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**AN ADVANCED PROTOTYPE SYSTEM FOR LOCATING AND MAPPING
OF UNDERGROUND OBSTACLES**

The Ohio State University

PH.D. 1982

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AN ADVANCED PROTOTYPE SYSTEM FOR LOCATING AND
MAPPING OF UNDERGROUND OBSTACLES

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By

Andrew Joseph Terzuoli, Jr., B.S., M.S..

The Ohio State University

1982

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ACKNOWLEDGEMENT

I would like to thank Professor Leon Peters, Jr. for all of his advice, help and encouragement during the course of this work. I am also indebted to the reading committee: Professors D.L. Moffatt and J.H. Richmond, for their interest and support of my work. I would like to thank the project principal investigator, Mr. Ross Caldecott for his leadership and advice and understanding during the course of this work. I am very indebted to Ms. Linda Eberle, a student research assistant, whose dedication and diligence contributed enormously to the software development and data reduction efforts. Another student research assistant, Mr. Adam Schackleford contributed greatly to the map display and plotting algorithms.

I would like to thank the Electric Power Research Institute for supporting this work and particularly Dr. W.E. Blair, the project manager. I would also like to thank the Industry Adviser, Mr. Michael Patena of the Cleveland Electric Illuminating Co. for his enthusiasm confidence and genuine interest in this work.

I am very grateful to all those at the ElectroScience Laboratory who contributed directly or indirectly to this work or its associated research project and for their frequent informal discussions and suggestions.

I would also like to thank the editorial, typing and drafting staff for their cooperation and help, in particular: Mr. J.M. Gibson and Ms. Sally Owens.

I am very grateful to the ElectroScience Laboratory and Electrical Engineering Department administration for their support during my graduate studies, especially Professors H.C. Ko and C.H. Walter.

And last, but definitely not least, I am thankful to God, for everything.

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A.J. Terzuoli, Jr., L. Peters, Jr., "The VSWR Properties of Dihedral Corrugated Horns", IEEE Trans. on Ant. and Prop., Vol. AP-26, No. 2, March 1978.

A.J. Terzuoli, Jr., J.H. Richmond, and L. Peters, Jr., "Shielding Effectiveness of Corrugations in Corrugated Horns", IEEE Trans. on Ant. and Prop., Vol. AP-26, No. 2, March 1978.

R. Caldecott, A.J. Terzuoli, Jr., and J.P. Hall, "An Underground Mapping System Using Impulse Radar", Proc. of the 7th IEEE/PES Trans. and Distr. Conference, April 1-6, 1979.

R. Caldecott, L. Peters, Jr., A.J. Terzuoli, Jr., J.D. Young and I.L. Volakis, "A Re-Examination of the Potential of Video Pulse Radars for the Detection of Tunnels in Hard Rock Media", Tunnel Detection Symposium, Colorado School of Mines, Golden Colorado, July 21, 1981.

A.J. Terzuoli, Jr., L.M. Eberle, R. Caldecott and L. Peters, Jr., "An Advanced Prototype System for Locating and Mapping of Underground Obstacles", 1982 Joint International IEEE/APS Symposium at University of New Mexico, Albuquerque, May 24-28, 1982.

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CHAPTER I

INTRODUCTION

Locating buried utility lines, building foundations and other underground objects poses a serious problem whenever construction or maintenance work requires excavation. A broken pipe or cable, besides being costly both to repair and through loss of service, poses a very real hazard to life and property. Conventional maps of buried objects may not exist at all and are often inaccurate, incomplete or out of date. The only other aids available to the contractor are visual inspection of the site; the presence of utility lines may sometimes be inferred from the existence of such things as manholes, gratings and curb boxes.

It was the above situation, coupled with the increasing use of plastic pipe by utilities, which led to a study at the ElectroScience Laboratory of the possibility of using radar methods for the location of underground objects. Development along these lines continued for

several years and has already led to a portable instrument capable of locating plastic as well as metal pipe. The principal features of this instrument are advanced well beyond the primitive metal detector. It comprises a signal source, a hand-held antenna and an indicating device, in this case a matrix of light emitting diodes resembling an oscilloscope display. A single trace appears on the display screen, which in the absence of any reflections would be a straight line. Deflection of the trace indicates the presence of a buried object whose depth is proportional to the distance between the point of deflection and the left edge of the screen. The display is thus similar to the A-scope of conventional radar. However, the blip corresponding to the scattered pulse caused by the presence of the pipe is obscured by many false scattered pulses. The goal of the research to be reported in this dissertation is to develop mapping techniques that would make it practical to clearly detect the presence and position of the pipe(s) and to eliminate (or identify) any undesired scatterers from the map.

Given an instrument capable of detecting nonmetallic as well as metallic objects underground, a logical extension is to use the instrument for preparing underground maps. The objective of the present program has been to interface an impulse radar, similar to the one previously developed, with a minicomputer to form an underground mapping system. A self-contained system has been built which is capable of producing a plan view of underground objects at a particular depth or

range of depths, or a sectional view below a given line on the surface of the ground. These views are produced by grey-scale plotting, a method giving the appearance of a coarse silk-screen picture.

In this work, the problem of detection and location of underground obstacles is pursued. By the word "obstacle" we mean any localized anomaly that can be characterized by a dielectric and/or conductive discontinuity from the ambient underground medium. Typically we are considering such practical structures as rocks, pipes, etc., found in a reasonably verifiable depth of say 0-15 feet. The goal of this work is then to produce maps (both plan views and vertical sections) of reasonable clarity such that all underground objects will be apparent and localized to a small region of the map.

Figure 1.1 depicts an artist's rendition of the entire Underground Mapping System. It is totally enclosed in a high-cube van type truck, with the antenna mounted on a locally mobile handcart. The cart and antenna are transported within the van. Electric power (@ 120V, 60Hz, AC) is supplied by an on-board gasoline generator. The van contains its own air conditioning/heating and lighting supplied by the generator. Housed within the van is a Hewlett-Packard 1000 Computer, a graphics printer and a graphics CRT computer terminal. The radar unit is comprised of a spark gap type pulser transmitter, a series of amplifiers, limiters and attenuators making up the receiver section, and a Tektronix Sampling Oscilloscope which digitizes the data and sends it to the computer. Maps are produced on either the printer or the CRT by taking advantage of their graphics capability.

ARTIST'S RENDITION OF THE UNDERGROUND
MAPPING SYSTEM

Figure 1.1. Artist's Rendition of System.

The antenna is comprised of two orthogonal folded over loaded dipoles: one used for transmit and the other for receive, thereby reducing direct coupling. The antenna cart is non-steerable and designed for straight passes only. The cart is constructed of non-metallic components (fiberglass, plastic and wood). A digital shaft encoder position sensor is mounted on the cart thus allowing the computer to record data at pre-determined antenna locations.

Map data are usually recorded for a rectangular area on the ground although long single straight passes are possible and in fact have been made. Data is taken (under full control of the computer) by systematically pushing the cart over the chosen area to be mapped in a raster scan fashion. Figure 1.2 shows a simplified block diagram of the system depicting flow of data from the transmitter to the antenna and back to the receiver. The data after being digitized by the scope is preprocessed by the computer and stored on floppy disc. After the entire map is taken, the data is retrieved from the disc, processed by the computer and sent to the printer to form a map.

Presently, the system best detects discrete anomalies comprised of media discontinuities. The data processing has been designed to accentuate these anomalies in three steps. The clutter reduction technique consists of first establishing a set of sample statistics and then determining a suitable clutter reduction threshold. This threshold is used to extract the statistically significant signals from the clutter. Slant range effects make the location in depth of the targets uncertain. Compensating for this effect necessitates a knowledge of the parameters of the soil. A method of extracting these parameters from

UNDERGROUND MAPPING SYSTEM BLOCK DIAGRAM

Figure 1.2. System Block Diagram.

the data has been developed. The data is then correlated with an expected response from a target buried at a certain depth in soil with those parameters for all possible depths. The system produces resonances which appear as multiple targets as a function of depth in the maps. However, only the shallowest of these is actually the target. To eliminate these false targets, an ideal target return is correlated with the data.

A. RELATED ELECTROMAGNETIC RESEARCH

The ElectroScience Laboratory has been working with time domain radar and its applications to underground detection and location for many years.

Davis (1979) and Chan (1979) in their dissertation introductions present excellent overviews of the earlier work done at this laboratory by Kennaugh, Moffatt, Peters, Young and Caldecott. Chan (1979) also presents an excellent summary of electromagnetic probing techniques starting as far back as the work of Melton (1937). Sandler (1975) presents techniques for image processing.

Underground mapping using an impulse radar was started at the ElectroScience Laboratory around 1977. Stapp (1978) in his MS. thesis developed a technique to gray scale map tunnels using a relatively coarse data grid. He developed some excellent interpretation techniques and compared his actual maps from those generated from synthetic data generated by the technique of Davis (1979). Volakis (1979) extended this tunnel mapping technology further by developing a technique to

extract antenna and system resonances from the data. His maps were consequently an improvement over those of Stapp.

B. OVERVIEW OF RELATED SEISMIC RESEARCH

Geophysicists have been using surface techniques for underground exploration for quite some time. Their interests of course usually lie in probing for natural geological anomalies such as faults, oil wells, gas wells, etc., and consequently are concerned with depths much greater than those that were considered for this work. They usually are interested in producing vertical section maps as opposed to plan views.

Two excellent texts on the fundamentals of the seismic data collection techniques and processing are Robinson and Treitel (1980) and Waters (1978). A good review paper dealing with earlier techniques of seismic data processing (through 1971) is given by Schneider (1971). Processing of seismic data has been underway since early in this century. Roman (1934), for example, performed one of the earlier works on analysis of vertical sections of seismic data. The difference between acoustic reflection and diffraction mechanisms were studied from a two dimensional model by Angona (1960) in an effort to better characterize reflection profiles from faults.

Jon F. Claerbout and his students at Stanford University have done a considerable amount of work in the area of off-line processing of arrays of seismic signals and resulting image reconstruction of layers and anomalies. The problem of an inhomogeneous medium and resulting velocity variation is treated by Claerbout (1970) using a numerical solution to the multidimensional scalar wave equation (for single

frequencies). They address the question of imaging in Claerbout (1971,a) using a numerical holographic technique--that is constructing in a computer a map of the locations of reflecting interfaces. Other schemes for mapping of seismic reflectors are shown to reduce to a single mapping formula by Claerbout (1971,b). Most of the work done was designed for the geometry of a single source with a line of receivers. In Claerbout and Doherty (1972) they extend their techniques to the geometry where a source and receiver move together along the surface as in marine profiling or the work described herein.

Hilterman (1970) presented a mathematical technique for modeling, in the time domain, the acoustic far field of some ideal geologic structure. As in the case of this study, all depth profiles are recorded as a function of time delay from the transmitted pulse. The problem of conversion to the depth domain has always been of great interest and challenge. Paturet (1971) considered three techniques depending on the particular velocity profile at hand. Using a water tank model, French (1974) vividly compared the advantage in clarity of 3-D migration or "focussing" vs. 2-D focussing. Using a collinearity function based on a certain number of samples in a time gate, Sattlegger and Stiller (1974) were able to enhance the signal to noise ratio for vertical sections.

Largely motivated by the previously mentioned work of Claerbout, there has been a tremendous interest in off-line data reduction techniques known as migration and velocity analysis. The following articles pertain to this general area and numerous variations thereof: Gardner, French and Matzuk (1974); Sattlegger (1975); Dohr and

Stiller (1975); Doherty and Claerbout (1976); Riley and Claerbout (1976); Loewenthal, Roberson and Sherwood (1976); Hubral (1977); Stolt (1978); Schultz and Claerbout (1978); Schneider (1978); Gazdag (1978); Phinney and Jurdy (1979); Clayton and Engquist (1980).

It should be noted that the focussing process developed in this dissertation is similar to the seismic concepts of migration and velocity analysis. That is, the essential feature of summing along the hyperbolic arcs of the vertical section maps done in migration, is essentially done by the focussing process described here. However, in this work the focussing is developed from the point of view of a spatial correlation. This technique reduces to migration when only specular reflections are considered but is general enough to consider the entire electromagnetic response of targets.

C. STRUCTURE OF THIS DISSERTATION

The structure of this document is as follows:

In Chapter II we present some basic geometrical considerations of the mapping system and the types of maps produced. We also present a discussion of the basic physical mechanisms expected of an ideal impulse radar and some degrading mechanisms.

In Chapter III we discuss the motivation behind the statistical clutter reduction technique and then present it as an effective process.

In Chapter IV we present a spatial correlation technique known as focussing which is a generalization of the migration technique and then demonstrate its effectiveness.

In Chapter V we present a technique to eliminate the effect of the antenna resonances.

In Chapter VI we present three actual street locations where this system was tried and worked very successfully.

Chapter VII summarizes major achievements of this work and makes recommendations for continued research.

CHAPTER II

GENERAL CONCEPTS OF UNDERGROUND MAPPING

A. THE PROCEDURE OF TAKING AND DISPLAYING A MAP

Consider an arbitrary volume of earth as shown in Figure 2.1 within which may be buried objects of unknown number and orientation. We seek to display with minimal uncertainty the location of all localized objects beneath the surface using a time domain impulse radar from above the surface. Since no prior knowledge of the target whatsoever is assumed, a raster scan of the volume is performed using the mobile downward-looking impulse radar unit mounted on a cart as shown. The scan is performed as follows: the cart is pushed in one direction as the radar transmits impulses into the ground at predetermined intervals, usually one pulse every 5-10 cm (2-4 inches) along the ground, thus defining and recording one linear pass of data. The next pass of data is taken parallel to the first with the interval between passes also being predetermined, usually 15-30 cm (6-12 inches). By continuing this procedure, a raster scan of the volume of earth under consideration is accomplished.

Figure 2.1. Arbitrary volume of earth considered by mapping system.

By consideration of the above described technique, it is obvious that volumes of earth with rectangular surface areas are best dealt with. Accordingly, for ease in future map display, a rectangular coordinate system is set up as shown with the origin being the starting point of the raster scan (starting point of the first pass). The x direction is always chosen to be the direction of cart travel, and the y direction is chosen to be orthogonal to cart travel. The x, y, and vertical UP directions always form a right-handed system. This convention enables a map to be displayed by a conventional printer in the same sense as it was recorded. The longest length of the surface area to be scanned is called x since it is easier to push the cart in the longer direction than to establish a large number of passes.

Once all the data is acquired and recorded, a map may then be produced. Maps are displayed in three basic ways: full plan views, plateau plan views, and depth profile cuts, as defined below.

Figure 2.2 shows a volume of earth containing a target; also shown is a plan view map that depicts what one would see "looking down" into the ground. It merely tells whether something is beneath the surface or not; no depth information whatsoever is conveyed. It is a useful first map of an area giving a quick bird's eye view. The map is produced with the origin in the upper left corner and the direction of cart travel going down the page. Note that the full pipe "T" junction illustrated appears with its correct orientation.

Figure 2.3 portrays a series of plateau plan views with four plateaus being shown. The number of plateaus, however, is arbitrary and

FULL PLAN VIEW

Figure 2.2. Schematic illustration of plan view.

PLATEAU PLAN VIEWS

Figure 2.3. Schematic illustration of plateau plan views.

may in fact be of unequal thickness. The idea is to divide the volume of consideration up into a number of smaller volumes representing different depth ranges and to produce a plan view of each depth range separately. This scheme provides limited depth information; namely the depth range of a target, and still retains much of the bird's eye view nature of the plan views. In this case the T junction is illustrated to be somewhere in the middle of the volume under consideration so that the first plateau plan view (most shallow) might show nothing. The second view might show the target. The third might show a fuzzy outline of the target and could in fact be depicting the bottom of the trench in which the pipe is buried. The fourth (deepest) plateau plan view might once again show nothing since it could be below all targets.

The next type of map one may produce is the vertical cut. It gives the most accurate depth information but no bird's eye view feeling is conveyed. As shown in Figure 2.4, its relation to the actual volume of earth is as follows. One particular value of y is chosen corresponding to one pass made by the cart in the raster scan recording scheme. Envision making a slice or cut in the volume for that value of y . If a target is intersected, its cross section will appear. The map then will depict the cross sections of targets as they would intersect that value of y . The accuracy of the depth information conveyed is then limited solely by the range resolution of the radar.

The display convention for the map has been to use a gray level intensity scale system consisting of 14 shades of gray with the darkest shade corresponding to the strongest radar return -- hence maximum likelihood of a target. The earlier maps were generated by a computer

VERTICAL PROFILE CUT

Figure 2.4. Schematic illustration of vertical cut.

line printer for reasons of both speed and economy. The 14 level gray scale is generated on the line printer by using 14 different combinations of letters and letter overstrikes, depending on their apparent darkness. Consequently, the maps consist of various localized groups of printed characters which in themselves have no particular meaning with regard to target presence or target identity. Only their position and relative intensity or darkness is of any interest.

The following discussion presents a series of actual sample maps taken over a gas pipe T junction buried about one meter below a grass surface. At that time, the entire system of pipes shown was buried; however, for this example only one of the T junctions was mapped. It should be noted the manner in which the pipes were buried; namely, a trench was dug about one meter deep and about 30 centimeters (1 foot) wide. The pipe whose diameter is about 10 centimeters (4 inches) was placed at the bottom of the trench. The removed soil was then replaced into the trench, the soil being well homogenized by that point, and compacted. The resulting interface between the undug (hence unhomogenized) soil and the newly compacted soil define the walls of the trench which in many respects forms a larger electromagnetic target than the pipe itself. In the following maps this fact becomes quite evident. Figure 2.5 shows a plan view sketch of the pipe layout with the mapped area of a T junction outlined and Figure 2.6 shows what an expected plan view map might look like. A full depth plan view map of this area generated by this system is shown in Figures 2.7(a),(b), and (c). Figure 2.7(a) is generated from "raw" or unprocessed data. The large

Figure 2.5. Ideal plan view of gas pipe network.

Figure 2.6. Ideal plan view of the gas T junction that was mapped.

a). Plan view from raw data.

Figure 2.7. Set of plan views of gas T junction.

b). Plan view from clutter reduced data.

Figure 2.7. (Continued).

c). Plan view from focussed data.

