Shapiro also compared the Monte Carlo simulations of the problem with perturbation solutions. In comparing the two techniques, the reliability of the perturbation expressions is investigated assuming that Monte Carlo results exactly represent the solution. In four test examples, Anderson and Shapiro observed good agreement between the results of the two techniques. Other investigators using perturbation methods include Gutjahr, et al.(1978) and Gelhar et al(1979).

The other group of first and second moment methods are based on Taylor series expansion of the governing equation around the expected values of the solution's parameters and independent variables. These series expansions are then used to deduce the probabilistic moments of the dependent variable. This is the most general approach and can be used to examine a broad range of groundwater flow problems with many different grid geometries, soil properties and boundary conditions. Non-homogeneity, Anisotropy, and spatial variability of the parameters can be studied through a proper

choice of covariance matrices. In this respect, this method is a useful alternative to the previous methods. This approach has been extensively used by Wilson and Dettinger (1978) in various groundwater models. These authors also tackled the issue of information uncertainty by adopting Kalman filter technique. Filter techniques utilize prior information about the parameters and piezometric heads to improve estimates of parameters, as well as the system state. This technique is used in the present research and is therefore studied in a better perspective in chapter 3. The basic assumption underlying first and second moment analyses is that higher moments, skewness for example, are negligible. Random variables fulfilling this assumption are those which are normally distributed. Since it is impossible to check this assumption using first and second moment analyses, other methods such as Monte Carlo simulation should be used whenever possible to check it.

Lee, I.M. (1987) used the First Order Second Moment approach to predict the groundwater surface in a shallow soil cover on a sloping impervious bedrock. His work is worth mentioning here because of its relevance to the present study. Lee derived a 2-d flow equation using the Darcy's law and the continuity equation. The flow equation is derived to incorporate effect of bedrock topography. The flow domain is discretized and an explicit finite difference scheme is used to solve the governing equation. The effects of spatial variabilities of permeability, specific yield, depth of soil cover and topography of ground surface are studied in depth. The effect of temporal variability of recharge on the variability of predicted groundwater surface is also studied. Lee applied the stochastic model to a hillslope in Alaska. Since measurements of the parameters are available only at a few locations in the site, Lee used Kriging theory to account for the variabilities of the parameters. Kriging theory provides the best linear unbiased estimator of a parameter and its minimum variance from a limited number of

measured data. The groundwater flow model, the Krigin theory, and the random field theory are combined to develop a stochastic solution for the groundwater surface. Lee observed significant variations of the groundwater surface due to the spatial variations of the aquifer parameters.

1.4 OBJECTIVES AND SCOPE OF STUDY

The objective of the present research was to develop a model to predict porewater pressures in a shallow soil cover on a sloping impervious bedrock. The state-of-the-art, as reviewed in previous sections, shows that the physics of flow problem is overlooked in light of computational techniques. Simple processes such as evapotranspiration and infiltration in the unsaturated zone are either totally neglected or highly simplified. Comprehensive approaches including that of Freeze(1971) are too costly to operate and far from practical applications. It was therefore the aim of the present research to develop a simple physically based model that accounts for all the salient flow processes. For this purpose, appropriate individual models were studied for evapotranspiration, infiltration, and drainage. The existing literature on the prediction of evapotranspiration is characterized by extensive data requirement. In this study, for the sake of practicality, the evapotranspiration model was chosen to require only regularly observed meteorological data. A mass balance model was developed to represent the flow process in the unsaturated zone. Drainage in the saturated zone was computed using a kinematic storage model. The validity of the kinematic storage model was checked by comparing its results with those of a more sophisticated model. The recharge from the unsaturated zone couples the mass balance model and the kinematic storage model thus yielding a comprehensive saturated-unsaturated flow model.

The stochastic nature of the flow model was examined by identifying the uncertain parameters in all the flow processes. The uncertain parameters are high in number. Lack of knowledge of their probability distributions precludes the usage of full distribution methods. So, the first-order second-moment (FOSM) method was chosen in the present study. Using this method, the uncertainty of groundwater predictions was studied as a function of uncertainties in the parameters involved in all the flow processes. The latter part of the research concentrated on updating the model predictions utilizing the few in-situ measurements that might be available on the groundwater system. Thus the study was broadly three-fold:

1. Development of a comprehensive flow model coupling the individual models of evapotranspiration, flow in the unsaturated zone, and drainage
2. Development of a stochastic model using FOSM to obtain the uncertainty in model predictions
3. Using updating techniques to utilize in-situ measurements that might reduce uncertainty in the model predictions

Chapter 2 describes the development of the flow model. The model is applied to two field sites where the required data is available. The stochastic model is developed in chapter 3 and the model is applied to a specific site. Since measured data are available at the site, updating technique is illustrated in chapter 4. Chapter 5 summarizes and concludes the study.

CHAPTER II

DEVELOPMENT OF GROUNDWATER MODEL

2.1 INTRODUCTION

The movement of groundwater surface is one of the important factors in evaluation of landslide hazards. Many solutions have been presented in the literature to predict the movement of groundwater surface (Sloan et al. 1983; Nieber, 1982; Nieber and Walter, 1981; Sangrey et al. 1984). Most of these solutions either totally neglected or highly simplified the effect of the unsaturated zone on groundwater movement. Field investigations by Harr and Yee (1975) underscored the contribution of unsaturated flow to the piezometric levels. The fluctuation in the piezometric levels on a slope is the net result of rainfall entering the ground, the interception and evapotranspiration losses, flow in the unsaturated zone and drainage loss in the saturated zone. In this chapter, these individual components are modeled and then coupled together to yield a comprehensive groundwater model.

2.2 EVAPOTRANSPIRATION MODEL

Evapotranspiration is an important part of the water balance of vegetated areas. Various studies including the work of Patric and Stephens (1967) on Alaskan hillside slopes emphasized the relationship between soil-water loss and evaporative demand. Evapotranspiration models vary from complex (Penman, 1948; Shuttleworth, 1979) to

simple (Thornthwaite, 1957; Eagleson, 1978). Unfortunately, most of these models require extensive data. The need for a simple, yet reliable method of estimating evapotranspiration on an hourly or daily basis using only regularly recorded climatological data has been appreciated for a long time by hydrologists and engineers. In the present research the model by Priestley and Taylor (1972) is chosen because it is simple and contains parameters that can be obtained or estimated from regular weather records.

The model by Priestley and Taylor (1972) is based on "equilibrium evapotranspiration" concept. The equilibrium evaporation rate is the evaporation which would occur in the limit as the air above the surface becomes saturated or when the relative humidity at the surface is equal to that in the air. It is therefore a lower bound for the potential evapotranspiration rate. The equilibrium evaporation is expressed as

$$E_e = \frac{s}{L(s+\gamma)} (R_n - G - M) \quad (2.1)$$

where,

s = the slope of saturation vapour pressure curve,

γ = the psychrometric constant,

L = the latent heat of vaporization of water,

R_n = the net radiation flux density,

G = the soil heat flux density, and

M = the rate of latent, sensible, and photosynthetic
energy storage in the canopy

Priestley and Taylor proposed the following relationship to determine the maximum evapotranspiration limited by atmospheric demand

$$E_{\max} = \alpha E_e \quad (2.2)$$

where α is an experimentally determined coefficient. For a variety of smooth surfaces with non-limiting soil water conditions and dry foliage, $\alpha = 1.26 \pm 0.2$ (Priestley and Taylor, 1972). In forested areas, a variety of studies have given $0.6 < \alpha < 1.1$ (Black, 1979; McNaughton and Black, 1973). However, below some critical value of water storage in the root zone, E becomes limited by the available soil moisture. Then, a linear decrease in E with root zone moisture content (θ_r) can be assumed, or

$$E_s = \beta \theta_r \quad (2.3)$$

where β is another site characteristic. Thus, this is a two-phase model wherein, for period of rainfall,

$$E = E_{\max} = \alpha E_e \quad (2.4)$$

and for period without rain,

$$E = \text{Lesser of } E_{\max}, E_s \quad (2.5)$$

The advantage of the model is that it requires only a mean temperature, precipitation and radiation flux density (R_n). In forests, daily $(G+M) < 0.05R_n$, and therefore can be ignored (Spittlehouse and Black, 1981). If R_n is not available it can be obtained with sufficient accuracy from daily total solar radiation, canopy solar radiation reflectivity and net long-wave radiation calculated from daily mean temperature (Aase and

Idso, 1978). A detailed procedure for computing equilibrium evapotranspiration is given in Appendix A.

However, it is difficult to obtain the critical root zone moisture content below which Eq. 2.3 is to be used. Therefore, a single representation of evapotranspiration is preferred over Eqs. 2.3 and 2.4. Mawdsley and Ali (1985) gave a mathematical representation for α so that Eq. 2.4 can be used for any period with or without rainfall. They used different data sets to obtain the relationship between α and antecedent precipitation index (API) as

$$\alpha = 0.123 \text{ API} - 0.0029 \text{ API}^2 - 0.0000056 \text{ API}^3 \quad (2.6)$$

$$\alpha = 1.26 \quad \text{API} < 20 \quad 0 \leq \text{API} \leq 20$$

The Antecedent Precipitation Index (API) is expressed as

$$\begin{aligned} \text{API}_t &= P_{t-1} + R P_{t-2} + R^2 P_{t-3} + \dots \dots \dots + R^{29} P_{t-30} \\ &= P_{t-1} + R \text{API}_{t-1} \end{aligned} \quad (2.7)$$

where P_{t-1} is the precipitation one day before the time under consideration, P_{t-2} is the precipitation two days before and so on. R is known as recession constant which is a function of evapotranspiration, soil permeability, and the physiographic characteristics of the catchment. In the light of experience, Kohler and Linsley (1951) suggested a value of 0.9 for R . However, R is found to vary within a narrow range of 0.85-0.90.

2.3 DRAINAGE MODEL

There have been several solutions to the subsurface flow problem (Nieber and Walter, 1981; Beven, 1981; Sloan et al. 1983; Okimura and Ichikawa, 1985). They represent a range of mathematical sophistication, with the two-dimensional finite element model being the most sophisticated and the simple storage models being the least sophisticated. In view of the present research, a physical model is chosen so that the model can be incorporated into complex watershed models. Physical models are computationally economical to use. The primary aim is to choose a model such that the prediction of the hydrologic response of any watershed would be placed on a more rational, physically correct, and less empirical footing.

To study the applicability of simple models for the flow in a shallow soil cover on a sloping bed, Sloan and Moore (1984) compared five models which represent a range of refinement. The models are

1. A two-dimensional finite element model (Nieber and Walter, 1981) based on the Richard's two-dimensional equation for saturated and unsaturated flow
2. A one-dimensional finite element model (Nieber, 1982) based on Richard's one-dimensional equation, in which it was assumed that there was no flow normal to the hillslope gradient
3. A kinematic wave subsurface flow model (Beven, 1981 and 1982) in which it was assumed that the hydraulic gradient was equal to the slope of the bed, and
4. Two simple storage-discharge models, one of which assumed kinematic wave approximation, and the other used the Boussinesq assumption (Sloan et al. 1983)

Comparison of the predicted water levels from the above models is shown in Fig. 2.1. Sloan and Moore concluded that the predictive accuracy of the simple kinematic stor-

age model is as good as that of the complex models. Sloan and Moore also warned against using complex models to evaluate simpler models because their results indicated that the assumptions inherent in the complex models can be as limiting as the assumptions involved in the simpler models.

In view of the above discussion, the kinematic storage model (Sloan, et al. 1983) is chosen in the present research. This model uses the mass balance equation with the entire hillslope segment being the control volume. The idealized plane hillslope segment has an impermeable boundary or bed, of slope η and length L_s , and a soil profile of constant thickness, D (Fig. 2.2). The mass balance equation can be expressed in mixed finite difference form as

$$\frac{S^{(1)} - S^{(0)}}{t^{(1)} - t^{(0)}} = IL_s - \frac{(q^{(1)} + q^{(0)})}{2} \quad (2.8)$$

where S is the drainable volume of water stored in the saturated zone per unit width, t is time, q is the discharge from the hillslope per unit width, I is the rate of water input to the saturated zone from the unsaturated zone per unit area, and superscripts 0 and 1 refer to the beginning and end of the time period, respectively.

The model assumes that the water table has a constant slope between the upslope and downslope boundaries of the sloping soil mass, as is shown in Fig. 2.2. The hydraulic gradient is assumed to be equal to the slope of the impermeable bed. For a plane hillslope (Fig. 2.3a), the drainable volume of water stored in the saturated zone of the hillslope is

Figure 2.1: Comparison of predicted groundwater surfaces (Sloan and Moore, 1984) a) at $t=0$, b) at $t=1000$ min.

Figure 2.2: Conceptual representation of the hillslope segment (Sloan and Moore, 1984)

$$S = \frac{H_0 \theta_d L_s}{2} \quad (2.9)$$

where H_0 is the saturated thickness normal to the hillslope at the outlet, and θ_d is the drainable porosity of the soil. The boundary condition at the outlet is assumed to allow free discharge that can be expressed as

$$q = H_0 v \quad (2.10)$$

where

$$v = K_{sat} \sin \eta \quad (2.11)$$

Eqs. 2.8, 2.9 and 2.10 allow the hydraulic head at the outlet end of each time increment, Δt , to be expressed explicitly as

$$H_0^{(1)} = [H_0^{(0)}(L_s \theta_d - v \Delta t) + 2L_s I \Delta t] / (L_s \theta_d + v \Delta t) \quad (2.12)$$

When the saturated zone rises so that the water table intersects the soil surface, Eq. 2.9 and Eq. 2.12 must be modified, so that

$$S = D \theta_d (L_s + L_{sat}) / 2 \quad (2.13)$$

$$q = I L_{sat} + D v \quad (2.14)$$

where L_{sat} is the saturated slope length.

In the case of converging slopes such as the one shown in Fig. 2.3b, Eq. 2.9 is not valid to compute drainable volume. Sloan and Moore's model can be modified in such cases, so that

a) Plane hillslope

b) Converging hillslope

Figure 2.3: Plane and converging slope conditions

$$S = \frac{\pi L_s^2 H_0 \theta_d}{2} \left(\frac{\theta}{360} \right) \quad (2.15)$$

However, in general, some segments of the hillslope may be plane and some others converging. In such cases, equation similar to (2.15) must be developed to compute the control volume of the entire hillslope. Section 2.5 illustrates the application of the model to a watershed.

