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A PRELIMINARY ANALYTICAL AND EXPERIMENTAL
INVESTIGATION OF HELICOPTER
ROTOR BOUNDARY LAYERS

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By

Dwight Allen Blaser, B.S.M.E., M.Sc.

* * * * *

The Ohio State University
1971

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NOMENCLATURE

a_n	the coefficients of the Tchebychef polynomials used to curve-fit the function $1/\Delta$ in equation (2-50).
A_1, A_2, A_3	coefficients in equation (2-78), used to approximate the potential flow and the surface curvature of airfoils.
c	the chord of the airfoil; units of length.
c_p	pressure coefficient, $p/\text{dynamic pressure}$.
d	hot-wire sensor diameter, units of length.
$f_{11}, f_{12}, f_{13},$ f_{14}, f_{15}	coefficients in the x-momentum-integral equation defined by equations (2-40) through (2-44).
$f_{21}, f_{22}, f_{23},$ f_{24}, f_{25}	coefficients in the z-momentum-integral equation defined by equations (2-45) through (2-49).
F, G, H	velocity profile functions defined by equations (2-19).
$g_1, g_2,$ h_1, h_2	potential flow functions defined by equations (2-24) through (2-27).
k	a constant used to represent a linear adverse pressure gradient in equation (2-74); units of length^{-1} .
K	the curvature of the airfoil surface; units of length^{-1} .
p	pressure; units of $\text{force}/\text{length}^2$.
$\vec{q}$	the velocity vector relative to the rotating blade; units of $\text{length}/\text{time}$.

NOMENCLATURE (cont'd)

r	the radial distance from the axis of rotation to a point on the blade; units of length.
Re	the Reynolds number based on blade chord and the free stream chordwise velocity.
Re_w	the Reynolds number used in the hot-wire calibration based on calibration jet exit velocity and hot-wire sensor diameter.
T_n, T'_n	Tchebychef polynomials and their derivatives, respectively.
u, v, w	local velocities in the x, y, z system of coordinates; units of length/time.
U, V, W	potential velocities in the x, y, z system of coordinates; units of length/time.
V_A, V_B	components of velocity normal to hot-wire sensors A and B, respectively; units of length/time.
v_A, v_B	output voltages of anemometers A and B, respectively; units of volts.
X, Y, Z	the Cartesian coordinate system that rotates with the blade, all units of length.
x	the distance from the leading edge of the blade, measured in the chordwise direction along the surface of the blade; units of length.
y	the distance from the surface of the blade, measured along a normal to the surface; units of length.
z	the distance from the axis of rotation, measured along the span of the blade; units of length.

NOMENCLATURE (cont'd)

α	the angle between the normal to the surface of the blade and a normal to the chord line of the blade.
α_1, α_2	coefficients in equation (2-78), used to approximate the potential flow and the surface curvature of airfoils.
β_1, β_2	velocity profile parameters defined by equations (2-21) and (2-23).
γ	the outflow angle, or the angle between the local flow velocity vector and the chord line of the blade measured positively in the increasing span direction.
δ	three-dimensional rotating boundary layer thickness; units of length.
δ_B	two-dimensional non-rotating boundary layer thickness; units of length.
δ_x^*, δ_z^*	boundary layer displacement thicknesses in the u and w profiles, respectively; units of length.
$\bar{\delta}_x^*, \bar{\delta}_z^*$	ratios of displacement thickness to the boundary layer thickness.
Δ	boundary layer thickness parameter, $\Omega\delta^2/\nu$.
Δ_x, Δ_z	partial derivatives of Δ with respect to x and z, respectively; units of length^{-1} .
ϵ	wall shear stress parameter.
ϵ_x, ϵ_z	partial derivatives of ϵ with respect to x and z, respectively; units of length^{-1} .
η	a dimensionless distance above the blade surface measured along a normal to the surface, y/δ .

NOMENCLATURE (cont'd)

θ	blade pitch angle or, the angle between a normal to the blade chord and the axis of rotation.
$\theta_x, \theta_z,$ θ_{xz}, θ_{zx}	boundary layer momentum thicknesses in the u and w profiles; units of length.
$\bar{\theta}_x, \bar{\theta}_z,$ $\bar{\theta}_{xz}, \bar{\theta}_{zx}$	ratios of momentum thickness to boundary layer thickness.
Λ_1, Λ_2	velocity profile parameters defined by equations (2-20) and (2-22).
μ	the viscosity; units of mass/length-time.
ν	the kinematic viscosity, μ/ρ ; units of length ² / time.
ρ	the density of the fluid; units of mass/length ³ .
τ_B	two-dimensional, non-rotating surface shear stress; units of force/length ² .
τ_{or}	component of wall shear stress along the radial direction; units of force/length ² .
$\tau_{o\theta}$	component of wall shear stress along the tangential direction; units of force/length ² .
τ_{ox}	component of wall shear stress along the x chordwise coordinate; units of force/length ² .
$\bar{\tau}_{ox}$	dimensionless chordwise wall shear stress, $\tau_{ox}\delta/\mu U$.
τ_{oz}	component of wall shear stress along the z spanwise coordinate; units of force/length ² .

NOMENCLATURE (cont'd)

ϕ	velocity potential for plane steady flow past the blade placed in a parallel stream of unit speed; units of length.
$\vec{\Omega}$	the rotational velocity vector; units of time^{-1} .
Ω	the magnitude of the rotational velocity vector or, the rotor speed; units of time^{-1} .

The following two additional symbols were used:

$O()$	meaning "of order", i.e. $O(1)$ and $O(\delta)$.
$\sum$	used to represent a summation of terms which follow the symbol.

CHAPTER I

INTRODUCTION

1.1 General Background Information

The design of rotor blades has relied heavily upon the use of two-dimensional airfoil data. By assuming that individual blade elements produce lift independently, a so called "blade element theory" was developed in the early stages of rotor design. With this theory, it was possible to make estimates of the rotor thrust and power requirements without requiring a thorough knowledge of the mechanisms by which lift and drag were being produced. Since that time many refinements to this theory have been incorporated into rotor designs including tip vortex effects, wake interaction, and non-steady flow effects, however, the reliance upon two-dimensional airfoil data has not been eliminated.

Experimental rotor data have given indications that rotor lift coefficients, much greater than those predicted by two-dimensional airfoil data, exist in certain regions at the rotor disk plane. Additionally, the on-

set of blade stall has been qualitatively determined to occur at a higher blade loading condition than expected two-dimensionally. Since blade stall limits the performance of the helicopter because of increased power requirements, aircraft roughness, vibration, and control loads a knowledge of the mechanism by which rotor blade stall develops is needed. Blade stall, however, depends on the nature of the boundary layer which exists on an airfoil.

For a helicopter in steady forward flight there are many actions occurring simultaneously which tend to promote three-dimensionality and time dependence within the rotor boundary layer. The primary action is the chordwise boundary layer flow in the presence of a given chordwise pressure distribution. A secondary flow, the spanwise flow component, exists. This secondary flow forces the total boundary layer flow to be examined as a vector quantity having magnitude and direction. The magnitude and direction of the flow at the edge of the boundary layer is continually changing as do the spanwise pressure gradients which exist. Due to rotation, the fluid particles within the boundary layer experience centrifugal and Coriolis accelerations. Compressible flow effects can occur near the tips of the advancing blade and a reversed flow occurs in the inboard regions of the retreating blade. Very rapid changes in angle-of-attack

can lead to unusual pressure gradient actions in the boundary layers. Periodic stall flutter on retreating blades has also been given recent intensive study. To further complicate the flow, the blades interact with their own tip vortices from the preceding revolution.

The simultaneous occurrence of all of these actions has made rotor boundary layers so complex that until recently investigation of the very nature of these boundary layers has been discouraged. It is the purpose of this investigation to initiate such an investigation by examining the basic nature of the boundary layers on a hovering rotor blade.

1.2 Approach

The investigation consists of an analytical phase and an experimental phase directed towards determining the basic nature of a boundary layer on a helicopter rotor in hover.

The analytical phase of the work consists of applying momentum-integral techniques to the solution of the three-dimensional boundary layer problem. Initially the flat plate or zero pressure gradient case was considered. Then, following a successful solution to this case, the method was extended to include non-zero pressure gradients and airfoils of finite thickness with

blunt leading edges. Due to the complexity of the potential flow, only airfoils at zero lift were examined. The solution is presented for a flat plate, an NACA 0012 airfoil and an NACA 0015 airfoil. The analytical solution obtained by this method is compared with the full computer solution obtained by Dwyer and McCroskey.

The experimental phase consists of hover tests using The Ohio State University hover stand. The investigation employed three experimental techniques to measure flow and pressure characteristics on the rotor blades.

The first experimental technique involved a flow visualization process developed at The Ohio State University during a period prior to the initiation of this study. The technique is an ammonia-azo process which allows visual observation of the flow direction near the surface of a rotating airfoil.

The second experimental technique involved the use of miniature diaphragm pressure transducers submerged within the blade surface. These transducers were used to measure the surface static pressures at various blade positions and hover conditions.

The third and most informative experimental technique employed hot wire anemometry. Dual sensor probes were mounted at various distances above the blade surface of the rotor blade to indicate the magnitude and direc-

tion of the local flow.

The experimental data is compared with existing two-dimensional data where possible. A comparison of the theoretical model and the hot wire data is made for the zero lift condition.

CHAPTER II

THEORETICAL ANALYSIS

2.1 Introduction

Although the boundary layer theory formulated by L. Prandtl in 1904 is of great importance in practical aerodynamics, the difficulties presented by the non-linearity of the boundary layer equations has limited the classes of solutions to flows which exhibit special symmetry. Most of the cases which have been treated to date are two-dimensional in character; solutions of three-dimensional boundary layer flows are quite rare.

One of the practically important types of three-dimensional boundary layer flows is the flow on rotating blades such as turbine, helicopter, and propeller blades. In the case where the blade is rotating steadily in an incompressible fluid, the boundary layer flow is governed by the chordwise and spanwise pressure distributions and the centrifugal and Coriolis accelerations. These actions couple the chordwise and spanwise equations of motion and force them to be solved simultaneously.

The theoretical portion of this study involves the development of an approximate method for calculating the laminar boundary layer flow over a cylindrical blade which is rotating steadily about an axis perpendicular to its leading edge.

2.1.1 Review of Literature

Many investigators have made notable contributions to the field of rotating boundary layer flows. The assumptions and results of a few of these researchers will be summarized below.

In order to examine the boundary layer flow, the inviscid or potential flow that serves as an outer boundary condition to this region of flow must be known. Sears (ref. 1) found a simple but powerful transform that derives the potential flow for the rotating cylinder from the two-dimensional chordwise potential flow across the same cylindrical section. The original result was for the case of infinite blades rotating steadily without circulation; however, Sears and Fogarty (ref. 2) and McCroskey and Yaggy (ref. 3) have extended this solution to include the effects of translation of the axis of rotation (forward flight) and circulation about the cylindrical section (rotors with lift).

Following the potential solution by Sears, a number of his co-workers began examining the boundary layer of

a cylindrical blade rotating steadily in an incompressible fluid. Among these were Fogarty, Tan, Graham, Rott, Smith, and Liu. In each case (except Graham) the curvature of the blade was neglected and the surface normal was assumed to be parallel to the axis of rotation.

Fogarty (ref. 4) uncoupled the motion equations by introducing the assumption of small cross-flow, or that $W/U \ll 1$. His results indicated that the effects of rotation were small; however, his assumptions limited the validity of his theory to several chord lengths from the axis of rotation where the flow is nearly two-dimensional.

Tan (ref. 5) removed Fogarty's assumption of small cross-flow by expanding his solution in powers of x/z and obtaining successive approximations to the differential equations for the case of a rotating flat plate. Unfortunately, this leads to the solution of a large number of differential equations and thus involves a great amount of work to obtain accurate results at large distances from the leading edge.

Graham (ref. 6) employed the small cross-flow assumption and then used momentum-integral techniques to obtain approximate calculations of the boundary layer on rotating cylinders. Due to the small cross-flow assumption, the chordwise momentum-integral equation became identical to the standard two-dimensional flow case.

This forced the separation point (based on zero chordwise velocity gradient at the wall) to coincide with two-dimensional calculations. Graham's solution did serve to estimate the cross-flow profile shape for a number of examples including that of a circular cylinder and a laminar airfoil section.

Rott and Smith (ref. 7) followed the technique employed by Tan (ref. 5) and examined the wedge-type flows characterized by the Falkner-Skan potential flow. Calculations were obtained for both positive and negative chordwise pressure gradients. The results indicated spanwise outflow for all cases; however, the greatest outflow was found for retarded flow.

Liu (ref. 8) further complicated the problem by allowing the axis of rotation to be arbitrarily located with respect to the leading edge of the blade. A perturbation procedure was set up expressing the velocity components in terms of locations on the blade and location of the axis of rotation relative to the blade. Following a great amount of algebraic manipulation, the equations were formulated and numerical solutions were obtained through the $(x/z)^2$ order terms. The results indicated that the development of the boundary layer from the leading edge of a flat plate is apparently independent of the location of the axis of rotation. For the case of a cylindrical blade of small thickness, the deviations of

the results from infinite-cylinder flow are found to be caused mainly by the modification of the outstanding pressure gradient due to the location of the axis of rotation.

Banks and Gadd (ref. 9) used momentum-integral techniques to calculate the boundary layer on a flat sector with a linear adverse external-velocity gradient. The effects of rotation were found to postpone separation of the tangential boundary layer flow. This is one of the clearest indications of the stabilizing nature of rotation on the boundary layer.

Velkoff (ref. 10) performed an integral analysis and was the first to include the effect of translation. His results indicated a general thinning of the boundary layer for an outward radial flow and thickening for an inward radial flow.

The first major contribution to the analysis of three-dimensional boundary layers on helicopter rotors in forward flight was made by McCroskey and Yaggy (ref. 3). They performed a regular perturbation expansion of the unsteady, three-dimensional incompressible boundary layer equations for small cross-flow. The results reveal that the effects of rotation can be large in regions of incipient separation, but elsewhere, the boundary layer generally resembles the viscous flow over a swept wing. The detailed structure of the spanwise flow was found to

depend upon whether the primary flow is accelerating or decelerating.

A direct numerical integration of the three-dimensional or two-dimensional time-dependent boundary layer equations was performed by Dwyer and McCroskey (ref. 11). The three-dimensional hover analysis included the flow near the hub for thin airfoils and circular cylinders. The time-dependent investigations simulated the unsteady effects occurring in forward flight on the outer portions of the rotor. A brute-force method was used assuming velocity profiles and iterating until a local solution was obtained. Each major action was examined independently to determine the order of magnitude of its contribution to the boundary layer flow.

Young and Williams (ref. 12) and Young (ref. 13) conducted a theoretical study to determine the effects of rotation, downflow, and forward flight on the development of the laminar boundary layer on a helicopter rotor. An 11.9 percent thick symmetrical Joukowski airfoil was examined. The essential feature of the analysis was the scaling of the chordwise coordinate so that the separation line is invariant with span and time. The transformed equations were expanded in an asymptotic series in span and solved by the Hartree-Wormsley method. The effects of rotation were found to delay separation the greatest amount near the axis of rotation. Forward

flight caused an oscillation about this separation line, so that the delay is greatest in the first and fourth quadrants.

Beyond the above listed investigations, little, if anything, has been done to explain why actual rotors have been observed to perform better than would be expected on the basis of steady-state, two-dimensional section characteristics of the blades as illustrated by Himmelskamp (ref. 14) and Harris (ref. 15).

