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CHARACTERIZATION OF ELECTROMAGNETIC BACKSCATTER FROM
MOVING TRACKED VEHICLES

The Ohio State University

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CHARACTERIZATION OF ELECTROMAGNETIC
BACKSCATTER FROM MOVING TRACKED VEHICLES

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By

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1982

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CHAPTER I

INTRODUCTION

The characterization of the electromagnetic backscatter from radar targets is of vital interest to target classification for two reasons. The first is that the classification process depends on features in the backscatter signal which are characteristic of different classes of targets. The second reason is that complete electromagnetic models are often very costly and from a time and cost viewpoint it behooves the researcher to find backscatter mechanisms that most faithfully characterize the targets of interest.

Each radar target has a radar cross section determined by the superposition of all of its constituent scattering mechanisms. This cross section will determine the magnitude of the backscattered signal. Moving subcomponents will modulate the return signal and these subcomponent modulations can often be used to characterize the target. When the target is a ground vehicle, such as a tracked vehicle, the moving subcomponents that are the most distinctive are the tracks. The rolling motion of the tracks is periodic and will modulate the backscatter such that distinctive harmonics will appear in the spectrum.

In surveying the literature, the following papers are of particular interest to this subject. In 1976, Jean Van Bladel [1] published a treatise on the topic of fields in the presence of rotating bodies. His paper showed, using a quasi-stationary approximation, that distinct harmonics do appear in the spectrum and that they arise from the periodic modulation of the scattered field. C.W. Chuang [2] applied the geometric theory of diffraction to model the backscatter, at normal incidence, from rotating n-sided cylinders. These cylinders were used to simulate the effect of tracks rolling around the sprocket of a tank. His results showed that the spectral harmonics are separated in frequency by an amount equal to the tread flip rate of the vehicle. Finally, a study was conducted by Stewart [3] on identifying periodically amplitude modulated (PAM) targets. Stewart's work was an attempt to formulate a general classification procedure based upon a set of 40 spectral features that would be valid at all aspect angles. The only electromagnetic model used was Chuang's model for track backscatter at the nose-on (180°) and tail-on (0°) aspect angles. The model did not sufficiently explain the apparent anomalies in the spectrum such as the occasional disappearance, smearing and spreading of harmonics. Since the behavior of the PAM harmonics was poorly understood, they were rejected as a source for features in the target classification process.

This study will focus on a more comprehensive model that will explain the disappearance, smearing and spreading of the PAM harmonics that are characteristic of tracked vehicles. Then, with a greater understanding of their behavior, PAM harmonics can be used as a source of features for target classification.

In Chapter II, a model for track backscatter will be developed based on the backscatter from an 11-sided conducting cylinder. The 11 sides correspond to the treads rolling around an 11 toothed sprocket. The interaction between the cylinder and the ground, the effects of acceleration, the addition of a second cylinder to simulate two tracks and the influence of fenders will be included.

Chapter III will review basic pattern recognition and classification principles and will select features, based upon the PAM harmonics, for classification.

Chapter IV will be a description of the radar system used for the experiments. The types of vehicles used, such as tanks, trucks, APCs, etc., will be described along with modifications on the tank to help isolate the source of the PAM harmonics. The test track will be discussed and some typical plots of the data will be shown.

Chapter V will show the comparison between the model and the experiments. The proposed features will be extracted from the data and feature plots will be made. From these plots, the classifiability of tracked vehicles will be discussed based upon the PAM harmonics.

Chapter VI will conclude with a summary of the significance of the work and with suggestions for future efforts.

CHAPTER II

THE HIGH FREQUENCY ELECTROMAGNETIC MODEL

A. INTRODUCTION

This chapter deals with the development of a high frequency model for the nose-on and tail-on backscatter of tracked vehicles using the geometrical theory of diffraction (GTD). Modeling is necessary to the target classification process because an intelligent selection of features for classification is based on understanding the physical processes that generated them. GTD is a well known and useful technique for modeling electromagnetic scattering and diffraction at high frequencies, i.e., where the major target dimensions are a wavelength or more in extent. Since accurate modeling is critical to the classification process, careful attention is given to the GTD model in this chapter.

First, a description is given of the geometrical theory of diffraction and its application to modeling the backscatter from a track like structure. Secondly, a previous modeling effort by Chuang [2] is discussed. Chuang's model showed that a single rotating track like structure will produce a periodic amplitude modulation (PAM) of the backscattered signal. The weakness in the model is that it lacked

adequate sophistication to explain many phenomena observed in experimental data.

Thirdly, Chaung's model is modified to include the interaction between the cylinder and the ground plane. This mechanism significantly affects the shape of the PAM spectral envelope; because of the proximity of the ground plane the envelope becomes asymmetric. In addition to including the cylinder-ground plane interaction, acceleration was added to the model resulting in the splitting of the spectral harmonics. The splitting increases with increasing frequency.

Lastly, the model is modified to include the addition of a second track collocated with the first. The second track's backscatter interferes constructively and destructively with that of the first at the receiver. The result is that the spectral harmonics are modulated in such a way as to remove some of them from the spectrum. The degree of interference is proportional to the difference in phase between the tracks. The two track model also permits the simulation of the effect of the tracked vehicle turning. This turning, as a consequence of different track velocities, will again cause the spectral harmonics to split.

The additions cited above prove useful in helping to explain the apparent anomalies in the measured data, namely; the splitting and spreading of harmonics and the disappearance of harmonics in the PAM spectrum.

B. A TRACKED VEHICLE MODEL

In modeling the backscatter of any radar target, the overall goal must be kept in mind as to what principally is sought in the final solution. Backscatter models can be based on finding the radar cross section, the resonances on the target, or moving subcomponent modulations, to mention a few. The goal of this chapter is to locate and model those mechanisms that would best lend themselves to target classification.

Every moving radar target produces a skin line doppler. The skin line is defined as that portion of the target's spectral signature that contains the energy scattered by the skin of the target. Typically this energy is concentrated at a single doppler frequency. The doppler frequency is given by

$$f_d = \frac{2Vf_0}{c} \quad (II-1)$$

where V = the component of the velocity along the viewing aspect,

f_0 = the radar frequency, and

c = the speed of light in free space.

Additionally, some targets can have moving subcomponents which will modulate the skin line doppler. Moving subcomponents might consist of rolling wheels, moving tracks, a rotating turret or engine vibrations.

Classification by comparing skin line doppler returns would be difficult because vehicles belonging to different classes could still have similar radar cross sections. But classification using the

subcomponent modulations is feasible because of the more distinctive nature of the subcomponents. The running gear of tracked vehicles differs markedly from that of wheeled vehicles.

Stewart [3] pointed out the fact that tracked vehicles will modulate the radar return periodically in a fashion similar to that described in Van Bladel [1], i.e., that the periodic motion of the tracks will produce a series of harmonics in the spectrum about the doppler frequency. At x-band frequencies these modulations have only been observed at the cardinal aspects of 0° (tail-on) and 180° (nose-on). The model in this chapter will center on the backscatter from tracks at the 0° and 180° aspect angles.

Figure 2.1 depicts the back or tail-on view of an M-60 tank. A significant portion of the physical structure can be seen to be the tracks. As one looks more closely at a tank track (Figure 2.2) from a 0° aspect, it is observed that the flat treads around the sprocket are similar to the flat sides of an 11-sided polygonal cylinder viewed radially. When the radar is operating at x-band frequencies, as is the case for our experiments, the wavelength of the signal is on the order of an inch and so the details of the tread which are much smaller than an inch will not be resolvable. Thus, approximating the track around the sprocket as an undecagonal cylinder is a reasonable approach to simplifying the model for the track. Using the geometrical theory of diffraction (GTD) methodology, the total backscattered field is composed of the fields reflected from the plates and all the diffractions from the plate junction edges.

Figure 2.1. The back view of an M-60 tank.

Figure 2.2. Close up of track and a similar 11-sided cylinder.

C. THE GEOMETRICAL THEORY OF DIFFRACTION

The GTD was first introduced by Joseph B. Keller in 1953 [4] as a means for treating the scattering by corners, edges and tips. In the high frequency regime, electromagnetic fields, can be depicted as rays and conform to the usual optical laws of reflection and refraction. But when rays strike corners, edges and tips, a modification has to be made to account for the diffraction effects.

When an object is large in terms of wavelength, it's scattered and diffracted fields are found to be essentially localized to certain specific parts of the object, e.g., points of specular reflection, the edges, etc. GTD shows that when a ray strikes an edge, a diffracted field results which radiates all around the edge but is modified in intensity by a diffraction coefficient whose value is dependent on the geometry. Since these effects occur locally, it is sufficient to solve canonical problems that describe the local geometry and then sum up these local effects.

The canonical problem that is relevant to modeling the backscatter from an undecagonal cylinder is the infinite wedge. The wedge will prove useful because the plates on the sides of the undecagonal cylinder join to form 11 wedges whose interior angles (WA), measured from plate to plate, are given by

$$WA = (2-n)\pi = \frac{9\pi}{11} . \quad (II-2)$$

The canonical two dimensional wedge configuration is illustrated in Figure 2.3. A line source is located at the point "0" defined by the

I.S.B. — THE INCIDENT SHADOW BOUNDARY
R.S.B. — THE REFLECTION SHADOW BOUNDARY

Figure 2.3. Conducting wedge in the presence of a line source.

cylindrical coordinates (ρ', ϕ') , and the field point is at "P" with coordinates (ρ, ϕ) . Using the uniform geometrical theory of diffraction (UTD) solution of reference [5] the field at point P consists of the superposition of fields associated with the direct ray along the path OP, the reflected ray along the path OQP and, as Keller describes, a diffracted ray along the path OQE diffracted by the edge Q_E . The rays follow paths governed by the Generalized Fermat's Principle, i.e., in geometrical optics, the field point consists of rays from the source following paths of shortest time traversal. These include the direct ray as well as rays constrained to include the points of reflection and diffraction. In essence, Keller showed that the total field at point P could be described as,

$$\vec{E}^t = \vec{E}^i U^i + \vec{E}^r U^r + \vec{E}^d \quad (\text{II-3})$$

where

- $\vec{E}^i$ = the incident electric field at P,
- $\vec{E}^r$ = the reflected field via the ray path OQP,
- $\vec{E}^d$ = the field diffracted at the point Q_E , and
- U^i, U^r = unit step functions that are unity in the incident and reflected regions and zero outside.

The field diffracted at the edge can be written as,

$$\vec{E}^d = \vec{E}^i(Q_E) \cdot \vec{\bar{D}}(\phi, \phi') \frac{1}{\sqrt{\rho}} e^{-jk\rho} \quad (\text{II-4})$$

where

$\vec{\bar{D}}(\phi, \phi')$ = the dyadic diffraction coefficient.

In matrix form, the diffracted field equation can be written as [8],

$$\begin{bmatrix} E_{\parallel}^d \\ E_{\perp}^d \end{bmatrix} = \begin{bmatrix} -D_s & 0 \\ 0 & -D_h \end{bmatrix} \cdot \begin{bmatrix} E_{\parallel}^i(Q_E) \\ E_{\perp}^i(Q_E) \end{bmatrix} \cdot \frac{e^{-jk\rho}}{\sqrt{\rho}} \quad (\text{II-5})$$

where

$E_{\parallel}^i(Q_E)$ = the component of the incident field parallel
to the edge,

$E_{\perp}^i(Q_E)$ = the component of the incident field perpendicular
to the edge,

$E_{\parallel}^d$ = the component of the diffracted field parallel
to the edge, and

$E_{\perp}^d$ = the component of the diffracted field perpendicular
to the edge.

The diffraction coefficient is given by,

$$D_{s,h} = \frac{e^{-j\pi/4}}{\sqrt{2\pi k}} [R(\phi-\phi') \mp R(\phi+\phi')] \quad , \quad (II-6)$$

where

$$R(\beta) = \frac{(1/n)\sin\pi/n}{\cos\pi/n - \cos\beta/n} \quad (II-7)$$

note that $n = 2-WA/\pi$ where WA is the wedge angle. The s and h subscripts on the diffraction coefficient D are merely conventions representing the acoustically soft boundary condition (electric field parallel to the edge) and the acoustically hard boundary condition (electric field perpendicular to the edge) that were earlier established in the field of acoustic diffraction.

Keller's solution was a significant achievement in that he showed that the diffracted field $\bar{E}^d$ behaves ray optically away from the edge and is proportional to the incident field at Q_E by a diffraction coefficient which varies with ϕ and ϕ' . Although Keller's diffraction coefficient gives accurate results away from the reflection and incident shadow boundaries, it becomes singular at them and fails to yield the correct results.

Pathak and Kouyoumjian [6] approached the wedge diffraction somewhat more rigorously to deduce a uniform nonsingular diffraction coefficient. The $E_{||}$ components of the field can be derived by convolving the source field with the Green's function in cylindrical coordinates. This method is basically equivalent to the impulse response in circuit theory. As one can find a circuit output by convolving the source with the impulse response of the circuit, one can find the radiated field by convolving the source field with the appropriate impulse response which is the Green's function solution. It is well known that the Green's function for a wedge imbedded in a cylindrical coordinate system is a series involving cylindrical Hankel and Bessel functions of fractional order. The Bessel and Hankel functions may be replaced by their integral equivalents given in [8].

The series is then summed leaving integral representations for the Green's function. The integrals are then approximated asymptotically by integrating along paths of steepest descent using the Pauli-Clemmow method [7], and the leading terms are retained. The resulting expression for the diffracted field can be written in the form of Equation (II-4) which makes it possible to deduce the new dyadic

diffraction coefficient $\bar{D}$. The new coefficients for the two dimensional wedge problem are;

$$D_{sh}(\phi, \phi') = \frac{-e^{-j\pi/4}}{2n\sqrt{2\pi k}} \times \left[\cot\left(\frac{\pi+(\phi-\phi')}{2n}\right) F[kL^+ a^+(\phi-\phi')] \right. \\ + \cot\left(\frac{\pi-(\phi-\phi')}{2n}\right) F[kL^+ a^-(\phi-\phi')] \mp \left\{ \cot\left(\frac{\pi+(\phi+\phi')}{2n}\right) F[kL^+ a^+(\phi+\phi')] \right. \\ \left. \left. + \cot\left(\frac{\pi-(\phi+\phi')}{2n}\right) F[kL^+ a^-(\phi+\phi')] \right\} \right] , \quad (II-8)$$

where

$$F(x) = 2j\sqrt{x} e^{jx} \int_{\sqrt{x}}^{\infty} \frac{e^{-j\tau^2}}{\sqrt{\tau}} d\tau , \quad (II-9)$$

and

$$a^{\pm}(\beta) = 2\cos^2\left(\frac{2n\pi N^{\pm} - \beta}{2}\right) . \quad (II-10)$$

Note that N^+ and N^- are integers which most nearly satisfy the equations

$$2\pi N^+ - \beta = \pi , \text{ and} \quad (II-11a)$$

$$2\pi N^- - \beta = -\pi . \quad (II-11b)$$

In addition, "k" is the wavenumber of the medium and the distance parameter is given in general by

$$L^i = \frac{\rho\rho'}{\rho+\rho'} \quad (\text{II-12})$$

These coefficients are uniform and as a result provide accurate values all around the wedge. In addition, they reduce to Keller's form of Equation (II-6) away from the incident and reflection shadow boundaries.

In the case of grazing incidence where $\phi'=0$ or $n\pi$, the diffraction coefficient is slightly modified. In this case, $D_s=0$ and D_h is modified by a factor of $1/2$. The need for the factor $1/2$ may be seen by considering grazing incidence to be the limit of oblique incidence. At grazing incidence, the incident and reflected fields merge so that one half of the total field propagating along the face of the wedge toward the edge is the incident field and the other half is the reflected field [8]. With this basic knowledge of GTD and the solution to the canonical wedge problem, Chuang [2] developed a model for the track using the infinite N-sided cylinder.

D. CHUANG MODEL

Chuang's model uses a variation of GTD called the self-consistent GTD which basically includes all of the orders of diffraction around the cylinder. He modeled the backscatter of the cylinder over a ground plane including the reflected field but did not include the multiple diffraction terms between the cylinder and the ground. The spectrum resulting from Chuang's analysis appears in Figure 2.4.

Figure 2.4. Chuang's model for the spectrum of an 11-sided cylinder at x-band. Velocity of rolling cylinder equals 13.5 ft/sec.

Two things to be learned from Chaung's model are the following:

1. The spectrum drops off sharply when $f > 2f_D$, where f_D is the doppler frequency.
2. The spacing of the harmonic lines is exactly equal to the tread flip rate (TFR), i.e., the velocity of the vehicle divided by the tread width.

In applying this model to cardinal cut measured data, Stewart [3] found that even though the harmonic spacing could be accurately predicted often in the data, some of the harmonics were not present. This effect could not be predicted by Chuang's model because of its simplicity. Provided that the model is indeed accurate, this would mean

that the harmonics are not reliable enough to use as spectral features for target classification.

The Chuang model was, therefore, modified to include the addition of acceleration to the moving and rotating cylinder, the addition of the multiple interactions between the cylinder and the ground, and the addition of a second cylinder to model the two tracks. These modifications result in showing how the spectral harmonics can wax and wane and how they will split and spread as observed in measured data, thus proving that given sufficient accuracy in the model, the harmonics can be used as spectral features for classification. The following sections describe the new tank track model.

E. TRACK/GROUND INTERACTION MODEL

Figure 2.5 shows the infinite undecagonal cylinder rotating and moving above a conducting ground plane depicting the effect of the treads flipping around the sprocket. The translational velocity of the cylinder is given by

$$v = v_0 + at. \quad (\text{II-13})$$

and the corresponding angular velocity is;

$$\omega = \frac{v_0}{r} + \frac{at}{r} = \omega_0 + \alpha t. \quad (\text{II-14})$$

Note that this model includes both the velocity and the acceleration of the cylinder, simulating tracked vehicle movement.

Figure 2.5. Cross section of an infinite undecagonal cylinder above a ground plane.

Let us systematically develop the GTD solution to the backscatter from the cylinder above the ground plane by looking at the dominant contributions which asymptotically approximate the total scattered field. The four basic scattering mechanisms are depicted in Figures 2.6 and 2.7 and are listed below:

1. DIRECT BACKSCATTER (see Figure 2.6a)

The set of rays coming directly from the transmitter, at an elevation angle θ , which diffract off of the cylinder and return at the same angle θ .