2.4 UNSATURATED FLOW MODEL

It is a well known fact that soil moisture storage in the unsaturated zone can provide a large buffer that regulates flow to the saturated zone. Harr and Yee (1975) in their field investigations observed the significant effect of unsaturated flow on the piezometric levels. It has been established by Campbell (1984) that the infiltration and percolation rates in the unsaturated zone are a function of the degree of saturation of the soil and not the rainfall rates. In other words, recharge to the groundwater table is not directly related to precipitation except through its relation to soil moisture. Nevertheless, many existing groundwater models assume that infiltration and percolation rates are a constant function of precipitation (for example, Sangrey et al. 1984). It is to be understood that the relationship between recharge and precipitation is only indirect and occurs via soil moisture content and affects not one but two variables, infiltration and percolation. In this section, a physically based mass balance model is developed for the flow in the unsaturated zone. The development of the model requires coupling of the evapotranspiration and drainage models discussed in the previous sections. It is assumed that flow in the unsaturated zone occurs only in the vertical direction. This is justifiable from Harr and Yee's observations at Mary's peak where they observed the unsaturated flow to be vertical throughout a particular storm period.

The unsaturated zone can be subdivided into three zones according to the predominant forces involved in causing the movement of soil moisture from surface to the water table (Eagleson, 1978). The division is as follows.

Soil moisture zone: Alternating wetting and drying of the soil surface during consecutive storm and interstorm periods cause a penetration of the soil by an unsteady wavelike diffusion of soil moisture. The diffusion is governed by complementary effects of capillarity and gravity during storm periods. During interstorm periods, capillarity opposes gravity. As depth of penetration increases, diffusion reduces the soil moisture gradients and thus reduces the effect of capillarity until moisture movement becomes dominated by gravity. The depth at which capillary forces become negligible determines the 'penetration depth' of the surface process and defines the thickness of the soil moisture zone. The presence of vegetation roots in this zone adds another mechanism for the soil moisture movement.

Percolation zone: Soil moisture moves out of the bottom of soil moisture zone and percolates downward under the domination of gravity forces until it encounters the increasing soil moisture gradients lying above the water table. At some depth the capillary forces, here upward at all times, will again dominate thus defining the bottom of 'percolation zone' or 'intermediate zone'.

Capillary zone: Between the water table and the percolation zone is a "capillary fringe" in which gravity and capillarity jointly govern the soil moisture movement.

This division of the unsaturated zone as shown in Fig. 2.4, is admittedly artificial, there being a continuous change of suction pressures in the entire unsaturated domain. Evapotranspiration as governed by the model developed in section 2.2, acts at the top of the unsaturated zone in combination with rainfall in the wet season and capillary rise in the dry season. Drainage, as governed by the model developed in section 2.3, contin-

uously takes place in the saturated zone. The fluctuation of the groundwater table obtained from the drainage model affect the thickness of the unsaturated zone. This results in continuous variation of the thicknesses of the three zones, L_1 , L_2 , and L_3 . However, Childs and Pouloussilis (1962) have shown it to be reasonable to assume that a rising water table tends to compress the unsaturated moisture profile into a shorter space, and conversely a falling water table extends it. Accordingly, it is assumed in the present research that the fluctuations of the water table result in proportionate elongations or compressions of the three zones. The following assumptions and conditions are made in developing the unsaturated flow equations.

1. The vertical soil column is of unit cross section
2. The flow is vertical
3. Rainfall accounted for evapotranspiration forms the special case for downward moisture flux (v_0) at the top
4. The moisture content distribution is linear in each zone (Fig. 2.5)
5. The gradients causing flow in the percolation zone are purely gravitational

The following water balance equation, based on conservation of mass in the soil, predicts expected water content in the three zones at various times.

$$\theta_j^{(1)}(L_j) = \theta_j^{(0)}(L_j) + (v_{j-1} - v_j - E_j)\Delta t \quad (2.16)$$

where,

$\theta_j^{(1)}, \theta_j^{(0)}$ = mean water contents in the j th zone

at times 1 and 0 respectively;

L_j = thickness of j^{th} zone;

E_j = evapotranspiration rate in the j^{th} zone

Figure 2.4: Division of the unsaturated zone

v_{j-1}, v_j = downward moisture fluxes at top and bottom
of the j^{th} zone respectively.

Downward moisture flux in Eq. 2.16 is evaluated by the Darcy equation for the unsaturated flow, which in this case is,

$$v_j = 0.5 K(\theta) \left[\left(\frac{\psi_j - \psi_{j-1}}{L_j} + 1 \right) + \left(\frac{\psi_{j+1} - \psi_j}{L_{j+1}} + 1 \right) \right] \quad (2.17)$$

where,

$K(\theta)$ = hydraulic conductivity as a function of
water content at the bottom of j^{th} zone.

ψ_{j-1}, ψ_j = suction potentials at top and bottom
of the j^{th} zone respectively.

Eq. 2.16 when applied to the soil moisture zone and percolation zone yields

$$\theta_1^{(1)}(L_1) = \theta_1^{(0)}(L_1) + \{(q - \alpha E_{eq}) - 0.5 K(\theta_2) \left[\frac{\psi_1 - \psi_0}{L_1} + 2 \right]\} \quad (2.18)$$

$$\begin{aligned} \theta_2^{(1)}(L_2) = \theta_2^{(0)}(L_2) + \{ & 0.5 K(\theta_2) \left[\frac{\psi_1 - \psi_0}{L_1} + 2 \right] \\ & - 0.5 K(\theta_2) \left[\frac{\psi_3 - \psi_1}{L_3} + 2 \right] \} \end{aligned} \quad (2.19)$$

The idealization shown in Fig. 2.5 is in accordance with the moisture profiles studied by Dunne(1978). He observed that in the cases where initial water table is shallow, saturation occurs due to a rising water table. This is particularly true when rainfall rates are less than permeability of the soil. It is to be noted that the moisture content in capillary zone (θ_3) need not be explicitly expressed as in Eq. 2.18 or 2.19 because it is

Figure 2.5: Idealization of moisture profile in the unsaturated zone

equivalent to the mean of θ_2 and moisture content at saturation θ_{sat} (Fig. 2.5). To obtain solutions of Eq. 2.18 and 2.19, the functions of $K(\theta)$ and $\psi(\theta)$ need be defined in a mathematically tractable form. Many functional forms have been suggested in the literature. A power curve has been proved to be a convenient descriptor for the moisture characteristic curve. Campbell (1974) suggested a method that allows the conductivity function to be estimated from this power curve with a matching factor of the saturated hydraulic conductivity K_{sat} . According to this method, the power curve representing the moisture characteristic is

$$\psi = \psi_s W^{-b} \quad (2.20)$$

in which the soil wetness W is equal to θ/θ_{sat} , where θ_{sat} is the saturated water content. ψ_s is the suction pressure at saturation and the exponent b is empirical and must be estimated. Using Eq. 2.20, Campbell derived a simple formula for k , the relative conductivity, as $k = W^{2b+2}$. But from the analysis of several soils, best results were obtained where the exponent was increased by 0.7. Consequently, Campbell suggested

$$k = \frac{K}{K_{sat}} = W^{2b+3} \quad (2.21)$$

Eq. 2.21 has been proved to be reasonably accurate over a wide range of b values (0.17-13.6) and W values.

Substituting Eq. 2.20 and 2.21 in 2.18 and 2.19 yields,

$$\theta_1^{(1)}(L_1) = \theta_1^{(0)}(L_1) + \{(q - \alpha E_{eq}) - 0.5 K_{sat} W_2^{2b+3} [\psi_s \frac{(W_2^{-b} - W_1^{-b})}{L_1} + 2]\} \quad (2.22)$$

$$\begin{aligned}\theta_2^{(1)}(L_2) = \theta_2^{(0)}(L_2) + \{0.5 K_{\text{sat}} W_2^{2b+3} [\psi_s \frac{(W_2^{-b} - W_1^{-b})}{L_1} + 2] \\ - 0.5 K_{\text{sat}} W_2^{2b+3} [\psi_s \frac{(1 - W_2^{-b})}{L_3} + 2]\}\end{aligned}\quad (2.23)$$

where, $W_1 = \frac{2\theta_1 - \theta_2}{\theta_{\text{sat}}}$, $W_2 = \frac{\theta_2}{\theta_{\text{sat}}}$.

The recharge T to be input in the drainage model (Eq. 2.12) is equivalent to flux v_2 (Fig. 2.4) accounted for change of moisture in the capillary zone ($\Delta\theta_3$); that is,

$$I = v_2 - (\Delta\theta_3)L_3 \quad (2.24)$$

where,

$$v_2 = 0.5 K_{\text{sat}} W_2^{2b+3} [\psi_s \frac{(1 - W_2^{-b})}{L_3} + 2]$$

and,

$$\begin{aligned}\Delta\theta_3 &= \frac{(\theta_2^{(1)} + \theta_s)}{2} - \frac{(\theta_2^{(0)} + \theta_s)}{2} \\ &= \frac{(\theta_2^{(1)} - \theta_2^{(0)})}{2}\end{aligned}\quad (2.25)$$

The terms L_1 , L_2 and L_3 in Eq. 2.22 and 2.23 must be updated at the end of every time step using head values from drainage model (Eq. 2.12). Thus,

$$(L_1)_{t+\Delta t} = (L_1)_t - (H_0^{(1)} - H_0^{(0)}) \left(\frac{L_1}{L_1 + L_2 + L_3} \right)$$

$$\begin{aligned}
 (L_2)_{t+\Delta t} &= (L_2)_t - (H_0^{(1)} - H_0^{(0)}) \left(\frac{L_2}{L_1 + L_2 + L_3} \right) \\
 (L_3)_{t+\Delta t} &= (L_3)_t - (H_0^{(1)} - H_0^{(0)}) \left(\frac{L_3}{L_1 + L_2 + L_3} \right)
 \end{aligned}
 \tag{2.26}$$

Therefore, the recharge terms T and head values H_0 couple the unsaturated flow model (Eq. 2.22 and 2.23) and the saturated model (Eq. 2.12) thus yielding the complete saturated-unsaturated flow model. When the entire unsaturated domain is saturated, Equation 2.26 reduces to $L_1 = L_2 = L_3 = 0$. While drainage model (Eq. 2.12) continues to operate in such cases, recharge to the water table is taken to be equal to rainfall limited by the saturated permeability of soil.

2.5 APPLICATION OF THE GROUNDWATER MODEL

The model developed in the previous section is applied to two sites. The first site is the Perkins Creek watershed located in the Siuslaw National Forest of the Oregon Coast Range. The second site is the Kennel Creek watershed located in Northeast Chicagof Island, Alaska, just south of Freshwater Bay. These sites are selected because of the following reasons.

1. The soil properties are homogeneous at both the sites. The soil cover at both the sites is shallow and is underlain by bedrock.
2. Extensive groundwater monitoring system has been employed at both the sites and the piezometric levels have been observed through several storms (Sidle, 1985; Pierson, 1977).

3. Numerical models have been developed and applied to the Kennel Creek watershed (Lee, 1987). The results from such models can be used to assess the validity of the present model.

2.5.1 Perkins Creek watershed

This watershed is located in the southern part of the Mapleton Ranger District of the Siuslaw National Forest (Fig. 2.6). The topography of the study area consists of narrow ridges, straight steep slopes ranging from about 35 to 55 degrees, and narrow stream valleys. For the purpose of applying the present model, two drainage basins in the watershed are chosen. Topographic maps of these basins are shown in Fig. 2.7. Piezometer locations are numbered. Dashed lines in the figure delineate the drainage area above each piezometer site. Dotted lines denote the drainage volumes for piezometric locations 2D and 3D. The average gradients of the longitudinal axes in the basins range from 36 to 41 degrees; depression side slopes are somewhat steeper, ranging from about 40 to 50 degrees.

The catchments lie within an old-growth forest stand of scattered mature Douglas Fir, which are being succeeded by Western Hemlock. These two species are distributed randomly throughout the watershed. The catchments are observed to be underlain by massive sandstone beds. Fig. 2.8 shows the particle size distribution of the soil cover. The high degree of aggregation gives these soils low bulk densities (0.8 to 1.3 g/cm^3) and high total porosities (55 to 70%) (Harr and Yee, 1975).

Pierson (1977) reported groundwater levels at piezometer locations 2D and 3D for significant storm periods between January 3 and December 11, 1976. Therefore, the model developed in the previous section is applied to the piezometric locations 2D and

Figure 2.6: Location of Perkins Creek watershed

Figure 2.7: Topographic maps of the Perkins Creek catchments

Figure 2.8: Soil composition at the Perkins Creek catchments

3D. The unsaturated soil properties to be used in the present model are taken from the results published by Clapp and Hornberger (1978). The soil classification for the present site falls under sandy loam, for which Clapp and Hornberger suggested the moisture characteristic, 'b' to be 4.90 and the suction at saturation, ψ_s , to be 21.8. The parameters required by the model for the two locations 2D and 3D are tabulated in Table 2.1.

Table 2.1
Parameters of the piezometer sites 2D and 3D at Perkins Creek

site	soil depth (m)	slope angle (deg)	slope length (m)	b	ψ_s (cm)	K_{sat} (cm/hr)	α
2D	1.7'	35'	142'	4.90*	21.8*	24'	0.85**
3D	2.7'	33'	144'	4.90*	21.8*	44'	0.85**

* Clapp and Hornberger (1978)
/ Measurements made by Pierson (1977)
** Black, T.A. (1979)

Figures 2.9 and 2.10 show the model results compared with field observations made by Pierson (1977). It can be seen that the results agree fairly well. The discrepancies between the model results and field observations are suspected to be mainly due to the simplifications in the present model, such as the shape of the groundwater profile in the drainage model and the mean values of material parameters used. Fig. 2.11 shows typical longitudinal profiles in the two catchments as observed by Pierson (1977). The correctness of the drainage model predictions depends on how much the observed profiles deviate from the assumed triangular shape.

Figure 2.9: Comparison of model results with observations at Perkins Creek for storm 1

Figure 2.10: Comparison of model results with observations at Perkins Creek for storm 2

Figure 2.11: Longitudinal saturated profiles at Perkins Creek

2.5.2 Kennel Creek watershed

This watershed is located on Northeast Chicagof Island, Alaska (Fig. 2.12) within a 40-ha unit designated for future timber harvest. The soil series at the site is classified as Kupreanoff series (Sidle, 1985). Overstory vegetation on the site is observed to be primarily Western hemlock and Sitka spruce with lesser amounts of Alaska-cedar (Sidle, 1985). The ground cover consists of various mosses and small forbs. Slope gradient in the general area ranges from 26 to 42 degrees. The entire landform was buried under ice during the Pleistocene period. Compact till was deposited on isolated areas of the underlying bedrock during retreat of the ice. Residual soils on the hillslope are shallow with mineral horizon generally less than 50 cm., and are developed over weathered graywacke, sandstone, granodiorite, limestone, or till. A fairly thick organic horizon ranging in depth from 10 to 50 cm. overlies the gravelly loam mineral soil with coarse fragment content as high as 50 percent. Soils developed in small hillslope depressions are frequently composed of colluvial material transported from past mass-wasting.

