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ADDITIVITY OF THE STILES-CRAWFORD EFFECT
FOR A FRAUNHOFER IMAGE.

The Ohio State University, Ph.D., 1973.
Biophysics, general

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ADDITIVITY OF THE STILES-CRAWFORD EFFECT
FOR A FRAUNHOFER IMAGE

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

by

Bruce Alan Drum, B.S.

* * * * *

The Ohio State University
1973

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ACKNOWLEDGMENTS

I wish to thank the following people for their assistance in preparing this dissertation:

Regents Professor Glenn A. Fry, for suggesting the topic, and for his assistance in directing me to pertinent literature;

Professor Robert Kouyoumjian, for helpful discussions on dielectric waveguides and diffraction theory;

Professors Stanley W. Smith and Richard M. Hill, the other members of my committee, for their continued interest in my graduate education;

Susan Heaphy and Brian Tsou, for helping me out in the lab;

Professor Carl R. Ingling, Jr., my adviser, for working so closely and constantly with me during the last four years. His example, and the countless hours he spent with me have greatly influenced nearly every aspect of my graduate education, and have made it a very positive and enjoyable experience.

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Vision Res., 13, 1151 (1973).
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Carl R. Ingling, Jr. and Bruce A. Drum, How Neural Adaptation Changes
Chromaticity Coordinates. J. Opt. Soc. Am., 63, 369 (1973).

FIELDS OF STUDY

Major Field: Biophysics

Studies in Visual Biophysics. Professor Carl R. Ingling, Jr.

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INTRODUCTION

The cone receptors of the human retina are less sensitive to light which enters the eye through the edge of the pupil than to light which enters through the center. This phenomenon is known as the Stiles-Crawford effect after Stiles and Crawford,¹ who first observed it by measuring visual thresholds for various positions of a small artificial pupil moved horizontally across the natural pupil (see Fig. 1). A target seen through the edge of the fully dilated pupil can appear nearly one log unit dimmer than the same target seen through the center of the pupil. The brightness ratio of a light seen through any point in the pupil to the same light seen through the center is called the relative luminous efficiency, η , of that point.

Subsequent studies (see Enoch,²⁻⁴ Pirenne⁵ and Stiles⁶ for reviews of this subject) have reported that color perception and visual acuity as well as brightness are functions of the angle at which light strikes the retina and that all of these effects are due primarily⁷ to physical and optical properties of the visual receptors themselves. Briefly, the receptors are thought to behave as cylindrical dielectric waveguides which catch light only at angles close to normal incidence. Since all the cones in a normal retina point approximately toward the center of the pupil,⁸ only light which passes through the center of the pupil can strike them at normal incidence.

FIGURE 1

The Stiles-Crawford Effect. After Stiles and Crawford.¹ The horizontal axis is distance in millimeters from the center of the pupil. The vertical axis is the relative luminous efficiency, η .

One empirical question which can be asked about the Stiles-Crawford effect is that of additivity; can the relative luminous efficiency of any given pupil be predicted by averaging the efficiencies of all the points within it? Enoch^{2,9} studied additivity both for a Maxwellian beam, in which the luminous source is imaged in the plane of the pupil, and a non-Maxwellian beam, in which the source is imaged directly on the retina. Enoch concluded that in both conditions additivity was valid, although he noted that when artifacts due to blur were eliminated, brightness for the non-Maxwellian beam was not reduced quite as much as additivity would predict. He suggested that the answer to this discrepancy might lie in the physical distribution of light on the retina. Enoch and Fry¹⁰ later hypothesized that the Stiles-Crawford effect may disappear completely if an aberration-free Fraunhofer image (diffraction pattern in the geometrical image plane) is sharply focused on the retina through a pupil which is circularly symmetric about the center of the natural pupil. A detailed discussion of the theory behind this hypothesis is given below.

In this paper, the Stiles-Crawford effect is investigated both theoretically and experimentally for a Fraunhofer image accurately focused on the retina. The hypothesis that under these conditions the Stiles-Crawford effect is absent or considerably reduced is compared with the hypothesis that the Stiles-Crawford effect is additive for all conditions of focus. These will be referred to respectively as the non-additivity hypothesis and the additivity hypothesis.

THEORY

Phase Behavior of a Coherent Converging Beam Near Focus

The imaging of a point source of light by an aberration-free convex lens can be described in terms of either geometrical optics or physical optics. According to geometrical optics, all rays from the source which pass through the lens converge to a point in the focal plane, then diverge again beyond it. These rays, which define the direction of propagation of the light from the source, pass undeviated through the focal plane. In other words, light which leaves the lens in a given direction will still be traveling in the same direction at the focal point. A physical optics description, however, appears to contradict this picture. Linfoot and Wolf¹¹ solved Maxwell's equations for the surfaces of constant phase (wave fronts) of a coherent beam converging to focus after diffraction at a circular aperture. The wave fronts are normal to the direction of propagation of the light. Linfoot and Wolf showed that for a number of wavelengths on either side of the geometrical focal plane these phase fronts become essentially planar (except for discontinuities at the minima of the Fraunhofer diffraction pattern) and normal to the optical axis. Within this region, then, all the energy in the beam travels in the direction of the optical axis, regardless of the size of the

diffracting aperture. Figures 2 and 3 show a comparison of the predictions of geometrical optics and physical optics. Measurements of phase distribution close to focus in the microwave region of the spectrum show excellent agreement with the physical optics prediction.¹²

Enoch and Fry¹⁰ considered the Stiles-Crawford effect in conjunction with the above theory and concluded that the Stiles-Crawford effect should be absent for an aberration-free Fraunhofer image of a point source focused exactly on the retina. They reasoned that since the wave fronts would be flat and normally incident on the retina, the receptors would not be able to distinguish them from wave fronts actually coming from the center of the pupil. A closer examination of the physical optics solution, however, casts some doubt on this hypothesis.