2. DIFFRACTED-REFLECTED (see Figure 2.6b)

The set of rays that come directly from the transmitter, at the angle θ , which diffract at a depression angle $-\theta$ and then reflect off of the ground plane back to the receiver.

3. REFLECTED-DIFFRACTED (see Figure 2.7a)

The set of rays that reflect off of the ground plane and arrive at the cylinder at the depression angle $-\theta$, and diffract at the angle θ back to the receiver.

4. REFLECTED-DIFFRACTED-REFLECTED (see Figure 2.7b)

The set of rays that reflect off of the ground plane, arrive at the angle $-\theta$, diffract back at $-\theta$ and reflect back to the receiver.

Figure 2.6. The direct and the diffracted-reflected backscatter terms.

Figure 2.7. The reflected-diffracted and the reflected-diffracted-reflected terms.

These four terms constitute the dominant GTD mechanisms associated with this structure. The rays incident from the angles $\pm\theta$ can diffract between the cylinder and ground plane an infinite number of times before finally returning to the receiver. However, single and double diffraction, plus reflections, dominate this GTD solution. The specific diffraction mechanisms are seen in Figures 2.8 through 2.11 and are:

1. Rays approaching an arbitrary edge ℓ from the $\pm\theta$ directions will diffract back towards the receiver in the $\pm\theta$ directions (see Figure 2.8).
2. Rays approaching an arbitrary edge ℓ from the $\pm\theta$ direction can diffract towards the $\ell + 1$ and $\ell - 1$ edges, along the surface of the plates, and diffract off of those edges in the $\pm\theta$ directions. The diffraction coefficients at edges $\ell + 1$ and $\ell - 1$ are modified by the factor $1/2$ because of grazing incidence on the second edge (see Figure 2.9).
3. Rays approaching an arbitrary edge ℓ from the $\pm\theta$ direction can diffract to the ground plane and reflect back up to the edges $\ell - 1, \ell$ and $\ell + 1$. These rays in turn diffract back in the $\pm\theta$ directions (see Figure 2.10).

Figure 2.8. Various single diffraction mechanisms possible for one edge of the rotating undecagonal cylinder.

Figure 2.9. Various double diffraction mechanisms possible for one edge of the rotating undecagonal cylinder.

Figure 2.10. Various double diffraction mechanisms, with an additional reflection, possible for one edge of the rotating undecagonal cylinder.

Figure 2.11. Various single diffraction mechanisms, after two reflections, for one edge of the rotating undecagonal cylinder.

4. Rays approaching an arbitrary plate from the $\pm\theta$ directions can reflect off of the plate towards the ground plane and again reflect back towards edges $\ell - 2$, $\ell - 1$, ℓ and $\ell + 1$. These rays in turn will diffract from those edges back in the $\pm\theta$ directions toward the receiver. Reciprocity applies in this circumstance so that the ray paths can be reversed (see Figure 2.11).

Additionally, the last two terms, those including interaction of the cylinder with the ground plane, represent the effect of the coupling of the cylinder to its image. These terms prove to be quite significant, i.e., they modify the envelope of the spectrum. A computer program, called "TRACK", was written to manage all of these diffraction mechanisms and is included in Appendix I.

The last four sets of figures depicted the 20 possible mechanisms (4 from reciprocity) necessary to be included in the cylinder backscatter. The details of the determination of the variables to be used in the diffraction coefficient and the phase delay can be illustrated with four examples. The first example will involve the terms depicted in Figure 2.8. These four terms have the same basic diffraction equation, for each edge ℓ , differing only by the input variables. The basic equation and variables are given by

$$H_{\perp\ell}^d = -D_h^K(\phi_\ell', \phi_\ell) H_{\perp}^i \frac{e^{-jks}}{\sqrt{s_\ell}} \quad (\text{II-15})$$

where

$H_{\perp \ell}^d$ = the diffracted H field from edge ℓ ,

ϕ_{ℓ} = the polar angle of the field point with edge ℓ as the axis,

ϕ'_{ℓ} = the polar angle of the source point with edge ℓ as the axis,

S_{ℓ} = the distance to the field point appropriately measured
from edge ℓ , and

D_h^K = the hard Keller edge diffraction coefficient .

The incident field is given by

$$H_{\perp}^i = H_0 e^{jks'_{\ell}} \quad , \quad (\text{II-16})$$

with

S'_{ℓ} = the distance to the source point appropriately measured
from edge ℓ .

Figure 2.12 depicts the cylinder and the single diffraction variables from which one can properly determine the equations for all of the variables. The variables to be determined are:

$$\xi = \pi/11,$$

θ', θ = aspect angles of the incident and diffracted fields,

r_0 = radius of the cylinder,

ω = instantaneous angular speed of the cylinder,

α = instantaneous angular acceleration of the cylinder,

Figure 2.12. Geometry describing the single diffraction mechanism.

$$\beta'(t) = \omega t + 1/2 \alpha t^2 = \text{angle of cylinder rotation}, \quad (\text{II-17})$$

$$\phi'_\ell = \pi/2 - 2(\ell-1)\xi - \beta(t) + \theta', \quad (\text{II-18})$$

$$\phi_\ell = \pi/2 - 2(\ell-1)\xi - \beta(t) - \theta, \quad (\text{II-19})$$

$$d'_\ell = r_0 \cos(\theta - (2\ell-1)\xi - \beta(t)), \quad (\text{II-20})$$

$$d_\ell = r_0 \cos(\theta + (2\ell-1)\xi + \beta(t)), \quad (\text{II-21})$$

$$S'_\ell = S' - d'_\ell, \quad (\text{II-22})$$

$$S_\ell = S - d_\ell, \text{ and} \quad (\text{II-23})$$

H = height of axis above the ground.

The distances S' and S are the distances from the coordinate origin to the receiver along the paths determined by the aspect angles. The distance S can be related to S' by

$$S = S' + 2H \sin\theta'. \quad (\text{II-24})$$

The total field due to the single diffraction terms is the sum of the contributions from each edge where the incident and diffracted rays remain within the angles $0 \leq \phi'_\ell, \phi_\ell \leq n\pi$. The terms depicted in Figure 2.9 have again the same basic diffraction equation but differ in the values of the input variables. The basic double diffraction from edge ℓ to edges $\ell \pm 1$ is given by

$$H_{l, l \pm 1}^{dd} = \frac{H_0}{2} D_h^K(\phi'_l, 0) D_h^{K-P}(0, \phi_{l \pm 1}) \frac{e^{jk(S'_l - S_{l \pm 1} - W)}}{\sqrt{WS_{l \pm 1}}} \quad (\text{II-25})$$

where

D_h^K = hard Keller edge diffraction coefficient,

D_h^{K-P} = hard Kouyoumjian-Pathak or UTD edge coefficient, and

W = Width of one plate.

The variables and the geometry are depicted in Figure 2.13. The variables ϕ'_l , $\phi_{l \pm 1}$, S'_l and $S_{l \pm 1}$ are already defined in Equations (II-16, 17, 20, and 21) and are therefore not shown in Figure 2.13. Also it is not necessary to show the double diffraction to edge $l+1$ because of its similarity to double diffraction to edge $l-1$. The D_h^K coefficient is Keller's diffraction coefficient and is used at edge l because of the incident plane wave. The D_h^{K-P} coefficient is the Kouyoumjian-Pathak (UTD) coefficient, Equation (II-6), and is used at edges $l \pm 1$ because the incident field is along the shadow boundary and in general may not be planar. All the necessary variables can be determined in a similar fashion to those given in Equations (II-15) through (II-21). The total field due to double diffraction will be the sum of the contribution from all of the edges where $0 < \phi'_l, \phi_{l \pm 1} < n\pi$. The computer subroutine, called "HARCYL", was written to calculate all of the single and double diffraction terms and it appears in Appendix II.

Figure 2.13. Geometry describing the double diffraction mechanisms.

The diffraction terms depicted in Figure 2.10 can be calculated with one basic example. The three equations appropriate are given by

$$H_{\perp \ell, \ell-1}^{\text{drd}} = H_0 D_h^K(\phi'_\ell, GL) D_h^{K-P}(YL, \phi_{\ell-1}) \frac{e^{jk(S'_\ell - S_{\ell-1} - P2)}}{\sqrt{S_{\ell-1}^{P2}}} \quad , \quad (\text{II-26})$$

and

$$H_{\perp \ell, \ell}^{\text{drd}} = H_0 D_h^K(\phi'_\ell, G) D_h^{K-P}(G, \phi_\ell) \frac{e^{jk(S'_\ell - S_\ell - SL)}}{\sqrt{S_\ell SL}} \quad , \quad (\text{II-27})$$

$$H_{\perp \ell, \ell+1}^{\text{drd}} = H_0 D_h^K(\phi'_\ell, XL) D_h^{K-P}(GLP1, \phi_{\ell+1}) \frac{e^{jk(S'_\ell - S_{\ell+1} - P1)}}{\sqrt{S_{\ell+1}^{P1}}} \quad (\text{II-28})$$

where

H^{drd} = hard diffraction-reflection-diffraction field,

D_h^K = hard Keller diffraction coefficient, and

D_h^{K-P} = hard Kouyoumjian-Pathak diffraction coefficient.

By studying the geometry appearing in Figure 2.14 one may again list the values of all the pertinent variables, not previously defined, as follows:

$$G = 2\pi - 2(\ell-1)\xi - \beta(t), \quad (\text{II-29})$$

$$GP = 2\pi - 2\ell\xi - \beta(t), \quad (\text{II-30})$$

$$GM = 2\pi - 2(\ell-2)\xi - \beta(t), \quad (\text{II-31})$$

$$SL = H - r_0 \cos((2\ell-1)\xi + \beta(t) - 3\pi/2), \quad (\text{II-32})$$

$$SLD1 = H - r_0 \cos((2\ell-1)\xi + \beta(t) - 3\pi/2), \quad (\text{II-33})$$

Figure 2.14. Geometry describing the double diffraction plus reflection mechanisms.

$$SLM1 = H - r_0 \cos((2\ell-3)\xi + \beta(t) - 3\pi/2), \quad (II-34)$$

$$P2 = \sqrt{W^2 + 4SL^2 - 4SLW \cos(G)}, \quad (II-35)$$

$$P1 = \sqrt{W^2 + 4SLP1^2 - 4SLP1W \cos(GP)}, \quad (II-36)$$

$$DEL2 = \text{TAN}^{-1} (W \cos(AB) / (SLM1 + SL)), \quad (II-37)$$

$$DEL1 = \text{TAN}^{-1} (W \cos(AA) / (SL + SLP1)), \text{ and} \quad (II-38)$$

$$XL = G + DEL1, \quad (II-39)$$

where

AA = angle between the plate ℓ , $\ell+1$ and the x axis, and

AB = angle between the plate ℓ , $\ell-1$ and the x axis.

From the above set of variables, all of the other variables may be calculated; again, variables depicted in the preceeding figures are not repeated here. The total diffracted field from these terms is the sum of these contributions from all edges as described in the previous two examples.

The last example illustrates the details of the diffraction terms appearing in Figure 2.11. The pertinent equation is given below and the associated geometry is shown in Figure 2.15.

$$H_{\ell-2}^{rd} = H_0 D_h^K(Y_L, \phi_{\ell-2}) \frac{e^{jk(R' - S_{\ell-1} - x - y)}}{\sqrt{S_{\ell-1}(x+y)}} \quad (II-40)$$

The details of the geometry are much more complicated in that the point of reflection (point 1) is variable depending on the plate position and

Figure 2.15. Geometry describing the reflection-reflection-diffraction terms.

angle of incidence. Using the law of sines and other trigonometric identities one may determine the path lengths and angles which are given below to be:

$$BE = \pi/2 + 2\ell\xi + \beta(t) - \theta', \quad (\text{II-41})$$

$$GP = 2\pi - 2\ell\xi - \beta(t), \quad (\text{II-42})$$

$$G = 2\pi - 2(\ell-1)\xi - \beta(t), \quad (\text{II-43})$$

$$DEL = GP - BE, \quad (\text{II-44})$$

$$HN = \pi - G - DEL, \quad (\text{II-45})$$

$$X = SLM1/\cos(DEL), \quad (\text{II-46})$$

$$Y_0 = x\sin(HN)/\sin(\pi-HN-2DEL) = \text{Length between 2-3}, \quad (\text{II-47})$$

$$D = x\sin(2DEL)/\sin(\pi-HN-2DEL), \quad (\text{II-48})$$

$$C = D - W, \quad (\text{II-49})$$

$$YP = c\sin(2\xi)/\sin(BE), \quad (\text{II-50})$$

$$Y = Y_0 + YP, \quad (\text{II-51})$$

$$Z = \text{length from edge } \ell \text{ to point 1} \\ = c\sin(2\xi+BE)/\sin(BE), \text{ and} \quad (\text{II-52})$$

$$TA = \sqrt{r_o^2 + z^2 - 2r_o z \cos(AL)} \quad (\text{II-53})$$

All other necessary variables have been previously defined. The ray reflected at point 1 may go to edge $\ell-1$ as depicted or it may go to edges ℓ , $\ell+1$ or $\ell+2$ depending on the angle BE . Any other possibilities are not considered because these four terms represent the most significant contributions. The total diffracted field from the reflection-reflection-diffraction mechanism is the sum of all the possible contributions from the edges and in this case reciprocity applies, i.e., the reverse paths will be followed so that the diffracted

field will be doubled. The subroutine, called "HIGHER", was written to include the higher order diffraction terms of the last two examples, and it appears in Appendix III.

Again, there are an infinite number of diffraction possibilities because of the concave nature of the structure. but the diffraction-reflection-diffraction and the reflection-reflection-diffraction terms just mentioned represent the major contributions. The total diffracted field is the sum of all the terms in the above four examples.

The computer program "TRACK" reads a given vehicle velocity, acceleration and elevation angle of the radar and then calculates the backscatter from the undecagonal cylinder after it makes one complete revolution, i.e., moves forward one circumference in distance. The program plots the envelope of the backscattered field for unit incidence as a function of time and then plots the resulting spectrum. The time domain waveform and its corresponding spectrum appears in Figure 2.16 where the velocity $V = 13.5$ ft/sec (9 mph) and the acceleration $a = 0$ ft/sec² were used. The lobes or spikes in the time domain plot represent specular flashes from the plates as the cylinder rotates. Because of the cylinders velocity the spectrum is shifted by the doppler frequency of 220 Hz (see Equation (II-1)) and the envelope of the spectrum deviates from Chuang's spectrum as a result of adding in the effects of ground reflections.

Figure 2.16. Backscatter for one full revolution and the corresponding spectrum with the effects of ground interaction.

F. ACCELERATION EFFECTS

Figure 2.17 shows the spectral behavior when acceleration is introduced. The spectral harmonics split and the splitting effect increases with frequency. This phenomena of spectral splitting can be explained in an approximate way by first looking at the generic expression for the modulated backscattered signal. Since the periodic rotation of the cylinder is modulating the doppler return one can write an expression to describe the modulated signal as

$$S(t) = M(t) \cos \omega_d(t), \quad (\text{II-54})$$

where

$M(t)$ = the periodic amplitude modulation introduced as the cylinder rotates, and

$$\omega_d(t) = \sum_{k=0}^{\infty} A_k \cos k \omega_T t$$

$\omega_d(t)$ = the doppler radian frequency.

In the case where the velocity is constant the modulation is periodic, and the effect is called Periodic Amplitude Modulation (PAM). Under these circumstances $M(t)$ may be expanded in a Fourier series and the spectrum is easily seen to consist of harmonics evenly spaced about the carrier. Let us take one harmonic of the spectrum and introduce acceleration as a perturbation of the doppler frequency. The instantaneous doppler frequency and tread flip rates are given by

$$f_d^i = \frac{2(V_0 + at)}{c} f_0, \text{ and} \quad (\text{II-55})$$

$$f_T^i = (V_0 + at)/W, \quad (\text{II-56})$$

where

V_0 = initial vehicle velocity,

a = vehicle acceleration,

f_0 = radar carrier frequency,

c = the velocity of light in free space, and

W = the tread width.

Now observing the k^{th} harmonic in the spectrum one finds

$$S_k(t) = A_k \cos 2\pi(kf_T t + \frac{k a t^2}{2W}) \cos 2\pi(f_d t + \frac{a f_0 t^2}{c}) \quad . \quad (\text{II-57})$$

This equation describes the result of the product of two FM signals.

The equation may be further simplified to give

$$\begin{aligned} S_k(t) = & \frac{A_k}{2} \cos 2\pi[(f_d - kf_T)t + a(\frac{f_0}{c} - \frac{k}{2W})t^2] \\ & + \frac{A_k}{2} \cos 2\pi[(f_d + kf_T)t + a(\frac{f_0}{c} + \frac{k}{2W})t^2] \quad . \end{aligned} \quad (\text{II-58})$$

From the form of the above equation one observes that we now have the sum of two FM signals. The difference terms yield the harmonics below the doppler and the sum terms yield the harmonics above. The factors involving t^2 represent linear FM modulation on each harmonic and these

Figure 2.17. The effects of acceleration on the spectrum.

terms generate the sidebands or "splitting". The slope of the modulation term is $2a(\frac{f_0}{c} \pm \frac{k}{2W})$ and the splitting increases with increasing slope. So as k increases, the slope decreases for the lower harmonics and increases for the higher harmonics thus explaining why the splitting increases for the higher harmonics.

G. THE ADDITION OF A SECOND TRACK

Since the tracked vehicles of interest have two tracks their corresponding backscatter can constructively and destructively interfere with each other at the receiver. Therefore, it is necessary to add a second cylinder collocated with the first but variably out of phase with the first so as to simulate two tracks. When the two cylinders rotate at different rates, then one can simulate the effect of the tracked vehicle turning. The maximum amount that the two 11-sided cylinders can be out of phase is one half of a tread width in circumference or by $\pi/11$ radians. Figure 2.18 shows the two cylinders collocated but out of phase by half a tread width.

Looking at the backscatter from two 11-sided cylinders in free space, the total field will be the superposition of the fields from each cylinder. The backscatter of one cylinder measured at the radar output is given by

$$S(t) = M(t) \cos \omega_d t , \quad (II-59)$$

Figure 2.18. Two cylinders collocated.

where

$M(t)$ = the periodic modulation of the cylinder and

ω_d = the doppler frequency in radians.