2.1.2 Coordinate Systems

The present analysis is restricted to blades which are rotated about an axis normal to the axis of the blade. The blade is assumed infinitely long so that there are no end effects. The coordinate system is shown in Figures 1 through 3. The blade chord lies in the X-Z plane with the Z-axis located at the blade leading edge. The Y-axis is the axis of rotation and is located at the leading edge of the blade. The axes are fixed with respect to the blade so that they rotate with constant angular speed Ω . Note that they form a left-hand coordinate system so that the standard two-dimensional terminology, u and v , for the primary flow and w for the cross-flow may be retained.

Figure 1. Coordinate Systems.

Figure 2. Chordwise or Primary Flow.

Figure 3. Spanwise or Secondary Flow.

For boundary layer calculations, it is necessary to define the curvilinear surface coordinate system (x,y,z) . The z axis coincides with the elements of the cylinder while the x and y axes are parallel and perpendicular to the blade surface respectively as shown in Figure 2. The boundary layer velocity components corresponding to the curvilinear coordinate system (x,y,z) will be denoted by (u,v,w) .

The relationships between the Cartesian and curvilinear coordinates can be obtained from Figure 2 and are given below.

$$X = \int_0^x \cos \alpha \, dx - y \sin \alpha \quad (2-1)$$

$$Y = \int_0^x \sin \alpha \, dx + y \cos \alpha \quad (2-2)$$

$$Z = z \quad (2-3)$$

The angle $\alpha(x)$ represents the slope of the surface of the airfoil with respect to the X -axis or midchord line.

2.1.3 Boundary Layer Equations

The boundary layer equations are obtained from the three-dimensional Navier-Stokes equations for a viscous incompressible fluid. Throughout this analysis the flow is assumed to be laminar. Fogarty (ref. 4) presents

a derivation of these equations valid over the region of the blade where the blade surface is nearly normal to the axis of rotation. This means that the equations are not valid near the stagnation point of an airfoil-shaped blade. This restriction was not imposed by Graham (ref. 6) and thus the essential steps of her derivation are described below.

The time dependence in the Navier-Stokes equations, since the blade is rotating steadily, is eliminated by using coordinates fixed to the blade. With this transformation, centrifugal and Coriolis accelerations appear. In a second transformation, the equations are written in curvilinear coordinates x, y, z where x and z lie on the blade and y is normal to the surface. (See Figure 1). These Navier-Stokes equations are reduced to boundary layer equations by the usual assumptions consistent with confining the viscous effects to a thin layer over the blade surface. That is, within this layer of thickness δ , the following orders of magnitude are assumed.

$$\begin{array}{lll} x = O(1) & u = O(1) & \\ y = O(\delta) & v = O(\delta) & \delta = O(\sqrt{\nu}) \\ z = O(1) & w = O(1) & \end{array}$$

Then if the radius of curvature of the blade surface is large compared to the boundary layer thickness, the following boundary layer equations are obtained by neglecting terms $O(\delta)$ and higher,

$$uu_x + vu_y + wu_z - 2\Omega w \cos\alpha - \Omega^2 x \cos\alpha = -p_x/\rho + vu_{yy} \quad (2-4)$$

$$- Ku^2 + 2\Omega w \sin\alpha + \Omega^2 x \sin\alpha = -p_y/\rho \quad (2-5)$$

$$uw_x + vw_y + ww_z + 2\Omega u \cos\alpha - \Omega^2 z = -p_z/\rho + vw_{yy} \quad (2-6)$$

where K is the blade surface curvature. In like manner the continuity equation becomes

$$u_x + v_y + w_z = 0 \quad (2-7)$$

Since the left hand side of equation (2-5) is $O(1)$, the pressure can vary at most by an amount $O(\delta)$ within the boundary layer. It is reasonable then to neglect this variation and determine the pressure solely by the potential flow outside the boundary layer. (An identical argument is made for two-dimensional equations governing the boundary layer flow over a curved surface).

The easiest way to determine the relationships between the pressure gradients p_x and p_z and the potential velocity components U and W is to examine the boundary layer equations near the outer edge of the boundary layer. In this region it may be assumed that $v = 0$, $\partial/\partial y = 0$, $u = U$, and $w = W$, so that equations (2-4) and (2-6) become

$$\begin{aligned}
 -p_x/\rho &= UU_x + WU_z - 2\Omega W \cos\alpha - \Omega^2 x \cos\alpha \\
 &= UU_x + WW_x - \Omega^2 x \cos\alpha
 \end{aligned}
 \tag{2-8}$$

$$\begin{aligned}
 -p_z/\rho &= UW_x + WW_z + 2\Omega U \cos\alpha - \Omega^2 z \\
 &= UU_z + WW_z - \Omega^2 z
 \end{aligned}
 \tag{2-9}$$

The second portions of equations (2-8) and (2-9) follow from the potential flow solution given in section 2.3.1 as equation (2-71). Substitution of the pressure gradient expressions into equations (2-4) and (2-6) yield the following final form of boundary layer equations.

$$\begin{aligned}
 uu_x + vu_y + wu_z - 2\Omega w \cos\alpha &= UU_x + WU_z - 2\Omega W \cos\alpha \\
 &\quad + vu_{yy}
 \end{aligned}
 \tag{2-10}$$

$$\begin{aligned}
 uw_x + vw_y + ww_z + 2\Omega u \cos\alpha &= UW_x + WW_z + 2\Omega U \cos\alpha \\
 &\quad + vw_{yy}
 \end{aligned}
 \tag{2-11}$$

2.1.4 Momentum-Integral Equations

Due to the great difficulties involved in solving equations (2-7), (2-10), and (2-11) an alternative approximate method was examined. In this approximate method, the boundary layer equations are not satisfied at each point in the boundary layer; however, the average value of each equation across the boundary layer is maintained. The essential steps of the derivation of the

three-dimensional momentum-integral equations are described below.

The y -component of the boundary layer velocity, v , is eliminated from equations (2-10) and (2-11) by making use of the continuity equation (2-7) in the following form.

$$v = - \int_0^y (u_x + w_z) dy \quad (2-12)$$

Then equations (2-10) and (2-11) are integrated across the boundary layer from $y=0$ to $y=h$, where h is a constant such that $h \geq \delta$. After making the following standard definitions for displacement and momentum thicknesses and shear stresses,

$$\begin{aligned} \delta_x^* &= \int_0^h \left(1 - \frac{u}{U}\right) dy & \delta_z^* &= \int_0^h \left(1 - \frac{w}{W}\right) dy \\ \theta_x &= \int_0^h \frac{u}{U} \left(1 - \frac{u}{U}\right) dy & \theta_z &= \int_0^h \frac{w}{W} \left(1 - \frac{w}{W}\right) dy \\ \theta_{xz} &= \int_0^h \frac{w}{W} \left(1 - \frac{u}{U}\right) dy & \theta_{zx} &= \int_0^h \frac{u}{U} \left(1 - \frac{w}{W}\right) dy \\ \tau_{ox} &= \mu \frac{\partial u}{\partial y} \Big|_{y=0} & \tau_{oz} &= \mu \frac{\partial w}{\partial y} \Big|_{y=0} \end{aligned} \quad (2-13)$$

the momentum-integral equations may be written in the form given below.

$$\frac{\partial}{\partial x}(U^2 \theta_x) + \frac{\partial}{\partial z}(UW \theta_{xz}) + UU_x \delta_x^* + WW_x \delta_z^* = \tau_{ox}/\rho \quad (2-14)^{20}$$

$$\frac{\partial}{\partial x}(UW \theta_{zx}) + \frac{\partial}{\partial z}(W^2 \theta_z) + UU_z \delta_x^* + WW_z \delta_z^* = \tau_{oz}/\rho \quad (2-15)$$

Equations (2-14) and (2-15) will be referred to as the momentum-integral equations throughout the remaining sections. It is reminded that the solution of these equations will indicate an averaged behavior of the boundary layer flow.

2.2 Method of Solution

Since solutions of the three-dimensional momentum-integral equations are rare and quite complex, it would seem appropriate to examine the ability of these equations to accurately describe three-dimensional boundary layer flows.

Although the validity of the momentum-integral method is well established in two-dimensional boundary layer flow, the use of this method in three-dimensional boundary layer flow demands the assumptions that the velocities in the boundary layer preserve, essentially, the corresponding directions at the stream lines in the external flow (ref. 16, p. 258). This demand is compatible with the requirement that only a weak secondary flow exist in the boundary layer. A portion of the

effort in this analysis was devoted to making predictions of boundary layer separation. Near separation the primary flow is diminished and the secondary flow becomes proportionately stronger. For this reason, the results of a three-dimensional momentum-integral analysis can only be used qualitatively to predict separation.

This limitation near flow separation does not imply that an integral technique is worthless in this flow region. Before this technique can be criticized, a thorough examination of the alternative techniques must be undertaken. For example, a finite difference technique could be applied to the complete Navier-Stokes equations similar to Dwyer and McCroskey (ref. 11). However, as separation is approached and the differences (derivatives) become larger and larger, the grid spacing would need to be lessened to a point where the calculations would no longer be possible without great expense. Thus, even the so-called "exact" methods of solution of boundary layer problems have drawbacks near separation,

2.2.1 Extension of the Pohlhausen Technique

Pohlhausen (ref. 17) was first to introduce a general method of solution of the two-dimensional momentum-integral equation. A complete description of the

method in a more modern form is presented by Schlichting (ref. 16, p.243), however, a brief outline of the Pohlhausen method will follow.

First, the two-dimensional momentum-integral condition results in the following single, ordinary differential equation.

$$U^2 \frac{d\theta}{dx} + (2\theta_x + \delta_x^*) U U_x = \tau_{ox} / \rho \quad (2-16)$$

The technique introduced by Pohlhausen was initiated by assuming a velocity profile function which contained an undetermined parameter. The parameter was determined at each location on the wall surface by solving the ordinary differential equation.

In contrast, the three-dimensional momentum-integral condition results in two partial differential equations (equations 2-38 and 2-39). The extension of Pohlhausen's technique involves assuming profile functions for the two velocity components, u and w , which may contain a total of two undetermined parameters. These parameters are subsequently determined by solving the partial differential equations over the wall surface. The following sections are devoted to the systematic definition of the velocity profile functions and the two profile parameters, the derivation of the partial differential equations, and the integration technique employed to obtain a solution to these equations.

2.2.2 Velocity Profiles

The momentum-integral method hinges on an approximate description of the velocity profiles within the boundary layer. To insure that the approximate velocity profiles closely represent those that actually occur, a number of conditions derived from the differential equations and boundary conditions are imposed at the surface as well as at the junction with the external flow. Those conditions satisfied in this analysis are as follows.

Conditions at the surface:

$$u = 0$$

$$w = 0$$

$$u_{yy} = \frac{1}{\nu} \left(\frac{1}{\rho} p_x - \Omega^2 x \cos \alpha \right)$$

$$w_{yy} = \frac{1}{\nu} \left(\frac{1}{\rho} p_z - \Omega^2 z \right)$$

Conditions at the junction with the external flow:

$$u = U$$

$$w = W$$

$$u_y = 0$$

$$w_y = 0$$

$$u_{yy} = 0$$

$$w_{yy} = 0$$

These conditions insure that, at least at both edges of the boundary layer, the approximate velocity profiles have the appropriate behavior. Consequently the greater the number of conditions the better the approximation.

The accuracy of the approximation is also dependent

to some extent on the characteristic parameters selected by the investigator since these parameters allow the profiles to assume different forms. In two-dimensional boundary layer problems, it is conventional to use some measure of the boundary layer thickness as one such parameter. In three-dimensional boundary layer problems, there are two such profiles which have directed some investigators into choosing two thicknesses defining the distances above the surface at which the $u=U$ and $w=W$ respectively. One such investigation was performed by Cooke (ref. 18). On the other hand, if it is assumed that a single boundary layer thickness defines a certain viscous region, that is, that both velocity profiles are equally thick, then it is logical to seek the second parameter at the other extremity of the boundary layer; for example, many investigators have chosen the ratio of the wall frictional forces such as Prandtl (ref. 19) and Taylor (ref. 20). In the present analysis, a single boundary layer thickness, δ , and a wall shear stress parameter, ϵ , were selected to represent the growth of the boundary layer on a rotating cylinder. The wall shear stress parameter will be defined by the boundary conditions imposed at the surface.

The velocity profiles for the chordwise and spanwise components of boundary layer flow are given below.

$$\frac{u}{U} = F(\eta) + \Lambda_1 G(\eta) + \beta_1 H(\eta) \quad (2-17)^{25}$$

$$\frac{w}{W} = F(\eta) + \Lambda_2 G(\eta) + \beta_2 H(\eta) \quad (2-18)$$

The following definitions are in order.

$$\eta = y/\delta$$

$$F(\eta) = 2\eta - 2\eta^3 + \eta^5 = 1 - (1+\eta)(1-\eta)^3$$

$$G(\eta) = \frac{1}{6}(\eta - 3\eta^2 + 3\eta^3 - \eta^4) = \frac{1}{6}(1-\eta)^3 \quad (2-19)$$

$$H(\eta) = 2(\eta - 6\eta^3 + 8\eta^4 - 3\eta^5) = 2\eta(1+3\eta)(1-\eta)^3$$

The profile parameters Λ_1 , β_1 , Λ_2 , and β_2 are functions of the boundary layer parameters δ and ϵ , and the potential flow according to the following relations.

$$\Lambda_1 = \left(\frac{UU_x + WW_x}{\Omega U} \right) \frac{\Omega \delta^2}{v} \quad (2-20)$$

$$\beta_1 = - \frac{W}{U} \epsilon \quad (2-21)$$

$$\Lambda_2 = \left(\frac{UU_z + WW_z}{\Omega W} \right) \frac{\Omega \delta^2}{v} \quad (2-22)$$

$$\beta_2 = \frac{U}{W} \epsilon \quad (2-23)$$

To simplify the presentation as much as possible, the following definitions will be used.

$$h_1(x, z) = (\Omega^2 x \cos \alpha - p_x / \rho) / \Omega U = (UU_x + WW_x) / \Omega U \quad (2-24)$$

$$h_2(x, z) = (\Omega^2 z - p_z / \rho) / \Omega W = (UU_z + WW_z) / \Omega W \quad (2-25)$$

$$g_1(x, z) = -W/U \quad (2-26)^{26}$$

$$g_2(x, z) = U/W \quad (2-27)$$

$$\Delta = \Omega \delta^2 / \nu \quad (2-28)$$

Thus equations (2-20) through (2-23) may be written in the simplified form given below.

$$\begin{aligned} \Lambda_1 &= h_1 \Delta & \beta_1 &= g_1 \epsilon \\ \Lambda_2 &= h_2 \Delta & \beta_2 &= g_2 \epsilon \end{aligned} \quad (2-29)$$

The profile shape parameters Λ_1 and Λ_2 evolve directly from the second boundary conditions at the surface. These conditions are obtained by evaluating the chordwise and spanwise Navier-Stokes equations at $y=0$, and are analogous to the condition used by K. Pohlhausen (ref. 17) in the development of a general two-dimensional momentum-integral method outlined earlier.

These parameters allow the shape of the velocity profiles to be influenced by their corresponding pressure gradients and will be referred to as "Pohlhausen" profile shape parameters. For example, if the pressure gradient is strongly adverse, these parameters will force the profiles to contain inflexion points that are found in exact two-dimensional flows and are expected to occur in certain three-dimensional boundary layer flows.

The "symmetry" in the ϵ portions of the profile representations of the chordwise and spanwise velocities was not accidental. Indeed, the first attempt at calculating the boundary layer on a rotating flat plate at zero incidence incorporated profiles quite similar to those used by Prandtl (ref. 19). In this initial analysis ϵ was defined as the ratio of the spanwise to the chordwise wall shear stresses and was included only in the spanwise profile. This led to "non-symmetric" forms for the two profiles. Numerical integration of the equations was performed and the results were compared with those of previous investigators. Near the leading edge of the flat plate the results compared favorably; however, at a value of $x/z=0.5$ the solution indicated that a singularity may exist. These calculations were then repeated using rotating polar coordinates where ϵ was defined as the ratio of the radial to the tangential wall shear stresses. In this case the numerical integration was stable for all values of x/z .