The wave equation which specifies the phase and intensity distributions near focus for coherent light converging from a diffracting aperture represents the field as a superposition of plane waves originating at points in the aperture and propagating in the directions of the geometrical rays.¹³ The resultant intensity and phase of any point is determined by the summation of the complex amplitudes of all the elemental plane waves at that point. The phase, amplitude and direction of each component is unaffected by the presence of other components and by the characteristics of the resultant wave. In other words, while the geometrical optics model is inadequate to describe the shape and direction of the resultant wave front near focus, it does accurately describe the directions of the elemental plane wave

FIGURE 2

Geometrical Optics Model of Wave Fronts Near Focus. Wave fronts are spherical arcs contained within the geometrical cone of rays and concentric with the focal point.

FIGURE 3

Physical Optics Model of Wave Fronts Near Focus. Adapted from Linfoot and Wolf.¹¹ Wave fronts are parallel to the focal plane except at points of zero intensity (the Airy dark rings) where they undergo an abrupt phase transition of π . The energy distribution is not uniform over a wave front surface. Note also that the energy is not contained within the geometrical cone of rays.

components. The validity of Enoch and Fry's hypothesis therefore depends primarily on whether the retinal receptors are sensitive to the resultant wave or to the individual plane wave components.

Dielectric Waveguide Model of Visual Receptors

Figure 4 is a diagram of foveal cone receptors, showing indices of refraction of the different regions as well as their dimensions and configuration in the retina. The inner segments of the cones are narrow circular cylinders tapering to the still narrower outer segments which contain the photo-sensitive pigment. Their diameters are close to the wavelength of visible light and their refractive index is slightly greater than that of the surrounding medium, permitting them to act as dielectric waveguides.¹⁶ Light in a dielectric waveguide can propagate only in a limited number of discrete modes, corresponding to discrete angles of reflection within the waveguide. The set of allowable modes depends on the diameter of the waveguide, the indices of refraction of the guide and the surrounding medium, and the wavelength and angle of incidence of the incoming light.¹⁷ Nearly all the light normally incident on the end of a receptor is propagated along its entire length (except for absorbed light) and is effectively "funnelled" into the pigment-bearing outer segment. Because the refractive index of the receptor differs so little from that of the surrounding medium, however, light which differs only slightly from normal incidence may exceed the critical angle of total internal reflection, and be progressively lost through the walls as

FIGURE 4

Schematic Diagram of Foveal Cones, giving refractive indices and typical dimensions of various parts of the receptor layer. (Refractive indices are taken from Barer.¹⁴ Dimensions are derived from Polyak.¹⁵) Abbreviations are: NR (neural retina), ILM (inner limiting membrane), IM (interreceptor medium), IS (inner segment), E (ellipsoid), OS (outer segment), PE (pigment epithelium). Numbers in parentheses are refractive indices.

The diagram is simplified in that the receptor nuclei which extend beyond the inner limiting membrane are not shown, nor are the organelles in the inner segments, such as mitochondria. The visual pigment is found throughout the lengths of the outer segments, organized in orderly stacks of transverse membranes. These structural details modify light transmission in the receptors in ways which are not presently understood.

it travels along the receptor, both before and after it reaches the outer segment, with the obvious result that less of the incident light is absorbed by the photopigment.^{3,4}

The waveguide theory which is usually invoked to describe the transmission of light in visual receptors is strictly true only for infinitely long cylinders. Receptors, however, are not infinitely long; their length-to-width ratio is only about 70/1, and the first cylindrical section, the inner segment, is only about one-third as long as the whole receptor. Consideration of how modes are established as light enters the receptors may therefore be important for understanding light transmission in the retina. Although the behavior of light as it enters the end of a dielectric waveguide has not been investigated in detail,^{17,18} some general statements can be made. Modal propagation theory applies not only to waveguides, but to light propagation in any optical system. In particular, free propagation (propagation in a homogeneous medium) allows an infinite number of modes, corresponding to all possible angles of propagation. Consider a plane wave normally incident on the matrix of inner segments. The wave undergoes refraction and diffraction as it passes through the inner limiting membrane into the receptor layer, and typical waveguide modes begin to appear in the inner segments. Modal propagation cannot be observed immediately, however, because there must be continuity of phase at the interface. There is a region of transition between free propagation and modal propagation, during which modes that cannot be propagated in the waveguide are lost. Although the length of this

transition region is not known, it should be strongly dependent on the size of the refractive index difference between the receptors and their surrounding medium. For a difference of only two per cent, this distance might well be a substantial fraction of the length of an inner segment.¹⁹

Waveguide Response to Focused Incident Light

How does the case of a plane wave apply to the region of focus of a Fraunhofer image and what are the implications for directional sensitivity? As stated above, the phase fronts in the focal region of a Fraunhofer point image are planar except at points of zero intensity (the Airy dark rings).

If the receptors can analyse the wave fronts into their directional components, the resulting waveguide modal patterns should be exclusive functions of these components regardless of the shape and direction of the composite wave fronts. Since the directions of the component waves are constant at all positions along the optical axis, directional sensitivity of the receptors should be completely independent of the focal position of the incident light.