Since the modulation is periodic, we can expand the modulation in a fourier series and the backscatter of Equation (II-59) then becomes

$$S(t) = \sum_{k=0}^{\infty} A_k \cos k\omega_T t \cos \omega_d t, \quad (\text{II-60})$$

where

A_k = the Fourier coefficients, and

ω_T = the tread flip rate in radians.

Now letting the phase delay of one cylinder be expressed as a time delay t_d in the modulation waveform and by using superposition one can express the total field as

$$S(t) = \sum_{k=0}^{\infty} A_k [\cos k\omega_T t + \cos k\omega_T(t-t_d)] \cos \omega_d t \quad . \quad (\text{II-61})$$

Taking the Fourier transform of both sides of Equation (II-60), one can obtain the backscatter spectrum. The resulting spectrum is

$$S_T(\omega) = \sum_{k=0}^{\infty} \pi/2 A_k [\delta(\omega+\omega_d-k\omega_T) + \delta(\omega+\omega_d+k\omega_T) + \delta(\omega-\omega_d-k\omega_T) + \delta(\omega-\omega_d+k\omega_T)] \\ \times [1 + e^{-jk\omega_T t_d}] \quad . \quad (\text{II-62})$$

One may now write the expression for the positive frequencies of $S_T(\omega)$ as

$$S_T^+(\omega) = \sum_{k=0}^{\infty} \pi/2 A_k [\delta(\omega-\omega_d-k\omega_T) + \delta(\omega-\omega_d+k\omega_T)] [1 + e^{-jk\omega_T t_d}] \quad , \text{ or} \\ = \sum_{k=0}^{\infty} \pi A_k e^{\frac{-jk\omega_T t_d}{2}} \cos\left(\frac{k\omega_T t_d}{2}\right) [\delta(\omega-\omega_d-k\omega_T) + \delta(\omega-\omega_d+k\omega_T)] \quad . \quad (\text{II-63})$$

Taking the magnitude of Equation (II-62) with A_k a real constant, one obtains

$$|S_T^+(\omega)| = \sum_{k=0}^{\infty} \pi |A_k| \left| \cos\left(\frac{k\omega_T t_d}{2}\right) \right| [\delta(\omega-\omega_d-k\omega_T) + \delta(\omega-\omega_d+k\omega_T)] \quad , \text{ or} \\ = \sum_{k=0}^{\infty} \pi/2 |A_k| |E(k, t_d)| [\delta(\omega-\omega_d-k\omega_T) + \delta(\omega-\omega_d+k\omega_T)] \quad , \quad (\text{II-64})$$

where

$$E(k, t_d) = 2 \cos \left(\frac{k \omega_T t_d}{2} \right) \quad . \quad (\text{II-65})$$

Thus, the magnitude of the spectrum for two tracks is just the magnitude of the spectrum for one track multiplied by $|E(k, t_d)|$.

Since the tread flip rate is 11 times the angular velocity of the cylinder, one obtains $\omega_T = 11\omega_c$. Also, since $\omega_c t_d$ is the phase delay of the second track we may set it to be some scalar multiple of a tread width such that

$$\omega_c t_d = a \frac{2\pi}{11} , \quad (\text{II-66})$$

then one obtains

$$E(k, t_d) = 2 \cos (ka\pi) \quad . \quad (\text{II-67})$$

Equation (II-67) reveals that when $a = 1/n$, $n = 1, 2, 3, 4, \dots$, i.e., where the tracks are out of phase by $1/n$ tread widths then the spectrum goes to 0 at every $\pm \frac{nm}{2}$ harmonic, where $m = 1, 3, 5, \dots$. As an example, when $n = 2$, the cylinders are out of phase by half a tread width and the harmonics $\pm 1, \pm 3, \pm 5, \dots$ are multiplied by 0. Figures 2.19 through 2.21 show how the spectrum is modulated, through interference, when the tracks are out of phase by a $\frac{1}{20}$, $\frac{1}{16}$, $\frac{1}{12}$, $\frac{1}{8}$, $\frac{1}{4}$ and $\frac{1}{2}$ tread widths.

(a)

(b)

Figure 2.19. Interference effects due to the addition of a second

track (a) delay = $\frac{1}{20}$ tread width

(b) delay = $\frac{1}{16}$ tread width.

Figure 2.20. Interference effects due to the addition of a second

track (a) delay = $\frac{1}{12}$ tread width

(b) delay = $\frac{1}{8}$ tread width.

Figure 2.21. Interference effects due to the addition of a second

track (a) delay = $\frac{1}{4}$ tread width

(b) delay = $\frac{1}{2}$ tread width.

The envelope of the spectrum is seen to change very rapidly for small changes in the phase between the two tracks. Small changes can easily occur as the vehicle is turning right or left. Therefore, it can be seen that the addition of a second cylinder to the track model accounts for interference effects in the received signal. This interference will result in removing harmonics in the spectrum.

With the addition of two tracks one may also vary the speed of each and simulate the vehicles turning. The radius of curvature for the turn is determined by the difference in velocity between the tracks.

Figure 2.22 shows the spectrum resulting when the vehicles's overall velocity is $V_V = 13.5$ ft/sec and the velocity of track 1 is $V_1 = .99V_V$ and that of track 2 is $V_2 = 1.01V_V$. Again one observes spectral splitting but in this case it is the result of the superposition of two spectra. Thus, one can see that many factors can influence the shape of the spectrum. A summary of the additions altering the spectrum in Figure 2.4 is given below:

1. Multiple diffractions between the cylinder and the ground plane modulate the harmonics producing an asymmetry about the doppler frequency.
2. Acceleration is found to cause the harmonics to split. The splitting is caused by an increasing frequency modulation above the doppler and a decreasing frequency modulation below the doppler.

Figure 2.22. Spectrum with two tracks at different velocities. $V_V = 13.5$ ft/sec., $V_1 = .99V_V$ ft/sec., $V_2 = 1.01V_V$ ft/sec.

3. The addition of a second track demonstrates two things:
 - a. As the tracks change in phase relative to each other, the envelope of the spectral harmonics changes by the factor $2\cos\left(\frac{k\omega_T}{2}\right)t_d$
 - b. The tracks can move at different velocities, thus simulating the vehicle turning. The result is again the splitting of spectral harmonics based on the principle of superposition of the spectrum from each track.

Figure 2.23. Spectrum with two tracks where: $V_T = 13.5$ ft/sec.,
 $V_1 = .99V$, $V_2 = 1.01V$, $a = 2$ ft/sec.², and delay
 $= 1/4$ tread width.

Any combination of the above effects will produce various distortions of the spectrum given in Figure 2.16. Figure 2.23 shows the spectrum where the tracks are initially out of phase, they move at different velocities and include acceleration. It is obvious that even at the cardinal angles, distinct harmonics cannot be easily detected if the vehicles are turning or accelerating.

H. THE EFFECT OF FENDERS ON THE MODEL

Most tracked vehicles have fenders covering portions of the tracks. If the fenders are metallic they will act to shield the radar energy from the track. When the tracked vehicle is viewed along the 0° or 180°

aspect, the fender typically will shield the upper portion of the track as it rolls around the sprocket. Without a detailed electromagnetic analysis, one can estimate the influence of the fender by considering the work of Graf [9].

Graf pointed out that doppler lines in the backscatter spectrum of rotating targets may also be viewed as being produced by each scattering center. Thus, scattering centers far from the axis of rotation produce greater doppler shifts because they are moving at higher angular velocities. In the case of the track model, the 11-sided cylinder's angular velocity is proportional to the translational velocity. Thus, as the cylinder moves, the scattering centers on the top half of the cylinder move faster relative to the radar than do the scattering centers on the bottom half. Therefore, the scattering centers on the top half of the cylinder produce the PAM harmonics above the skin line doppler and the scattering centers on the bottom half of the cylinder produce the PAM harmonics below the skin line doppler. If a fender shields some of the scattering centers on the top half of the cylinder from the radar signal, the effect will be to attenuate PAM harmonics in the upper sideband of the skin line doppler.

I. THE BEAMWIDTH OF THE PAM EFFECT

The model used thus far assumes aspect angles along the cardinal directions; however, the radar could be arbitrarily oriented with respect to the vehicle in which case one must ascertain the validity of the two-dimensional model for oblique incidence. This can be

accomplished by reducing the infinite cylinder to a finite cylinder whose width corresponds to the track width. The backscatter from a finite cylinder using GTD is far too complicated to rigorously solve here. A simpler solution called the Kirchoff or physical optics approximation can be used which assumes that the induced surface currents on the surface of the cylinder can be approximated by

$$\vec{J}_s = 2\hat{n} \times \vec{H}^i \quad (II-68)$$

where

$\hat{n}$ = the normal to the surface, and

$\vec{H}^i$ = the incident magnetic field intensity.

Although this approximation is less accurate than the GTD applied previously. Ross's work [10] indicates that physical optics will provide a sufficiently accurate closed form expression to indicate the width of the backscatter main beam. Since, as the cylinder rotates, the primary energy backscattered comes from the specular reflections from each face, one at a time, it is sufficient to calculate the physical optics backscatter for one plate. The width of the main beam for one plate will approximate that of the whole cylinder. The width of the main beam is determined by the width of the plate in an inverse sense. Figure 2.24 shows the plate in a spherical coordinate system. The incident field of arbitrary polarization is given by

$$\vec{H}^i = H_0(\cos\delta\hat{\theta} + \sin\delta\hat{\phi})e^{jkr} \quad , \quad (II-69)$$

Figure 2.24. A flat conducting plate with incident field of arbitrary polarization.

where

δ = determines the direction of polarization.

The scattered field equation using the physical optics currents can be found in many basic electromagnetics books (Harrington [11]) and is the integral of the product of the induced currents times the free space Green's function evaluated over the surface of the plate. The integral is given by

$$\vec{E}^s(r, \theta, \phi) = \frac{-j\omega\mu}{4\pi r} e^{-jkr} \int_{-a}^a \int_{-b}^b \hat{n} \times \vec{H}^i e^{jk(y' \sin\theta \sin\phi + z' \cos\theta)} ds . \quad (II-70)$$

The backscattered H-field can be found using Maxwell's equations and the final solution is given by

$$\begin{aligned} \overline{H^S}(r, \theta, \phi) = \frac{jke^{-jkr}}{4\pi r} 2H_0 [\hat{\theta} \cos\delta \sin\theta \cos\phi + \hat{\phi} \sin\delta \cos\phi \sin\theta] \\ \times ab \frac{\sin(ka \cos\theta)}{ka \cos\theta} \frac{\sin(kb \sin\theta \sin\phi)}{kb \sin\theta \sin\phi} \end{aligned} \quad (II-71)$$

Now calculating the radar cross section, as defined in Harrington [12], one finds

$$\begin{aligned} \sigma &= \lim_{r \rightarrow \infty} (4\pi r^2 \frac{|\overline{H^S}|^2}{|H^i|^2}) \quad , \text{ or} \\ \sigma &= \frac{4\pi(ab)^2}{\lambda^2} \cos^2\phi \sin^2\theta \left[\frac{\sin(ka \cos\theta)}{ka \cos\theta} \frac{\sin(kb \sin\theta \sin\phi)}{kb \sin\theta \sin\phi} \right]^2 \quad , \end{aligned} \quad (II-72)$$

where

λ = the wavelength in free space.

Being consistent with the GTD model it is assumed that the incident field is vertically polarized. Thus, one can set $\phi = 0$ and σ reduces to,

$$\sigma = \frac{4\pi(ab)^2}{\lambda^2} \sin^2\theta \left[\frac{\sin(ka \cos\theta)}{ka \cos\theta} \right]^2 \quad (II-73)$$

The half-power points are, with $a = 15\lambda$ as seen in Figure 2.25, at $\theta_{3dB} \approx 1^\circ$. Thus, for the model itself, the PAM harmonics could not be seen much beyond $\pm 1^\circ$. The above analysis has been carried out for a perfectly conducting flat plate. In reality, treads are far from flat and often include rubber pads on their surface. This being the case,

Figure 2.25. Normalized backscatter cross section for $\phi = 0^\circ$, $a = 15\lambda$.

the actual beamwidth will probably be broader than $\pm 1^\circ$. Even allowing for the fact that the treads are not flat plates, the PAM harmonics are fairly localized to the cardinal aspects of 0° and 180° .

In summary, this chapter has shown that the tracks of tracked vehicles will produce modulations on the target skin line that can prove useful as features for classification. The tracks can be modeled by 11-sided cylinders at the nose-on and tail-on aspects. The track backscatter will constructively and destructively interfere at the receiver such that harmonics will be suppressed or removed in the spectrum. Additionally, the acceleration and turning of the tracked vehicle will result in smearing and splitting of the spectral harmonics. All of these effects can be shown by the GTD model, thereby lending insight into the feature selection and target classification process.

CHAPTER III

PATTERN RECOGNITION AND TARGET CLASSIFICATION

A. INTRODUCTION

The successful classification of radar targets is a twofold process. First, one must have a good understanding of the electromagnetics, as discussed in Chapter II, to determine features that are unique to specific classes of targets. Secondly, a knowledge of basic pattern recognition principles and classification techniques is necessary to make the best use of the features found through electromagnetic theory.

This chapter will review the basic principles underlying pattern recognition and classification. Then a discussion will be presented on the previous work done for tracked vehicles. Thirdly, features will be selected and assessed for tracked vehicles. Lastly, an algorithm will be proposed for extracting features and making a classification.

B. PATTERN RECOGNITION AND CLASSIFICATION REVIEW

The subject of pattern recognition and classification has been extensively dealt with in the literature (see, for example, Duda and Hart [12] and Tou and Gonzalez [13]).

The basic problem in pattern recognition is graphically represented in Figure 3.1. A measurement system, in our case a radar, makes measurements of a physical object or target. The measurements typically contain information such as the magnitude and phase of the target's backscatter and they are also corrupted by noise. The sources of noise can include ground clutter, external electromagnetic interference, thermal noise, intermodulation harmonics, power supply leakage and distortion due to system nonlinearities. Each measurement can be viewed as a vector $\bar{y}$ in measurement space and often contains more information than is necessary for a classification of the object measured.

The next step is to select features in the measurement space that are germane to the measured targets. A feature selection procedure may begin first with an analog to digital conversion, a transformation of the data (e.g., the Fourier transform) and a dimensionality reduction (e.g., considering only certain frequency bands in the spectrum). Features are then extracted from the remaining data and they constitute the components of the vector $\bar{x}$ in feature space. Patterns are then observed in feature space which, if the proper features are selected, should be distinctive for each class of targets. An example of a pattern in feature space might be the clustering of the feature vectors in different regions. Once patterns are observed, a pattern recognition

Figure 3.1. Feature extraction and pattern classification.

decision rule can be devised that, when applied to vectors in feature space, will decide to which class to assign the target or object under measurement. The best decision rule is the one that minimizes the probability of error in the decision.

One way to quantitatively view the classifier is to use Bayes decision rule. Although in practice a Bayesian model may not be appropriate, i.e., enough samples may not be taken to constitute a good statistical base, nonetheless, it serves to demonstrate basic principles of the theory which can give insight into less probabilistic problems.

Suppose there exists c classes, and ω_i represents the i th class. There is an a priori probability $P(\omega_i)$, which represents our prior knowledge of how often elements of each class are likely to occur. A feature vector $\bar{x}$ from the feature space is a random variable whose state-conditional probability density is $p(\bar{x}|\omega_j)$ for each class ω_j . If

the a priori probability and the state-conditional probability are assumed known, then we can use Bayes rule to derive the probability of occurrence of the class ω_j given $\bar{x}$. The rule gives,

$$P(\omega_j|\bar{x}) = \frac{p(\bar{x}|\omega_j)P(\omega_j)}{p(\bar{x})} \quad (\text{III-1})$$

where

$$p(\bar{x}) = \sum_{j=1}^c p(\bar{x}|\omega_j)P(\omega_j) \quad (\text{III-2})$$

Bayes rule above yields the a posteriori probability $P(\omega_j|\bar{x})$, and this can then be used to determine which class is present. As an example, let us look at a two-class case. If the random variable $\bar{x}$ is one dimensional, we can plot two hypothetical class-conditional probability density functions as seen in Figure 3.2a. If we select the a priori probabilities to be $P(\omega_1) = 2/3$ and $P(\omega_2) = 1/3$, a typical plot in Figure 3.2b would result. For a given $\bar{x}$ the probability of error can be computed as,

$$P(\text{error}|\bar{x}) = \begin{cases} P(\omega_1|\bar{x}) & , \text{ if } \omega_2 \text{ is chosen} \\ P(\omega_2|\bar{x}) & , \text{ if } \omega_1 \text{ is chosen} \end{cases} \quad (\text{III-3})$$

Using this, we can minimize the average probability of error,

$$P(\text{error}) = \int P(\text{error}|\bar{x})p(\bar{x})d\bar{x} \quad (\text{III-4})$$

(a)

(b)

Figure 3.2. Hypothetical class-conditional probability density functions and their resultant a posteriori probabilities using Bayes rule.

Equation (III-4) may be generalized to the Bayes decision rule which yields the minimum probability of error,

$$\text{Decide } \omega_i \text{ if } P(\omega_i|\bar{x}) > P(\omega_j|\bar{x}) \text{ for all } j \neq i \quad (\text{III-5})$$

There are many different ways to represent pattern classifiers. One mentioned is in terms of discriminant functions $g_i(\bar{x})$ where $i = 1, 2, \dots, c$. The classifier in this case is said to assign a feature vector $\bar{x}$ to the class ω_i if

$$g_i(\bar{x}) > g_j(\bar{x}) \text{ for all } j \neq i \quad (\text{III-6})$$

Thus, the classifier is a machine which computes c discriminant functions and assigns the feature vector $\bar{x}$ to the class associated with the largest discriminant.

The choice of discriminant functions is not unique. We can always multiply these functions by a positive constant or bias them with an additive constant and not change the resultant decision. In general we can transform these discriminant functions with a monotonically increasing function, and the resulting classifications are unchanged. The minimum probability of error can be achieved with any of the following discriminant functions;

$$g_j(\bar{x}) = P(\omega_j|\bar{x}) \quad (\text{III-7})$$

$$g_j(\bar{x}) = p(\bar{x}|\omega_j)P(\omega_j) \quad (\text{III-8})$$

$$g_j(\bar{x}) = \log p(\bar{x}|\omega_j) + \log P(\omega_j) \quad (\text{III-9})$$

Even though the discriminant functions can be written in different forms, the decision rules are equivalent. A decision rule in effect divides up the feature space into c decision regions, $R_1, R_2, \dots, R_c$, where $g_j(x) = g_i(x)$ defines the decision boundaries to these regions. These boundaries are surfaces in the n -dimensional feature space.