Since the calculations cannot depend upon the coordinate system used to represent the physical problem, it was apparent that the differences in the results must be attributed to the two definitions of the parameter ϵ . To confirm these conclusions, ϵ was defined to be the ratio of the radial to the tangential wall shear stresses; however, the calculations were made using a Cartesian

coordinate system. This forced the chordwise and spanwise profiles to be "symmetric" with respect to the parameter ϵ . The results were identical with those obtained using polar coordinates as would be expected.

The velocity profile functions given by equations (2-17) and (2-18) are generalizations of those used for the initial flat plate analysis. For this reason, ϵ is no longer the ratio of radial to tangential wall shear stresses unless $\Delta=0$. Further discussion of this parameter for a flat plate is included with the results.

2.2.3 Formulation of the Mathematical Equations

The mathematical problem was formulated as follows. The momentum-integral equations (equations 2-14 and 2-15) have allowed the choice of two free parameters in the velocity profiles. These parameters are δ , the boundary layer thickness and ϵ , a wall shear stress parameter. Throughout this analysis, $\Delta=\Omega\delta^2/\nu$, a dimensionless form of boundary layer thickness will be used in place of δ for the first parameter.

Substitution of the velocity profile expressions (equations 2-17 through 2-19) into the equations (2-13) yields the following polynomials for the displacement and momentum thicknesses and the wall shear stresses.

$$\bar{\delta}_x^* = \frac{\delta_x^*}{\delta} = \frac{3}{10} - \frac{1}{120} \Lambda_1 - \frac{1}{5} \beta_1 \quad (2-30)^{29}$$

$$\bar{\theta}_x = \frac{\theta_x}{\delta} = \frac{37}{315} \left(1 - \frac{1}{111} \Lambda_1 - \frac{16}{37} \beta_1 - \frac{19}{444} \Lambda_1 \beta_1 - \frac{5}{5328} \Lambda_1^2 - \frac{208}{407} \beta_1^2 \right) \quad (2-31)$$

$$\bar{\theta}_{xz} = \frac{\theta_{xz}}{\delta} = \frac{37}{315} \left(1 - \frac{71}{1776} \Lambda_1 + \frac{55}{1776} \Lambda_2 - \frac{79}{74} \beta_1 + \frac{47}{74} \beta_2 - \frac{19}{888} \Lambda_1 \beta_2 - \frac{19}{888} \Lambda_2 \beta_1 - \frac{5}{5328} \Lambda_1 \Lambda_2 - \frac{208}{407} \beta_1 \beta_2 \right) \quad (2-32)$$

$$\bar{\delta}_z^* = \frac{\delta_z^*}{\delta} = \frac{3}{10} - \frac{1}{120} \Lambda_2 - \frac{1}{5} \beta_2 \quad (2-33)$$

$$\bar{\theta}_z = \frac{\theta_z}{\delta} = \frac{37}{315} \left(1 - \frac{1}{111} \Lambda_2 - \frac{16}{37} \beta_2 - \frac{19}{444} \Lambda_2 \beta_2 - \frac{5}{5328} \Lambda_2^2 - \frac{208}{407} \beta_2^2 \right) \quad (2-34)$$

$$\bar{\theta}_{zx} = \frac{\theta_{zx}}{\delta} = \frac{37}{315} \left(1 + \frac{55}{1776} \Lambda_1 - \frac{71}{1776} \Lambda_2 + \frac{47}{74} \beta_1 - \frac{79}{74} \beta_2 - \frac{19}{888} \Lambda_1 \beta_2 - \frac{19}{888} \Lambda_2 \beta_1 - \frac{5}{5328} \Lambda_1 \Lambda_2 - \frac{208}{407} \beta_1 \beta_2 \right) \quad (2-35)$$

$$\bar{\tau}_{ox} = \frac{\tau_{ox}^\delta}{\mu U} = 2 \left(1 + \frac{1}{12} \Lambda_1 + \beta_1 \right) \quad (2-36)$$

$$\bar{\tau}_{oz} = \frac{\tau_{oz}^\delta}{\mu W} = 2 \left(1 + \frac{1}{12} \Lambda_2 + \beta_2 \right) \quad (2-37)$$

Note that the "symmetric" forms of the two velocity profiles forces symmetry in the above expressions. For example, $\bar{\theta}_{zx}$ can be obtained from $\bar{\theta}_{xz}$ by interchanging subscripts 1 and 2.

Substitution of equations (2-30 through 2-37) into the two momentum-integral equations results in a pair of equations from which Δ and ϵ are to be obtained as functions of x and z . After a great amount of algebraic manipulation, the equations were written in the following form.

$$f_{11} \Delta_x + f_{12} \Delta_z + f_{13} \epsilon_x + f_{14} \epsilon_z = f_{15} \quad (2-38)$$

$$f_{21} \Delta_x + f_{22} \Delta_z + f_{23} \epsilon_x + f_{24} \epsilon_z = f_{25} \quad (2-39)$$

Where the functions f_{ij} are complex functions of x , z , Δ , ϵ , and the potential flow. These functions are listed below.

$$f_{11} = \frac{U}{\Omega} \left(\bar{\theta}_x + 2 \Lambda_1 \frac{\partial \bar{\theta}_x}{\partial \Lambda_1} \right) \quad (2-40)$$

$$f_{12} = \frac{W}{\Omega} \left[\bar{\theta}_{xz} + 2 \left(\Lambda_1 \frac{\partial \bar{\theta}_{xz}}{\partial \Lambda_1} + \Lambda_2 \frac{\partial \bar{\theta}_{xz}}{\partial \Lambda_2} \right) \right] \quad (2-41)$$

$$f_{13} = 2 \frac{U}{\Omega} \Delta g_1 \frac{\partial \bar{\theta}_x}{\partial \beta_1} \quad (2-42)$$

$$f_{14} = 2 \frac{W}{\Omega} \Delta \left(g_1 \frac{\partial \bar{\theta}_{xz}}{\partial \beta_1} + g_2 \frac{\partial \bar{\theta}_{xz}}{\partial \beta_2} \right) \quad (2-43)$$

$$\begin{aligned}
f_{15} = & -2 \Delta \left[\frac{U}{\Omega} x (2 \bar{\theta}_x + \bar{\theta}_x^*) + \frac{W}{\Omega U} x \bar{\theta}_z^* + \left(\frac{W}{\Omega} z + \frac{WU}{\Omega U} z \right) \bar{\theta}_{xz} \right]^{31} \\
& - 2 \Delta^2 \left[\frac{U}{\Omega} \frac{\partial \bar{\theta}_x}{\partial \Lambda_1} \frac{\partial h_1}{\partial x} + \frac{W}{\Omega} \left(\frac{\partial \bar{\theta}_{xz}}{\partial \Lambda_1} \frac{\partial h_1}{\partial z} + \frac{\partial \bar{\theta}_{xz}}{\partial \Lambda_2} \frac{\partial h_2}{\partial z} \right) \right] \\
& - 2 \Delta \epsilon \left[\frac{U}{\Omega} \frac{\partial \bar{\theta}_x}{\partial \beta_1} \frac{\partial g_1}{\partial x} + \frac{W}{\Omega} \left(\frac{\partial \bar{\theta}_{xz}}{\partial \beta_1} \frac{\partial g_1}{\partial z} + \frac{\partial \bar{\theta}_{xz}}{\partial \beta_2} \frac{\partial g_2}{\partial z} \right) \right] \\
& + 2 \bar{\tau}_{ox}
\end{aligned} \tag{2-44}$$

$$f_{21} = \frac{U}{\Omega} \left[\bar{\theta}_{zx} + 2 \left(\Lambda_1 \frac{\partial \bar{\theta}_{zx}}{\partial \Lambda_1} + \Lambda_2 \frac{\partial \bar{\theta}_{zx}}{\partial \Lambda_2} \right) \right] \tag{2-45}$$

$$f_{22} = \frac{W}{\Omega} \left(\bar{\theta}_z + 2 \Lambda_2 \frac{\partial \bar{\theta}_z}{\partial \Lambda_2} \right) \tag{2-46}$$

$$f_{23} = 2 \frac{U}{\Omega} \Delta \left(g_1 \frac{\partial \bar{\theta}_{zx}}{\partial \beta_1} + g_2 \frac{\partial \bar{\theta}_{zx}}{\partial \beta_2} \right) \tag{2-47}$$

$$f_{24} = 2 \frac{W}{\Omega} \Delta g_2 \frac{\partial \bar{\theta}_z}{\partial \beta_2} \tag{2-48}$$

$$\begin{aligned}
f_{25} = & 2 \bar{\tau}_{oz} - 2 \Delta \left[\frac{W}{\Omega} z (2 \bar{\theta}_z + \bar{\theta}_z^*) + \frac{U}{\Omega W} z \bar{\theta}_x^* \right. \\
& + \left. \left(\frac{U}{\Omega} x + \frac{UW}{\Omega W} x \right) \bar{\theta}_{zx} \right] - 2 \Delta^2 \left[\frac{U}{\Omega} \left(\frac{\partial \bar{\theta}_{zx}}{\partial \Lambda_1} \frac{\partial h_1}{\partial x} \right. \right. \\
& + \left. \frac{\partial \bar{\theta}_{zx}}{\partial \Lambda_2} \frac{\partial h_2}{\partial x} \right) + \frac{W}{\Omega} \frac{\partial \bar{\theta}_z}{\partial \Lambda_2} \frac{\partial h_2}{\partial z} \left. \right] - 2 \Delta \epsilon \left[\frac{U}{\Omega} \left(\frac{\partial \bar{\theta}_{zx}}{\partial \beta_1} \frac{\partial g_1}{\partial x} \right. \right. \\
& + \left. \frac{\partial \bar{\theta}_{zx}}{\partial \beta_2} \frac{\partial g_2}{\partial x} \right) + \frac{W}{\Omega} \frac{\partial \bar{\theta}_z}{\partial \beta_2} \frac{\partial g_2}{\partial z} \left. \right]
\end{aligned} \tag{2-49}$$

2.2.4 Numerical Integration Technique

Equations (2-38) and (2-39) may be described mathematically as two coupled, quasi-linear, first order, partial differential equations. The solutions of these equations involve the determination of the functions $\Delta(x,z)$ and $\epsilon(x,z)$. These two functions cannot be determined explicitly due to the nonlinearity of the differential equations. For this reason, an approximate integration technique was developed which determines values of Δ and ϵ at a discrete number of locations on the blade surface. This technique will be described in detail since it provides the results for the theoretical analysis portion of this study.

The grid network used throughout this analysis is illustrated in Figure 4. A uniform spanwise grid spacing, $\Delta z = 0.5$ chord, was chosen to provide a reasonable number of spanwise locations for data interpolation. The spanwise locations were located from 0.5 chord to 20 chords to map the flow over the entire portion of most rotor blades. The singularity of the flow in the vicinity of the axis of rotation prevents calculations at spanwise positions near $z=0$. This was not considered to be a drawback to the results since most rotors do not extend their blade sections into this region due to inefficient lift production in the hub interference area. The chordwise grid spacing, Δx , was continually adjusted to maintain a

Figure 4. Calculation Grid Network.

specified computing accuracy during the numerical integration and will be discussed later.

The problem was formulated as an initial value problem. The values of $\Delta(0, z_i)$ and $\epsilon(0, z_i)$, where z_i denotes the distance to the i^{th} spanwise grid point, are known from power series expansions for small x discussed in section 2.2.5. For example, for a sharp leading edged blade, both $\Delta(0, z_i)$ and $\epsilon(0, z_i)$ vanish for all spanwise grid points. The values of $\Delta_z(0, z_i)$ and $\epsilon_z(0, z_i)$ can also be evaluated from these power series. With the leading edge values known, the calculations may be stepped a distance Δ_x in the chordwise direction by using equations (2-38) and (2-39) to calculate $\Delta_x(0, z_i)$ and $\epsilon_x(0, z_i)$ and employing a Runge-Kutta integration routine.

With the values $\Delta(\Delta x, z_i)$ and $\epsilon(\Delta x, z_i)$ known, the next step becomes more complicated since $\Delta_z(\Delta x, z_i)$ and $\epsilon_z(\Delta x, z_i)$ are no longer known as they were at the leading edge. To determine these new values of Δ_z and ϵ_z , a numerical curve-fitting routine was developed. Once $\Delta_z(\Delta x, z_i)$ and $\epsilon_z(\Delta x, z_i)$ are known, equations (2-38) and (2-39) are again used to calculate $\Delta_x(\Delta x, z_i)$ and $\epsilon_x(\Delta x, z_i)$. Then the Runge-Kutta integration routine steps the calculations to a distance of $2\Delta x$ from the leading edge. Repetition of these calculations results in an approximate solution to the momentum-integral equations.

The numerical integration along the chordwise direc-

tion at each spanwise grid point was performed on an IBM 360/75 computer. An IBM application program was selected from the System/360 Scientific Subroutine Package. This subroutine, labeled DHPCG, uses Hamming's modified predictor-corrector method for the solution of general initial-value problems (ref. 21). It is a fourth order integration procedure which can estimate the local truncation error at each step and properly adjust the step size Δx . Hamming's predictor-corrector method itself is not self-starting; therefore, DHPCG employs a special Runge-Kutta procedure to obtain starting values. DHPCG is a double-precision subroutine which carries at least sixteen significant digits throughout all calculations. Double-precision was required primarily for initiating the calculations near the stagnation point of blunt leading edge blades where the equations are singular.

The numerical differentiation routine developed to calculate Δ_z and ϵ_z at each chordwise position will now be discussed. A typical spanwise variation of the parameter Δ for a rotating flat plate is illustrated in Figure 5 for fixed chordwise positions. Notice that the variations are greatest for small values of z . In order to employ a standard polynomial least-square curve fit routine, it was found convenient to first invert the parameter and then apply a curve-fitting technique. In this manner, the following approximation was used,

Figure 5. Variation of the Parameter Δ on a Flat Plate.

$$\frac{1}{\Delta} = \sum_{n=1}^N a_n T_n(z) \quad (2-50)$$

where $T_n(z)$ is the Tchebychef polynomial of order n , and a_n is the coefficient of $T_n(z)$ determined from a least-square-error criterion. To obtain Δ_z , equation (2-50) was differentiated yielding,

$$\Delta_z = -\Delta^2 \sum_{n=1}^N a_n T'_n(z) \quad (2-51)$$

The parameter ϵ had a similar variation with span and consequently, ϵ_z was determined according to an expression equivalent to equation (2-51). The standard deviations for the curve fit expressions ranged from 10^{-3} to 10^{-5} for both Δ and ϵ .

2.2.5 Leading Edge Starting Values

At the leading edge, the flow is two-dimensional in character and the solutions become identical to the two-dimensional momentum-integral solutions. In solving equations (2-38) and (2-39) for Δ_x and ϵ_x , the resulting right hand sides of both equations are quite complex. To evaluate the nature of the solution in the vicinity of the leading edge a power series approximation technique was employed. The form of these series varies widely depending upon whether the airfoil has a "sharp", or a "blunt",

leading edge.

For the case of a sharp leading edge, the first term in the series representations of the functions Δ and ϵ are as follows:

$$\Delta = 34.0540 \ x/z + 0(x^3) \quad (2-52)$$

$$\epsilon = -0.907491 \ x/z + 0(x^3) \quad (2-53)$$

These series expansions were used to evaluate Δ_x and ϵ_x at the leading edge. Beyond the leading edge, the complete form of equations (2-38) and (2-39) were evaluated to obtain Δ_x and ϵ_x .