For the purpose of applying the model, drainage basin 'A' (Fig. 2.12) is chosen since piezometric levels are monitored at this location (Sidle, 1985). The topographic map of the drainage basin is shown in Fig. 2.13. Piezometric locations are numbered. The present mass balance model is applied to the piezometric locations 4-1 and 4-3 to check the predictions against the field observations made by Sidle (1985). At these two locations Sidle reported organic layers of about 27 cm. depth. It was observed in the field that water is held in the organic layer before the rainfall reaches the mineral soil (Personal communication with Wu). In other words, the organic layer gets saturated upto its storage capacity before transferring rainfall to the mineral soil. In the present study, the storage capacity of the organic layer is taken to be 50%. This is in accordance with

Figure 2.12: Location of Kennel Creek watershed

the porosity values reported for the top layers at the site (Schroeder, 1983). This is also in agreement with the tests conducted by Wu (1976) on the organic samples at the Maybeso valley site in Prince of Wales Island, Alaska. Thus in the present study, rainfall is applied at the top of the mineral soil after accounting for the storage capacity of the organic layer. To compute recharges using the present mass balance approach, 'b' is chosen to be 4.38 in Eq. 2.22 and 2.23. This is in accordance with the unsaturated soil properties suggested by Clapp and Hornberger (1978) for various soil types. Equilibrium evapotranspiration (E_e) for the site is found to be 0.01 cm/hr based on the temperature record for the site. α is chosen to be 0.85 (Spittlehouse and Black, 1981). Figures 2.14 and 2.15 show the model results compared with field observations made by Sidle (1985) at piezometric sites 4-1 and 4-3. It can be seen that the theoretical models predict the piezometric levels reasonably well at both the sites. The figures also show the predictions made by 2-d finite difference model to be studied in detail in the next section. In the present context, it is to be noticed that the mass balance model predicts as effectively as the more sophisticated 2-d model.

Figure 2.13: Topographic map of the Kennel Creek basin

Figure 2.14: Comparison of mass balance model and 2-d model with field observations at piez. 4-3

Figure 2.15: Comparison of mass balance model and 2-d model with field observations at piez 4-1

2.6 COMPARISON WITH 2-D FINITE DIFFERENCE MODEL

Lee (1987) developed a numerical model to study the porepressure variations in Kennel Creek watershed. Lee derived a 2-d flow equation using the Darcy's law and the continuity equation considering the effect of bedrock topography. The saturated flow domain is discretized and an explicit finite difference scheme is used to solve the governing equation. Lee assumed recharge to be instantaneous and therefore considered no phase lag between recharge and the occurrence of rainfall. The results from Lee's study can be used to assess the validity of the present model. Accordingly, this section is focussed on the following three aspects.

1. Evaluating the present drainage model against 2-d numerical model for the same recharge
2. Demonstrating the effect of the unsaturated zone by comparing the recharge from the present model with that used in the 2-d model
3. Comparing the 2-d model and the present mass balance model in light of field observations available for a particular storm at the site

2.6.1 Comparison with component models

To evaluate the drainage model, two representative segments B and C are chosen (Fig. 2.16). The segment B represents plane slope conditions and the segment C represents converging slope conditions. Eq. 2.9 is used to calculate the control volume in the former case and Eq. 2.15 is used in the latter. Same values of mean permeability (0.0349m/hr) and specific yield (34 percent) as used in the 2-d model are used in the drainage model. These are the mean values of measurements made by Sidle (1985). The recharge values for both the models are computed according to the following regression relationship used by Lee (1986).

$$R = -0.012 + 0.7P \quad (2.27)$$

The regression coefficients in Eq. 2.27 are obtained from case studies by Johnson (1977). Fig. 2.17 shows the comparison of 2-d numerical model and Sloan and Moore's model for the plane slope. Fig. 2.18 shows the comparison of 2-d numerical model and Sloan and Moore's model for the converging slope. It can be seen that the models agree fairly well under both conditions. Thus, it is believed that the present drainage model simulates the groundwater flow as effectively as the more sophisticated 2-d model. However, the following drawbacks should be noticed while using the present drainage model.

1. The use of mean hydraulic conductivities and mean hydraulic gradients ignores spatial variations, particularly for large control volumes.
2. The assumption that the water table has a constant slope between the upslope and downslope boundaries of the sloping soil mass may be violated by local depressions.

Fig. 2.19 compares the recharges obtained by the unsaturated flow model developed in section 2.4 and standard regression techniques such as Eq. 2.27. For the sake of comparison, piezometric location 4-3 is chosen. Fig. 2.19 demonstrates the following aspects.

1. Recharge is a function of antecedent moisture contents. The antecedent moisture conditions may produce recharge even before the storm begins. On the other hand, the standard regression procedures ignore this component
2. Recharge is continuous and occurs even during the interstorm periods. Regression procedures show no recharge during the interstorm periods.

Figure 2.16: Plane and converging segments in the Kennel Creek basin

Figure 2.17: Comparison of kinematic and 2-d models under plane slope conditions

Figure 2.18: Comparison of kinematic and 2-d models under converging slope conditions

Figure 2.19: Comparison of recharges obtained by mass balance model and other standard regression procedures

3. The unsaturated zone maintains a lag between the rainfall occurrence and the corresponding recharge. This lag, for a typical rainfall such as this, can be as great as 4 hours
4. Even after the cessation of rainfall, recharge may be considerable because of the increased moisture content in the percolation zone

Thus, it is concluded that recharge is related to rainfall only as far as soil moisture conditions are related to rainfall. Neglecting soil moisture conditions in the unsaturated zone might lead to misleading results.

Figures 2.14 and 2.15 show the comparison of the present mass balance model and the 2-d numerical model in light of field observations made by Sidle (1985). It can be noticed that the predictive accuracy of the mass balance model is as good as the 2-d model. This shows that the assumptions inherent in the development of complex models, such as that of instantaneous recharge, can be as limiting as the assumptions involved in the simpler models.

2.6.2 Spatial Variations

The groundwater predictions in the previous section were based on mean values of the parameters. The material was assumed to be homogeneous throughout the watershed. Due to one-dimensional nature of the present model, the effect of spatial variations of the material parameters cannot be examined. However, the variances of recharge and material parameters can be used in more sophisticated models to assess the effect of spatial variations.

The 2-d program developed by Lee(1986) for the Kennel Creek is used to evaluate the variances of head predictions due to the spatial variations of the material parame-

ters. The flow domain of the Kennel Creek as discretized by Lee is shown in Fig. 2.20. For the purpose of illustration, the variances of predicted heads in the element A (Fig. 2.20) are evaluated assuming variations of material parameters in the element B. Element B is chosen to be in the depression zone since spatial variations in depressed zones affect groundwater regime more than those in elevated zones. The parameters considered to be variable in the element B are: permeability, recharge, and depth of soil cover. The variance of head at A (H_A) due to permeability variation at B can be expressed as (Ang and Tang, 1984)

$$\text{Var}(H_A) = \left[\frac{\partial H_A}{\partial K_B} \right]^2 \text{Var}(K_B) \quad (2.28)$$

$\text{Var}(K_B)$ is obtained by taking the variance of all the permeability measurements made in the site (0.0019) (Sidle, 1985). The sensitivity, $\frac{\partial H_A}{\partial K_B}$ is obtained by running the program with different values of K_B around its mean (0.0349 m/hr). Similar to Eq. 2.28, the variance of H_A due to variation of recharge (I) at B can be expressed as

$$\text{Var}(H_A) = \left[\frac{\partial H_A}{\partial I_B} \right]^2 \text{Var}(I_B) \quad (2.29)$$

I_B , in itself, is a function of several random variables (Eq. 2.24). Therefore, variance of I_B can be expressed as (Ang and Tang, 1984)

$$\begin{aligned} \text{Var}(I_B) = & \left[\frac{\partial I_B}{\partial b} \right]^2 \text{Var}(b) + \left[\frac{\partial I_B}{\partial \psi_s} \right]^2 \text{Var}(\psi_s) + \left[\frac{\partial I_B}{\partial K_{\text{sat}}} \right]^2 \text{Var}(K_{\text{sat}}) \\ & + 2 \left[\frac{\partial I_B}{\partial b} \right] \left[\frac{\partial I_B}{\partial K_{\text{sat}}} \right] \text{Cov}(b, K_{\text{sat}}) \end{aligned} \quad (2.30)$$

Figure 2.20: Discretization of the Kennel Creek basin (after Lee, 1986)

For the purpose of illustration, the means and variances of the independent random variables in Eq. 2.30 are obtained from the existing literature (Clapp and Hornberger, 1978). These values are later tabulated in Table 3.1. The present mass balance model is used to obtain the sensitivities of I_B with respect to the random variables around their mean values. The variance of H_A due to variation in depth of soil cover, (D_B) at B can be expressed as

$$\text{Var}(H_A) = \left[\frac{\partial H_A}{\partial D_B} \right]^2 \text{Var}(D_B) \quad (2.31)$$

Measurements of depth of soil cover are available at various locations at the site (Sidle, 1985). Mean (0.685m) and variance (0.02636) of these measurements are used in the present analysis. The sensitivity, $\frac{\partial H_A}{\partial D_B}$ is again obtained by running Lee's 2-d program with different values of D_B around its mean. Variances of H_A calculated for every time step using Eq. 2.28, 2.29 and 2.31 are compared in Fig. 2.21. The comparison shows that the predicted head values are most affected by the variations of recharge and least affected by the variations in depth of soil cover for this particular problem.

Figure 2.21: Variance of head due to variation of parameters in the neighborhood element

CHAPTER III

DEVELOPMENT OF THE UNCERTAINTY MODEL

3.1 INTRODUCTION

The deterministic solutions developed in the previous chapter had to assume that the hydrogeologic parameters such as permeability and porosity are constant and known. But, it is well known that most of these parameters are not amenable to exact quantification, and can only be estimated approximately. Any model, however elaborate, that ignores the uncertainty in the parameters might lead to conclusions far from reality. Also, errors can arise in the solutions because of the assumptions and simplifications in the formulation of the model. These errors are generally classified as model error. Thus, the uncertainty associated with predictions of the present groundwater model is mainly due to two factors:

1. Uncertainty in the slope geometry and material parameters such as permeability and porosity
2. Uncertainty in the model structure due to assumptions and simplifications in the formulation

The deterministic approach developed in the previous chapter predicts mean values of groundwater level using mean values of the material parameters. In this chapter, the uncertainty of the mean groundwater response is modeled as a function of uncertainties in the mean values of parameters. Soil is assumed to be homogeneous throughout the watershed. In other words, spatial variations in the mean values of the parameters

are not considered. To obtain the total uncertainty of groundwater response at a point, uncertainties due to spatial variations as described in section 2.6.2 should be added to those developed in the present chapter.

A variety of procedures are available to treat the effects of uncertainty on groundwater flow. The various approaches can be divided into two main groups: full distribution analyses and first and second moment analyses. Full distribution methods include a complete specification of the probabilistic properties of all non-deterministic inputs and attempt to specify completely the probability distribution of the resulting flow. First order second moment methods assume that the first two moments of a random variable are sufficient to characterize it. On the basis of this assumption, they consider only the mean and variance-covariance of the dependent variables in the model. It is often difficult to obtain the probability distributions of the input parameters. Therefore, the results obtained using full distribution analyses can be deceptive because they depend on the exact distributions selected. For this reason, the first order second moment (hereafter FOSM) approach is selected in the present study. Section 3.2 reviews the general FOSM analysis. Section 3.3 deals with the application of FOSM analysis to the present groundwater model. The uncertainty model is applied to a field site in section 3.4.

3.2 REVIEW OF FOSM ANALYSIS

First order analysis of the moments of a random function is a simple way of estimating its probabilistic properties. The FOSM analysis is characterized by the following two features:

1. Under the assumption of statistical homogeneity for the medium, the random variable x is characterized by the first two moments, mean and variance, i.e.

$$x(t) = [\bar{x}(t), \sigma_{x(t)}^2]$$
2. Only the first order terms in the Taylor's expansion are used in dealing with functional relationships among random variables

Let f be a generic function of the random variable x . The function can be expanded in a Taylor series about the expected value of x , $\bar{x}$ as,

$$f(x) = f(\bar{x}) + f^{(1)}(x - \bar{x}) + \frac{1}{2} f^{(2)}(x - \bar{x})^2 + \frac{1}{6} f^{(3)}(x - \bar{x})^3 + \dots \quad (3.1)$$

with $f^{(k)}$ equal to the k th derivative of f with respect to x , evaluated at $\bar{x}$, the expected value of x . Neglecting higher order terms, the mean or expected value of f is calculated to be

$$E[f] = \bar{f} = E[f(\bar{x}) + f^{(1)}(x - \bar{x})] \quad (3.2)$$

$E[f]$ denotes the expected value of f . Since the expected value is a linear operation, Eq. 3.2 can be further simplified as

$$\begin{aligned} \bar{f} &= f(\bar{x}) + f^{(1)}E[x - \bar{x}] = f(\bar{x}) + f^{(1)}[E(x) - \bar{x}] \\ &= f(\bar{x}) \end{aligned} \quad (3.3)$$

The variance of f is the second moment around the mean value of f , i.e.
 $\sigma_f^2 = E[(f - \bar{f})^2]$. Using the Taylor series for f (Eq. 3.1), the variance is

$$\begin{aligned}
\sigma_f^2 &= E[\{f(\bar{x}) + f^{(1)}(x - \bar{x}) - f\}^2] \\
&= E[\{f(\bar{x}) + f^{(1)}(x - \bar{x}) - f(\bar{x})\}^2] \\
&= E[\{f^{(1)}(x - \bar{x})\}^2] = [f^{(1)}]^2 E[(x - \bar{x})^2] = [f^{(1)}]^2 \sigma_x^2
\end{aligned} \tag{3.4}$$

Very often, the functional relationship f involves a vector of random variables, y . The Taylor expansion of a vector with Vetter's (1973) notation is

$$f(y) = f(y_0) + \sum_{k=1}^{\infty} \frac{1}{k!} [D_{y^T}^k f] \cdot (y - y_0)^k \tag{3.5}$$

where f is a vector function of the vector y , and y_0 is the value of y around which the function is expanded. $[D_{y^T}^k f]$ is the k^{th} derivative of f with respect to the transpose of y , evaluated at y_0 , and $(y - y_0)^k$ is the result of k Kronecker products of $(y - y_0)$ with itself. It can be noticed that Eq. 4.6 is equivalent to Eq. 4.2 with a different notation. Neglecting higher order terms in Eq. 4.6, the expected value of f , i.e., $E[f]$ can be written as

$$\begin{aligned}
E[f] &= f = E[f(\bar{y})] + E[(D_{y^T} f)(y - \bar{y})] \\
&= f(\bar{y}) + (D_{y^T} f) E(y - \bar{y}) = f(\bar{y})
\end{aligned} \tag{3.6}$$

where the function is expanded around the expected value of y , $y_0 = E[y] = \bar{y}$. The covariance of f can be approximated to first order as

$$\begin{aligned}
\text{Cov}(f) &= E[(f - \bar{f})(f - \bar{f})^T] \\
&= E[\{f + (D_{y^T} f)(y - \bar{y}) - \bar{f}\} \{f + (D_{y^T} f)(y - \bar{y}) - \bar{f}\}^T] \\
&= E[(D_{y^T} f)(y - \bar{y})(y - \bar{y})^T (D_{y^T} f)^T] \\
&= (D_{y^T} f) E[(y - \bar{y})(y - \bar{y})^T] (D_{y^T} f)^T \\
&= (D_{y^T} f) \text{Cov}(y) (D_{y^T} f)^T
\end{aligned} \tag{3.7}$$

where $\text{Cov}(y)$ is the covariance of y and $(D_{y^T} f)$ evaluated at the expected value of y . It can be seen from Eq. 3.7 that the covariance of f is a function of the variability of y and the sensitivity of f to y in the neighborhood of $\bar{y}$. Thus the components of f that are insensitive to y will be little affected by uncertainty in y .