Again, looking at the error probability for the two-class case, suppose the classifier has partitioned the decision space into two regions; R_1 for class ω_1 and R_2 for class ω_2 . There are two ways in which a classification error can occur, one where x falls in R_1 and class ω_2 is present or x falls in R_2 and class ω_1 is present. The average probability of error can then be found to be

$$P(\text{error}) = \int_{R_1} p(x|\omega_2)P(\omega_2)dx + \int_{R_2} p(x|\omega_1)P(\omega_1)dx \quad (\text{III-10})$$

This is shown in Figure 3.3 where the error is the area under the tails of the density functions. If the discriminant function is given by $p(x|\omega_j)P(\omega_j)$ then the optimum decision boundary will fall where $g_1(x) = g_2(x)$. In this case the decision boundary should be moved to the left so that $p(x|\omega_1)P(\omega_1) = p(x|\omega_2)P(\omega_2)$. This is equivalent to minimizing the area under the tails of the density functions thus minimizing the error.

Here we can see how Bayes rule serves to show us the ideal classifier and some example discriminant functions. Unfortunately, in pattern recognition applications we rarely if ever have this kind of complete knowledge about the probabilistic structure of the problem, i.e., we often do not know the a priori statistics. Additionally, the

Figure 3.3. Components of the probability of error.

Bayesian case deals with unimodal probability densities (have a single local maximum), whereas real world problems often involve multimodal densities. Therefore, we should examine some nonparametric procedures that can be used without assuming the form of the underlying densities.

C. NONPARAMETRIC TECHNIQUES

Since, in general, the statistics of real world data are not known a priori, we must resort to one of two approaches to circumvent this problem. One solution is to make enough measurements to determine some statistical parameters such as the mean and variance. These parameters can be used to approximate the statistics and then use a statistical technique like Bayes decision rule. This approach is called a parametric technique and though ideal, is not cost effective because of the large data base necessary to establish the statistics of the data.

The second basic approach is to use nonparametric methods. In other words, use techniques that are not dependent on the statistical parameters. Several techniques are available and are discussed in [12,13].

We will review two techniques whose basic elements seem to be appropriate for our case; the Nearest Neighbor (NN) rule and the Fisher linear discriminant. To appreciate the value of the NN rule and the Fisher linear discriminant, we must discuss the fundamentals of minimum-distance pattern classification.

D. MINIMUM-DISTANCE PATTERN CLASSIFICATION

Looking at the two-dimensional plot of a feature vector in feature space (Figure 3.4) we wish to determine to what class ω_i to assign the feature vector $\bar{x}$. An intuitively appealing approach would be to make the classification by using distance functions. The reason for the use of distance functions is that the most obvious way of establishing a measure of similarity between feature vectors, which are points in Euclidean space, is by determining their proximity. As in the case of Figure 3.4, it would seem that the feature vector $\bar{x}$ belongs to the class ω_1 solely on the basis that it is closer to the patterns of that class. For distance functions to work well, it is important for the feature vectors in different classes to tend to cluster. The general technique of using distance functions can be termed "minimum-distance pattern classification". When the feature vectors of each class tend to cluster tightly, we may choose one feature vector of each class to be

Figure 3.4. Patterns classifiable by proximity concept.

the prototype for the class. If we have M classes, $\omega_1, \omega_2, \dots, \omega_M$, representable by the prototype vectors $z_1, z_2, \dots, z_M$, the Euclidean distance between an arbitrary feature vector $\bar{x}$ and a prototype is given by

$$D_i = \|\bar{x} - \bar{z}_i\| = \sqrt{(\bar{x} - \bar{z}_i)^t (\bar{x} - \bar{z}_i)} \quad . \quad (\text{III-11})$$

Thus we can calculate the Euclidean distance for each prototype and if $D_i < D_j$ for all $j \neq i$, then assign the feature vector $\bar{x}$ to the class ω_i . This can be called single prototype minimum distance pattern classification.

By squaring all the terms in Equation (III-11) we get

$$D_i^2 = (\bar{x} - \bar{z}_i)^t (\bar{x} - \bar{z}_i) = \bar{x}^t \bar{x} - 2\bar{x}^t \bar{z}_i + \bar{z}_i^t \bar{z}_i \quad (\text{III-12})$$

Here we see that $\bar{x}^t \bar{x}$ is independent of i so that, without altering the classifiability, we may define

$$d_i(\bar{x}) = \bar{x}^t \bar{z}_i - 1/2 \bar{z}_i^t \bar{z}_i \quad (\text{III-13})$$

and assign $\bar{x}$ to ω_i when $d_i(\bar{x}) > d_j(\bar{x})$ for all $i \neq j$. This is simply a linear decision function that is equivalent to a minimum-distance pattern classification. The decision boundary of a two class example in which each class is depicted by a single prototype is shown in Figure 3.5.

Figure 3.5. Decision boundary of two classes characterized by single prototypes.

Instead of using single prototypes we can modify the above method so that each class will tend to cluster about, not one, but N_i prototypes $\bar{z}_i^1, \bar{z}_i^2, \dots, \bar{z}_i^{N_i}$, where N_i is the number of prototypes in the i th class. Under these conditions we can design a classifier similar to the preceding one. Let the distance function between an arbitrary feature vector $\bar{x}$ and class ω_i be denoted by

$$D_i = \min_{\ell} \|\bar{x} - \bar{z}_i^{\ell}\|, \ell = 1, 2, \dots, N_i \quad (\text{III-14})$$

that is, D_i is the smallest of the distances between the feature vector $\bar{x}$ and the N_i prototypes. Compute D_i for all the classes and decide $\bar{x}$ belongs to class i when $D_i < D_j$ for all $j \neq i$. As in the preceding

development, Equation (III-14) may be simplified to be

$$d_i(\bar{x}) = \max_{\ell} \{(\bar{x}^t \bar{z}_i^{\ell} - 1/2 (\bar{z}_i^{\ell})^t \bar{z}_i^{\ell})\}, \ell = 1, 2, \dots, N_i \quad (\text{III-15})$$

and in this case, $\bar{x}$ belongs to ω_i when $d_i(\bar{x}) > d_j(\bar{x})$ for all $j \neq i$.

The above concepts naturally lead us into a classification technique known as the Nearest-Neighbor rule.

E. NEAREST NEIGHBOR RULE

The Nearest Neighbor rule, first proposed by Fix and Hodges [14], is a generalization of the above concepts. Even though using a small number of prototypes and measuring Euclidean distances is intuitively appealing, we are not limited to that distance measure. Suppose we have a set of known patterns $\{\bar{s}_1, \bar{s}_2, \dots, \bar{s}_N\}$ where it is assumed that each pattern belongs to one of the classes $\omega_1, \omega_2, \dots, \omega_M$. We may define a "Nearest Neighbor" (NN) classification rule which assigns a feature vector $\bar{x}$ of unknown classification to the class of its nearest neighbor, where we say $\bar{s}_i \in \{\bar{s}_1, \bar{s}_2, \dots, \bar{s}_N\}$ is a nearest neighbor to $\bar{x}$ if

$$D(\bar{s}_i, \bar{x}) = \min_{\ell} \{D(\bar{s}_{\ell}, \bar{x})\}, \ell = 1, 2, \dots, N. \quad (\text{III-16})$$

Here D is any distance measure definable over the feature space. Some examples of $D(\bar{s}_i, \bar{x})$ are the Euclidean distance previously cited (Equation (III-11)) or perhaps a weighted form of the Euclidean distance where $D(\bar{s}_i, \bar{x}) = \|\bar{P}\bar{s}_i - \bar{x}\| = (\bar{P}\bar{s}_i - \bar{x})^t (\bar{P}\bar{s}_i - \bar{x})$ and P is an arbitrary matrix.

This scheme may be referred to as the 1-NN rule since we make the classification when $\bar{x}$ is nearest to one neighbor. We could also extend this concept and define the q-NN rule where we determine the q nearest neighbors to $\bar{x}$ and make a classification based on majority vote. To avoid ties, q should be odd. The 1-NN rule is equivalent to the multiprototype case in Equation (III-14) if D is the Euclidean distance.

In general, when given M classes, the 1-NN rule is superior to the q-NN rule if all the distances between features of a class are smaller than the distances between features of different classes. Also, the q-NN rule takes more computation time and is therefore not as cost effective. Cover and Hart [15] showed that if the number of samples is large the error probability of the 1-NN is bounded as follows:

$$P_B < P_{e_1} < P_B \left(2 - \frac{M}{M-1} P_B\right) \quad (\text{III-17})$$

where

P_B = Bayes probability of error

P_{e_1} = 1-NN probability of error

M = the number of pattern classes.

Thus, the 1-NN rule can never be better than the Bayes rule in probability of error and never worse than twice the Bayes probability of error.

F. FISHER LINEAR DISCRIMINANT

Another nonparametric technique is called the Fisher linear discriminant. This method has been developed in the interest of reducing the dimensionality of the feature space because methods that are computationally and analytically manageable for low-dimensional spaces can become unwieldy and impractical for higher dimensions.

We can reduce the dimensionality from d dimensions to e dimensions where, $e < d$. The smaller the subspace is, the greater the savings in complexity. Naturally, a trade off exists where as the complexity is reduced a risk is run of an increasing probability of error in detection. If the separation of pattern classes can be preserved, it would be ideal to reduce the dimensionality from d dimensions to one dimension by projecting the d -dimensional data onto a line.

The orientation of the line is critical because samples that cluster well in d -dimensional space may overlap for the wrong projections. Figure 3.6 shows two cases of the projection of some samples in pattern space onto a line. In Figure 3.6a, the clustered samples project in such a way that there is good separation between feature classes and in Figure 3.6b the projection is poorly chosen because classes overlap. By moving the vector $\bar{W}$ around, we should be able to find the optimum orientation.

Suppose we have a set of n d -dimensional samples $\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n$, n_1 in the subset X_1 labelled ω_1 and n_2 in the subset X_2 labelled ω_2 . If we form a linear combination of the components of $\mathbf{x}$, we obtain the scalar

Figure 3.6. Projection of samples in pattern space onto a line.

$$y = \bar{W}^t \bar{x} \quad (\text{III-18})$$

and a corresponding set of n samples $y_1, y_2, \dots, y_n$ divided into the subsets Y_1 and Y_2 . If $\|\bar{W}\| = 1$, then each y_i would be the projection of the corresponding $\bar{x}_i$ onto a line in the direction of $\bar{W}$. The magnitude of $\bar{W}$ is not important but the orientation should be chosen so that the different classes cluster on the line.

A means for determining the degree of separation of the projected samples is necessary so that we can optimize the orientation of $\bar{W}$. A measure of the separation between classes can be determined by the class means given by

$$\bar{m}_i = \frac{1}{n_i} \sum_{x \in X_i} \bar{x} \quad (\text{III-19})$$

Then the sample mean for the projected points is given by

$$\tilde{m}_i = \frac{1}{n_i} \sum_{y \in Y_i} y \quad (\text{III-20})$$

$$= \frac{1}{n_i} \sum_{x \in X_i} \bar{W}^t \bar{x} = \bar{W}^t \bar{m}_i \quad (\text{III-21})$$

So now the separation is, in the 2-dimensional case, $|\tilde{m}_1 - \tilde{m}_2| = |\bar{W}^t (\bar{m}_1 - \bar{m}_2)|$. The separation between means in and of itself would not be sufficient to determine the projected sample separation. We must also weigh the difference of the means against a measure of the projected

variance. This "scatter" is defined as

$$\tilde{s}_i^2 = \sum_{y \in Y_i} (y - \tilde{m}_i)^2 \quad . \quad (\text{III-22})$$

An estimate of the variance of the pooled data is $(\tilde{s}_1^2 + \tilde{s}_2^2)/n$, and $\tilde{s}_1^2 + \tilde{s}_2^2$ is called the total within-class scatter of the projected samples. The Fisher linear discriminant is defined as the linear function $\bar{w}^t \bar{x}$ for which the criterion function

$$J(\bar{w}) = \frac{|\tilde{m}_1 - \tilde{m}_2|^2}{\tilde{s}_1^2 + \tilde{s}_2^2} \quad (\text{III-23})$$

is a maximum.

To obtain J as an explicit function of $\bar{w}$, we define the scatter matrices s_i and s_W by

$$s_i = \sum_{x \in X_i} (\bar{x} - \bar{m}_i)(\bar{x} - \bar{m}_i)^t \quad (\text{III-24})$$

and

$$s_W = s_1 + s_2 \quad (\text{III-25})$$

then using Equation (III-22), we get

$$\begin{aligned}
\tilde{s}_i^2 &= \sum_{x \in X_i} (\bar{w}^t \bar{x} - \bar{w}^t \bar{m}_i)^2 \\
&= \sum_{x \in X_i} \bar{w}^t (\bar{x} - \bar{m}_i) (\bar{x} - \bar{m}_i)^t \bar{w} \\
&= \bar{w}^t s_i \bar{w}
\end{aligned} \tag{III-26}$$

so that,

$$\tilde{s}_1^2 + \tilde{s}_2^2 = \bar{w}^t s_W \bar{w} \tag{III-27}$$

similarly,

$$\begin{aligned}
(\tilde{m}_1 - \tilde{m}_2)^2 &= (\bar{w}^t \bar{m}_1 - \bar{w}^t \bar{m}_2)^2 \\
&= \bar{w}^t (\bar{m}_1 - \bar{m}_2) (\bar{m}_1 - \bar{m}_2)^t \bar{w} \\
&= \bar{w}^t s_B \bar{w}
\end{aligned} \tag{III-28}$$

where

$$s_B = (\bar{m}_1 - \bar{m}_2) (\bar{m}_1 - \bar{m}_2)^t \tag{III-29}$$

The matrix s_W is called the within-class scatter matrix and the matrix s_B is called the between-class scatter matrix.

In terms of s_B and s_W , the criterion function J can be written as,

$$J(\bar{W}) = \frac{\bar{W}^t s_B \bar{W}}{\bar{W}^t s_W \bar{W}} \quad (\text{III-30})$$

This expression in mathematical physics is called the Rayleigh quotient and is maximum when

$$\bar{W} = s_W^{-1} (\bar{m}_1 - \bar{m}_2) \quad (\text{III-31})$$

Thus, we have obtained Fisher's linear discriminant which maximizes the ratio of between-class scatter to within-class scatter. The problem has been converted from one of d -dimensional complexity to one-dimensional complexity. Since the mapping is many-to-one, we cannot in theory reduce the minimum achievable probability of error with this method, in fact we can make it much worse if the clustering is not preserved in the transformation.

The above two techniques have been discussed because the elements in each lend themselves well to classification of ground tactical vehicles. Being nonparametric techniques, they will help to reduce the cost of the large data base necessary for parametric analysis. The particular nonparametric method used will be dictated by the plots of the features and their clustering properties in feature space.

G. PREVIOUS PATTERN CLASSIFICATION OF TRACKED VEHICLES

The most significant attempt to apply pattern classification to tracked vehicles is by Stewart [3]. Stewart attempted to classify tracked vehicles with an algorithm applicable at all aspect angles. His work separates the classes into two groups; feature selection and classification at the cardinal aspects of 0° and 180° and feature selection and classification at all other aspects. Because of the distinctive nature of the PAM harmonics, Stewart tried to use Chuang's model to predict the behavior of the harmonics and found that several sources of uncertainty arose in the actual data that did not exist in Chuang's model [3:37]. Therefore, the harmonics were not considered as a viable feature. Instead, Stewart elected to search for 40 features that were invariant to target parameters such as range, elevation angle, azimuth angle and velocity. These features were then used for classification at all aspects. Since many of the apparent anomalies in the PAM harmonics are now better understood, features can be selected that will be useful for classification of tracked vehicles along the cardinal aspects of 0° and 180° .

H. FEATURE SELECTION FOR TRACKED VEHICLES

The new model for the backscatter from tracked vehicles lends further insight into the behavior of PAM harmonics thus aiding the feature selection process. Equation (II-63) shows that even though harmonics may disappear due to interference, the power is preserved in the sidebands. With the increased understanding of the behavior of the PAM

harmonics, features related to them will be selected for target classification.

The first step in the feature extraction process, for this particular application, is to divide the backscatter spectrum into six bands as depicted in Figure 3.7. Band B_1 is defined as the band including clutter and artifacts such as 60 cycle noise. This band will not be used in the feature extraction process. Next, the doppler frequency (f_d) is found and narrow bands about f_d and $2f_d$, called B_3 and B_5 , are established and are assigned the same width. Band B_6 includes all frequencies between B_5 and $4f_d$. The sidebands B_2 and B_4 are the sidebands adjacent to the skin line doppler and are therefore the bands in which to search for the PAM harmonics.

Since the time domain data will be digitized and processed via the fast fourier transform (FFT), the features will be described in discrete notation. The discretized spectrum will be referred to as $s(n)$.

In order to have a basis for comparing features over a large ensemble of measurement data, it is necessary to choose some reference for normalization. Using the skin line doppler is not adequate since the skin line of all vehicles is not equal, i.e., small vehicles, after normalization by the skin line, would tend to exhibit a higher noise level in the sidebands. Thus, the reference for normalization will be found from a scalar multiple of the rms noise level in band B_6 . The rms noise level is given as

Figure 3.7. Division of spectrum into six bands for feature extraction.

$$N_6 = \sum_{n \in B_6} [s(n)]^2 / \Delta B_6 \quad (\text{III-32})$$

where

ΔB_6 = the width of the band B_6 .

The normalized spectrum will be referred to as $\hat{s}(n)$. With the spectrum normalized, three features may be defined and extracted for use in classification. The first feature is defined as the total number of harmonics detected in bands B_2 and B_4 . A harmonic is said to be present wherever the signal-level within a band exceeds a specified threshold level. The threshold level will be a scalar multiple (c) of the rms noise in B_6 . Thus feature one is the total number of harmonics and is defined as

$$f(1) = T_2 + T_4 \quad (\text{III-33})$$

where

T_2 = total number of harmonics in B_2 above the threshold cN_6

T_4 = total number of harmonics in B_4 above the threshold cN_6 .

