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Lewis, David Kent

**CHARACTERIZATION OF FLAWS AND STRUCTURAL INTEGRITY BY
ULTRASONIC SPECTROSCOPY**

The Ohio State University

PH.D. 1986

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**CHARACTERIZATION OF FLAWS AND STRUCTURAL INTEGRITY
BY ULTRASONIC SPECTROSCOPY**

DISSERTATION

**Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of the Ohio State University**

By

David Kent Lewis, B.S., M.S.

The Ohio State University

1986

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PATENT

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2. "Models for the Frequency Dependence of Ultrasonic Scattering from
Real Flaws," (with Laszlo Adler), presented at the ARPA/AFML
Review of Progress in Quantitative NDE, Asilomar Conference
Grounds, California, 1976.
3. "Identification of Flaws from Scattered Ultrasonic Fields as
Measured at a Planar Surface," (with Peter Szilas, Dale W.
Fitting, and Laszlo Adler), presented at the ARPA/AFML Review of
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4. "Flaw Location and Characterization in Anisotropic Materials by
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Laszlo Adler), presented at the Spring Conference of the American
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5. "Spectrum Analysis of Elastic Wave Scattering from Cracks in
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6. "Frequency Dependence of Ultrasonic Wave Scattering from Cracks,"
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9. "Characterization of Elliptical Cracks in the Time and Frequency Domains," (with F. Cohen-Tenoudji, Gerard Quentin, and Laszlo Adler), presented at the Ultrasonics International Meeting, Brighton, England, 1981.

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11. "Diffraction of Ultrasonic Waves by Griffith Cracks and Surface Breaking Cracks," (with Dale W. Fitting and Laszlo Adler), Proceedings of the ARPA/AFML Review of Progress in Quantitative NDE, LaJolla, California, 1980.
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INTRODUCTION

Introduction

This thesis is a compilation of experimental work aimed at the characterization of embedded flaws. The flaws were both planar cracks and spheroidal voids or inclusions. The work was conducted at the Physics department of the University of Tennessee in Knoxville, the Metals and Ceramics Division of the Oak Ridge National Laboratory in Tennessee, the Solid Physics Group of the University of Paris VII in France, and the Welding Engineering department of the Ohio State University in Columbus.

In the course of the work, it became necessary to investigate the effect of rough surfaces on the samples which contained the flaws, the effect of porosity in the elastic volume containing the flaws on the ultrasonic wave and the effect of a time dependent change in the elastic properties of the sample.

Since all of these secondary effects influenced the final form of the elastic wave scattered from the inclusions, some method of taking into account the presence of these conditions was necessary. To this end, various experimental systems were designed and built, data acquisition systems were designed and implemented, and methods of

processing the acquired data were written. The work drew heavily on previous work and ideas of others in the field, whose help is gratefully acknowledged.

This thesis details scattering from cracks and the inversion methods for determining the crack size and orientation from the scattered field in Chapter III.

Chapter IV covers the same determinations for embedded spheroids, both solid inclusions and voids. The effect of a randomly rough surface on the ultrasonic field used to investigate both is studied in Chapter V.

Chapter VI covers the study of porous volumes containing flaws, and can be applied as a method of studying porosity itself, though at present for the experimental procedure used, no method of determining the pore size exists without applying spacial averaging, which was avoided in the present work.

The case of flaw characterization in a changing elastic field is presented in Chapter VII in studies of the frequency dependent attenuation and phase velocity through a thin section of curing epoxy at constant temperature. The experimental system and methods of analysis used are presented in Chapter II, and Chapter I will consist of historical background for the methods and ideas used in this work.

CHAPTER I

BACKGROUND

Embedded Cracks

Investigation of material flaws by wideband ultrasound was suggested by Gericke¹ in 1963. He demonstrated the inability of conventional backscattered amplitude methods in determining the size of a flat crack, unless the beam was at normal incidence to the crack face.

Adler and Whaley² developed a model for scattering from flat discontinuities in 1972, and found they could determine the relative separation of the nearest and farthest points on the scatterer from the orientation of the crack face and the separation of the minima in the modulation envelope of the spectral density of the scattered wave. An analysis of the effects of equivalent time signals separated by some time interval on the spectrum relative to a single signal was done by Simpson³ in the same year, and provided a theoretical basis for analyzing nonnormally orientated cracks. This research proceeded into more elegant analysis methods covered in later chapters, and the method of spectral analysis of scattered ultrasonic waves was firmly established.

Theoretical consideration on the scattering of elastic waves from obstacles had begun some years earlier with the derivation of the exact

solution for scattering by a rigid half plane by Fridman⁴ in 1944 and the corresponding solution for a traction free half plane by Maue⁵ in 1953. An application of a separable coordinate system other than the spherical or Cartesian systems was done by Filipczynski⁶ in 1961 for normally incident longitudinal waves. He derived both the scattered longitudinal and the mode converted shear wave components in the far field for the long wavelength case.

To solve the diffraction problem for an aperture in optics in the short wavelength or ray regime, Keller^{7,8} postulated a geometrical theory of diffraction. This idea was adopted by Adler and Lewis^{9,10} to study the circular crack further, and the methods of geometrical diffraction were applied to elastodynamics in 1977 by Achenbach and Gautesen.¹¹ The next year, Domany, Krushansl, and Teitel¹² solved the long wavelength scattering for circular cracks in what they termed the quasi-static approximation, in that the solution involved an expansion in frequency terms, the limit of zero frequency yielding the steady state stress and displacement solutions.

The diffraction corrections for geometrical elastodynamic diffraction were worked out and tested against experiment in 1978 by Adler, Szilas, Fitting, and Lewis¹³ and in 1979 by Achenbach, McMaken, Adler, and Lewis.¹⁴ They used the exact solutions for the half plane from Maue in the method outlined by Keller and found a good agreement to actual scattering in the short wave limit.

In addition, the latter suggested and verified an inversion model for scattering from general elliptical flat cracks. In the next year,

deBilly, Cohen-Tenoudji, Quentin, Adler, and Lewis¹⁵ considered the case of a circular crack whose face exhibited a fracture surface, in connection with their studies of randomly rough surfaces.

Both rough and smooth faced elliptical cracks large relative to the wavelength of the incident elastic field will be discussed at length later in this work. An excellent survey of some of these early theoretical works is given by Kraut.¹⁶

Surface Cracks

Surface breaking cracks are by their very nature perhaps the most dangerous of all material defects. They occur in that part of a material which is usually subjected to the maximum stresses. They are difficult to characterize, since in the absence of stress, they are often closed, rendering liquid penetrant methods inaccurate, radiographic or X-ray methods useless in many cases, and microphotography unable to determine their depth. Since they are best described as a discontinuity in the materials physical properties, ultrasound has been used to characterize their depth and orientation. It is even conceivable that applications of determination of the properties of both rough and periodic surfaces can be used to distinguish between brittle crack surfaces, and the more periodic structures of a fatigue crack. This section will examine the progress already made in the characterization of surface cracks, and the

direction taken by other researchers currently active in the field to extend this ability.

The first approach to the problem of determining the characteristics of surface cracks used the scattering of waves from both the crack and the surface. Silk and Liddington¹⁷ in 1975, and Liddington, Silk, and Montgomery¹⁸ in 1976 considered the interaction of surface cracks with bulk waves. Their experiments used a two transducer arrangement on the cracked surface to launch steeply angled ultrasonic beams into the samples. The time delay between the signal diffracted from the crack tip and the signal specularly reflected from the opposite flat smooth side of the sample yielded accuracies of ± 50 microns for cracks of 0.5 to 30 millimeters normal to the surface.

Achenbach and Norris¹⁹ considered the bulk wave problem in 1980. They considered a diffraction problem, and looked at the interference between the crack tip and the surface edge. Their model predicted a periodicity in the scattered amplitude's frequency spectrum which was not sensitive to the angular orientation of the crack to the surface, and the magnitude of the scattered amplitude spectrum to be dependent on the angular orientation of the crack face. Since their model predicted periodicity, they were able to develop an inversion method to yield the crack depth directly.

Rayleigh wave scattering was employed in 1978 by several researchers, since their use meant that no flat smooth parallel surface was required on the opposite side of the specimen. These surface waves are nondispersive, but diminish rapidly in amplitude with increasing

depth into the sample. Their properties are worth a digression, since these properties bear directly on one's ability to measure a defects parameters accurately.

First predicted theoretically by Lord Rayleigh²⁰ in 1885, they were regarded as somewhat of a mathematical curiosity until the 1950's. Goos and Hanchen²¹ had found a change in the amplitude spectrum of optical waves at nonnormal incidence in 1947. Schoch²² proposed a theoretical explanation for this change in 1952, and in 1953,²³ reported the same effect in acoustic waves. Using a Schlieren system to visualize the ultrasonic beam, he showed that the energy redistribution in the beam was similar to the optical case.

Neubauer²⁴ measured the reflection coefficient as amplitude versus incident angle in 1973, and found that a significant portion of the incident beam was converted to a Rayleigh wave. In the same year, Bertoni and Tamir²⁵ analyzed the problem as the effect of a finite beam, rather than the usual assumption of an infinite plane wave, and predicted the amplitude redistribution in the beam for Rayleigh incidence. Breazeal, Adler, and Scott²⁶ confirmed the theory in 1977. For an excellent discussion and summary of both Rayleigh and Lamb waves, see the book by Viktorov²⁷ listed in the references.

In 1981, Nayfeh, Chimenti, Adler, and Crane²⁸ investigated the effects of a thin layer bonded to a surface on the Rayleigh wave. They developed a theory incorporating the change in material properties at the layer-sample interface, and found that the resultant surface wave was dispersive, similar to a Lamb wave. This allowed cracks in a

coating to be detected, and, since the bond coupling influenced the wave equation, gave a method for determining bond integrity.

In 1982, Adler, Bolland, deBilly, and Quentin²⁹ found that in immersed systems, Rayleigh waves were backscattered, and developed a method for determining the surface wave velocity using a single transducer. They also observed surface wave generation for any angle of incidence by a finite beam.

Budianski and Rice³⁰ considered scattering in the long wavelength limit, using Rayleigh waves in 1978, while the short wavelength limit was investigated both theoretically and experimentally by Tittmann, Cohen-Tenoudji, deBilly, Jungman, and Quentin³¹ in the same year. Tittmann, Cohen-Tenoudji, deBilly, and Quentin³² also considered the case of a crack whose dimensions were of the same general size as the Rayleigh wavelength. These and other researchers used interdigital transducers deposited on the surface, angled wedges coupled to standard contact ultrasonic transducers to introduce the beam at the Rayleigh angle, and immersed pairs of transducers to launch the beam into the sample surface and to pick up the scattered "leaky" Rayleigh wave.

Achenbach, Gautesen, and Mendelsohn³³ and Angel and Achenbach³⁴ considered the surface crack problem using a full elastodynamics approach to scattering in 1980, and predicted the transmission and reflection coefficients of an incident Rayleigh wave as a function of crack depth for any arbitrary angle of incidence. Dong and Adler³⁵ confirmed the theory for surface cracks in titanium of .4 to 1.5 millimeters both as to coefficient amplitude as a function of angle for

fixed frequency and as to scattered frequency spectrum for fixed angle of incidence and scattering. Finally, the case of an edge crack has been considered by Achenbach, Keer, and Mendelsohn³⁶ in an elastodynamic analysis, though no experimental confirmation has been attempted as far as I am aware.

Embedded Spheroids

The scattering of elastic waves by spherical inclusions was worked out in exact form for incident longitudinal waves by Ying and Truell³⁷ in 1956. The same problem for incident transverse or shear waves was treated in detail by Einspruch, Witterholt, and Truell³⁸ in 1960. Both used an expansion of partial plane waves and matched the boundary conditions at the inclusion surface exactly. The solution incorporated mode conversion by means of the stress boundary conditions on the inclusion.

Kraft and Franzblau³⁹ considered these papers in 1971, and found the scattering cross sections for both cases of both shear and longitudinal components for a stress free void. All these methods made use of eigenfunctions for the Helmholtz problem in the spherical coordinate system. Since the wave functions for more general spheroidal shapes are not as readily available as the spherical functions, and since most practical scattering problems are not nearly as neat as a solution in a separable coordinate system, various approximations have been used to approach more complicated problems.

A numerical approach to the scattering problem is the transition or T matrix theory. Beginning in 1969, Waterman,^{40,41} using the boundary surface formulation, expanded all the field variables in terms of eigenfunctions appropriate to the geometry at hand.

Using these expansions for the incident displacement field and the Green's function solution to the impulse response problem, a sphere is designated around the scattering center and the expansion coefficients determined for the Green's function. Next the scatterer's surface is considered and the coefficients of the incident displacement field lead to the coefficients of the two modes of scattered fields. The original papers consider the cases of voids and rigid, elastic and fluid filled spheres.

Varadan and Varadan⁴² have extended these ideas to include oblate and prolate spheroids, and to approximate symmetric nearly spheroidal inclusions in an elastic medium.

The appeal of the T matrix approach is that once solved, the solution for any incident wave and any material properties of the inclusion is immediately available by putting the physical values in the coefficients. The draw back is the amount of computer storage needed to maintain all these coefficients. The theories also are limited by the terms needed for good convergence, and in practice form a numerical solution in the long wavelength regime.

The use of perturbation methods was applied to elastic scattering by Gubernatis, Domany, Krumhansl, and Huberman⁴³ in 1979. They assumed a soft scatterer, that is a scatterer whose elastic properties varied

only slightly from those of the host medium. They further clarified some of the more formal aspects of the approximation the next year,⁴⁴ and Adler and Lewis⁴⁵ compared the overall scattering field for scattering to an arbitrary angle within 60 degrees of the back direction from normal incidence.

In 1979, Lewis and Adler⁴⁶ discussed the experimental aspects for applying the by then named Born approximation for two transducer scattering, and Rose and Krumhansl⁴⁷ worked out a way to characterize the flaw from the back scattering spectra. Elsley and Addison⁴⁸ and Elsley, Ahlberg, and Richardson⁴⁹ looked at the effects of noise in the received signal and the limited bandwidth of most transducers in 1980, and found good agreement for the flaw characterization inversion within the range $0.5 < ka < 2.5$.

The same year, Rose, Varadan, Varadan, Elsley, and Tittman⁵⁰ considered the case of nonspheroidal voids, using new diffusion bonded samples.

Rough Surfaces

The original treatment of scattering of scalar waves from a periodic rough surface was published by Lord Rayleigh in 1926.⁵¹ He considered a periodic sine profile, and reached a solution by using an infinite sum of partial wave functions to match the boundary conditions at the boundary. Meecham⁵² extended this treatment to cover arbitrary

two dimensional roughness using an extension of the same technique in 1956, based on Baker and Copson's⁵³ treatment of Huygen's principle of scattered spherical wavelets from each part of the boundary.

Clay⁵⁴ considered coherent scattering of acoustic waves from a randomly rough surface in 1966, using the Fresnel phase approximation for short wavelengths relative to the surface irregularities. In 1970, Clay and Medwin⁵⁵ tested their corrected theory against experiment again in the short wavelength approximation. Melton and Horton^{56,57} examined this correction allowing different irregularities than Gaussian, and considering backscattering.

Boyd and Deavenport⁵⁸ considered the whole problem of both coherent and incoherent or long wavelength scattering in 1973, using a modified Green's function solution and integral expansion of the scattered field. They were able to predict the distribution of energy more accurately than was possible before, and could cover many different distributions of irregularities than Gaussian, though at considerable mathematical complexity.

From an alternate point of view, Kupradze⁵⁹ and Vekula⁶⁰ in 1965 and 1967 respectively had realized that since the solutions of the Helmholtz equation were harmonic, one could formulate solutions to it in terms of a potential in the same way as Newtonian potentials solve Laplace's equation. Following this approach, Melton⁶¹ was able to solve the scattering problem in 1973 for any statistically rough continuous surface as an infinite series of potentials in analogy to the Born expansion for scattering. DeBilley, Dohen-Tenoudji, Jungman, and

Quentin⁶² applied this method to characterize rough surface in 1976, determining both the average roughness and the autocorrelation length for several surfaces.

In this thesis we present work done with this group and Adler on methods developed for determining both the scattering amplitude and the phase experimentally in reflection from and transmission through randomly rough surfaces in case materials. The application of this method for the determination of the roughness of an embedded fracture surfaced crack is also presented.

Porosity

Recently, much interest has been evident in the experimental determination of porosity in cast materials and composite structures. The relative ease of construction of complex shapes and their high strength relative to welded or machined parts points to increasing use in the future. The formation of porosity influences their overall strength, and high concentrations of pores can lead to failure, so some way must be found to characterize the porosity in three dimensions in any arbitrarily shaped sample. Since ultrasound directly measures the material properties of the material under investigation, it seems natural to include a mapping of porosity in cast and composite materials. The work in this thesis concentrates on the problem of nondestructively determining the distribution of pores in cast aluminum.

The study of grain noise in casts is a logical starting place, since this scattering is typically Rayleigh scattering, and has been extensively studied. Mason and McSkimin⁶³ in 1948, studied grain scattering for the cases of kd smaller than, equal to, and larger than one, where k is the wave number and d is a typical grain diameter.

A detailed study of grain scatter induced attenuation was given by Papadakis⁶⁴ in 1965. He found expressions for the long and short wave length relative to the grain boundary, and suggested an interpolation scheme for the intermediate regime. The analysis used the elastic modulus and density of the material, and allowed no aggregate samples containing grains of differing sizes and compositions.

Evans, Tittmann, Ahlberg, Kuri-Yakub, and Kino⁶⁵ extended this idea in 1978 using a numerical method. This formulation allowed the testing of alloys and aggregate ceramics, and was compared to experiment by Tittmann, Ahlberg, Cohen-Tenoudji, and Quentin.⁶⁶

Thompson, Wormley, Rose, and Thompson⁶⁷ studied pores in glass using backscattered ultrasound. They tried to find a characteristic signature based on the size of the visible flaws.

Gubernatis and Domany⁶⁸ applied Foldy's⁶⁹ 1948 results to work out a theory which could yield accurate results for simple approximations. This was further worked out by Rose⁷⁰ for the case of forward scattering through a region containing a statistical distribution of pores of approximately the same diameter.

This was compared to experiment by Wang, Csakany, Adler, and Mobley⁷¹ in 1983 for a set of aluminum cast samples whose porosity was

further checked by density measurements and microphotography. The work reported on in this thesis is an extension of that work applied to the case of arbitrarily shaped samples containing a statistical distribution of identical pores. The samples are assumed accessible on only one side.

Thermosetting Polymer

The use of thermosetting resins for the base of matrix of composite materials is a relatively recent development. It is widely applied because of its ease of fabrication. The ability to form shapes nearly impossible by any other method and to achieve strength in certain critical directions while saving material and weight by lessening the strength in other, noncritical directions points to even wider use in the future.

In the fabrication process, the resin transforms from a viscous liquid to a solid. The strength and toughness can be controlled by controlling the curing temperature. Since the material properties of the resin are clearly changing during this time, ultrasound should be able to monitor the curing process.

Previous work on this problem has involved the use of single frequency monitoring of the attenuation and phase velocity of an ultrasonic wave in or through the curing medium. Sofer and Houser⁷² and Sofer, Dietz, and Houser⁷³ used a tone burst through a sample of epoxy

undergoing cure, and monitored the attenuation and phase velocity of their chosen frequency, in this case, 2 MHz.

In another approach, Papadakis⁷⁴ embedded glass and wire in a curing polymer, and looked at the amount of signal leaking into the epoxy as the cure proceeded. Lindrose⁷⁵ also used a tone burst in a pulse-echo experiment. His results showed a clear relationship between the time of cure and the elastic modulus of the sample.