Consideration of some basic optical principles leads to the prediction that the above model is probably correct. In general, optical systems which exhibit wave interference phenomena such as reflection, refraction and diffraction are capable of analysing incident waves into their elementary components. For example, if a plane mirror replaced the retina at the focal plane of the eye, the reflected

beam would certainly not be collimated because of the flat wave fronts at the focal plane, but would exactly retrace the path of the incident converging beam. On the other hand, systems which absorb light respond only to the properties of the resultant waves. Although dielectric waveguides do absorb some light, modal propagation theory is based on total internal reflection and interference phenomena. Modal patterns in the receptors are therefore probably functions of the directions of elementary wave components in the incident light. However, if the modal patterns formed in the inner segments are functions of the resultant wave fronts which determine the direction of energy flow of the incident light, and if the region of planar wave fronts is longer than the region of transition to modal propagation, the modal patterns formed in the inner segments should be the same as those formed from plane waves of normal incidence. If the focal region is too short, the wave fronts may begin to diverge before the modes are sufficiently well defined. Some modes would then correspond to an oblique angle of incidence greater than the critical angle of total internal reflection and thus a lower efficiency of transmission. The depth of focus of the image, defined as the region of plane wave fronts, may therefore be a parameter of critical importance.

Depth of Focus of a Fraunhofer Image

The depth of focus around a Fraunhofer image plane is usually defined as the region in which the axial intensity is within 80 per cent of its maximum value at the geometrical focal point.²⁰ Figure 5,

FIGURE 5

Depth of Focus in Relation to the Relative Intensity Distribution Near Focus for a Circular Aperture. Adapted from Linfoot and Wolf.¹¹ The normalized coordinates,

$$u = \frac{2\pi}{\lambda} \frac{a^2}{f^2} z \quad 1$$

and

$$v = \frac{2\pi}{\lambda} \frac{a}{f} \sqrt{x^2 + y^2} \quad 2$$

allow the intensity distribution to be plotted independently of the wavelength and the angle of the geometrical cone of rays. The dotted vertical line marks the boundaries of the focal region. See text for further explanation.

adapted from Linfoot and Wolf,¹¹ shows lines of equal intensity around the image of a point source focused through a circular aperture. The horizontal axis is the optical axis of the system and the vertical axis is an edge-on view of the geometrical image plane. The shaded area designates the geometrical cone of rays. The normalized dimensionless coordinates, u and v , are functions of wavelength λ , focal length f , and aperture radius a . The vertical dotted line, which marks the boundaries of the focal region as defined above, occurs at about $u = \pm\pi$. The parallel tubular structure of the equal intensity lines close to focus suggests that the wave fronts there are flat, since energy propagated in the v direction would cause the lines to be curved. Still, the standard definition of depth of focus does not specify directly a region of flat wave fronts, which would be a more suitable definition for the present discussion. Fortunately, as will be shown below, the two definitions are reasonably compatible.

Figure 6, adapted from Farnell,²¹ shows wave fronts close to focus (similar to Fig. 3) for an $f/2$ lens. Axial coordinates are given both in units of u and number of wavelengths. The solid vertical line at $u = \pi$ marks the standard depth of focus. It is apparent that although the flat wave front criterion is slightly more stringent (dotted vertical line), the two definitions are in fairly good agreement, at least for an $f/2$ lens. This comparison can be generalized to any f -number by application of a relation derived by Linfoot and Wolf.¹¹ They showed that differences between the phase fronts of the converging beam and those of hypothetical plane waves defined to have the same phase on axis are independent of f -number as a function of u ,

FIGURE 6

Depth of Focus in Relation to the Phase Distribution Near Focus for a Circular Aperture. Adapted from Farnell.²¹ The axis calibration in number of wavelengths applies only to an $f/2$ aperture, whereas the u axis applies to any f -number. The shape of the wave fronts is invariant for a given u . The dotted vertical line marks the boundary of the conventional depth of focus. The solid vertical line is the boundary of the region of flat wave fronts, arbitrarily set at $u = 3/4 \pi$.

but not as a function of distance (for example, number of wavelengths). In other words, the degree of flatness of the wave fronts is the same for a given value of u regardless of f-number, even though the number of wavelengths from focus at constant u varies with f-number.

Figure 5 can now be used to calculate, for example, the depth of focus (flat wave front criterion) at the fovea of a normal well-corrected eye. Assume a focal length f , of 16.6832 mm,²² pupil radius a , of 3 mm and wavelength λ , of 5.8×10^{-4} mm. According to Fig. 6, the limits of the depth of focus are at $u = \pm 0.75\pi$, and from Fig. 5,

$$u = \frac{2\pi}{\lambda} \frac{a^2}{f^2} z.$$

Solving for z (distance along optic axis) and substituting,

$$\text{depth of focus} = \Delta z = \frac{u\lambda}{\pi} \frac{f^2}{a^2} \approx 13\mu.$$

This is more than half the length of a cone inner segment (see Fig. 4).

Factors Limiting Retinal Image Quality in an Experimental Test

The question of how retinal receptors respond to a Fraunhofer image focused at the inner segments through an axially symmetric pupil can be approached experimentally if a sufficiently good image can be focused on the retina. To the extent that the receptors respond to the resultant flat wave fronts in the focal region, sensitivity should be independent of pupil area; in other words, the Stiles-Crawford effect should be non-additive. If the receptors respond to the component wave fronts from all the points in the pupil, however, additivity

of the Stiles-Crawford effect should hold exactly for any pupil, regardless of focus.