The blunt leading edge case was more difficult since the right hand sides of the equations for Δ_x and ϵ_x at the leading edge are indeterminate, that is, the numerators and denominators vanish at the stagnation line along the leading edge. Once again, a power series expansion was used and the first terms were evaluated and given by equations (2-54) and (2-55).

$$\Delta = \frac{7.05232 \ \Omega}{U_x(0,z)} + 0(x^2) \quad (2-54)$$

$$\epsilon = \left(-0.0578311 + 0.102213 \frac{z \ W_{xx}(0,z)}{U_x(0,z)} \right) \frac{x}{z} + 0(x^3) \quad (2-55)$$

Note that for the blunt leading edge airfoil the initial value of Δ and the initial chordwise slope of ϵ depend

upon the form of the potential flow and consequently, the shape of the blade surface at the leading edge.

However, the calculation of the value of Δ_x near the leading edge requires the second term in the power series expansion. The algebra involved in computing this second term is rather complex, thus, a second and more approximate method was used. First, calculations were initiated with $\Delta_x(x_\epsilon, z) = 0$, where x_ϵ is a small value of x on the order of 0.01. The complete expressions were used for $x > x_\epsilon$. The stability of the computer solution for such a calculation is illustrated in Figure 6 by examining the variation in Δ_x . From the power series method, it is known that the second term in the power series expansion of Δ is of order x^2 as shown in equation (2-54). Thus Δ_x should start with zero at the leading edge and vary linearly for $x \leq x_\epsilon$. The slope of this linear portion of curve was then determined by extrapolating the well-behaved portion of the curve back to the origin. The second set of calculations were initiated with the slope of Δ_x for $x \leq x_\epsilon$ being forced to follow the dashed line. The results of this second calculation again show some instability in the technique near the leading edge; however, at a distance of x_ϵ behind the leading edge, the results appear to be independent of this instability since both runs agree quite accurately.

In summary, it was found that considerable insight

Figure 6. Stability of Numerical Technique Near the Stagnation Line of an NACA 0012.

to the analysis could be obtained from a power series solution in the vicinity of the leading edge; however, the major disadvantage of the power series method is the vast amount of algebraic manipulations that must be performed.

2.3 Potential Flow Solution

The potential flow solution can be viewed as a solution to the Navier-Stokes equations (equations 2-4 through 2-6) for the special case where viscous effects are neglected. In this type of idealistic flow the "no-slip" condition at the blade surface is relaxed and the convective motions of the flow are governed solely by the existing pressure gradients.

Although the potential flow is at best a first approximation for air flows, it is important since it represents the outer condition for the viscous flow. It is this potential solution that must be matched by the boundary layer solution at the outer edge of the boundary layer.

2.3.1 General Solution of Sears

The potential flow solution for the case of an infinitely long cylinder, steadily rotating about an axis normal to it was first solved by Sears (ref. 1). The

results are given by equations (2-56 through 2-58) which are the three components of the potential velocity along the Cartesian coordinates shown in Figures 1 through 3.

$$U = \Omega Z \frac{\partial \phi}{\partial X} \quad (2-56)$$

$$V = \Omega Z \frac{\partial \phi}{\partial Y} \quad (2-57)$$

$$W = \Omega(\phi - 2X) \quad (2-58)$$

The function $\phi(X,Z)$ denotes the velocity potential for plane, steady flow past the cylinder placed in a parallel stream of unit speed.

For this analysis it is necessary to have the potential velocity components along the curvilinear or boundary layer coordinates. To obtain approximate expressions along these coordinates, a portion of the derivation presented in Sears' paper will be repeated to make the necessary adjustments to the curvilinear coordinate system.

In any coordinate system rotating with the blade, the potential flow must satisfy the following expressions:

$$\text{div } \vec{q} = 0 \quad (2-59)$$

$$\text{curl } \vec{q} = 2 \vec{\Omega} \quad (2-60)$$

where $\vec{q}$ is the velocity vector relative to the rotating blade and $\vec{\Omega}$ is the angular velocity vector of the blade.

Equation (2-59) is the expression for continuity while equation (2-60) expresses the fact that the potential flow is irrotational with respect to some non-rotating set of coordinates.

Using the equations developed by Mager (ref. 22) for representing equations (2-59) and (2-60) in terms of orthogonal curvilinear coordinates, the following four equations were obtained.

$$\frac{U_x}{1+Ky} + V_y + \frac{KV}{1+Ky} + W_z = 0 \quad (2-61)$$

$$W_y - V_z = 2\Omega_x = 2\Omega \sin\alpha \quad (2-62)$$

$$U_z - \frac{W_x}{1+Ky} = 2\Omega_y = 2\Omega \cos\alpha \quad (2-63)$$

$$\frac{V_x}{1+Ky} - U_y - \frac{KU}{1+Ky} = 2\Omega_z = 0 \quad (2-64)$$

K is the curvature of the blade surface and may be represented in terms of the physical data by the following equation.

$$K = - \frac{d\alpha}{dx} \quad (2-65)$$

The assumption imposed at this point in the analysis is that the radius of curvature of the blade is large in comparison with the boundary layer thickness, or that $1/K \gg \delta$. Also, the primary use of the potential flow is to provide the outer condition for the boundary layer

flow; therefore, the region of interest in the potential flow is restricted to that region very near the blade surface. With this in mind, equations (2-61 through 2-64) can be simplified to equations (2-66 through 2-69) by assuming that $1+Ky \approx 1$ and $V \approx 0$.

$$U_x + V_y + W_z = 0 \quad (2-66)$$

$$W_y = 2\Omega \sin\alpha \quad (2-67)$$

$$U_z - W_x = 2\Omega \cos\alpha \quad (2-68)$$

$$U_y = 0 \quad (2-69)$$

One further assumption made in the analysis is that the spanwise flow component, W , is not a function of x . From equation (2-67), it can be seen that this implies that $\sin\alpha$ is very small which is satisfied only by thin airfoils. Near the leading edge of blunt airfoils this assumption is not valid; however, the magnitude of the spanwise flow in this region is quite small. For this reason it is believed that this assumption will not greatly affect the results of the momentum-integral technique.

One major result of the preceeding assumption may be observed by examining equation (2-58). The potential function ϕ , even in curvilinear coordinates is only a function of x and y , because there was no "mixing" of the

z coordinate in the coordinate transformations. Thus, with the assumption that $W_y \approx 0$, the spanwise potential flow becomes solely a function of the chordwise coordinate. With this in mind, the continuity equation for the potential flow becomes,

$$U_x + V_y = 0 \quad (2-70)$$

and the irrotational flow condition results in the following single component condition.

$$U_z - W_x = 2\Omega \cos\alpha \quad (2-71)$$

The continuity equation expresses the fact that if a pressure gradient exists in the chordwise flow, the resulting acceleration or deceleration of the primary flow will necessarily be balanced by a deceleration or acceleration in the flow normal to the surface. It must be re-emphasized that in a momentum-integral analysis, this potential velocity component normal to the surface is completely neglected. Thus, the continuity equation is not satisfied for the potential flow. An intuitive physical description of equation (2-71) is that it guarantees that the potential flow relative to fixed coordinates has no component of rotation about the y-axis.

2.3.2 Flat Plate (Zero Pressure Gradient)

The simple case of a flat plate blade at zero angle of attack provides an ideal starting point for this analysis. Subsequently, additional effects of pressure gradients and blade thickness will be included. The potential flow for this case is almost trivial since without viscous effects and a pressure gradient the flow is not influenced by the presence of the blade.

Therefore, the potential streamlines are circles relative to the rotating coordinates and the velocity components are as follows:

$$U = \Omega z \quad (2-72)$$

$$W = -\Omega x \quad (2-73)$$

The chordwise profile can be obtained from equation (2-56) by recognizing that $\phi=1$. The spanwise profile can be obtained from equation (2-71) by recognizing that $\alpha=0$.

2.3.3 Flat Plate (Adverse Pressure Gradient)

Prior to examining an airfoil with thickness, it was recognized that for many airfoils the two-dimensional chordwise velocity is decelerated at approximately a constant rate over the aft 50% to 70% of the airfoil. To make the progression from a flat plate to an airfoil less

abrupt, the second case examined in this study is that of a flat plate with a linearly decelerating chordwise flow. For this case, the chordwise velocity component, U , is given by the form,

$$U = \Omega z (1 - kx) \quad (2-74)$$

where k is to be taken as a given constant. The spanwise component, W , is obtained by integrating equation (2-71) while assuming that $\alpha=0$. This spanwise component is given by equation (2-75).

$$W = - \Omega x (1 + \frac{1}{2} kx) \quad (2-75)$$

2.3.4 NACA 0012 at Zero Incidence

The most common method used to calculate the two-dimensional potential flow about an airfoil is the method introduced by Theodorsen (ref. 23). This method involves the complex transformation of an arbitrary airfoil shape into a circular cylinder using approximate numerical techniques. The potential flow calculations for an NACA 0012 using this method have been tabulated by Abbott and von Doenhoff (ref. 24). The chordwise velocity profile was obtained from this latter reference and is plotted in Figure 7. It should be noted that the abscissa is the distance from the leading edge along the airfoil contour.

Since these data were obtained using an approximate

Figure 7. Chordwise Potential Flow for an NACA 0012 Airfoil at Zero Lift.

Figure 8. Surface Geometry for an NACA 0012 Airfoil.

numerical technique, only a discrete number of data points are known; however, a solid curve is indicated. Unfortunately, the momentum-integral analysis requires that the first two derivatives of this velocity component be supplied, as well as the velocity itself. Since $U/\Omega z$ is such a rapidly varying function of z near the leading edge, a polynomial curve fit routine cannot match this data without many oscillations about the curve. These oscillations would produce first and second derivatives that could be in error by an order of magnitude. To maintain a "smooth" functional curve fit that would produce continuous derivatives with a minimum amount of numerical calculations, the following technique was developed.

The solid line in Figure 7 can be grossly approximated by two straight lines. The first line would pass through the origin and represent the two-dimensional potential velocity in the vicinity of a stagnation point as given by equation (2-76).

$$\frac{U}{\Omega z} = A_1 x \quad (2-76)$$

The second straight line would be analogous to the potential flow of section 2.3.3. This represents a linearly decelerating flow and may be given by equation (2-77).

$$\frac{U}{\Omega z} = A_2 x + A_3 \quad (2-77)$$

The Kutta condition at the trailing edge which forces the velocity to return to zero will be ignored in this approximation since the laminar flow should separate prior to reaching this point.

In general, the actual velocity distribution is a combination of these two straight lines. It was proposed that the complete curve (barring the trailing edge) could be approximated by the following combination of the above straight lines.

$$\frac{U}{\Omega z} = A_1 x e^{-\alpha_1 x} + (A_2 x + A_3)(1 - e^{-\alpha_2 x}) \quad (2-78)$$

Note that when x is small, the second half of this expression is small and the first half is approximately $A_1 x$. Also, when x is large, the first half of this equation is small and the second half is approximately $A_2 x + A_3$. In order to determine the best set of coefficients A_1 , A_2 , A_3 , α_1 , and α_2 , to match the 0012 velocity distribution, the following conditions were enforced.

$$\begin{aligned} \frac{d}{dx} \left(\frac{U}{\Omega z} \right) \Big|_{x=0} &= 70.0 \\ \left(\frac{U}{\Omega z} \right) \Big|_{x \rightarrow \infty} &= -0.267 x + 1.245 \\ \frac{d}{dx} \left(\frac{U}{\Omega z} \right) \Big|_{x=.133} &= 0 \\ \left(\frac{U}{\Omega z} \right) \Big|_{x=.133} &= 1.188 \end{aligned} \quad (2-79)$$

The coefficients satisfying conditions (2-79) are as follows.

$$\begin{aligned} A_1 &= 41.0 & \alpha_1 &= 38.42 \\ A_2 &= -0.267 & \alpha_2 &= 23.32 \\ A_3 &= 1.245 \end{aligned} \quad (2-80)$$

The final functional representation of the chordwise velocity distribution for an NACA 0012 airfoil at zero lift is given by equation (2-78) with the coefficients defined in (2-80). This approximation is plotted as a broken line in Figure 7. The agreement is accurate to within 2% over the major portion of the distribution. The accuracy is much better than this for the adverse pressure gradient region which is most critical for boundary layer calculations.

In order to determine an expression for the spanwise profile, equation (2-71) must be integrated. To accomplish this integration, $\cos\alpha$ must be represented in a functional form. The data for $\cos\alpha$ were obtained from the expression given by NACA which represents the thickness distribution for an NACA 0012 airfoil. This expression is listed by Abbott and von Doenhoff (ref. 24). The data are plotted in Figure 8. Notice the resemblance of the form of this data to that of the velocity distribution data in Figure 7. This prompted the use of

the same form for the functional curve fit as given by equation (2-78). In this case, the following conditions were enforced.

$$\begin{aligned}
 \frac{d}{dx}(\cos\alpha) \big|_{x=0} &= 63.02 \\
 \cos\alpha \big|_{x \rightarrow \infty} &= -0.014x + 1.004 \\
 \frac{d}{dx}(\cos\alpha) \big|_{x=.315} &= 0 \\
 \cos\alpha \big|_{x=.315} &= 1.0
 \end{aligned}
 \tag{2-81}$$

The coefficients satisfying conditions (2-81) are as follows.

$$\begin{aligned}
 A_1 &= -1.406 & \alpha_1 &= 19.91 \\
 A_2 &= -0.014 & \alpha_2 &= 64.17 \\
 A_3 &= 1.004
 \end{aligned}
 \tag{2-82}$$

The approximate functional curve fit is also shown in Figure 8. The accuracy of this was found to be within 1% of the actual surface data.

With both U and $\cos\alpha$ given in explicit functional forms, the spanwise velocity component was determined by mechanically integrating equation (2-71). All necessary derivatives of these functions were obtained by differentiation of the approximate expressions.

2.3.5 NACA 0015 at Zero Incidence

Due to the similarity of the NACA 0015 airfoil to the NACA 0012 airfoil, the chordwise potential flow distribution may be approximated by the same functional form that is given by equation (2-78). The conditions imposed to obtain the coefficients are as follows.

$$\begin{aligned}
 \frac{d}{dx} \left(\frac{U}{\Omega z} \right) \Big|_{x=0} &= 60.0 \\
 \left(\frac{U}{\Omega z} \right) \Big|_{x+\infty} &= -0.364 x + 1.323 \\
 \frac{d}{dx} \left(\frac{U}{\Omega z} \right) \Big|_{x=.150} &= 0 \\
 \left(\frac{U}{\Omega z} \right) \Big|_{x=.150} &= 1.233
 \end{aligned}
 \tag{2-83}$$

With these conditions, the coefficients were calculated and listed as follows.

$$\begin{aligned}
 A_1 &= 35.71 & \alpha_1 &= 31.79 \\
 A_2 &= -0.364 & \alpha_2 &= 18.35 \\
 A_3 &= 1.323
 \end{aligned}
 \tag{2-84}$$

Once again, both the theoretical data obtained from Abbott and von Doenhoff (ref. 24), and the approximate functional curve fit are plotted in Figure 9. The

Figure 9. Chordwise Potential Flow for an NACA 0015 Airfoil at Zero Lift.

Figure 10. Surface Geometry for an NACA 0015 Airfoil.

accuracy was found to be within 2% over the major portion of the distribution. The functional fit of $\cos\alpha$ for the NACA 0015 was forced to satisfy the following conditions.

$$\frac{d}{dx}(\cos\alpha) \big|_{x=0} = 40.4$$

$$\cos\alpha \big|_{x \rightarrow \infty} = -0.021 x + 1.007$$

(2-85)

$$\frac{d}{dx}(\cos\alpha) \big|_{x=.322} = 0$$

$$\cos\alpha \big|_{x=.322} = 1.0$$

The coefficients in equation (2-78) were obtained from these conditions and given below.

$$A_1 = -0.201 \qquad \alpha_1 = 8.96$$

$$A_2 = -0.021 \qquad \alpha_2 = 40.32 \qquad (2-86)$$

$$A_3 = 1.007$$

The accuracy of this approximation is approximately 2% over the major portion of the curve. This may be observed in Figure 10.