Figure 2.7. (Continued).

rectangular gray area represents the entire mapped area whereas the darker T shaped section, corresponding to the expected target region of Figure 2.6, indicates the presence of a target, in this case the T junction pipe and its trench. As mentioned earlier, it is a plan view "looking down" conveying no depth information. Figures 2.7(b) and 2.7(c) indicate various degrees of processing designed to accentuate the radar returns from more or less localized objects such as pipes. The motivation behind processing and details of the various processing techniques are the subject of this work and are explained elsewhere in this document. In Figure 2.7(b) an attempt has been made to reduce background clutter and the noticeable effect of a darker outline in the T junction pipe region relative to the rest of the map. Various regions of pure white now appear indicating true no-target regions. Other areas of darkness do not appear to be clutter but could be false targets or simply targets of no interest. In Figure 2.7(c) an attempt has been made at a spatial summation of target responses that appear from the same target at different but adjacent points in the raster scan. This technique has the effect of focussing in on localized objects such as pipes and rocks but to defocus and hence de-emphasize the returns from distributed targets such as trenches. Consequently, Figure 2.7(c) seems to portray a further darkening of the outline of the T junction pipe and larger white regions. Some additional areas of apparent target locations (dark areas) may be randomly buried objects. As a matter of fact, one object shown in the lower area of the map was verified to be a rock.

The next series, Figures 2.8(a), (b), (c), and (d), represent a set of four plateau plan views of the same area. As introduced earlier (see Figure 2.3), four plan views were generated, each representing an equal segment of the full depth range of the radar. For this map the full depth range was set to 0-3m, hence the depth range of each of the plateau plan views is as follows:

<u>Figure</u>	<u>Depth Range</u>
2.8(a)	0-.75m
2.8(b)	.75-1.5m
2.8(c)	1.5-2.25m
2.8(d)	2.25-3m

Each of these maps have been subjected to the clutter reduction process described earlier. Recall that the pipe is buried about 1m deep. In Figure 2.8(a) (depth range above the pipe) we notice a vague outline of the T junction but a relatively high level of equally strong targets. The outline of the T junction in this map is most likely due to effects from the top of the trench whereas the other dark areas probably correspond to roughness of the grassy surface. In Figure 2.8(b) (depth range includes pipe) a very strong outline of the T junction appears. The general level of other target areas in this case appear to be much weaker than the pipe response itself. In Figure 2.8(c) (depth range just below pipe) we notice a somewhat spotty outline of the T junction but rather strong in places whereas other dark areas are not particularly prominent. It is felt that the vague T outline here corresponds to the places where the power shovel that dug the trench must have scooped a bit deeper than planned. In Figure 2.8(d) (depth

a). Depth range 0-.75m.

Figure 2.8. Set of plateau plan views of the gas T junction for various depth ranges.

b). Depth range .75-1.5m.

Figure 2.8. (Continued).

c). Depth range 1.5-2.25m.

Figure 2.8. (Continued).

d). Depth range 2.25-3m.

Figure 2.8. (Continued).

range well below the pipe) virtually no trend of any kind is noticed. There appears to be a random blend of dark and light regions.

As introduced earlier (Figure 2.4), the final series of sample maps will be vertical profile cuts taken along five equally spaced passes along the T junction. Figure 2.9 once again shows a plan view of the T junction with "cut" marks showing the location of the five vertical cuts. The vertical cut maps depict the profile for the particular value of y shown ($y = 0, 1.25, 2.5, 3.75, 5\text{m}$) with x being the variable indicating distance along the ground. The vertical cut maps appear in Figure 2.10: a($y=0$), b($y=1.25\text{m}$), c($y=2.5\text{m}$), d($y=3.75\text{m}$), and e($y=5\text{m}$). They have only been subjected to clutter reduction. In the following discussion, the transition from the pipe orientation as seen (Figure 2.9) and the vertical cut location indicated to the actual vertical cut map should be noted carefully. In Figure 2.10a we show the vertical cut for $y=0$. According to the plan view (Figure 2.9), only the secondary pipe of the junction is intersected. The x location of this intersection is noted from Figure 2.9. We look on Figure 2.10a at this same x location and we note an apparent "U" shaped structure with a bottom depth of about 1m. This is assumed to be the trench of the secondary pipe since, in general, the trench is a larger target than the pipe and hence produces a stronger radar return. It should be noted again, however, that the focussing process, if it were applied, would accentuate the pipe and de-emphasize the trench. However, this discussion of a series of five sequential vertical cuts showing a pipe junction is more vivid when trenches are observed. Figure 2.10b shows the vertical cut for $y=1.25\text{m}$. The plan view (Figure 2.9) indicates not

Figure 2.9. Ideal plan view of gas T junction showing locations of vertical cuts.

a). Vertical cut map of gas T junction ($y=0$).

Figure 2.10. Set of vertical cut maps of gas T junction for different values of y .

b). Vertical cut map of gas T junction ($y=1.25$ m).

Figure 2.10. (Continued).

c). Vertical cut map of gas T junction ($y=2.5$ m).

Figure 2.10. (Continued).

d). Vertical cut map of gas T junction ($y=3.75$ m).

Figure 2.10. (Continued).

e). Vertical cut map of gas T junction ($y=5$ m).

Figure 2.10. (Continued).

only the presence of the secondary pipe but also the primary pipe appearing on the far right. From the vertical cut it is seen that the cross section of the trench corresponding to the secondary pipe has moved to the right as expected. We also notice a dark area at the far right where the primary pipe is expected to be. Note that the primary pipe has not fully entered this vertical cut. In the next vertical cut shown in Figure 2.10c (for $y=2.5\text{m}$), an interesting phenomenon is observed. According to the plan view (Figure 2.9), the vertical cut should show the intersection of the primary and secondary pipe of the T junction. An inspection of the vertical cut for this case shows a wide "U" shaped response corresponding to the combined trenches of the primary and secondary pipes. Figure 2.10d shows the vertical cut for $y=3.75\text{m}$. This time from the plan view (Figure 2.9) we expect only to see a response from the primary pipe somewhat to the left of the previous combined trench. The vertical cut this time shows a localized dark region at the x location corresponding to the location of the primary pipe. It is not known at this time why the trench image is not as vivid for the primary pipe as it was for the secondary pipe. For the last vertical cut shown in Figure 2.10e (corresponding to $y=5\text{m}$) the plan view (Figure 2.9) indicates that the primary pipe will appear further to the left. An inspection of the vertical cut does show that, once again, the trench image does not appear whereas a localized image is present. It is postulated that this may be caused by local changes in the moisture content of the ground.

When the above series of five vertical cuts is considered together in conjunction with the plan view, one gets a feeling for the three

dimensional nature of the volumetric region of the earth under consideration. The vertical cuts show the depth location of the trenches and pipes accurately and furthermore, show how the cross-section of the pipes change as the vertical cut location is changed.

B. THE CHARACTER OF THE RAW DATA

As indicated previously in Figures 2.7(a), (b), and (c), maps produced from "raw" data are less appealing than those for which the data has been "processed". But before the details of the various processing techniques are considered, it is informative to first consider how the raw data becomes so unappealing.

Figure 2.11a depicts in a very simple manner the basic underground radar situation applicable to this system. For an ideal antenna and a clutter free environment, the radar would observe an ideal signal caused by the target alone. Also depicted in a simple schematic way, however, are three factors introducing degradation to this ideal radar response:

- 1) unknown attenuation with depth
- 2) uneven ground (ground clutter) and buried clutter
- 3) antenna resonance

Fortunately, these three factors can be characterized and dealt with in a systematic way. Nevertheless, it is this degraded data that is referred to here as "raw" data and must be processed to minimize the effects of these degrading factors. But before one attempts to compensate for these degradations, it is instructive to consider the

a). Antenna setup.

Figure 2.11. A basic underground radar.

b). An actual received waveform.

Figure 2.11. (Continued).

exact manifestation of each of the degrading factors in our data.

When a radar response is taken at a point on the ground, a waveform consisting of the amplitude of the received signal as a function of time is recorded. A sequence of these waveforms is then used to produce a map. The pulser output (transmitted signal) is approximately a band limited Gaussian pulse. This pulse is multiply reflected, distorted by the various mechanisms listed above, and is attenuated in a frequency dependent manner by the antenna. Thus, an actual received waveform is distorted and is generally of the form shown in Figure 2.11b. It should be observed that targets of interest to the utility companies usually possess no resonances in the frequency band that can propagate through the earth. Thus the antenna resonance would be the major feature in the received waveform, and the reflected pulse would actually be a filtered (by the earth) replica of the transmitted waveform. Of course, scattered fields from other objects (clutter) may introduce other resonances and additional waveform distortion. The assumed scattered waveform is then an attenuated and delayed replica of the transmitted waveform.

It was the goal of this work to process the returned signals such that an intelligible map could be produced. It has proven extremely valuable to view the processing as eliminating each of these factors one by one rather than lumping them together. In the following few paragraphs of this chapter is presented a series of elementary arguments which are presented in this way so as to provide motivation for the

simple but effective manner in which the data processing was approached and has proven very successful.

Figure 2.12 portrays an idealization of our pulse radar system. Shown schematically are transmitting and receiving antennas looking beneath the ground at a target buried a distance d below the surface. It is assumed that the velocity of propagation of electromagnetic waves is v . As shown, our transmitted signal is an unmodulated video pulse occurring at time $t=0$. For an ideal target located a distance d below the earth, the ideal received signal would of course be simply a pulse delayed in time by $2d/v$. For two targets at different depths as shown in Figure 2.13, the ideal received signal naturally consists of two pulses but the pulse corresponding to the deeper target is considerably more attenuated, where the exact degree of attenuation is unknown a priori. This fact makes it impossible to use the same target detection threshold on targets of different depths. The detection scheme must therefore depend on the time position of interest within the waveform and must be independent of a priori knowledge of attenuation. Figure 2.14 once again depicts an idealized pulse radar illuminating a buried target; this time in the presence of clutter. The clutter has been depicted as disturbances smaller than the target between the antenna and the target and also in the vicinity of the target. If one envisions each of the clutter objects as independent scatterers and the transmit signal is a pulse of short duration, one would expect that the received signal would consist of a series of pulses of different magnitudes, polarities and time delays as shown, corresponding to the size, nature and depth of each piece of clutter.

IDEAL PULSE RADAR

Figure 2.12. An idealization of the pulse radar system.

Figure 2.13. Idealized impulse radar with two targets at different depths.

CLUTTER AND IDEAL PULSE RADAR

Figure 2.14. Idealized impulse radar in the presence of clutter.

For most of this work, the antenna used for this system was of the form of a folded dipole as depicted in Figure 2.15. Of course like any dipole, it is a resonant structure such that the resonant frequency corresponds to the full antenna length being $\lambda/2$. When the antenna is used to radiate a video pulse of short duration, however, this resonant effect is best described in terms of reflections.

The antenna is fed with a balanced pulse (via a balun). Consider only the positive side of the antenna as shown. After the pulse arrives at the central feed region of the antenna, it traverses the arms of the antenna; then it reflects back and re-traverses those arms, etc. If the reflection time is T_r , the transmitted signal will be a series of pulses of a polarity dictated by the impedance of the reflection point and separated by the reflection time T_r . The total number of pulses of course depends on the number of reflections that remain significant despite the attenuation in the antenna.

The above mentioned antenna attenuation actually decreases the sharpness of the resonance; hence the actual transmitted signal is smoother and more oscillatory as shown and in fact is a damped sinusoid.

In consideration of the above argument, the received signal due to a target buried a distance d below the ground in the absence of clutter will be an attenuated replica of the oscillatory transmit signal but delayed by $2d/v$, as depicted in Figure 2.16

In summation, when the effects of antenna resonance, clutter and attenuation are introduced to the idealized pulse radar, as shown in Figure 2.11b, the actual received signal is generally an unrecognizable waveform whose general resemblance to all the above

RESONANCE IN THE ANTENNA

Figure 2.15. The effect of resonance and absorber loading of the antenna.

IDEAL PULSE RADAR CONSIDERING
ANTENNA RESONANCE

Figure 2.16. The idealized impulse radar after antenna effects.

mentioned degrading characteristics is most notable. In particular, we notice a general oscillatory character attributable to antenna resonance. Also notable is a general decrease in amplitude with time corresponding to increased attenuation with target depth. Finally, we notice a general randomness associated with clutter. This type of signal is commonly referred to as "raw" data and in general must be processed off line.

CHAPTER III

STATISTICAL PROCESSING OF SIGNALS IN TIME

As mentioned in a previous section, when a radar response is taken at a point on the ground, a waveform consisting of the amplitude of the received signal as a function of time is recorded. A sequence of these waveforms is then used to produce a map. For the plan views indicating simple target presence beneath the ground, some parameter indicating the degree of oscillation of the waveform is determined. This parameter can typically be the area under the absolute value of the signal, its RMS value, its maximum value, etc.

Since an area of the ground with a target presence will generally produce a greater signal swing than a target free area, it would appear that raw data would be reasonably acceptable for plan views, although we recall from Figures 2.7(a), (b), and (c) that this is not quite true. For the vertical cut, however, the argument is quite different. The depth cut is produced by simply indicating the magnitude of the signal as a function of time (hence depth) using a gray intensity scale. When a sequence of signals along the ground is displayed in this way, the

vertical cut map is produced. As defined previously, it depicts distance along the ground versus depth. Clearly for this map, raw waveforms containing the clutter, resonance and attenuation effects previously described, would produce a vertical cut that would be most ambiguous if at all intelligible. An example of such a map from raw data is shown in Figure 3.1, whereas Figure 3.2 indicates the same map with a small degree of processing applied; in particular the clutter reduction which is to be discussed shortly. Hence, in order for the vertical cuts to be unambiguous, the signals should be processed. The vertical cut is frequently of more significant use than the plan view because of its accuracy in revealing depth information of the target. In the light of this importance of the vertical cut, a significant part of this work has been devoted to processing of the raw signals.

Clearly, if the signals from the idealized pulse radar of Figure 2.12 were used to make a map, the vertical cut would show the precise target location with no ambiguity and utmost clarity!

The goal of the processing then, is to approach the ideal pulse radar signals as nearly as possible by removing the unwanted effects previously described. Of course this entails various approximations and trade-offs; but in the end, it is possible to produce a signal consisting of a single contiguous pulse existing over a small time interval representing a target at the corresponding depth. Such signals can then be made into vertical cut maps that show target depth reasonably clearly.

In the processing, we seek to eliminate and/or compensate for each of the previously described degrading factors one by one. Two will be

Figure 3.1. Example raw vertical cut map showing clutter.

Figure 3.2. Vertical cut of Figure 3.1 with clutter reduction applied.

treated in this section. They are: clutter reduction and a signal normalization technique that compensates for attenuation with depth. Both of these processes must function accurately without a priori knowledge of the degree of clutter or ground attenuation; hence a statistical technique is used. Two other deterministic processes, described in another section, are used in conjunction with the statistical process to produce the final maps. They are the focussing technique previously mentioned and a technique to compensate for signal distortion due to the antenna.

A. CLUTTER REDUCTION

If we record pulse radar return waveforms along the ground in the vicinity of a target such that they would be expected to be similar, we would find that they vary unpredictably; nevertheless, being valid radar returns they must contain information.

Within the realm of standard communication theory, any signal containing information that is not known a priori (hence unpredictable) can be classified as a random signal. Note, however, that we would not know the signal was truly random unless we had others to compare it with. As portrayed in Figure 3.3, a group of time-signals (or waveforms) which we examine and conclude some randomness is present, we define to be an ensemble of signals. Assume the ensemble contains N signals.

In this mapping system, our ensemble of signals is generated by recording waveforms taken along a linear path on the ground. This is done rather systematically so that even though each signal is randomly

Figure 3.3. An ensemble of time signals.

perturbed, any trends produced by true targets can eventually be extracted as a coherent component of the received signal. With a waveform ensemble taken in this way, assuming the reflections from true targets remain fixed and the ground conditions stable, it is perceived that the randomness is created by side effects such as: variations in the local terrain of the antenna (since the antenna is being moved), variations in signal paths from the antenna to the target, multipath effects, effects of the varying antenna ground interface, etc. Even though all of these effects do contain a form of information, they are nevertheless unwanted and are referred to here as clutter. The clutter reduction technique described herein is designed to remove the random effects of clutter. The technique is intended to bring forth the target response and hence strives to recreate in so far as possible pulse returns as anticipated in the ideal pulse radar situation described earlier. The technique is intended for use in obtaining detection and location information of targets only. Any identification efforts must be done on the original waveform where resonance information will still be present in the form of signal oscillations.

Consider once again the spatial ensemble of N signals. As shown in Figure 3.4, each of the waveforms is sampled by the receiver and stored as a series of consecutive samples. We denote the sample times as t_i . We define a random variable for each of the sample times (t_i) comprised of the values of all of the N waveforms obtained as the antenna is moved over the surface at that particular sample time as shown in Figure 3.4. If each waveform is $u_n(t)$; $n=1, \dots, N$ we can denote the above random variable at time t_i as:

Figure 3.4. The random variable $u(t_i)$ defined from the waveform $u_n(t); n=1, \dots, N$.

$$u(t_i) : \{u_1(t_i), \dots, u_N(t_i)\} \quad (1)$$

representing that the random variable may take on the N different values in the brackets $\{\}$, namely the particular value of each of the N waveforms at the time t_i . Even though two adjacent random variables say $u(t_i)$ and $u(t_{i+1})$ would be correlated, the concept of defining a random variable for each sample time rather than a time interval is important for this process since it enables target decisions to be made arbitrarily in time without regard to time duration.

Since each of the waveforms and hence each of their sample time random variables $u(t_i)$ are partially due to a summation of various physical random processes, the Central Limit Theorem of statistics (Bendat and Piersol, 1966) suggests that we represent each random variable as having a Normal (or Gaussian) distribution.

The probability density function for any Normal random variable x can be written as

$$p(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-1/2 \left(\frac{x-\mu}{\sigma}\right)^2} \quad (2)$$

where μ = expected value or mean of the random variable x

σ = standard deviation of the random variable x

given by

$$\mu = E[x] = \int_{-\infty}^{\infty} xp(x)dx \quad (3)$$

$$\sigma^2 = E[(x-\mu)^2] = E[x^2] - \mu^2 \quad (4)$$

Since the probability density function $p(x)$ completely describes the random variable x , an adequate estimate of μ and σ would enable us to completely characterize the random variable. Estimates for μ and σ based on actual sample values of x rather than the definition is the subject of estimation theory in statistics. Nevertheless, it is sufficient to say that if the random variable is represented by the finite values $x: \{x_1, \dots, x_N\}$, then μ and σ can be estimated by the sample mean $\bar{x}$ and sample standard deviation respectively as follows:

$$\mu \approx \bar{x} = 1/N \sum_{n=1}^N x_n \quad (5)$$

$$\sigma^2 \approx 1/(N-1) \sum_{n=1}^N (x_n - \bar{x})^2$$

Now for each of our waveforms, it is assumed that the associated random variables $u(t_i)$ are normally distributed. Each one has its own probability density function similar in form to the one above and further has its own sample mean $\bar{u}(t_i)$ and sample standard deviation $\sigma(t_i)$ which can be calculated from the N waveforms sampled at time t_i . If the sample mean $\bar{u}(t_i)$ and sample standard deviation $\sigma(t_i)$ are considered as a function of t_i , two new time sequences will be generated. If these time sequences are considered time samples of waveforms, the corresponding waveforms would be the ensemble average (or sample mean) waveform and standard deviation waveform, respectively.