Rokhlin, Hefets, and Rosen⁷⁶ used surface waves through a region joined by curing epoxy to monitor the phase velocity and attenuation through the joined region. Rokhlin⁷⁷ also considered a thin film of epoxy on the surface of a single block in the same way.

Hunston⁷⁸ used an interdigital surface wave transducer to generate a shear wave on the surface, and monitored the attenuation through a surface region covered by curing polymer. Winfree and Parker⁷⁹ used a single frequency reflection from a glass-polymer surface to infer the velocity and time rate of change of velocity in the epoxy as a function of the time of cure.

The work reported in this thesis is an attempt to understand the frequency dependent as well as the time dependent relationship of the attenuation and phase velocity through a thin sample of curing epoxy.

A method of measuring both ultrasonic parameters has been developed, and the data arranged as attenuation and phase velocity surfaces as functions of both time and frequency at a single temperature.

CHAPTER II

EXPERIMENTAL METHOD

Experimental System

Most of the scattered waveforms used in this work were obtained using an immersion system. In this technique, the transducer or transducers and samples were immersed in a water bath, with the transducer to sample separation typically more than ten centimeters. The chief advantage of an immersion system is good coupling between the transducer, water, and sample surfaces. The separation allows one to avoid the frequency dependent effects of the near field from a transmitter. The transducers were commercially available lithium sulphate crystals, cut for longitudinal wave generation, and damped by a backing layer so as to broaden the bandwidth, i.e., lower the resonance or Q factor.

The two immersion techniques used were a single transducer to both send and receive the displacement wave, a pulse-echo arrangement, and a two transducer arrangement with both capable of sending and receiving the wave, a pitch-catch arrangement. Alignment of the experimental system was performed by maximizing the received waveform amplitude. An analog spectral analyzer system assured optimum high frequency response.

Since these transducers could be modeled as a piston in an infinite baffle,⁸⁰ the radiation pattern from them diverges from an ideal beam and the lobe structure of the generated field is the Fourier transform of the effective aperture. Thus, the high frequency components of the wave are least affected by this beam divergence, and can be expected to yield the best directivity. The analog spectrum analyzer system is then essential for accurate alignment of a broadband ultrasonic spectroscopy system, since the maximization of the received voltage waveform alone essentially measures the low frequency components of the signal.

The original system used in this work consisted of a balanced mixer driven in parallel by the received signal and a square gate pulse from an oscilloscope. The mixer passed signals only when the voltage across it was higher than its threshold value, so that part of the measured signal which coincided with the positive gate pulse was passed, and all else was suppressed.

The next improvement was the development of the stepless gate, which performed the same function, and did not result in a voltage step in the selected signal. This time gated signal was then passed to an analog spectrum analyzer.

The analyzer measured the intensity of the signal in a variable bandpass filter, and displayed this value in its display. Normally driven by a ramp voltage which changed the range of the filter, the analyzer also had a manual control and X and Y output port. These were connected to an X Y plotter, and the amplitude spectrum of the gated signal could be recorded on a graph. A single frequency generator was used to calibrate the displayed results.

The measurement system was further improved by using a boxcar integrator which measured the signal voltage in a small increment of time, and stepped that increase through a selected interval. This resulting stored representation of the received signal was then fed slowly to an analog to digital converter, and stored in a mini computer. Another method was the use of a storage scope which then fed a similar A/D converter. In both methods, the result was to slow the MHz signal down to the resolution speed of the converter.

This digitization allowed the use of the fast Fourier transform,⁸¹ or FFT, to be used on the signal. The improvements were two fold, in that the normalization of the received signal as described in the Signal Processing Section could be performed easily by the computer, and that both the amplitude and the phase of the signal could now be determined.

The last step was the incorporation of a high speed digitizer coupled in parallel to the mini computer. The sampler has a nominal sampling rate of 100 MHz, yielding a measurement of the same signal in 10 nanosecond intervals. It is an eight bit machine and achieves an effective resolution of 256 by the use of twos complement to represent negative numbers. This system allows high speed sampling of the waveform and overcomes problems with time jitter which the previous systems were prone to, since the entire waveform is measured at once rather than a tiny piece of successive signals.

This configuration also allowed time correlation⁷⁹ of successive signals. In this procedure, the initial signal is captured and an interval with some characteristic shape identified in the computer.

Successive signals are then captured, digitized, and time shifted by a single sample interval step for as many times as are specified by the operator. In this way, the time error can be reduced to $\pm$ one half sample interval, or 5 nanoseconds in this case.

The transducers were excited by either a pulse of a few nanoseconds duration, or a gated single frequency waveform. The pulse or shock excitation produces a very broadband signal generated by the transducers. The pulser-receiver allowed some tuning of the pulse in that the voltage of the spike could be lowered in three steps by an energy switch, and the duration of the decay could be varied by means of a damping control.

The single frequency system consisted of a sine wave generator coupled to a gating amplifier. The gate width could be varied from zero to a few microseconds, and the rise and fall time of the gated signal was typically a few hundred nanoseconds. Fed to a quartz transducer, this generated a very narrow range of frequencies, with the normal sidebands, and with a broadband transducer, a slightly wider bandwidth with less sidebands, in a fair approximation of a single frequency.

The amplifiers were, in the original system, a low gain oscilloscope module feeding a high gain, high noise tube amplifier with a bandwidth from a few kHz to a few MHz, and in subsequent systems, a solid state amplifier with a nominal 40 dB gain and a bandwidth from a few kHz to more than 35 MHz.

A dynamic range of slightly more than 60 dB was possible for a signal of optimal amplitude, and signal averaging allowed an increase in

gain above the noise floor as described in the section on processing. A general schematic of both the analog and final digital systems is shown in Fig. 1.

The combination of the two allows fast and accurate alignment via the analog system, and capture and storage of the scattering signal for later processing. In addition, the comparison of the analog and digital spectra, before normalization, gives a constant check of the system. It should be noted that the pulser receivers used not only amplified the received signals, but also limited the output voltage to ± 5 volts. This is crucial in using a digitizing system, since the digitizer is usually not very well protected from incoming overload, and the several hundred volts of the initial pulse can, and do, burn out the front end of the digitizer. In this system, a time delay in addition to the internal delay of the digitizer was necessary in order to capture the scattering waveform, since the wave speed in water is much slower than in most sample materials.

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Signal Processing

The development of the Fast Fourier Transform algorithms has allowed the methods of modern signal processing to be used. Extremely complex waveforms can be analyzed without the necessity of the solution of an analog integral, but the discrete or digital nature of the method poses grave problems which are sometimes hidden from the user.

The continuous or classical Fourier transform used an infinite sum of sinusoidal waves to describe a waveform. Each pure wave is multiplied by its corresponding coefficient and added to the sum in a definite phase relationship to the origin of time. Probably the best known and least loved example is the square wave, composed of odd order harmonics of the fundamental with a discrete spectrum of coefficients.

The frequency distribution in an arbitrary waveform can have a continuous spectral density and phase in which case it becomes meaningless to talk of a fundamental frequency. The essential thing here is that the waveform being analyzed can have any shape, and the integral transform will match that shape up to some sum of weighed sinusoidal waves.

The situation for discrete sampling is much different. Consider a digitizing system which measures the amplitude of a wave form exactly once in a definite period Δt . The Fast Fourier Transform then matches a sinusoid of discrete frequency to the measured points. The nature of the duality of time and frequency will not be explored here, but the description is as follows.

For a time record of length T measured every Δt units, there is a frequency band of width F composed of discrete points separated by an interval Δf . The frequency interval is of width $1/T$, and the width of the frequency band is $1/\Delta t$.

The Fast Fourier Transform uses only those discrete frequencies lying Δf apart from 0 to F to describe the waveform. Furthermore, the time and frequency records being examined are assumed by the algorithm to be repetitive, as illustrated by Fig. 2. The time record is assumed to be identically repeated with a frequency of Δf , and the frequency bandwidth, from $-F/2$ to $F/2$ is assumed to repeat with an interval $1/\Delta t$.

Aliasing

Consider the following situation. A wave form of known frequency is measured every Δt time units. Figure 3 illustrates the effect of digitizing the continuous waveforms. The case of the period equal to 4 sample intervals yields a good approximation to the actual waveform. Similarly, Figs. 3b and 3c yield a waveform that can only correspond to the correct frequency.

Fig 2. TIME-FREQUENCY RELATIONSHIP THROUGH THE FAST FOURIER TRANSFORM

Fig 3. SAMPLE INTERVAL EFFECTS

The case of the period equal to 2 sample intervals is shown in Fig. 3d, and the points fit perfectly, leaving no error in interpretation. Once this frequency is exceeded, however, digitizing problems lead to an incorrect interpretation as to what exactly was measured.

While the period of the waveform illustrated in Fig. 3e is 1.6 times the sample interval, the best fit to only the digitized values becomes $2 \frac{2}{3}$ intervals. As we increase the frequency, the apparent frequency decreases and the apparent period is related to the actual period logarithmically.

When we reach a period equal to the sample interval, the apparent period is infinite, corresponding to zero frequency. Increasing the real frequency still further results in an apparent increase, until we reach a period of $1/2$ the sample interval.

This effect is the basis of the Niquist criteria, which states that the highest frequency making up the waveform under investigation must be less than or equal to $1/2$ of the bandwidth of spectral density space.

Figure 4 illustrates a case of aliasing resulting from a spectrum containing energy up to frequencies of $3/2$ the allowed bandwidth. The solid line is the apparent, but hopelessly incorrect amplitude spectrum. Thus, the classic compromise must be made, since a higher sampling rate will yield a wider frequency bandwidth, but a correspondingly cruder frequency resolution for a fixed number of points. In all cases, the Niquist criteria must be observed, and the sample rate increased if there is any doubt.

Fig 4. SPECTRAL RESULT OF UNDER SAMPLING

Deconvolution

A typical ultrasonic spectroscopy system can be described mathematically as a series of operators acting both on the amplitude or density spectrum and the phase spectrum as functions of frequency. Consider a delta function impulse emitted by an ultrasonic pulser. This ideal pulse has a frequency spectrum with amplitude equal to 1 for all frequencies, and a phase of 0, again for all frequencies.

The change in the signal caused by the connecting cables and the transformation of electrical to mechanical energy by the transducer can then be represented as $A(\omega)\Delta(\omega)$, where $\Delta(\omega)$ is the complex spectral density of the delta function, $\delta(t)$, i.e., its Fourier transform. There are further effects in traveling through the coupling medium, striking the surface of the sample, traveling on or through the sample, striking the scatterer, etc. These interactions make it impossible to strike the scatterer with an impulse wave, but if we can find the response of the system without the scatterer, we can extract the impulse response in the following manner.

If the system has a response to an impulse function, $\delta(t)$, which we call $h(t)$, when the impulse response of some scattering, $g(t)$ will be modified. This modification is described by the convolution integral, since as $g(t)$ passes through the system, $h(t)$ will act on it continuously.

Thus, the measured response will be

$$f(t) = \int d\tau [g(\tau)h(t - \tau)] \quad (1)$$

A simple method of analyzing this integral is by use of the Fourier transforms of f , g , and h . We will call them $F(\omega)$, $G(\omega)$, and $H(\omega)$, respectively.

Then

$$F(\omega) = G(\omega)H(\omega) \quad (2)$$

so if we can measure H , then with the F we've been forced to measure, we can obtain $G(\omega)$ by a complex division. The problem then is the determination of $H(\omega)$.

What we wish to do is generate an impulse at the scatterer, but that is presently impossible. However, we can consider a delta function in material properties, as at the interface of a smooth flat solid and the coupling medium.

The difference between an impulse response and our systems response is then just our system's response, so we consider this reflection as $H(\omega)$. There is considerable discussion on this point among researchers. Some use the surface reflection, and some use the reflection from the back side of the sample, since this also incorporates the response of the sample. However, this requires a flat smooth parallel surface, not always attainable, and measures the effects of a longer path of the sample than the scattering case, whereas the method previously described, measures no effect at all.

Since we will also be concerned with the response of a rough front surface, as well as the volumetric response itself, we will always use the front surface reflection as the system response.

The bandwidth of a typical transducer is not infinite as assumed in an impulse function. When we divide one frequency spectrum by itself, we will always receive an amplitude of 1 and a phase of 0 but if we measure the reflection spectrum of the same surface several times, the very small differences between the j th and k th spectra can result in the misleading result of dividing one small number by an even smaller one.

To overcome this, we use a Weiner filter as suggested by Murakami, Khuri-Yakub, Kine, Richardson, and Evans.⁸² The rationale of the filter is simply that if we have trouble with the magnitude of the denominator becoming too small in our complex division to determine the impulse response, then we can add a small number to that denominator to overcome this problem. If that number is real, then the filter will have no effect on the phase. Furthermore, if the number is in some way proportional to the noise level, then we will essentially be diminishing the effects of the noise on our results.

We can thus relate the constant to the maximum of our amplitude spectrum, and the impulse response is then found as

$$G(\omega) = W_Q(F(\omega), H(\omega)) \quad (3a)$$

$$= FH^*/(HH^* + (Q|H|_{\max})^2) \quad (3b)$$

so if Q is zero, we have the simple division of two complex numbers.

CHAPTER III

EMBEDDED CRACKS

Introduction

Ultrasonic inspection has been used to detect and attempt to size flaws since the first commercial transducers became available. Generally a single frequency transducer was used, and the echo amplitude used to locate the depth of the flaw and to estimate its cross sectional area. The assumption here is of necessity that the flaw is flat and oriented normal to the face of the work piece.

A new approach stated by Gericke¹ was the development of a contact ultrasonic spectrum analysis system to characterize the geometry of hidden flaws in solids. He used a broad band, rather than a single frequency ultrasonic pulse, and he found that the spectral components of the echoes were strongly affected by the geometry of the discontinuity. Whaley and Cook⁸³ developed a spectrum analysis immersion system and Adler and Whaley² carried out a systematic study of the spectral response of scattered waves from circular reflectors in water by using this system. They developed a model to measure the size and orientation of circular reflectors. In this model, the edges of disks are assumed to be Huygens sources of the scattered field in

analogy to the Sommerfeld⁸⁴ approach to diffraction of light waves by an aperture in a screen.

They showed that the amplitude of the ultrasonic beam scattered from the flaw was highly dependent on the orientation as well as the size of the flaw. Additionally, their simple model showed astonishingly good results when used to estimate both the size and orientation of flat cracks.

Mathematically, ultrasonic wave scattering in solids is treated by using the equations of elastodynamic theory,⁸⁶ with boundary conditions appropriate to the problem. Exact solutions exist only for a few idealized configurations. Several approximate solutions have been developed recently.

The problem is classified as to whether the discontinuities are bounded by smooth closed surfaces such as for spheroids, or by planar surfaces having sharp edges, as for crack-like discontinuities. Another classification of the theory is based on the applicability in the region of ka , where k is the wave number and a is some dimension of the discontinuity

For crack-like planar defects in the region $ka > 1$, Adler and Lewis^{9,10} used Keller's^{7,8} geometrical diffraction theory for optics to analyze their experimental data. Achenbach and Gautesen,¹¹ and extended the geometrical diffraction theory to the three-dimensional diffraction by cracks in solids, where wave motions are governed by a scalar and a vector wave equation whose solutions are coupled by the boundary conditions on the diffracting obstacles.

Geometrical diffraction theory is based on the use of canonical solutions, which are asymptotic results for diffraction of a plane wave by a semi-infinite crack.⁵ In GTD, these canonical solutions are appropriately adjusted to account for curvature of incident wave fronts and curvature of crack edges, and for finite dimensions of the crack. Domany, Krumhansl, and Teitel¹² also used elastodynamics in the region $ka < 1$, or the quasi static region, i.e., very low frequency, using the same sort of surface integral solution. These approximations have been compared to available exact solutions in their region of applicability and found to be in excellent agreement.

This section will give some of the details of flat crack scattering approximations for $ka > 1$, and detail the experimental confirmation of the elastodynamic model. Imaging inversion methods will also be discussed. For a comprehensive overview the scattering of ultrasound by embedded cracks, see Doyle and Scala⁸⁶ or Kraut.¹⁶

Theory

Keller's theory holds for $ka > 1$ for planar discontinuities with sharp edges. Originally developed for electromagnetic waves, the theory assumes that every time an incident ray strikes an edge, a family of so-called diffracted rays is produced. These diffracted rays are not predicted by either conventional geometrical acoustics or optics. There is an infinite number of diffracted rays, all straight lines, produced by each incident ray.

To obtain a quantitative value for the diffracted wave field, the following assumptions are made:

1. There is an amplitude A and phase ψ associated with each ray. The incident ray is given by $u = Ae^{ik\psi(s)}$ where k is the wave number and s is the distance along the ray.
2. The amplitude of each diffracted ray is proportional to the amplitude of the incident ray and the proportionality factor in the diffraction coefficient. This diffraction coefficient can be calculated from an exactly soluble problem that is similar to the problem being studied.
3. The incident wave can be represented as plane waves

The total diffracted field at a give point is the sum of the fields due to all diffracted rays passing through that point, and that is given by Keller⁸ as:

$$u = \sum_j \frac{Ae^{ik(\psi_j + s_j) + i\pi/4}}{(8\pi k)^{1/2} \sin \beta_j} \left(\sec \frac{(\theta_j - \alpha_j)}{2} \pm \csc \frac{(\theta_j + \alpha_j)}{2} \right) \\ \times s_j \left(1 - \frac{s_j [\cos \delta_j + \rho_j \beta_j \sin \beta_j]}{\rho_j \sin^2 \beta_j} \right)^{-1/2} \quad (4)$$

Figure 5 shows particular incident and diffracted rays. β is the angle between the incident ray and the tangent to the aperture, $\dot{\beta}$ is the derivative of β with respect to the arc length along the edge of the aperture, ρ is the radius of curvature of the aperture edge at the point of interaction, δ is the angle between the diffracted ray and the normal to the edge in the plane of the screen, α and θ are the projections of the angles of incidence and diffraction, respectively, in the plane normal to both the edge and the screen, s is the distance along the diffracted ray to the observation point, and ψ is the value of the phase of the incident ray at the edge. The $\pm$ sign is used as $+$ in the case $u = 0$ in the aperture and as $-$ in the case $\partial u / \partial n = 0$ in the aperture.

Keller's is a scalar theory and does not include the effect of mode conversion. Since the S and L components of the scattered wave could be separated in time, due to their differing velocities, and separately analyzed, Eq. (4) could be used to describe the scattered S wave field by the replacement of k , the L wave number, by K , the S wave number. This expression is general for any two-dimensional discontinuity with a smoothly varying edge, and has been investigated for two special cases: (1) the circular aperture-like discontinuity and (2) the elliptical-like discontinuity.

Fig 5. GEOMETRY OF KELLER'S APERTURE

Circular Discontinuity

For normal incidence on a circular edge, only two points contribute diffracted rays to the observation point, the nearest and farthest points, thus, Eq. (4) reduces to:

$$u = \frac{A(k)a}{\sin \theta} \left\{ \frac{1 - \cos \theta \sin 2(ka \sin \theta)}{ka \sin \theta} \right\}^{1/2} \quad (5)$$

where $A(k)$ is the frequency spectrum of the incident ray as seen by the receiver, the so-called transfer function, a is the radius of the discontinuity, θ is the angle from the axis normal to the discontinuity to the line from the discontinuity to the observation points.