2.4 Boundary Layer Results and Discussion

The analytical boundary layer results are presented in four sections each representing a different blade shape or potential flow. Throughout the discussion, reference is made to the analytical calculations of Dwyer and McCroskey (ref. 11) as much as possible. It should be noted that the data presented in reference 11 were obtained through a combination finite difference-relaxation technique applied directly to the complete Navier-Stokes equations. For this reason, their results do not reflect the assumptions that were necessary in this analysis with regards to the derivation and method of solution of the momentum-integral equations. Therefore, the comparisons with reference 11, made throughout this section, may be interpreted as an investigation of the accuracy of the momentum-integral method for three-dimensional rotating boundary layer flows.

2.4.1 Flat Plate, Zero Pressure Gradient

The boundary layer equations for a rotating flat plate exhibit symmetry with respect to the variable x/z . The momentum-integral equations for this case can be reduced to ordinary differential equations for $\Delta(x/z)$ and $\epsilon(x/z)$. Therefore, the numerical differentiation routine discussed in section 2.2.4 for solving the general partial differential equations is not needed for the

rotating flat plate problem. This similarity presents an ideal check on the accuracy of this differentiation routine since the spanwise derivatives can be calculated exactly according to the following expressions.

$$\Delta_z = - \frac{x}{z} \Delta_x \quad (2-87)$$

$$\epsilon_z = - \frac{x}{z} \epsilon_x \quad (2-88)$$

The flat plate problem was solved twice. First, similarity was employed and the resulting ordinary differential equations were solved. Second, similarity was ignored and the numerical differentiation technique was used to calculate spanwise derivatives. The results of these two solutions agreed to within less than 1%. This accuracy check gives support to the overall technique employed to approximately solve the partial differential equations.

A physical picture of the lines of similarity are shown in Figure 11. Lines of constant x/z are rays emanating from the origin or axis of rotation. All of the results for the flat plate case are presented graphically by plotting each parameter versus x/z . From these plots, the boundary layer data can be examined over the complete blade surface. It must be kept in mind that a small value of x/z could be interpreted as either a position near the leading edge ($x \approx 0$), or a position at a large distance from

Figure 11. Lines of Similarity for a Rotating Flat Plate.

the axis of rotation along the span ($z \rightarrow \infty$). Likewise a large value of x/z could mean a large value of x or a small value of z . Thus, Figure 11 must be kept in mind throughout the interpretation of the analytical results.

The variation in the parameters Δ and ϵ over the flat plate surface are shown in Figure 12. The square of the boundary layer thickness is illustrated by the function Δ , while ϵ is related to the ratio of the radial to tangential shear stresses according to equation (2-89).

$$\epsilon = \frac{\tau_{or}}{\tau_{o\theta}} - \frac{\Delta}{12} \quad (2-89)$$

As x/z becomes greater than 1.3, the boundary layer begins to thin instead of the usual steady growth observed in two-dimensional flows. This thinning may be attributed to the strong component of centrifugal force in the x -direction which exerts a strong stabilizing force on the primary flow. Since ϵ is a mixture of physical parameters and the boundary layer thickness parameter, it is difficult to attach a great deal of physical insight to the variation of ϵ .

Figures 13 and 14 present the distributions of the velocity profile parameters over the surface of the flat plate rotor blade. The Pohlhausen parameter in the chord-wise velocity profile, (Λ_1) , is continually increasing. This is indicative of the fact that rotating flows with zero pressure gradient experience an acceleration due to

Figure 12. Profile Parameter Variation for a Rotating Flat Plate.

Figure 13. Pohlhausen Parameter Variation for a Rotating Flat Plate.

Figure 14. Wall Shear Stress Parameter Variation for a Rotating Flat Plate.

the x-components of centrifugal and Coriolis forces. The value of Λ_2 , the Pohlhausen parameter in the spanwise velocity profile, is negative and quite large near the leading edge ($x/z=0$). This will promote a reverse flow region in the spanwise profile near the leading edge. Farther aft of the leading edge, Λ_2 decreases in magnitude and this reverse flow region may disappear. A similar variation is observed for the shear parameters β_1 and β_2 , however, the signs are reversed. This indicates that β_1 has a decelerating effect on the chordwise flow while β_2 has a stabilizing effect on the spanwise flow. It should be noted that the symmetry of the profile parameter definitions for the flat plate case forces $|\Lambda_1| = |\Lambda_2|$ and $|\beta_1| = |\beta_2|$ at $x/z=1$.

The total effects of the profile parameters are best observed by examining the chordwise and spanwise velocity profiles at various positions on the blade. Figure 15 illustrates the development of the chordwise velocity profiles for x/z values of 0, 1, and 2. The ordinate used in this figure, and all subsequent velocity profiles, is a stretched boundary layer coordinate. It is a standard grouping of physical constants that is used for nearly all two-dimensional flat plate analyses (ref. 16 p. 122). Near the leading edge, the velocity profile is the standard two-dimensional profile. The primary or chordwise profiles become "fuller" as x/z is increased. This indi-

Figure 15. Chordwise Velocity Profiles for a Rotating Flat Plate.

Figure 16. Spanwise Velocity Profiles for a Rotating Flat Plate.

cates that the fluid in the lower portion of the boundary layer is being accelerated. Again, it is the centrifugal force that promotes this acceleration. The effect is greater for large values of x/z because the x -component of centrifugal force increases with increasing x/z .

Next, let us consider the secondary or spanwise profiles in Figure 16. These profiles are quite unlike the primary flow profiles as is expected by the large differences in the values of Λ_1 and Λ_2 near the leading edge. Indeed, the spanwise profile does exhibit a flow reversal near the leading edge. It should be noted that the spanwise velocity is normalized with the absolute value of the potential spanwise velocity. This normalization allows the retention of positive profiles as outflow and negative profiles as inflow. Near the leading edge, the centrifugal force pumps the lower levels of the boundary layer outward. However, the major portion of the boundary layer is still governed by the potential flow, W , which pulls the spanwise profile inward. As the value of x/z becomes large, the centrifugal force vector shifts to accelerate the chordwise flow and the spanwise outflow is decreased, until the outflow completely disappears. It is in this region that both the spanwise and chordwise velocity profiles become similar in form and appear to be quite stable.

The velocity profiles on a rotating flat plate are

compared with those of reference 11 in Figures 17 and 18. The profiles are compared for a position near the leading edge ($x/z=.0083$), and a position considerably aft of the leading edge, ($x/z=0.7$). These comparisons represent the first indications that the momentum-integral method is reasonably accurate for predicting the flow over a rotating flat plate. The agreement with reference 11 is remarkable. Note that the profiles match those of Dwyer and McCroskey with a little better accuracy for $x/z=.7$ than for $x/z=.0083$. This is reasonable since the flow is being accelerated due to rotational effects and the momentum-integral method is known to give better results in two-dimensional accelerated flows.

A second comparison with Dwyer and McCroskey's results for a flat plate is made in Figure 19. In this figure, the ratio of the chordwise wall shear stress to the equivalent two-dimensional wall shear stress is plotted versus x/z . For two-dimensional flow over a flat plate using the momentum-integral method, the shear stress is given by equation (2-90).

$$\tau_B = \frac{2\mu\Omega z}{\delta_B} \quad (2-90)$$

Thus, this shear ratio is given by the following equation,

$$\frac{\tau_{0x}}{\tau_B} = \frac{\delta_B}{\delta} \left(1 + \frac{\Lambda}{12} + \beta_1 \right) \quad (2-91)$$

Figure 17. Comparison of Chordwise Profiles with Reference 11 for a Flat Plate.

Figure 18. Comparison of Spanwise Profiles with Reference 11 for a Flat Plate.

Figure 19. Chordwise Shear Stress Variation for a Rotating Flat Plate.

where δ_B/δ is the ratio of two-dimensional to three-dimensional boundary layer thickness.

The two methods agree to within 3% for $x/z \leq 0.8$. It was shown in reference 11 that a fourth-order perturbation analysis was accurate only for $x/z \leq 0.5$, however, beyond this value of x/z , the perturbation solution diverged very rapidly. The momentum-integral technique, representing an averaged boundary layer behavior, maintains the correct trend through much larger values of x/z .

The direction of the boundary layer flow near the surface is indicated by the ratio of the spanwise and chordwise wall shear stresses. This ratio is plotted in Figure 20. More accurately, τ_{oz}/τ_{ox} represents the slope of the limiting streamline on the blade surface according to equation (2-92).

$$\left(\frac{dz}{dx} \right)_s = \lim_{y \rightarrow 0} \frac{w}{u} = \frac{\tau_{oz}}{\tau_{ox}} \quad (2-92)$$

The subscript s is used to indicate that the derivative is taken along a surface streamline. From Figure 20, it is observed that the flow near the surface is outward for $x/z < .92$ while for $x/z > .92$ this flow is inward. Thus at $x/z = .92$, the spanwise velocity profile is a separation type of profile where the velocity gradient at the wall is zero. This does not mean that flow separation occurs in the two-dimensional sense, since τ_{oz} represents only

Figure 20. Ratio of Spanwise to Chordwise Wall Shear Stresses for a Rotating Flat Plate.

one component of the wall shear stress vector.

In general, the rotating flat plate results have shown that the momentum-integral method provides an accurate and stable technique to the solution of the three-dimensional rotating boundary layer problem.

2.4.2 Flat Plate, Adverse Pressure Gradient

The flat plate results are, by no means, conclusive evidence of the power of the momentum-integral technique. Since most airfoils exhibit an adverse chordwise pressure gradient, a second case is now considered.

In this second case, the airfoil is still assumed to have a sharp leading edge; however, the chordwise potential flow is to be linearly decelerated according to equation (2-74). The value of k was chosen to give a deceleration similar to that of an NACA 0012 airfoil. Thus, from section 2.3.4, k was chosen to be 0.267.

This hypothetical potential flow is included in this analysis as a bridge between the simple case of a flat plate airfoil which has zero pressure gradient and the complex case of a real airfoil which has surface curvature and a variable pressure gradient. The results are presented in Figure 21. This figure indicates the location of flow separation on a rotating flat plate airfoil which has a superimposed linearly decelerated potential

Figure 21. Comparison of Separation Lines for a Linearly Decelerated Flow, Rotating and Non-rotating Cases.

flow. Note that the stabilizing effects of rotation tend to delay the point of separation from the normal two-dimensional case.

The definition of the point of separation of three-dimensional boundary layer flows has long been a controversial subject. Since universal agreement has not been reached, the definition for separation used throughout this analysis is that point where the chordwise wall shear stress vanishes. This definition is compatible with that used by reference 11.

For this case, the similarity described by equations (2-87) and (2-88) is not applicable. Thus, the numerical differentiation technique described in section 2.2.4 was necessary for the solution presented in Figure 21.

The results of the linearly decelerated flow case are indicative of the results obtained by Banks and Gadd (ref. 9). Their analysis also included a linearly decelerated flow, however, the deceleration was along the tangential, not the chordwise, direction. The tangential component of the potential flow in their analysis is given by equation (2-93).

$$U = \Omega r(1-k\theta) \quad (2-93)$$

Using equation (2-93) and assuming that the radial component of potential velocity was zero, similarity

with respect to θ is maintained. The results of reference 9 indicate that for values of k less than 0.548, the boundary layer was completely stabilized against separation.

Since $\theta = \tan^{-1}(x/z)$, equation (2-93) indicates that for a fixed value of x , the adverse pressure gradient effect decreases for increasing values of z . This was a convenient choice since the favorable effects of rotation decrease with increasing values of z . This could explain the complete stabilization of the boundary layer reported in reference 9. The linear adverse pressure gradient examined in this study does not exhibit this trend as can be observed by the differences in equations (2-74) and (2-93).

2.4.3 NACA 0012 at Zero Incidence

The third case considered in this analysis was that of an NACA 0012 airfoil at zero angle of attack. The potential flow for this airfoil was discussed in section 2.3.4. This airfoil shape is used for a number of helicopter rotors and was also used throughout the experimental portion of this study. This section is devoted solely to presenting the analytical results; the experimental comparisons will be discussed in chapter IV.

Again, similarity is not present for this case as it was for the flat plate, zero pressure gradient example. The boundary layer solutions for an airfoil with thickness

presented the problem of initiating the calculations at the stagnation line along the leading edge. The power series method, coupled with the numerical technique discussed in section 2.2.5, was used to alleviate the singularity problems in this region of the blade.

Chordwise and spanwise velocity profiles are shown in Figures 22 and 23 for a rotating NACA 0012 airfoil. The results of the momentum-integral analysis are plotted as solid lines for a chordwise position $x/c=0.6$ and spanwise positions of $z/c=1$ and $z/c=\infty$. This latter spanwise position corresponds to the two-dimensional chordwise flow profile. The chordwise profiles will be examined first.

The stabilizing effects of rotation are again depicted in the chordwise profiles since the profile at $z/c=1$ is considerably "fuller" than the equivalent profile at $z/c=\infty$. The results of the finite difference solution by Dwyer and McCroskey (ref. 11) are included for comparison. It must be noted that while their results are for identical spanwise positions, their chordwise position was at $x/c=0.63$. The comparison is still quite good, especially for the $z/c=1$ case. It is ironic that the momentum-integral method appears to be more accurate for the three-dimensional rotating case than for the two-dimensional non-rotating case. This can be resolved since the three-dimensional flow has the added stability of the centrifugal and Coriolis accelerations. In fact,

Figure 22. Comparison of Chordwise Velocity Profiles with Reference 11 for an NACA 0012 Airfoil.

Figure 23. Comparison of Spanwise Velocity Profile with Reference 11 for an NACA 0012 Airfoil.

the chordwise shear stress was actually increasing at $x/c=0.6$ and $z/c=1.0$. Therefore, the two profiles in Figure 22 represent a decelerated flow at $z/c=\infty$ and an accelerated flow at $z/c=1$.

The spanwise profile at $x/c=0.6$ and $z/c=1.0$ is shown in Figure 23. The data from reference 11 are again included for comparison. The spanwise flow exhibits considerably more outflow than was observed in the profiles on a rotating flat plate. This greater outflow can be attributed to the deceleration in the chordwise potential flow. As the chordwise flow slows down, continuity must still be preserved, therefore, the spanwise flow acts as a flow relief and reduces the rate of boundary layer growth.

The separation line determined from the momentum-integral analysis for the 0012 airfoil is illustrated in Figure 24. This is the first real discrepancy between the results of reference 11 and those presented in this study. The two-dimensional asymptote from the momentum-integral analysis occurs at $x/z=0.77$ while that reported in reference 11 occurs at $x/c=0.70$. The delaying effect of rotation appears to be quite similar for both methods which indicates that the two-dimensional separation lines are displaced.

The reason for the disagreement in the two-dimensional separation lines is unknown to the author. The potential flow used in reference 11 was not described in detail and

Figure 24. Comparison of Separation Lines with Reference 11 for an NACA 0012 Airfoil.

it is not known how the derivatives of the potential flow components were obtained. To check the capability of the momentum-integral analysis to predict separation, calculations were made for two-dimensional flow over a circular cylinder. The results indicated that laminar separation on the circular cylinder occurs at an angle of 107.5° from the stagnation point. The value reported in Schlichting (ref. 16, p. 253) as a so-called "exact" location of laminar separation is 108.8° . This value was obtained by a power series method which included terms through x^{11} . This example gave strong evidence that the momentum-integral analysis could predict separation to within $1\frac{1}{2}$ of the "exact" value.

This evidence is not considered to be conclusive that the separation line on an NACA 0012 airfoil is at $x/c=0.77$. Indeed, there could be some discrepancy in the assumed potential flow for this analysis and that of reference 11.