Figures 3.5 and 3.6 illustrate the above argument where for ease of illustration the waveforms $u_n(t)$ have been represented continuously rather than as time samples, and the sample time variable t_i has been replaced by the continuous time variable t . The ensemble average (or

CLUTTER REDUCTION DEFINITIONS

Figure 3.5. Definition of ensemble average waveform.

Figure 3.6. Definition of standard deviation waveform.

sample mean) waveform $\bar{u}(t)$ and standard deviation waveform $\sigma(t)$ are hence defined. It should be emphasized that the continuous nature of those waveforms is for convenience only and should be thought of as a time sequence of random variables and random variable parameters as discussed above. Note that Figure 3.5 starts with the N raw waveforms $u_n(t)$: $n=1, \dots, N$ and defines the ensemble average waveform (with a sum over the ensemble) as:

$$\bar{u}(t) = 1/N \sum_{n=1}^N u_n(t) \quad (6)$$

so that time as a variable is preserved. Now in light of the definition of the sample standard deviation, Figure 3.6 begins with an ensemble of N zero mean waveforms $v_n(t)$: $n=1, \dots, N$ given by

$$v_n(t) = u(t) - \bar{u}(t) . \quad (7)$$

The zero mean waveform is a raw waveform with the ensemble average waveform removed. It is normally felt that in a situation where the raw waveforms correspond to pulse radar returns that the zero mean waveforms will have some coherent clutter (that is present to all waveforms) removed. Note that the zero mean waveforms derive their name from the property that their ensemble average waveform is identically zero as follows:

$$\begin{aligned}
\bar{v}_n(t) &= 1/N \sum_{n=1}^N v_n(t) = 1/N \sum_{n=1}^N [u_n(t) - \bar{u}(t)] \\
&= 1/N \sum_{n=1}^N u_n(t) - 1/N \sum_{n=1}^N \bar{u}(t) \\
&= \bar{u}(t) - \bar{u}(t) \equiv 0.
\end{aligned} \tag{8}$$

This simplifies calculation of the standard deviation waveform and it is given by:

$$\sigma(t) = \sqrt{1/(N-1) \sum_{n=1}^N v_n^2(t)} \tag{9}$$

where once again the sum is in the ensemble as shown so that time is preserved as a variable. It should be noted that in the same way that the continuous waveforms $u_n(t)$ are representative of a time sequence of random variables

$$u(t_i), \text{ for all } i, \tag{10}$$

the zero mean waveforms $v_n(t)$ are representative of a time sequence of random variables

$$v(t_i), \text{ for all } i. \tag{11}$$

Further we can denote the particular random variable at time t_i as:

$$v(t_i): \{v_1(t_i), \dots, v_N(t_i)\}, \tag{12}$$

indicating that it may assume the N different values shown, which are samples of the N waveforms $v_n(t)$ at time t_i .

It was mentioned previously that each random variable at time t_i , $u(t_i)$ has its own normal probability distribution function associated with it, namely

$$p[u(t_i)] = \frac{1}{\sigma(t_i) \sqrt{2\pi}} e^{-1/2 \left[\frac{u(t_i) - \bar{u}(t_i)}{\sigma(t_i)} \right]^2} \quad (13)$$

where $\bar{u}(t_i)$ = ensemble average at time t_i and $\sigma(t_i)$ = standard deviation at time t_i . As shown in Figure 3.7a, this function has its maximum value at $\bar{u}(t_i)$, and $\sigma(t_i)$ is the distance from the maximum value to the inflection point. However, the probability density function for each random variable $v(t_i)$ is

$$p[v(t_i)] = \frac{1}{\sigma(t_i) \sqrt{2\pi}} e^{-1/2 \left[\frac{v(t_i)}{\sigma(t_i)} \right]^2} \quad (14)$$

where their ensemble average is zero, because they are zero mean random variables. As shown in Figure 3.7b, this function has its maximum value at zero and is typically easier to deal with analytically.

The utility of the probability density function is in determining the probability of the random variable being within a certain interval. For an arbitrary random variable x , the probability of x assuming values contained within the interval $[a,b]$ is expressed as:

$$P[a < x < b] = \int_a^b p(x) dx \quad (15)$$

That is, this probability is the area under the probability density function in the interval $[a,b]$. With this notion of area representing

a). Probability density function for raw data $u(t_i)$.

Figure 3.7. Probability density function for data prior to clutter reduction.

b). Probability density function for zero mean data $v(t_i)$.

Figure 3.7. (Continued).

a). Histogram for $v(t_i)$.

Figure 3.8. Figures showing target/clutter decision points.

probability, let us consider the meaning of the probability density function for the zero mean waveform random variable $v(t_i)$. We recall that this random variable may assume the amplitude of any of the signals $v_n(t_i)$, $n=1, \dots, N$, at time t_i . Statistically speaking, the "larger" amplitudes are more likely to correspond to target returns and the "smaller" amplitudes are likely to correspond to clutter or various other unwanted effects. However, if all of the N amplitudes were plotted on a histogram as shown in Figure 3.8a for a very large sample number N , the Law of Large Numbers of statistics (Bendat and Piersol, 1966) tells us that the histogram would resemble the probability density function $p[v(t_i)]$ of Figure 3.7b.

We can now begin to establish quantitatively the previously mentioned concepts of target vs. clutter, and probability being area under the probability density function. Since targets would normally occupy a small part of a given volume of earth under consideration, one would expect the "a priori" probability of a target being present to be reasonably small. Hence target probability might be associated with regions of smaller area of the probability density function. Furthermore, if we accept the premise that larger signal amplitudes at any given time might correspond to a target radar return, we are led to associate certain regions of the probability density function (or histogram of Figure 3.8a) with targets and other regions with clutter. In particular the leftmost and rightmost sections of Figure 3.8a are likely to be targets since they are regions of large signal amplitude and small area of the function. The central region being of larger area and of relatively nominal amplitude is likely to correspond to clutter.

This hypothesis forms the basis of the statistical process to be used. Motivated by the above discussion, we can postulate a technique to decide between a clutter and target signal at each point t_i . At each point t_i , we assume the existence of a normal probability density function representing the signal random variable $v(t_i)$ for time t_i . Without creating an amplitude histogram, we can claim that this amplitude distribution is given by the probability density function $p[v(t_i)]$, which is completely defined by the standard deviation $\sigma(t_i)$ which in turn can be calculated easily. Then, as described in the next section, by judiciously choosing a threshold θ , we establish a target/clutter decision point given by $\pm\theta\sigma(t_i)$. This is depicted in Figure 3.8b. The shaded area of the target regions would then correspond to the probability of a target being present. Then signal amplitudes (at this time t_i) within the region $\pm\theta\sigma(t_i)$ are hence considered to be clutter and can be set to zero.

It should be noted once again that this process of clutter reduction is applied separately to every time point t_i and every waveform in the ensemble. Accordingly, in order to fully illustrate its effect, it is beneficial to once again consider the zero mean waveforms to be represented continuously as

$$v_n(t); n=1,\dots,N. \quad (16)$$

As shown in Figure 3.9, let us trace one particular waveform through the clutter reduction process. We shall consider the n^{th} zero mean waveform defined as:

$$p[v(t_i)] = \frac{1}{\sigma(t_i)\sqrt{2\pi}} e^{-1/2 \left[\frac{v(t_i)}{\sigma(t_i)} \right]^2}$$

b). Probability density function for $v(t_i)$.

Figure 3.8. (Continued).

ONE WAVEFORM TRACED THRU CLUTTER REDUCTION

Figure 3.9. One waveform traced through clutter reduction.

$$v_n(t) = u_n(t) - \bar{u}(t). \quad (17)$$

Consider two "waveforms" given by $\pm\theta\sigma(t)$ which would define the envelope of the target/clutter decision boundary, represented continuously in time. The waveform $v_n(t)$ is then compared to this envelope given by $\pm\theta\sigma(t)$. All parts of the waveform $v_n(t)$ within the envelope are set to zero, whereas all parts outside the envelope are retained. Hence a clutter reduced waveform $\underline{v}_n(t)$ is defined as follows:

$$\underline{v}_n(t) = \begin{cases} 0; & |v_n(t)| \leq \theta\sigma(t) \\ v_n(t); & |v_n(t)| > \theta\sigma(t) \end{cases} \quad (18)$$

which is depicted as a series of pulses. Consequently this has taken us one step closer to the ideal pulse radar signal. This process is repeated for all waveforms and a new ensemble of clutter reduced waveforms is then created:

$$\underline{v}_n(t); n=1, \dots, N; \quad (19)$$

which, once again, is a convenient notation for a new time sequence of samples:

$$\underline{v}_n(t_i); \text{ for all } t_i; n=1, \dots, N. \quad (20)$$

B. SIGNAL NORMALIZATION IN TIME

As depicted in Figure 3.9, the clutter reduced waveform $v_n(t)$ will still exhibit the effects of attenuation in time or depth. Thus a form of attenuation compensation is needed so that shallow and deep targets can be displayed uniformly on the depth profile map. As mentioned previously, the ground attenuation is by and large unknown; hence a statistical approach is convenient. A combined sample time and continuous time representation of the waveforms will be used for this discussion. As will be demonstrated next, the standard deviation waveform $\sigma(t)$ which was calculated and used for clutter reduction is extremely useful for signal normalization.

Let us consider the root mean square (RMS) of a time sampled* signal $v(t_i)$ given by:

$$V_{RMS} = \sqrt{1/M \sum_{i=1}^M v^2(t_i)} \quad (21)$$

where we assume we have a total of M time samples. This equation essentially gives a form of a "characteristic signal level," as portrayed in Figure 3.10. When $\pm V_{RMS}$ are allowed to form an amplitude envelope for the continuous signal $v(t)$, we find that time amplitude peaks (both positive and negative) of $v(t)$ will exceed the characteristic level V_{RMS} . This is truly an elementary point. But let us now consider the form of the equation for the standard deviation of the ensemble of zero mean waveforms $v_n(t)$; $n=1, \dots, N$, given by:

*This is used for discussion purposes only and the waveform so chosen does not include any attenuation characteristics associated with depth.

$$\text{RMS SIGNAL LEVEL, : } V_{\text{RMS}} = \sqrt{\frac{1}{M} \sum_{i=1}^M v^2(t_i)}$$

Figure 3.10. Definition of RMS.

$$\sigma(t) = \sqrt{\frac{1}{N-1} \sum_{n=1}^N v_n^2(t)} \quad . \quad (22)$$

Even though this is a "purely" statistical quantity, its resemblance to the equation for V_{RMS} is striking. The important difference is, of course, that the equation for V_{RMS} uses a sum in time, whereas the equation for $\sigma(t)$ uses an ensemble sum. Consequently, one may interpret the standard deviation $\sigma(t)$ as a "characteristic signal level of the ensemble as a function of time." As pointed out in the clutter reduction section when $\pm\sigma(t)$ are allowed to form an envelope, then for each and every continuous signal $v_n(t)$, $n=1, \dots, N$, we find that the time peaks of the ensemble will exceed the characteristic level $\sigma(t)$ for the appropriately corresponding time. But this property now takes on new meaning in the light of the similarity between V_{RMS} and $\sigma(t)$.

Furthermore, it can be argued that an ensemble characteristic signal level as a function of time, when time is related to target depth in a lossy medium, would typically be expected to be large for early time and smaller for later time. In fact, then, $\sigma(t)$ is representative of the overall attenuation that the ensemble of signals experiences as a function of target depth in the ground.

Suppose now, there existed a new ensemble of signals: $\hat{v}_n(t)$, $n=1, \dots, N$, such that their standard deviation $\hat{\sigma}(t)$ was constant as a function of time. The previous discussion would lead us to believe that this new ensemble is free from any effects of signal attenuation in time. Furthermore, if we could generate such an ensemble of signals

from our original ensemble $v_n(t)$, $n=1,\dots,N$, the goal of compensating for attenuation would have been achieved. Let us consider an ensemble of signals $\hat{v}_n(t)$, defined by:

$$\hat{v}_n(t) = \frac{v_n(t)}{\sigma(t)} \quad (23)$$

where $v_n(t)$ is the ensemble of zero mean signals and $\sigma(t)$ is their standard deviation waveform. The ensemble average waveform for this new ensemble is given by:

$$\begin{aligned} \overline{\hat{v}}(t) &= 1/N \sum_{n=1}^N \hat{v}_n(t) \\ &= 1/\sigma(t) \left[1/N \sum_{n=1}^N v_n(t) \right] \\ &= 1/\sigma(t) [\overline{v}(t)] \\ &= 0 \end{aligned} \quad (24)$$

since $v_n(t)$ are zero mean signals. Then their standard deviation waveform is given by:

$$\begin{aligned}
\sigma(t) &= \sqrt{\frac{1}{N-1} \sum_{n=1}^N \hat{v}_n^2(t)} \\
&= \sqrt{\frac{1}{N-1} \sum_{n=1}^N \frac{v_n^2(t)}{\sigma^2(t)}} \\
&= 1/\sigma(t) \sqrt{\frac{1}{N-1} \sum_{n=1}^N v_n^2(t)} \\
&= 1/\sigma(t) [\sigma(t)] \\
&= 1.
\end{aligned} \tag{25}$$

Thus the ensemble of signals

$$\hat{v}_n(t) = \frac{v_n(t)}{\sigma(t)}, \quad n=1, \dots, N \tag{26}$$

has its standard deviation waveform

$$\hat{\sigma}(t) = 1, \tag{27}$$

a constant in time. This normalized set of signals is then free of all effects of attenuation in time since its ensemble characteristic value as a function of time, $\hat{\sigma}(t)$, is constant.

The preceeding discussion produced a set of normalized signals $\hat{v}_n(t)$ from the zero mean signals $v_n(t)$ and their standard deviation. The theory was presented in that manner for purposes of clarity. Actually, however, the implementation of the signal normalization may be

carried out in conjunction with the clutter reduction. Figure 3.11 illustrates this combination. First, the standard deviation waveform $\sigma(t)$ is calculated from the ensemble of zero mean signals $v_n(t)$, $n=1, \dots, N$. Then each and every waveform $v_n(t)$ of the ensemble is clutter reduced based on $\sigma(t)$ as described in the previous section. Next it is normalized via $\sigma(t)$ to produce the normalized clutter reduced waveform $\hat{v}_n(t)$ shown. Even though the signal normalization was presented as being applied to ordinary zero mean waveforms, its effect is the same when applied to clutter reduced waveforms because the clutter reduction simply has the effect of setting to zero certain time intervals of the signal. Namely since

$$\underline{v}_n(t) = \begin{cases} 0 & ; |v_n(t)| < \theta\sigma(t) \\ v_n(t) & ; |v_n(t)| > \theta\sigma(t) \end{cases} \quad (28)$$

and

$$\hat{v}_n(t) = \frac{v_n(t)}{\sigma(t)} = \begin{cases} 0 & ; |v_n(t)| < \theta\sigma(t) \\ \frac{v_n(t)}{\sigma(t)} & ; |v_n(t)| > \theta\sigma(t) \end{cases} \quad (29)$$

we can see that each signal is normalized according to the original discussion in the time intervals for which $|v_n(t)| > \theta\sigma(t)$. This means that the amplitude envelope of the new normalized clutter reduced signals $\hat{v}_n(t)$ would be the same as the ordinary normalized signals $\hat{v}_n(t)$. It should be noted that the new standard deviation of the normalized clutter reduced signals can not be shown to be one in general because the regions of clutter reduction (zeroing of the signal) are not

Figure 3.11. Normalized clutter reduction of a waveform.

the same from signal to signal over the ensemble. However, for each signal $v_n(t)$ separately in the regions where $|v_n(t)| > \theta\sigma(t)$, the normalization is as expected.

C. DETERMINATION OF THE CLUTTER REDUCTION THRESHOLD

In the clutter reduction section, a threshold θ was introduced, such that $\theta\sigma(t)$ was used as a decision point between clutter and target. It is the subject of this section to relate this threshold θ to some physically meaningful quantity in order to optimally evaluate it.

In classical Bayes decision theory, a similar problem is posed. There is typically a transmitter capable of sending any number n of different known signals. Between the transmitter and the decision point of the receiver, the signals are subjected to random noise. If the signals are sent one at a time and if the statistics of the random noise is known, the receiver can be made to optimally decide which of the n signals was sent. The only additional information that must be supplied to the receiver is the a priori probability of each of the n signals actually being sent.

In our situation the random noise is of course replaced by random clutter of assumed Normal statistics. However, since this system is intended to scan over arbitrary areas of ground, possibly containing targets from 0 - 15 feet deep, it is in general impossible to set up a repertoire of n known signals to be received each with a given a priori probability. Hence, a Bayes system is not feasible here. Also as mentioned earlier, this system does not attempt to discriminate between different target signals; it simply seeks to determine whether or not a

target of any type is present. Nevertheless, the Bayes concept of a priori probability of signal transmission is carried over to this system simply as an a priori probability of a target of any type being present. This number is actually quite easy to estimate for a given map site.

The section on signal normalization provides a simplification in the determination of the threshold θ . Recall that Figure 3.8b represents the probability density function of the zero mean random variable $v(t_i)$. Since the corresponding normalized random variable

$$\hat{v}(t_i) = \frac{v(t_i)}{\sigma(t_i)} \quad (30)$$

is also zero mean with unit standard deviation, and Normally distributed, its probability density function $p[\hat{v}(t_i)]$ would be

$$p[\hat{v}(t_i)] = \frac{1}{\sqrt{2\pi}} e^{-1/2 \hat{v}^2(t_i)} \quad (31)$$

as shown in Figure 3.12. Note that in this case the CLUTTER/TARGET decision point is simply $\pm\theta$. From this function then the target probability (P_T) could be calculated via

$$P_T = \int_{-\infty}^{-\theta} p[\hat{v}(t_i)] d\hat{v}(t_i) + \int_{+\theta}^{\infty} p[\hat{v}(t_i)] d\hat{v}(t_i) \quad (32)$$

Using the previously defined expression for $p[\hat{v}(t_i)]$, which is an even function of $\hat{v}(t_i)$, we have:

$$P_T = 2 \int_{-\infty}^{-\theta} \frac{1}{\sqrt{2\pi}} e^{-1/2 v^2(t_i)} d\hat{v}(t_i) \quad (33)$$

$$p \left[\hat{v}(t_i) \right] = \frac{1}{\sqrt{2\pi}} e^{-1/2 \hat{v}^2(t_i)}$$

Figure 3.12. Probability density function of $\hat{v}(t_i)$.