Elliptical Discontinuity

Since most smooth two-dimensional discontinuities may be approximated by some kind of elliptical symmetry, Eq. (4) was also solved for an elliptical discontinuity. It reduces to:

$$u = A(k)\rho(\phi) \left(\frac{1 - \cos \theta \sin 2(ka(\phi) \sin \theta)}{k\rho(\phi) \sin \theta} \right)^{1/2} \quad (6)$$

where

$$a(\phi) = eb[1 + (\epsilon^2 - 1)(1 + \epsilon^2 \tan^2 \phi)^{-1}]^{-1/2} \quad , \quad (7a)$$

$$\rho(\phi) = \epsilon^{-1} b [1 + (\epsilon^2 - 1)(1 + \epsilon^2 \tan^2 \phi)^{-1}]^{3/2} . \quad (7b)$$

In Eq. (7), b is the diameter along the major axis of the ellipse, ϵ the ratio of the major to minor axes, and ϕ the azimuthal angle in the plane of the ellipse from the major axis to the point of origin of the diffracted rays on the edge. The above equation is also for normal incidence only.

Elastodynamic Theory

Since Keller's theory was an application of optics to ultrasonic scattering, there was no provision for mode conversion. Achenbach and Gautesen¹¹ used the same general approach to the elastodynamic problem.

Let $u_i(x_j, t)$ be the components of the displacement vector in a rectangular Cartesian coordinate system with coordinates x_i , $i = 1, 2, 3$. The components of the stress tensor τ_{ij} are related to the gradients of the displacement vector by Hooke's law:

$$\tau_{ij} = \lambda u_{k,k} \delta_{ij} + \mu(u_{i,j} + u_{j,i}) \quad (8)$$

Here $u_{i,j} = \partial u_i / \partial x_j$ and repeated indices indicate a summation. Also λ and μ are Lamé's elastic constants, and δ_{ij} is the Kronecker delta.

In the usual manner, the displacement vector is expressed in terms of a scalar potential ϕ and a vector potential with components ψ_k by

$$u_i = \phi_{,i} + e_{ijk}\psi_{k,j} \quad (9)$$

with

$$\psi_{k,k} = 0 \quad (10)$$

Here e_{ijk} is the alternating symbol of Cartesian tensor notation. It is easy to show by direct substitution that the representation (9) satisfies the displacement equation of motion if ϕ and ψ_k are solutions of the wave equation.

Although these wave equations are uncoupled, the potentials are coupled through the boundary conditions. This coupling is the major cause of difficulty in solving elastodynamic problems.

Keller's theory of geometrical diffraction applied to elastodynamics states that two conical surfaces of diffracted rays are generated when an incident ray strikes an edge. The surface of the inner cone consists of rays of longitudinal motion, while the surface of the outer cone is composed of rays of transverse motion. The half-angles of the cones are related by Snell's law.

For an incident longitudinal ray, the diffracted longitudinal rays make the same angle ϕ_L with the tangent to the edge as the incident ray, and the diffracted rays of transverse motion are under an angle ϕ_T with the edge, where

$$c_L \cos \phi_T = c_T \cos \phi_L \quad (11)$$

For a straight diffracting edge, and an incident longitudinal ray, the diffracted displacement fields are related quantitatively to the incident field by

$$\underline{u}_d^L = e^{i\omega S_1/c_L} [S_1(1 + S_1/R_1)]^{-1/2} D_L \hat{i}_L^d A e^{i\omega(S_0/c_L - t)} \quad (12a)$$

$$\underline{u}_d^T = e^{i\omega S_2/c_T} [S_2(1 + S_2/R_2)]^{-1/2} D_T \hat{i}_T^d A e^{i\omega(S_0/c_L - t)} \quad (12b)$$

Here $A e^{i\omega(\frac{S_0}{c_L} - t)}$ defines the amplitude and the phase of the incident field at the point of diffraction, and D_L and D_T are diffraction coefficients which relate the diffracted field to the incident field. Also, S_1 and S_2 are the smaller of the principal radii of curvature of the diffracted wave front, or equivalently the distances along the diffracted rays from the points of diffraction to the observation points. The unit vectors $\hat{i}_L$ and $\hat{i}_T$ relate the direction of displacement of the diffracted field to the direction of displacement of the incident field.

For diffraction by a stress free crack, the canonical solution for diffraction by a semi infinite crack is used to obtain the diffraction coefficients.

For a longitudinal wave incident on a Griffith crack of width $2d$, Achenbach and Gautesen using both symmetric and antisymmetric solutions, calculate the scattering amplitude as

$$u_k^S(\underline{r}) = \int_S ds \, n_j(\underline{r}') \Delta u_1(\underline{r}') \, \tau_{1j,k}^G(\underline{r} - \underline{r}') \quad (13)$$

dependent on the j th component of the normal to the crack surface n_j , the stress components of the solution for an unbounded medium evaluated at the crack surface τ_{1j}^G , and the crack opening displacement or discontinuity at the crack $\Delta u_1(\underline{r}')$.

Figure 6 illustrates the physical situation. The crack of width $2d$ lies in the x_2 constant plane, and a plane longitudinal wave is incident at an angle θ_0 , and the scattering amplitude is calculated for θ .

Since this elastodynamic field solution yields the vector displacement $\underline{u}$ for all space, the longitudinal and transverse or shear components are determined in local coordinates at the point of detection.

For ellipsoidal cracks, the boundary of the crack is closed on itself, and the stress term becomes unwieldy. When the approximation of far field scattering, simplifications allow solutions as follows.

In Eq. (13), n is the outward normal (pointing from the A^+ to the A^- face). Also, $\Delta u_1^S(X)$ is the crack opening displacement, defined by

$$\Delta u_1^S(X) = (u_1^S)^{A^+} - (u_1^S)^{A^-} \quad , \quad (14)$$

Fig 6. COORDINATE SYSTEM

and $\tau_{ij,m}^G$ is a tensor of rank three, which represents the stress components due to a unit load in the x_m direction applied at the point defined by $x = x_Q$.

x_S , x_Q , and X denote the position vectors of S , Q , and any points on the crack, and $x_S = |x_S|$, $x_Q = |x_Q|$, and $X = |X|$, then $x_S, x_Q > X$. Defining the unit vector $\hat{x}_Q = x_Q/x_Q$, we can write $|x_Q - X| = x_Q - (\hat{x}_Q \cdot X)$, $x_Q > X$. The expression for $\tau_{ij,m}^G$ then simplifies considerably. An explicit expression can be found in Achenbach et al.¹⁴ in the following form

$$\begin{aligned} \tau_{ij,m}^T = & ik_L b_{ij,m}^{G,L}(\hat{x}_Q) G_L(x_Q) \exp(-ik_L \hat{x}_Q \cdot X) \\ & + ik_T b_{ij,m}^{G,T}(\hat{x}_Q) G_T(x_Q) \exp(-ik_T \hat{x}_Q \cdot X) \end{aligned} \quad (15)$$

where for $\beta = L$ or $\beta = T$

$$G_\beta(x_Q) = (1/4\pi x_Q) \exp(ik_\beta x_Q) \quad (16a)$$

$$k_\beta = \omega/c_\beta; \quad c_L^2 = (\lambda + 2\mu)/\rho; \quad c_T^2 = \mu/\rho \quad (16b)$$

and

$$b_{ij,m}^{G,L}(\hat{x}) = (\lambda + 2\mu)^{-1} (2\mu \hat{x}_i \hat{x}_j + \lambda \delta_{ij}) \hat{x}_m \quad (17a)$$

$$b_{ij,m}^{G,T}(\hat{x}) = \delta_{im}\hat{x}_j + \delta_{jm}\hat{x}_i - 2\hat{x}_i\hat{x}_j\hat{x}_m \quad (17b)$$

It is easily checked that the longitudinal and transverse parts of the far field displacements have only components parallel and normal to x_Q , respectively. Substitution of Eq. (15) into (13) yields the scattered field as

$$\begin{aligned} u_m^S(x_Q) &= -ik_L b_{ij,m}^{G,L}(\hat{x}_Q) G_L(x_Q) n_j I_i^L(\hat{x}_Q) \\ u_m^S(x_Q) &= -ik_L b_{ij,m}^{G,L}(\hat{x}_Q) G_L(x_Q) n_j I_i^L(\hat{x}_Q) \end{aligned} \quad (18a)$$

$$-ik_T b_{ij,m}^{G,T}(x_Q) G_T(\hat{x}_Q) n_j I_i^T(\hat{x}_Q) \quad L,T \quad (18b)$$

$$I_i^\beta(\hat{x}_Q) = \int_{A^+} \exp(-ik_\beta \hat{x}_Q \cdot X) \Delta u_i^S(X) dA(X), \beta = L,T$$

In the physical elastodynamics approximation, only the leading contributions arising from the incident wave and the specularly reflected waves are included in Δu^S . Thus, the effect of the crack edge of the displacement discontinuity across the crack faces is neglected. This is acceptable if the wave length is sufficiently smaller than the cross-sectional dimensions of the crack.

On the faces of the crack, the spherical wave emitted at S is locally approximated by a plane wave

$$\underline{u}^i = -A x_s G_L(\hat{x}_s) \exp(-ik_L x_s \cdot X), \quad x_x > x \quad (19)$$

Then

$$\frac{\{u_m^s(x_Q)\}^\beta}{G_L(x_s)G_\beta(x_Q)} = -Ab_{1j,m}^{G,\beta}(\hat{x}_Q)n_j I(k_\beta) \quad (20a)$$

where $\beta = L$ for the longitudinal wave and $\beta = T$ for the transverse wave,

$$I(k_\beta) = ik_\beta \int_{A^+} \exp(-ik_\beta q \cdot X) dA(X) \quad (20b)$$

and

$$q = \hat{x}_s + \hat{x}_Q \quad \text{for } \beta = L \quad ; \quad (21a)$$

$$q = (c_T/c_L)\hat{x}_s + \hat{x}_Q \quad \text{for } \beta = T \quad (21b)$$

The far field dependence on the crack occurs only through the function $I(k_\beta)$, which depends on the wave number k and the vector q .

It is convenient to express the area integral in terms of the ξ_i coordinate system, whose origin 0 is located in the plane of the crack. Writing

$$X = X_0 + \xi; \quad q = \hat{x}_q + \hat{x}_s = \frac{(X_0 + \xi_Q)}{X_Q} + \frac{(X_0 + \xi_s)}{X_s} \quad (22a,b)$$

we find for $\beta = L$,

$$I(k_L) = ik_L e^{-ik_L X} \int_{A^+} e^{-ik_L q \cdot \xi} d\xi_1 d\xi_2 \quad (23)$$

where

$$\chi = (\hat{x}_0 + \hat{x}_s) \cdot x_0 \quad (24)$$

The area integral can be reduced to a line integral over the crack edge C by noting that $I(k_L)$ can also be expressed in the form

$$I(k_L) = - \frac{\exp(-ik_L \chi)}{q_1^2 + q_2^2} \int_A \frac{\partial}{\partial \xi_1} [q_1 e^{-ik_L q \cdot \xi}] d\xi_1 d\xi_2; \quad i = 1, 2 \quad (25)$$

where q_1 and q_2 are the components of q in the ξ_i system. An application of the divergence theorem subsequently yields

$$I(k_L) = - \frac{\exp(-ik_L \chi)}{q_1^2 + q_2^2} \int_C v_i q_i e^{-ik_L q \cdot \xi} ds \quad (26)$$

where v_i ($i = 1, 2$) is the outward unit normal to the crack edge, and s is the arc length measured along the crack edge. An analogous expression can be derived for $I(k_T)$. By means of Eqs. (26) and (20), the scattered longitudinal field is expressed as radiation generated by a superposition of sources over the edge of the crack. This kind of representation seems to be analogous to the method of equivalent currents, which has also been explored by several authors in electromagnetic scattering theory.

Crack of Elliptical Shape

For an elliptical crack of major axis b and minor axis a , the line integral over the edge C given by Eq. (26) can be evaluated explicitly. Assuming that the origin is taken in the plane of the crack, we have $X_0 = 0$ and therefore $\chi = 0$.

We introduce new variables ϕ and ψ by the relations

$$(\xi_1, \xi_2) = [b \cos (\phi + \psi), a \sin (\phi + \psi)], \quad 0 \leq \phi \leq 2\pi \quad (27a)$$

$$(\cos \psi, \sin \psi) = \rho^{-1}(bq_1, aq_2) \quad (27b)$$

$$\rho = (b^2q_1^2 + a^2q_2^2)^{1/2} \quad (27c)$$

It then follows that

$$(v_1, v_2)ds = [a \cos (\phi + \psi), b \sin (\phi + \psi)]d\phi \quad (28)$$

and Eq. (26) becomes

$$\begin{aligned} I(k_L) = & - \frac{\rho}{q_1^2 + q_2^2} \int_0^{2\pi} \left[\frac{a}{b} \cos (\phi + \psi) \cos \psi \right. \\ & \left. + \frac{b}{a} \sin (\phi + \psi) \sin \psi \right] e^{-ik_L \rho \cos \phi} d\phi \end{aligned} \quad (29)$$

These theories are all based on the steady state physical situation, so some care must be exercised in their use. One could employ Laplace transforms on the wave functions (see for example Pao and Mow),⁸⁷ or accept the solution as a valid approximation, as is done here, and use time mapping to measure the experimental results.

Since elastodynamics yields different wave speed for all possible modes in general, it is important to measure not only at the proper place and orientation, but also at the proper time.

The Direct Experiment

The "Griffith" cracks are fabricated into a diffusion bonded titanium alloy by Rockwell Science Center. The length of the crack is 2388 μ m. Four cracks were obtained of the same length but with orientations of 0°, 30°, 60°, and 90° relative to the surface of the sample.

The Griffith crack in Fig. 7 is oriented 60° from the surface of the sample. The scattered angles (measured in the titanium alloy) are in 15 degree intervals. There is an asymmetry in the spectrum because of the tilted cracks. For a crack where the plane is parallel with the sample surface, the scattered spectra is symmetrical.

The experimentally observed amplitude spectrum is compared to calculate values of the scattered amplitude as a function of frequency. On Fig. 8, the orientation of the Griffith crack is shown.

Fig 7. A GRIFFITH CRACK AND ANGULAR SCATTERING SPECTRA FROM IT

Fig 8. COMPARISON OF EXPERIMENTAL AND THEORETICAL AMPLITUDE SPECTRA FOR A GRIFFITH CRACK

The experimental obtained amplitude spectrum (by Fourier analyzing the time domain signal) is compared to the theoretical value of the amplitude spectrum. The comparison between theory and experiment is rather poor for the case when the wave scatters along the surface of the Griffith crack. Clearly, the theoretical calculation has to include surface wave generation in this case. The agreement between experiment and theory is reasonable good, however, for the angular distribution of the scattered amplitude for a 10 MHz L-wave incident at 150° as shown on Fig. 9. Due to the geometry of the sample, only a limited number of data points were taken.

The Inverse Problem

The previous discussion was concerned with the direct problem, that is the computation of the scattered field when the size, shape, and orientation of the crack are known. Now we consider the inverse problem for plane waves incident on penny-shaped cracks, for the special case that the incident wave is known to be in a plane of symmetry of the crack. For a given point of observation, the unknowns then are θ_0 , a , and b .

The frequency spectrum of these longitudinal signals contains a considerable amount of information on the crack. By appropriate processing of the experimental data one obtains, in fact, the amplitudes corresponding to single harmonic waves as functions of the frequency.

Fig 9. ANGULAR DIFFRACTION PATTERN FOR A LONG CRACK

Comparison of this experimental information with the analytical fields on the primary diffracted body wave rays provides a way to solve the inverse problem.

For oblique incidence the diffraction coefficients depend on the angle of incidence, i.e., $D_L^L = D_L^L(\theta_j; \theta_1^1)$. At the points A_1 and A_2 on the diffracted L rays in the plane of symmetry the displacements are

$$(u_L)_j = U_{A_1 L}^{j j} \quad . \quad (30)$$

where

$$U_A^j = \exp(i\omega S_j/c_L)(\omega S_j/c_L)^{-1/2}(1 + S_j/C)^{-1/2}D_L^L(\theta_j; \theta_1^j)U_0^j \quad . \quad (31)$$

In Eq. (30), $j = 1, 2$ for the rays O_1A_1 and O_2A_2 , respectively, and

$$\theta_1^1 = \pi/2 + \theta_0, \theta_1^2 = \pi/2 - \theta_0 \quad . \quad (32)$$

The displacements on the corresponding L rays in the fluid are

$$(u_L)_j = \exp(i\omega S_j/c_F)(1 + S_j/E)^{-1/2}T(\theta_L^j)U_{A_1 F}^{j j} \quad , \quad (33a)$$

where

$$\theta_L^1 = \pi - (\theta_1 + \theta_0), \theta_L^2 = \theta_2 - \theta_0 \quad . \quad (33b)$$

The interference patterns for the first arriving longitudinal wave in the fluid are generated by phase differences and amplitude differences on the direct rays from the two crack tips. The differences in the incident fields are generated by amplitude and phase differences on the direct rays from the two crack tips. The differences in the incident fields are represented by

$$U_0^1 = U_0 \exp[i(\omega a/c_L) \sin \theta_0] \quad , \quad (34a)$$

$$U_0^2 = U_0 \exp[-i(\omega a/c_L) \sin \theta_0] \quad . \quad (34b)$$

Now adding the primary diffracted longitudinal fields from the points O_1 and O_1 , we obtain

$$\begin{aligned} u_L = & H_1 \exp[i\omega(S_1/c_L + \bar{S}_1/c_F + a \sin \theta_0/c_L) + i\pi/4] U_0 i_F^1 \\ & + H_2 \exp[i\omega(S_2/c_L + \bar{S}_2/c_F - a \sin \theta_0/c_L) + i\pi/4] U_0 i_F^2 \quad , \quad (35) \end{aligned}$$

where

$$H_j = \frac{\text{sgn}(\cos \theta_j) T(\theta_j^L) |D_L^L(\theta_j; \theta_j^j)|}{(\omega S_j/c_L)^{1/2} (1 + S_j/C)^{1/2} (1 + \bar{S}_j/E)^{1/2} (1 + \bar{S}_j/\bar{C})^{1/2}} \quad . \quad (36)$$

We now introduce rays $\overline{OA}$ and $\overline{AB}$, which connect the center of the crack with the point of observation in the fluid. From the geometry, it is not difficult to see that

$$S_2 = S + a \cos \theta_2 - \bar{A}_2 \bar{A} \cos(\theta - \theta_0) \quad , \quad (37a)$$

$$S_1 = S - a \cos \theta - \bar{A}_1 \bar{A} \cos(\pi - \theta_1 - \theta_0) \quad . \quad (37b)$$

$$\bar{S}_2 = \bar{S} + \bar{A}_2 \bar{A} (c_F/c_L) \cos(\theta_2 - \theta_0) \quad (37c)$$

$$\bar{S}_1 = \bar{S} - \bar{A}_1 \bar{A} (c_F/c_L) \cos(\pi - \theta_0 - \theta_1) \quad . \quad (37d)$$

If the crack is sufficiently far from the interface, the following approximations can be made:

$$\theta_2 \sim \pi - \theta_1 - \theta \quad , \quad (38a)$$

$$i_F^1 = i_F^2 = i_F \quad , \quad (38b)$$

and consequently

$$u_L = F(\theta, \theta_0) \exp[i\omega(S/c_L + \bar{S}/c_F) + i\pi/4] U_0 i_F \quad (39)$$

where

$$\begin{aligned} F(\theta, \theta_0) = & H_1 \exp[-i(\omega a/c_L)(\cos \theta - \sin \theta_0)] \\ & + H_2 \exp[i(\omega a/c_L)(\cos \theta - \sin \theta_0)] \quad . \end{aligned} \quad (40)$$

Of particular interest is the absolute magnitude of F ,

$$|F| = \{|H_1|^2 + |H_2|^2 + 2|H_1||H_2|\sin[2(\omega a/c_L) \times (\cos \theta - \sin \theta_0)]\}^{1/2} . \quad (41)$$

This result implies that the amplitude of the primary diffracted field is modulated with respect to ω/c_L with period

$$P = \pi/a|\cos \theta - \sin \theta_0| . \quad (42)$$

The theoretical amplitude spectra are plotted by the solid lines while the experimentally obtained amplitude spectra are shown by the dots. The frequency varies from 2 MHz to about 14 MHz (usable energy range available). A multiplane goniometer (see Fig. 10) was used to measure the scattered intensity at various diffraction angles. The diffraction angles (θ_L') are 35°, 45°, 55°, 60°, and 65°, respectively, in the solid. In all cases, the positions of maxima and minima of the spectra agree very well for theory and experiment. The locations of the maxima are significant for the inversion process (see Fig. 11).