If the first alternative is true, the effect must be extremely sensitive to image degradation, since Enoch concluded in a previous study that additivity held.^{2,9} The retinal image must therefore be carefully optimized to minimize possible spurious additivity. As will be explained below, even a perfect Fraunhofer image on the retina may not result in complete non-additivity under the first hypothesis. Any significant non-additivity may therefore be taken as evidence that the receptors are sensitive to the resultant wave fronts.

The practical problems involved in performing the experiment are (1) precise control of focus, (2) elimination of optical aberrations, and (3) elimination of artifacts due to neural aspects of vision such as the effects of blur on apparent brightness. The optical problems are especially difficult because the eye is an imperfect optical system which, besides having considerable amounts of chromatic, spherical and off-axis aberration along with varying amounts of astigmatism, often does not have a well defined optical axis and contains minor random imperfections which degrade the image.

In addition, the eye is a living system in which the pupil, the lens, the retina and the eye as a whole are in states of continuous motion, which means that both the quality and position of the image on the retina are constantly changing. While major aberrations can usually be corrected, irregular ones often cannot.

Spherical Aberration.

The two major sources of aberration in the normal eye are chromatic aberration, which can be eliminated by using monochromatic light, and spherical aberration. There is some disagreement about the best way to measure spherical aberration in the human eye and about whether it is symmetric with respect to the optic axis.^{23,24} Ivanoff²⁵ measured the spherical aberration along the horizontal meridian of ten eyes by a parallax method, with the achromatic axis as the center. A later reinterpretation of the data²³ corrected the erroneous appearance of large changes near the axis, but still showed very large individual differences, changes with accommodation, and asymmetries about the axis. Koomen, Scolnik and Tousey²⁴ criticize Ivanoff's use of the achromatic axis, claiming that most of the asymmetry in his curves would have been eliminated by referring instead to the axis of maximum symmetry as had been done in their study.²⁶ Although the two axes coincide for an averaged population, they can differ substantially in individuals and can change with respect to each other as a function of accommodation. Koomen et al. also advocate the use of annular pupils, pointing out that data only along the horizontal meridian is subject to "local imperfections in the eye, and may not be representative of the spherical aberration of the whole eye.

Koomen et al.'s method probably does not completely specify spherical aberration in most cases, giving instead the best average value for each annular zone. It is nevertheless the best method for

minimizing spherical aberration in the present study because of the additional requirement of a large, axially symmetric pupil.

Annular Pupil.

Besides minimizing spherical aberration, an annular pupil has a number of other important effects on the retinal image. These effects can perhaps best be illustrated by comparing diffraction by an annular aperture with that by a circular aperture of the same outer diameter and focal length.

The most important comparison for the present experiment is of course that of the shapes of the wavefronts near focus. Although, to my knowledge, this has not been worked out for the case of visible light, Whitford and Pavlasek²⁷ have investigated the problem both theoretically and experimentally in the microwave region. Their calculations show that for a relatively large central obstruction, the wave fronts near focus from an annular aperture are not only flatter than those from a circular aperture, but are also flatter through a greater distance. The experimental results agree very well with theory.

The finding of a more extended region of flat wave fronts for an annular pupil was predictable from earlier calculations of the intensity distribution near focus for an annular aperture.^{28,29} Figure 7 shows equal intensity contours in the focal region of an annular aperture with ϵ (the ratio of the inner to outer diameter) equal to 0.707. Comparison with Fig. 5 reveals that the tubular appearance

FIGURE 7

Relative Intensity Distribution Near Focus for an Annular Aperture.

After Linfoot and Wolf.²⁸ $\epsilon = 0.707$. The shaded area denotes the hollow cone of rays. Note the extensive central tubular structure, approximately twice the depth of that for a circular aperture (see Fig. 5). The depth of the region of flat wave fronts is correspondingly increased.

of the intensity distribution near the focal plane extends at least twice as far as that for the circular aperture, with a corresponding increase in the depth of focus. This implies that if the Stiles-Crawford effect disappears at focus, it should disappear over a greater range for an annular pupil than for a circular pupil of the same outer diameter.

It is also of interest to compare the intensity distributions of the two apertures in the geometrical focal plane. Figure 8 compares the relative intensity distribution on the retina for Fraunhofer point images from a circular pupil and an annular pupil of equal outer diameter. The curves are computed from equations given by Born and Wolf,³⁰ with the horizontal coordinate converted from normalized distance v , to real distance $r = \sqrt{x^2 + y^2}$, in microns. For the circular pupil,

$$\frac{I}{I_0} = [2J_1(v)/v]^2 \quad 3$$

and for the annular pupil,

$$\frac{I}{I_0} = \frac{1}{(1-\epsilon^2)^2} \{ [2J_1(v)/v] - \epsilon^2 [2J_1(\epsilon v)/\epsilon v] \}^2 \quad 4$$

where

$$v = \frac{2\pi}{\lambda} \frac{a}{f} r$$

(see eq. 1, Fig. 5) and $J_1(v)$ is a Bessel function of the first order. Parametric values of the curves in Fig. 8 are: $\lambda = 580$ nm,

FIGURE 8

Relative Intensity Distributions of Point Images in the Fraunhofer Plane for Circular and Annular Pupils, calculated from eqns. 3 and 4 respectively. The horizontal axis is radial distance in microns. Parametric values are: $\lambda = 580$ nm, $a = 3$ mm, $f = 16.6832$ mm, and $\epsilon = 5/6$ (for the annular pupil). See text for further discussion.

$\epsilon = 5/6$, $a = 3$ mm, and $f = 16.6832$ mm, corresponding to the parameters of the experimental annular pupil described later. Two striking differences are apparent between the two curves; the annular pupil produces a considerably narrower central maximum and much brighter secondary maxima than does the circular pupil. These differences are sufficiently large to be reflected in the appearance of extended visual targets.