Since similarity does not exist on the airfoil, the ratio of the chordwise wall shear stress to the equivalent two-dimensional shear stress must be presented in two separate figures.

In Figure 25, this shear ratio is presented for a fixed chordwise position and a variable spanwise position. The chordwise positions are at $x/c=0.6$, 0.7 , and 0.75 for the momentum-integral analysis while the data from refer-

Figure 25. Spanwise Distribution of Chordwise Shear on a Rotating NACA 0012 Airfoil.

Figure 26. Chordwise Distribution of Chordwise Shear on a Rotating NACA 0012 Airfoil.

ence 11 are at $x/c=0.63$. For large values of z , $x/z \rightarrow 0$, this shear ratio approaches unity. As the spanwise location is decreased, the shear ratio grows indicating that the rotating flow is not approaching separation as fast as the non-rotating case.

The data from reference 11 at $x/c=0.63$ compare quite favorably with the momentum-integral results at $x/c=0.7$. This agreement can be explained by re-examining the separation lines in Figure 24. At $x/c=0.63$, the results of Dwyer and McCroskey (ref. 11) are within a distance of $.07c$ of its separation point. At $x/c=0.7$, the momentum-integral analysis is also within a distance of $.07c$ of its predicted separation point. Therefore, it is not unusual that the shear ratios from the two analyses agree at these two different chord positions.

In Figure 26, the shear ratios are plotted for fixed spanwise locations and variable chordwise positions. The discrepancy between the results in this analysis and those of reference 11 at $z/c=1$ can again be traced to the difference in the predicted two-dimensional separation points. The rapid rise in the shear ratio for increasing x , (for fixed z you move directly along the chord), is indicative of the fact that the two-dimensional chordwise wall shear vanishes at a smaller value of x than the rotating three-dimensional shear.

Aside from the reported displacement of the two-

dimensional separation line, the three-dimensional momentum-integral analysis was found to agree quite well with the more complex method of reference 11 for the NACA 0012 rotating boundary layer calculations.

2.4.4 NACA 0015 at Zero Incidence

The final case considered in this analysis is an NACA 0015 airfoil. The momentum-integral technique was proven to be a useful analytical tool by comparing the results of sections 2.4.1 and 2.4.3 with those of reference 11. Since the NACA 0015 airfoil is also used on a number of helicopter rotors, a small portion of the boundary layer results for this airfoil are included.

The chordwise and spanwise velocity profiles are shown in Figures 27 and 28, respectively. The results are quite similar to those for the 0012 airfoil. The chordwise profile for $z/c=\infty$ and $x/c=0.6$ more nearly resembles a separation profile near the blade surface than the corresponding profile in Figure 22. The region of spanwise outflow appears to persist for a greater distance above the surface than for the 0012 airfoil.

The location of the separation line for the 0015 airfoil is presented in Figure 29. The results from the 0012 are included for comparison. The 0015 airfoil is thicker and has a steeper chordwise pressure gradient as

Figure 27. Chordwise Velocity Profiles for an NACA 0015 Airfoil.

Figure 28. Spanwise Velocity Profile for an NACA 0015 Airfoil.

Figure 29. Comparison of Separation Lines for Rotating NACA 0012 and 0015 Airfoils.

illustrated by the potential flow calculations in sections⁸⁵
2.3.4 and 2.3.5. Therefore, the boundary layer separates
at a smaller value of x/c on the 0015 than for 0012 air-
foil.

CHAPTER III

EXPERIMENTAL STUDIES

The experimental portion of this research was designed to be a preliminary study of the feasibility and usefulness of three experimental techniques and was not intended to be a conclusive study. The measurements include flow visualization, surface static pressure data, and boundary layer flow magnitude and direction data. The purpose of this chapter is to describe the experimental equipment and instrumentation, the calibration techniques, the testing procedure, and the subsequent results of this portion of the study.

3.1 Testing Equipment

3.1.1 Rotor Stand Test Module

The rotor test stand used to simulate model rotor hover conditions is shown in Figures 30 and 31. A single blade rotor was used throughout this study. The rotor is powered by two air motors which are connected to a 100psi supply line through a control valve. The rotor speed is

Figure 30. Hover Test Stand.

Figure 31. Single Blade Rotor on Hover Test Stand.

controlled by regulating the air flow rate to these motors. A tachometer is connected to the rotor shaft by a timing belt to prevent slippage errors.

A hollow drive shaft permits electronic instrumentation cables to be fed from the rotor hub instrumentation systems to the slip ring unit at the base of the rotor stand. Cables are attached to the stationary portion of the slip ring unit and fed through electrical conduit to the data console located behind the safety wall. The slip ring unit consists of fifteen rings and uses carbon graphite brushes.

A wire fence and a safety wall were erected as a precaution against any objects which may be accidentally thrown from the rotor.

3.1.2 Rotor Blades

All of the rotor blades used in this study had the same construction and were originally the same length. Rotor tests were conducted in a flow channel during the latter stages of this study which required a reduction in the rotor length.

To minimize the mutual interference between various pieces of instrumentation, three separate blades were fabricated. Each blade was designed to incorporate the instrumentation required to obtain one type of experimental data. The blades were constructed with birch forward

sections and balsa trailing sections. The physical data for the three rotor blades is given in Table 1.

Description	Rotor Blade		
	Visualization	Pressure	Velocity
Airfoil Designation	NACA 0012	NACA 0012	NACA 0012
Span, ft	4.0	3.50	4.0
Chord, ft	0.75	0.75	0.75
Twist, deg	None	None	None
Disk Area, ft ²	50.2	38.5	50.2
Solidity	0.0596	0.0682	0.0596
Effective Root Cutout, %R	8.9%	9.3%	8.9%
Lock Number	2.12	1.61	1.82

Table 1. Physical Rotor Data

The flow visualization rotor blade is shown in Figure 32. This blade was constructed for use in a tip flow study (ref. 25). The screws near the tip permit interchangeability of blade tips.

Figure 32. Rotor Blade for Flow Visualization Studies.

3.1.3 Pressure Transducers

The pressure measurements were obtained from sub-miniature pressure transducers purchased from Sensotec Division of Scientific Advances Incorporated. The transducers utilize semiconductor type strain gages bonded to stainless steel diaphragms and are designed to operate in severe environments. The range of each transducer is 0-25psia over which the linearity and hysteresis was quoted at $\pm 0.5\%$ of full scale. The transducers were calibrated a number of times to guarantee correct sensitivities and linearities.

Since the transducers represented a large investment, it was decided that the removal of the transducers after the completion of the rotor pressure measurements would be a high priority concern. To accomplish this feat, a mounting pad was constructed for each transducer. A typical transducer-pad assembly is shown in Figure 33. The transducers were epoxied onto the pads in such a way that the diaphragm was cantilevered. This prevented the transducer from sensing the strain of the blade. The pad is then attached to the rotor blade with a single screw. In this manner, the transducer was mechanically attached to rotor blade and could be removed easily.

Figure 34 illustrates the transducers during the mounting process. On the right, the transducers are

Figure 33. Pressure Transducer and Mounting Pad Assembly.

Figure 34. Pressure Blade Showing Transducer Mounting Technique.

visible including the wiring and solder tabs. On the left, the transducers have been covered with thin strips of veneering. Each strip was individually tailored and in each a small hole was drilled prior to attachment with epoxy. After all of the veneering strips were cemented in place, the blade surface was smoothed using very fine sand paper. Finally, the small pressure taps above the transducers were covered and the blade was painted to assure a smooth surface. Although the steps outlined in the mounting procedure were brief, mounting was a very tedious task due to the ease with which the thin diaphragms could be damaged.

The sensitivities of the transducers vary from 1 mv/psi to 2 mv/psi. The output signals that must be transmitted through the slip rings are on the order of 1 mv. Since the available slip ring unit has a background ring noise of this same order, signal amplification was required. An amplifier package was constructed and is shown in Figure 35. This unit consists of twelve amplifiers, one for each transducer, and a power regulator for controlling the input transducer power. The transducers were paralleled for common power. This amplifier unit was mounted on the rotor hub for signal amplification prior to signal transmission by the slip rings.

Figure 35. Amplifier Package for Twelve Pressure Transducers.

3.1.4 Hot-wire Anemometers

The anemometers used to measure velocities were purchased from DISA S and B incorporated. They are constant temperature or constant resistance anemometers whose operation can be explained as follows:

The anemometer probe consists of a fine wire sensor whose resistance is a linear function of temperature. This probe is incorporated as one leg of a Wheatstone bridge network. For purposes of description, let us assume the bridge is balanced. Then, if the sensor temperature changes, the bridge will be unbalanced and a voltage will be present across the output terminals of the bridge. This voltage is amplified by a high performance d-c differential amplifier. The signal is then fed into a single-ended d-c amplifier for additional gain. This final amplification stage also acts as a d-c level power amplifier since it is placed in a Darlington configuration with a third amplifier. Finally, the current from the output of the Darlington pair is fed to the bridge with a sign to cause the resistance of the probe to change and balance the bridge network. The anemometer output voltage is an indication of the power required to balance the bridge. The value of probe resistance which balances the bridge can be adjusted by varying the value of the potentiometer used as the bridge

leg adjacent to the probe. This leg is termed the "over-heat" resistor since its value determines the difference between the probe temperature and ambient temperature at zero flow.

As was the case with the pressure transducers, the slip ring noise would mask the anemometer signal unless the afore-mentioned amplification were mounted on the rotor hub. Thus, only the printed-circuit boards used in the commercial anemometers were purchased. Two of these boards were customized into a dual anemometer package that could be mounted on the rotor hub and are shown in Figure 36. The two boards were mounted vertically facing one another to minimize centrifugal effects.

The operation of the anemometer depends on the heat transfer characteristics between the sensor and the stream being measured. These heat transfer characteristics cause the output voltage to be a non-linear function of stream velocity at the sensor. As a result, the anemometer is extremely sensitive to low velocities and has a very low sensitivity for high velocities. For boundary layer measurements, a wide range of velocities will be encountered; therefore, linearizers were incorporated to make the sensitivity nearly uniform over the complete velocity range. One of the linearizers is shown in Figure 37.

The first hot-wire probe that was used in this study was a boundary layer X-configuration probe. This probe is

Figure 36. Dual Constant Temperature Anemometer Package.

Figure 37. Anemometer Linearizer Unit.

shown in Figures 38 through 40. The probe was designed to be mounted above the blade surface with the sensors extending into the boundary layer. The X-configuration of the two sensors is clearly defined in Figure 39. This planar orientation of the two sensors is standard for measurement of both flow magnitude and direction except for the vertical spacing between the two wires. Standard probes have a spacing of .05 inch while the wires for this probe were placed as close as physically possible and are shown in Figure 40 to be about .009 inch. It should be noted that the photographs in Figures 39 and 40 have an amplification of about 40.

This probe was calibrated and tests were initiated on the hover stand. After a few minutes of operation, the sensors were found to be destroyed. New wires were placed on the probe, the probe was recalibrated, hover tests were initiated, and again the sensors were destroyed. After examining the sensors under a microscope, the damage in each case was observed to be a break in the wires at the center of the X. This indicated that either the wires came in physical contact or there was sufficient electronic interference to burn the wires.

To eliminate both of the methods in which the wires were predicted to be damaged, a second probe was ordered. This probe was identical to the first except that the wires were placed in a V-configuration instead of an X-configuration. This permitted the wires to be placed at

Figure 38. X-configuration Hot-wire Boundary Layer Probe.

Figure 39. X-configuration with about 40
Magnification.

Figure 40. Side View of X-probe Showing Wire Spacing.

Figure 41. V-configuration with about 20
Magnification.

the same vertical height which eliminates the measurement error due to the sensors being located at two different boundary layer heights. A photograph of this V-configuration is shown in Figure 41 with a magnification of about 20. The wire-to-wire interference for this type of probe is considerably reduced. All of the anemometer data reported in this study was obtained from this V-configuration probe. None of the problems encountered with the X-configuration probe occurred with the V-configuration. In fact, all of the hover data was obtained with a single set of sensors on the V-probe.

3.2 Calibration Systems

3.2.1 Rotor Stand

The hover stand rotor speed was monitored by a tachometer which was driven by the rotor shaft. To calibrate an accurate scale for the tachometer, the following technique was used.

A gear was mounted on the rotor shaft, and a magnetic pickup was rigidly attached to the rotor stand and placed within approximately 1/8 inch of the gear teeth. As the gear teeth pass by the magnetic pickup, they cut magnetic field lines and generate small pulses of electrical current. The frequency of these current, or voltage, pulses can be related directly to the rotor speed.

The pulse frequency could be obtained from an oscilloscope trace of the pulses and a visual count; however, this makes it very difficult to set a desired rotor speed. For this reason, an electronic EPUT (events-per-unit-time) counter was incorporated into the calibration. This meter electronically counts the number of pulses for a preset time period and presents the results on a digital display.

With this technique, the rotor speed was calibrated within an accuracy of .5%; however, during testing it was not possible to maintain a desired rotor speed more accurately than approximately $\pm 2\%$.

3.2.2 Pressure Transducer

The pressure transducers were calibrated a number of times using three different techniques.

First, the transducers were calibrated by the manufacturer before they were mounted on the specially designed pads. This calibration was performed in a pressurized chamber and was presented as full-scale output millivolts. The calibration was performed at three temperatures from 30°F to 130°F in order to attest the accuracy of the temperature compensation network. Although it varies for each transducer, the maximum sensitivity drift with temperature was only 0.01 mv/°F while the sensitivity magnitudes varied from 1 mv/psi to 2.5 mv/psi.

The transducers were re-calibrated in a pressurized container shown in Figures 42 and 43. The container was sealed by "o-rings" and the cables were removed through a tubing connection in the side. The pressure was controlled by a needle valve while the chamber pressure was recorded by a static pressure port and a mercury or water manometer. A vacuum pump was also used to complete the calibration from -8psig to +8psig. The results of this calibration were within 2% of the data supplied by the manufacturer. This calibration was performed just prior to submerging the transducer-mounting pad assemblies in the rotor blade.

Due to the complexity of the transducer-mounting and subsequent surface smoothing, a third calibration was performed. This calibration was conducted after the transducers were mounted in the rotor blade surface and surface smoothing was completed. This final calibration technique is depicted in Figure 44. A small tee-connection was used. One tube was connected to the vacuum supply line, one to the manometer, and a small piece of pliable rubber tubing was placed on the third. This rubber tubing could be pressed against the blade surface and a vacuum could be maintained within the tee-connector. Using this device, the transducers could be individually calibrated at any time prior to, or after, the tests.

The results of this final calibration are presented

Figure 42. Pressure Transducer Calibration Chamber.

Figure 43. Pressure Transducer Calibration Apparatus.

Figure 44. Calibration Device for Pressure Transducers Mounted in Rotor Blade.

Figure 45. Calibration of a Typical Pressure Transducer.

in Figure 45 for a typical transducer. Since all of the transducers were mounted on the upper surface of the blade (the suction side), only negative gage pressures were used during this calibration. The slope of the straight line drawn through the data points was determined by a least-square-error method. The data are linear to within 1% and the transducer sensitivities were invariably repeatable to within 2%.

Finally, it should be noted that throughout each calibration, the complete transducer electronics system was checked.

3.2.3 Hot-wire Anemometer

The calibration of the hot-wire anemometer system was one of the most difficult tasks of the complete study. In order to calibrate a V-configuration probe which is to be used for measurement of flow magnitude and direction, it is obviously a necessity to have a laboratory standard flow with a known magnitude and direction.

To produce such a standard flow, the device shown in Figure 46 was designed, constructed and calibrated. The body consists mainly of a cylindrical chamber made from four-inch aluminum pipe with a nozzle at one end. The chamber has a vertical plate to deflect the incoming flow, a set of five screens to reduce turbulence, and two ports

Figure 46. Hot-wire Calibration Device.

for the measurement of chamber pressure and temperature. The nozzle is conical with faired inlet and exit regions. The exit diameter is 0.375 inch.