From Eq. (41), it follows that the amplitude of the diffracted field is modulated with respect to ω/c_L with period $P = \pi/a(\cos \theta - \sin \theta_0)$. For this case, the plane of the crack is parallel with the surface and hence, $\theta_0 = 0$. From the amplitude spectrum (see Fig. 11), we can then obtain the radius of the crack. (If $\theta_0 \neq 0$, then from two

Fig 10. MULTIPLANE ULTRASONIC GONIOMETER

Fig 11. L-L SCATTERING FROM AN ELLIPTICAL CRACK; NORMAL INCIDENCE, 30° SCATTERING ANGLE

measurements both θ_0 and a can be obtained.) The radius a of the crack is then given as

$$a = c_L / 2 \sin(\theta'_L) (\Delta f_{\max})_{\text{av}} \quad , \quad (43)$$

where $\theta'_L = (\pi/2) - \theta$ and $(\Delta f_{\max})_{\text{av}}$ is the average frequency spacing between two consecutive maxima.

In Table 1, the result of the size determination from the spectral components of the diffracted wave is given. The agreement between actual crack size ($a = 2500\mu$) and the predicted value is excellent. The largest deviation is 3.7 percent.

In the same way, the variable edge separation for an elliptical can be measured on both the major and minor axes results for various ellipses are shown in Table 2.

Weld sample number 2 contained only planar discontinuities. Three artificial flaws, nominally 1/16", 1/8", and 1/4" in diameter were machined into the weld, along its center line (Fig. 12). The plane of the defects was parallel to the sample surface. Both the top and bottom surfaces of the specimen were ground flat.

The flaws were then insonified at 0° , 10° , and 20° in the weld metal assuming a longitudinal velocity of 5939 m/s measured in the normal or 0 degree direction. Welds are typically anisotropic, however, so the assumption led to an increase in the apparent size of the crack as discussed previously in this work. Figure 13 shows the superimposed spectra for all three cases and Table 3 summarizes the inversion results.

Table 1. The result of the size determination from the spectral components of the diffracted wave.

Diffraction angle θ_L'	$(\Delta f_{\max})_{av}$	Computed radius of the crack in μ
35	2.18	2530
40	1.87	2630
45	1.83	2450
50	1.68	2460
55	1.60	2410
60	1.47	2500
65	1.39	2510

Table 2. Determination of Major (B) and Minor Axis (A) of elliptical cracks in titanium from scattered amplitude.

Aspect ratio	(b) Major axis (μ)		(a) Minor (μ)	
	Actual	Measured	Actual	Measured
1	2500	2498	2500	2498
2	2500	2587	1250	1291
4	2500	2358	625	830
8	2500	--	312	285

Fig 12. WELD SAMPLE No. 2 CONTAINING PLANAR DISCONTINUITIES
1/16", 1/8" AND 1/4" IN DIAMETER

Fig 13. BACKSCATTERING SPECTRA FROM SUSPECT REGIONS IN WELD No.2

Table 3. Scattering estimates from penny cracks in a weldment.

10-degree Measurement (mm)	20-Degree Measurement (mm)	Diameter (mm)
1.8	2.06	1.6
3.5	3.6	3.2
6.36	7.3	6.35

Crack Imaging

Another imaging method, using the idea of Langston and Haines,⁸⁸ starts with the Kirchhoff-Helmholtz integral solution to the scattering problem and, by a change of variable, changes the scattering integral into the Fourier transform of the voids dimension perpendicular to the plane of the normal of the void and the incident wave vector. The integral is

$$F(f) = \frac{\cos(\theta) R(\theta)}{A_{ca} R(0)} \iint_A \exp\left\{-\frac{14\pi f z}{c}\right\} dA \quad (44)$$

where θ is the angle of the incident wave to the normal of the crack surface, $R(\theta)$ is the reflection coefficient at the interface, c is the speed of the wave in the material, f is the frequency, z is the distance along the surface in the wave vector-direction, A_{ca} is the amplitude spectrum at normal incidence of a known surface area, and da is some differential area on the surface. By making the transformation $ct = 2z$, and performing the integration perpendicular to the wave direction, we arrive at the expression

$$F(f) = K(\theta) \int l(t) \exp\{-i2\pi ft\} dt \quad (45)$$

so the width of the surface perpendicular to the wave vector-normal plane is just

$$l(t) = C(\theta)F^{-1}\{F(f)\} \quad (46)$$

When this inversion is made, the resulting waveform in the time domain can be scaled into the distance domain to provide the distance from the nearest to the farthest point of the surface and the width of the surface at any distance between these two points.

In the limit of an infinite frequency band, the inversion would provide an exact value of the length and width, but FFTs cannot process an infinite frequency band, nor can any transducer provide one, so the results of an inversion of a single spectrum suffer from poor resolution.

To improve this, two techniques are used. First, the band width is extended by taking another R-F sample at the same orientation with a different transducer which has a higher mid frequency, choosing some frequency in the overlap area of the transducers, and matching both the amplitude and phase spectra at that point.

In the region where $ka \ll 1$, where a is the maximum surface dimension in the direction of the wave, the noise dominated spectrum is discarded and replaced by the theoretically correct parabolic curve also matched to the data spectrum. When this augmented complex spectrum is inverted, the resolution in the time/distance domain is improved.

The first test was the relative power reflected from each crack face in normal incidence. The power differed by 12 dB, implying an area ratio of 4:1 which is exactly correct. Waveforms were then taken at an inclination of 17° in the metal of the circular disc and of the ellipse

both along the minor and major axis. The circular disc image was circular, and calculations give the dimension along the wave vector as 5.5 mm while the signal matched a half circle of 2.58 mm giving an error of 10 percent along the wave, and 3 percent across the disc. For the ellipse A_{ca1} was taken as 4, for all frequencies using the circle as reference, and along the major axis the resulting image, gave a very good likeness of an ellipse, and implied a length of 5.16 mm. The minor axis, calculated from the length to height ratio of the inverse signal, is 1.38 mm. These results are within 3 percent and 10 percent respectively, and are given in Table 4.

Table 4. Deconvolution results (pulse-echo at 17°).

Aspect Ratio	Actual Size (mm)	Measured Size (mm)
1	5.00	5.50
4	5.00	5.16
4	1.25	1.38

CHAPTER IV

SPHEROIDS

Introduction

An elastodynamic solution for scattering from spherical inclusions, voids, elastic, and rigid inclusions, is presented in detail by Ying and Truell³⁷ for incident longitudinal waves. The corresponding solution for incident transverse waves is given by Einspruch, Witterholt, and Truell.³⁸

In both cases mode conversion is a natural result of matching both the displacement and normal stresses at the sphere boundary.

Kraft and Franzblau³⁹ considered scattering by generalized elastic waves by cavities, and calculated the scattering cross sections for both incident shear and longitudinal waves as functions of the wave number times the sphere radius, ka , and the ratio of longitudinal to shear wave speeds or the Poisson ratio of the host medium.

Since this problem is so thoroughly available in so many places, it will not be detailed here.

The method used, assumption of a divergenceless scalar and a sourceless vector potential in the separable wave equation, with connection in the boundary conditions, is applicable to any problem which can be formulated in any of the 11 separable coordinate systems.

Since the eigenfunctions needed for all but the spherical system are not readily available for the most part, most nonspherical problems are approached by means of various approximations.

Born Approximation

The basic scattering integral equation from a spheroidal inclusion can be represented as

$$u_i(r) = \int_{R_1} dR_1 G[u_m, u_{l,m}] \quad (47)$$

where the displacements and stresses, through $u_{l,m}$ and C_{ijkl} , are stipulated in the volume R_1 .

If we use a reasonable guess as to expressions for these quantities, we can form an iteration of integrals with

$$u_i^{(n)}(r) = \int_{R_1} dR_1 G[u_m^{(n-1)}, u_{l,m}^{(n-1)}] \quad (48)$$

and form the Born series if the answer converges with increasing n .

The first iteration, or first Born approximation, has been applied to scattering from spheroids and found to fit rather poorly in the forward direction for voids, hardly surprising, since the perturbation of the elastic constants are zero.

For a void, or tractionless boundary, the expression is very simple, and for an elastic inclusion, the derivative of the incident field is easy to find.

The approximation then consists of replacing the unknown displacement and stress at the scatterer by the fields that would be there if the flaw were not.

Again the boundary conditions lead to mode conversion, and the longitudinal to longitudinal or L-L scattering amplitude coefficients, longitudinal to shear or L-S scattering coefficients, and S-S and S-L coefficients for spheroids have been worked out and experimentally verified for a number of scatterers.

Theory

Consider the equation of motion in an elastic solid for two cases, a displacement $\underline{u}$ with its corresponding body force or driving force $\underline{f}$ and another displacement field $\underline{v}$ with its driving force $\underline{g}$.

$$C_{ijkl}u_{k,lj} + f_i = \rho \ddot{u}_i \quad (49a)$$

$$C_{ijkl}v_{k,lj} + g_i = \rho \ddot{v}_i \quad (49b)$$

Algebraic manipulation yields the combined equation

$$\begin{aligned}
& C_{ijkl}(v_i u_{k,lj} - u_i v_{k,lj}) + \rho(v_i \bar{u}_i - u_i \bar{v}_i) \\
& + (v_i f_i - u_i g_i) = 0
\end{aligned} \tag{50}$$

Integrating over some region R (see Fig. 14) bounded by a surface S and using Green's theorem, we obtain

$$C_{ijkl} \int_S ds [v_i u_{k,l} - u_i v_{k,l}] n_j + \int_R dR [v_i f_i - u_i g_i] = 0 \tag{51}$$

assuming that both displacement fields are time dependent as $\exp(-i\omega t)$. If Eq. (1b) is an impulse field, i.e. v_i is the response of the medium described by C_{ijkl} and ρ to an impulse $\delta_{im} \delta(\underline{r} - \underline{r}')$, then $v_i(\underline{r})$ is the response of the medium at $\underline{r}$ to an impulse at $\underline{r}'$, a Green's function form, and our equation reduces to

$$\begin{aligned}
u_m(\underline{r}) = & C_{ijkl} \int_S ds n_j [g_{im}(\underline{r}, \underline{r}') u_{k,l}(\underline{r}') - u_i(\underline{r}') g_{km,l}(\underline{r}, \underline{r}')] \\
& + \int_R dR f_i(\underline{r}') g_{im}(\underline{r}, \underline{r}')
\end{aligned} \tag{52}$$

where $g_{im}(\underline{r}, \underline{r}')$ is the response function previously denoted by v_i .

This integral solution for the displacement at $\underline{r}$, $u_i(\underline{r})$ depends on the impulse displacement at $\underline{r}$ due to an impulse at $\underline{r}'$, $g_{im}(\underline{r}, \underline{r}')$ the impulse tractions at $\underline{r}'$ due to an impulse at $\underline{r}$, $C_{ijkl} g_{km,l}(\underline{r}, \underline{r}') n_j$, and the volume integral of the driving field at $\underline{r}'$ within R with the impulse response.

$$R = R_1 + R_2$$

$$S = S_1 + S_2$$

$$u_i = u_i^0 + u_i^s$$

Fig 14. THE SCATTERING PROBLEM

We proceed by assuming the inclusion is a perturbation in an otherwise undisturbed field.

Let C_{ijkl} , the elastic stiffness tensor in R_1 be defined

$$C'_{ijkl} = C_{ijkl} + \delta C_{ijkl} \quad (53a)$$

where C_{ijkl} is the stiffness in R_2 .

Also ρ' , the density of material contained in R_1 is defined as

$$\rho' = \rho + \delta\rho \quad (53b)$$

and that the variation of both quantities is negligible with respect to the quantities themselves.

If we identify the surface integral in Eq. (5) with an incident displacement field u_i^0 , which satisfies the homogenous field equation of motion

$$C_{ijkl} u_{k,lj}^0 + \rho \omega^2 u_i^0 = 0 \quad (54)$$

then the scattered field becomes

$$\begin{aligned} u_i(\underline{r}) = & \delta\rho\omega^2 \int_{R_1} dR_1 g_{im} u_m(\underline{r}') \\ & + \delta C_{ijkl} \int_{R_1} dR_1 g_{ij,k}(\underline{r} - \underline{r}') u_{l,m}(\underline{r}') \end{aligned} \quad (55)$$

where we have used the reciprocity relations for $g(\underline{r}, \underline{r}')$. As the surface S goes to infinity, we can use asymptotic approximations for $\underline{r} - \underline{r}'$. Our equation becomes in the far field limit

$$u_i = \text{Const } \hat{r}_i \hat{r}_j f_j(\underline{k}) \frac{e^{ikr}}{r} + (\delta_{ij} - \hat{r}_i \hat{r}_j) f_j(\underline{k}) \frac{e^{iKr}}{r} \quad (56)$$

where

$$\begin{aligned} f(\underline{\eta}) = & \frac{\eta^2}{4\pi\rho\omega^2} \left[\delta\rho\omega^2 \int_{R_1} dR_1 u_i(\underline{r}') \bar{e}^{i\eta \cdot \underline{r}'} \right. \\ & \left. + i\eta \hat{r}_j \delta C_{ijkl} \int_{R_1} dR_1 \epsilon_{kl} e^{-i\eta \cdot \underline{r}'} \right] \end{aligned} \quad (57)$$

Finally, we can identify the frequency dependent $F(k)$ in terms of the far field longitudinal and shear scattering amplitudes as

$$A_i(\underline{k}) = \hat{r}_i \hat{r}_j f_j(\underline{k}) \quad (58a)$$

for longitudinal, and

$$B_i(\underline{k}) = (\delta_{ij} - \hat{r}_i \hat{r}_j) f_j(\underline{k}) \quad (58b)$$

for shear, so that

$$A_i^{L-L} = \hat{r}_i \frac{k^2}{4\pi} \left[\frac{\delta\rho}{\rho} \cos \theta - \frac{\delta\lambda + \frac{\partial\delta\mu}{\partial\mu} \cos^2 \theta}{\lambda + \delta\mu} \right] S(k, k, \theta, \phi) \quad (59a)$$

$$B_1^{L-S} = \hat{\theta}_1 \frac{k^2}{4\pi} \left[\frac{k}{K} \frac{\delta\mu}{\mu} \sin \theta \cos \phi - \frac{\delta\rho}{\rho} \sin \theta \right] S(k, K, \theta, \phi) \quad (59b)$$

$$A_1^{S-L} = \hat{r}_1 \frac{k^2}{4\pi} \left[\frac{\delta\rho}{\rho} \sin \theta \cos \phi - \frac{K}{k} \frac{\delta\mu}{\lambda + \delta\mu} \sin \theta \cos \phi \right] S(K, k, \theta, \phi) \quad (59c)$$

$$\begin{aligned} B_1^{S-S} = & \hat{\theta}_1 \frac{k^2}{4\pi} \left[\frac{\delta\rho}{\rho} \cos \theta \cos \phi - \frac{\delta\mu}{\mu} \cos \theta \cos \phi \right] S(K, k, \theta, \phi) \\ & + \hat{\phi}_1 \frac{k^2}{4\pi} \left[-\frac{\delta\rho}{\rho} \sin \phi - \frac{\delta\mu}{\mu} \cos \theta \sin \phi \right] S(K, k, \theta, \phi) \end{aligned} \quad (59d)$$

where

$$S(\alpha, \beta, \theta, \phi) = \int_{R_1} dR_1 e^{i(\alpha\hat{z} - \beta\hat{r}) \cdot \underline{r}'} \quad (60)$$

for scattering of wave vector β in the direction $\hat{r}$ resulting from an incident wave vector α along the axis of scattering symmetry $\hat{z}$.

The Green's function for elastic space is very complicated at best, but if we specialize to $L + L$ scattering in isotropic media only, then we know that the scattered field will asymptotically reduce to

$$u_m^S(r) \xrightarrow{r \rightarrow \infty} S(\underline{k}) e^{ikr}/r \quad (61)$$

while our specialized Green's function becomes

$$G_{ij}(\underline{r}, \underline{r}') = 1/[4\pi\rho\omega^2] \left[\frac{e^{ik|\underline{r}-\underline{r}'|}}{|\underline{r}-\underline{r}'|} \right]_{,ij} \quad (62)$$

where

$$k^2 = \frac{\rho \omega^2}{(\lambda + 2\mu)} \quad (63)$$

Now as $\frac{r}{r'} \rightarrow \infty$

$$\frac{1}{|\underline{r} - \underline{r}'|} \rightarrow \frac{1}{|\underline{r}|} \quad (64a)$$

$$|\underline{r} - \underline{r}'| \rightarrow |\underline{r}| - \underline{r} \cdot \underline{r}' \quad (64b)$$

the far field scattering conditions which were arrived at by Max Born, and

$$\left[\frac{e^{ik|\underline{r} - \underline{r}'|}}{|\underline{r} - \underline{r}'|} \right]_{,i} \rightarrow \frac{ike^{ikr}}{r} \underline{e}_i e^{-ik \cdot \underline{r}'} \quad (65a)$$

$$\left[\frac{e^{ik|\underline{r} - \underline{r}'|}}{|\underline{r} - \underline{r}'|} \right]_{,ij} \rightarrow \frac{-k^2 e^{ikr}}{r} \underline{e}_i \underline{e}_j e^{-ik \cdot \underline{r}'} \quad (65b)$$

the Kirchhoff limiting conditions.

The last term in Eq. (55) involves the derivative of the function $\theta(\underline{r}')$ which is the Dirac delta function $\delta(\underline{r} - \underline{r}')$ so we can neglect the term for large r and arrive at

$$u_m^s \rightarrow \frac{e^{ikr}}{r} \int e^{-ik \cdot \underline{r}'} [\text{other terms}] k^2 \theta(\underline{r}') d^3 r' = \frac{e^{ikr}}{r} S(\underline{k}) \quad (66)$$

Now S is just the back scattered amplitude spectrum obtained from our experiment and the integral is just the Fourier transform of $\theta(\underline{r}')$, so by an inverse transform, we can obtain $\theta(\underline{r}')$, the characteristic function. Using an inverse sine transform of the deconvolved amplitude spectrum scattered from the flaw, we finally obtain⁸⁹

$$\theta(r) = \text{constant} \int S(k) \frac{\sin 2kr}{2kr} dk \quad (67)$$

Sample Preparation

Several different types of samples were made for this section of the work. Discs of diffusion bonded titanium incorporating machined voids in their centers were provided by the Rockwell Science Center for this as well as the preceding embedded crack section. Among them, and available periodically, were spheres of 800 and 1,200 micron diameters, oblate, and prolate spheroids with an 800 micron symmetry axis and aspect ratios of 2:1, 1:2, and 1:4.

Blocks of aluminum cast using various methods to achieve differing degrees of surface roughness and volume porosity were provided by the Boeing Corp. and the Ohio State University's Metallurgy foundry.

The Boeing samples were very fine grained and essentially free of porosity and grain boundary scattering effects, discussed later in this work.

For this part of the work, the samples were smoothed and polished to provide a simple slip boundary condition in immersion, though as described in previous work,⁹⁰ they were also used because of their surface conditions.