3.3 GROUNDWATER UNCERTAINTY MODEL

In the previous chapter, equations are developed to model the flow in both unsaturated and saturated zones. These equations are rewritten here for the sake of clarity. Water contents in the unsaturated zone are given by

$$\begin{aligned}
\theta_1^{(1)}(L_1) &= \theta_1^{(0)}(L_1) + \{(q - \alpha E_{eq}) - 0.5 K_{sat} W_2^{2b+3} [\psi_s \frac{(W_2^{-b} - W_1^{-b})}{L_1} + 2]\} \\
\theta_2^{(1)}(L_2) &= \theta_2^{(0)}(L_2) + \{0.5 K_{sat} W_2^{2b+3} [\psi_s \frac{(W_2^{-b} - W_1^{-b})}{L_1} + 2] \\
&\quad - 0.5 K_{sat} W_2^{2b+3} [\psi_s \frac{(1 - W_2^{-b})}{L_3} + 2]\}
\end{aligned}$$

and the groundwater level in the saturated zone is given by

$$H_0^{(1)} = [H_0^{(0)}(L_s \theta_d - v \Delta t) + 2L_s I \Delta t] / (L_s \theta_d + v \Delta t)$$

$$\text{where, } I = 0.5K_{\text{sat}} W_2^{2b+3} \left[\psi_s \frac{(1 - W_2^{-b})}{L_3} + 2 \right] - \left[\frac{\theta_2^{(1)} - \theta_2^{(0)}}{2} \right] L_3 .$$

The slope geometry needed by the drainage model is assumed to be known without uncertainty. The parameters considered to be uncertain in the above equations are listed below:

1. Evapotranspiration parameter, α ,
2. Suction pressure at saturation, ψ_s ,
3. Empirical coefficient in moisture characteristic relationship, b
4. Coefficient of permeability at saturation, K_{sat} ,
5. Drainage porosity, θ_d .

α , ψ_s and b belong to the unsaturated flow formulation and θ_d belongs to saturated flow formulation. Permeability coefficient at saturation, K_{sat} appears in both the formulations. The vector of random variables which is otherwise known as the augmented state vector (Schweppe, 1973) can now be written as

$$u(t) = \begin{pmatrix} s(t) \\ b \\ K_{\text{sat}} \\ \theta_d \\ \psi_s \\ \alpha \end{pmatrix} \quad (3.8)$$

where the state vector, $s(t)$, is given by

$$s(t) = \begin{pmatrix} \theta_1 \\ \theta_2 \\ H \end{pmatrix} \quad (3.9)$$

In this study, the evapotranspiration parameter, α is assumed to be uncorrelated with the rest of the parameters. Clapp and Hornberger (1978) suggested that ψ_s is not correlated with either b, K_{sat} or θ_d . So in this study, only correlation among b, K_{sat} and θ_d is considered. The variance matrix of $u(t)$ can be expressed as

$$\Sigma_{u(t)} = \begin{pmatrix} \Sigma_s & \Sigma_{sb} & \Sigma_{sK} & \Sigma_{s\theta} & \Sigma_{s\psi_s} & \Sigma_{s\alpha} \\ \Sigma_{sb}^T & \Sigma_b & \Sigma_{bK} & \Sigma_{b\theta} & - & - \\ \Sigma_{sK}^T & \Sigma_{bK} & \Sigma_K & \Sigma_{K\theta} & - & - \\ \Sigma_{s\theta}^T & \Sigma_{b\theta} & \Sigma_{K\theta} & \Sigma_\theta & - & - \\ \Sigma_{s\psi_s}^T & - & - & - & \Sigma_{\psi_s} & - \\ \Sigma_{s\alpha}^T & - & - & - & - & \Sigma_\alpha \end{pmatrix} \quad (3.10)$$

where Σ denotes the covariance matrix and T represents transpose of the matrix. In accordance with Eq. 3.7, the covariance matrix of the predictive model can be written as below.

$$\Sigma_{u(t+\Delta t)} = \begin{pmatrix} Q & B & K & T & P & A \\ - & I & - & - & - & - \\ - & - & I & - & - & - \\ - & - & - & I & - & - \\ - & - & - & - & I & - \\ - & - & - & - & - & I \end{pmatrix} \Sigma_{u(t)} \begin{pmatrix} Q^T & - & - & - & - & - \\ B^T & I & - & - & - & - \\ K^T & - & I & - & - & - \\ T^T & - & - & I & - & - \\ P^T & - & - & - & I & - \\ A^T & - & - & - & - & I \end{pmatrix} \quad (3.11)$$

where $Q_{(3,3)}, B_{(3,1)}, K_{(3,1)}, T_{(3,1)}, P_{(3,1)}$ and $A_{(3,1)}$ are sensitivity matrices of s , b , K_{sat} , θ_d , ψ_s and α respectively. If f denotes the functional relationship between $s(t)$ and the uncertain parameters, then the sensitivity matrices can be obtained as

$$\begin{aligned}
 Q &= \frac{\partial s(t + \Delta t)}{\partial s(t)} = \frac{\partial}{\partial s(t)} f[s(t), b, K_{sat}, \theta_d, \psi_s, \alpha] \\
 B &= \frac{\partial s(t + \Delta t)}{\partial b} = \frac{\partial}{\partial b} f[s(t), b, K_{sat}, \theta_d, \psi_s, \alpha] \\
 K &= \frac{\partial s(t + \Delta t)}{\partial K_{sat}} = \frac{\partial}{\partial K_{sat}} f[s(t), b, K_{sat}, \theta_d, \psi_s, \alpha] \\
 T &= \frac{\partial s(t + \Delta t)}{\partial \theta_d} = \frac{\partial}{\partial \theta_d} f[s(t), b, K_{sat}, \theta_d, \psi_s, \alpha] \\
 P &= \frac{\partial s(t + \Delta t)}{\partial \psi_s} = \frac{\partial}{\partial \psi_s} f[s(t), b, K_{sat}, \theta_d, \psi_s, \alpha] \\
 A &= \frac{\partial s(t + \Delta t)}{\partial \alpha} = \frac{\partial}{\partial \alpha} f[s(t), b, K_{sat}, \theta_d, \psi_s, \alpha]
 \end{aligned} \tag{3.12}$$

Expansion of Eq. 4.12 yields the following covariance matrices of the predictive model.

$$\begin{aligned}
 \Sigma_{s(t+\Delta t)} &= Q\Sigma_s Q^T + B\Sigma_b^T Q^T + K\Sigma_{sK}^T Q^T + T\Sigma_{s\theta}^T Q^T + P\Sigma_{s\psi_s}^T Q^T + A\Sigma_{s\alpha}^T Q^T \\
 &+ Q\Sigma_{sb} B^T + B\Sigma_b B^T + K\Sigma_b K^T + T\Sigma_{b\theta} B^T \\
 &+ Q\Sigma_{sK} K^T + B\Sigma_{bK} K^T + K\Sigma_K K^T + T\Sigma_{K\theta} K^T \\
 &+ Q\Sigma_{s\theta} T^T + B\Sigma_{b\theta} T^T + K\Sigma_{K\theta} T^T + T\Sigma_{\theta} T^T \\
 &+ Q\Sigma_{s\psi_s} P^T + P\Sigma_{\psi_s} P^T \\
 &+ Q\Sigma_{s\alpha} A^T + A\Sigma_{\alpha} A^T
 \end{aligned}$$

$$\Sigma_{sb(t+\Delta t)} = Q\Sigma_{sb} + B\Sigma_b + K\Sigma_{bK} + T\Sigma_{b\theta}$$

$$\Sigma_{sK}(t + \Delta t) = Q\Sigma_{sK} + B\Sigma_{bK} + K\Sigma_K + T\Sigma_{K\theta}$$

$$\Sigma_{s\theta}(t + \Delta t) = Q\Sigma_{s\theta} + B\Sigma_{b\theta} + K\Sigma_{K\theta} + T\Sigma_\theta$$

$$\Sigma_{s\psi_s}(t + \Delta t) = Q\Sigma_{s\psi_s} + P\Sigma_{\psi_s}$$

$$\Sigma_{s\alpha}(t + \Delta t) = Q\Sigma_{s\alpha} + A\Sigma_\alpha \quad (3.13)$$

It can be seen from Eq. 3.13 that the uncertainty in $s(t + \Delta t)$ is a function of uncertainties and sensitivities of the parameters.

3.4 APPLICATION OF THE UNCERTAINTY MODEL

The uncertainty model developed in the previous section is now demonstrated by applying to the piezometric location 4-3 in Kennel Creek watershed. To use the uncertainty model, one needs the variances of parameters to estimate the uncertainty of model predictions. However, for the particular problem, these values are not known exactly and therefore have to be estimated.

Clapp and Hornberger (1978), in their extensive study, obtained the average values and standard deviations of moisture characteristic parameter, b , and suction at saturation, ψ_s , for 11 different soil textural classes. Their results seemed to conform reasonably well with those of Holton, et al. (1968) who studied the properties of 1800 different soils. Therefore, for the sake of the present study, the values reported by Clapp and Hornberger are chosen. The evapotranspiration parameter, α , is studied extensively by Black, T.A. who suggested the range $0.6 < \alpha < 1.1$. The variance of α used in this

study is calculated from this range. The various values chosen in this study are tabulated in Table 3.1. The means and variances of permeability and porosity are obtained from measurements made by Sidle (1985) and Schroeder (1983). Even though these measurements are made at different locations in the site, they are assumed to represent a single homogeneous material. It is to be recalled here that spatial variations are ignored in the present FOSM analysis.

Table 3.1
Variances used for the uncertain parameters

Uncertain parameter	Mean	Variance
Moisture characteristic parameter (b)	4.38 [*]	2.16 [*]
Suction at saturation (ψ_s)	9.00 [*] cm	150.00 [*]
Evapotranspiration parameter (α)	0.85 [†]	0.0625 [†]
Permeability at saturation (K_{sat})	0.05 ^{**} m/hr	0.0019 ^{**}
Porosity (θ_d)	0.34 ^{***}	0.00034 ^{***}

* Clapp and Hornberger (1978)

† Black, T.A. (1979)

** Measured at the site (Sidle, 1985)

*** Schroeder, W.L. (1983)

Correlation coefficients among various parameters are not available to calculate the covariances. In the present study, strong positive correlation is assumed between permeability and porosity, and therefore the coefficient is chosen to be 0.8. Clapp and Hornberger's studies showed a positive correlation between the parameter 'b' and mean clay fraction in the soil type. On the other hand, mean clay fraction is negatively correlated with permeability and porosity. Therefore, it is assumed that a negative correlation exists between 'b', and permeability and porosity, and a correlation coefficient of -0.8 is chosen. Clapp and Hornberger observed no correlation between the suction at saturation, ψ_s , and any other parameters. Accordingly, in the present study, the parameters ψ_s and α are assumed to be uncorrelated with the rest of the parameters.

Figures 3.1, 3.2, and 3.3 show the results of the uncertainty model. For this particular problem, the initial values of θ_1 and θ_2 are not known exactly. Only the groundwater level, H , is known. Therefore, a coefficient of variation of 0.5 is chosen for θ_1 and θ_2 to represent the initial uncertainty. However, to show the effect of uncertainty in the initial conditions, computations are also made with no uncertainty in the initial conditions. The graphs show that in general, the uncertainty of h is considerably greater than that of θ_1 or θ_2 . This shows that h is more sensitive to the uncertainty in parameters than θ_1 or θ_2 . The graphs also show that the uncertainty in the initial conditions make the early predictions of θ_1 and θ_2 unreliable. However, the initial uncertainty has little effect on the latter predictions of θ_1 and θ_2 . The variances of head predictions are considerably affected by the initial variations of θ_1 and θ_2 (Fig. 4.3).

These uncertainty computations are done with the variance of parameters kept constant. The uncertainty in the parameters has not changed during the prediction since

Figure 3.1: Uncertainty in θ_i at piezometric location 4-3

Figure 3.2: Uncertainty in θ_2 at piezometric location 4-3

Figure 3.3: Uncertainty in H at piezometric location 4-3

no additional knowledge of the parameters is available during this period. If the behaviour of the system is monitored during the prediction period, and if there is a technique that can make inferences about the parameters from the system behaviour, then the variances in model predictions will be considerably reduced. The next chapter deals with such inference techniques.

CHAPTER IV

BAYESIAN UPDATING

4.1 INTRODUCTION

The uncertainty of groundwater predictions, described in the previous chapter, is mainly due to the variability of parameters involved. There are two ways to improve our knowledge, or reduce the uncertainty, about the parameters of the groundwater system. The first method is to measure them directly while the second one is to make inferences about them from real data of the input-output behavior of the system. Very often, it is costly and also difficult to measure the needed parameters. Therefore it is advantageous to develop a technique that estimates the parameters using any observations made on the system. One such technique is Bayesian updating. Section 4.2 describes the general Bayesian updating theory. The theory is applied to the present problem in section 4.2.

4.2 GENERAL BAYESIAN UPDATING THEORY

The four quantities that are important from the viewpoint of any estimation theory are :

1. The variables to be estimated : augmented state vector, u
2. The observations on the system : w

3. The mathematical model describing how the observations are related to the variables to be estimated
4. The uncertainties present

These quantities can be related by the following equation.

$$w = \Xi u + v \quad (4.1)$$

where Ξ is a known matrix relating the observations and variables to be estimated and v is an unknown observation disturbance. If $\hat{u}$ denotes an estimate of the mean value of u , then the objective of estimation theory is to find an operator Φ , such that

$$\hat{u} = \Phi w \quad (4.2)$$

Φ is an estimator determined by Ξ and uncertainty models for u and v . The function chosen for Φ should minimize $(u - \hat{u})$.

With the estimation theory as background, the updating theory can be defined by the following two models.

1. A priori model : the model for u which models the uncertainty before observations are made
2. A posteriori model : the model for $u - \hat{u}$ which models the uncertainty in u after the observations have been made and processed to yield the estimate $\hat{u}$.