A probe positioning device is shown attached to the top of the nozzle. The V-configuration probe is mounted in the small clamp directly in front of the nozzle. The flow direction relative to the probe body can be adjusted by rotating the probe. A protractor was mounted on the horizontal plate above the positioning device to record the probe's angular position. The speed of the flow at the nozzle exit is controlled by the needle valve located in the 100psi supply line. An air filter was installed in this supply line to maintain a clean supply of air. Since considerable effort was spent calibrating this nozzle, Appendix A is devoted to a discussion of the method and results of this calibration.

The V-configuration probe calibration was performed by setting a nozzle exit velocity and recording the output voltages of each anemometer at probe angular positions from -45° to $+45^{\circ}$ in increments of 5° . A complete calibration involved repetition of this process for a number of exit velocities to cover the range expected during testing.

An attempt was made to correlate the results of this calibration with the "cosine" law used by Champagne, Sleicher and Wehrmann (ref. 26) with a single wire probe. After many unsuccessful attempts of correlation with this

simple law, it was concluded that another method must be used to maintain the accuracy of the calibration. A numerical technique was developed to correlate the calibration data and will be described next.

Figure 47 illustrates the V-configuration, denoting the left sensor as B and the right sensor as A. The stream velocity is of magnitude V and approaches the probe with an outflow angle γ . The components of velocity normal to sensors A and B are denoted by V_A and V_B , respectively. These velocities may be related to the flow magnitude V and direction γ by equations (3-1) and (3-2).

$$\begin{aligned} V_A &= V \cos(45^\circ - \gamma) \\ &= \frac{V}{\sqrt{2}} (\cos \gamma + \sin \gamma) \end{aligned} \quad (3-1)$$

$$\begin{aligned} V_B &= V \sin(45^\circ - \gamma) \\ &= \frac{V}{\sqrt{2}} (\cos \gamma - \sin \gamma) \end{aligned} \quad (3-2)$$

Rearranging these expressions yields,

$$V \sin \gamma = \frac{V_A - V_B}{\sqrt{2}} \quad (3-3)$$

$$V \cos \gamma = \frac{V_A + V_B}{\sqrt{2}} \quad (3-4)$$

which can be solved directly for the flow magnitude and direction in the form given by equations (3-5) and (3-6).

Figure 47. V-configuration Hot-wire Probe Geometry.

$$V = \left[\left(\frac{V_A - V_B}{\sqrt{2}} \right)^2 + \left(\frac{V_A + V_B}{\sqrt{2}} \right)^2 \right]^{1/2} \quad (3-5)$$

$$\tan \gamma = \frac{V_A - V_B}{V_A + V_B} \quad (3-6)$$

Since linearizers were incorporated into the calibration, the components of velocity V_A and V_B are approximately proportional to the output voltages of anemometers A and B. With this in mind, the output voltages were grouped according to equations (3-5) and (3-6) and the following definitions were made,

$$V\text{-MAGNITUDE} = \left[\left(\frac{V_A - V_B}{\sqrt{2}} \right)^2 + \left(\frac{V_A + V_B}{\sqrt{2}} \right)^2 \right]^{1/2} \quad (3-7)$$

$$V\text{-DIRECTION} = \frac{V_A - V_B}{V_A + V_B} \quad (3-8)$$

where v_A and v_B are the output voltages of anemometers A and B, respectively.

Calibration produces a data array of voltages v_A and v_B for various values of flow magnitude V , and flow direction γ . To correlate these results, the voltage parameters $V\text{-MAGNITUDE}$ and $V\text{-DIRECTION}$ were calculated according to equations (3-7) and (3-8). Also, the flow magnitude was dimensionalized into the wire Reynolds number defined by,

$$Re_w = \frac{Vd}{\nu} \quad (3-9)$$

where d is the wire diameter.

A computer program was devised to automatically perform a complete correlation of the resulting calibration data. This includes a regression analysis for subsequent data interpolation and computerized plotting of the data to show "goodness of fit". These plots are included in Figures 48 through 52. The solid lines in these figures represent the regression curve-fit while the symbols are the calibration data.

Figure 48 illustrates the variation of these voltages parameters with wire Reynolds numbers and flow direction. During calibration, the flow direction is fixed and the probe is rotated, therefore, Figures 48 through 52 refer to γ as the probe body angle.

Figure 49 illustrates the variation of V-MAGNITUDE with wire Reynolds number. The variation of these data about a single straight line indicate that V-MAGNITUDE, while primarily a linear function of Re_w , has a slight variation with the probe body angle.

Figure 50 illustrates the variation of V-DIRECTION with the probe body angle. Again, V-DIRECTION is primarily a function of probe body angle; however, the data scatter indicate a slight variation with wire Reynolds number. It should be noted that if each wire identically obeyed the "cosine" law, V-DIRECTION could be represented as the tangent of the probe body angle. Therefore, Figure

PROBE CODE.....TSI-SFCE
DATE.....03/19/71

Figure 48. V-DIRECTION versus V-MAGNITUDE

PROBE CODE.....TSI-SFCE
DATE.....03/19/71

Figure 49. V-MAGNITUDE versus Wire Reynolds Number.

PROBE CODE.....TSI-SFCE
DATE.....03/19/71

Figure 50. V-DIRECTION versus Probe Body Angle.

PROBE CODE.....TSI-SFCE
DATE.....03/19/71

Figure 51. V-MAGNITUDE versus Probe Body Angle.

PROBE CODE.....TSI-SFCE
DATE.....03/19/71

Figure 52. V-DIRECTION versus Wire Reynolds Number.

50 would be a unique tangent curve, not a band of curves as were obtained in the actual calibration.

Figures 51 and 52 are cross-plots of Figures 49 and 50. They represent the secondary variations of V-MAGNITUDE and V-DIRECTION. These two figures best illustrate the accuracy of the regression analysis.

During data reduction, the regression analysis was solved in reverse. That is, the numerical curve-fit equations were solved for values of Re_w and γ when given the voltages v_A and v_B .

3.3 Testing Procedures

3.3.1 Visualization Tests

The ammonia feed apparatus used for the visualization tests is shown in Figure 53. The pressurized tank of gaseous ammonia was bubbled through a liquid ammonia solution and then fed to the top of the rotor stand as shown in Figure 31. The ammonia then passed through a transfer ring, through a tube inserted in the rotor blade, and out the small orifices in the blade surface.

The technique used in these tests is almost identical to that described in reference 27. The only difference is that the tests in this study incorporated a revolute film purchased from Charles Bruning Company while those of

Figure 53. Ammonia Feed Apparatus for Visualization Tests.

reference 27 used regular ozalid paper. The revolute film comes in a variety of colors, has a more uniform coating of diazo dye, is more easily attached and removed from the blade, and gives a more permanent trace than the ozalid paper.

The surface traces are obtained by bringing the rotor to the desired hover condition and forcing ammonia to flow into the rotating boundary layer by opening the valve on the tank of gaseous ammonia. After the trace is formed, the valve is closed, the rotor is stopped, and the plastic revolute film is removed. This film represents a permanent surface trace for the set rotor conditions of the test.

3.3.2 Pressure Measurements

Each pressure transducer has a non-zero output voltage at atmospheric pressure condition. This offset voltage varies from test date to test date due to barometric pressure variations. The offset voltages at atmospheric conditions were constant throughout all tests and were subtracted from subsequent outputs to represent surface static pressures above the barometric ambient.

The ambient temperature, barometric pressure, and wet-bulb temperature were recorded during each test to determine the local air density. The air density was used to calculate the local free stream dynamic pressure

and subsequently dimensionalize the data into pressure coefficient form.

All of the pressure data were obtained at 800rpm and consisted of three consecutive independent tests.

3.3.3 Velocity Measurements

The velocity measurements represent the most significant experimental contribution of this study; thus, the anemometer testing procedure will be examined in detail.

The V-configuration probe was mounted above the blade surface and oriented according to Figures 54 and 55. Figure 54 illustrates the probe as it is viewed from the rotor blade tip. The anemometer amplifier package can be seen in the background. Figure 55 shows the probe as seen from the blade leading edge. The plane of the V-configuration sensors is made parallel to the blade surface by the apparatus pictured in Figure 56. It consists of a traversing microscope and associated mechanism to attach the microscope rigidly to the blade. By viewing the probe and probe surface simultaneously through the 100X lens of the microscope, the probe could be aligned accurately.

The vertical position of the probe is adjusted by rotating a nut attached on the bottom surface of the blade. A lock nut was used to maintain a fixed probe position

Figure 54. Hot-wire Probe Mounted on Rotor Blade
as Viewed from the Blade Tip.

Figure 55. Hot-wire Probe Mounted on Rotor Blade
as Viewed from the Leading Edge.

Figure 56. Traversing Microscope Attached to Rotor Blade during Probe Height Measurement.

during the tests. After the probe is locked in the desired position, the traversing microscope was used to measure the perpendicular distance from the blade surface to the sensor plane. This traversing mechanism permitted a readability to the nearest 0.0001 inch. Measurements were made from 0.1 inch to 0.002 inch to cover the complete boundary layer flow.

The tests were conducted in the following sequence. The chordwise and spanwise positions were chosen and the probe aligned at this surface position by the technique just described. The probe was set at a position near 0.1 inch to represent a condition outside the boundary layer. At this probe position, tests were conducted at four rotor speeds (100, 200, 300, and 400rpm), and four pitch angles (0° , 5° , 10° , and 15°). The probe was then lowered and tests repeated until data was obtained across the entire boundary layer. This sequence of tests required the fewest adjustments of the probe and minimized the possibility of accidental probe damage from contact with the sensors.

3.4 Test Results and Discussion

3.4.1 Ammonia Surface Traces

The ammonia visualization traces give an indication of the flow direction in the lower portion of the boundary layer. The traces are shown in Figures 57 through 59. The data in these figures were obtained at a pitch angle of 12° and rotor speeds of 250, 700, and 990rpm, respectively. Data at pitch angles of 13.5° and 15° are presented in Appendix B; however, these data only serve to confirm the results of the data in Figures 57 through 59.

The leading and trailing edges are marked in each figure. The spanwise positions are given 50, 60, 70, 80, 90, and 99% radius. Orifices were located at chordwise positions of 0, 5, 10, 15, 20, 25, 30, 35, 40, 50, 60, and 75% chord. In some of the data, traces were not formed at the orifices near the trailing edge. Due to the large negative pressures on the upper surface near the leading edge, most of the ammonia would escape in this region.

The 250rpm data at 50% radius indicate a region of outflow from 5% to 10% chord. This action is similar to that reported by reference 27, in which flow discontinuities were hypothesized to be standing laminar separation bubbles. Note that no discontinuities can be observed at any of the

Figure 57. Ammonia Trace Data; $\theta=12^\circ$, 250rpm.

Figure 58. Ammonia Trace Data; $\theta=12^\circ$, 700rpm.

Figure 59. Ammonia Trace Data; $0-12^\circ$, 990rpm.

other radial positions for any of the rpm conditions. Since the discontinuity is believed to be a laminar flow phenomenon, it disappears when the flow transitions prior to the formation of the separation bubble. The tip Reynolds numbers corresponding to 250, 700, and 990rpm are 0.66, 1.83, and 2.59 million, respectively. This indicates that the local free stream Reynolds number at the location of the discontinuity was 0.33 million. All other data have higher Reynolds numbers.

The traces at 60% and 70% radius at all of the rpm conditions indicate a small amount of outflow in the region prior to the 25% chord position. The flow appears to be very stable over the complete blade. Even the flow following the discontinuity is well behaved and appears to be strongly attached to the airfoil.

The flow near the tip has a strong tendency to flow inward. This action is caused by the motion of the tip vortex. This tip vortex is created by unequal pressures on the upper and lower surfaces which cause a net flow around the tip from lower to upper surface. Note that the effect of this spanwise tip flow is not evident at the 99% radius trace until about the 20% chord positions. This could indicate that the tip vortex is being formed in the first 20% chord (ref. 25).

In summary, the ammonia trace visualization data only indicate the overall gross flow effects. At Reynolds

numbers below 0.33 million, this data indicate the occurrence of standing laminar separation bubbles. It is this flow phenomena that is hoped to be examined using the hot-wire technique.

3.4.2 Surface Static Pressures

The surface static pressure data are shown in Figures 60 and 61. These data were obtained at 800rpm and pitch angles of 0° , $\pm 5^\circ$, $\pm 10^\circ$, and $\pm 15^\circ$. Since transducers were located only on the upper surface, the negative pitch angles were used to indicate the pressure on the lower surface of the blade. The data are presented in pressure coefficient form at 70% and 93% radius positions.

The solid curves are theoretical two-dimensional pressure distributions. In order to calculate this two-dimensional distribution, the local angle of attack for each hover condition had to be estimated. This estimation was made using simple blade element theory as described in reference 28, and does not include the three-dimensional effects of the rotating case.

Figure 60 presents the data at 70% radius. Note that the pressure coefficients are greater than the two-dimensional theory in the region from the leading edge to the 25% chord position. Aft of this position the data fall below the two-dimensional theory. This same trend

Figure 60. Surface Static Pressure Data at 70% Radius.

Figure 61. Surface Static Pressure Data at 93% Radius.

occurs on both upper and lower surfaces.

The data at 93% radius are illustrated in Figure 61. At this location, the data aft of the 25% chord position on the upper surface of the blade also fall below the theory. The pressure coefficients near the leading edge are not greatly under-estimated by the two-dimensional theory. The effects of tip proximity could be evident in the 93% radius pressure data.

The number of transducers used in this study greatly limited the usefulness of the results. With only six transducers per spanwise position, it is difficult to accurately define the chordwise pressure distributions; however, the data in Figures 60 and 61 indicate the same trends as the two-dimensional theory. Again, it is not known how accurate this comparison should be since the accuracy of the theory is questionable.

3.4.3 Boundary Layer Velocity Data

The velocity data are presented for discussion in Figures 62 and 63. This represents a relatively small portion of the hot-wire data; however, the remainder of this data are included as Appendix C.

Figures 62 and 63 represent boundary layer velocity data at a fixed spanwise position of 72% radius, fixed pitch angle of 10° , and rotor speeds of 100rpm and 400rpm.

Figure 62. Velocity Data at 72% Radius, $\theta=10^\circ$, and 100rpm.

100 RPM
 $\theta=10^\circ$
 142

Figure 63. Velocity Data at 72% Radius, $\theta=10^\circ$, and 400rpm.

400 RPM
 $\theta=10^\circ$

The graphs from left to right represent increasing chordwise positions of 10%, 15%, 20%, and 25% chord. The top row of graphs are chordwise velocity profiles, the middle row are spanwise velocity profiles, and the bottom row are outflow angles. Since all of the measured velocities are non-dimensionalized by their corresponding potential flow component, the experimental values of the chordwise potential flow U , and the spanwise potential flow W are also listed at each position. The data are all plotted versus actual vertical distance above the blade surface.

First, the data at 100rpm will be examined. At 10% chord, the chordwise profile exhibits a linear region near the surface and then rapidly approaches the potential flow value. The spanwise flow exhibits a slight overshoot type of profile. The outflow angle varies from a small value at the outer edge of the boundary layer to a relatively large value near the surface. This large outflow may be attributed to the centrifugal pumping action which occurs as the air particles become trapped in the boundary layer.

The top row of graphs depicts the development of the chordwise boundary layer. Notice that the profile becomes less and less "full" as the chordwise position is increased from 10% to 20% chord. At 25% chord, the chordwise profile seems to have inherited a new shape. The explanation of this chordwise boundary layer development

can be explained as follows.