Each point on an extended self-luminous surface can be treated as an independent point source, and thus corresponds to a diffraction image in the Fraunhofer plane.^{2,9} The total irradiance distribution in the image plane is found by integrating the product of the point spread function (intensity function of the diffraction pattern) and the source radiance over the surface of the source. Within a patch of uniform intensity, the form of the point spread function makes no difference in the image, but at a discontinuity it becomes quite important. Steel³¹ has computed the irradiance distribution across a straight, sharp intensity boundary for annular apertures. Curves for a circular aperture and an annular aperture with $\epsilon = 0.5$ are shown in Fig. 9. It is evident that the circular pupil produces a much sharper boundary. This result applies to any sharp boundary that has a small curvature compared to the size of the point spread function. The visual effect is that a border seen through an annular pupil appears more blurred than the same border seen through a circular pupil of the same diameter.

FIGURE 9

Relative Intensity Distributions Across the Fraunhofer Image of an Extended Straight Intensity Border for Circular and Annular Apertures. Adapted from Steel.³¹ The image is considerably sharper for the circular than for the annular aperture. $\epsilon = 0.5$ for the annular aperture. The image would be degraded much more for an annulus with $\epsilon = 5/6$.

Blur.

It is well known that the quality of borders in the visual field greatly influences the perception of contrast and brightness.³² In general, areas enclosed by blurred, fuzzy edges look dimmer and harder to see than areas enclosed by sharp edges. It is therefore important to control for border differences in any test which compares the contrast or brightness of different visual fields. For example, in the present experiment differences in blur between test fields could cause differences in thresholds or brightness difficult to distinguish from differences in the Stiles-Crawford effect.^{2,9} Blur may arise from several causes, but only two are relevant here: dioptric blur, caused either by defocusing or by residual aberrations, and blur caused by diffraction. The effects of dioptric blur could mimic the change in the Stiles-Crawford effect predicted by the non-additivity hypothesis, and so must be controlled to avoid an artifactual non-additive result. Blur due to diffraction applies only near focus, and is mainly a function of the size and shape of the pupil. Although its effects are usually negligibly small, they are appreciable in the case of an annular pupil (see Fig. 9). In a direct brightness matching test, borders seen through an annular pupil appear more blurred, and therefore dimmer, than those seen through a circular pupil. Similarly, thresholds taken with an annular pupil may be higher than thresholds taken with a circular pupil. Fortunately, threshold methods exist, such as flicker^{2,9} and increment³³ thresholds, which are relatively insensitive to blur. A discussion of steps taken to eliminate the

effects of blur in the present experiment is included in the Apparatus and Methods section below.

APPARATUS AND METHODS

Description of Apparatus

The apparatus was similar in principle to that used by Enoch in his study of additivity.^{2,9} It provided a situation very similar to free viewing in that a luminous source was conjugate to the retina, and simultaneously allowed an artificial pupil to be projected into the plane of the natural pupil. It was slightly different than free viewing in that the eye viewed an image of the source instead of the source itself, but the focused retinal image was still a Fraunhofer image of the source. The apparatus is shown schematically in Fig. 10. A magnified image of a high pressure mercury arc was focused on the back of a flashed opal glass diffuser. The diffuser acted like a self-luminous light source and could therefore be treated as a surface made up of independent point sources of coherent light.^{2,9} The combination of a 580 nm interference filter (15 nm half-width) and a short wavelength blocking filter (Wratten 21) interposed between the condensing lens and the diffuser isolated the 578 nm yellow mercury line from the source. Intensity was controlled by a circular neutral density wedge and fixed neutral density filters, also placed behind the diffuser.

The optical system in front of the diffuser (source) can be conceptually divided into two systems. For the first, which I will call

FIGURE 10

Schematic Diagram of Apparatus. See text for description.

the pupil system, an artificial pupil, consisting either of an annulus with $\epsilon = 5/6$ and $a = 3$ mm, a small angular section of that annulus, or a 1 mm diameter circle, was conjugate to the pupil of the eye. For the retina system, a 1/4 inch diameter circular aperture lay in contact with the front surface of the diffuser. L_1 formed a minified image of this aperture in the plane of L_2 . L_3 , which acted as a simple magnifier, could be adjusted to make the source image exactly conjugate to the subject's retina. Since L_2 was in an image plane, it had no power in the retina system. In the pupil system, light from the artificial pupil immediately in front of L_1 was collimated by L_2 and focused in the plane of the natural pupil by L_3 . The pupil rays between L_2 and L_3 were accurately collimated, so that the source image could be focused on the retina by identical movement of L_3 and the eye along the optic axis without changing the size of the pupil image or its position in the natural pupil. L_2 and L_3 had identical focal lengths, giving the pupil image unit magnification. The subject immobilized his head by biting a dental impression mounted on an xyz positioner with cross and vertical feeds graduated in thousandths of an inch. A background field formed by placing an aperture in front of a white diffusing bulb was directed into the eye by a beam combiner and a thin glass plate. Fixation points were also reflected from the glass plate.

Apparatus Alignment

All elements in the test channel of the apparatus were aligned along an axis defined by a laser beam (see Fig. 11). The beam was

FIGURE 11

Schematic Diagram of Laser Alignment of Apparatus. See description in text. The order of alignment of system elements was: L_1 , L_2 , L_3 , pupil aperture, field aperture, diffuser. All elements were initially removed from the channel, but were allowed to remain in place after alignment. The field aperture was hung directly on the diffuser, so both were aligned simultaneously.

leveled by directing it through two colinear pinholes which had first been centered on the same reference point. A third pinhole was then centered in the beam at the far end of the channel as a further reference. Neither the beam nor any of the three colinear pinholes were moved for the rest of the alignment procedure.