The total number of harmonics in the sidebands can vary depending on the phase delay between the tracks and the spacing between treads of the tracked vehicle. Table III-1 below, shows the range of the total number of harmonics in the sidebands for APCs and tanks as a function of the phase delay between the tracks.

TABLE III-1
TOTAL NUMBER OF HARMONICS IN THE BANDS B_2 AND B_4
AS A FUNCTION OF THE VEHICLE TYPE AND THE
PHASE DELAY BETWEEN TRACKS

	TREAD SPACING	MAX. NO. OF HARMONICS PHASE DELAY=0. TREAD	MIN. NO. OF HARMONICS PHASE DELAY=.5 TREAD
APC	6 1/8 in.	18	8
TANK	7 in.	16	8

It can be seen in Table III-1 that both the tank and APC can produce an identical number of harmonics in the spectrum. Therefore, the feature $f(1)$ is not sufficient to distinguish tanks from APCs. The feature $f(1)$, however, should prove useful in distinguishing tracked from wheeled vehicles.

The second feature is based upon the total power in the harmonics. It was noted in Equation (II-64) that even though interference from tracks out of phase might cause the cancelling of harmonics in the spectrum, the total power content in the harmonics is preserved. Let us define the peak signal level of each detected harmonic as

$$P(i,j) = \underset{n \in B_j}{i^{\text{th}} \text{ local Max}}(\hat{S}(n)), \quad i=1,2,\dots,M_j \quad (\text{III-34})$$

That is, if there are M_j harmonics detected in band B_j then $P(i,j)$ is the peak value of the normalized i^{th} harmonic in band B_j . Then we may define the total sum of the signal strengths in the harmonics as

$$f(2) = \sum_{j=2,4} \sum_{i=1}^{M_j} P(i,j). \quad (III-35)$$

Because the energy in the sidebands is preserved, the second feature should prove to be fairly stable even for fluctuations in the total number of harmonics detected.

The third feature will be based upon the frequency separation of the PAM harmonics. Chuang demonstrated that the PAM harmonics, with the exception of those cancelled by interference effects, are located at the frequencies $f_d \pm nTFR$, where TFR is the tread flip rate and n is an integer such that $n = 1, 2, \dots, N \leq f_d/TFR$. The tread flip rate is determined by the vehicle velocity and the tread spacing and is given by

$$TFR = \frac{\text{Vehicle Velocity}}{\text{Tread Spacing}} \quad (III-36)$$

The vehicle velocity, V , may be ascertained from the skin line doppler and is given by

$$V = \frac{f_d C}{2f_0} \quad (III-37)$$

where

f_d = the skin line doppler

c = the speed of light in free space

f_0 = the radar operating frequency.

Thus, if the frequency difference between harmonics in the spectrum is known then the tread spacing is known and the vehicle can be properly classified.

In order to define the third feature, the frequency of each harmonic must be determined. Since we have previously designated the detected PAM harmonics as $P(i,j)$, we may also define the frequency corresponding to each harmonic peak as

$$R(i,j) = n \text{ where } \hat{S}(n) = P(i,j) \quad (\text{III-38})$$

Figure 3.8 shows an example of four harmonics $P(i,j)$ and their corresponding frequencies $R(i,j)$. All of the harmonics are separated from the skin line doppler by an integral multiple of the tread flip rate.

The third feature will be composed of a set of subfeatures designated $f(3,i,j)$. These subfeatures will be defined as the difference frequency between each detected harmonic and the skin line doppler. They are given as

$$f(3,i,j) = |R(i,j) - f_d| \quad (\text{III-39})$$

Figure 3.8. Tracked vehicle backscatter spectrum with four PAM harmonics.

Thus, the total number of subfeatures is equal to the total number of detected harmonics, $T_2 + T_4$, and varies for each new radar measurement.

I. DECISION RULES FOR THE EXTRACTED FEATURES

The features proposed in the previous section were the total number of detected harmonics ($f(1)$), the total signal strength in the detected harmonics ($f(2)$) and a set of subfeatures ($f(3,i,j)$) which consist of the frequency differences between each PAM harmonic and the skin line doppler f_d . Because of the corruptive influence of noise and the wide range of the possible number of harmonics detectable, a decision rule will not be discussed for the first two features until feature plots are made using experimental data. Patterns will then be observed in feature space and the appropriate decision rules will be assigned.

A decision rule can presently be developed, however, for the subfeatures $f(3,i,j)$. As was previously stated, the subfeatures $f(3,i,j)$ are the difference frequencies between each detected harmonic and the skin line doppler. If noise spikes have not been mistakenly detected as PAM harmonics, then each subfeature frequency should be an integral multiple of the tread flip rate. The two tread flip rates possible, one for an APC and one for a tank, are

$$TFR_{apc} = \frac{\text{Vehicle Velocity}}{\text{APC tread spacing}} \quad (\text{III-40})$$

$$TFR_{tank} = \frac{\text{Vehicle Velocity}}{\text{Tank tread spacing}} \quad (\text{III-41})$$

Thus, if an APC is present, each $f(3,i,j)$ should be an integral multiple of TFR_{APC} . If a tank is present, each $f(3,i,j)$ should be an integral multiple of TFR_{tank} . Perturbations will be introduced into the subfeatures through measurement inaccuracy or through random noise spikes exceeding the threshold and being erroneously labeled as harmonics.

With Equations (III-40) and (III-41) acting as prototype features, a comparison can be made of the prototypes with each subfeature, one at a time. The comparison can be accomplished by testing to see which prototype most nearly divides into each subfeature. A convenient tool to accomplish this test is called the modulus and is given for each prototype as,

$$\begin{aligned} g_1(i,j) &= \text{mod}(f(3,i,j), TFR_{apc}) \\ &= |f(3,i,j) - \ell TFR_{apc}| \end{aligned} \quad (III-42)$$

where

ℓ = an integer closest in value to the quantity

$$\frac{f(3,i,j)}{TFR_{apc}}$$

and

$$\begin{aligned} g_2(i,j) &= \text{mod}(f(3,i,j), TFR_{tank}) \\ &= |f(3,i,j) - k TFR_{tank}| \end{aligned} \quad (III-43)$$

where

k = an integer closest in value to the quantity

$$\frac{f(3,i,j)}{TFR_{tank}}$$

Thus, if TFR_{apc} exactly divides into $f(3,i,j)$, then $g_1(i,j) = 0$, or if TFR_{tank} exactly divides into $f(3,i,j)$ then $g_2(i,j) = 0$. Since perturbations will occur in the subfeatures, such as noise spikes or measurement errors, then g_1 and g_2 will be nonzero quantities. The smallest g will reveal to which class the subfeature belongs, i.e., if $g_1 < g_2$ then the subfeature $f(3,i,j)$ belongs to the class of APCs. If $g_2 < g_1$ then the subfeature $f(3,i,j)$ belongs to the class of tanks. A threshold may also be established such that if g_1 and g_2 exceed it then the subfeature $f(3,i,j)$ is not divisible by either prototype and the subfeature does not belong to either class. The threshold will be chosen empirically to minimize the probability of error in classification. All three features and decision rules will be used and discussed in detail in Chapter V.

CHAPTER IV
RADAR MEASUREMENTS OF MOVING
GROUND TACTICAL VEHICLES

A. INTRODUCTION

This chapter will discuss the X-band radar, and a series of experiments that were designed to obtain backscatter measurements of moving ground tactical vehicles. Particular attention is given to the measurements that were designed to find the backscatter at the cardinal aspect angles of 0° and 180° .

The various vehicles used in the experiment will be discussed along with the modifications that were made on the tank for the purpose of isolating the source of the PAM harmonics.

The chapter will close with several plots of the backscatter spectrum obtained for the tank, APC, and truck.

B. X-BAND RADAR SYSTEM

The X-band radar is a pulsed, coherent radar with a 20 KHz pulse repetition frequency (PRF). The radar was operated with the range gate set to cover the entire vehicle maneuvering area. The radar system schematic (taken from Walton [16]) appears in Figure 4.1. The details of its operation are explained in reference [16].

The entire experimental region is within the antenna main beam and the range gate of the radar. The result is that the output of the quadrature phase detector is proportional to the phasor sum of all the scatterers in the vehicle maneuvering area. This signal is filtered by a low pass filter whose frequency cutoff is 5 KHz and the result is stored on analog tape. Thus, the DC value represents the phasor sum of all the stationary scatterers in the radar range. When a moving target enters into the main beam and range gate of the radar, a rotating component of the phasor appears and this term is easily separated from the DC clutter.

The X-band antenna is a vertically polarized horn. It has a 20 dB gain and a beamwidth (3dB) of $\pm 8^\circ$. The entire radar system was mounted on a truck with the horn located on a deployable 40 ft. boom. The truck enabled the radar system to go on sight and provided for a variable elevation angle of the antenna with respect to the target area. Figure 4.2 shows a photograph of the radar truck on sight.

Figure 4.2. Radar van with X-band horn on a deployable 40 ft. boom.

C. MOVING GROUND TACTICAL VEHICLES

The vehicles used for the X-band experiments fall into two categories; tracked vehicles and a wheeled vehicle. The two tracked vehicles were a M113 armored personnel carrier (APC) and an M-60 tank. The two tracked vehicles appear in Figures 4.3 and 4.4. Table IV-1 gives some critical dimensions that will be important later in calculating the tread flip rates (TFR) and the beamwidth of the track component of the backscattered field.

The wheeled vehicle is an M35 five ton truck and it appears in Figure 4.1.

TABLE IV-1

CRITICAL TRACKED VEHICLE DIMENSIONS

	TRACK WIDTH	TREAD SPACING
M-60 TANK	28 in.	7 in.
M113 APC	15 in.	6 1/8 in.

Figure 4.3. Back view of an M-60 tank.

Figure 4.4. An M113 armored personnel carrier.

Figure 4.5. An M35 five ton truck.

D. EXPERIMENTAL VEHICLE MODIFICATIONS

Since PAM harmonics will be considered as features for target classification and since they are presumed to originate from the track moving around the sprocket, a modification of the tank structure will help to verify the PAM harmonics actual source. Thus, in addition to experimental runs with tanks, APC's and trucks, a tank will be run that has aluminum skirts attached to the rear fenders. Figure 4.6 shows the rear view of a tank with skirts attached. These skirts are 3 feet high and cover the most significant portion of treads around the rear sprocket. If the treads moving around the sprocket are the major source of the PAM harmonics, then the fender skirts should reduce or attenuate their presence in the backscatter spectrum.

E. THE EXPERIMENTAL SCENARIO

A careful layout of the pattern that the vehicles followed while maneuvering was necessary to insure that the vehicles indeed were observed along the cardinal aspects. Figure 4.7 shows the pattern for the vehicles to follow. Vehicles entered on leg 1 of the track and proceeded at approximately 10 mph. The track is oriented such that the vehicle moves at aspects of $n45^\circ$, where $n = 0, 1, \dots, 8$, and then turns in a circle around the center. The multiple aspects were used by Gross, et. al., [17] for VHF, UHF and X-band measurements in a global search for features over a broad range of frequencies. The aspect angles of importance to this study are those along legs 7 and 15 or aspects of 0° and 180° .

Figure 4.6. M-60 tank with skirts mounted to shield the sprockets.

Figure 4.7. Experimental track.

The vehicles did not always travel in a straight line down the legs nor did they maintain a consistent velocity. Therefore, we can expect variations in the detected PAM harmonics such as smearing, splitting and even total disappearance of the harmonics if the vehicles veer off of the prescribed path.

In addition to the radar system already described, a video tape was made of each of the experimental runs with a time code recorded on the audio track (for coordination with the radar data). The camera was coincident with the transmitter position so that a verification of the true aspect angle as a function of time can be made.

F. EXPERIMENTAL RESULTS

The experimental analog data tapes were sampled at a 5 K samples/sec. Data for vehicles moving along aspects of 0° and 180° were stored on the computer for processing. The spectrum of the data was generated via the fast Fourier transform and any artifacts consistently found in the data were removed.

Some example spectra for the vehicles appears in Figures 4.8 through IV-11. The data base was rather small for the monostatic backscatter of vehicles along the 0° and 180° aspect angles. In fact, the data used in this study consisted of 12 records of APC backscatter, 12 records of five ton truck backscatter and 48 records of tank backscatter. The tank was run four times. The first and third runs were for the tank in its normal configuration and are labelled runs A and C, respectively. The second run was the tank without a right rear

fender and is labelled run B. The fourth run is the tank with skirts attached and is labelled run D. And the APC and truck runs are labelled runs E and F, respectively.

The number of data points transformed in each case was 2048. Therefore, at a 5 K samples/sec, each record was approximately .5 seconds long. The record length is short enough to ensure that the the vehicles did not change their orientation or velocity significantly thus giving a more stable spectrum.

Figure 4.8 shows two records of the backscatter spectrum for a tank at the 0° and 180° aspect angles. Artifacts have been removed at 158 Hz, 316 Hz, and 747 Hz. The skin line doppler is seen to be approximately 220 Hz and PAM harmonics can also be detected at both aspect angles. Often the harmonics are as much as 20 dB above the noise level.

Figure 4.9 shows two records of the backscatter spectrum for a tank with attached skirts. The aspects are 0° and 180° . The harmonics have virtually disappeared in Figure 4.9a due to the presence of fender skirts but can be seen in Figure 4.9b at the 180° aspect.

Figure 4.10 shows two records of the backscatter spectrum for an APC at the 0° and 180° aspects. Harmonics again can be seen at both aspects but with attenuation of the harmonics in the upper sidebands. This attenuation can be accounted for by the presence of the APC fenders as is discussed in Chapter II.

Figure 4.11 shows two records of the backscatter spectrum for a 5 ton truck at the 0° and 180° aspects. The doppler frequency (f_d) line

Figure 4.8. Spectrum of the backscatter from a normal tank.

Figure 4.9. Spectrum of the backscatter from a tank with fender skirts.

Figure 4.10. Spectrum of the backscatter from an APC.

Figure 4.11. Spectrum of the backscatter from a 5 ton truck.

and the $2f_d$ line can be clearly seen and since tracks are not present there is less energy content in the upper and lower sidebands.

The data is seen to have a good dynamic range, i.e., the skin line can be as much as 40 dB above the noise level. The PAM harmonics are also clearly discernable in the examples given. Therefore, the data taken in the experimental runs should prove reliable in testing the classifiability of the vehicles based upon the PAM harmonic features.

The next chapter will consist of feature extraction using experimental data, the resulting feature plots, decision rules, and a discussion of the classifiability of the vehicles using the prescribed features of Chapter III.

CHAPTER V

CLASSIFIABILITY USING PAM HARMONICS

A. INTRODUCTION

This chapter will discuss the classifiability of the radar targets based upon the extracted features discussed in Chapter III. Since the features in Chapter III are all related to the PAM harmonics, then their clustering properties in feature space will help to demonstrate how well tracked vehicles are characterized by the track backscatter. Since the data set is rather small, the results of this chapter will merely serve to indicate classifiability rather than test rigorously the robustness of any particular classifiers.

First, the spectra of various vehicles will be divided into bands, as indicated in Chapter III. Then the features will be extracted and plotted in feature space. Decision regions will then be established and probabilities of classification and misclassification will be computed. The probabilities can then serve as a measure for indicating classifiability.

B. EXTRACTION OF FEATURES FROM DATA

The feature extraction process begins with dividing the spectrum into bands $B_1, B_2, \dots, B_6$ as depicted in Figure 3.7. The doppler or skin line frequency is found and from it the vehicle velocity and the prototype tread flip rates TFR_{tank} and TFR_{APC} are then determined. The RMS noise level is then found in band B_6 and from it a threshold level is determined. If any signal level in bands B_2 or B_4 is found to be above the threshold, it is designated a PAM harmonic. Each harmonic has a peak value of $P(i,j)$ and a corresponding frequency $R(i,j)$.

The features, described in Chapter III, are then extracted and have been defined as,

1. The total number of harmonics detected in bands B_2 and B_4 is the feature $f(1)$.
2. The sum of the amplitudes of the detected harmonics is the feature $f(2)$.
3. A set of subfeatures is finally extracted and consists of the various difference frequencies between the detected harmonics and the doppler frequency. The number of subfeatures depends on how many harmonics are detected and they are designated as $f(3,i,j)$

Figures 5.1 through 5.12 show the spectra of several vehicles after the bands B_1, B_3, B_5 and B_6 have been removed. The threshold for harmonic detection was empirically determined to minimize the error in harmonic detection and was set at 40 times the rms noise level in band

B₆. The vertical scale is linear and the values of the doppler frequency, vehicle velocity, TFR_{tank} , TFR_{APC} , total number of detected harmonics, and the sum of the harmonic amplitudes is given beside each data record.

Figures 5.1 through 5.6 show the spectra for three different runs of the tank, labelled Tanks A, B and C. Both cardinal cuts were taken for each run, i.e., the vehicle was receding at 0° and approaching at 180°. In all cases, PAM harmonics are visible in the sidebands. Figures 5.7 and 5.8 show the spectra for a tank with skirts (tank D) mounted on the back of the tank to shield the sprocket area. The sidebands have changed shape considerably and the PAM harmonics are less visible. This result helps to verify that the PAM harmonics indeed originate from the track rolling around the sprocket. Figures 5.9 and 5.10 show the spectra of an APC (APC E) Again PAM harmonics are observable in the sidebands. Lastly, Figures 5.11 and 5.12 show the spectra for a 5 ton truck (truck F). Here the sidebands are noticeably different than that of tracked vehicles.

C. FEATURE PLOTS

Once the features $f(1)$, $f(2)$ and $f(3,i,j)$ are extracted, plots can be made in feature space to observe patterns, determine decision rules and observe the general classifiability of tracked vehicles when characterized by track backscatter. Since the subfeatures, $f(3,i,j)$, vary in number according to the number of harmonics detected, separate plots will be made for the pair of features $f(1)$, $f(2)$ and transformations of the subfeatures $f(3,i,j)$.

Figure 5.1. The backscatter spectrum of tank A at the 0° aspect after the bands B₁, B₃, B₅ and B₆ have been removed.

Figure 5.2. The backscatter spectrum of tank A at the 180° aspect after the bands B₁, B₃, B₅ and B₆ have been removed.

Figure 5.3. The backscatter spectrum of tank B at the 0° aspect after the bands B₁, B₃, B₅ and B₆ have been removed.