Thus, the choice of estimator Φ in Eq. 4.2 depends on the a priori model. The a posteriori model depends on which estimator, i.e. which Φ , is chosen. There are many different ways to choose the estimators. Broadly, they fall under three categories:

1. Bayesian model : u and v are random vectors
2. Fisher model : u is completely unknown; and v is a random vector

3. Weighted least squares : no models for u and v
4. Unknown-but-bounded model : u and v are constrained to lie in specified sets

In the present study, the state vector u is random. Therefore, Bayesian model is chosen.

The Bayesian model is defined by the following conditions:

$$w = \Xi u + v \quad (4.3)$$

$$E\{u\} = m_u$$

$$E\{(u - m_u)(u - m_u)^T\} = \Psi$$

$$E\{v\} = 0$$

$$E\{vv^T\} = \Omega$$

$$E\{uv^T\} = 0$$

Schweppe (1973) shows that the estimator

$$\hat{u} = \Phi w + \Phi_0 \quad (4.4)$$

yields the minimum error covariance matrix for all Φ and Φ_0 when

$$\Phi = \Sigma_0 \Xi^T \Omega^{-1}$$

$$\Phi_0 = \Sigma \Psi^{-1} m_u \quad (4.5)$$

$$\Sigma_0 = (\Xi^T \Omega^{-1} \Xi + \Psi^{-1})^{-1}$$

m_u and Σ_0 in Eq. 4.5 are defined as

$$m_u = E\{\hat{u}\}$$

and

$$\Sigma_0 = E\{(u - \hat{u})(u - \hat{u})^T\}$$

In the above formulations, E denotes the expected value and superscript T denotes the transpose of a matrix. Matrix manipulations of Eq. 4.4 and 4.5 yield

$$\hat{u} = m_u + \Psi \Xi^T [\Xi \Psi \Xi^T + \Omega]^{-1} [w - \Xi m_u] \quad (4.6)$$

$$\Sigma_0 = \Psi - \Psi \Xi^T (\Xi \Psi \Xi^T + \Omega)^{-1} \Xi \Psi \quad (4.7)$$

Equations 4.6 and 4.7 represent the updated means and covariances of the state vector u . Thus in a dynamic system, if $\Sigma_{u(t+\Delta t|t+\Delta t)}$ represents the covariance matrix at time $(t + \Delta t)$ given measurements w of the state vector at time $(t + \Delta t)$, the minimum variance estimate for the state of the system is given by

$$\hat{u}(t + \Delta t|t + \Delta t) = \hat{u}(t + \Delta t|t) + K_u [w(t + \Delta t) - \Xi \hat{u}(t + \Delta t|t)] \quad (4.8)$$

with a covariance

$$\Sigma_{u(t+\Delta t|t+\Delta t)} = \Sigma_{u(t+\Delta t|t)} - K_u \Xi \Sigma_{u(t+\Delta t|t)} \quad (4.9)$$

where K_u , known as the Kalman gain matrix, can be written as

$$K_u = \Sigma_{u(t+\Delta t|t)} \Xi^T [\Omega + \Xi \Sigma_{u(t+\Delta t|t)} \Xi^T]^{-1} \quad (4.10)$$

4.3 APPLICATION OF BAYESIAN UPDATING

The theory developed in section 4.2 is now applied to the piezometric location 4-3 in Kennel Creek. In the present problem, only values of head H are measured and no knowledge is available about the moisture contents. The sensitivity of the parameter estimates to measured values of head depends on the sensitivity matrices B , K , T , P , and A (Eq. 3.12) through the covariance matrices. As a general rule, if the model predictions are insensitive to parameters, or if the measurement error Ω (Eq. 4.3) is significant, then the head measurements will not be very informative about parameters. On the other hand, if the model is sensitive to parameters compared to measurement error, then the parameters are more identifiable.

The error in the observations of head can be considered negligible when compared with the parameter uncertainty and model error. So, in the present study, the measurements are assumed to be error-free, i.e., Ω in Eq. 4.10 is 0. For each time step, the variance matrix of the state vector is first obtained using Eq. 3.11. Since measurements are made only on head H at the location 4-3, the matrix Ξ is expressed as

$$\Xi = [0, 0, 1, 0, 0, 0, 0, 0] \quad (4.11)$$

Using Ξ and the variance matrix $\Sigma_{u(t+\Delta t)}$, Kalman gain matrix is computed using Eq. 4.10. The means and variances of the state vector and the parameters are then updated using Eq. 4.8 and 4.9.

Figures 4.1 and 4.2 show the effect of updating on the variances of θ_1 and θ_2 at the end of every time step. The reductions in variance due to updating are shown by dotted lines. Figures 4.1 and 4.2, when compared with Figures 3.1 and 3.2, underscore the advantage of updating. Within ten hours of updating, the variances of θ_1 and θ_2 are reduced to negligible quantities. However, they are not totally reduced to zero

because of the presence of uncertainties in the parameters. Table 4.1 shows the initial and updated moments of the parameters.

Table 4.1
Parameters before and after updating

Parameter	Before updating		After updating	
	Mean	Variance	Mean	Variance
Moisture characteristic parameter (b)	4.38	2.16	4.00	0.0000
Suction at saturation (ψ_s) cm	9.00	150.00	1.00	1.6370
Evapotranspiration parameter (α)	0.85	0.0625	0.8496	0.0625
Porosity (θ_d)	0.34	0.0003	0.3110	0.6793×10^{-5}
Coefficient of permeability (K_{sat}) m/hr	0.05	0.0019	0.0505	0.000011

Out of all the parameters, the evapotranspiration parameter is least affected by updating. This lesser identifiability of α is due to its lesser influence on the measurements of head values. The parameters, b and ψ_s , are significantly affected by updating. Figures 4.3 and 4.4 show the effect of updating on means and variances of the parameters, b and ψ_s . It is observed that the variances of the parameters are reduced significantly within 10 hours of updating.

The identified values of the parameters can now be utilized to predict the groundwater levels for a future storm period at the specific site. The parameters can also be utilized to predict the groundwater levels in any other watershed with similar geological characteristics; however, differences in topographical characteristics might intro-

Figure 4.1: Variance reduction for due to updating

Figure 4.2: Variance reduction for due to updating

Figure 4.3: Update values of ' b '

Figure 4.4: Updated values of suction at saturation

duce new uncertainties into the system. To show the improvement in our knowledge of the parameters, the updated parameters are used to predict the piezometric levels at location 4-3 (Fig. 4.5). It can be noticed that the predictions with updated parameters agree more closely with observations than the previous predictions. The efficiency of updating can be considerably increased if more measurements are taken on the state of the system. This is to say, in the present problem, the uncertainty in parameter estimates can be greatly reduced if water contents in the unsaturated zone are also measured along with the piezometric levels. However, in practice, the model error makes it impossible to eliminate the parameter uncertainty totally. Figure 4.6 shows the predictions made at piezometric location 4-1 using the updated parameters at 4-3. It can be seen that the predictions with the new parameters agree more closely with observations than the previous predictions. However, in general, if the groundwater system has large spatial variations, the updated model at one location cannot be used to predict the groundwater response at another location. This is because the variations used to update the parameters did not include the effects of spatial variations described in section 2.6.2. Thus, the better agreement at 4-1 can be attributed to spatial homogeneity at the site. The spatial variations of the parameters usually make it impossible to identify unique values for the parameters in any watershed.

Figure 4.5: Predictions at piez 4-3 using updated parameters

Figure 4.6: Predictions at piez 4-1 using updated parameters

CHAPTER V

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.1 SUMMARY AND CONCLUSIONS

A comprehensive unsaturated-saturated flow model was developed to predict porewater pressures in a shallow soil cover on a sloping impervious bedrock. The porewater pressures were predicted as a function of rainfall, evapotranspiration, infiltration in the unsaturated zone, and drainage in the saturated zone. Appropriate individual models were studied for evapotranspiration, infiltration, and drainage. The evapotranspiration model was chosen to require only regularly observed meteorological data. A mass balance model was developed to represent the flow process in the unsaturated zone. Drainage in the saturated zone was computed using a kinematic storage model.

The validity of the kinematic storage model was checked by comparing its results with those of a more sophisticated 2-d model. The comparisons showed that the kinematic storage model simulates the saturated flow as effectively as the more sophisticated 2-d model. It was concluded that recharge is related to rainfall only as far as soil moisture conditions are related to rainfall. Neglecting soil moisture conditions in the unsaturated zone might lead to misleading results.

The unsaturated-saturated flow model developed was applied to two specific field sites. The first site is the Kennel Creek watershed located on Northeast Chicagof Island, Alaska and the second site is the Perkins Creek watershed located in the Siuslaw

National Forest of the Oregon Coast Range. Piezometric levels for various storm periods were available for these two sites. Model results showed reasonable agreement with the field observations. The stochastic nature of the flow model was examined by identifying the uncertain parameters in the model. The uncertain parameters considered are

1. Evapotranspiration parameter
2. Suction pressure at saturation
3. Moisture characteristic parameter
4. Permeability coefficient at saturation
5. Drainage porosity

The uncertainty of groundwater predictions was studied as a function of uncertainties in these parameters. The First Order Second Moment (FOSM) approach was chosen for this purpose. The stochastic model was applied to the Kennel Creek watershed site. Uncertainty in the antecedent moisture conditions was found to have considerable effect on the uncertainty of groundwater level predictions.

Bayesian Updating was used to study the reductions in prediction uncertainty given the input-output measurements of the groundwater system. The piezometer level observations made at the Kennel Creek watershed were used to update the hydrogeological parameters and moisture profile at the site. Considerable reductions in variances of the parameters were obtained due to updating. However, the parameters could not be totally identified because of the insufficient measurements on the groundwater system.

5.2 RECOMMENDATIONS

The model developed in this study is general in nature and can be applied to any watershed. The generality of the model demanded several simplifications. The author believes that simplifications are essential to build any model and should be given due importance in future investigations.

The primary aim throughout this study has been in terms of practicality. Therefore, the model was developed to require the least number of parameters. For instance, the evapotranspiration model was so chosen as to require only one site-dependent parameter, α , apart from regularly observed meteorological data. The dependence of α on vegetation and soil types should be studied in depth to gain confidence in the model. The division of unsaturated domain into three zones is quite artificial there being a continuous change of suction pressures in the entire domain. The effect of raining and declining water table on the unsaturated moisture profile must be further ascertained. Also, the universal validity of the moisture characteristic relationship adopted in this study has yet not been established.

The kinematic storage model adopted in this study includes many simplifying assumptions. Even though the assumption that the water table has a constant slope, is in general satisfactory, field observations by Pierson showed some exceptions. Piezometric levels in some downhill sections were observed to be concave upward. Future studies might attempt to relax the assumptions in the kinematic storage model.

The First Order Second Moment method used in this study is highly general in nature. However, it is to be noted that the method is an approximate linearization around the expected values of the parameters. Most of the groundwater problems such

as the present one are highly non-linear in nature. So, to be appropriate, the linearization must be conducted around those values which are most likely to cause the slope failure. For this purpose, preliminary slope stability analyses can be conducted to determine the most critical groundwater regime. Also, the basic assumption underlying the FOSM approach is that higher moments, skewness for example, are negligible for the uncertain variables concerned. Random variables fulfilling this assumption are those which are normally distributed. Thus, unless the hydrogeological parameters are normally distributed, the uncertainty predictions of FOSM approach may not be accurate. Also, previous investigations showed that FOSM approach may not work well with parameters having high variances. Other methods such as Monte Carlo simulation should be used whenever possible to check the FOSM predictions.

Probably, the most limiting feature of the present model is its inability to deal with spatial variation of the soil properties. Using mean properties of parameters might result in considerable inaccuracies especially when the flow domains considered are large. In this respect, the author feels that this study offers a challenging scope for future investigations.

Appendix A

COMPUTATION OF EQUILIBRIUM EVAPOTRANSPIRATION

The equilibrium evapotranspiration (E_e) is the expected rate of evapotranspiration from a moist surface of infinite extent. Priestley and Taylor (1972) expressed E_e as

$$E_e = \frac{s}{L(s + \gamma)} (R_n - G - M) \quad (A.1)$$

As can be seen in Eq. 2.1, E_{eq} is a function of several meteorological variables. However, all these variables can be estimated using regularly observed data such as temperature. Procedures to estimate individual variables are described below.

Latent Heat of Vaporization (L) :

Latent heat of vaporization (L) changes with temperature and is virtually unchanged by atmospheric pressure. Brunt (1952) expressed L as

$$L = 595 - 0.51 T, \text{ cal. g}^{-1}. \quad (A.2)$$

where T is temperature in °C .

Slope of Saturation Vapor Pressure Curve (s) :

Bosen (1960) presented the following expressions which were used to estimate the saturated vapor pressure (e^0) over water :

For $60^\circ\text{F} < T < 130^\circ\text{F}$,

$$e^0 = (0.0041T + 0.676)^8 - 0.000019|T + 16| + 0.001316 \quad (A.3)$$

where e^0 is in inches of Hg.

For $-51^\circ\text{C} < T < 54^\circ\text{C}$,

$$e^0 = 33.8639[(0.00738T + 0.8072)^8 - 0.000019|1.8T + 48| + 0.001316] \quad (A.4)$$

where e^0 is in mb.

Using these formulae, the slope of the saturation vapor pressure curve, s , can be evaluated as

$$s = \frac{de^0}{dT} = 0.0328(0.0041T + 0.676)^7 - 0.000019 \quad (A.5)$$

for $T \geq -16^\circ\text{F}$, s in inches $\text{Hg } ^\circ\text{F}^{-1}$.

and

$$s = \frac{de^0}{dT} = 33.8639[0.05904(0.00738T + 0.8072)^7 - 0.0000342] \quad (A.6)$$

for $T \geq -23^\circ\text{C}$, s in $\text{mb } ^\circ\text{C}^{-1}$.

Psychrometric Constant, (γ):

Psychrometric constant is a property of dry air and is directly dependent upon air pressure or more specifically altitude, and to a much smaller extent upon air temperature. The following expression is given by Brunt (1952).

$$\gamma = \frac{c_p P}{0.622 L} = \frac{\text{mb}}{^\circ\text{C}} \text{ or } \frac{\text{in. Hg}}{^\circ\text{F}} \quad (A.7)$$

where c_p is the specific heat of air at constant pressure, and P is the atmospheric pressure. For the sake of convenience, Burman, R.D. (1962) compiled the variations of $\left(\frac{s}{s+\gamma}\right)$ with elevation and temperature (Table A-1).