Weld samples were fabricated to achieve the assumed elastic boundary conditions for a soft scatter in accordance with the development of the Born approximation and subsequent inversion technique.

A robot welder using a filler wire was used for a repeatable four pass weld.

After the base and second pass, small pieces of 1/8" tungsten wire were placed in the weld pool, the third pass laid, more tungsten wire and shot laid in, and the cover pass completed. The weld was then machined smooth on both sides, cut off on one side, and cylindrical holes machined in parallel to the sample face (see Fig. 15).

Finally, a thermosetting polymer was mixed, degassed, and a small lead shot dropped into the resin during the curing processing.

Experimental Procedure

The experiment proceeded in three steps, the testing of equipment and samples against the exact theory of scattering for spheres, the comparison to experiment of the Born approximation for scattering for general spheroids using a two transducer arrangement, and the test of the Born inversion using backscattering only, and a single transducer.

Fig 15. WELD SAMPLE WITH INCLUSIONS

The incident wave was a longitudinal wave in all cases in the interests of scattering amplitude, but both the longitudinal and shear scattered waves were measured at various angles to try simulation of an ultrasonic array.

The semi-precision goniometer shown in Fig. 10 of the previous section was used for this.

The transducers were a matched production pair with very similar transmission and reception characteristics.

Finally, a single transducer with X, Y, and θ freedom was used to test the approximation at various angles of incidence.

Results and Conclusion

The experimental system, calibrated on the sphere scattering, was applied to scattering at various angles from spheroids. As discussed, the Born approximation was developed on the assumption that the material properties of the scattering inclusion and the host material were only slightly different, while it's hard to be more different from an elastic solid than a void is, so the scattering results were far from exact, roughly worsening with increasing difference from backscatter.

Figure 16 illustrates the results typical for a spheroid. The backscattered amplitude spectra though were surprisingly good, so testing of the inversion began.

Fig 16, LINEAR AMPLITUDE DISTRIBUTION OF BOTH L AND S WAVES
AS A FUNCTION OF FREQUENCY AT TWO SCATTERING ANGLES
FOR A 1:4 OBLATE SPHEROIDAL CAVITY IN TITANIUM

INVERSION RESULTS

Spectra Correction

Once the scattered waveform has been Fourier transformed and normalized to its resulting complex amplitude impulse spectrum, both low and high frequency corrections must be made before the spectrum can be inverted to yield the characteristic function of the inclusion. In the low frequency regions, the scattered power should not exhibit a dc component, so the Fourier amplitude $A(v) = \alpha(v) + i\beta(v)$ is approximated by an exponential e^{ika} . More specifically, the real part of the spectrum is matched both in amplitude and slope at the low frequency limit v_0 to a trial function⁹¹

$$a(v < v_0) + a_2v^2 + a_4v^4 \quad (68a)$$

$$b(v < v_0) + \alpha_3v^3 + \alpha_5v^5 \quad (68b)$$

The high frequency limit v_1 is chosen at least five times as high as v_0 , and the Hanning window set so that $[a^2(v_1) + b^2(v_1)]^{1/2}$ is 24 dB down from the spectrum's maximum.

Time Shifting

Next the correct time of the signal must be set. The theory assumes that the timing of the problem is such as $t = 0$ when the incident impulse reaches the center of the scatterer, so the band limited spectrum is time shifted until the time is correct. For this purpose, the area function is calculated, and the time shifted until the maximum area corresponds to the zero fo time in the problem.

The digital gating technique and equivalence of either the amplitude and phase or complex amplitude representations are illustrated in Fig. 17.

The inversion assumes that the incident wave passes the scattering center at $t = 0$, so the wave forms must in general be shifted to align the phase properly in order to accurately determine the area function.

The area function is defined as:

$$\alpha(z) = \text{constant} \int_{-\infty}^{\infty} \frac{A_0(k)}{k^2} (e^{-12kz}) dk \quad (69)$$

and is the area of a plane through the scatterer at a distance z , so if our measured spectrum is from an r.f. waveform offset from the correct time by an amount Δt , where $\Delta z = 1/2v\Delta t$, we will arrive at:

$$\bar{\alpha}(z) = \int \frac{A_0(k)e^{-12kz}}{k^2} dk \quad (70a)$$

Fig 17. FOURIER SPECTRAL ANALYSIS

$$= \int \frac{A_0(k) e^{-i\omega\Delta t - i2kz}}{k^2} dk \quad (70b)$$

$$= \alpha(z - \Delta z) \quad (70c)$$

Since our shifted spectrum will be of the form,

$$A_0(v) = A(v) e^{-i\omega\Delta\tau} \quad (71a)$$

$$= \rho(v) e^{i[\sigma(\omega) - \omega\Delta\tau]} \quad (71b)$$

the correction will introduce a slope in the phase spectrum, but no change in the modulus, and a time shift of $\Delta\tau$ in the r.f. waveform.

If we define the Fourier transform of some function of time $f(t)$ as

$$F(v) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt \quad (72a)$$

$$= a(v) + ib(v) \quad (72b)$$

$$= \alpha(v) e^{i\beta(v)} \quad (72c)$$

then we can look at the complex spectrum of $f(t)$ as having real and imaginary parts or as having an amplitude and a phase. The representations are equivalent in every way.

If we now look at the transform of the same function shifted in time by a constant amount Δt , then we have

$$\int f(t - \Delta t) e^{-i\omega t} dt = \int f(\tau) e^{-i\omega(\tau + \Delta t)} d\tau \quad (73a)$$

$$= e^{-i\omega\Delta t} \int f(\tau) e^{-i\omega\tau} d\tau \quad (73b)$$

$$= e^{-i\omega\Delta t} F(\nu) \quad (73c)$$

$$= \{a(\nu) \cos[2\pi\nu\Delta t] + b(\nu) \sin[2\pi\nu\Delta t]\} \\ + i\{b(\nu) \cos[2\pi\nu\Delta t] - a(\nu) \sin[2\pi\nu\Delta t]\} \quad (73d)$$

$$= \alpha(\nu) e^{i[B(\nu) - 2\pi\nu\Delta t]} \quad (73e)$$

so that the resulting real and imaginary parts of the spectra are now coupled and modulated, while the amplitude is the same as before and the phase has a linear term in ν added to it, corresponding to changing the overall slope of the phase, while preserving all the other frequency dependent structure intact. Thus, we can conclude that the amplitude of a Fourier transform is independent of the absolute time, while the slope of the phase is linearly dependent on any absolute time offset.

By introducing a compensating phase shift into the deconvolved spectrum, we have a time corrected, band width limited experimental spectrum which can be inverted to yield the characteristic function. This procedure is illustrated in Fig. 18.

Fig 10. INVERSION ROUTINE FUNCTIONS

Characteristic Radius Estimation

The characteristic function should be one up to the radius of the flaw, and zero thereafter, but the band limiting prohibits this, so that the function instead starts near one and tails off towards zero. Two complementary methods are used to establish the estimate a . In one method, the characteristic function $\theta(r)$ is integrated from $r = 0$ to $r_{\max}$, and divided by the maximum $\theta_{\max}$. In the other, that point at which θ has fallen to the value of $1/2 \theta_{\max}$ is used to establish the estimated radius. Due to the nonideal nature of the spectrum being processed, the two seldom agree exactly, so the latter has been used in determining the estimated radius since it is much more accurate.⁸⁹

As can be seen in Table 5 and the graphic representation of this data in Fig. 19, the inversion routine yields excellent results both for elastic inclusions and voids in a wide range of materials.

Furthermore, for normal incidence relative to the axis of symmetry of a cylinder, it seems remarkably insensitive to the overall shape of the scatterer, yielding the characteristic radius in the direction of observation.

As found by Elsley, Ahlberg, and Richardson,⁴⁹ the inversion requires a bandwidth described by $0.5 \leq ka \leq 2.5$ where k is the incident wave number and a the characteristic radius.

Since the properties of the materials used in this study varied rather widely,, the same data in terms of the velocity divided by the radius is retabulated in Table 6, and presented in revised form in Fig. 20.

Table 5. Born inversion estimate of inclusion radii.

Sample	Actual Radius (μm)	Estimated (μ)
Cylindrical inclusion in weld	1587.	1522.
Cylindrical void in weld	698.	537.
Spherical Void in titanium	600.	619.
Spherical void in titanium	400.	425.
Cylindrical void in AL	356.	348.
Spherical inclusion in plastic	307.	291.
Oblate void in titanium	200.	243.
Spherical inclusion in plastic	87.	161.

Fig19. EXPERIMENTALLY ESTIMATED vs ACTUAL RADII OF THE INCLUSIONS

Table 6. Normalized Born estimates.

Sample	v/a	Calc
Cylindrical inclusion in weld	3.37	.959
Spherical inclusion in plastic	8.60	.948
Cylindrical void in weld	9.48	.769
Spherical void in titanium	10.57	1.032
Spherical void in titanium	15.85	1.063
Cylindrical void in AL	18.03	.978
Spherical inclusion in plastic	30.35	0.851
Oblate void in titanium	31.7	1.215

Fig 20. NORMALIZED RADII OF THE ESTIMATES vs. WAVE SPEED / ACTUAL RADII

The normalized estimate, i.e. the measured estimate divided by the actual radius is accurate to within 25 percent in all cases considered.

Even though the available samples were scarce, they covered a wide range of size of inclusion and wave speed.

CHAPTER V

ROUGH SURFACE

Introduction

The effects of a rough surface on the incident wave are important since the flaw characterization schemes described so far, depend on the signal back scattered from the flaw.

If the face of the sample affects this signal, results can be expected to be poor.

However, if the surface can be characterized, and its effects on both the incident and scattered wave characterized as well, then the characterization methods are still useful, and the roughness of the surface can also be easily determined. We thus have the problem of first determining the surface condition of a sample to be characterized, then the problem of predicting the effect on the incident wave that this surface will have, and finally using these results to modify the scattered signal from a flaw so that the surface conditions do not adversely affect the inversion results. The first treatment of matter waves at a nonplane interface was by Lord Rayleigh.⁵²

He considered a periodic surface with a sine profile. His exact solution by matching boundary conditions using an infinite sum of plane

waves was extended to arbitrary periodic surfaces by Meechum⁵² and to randomly rough surfaces by Rice.⁹³

The harmonic method, based on an assumed potential at the surface to approximate boundary conditions, uses this potential to arrive at a succession of integrals which give a solution approaching the exact solution in the limit of the sum.

This method has been used by Lapin⁹⁴ and by Boyd, Davenport, and Welton.^{58,61,95} We will follow this potential formulation to arrive at the scattered intensity from a randomly rough surface which has a Gaussian distribution of irregularities.

The theory will be compared both to sample surfaces and also to the rough fracture surface of an embedded crack.

We will also consider a simple model for predicting the transmission coefficient through a rough surface, and compare its predictions to experiment.

Theory

We define a general potential $\Omega(\rho)$ as existing on the surface defined by $\zeta(x,y)$ as shown in Fig. 21, at position $\zeta_0(\rho_0)$. If we make the following approximations, that at every point on the surface ζ there is a unique normal $\underline{n}$, that ζ is at least piecewise continuous and single valued, then we can write an expression for the scattered pressure as

Fig 21. A RANDOMLY ROUGH SURFACE

$$p(r_1) = \int_s ds \Omega(\rho) \frac{\partial}{\partial n} \left[\frac{e^{ikr_1}}{r_1} \right] \quad (74)$$

since $\frac{e^{ikr}}{r}$ is a characteristic solution of the Helmholtz Equation.

If we now write the pressure at some point ρ_0 on the surface at $\zeta_0(\rho_0)$ as a negative of the pressure that would exist there if the surface were not present times a shadowing function whose form depends on the surface, then we can write.

$$P_0 = -S(\rho_0) P_{in}(\rho_0) \quad (75)$$

where S represents the probability that the point ζ_0 could be insonified by both receiver and transmitter. For our pulse-echo experiment, this function takes a simple form depending on the angle of incidence, and if we further specialize to normal incidence, the value goes to one.

If we now approximate our unknown potential as

$$\Omega(\rho_0) = \Omega_0 + \Omega_1 + \Omega_2 + \dots \quad (76)$$

where

$$\Omega_0 = \frac{1}{2\pi} P_{in}(\rho_0) \quad (77a)$$

$$\Omega_1 = \frac{1}{2\pi} \int_s ds \Omega_0 \frac{\partial}{\partial n} \left[\frac{e^{ikr_1}}{r_1} \right] \quad (77b)$$

$$\Omega_2 = \frac{1}{2\pi} \int_{s'} ds' \Omega_0 \frac{\partial}{\partial n'} \left[\frac{e^{ikr'}}{r'} \right] \quad (77c)$$

the scattered pressure becomes

$$P_s = \frac{1}{2\pi} \int_s ds P_{in}(p_0) \frac{\partial}{\partial n} \left[\frac{e^{ikr_1}}{r_1} \right] + \dots \quad (78)$$

This first term is the single scattering term, which assumes that there is no contribution to the pressure at z_0 from scattering at other points on the surface. Successive terms account for multiple scattered initial pressure, i.e., the contribution at r_1 from scattering off the surface at r_1' , etc.

If the height of the surface about the mean height $\bar{z}$, defined as

$$\bar{z} = \int_s ds \, z(x,y) \quad (79)$$

is small and the distance to the receiver is large with respect to the wave length then

$$\frac{\partial}{\partial n} \left(\frac{e^{ikr_1}}{r_1} \right) \rightarrow ik \frac{e^{ikr_1}}{r_1} \frac{n}{r_1} = \underline{e}_1 \quad (80)$$

the Kirchoff limit and we can write

$$P_1 = \text{Constant} \int dsurf P_0 e^{ik2\sqrt{(z^2 - \bar{z}^2)}} \times [\text{surface terms}] + \dots \quad (81)$$

If we now assume a Gaussian beam profile for the incident pressure, the intensity becomes

$$\langle I_s \rangle = pp^* \quad (82a)$$

$$= \text{constant} \int dsurf e^{-4k^2 h^2} \times [\text{same surface terms}]$$

$$+ \text{constant} \int dsurf e^{-1k^2 h^2} \sum_{m=1}^{\infty} \frac{(4k^2 h^2)^m}{m!}$$

$$\times [\text{cross surface terms}] \quad (82b)$$

where the same surface terms are the result of multiplying surface increments corresponding to the same point and the cross surface terms the results of surface increments displaced with respect to each other. Since our surface is random, we can interpret the same surface terms as being the contribution from coherent scattering, while the cross surface terms are the contribution of incoherent or multiple scattering. In the low frequency limit, only the coherent scattering will be of significance,⁶¹ and we can write the scattered intensity as

$$\langle I_s \rangle = I_0 e^{-g} \quad (83)$$

where I_0 is the intensity that would have been received if the surface had been smooth, and g in the Rayleigh-Beckman parameter specialized to back scattering at normal incidence

$$g = (2kh)^2 \quad (84)$$

where h is the r.m.s. roughness of the surface

$$h = (\zeta^2 - \bar{\zeta}^2)^{1/2} \quad (85)$$

We can then determine this parameter of the surface from our low frequency scattered spectrum, or predict the scattered spectrum if we know the r.m.s. roughness of a surface, since

$$k^2 h^2 = \text{constant} * \log\left[\frac{\langle I_s \rangle}{I_0}\right] \quad (86)$$

or proportional to the normalized scattered spectrum in dB.

R.M.S. Roughness

If we assume that the rough surface has a distribution of irregularities that is Gaussian about some mean value $\bar{\zeta}$ and set that value as our $z = 0$ value, then the probability that some point will be

$$P(\zeta) = C e^{-1/2 \frac{(\zeta^2 - \bar{\zeta}^2)}{\sigma^2}} \quad (87)$$

If we set $C = (2\pi\sigma)^{-1}$ then the expectation values of the various moments of the surface distribution will be

$$\langle \zeta^n \rangle = \int_{-\infty}^{\infty} \zeta^n \underline{p}(\zeta) d\zeta \quad (88)$$

and in particular

$$\langle \zeta \rangle = \bar{\zeta} \quad (89a)$$

$$= \langle \zeta^2 \rangle - \langle \zeta \rangle^2 \quad (89b)$$

$$= \sigma^2 \quad (89c)$$

the variance of the distribution and σ , also called h , is the standard deviation, or root mean square, of the distribution.

Low Frequency Correction

Since we are renormalizing the probability in terms of z , σ , or h , will be the R.M.S. value of the irregularities, or ζ and we can look at the characteristic function of the distribution.

$$\phi(k) = \int dz e^{ikz} p(z) = F^{-1}(P(z)) \quad (90a)$$

$$= 1 + ik\langle z \rangle - (k^2/2!) \langle z^2 \rangle + \dots + \frac{(ik)^n}{n!} \langle z^n \rangle + \dots \quad (90b)$$

$$= e^{-1/2k^2\langle z^2 \rangle}$$

(90c)

for a true Gaussian.

Now, since we have Fourier transformed the normalized intensity spectrum to arrive at the probability distribution, we should have obtained again our intensity spectrum in the characteristic function for the system. From this we can obtain a correction for over intensity spectrum for low frequencies, i.e., where $k^2\langle z^2 \rangle < 1$. This is necessary since our use of the Weiner filter used in the deconvolution of our scattered spectrum has limited our spectrum to the usable band width of our transducers, and they do not extend to frequencies much lower than 1MHz.

Thus, we correct the low frequency of our intensity spectrum with an approximation

$$\frac{\langle I \rangle}{I_0} = 1 - \alpha k^2 \quad (91)$$

such that the approximation matches the experimental spectrum at our low frequency limit and that at zero frequency, the power spectrum is 0 dB, which implies that for very low frequencies, a rough and a smooth surface scatter equal intensity.

In the high frequency regime, $ka > 1$, we can assume that all the field is due to incoherent scattering only. Again for normal incidence, we find the scattered intensity as

$$\langle I \rangle = \frac{AL^2}{16\pi} r_0^2 r_1^2 h^2 \quad (92)$$

where h is the R.M.S. roughness still, L is the correlation length, and A is the area of the rough surface insonified.

For a smooth surface, the scattered intensity back to our transducer is just

$$I_0 = \frac{A^2 k^2}{\pi^2 r_0^2} \quad (93)$$

so the normalized intensity leads to the correlation length as

$$L = 4hka \sqrt{\frac{\langle I \rangle}{I_0}} \quad (94)$$

so the normalized amplitude should behave in the high frequency limit as inverse frequency.

For the rough surface experiments described here, the high frequency limit was at or above the bandwidth of the transducers, and so the autocorrelation lengths were not calculated.

Since a rough surface both scatters incident energy in all directions and couples to the surface creating some component of surface wave, a simple model based on conservation of energy cannot be expected to hold.

Figure 22 shows the problem of determining an inclusions size when the sample surface is rough.

Fig 22. THE ROUGH SURFACE PROBLEM

The transmission coefficient both into and out of the sample is necessary for accurate determination of the inclusions characteristics.

The transmission coefficient for a randomly rough surface has not been worked out rigorously, but a first order approximation has been formulated by Nagy.⁹⁶ He assumes an average surface $\bar{\zeta}$ and a distribution about that level as

$$h(x, y) = \zeta(x, y) - \bar{\zeta} \quad (95)$$

An incident plane wave in material 1 can be represented as

$$E_0(\underline{x}, t) = E(\underline{x})e^{i\underline{k}_1 \cdot \underline{x}} \quad (96)$$

If we assume the roughness of the surface is a perturbation on the well known flat smooth surface case, we can write the frequency spectrum of the reflected energy as

$$F_r(\underline{k}) = \frac{1}{\text{Area}} \int_s dse^{i\phi_r(x,y)} R_0 E_0 e^{-i\underline{k}_1 \cdot \underline{x}} \quad (97)$$

where R_0 is the reflection coefficient for the smooth case, $\phi_r(x, y)$, is the assumed phase perturbation and Area is the area of the surface insonified by the incident beam.