Figure 5.4. The backscatter spectrum of tank B at the 180° aspect after the bands B₁, B₃, B₅ and B₆ have been removed.

Figure 5.5. The backscatter spectrum of tank C at the 0° aspect after the bands B₁, B₃, B₅ and B₆ have been removed.

Figure 5.6. The backscatter spectrum of tank C at the 180° aspect after the bands B₁, B₃, B₅ and B₆ have been removed.

Figure 5.7. The backscatter spectrum of tank D at the 0° aspect after the bands B₁, B₃, B₅ and B₆ have been removed.

Figure 5.8. The backscatter spectrum of tank D at the 180° aspect after the bands B₁, B₃, B₅ and B₆ have been removed.

Figure 5.9. The backscatter spectrum of APC E at the 0° aspect after the bands B₁, B₃, B₅ and B₆ have been removed.

Figure 5.10. The backscatter spectrum of APC E at the 180° aspect after the bands B₁, B₃, B₅ and B₆ have been removed.

Figure 5.11. The backscatter spectrum of truck F at the 0° aspect after the bands B₁, B₃, B₅ and B₆ have been removed.

Figure 5.12. The backscatter spectrum of truck F at the 180° aspect after the bands B₁, B₃, B₅ and B₆ have been removed.

Figure 5.13 shows a plot of the features $f(1)$ and $f(2)$ for four tanks, an APC and a truck at the 0° aspect angle. The tracked vehicles in general seem to cluster at the higher harmonic levels. The reason that PAM harmonics were counted at all for the truck is due to spurious noise components rising above the harmonic threshold. With the clustering properties observed in Figure 5.13 a decision boundary can be established and we can roughly determine the probabilities of classification and misclassification. Again, since the data base is so small, the probabilities calculated can only serve as indicators rather than as sound statistical results.

If a decision boundary is taken to be the horizontal line that intersects the $f(1)$ axis at 4.5, then Table V-1 shows the resulting probabilities. Only two classes can be established at this point: tracked vehicles and wheeled vehicles.

It is seen that the unmodified tanks can be correctly classified as tracked vehicles the majority of the time. If the decision is to be made by majority vote after six records, then the unmodified tanks could be correctly classified everytime. The tank with skirts was never correctly classified at that aspect owing to the presence of the skirts. The APC could only be correctly classified at this aspect once out of six tries. The truck clustered well enough in the lower harmonics to allow classification five out of six times.

Figure 5.14 shows the feature plot for the same vehicles, but at the 180° aspect. If the decision boundary is taken to be the horizontal line that intersects the $f(1)$ axis at 3.5, then the resulting probabilities can be seen in Table V-2. Here the overall results have

Figure 5.13. Features $f(1)$ and $f(2)$ for six targets at the 0° aspect.

Figure 5.14. Features $f(1)$ and $f(2)$ for six targets at the 180° aspect.

TABLE V-1
PROBABILITIES RESULTING FROM FIGURE 5.13

VEHICLE	PROBABILITY OF	
	CLASSIFICATION	MISCLASSIFICATION
A	5/6	1/6
B	5/6	1/6
C	1	0
D	0	1
E	1/6	5/6
F	5/6	1/6

TABLE V-2
PROBABILITIES RESULTING FROM FIGURE 5.14

VEHICLE	PROBABILITY OF	
	CLASSIFICATION	MISCLASSIFICATION
A	5/6	1/6
B	1	0
C	1	0
D	2/6	4/6
E	1	0
F	5/6	1/6

improved in that the vehicles are more easily classified. The normal tanks are classified as tracked vehicles virtually everytime. The tank with skirts still proved difficult to classify correctly. The APC was classified correctly everytime. The disparity in classification for the APC at the two aspects is due in part to the fact that the APC's rear fenders shield the tracks more than the front fenders do, thus removing more lines in the spectrum.

The plots of $f(1)$ vs. $f(2)$ do indicate, for this limited data base, that tracked vehicles can be separated from wheeled vehicles based upon the PAM harmonics in the spectrum. The tanks are seen to be the most classifiable of the two tracked vehicles. But the features $f(1)$ and $f(2)$ are not sufficient for distinguishing tanks from APCs or, for that matter, distinguishing one type of tracked vehicle from the next. The subfeatures $f(3,i,j)$ will provide for discrimination between tanks and APCs.

The subfeatures $f(3,i,j)$ do not lend themselves well to plotting because they vary with each data record. Rather than plot them as they are and then consider any particular decision rule, advantage can be taken of the prototype features given by Equations (III-40) and (III-41). If an APC or a tank is present, then one of the two prototypes should evenly divide into each subfeature $f(3,i,j)$. The closeness of the division can be gauged by taking the modules of the subfeatures and both prototypes. (This method has been discussed in detail in Chapter III.) If the module is below a given threshold value, then that particular subfeature $f(3,i,j)$ is declared to be a valid PAM harmonic. Thus, for one record, all of the subfeatures $f(3,i,j)$ are

divided by the prototype TFR_{tank} . The number of subfeatures that prove to be even multiples of TFR_{tank} are then totaled. The same process is repeated using the prototype TFR_{APC} . The number of subfeatures that are even multiples of TFR_{APC} are also totaled. These two totals are then plotted in Figures 5.15 and 5.16 for the vehicles A, B, C, D, E and F.

In Figure 5.15, the vehicles are all receding at the 0° aspect. It can be seen that there are clustering properties such that three decision boundaries may be established. If the total number of subfeatures divisible by TFR_{APC} is greater than the total number of subfeatures divisible by TFR_{tank} then an APC is declared to be present. If the total number of subfeatures divisible by TFR_{tank} is greater than the total number of subfeatures divisible by TFR_{APC} , then a tank is declared to be present. If there is a tie no decision can be made and if both totals fall below 2 then a tracked vehicle is assumed to not be present. Again, the probabilities of classification and misclassification can be established for vehicles at 0° and the probabilities are listed in Table V-3. Table V-3 shows that APCs and tanks can be correctly classified as APCs or tanks the majority of the time. The vehicle D is the tank with skirts covering the rear sprockets and due to the presence of the skirts cannot be properly classified the majority of the time.

In Figure 5.16, the vehicles are all approaching at the 180° aspect. The same decision boundaries can be established as in Figure 5.15 and Table V-4 shows the resulting probabilities and ties. Table V-4 shows that the APCs and tanks can readily be properly classified by

Figure 5.15. The total number of subfeatures divisible by the prototype features for six vehicles at the 0° aspect.

Figure 5.16. The total number of subfeatures divisible by the prototype features for six vehicles at the 180° aspect.

TABLE V-3
PROBABILITIES RESULTING FROM FIGURE 5.15

VEHICLE	PROBABILITY OF		
	CLASSIFICATION	MISCLASSIFICATION	TIES
A	4/6	0	2
B	5/6	1/6	0
C	1	0	0
D	1/6	5/6	0
E	5/6	1/6	0
F	2/6	4/6	0

TABLE V-4
PROBABILITIES RESULTING FROM FIGURE 5.16

VEHICLE	PROBABILITY OF		
	CLASSIFICATION	MISCLASSIFICATION	TIES
A	5/6	0	1
B	1	0	0
C	1	0	0
D	5/6	1/6	0
E	4/6	1/6	1
F	1	0	0

majority vote and that the two tracked vehicles can be easily discriminated.

Based on the properties of the features $f(1)$, $f(2)$ and $f(3,i,j)$ it appears that the features $f(1)$ and $f(2)$ can be used to distinguish tracked vehicles from wheeled vehicles. Then after that decision has been made, the features $f(3,i,j)$ can be used in cascade with $f(1)$ and $f(2)$ to distinguish between different types of tracked vehicles.

D. CONCLUSION ON CLASSIFIABILITY

The previous tables of probabilities of classification and misclassification indicate that the features $f(1)$ and $f(2)$ can serve well to discriminate wheeled vehicles from tracked vehicles. The tables also indicate that the subfeatures $f(3,i,j)$ can serve well for discriminating between different types of tracked vehicles such as APCs and tanks. Since all of the above features are extracted from the PAM harmonics, then the features show that the backscatter from tracked vehicles can be adequately characterized by the backscatter from the tracks when viewed from the 0° or 180° aspect angles.

CHAPTER VI

SUMMARY AND CONCLUSIONS

This study showed that the electromagnetic backscatter of moving tracked vehicles can be characterized by the backscatter of the tracks alone. This is so because the tracks periodically amplitude modulate the backscattered return producing harmonics in the spectrum that can be reliably used for classification.

The electromagnetic backscatter from a track is first approximated by the backscatter from an infinite 11-sided rolling polygonal cylinder. The cylinder sides represent the track treads rolling around the sprocket; as is the case for tanks and APCs. Two cylinders are then used representing the two tracks and are allowed to move with a variable velocity, acceleration and independently of each other so as to simulate the tracked vehicle moving, accelerating and turning. The electromagnetic backscatter from the cylinders is then computed by means of a high frequency technique called the geometric theory of diffraction (GTD). GTD is used to calculate not only the direct backscatter but also the interaction between the cylinders and the ground plane. The

rolling motion of the cylinders was shown to periodically amplitude modulate (PAM) the radar return and as a consequence produced PAM harmonics in the spectrum. The harmonics are clearly visible when the vehicle moves at constant velocities but are frequency or amplitude modulated as the vehicle accelerates or turns. The frequency modulation from acceleration splits the harmonics. Whereas the amplitude modulation from the vehicle turning or the tracks being out of synchronism, will cancel various harmonics in the spectrum.

Previous models did not include the above behavior of the harmonics and consequently the models significantly diverged from the experimental data. The mechanisms included in this study produced a PAM model matching closely the experimental results and thereby provided viable features for classification. Several features for classification were extracted from the PAM harmonics. The first feature $f(1)$ was defined as the total number of PAM harmonics detected in the spectrum of the backscatter. The second feature $f(2)$ was defined as the sum of the detected harmonic amplitudes. And the last was a set of subfeatures $f(3,i,j)$ which were defined as the frequency differences between each harmonic and the skin line doppler.

Experiments were run with an X-band radar system in which a tank, APC and a truck were observed along the 0° and 180° aspects to the radar. The tank was also modified by hanging conducting skirts around the rear drive sprockets. The skirts were designed to shield the track modulations and thus verify the hypothesis that the PAM harmonics originated at the tracks. Several data records were then digitized and and fourier transformed for computer analysis.

The features previously defined were extracted from the experimental data and were plotted in feature space. The features $f(1)$ and $f(2)$ were plotted and the subfeatures $f(3,i,j)$ were first modified by a set of prototypes summed and then plotted. Clustering properties were observed in the feature plots and decision boundaries were then drawn to maximize the probability of correct classification for each vehicle. It was noted that the features $f(1)$ and $f(2)$ served to discriminate tracked from wheeled vehicles and the subfeatures $f(3,i,j)$ served to discriminate between different classes of tracked vehicles. Thus, the two sets of features could be used in cascade to properly classify tanks, APCs and trucks.

Tables of probabilities were then constructed from the feature plots and, though the data base was limited, the probabilities indicated that tracked vehicles can be reliably classifiable based upon features extracted from the PAM harmonics. Thus the PAM harmonics, resulting from the track backscatter, do characterize well the backscatter from moving tracked vehicles.

It is recommended that future work be focused on three areas. First of all, that a larger data base be established. Even though the above data base served to indicate classifiability, it was not large enough to draw solid statistical conclusions on the expected probabilities of classification and misclassification. Secondly, with a larger data base, various classifiers should be tested to maximize the probability of classification for each vehicle. And lastly, since previous efforts to classify tracked vehicles at all aspect angles were based upon a set of features empirically derived, it is recommended that

a three dimensional electromagnetic model be devised for all aspects. Features could then be selected based upon an understanding of the physics involved and thus maximize the probability of classification.

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APPENDIX I

The computer program, "TRACK", is written to calculate the backscatter from a rotating N-sided polygonal cylinder.

```

C      THIS PROGRAM FINDS THE KELLER SINGLE DIFFRACTION
C      AND THE DOUBLE DIFFRACTION TERMS PLUS THE MULTIPLE
C      DIFFRACTION DIFFRACTION TERMS BETWEEN TWO 11 SIDED
C      CYLINDERS COLLOCATED AND THE GROUND PLANE.
C
C      VARIABLES TO ENTER ARE THE VELOCITY, ACCELERATION, THE
C      DELAY BETWEEN THE CYLINDERS, WETHER TO USE A TAPERED
C      WINDOW AND WHICH MODES OF PLOTTING.
C
      COMMON NS,SKA,VT,AT,TS
      COMPLEX AS(2048)
      DIMENSION AG(2048),T(2048),A(4096),S(1024),F(2048),F1(2048)
C,F2(2048)
      NS=11
      SKA=53.
      TYPE 18
18      FORMAT($,' DO YOU WANT THE TIME PLOT?(0,1)')
      ACCEPT 38,MT
      TYPE 19
19      FORMAT($,' DO YOU WANT HIGHER ORDER TERMS?(0,1)')
      ACCEPT 38,MM
      TYPE 20
20      FORMAT($,' VELOCITY OF THE TANK=')
      ACCEPT 33,VT
      TYPE 25
25      FORMAT($,' ACCELERATION OF THE TANK=')
      ACCEPT 33,AT
      TYPE 35
35      FORMAT($,' PHASE DIFF BETWEEN TRACKS(DEL*ZET:DEL=.1,...,1.)=')
      ACCEPT 33,DEL
      NP=238
33      FORMAT(F7.4)
      TYPE34
34      FORMAT($,' THE ANGLE OF THE SOURCE ABOVE GROUND(0 TO 180)=')
      ACCEPT 33,PHIP
      PHI=PHIP
      TYPE 39
39      FORMAT($,' DO YOU WISH A COSINE TAPER?(1=YES,0=NO)')
      ACCEPT 38,NHA
      TYPE 37
37      FORMAT($,' WHAT PERCENT TAPER ON THE TIME WINDOW?')
      ACCEPT 38,NN
38      FORMAT(I3)
      TYPE 40
40      FORMAT($,' DO YOU WISH DIFFERENT TRACK VELOCITIES?')
      ACCEPT 38,ND
      PI=3.141592654
      TPI=2.*PI
      PHI=PHI*PI/180.
      PHIP=PHIP*PI/180.
      TPI=2.*PI
      RH=7.
      RA=1.03
      SS=SKA*RH*SIN(PHIP)/RA
      DO 100 I=1,2048
100     AS(I)=CMPLX(0.,0.)
C-----
C      CALL HARD CYLINDER DIFFRACTION FOR EACH COMBINATION OF DISTATIC
C      ANGLES THAT ARISE DUE TO HAVING AN IMAGE ABOVE A GROUND PLANE
C      THE SAME PROCESS FOR HIGHER ORDER DIFFRACTION.
C      ADDITIONALLY, BY SPECIFYING THE PHASE SHIFT ON WHICH TO START THE
C      ROTATION OF THE CYLINDER, WE CAN ADD IN THE EFFECT OF THE SECOND
C      TRACK BEING OUT OF PHASE WITH THE FIRST.

```



```

C-----
      V1=.99*VT
      V2=1.01*VT
      CALL HARCYL(AS,T,PHIP,PHIP,0.,0.,0.)
      WRITE(6,500) VT,TS
500    FORMAT(2(F10.4,2X))
C
C    USE THESE TERMS IF THE ELEVATION ANGLE IS GREATER THAN 0 DEGREES.
C
C      CALL HARCYL(AS,T,PHIP,-PHIP,0.,SS,0.)
C      CALL HARCYL(AS,T,-PHIP,-PHIP,SS,SS,0.)
C      IF(MM.EQ.1)CALL HIGHER(AS,T,PHIP,PHIP,0.,0.,0.)
      IF(DEL.GT.0.)CALL HARCYL(AS,T,PHIP,PHIP,0.,0.,DEL)
      IF(DEL.GT.0..AND.MM.EQ.1)CALL HIGHER(AS,T,PHIP,PHIP,0.,0.,DEL)
      WRITE(6,500)VT,TS
C      CALL HIGHER(AS,T,PHIP,-PHIP,0.,SS,0.)
C      CALL HIGHER(AS,T,-PHIP,-PHIP,SS,SS,0.)
      IF(NHA)CALL TAPER(AS,NN)
      IF(V1.EQ.0) GO TO 600
      TS1=TPI*RA/V1
600    IF(AT.NE.0.) TS1=(-V1+(V1**2+4.*PI*AT*RA)**.5)/AT
      IF(V2.EQ.0) GO TO 800
      TS2=TPI*RA/V2
800    IF(AT.NE.0.) TS2=(-V2+(V2**2+4.*PI*AT*RA)**.5)/AT
      DO 4 J=1,2048
      AG(J)=CABS(AS(J))
      A(2*J-1)=REAL(AS(J))
      A(2*J)=AIMAG(AS(J))
      F1(J)=FLOAT(J-1)/TS1
      F2(J)=FLOAT(J-1)/TS2
4      CONTINUE
      IF(MT)CALL PLTPKG(T,AG,2048,2048,1,0,1)
C
C    CALCULATE THE FOURIER TRANSFORM OF THE BACKSCATTER
C
      CALL FORT(A,11,S,-1,IFERR)
      DO 6 J=1,2048
      AS(J)=CMPLX(A(2*J-1),A(2*J))+.0001
      AG(J)=20.*ALOG10(CABS(AS(J)))+70.
6      IF(AG(J).LT.0.)AG(J)=0.
      CALL VPLOTS(0,0,0)
      CALL PLOT(2.5,3.,-3)
      CALL FACTOR(.725)
      CALL FFAXIS(0.,0.,9HFREQUENCY,-9,5.,0.,0.,100.,.5,0,1.,4)
      CALL FFAXIS(0.,0.,9HMAGNITUDE,9,3.7,90.,-70.
      *,21.7,.461,-1,1.,4)
      CALL LINE(F1,0.,100.,AG,0.,21.7,240,0,0)
      CALL LINE(F2,0.,100.,AG,0.,21.7,240,0,0)
      CALL PLOT(0.,0.,+999)
      CALL EXIT
      END
      SUBROUTINE TAPER(X,NN)
      COMPLEX X(2048)
C---THIS SUBROUTINE APPLIES A COSINE TAPER TO THE DATA
      PER=FLOAT(NN)/100.
      PER=2048.*PER
      NL=PER/2.
      NH=2048-NL
      A=2.*3.141592654/PER
      DO 1 I=1,2048
      IF(I.GT.NL.AND.I.LT.NH) GO TO 1
      J=I-1
      IF(I.GE.NH)J=I-NH+NL-1