All lenses were aligned by simultaneously centering both the transmitted and reflected beams on the appropriate reference pinholes. Aligned lenses could be left in place without interfering with the subsequent alignment of other elements. Although the annular pupil aperture obstructed the central part of the laser beam, a sharp image of the annulus was nevertheless formed around the last pinhole from diffracted light, permitting it to be centered directly. The orientation of the diffuser was adjusted by centering the reflected beam. Finally, the field aperture was centered around the maximum of the beam image on the diffuser.

Light from the plane of the pupil aperture had to be accurately collimated by L_2 in order for the size and position of the pupil image to be independent of the focusing procedure. Collimation was achieved by direct measurement of the size of the pupil image. The pupil aperture was diffusely illuminated from behind to make the depth of focus as shallow as possible and the pupil image was observed through a Bausch & Lomb 7X comparator. The position of L_2 was adjusted until the image size was equal to the aperture size for widely divergent positions of L_3 .

Subject Alignment

Alignment of the subject consisted of (1) positioning the artificial pupil image in the plane of the natural pupil, (2) centering the eye on the optical axis of the system, and (3) focusing the image of the test spot on the retina. Since the position and quality of the retinal image were so important in this experiment, special measures were taken to make each of these adjustments as accurately as possible.

A vignetting test was used to determine when the pupil image was in the plane of the natural pupil. The subject decentered himself while looking at the test spot until it began to dim as the pupil image was occluded by the edge of the natural pupil. The test spot dimmed uniformly only if the image plane coincided with the pupil plane. If the image was behind the pupil plane, the test spot appeared to dim first on the occluded side; if the image was in front of the pupil plane, it dimmed first opposite the occluded side. This adjustment could be made with a precision of ± 0.01 inches. Once the correct position was found, a sliding spacer arm on the focusing lens L_3 (see Fig. 10) was moved into contact with the bite bar and retightened. L_3 was then released to move in synchrony with the bite bar along the z (depth) axis, but not along the x (lateral) or y (vertical) axes. This adjustment needed to be made only once for each subject.

The pupil image was centered with respect to the axis of greatest symmetry in the eye, as suggested by Koomen et al.^{24,26} The subject observed a defocused image of the test field on the axis of the channel and centered his eye to make the blur pattern circularly

symmetrical. This procedure was also very precise, being reproducible to within ± 0.002 inches on both the x and y axes.

Best focus was determined with a vernier optometer similar to one described by Moses.³⁴ The test spot was replaced by a horizontal slit, half of which was polarized horizontally, the other half vertically. The annular pupil was replaced by one of identical shape, but with the top half polarized vertically and the bottom half horizontally (or vice versa). One half of the line was thus seen through the top half of the pupil and the other half through the bottom. When the image of the line was not exactly focused on the retina, the two half-lines appeared vertically displaced from each other in accordance with Scheiner's principle.³⁵ Focus was then adjusted until no vertical displacement could be detected.

This method has two distinct advantages over Scheiner principle optometers which use doubling of a point source seen through two small adjacent artificial pupils as a focusing criterion. First, vernier acuity is as much as thirty times more sensitive than point resolution acuity; the limits are approximately two seconds of arc and one minute of arc respectively.³⁶ Second, the optometer pupil can be made exactly the same shape as the pupil for the test condition, eliminating errors due to differences in aberrations between the two pupils.

Settings of focus with the vernier optometer proved to be quite difficult because of image imperfections presumably produced by diffraction from flaws in the pupil and by aberrations in the eye, but still were reproducible to within ± 0.025 inches on the z axis, cor-

responding to ± 0.03 diopters or $\pm 8.1\mu$ at the retina. One particularly troublesome image defect was identified as diffraction of the slit by the dividing line between the juxtaposed polaroids in the pupil. This was minimized by cementing the polaroids between two thin glass plates with Canada balsam to make a "sandwich", thus reducing the refractive index difference between the polaroids and the border.

Calibration and Measurement

To calibrate the circular neutral density wedge, a photomultiplier tube (PMT) was positioned so that it received only light which had passed through the wedge. Photocurrents for a number of angular wedge positions spanning the total density range were read directly from a digital picoammeter. Relative density was computed as $\log_{10}(1/\text{photocurrent})$ and plotted versus wedge position to obtain the calibration curve.

The accuracy of the calibration was tested with a null method. Photocurrent readings were taken with and without a fixed filter at different wedge positions. Any discrepancies in the calculated fixed filter densities for different wedge positions could then only be ascribed to inaccuracy of either the picoammeter or the PMT. The calibration was found to be accurate to within 5 per cent over the full range of the wedge.

The relative areas of the three artificial pupils used in the experiment (annular, annular section, circular) were determined by placing each pupil in turn directly on a small integrating diffuser

in front of the PMT, where they were uniformly illuminated by a constant intensity point source. The pupil area was then directly proportional to the photocurrent. Relative areas were, respectively, 13.00, 1.00, and 1.594. Thresholds set with different artificial pupils were compared by subtracting $\log_{10}$ (relative pupil area) from the threshold wedge density.

The mercury arc lamp exhibited intensity fluctuations of up to 6 per cent. It was therefore monitored during experimental sessions with a small barrier layer photocell and the picoammeter. Each threshold was then corrected for lamp brightness by subtracting $\log_{10}$ (photocurrent) from the threshold wedge density.