For normal incidence along the z direction

$$F(k_x = 0, k_y = 0, k_z) = \frac{R_0 E_0}{\text{Area}} \int_s dse^{i\phi_r} dx dy \quad (98a)$$

assuming that the perturbation is very small, we can expand this equation as

$$= \frac{R_0 E_0}{Area} \left[1 + i \int_s ds \phi_r - 1/2 \int_s ds \phi_r^2 + \dots \right] \quad (98b)$$

The second term is just the mean value of the surface irregularity, assumed zero, so

$$R = R_0 [1 - 1/2 \phi_r^2] \quad (99)$$

Physically, the phase difference is the deviation from flatness times the wave vector, as illustrated in Fig. 23, so we have

$$R = R_1 [1 - 2h^2 k_1^2] \quad (100)$$

and the transmission coefficient

$$T = T_0 [1 - 1/2 \phi_{tr}^2] \quad (101a)$$

$$= T_0 [1 - 1/2 h^2 (k_2 - k_1)^2] \quad (101b)$$

This theory predicts that the transmission coefficient is the same in either direction while the reflection coefficients differ depending on the physical properties of the two materials. For a random roughness

Fig 23. THE ROUGH SURFACE TRANSMISSION MODEL

with a Gaussian distribution of irregularities, these coefficients become

$$R = R_0 e^{-2h^2 k_1^2} \quad (102a)$$

$$T = T_0 e^{-1/2 h^2 (k_1 - k_2)^2} \quad (102b)$$

Thus, if we can characterize a surfaces roughness via backscattering, we can approximate its transmission coefficient also.

Sample Preparation

The samples examined in this section were all case aluminum blocks, sectioned to assure parallel sides, and polished smooth on all but one side. The commercial casts were provided by the Boeing Corporation, and were cast components of an aircraft section. They typically exhibited very low porosity and low R.M.S. roughness. Other casts were made by the foundry of the Ohio State University's Metallurgy department in sand casts, and exhibited uniform roughness in many cases, but very high porosity.

The commercially available powders were of A-357 and C-355 aluminum.

Data Analysis

In considering the change in an original ultrasonic field $f(v)$ caused by its interaction with any medium, we can represent the field after the interaction as the original complex field, composed of amplitude and phase as functions of frequency, times a complex operator representing the interaction. If we represent the beam spreading and attenuation of $f(v)$ in medium 1 by $g_1(v, x)$ at x and the interaction involved in crossing a rough boundary into medium 2 by τ_{12} , then $\tau_{12}g_1f$ is the amplitude and phase of the resultant field just inside the rough surface and $\rho_{12}g_1f$ that just outside it reflecting back to transducer 1. Representing the response functions of the transmitter and receiver as γ_t and γ_R respectively, $\gamma_R g_1(d')t_{21}g_2(h)\tau_{12}g_1(d)f$ is the receiver response resulting from a field f traveling a distance d in medium 1, crossing a rough surface into medium 2, suffering beam spread and attenuation represented by $g_2(h)$, crossing a smooth boundary back into medium 1, traveling a distance d' and being partially converted back into electrical energy by the receiver. Similarly, $\gamma_T g_1(d)\rho_{12}g_1(d)f$ is the surface pulse-echo signal and $\gamma_T g_1(d)\tau_{21}t_2(h)r_{21}g_2(h)\tau_{12}g_1(d)f$ is the pulse-echo from the opposite smooth face of material 2. The extensions and much more complicated cases involving nonnormal angular incidence resulting in mode conversion, the creation of expected surface waves, etc., is obvious.

Now consider the situation shown in Fig. 24. For normal incidence with a parallel sided smooth sample we can represent signal A as the

Fig 24. SCHEMATIC OF THE SURFACE PROBLEM

surface pulse-echo signal, $A = \gamma_T g_1(d) r_{12} g_1(d) f$. Similarly,

$C = \gamma_T g_1(d) t_{21} g_2(h) r_{21} g_2(h) t_{12} g_1(d) f$, so deconvolving C by A yields

$$\frac{C}{A} = \frac{t_{21} g_2(h) r_{21} g_2(h) t_{12}}{r_{12}} \quad (103)$$

which is independent of the transmitters response and all damping effects of medium 1. Further, t_{12} , t_{21} , r_{12} , and r_{21} can be calculated from elementary theory and eliminated leaving $(g_2(h))^2$ which contains the attenuation of material 2 with respect to frequency due to the attenuation of the material and the signal losses due to beam spreading. Calculating and eliminating the beam spread losses,⁹⁵ we arrive at the attenuation of material 2 with respect to frequency. If we consider instead the pitch catch system of signals:

$$B = \gamma_T g_1(d') t_{21} g_2(h) t_{12} g_1(d) f \quad (104)$$

$$J = \gamma_T g_1(d') g_1(h) g_1(d) f \quad (105a)$$

$$= \gamma_T g_1(d' + h + d) f \quad (105b)$$

then

$$\frac{B}{J} = \frac{t_{21} g_2(h) t_{12}}{g_1(h)} \quad (106)$$

Similarly, when we introduce a rough surface, possible since the samples, smooth on all but one side were approximately cubic, we can find the corresponding rough surface coefficients as

$$\frac{D}{A} = \frac{\rho_{01}}{r_{01}} \quad (107a)$$

$$\frac{E}{B} = \frac{T_{01}}{t_{01}} \quad (107b)$$

$$\frac{H}{B} = \frac{T_{10}}{t_{10}} \quad (107c)$$

$$\frac{I}{C} = \frac{\rho_{10}}{r_{10}} \quad (107d)$$

as well as many other possible combinations.

Experimental Procedure and Results

For calibration of the experiment, a sample was micro profiled using Battelle Laboratories profilimeter. The scan's resulting R.M.S. roughness was determined as well as the ultrasonic result. To assure there was no reason not to assume that the superposition principle would hold, the surface was then checked by a set of narrow band quartz transducers excited by a short pure wave train, and the discrete results compared to the broadband spectrum obtained. This was then fit to a decreasing frequency squared approximation in the least square sense, and this is shown on Fig. 25.

Fig 25. REFLECTION COEFFICIENT FOR A RANDOMLY
ROUGH SURFACE THEORITICAL AND EXPERIMENTAL

In the investigation of embedded cracks and statistically rough surfaces, the question of the effect a rough surface would have were it on the crack itself. To determine this, a Ti-6Al-4V sample was made from two discs. A penny shaped crack 5.1 cm in radius and 1.25 cm in depth was machined into one of the discs and a hole was drilled through the other. A plug of the same radius was made from one face of a fatigue fracture specimen and driven into the hole so that it was flush with the inner face, and then the two discs were diffusion bonded together.

Scattering studies were made from both sides of the sample to infer differences between the two cases. For normal incidence, the backscattered spectra differed markedly for the two sides of the samples (Fig. 26). Comparison of these spectra, normalized with respect to the reflection from the sample surface, yielded a back scatter spectrum of unity for side B, with the amplitude proportional to the area of the crack face, while the scattering from side A yielded a spectrum proportional to the frequency squared up to approximately 5 MHz, smoothly changing to a constant value for higher frequencies. A fit of the low frequency data to a frequency squared curve yielded an R.M.S. roughness of 131 microns.

The R.M.S. value determined from 320 individual surface height measurements of the complementary side of the rough surface in the region of the fracture plug gave a value of 101 microns in excellent agreement. Additionally, the rough crack was investigated using a series of single frequency transducers, and these measurements followed

Fig 26. RF WAVEFORMS AND FREQUENCY SPECTRA OF BACKSCATTERED WAVES FROM SMOOTH AND ROUGH CRACKS

the broadband results closely. The scattering to various angles from normal incidence longitudinal waves was studied to size the two cracks in a two transducer pitch-catch arrangement, and the inversion described previously for flat cracks yielded results with 3 percent of the actual size in both cases.

In this same arrangement, the total scattered power was compared for both sides, and differed radically between the smooth and rough sides as shown in Fig. 27. It should be noted that the crack had been mislabeled so that the rough side B was actually the smooth side and vice versa as shown by Fig. 25, and that subsequent etching of the sample surfaces confirmed this mixup. This work is described in considerably more detail elsewhere.⁹⁸

From the expressions for the predicted Transmission coefficient through a surface characterized by an R.M.S. roughness h , we see that the coefficients relate to each other as

$$\ln R = \ln R_0 - 2h^2 \frac{\omega^2}{v_1^2} \quad (108)$$

$$\ln T = \ln T_0 - h^2 \omega^2 \frac{(v_2 - v_1)^2}{(v_1 v_2)^2} \quad (109a)$$

$$= \ln T_0 - 1/2 \frac{(v_2 - v_1)^2}{v_2^2} \ln \frac{R}{R_0} \quad (109b)$$

Since the longitudinal wave speeds are different by approximately a factor slightly greater than 4, we expect far less effect in the amplitude of the transmission coefficients than on the reflection.

Fig 27. ANGULAR VARIATION OF THE SCATTERED INTEGRATED POWER

For the samples provided, the surface characterization worked very well when compared to profile measurements, but the porosity of most of the samples made transmission coefficient measurements impractical. Figure 28 shows the volume effect in a typical case for three different depths into the material, all through the same randomly rough surface.

The sample was milled as closely as possible to parallel with the cast surface, but the very high porosity washed the transmitted signals out too quickly for an appreciable bandwidth to be left.

The samples obtained from Boeing, however, were much less attenuating, so measurements of the transmission coefficients were possible.

Figure 29 shows that even in this case, the volume effects are much greater than the surface effect.

Line 1 is the result of B/E in Fig. 24 and is the amplitude of the transmission coefficient through the rough surface from the metal into the water. Line 2 is the smooth sample no sample case denoted B/J, and shows the volume effects clearly. Line 3 then shows both the surface and volume effects behaving as the product of frequency dependent linear operators.

It is interesting to compare the range of surface height irregularities, and Fig. 30 shows the normal reflection coefficient from a rough surface relative to that from a smooth surface for three of the samples.

A least squares fit to $A - Bf^2$ yields the R.M.S. roughnesses as 14.37, 18.29, and 20.33 microns respectively.

Fig 28. TRANSFER FUNCTION FOR THE ROUGH SURFACE OF A C-355 Al CAST

Fig 29. TRANSMISSION COEFFICIENTS THROUGH SMOOTH AND RANDOMLY ROUGH SURFACED CASTS

Fig 30. REFLECTION COEFFICIENT FOR 3 ROUGH SURFACES

The Phase for all the reflection and transmission coefficients was approximately zero as shown in Fig. 31 for the linear amplitude and phase of τ_{10} for sample 5.

Finally, measurements of the transmission coefficients, as shown in Fig. 32 for sample 7, yielded values of approximately the correct magnitude, and equality within the experimental precision possible.

We can thus conclude that a rough surface poses no problem for the inclusion characterization schemes discussed so far, but that a rough face on an inclusion can give angular scattering results which are very much changed due to the much greater effect on reflection from a rough surface than from transmission through it.

Fig 31. TRANSMISSION COEFFICIENT THROUGH A RANDOMLY ROUGH SURFACE

Fig 32. TRANSMISSION COEFFICIENTS THROUGH A ROUGH SURFACE

CHAPTER VI

POROSITY

Introduction

Since modern design methods make use of the closest safety tolerances that can be calculated in order to reduce costs and since the method of casting rather than fabricating complicated parts is both less costly and stronger, the quality of castings is of utmost importance.

Porosity, i.e., voids and gas bubbles, in castings significantly affect their mechanical properties, so a reliable method of determining the concentration of pores, and their size might be used as a method of judging whether a particular cast is acceptable or not for a given application.

Piwonka and Flemings⁹⁹ have shown that porosity may occur in castings via two main mechanisms, gas evolution or capture during solidification and shrinkage during solidification. Nondestructive evaluation techniques have been successfully used to detect and characterize large pores on the order of 1 mm and larger in many alloys, but for smaller pores, only destructive techniques have been available.

Recently a new method of characterizing porosity has been developed. Using the techniques of grain size determination and

porosity characterization, a method of determining the concentration of pores in the volume and the mean pore size in an alloy of aluminum has been developed.

As developed by Rose,⁷⁰ based on the idea of the forward scattering cross section for spheroidal voids as a function of ka as worked out by Gubernatis and Domany,^{68,100} it separates the frequency spectrum of the propagating wave into two regions, $ka < 1$ where Rayleigh scattering predominates, and $ka > 1$ where the cross section becomes approximately constant as in Fig. 33.

Thus, a thorough transmission experiment can determine both the pore size and the volume fraction of the pores, also called the concentration.

Experimentally adapted for pulse-echo by Wang, Csakany, Adler, and Mobley,⁷¹ the reflection from the opposing parallel face is treated as a thorough transmission of twice the sample size.

This method has been shown to be an accurate method for determining the state of a porous elastic solid, but depends on two flat smooth parallel surfaces or on two transducer measurements.

Since for most applications this is an unrealistic restriction, the problem was approached from the point of view of volumetric back scattering. Since the pores are considered to be virtually identical as to size and shape, monitoring of the continuous back scattering signal from various depths was undertaken in hopes that the back scattered spectra would exhibit this characteristic attenuation function.

Fig 33. REDUCED SCATTERING CROSS SECTION

In order to do this, a computer program was written which computed the energy spectrum of the back scattered ultrasonic wave for discrete time intervals. Since the effects of through transmission on the initial energy spectrum were known, and the method accurate, it was hoped that a continuous surface of energy as a function of both frequency and depth into the porous medium would yield the attenuation spectrum and thus the porosity of the sample under investigation. In the course of analysing the backscattered waveforms, it was realized that interference effects overwhelmed the hoped for data, but gave a check on the pore separation.

Since no theory could be found which predicted the behavior of the backscattered energy spectrum with respect to frequency as a function of time, an attempt was made to model the porous medium, taking into account both the interference expected as a result of the regular pore spacing, and the effect of the statistical nature of the distribution of the pore locations.

Porosity Scattering Background

Consider an elastic space uniform and homogeneous everywhere except between two parallel planes identified by $z = z_0$ and $z = z_1$. Between these planes there exists a random distribution of identical spherical pores of radius r_0 .

We assume that the homogeneous host material does not attenuate the incident ultrasonic wave for simplicity. If we assume a frequency dependent attenuation coefficient due to the scattering by the porosity $\alpha_p(k)$, then the displacement of a plane longitudinal wave propagating along z will be

$$u(k, z) = u(k, z_0) \exp(i(k(z - z_1) - \omega t)) \exp(-\alpha_p(z - z_0)) \quad (110)$$

If we look at the Fourier transform of the displacement $U(k, z)$, we can find the attenuation as

$$\alpha_p(k) = -(1/(z - z_0)) \ln(U(k, z)/U(k, z_0)) \quad (111)$$

so the attenuation coefficient is calculable from the experimentally measured spectra of the wave at different points in its propagation.

Gubernatis and Domany showed that for a collection of dilute pores whose radii obey some statistical distribution which peaks about some mean radius a_0 , the reduced cross section for scattering could be used to determine that mean radius.

They define the reduced scattering cross section as

$$\alpha_p(k) = 2\pi a_0^2 \gamma(k) \quad (112)$$

where the reduced cross section $\gamma(k)$ essentially determines the effectiveness of the pores cross section as a scatter.

Since the total scattering cross section gives the proportion of the incident energy scattered out of the beam, it will determine the attenuation.

They calculated the scattering cross section for various values of the ratio of shear to longitudinal velocities for ka_0 from 0 to 10. The cross section approached a constant for large ka_0 relative to 1, and obeyed Rayleigh scattering predictions for small ka_0 .

Furthermore, they found that the reduced cross section exhibited different behavior around $ka_0 = 1$ depending on Poisson's ratio for the host medium.

The greater this peak effect, i.e., for smaller ratios of shear to longitudinal wave speeds, the more accurately the pore size could be determined.

Rose considered the effects of grain scattering in light of their work, considering that at high frequencies, the grain scattering cross section increases as frequency squared rather than assuming a constant value.

He further separated the concentration and pore size explicitly to directly determine these key characteristics. For the material investigated by Wang, Csakany, Adler, and Mobley, and this work, the Poisson's ratio yielded a cross section which did not peak to any significant degree at $ka_0 = 1$, so the turnover point was chosen as that frequency at which the attenuation stops behaving as frequency to the fourth power and becomes constant.

They used beam spread corrections from Seki, Granato, and Truell⁹⁵ to keep the diffractive power losses from giving a falsely high and frequency dependent scattering attenuation due only to this mechanism.

All the cast samples were polished and microphotographed and their density determined. Only those samples whose density measurements and microphotographs gave the same porosity were used in the experiments.

Theory

Consider the generalized model in Fig. 34. An infinite homogenous isotropic elastic solid has a region between $z = 0$ and $z = Z$ that extends to infinity in the x and y directions, and that region contains pores of some average radius a_0 randomly distributed such that the volume fraction of pores is the same small value anywhere within the region. An infinite plane wave is incident along the z direction, and it passes through the $z = 0$ to Z region.

For wave numbers such that ka_0 is much less than one, the attenuation of the wave will go as k to the fourth power. For wave numbers such that ka is much greater than one, the attenuation will be nonzero, but constant.

Following Wang, Csakany, Adler, and Mobley, we interpolate back from the constant or high frequency region, and term the frequency at which the attenuation begins to fall as the point frequency f_p . Below that, the attenuation will go to zero as frequency to the fourth power.

Fig 34. THE FORWARD SCATTERING PROBLEM

If we know the material constants of the host material, we can determine the pore radius as

$$a = \frac{v}{2\pi} \frac{1}{f_p} \quad (113)$$

where v is the wave speed, and f_p is the turning point frequency.

The porosity concentration c is then

$$c = \frac{v}{2\pi K} \frac{a_p}{f_p} \quad (114)$$

where K is a constant depending on the material properties of the material.

Now consider the frequency dependence of the energy that is scattered out of the wave.

The back scattering spectrum for a sphere is just the cross section for scattering.

Using the operator notation introduced in the rough surface section, we will represent the spectrum backscattered from a pore as $H(\omega)$, and the attenuation effects of propagation through the porous medium as $G(\omega, z)$.

If there are N pores per unit volume, then comparing the back scattering from z_1 to $z_1 + dz$ and z_2 to $z_2 + dz$, we find a ratio as

$$\frac{G(\omega, z_1)H(\omega)G(\omega, z_2)F(\omega)}{G(\omega, z_1)H(\omega)G(\omega, z_1)F(\omega)} \quad (115)$$

where $F(u)$ represents the collective effects of the rest of the measurement system

Assuming the operators are linear, and can be computed, this should give us the same results as for the forward transmission case, since all other effects cancel except the propagation loss operator.

We assume dz is so small that the differences between scattering between different pores is negligible, and that the infinite extent of the number of pores affected, due to the extent of the infinite plane wave, yields no phase effect.

The method then will be to sequentially gate the backscattered time signal such that the gates overlap, and create an amplitude surface as a function of both frequency and time or distance into the porous medium. When the amplitude surface is expressed in dB, the slope along distance will yield the attenuation for a particular frequency as a function of distance. Since the model described assumes a constant change, we can fit the dB verses distance to a first order curve, and the approximation slopes should give use the negative of the attenuation function as a function of frequency. The method and mechanics will be described shortly, but we must first consider something else. Figure 35 shows the case of two nearby pores and in incident wave along z .

The pores are identical, so both their forward and back scattering cross sections are the same for the same frequency. We assume that except for a phase change introduced by the scattering, the phase of that part of the wave interacting with each pore depends only on propagation distance, and that at some location 0 they were the same.