Table A.1

Variation of $\frac{s}{(s+y)}$ with elevation and temperature

		Elev., m = 0	500	1000	1500	2000	2500	3000
		Elev., ft = 0	1640	3281	4921	6562	8202	9843
°C	°F							
.0	32.	.401	.414	.428	.443	.458	.475	.494
5.0	41.	.477	.491	.505	.520	.536	.552	.570
10.0	50.	.551	.564	.578	.593	.608	.624	.641
15.0	59.	.620	.632	.645	.659	.673	.688	.703
20.0	68.	.681	.693	.705	.717	.730	.743	.757
25.0	77.	.735	.745	.756	.767	.778	.790	.801
30.0	86.	.781	.790	.799	.809	.818	.828	.838
35.0	95.	.820	.828	.835	.844	.852	.860	.869
40.0	104.	.852	.858	.865	.872	.879	.886	.893
45.0	113.	.878	.884	.889	.895	.901	.907	.913
50.0	122.	.900	.904	.909	.914	.919	.924	.929

Net Radiation Flux Density (R_n):

The net radiation flux density can be estimated from net short wave radiation received by a field and net outgoing thermal radiation. It is expressed as

$$R_n = 0.77 R_s - R_b \quad (\text{A.8})$$

where R_b is total solar radiation given in standard tables such as Table A-2 and R_b is the net outgoing thermal radiation.

Based on Stefan-Boltzmann law, Aase and Idso (1978) obtained an expression for R_b as,

$$R_b = \epsilon' \sigma T^4 \quad (\text{A.9})$$

$$\epsilon' = -0.02 + 0.261 \exp[-7.77 \times 10^{-4} (273 - T)^2] \quad (\text{A.10})$$

where T is temperature in Kelvins.

Table A.2
Mean Solar Radiation

Latitude °N	Month											
	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
60	58	152	319	533	671	763	690	539	377	197	87	35
55	100	219	377	558	690	780	706	577	430	252	133	74
50	155	290	429	617	716	790	729	616	480	313	193	126
45	216	365	477	650	729	797	748	648	527	371	260	190
40	284	432	529	677	742	800	755	674	567	426	323	248
35	345	496	568	700	742	800	761	697	603	474	380	313
30	403	549	600	713	742	793	755	703	637	519	437	371
25	455	595	679	720	742	780	745	703	660	561	486	423
20	500	634	652	720	726	760	729	697	680	597	537	474
15	545	673	671	713	706	733	706	684	697	623	580	519
10	584	701	681	707	684	700	681	665	707	648	617	565
5	623	722	690	700	652	663	645	645	710	665	650	606
0	652	740	694	680	623	627	616	623	707	684	680	619

Latitude °S	Month											
	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
5	648	758	690	663	590	587	577	590	693	690	727	677
10	710	772	681	640	571	543	526	558	680	690	727	710
15	729	779	665	610	516	497	497	519	657	687	747	739
20	748	779	645	573	474	447	445	481	630	677	753	761
25	761	779	626	533	419	400	406	439	600	665	767	777
30	771	772	600	497	384	353	358	390	567	648	767	793
35	774	754	568	453	335	300	310	342	530	629	767	806
40	774	729	529	407	281	243	261	290	477	603	760	813
45	774	704	490	357	229	183	203	235	447	571	747	813
50	761	669	445	307	174	127	148	177	400	535	727	806
55	748	630	397	250	123	77	97	123	343	497	707	794
60	729	588	348	187	77	33	52	74	283	455	700	787

Appendix B
COMPUTER PROGRAM LISTING

```

C*****
C*****
C   COMPUTER PROGRAM FOR STOCHASTIC GROUNDWATER FLOW
C
C           BY
C
C           LAKSHMI N. REDDI
C*****
C*****
C
C
C----- DEFINITIONS OF PRINCIPAL VARIABLES -----
C
C   ICHOIS   =   INDICATOR FOR CHOICE OF ANALYSIS
C
C               0,   MEAN ANALYSIS
C               1,   COVARIANCE ANALYSIS
C               2,   COVARIANCE ANALYSIS WITH UPDATING
C
C   GRID     =   C/S AREA OF UNSATURATED SOIL COLUMN
C
C   DB       =   DEPTH TO BEDROCK
C
C   SL       =   SLOPE LENGTH
C
C   GRAD     =   GRADIENT OF BEDROCK
C
C   DT       =   LENGTH OF EACH TIME STEP
C
C   NUMT     =   NUMBER OF TIME STEPS
C
C   EEQ      =   EQUILIBRIUM EVAPOTRANSPIRATION RATE (SEE APPENDIX A)
C
C   TS       =   WATER CONTENT AT SATURATION
C
C   PM,PCVR  =   MEAN AND VARIANCE OF THE PARAMETER 'B'
C
C   APS,KCVR =   MEAN AND VARIANCE OF SATURATED PERMEABILITY
C
C   PORS,LCVR=   MEAN AND VARIANCE OF POROSITY
C
C   AM,ACVR  =   MEAN AND VARIANCE OF EVAPOTRANSPIRATION PARAMETER
C
C   RS,SCVR  =   MEAN AND VARIANCE OF SUCTION AT SATURATION

```

C PKCVR = COVARIANCE BETWEEN PERMEABILITY AND 'B'
 C PLCVR = COVARIANCE BETWEEN POROSITY AND 'B'
 C KLCVR = COVARIANCE BETWEEN PERMEABILITY AND POROSITY
 C TL1OLD,TL2OLD,TL3OLD = HEIGHTS OF THE THREE ZONES IN THE
 C UNSATURATED DOMAIN
 C TOLD(1),TOLD(2),HOLD = INITIAL WATER CONTENTS IN THE FIRST
 C TWO ZONES AND GROUNDWATER LEVEL
 C H1CVR,H2CVR,H3CVR = INITIAL VARIANCES IN THE WATER CONTENTS
 C AND GROUNDWATER LEVEL
 C ARAIN = RAINFALL AT THE SITE FOR EACH TIME STEP
 C ZM = OBSERVATIONS MADE, IF ANY, ON THE GROUNDWATER LEVELS

C----- DATA ARE INPUT IN THE FOLLOWING ORDER -----

C
 C 1) TITLE (18A4)
 C 2) ICHOIS (I10)
 C 3) GRID,DB,SL,GRAD,DT,NUMT (5F10.4,I10)
 C 4) EEQ,TS (2F10.4)
 C 5) PM,PCVR (2F10.4)
 C 6) PS,KCVR,PKCVR,KLCVR (4F10.4)
 C 7) PORS,LCVR,PLCVR (3F10.4)
 C 8) AM,ACVR (2F10.4)
 C 9) RS,SCVR (2F10.4)
 C 10) TL1OLD,TL2OLD,TL3OLD (3F10.4)
 C 11) TOLD(1),TOLD(2),HOLD (3F10.4)
 C 12) H1CVR,H2CVR,H3CVR (3F10.4)
 C 13) ARAIN,ZM (2F10.4) -- ONE CARD FOR EACH TIME STEP

C-----

C
 C NOTE : IMSL ROUTINES VMULFF,VMULFP,LGINF ARE USED IN THE
 C PROGRAM

C
 C*****

```

C
C
C   MAIN PROGRAM
C
    REAL KCVR(100,100),LCVR(100,100),KLCVR(100,100)
    COMMON/SIZE/GRID,DB,SL,GRAD,DT
    COMMON/NUMBER/NUMT
    DIMENSION TOLD(100),TNEW(100),S(100,100)
    DIMENSION RAIN(1000)
    DIMENSION ARAIN(1000),ZM(1000)
    DIMENSION TIME(3000),SCVR(100,100)
    DIMENSION PCVR(100,100),PLCVR(100,100)
    DIMENSION ACVR(100,100),PKCVR(100,10),SVAR(100,100)
C
C   TITLE
C
    READ(5,1111) TITLE
1111 FORMAT(18A4)
    WRITE(6,1001) TITLE
1001 FORMAT(18A4)
    READ(5,9) ICHOIS
    9 FORMAT(I10)
C
C   DEFINE PARAMETERS
C
    READ(5,1) GRID,DB,SL,GRAD,DT,NUMT
    1 FORMAT(5F10.4,I10)
    WRITE(6,101) GRID,DB,SL,GRAD,DT,NUMT
101 FORMAT(1H1,4X,'GRID SIZE',3X,F10.4,/,
1      5X,'DEPTH TO BEDROCK',3X,F10.4,/,
2      5X,'SLOPE LENGTH',3X,F10.4,/,
3      5X,'GRADIENT OF BEDROCK',3X,F10.4,/,
4      5X,'LENGTH OF EACH TIME STEP',3X,F10.4,/,
5      5X,'NUMBER OF TIME STEPS',3X,I10,/)

```


C

```

      READ(5,81) EEQ,TS
81  FORMAT(2F10.4)
      WRITE(6,82) EEQ,TS
82  FORMAT(1H1,4X,'EQ. EVAPOTRANSPIRATION',3X,F10.4,/,
6      5X,'SATURATION WATER CONTENT',3X,F10.4,/)

```

C

C INPUT DATA OF COEFFICIENT (M)

C

```

      WRITE(6,102)
102  FORMAT(//,1X,'INPUT DATA OF COEFFICIENT(M)',//)
      READ(5,4) PM,PCVR(1,1)
4  FORMAT(2F10.4)
      WRITE(6,103) PM,PCVR(1,1)
103  FORMAT(2F10.4)

```

C

C INPUT DATA OF SATURATED PERMEABILITY

C

```

      WRITE(6,91)
91  FORMAT(//,1X,'INPUT DATA OF PERMEABILITY',//)
      READ(5,11) APS,KCVR(1,1),PKCVR(1,1),KLCVR(1,1)
11  FORMAT(4F10.4)
      WRITE(6,92) APS,KCVR(1,1),PKCVR(1,1),KLCVR(1,1)
92  FORMAT(4F10.4)
      PS=APS*GRID

```

C

C INPUT DATA OF POROSITY

C

```

      WRITE(6,93)
93  FORMAT(//,1X,'INPUT DATA OF POROSITY',//)
      READ(5,12) PORS,LCVR(1,1),PLCVR(1,1)
12  FORMAT(3F10.4)
      WRITE(6,94) PORS,LCVR(1,1),PLCVR(1,1)
94  FORMAT(3F10.4)

```

```
C
C   INPUT DATA OF ALPHA
C
      WRITE(6,106)
106  FORMAT(//,1X,'INPUT DATA OF ALPHA',//)
      READ(5,50) AM,ACVR(1,1)
      50  FORMAT(2F10.4)
      WRITE(6,107) AM,ACVR(1,1)
107  FORMAT(2F10.4)
C
C   INPUT DATA OF SUCTION AT SATURATION (RS)
C
      WRITE(6,108)
108  FORMAT(//,1X,'INPUT DATA OF RS',//)
      READ(5,109) RS,SCVR(1,1)
109  FORMAT(2F10.4)
      WRITE(6,110) RS,SCVR(1,1)
110  FORMAT(2F10.4)
C
C   DEFINE INITIAL CONDITIONS
C
      READ(5,95) TL1OLD,TL2OLD,TL3OLD
      95  FORMAT(3F10.4)
      WRITE(6,97)
      97  FORMAT(//,1X,'INITIAL HEIGHTS OF THE ZONES',//)
      WRITE(6,98) TL1OLD,TL2OLD,TL3OLD
      98  FORMAT(1X,3F10.4)
      TL1=TL1OLD
      TL2=TL2OLD
      TL3=TL3OLD
      READ(5,96) TOLD(1),TOLD(2),HOLD
      96  FORMAT(3F10.4)
      WRITE(6,113)
113  FORMAT(//,1X,'INITIAL VALUES AT TIME ZERO',//)
```

```

        WRITE(6,114) TOLD(1),TOLD(2),HOLD
114  FORMAT(1X,3F10.4)
        HNEW=HOLD
        TOLD(1)=TOLD(1)*GRID
        TOLD(2)=TOLD(2)*GRID
        TNEW(1)=TOLD(1)
        TNEW(2)=TOLD(2)
        TSAV=TOLD(2)
C
C   DEFINE INITIAL VARIANCES IN WATER CONTENTS AND/OR GROUNDWATER
C   LEVEL
C
        READ(5,192) H1CVR,H2CVR,H3CVR
192  FORMAT(3F10.4)
        WRITE(6,193)
193  FORMAT(//,1X,'INITIAL VARIANCES OF STATE VECTOR',//)
        WRITE(6,194) H1CVR,H2CVR,H3CVR
194  FORMAT(1X,3F10.4)
C
C   NUMBER OF TIME STEPS AND RAINFALL RATE
C   AT EACH TIME STEP
C
        DO 170 K=1,NUMT
            TIME(K)=0.0
            TIME(K)=K*DT+TIME(K)
170  CONTINUE
        DO 180 I=1,NUMT
            READ(5,175) ARAIN(I),ZM(I)
            RAIN(I)=ARAIN(I)*GRID
175  FORMAT(2F10.4)
180  CONTINUE
C
C   COMPUTE MEANS AND/OR VARIANCES OF THE STATE VECTOR
C   DEPENDING UPON THE CHOICE MADE (ICHOIS)

```

C

```

      DO 160 K=1,NUMT
      CALL TMEAN(TOLD,TNEW,TSAB,RAIN,PM,AM,TL2OLD,TL2,TL1OLD,
    *TL1,TL3OLD,TL3,HOLD,HNEW,TIME,K,PS,PORS,RS,EEQ,TS,ICHOIS)
      IF (ICHOIS .EQ. 0) GO TO 190
      CALL TVARI(TOLD,TSAB,RAIN,PM,AM,PCVR,KCVR,LCVR,ACVR,PKCVR,SCVR,
    *PLCVR,KLCVR,RS,EEQ,TS,H1CVR,H2CVR,H3CVR,
    *TL1OLD,TL1,TL2OLD,TL2,TL3OLD,TL3,HOLD,HNEW,K,PS,PORS,SVAR)
190 DO 299 I=1,2
      TOLD(I)=TNEW(I)
299 CONTINUE
      IF(ICHOIS .EQ. 0 .OR. ICHOIS .EQ. 1) GO TO 191
      CALL FILTER(S,K,ZM,TOLD,HNEW,PM,PS,PORS,AM,RS,SVAR)
191 HOLD=HNEW
      TL1OLD=TL1
      TL2OLD=TL2
      TL3OLD=TL3
160 CONTINUE
111 STOP
      END

```

C*****

C

```

C      THE FOLLOWING SUBROUTINE COMPUTES THE MEAN HEADS
C      AT EACH NODAL POINT

```

C

C*****

C

C

```

      SUBROUTINE TMEAN(TOLD,TNEW,TSAB,RAIN,PM,AM,TL2OLD,TL2,TL1OLD,
    *TL1,TL3OLD,TL3,HOLD,HNEW,TIME,K,PS,PORS,RS,EEQ,TS,ICHOIS)
      COMMON/SIZE/GRID,DB,SL,GRAD,DT
      COMMON/NUMBER/NUMT
      DIMENSION TOLD(100),TNEW(100)
      DIMENSION TIME(1),RAIN(1000)