```

```
1      X(I)=X(I)*(.5-.5*COS(A*J))  
      CONTINUE  
      RETURN  
      END
```

APPENDIX II

The subroutine, "HARCYL", is written to calculate the single and double diffraction terms for the backscatter of an N-sided polygonal cylinder.

```

      SUBROUTINE HARCYL(AH,T,PHIP,PHI,SSP,SS,DEL)
C
C-----THE PARAMETERS PASSED ARE DEFINED AS FOLLOWS:
C-----AH- THE COMPLEX DATA ARRAY
C-----T - THE TIME BASE
C-----PHIP-THE ANGLE OF THE INCIDENT FIELD
C-----PHI- THE ANGLE OF THE RECEIVER
C-----SSP- THE PHASE DELAY FROM THE SOURCE
C-----SS- THE PHASE DELAY RELATIVE TO THE RECEIVER
C-----DEL- THE PHASE DELAY BETWEEN THE TRACKS,I.E.,THE PHASE BETWEEN
C----- THE PLATES
C
C      THIS SUBROUTINE GENERATES THE SINGLE DIFFRACTION AND
C      DOUBLE DIFFRACTION TERMS NECESSARY TO COMPUTE THE BACKSCATTER
C      FOR AN N SIDED POLYGONAL CYLINDER.
C
C      YOU MUST PASS THE NO. OF SIDES, KA,VEL.,ACCEL..
C
      COMMON NS,SKA,VT,AT,TS
      COMPLEX AH(2048),HA,TEMP1,T1,T2,D(4)
      DIMENSION T(2048)
      WRITE(6,500) VT
      FORMAT(F13.5)
500    RA=1.03
      PI=3.141592654
      TPI=2.*PI
      ZET=PI/FLOAT(NS)
      FNS=FLOAT(NS)
      WN=(2.+FNS)/FNS
      SMA=.0004
      RK=TPI
      WW=2.*SKA*SIN(ZET)
      W=WW/TPI
      EPSR=6.
C
C-----CALCULATE THE REFLECTION COEFFICIENT GAMMA FOR THE
C-----GROUND- IN THIS CASE IT IS GRAVEL WITH EPSR=6
C
      GAMMA=(EPSR*SIN(PHIP)-(EPSR-COS(PHIP)**2)**.5)/(EPSR*SIN(PHIP)-(EPSR
C-COS(PHIP)**2)**.5)
      IF(PHIP.EQ.PHI.AND.PHI.GE.0.) GAMMA=1.
      GAMMA=1.
C
C---IF WE LOOK AT THE BISTATIC SCATTERING FOR THE GROUND TO THE
C---CYLINDER TO THE RECEIVER IT IS IDENTICAL TO THE OPPOSITE
C---PATH SO SIMPLY MULTIPLY BY 2.
C
      IF(PHIP.NE.PHI)GAMMA=2.*GAMMA
      IF(PHIP.EQ.PHI.AND.PHI.LT.0.) GAMMA=GAMMA**2.
C
C----- FACTOR IN THE EFFECTS OF VELOCITY AND ACCELERATION
C----- ON THE PERIOD OF ROTATION OF THE CYLINDER
C
      IF(VT.EQ.0.) GO TO 40
      TS=TPI*RA/(VT)
40    IF(AT.NE.0.)TS=(-VT+(VT**2+4.*PI*AT*RA)**.5)/AT
      DO 4 J=1,2048
      HA=CMPLX(0.,0.)
      T(J)=TS*(J-1.)/2048.
      TH=(VT*T(J)+AT*T(J)**2/2.)/RA+DEL*ZET+.0004
C
C-----NOW REVERSE THE DIRECTION OF SPIN
      TH=TPI-TH

```

```

C
C-----CALCULATE THE DOPPLER SHIFT
C
      DT=(COS(PHI)+COS(PHIP))*SKA*(VT+.5*AT*T(J))*T(J)/RA
      DO 5 I=1,NS
      Z1=2.*I*ZET-(PHI+PHIP)/2.
      IF(Z1.LT.0.) Z1=Z1+TPI
      IF(Z1.GT.TPI) Z1=Z1-TPI
      DIF=ABS(TH-Z1)
      IF(DIF.LT.SMA) GO TO 45
      GO TO 46
45      IF(TH-Z1) 47,48,48
47      TH=TH+DIF+SMA
      GO TO 46
48      TH=TH+SMA-DIF
46      AN=PI/2.-2.*(I-1.)*ZET+TH+PHIP
      BN=PI+2.*ZET-AN
      CN=PI-AN
      DN=PI-BN
      EN=PI/2.-2.*(I-1.)*ZET+TH+PHI
      FN=PI+2.*ZET-EN
      S1=SKA*COS((2.*I-1.)*ZET-TH-PHIP)+SSP
      S2=SKA*COS((2.*I-1.)*ZET-TH-PHI)+SS
      SNM1=SKA*COS((2.*I-3.)*ZET-TH-PHI)+SS
      SNP1=SKA*COS((2*I+1.)*ZET-TH-PHI)+SS
      SS1=S2+SNM1-WW
      SS2=S2+SNP1-WW

C
C THIS SECTION COMPUTES THE SINGLE DIFFRACTION TERM
C
      IF(AN.LT.0.) AN=AN+TPI
      IF(AN.GT.TPI) AN=AN-TPI
      IF(AN.GE.WN*PI) GO TO 5
      IF(EN.LT.0.) EN=EN+TPI
      IF(EN.GT.TPI) EN=EN-TPI
      IF(EN.GE.WN*PI) GO TO 5

C
C----- KELLER SINGLE DIFFRACTION TERMS
C
      CS1=COS(PI/WN)
      CS2=COS((EN-AN)/WN)
      CS3=COS((EN+AN)/WN)
      EX=S1+S2-PI/4.
      TEMPL=(1./((CS1-CS2)+1./((CS1-CS3))*CMPLX(1.,0.))
      TEMPL=TEMPL*SIN(PI/WN)*CEXP(CMPLX(0.,EX))/(WN*(TPI*WK)**.5)
      HA=HA+TEMPL

C
C-----THIS SECTION COMPUTES THE DOUBLE DIFFRACTION FROM THE N-1 EDGE
C
      IF(CN.LT.0.) CN=CN+TPI
      IF(CN.GT.TPI) CN=CN-TPI
      IF(CN.GE.PI*WN) GO TO 7
      CALL WD(D,W,0.,CN,1.,WN)
      T1=(0.,0.)
      DO 8 K=1,4
      T1=T1+D(K)
8      CONTINUE
      CALL WD(D,W,EN,0.,1.,WN)
      T2=(0.,0.)
      DO 9 K=1,4
      T2=T2+D(K)
9      CONTINUE
      HA=HA+.5*T1*T2*CEXP(CMPLX(0.,SS1))/(W**.5)

```

```

C
C-----THIS SECTION COMPUTES THE DOUBLE DIFFRACTION FROM THE N+1 EDGE
C
7      IF(FN.LT.0.) FN=FN+TPI
      IF(FN.GT.TPI) FN=FN-TPI
      IF(FN.GE.WN*PI) GO TO 5
      IF(DN.LT.0.) DN=DN+TPI
      IF(DN.GT.TPI) DN=DN-TPI
      IF(DN.GE.WN*PI) GO TO 5
      CALL WD(D,W,0.,DN,1.,WN)
      T1=(0.,0.)
      DO 13 K=1,4
      T1=T1+D(K)
13     CONTINUE
      CALL WD(D,W,FN,0.,1.,WN)
      T2=(0.,0.)
      DO 14 K=1,4
      T2=T2+D(K)
14     CONTINUE
      HA=HA+.5*T1*T2*CEXP(CMPLX(0.,SS2))/(W**.5)
5      CONTINUE
      AH(J)=AH(J)+GAMMA*HA*CEXP(CMPLX(0.,DT))
4      CONTINUE
      RETURN
      END

```

APPENDIX III

The subroutine, "HIGHER", is written to calculate the track/ground plane interaction mechanisms for the backscatter of an N-sided polygonal cylinder.

```

      SUBROUTINE HIGHER(AH,T,PHIP,PHI,SP,S,DEL)
C     THIS SUBROUTINE GENERATES THE MULTIPLE DIFFRACTIONS FROM THE
C     CYLINDER TO THE GROUND PLANE AND BACK.
C
      COMMON NS,SKA,VT,AT,TS
      COMPLEX AH(2048),HA,TENP1,T1,T2,D(4)
      DIMENSION T(2048),Z1(16)
      RH=7.
      RA=1.03
      PI=3.141592654
      TPI=2.*PI
      ZET=PI/FLOAT(NS)
      AL=PI/2.-ZET
      FNS=FLOAT(NS)
      WN=(2.+FNS)/FNS
      XY1=.03
      XY2=.075
      SMA=.0004
      RK=TPI
      WW=2.*RA*SIN(ZET)
      W=WW/TPI
      TE1=PI
      TE2=PI+4.*ZET
      EPSR=.6.
      PP=ABS(PHIP)

C-----ADD IN THE REFLECTION COEFFICIENT FOR THE GROUND, IN THIS
C-----CASE IT IS GRAVEL WITH ER=6*E0
C
      GAMMA=(EPSR*SIN(PP)-(EPSR-COS(PP)**2)**.5)/(EPSR*SIN(PP)+(EPSR
C-COS(PP)**2)**.5)
      IF(PHIP.EQ.PHI.AND.PHI.GE.0.) GAMMA=1.
      IF(PHIP.NE.PHI) GAMMA=2.*GAMMA
      IF(PHIP.EQ.PHI.AND.PHI.LT.0.) GAMMA=GAMMA**2.
      GAMMA=1.

C-----FACTOR IN THE EFFECTS OF VELOCITY AND ACCELERATION
C-----ON THE PERIOD OF ROTATION OF THE CYLINDER
C
      IF(VT.EQ.0.) GO TO 40
      TS=TPI*RA/(VT)
40    IF(AT.NE.0.) TS=(-VT+(VT**2+4.*PI*AT*RA)**.5)/AT
      DO 4 J=1,2048
      HA=CMPLX(0.,0.)
      T(J)=TS*(J-1.)/(2048.)
      TH=(VT*T(J)+AT*T(J)**2/2.)/RA+DEL*ZET+.0008
      TTT=180.*TH/PI

C-----CALCULATE THE DOPPLER SHIFT
C
      DT=(COS(PHI)+COS(PHIP))*SKA*(VT+.5*AT*T(J))*T(J)/RA
      L=0
      DO 6 I=1,NS
      Z1(1)=-2.*I*ZET+(PHIP-PHI)/2.
      Z1(2)=-PI/4.-(2.*I-1)*ZET+(PHI+PHIP)/4.
      Z1(3)=-PI-2.*(I-1)*ZET+(PHIP-PHI)/2.
      Z1(4)=-PI/4.-2.*I*ZET+(PHI+PHIP)/4.
      Z1(5)=-PI-2.*I*ZET+(PHIP-PHI)/2.
      Z1(6)=PI/4.-(2*I-1)*ZET+(PHIP+PHI)/4.
      Z1(7)=-2*(I-1)*ZET+(PHIP-PHI)/2.
      Z1(8)=PI/4.-2.*I*ZET+(PHIP+PHI)/4.
      Z1(9)=PI/4.-2.*I*ZET+(PHIP-PHI)/2.
      Z1(10)=-3.*PI/4.-2.*I*ZET+(PHIP-PHI)/2.