Fig35. THE PHASE INTERACTION

The wave incident on pore 1 travels a distance z_1 , interacts with the pore, and travels a distance $Z - z_1$ in the forward direction and z_1 in the back direction. A similar analysis for pore 2 shows that the waves scattered in the forward direction arrive at Z with their relative phase unchanged, while the back scattered waves arrive at 0 again with a phase change proportional to twice the separation s .

Since the underlying theme of this work is phase modulation, the possible effects of this phase change in a random collection of pores must be considered.

Theoretical Model

Consider an infinitely long string with an infinite number of beads on it such that there are on the average N beads per length L on the string.

If we take an interval Δx less than L/N long, we can try to describe the probability of finding M beads in our interval by using the normal or Gaussian distribution.

We represent the distribution as

$$h(M) = C \exp((-M/C')M_0)^2 \quad (116)$$

where C , C' , and M_0 are yet to be determined.

Then the probability of not finding any beads in Δx is

$$h(0) = C \quad (117)$$

the probability of finding one bead is

$$h(1) = C \exp((-1/C')Mo)^2 \quad (118)$$

the probability of finding half a bead, or finding 500 in 1,000 tries for example, is

$$h(.5) = C \exp((.5/C')Mo)^2 \quad (119)$$

and so on.

Now consider another probability distribution, the number of pores we'll encounter if we look a distance X away from any arbitrary pore. That is, we find a pore at some position we call 0, and look along increasing X , letting the probable number of beads encountered be

$$j(X) = C'' X \quad (120)$$

For our assumptions, C'' is just N/L as described above, and we parameterize it as $1/X_0$, the inverse of the average distance between beads.

Therefore, if we look a distance X_0 away from our pore, we'll find X_0/X_0 or 1 bead on the average, so

$$j(X) = X/X_0 \quad (121)$$

Now, if we interpret the product of these probabilities as the chance that at a distance X we will find a number of beads M inside the interval assumed in $h(M)$, then we can write the probability of finding a single bead in the interval during displacement a distance X away from the bead defining our origin as

$$p_1(X) = C(X/X_0)\exp(-(X/(C')X_0))^2 \quad (122)$$

where we have replaced M , the number encountered in a distance X , by X , and M_0 , the number we expect to have encountered in a standard distance X_0 , by X_0 .

We can evaluate the unknown parameters by requiring that the most likely place to find the next bead is in an interval situated a distance X_0 away from the reference bead, that is that the maximum value of $p_1(X)$ occurs at X_0 . Differentiating with respect to X and setting the result equal to zero yields

$$1 - 2X^2/(C'X_0)^2 = 0 \quad (123)$$

so it must be that

$$C' = 2^{1/2} \quad (124)$$

We require that the next pore be found somewhere, so the integral of our probability of finding the next pore at a distance X for all X from 0 to infinity must be equal to 1.

Thus we find that it must also be that

$$C = 1/X_0 \quad (125)$$

Then we have the following situation. Given that we set X equal to zero at any arbitrary pore, and move an interval ΔX wide through X equal zero to any positive value, the probability of finding the next bead in ΔX at a position X is

$$p_1(X) = (X/(X_0^2)) \exp(-(1/2)(X/X_0)^2) \quad (126)$$

Thus, the probability of finding the next bead at 0 is 0, since the next bead cannot be the original reference bead.

We are most likely to find the next bead X_0 away, and less and less likely to find it farther away from X_0 .

Now consider the second bead we encounter. We would expect it to be a distance $2X_0$ away from our reference, but it too is governed by our probability distribution.

If we do find the first succeeding bead at X_0 , then we are most likely to find the second succeeding bead at $2X_0$, and we will find the first bead at X_0 $p_1(X_0)$ of the time. On the other hand, we may find the

first next bead at $.5 X_0$, so that $p_1(.5 X_0)$ of the time we will be most likely to find the second next bead at $1.5 X_0$, and so on.

In fact, the second next bead's position is governed by the convolution integral as

$$\begin{aligned}
 p_2(X) &= p_1(0) p_1(X - 0) + p_1(dX) p_1(X - dX) \\
 &\quad + p_1(2dX) * p_1(X - 2dX) + \dots \\
 &= \int (p_1(Y) p_1(X - Y) dY) \quad Y = 0 \rightarrow \infty \quad (127)
 \end{aligned}$$

a very difficult integral.

We can solve it simply though if we can find the Fourier transform of $p_1(X)$, since the convolution theorem which is employed for deconvolution can also be used to convolve these probability functions.

Assume that we can find the transform of $p_1(X)$, and assume that it is $P_1(K)$, and that we can represent this operation as

$$P_1(K) = F(p_1(X)) \quad (128)$$

and

$$p_1(X) = F^{-1}(P_1(K)) \quad (129)$$

Then

$$P_2(K) = F(p_1(X)) \quad (130a)$$

$$= F(\int p_1(Y)p_1(X - Y)dY) \quad (130b)$$

$$= P_1(K)P_1(K) \quad (130c)$$

by the convolution theorem and

$$p_2(X) = F^{-1}((P_1(K))^2) \quad (131)$$

In fact, for the j th next bead, the probability of finding it at X is proportional to

$$p_j(X) = F^{-1}(P_1(K)^j) \quad (132)$$

For simplicity, we assign meaning only to X values greater than zero, and expand the first probability as an odd function.

For the dimensionless complementary variables $\zeta = X/X_0$ and $k = X_0 K$, we have

$$P_1(k) = -ik \exp(-(1/2)k^2) \quad (133)$$

so that for the L th bead, we have the probability distribution as

$$p_L(z) = \frac{1}{x_0} \frac{z^L}{L(L+1/2)} e^{-\frac{1}{2} \frac{z^2}{L}} \quad (134)$$

Distributions for the first, second, third, and fourth next beads from our arbitrary reference are shown in Fig. 36.

The distributions have been modified by a weighing factor, found from integrating each of the distributions from $x = 0$ to infinity. The integrals of the normalized distributions are all equal to one, since we must always find the j th bead somewhere. The inset of Fig. 36 shows the first few of these weighing factors versus the respective distribution number. From these plots we see that we can be less and less sure of a guess as to where the j th bead will be. The probabilities for each position decrease, and the spread of uncertainty becomes greater and greater.

This simple model can now be related to the physical situation in the following manner. We imagine a cylindrical volume in the midst of a randomly porous medium. This volume is made up of the area of the ultrasonic beams wavefront times the product of the wave speed and the time of the backscattered waveform divided by 2.

If the beam has a radius R , and we analyze that part of the waveform from t_0 to t_1 , or Δt , the volume we are analyzing is

$$Vol = \pi R^2 (v \Delta t / 2) \quad (135)$$

the beginning of the time window corresponding to the waves scattered back from $vT/2$, and the end of the time window to the waves backscattered from $v(T/2 + \Delta t/2)$.

Fig 36. GAUSSIAN EXPECTATION FUNCTIONS

We assume that there is no multiple scattering, so that each pore scatters a wave whose characteristics have not changed in relation to any other pore within a few expectation distances. Thus, the backscattered amplitude spectrum from any pore is virtually identical to that of any other nearby pore, in both amplitude and phase. Over a long range, the spectral content of the incident wave fronts will change in accordance with the predictions of Rose, confirmed by Wang, et al.

Now, in a representative volume for a time window of 1 microsecond, and a beam radius of 1/2 inch, with a pore concentration of 1 percent, and pore radii of 100 microns, we expect to find approximately 7,800 pores for a longitudinal wave, and 3,600 for a shear wave because of its lower wave speed. This means we have enough pores to provide good statistics. We must now relate the simple model given above for the beads on a string to the experimental case of pores randomly arranged in a volume. If the pores can also be described by a normal distribution, and we assume they can, then they too have some average separation from each other in three dimensions.

We take the space to be describable in spherical coordinates. For a random distribution, we expect that there will be no angular dependence in the distribution. We then describe the position of the next nearest pore to some arbitrary reference pore by a similar distribution function, exchanging r for X and r_0 for X_0 .

A full space integral then yields the required value of 1 for

$$\iiint \left(\frac{r}{r_0}\right) e^{-\frac{1}{2}\left(\frac{r}{r_0}\right)^2} r^2 \sin \theta \, dr d\theta d\phi = 1 \quad 146$$

(136)

for θ from 0 to π , ϕ from 0 to 2π , and r from 0 to ∞ . This will yield the normalization constants exactly as before, and our normalizing distance r_0 , the average separation, is found from the pore size, and the concentration.

Since we are only interested in the forward direction, we restrict the solid angle to θ from 0 to $\pi/2$ and ϕ from 0 to 2π . This tells us that the average separation in the z or wave propagation direction is $1/4$ that in all space. We can thus rescale our original distributions, and describe the probability of finding the j th nearest local neighbor in the z direction by $p(z)$, where $z_0 = 1/4 r_0$. This is illustrated in Fig. 37 for arbitrary θ and ϕ in the forward solid angle, and for clarity the pore is assumed to lie on the surface of the hemisphere $r = r_0$.

Now consider the limits of the argument. By choosing the normal distribution, we chose a prescribed uncertainty in the pores position.

If we had chosen instead a very sharply peaked distribution, as in an ordered lattice for example, then the spheres would be exactly $(1/4)r_0$ apart in z , and we would get, in the local neighborhood, exactly the same spectral density backscattered with a phase change corresponding to a path difference of $2(1/4)r_0$, repeated over and over.

If that distance causes a phase change of $2k\langle z \rangle$, then summing up the received signals, we would find

Fig 37. THE LOCAL PROBLEM

$$\begin{aligned}
 R(\omega) = & A(\omega)p_0(z) + A(\omega)p_1(z) + A(\omega)p_2(z) + \dots \\
 & + A(\omega)\delta(z) + A(\omega)\delta(z - \langle z \rangle)e^{i2kz} + A(\omega)\delta(z - 2\langle z \rangle)e^{i2kz}
 \end{aligned}
 \tag{137a}$$

$$= A(\omega)[1 + e^{i2k\langle z \rangle} + e^{i4k\langle z \rangle} + \dots] \tag{137b}$$

$$\begin{aligned}
 = & A(\omega)[e^{i1k\langle z \rangle}c_1(e^{i1k\langle z \rangle} + \bar{e}^{i1k\langle z \rangle}) \\
 & + e^{i2k\langle z \rangle}c_2(e^{i2k\langle z \rangle} + \bar{e}^{i2k\langle z \rangle}) + \dots]
 \end{aligned}
 \tag{137c}$$

$$= A(\omega)[e^{i1k\langle z \rangle}c_1 2 \cos(k\langle z \rangle) + e^{i2k\langle z \rangle}c_2 2 \cos(2k\langle z \rangle) + \dots] \tag{138d}$$

Considering the first interaction only, we would find the backscatter spectrum from a simple sphere, modulated by a cosign function with minima at $k\langle z \rangle = n\pi/2$.

The phase has been changed by a linear time, so if

$$R(\omega) = \rho(\omega)e^{i\sigma(\omega)} \tag{138a}$$

where

$$\rho(\omega) = |R(\omega)| \tag{138b}$$

and

$$\sigma(\omega) = \tan^{-1} \left(\frac{\text{Imaginary}(R(\omega))}{\text{Real}(R(\omega))} \right) \quad (138c)$$

then for $A(\omega) = \alpha(\omega)e^{i\beta(\omega)}$ we find

$$\rho(\omega) = \alpha(\omega)2 \cos(k\langle z \rangle) \quad (139a)$$

and

$$\sigma(\omega) = \beta(\omega) + kz \quad (139b)$$

On the other hand, if the pores were totally random, with no order at all, the phase differences would be random, and the sum would yield

$$R(\omega) = A(\omega)[P_0 + P_1'(dz)e^{i2(kdz)} + P_2'(2dz)e^{i2(kdz)} + \dots] \quad (140a)$$

where $P_j'(z)$ is the probability distribution for the nonordered set, and is assumed equal for all values of z so for the nonordered case

$$R(\omega) = A(\omega)[1 + c \int_0^\infty e^{ikz} dz] \quad (140b)$$

But this is just the stationary phase integral and it sums to 1, so we would find no interference at all.

For our intermediate case with a finite uncertainty, which tends to increase with succeeding next pore position, we expect some local interference for the first few sequences, finally decaying into chaos.

This should yield smaller and much broader minima in the spectrum received.

In fact, for the distribution chosen as the scattering model, we would find for the j th next pore a scattering spectrum as

$$R(\omega) = A(\omega) \left[1 + \sum_n \int dz C_n \frac{z}{z_0} C^{-1/2(z/z_0 C'_n)^2} e^{12kz} \right] \quad (141a)$$

$$= A(\omega) \left[1 + \sum_n \frac{C_n C'_n{}^2}{z_0} 1kz_0 e^{-\left(\frac{kz_0 C'_n}{2}\right)^2} \right] \quad (141b)$$

so that we would find instead of a cosign modulation, that the spectrum $A(\omega)$ would be modulated by the compliment to a Gaussian distribution whose minimum would occur when $C_n k(z)/2$ equaled one.

In choosing the expectation function as our distribution, we have given up control over the spread in uncertainty in the interest of mathematical simplicity.

Had we chosen to center a Gaussian distribution centered about the expectation distance, we would have found the characteristic cosign modulation, less deep and more blurred as the uncertainty increased.

This distribution, however, presents mathematical difficulties in that the probabilities would not go to zero at the 0th pore position.

Our present choice and the resulting modulation can still be interpreted in terms of a cosign modulation in the following manner.

Consider the interference between the zeroth and first pore. The interference between positions results in a rectified cosign envelope, so we will have an envelope composed of cosigns with minima at $kx_0 = n\pi/2$, and their superposition results in a Gaussian shape. Summing the contributions results in a dip in attenuation at the first minima, and the succeeding ones average out to a constant value.

This model then predicts that a higher than normal attenuation will be observed in the frequency spectrum of the attenuation coefficient at positions corresponding to the order of interference. The peaks in the coefficient will occur for increasing frequency for decreasing separation.

Now consider a more controllable but mathematically more cumbersome model. Suppose that the pores are most likely to be located at multiples of the expectation distance z_0 , but that the position about that distance is describable by a Gaussian distribution. The first pore is then located at z_0 , and its positional probability is described by

$$p_1(z) = \left(\frac{1}{2\pi\sigma^2}\right)^{-1/2} - \frac{1}{2} e^{-\frac{(z-z_0)^2}{2\sigma^2}} \quad (142)$$

The next pore should be at $2z_0$, but its locational probability will be governed by the actual position of pore supposedly at z_0 . This again results in a convolution integral, and the resulting transform for any pore is then

$$p_1(k) = \left(\frac{1}{2\pi}\right)^{1/2} e^{-1/2k^2} \quad (143a)$$

where

$$K = \sigma k \quad (143b)$$

and

$$P_n(K) = \left(\frac{1}{2} \pi\right)^{n/2} e^{-n/2 K^2} \quad (144)$$

so the resulting probability distributions are

$$P_n(Z) = \left(\frac{1}{2} \pi\right)^{n/2} e^{-1/2n \frac{(z - nZ_0)^2}{\sigma^2}} \quad (145)$$

These are shown in Fig. 38.

This model allows us to vary the uncertainty in position for one of the positional probabilities, but once varied, all succeeding probabilities are also changed. It is also less philosophically satisfying, since we've argued that the probabilities apply in the forward direction only, while for all pores, there is a finite probability that they will lie in the negative z direction. This model will yield the rectified cosign modulation resulting in regularly spaced peaks in the attenuation coefficient as a function of frequency, and the width of these peaks is dependent on the uncertainty parameter σ .

Fig 38. GAUSSIAN PROBABILITY FUNCTIONS

Sample Preparation

To test the porosity scattering theory, castings of A357 aluminum were custom made in the Ohio State University's Foundry under controlled conditions.

The sample composition was 6.5-7.5 percent Si, 0.4-0.7 percent Mg, 0.2 percent of both Fe and Cu, 0.1-0.2 percent Ti, 0.1 percent of both Mn and Zn, 0.04-0.09 percent Be, 0.15 percent other adultrants by weight, and the balance aluminum.

To produce degrees of porosity of from 0-6 percent, various parameters were changed from one run to the next.

To produce the nonporous reference samples, the alloy was melted, poured, and solidified in a vacuum chamber at a pressure of approximately 1 mm or mercury.

Then melting was carried out in both induction and gas-fired crucible furnaces, with the alloy exposed to air, and moisture was introduced by placing a wet towel on top of the case, by using a moistened brick to support the cast, or by spraying moisture onto the model surface.

The castings were then sectioned, and machined into rectangular blocks with parallel smooth opposing surfaces.

Experimental Procedure

The data strings in each experiment consisted of 1,000 separate samples, time correlated with each other. The first sample of 2,048 voltage points 10 nanoseconds apart was displayed and a characteristic feature of the waveform identified typically consisting of approximately one micro second. Succeeding samples were then correlated about this feature.

This was possible since the experimental sample was held fixed throughout the experiment. No time offset distribution was recorded since the absolute time was of no importance, and the front surface reflection was within the 20.48 micro second data string. This signal averaging procedure allowed an effective signal to noise improvement of 30 dB over the gain of the ultrasonic amplifiers. Data was taken at normal incidence, resulting in longitudinal back scattering only, and at 20 degrees to normal, past the longitudinal critical angle and corresponding to a 45 degree shear wave, which led to shear scattering only in the theoretical model used.

The analysis of the data used a variable width gate with a cosign squared shape, known as a Hanning window, and a square gate of paired step function of variable width. This gate was stepped through the data strings, and for each position, the frequency spectrum of the signal in the gate was calculated. The procedure is illustrated in Fig. 39 for a backscattered shear wave.

Fig 39. ORIGINAL SIGNAL AND SUCCESSIVE GATING SCHEME

The type and width of the gate make a good deal of difference in the resolution and gate generated noise in analyzing a continuous signal. To examine the effects of the gate, a continuous signal of cosines of different frequencies was processed using both gates for various gate widths.

Figure 40 shows the effect of the Hanning window clearly. The increment of the signal gated in a Hanning window has very little gate noise, or $\sin(x)/x$ ringing, but the resolution is only half as good as a square gate. For the normal scattering pulse used in an ultrasonic spectroscopy system, the square gate is always used since the signals are almost always less than one microsecond in duration, and no gate noise is generated because the signal begins and ends within the gate.

Since we were not aware of any criterion for choosing a gate shape and duration to optimize anything, the data were analyzed using many combinations to search for some identifiable characteristic in the back scattered signal. As each successive gate was fast Fourier transformed, its amplitude spectrum was added to a growing surface of scattering data that formed the amplitude surface for each experiment.

These surfaces were then averaged with the nearest and next nearest neighbor values weighed according to their distance and the averaged point weighed to count as much as all the others combined. Since the gates overlapped in distance and the frequency spectra should be smooth, this should have only a light smoothing effect.

These surfaces were then approximated by a surface of linear fits along time or distance. Random lines were checked against their

Fig 40. GATE WIDTH AND GATE SHAPE EFFECTS

corresponding approximations, and the index of determination for each line was calculated as well as the slope for each line. The amplitude surface in dB yrs. frequency and distance is shown in Fig. 41 along with the approximation to that surface in Fig. 42.

Results and Conclusions

The scattering cross section for a spherical void as a function of ka in the longitudinal case and Ka in the transverse case can be found in Kraft and Franzblau, and the extension to a porous medium of spherical voids has been given in the longitudinal case by Gubernatis and Domany. Using their reasoning, we can expect the shear scattering to behave as the single pore also to within a constant term. Thus, if all the interference is washed out by randomness, we hope to find an attenuation coefficient for the longitudinal case as previously found in smooth parallel sided samples by Wang et al.