```

```

VEL=(PS*GRAD)/GRID
WTR=TOLD(1)/TS
WT=TOLD(2)/TS
WTS=(WTR+WT)/2.
TOLD(3)=(TOLD(2)+TS)/2.
WT3=TOLD(3)/TS
WTB=(WT+WT3)/2.
C
C *** KINEMATIC MODEL FOR DRAINAGE ***
C   CONTROL VOLUME FOR THE SPECIFIC SITE SHOULD BE INCLUDED
C   IN EQUATION 2.8. HERE PIEZ. 4-3 AT KENNEL CREEK IS CHOSEN
C   AS AN EXAMPLE
C
RECH=.5*(PS*WTB**(2*PM+3))*(2.+RS*(1.-WT**(-PM)))/TL3)
RECHA=RECH-TL3*(TOLD(2)-TSAV)/2.
RECH1=TL3*(TOLD(2)-TSAV)/2.
IF (ICHOIS .EQ. 2 .AND. RECH1 .LT. 0.) RECHA=RECH+RECH1
OO=3.14*(SL**2)*RECHA/6.
IF (HNEW .GE. DB) FLAG=1.0
IF (FLAG .EQ. 1.0) OO=0.0
HNEW=(HOLD*(3.14*(SL**2)*PORS/12.-VEL)+OO/10000.)/
*(3.14*(SL**2)*PORS/12.+VEL)
IF (FLAG .EQ. 1.0) GO TO 100
TL1=TL1OLD-(HNEW-HOLD)/3.
TL2=TL2OLD-(HNEW-HOLD)/3.
TL3=TL3OLD-(HNEW-HOLD)/3.
TL3OLD=TL3
TL1OLD=TL1
TL2OLD=TL2
AA=(RS*(TOLD(2)**(-PM)-TOLD(1)**(-PM)))/(TL1*TS**(-PM))+2.
AAA=.5*PS*AA*WTS**(2*PM+3)
AK=RAIN(K)
TNEW(1)=TOLD(1)+(AK-AM*EEQ-AAA)/TL1
IF(TNEW(1) .GT. TS) TNEW(1)=TS

```

```

      IF(TNEW(1) .LE. 0.) TNEW(1)=300.0
      BB=(RS*(WT**(-PM)-WTR**(-PM))/TL1)+2.
      BBB=.5*(PS*((WT+WTR)/2.)*(2*PM+3))*BB-RECH
      TNEW(2)=(TOLD(2)*TL2+BBB)/TL2
      TSAV=TOLD(2)
      IF(TNEW(2) .GT. TS) TNEW(2)=TS
      IF(TNEW(2) .LE. 0.) TNEW(2)=300.0
      WRITE(6,102) TIME(K)
102  FORMAT(///,1X,'TIME IS',5X,F10.4,/)
      ATN1=TNEW(1)/GRID
      ATN2=TNEW(2)/GRID
      WRITE(6,118)
118  FORMAT(/,1X,'PREDICTED WATER CONTENTS AND H',/)
      WRITE(6,109) ATN1,ATN2,HNEW
109  FORMAT(3F10.4)
      GO TO 111
100  HNEW=HNEW+(RAIN(K)-AM*EEQ)/10000.
      WRITE(6,110) HNEW
110  FORMAT(1X,'FULL SATURATION',1X, 'HNEW=',2X,1F10.4)
111  RETURN
      END

C
C
C*****
C   THE FOLLOWING SUBROUTINE COMPUTES THE SENSITIVITY MATRICES
C   AND VARIANCE MATRIX OF THE AUGMENTED STATE VECTOR
C*****
C
C
      SUBROUTINE TVARI(TOLD,TSAV,RAIN,PM,AM,PCVR,KCVR,LCVR,ACVR,
      *PKCVR,SCVR,PLCVR,KLCVR,RS,EEQ,TS,H1CVR,H2CVR,H3CVR,
      *TL1OLD,TL1,TL2OLD,TL2,TL3OLD,TL3,HOLD,HNEW,K,PS,PORS,SVAR)
      REAL KCVR(100,100),LCVR(100,100),KLCVR(100,100)
      COMMON/SIZE/GRID,DB,SL,GRAD,DT

```

```

COMMON/NUMBER/NUMT
DIMENSION TOLD(100)
DIMENSION DHDH(100,100),DHDP(100,100),DHDS(100,100)
DIMENSION DHDA(100,100),AB(100,100),ABC(100,100)
DIMENSION HCVR(100,100),HPCVR(100,100),HACVR(100,100)
DIMENSION RAIN(1000),HSCVR(100,100)
DIMENSION DHDK(100,100),DHDL(100,100)
DIMENSION HKCVR(100,100),HLCVR(100,100),PCVR(100,100)
DIMENSION ACVR(100,100),SCVR(100,100),PLCVR(100,100)
DIMENSION PKCVR(100,100),SVAR(100,100),SAVE(100,100)

C
C
WTR=TOLD(1)/TS
WT=TOLD(2)/TS
WTA=(TOLD(1)+TOLD(2))/(2.*TS)
PSS=PS/GRID
TSS=TS/GRID
TOLD(3)=(TOLD(2)+TS)/2.
W23=(TOLD(2)+TOLD(3))/(2.*TS)

C
C -----
C   INITIAL VARIANCES AND COVARIANCES
C -----
C
DO 999 I=1,3
DO 999 J=1,3
999 HCVR(I,J)=0.0000
HCVR(1,1)=H1CVR
HCVR(2,2)=H2CVR
HCVR(3,3)=H3CVR
DO 998 I=1,3
HPCVR(I,1)=0.0
HKCVR(I,1)=0.0
HLCVR(I,1)=0.0

```

```

HSCVR(I,1)=0.0
998 HACVR(I,1)=0.0
  IF(K .EQ. 1) GO TO 111
  PCVR(1,1)=SVAR(4,4)
  PKCVR(1,1)=SVAR(4,5)
  KCVR(1,1)=SVAR(5,5)
  LCVR(1,1)=SVAR(6,6)
  ACVR(1,1)=SVAR(7,7)
  SCVR(1,1)=SVAR(8,8)
  PLCVR(1,1)=SVAR(6,4)
  KLCVR(1,1)=SVAR(6,5)
  DO 996 I=1,3
    HPCVR(I,1)=SVAR(I,4)
    HKCVR(I,1)=SVAR(I,5)
    HLCVR(I,1)=SVAR(I,6)
    HACVR(I,1)=SVAR(I,7)
    HSCVR(I,1)=SVAR(I,8)
996 CONTINUE
  DO 997 I=1,3
    DO 997 J=1,3
897 HCVR(I,J)=SVAR(I,J)
C
C   SENSITIVITY MATRICES SET UP HERE
C
111 DHDH(3,1)=0.0
    D1=(RS*(1.-WT**(-PM))/TL3)+2.
    D2=SL*PORS+PSS*GRAD
    D11=(0.5*PSS*(2.*PM+3.)*(W23**(2.*PM+2.))*D1)/(2.*TSS)
    D22=((0.5*PSS*W23**(2.*PM+3.))*RS*PM*WT**(-PM-1.))/TL3
    DHDH(3,2)=(D11+D22-TL3/20000.)*2.*SL/D2
    D3=SL*PORS-PSS*GRAD
    DHDH(3,3)=D3/D2
    DHDA(3,1)=0.0

```



```

KKK1=SL*(W23**(2.*PM+3.))*D1
KKK2=(KKK1*PSS)-((TOLD(2)-TSAV)*TL3/20000.)
DHDK(3,1)=((-2.*SL*PORS*HOLD*GRAD)+(KKK1*D2)-
*(KKK2*GRAD))/(D2**2)
D33=(EXP((2.*PM+3.)*ALOG(W23)))*2.*ALOG(W23)*D1
D44=(EXP(-PM*ALOG(WT)))*(W23**(2.*PM+3.))*ALOG(WT)*RS/TL3
DHDP(3,1)=(D33+D44)*SL*PSS/D2
DHDS(3,1)=(SL*PSS*(W23**(2.*PM+3.))*(1.-WT**(-PM)))/
*(TL3*D2)
D55=(0.5*PSS*(W23**(2.*PM+3.))*D1)-((TOLD(2)-TSAV)*TL3/20000.)
DHDL(3,1)=(2.*HOLD*PSS*SL*GRAD-(2.*D55*SL**2))/(D2**2)
X1=(RS*(WT**(-PM)-WTR**(-PM))/TL1)+2.
X2=(2.*PM+3.)*0.5*PSS*(WTA**(2.*PM+3.))*X1/(2.*TSS)
X3=0.5*PSS*(WTA**(2.*PM+3.))*RS*PM*(WTR**(-PM-1.))/(TL1*TSS)
DHDH(1,1)=1.-(X2+X3)/TL1
X4=(2.*PM+3.)*(WTA**(2.*PM+2.))*X1/(2.*TSS)
X5=(WTA**(2.*PM+3.))*(WT**(-PM-1.))*RS*PM/(TL1*TSS)
DHDH(1,2)=-0.5*PSS*(X4-X5)/TL1
CONS1=((RAIN(K)-AM*EEQ)/10000.)-0.5*PSS*(WTA**(2.*PM+3.))*X1
DHDHN=CONS1/(2.*((TL1OLD+HOLD/3.)-HNEW/3.))**2)
DHDH(1,3)=-DHDHN+DHDHN*DHDH(3,3)
DHDA(1,1)=-EEQ/(TL1*10000.)
X11=-0.5*(WTA**(2.*PM+3.))*X1/TL1
DHDK(1,1)=X11+DHDHN*DHDK(3,1)
X22=(EXP((2.*PM+3.)*ALOG(WTA)))*2.*ALOG(WTA)*X1
X33=RS*(WTA**(2.*PM+3.))*(-ALOG(WT)*EXP(-PM*ALOG(WT))
*+ALOG(WTR)*EXP(-PM*ALOG(WTR)))/TL1
X44=-0.5*PSS*(X22+X33)/TL1
DHDP(1,1)=X44+DHDHN*DHDP(3,1)
X55=-0.5*PSS*(WTA**(2.*PM+3.))*(WT**(-PM)-WTR**(-PM))/
*(TL1**2)
DHDS(1,1)=X55+DHDHN*DHDS(3,1)
DHDL(1,1)=DHDHN*DHDL(3,1)
Y1=(2.*PM+3.)*(WTA**(2.*PM+2.))*X1/(2.*TSS)

```

```

Y2=(WTA**(2.*PM+3.))*RS*PM*(WTR**(-PM-1.))/(TL1*TSS)
DHDH(2,1)=0.5*PSS*(Y1+Y2)/TL2
Y3=(2.*PM+3.)*(WTA**(2.*PM+2.))*X1/(2.*TSS)
Y5=(2.*PM+3.)*(W23**(2.*PM+2.))*D1/(2.*TSS)
Y4=RS*PM*(WTA**(2.*PM+3.))*(WTA**(-PM-1.))/(TL1*TSS)
Y6=(W23**(2.*PM+3.))*(WT**(-PM-1.))*RS*PM/TL3
DHDH(2,2)=1.+(Y3-Y4-Y5-Y6)*0.5*PSS/TL2
CONS2=(0.5*PSS*(WTA**(2.*PM+3.))*X1)-(0.5*PSS*
*(W23**(2.*PM+3.))*D1)
DHDHN2=CONS2/(2.*(TL2OLD+HOLD/3.-(HNEW/3.))*2)
DHDH(2,3)=-DHDHN2+DHDHN2*DHDH(3,3)
DHDA(2,1)=0.0
Y22=(0.5*(WTA**(2.*PM+3.))*X1)-(0.5*(W23**(2.*PM+3.))*D1)
DHDK(2,1)=(Y22/TL2)+DHDHN2*DHDK(3,1)
Y33=(EXP((2.*PM+3.)*ALOG(WTA)))*2.*ALOG(WTA)*X1
Y55=(EXP((2.*PM+3.)*ALOG(W23)))*2.*ALOG(W23)*D1
Y44=(-ALOG(WT)*EXP(-PM*ALOG(WT))+ALOG(WTR)*EXP(-PM*ALOG(WTR)))
**RS*(WTA**(2.*PM+3.))/TL1
Y66=RS*(W23**(2.*PM+3.))*EXP(-PM*ALOG(WT))*ALOG(WT)/TL3
Y77=0.5*PSS*(Y33+Y44-Y55-Y66)/TL2
DHDP(2,1)=Y77+DHDHN2*DHDP(3,1)
Y88=0.5*PSS*(WTA**(2.*PM+3.))*(WT**(-PM)-WTR**(-PM))/(TL1*TL2)
Y99=0.5*PSS*(W23**(2.*PM+3.))*(1.-WT**(-PM))/(TL2*TL3)
DHDS(2,1)=(Y88-Y99)+DHDHN2*DHDS(3,1)
DHDL(2,1)=DHDHN2*DHDL(3,1)
DHDH(1,1)=DHDH(1,1)+DHDHN2*DHDH(3,1)
DHDH(1,2)=DHDH(1,2)+DHDHN2*DHDH(3,2)
DHDH(2,1)=DHDH(2,1)+DHDHN2*DHDH(3,1)
DHDH(2,2)=DHDH(2,2)+DHDHN2*DHDH(3,2)

```

C

C

C COVARIANCE OF WATER CONTENTS

C

C DHDH*HCVR*DHDHT = (1)

```

C
CALL VMULFF(DHDH,HCVR,3,3,3,100,100,AB,100,IER)
CALL VMULFP(AB,DHDH,3,3,3,100,100,HCVR,100,IER)

C
C
C (1) + (DHDP*HPCVRT*DHDHT = (2)
C
CALL VMULFP(DHDP,HPCVR,3,1,3,100,100,AB,100,IER)
CALL VMULFP(AB,DHDH,3,3,3,100,100,ABC,100,IER)
DO 210 I=1,3
DO 210 J=1,3
HCVR(I,J)=HCVR(I,J)+ABC(I,J)
210 CONTINUE

C
C
C (2)+DHDA*HACVRT*DHDHT = (3)
C
CALL VMULFP(DHDA,HACVR,3,1,3,100,100,AB,100,IER)
CALL VMULFP(AB,DHDH,3,3,3,100,100,ABC,100,IER)
DO 220 I=1,3
DO 220 J=1,3
HCVR(I,J)=HCVR(I,J)+ABC(I,J)
220 CONTINUE

C
C
C (7)+(DHDK*HKCVRT*DHDHT) = (8)
C
CALL VMULFP(DHDK,HKCVR,3,1,3,100,100,AB,100,IER)
CALL VMULFP(AB,DHDH,3,3,3,100,100,ABC,100,IER)
DO 270 I=1,3
DO 270 J=1,3
HCVR(I,J)=HCVR(I,J)+ABC(I,J)
270 CONTINUE

C
C
C (8)+(DHDL*HLCVRT*DHDHT) = (9)
C
CALL VMULFP(DHDL,HLCVR,3,1,3,100,100,AB,100,IER)