This integral is evaluated numerically, and the results are shown in Figure 3.13.

As mentioned previously, it is a good assumption that the statistics of the system are Normal; hence it is expected that the a priori target probability will be very close to the theoretical target probability P_T calculated from the probability density function. Consequently, Figure 3.13 can be used to relate the a priori target probability to the desired threshold θ ; hence θ is determined. It should be mentioned that the a priori target probability will usually be different for different depths in the ground. Therefore it would be expected that the threshold would be a function of time, that is $\theta(t_i)$.

The following illustrates this clutter reduction technique on an actual received signal. Figure 3.14 is a sketch of an actual received signal with the ensemble average already removed. Figure 3.15 shows the standard deviation waveform of the ensemble of signals from which Figure 3.14 was obtained. Finally, Figure 3.16 shows the effect of the clutter reduction on this waveform. One may notice that only peaks of the original signal now comprise the clutter reduced signal. Hence we are one step closer to the signal that would be returned from the ideal pulse radar.

Figure 3.13. Threshold level (θ) vs. target probability.

TARGET WAVEFORM - ENSEMBLE AVERAGE REMOVED

Figure 3.14. A typical received signal with ensemble average removed.

STANDARD DEVIATION WAVEFORM

Figure 3.15. A typical standard deviation waveform.

TARGET WAVEFORM — STATISTICALLY COMPRESSED

Figure 3.16. A typical clutter reduced waveform.

CHAPTER IV

SPATIAL PROCESSING OF SIGNALS

As mentioned in the previous section, the vertical cut map is of considerable interest in practice because of its potential accuracy in determining target depth. However, even a vertical cut having undergone statistical processing such as the one shown in Figure 3.2 does not yet reveal depth information very accurately. Assuming that the darker areas of the map indicate a target region, we notice an inverted hyperbolic shape characteristic of each of the dark regions. This effect makes location of the target somewhat vague not only in depth but also in position along the ground. Compensating for this effect is rather straight forward after a simple explanation of the process generating these hyperbolas.

A. SPATIAL SPREADING OF TARGET IMAGE

Figure 4.1a depicts an idealized vertical profile of a target buried beneath the ground. Its coordinates are distance along the ground vs. depth. The target diameter is assumed to be significantly

a). Actual setup.

b). Reconstructed from data.

Figure 4.1. Idealized vertical profiles for two antenna positions.

less than its depth. The antenna is assumed to transmit into the earth and receive reflected signals from below the surface. Similar to our mapping system, the antenna is moved along the ground in the x direction in such a way that radar returns are taken at equal intervals along the ground. It is assumed for the moment that the target is orthogonal to the plane formed by the different antenna positions, so that there is at least one antenna position that is closer to the target than the others. For the purposes of illustration it can safely be assumed that if we are not following the target, there is one radar return that is taken "directly above" the target. This is a reasonably accurate assumption if the radar returns are taken about 5-10cm apart along the ground. Figure 4.1a shows the actual location of a target with two waveforms being taken: one directly above the target, and the other displaced along the ground by a distance Δx . The actual distances from the antenna to the target are d_0 and d_1 , respectively, where

d_0 = "actual" target depth, and

$$d_1 = \sqrt{d_0^2 + \Delta x^2} = \text{slant range} .$$

Using the radar impulse reflection arguments of Section II, we can reconstruct the location of the target as seen by each of the waveforms. Figure 4.1b shows the apparent target location when it is assumed that the target is directly below each antenna position. The apparent target depths of each reconstructed image correspond to the actual target range from Figure 4.1a. Only the waveform taken directly above the target then has its apparent depth equal to the actual depth d_0 . The other

apparent target location will appear deeper, namely as the slant range d_1 .

Figures 4.2a and 4.2b represent a generalization of this argument for an arbitrary number of antenna waveform locations. In Figure 4.2a the actual target and location are shown with radar waveforms being recorded at a total of $2N+1$ antenna positions. The antenna is assumed to be positioned such that the middle waveform of the group is directly above the target and with $2N$ waveforms equally and symmetrically spaced on each side. The actual depth (i.e., direct distance to the target) is once again d_0 , and the other distances (slant ranges) are denoted as d_n , $n=1\dots N$, where

$$d_n = \sqrt{d_0^2 + (n\Delta x)^2} = \text{slant range} \quad (34)$$

Once again using the impulse reflection arguments of Section II, Figure 4.2b represents the target locations reconstructed from each of the waveforms recorded. Note again that the apparent depth of the target seen at each antenna position corresponds to the actual slant range shown in Figure 4.2a. However, for the reconstruction, each apparent target location must be assumed to be directly below its antenna position since the actual target location is not known a priori. The concepts of Figures 4.2a and b are once again used in Figure 4.3 where here the locus of apparent target locations as seen by antenna positions on either side of the target is shown as a continuous hyperbolic arc. As before, the direct distance to the target (actual target depth) is d_0 . Waveforms having a slant range to the target are shown at antenna

a). Actual setup.

b). Reconstructed from data.

Figure 4.2. Ideal vertical profile for $2N+1$ antenna positions.

Figure 4.3. Locus of apparent target locations.

positions a distance Δx from the waveform taken directly above the target (center waveform).

In the discussion that follows we shall form a model of the ground comprised of an air/dielectric interface with the relative dielectric constant of the earth denoted as ϵ_r . As the remainder of this chapter unfolds, this will be seen to be a reasonably good approximation. Since the antenna transmission and reception takes place above the ground level, the radar return signal therefore takes a two way path into and out of the dielectric interface. Consequently for antenna positions involving slant range, the maximum slant angle at which the antenna can receive radar returns from the target is the critical angle given by Snell's law.* This enables us to determine the maximum extent of the hyperbolic arcs for any given actual depth d_0 and ϵ_r , as follows:

From Snell's law we have

$$\sin \theta_c = \frac{1}{\sqrt{\epsilon_r}} \quad \text{or} \quad \tan \theta_c = \frac{1}{\sqrt{\epsilon_r - 1}} \quad .$$

From the geometry we have

(35)

$$\tan \theta_c = \frac{d_0}{\Delta x_{\max}}$$

$$\text{therefore, } \Delta x_{\max} = \frac{d_0}{\sqrt{\epsilon_r - 1}} \quad .$$

This is the maximum offset distance along the ground which will be considered by the data processing. One can still "see" the target beyond this point with an antenna at the surface. However, the lateral wave now also comes into play (see Davis (1979) for a further discussion).

*See section on Estimation of Dielectric Constant.

Figure 4.4 depicts the full extent of the hyperbolic arcs for different actual target depths. Shown are a parametric family of curves representing lateral distance vs. apparent depth with the parameter being actual depth d_0 . The parametric equation for this family of hyperbolic arcs is then

$$d_A = \begin{cases} \sqrt{d_0^2 + (\delta x)^2} & ; \quad |\delta x| < \frac{d_0}{\sqrt{\epsilon_r - 1}} \\ \text{Not considered;} & |\delta x| > \frac{d_0}{\sqrt{\epsilon_r - 1}} \end{cases} \quad (36)$$

where δx = offset distance along the ground from the center
(curve independent variable)

d_0 = actual target depth at center of curve (parameter
of the curves)

d_A = apparent depth of target due to slant range
(curve dependent variable).

In summary, then, these hyperbolic arcs represent the locus of points for which an impulse radar target return will lie for an actual target depth located at the center of the arc. The following will show that this family of curves, when transformed to time instead of depth, comprises an impulse response of an ideal target at various depths due to an ensemble of impulses on either side of the target. Improved target responses can then be "sifted" out of these ensembles of raw waveforms by using spatial correlation over predetermined contours.

Figure 4.4. Locus of apparent target locations as a function of target depth.

This process is referred to herein as "FOCUSSING" because it tends to concentrate the target radar returns into a tighter and denser distribution.

B. THE TRUE TIME DOMAIN IMPULSE RESPONSE

In the preceeding discussion the spatial spreading of the target image was shown to be dependent upon the average relative dielectric constant of the earth (ϵ_r) and the actual target depth (d_0). But under the basic premise of this work, the actual target depth and the ϵ_r of the earth are unknown a priori. Via an earlier discussion, all that is known is the time delay of the ideal returned pulse. Consequently, it is necessary to convert to time rather than depth. However, the introductory remarks using depth are necessary because only distance can be triangulated using actual range and slant range.

As shown in Figure 4.5, we consider a reference waveform at $x = X_0$ and we postulate the existence of an impulse radar return at $t = T_0$, which corresponds to the maximum round trip path to and from the target. We consider waveforms to be located on either side of X_0 . It would be expected that these waveforms within the critical angle would also have impulse returns, but for $t > T_0$. Since the hyperbolic arcs as a function of time are similar to those in depth, for simplicity in Figure 4.5 only two waveform locations are shown: the central waveform location with direct distance to the target at $x = X_0$ and one waveform location at $x = X_0 - n\Delta x$ with slant range to the target. Since the arcs are symmetric about the central waveform, only waveforms to one side need to be considered. In the n^{th} waveform, then, the round trip time

THE FOCUSING PROCESS

Figure 4.5. Illustration of the focussing algorithm.

location T_n of the impulse return corresponding to that at T_0 in the central waveform is (via distance triangularization) given by:

$$\left(\frac{vT_n}{2}\right)^2 = \left(\frac{vT_0}{2}\right)^2 + (n\Delta x)^2 \quad (37)$$

where

$$v = \frac{1}{\sqrt{\mu_0 \epsilon_r \epsilon_0}} = \frac{c}{\sqrt{\epsilon_r}} = \text{velocity of wave in earth}$$

and

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = \text{velocity of wave in air} .$$

Therefore,

$$T_n = \sqrt{T_0^2 + \left(\frac{2n\Delta x}{c}\right)^2} \epsilon_r \quad ; \text{ for all } n$$

such that

$$|n\Delta x| \leq \frac{d_0}{\sqrt{\epsilon_r - 1}} = \frac{c T_0}{2 \sqrt{\epsilon_r (\epsilon_r - 1)}} .$$

Once again, these equations represent the locus of points in time of impulse radar returns corresponding to the impulse radar return in the central waveform at $t = T_0$, where $T_0 < T_n$. It is an ensemble impulse response functionally depending only on the distance between waveforms, Δx , and the ground relative dielectric constant, ϵ_r . The initial target time, T_0 , is assumed to be a parameter of this family of curves.

It is important to mention at this time that it is well known and has been verified experimentally (see Davis (1979)), that other modes of propagation at the air-earth interface exist in addition to the direct path. In particular, the "over and down" mode is applicable in this

situation. While the previous discussion was limited to direct path transmission within the critical angle from the antenna to the target and back to the antenna, rigorously speaking, surface waves to and from the antenna allow the target to be seen even when the direct path from the antenna to the target is beyond the critical angle. Figure 4.6 illustrates this effect. Figure 4.6 simply depicts valid reciprocal paths for the air-earth interface; all those paths intersecting the interface at less than the critical angle support direct path round trip transmission from the antenna to the target. As already mentioned, for all paths intersecting the interface at greater than the critical angle, we will ignore the direct path round trip transmission from the antenna to the target. The path intersecting the interface at the critical angle in addition to supporting direct path round trip transmission, is also capable of coupling to a lateral wave along the air-earth interface. As depicted in Figure 4.6 then, any antenna position whose direct paths to the target intersects the interface at greater than the critical angle has a round trip transmission path comprised of the surface wave path along the ground coupled with the direct path intersecting the interface at the critical angle. This is supported by Fermat's Principle (Principle of Least Time) which simply states that the path of any wave will be such as to minimize the total transit time. This was verified experimentally by Davis (1979) using time of arrival measurements of an impulse transmitted from below the ground and received at various points along the surface of the ground. For receiving antenna positions having a direct path intersecting the interface well outside the critical angle,

Figure 4.6. Valid reciprocal paths for air/earth interface.

the measured time delay was shown to correspond to a path comprised of a surface path coupled to a path to the target intersecting the interface at the critical angle as given by Snell's Law.

We are now in a position to extend, upon modification, the equation for the round trip time T_n to include those radar returns provided by the lateral wave effect. Let us first denote the round trip time corresponding to the antenna target path intersecting the ground at the critical angle as $T_{n \text{ crit}}$, where that waveform number is n_{crit} . This value is obtained from the equation for T_n when

$$|n\Delta x| = \frac{cT_0}{2\sqrt{\epsilon_r(\epsilon_r-1)}}$$

and is given by

$$T_{n \text{ crit}} = T_0 \sqrt{1 + \frac{1}{\epsilon_r - 1}} \quad .$$

Then the values for T_n corresponding to paths intersecting the ground at greater than the critical angle are given by:

$$T_n = T_{n \text{ crit}} + 2(n - n_{\text{crit}}) \frac{\Delta x}{V_s} \quad ; \quad \text{for all } n$$

such that

$$|n\Delta x| > \frac{d_0}{\sqrt{\epsilon_r - 1}} = \frac{cT_0}{2\sqrt{\epsilon_r(\epsilon_r - 1)}}$$

where V_s = velocity of the surface wave.

This of course is simply a linear equation in Δx . Therefore, one would expect the net impulse response of an ideal target, due to an ensemble of impulses on either side of the target, to be comprised of hyperbolic arcs within the critical angle region then becoming straight lines beyond the critical angle region as depicted in Figure 4.7. Theoretically, these curves can go on without limit in x .

It is the purpose of this section to introduce the spatial focusing processing which is intended to correct for the spreading of the target images as explained herein. It is a generally accepted fact, and has been shown empirically during the course of this work, that it is far superior to perform some clutter reduction prior to any processing which is designed to enhance target images.* Consequently, all of the maps go through the previously described clutter reduction process prior to spatial processing (focusing). In this regard it has been found that the target returns appear to terminate when the slant range corresponds to an antenna to target direct path intersecting the ground at the critical angle. In other words, after clutter reduction the only visible radar returns on the maps seem to be located within the hyperbolic arc region. There does not seem to be a linearizing of the curves as would be indicative of the surface waves.

This seems to imply that the signal levels of the target returns involving surface waves, when compared with those launched directly

*This would not be true for the case where an undesired natural resonance is to be removed. See Volakis (1979).

into the ground (i.e., region inside the critical angle), appear to be within the "clutter level" and are hence eliminated by the clutter reduction. The exact phenomenon of the surface waves, their launching, and their coupling to the direct path wave was not the topic of this study. Consequently, their apparent lack of appearance in the clutter reduced data is accepted as an empirical fact, and the processing proceeded only considering the direct path waves inside the critical angle. It is conjectured, however, that the launching efficiency of the wave in the earth via the surface wave is much less than the wave in the earth directly from the antenna; hence the significant difference in signal level. Thus in performing the Focussing process, once again only those target responses whose direct antenna to target path intersects the ground at less than the critical angle need be considered. The hyperbolic arcs of Figures 4.3, 4.4, and 4.5 therefore represent the complete impulse response for clutter reduced data.

It should be noted at this point that there are various techniques available for measuring the relative dielectric constant ϵ_r of the earth, and in fact one such technique has been developed here at the ElectroScience Laboratory by Hayes (1979). However, there is a way of extracting an estimated value of an average uniform dielectric constant from our unfocused data. From Figure 4.7 let the crest of the hyperbolic region be located at depth time T_0 and the distance from the center of the hyperbola to the end of the critical angle region be $n_{crit}\Delta x$ which we shall denote as Δx_{max} . Then just at the critical angle we must have

$$|\Delta x_{max}| = \frac{cT_0}{2\sqrt{\epsilon_r(\epsilon_r-1)}}$$

Figure 4.7. Locus of apparent target locations including lateral wave effect.

from which we can solve for an estimated value of ϵ_r from entirely measurable quantities as

$$\epsilon_r = \frac{1}{2} + \frac{1}{2} \sqrt{\left(\frac{cT_0}{|\Delta x_{\max}|}\right)^2 + 1} .$$

This of course would then yield that the corresponding depth d_0 of the target would be

$$d_0 = \frac{cT_0}{2\sqrt{\epsilon_r}} .$$

C. THE FOCUSSED PROCESS

We shall now present the FOCUSSED process which is a two-dimensional cross-correlation in space and time of the impulse response of an ideal target at various depths due to an ensemble of impulses transmitted along the ground with the actual radar returned data as a function of space and time. The resulting map is then comprised of a two-dimensional correlation function (in space and time) having the usual properties of reduced clutter (in space) and reduced noise (in time). The process is performed on one vertical section at a time and hence will be explained using the idealized vertical cuts previously used. Recall however, that the relative dielectric constant ϵ_r is still an unknown parameter a priori. Consequently for the initial discussion of this process, ϵ_r is assumed to be uniform throughout the ground and is estimated based on other empirical facts. This estimated value of ϵ_r is then used to calculate the shape of the correlation contours previously introduced.

As depicted in Figure 4.8, let us consider an idealized vertical profile map comprised of N waveforms as is typically produced by our system after the clutter reduction process. Using the notation of the previous chapter let us denote each clutter reduced waveform as

$$\underline{v}_n(t_i); \quad n=1,\dots,N$$

corresponding to antenna positions along the ground in the x direction given by

$$x_n = n\delta x; \quad n=1,\dots,N.$$

Recall that t_i are the sample time locations within each waveform. As is illustrated in Figure 4.9 we form a discrete function $f(x_n, t_i)$ containing all of the clutter reduced waveforms $\underline{v}_n(t_i); n=1,\dots,N$, such that the value of $f(x_n, t_i)$ at (x_n, t_i) is the value of the waveform $\underline{v}_n(t_i)$ at the time point t_i ; that is, $f(x_n, t_i) = \underline{v}_n(t_i)$. This provides us with a discrete three dimensional vertical profile map suitable for correlation in space and time whose height above the (x, t) plane represents the magnitude of the returned signal as a function of distance along the ground (x) and depth (time, t). From the discussions associated with Figures 4.4 and 4.5 we recall that we have available, in a sense, an impulse response in space and time from which a test function can be generated to be correlated (or sifted) through the three dimensional vertical profile map of Figure 4.9. In Figure 4.10 we have depicted such a test function. It is a function of time (depth) and space (x) corresponding to distance along the ground. Note that the apparent spreading is due to the fact that we are depicting a three dimensional function projected into two dimensions. The value of this

Figure 4.8. Idealized vertical profile map schematically showing typical hyperbolic response.