For the shear case, we expect to find a frequency to the fourth power behavior below $Ka = 1$, and a slow tapering for higher values.³⁷ The backscattering data does not fit any of these models, however, and is not very consistent in results in any sample.

Figure 43 is the superposition of the back scattering coefficients of four closely spaced regions of sample 1830. These are illustrated schematically in the inset figure. There is some rough agreement in that we expect a fourth power drop in the amplitude we receive from

Fig 41. ATTENUATION SURFACE

Fig 42. APPROXIMATION SURFACE

Fig 43. BACKSCATTER SURFACE SLOPE vs. FREQUENCY

backscattering, and a lower apparent attenuation at $nKRo$ as well as a higher apparent value at $((2n - 1)/2)KRo$, but the structure doesn't fit either the cosign or Gaussian rule we looked for.

The corresponding longitudinal wave case is shown in Fig. 44 and again, there seems to be some semblance of what we would expect for interference backscattering, but the results are too inconsistent to match to any model. There is also the problem that knowing the average pore separation doesn't do any good towards characterizing the porosity concentration unless one knows the average pore size. The backscattered results in all cases were unable to locate either the $ka = 1$ or $Ka = 1$ frequency.

Another shear wave scattering case is shown in Fig. 45, and again, the attenuation behaves almost as expected, yet does not fit either of our proposed models. The other cases considered by Wang et al. yielded similar results when the results quoted there were used in the calculation of nka .

This study has shown that the interference effects for backscattering from a porous medium clearly predominate over the forward scattering effects in a determination of the attenuation coefficient as a function of frequency.

Fig 44. BACKSCATTER SURFACE SLOPE vs. FREQUENCY

Fig 45. BACKSCATTER SURFACE SLOPE vs. FREQUENCY

CHAPTER VII

POLYMER

Introduction

Thermosetting resins are widely used as the matrix for composite materials and as structural adhesives. Fabrication of composite materials consists of the polymerization of resin impregnated fiber fabrics under proper thermal and pressure conditions. In the curing reaction, the thermoset resin transforms from the liquid to the gelled state and then vitrifies to the gelled glass state.

Figure 46 is a typical Time-Temperature-Transformation diagram, For temperatures below T_{g0} , the resin is solid and ungelled. It may thus be mixed and stored for later use as long as the temperature remains lower than T_{g0} . Above T_{g0} , the mixture liquifies but if the temperature is less than T_g , no crosslinking (or conversion) will occur. Above T_g , gelation eventually occurs and the conversion proceeds until the gelled glass state is reached. It must be emphasized that this does not imply total cross linkage. The degree of conversion is determined from the coinversion line which intercepts the vitrification line at the curing temperature. Raising the temperature causes the resin's state to follow the conversion percentage line to the new

Fig 46. THE THERMOSETTING PROCESS:
TIME-TEMPERATURE-TRANSFORMATION CURE DIAGRAM

temperature, after which curing proceeds to the new state of vitrification. At T_{g_0} , at least 99 percent of the cross linkages are complete. This, in general, yields the highest elastic modulus in the vitrified state, but this most completely converted state is also more brittle, i.e., less tough, than a less completely converted final state. The curing temperature must be chosen then to attain the optimized state of cohesiveness and strength for a given application.

Ultrasonic measurement would seem to be an ideal method of monitoring the curing process, since it allows determination of the material constants of the substance under investigation. Measurements of the frequency, time, and temperature dependent phase velocity and attenuation can therefore yield important information on the extent of the thermostat curing reaction and the mechanical properties of the material. These measurements correspond to the real and imaginary parts of the complex wave propagation vector.

The application of ultrasonic elastic waves to monitor polymer curing was first suggested by Sofer and Hauser,⁷² and Sofer, Dietz, and Hauser.⁷³ They used a 10 microsecond pulse of about 2 MHz signals to monitor the attenuation and phase velocity in curing thermosetting polymers. Papadakis⁷⁴ used glass blocks and tungsten wire embedded in curing resins to monitor the amount of signal lost into the curing epoxy for both bulk modes, longitudinal and transverse, and the magnet-restrictive wire modes.

Lindrose⁷⁵ used 1 MHz longitudinal and shear waves in pulse-echo experiments over an extended time to determine the moduli of the curing

thermoset plastic, and found a definite time of cure dependent relationship. Rokhlin, Hefets, and Rosen⁷⁶ generated a 1.2 MHz bulk wave in one of two identical half spaces joined by curing epoxy. The converted interface wave was then monitored for several hours at differing temperatures and the attenuation, group, and phase velocities determined.

Rokhlin⁷⁷ also considered a thin layer of structural adhesive under the same sort of conditions. Hunston⁷⁸ directly generated guided waves in a thin layer of a delay line to monitor the attenuation during the process for a 2.9 MHz shear wave pulse. Winfree and Parker⁷⁹ used a 20 MHz longitudinal wave pulse to monitor the reflectivity of a glass epoxy interface during the curing reaction to determine the change in velocity over time and the rate of change of the velocity as a function of the curing temperature.

This work is an attempt to understand the interaction of ultrasonic waves with thermosetting resins during the curing process. The time and frequency behavior of the wave vector is investigated using spectral analysis of an ultrasonic wave pulse propagating through a fixed distance of thermosetting resin undergoing the curing process at a fixed temperature. The phase velocity and attenuation as functions of frequency and time are determined. In addition, a determination of the group velocity of a broadband pulse as a function of time is made.

The reflection and transmission coefficients at the aluminum buffer rod-epoxy interface depend on the phase velocity. Since this is frequency and time dependent, the changes are incorporated in the calculation of the attenuation coefficient.

Theory

The idea of using the phase spectra to determine the phase velocity of an ultrasonic signal through a fixed length was suggested by Sachse and Pao.¹⁰¹ If we represent the material displacement at a distance x and time t as

$$u(x, t) = (2\pi)^{-1/2} \int_{-\infty}^{\infty} F(x, \omega) e^{-i\omega t} d\omega \quad (146)$$

where

$$F(x, \omega) = F_0(0, \omega) e^{ik(\omega)x} \quad (147a)$$

with

$$F_0(0, \omega) = |F_0(0, \omega)| e^{i\phi_0(\omega)} \quad (147b)$$

the complex Fourier spectrum of the incident ultrasonic signal, ϕ_0 the initial phase angle.

We represent the wave number k as a complex function of the real frequency as

$$k(\omega) = k_1(\omega) + i\alpha(\omega) \quad (148)$$

In this representation α is the frequency dependent ultrasonic attenuation, and k_1 is the real wave number related to the phase velocity through the definition

$$\omega = k_1(\omega) v_p(\omega) \quad (149)$$

The $F(x, \omega)$ in Eq. (147) can then be rewritten as

$$F(x, \omega) = |F_0(0, \omega)| e^{-\alpha(\omega)x} e^{ik_1(\omega)x} e^{i\phi_0(\omega)} \quad (150)$$

where the phase spectrum of the signal at x and t is

$$\phi(x, \omega) = k_1(\omega)x + \phi_0(0, \omega) \quad (151)$$

If a wave travels from $x = 0$ to L , then we can experimentally obtain the complex frequency spectra $F(L, \omega)$ so the attenuation becomes

$$\alpha(\omega) = \frac{1}{L} \ln \left[\frac{|F(L, \omega)|}{|F(0, \omega)|} \right] \quad (152)$$

Similarly, the phase spectrum at L

$$\phi(L, \omega) = \frac{\omega}{v_p(\omega)} L + \phi_0(0, \omega) \quad (153)$$

yields the phase velocity as

$$v_p(\omega) = \frac{\omega L}{\phi(\omega) - \phi_0(\omega)} \quad (154)$$

Experimentally, we compare two transmitted signals which pass over the same distance L , so there equations must be modified.

If we represent the signal which passed through water, with low attenuation and no dispersion as

$$F_r = |F_0(\omega)| e^{-\alpha_r L} e^{i\phi_r(L, \omega)} \quad (155)$$

then

$$\phi_r(L, \omega) = \frac{\omega}{v_r} L + \phi_0(0, \omega) - \omega(t_{r,0} + t_{r,off}) \quad (156)$$

where $\phi_0(0, \omega)$ is the initial phase of the wave used in Eq. (147), $t_{r,0}$ is the time delay of the beginning of each 20.48 microsecond time record, and $t_{r,off}$ is the offset from that introduced by the time correlation algorithm, i.e., the delay from $t_{r,0}$ to the centroid of the averaged time record. This last offset is necessary to minimize the first order slope of the phase spectrum of the Fourier transform resulting from the analysis of the averaged wave form. This is illustrated in Fig. 47 for an arbitrary signal pulse.

We obtain the same equations for signals propagating through the curing epoxy resin as

$$F_E(L, \omega) = |F_0(\omega)| e^{-\alpha_E(\omega)L} e^{-i\phi_E(L, \omega)} \quad (157)$$

Fig 47. PULSE OFFSET DETERMINATION

and

$$\phi_E(L, \omega) = \frac{\omega}{v_E(\omega)} L + \phi_0(0, \omega) - \omega(t_{E,0} + t_{E,off}) \quad (158)$$

To assure the bandwidth limits of the ultrasonic transducers are not exceeded, we deconvolve the system response from the experimental response using a Weiner filter which acts only on the modulus of the resulting impulse response frequency spectra. We denote this operation as

$$W_Q(F_r, F_E) = (F_E F_r^*) / [F_r F_r^* + (Q|F_r|_{\max})^2] \quad (159)$$

where F_r^* is the complex conjugate of the complex reference spectrum.

For our experimental parameters of L equal to 3.175 mm and Q equal to 0.2236, we can represent each deconvolved impulse response as

$$A(\omega) = W_Q(F_r, F_E) \quad (160a)$$

$$= |A(\omega)| e^{i\Delta\phi(\omega)} \quad (160b)$$

where

$$\Delta\phi = \phi_E - \phi_r \quad (161)$$

$$= \omega L \left(\frac{1}{v_E(\omega)} - \frac{1}{v_r} \right) - \omega [(t_{E,0} - t_{r,0}) - (t_{E,off} - t_{r,off})] \quad (162)$$

Assuming a constant phase velocity of ultrasound over the range 2 to 15 MHz in water at 41°C, we obtain the frequency depend phase velocity in the epoxy as

$$v_E(\omega) = \omega L / [\Delta\phi(\omega) + \omega [(t_{E,0} - t_{r,0}) - (t_{E,off} - t_{r,off}) + \omega L / v_r]] \quad (163)$$

Since the apparent attenuation now involves the transmission coefficient difference losses at the interface of the buffer rods and the sample, one must take this difference into account in the data analysis. The transmission coefficient into the epoxy at normal incidence is, from Krautkramer and Krautkramer¹⁰²

$$D_{A1} = (2\rho_{A1}v_{A1}) / (\rho_{A1}v_{A1} + \rho_E v_E(\omega)) \quad (164a)$$

and into the aluminum from the epoxy

$$D_{E1} = (2\rho_E v_E(\omega)) / (\rho_{A1}v_{A1} + \rho_E v_E(\omega)) \quad (164b)$$

so that the total transmission coefficient is

$$T_E = \frac{(4\rho_{A1}v_{A1}\rho_E v_E(\omega))}{(\rho_{A1}v_{A1} + \rho_E v_E(\omega))^2} \quad (164c)$$

with a similar, though nonfrequency dependent expression for the total transmission coefficient for the water or reference case which we denote T_r .

Then the attenuation is

$$\alpha_E(\omega) = -\frac{1}{L} \ln\left[\frac{|A(\omega)| T_r}{T_E(\omega)}\right] + \alpha_r \quad (165)$$

and involves the derived phase velocity through T_E .

Sample Preparation

Epon 815 was selected for this study. It is based on Diglycidyl Ether or Bisphenol A (DGEBA). N-Aminoethylpiperazine (AEP), a low viscosity liquid at room temperature, was selected as the curing agent or hardener. Two aluminum buffer rods are mounted in a cylindrical support and can be moved axially to produce a known gap between the inside ends. Ultrasonic transducers were coupled to the buffer rods to transmit and receive longitudinal wave pulses. Since the attenuation became so large during the experiment, only the transmitted, or pitch-catch method was used to acquire data. The apparatus was filled with water and temperature stabilized at 41°C. The data acquired was used as the reference of the experimental setup throughout. Next the apparatus was filled with epoxy resin, temperature stabilized and the data acquisition repeated. After thorough cleaning, the apparatus was filled

with hardener, temperature stabilized and measurements repeated. Finally, the component amounts of prewarmed epoxy and hardener were well mixed. The mixture was degassed in vacuum for several minutes to avoid pore formation, then put into the apparatus and temperature stabilized. Data was acquired in less than the first 20 seconds of every minute for 168 minutes.

Experimental Procedure

The experimental system, shown schematically in Fig. 48, consisted of a pulser to excite the initial wave pulse in the transmitting transducer, the sample apparatus previously described, a receiving transducer coupled to an unsynchronized amplifier, a 100 MHz digitizer and a laboratory minicomputer. The pulser-receiver units were Panametrics capable of a nominal 40 dB gain from 10 kHz to 50 MHz. The transducers were commercially available piezoelectric units with a nominal resonance of 15 MHz and very broad band characteristics. The digitizer, a Biomation 8100, is an 8 bit machine with a sample rate of 100 MHz. The resolution is from -127 to 128, using the two's complement representation to record negative numbers. The input voltage can be varied from .05 to 5 volts peak to peak by internal attenuation. The unit was externally triggered by the synchronizing pulse from the pulser. Delay past the normal 999 sample intervals was attained by an auxiliary digital delay unit. The Biomation is completely controllable

Fig 48. EXPERIMENTAL SYSTEM

via an RS-232 interface. The acquired data is transferred to the minicomputer, a Digital Equipment PDP 11/34, by a hand shaking arrangement.

The computer's internal clock was used as the master clock in the experiment, initiating data acquisition at the beginning of each minute. The first digitized wave form was stored as a reference, and each succeeding wave form time correlated against it. Each resulting time offset was also stored and the resulting distribution used to find the best approximation of the correct offset, since there was no reason to assume the time of the first signal was any more exact than any of the succeeding ones. Typical data acquisition and averaging time was less than 20 seconds. The resulting averaged wave form was then stored on magnetic disc for later analysis. The signal was continually changing, and the time correlation window was only 1.5 to 2.5 out of the total time record of 20.48 microseconds, so some of the correlations failed, and this data was identified by strong interference patterns. The input attenuation was also changed many times during the experiment to optimize the resolution of the digitizer. Since the computer controlled the beginning of data acquisition, some mismatch of the actual to the nominal attenuation resulted. These were noted during the course of the experiment, and corrected during the analysis.

All measurements were performed at the temperature of 41°C. For temperature stabilization a water bath with a constant temperature water bath circulator was used. An aluminum block was immersed to about two-thirds of its height. The experimental apparatus was atop the block and the sample temperature continuously monitored by a thermocouple.

Data Analysis

Each time record of the transmission waveform was digitally gated to isolate the signal of reference.

These signals were then fast Fourier transformed. The transmission spectra for the experimental setup with water in the sample holder was used as the impulse response of the system, and this spectrum was used as the normalizing function to deconvolve each of the experimental spectra. The linear amplitudes and phase spectra were stored in an array as the amplitude or phase as a function of both frequency and time into the experiment. The amplitude spectral surface shown in Fig. 49 was then converted into the dB scale, and limited to the dynamic range of the system, about 50 dB down from the reference spectrum.

During the fast Fourier transform, each signal has been time shifted to minimize phase errors, as explained in the theory, so the phase was slope corrected, i.e., modified by an exponential factor linear in frequency, to retain the proper phase information. The real part of the wave propagation vector was then calculated and from this, the phase velocity surface was determined, and this is shown in Fig. 50. Finally, the transmission coefficient, incorporating the phase velocity, was calculated as a function of frequency, and the attenuation surface versus frequency and time was calculated. This is shown in Fig. 51.

Classical relaxation theory predicts that one can write the attenuation coefficient as a function of frequency as¹⁰³

Fig 49. TRANSMISSION AMPLITUDE SURFACE

Fig 50. PHASE VELOCITY SURFACE

Fig 51. ATTENUATION SURFACE

$$\alpha = A\omega t / (1 + (\omega t)^2) + R\omega^2 \quad (166)$$

where ω is the radial frequency, t is the relaxation time, A is the relaxation constant, and R is the viscosity constant. This theory predicts that α will behave as ω^2 for low frequencies, and as $\text{const} + R\omega^2$ for high frequencies. This basic theory as well as more comprehensive relaxation theories which use continuous or discrete distributions of relaxation times, does not describe experimental data for highly viscous liquids very well.

A more modern theory by Isokovich and Chaban¹⁰⁴ describes a highly viscous liquid as a microhomogenous medium. They have used the theory to successfully explain anomalous absorption and dispersion for a few highly viscous liquids such as glycerine. They predict a ω^2 behavior for low frequencies, as does the classical theory, and a high frequency behavior as $\omega^{1/2}$.

The attenuation curves as a function of frequency for each time in the experiment were fit by a polynomial fitting routine to linear and second order curves for a best fit in the least squares sense. In all cases, the linear fit was clearly superior to the quadratic, leading to the conclusion that the attenuation throughout the experiment is a first order function of the frequency.

The slopes of each attenuation curve as a function of time into the experiment is shown in Fig. 52. This linear behavior of the attenuation coefficient for all stages of the curing process is poorly understood on

FIG 52. ATTENUATION SLOPE DIAGRAM

the molecular level. This so-called hysteresis behavior has been found previously for other highly viscous liquids by Meister, Marhoeffer, Sciamanda, Cotter, and Litovitz,¹⁰⁵ in solid polymers by Kline¹⁰⁶ and by Adachi, Harrison, Lamb, North, and Pethrick¹⁰⁷ and in biological tissue to Kadaba, Bhagat, and Wu,¹⁰⁸ and Dunn.¹⁰⁹ Cross sections of the attenuation surface at various times into the experiment are shown in Fig. 53. The ragged tops at 30 and 40 minutes are due to the noise limits of the equipment. Figure 54 shows other cross sections of the attenuation surface as a function of time for various frequencies. The maximum in attenuation does not seem to change in frequency as a function of time.

Cross sections of the phase velocity surface as a function of frequency for various times into the cure are shown in Fig. 55. The dispersive nature of the medium is clearly seen in all the velocity curves. For comparison, the phase velocity as a function of frequency for water, hardener alone, and epoxy resin alone are shown in Fig. 56.

Cross sections of the phase velocity surface as functions of time for various frequencies are shown in Fig. 57. It can be seen that the phase velocity changes with time of cure as an S shaped curve, and that each curve is also dependent on frequency. Note that for all frequencies, the mixture of hardener and epoxy has a lower phase velocity initially than either of the components alone, and that the final state phase velocity is higher than either alone.

This data suggests that the epoxy thermosetting resin behaves as a polymer glass in the frequency range studied in all stages of the curing

Fig 53. ATTENUATION vs FREQUENCY FOR VARIOUS TIMES OF CURE

Fig 54. ATTENUATION THROUGHOUT THE CURE FOR VARIOUS FREQUENCIES

Fig 55. PHASE VELOCITY vs FREQUENCY FOR VARIOUS TIMES OF CURE

Fig 56. PHASE VELOCITY vs. FREQUENCY FOR EPOXY AND HARDENER

Fig 57. CROSS SECTIONS OF THE PHASE VELOCITY SURFACE

reaction. The ultrasonic wave is affected by the liquid-solid transformation as if it were a transformation in a glassy state, in this experiment for one temperature. The transformation rate is of course temperature dependent.

Neither relaxation nor microhomogeneous behavior was observed, though it must be emphasized that the frequency range covered only about three octaves in this experiment. For a more confident conclusion, it would be preferable to make measurements over the range of a few decades.

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