```

```

      CALL VMULFP(AB,DHDH,3,3,3,100,100,ABC,100,IER)
      DO 280 I=1,3
      DO 280 J=1,3
      HCVR(I,J)=HCVR(I,J)+ABC(I,J)
280  CONTINUE
C
C      (9) + DHDS*HSCVRT*DHDHT = (10)
C
      CALL VMULFP(DHDS,HSCVR,3,1,3,100,100,AB,100,IER)
      CALL VMULFP(AB,DHDH,3,3,3,100,100,ABC,100,IER)
      DO 290 I=1,3
      DO 290 J=1,3
      HCVR(I,J)=HCVR(I,J)+ABC(I,J)
290  CONTINUE
C      DO 341 I=1,3
C      IF(HCVR(I,I) .LT. 0.0) HCVR(I,I)=0.0000
C      IF(HCVR(3,3) .LT. 0.0) HCVR(3,3)=0.0000
C 341  CONTINUE
C
C -----
C      COMPUTE HPCVR
C -----
C      DHDH*HPCVR = (1)
C
      CALL VMULFF(DHDH,HPCVR,3,3,1,100,100,AB,100,IER)
      DO 410 I=1,3
      DO 410 J=1,3
      HPCVR(I,J)=AB(I,J)
410  CONTINUE
C
C      (1)+DHDP*PCVR = HPCVR
C
      CALL VMULFF(DHDP,PCVR,3,3,1,100,100,AB,100,IER)
      DO 420 I=1,3

```

```

DO 420 J=1,3
HPCVR(I,J)=HPCVR(I,J)+AB(I,J)
420 CONTINUE
C
C      (2)+(DHDK*PKCVRT) = (3)
C
      CALL VMULFP(DHDK,PKCVR,3,1,1,100,100,AB,100,IER)
DO 430 I=1,3
DO 430 J=1,3
HPCVR(I,1)=HPCVR(I,1)+AB(I,1)
430 CONTINUE
C
C      (3) + DHDL*PLCVR = HPCVR
C
      CALL VMULFF(DHDL,PLCVR,3,1,1,100,100,AB,100,IER)
DO 440 I=1,3
HPCVR(I,1)=HPCVR(I,1)+AB(I,1)
440 CONTINUE
C
C -----
C      COMPUTE HKCVR
C -----
C
C      DHDH*HKCVR = (1)
C
      CALL VMULFF(DHDH,HKCVR,3,3,1,100,100,AB,100,IER)
DO 510 I=1,3
HKCVR(I,1)=HKCVR(I,1)+AB(I,1)
510 CONTINUE
C
C      (1)+(DHDP*PKCVR) = (2)
C
      CALL VMULFF(DHDP,PKCVR,3,1,1,100,100,AB,100,IER)
DO 520 I=1,3

```

```

      HKCVR(I,1)=HKCVR(I,1)+AB(I,1)
520 CONTINUE
C
C      (2)+(DHDK*KCVR) = (3)
C
      CALL VMULFF(DHDK,KCVR,3,1,1,100,100,AB,100,IER)
      DO 530 I=1,3
      HKCVR(I,1)=HKCVR(I,1)+AB(I,1)
530 CONTINUE
C
C      (3) + DHDL*KLCVR = HKCVR
C
      CALL VMULFF(DHDL,KLCVR,3,1,1,100,100,AB,100,IER)
      DO 540 I=1,3
      HKCVR(I,1)=HKCVR(I,1)+AB(I,1)
540 CONTINUE
C
C -----
C      COMPUTE HLCVR
C -----
C
C      DHDH*HLCVR = (1)
C
      CALL VMULFF(DHDH,HLCVR,3,3,1,100,100,AB,100,IER)
      DO 610 I=1,3
      HLCVR(I,1)=HLCVR(I,1)+AB(I,1)
610 CONTINUE
C
C      (1)+(DHDL*LCVR) = (2)
C
      CALL VMULFF(DHDL,LCVR,3,1,1,100,100,AB,100,IER)
      DO 620 I=1,3
      HLCVR(I,1)=HLCVR(I,1)+AB(I,1)
620 CONTINUE

```

```

C
C      (2) + DHDP*PLCVR = (3)
C
C      CALL VMULFF(DHDP,PLCVR,3,1,1,100,100,AB,100,IER)
C      DO 630 I=1,3
C      HLCVR(I,1)=HLCVR(I,1)+AB(I,1)
630 CONTINUE

```

```

C
C      (3) + DHDK*KLCVR = HLCVR
C
C      CALL VMULFF(DHDK,KLCVR,3,1,1,100,100,AB,100,IER)
C      DO 640 I=1,3
C      HLCVR(I,1)=HLCVR(I,1)+AB(I,1)
640 CONTINUE

```

```

C
C -----
C      COMPUTE HACVR
C -----

```

```

C
C      DHDH*HACVR = (1)
C
C      CALL VMULFF(DHDH,HACVR,3,3,1,100,100,AB,100,IER)
C      DO 470 I=1,3
C      HACVR(I,1)=AB(I,1)
470 CONTINUE

```

```

C
C      (1)+DHDA*ACVR = HACVR
C
C      CALL VMULFF(DHDA,ACVR,3,1,1,100,100,ABC,100,IER)
C      DO 480 I=1,3
C      HACVR(I,1)=HACVR(I,1)+ABC(I,1)
480 CONTINUE

```

```

C
C -----

```

```

C   COMPUTE HSCVR
C   -----
C
C   DHDH*HSCVR = (1)
C
C   CALL VMULFF(DHDH,HSCVR,3,3,1,100,100,AB,100,IER)
C   DO 910 I=1,3
C   HSCVR(I,1)=AB(I,1)
C   910 CONTINUE
C
C   (1) + DHDS*SCVR = HSCVR
C
C   CALL VMULFF(DHDS,SCVR,3,1,1,100,100,ABC,100,IER)
C   DO 920 I=1,3
C   HSCVR(I,1)=HSCVR(I,1)+ABC(I,1)
C   920 CONTINUE
C
C   -----
C   COMPUTE THE REST OF HCVR
C   -----
C
C   (10) + HPCVR*DHDPT = HCVR
C
C   CALL VMULFP(HPCVR,DHDP,3,1,3,100,100,ABC,100,IER)
C   DO 800 I=1,3
C   DO 800 J=1,3
C   HCVR(I,J)=HCVR(I,J)+ABC(I,J)
C   800 CONTINUE
C
C   (11) + HKCVR*DHDKT = (12)
C
C   CALL VMULFP(HKCVR,DHDK,3,1,3,100,100,ABC,100,IER)
C   DO 810 I=1,3
C   DO 810 J=1,3

```



```

      HCVR(I,J)=HCVR(I,J)+ABC(I,J)
810 CONTINUE
C
C      (12) + HLCVR*DHDLT = (13)
C
      CALL VMULFP(HLCVR,DHDL,3,1,3,100,100,ABC,100,IER)
      DO 820 I=1,3
      DO 820 J=1,3
      HCVR(I,J)=HCVR(I,J)+ABC(I,J)
820 CONTINUE
C
C      (13) + HSCVR*DHDST = (14)
C
      CALL VMULFP(HSCVR,DHDS,3,1,3,100,100,ABC,100,IER)
      DO 830 I=1,3
      DO 830 J=1,3
      HCVR(I,J)=HCVR(I,J)+ABC(I,J)
830 CONTINUE
C
C      (14) + HACVR*DHDAT = HCVR
C
      CALL VMULFP(HACVR,DHDA,3,1,3,100,100,ABC,100,IER)
      DO 840 I=1,3
      DO 840 J=1,3
      HCVR(I,J)=HCVR(I,J)+ABC(I,J)
840 CONTINUE
      IF(HCVR(1,1) .LT. 0.0) GO TO 747
      IF(HCVR(2,2) .LT. 0.0) GO TO 747
      IF(HCVR(3,3) .LT. 0.0) GO TO 747
      DO 309 I=1,3
      DO 309 J=1,3
309 SVAR(I,J)=HCVR(I,J)
      DO 710 I=1,3
      SVAR(I,4)=HPCVR(I,1)

```

```

SVAR(4,I)=HPCVR(I,1)
SVAR(I,5)=HKCVR(I,1)
SVAR(5,I)=HKCVR(I,1)
SVAR(I,6)=HLCVR(I,1)
SVAR(6,I)=HLCVR(I,1)
SVAR(I,7)=HACVR(I,1)
SVAR(7,I)=HACVR(I,1)
SVAR(8,I)=HSCVR(I,1)
710 SVAR(I,8)=HSCVR(I,1)
GO TO 757
747 DO 767 IO=1,3
DO 767 JO=1,3
767 SVAR(IO,JO)=SAVE(IO,JO)
DO 787 IP=1,3
SVAR(IP,4)=SAVE(IP,4)
SVAR(IP,5)=SAVE(IP,5)
SVAR(IP,6)=SAVE(IP,6)
SVAR(IP,7)=SAVE(IP,7)
787 SVAR(IP,8)=SAVE(IP,8)
757 SVAR(4,4)=PCVR(1,1)
SVAR(4,5)=PKCVR(1,1)
SVAR(5,4)=PKCVR(1,1)
SVAR(5,5)=KCVR(1,1)
SVAR(6,6)=LCVR(1,1)
SVAR(7,7)=ACVR(1,1)
SVAR(4,6)=PLCVR(1,1)
SVAR(5,6)=KLCVR(1,1)
SVAR(6,4)=PLCVR(1,1)
SVAR(6,5)=KLCVR(1,1)
SVAR(8,8)=SCVR(1,1)
WRITE(6,102)
102 FORMAT(/,1X,'VARIANCE MATRIX',/)
DO 311 I=1,8
311 WRITE(6,103) (SVAR(I,J),J=1,8)

```

```

103 FORMAT(8(2X,E10.4))
      DO 312 I=1,8
      DO 312 J=1,8
      SAVE(I,J)=SVAR(I,J)
312 CONTINUE
C
C
      RETURN
      END
C
C *****
C   SUBROUTINE COMPUTES THE UPDATED MEANS AND VARIANCES
C   USING BAYESIAN TECHNIQUE
C   NOTE: THIS ROUTINE WAS WRITTEN FOR UPDATING WHEN GROUNDWATER
C   LEVELS ARE KNOWN. WHEN OTHER PARAMETERS ARE KNOWN,
C   MODIFICATIONS HAVE TO BE MADE IN 'EM' MATRIX IN THE PROGRAM
C *****
C
C
      SUBROUTINE FILTER(S,K,ZM,TOLD,HNEW,PM,PS,PORS,AM,RS,SVAR)
C
      COMMON/SIZE/GRID,DB,SL,GRAD,DT
      COMMON/NUMBER/NUMT
      DIMENSION SVAR(100,100),EM(100,100),S(100,100),TOLD(100)
      DIMENSION XY(100,100),AINV(100,100),AB(100,100)
      DIMENSION AKAL(100,100),ZM(100),ZMINUS(100,100)
      DIMENSION ABC(100,100)
      DIMENSION SP(8),WK(14)
      S(1,1)=TOLD(1)/GRID
      S(2,1)=TOLD(2)/GRID
      S(3,1)=HNEW
      S(4,1)=PM
      S(5,1)=PS/GRID
      S(6,1)=PORS

```

```

S(7,1)=AM
S(8,1)=RS
DO 7 I=1,8
DO 7 J=1,8
7 EM(I,J)=0.0
EM(3,3)=1.0
DO 8 KZ=1,8
8 ZMINUS(KZ,1)=0.0
ZMINUS(3,1)=ZM(K)-S(3,1)
DO 9 IY=1,8
DO 9 JY=1,8
9 XY(IY,JY)=0.0
XY(3,3)=SVAR(3,3)
CALL LGINF(XY,100,8,8,0,AINV,100,SP,WK,IER)
CALL VMULFF(SVAR,EM,8,8,8,100,100,AB,100,IER)
CALL VMULFF(AB,AINV,8,8,8,100,100,AKAL,100,IER)
CALL VMULFF(AKAL,ZMINUS,8,8,1,100,100,ABC,100,IER)
C
C ----- UPDATE MEAN VALUES -----
C
DO 10 KL=1,8
10 S(KL,1)=S(KL,1)+ABC(KL,1)
IF(S(1,1) .LT. 0.0) S(1,1)=TOLD(1)/GRID
IF(S(2,1) .LT. 0.0) S(2,1)=TOLD(2)/GRID
IF(S(1,1) .GE. 0.5) S(1,1)=.4
IF(S(2,1) .GE. 0.5) S(2,1)=.4
IF(S(7,1) .LT. 0.0) S(7,1)=0.0
IF(S(7,1) .GE. 1.5) S(7,1)=1.5
IF(S(8,1) .LT. 1.0) S(8,1)=1.0
IF(S(4,1) .LT. 4.0) S(4,1)=4.0
IF(S(5,1) .LT. 1.5) S(5,1)=PS/GRID
IF(S(6,1) .LT. 0.2) S(6,1)=PORS
IF(S(6,1) .GT. 0.4) S(6,1)=0.4
WRITE(6,98)

```

```

98 FORMAT(/,1X,'UPDATED MEANS',/)
   WRITE(6,101) (S(IL,1),IL=1,8)
101 FORMAT(8(2X,E10.4))
   TOLD(1)=S(1,1)*GRID
   TOLD(2)=S(2,1)*GRID
   HNEW=S(3,1)
   PM=S(4,1)
   PS=S(5,1)*GRID
   PORS=S(6,1)
   AM=S(7,1)
   RS=S(8,1)
   CALL VMULFF(AKAL,EM,8,8,8,100,100,AB,100,IER)
   CALL VMULFF(AB,SVAR,8,8,8,100,100,ABC,100,IER)
C
C ----- UPDATE COVARIANCES -----
C
   DO 12 I=1,8
   DO 12 J=1,8
12  SVAR(I,J)=SVAR(I,J)-ABC(I,J)
   IF(SVAR(1,1) .LT. 0.0) SVAR(1,1)=0.0
   IF(SVAR(2,2) .LT. 0.0) SVAR(2,2)=0.0
   IF(SVAR(3,3) .LT. 0.0) SVAR(3,3)=0.0
   IF(SVAR(4,4) .LT. 0.0) SVAR(4,4)=0.0
   IF(SVAR(5,5) .LT. 0.0) SVAR(5,5)=0.0
   IF(SVAR(6,6) .LT. 0.0) SVAR(6,6)=0.0
   IF(SVAR(7,7) .LT. 0.0) SVAR(7,7)=0.0
   IF(SVAR(8,8) .LT. 0.0) SVAR(8,8)=0.0
   WRITE(6,99)
99  FORMAT(/,1X,'UPDATED COVARIANCES',/)
   DO 14 IP=1,8
14  WRITE(6,102) (SVAR(IP,JP),JP=1,8)
102 FORMAT(8(2X,E10.4))
C
C

```

RETURN
END

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