Figure 4.9. Idealized vertical profile map producing the discrete function $f(x_n, t_i)$.

Figure 4.10. Test function generated from the ensemble impulse response of an ideal target.

test function is set to unity over the locus of points corresponding to the time location of target responses for a target located at $x = 0$ and for various depths. This test function, denoted as $S(x_n, t_i, T_j)$, is actually a parametric family of functions depending upon the parameter T_j which is defined as the time location of the radar impulse return for the antenna position directly above the target (at $t = 0$). As before, within the x, t plane, the locus of points for which the function is unity (locus of the hyperbolic arcs) is given by

$$\left(\frac{vt_i}{2}\right)^2 = \left(\frac{vT_j}{2}\right)^2 + x_n^2$$

$$\text{for } |x_n| < \frac{cT_j}{2\sqrt{\epsilon_r(\epsilon_r-1)}} \quad .$$

The value of the function is defined to be zero for all

$|x_n| > \frac{cT_j}{2\sqrt{\epsilon_r(\epsilon_r-1)}} \quad .$ The test function is given by:

$$S(x_n, t_i, T_j) = \begin{cases} 1; & \text{for } \left(\frac{vt_i}{2}\right)^2 = \left(\frac{vT_j}{2}\right)^2 + x_n^2 \\ & \text{and } |x_n| < \frac{cT_j}{2\sqrt{\epsilon_r(\epsilon_r-1)}} \\ 0; & \text{for } \left(\frac{vt_i}{2}\right)^2 \neq \left(\frac{vT_j}{2}\right)^2 + x_n^2 \\ & \text{or } |x_n| > \frac{cT_j}{2\sqrt{\epsilon_r(\epsilon_r-1)}} \end{cases}$$

where x_n = offset distance along the ground from the origin
(independent variable)

- T_j = time (indicative of depth) measured from the ground surface corresponding to an actual target location (parameter)

t_i = time (indicative of depth) measured from the ground surface corresponding to possible apparent target locations (independent variable)

ϵ_r = ground relative dielectric constant, estimated and assumed uniform

$$v = \frac{c}{\sqrt{\epsilon_r}} = \text{velocity of waves in earth.}$$

We now form a correlation function $\phi(x_n, t_i)$ in space and time by correlating our test function $S(x_n, t_i, T_j)$ through our original map function $f(x_n, t_i)$ as follows

$$\phi(x_n, t_i) = \sum_{k \geq 0} \sum_{m=0}^N f(x_m, t_k) S(x_m - x_n, t_k, t_i) .$$

"Focussed" waveforms $\phi_n(t_i)$; $n=1, \dots, N$ may then be defined from the discrete correlation function $\phi(x_n, t_i)$ such that the value of each waveform $\phi_n(t_i)$ is the value of the correlation function $\phi(x_n, t_i)$ at the point (x_n, t_i) , that is

$$\phi_n(t_i) = \hat{\phi}(x_n, t_i) ; n=1, \dots, N.$$

A focussed vertical cut map may then be produced from this new ensemble of waveforms in the usual way. We shall now illustrate the benefits of the focussing process in an actual sample map.

Figure 4.11 portrays an ideal plan view of an area of the ElectroScience Laboratory parking lot. Shown are a storm drain and four drain pipes. The 10m by 3m sub-area which was mapped, shown enclosed in dotted lines, contains two pipes coming from the building as shown. Note the origin of the map and the x and y coordinate. Figure 4.12 is a plan view of this sub-area actually produced from this system from raw (un-processed) data. One can notice the darker regions where the two above mentioned pipes are known to be. Figure 4.13 is a representative vertical profile cut produced by the system from raw data. Note once again the origin of the map and the x and depth coordinates. As will be explained later, an estimated dielectric constant has been used to produce a depth scale. The radar responses of the two targets can be seen beginning at the appropriate depth (about 1 m). It should be observed that the target responses consist of three to four echo images in depth. These are due to re-radiation into the ground caused by the transmitted pulse reflecting back and forth on the antenna. Removal of these echos is explained in another section. In this section only the spatial variation of the target is considered; that is, in the x direction. The pipe responses can be seen to be comprised of a dark central region with a tail on either side. The dark central region is the radar response from the target corresponding to antenna/target paths intersecting the ground within the critical angle. A slight hyperbolic shape can be noted. The lighter portion of the tails have a linear shape characteristic of the surface wave effect previously mentioned. Figure 4.14 is the same vertical profile cut after the clutter reduction process has been performed. One notices a general reduction in

Figure 4.11. Ideal plan view of ESL parking lot.

Figure 4.12. Plan view of ESL lot produced by mapping system from raw data.

Figure 4.13. Vertical cut from raw data.

Figure 4.14. Vertical cut from clutter reduced data.

background clutter. Also note that the lighter portion of the target response tails corresponding to the surface wave effect have also disappeared. As was previously pointed out this indicates that the radar returns involving surface waves have a statistically weaker signal level than do the direct signals. It is conjectured that their launching efficiency is significantly less than the direct waves and/or that they are interacting with the rough ground surface. From the width of the target responses evident on this map in the x direction, one may estimate the average relative dielectric constant. For this map the average relative dielectric constant was found to be $\epsilon_r = 17$. Figure 4.15 illustrates the same vertical profile cut after the focussing process has been applied. One notes the increased density and increased strength of the radar signal returns in the regions corresponding to the pipe locations. Also one notes that the inverted hyperbolic shape of the darker regions of Figure 4.14 have been flattened out in Figure 4.15, thus fully compensating for the slant range effect. Finally, a further degree of clutter reduction is noted in Figure 4.15 over Figure 4.14 which is afforded by the correlation nature of the focussing process.

D. ESTIMATION OF GROUND RELATIVE DIELECTRIC CONSTANT - EXTENSION TO STRATIFIED MEDIA

In the preceeding discussion much has been said about the direct path cutoff being related to the critical angle as given by Snell's law. This is a convenient and precise simplification technique used in the Focussing process. Since Snell's law is usually used in conjunction

Figure 4.15. Vertical cut from focussed data.

with optical frequencies and abrupt interfaces, a derivation of a generalized Snell's law from Fermat's Principle is presented for a wave of arbitrary frequency and for a gradual interface (see Tai (1956), Kay (1959), Luneberg (1964)).

Consider a wave traveling in space along a path C as shown in Figure 4.16a, given by the curve $y(x)$ using the usual x and y coordinates. Let the velocity of wave propagation be only a function of the y coordinate as would be true for the case of stratified media and of a gradual air/earth interface. Fermat's Principle states that the time taken for an arbitrary wave to travel along the path C (or even any segment of it) shall be a minimum. The time T required for a wave to traverse the path C is given by the line integral

$$T = \int_C dt = \int_C \frac{ds}{v(y)} \quad .$$

As shown in Figure 4.16a, we have defined the infinitesimal arc length of the curve C as $ds = v(y)dt$. Resolving ds into dx and dy components and letting $y' = \frac{dy}{dx}$ we have

$$\begin{aligned} ds &= \sqrt{dx^2 + dy^2} \\ &= \sqrt{dx^2 + (y')^2 dx^2} \\ &= \sqrt{1 + (y')^2} \, dx \quad . \end{aligned}$$

Figure 4.16. Illustration of the arcs used for the calculus of variations.

Substituting into the integral for T we have

$$T = \int_c \frac{\sqrt{1+(y')^2}}{v(y)} dx .$$

We now represent the integrand as

$$F(y') = \frac{\sqrt{1+(y')^2}}{v(y)}$$

which is explicitly only a function of y' , although the velocity v is a function of y . The integral for T then becomes

$$T = \int_c F(y') dx$$

which we assume to be minimized for some particular curve $y_0(x)$. The Equation for T is referred to as a functional. In the Calculus of Variations (see, for example, Bliss (1925)) the curve which minimizes the functional for T is given by

$$\delta T = 0$$

where δT is referred to as the first variation by the functional for T, formally defined as

$$\delta T(y) = \lim_{\Delta y \rightarrow 0} \frac{[T(y+\Delta y) - T(y)]}{\Delta y} .$$

It can be shown (Bliss, (1925)) that the solution to $\delta T = 0$ is given by the solution to the Euler - Lagrange equation:

$$\frac{\partial F}{\partial y} - \frac{d}{dx} \left(\frac{\partial F}{\partial y'} \right) = 0$$

where F is the integrand in the equation for T . It can further be shown that in a case such as this, where F does not depend explicitly on x , the solution to $\delta T = 0$ is given by the first integral of the Euler - Lagrange equation, namely:

$$F - y' \frac{\partial F}{\partial y'} = K ,$$

where K is an arbitrary constant of integration. In performing $\frac{\partial F}{\partial y'}$, we differentiate with respect to y' explicitly giving

$$\frac{\partial F}{\partial y'} = \frac{1}{v(y)} \frac{y'}{\sqrt{1+(y')^2}} .$$

Substituting the expressions for $F + \frac{\partial F}{\partial y'}$ into the first integral equation and rearranging we obtain:

$$1 = Kv(y) \sqrt{1+(y')^2} .$$

As shown in Figure 4.16b, we now introduce the angle ϕ measured from the horizontal to the tangent of the curve $y(x)$ at the point (x,y) . From the figure then:

$$\tan \phi = \lim_{\substack{\Delta x \rightarrow 0 \\ \Delta y \rightarrow 0}} \left(\frac{\Delta x}{\Delta y} \right) = \frac{1}{y'}$$

therefore

$$y' = \frac{\cos \phi}{\sin \phi} .$$

Using this in the above equation we have:

$$1 = Kv(y) \sqrt{1 + \frac{\cos^2 \phi}{\sin^2 \phi}} .$$

$$\sin \phi = Kv(y) \sqrt{\sin^2 \phi + \cos^2 \phi} .$$

Therefore:

$$\frac{\sin \phi}{v(y)} = K$$

which is a general form of Snell's law applicable to any two arbitrary points on the curve $y(x)$ representing the path of the wave. For example, consider any two points (x_1, y_1) and (x_2, y_2) on the path of the wave as shown in Figure 4.16c. At these two points we define the angles ($<180^\circ$) ϕ_1 and ϕ_2 , respectively, from the vertical to the tangent at that point. Assume the relative dielectric constant in the region of each of the two points is ϵ_{r1} and ϵ_{r2} respectively, so that the local velocities would be

$$v(y_1) = \frac{c}{\sqrt{\epsilon_{r1}}}$$

and

$$v(y_2) = \frac{c}{\sqrt{\epsilon_{r2}}} .$$

Inserting these into the generalized Snell's law we have

$$\frac{\sin \phi_1}{(c/\sqrt{\epsilon_{r1}})} = \frac{\sin \phi_2}{(c/\sqrt{\epsilon_{r2}})}$$

or

$$\sqrt{\epsilon_{r1}} \sin \phi_1 = \sqrt{\epsilon_{r2}} \sin \phi_2$$

namely, the usual form for Snell's law which we have now proven is valid at any two points regardless of the dielectric constant behavior between them.

Now since Snell's law, which relates the angles formed by the vertical and the path tangent at any two points, is valid regardless of dielectric constant behavior between them, it must be true that the concept and value of the critical angle, determined exclusively by the local ϵ_r , must also be valid regardless of dielectric behavior between any two points. Applying this to a representative air/earth gradual interface as shown in Figure 4.17a, the critical angle θ_{2c} at point (x_2, y_2) representing the maximum allowable effective slant range look angle is obtained by letting $\phi_1 = 90^\circ$, $\phi_2 = \theta_{2c}$, and $\epsilon_{r1} = 1$; therefore $\sin \theta_{2c} = \frac{1}{\sqrt{\epsilon_{r2}}}$ as usual. Consequently a determination of the critical angle θ_{2c} at a point will provide an equation for the relative dielectric constant at that point. As is illustrated in Figure 4.17b, the critical angle can be determined from the width and depth of the hyperbolic arcs (on the vertical cuts) within the direct path slant range region, since $\tan \theta_c = \frac{n\Delta x}{d_0}$. But as was mentioned previously, the

Figure 4.17. A representative air-earth gradual interface.

target depth d_0 is unknown; all that is known is its corresponding impulse reflection time T_0 . However the reflection time T_0 depends on the velocity within the medium between the interface and the target, which in turn depends upon the dielectric constant variation between the interface and the target. So then, for determination of the dielectric constant at a point, the dielectric constant between the ground and that point enters into the equation for the critical angle only in the determination of the actual target depth (denoted here as D) as follows:

$$|n_{\text{crit}} \Delta x| = \frac{D(\tilde{\epsilon}_r)}{\sqrt{\epsilon_r - 1}}$$

where D is a function of dielectric constant history denoted as $\tilde{\epsilon}_r$. This simplification enables us to be concerned with the dielectric constant variation in the direct path only and to ignore the more difficult slant range paths which are not necessarily straight lines. In Figure 4.17 let us assume that the target response at time T_0 (corresponding to the still unknown target depth d_0) is the earliest target response in our map corresponding to the shallowest target. Other deeper targets may be present. In light of this, let us begin dealing with stratified media. We shall only deal with discrete layers of relative dielectric constant ϵ_r such that within each layer, ϵ_r is assumed to be uniform. Since this technique makes use of critical angle information gleaned from slant range radar returns, such as that depicted in Figure 4.17, it will be limited to discerning only dielectric layers within which a localized "target" return is received.

This is not much of a limitation, however, because even a dielectric interface itself will frequently present a radar return. Consequently let us consider a piecewise uniform stratified dielectric media as shown in Figure 4.18, where we assume that there are no targets present such as pipes, rocks, etc. Let us further assume that each dielectric interface has produced an ensemble of impulse radar target returns possessing the hyperbolic slant range effect as shown. We envision horizontal dielectric interfaces passing through the crest of each hyperbolic zone thus forming a stratified media formed of an arbitrary number of dielectric layers. Let us assume that the dielectric constant is uniform within each layer and denoted as ϵ_{r1} , ϵ_{r2} , etc., as shown. A process can then be followed to iteratively determine ϵ_{r1} , ϵ_{r2} , etc., successively in the order of increasing depth.

This extension to stratified media has not actually been carried out during the course of this work since the assumption of a homogeneous medium is reasonably good for depths of interest to utilities.

Figure 4.18. Illustration of the focussing for stratified media.

CHAPTER V

TIME PROCESSING OF SIGNALS

Once again, it should be mentioned that the vertical cut map has the greatest potential of rendering to the user the most accurate information regarding objects buried beneath a given area. The most obvious reason for this is simply accuracy in determining target depth. But if a relatively congested area is being mapped, it is typically true that, due to resolution limitations not only in depth but also along the ground, one may not be able to differentiate different targets even at different depths from the plan view alone. This is true because of the two-dimensional collapsing effect of the plan view. In situations such as these, it is frequently necessary to use the vertical cuts in conjunction with the plan view in order to determine the true location and orientation of buried objects.

Let us reconsider Figure 4.15 which shows a vertical profile cut map of a location where two pipes are known to be buried. Recall that the map has gone through the statistical clutter reduction and then the

spatial focussing processes, both described earlier. Once again assuming that the darker areas of the map indicate a target region, ideally one would desire to see two dark circular areas representing the intersection of each of the two pipes with the plane of the vertical cut. However, one notices a series of dark areas in depth corresponding to each of the target locations. The most shallow of the responses actually corresponds to each of the targets at their appropriate depth; the other responses correspond to echoes due to antenna resonance.

A. TIME DOMAIN DISCUSSION

In an earlier discussion referring to Figure 2.15, it was pointed out that, for the system at hand, a narrow (approximately 1 ns) pulse is being transmitted along a relatively long (approximately 4 feet) folded dipole antenna. Consequently, the pulse tends to bounce back and forth on the antenna creating an effective transmitted signal waveform much like that depicted in Figure 2.16. The returned signal waveform therefore has these same oscillatory characteristics as imparted by the antenna. Since the vertical cut is generated by simply displaying the magnitude of the returned signal as a function of time (depth) via the gray scale, all oscillatory characteristics present on the returned signal waveform will therefore appear as echoes in depth for each target depicted on the vertical cut. Of course from the basic impulse radar viewpoint, the depth of a target is determined from the time of arrival of the earliest portion of the returned signal as is indicated by the returned signal waveform amplitude as a function of time

(or depth). This would correspond to the shallowest point of the target, and in fact one would ideally expect to receive a returned signal mostly from the top portion of the target.

So in our quest to eliminate the unwanted antenna resonance and restore the map to a nearly ideal one, one must note that a truly ideal vertical cut would locate mainly the tops of targets. The apparent cross-sectional width of the target on the vertical cut is simply due to the gray scale display of the magnitude of the pulse whose width has been increased by the antenna resonance and other factors relating to underground propagation. Although the echoes can successfully be eliminated, the increase in pulse width is indicative of resolution limitations of the system and in general cannot be improved by off line processing. It should be noted, then, that the width of the target region on the vertical cut simply represents a resolution uncertainty and does not reliably indicate the width or diameter of the actual target. This resolution limitation simply indicates that if two or more targets were present within the width of the target region on the map, they all might appear as one target. The location of the top of the target (or target cluster) should, however, be reasonably accurate. Consequently in removing the echoes due to antenna resonance, the cross-sectional width representing inherent resolution limitations will be preserved.

A notable work performed in the area of elimination of resonances is that of Volakis (1979) where he used the Prony algorithm to find the pole of the antenna resonance and then deconvolved the pole in the time

domain. This approach is more elegant than the technique described herein but was not used because of time and computational limitations. Also, it has been reported that the pole deconvolution technique tends to decrease the signal to noise ratio.