```



```

      Z1(11)=-PI/4.-2.*(I-1.)*ZET+(PHIP+PHI)/4.
      Z1(12)=-PI/4.-2.*(I-1.)*ZET+PHI/2.
      Z1(13)=3.*PI/4.-2.*(I-1.)*ZET+PHI/2.
      Z1(14)=PI/4.-2.*(I-1)*ZET+(PHIP+PHI)/4.
      Z1(15)=PI/4.-(2*I+1)*ZET+(PHIP+PHI)/4.
      Z1(16)=-PI/4.-(2*I+1)*ZET+(PHIP+PHI)/4.
      DO 5000 M=1,16
      Z1(M)=AMOD(Z1(M),TPI)
      IF(Z1(M).LT.0.) Z1(M)=Z1(M)+TPI
      DIF=ABS(TH-Z1(M))
      IF(DIF.LT.SMA) GO TO 85
      GO TO 5000
85      IF(TH-Z1(M)) 87,88,88
87      TH=TH+DIF+1.5*SMA
      GO TO 5000
88      TH=TH+1.5*SMA-DIF
5000     CONTINUE
C
C---CALCULATE THE ANGLE TO THE SOURCE AND FIELD PT. FROM EDGES N,N-1,N+1,N+2
C
      AN=PI/2.-2.*(I-1.)*ZET-TH+PHIP
      BN=PI/2.-2.*(I+1.)*ZET-TH+PHI
      G=-2.*(I-1.)*ZET-TH
      EN=PI/2.-2.*(I-1.)*ZET-TH+PHI
      FN=PI/2.-2.*(I-2.)*ZET-TH+PHI
      FNPL=PI/2.-2.*I*ZET-TH+PHI
      FN=AMOD(FN,TPI)
      IF(FN.LT.0.) FN=FN+TPI
      FNPL=AMOD(FNPL,TPI)
      IF(FNPL.LT.0.) FNPL=FNPL+TPI
      AN=AMOD(AN,TPI)
      G=AMOD(G,TPI)
      IF(G.LT.0.) G=G+TPI
      EN=AMOD(EN,TPI)
      IF(EN.LT.0.) EN=EN+TPI
      BN=AMOD(BN,TPI)
      IF(BN.LT.0.) BN=BN+TPI
      GP=-2.*I*ZET-TH
      GP=AMOD(GP,TPI)
      IF(GP.LT.0.) GP=GP+TPI
C
C---THE ELEVATION ANGLE OF THE PLATES N,N-1,N+1
C
      AA=2.*I*ZET+TH+PI/2.
      AB=2.*(I-1.)*ZET+TH+PI/2.
      AC=2.*(I+1.)*ZET+TH+PI/2.
      AA=AMOD(AA,TPI)
      IF(AA.LT.0.) AA=AA+TPI
      AB=AMOD(AB,TPI)
      IF(AB.LT.0.) AB=AB+TPI
      AC=AMOD(AC,TPI)
      IF(AC.LT.0.) AC=AC+TPI
C
C---THE VERTICAL DISTANCES FROM EACH EDGE TO THE IMAGE EDGES,N,N-1,N+1,N+2
C
      SN=(RH-2.*RA*COS((2.*I-1.)*ZET+TH-1.5*PI))
      SNP1=(RH-2.*RA*COS((2.*I+1.)*ZET+TH-1.5*PI))
      SNM1=(RH-2.*RA*COS((2.*I-3.)*ZET+TH-1.5*PI))
      SNP2=(RH-2.*RA*COS((2.*I+3.)*ZET+TH-1.5*PI))
      H=SKA*SN/(RA*TPI)
C
C---THE PATH LENGTH OF THE DOUBLE DIFFRACTION BETWEEN ADJACENT EDGES
C

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P1=SQRT(WW**2.+SNP1**2.-2.*SNP1*WW*COS(GP))
P2=SQRT(WW**2.+SN**2.-2.*SN*WW*COS(G))
PP=SKA*P1/(RA*TPI)
PM=SKA*P2/(RA*TPI)
C
C--THE PHASE PATH FROM VARIOUS POINTS OF DIFFRACTION
C
S1=SKA*COS((2.*I-1.)*ZET+TH-PHIP)+SP
S2=SKA*COS((2.*I-1.)*ZET+TH-PHI)+S
S3=SKA*COS((2.*I-3.)*ZET+TH-PHI)+S
S4=SKA*COS((2.*I+1.)*ZET+TH-PHI)+S
S5=SKA*COS((2.*I+3.)*ZET+TH-PHI)+S
SS=S1+S2-SKA*SN/RA
SSM=S1+S3-SKA*P2/RA
SSP=S1+S4-SKA*P1/RA
AAA=TH*180./PI
C
C THIS SECTION COMPUTES THE SECOND ORDER DIFF BETWEEN THE EDGE N
C AND THE GROUND.
C
IF(AN.LT.0.) AN=AN+TPI
IF(AN.GE.WN*PI)GO TO 6
IF(EN.GE.WN*PI) GO TO 6
IF(G.GE.WN*PI) GO TO 6
GAMMA0=(EPSR-EPSR**.5)/(EPSR+EPSR**.5)
GAMMA0=1.
IF(PHI.NE.PHIP) GAMMA0=2.
ANG1=G
ANG2=AN
ANG3=G
ANG4=EN
A1=G+EN-TE1
A1=AMOD(A1,TPI)
A2=G+EN-TE2
A2=AMOD(A2,TPI)
DIF1=ABS(A1)
DIF2=ABS(A2)
IF(DIF1.GT.XY1.AND.DIF2.GT.XY1)GO TO 800
ANG1=G
ANG2=EN
ANG3=G
ANG4=AN
800 CALL WD(D,H,ANG1,ANG2,1.,WN)
T1=(0.,0.)
DO 8 K=1,4
T1=T1+D(K)
8 CONTINUE
CS1=COS(PI/WN)
CS2=COS((ANG3-ANG4)/WN)
CS3=COS((ANG3+ANG4)/WN)
EX=-PI/4.
T2=(1./(CS1-CS2)+1./(CS1-CS3))*CMPLX(1.,0.)
T2=T2*SIN(PI/WN)*CEXP(CMPLX(0.,EX))/(WN*(TPI*WK)**.5)
HA=HA+GAMMA0*T1*T2*CEXP(CMPLX(0.,SS))/(H**.5)
LL=0
C WRITE(7,500)LL,I,SS,HA,T2,AAA
C500 FORMAT(2X,2I5,6F11.4)
C
C FIRST CALCULATE THE DIFFRACTION OFF OF EDGE N WHICH REFLECTS OFF
C OF THE GROUND PLANE AND DIFFRACTS OFF OF THE EDGE N-1
C
HM=PI-G
HM=AMOD(HM,TPI)

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```

      IF(HM.LT.0.) HM=HM+TPI
      IF(HM.LE.PI/2..OR.HM.GE.PI) GO TO 22
      IF(FN.GE.WN*PI) GO TO 22
      DEL2=ATAN(2*WN*COS(AB)/(SNM1+SN))
      GAMMA1=(EPSR*COS(DEL2)-(EPSR-SIN(DEL2)**2)**.5)/(EPSR*COS(DEL2)+
C(EPSR-SIN(DEL2)**2)**.5)
      GAMMA1=1.
      IF(PHI.NE.PHIP) GAMMA1=2.
      GN=G-DEL2
      HN=HM-DEL2
      YN=PI+2*ZET-HN
      YN=AMOD(YN,TPI)
      IF(YN.LT.0.) YN=YN+TPI
      ANG1=GN
      ANG2=AN
      ANG3=YN
      ANG4=FN
      A1=FN+YN-TE1
      A1=AMOD(A1,TPI)
      A2=FN+YN-TE2
      A2=AMOD(A2,TPI)
      DIF1=ABS(A1)
      DIF2=ABS(A2)
      IF(DIF1.GT.XY2.AND.DIF2.GT.XY2) GO TO 1000
      ANG1=YN
      ANG2=FN
      ANG3=GN
      ANG4=AN
1000  CALL WD(D,PM,ANG1,ANG2,1.,WN)
      T1=(0.,0.)
      DO 10 K=1,4
      T1=T1+D(K)
10    CONTINUE
      CS1=COS(PI/WN)
      CS2=COS((ANG3-ANG4)/WN)
      CS3=COS((ANG3+ANG4)/WN)
      EX=-PI/4.
      T2=(1./(CS1-CS2)+1./(CS1-CS3))*CMPLX(1.,0.)
      T2=T2*SIN(PI/WN)*CEXP(CMPLX(0.,EX))/(WN*(TPI*WK)**.5)
      HA=HA+GAMMA1*T1*T2*CEXP(CMPLX(0.,SSM))/(PH**5)
      LL=1
C     WRITE(7,500)LL,I,SSM,HA,T2,AAA
C
C     NOW CALCULATE THE DIFFRACTION OFF OF EDGE N WHICH REFLECTS OFF
C     OF THE GROUND PLANE AND DIFFRACTS OFF OF THE EDGE N+1
C
22    IF(FNPL.GE.WN*PI) GO TO 6
      HP=PI+2.*ZET-G
      HP=AMOD(HP,TPI)
      IF(HP.LT.0.) HP=HP+TPI
      IF(HP.LE.PI/2..OR.HP.GE.PI) GO TO 6
      DEL1=ATAN(2.*WN*COS(AA)/(SN+SNP1))
      GAMMA2=(EPSR*COS(DEL1)-(EPSR-SIN(DEL1)**2)**.5)/(EPSR*COS(DEL1)+
C(EPSR-SIN(DEL1)**2)**.5)
      GAMMA2=1.
      IF(PHI.NE.PHIP) GAMMA2=2.
      GNPL=GP-DEL1
      HNPL=HP-DEL1
      XN=PI+2.*ZET-HNPL
      XN=AMOD(XN,TPI)
      IF(XN.LT.0.) XN=XN+TPI
      ANG1=XN
      ANG2=AN

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ANG3=GNP1
ANG4=FNPI
A1=FNPI+GNP1-TE1
A1=AMOD(A1,TPI)
A2=FNPI+GNP1-TE2
A2=AMOD(A2,TPI)
DIF1=ABS(A1)
DIF2=ABS(A2)
IF(DIF1.GT.XY2.AND.DIF2.GT.XY2) GO TO 2000
ANG1=GNP1
ANG2=FNPI
ANG3=XN
ANG4=AN
2000 CALL WD(D,PP,ANG1,ANG2,1.,WN)
T1=(0.,0.)
DO 12 K=1,4
T1=T1+D(K)
12 CONTINUE
CS1=COS(PI/WN)
CS2=COS((ANG3-ANG4)/WN)
CS3=COS((ANG3+ANG4)/WN)
EX=-PI/4.
T2=(1./(CS1-CS2)+1./(CS1-CS3))*CMPLX(1.,0.)
T2=T2*SIN(PI/WN)*CEXP(CMPLX(0.,EX))/(WN*(TPI*RK)**.5)
HA=HA+GAMMA2*T1*T2*CEXP(CMPLX(0.,SSP))/(PP**.5)
LL=2
C WRITE(7,500)LL,I,SSP,HA,T2,AAA
C
C FIND THE REFLECTION-REFLECTION-DIFFRACTION FROM THE NTH PLATE
C TO THE EDGE N-1
C
C
C--THE ANGLE BETWEEN THE PLATE N FROM THE SOURCE RAY
C
DEL3=ATAN(2.*WW*COS(AC)/(SNP1+SNP2))
TEST2P=GP+DEL3
TEST2P=AMOD(TEST2P,TPI)
IF(TEST2P.LT.0.) TEST2P=TEST2P+TPI
K=0
BE=PI/2.+2.*I*ZET+TH-PHIP
BE=AMOD(BE,TPI)
IF(BE.LT.0.) BE=BE+TPI
DEL1=ATAN(2.*WW*(COS(AA)+COS(AB))/(SNM1+SNP1))
DEL2=ATAN(2.*WW*(COS(AA)+COS(AC))/(SN+SNP2))
DEL1=AMOD(DEL1,TPI)
DEL2=AMOD(DEL2,TPI)
IF(DEL1.LT.0.) DEL1=DEL1+TPI
IF(DEL2.LT.0.) DEL2=DEL2+TPI
AMIN=GP-DEL1
AMAX=GP+DEL2
TEST1=GN-2.*ZET
TEST1=AMOD(TEST1,TPI)
IF(TEST1.LT.0.) TEST1=TEST1+TPI
C-----CHECK OUT TESTS FOR BE BECAUSE
C-----4 PRINTS TWICE IN DATA
TEST2=PI-HNP1
IF(BE.LT.AMIN.OR.BE.GT.AMAX) GO TO 6
IF(BE.GE.AMIN.AND.BE.LE.TEST1) GO TO 23
IF(BE.GE.GNP1.AND.BE.LE.GP) GO TO 24
IF(BE.GT.GP.AND.BE.LE.TEST2) GO TO 25
IF(BE.GE.TEST2P.AND.BE.LE.AMAX) GO TO 27
GO TO 6
23 DE=GP-BE

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      GAMMA3=(EPSR*COS(DE)-(EPSR-SIN(DE)**2)**.5)/(EPSR*COS(DE)+
C(EPSR-SIN(DE)**2)**.5)
      GAMMA3=1.
      HNEW=HM-DE
      HNEW=AMOD(HNEW,TPI)
      IF(HNEW.LE.0.) HNEW=HNEW+TPI
      X=SRH1/(2.*COS(DE))
      Y0=X*SIN(HNEW)/SIN(HNEW+2.*DE)
      DD=X*SIN(2.*DE)/SIN(HNEW+2.*DE)
      CC=DD-WW
      YP=CC*SIN(2.*ZET)/SIN(BE)
      Y=Y0+YP
      Z=CC*SIN(2.*ZET+BE)/SIN(BE)
      IF(Z.LT.0..OR.Z.GT.WW) GO TO 6
      TA=SQRT(RA**2+Z**2-2.*RA*Z*COS(AL))
      GA=ASIN(Z*SIN(AL)/TA)
      GA=AMOD(GA,TPI)
      IF(GA.LT.0.) GA=GA+TPI
      GAP=2.*ZET-GA
      GAP=AMOD(GAP,TPI)
      IF(GAP.LT.0.) GAP=GAP+TPI
      S6=SKA*TA*COS((2.*I+1.)*ZET+TH-PHIP-GAP)/RA+SP
      IF(K.EQ.1) S6=SKA*TA*COS((2.*I+1.)*ZET+TH-PHI-GAP)/RA+SP
      IF(K.EQ.1) S3=SKA*COS((2.*I-3.)*ZET+TH-PHIP)+S
      D1=S3+S6-SKA*(Y+X)/RA
      ANG1=PI+2.*ZET-HNEW
      ANG2=FN
      IF(K.EQ.1) ANG2=PI/2.-2.*(I-2.)*ZET-TH+PHIP
      ANG2=AMOD(ANG2,TPI)
      IF(ANG2.LE.0.) ANG2=ANG2+TPI
      ANG1=AMOD(ANG1,TPI)
      IF(ANG1.LE.0.) ANG1=ANG1+TPI
C
C
C      USE KELLERS DIFFRACTION TERMS HERE

      CS1=COS(PI/WN)
      CS2=COS((ANG2-ANG1)/WN)
      CS3=COS((ANG2+ANG1)/WN)
      EX=-PI/4.
      T1=(1./(CS1-CS2)+1./(CS1-CS3))*CMPLX(1.,0.)
      T1=T1*SIN(PI/WN)*CEXP(CMPLX(0.,EX))/(WN*(TPI*RK)**.5)
      HA=HA+GAMMA3*2.*T1*CEXP(CMPLX(0.,D1))
      LL=3
C      WRITE(7,500)LL,I,D1,HA,T1,AAA
      K=1
      BE=PI/2.+2.*I*ZET+TH-PHI
      BE=AMOD(BE,TPI)
      IF(BE.LT.0.) BE=BE+TPI
      IF(BE.GT.TEST2.AND.BE.LE.AMAX.AND.PHI.NE.PHIP) GO TO 27
      GO TO 6
C
C----NOW FIND THE REFLECTION FROM PLATE N TO EDGE N
C
24      DE=GP-BE
      GAMMA4=(EPSR*COS(DE)-(EPSR-SIN(DE)**2)**.5)/(EPSR*COS(DE)+
C(EPSR-SIN(DE)**2)**.5)
      GAMMA4=1.
      HNEW=HP-DE
      IF(HNEW.LT.HHP1) GO TO 6
      X=SN/(2.*COS(DE))
      Y=X*SIN(HNEW)/SIN(BE)
      Z=X*SIN(2.*DE)/SIN(BE)
      IF(Z.GT.WW.OR.Z.LT.0.) GO TO 6

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```

      TA=SQRT(RA**2+Z**2-2.*RA*Z*COS(AL))
      GA=ASIN(Z*SIN(AL)/TA)
      GA=AMOD(GA,TPI)
      IF(GA.LT.0.) GA=GA+TPI
      GAP=2.*ZET-GA
      GAP=AMOD(GAP,TPI)
      IF(GAP.LT.0.) GAP=GAP+TPI
      S6=SKA*TA*COS((2.*I+1.)*ZET+TH-PHIP-GAP)/RA+SP
      IF(K.EQ.1) S6=SKA*TA*COS((2.*I+1.)*ZET+TH-PHI-GAP)/RA+SP
      IF(K.EQ.1) S2=SKA*COS((2.*I-1.)*ZET+TH-PHIP)+S
      D1=S2+S6-SKA*(Y+X)/RA
      ANG1=PI+2.*ZET-HNEW
      ANG2=EN
      IF(K.EQ.1) ANG2=PI/2.-2.*(I-1)*ZET-TH+PHIP
      ANG2=AMOD(ANG2,TPI)
      IF(ANG2.LE.0.) ANG2=ANG2+TPI
      ANG1=AMOD(ANG1,TPI)
      IF(ANG1.LE.0.) ANG1=ANG1+TPI
C
C      USE KELLERS DIFFRACTION TERMS HERE
C
      CS1=COS(PI/WN)
      CS2=COS((ANG2-ANG1)/WN)
      CS3=COS((ANG2+ANG1)/WN)
      EX=-PI/4.
      T1=(1./(CS1-CS2)+1./(CS1-CS3))*CMPLX(1.,0.)
      T1=T1*SIN(PI/WN)*CEXP(CMPLX(0.,EX))/(WN*(TPI*WK)**.5)
      HA=HA+GAMMA4*2.*T1*CEXP(CMPLX(0.,D1))
C
C      LL=4
C      WRITE(7,500) LL,I,D1,HA,T1,AAA
      K=1
      BE=PI/2.+2.*I*ZET+TH-PHI
      BE=AMOD(BE,TPI)
      IF(BE.LT.0.) BE=BE+TPI
      IF(BE.GT.GP.AND.BE.LE.TEST2.AND.PHI.NE.PHIP) GO TO 25
      DIF=ABS(BE-GP)
      IF(DIF.LT..0001) GO TO 25
      GO TO 6
C
C      FIND THE REFLECTION-REFLECTION-DIFFRACTION TERM FROM THE NTH PLATE
C      TO THE N+1ST EDGE.
C
25      DE=ABS(BE-GP)
      GAMMA5=(EPSR*COS(DE)-(EPSR-SIN(DE)**2)**.5)/(EPSR*COS(DE)+
C(EPSR-SIN(DE)**2)**.5)
      GAMMA5=1.
      HNEW=GP-DE
      HNEW=AMOD(HNEW,TPI)
      IF(HNEW.LE.0.) HNEW=HNEW+TPI
      IF(HNEW.LT.GNP1.OR.HNEW.GT.GP) GO TO 6
      X=SNP1/(2.*COS(DE))
      Y=X*SIN(HNEW)/SIN(PI-BE)
      Z=X*SIN(2.*DE)/SIN(PI-BE)
      IF(Z.GT.WN) GO TO 6
      TA=SQRT(RA**2+Z**2-2.*RA*Z*COS(AL))
      GA=ASIN(Z*SIN(AL)/TA)
      GA=AMOD(GA,TPI)
      IF(GA.LT.0.) GA=GA+TPI
      S7=SKA*TA*COS((2.*I+1.)*ZET+TH-PHIP-GA)/RA+SP
      IF(K.EQ.1) S7=SKA*TA*COS((2.*I+1.)*ZET+TH-PHI-GA)/RA+SP
      IF(K.EQ.1) S4=SKA*COS((2.*I+1)*ZET+TH-PHIP)+S
      D1=S4+S7-SKA*(X+Y)/RA
      ANG1=HNEW

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      ANG2=FNPI
      IF(K.EQ.1) ANG2=PI/2.-2.*I*ZET-TH+PHIP
      ANG2=AMOD(ANG2,TPI)
      IF(ANG2.LE.0.) ANG2=ANG2+TPI
C
C      USE KELLERS DIFFRACTION TERMS HERE
C
      CS1=COS(PI/WH)
      CS2=COS((ANG2-ANG1)/WH)
      CS3=COS((ANG2+ANG1)/WH)
      EX=-PI/4.
      T1=(1./(CS1-CS2)+1./(CS1-CS3))*CMPLX(1.,0.)
      T1=T1*SIN(PI/WH)*CEXP(CMPLX(0.,EX))/(WH*(TPI*WK)**.5)
      HA=HA+GAMMA5*2.*T1*CEXP(CMPLX(0.,D1))
      LL=5
C
C      WRITE(7,500)LL,I,D1,HA,T1,AAA
C
      IF(BE.GE.TEST2P.AND.BE.LE.AMAX) GO TO 27
      K=1
      BE=PI/2.+2.*I*ZET+TH-PHI
      BE=AMOD(BE,TPI)
      IF(BE.LT.0.) BE=BE+TPI
      IF(BE.GT.GNPI.AND.BE.LE.GP.AND.PHI.NE.PHIP) GO TO 24
      GO TO 6
C
C      REFLECTION REFLECTION DIFFRACTION FROM THE NTH PLATE TO THE
C      N+2ND EDGE
C
27      DE=(BE-GP)
      GAMMA6=(EPSR*COS(DE)-(EPSR-SIN(DE)**2)**.5)/(EPSR*COS(DE)+
C(EPSR-SIN(DE)**2)**.5)
      GAMMA6=1.
      HNEW=GP-2*ZET-DE
      HNEW=AMOD(HNEW,TPI)
      IF(HNEW.LT.0.) HNEW=HNEW+TPI
      IF(HNEW.GE.WN*PI) GO TO 6
      ANG=PI-2.*DE-HNEW
      X=SNP2/(2.*COS(DE))
      Y0=X*SIN(HNEW)/SIN(ANG)
      DD=X*SIN(2.*DE)/SIN(ANG)
      CC=DD-WW
      YP=CC*SIN(2.*ZET)/SIN(BE)
      Y=Y0+YP
      Z=CC*SIN(BE-2.*ZET)/SIN(BE)
      IF(Z.LT.0..OR.Z.GT.WW) GO TO 6
      TA=SQRT(RA**2+Z**2-2.*RA*Z*COS(AL))
      GA=ASIN(Z*SIN(AL)/TA)
      GA=AMOD(GA,TPI)
      IF(GA.LT.0.) GA=GA+TPI
      S7=SKA*TA*COS((2.*I+1.)*ZET+TH-PHIP-GA)/RA+SP
      IF(K.EQ.1) S7=SKA*TA*COS((2.*I+1)*ZET+TH-PHI-GA)/RA+SP
      IF(K.EQ.1) S5=SKA*COS((2.*I+3)*ZET+TH-PHIP)+S
      D1=S5+S7-SKA*(X+Y)/RA
      ANG1=HNEW
      ANG2=BN
      IF(K.EQ.1) ANG2=PI/2.-2.*(I+1)*ZET-TH+PHIP
      ANG2=AMOD(ANG2,TPI)
      IF(ANG2.LE.0.) ANG2=ANG2+TPI
C
C      USE KELLERS DIFFRACTION COEFFICIENT
C
      CS1=COS(PI/WH)
      CS2=COS((ANG2-ANG1)/WH)
      CS3=COS((ANG2+ANG1)/WH)

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EX=-PI/4.
T1=(1./(CS1-CS2)+1./(CS1-CS3))*CMPLX(1.,0.)
T1=T1*SIN(PI/WN)*CEXP(CMPLX(0.,EX))/(WN*(TPI*RK)**.5)
HA=HA+GAMMA6*2.*T1*CEXP(CMPLX(0.,D1))
C LL=6
C WRITE(7,500)LL,I,D1,HA,T1,AAA
K=1
BE=PI/2.+2.*I*ZET+TH-PHI
BE=AMOD(BE,TPI)
IF(BE.LT.0.) BE=BE+TPI
IF(BE.GT.AMIN.AND.BE.LE.TEST1.AND.PHI.NE.PHIP) GO TO 23
6 CONTINUE
C
C MULTIPY BY 2 BECAUSE THE P-K COEFFICIENT IS OFF BY A FACTOR OF
C 2 ALONG THE SHADOW BOUNDARY
C
AH(J)=AH(J)+GAMMA*2.*HA*CEXP(CMPLX(0.,DT))
C LL=20
C WRITE(7,500)LL,I,DT,AH(J),HA,AAA
4 CONTINUE
RETURN
END

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