Viewed through the fully dilated pupil, the background field had a retinal illuminance of approximately 200 photopic trolands.

Image Control

Because precise control of the retinal image position was so important for obtaining trustworthy results in this experiment, a number of calculations were done to quantify the effects of movements of various parts of the optical system on the retinal image position.

As was already stated above, the retinal image had considerable depth of focus (about 30 μ for the annular pupil). It was nevertheless very important for the geometrical image to lie in a single plane. A three-dimensional image would have had the same effect as aberrations in that not all points could have been simultaneously focused on the receptors. The opal glass diffuser which acted as the luminous source

had some thickness and at least some of the light which reached the eye came from points behind its front surface. It was therefore of interest to compute the relative thicknesses of the diffuser and its image on the retina. This was done by solving the thin lens formula,

$$f = \frac{1}{i} + \frac{1}{o} \quad 5$$

(where f is focal length, i is image distance and o is object distance) for successive components of the system, for both the front and back surfaces of the diffuser. The 0.5 mm thick diffuser corresponded to a 0.25 μ thick retinal image, which was negligible compared to receptor lengths.

Similar calculations showed that movements of the subject's head on the bite bar caused insignificant changes of focus for the retinal image. Such a movement was optically identical to an equal movement of the virtual image at the subject's far point. For a given xyz setting, subjects could immobilize their heads in the z direction to within ± 0.006 inches. For subject BD (+1.32 diopters), this corresponded to a 0.25 μ movement of the retinal image. The change of angular subtense of the annular pupil with respect to the retina was also calculated for a ± 0.006 inch head movement, taking into account the refractive index and power of the eye, and found to be a negligible ± 4 minutes of arc.

The ratio between focusing lens movement and the corresponding retinal image movement is also very important because it is a major factor in the precision of focusing adjustments. This was calculated

by first finding the movement of the magnified virtual image caused by a given focusing lens adjustment, then solving for the retinal image motion as before. For subject BD, a lens adjustment of 0.001 inches corresponded to a retinal image movement of only 0.325 μ . This figure was relatively insensitive to differences in the subjects' refractive errors; differences of up to one diopter changed the ratio by only a few per cent.

Instrumental precision would have been of little value without an accompanying stability of the optics of the eye. Therefore, accommodation was paralysed and the pupil dilated with two or three drops of Mydriacyl (1% tropicamide) before each experimental session, and single additional drops at 20 minute intervals during the session. Paralysis stopped fluctuations in spherical aberration and astigmatism as well as accommodation. Each subject viewed a sensitive astigmatism target while paralysed and was found to have an insignificant amount of astigmatism.

The effects of blur on the brightness and threshold of a test object have already been discussed. Since blur varied both as a function of focus conditions and of pupil shape, stimulus conditions were arranged to minimize these effects. Ogle³³ studied the effect of dioptric blur on increment threshold as a function of stimulus size. His stimuli were foveally presented disks, flashed for 0.5 sec on a photopic background. The effect of blur on threshold was found to decline with increasing stimulus diameter. The threshold of a 25 min disk was not significantly affected by blur of less than 0.3 diopters. In the present experiment, a 32 min circular test flash was centered

on a 5 deg steady background. All thresholds were set well within ± 0.3 diopters of focus. In addition, the test flash duration was set at 50 msec to take full advantage of the retina's sensitivity to transient stimuli. For these stimulus conditions, differences in blur had no detectable effect on the test flash threshold.

Experimental Methods

The stimulus was a monochromatic (578 nm) circular field 32 min in diameter. It was superposed on a concentric 5 deg photopic background and surrounded by four small fixation points in a concentric diamond pattern. The test field was presented in 50 msec flashes at intervals of 1 sec. The stimulus configuration is shown in Fig. 12.

The additivity of the Stiles-Crawford effect for a focused image was tested indirectly by comparing thresholds of focused and defocused test flashes seen through the annular pupil. After the position of best focus was found, the subject set thresholds by method of adjustment at intervals of 0.012 diopters up to 0.06 diopters on either side of focus. Thresholds were also set at positions 0.12 diopters in front of and behind focus. All thresholds were set in random sequence to control for fatigue effects.

The subject initially increased the stimulus intensity until the test flash was easily visible, then decreased it again gradually until he failed to see the test flash for a criterion number of consecutive flashes. Threshold was taken to be the median of three such settings.

FIGURE 12

Stimulus Configuration. Subjects fixated in the center of the fixation array and saw 50 msec test flashes superposed on the uniform background at 1 sec intervals. The test flash was 578 nm in wavelength. Both the fixation points and the background were yellow tungsten light.

Thresholds were also measured at the focused position with only a small sector of the annular pupil exposed to determine the threshold expected by additivity. Thresholds measured for four sector positions at 90 deg intervals around the annulus were averaged to take into account any possible asymmetry of the Stiles-Crawford effect.

An additional threshold was taken at focus with a 1 mm circular pupil concentric with the annular pupil to determine the threshold expected by a complete failure of additivity; that is, an apparent complete disappearance of the Stiles-Crawford effect. Note that these comparisons are valid whether or not the pupils are centered precisely on the point of maximum efficiency of the Stiles-Crawford effect.