Let us consider a representative returned signal waveform $\phi(t)$ after clutter reduction and focussing as is depicted in Figure 5.1. The vertical cut map shown in Figure 4.15, for example, would typically be produced from an ensemble of signals such as these. We seek to correlate $\phi(t)$ with a test waveform such that the resultant waveform will be free of the effects of antenna resonance. Clearly, if one could somehow predict exactly the nature of the transmitted signal in the vicinity of the target, one could use that as a test signal and expect excellent results. In other words the test waveform should characterize as best as possible a realistic received signal (which is assumed to be a reflection of a realistic transmitted signal) representing all known spurious effects. Let us assume that the transmitted signal as it leaves the antenna has the form

$$e^{-\alpha_a t} \sin \omega_a t u(t)$$

where $\omega_a = \frac{2\pi}{T_a}$ is the antenna resonant frequency and α_a is the attenuation constant along the antenna. Then the idealized returned signal from a point target (neglecting clutter) would be of the form

Figure 5.1. A representative returned signal waveform after clutter reduction and focussing.

$$e^{-\alpha_g T_0} e^{-\alpha_a(t-T_0)} \sin \omega_a(t-T_0) u(t-T_0)$$

where α_g is the attenuation constant of the ground (round trip). That is, it begins at time T_0 (corresponding to the depth of the target) and continues on indefinitely. But as depicted in Figure 5.1, the signal after clutter reduction and focussing is assumed to terminate rather quickly because there will be a point where the signal will decay below the clutter level. In general the duration of the signal is unknown a priori and simply depends on the signal to clutter ratio. Let us arbitrarily say the clutter reduced signal terminates at time T_f , so that the actual returned signal might be represented as

$$\phi(t) = [e^{-\alpha_g T_0} e^{-\alpha_a(t-T_0)} \sin \omega_a(t-T_0)] [u(t-T_0) - u(t-T_f)] .$$

Actually, the exact analytical form of the returned signal is not critical in finding a test signal to extract antenna resonance. All that one really needs to know is the resonant frequency ω_a and the number of cycles that survive clutter reduction. The resonant frequency is, needless to say, easy to determine; but the number of significant cycles is in general impossible to predict a priori. This simply depends on the relative magnitudes of $\alpha_g, \alpha_a, \omega_a$ and the signal to clutter

ratio. Our experience with this system has indicated that one usually receives about two echoes. However, instances have been recorded where as many as five echoes were received. It is safe to say that if a signal comes back at all it will have at least one cycle. So as depicted in Figure 5.2, let us form our test signal $p(t)$ of unit magnitude to exist for only one cycle; namely, a pulse of width $\frac{T_a}{2}$ corresponding to the antenna resonant frequency $\omega_a = \frac{2\pi}{T_a}$. If we now define a new signal $s(t)$ to be the correlation of the test signal $p(t)$ with the square of the clutter reduced signal, i.e.,

$$s(t) = \int_{\tau=0}^{\infty} \phi^2(\tau) p(\tau-t) d\tau$$

as shown in Figure 5.3, we obtain a signal free from antenna resonance. Then a simple peak detecting process is applied which produces a pulse much narrower in time duration. Figure 5.4 shows the vertical cut map of Figure 4.15 after this antenna resonance elimination process has been applied. Its effect on improving the map clarity is obvious. Figure 5.5a shows an actual returned signal after clutter reduction and focussing where two targets are present. Figure 5.5b shows the same signal after the antenna resonance eliminator has been applied. Figure 5.6 shows the corresponding plan view of Figure 4.12 after full processing including the antenna resonance elimination. One notes that the plan view is also greatly improved in that the definition of the pipes is much clearer.

Figure 5.2. Test signal used to eliminate antenna resonance.

It should be noted once again that the apparent cross-sectional width of the targets of Figure 5.5 are more indicative of resolution uncertainty than of actual target width. As mentioned before, this is due to the narrow band effect of the antenna resonance.

B. FREQUENCY DOMAIN DISCUSSION

So far the effect of the antenna resonance has been discussed from the point of view of transmit pulse reflections along the antenna of period T_a . If instead we consider the antenna to be a resonant structure with a resonant frequency $\omega_a = \frac{2\pi}{T_a}$, it can then be considered to be effectively a bandpass filter centered at ω_a in series with the transmitter. This then would ensure all returned signals $\phi(t)$ to have a frequency spectrum $\phi(\omega)$ with a peak at ω_a , as portrayed in Figure 5.7. We wish to analyze the previously mentioned antenna resonance eliminator in the frequency domain. Let us denote the Fourier Transform of a signal $f(t)$ as:

$$F(\omega) = \mathcal{F}[f(t)] = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

Let us also denote the processes of convolution and correlation as

Figure 5.3. Schematic illustration of correlation of $p(t)$ with $\phi^2(t)$.

Figure 5.4. Vertical cut map after removal of antenna resonance.

a). Actual signal after clutter reduction and focussing.

b). Actual signal after removal of antenna resonance.

Figure 5.5. Illustrations of the antenna resonance removal.

Figure 5.6. Plan view after antenna resonance removal.

Figure 5.7. Schematic representation of a typical frequency spectrum of a returned signal.

$$f_1(t) * f_2(t) = \int_{-\infty}^{\infty} f_1(\tau) f_2(\tau - t) d\tau$$

$$f_1(t) \times f_2(t) = \int_{-\infty}^{\infty} f_1(\tau) f_2(\tau + t) d\tau$$

respectively, whose Fourier Transforms are

$$[f_1(t) * f_2(t)] = F_1(\omega) F_2(\omega)$$

$$[f_1(t) \times f_2(t)] = F_1^*(\omega) F_2(\omega)$$

respectively, where $F_1^*(\omega)$ denotes the complex conjugate of $F_1(\omega)$. It should be noted that if one or both of the time functions are even, convolution and correlation in the time domain are then identical.

Since the inverse Fourier Transform is given by

$$f(t) = \mathcal{F}^{-1}[F(\omega)] = 1/2\pi \int_{-\infty}^{\infty} F(\omega) e^{j\omega t} d\omega$$

by symmetry then

$$\mathcal{F}[f_1(t) f_2(t)] = 1/2\pi F_1(\omega) * F_2(\omega) .$$

Recall that the signal with antenna resonance removed was defined as:

$$s(t) = \int_{-\infty}^{\infty} \phi^2(\tau) p(\tau-t) d\tau$$

or

$$\begin{aligned} s(t) &= \phi^2(t) * p(t) \\ &= [\phi(t)\phi(t)] * p(t) . \end{aligned}$$

Consequently its Fourier Transform is

$$S(\omega) = \mathcal{F}[S(t)] = [1/2\pi \phi(\omega) * \phi(\omega)] P(\omega)$$

where if $p(t)$ is a pulse of width $\frac{T_a}{2}$ centered at the origin then

$$P(\omega) = \frac{\sin \frac{\omega T_a}{4}}{\frac{\omega T_a}{4}} .$$

Figure 5.8 shows the two significant parts of $S(\omega)$ which gives us insight as to how the antenna resonant frequency is eliminated. Note that during the self-convolution of $\phi(\omega)$ a relative null is placed at the antenna resonant frequency ω_a , but a secondary maximum is placed at $2\omega_a$ corresponding to the absolute value process in the time domain. This local maximum is then reduced by the multiplication by $P(\omega)$ which

Figure 5.8. Schematic showing the frequency spectrum of the signal $s(t)$ as derived from a product.

has a null at $2\omega_a$. The resultant frequency spectrum, shown in Figure 5.9, then has most of its energy concentrated between 0 and $2\omega_a$ with no major local maxima.

The exact fine details of amplitude and phase of the frequency spectra for each $\phi(t)$ of course is unique and is not considered in this schematic discussion; however, they are pretty much preserved. The basic feature of the filter is then to simply increase the lower frequency components at the expense of the resonant frequency.

It should also be noted that the final frequency spectrum is Gaussian in shape as is the frequency spectrum of the ideally transmitted Gaussian pulse. We thus have another indication of how the processing approaches the idealized impulse radar.

Figure 5.9. Final frequency spectrum of signal after antenna resonance removal.

CHAPTER VI

CASE STUDIES

This section presents the resulting maps at various locations that were mapped. They illustrate how the various degrees of processing affect the target responses for different target configurations, depths and soil conditions.

A. CASE 1 - KIOKA AND EASTCLEFT, UPPER ARLINGTON, OHIO

Figure 6.1 is a copy of the city engineer's drawing of the area mapped (shown boxed in). It includes an eight inch water line about one meter deep and a twelve inch storm sewer about one half meter deep. The effects of processing are seen best on the depth profile cuts; hence only one vertical cut is considered. Figure 6.2 depicts an idealized vertical profile cut, which was drawn from an engineer's drawing of an actual city street. Known to be present are an eight inch water line and a twelve inch storm sewer at the depths shown. Figure 6.3 is the vertical profile cut produced by the system from raw data. Weak

Figure 6.1. City Engineer's drawing.

Figure 6.2. Idealized vertical cut.

Figure 6.3. Vertical cut from raw data.

responses are noted in the target region. Figure 6.4 is the vertical profile cut produced from data which has been clutter reduced. A much more noticeable target response is now present, but the effects of slant range and antenna resonance are still present. Figure 6.5 represents the vertical profile cut produced from data focussed to correct for the slant range effect. Notice that the inverted hyperbolic target responses have now become somewhat flatter. Figure 6.6 is the final stage of processing for the vertical profile cut. The antenna resonance has been extracted, and now the target response corresponds perfectly with the idealized vertical cut of Figure 6.2.

Figure 6.4. Vertical cut after clutter reduction.

Figure 6.5. Vertical cut after focussing.

Figure 6.6. Vertical cut after antenna resonance removal.

B. CASE 2 - ELECTROSCIENCE LABORATORY PARKING LOT

Figure 6.7a is a schematic drawing representing an idealized plan view of the four pipes going to a storm grating in the parking lot of the ElectroScience Laboratory. Figure 6.7b is an idealized vertical cut along A-A and Figure 6.7c is an idealized vertical cut along B-B showing the pipes at their corresponding depths. The pipes are plastic corrugated drain pipes and concrete sewer pipes. Figure 6.8 is a plan view of the same area produced by the mapping system from raw data. Notice that the storm drain grating dominates the map and in fact resonates with the antenna structure, thereby making it statistically incompatible with the true radar reflections from the pipes. Hence, these signals from the grating were set to zero after the data had been collected. Figure 6.9a is a plan view from the raw data after the storm grating was eliminated. Notice that the storm grating is now gone. Figure 6.9b is a plan view after clutter reduction and focussing. We note that the pipe responses appear where they are expected to be. Figure 6.9c is a plan view after total processing including antenna resonance removal. Note that the grating has been sketched in by hand.

Figure 6.10a is a vertical cut along A-A produced from raw data. Note the resonance of the storm grating. Figure 6.10b is a vertical cut along A-A after clutter reduction and focussing. Figure 6.10c is a vertical cut along A-A after total processing including antenna resonance removal. Note that the pipes appear reasonably well when compared with the ideal of Figure 6.7b.

Figure 6.11a is a vertical cut along B-B produced from raw data. Figure 6.11b is a vertical cut along B-B after clutter reduction and focussing. Figure 6.11c is a vertical cut along B-B after total processing including antenna resonance removal. Note that the pipes compare well with the ideal of Figure 6.7c.

a). Plan view.

Figure 6.7. Idealized maps of the ElectroScience Laboratory parking lot.

b). Vertical cut along A-A.

Figure 6.7. (Continued).

c). Vertical cut along B-B.

Figure 6.7. (Continued).

Figure 6.8. Plan view from raw data.

a). Plan view after storm grating was eliminated.

Figure 6.9. Set of plan views produced by the mapping system.

b). Plan view after clutter reduction and focussing.

Figure 6.9. (Continued).

c). Plan view with total processing.

Figure 6.9. (Continued).

a). Vertical cut along A-A from raw data.

Figure 6.10. Set of vertical cuts along A-A.

b). Vertical cut along A-A after clutter reduction
and focussing.

Figure 6.10. (Continued).

c). Vertical cut along A-A after total processing.

Figure 6.10. (Continued).

a). Vertical cut along B-B from raw data.

Figure 6.11. Set of vertical cuts along B-B.

b). Vertical cut along B-B after clutter reduction
and focussing.

Figure 6.11. (Continued).

c). Vertical cut along B-B after total processing.

Figure 6.11. (Continued).

C. CASE 3 - HAMILTON AVE. AND E. 49th ST., CLEVELAND, OHIO

Figure 6.12 shows an engineer's drawing of a plan view of the site. The exact area that was mapped is contained in the shaded region. The area contains an abandoned twelve inch storm sewer, a forty-eight inch water line, a twenty-four inch gas line, an eight inch electric pipe type cable, and two electric ducts leading to a manhole casing.

Figure 6.13 is an engineer's vertical section along B-B illustrating how the two electric ducts go around the water line and enter the manhole casing. Figure 6.14 is an engineer's vertical section along A-A showing the depth location of the water line, gas line, storm sewer, electric cable pipe and the two electric ducts. Figure 6.15a is the vertical cut along A-A produced by the mapping system from raw data. Figure 6.15b is the vertical cut along A-A after clutter reduction. Target presence is now becoming evident. Figure 6.15c is the vertical cut along A-A after focussing (in two orthogonal directions) in which the antenna resonance is quite evident. Figure 6.15d is the vertical cut along A-A after antenna resonance removal. One notes that all the targets appear at their expected location with the exception of the 48" water line which fades heavily. It is conjectured that the diameter of the water line is large enough that the polarization of the electric field is not rotated significantly so as to couple to the receiver antenna.

Figure 6.16a is a plan view produced from raw data. Figure 6.16b is a plan view after clutter reduction. Figure 6.16c is a plan view after two orthogonal focussings and Figure 6.16d is a plan view after

total processing. One notes that responses from all the targets are present but because of the congestion of targets it is impossible to differentiate between the radar responses of each of the targets. When used, however, in conjunction with vertical cuts such as those of Figure 6.15, we are able to locate all the pipes.

IDEAL PLAN VIEW OF MAPPED AREA

FIGURE 3

Figure 6.12. Engineer's drawing of plan view.

SECTION "B"-"B"

IDEAL VERTICAL SECTION SHOWING SPLIT
IN ELECTRICAL DUCT
FIGURE 4

Figure 6.13. Engineer's drawing of vertical cut along B-B.

IDEAL VERTICAL SECTION "A"—"A" FROM FIGURE 2
FIGURE 4

Figure 6.14. Engineer's drawing of vertical cut along A-A.

a). Vertical cut along A-A from raw data.

Figure 6.15. Set of vertical cuts along A-A.

b). Vertical cut along A-A after clutter reduction.

Figure 6.15. (Continued).

c). Vertical cut along A-A after focussing.

Figure 6.15. (Continued).

VERTICAL SECTION "A" - "A"
PRODUCED BY MAPPING SYSTEM

d). Vertical cut along A-A after total processing.

Figure 6.15. (Continued).

a). Plan view from raw data.

Figure 6.16. Set of plan views produced by the mapping system.

b). Plan view after clutter reduction.

Figure 6.16. (Continued).

c). Plan view from after focussing.

Figure 6.16. (Continued).

d).. Plan view from total processing.

Figure 6.16. (Continued).

CHAPTER VII

SUMMARY AND CONCLUSIONS

This research was directed toward two major goals. The first sought to demonstrate that an impulse radar combined with a computer could be used to produce maps of underground objects. The second goal was the building of a self-contained system, including radar and computer mounted in a van. Both of these objectives have been accomplished.

The original motivation for this work was the successful construction of a portable pipe locator (now known as Terrascan) at the ElectroScience Laboratory. This portable device consisted of a handheld antenna and a battery operated radar. The display was similar to a radar A-scope. The idea was to add a computer to this device which would store the various A-scope responses, keeping track of where they were recorded, and then print a map by plotting the amplitude of the response versus the position at which it was recorded.

This concept resulted in the development of the prototype system. The radar remained very similar to that used in the pipe locator with the difference that the antenna was mounted on wheels rather than being handheld. This permitted the computer to sense the motion of the antenna. The first maps were produced by pushing this antenna back and forth over the ground and plotting the rms value of each waveform as a function of antenna position. The amplitude was represented by means of a grey scale; the larger the amplitude the darker the printing on the map.

These early maps were very shadowy in appearance. Reflections from pipes were mixed with reflections from the ground surface, reflections on the antenna structure and the effects of ground inhomogeneities. It was obvious that much more data processing was necessary if maps of good quality were to be produced.

The first step in improved data processing was to normalize all the reflected signals to their statistical average at the same depth. This had two benefits. Statistically insignificant signals were ignored, thus greatly reducing the clutter problem, and the depth dependence of the reflected signals was removed from the map. The map quality was greatly improved, but it became apparent that if much more data processing was to be done, greater computer capacity would be needed, both in terms of memory and speed. This problem was solved for the time being by using the minicomputer for data recording only. The data was then transferred to a larger computer for processing and printing of the map.

In time a number of other processing steps were added to the computer programs. Perhaps the most significant of these was the focussing algorithm which compensates for the apparently greater depth of a target when the antenna is not directly above it and looks at it obliquely. A side benefit of this focussing process is the ability to determine depth. The radar measures only transit time. The velocity of propagation, and hence the depth corresponding to a given time, depends on the local permittivity which is only known approximately. The focussing algorithm calculates this local permittivity by matching the range of an object when viewed obliquely with that calculated from the geometry.

After further development of a number of these processes, it was possible to produce clean looking maps of objects at depths down to about 2.5m (8 feet) in the local soil conditions. Greater or lesser depths might be achieved in other soils. Any further increase in depth was limited by the dynamic range of the radar receiver. This then was one limitation of the prototype system. Other limitations included the use of a separate computer for data processing and the fact that two passes must be made over an area to be sure that all pipe and cable targets had been seen. The latter was a consequence of the crossed dipole antenna used in the radar.

A. SUGGESTIONS FOR FURTHER PROCESSING

Most of the processing to date has been time domain amplitude processing and has concentrated on compensating for, or eliminating,

physical effects. Also, they tend to concentrate on localized phenomena. These techniques have been largely successful but have proven inadequate in dealing with such problems as a pipe of large diameter, or a map where the signal from a long pipe will fade in certain parts of the map. Also, shortcomings still remain in dealing with weak target returns in the presence of strong returns and with resolution limitations created by the very short transmit pulse resonating on the antenna. Consequently, new processing is suggested on the following areas:

1. Time Domain Amplitude Processing

The work already progressing should be continued. Some details of the statistical clutter reduction need refining in addition to some non-linear clipping techniques to deal with the dynamic range problem. It is envisioned that this should also be done statistically. A statistical technique has already been developed to compensate for the attenuation of the ground, but it was never fully implemented. This work needs to be completed.

2. Image Processing

A logical extension of the time domain amplitude processing dealing with physical phenomena would be large scale global correlations. This would tend to accentuate pipes or targets that exist in larger areas of the map. This would compensate for the fading of targets which is

probably due to effects of localized clutter and localized moisture variation. Some work in refining the images of the vertical sections has already been done. This work needs to be evaluated and possibly implemented.