RESULTS

Figure 13 shows typical relative thresholds as a function of distance from focus for subject BD. No systematic changes in threshold were observed over a range of ± 0.12 diopters from focus. The test spot was obviously blurred at both ends of the range when viewed under suprathreshold conditions, confirming that the focal plane was included. The square symbol at $z = 0$ is the average of four thresholds measured with a small segment of the annular pupil, at 90 deg intervals of pupil position. This threshold is expected to reflect the full Stiles-Crawford effect at the eccentricity of the annular radius, regardless of focus. Thresholds at the four orientations differed somewhat, indicating that the annulus was not exactly concentric with the point of peak efficiency, but the average threshold was nevertheless the same as that for the annular pupil. This result excluded the possibility that annular thresholds were uniformly lower over the tested range of focus. Threshold for the circular pupil (diamond symbol) was about 0.4 log units below the annular and eccentric thresholds.

Figure 14 shows comparable results for subject CI.

FIGURE 13

Log Relative Thresholds as a Function of Distance from Focus for Subject BD. All thresholds are computed relative to the annular threshold at best focus (T_f). Thresholds are shown for an annular pupil (o), an eccentric pupil consisting of a small section of the annular pupil ($\square$), and a centered circular pupil 1 mm in diameter ($\diamond$).

FIGURE 14

Log Relative Thresholds as a Function of Distance from Focus
for Subject CI. All thresholds are computed relative to the annular
 threshold at best focus (T_f). Thresholds are shown for an annular
 pupil (○), an eccentric pupil consisting of a small section of the
 annular pupil (□), and a centered circular pupil 1 mm in diameter (◇).

DISCUSSION

The data in Figures 13 and 14 show additivity of the Stiles-Crawford effect for all positions of focus. While this result was expected on the basis of the theoretical analysis, it must nevertheless be interpreted with caution. At least two other arguments consistent with the non-additivity hypothesis can be advanced to explain the additive results: (1) poor image quality, and (2) inappropriate waveguide parameters. These two possibilities will be considered in turn.

(1) Although special care was taken in the experiment to minimize aberrations in the retinal image, it is possible that the optics of the eye are not capable of forming a retinal image for which the wave fronts at focus are flat and parallel to the focal plane. This could happen if, for example, different regions in an annular zone of the eye lens had different power, and superimposed a number of images on the same retinal area but in slightly different focal planes. This type of aberration has been described as asymmetrical spherical aberration in the literature^{23,24} (also, see earlier discussion of spherical aberration) and is minimized for an annular pupil by centering the eye on the axis of greatest symmetry. In the presence of asymmetrical spherical aberration, wave fronts are undoubtedly distorted near focus, and in severe cases might not even be defined for the whole pupil, but

only for sections of constant power. Such wave fronts would not be parallel to the image plane. A ring of faintly outlined overlapping images which was apparent in the badly defocused (annular) blur circle of the test spot may well have been due to asymmetrical spherical aberration.

Assuming for the moment that the Stiles-Crawford effect is absent for an aberration-free image, it is evident that it could be restored by the introduction of enough asymmetrical spherical aberration. If the aberration is small, however, some attenuation should remain. Since asymmetrical spherical aberration is quite similar to astigmatism, it seems unlikely that large asymmetries would exist in the absence of astigmatism. Neither subject in the experiment showed any significant astigmatism. It is therefore doubtful that asymmetrical spherical aberration caused an artifactual additive result.

(2) In the analysis of waveguide properties of visual receptors given above, it was concluded that in order for an attenuation or disappearance of the Stiles-Crawford effect to occur, modal propagation must be firmly established within a distance less than the thickness of the flat wave front region. Since the flat wave front region for the annular pupil was calculated to be as thick as the lengths of the inner segments, this requirement was probably met. In the absence of any objective criteria either for wave front flatness or mode-forming distance, however, it remains possible (though unlikely) that the particular combination of waveguide parameters allows the wave fronts to diverge before they can be trapped by the receptor.

If the non-additivity hypothesis were correct, some residual non-additivity should have been detectable under the conditions of the experiment, even with limited intrusions from the two factors described above. No significant deviations from additivity were found.

In conclusion, the non-additivity hypothesis is rejected in favor of the additivity hypothesis. The trapping efficiency of a receptor is apparently not completely determined by parametric constraints and the direction of propagation of the incident light as it enters the receptor. Rather, the eventual modes of propagation in the receptors depend on the directions of the elemental plane wave components which make up the composite wave fronts, regardless of the direction of energy propagation. This interpretation is predictable from general principles of the superposition of waves. It is also the only explanation which does not predict at least some reduction of the Stiles-Crawford effect at focus. Finally, it has the intuitive elegance of confirming the geometrical optics prediction that the Stiles-Crawford effect is independent of focus.

SUMMARY

Additivity of the Stiles-Crawford effect over pupil area was tested for a Fraunhofer image sharply focused on the retina. Two specific hypotheses, based on different interpretations of dielectric waveguide theory, were compared: (1) that the Stiles-Crawford effect is absent at focus for a centered, symmetrical pupil, and (2) that additivity of the Stiles-Crawford effect holds for all positions of focus.

Additivity was tested indirectly by comparing increment thresholds of focused and defocused test flashes viewed through an annular pupil. The two hypotheses were also compared directly by comparing thresholds of focused test flashes viewed through the annulus to those viewed through (1) a small centered circular pupil, and (2) a small section of the annulus. The non-additivity hypothesis predicts that thresholds for the annular pupil should equal those for the circular pupil, while the additivity hypothesis predicts that they should equal those for the annular section pupil.

Additivity was found to hold precisely for all conditions of focus. This result was interpreted as a general refutation of the non-additivity hypothesis and not as an artifact due to inadequate image quality or to specific waveguide parameters of the visual receptors.

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