3. Modeling and Cross-correlation Techniques

Problems such as those mentioned with regard to large cylindrical pipes and fading need to be studied as part of a highly controlled experiment. Since most of the successful processing so far has been of the type that is matched to expected classes of signal returns, we need to determine the exact nature of an ideal target return for such targets as the large diameter pipe or multiple target situations. Here one would have a choice of either burying targets of known type in a relatively ideal environment or to model these configurations on the computer. It is envisioned that modeling on a large digital computer would provide maximum flexibility and will probably be quicker and cheaper in the long run.

Once the basic ideal returned signal type is determined, one can then proceed to broaden the matched filter concepts that have so far been reasonably successful for simple targets and target configurations. However, another avenue of processing has as yet remained untried. That is cross-correlation techniques. Various techniques exist employing Bayes decision criteria to essentially cross-correlate a repertoire of ideal expected target returns through the data. As in the focussing processing that already works successfully for small targets, one

essentially develops an ensemble impulse response in the region of the expected target. Also available, of course, are single signal correlation techniques.

4. Frequency Domain Processing

Although we have developed Fast Fourier Transform (FFT) techniques on the system already, we have not made significant use of its capabilities largely because of speed and data storage considerations. One would like to make extensive use of the FFT by transforming all of the data into the frequency domain. This would give us greater accuracy in extracting the effects of antenna resonance. Also there is the possibility of performing target identification studies statistically in the frequency domain which would augment any success that might be achieved in the cross-correlation techniques already mentioned.

5. The Focussing Process

The focussing process described herein was limited to modelling the earth as a lossless dielectric. In conjunction with the statistical derivation of the ground attenuation, one could extend the focussing to a lossy dielectric model.

APPENDIX A

DESCRIPTIONS OF THE MAIN COMPUTER MAPPING PROGRAMS

The software is organized in a modular building-block fashion. The main section of the programs are divided into subsections, each of which is executable on command. Each processing, recording or plotting function is performed by a suitable subroutine.

A. DATA TAKING COMMANDS

- SMAP Start map.
Enter map length in X and Y.
Enter waveform interval in X and Y.
Enter number of polarizations.
Enter disc drive logical unit number.
Enter number of nanoseconds per cm on oscilloscope sweep.
Enter number of mV per cm on oscilloscope vertical amplitude.
Enter the point on the oscilloscope sweep corresponding to ground level.
Enter the number of repeat waveforms to be averaged at each location.
Enter disc identification number.
Enter number of good tracks on disc.
Enter shaft encoder degrees per 10 m of travel.
- DUMP Dump map identification parameters on disc track 0.
- CMAP Continue map.
Retrieves map identification parameters used to resume mapping after an intermission.
- GO Commands the computer to begin recording waveforms. Used at the beginning of each pass in Y.

SETA Set attenuator.
Fixes the receiver rf attenuators constant as a function of depth.

MAN Manual stepped attenuator setting.
Allows predetermined attenuator setting for each time zone.

AUTO Automatic stepped attenuator setting.
Allows attenuation to be changed for each time zone as the waveforms are recorded.

SWEP Sweep oscilloscope.
Sweeps the sampling oscilloscope any number of times for test and calibration purposes. Will operate under all three attenuator modes.

PRID Print.
Prints out the numerical values of the waveform obtained during SWEP.

VOLT Calibrates the maximum voltage on the shaft encoder potentiometer.

CAL Calibrates the shaft encoder gear ratio.
A 10 m run is made and the program computes the correct ratio.
It is not necessary to do this calibration unless the cart gearing is changed. The calibration is predetermined during the SMAP operation (variable NDTM).

The sequence of events taking place during a data recording sequence is shown in the flow chart in Figure B.1. These events take place in response to the GO command or pressing of the start button.

The hierarchy of the various subroutines used by the Data Taking portion of the program is shown in Figure B.2.

B. DATA PROCESSING COMMANDS

DISK Enter logical unit number of disc from which data is to be read.
Enter logical unit number of disc to which data is to be written.

INI Initialize.
Read the map identification parameters from the "read" disc.
Also sets default map boundaries to the full map area.

XYB X Y Boundaries.
The boundaries may be set to any subsection of the map.
(Default is the full map)

- GAT Range Gate.
Allows any time interval of the waveform to be processed.
(Default is the full waveform)
- UNAT Unattenuate
Used to remove attenuator values if not done during data taking.
(Default is that attenuator values are removed during data taking)
- PCOM Pulse Compress.
For spiral antenna data only. Compensates for the frequency dispersion characteristics of the spiral antenna. Its effect is to compress the waveform in time.
- RCL Reduce Clutter.
Sets up the statistics used for clutter reduction and then proceeds to remove the ensemble average and clutter.
- XFO Focus in the X direction.
The process compensates for the spatial dispersion effect caused by the antenna's effectively large beamwidth. It correlates adjacent waveforms in the X direction and replaces each waveform with one corrected for this dispersion.
- YFO Focus in the Y direction.
Similar to XFO except that adjacent waveforms are taken in the Y direction.
- RNG Ring Eliminator.
Removes the multiple target effect due to "ringing" of the transmitted pulse on the antenna.

The hierarchy of the subroutines used in data processing is shown in Figure B.3.

C. MAP DISPLAY COMMANDS

- PLN Plan View.
Produces a plan view of the area of interest. The section to be displayed can be generated by the XYB and GAT commands to produce partial map or plateau plan views in depth.
- YCT Depth cut for one value of Y.
Produces profile depth cuts as a function of the X coordinate. This also can be governed by the XYB and GAT commands. The default is one map for each pass in Y for the full X range and full depth range.

D. WAVEFORM PLOTTING COMMANDS

INI Reads map identification parameters from the disc.

DISK Enter the logical unit of the disc drive.

MPON Multiplot "on".
Enter the maximum and minimum amplitudes (defaults to those obtained during MXMN). Plots axes.

MDRW Multiplot draw.
Enter Y coordinate, polarization number, X starting and stopping points, X increment. Plots up to 18 waveforms on axes drawn by MPON, each labeled with its X coordinate.

MXMN Find maximum and minimum.
Enter Y coordinate, polarization number, X starting and stopping points, X lincrement. Determines the maximum and minimum amplitudes of all waveforms considered.

PON Plot "on".
Enter maximum and minimum amplitude (defaults are those obtained by MXMN), X axis label, Y axis label. Plots labeled axis.

DRW Draw wave.
Enter X and Y coordinates, polarization number. Plots desired wave on axes drawn by PON. Up to six waves can be drawn by this command on one plot.

ERAS Erase.
Enter the number (1 to 6) of the wave plot to be erased from those plotted by PON and DRW.

DRS Draw standard deviation.
Plots standard deviation waveform stored on disc. Use PON before this command to give axes for this plot.

DRA Draw ensemble average.
Plots ensemble average waveform stored on disc. Use PON before this command to draw axes.

D. PRINCIPAL VARIABLE NAMES USED BY THE MAPPING PROGRAMS

IDIM number of points per waveform
NTZ number of time zones per waveform (for attenuation)
NTP IDIM/NTZ that is the number of points per time zone
NDIM $9 \times \text{NTP} + 1$
MDIM $11 \times \text{NTP} + 1$
ID(IDIM), JD(IDIM), KD(IDIM) are temporary integer waveforms
MV(IDIM) array containing millivolt settings used to sweep oscilloscope
IB1(NDIM) series of data read and sweep commands
NCOUT(NTZ) number of characters returned by INTAS
IATTN (NTZ) contains attenuation settings for each time zone
KV(128) holds 128 critical integer values used for program maintenance
IB2(MDIM,NTZ) actual ASCII series of commands sent to HP-IB
NVAR number of variables used in KV
IDX distance between waveforms in X direction in cm
IDY distance between waveforms in Y direction in cm
IY Y distance point counter
IX X distance point counter
NPO total number of polarizations employed
IXCM current X distance in cm
NDX total number of intervals in the X direction
NDY total number of intervals in the Y direction
NFL file number of waveform storage
ITPO time point on the waveform corresponding to ground level
IPO polarization counter (for multipolarization maps)
NSCM number of nsec/cm on the oscilloscope sweep
MVCN number of mV/cm on the oscilloscope amplitude
IAUTO attenuation function indicator; 0 = fixed constant attenuation,
-1 = fixed stepped attenuation, 1 = automatic stepped attenuation
NDTM number of degrees of rotation of the shaft encoder for 10 m travel
IDENT disc identification number
IUPP upper threshold for automatic attenuation
ILOW lower threshold for automatic attenuation
AJ and AI array containing waveform
EAVS ensemble average of waveform squared
EAV ensemble average of waveform
SIG standard deviation
INCR direction flag for pass, positive forward, negative return
IPS pass counter
NRPT number of repeats in reading wave while data taking
ISET attenuator setting if whole wave is set to one attenuation level
IWDSK logical unit number of disc where data will be written
IRDSK logical unit number of disc from which data will be read
NPT number of points in a time zone (= 32)
IST starting time point
ISP stopping time point
EMAX=SMAX=AMAX maximum value in waveform
EMIN=SMIN=AMIN minimum value in waveform
JMAX time point where maximum value occurred

JMIN time point where minimum value occurred
ITPO time point corresponding to ground surface (= 13)
ISUS flag to tell if statistics have been done
IXST starting X coordinate
IYST starting Y coordinate
IPST starting polarization
DMIN minimum depth in meters
DMAX maximum depth in meters
NSMO number of smooths
ER permittivity of soil

E. MAPPING PROGRAM SUBROUTINES

The subroutines described in this section are written in Fortran and their source listings are appended to the main programs that use them; thus main programs and subroutines are compiled together. Unless otherwise given with the individual subroutine description, the variable names used are as described in Section D. In the various call statements, arguments which are outputs from the subroutines are underlined, the remainder are inputs.

1. Subprogram GENIB

Fortran: CALL GENIB (IDIM, NTZ, JIB, IB1, IB2, NDIM, MDIM, IDBG, NCOUT, MV)

Use: Creates an ASCII array of commands to sweep the oscilloscope through four different time zones and reads the data from the oscilloscope while it is sweeping.

2. Subprogram WAVE

Fortran: CALL WAVE (ID, IDIM, NTZ, IB2, MDIM, NCOUT, IDBG, IAUTO, IUPP, ILOW, IATTN)

Use: Sweeps the oscilloscope for the four different time zones,
adjusting the attenuation prior to each time zone.

3. Subprogram SCOPE

Fortran: CALL SCOPE (ID, IDIM, MDIM, NTZ, NCOUT, NT, IST, ISP, IDBC)

Use: Issues command to sweep the oscilloscope for each time zone.

4. Subprogram LITE

Fortran: CALL LITE (IN)

Use: Controls the red, yellow and green control lights, and the beeper.

5. Subprogram FATTN

Fortran: CALL FATTN (AI, NTZ, IDIM, IATTN, IUPP, ILOW, IDBG)

Use: Determines attenuator settings to be applied to the next time
zone.

6. Subprogram UNATT

Fortran: CALL UNATT (AI, IDIM, NTZ, IATTN, MVCM)

Use: Removes attenuation from waveform

7. Subprogram ATTEN

Fortran: CALL ATTEN (INN, IDBG)

Use: Sends actual command to change the setting of the programmable
attenuator.

8. Subprogram WDISK

Fortran: CALL WDISK (IDENT, IX, IY, IPO, AI, IO, NFL, IWDSK, IDIM)

Use: Packs floating point data into integer array. Then stores X coordinate, Y coordinate, polarization number, file number, and identification code into first five elements of the integer array. The integer array is then written on the disc.

9. Subprogram FILNM

Fortran: CALL FILNM (IX, IY, IPO, NPO, NDX, NFL)

Use: Finds file number of a waveform.

10. Subprogram TRSEC

Fortran: CALL TRSEC (NFL, ITRK, ISEC, IDIM)

Use: Finds track and sector location of a given file number.

11. Subprogram SHAFT

Fortran: CALL SHAFT (IXCM, JXCM, IDX, IDBG, NDTM, IDIR)

Use: Determines actual location in centimeters from start of pass.

12. Subprogram CVOLT

Fortran: CALL CVOLT (IDBG)

Use: Calibrates shaft encoder to find maximum voltage.

13. Subprogram CSHAF

Fortran: CALL CSHAF (NDTM, IDBG)

Use: Determines how many degrees the shaft encoder rotates in a ten meter pass.

14. Subprogram POT

Fortran: CALL POT (ANG, INIT, IBUT, IDBG)

Use: Does the actual reading of the voltage from the shaft encoder and determines the angle of rotation relative to that at beginning of pass.

15. Subprogram ERR

Fortran: CALL ERR (NUM)

Use: Error message subroutine. Indicates in which subroutine an HP-IB error has occurred.

16. Subprogram MSSGE

Fortran: Miscellaneous message subroutine. Tells operator of problems or things he must do before the command he has attempted to give.

17. Subprogram RDISK

Fortran: CALL RDISK (IDENT, ID, AI, IDIM, IX, IY, IPO, IRDSK, NPO, NDX,
NFL)

Use: Reads waveform from disk, checks identification codes stored in first five positions of waveform (X coordinate, Y coordinate, polarization number, file number, identification code) by comparing them to those of waveform desired. If these do not match, an error message is written. Otherwise the data is unpacked and put into a real array.

18. Subprogram SMOTH

Fortran: CALL SMOTH (AI, AJ, IST, ISP, NSMO, IDIM)

Use: Smooths waveform making it less jagged.

19. Subprogram INTRP

Fortran: CALL INTRP (AI, AJ, IDIM, N1, N2, IDX, IDY)

Use: Interpolates data for making plan views.

N1: Initial number of points

N2: Number of points after interpolation

20. Subprogram FOCUS

Fortran: CALL FOCUS (AI, DXY, ER, ID, ICODE, IPO, ITPO, IXYST, IXYS,
IYX, NDX, NDX, NPO, IDBG, LUB, NSCM, IRDSK, IWDSK, IAV,
DEPMX, IDENT)

Use: Compensates for the inverse hyperbolic effects present in the maps which are due to the slant range from the antenna to the target. Its operation is to sum along hyperbolic contours which represent the locus of the target as would be seen by the antenna in the region of the target.

DXY	Distance between waveforms to be focussed. Can be in X or Y direction.
ICODE	Flag to indicate whether X focus or Y focus.
IXYST	Starting location for focussing.
IXYSP	Stopping location for focussing.
IYX	Particular pass to be focussed.
NDXY	Number of waves in direction to be focussed.
IAV	Indicates whether sums or averages are used.

21. Subprogram GRAPE

GRAPE represents one dimensional arrays of real numbers as lines of 16 level gray scale characters printed directly to paper by a HEWLETT PACKARD 2631G printer.

GRAPE prints a line of up to 122 gray scale characters which are formed in an 8x8 dot cell of printer dots. The characters are printed from left to right across the top of the page. A subset of the input array may be plotted by specifying the subscripts of the first and last elements in the array. The program is called repeatedly, once for each line of data to be represented. The absolute value of the input data is plotted. Error checking is included which looks for input data that exceeds the limits set in the calling window. If an element exceeds the limits, it is set equal to the limit (max or min) that it tried to exceed, an error message is printed on the terminal and the error flag is set to alert the calling program.

SUBROUTINE PARAMETERS:

RA : Input array of real numbers. The data to be plotted.
IFIRST: Subscript of the first element of RA to be plotted.
ILAST : Subscript of the last element of RA to be plotted.
RMAX : Maximum value assumed by any element in the input array.
RMIN : Minimum value assumed by any value in the input array.
INFL : Initialization flag. 1 = initialize.
IERFLG: Error occurrence flag. 1 = error was detected.

CALLING PROCEDURE:

- 1: Call GRAPE with the initialization flag set.
- 2: For each 'line' (array really) of data to be plotted call GRAPE with the subroutine parameters set appropriately. The lines of gray scale characters are not separated by any space.

22. Subprogram GRAY SCALE

GRAY SCALE represents a two dimensional array of real numbers as a two dimensional matrix of gray scale characters displayed on a HEWLETT PACKARD 2648A graphics computer terminal. The characters vary in reflectivity with the absolute values of their magnitudes. The characters are similar to normal characters; they differ in that they convey information with density instead of form.

GRAY SCALE accepts one row of the aforementioned 2D array of numbers at a time (one row per call after the initialization call). When all of the 2D array has been received, GS plots it on the screen as gray scale characters.

Two character sets can be used. The main difference between the two is their size. One size is formed from an 8x8 matrix of graphics dots while the other (half as small) is formed in a 4x4 matrix of dots. The two sets are similar in that they both display 16 different levels

of gray. Input elements of high magnitude are displayed as being whiter (brighter) than elements of less magnitude. The size of the characters limits the number of them that can be displayed on the screen of the 2648A to 16200 in a 180x90 matrix for 4x4 dot characters, and 4050 in a 90x45 matrix for 8x8 dot characters. The actual number of characters may be less than the maximum if the user so specifies.

Data may be displayed in one of two major modes, horizontally or vertically.

When horizontal plotting is selected, each input line of data is plotted as a horizontal row of gray scale characters. The input arrays are plotted from the top of the screen down, with the first element of each line plotted on the left.

Vertical plotting plots each input array as a vertical column of gray scale characters. The arrays are plotted from right to left with the first element of each line plotted at the top.

The length (in characters) of each plotted line and the number of those lines is user selectable.

If the matrix of gray scale characters is less than the screen size of the 2648A, then it can be positioned (with restrictions) anywhere on the screen.

The calling name of this subroutine is GSAR.

CALLING PARAMETERS:

INITFL : Initialization flag. 1 = initialize
IVHFLG : Plotting orientation select.
 >0 = horizontal plotting, <0 = vertical plotting.
IFIRST : The subscript of the first element in the input array to
 be plotted.

ILAST : The subscript of the last element in the input array to be plotted. The IFIRST and ILAST allow a subset of the input array to be plotted.
 IBOX : The size (in dots) of the individual gray scale character. This is equal to 8 or 4
 IORGX : The x coordinate of the block of gray scale characters on the 2548A screen.
 IORGY : The y coordinate of the block of gray scale characters on the 2648A screen.
 INMLIN: The number of lines of data to be plotted as lines of gray scale characters.
 RMAX : The maximum value assumed by any element of the input array (or the subset of it being used).
 RMIN : The MINimum equivalent of RMAX.
 RA : The input array of real numbers to be plotted as a line of gray scale characters.
 IERFLG: The error indication flag. 1 = error occurred.
 1: Call GSAR with the initialization flag set.
 2: Call GSAR once for each line of data to be plotted as a line of gray scale characters (call it INMLIN times).

ERROR TRAPPING

GRAY SCALE checks the input data before and during plotting. When an error is detected, an error message is written to the terminal (usually) and GRAY SCALE returns (usually) to the calling program with an error flag set.

MAX and MIN errors:

If an element in an input array exceeds the MAXimum set by RMAX in the calling window or an element in an input array is smaller than the MINimum set by RMIN in the calling window the offending element is set equal to the limit that it tried to exceed and the error flag is set.

ORIGIN ERRORS:

The calling program specifies where on the screen of the terminal to put the block of gray scale characters by passing absolute x and y coordinates.

There are restrictions on the origin that may be chosen which depend on the orientation of the graph (horizontal or vertical) and the size of the block of gray scale characters to be put at that origin.

Horizontal plotting requires an x coordinate (of the origin) to be either a multiple of the character cell size or zero and a y coordinate that is a non-zero multiple of the character cell size.

Vertical plotting requires x and y coordinates that are (non-zero multiples of the character cell size - 1).

If the above conditions are not met, an error message is written to the terminal and the program returns to the calling program with the error flag set.

SCREEN OVERFLOW ERRORS:

If the combination of the character cell size, the number of lines to be plotted, the number of characters per line, the x coordinate of the origin of the block of gray scale characters, and the y coordinate of the origin of the block of gray scale characters will produce a plot that exceeds the

physical limits of the 2648A terminal then an error message is written to the terminal (the error message pinpoints the offending parameters) and the program returns to the calling program with the error flag set.

23. Subprogram PLOTS

PLOTS is a waveform plotting subroutine for the HEWLETT PACKARD 2648A computer graphics terminal.

PLOTS accepts and stores up to six waveforms which can be plotted, erased (and purged) and replotted in any order. New waveforms may be added to the program's waveform buffers after the old ones are erased. The X and Y axes can be labeled with up to 20 characters, and each waveform may be labeled with up to 16 characters. Each of the six waveforms are plotted by the program with a different linetype although the user may override and specify the linetype. Axes are put on the graph with user selectable divisions and tic marks between divisions. A grid can be overlaid on the graph.

CALLING PARAMETERS

XMAX : Maximum on the X axis.
XMIN : Minimum on the X axis. The X minimum and maximum are only used to label the axes because an array of X values is not used to plot the Y values.
YMAX : The maximum value in the input array.
YMIN : The minimum value in the input array.
IDIVX : The number of labeled divisions on the X axis.
IDIVY : The number of labeled divisions on the Y axis.
ITICX : The number of tics between the labels on the X axis.
ITICY : The number of tics between the labels on the Y axis.
IGRS : The grid select flag. 1 = draw a grid, 0 = no grid.

INMPTS: The number of points in the Y array (RIA) to plot.
 IMX : A twenty element integer array that contains the Hollerith codes for the X axis label.
 IMY : A twenty element integer array that contains the Hollerith codes for the Y axis label.
 IWFMSG: An eight element integer array that contains the Hollerith codes for the waveform message.
 RIA : The input array of Y values to be plotted.
 ILNFL : The auto line select flag. This is used to override the automatic selection of a linetype.
 0 = use auto select, 1-6 selects linetype <ILNFL>.
 INITFL: Initialization flag. 1 = initialize, 0 = plot or erase.
 IERAS : Waveform erase flag. 0 = no erase, plot instead.
 1-6 = erase waveform number <IERAS>.
 ISTAT : A six element integer that contains the statuses of the six waveform buffers.

CALLING PROCEDURE

- 1: Call PLOTS with the initialization flag set.
- 2: Call PLOTS with the initialization flag reset for each waveform to be plotted or erased. Each call after initialization either erases a waveform or adds one. If the program is asked to plot more than six waveforms on the screen at once it simply returns to the calling program without doing anything. Grid selection must be made at initialization. The MAXes and MINs should remain constant after initialization. The status array (ISTAT) should not be altered by the calling program under normal circumstances.

24. Subprogram MPL

MPL is a subroutine that plots a set of up to 18 separate X-Y graphs on the screen of a HEWLETT PACKARD 2648A graphics computer terminal.

MPL plots the graphs in any of 18 'boxes' on the screen of the 2648A. Each graph is labeled by a three character numeric label which is specified by the calling program. The graphs may be overlaid, that is, the same position for two or more graphs may be used, which plots one graph right over the others. The graphs are plotted like ordinary

cartesian graphs except that they are rotated clockwise 90 degrees. Each graph is provided with a dotted line which indicates the relative position of 0 on the Y axis. MPL plots up to 128 Y values. The X values are linearly generated internally. The maximum and minimum values for the input array of Y values specified by the calling program may change for each graph though the location of the zero line may not be accurate if the maximum and minimum are changed.

MPL is called once to initialize and then once for each graph to be plotted.

CALLING VARIABLES

RA : The array of Y values to be plotted.
IM : An integer number that labels the graph being plotted.
RMAX : The largest value in RA.
RMIN : The smallest value in RA. If RMAX or RMIN are exceeded MPL allows the graph to overflow its "box" on the screen.
IPOS : The position on the screen at which to put the graph. From 1 to 18. The position of the 'boxes' on the screen is illustrated below.

9	8	7	6	5	4	3	2	1
18	17	16	15	14	13	12	11	10

INITFL: The initialization flag. 1 = initialize.
0 = no initialize.
IFIRST: The subscript of the first element of RA to be plotted.
ILAST: The subscript of the last element of RA to be plotted.
IFIRST and ILAST allow a subset of the RA to be plotted.
Only the elements that lie between IFIRST and ILAST (inclusive) are plotted.

CALLING PROCEDURE

- 1: Call MPL with INITFL = 1 to initialize the program.
- 2: Call MPL once for each graph to be plotted.

BIBLIOGRAPHY

- Angona, F.A., "Two-Dimensional Modeling and Its Application to Seismic Problems", Geophysics, Vol. 25, no. 2, April 1960, pp. 468-482.
- Bendat and Piersol, Measurement and Analysis of Random Data, Wiley, 1966.
- Bliss, Gilbert Ames, Calculus of Variations, LaSalle, Illinois: The Open Court Publishing Company, 1925.
- Burrell, G.A., L. Peters, Jr., and A.J. Terzuoli, Jr., "The Propagation of Electromagnetic Video Pulses with Applications to Subsurface Radar for Tunnel Detection", Report 4460-2, December 1966, The Ohio State University ElectroScience Laboratory, Department of Electrical Engineering; prepared under Contract DAAG53-76-C-0179 for U.S. Army Mobility Equipment Research and Development Command.
- Campbell, K.J. and A.S. Orange, "A Continuous Profile of Sea Ice and Freshwater Ice Thickness by Impulse Radar", Polar Record, 17-106, 31/41, 1974.
- Chan, L.C., "Subsurface Electromagnetic Target Characterization and Identification", Ph.D. Dissertation, The Ohio State University, June 1979.
- Chang, D.C., "Electromagnetic Sensing of Roof Thickness in a Coal Mine", 1978 International Symposium on Antenna and Propagation, August 1978.
- Claerbout, Jon F., "Coarse Grid Calculations of Waves in Homogenous Media with Application to Delineation of Complicated Seismic Structure", Geophysics, Vol. 35, no. 3, June 1970, pp. 407-418.
- Claerbout, Jon F., "Numerical Holography", Acoustical Holography, Vol. 3, Plenum Press, N.Y.-London, 1971a, Edited by A.F. Methrell.
- Claerbout, Jon F., "Toward a Unified Theory of Reflector Mapping", Geophysics, Vol. 33, no. 3, June 1971(b), p. 467-481.
- Claerbout, Jon F., "Synthesis of a Layered Medium From Its Acoustic Transmission Response", Geophysics, Vol. 33, no. 2, April 1978, pp. 264-269.
- Claerbout, Jon F., and Stephan M. Doherty, "Downward Continuation of Moveout-Corrected Seismograms", Geophysics, Vol. 37, 1972, pp. 741-768.

- Clayton, Robert W. and Bjorn Engquist, "Absorbing Boundary Conditions for Wave-Equation Migration", Geophysics, Vol. 45, no. 5, May 1980, pp. 895-904.
- Dakermadj, G. and W.T. Joines, "A Method for Measuring the Electrical Properties of Sea Water and Wet Earth at Microwave Frequencies", IEEE Trans., IM-26-2, pp. 124-127, June 1977.
- Davis, C.W., III, "A Computational Model for Subsurface Propagation and Scattering for Antennas in the Presence of a Conducting Half Space", Ph.D. Dissertation, The Ohio State University ElectroScience Laboratory, Department of Electrical Engineering, 1979.
- Doherty, Stephan M. and Jon F. Claerbout, "Structure Independent Velocity Estimation", Geophysics, Vol. 41, no. 5, October 1976, pp. 850-881.
- Dohr, Gerhard P. and P.K. Stiller, "Migration Velocity Determination: Part II, Applications", Geophysics, Vol. 40, no. 1, February 1975, pp. 6-16.
- French, William S., "Two-Dimensional and Three-Dimensional Migration of Model-Experiment Reflection Profiles", Geophysics, Vol. 39, no. 3, June 1974, pp. 265-277.
- Gabillard, R., P. Deganne and J.R. Wait, "Subsurface Electromagnetic Telecommunication--A Review", IEEE Transactions on Communications Technology, Vol. COM-19, December 1971.
- Gardner, G.H.F., W.S. French, and T. Matzuk, "Elements of Migration and Velocity Analysis", Geophysics, Vol. 39, no. 6, December 1974, pp. 811-825.
- Gazdag, Jeno., "Wave Equation Migration with the Phase-Shift Method", Geophysics, Vol. 43, no. 7, December 1978, pp. 1342-1351.
- Glenn, W.E., and S.H. Ward, "Statistical Evaluation of Electrical Sounding Methods, Part I: Experiment Design", Geophysics, Vol. 41, No. 6A, December 1976, pp. 1207-1221.
- Gudmanden, P., "Electromagnetic Probing of Ice", Proc. of the Arctic Basin Symposium, 1972.
- Hagedoorn, J.G., "A Process of Seismic Reflection Interpretation", Geophysical Prospecting, Vol. 2, 1954, pp. 85-127.
- Harrington, R.F., Time-Harmonic Fields, McGraw-Hill, 1961.

- Hayes, P.K., "An On-Site Method for Measuring the Dielectric Constant and Conductivities of Soils Over a One Gigahertz Bandwidth", M.Sc. Thesis, The Ohio State University ElectroScience Laboratory, Department of Electrical Engineering, 1979.
- Hilterman, Fred J., "Three-Dimensional Seismic Modeling", Geophysics, Vol. 35, no. 6, December 1970, pp. 1020-1037.
- Hubral, P., "Time Migration--Some Ray Theoretical Aspects", 1977.
- Kay, A.F., "Spherically Symmetric Lenses", IRE Trans. on Antennas and Propagation, Vol. AP-7, no. 1, page 32, January 1959.
- Keller, G.V. and F.C. Frischknecht, Electrical Methods in Geophysical Prospecting, Pergamon Press, 1966.
- Kennaugh, E.M. and R.L. Cosgriff, "The Use of Impulse Response in Electromagnetic Scattering Problems", The Ohio State University ElectroScience Laboratory, reprinted from 1985 IRE National Convention Record, Part I.
- Kennaugh, E.M. and D.L. Moffatt, "Transient and Impulse Response Approximations", The Ohio State University ElectroScience Laboratory, reprinted from the Proceedings of the IEEE, Vol. 53, no. 8, pp. 893-901, August 1965.
- Lajoie, J., J. Alfonso-Roche, and G.F. West, "EM Response of an Arbitrary Source on a Layered Earth: A New Computational Approach", Geophysics, Vol. 40, no. 5, October 1975, pp. 773-789.
- Lerner, R.M., "Ground Radar System", United States Patent 3,831,173, August 1974.
- Lines, L.R., R.W. Clayton, and T.J. Ulrych, "Impulse Response Models for Noisy Vibroseis Data", Geophysical Prospecting, 1980, Vol. 28, pp. 49-59.
- Loewenthal, D., R. Roberson, J. Sherwood, "The Wave Equation Applied to Migration", 1976.
- Luneberg, R.K., Mathematical Theory of Optics, University of California Press, 1964.
- Melton, B.S., "Electromagnetic Prospecting Method", United States Patent 2,077,707, April 1937.
- Moffatt, D.L., "Time Domain Electromagnetic Scattering from Highly Conducting Objects", Final Report 2971-2, May 1971, The Ohio State University ElectroScience Laboratory, Department of Electrical Engineering: prepared under Contract F19628-70-C-0125 for the Air Force systems Command, Bedford, Maryland.

- Moffatt, D.L. and R.K. Mains, "Detection and Discrimination of Radar Target", IEEE Transactions on Antennas and Propagation, Vol. AP-23, no. 3, May 1975.
- Moffatt, D.L. and L. Peters, Jr., "An Electromagnetic Pulse Hazard Detection System", The Ohio State University ElectroScience Laboratory, reprinted from the Proceedings of the 1972 North American Rapid Excavation and Tunneling Conference, June 1972.
- Moffatt, D.L. and R.J. Puskar, "A Subsurface Electromagnetic Pulse Radar", Geophysics, Vol. 41, no. 3, June 1976, pp. 506-518.
- Musgrave, Albert W., "Wave-Front Charts and Three Dimensional Migrations", Geophysics, Vol. 26, no. 6, December 1961, pp. 738-753.
- Parry, J.R. and S.H. Ward, "Electromagnetic Scattering from Cylinders Arbitrary Cross-Section in a Conductive Half-Space", Geophysics, Vol. 36, no. 1, February 1971, pp. 67-100.
- Paturet, Daniel, "Different Methods of Time-Depth Conversion With and Without Migration", 1971.
- Peters, L., Jr., and D.L. Moffatt, "Electromagnetic Pulse Soundings for Geological Surveying with Application in Rock Mechanics and Rapid Excavation Program", Report 3190-2, September 1972, The Ohio State University ElectroScience Laboratory, Department of Electrical Engineering; prepared under Contract H0210042 for U.S. Department of Interior, Bureau of Mines.
- Phinney, Robert A. and Donna M. Jurdy, "Seismic Imaging of Deep Crust", Geophysics, Vol. 44, no. 10, October 1979, pp. 1637-1660.
- Riley, Don. C. and Jon F. Claerbout, "2-D Multiple Reflections", Geophysics, Vol. 41, no. 4, August 1976, pp. 592-620.
- Robinson, Enders A. "Predictive Decomposition of Time Series with Application to Seismic Exploration", Geophysics, Vol. 32, no. 3, June 1967, pp. 418-484.
- Robinson, Enders A. and Sven Traitel, Geophysical Signal Analysis, Prentice Hall, 1980.
- Roman, I., "Analysis of Seismic Profiles", Trans. AIME, Geophysical Prospecting, 110, 493-527.
- Sandler, S.S., "Picture Processing and Reconstruction", D.C. Heath and Company, Lexington, Massachusetts, 1975.

- Sattlegger, J.W. and P.K. Stiller, "Section Migration, Before Stack, After Stack, or Inbetween", 1974.
- Sattlegger, J., "Series for Three-Dimensional Migration in Reflection Seismic Interpretation", 1962.
- Sattlegger, J., "Migration Velocity Determination: Part I, Philosophy", Geophysics, Vol. 40, no. 1, February 1975, pp. 1-5.
- Schneider, William A., "Developments in Seismic Data Processing and Analysis (1968-1970)", Geophysics, Vol. 36, no. 6, December 1971, pp. 1043-1073.
- Schneider, William A., "Integral Formulation for Migration in Two and Three Dimensions", Geophysics, Vol. 43, no. 1, February 1978, pp. 49-76.
- Schultz, Philip S. and Fon F. Claerbout, "Velocity Estimation and Downward Continuation by Wavefront Synthesis", Geophysics, Vol. 43, no. 4, June 1978, pp. 691-714.
- Sibul, Leon H. and Edward L. Titlebaum, "Signal Design for Detection of Targets in Clutter", Proceedings of the IEEE, Vol. 69, no. 4, April 1981, pp. 481-482.
- Stapp, D.O., "Method for Grey Scale Mapping of Underground Obstacles Using Video Pulse Radar Returns", M.Sc. Thesis, The Ohio State University ElectroScience Laboratory, Department of Electrical Engineering, 1978.
- Stolt, R.H., "Migration by Fourier Transform", Geophysics, Vol. 43, no. 1, February 1978, pp. 23-48.
- Tai, C.T., "The Electromagnetic Theory of the Luneberg Lens", Antenna Laboratory, The Ohio State University Research Foundation, Contract AF33(616)-3353, 31 August, 1956, AD109 371.
- Treitel, S. and E.A. Robinson, "Seismic Wave Propagation in Layered Media in Terms of Communication Theory", Geophysics, Vol. 31, no. 1, February 1966, pp. 17-32.
- Tribuzi, C.A., "An Antenna for Use in an Underground (HFW) Radar System", Report 784460-4, November 1977, The Ohio State University ElectroScience Laboratory, Department of Electrical Engineering; prepared under Contract DAAG 53-76-C-0179 for U.S. Army Mobility Equipment Research and Development Command, Ft. Belvoir, Virginia.

- Volakis, Ioannis Leonidas, "Improved Identification of Underground Targets Using Video Pulse Radars by Elimination of Undesired Natural Resonances", M.Sc. Thesis, The Ohio State University ElectroScience Laboratory, Department of Electrical Engineering, 1979.
- Ward, S.H., B.D. Smith, W.E. Glenn, L. Rijo, and J.R. Inman, Jr., "Statistical Evaluation of Electrical Sounding Methods, Part II, Applied Electromagnetic Depth Sounding", Geophysics, Vol. 41, no. 6A, December 1976, pp. 1222-1235.
- Waters, Kenneth H., Reflection Seismology, Wiley, 1978.
- White, R.E. and P.N.S. O'Brien, "Estimation of the Primary Seismic Pulse", 1974.
- Won, I.J., "A Wide-Band Electromagnetic Exploration Method--Some Theoretical and Experimental Results", Geophysics, Vol. 45, no. 5, May 1980, pp. 928-940.
- Yoshino, T. and T. Eto, "Radio Echo Sounding of Antarctic Ice", JARE Scientific Reports Special Issue No. 2, Polar Research Center, March 1971.