It appears that a laminar separation bubble is standing on the rotor blade in the vicinity of 20% to 25% chord for the hover conditions in Figure 62. The best indications of the existence of a separation bubble are the separation profile shape at 20% chord and the much "fuller" profile that occurs just 5% chord downstream. This data indicates that a very abrupt change occurred between 20% and 25% chord.

The hypothesis for explaining the data in Figure 62 is as follows. The chordwise flow is laminar at 10% chord; however, the steep adverse chordwise pressure gradients observed in Figures 60 and 61 are decelerating this flow. By the time the flow reaches the 20% chord position, the chordwise component of the wall shear stress has almost vanished. A laminar separation takes place shortly after the 20% chord position is passed. When the flow separates, it gains momentum from the free stream and re-attaches as a turbulent boundary layer. Therefore, the chordwise profile observed at 25% chord is a turbulent flow profile.

Next, the spanwise profile development can be observed as the flow progresses along the chord from 10% to 25% chord. The spanwise profile at 10% chord indicates a very small amount of overshoot, prior to approaching zero near the surface. As the flow progresses to 15% chord,

this profile shape becomes rather "jagged" which may be due to experimental error. However, at 20% chord, the data indicate an extremely smooth profile which exhibits a large amount of overshoot. This illustrates that as the chordwise flow approaches a separation profile, the spanwise flow is greatly accelerated to maintain continuity within the boundary layer. It is this spanwise relief that permits the rotating boundary layer to remain thinner than the associated two-dimensional boundary layer.

Before leaving the data of Figure 62, a major point must be made. The ammonia trace data of Figure 57, and the vast amount of similar data reported in reference 27, was interpreted only after the hypothesis of the existence of laminar separation bubbles on the rotating airfoils. Although this hypothesis was consistent with every trend that was illustrated by the ammonia data, it was still a hypothesis and not a proven fact. The data of Figure 62 present a significant contribution to the support of the laminar separation bubble hypothesis. First, the chordwise profiles indicate laminar separation and turbulent reattachment, which was discussed at length in reference 27. Next, the almost instantaneous increase in the spanwise outflow at 20% chord, (the vicinity of vanishing chordwise wall shear), is very reminiscent of the ammonia discontinuities.

The ammonia trace "surface streamlines" actually

turn sharply outward away from the potential streamlines at the discontinuities. The outflow angles in Figure 62 also corroborate this trend in the ammonia data. The outflow angle is actually a measure of the angle between the chord line and the streamline at a given boundary layer height. For example, in Figure 62 at 20% chord, the outflow angle reaches a peak value of about 36° at approximately .007 inch from the blade surface, while the outflow angle is only about 3° at .050 inch from the blade surface. Again, this large outflow in the boundary layer is indicative of the ammonia trace discontinuities.

Figure 63 presents data at the same conditions as Figure 62, however, at 400rpm. At this speed, the boundary layer thickness is considerably smaller; therefore, fewer data points were obtained within the boundary layer. The results at 400rpm do not indicate the existence of a laminar separation bubble. This increase in Reynolds number has probably initiated transition prior to laminar separation as was observed with the ammonia traces. The rather rapid growth in the boundary layer thickness can still be observed in the chordwise velocity profiles.

When using a hot-wire for velocity measurements close to a solid boundary, errors may be introduced if the effect of the boundary on the rate of heat loss from the wire is ignored. To account for this heat loss, the experimental data of Wills (ref. 29) were used. Wills obtained exper-

imental data on various diameter wires in a known boundary layer flow. His data were then used to calculate correction factors for heat losses to the surface. For constant temperature wires, this correction factor was found to be a function of the ratio of distance from the surface to the wire radius. This correction factor was lifted from reference 29 and was incorporated into the hot-wire data reduction scheme to account for wall proximity heat losses and associated velocity measurement errors.

CHAPTER IV

COMPARISON OF ANALYSIS AND EXPERIMENTS

This chapter is devoted to comparing the analytical and experimental portions of the study. The only portions of these individual studies that warrant comparison are those for which calculations and measurements were made for a rotating NACA 0012 airfoil at zero lift.

The experimental hot-wire data at $\theta=0^\circ$, (zero lift condition), were obtained at spanwise locations of $z/c=3.87$ and $z/c=4.92$. The chordwise locations at which velocity profiles were obtained are indicated in Figure 64. The relationships between z/c and percent radius are shown along with the relationships between x/c and percent chord. At $z/c=3.87$, the Reynolds numbers based on the blade chord are 0.158, 0.316, 0.474, and 0.632 million which correspond to rotor speeds of 100, 200, 300, and 400rpm, respectively. At these rotor speeds the Reynolds numbers at $z/c=4.92$ are 0.185, 0.369, 0.554, and 0.739, respectively.

Figures 65 through 70 present chordwise velocity profiles on a rotating NACA 0012 airfoil at zero lift. By plotting versus the "stretched" boundary layer coordinate, $y\sqrt{\frac{\Omega z}{v_x}}$, used in the analytical study, the data at

Figure 64. Location of Hot-wire Data on Rotor Blade.

Figure 65. Chordwise Profiles on an NACA 0012 at $x/c=.115$ and $z/c=3.87$.

Figure 66. Chordwise Profiles on an NACA 0012 at $x/c=.165$ and $z/c=3.87$.

Figure 67. Chordwise Profiles on an NACA 0012 at $x/c = .215$ and $z/c = 3.87$.

Figure 68. Chordwise Profiles on an NACA 0012 at $x/c = .265$ and $z/c = 3.87$.

Figure 69. Chordwise Profiles on an NACA 0012 at $x/c = .165$ and $z/c = 4.92$.

Figure 70. Chordwise Profiles on an NACA 0012 at $x/c = .265$ and $z/c = 4.92$.

different rotor speeds or Reynolds numbers can be combined into a single graph. The solid line in each of these figures indicates the velocity profile predicted by the momentum-integral analysis.

At each location on the blade, the experimental data indicate a "fuller" profile than the analytical calculations. The analytical calculations assumed that the flow was laminar. Due to variations in the blade surface roughness, the actual flow may be transitioning, or even turbulent. If this were the case, the profiles within the boundary layer would be "fuller" than the corresponding laminar flow profiles.

Another point must be made regarding the limitations of the analytical solutions. One of the major controlling factors in making boundary layer calculations is the potential or inviscid flow. It is this region which dictates whether a flow even occurs; however, its most important contribution is the acceleration or deceleration of the boundary layer flow. Throughout all of the analytical calculations, this potential flow was obtained from theoretical, two-dimensional calculations. Any inaccuracies occurring in the potential flow would alter the displacement thickness and ultimately the wall shear stress distribution. The disagreement between theory and experiment in Figures 65 through 70 could indicate an inaccuracy in the assumed potential flow.

Except for Figure 66, the experimental chordwise velocity data congregate quite well for the range of Reynolds numbers presented. Figure 66 indicates a considerable amount of data scatter. The data presented in this figure represent the very first rotor hot-wire data obtained in this study. Since the positioning of the probe was accomplished by visually viewing the probe through the microscope, the consistency of the data depended greatly on the technique used while adjusting the probe height. After this first profile was completed, the consistency of the data appears to be much improved as is indicated by Figures 65, 67, 68, 69, and 70.

The spanwise profiles are compared in Figure 71 for a chordwise position $x/c=0.115$ and a spanwise position $z/c=3.87$. The results indicate that no comparison can be made between experimental and theoretical spanwise profiles. Very near the blade surface, the theoretical profile is moving toward the experimental data; however, it soon reverses its direction. The theoretical profile is primarily inflow while all of the experimental data indicate outflow.

An explanation of the inconsistencies of Figure 71 can be found by examining the location of the axis of rotation in each case. Throughout the analytical study, the axis of rotation was placed at the blade leading edge as shown in Figure 1, to simplify the mathematics

Figure 71. Spanwise Profiles on an NACA 0012 at $x/c = .115$ and $z/c = 3.87$.

and to compare results with other investigators. The rotor blades used in the experimental portion of this study were rotated about their quarter-chord positions. This seems like a very minor point; however, one must keep in mind that the spanwise flow is less than 10% of the total flow in the positions near the leading edge where measurements were made. For this reason, the direction of this spanwise flow is very sensitive to small changes in the geometry.

For example, the potential streamlines of an NACA 0012 airfoil can be closely approximated by circular arcs near the leading edge of the blade as shown in Figure 72. Since all of the experimental data were obtained in the region in front of the quarter-chord, the experimental spanwise potential flow is outward while the analytical spanwise potential flow is inward. This explains why the momentum-integral profile shown in Figure 71 reverses its direction. The lower portion of the boundary layer is still pumped outward due to the large component of centrifugal force; however, the upper portion of the boundary layer is governed principally by the inward potential flow. This analytical curve in Figure 71 would be greatly changed if the direction of the spanwise potential flow were reversed.

A possible explanation for the differences in the analytical and experimental chordwise profiles is as follows. The experimental spanwise flow is outward as

Figure 72. Approximate Potential Streamlines on a Rotating NACA 0012 Airfoil Near the Leading Edge.

indicated in Figure 71, and outflow in a rotating system creates an equivalent favorable pressure gradient. All but the bottom level of the analytical profile is inflow as indicated in Figure 71, and inflow in a rotating system creates an equivalent adverse pressure gradient. These effective favorable and adverse pressure gradients due to the Coriolis forces could tend to create the differences in the chordwise profiles that were indicated in Figures 65 through 70.

To re-emphasize the differences between the potential spanwise flows used in the analytical calculations and those experimentally measured, the following list of values of W are included. The experimental values of W in Figure 71 are 5.1, 11.0, 16.8, and 23.6fps corresponding to rotor speeds of 100, 200, 300, and 400rpm, respectively. The analytical values of W used in the momentum-integral analysis are -.6, -1.2, -1.8, and -2.4fps, respectively. Thus, the analytical spanwise potential flow has the wrong sign and is an order of magnitude too small. These comparisons illustrate the probability of error introduced in the analytical calculations due to the incorrect description of the potential flow.

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

The results of this study are:

1. The momentum-integral method, combined with the numerical solution technique, provided an accurate engineering approach to three-dimensional rotating boundary layer calculations. The comparisons with the so-called "exact" solutions of reference 11 provided sound evidence of the accuracy of this momentum-integral technique.
2. The ammonia flow visualization technique using the revolute film provided by a local manufacturer produces higher quality traces than did the ozalid paper used in reference 22. The ammonia traces can only be used to indicate an averaged boundary layer flow direction; however, the technique is so simple and inexpensive that the results obtained per time and money spent greatly outweigh the limitations.
3. Pressure data obtained during hover conditions requires a larger number of transducers per span-

wise location in order to produce data worthy of detailed interpretation. Although the simple blade element theory of reference 28 is useful for beginning analyses, it will not predict local angles of attack with sufficient accuracy to calculate two-dimensional pressure distributions.

4. The hot-wire anemometer instrumentation provided the most significant results of this study. It is believed that data provided by this portion of the study will serve as a foundation for future analytical and experimental research.

5. The velocity data obtained at a Reynolds number of 158,000 and zero pitch indicate the presence of a laminar separation and a turbulent reattachment between 20% and 25% chord on an NACA 0012 airfoil.

6. The results of the hot-wire study have served to verify the hypothesis of the existence of laminar separation bubbles on rotors. Therefore, these results provide positive substantiation to the conclusions drawn from the ammonia trace studies of this research and those of reference 27.

5.2 Recommendations

The recommendations for areas in which this study could be improved or extended are listed as follows:

1. A number of extensions could be made to the momentum-integral analysis. Among these are the application of this method to rotors with lift, the modification of the theory to allow the axis of rotation to be spaced an arbitrary distance from the blade leading edge, and the addition of a turbulent shear stress variation to more accurately predict the boundary layer trends in real flows.
2. It is recommended that any experimental attempt to measure surface pressures on a rotating airfoil include a minimum of 10 transducers, densely spaced near the leading edge at each spanwise blade position.
3. Since only a small region of the blade was investigated using the hot-wire probe, it is the author's recommendation that the experimental procedures developed during this study be used to provide a complete set of boundary layer information over an entire blade. This continued investigation should include data in the blade tip region to provide information relative to the formation of the tip vortex.

4. New hot-wire sensors have been released by anemometer manufacturers. These sensors employ gold plating of the wires in the vicinity of the support needles to lessen the adverse heat transfer gradients in this region. The use of these new sensors on the V-configuration probe may allow the application of the "cosine" law during calibration and greatly reduce the vast amount of calibration data. It is hoped that further studies will investigate the improvements offered by this new sensor.

APPENDIX A
CALIBRATION OF THE HOT-WIRE FLOW JET

CALIBRATION OF THE HOT-WIRE FLOW JET

In order to use the hot-wire calibration device shown in Figure 46, the device itself had to first be calibrated. The desired calibration data was to be of the form "air velocity versus chamber pressure". The first approach was to use an anemometer probe which was calibrated by the vendor as a standard reference. However, this approach was abandoned due to insufficient data supplied by the hot-wire probe manufacturer.

It was therefore decided to calibrate the nozzle by the use of a pitot-static tube. A standard 1/8-inch diameter probe was obtained, and calibration data were generated. Upon data reduction, it was noted that the nozzle efficiency was near unity at high velocities, but decreased significantly at lower velocities. It was at this point that the validity of the data became suspect. Since the diameter of the pitot-static tube was one-third of the exit jet diameter, this method of measurement was believed to be altering the jet flow to the extent that the static pressure ports were prone to error. That is, the jet was forced to expand around the probe which presented diverging streamlines at the static pressure ports. The error introduced by this jet expansion is similar to

a misalignment of the probe as discussed by Goldstein (ref. 30).

The calibration was repeated with a much smaller total head probe, while atmospheric pressure was used as the exit static pressure. This total head probe was made from .032-inch O.D. and 0.020-inch I.D. tubing. The small size of the probe was expected to greatly reduce jet interference during the measurements.

The calibration data reduction indicated that nozzle efficiencies of 100% were evident over the full range of velocities from 0-500fps. It should be noted that the 100% efficiency is good only to within 0.5% due to instrumentation readability and accuracy. Therefore, the isentropic relations could be applied directly to the chamber-to-atmospheric pressure ratio to obtain the exit velocity. The resulting calibration curve is shown in Figure 73.

Additional tests were conducted to define the exit velocity as a function of distance from the exit plane. These data were required to define the allowable band of distances from the exit over which the probe can be calibrated with accurate results. Figure 74 presents the results of these tests. It should be noted that p_o refers to the jet center line total pressure. Three chamber pressure levels were selected arbitrarily to show what effect the pressure level had on the exit jet. It is

Figure 73. Exit Velocity versus Chamber Pressure for Hot-wire Calibration Device.

Figure 74. Center Line Pressure Decay versus Distance from Jet Exit Plane.

Figure 75. Velocity Profile of Jet at Various Distances from Jet Exit Plane.

observed that the total pressure in the stream is equal to the total pressure in the chamber from the exit plane to approximately 1.0 inch beyond the exit plane. The rate of center line total pressure decay increases with decreasing chamber pressure.

Finally, vertical and horizontal traverses of the exit jet were made using the small total head probe. These data were obtained at a chamber pressure of 50 inches of water, or a velocity of approximately 460fps. Figure 75 shows the ratio of local velocity to center line velocity as a function of position at various distances from the nozzle exit. It is observed that the velocity of the jet is nearly uniform at short distances from the exit, but starts to approach a parabolic type distribution at greater distances from the exit plane. For example, at 0.25 inch from the exit, the velocity is uniform over 75% of the exit area, whereas, at 0.75 inch from the exit, the velocity is uniform over only 44% of the exit area.

The results of this calibration are as follows:

1. The exit velocity of the calibration nozzle may be calculated from isentropic relations with an accuracy of 0.5%.
2. The hot-wire probe must be mounted within one inch of the exit plane to maintain the exit plane center line velocity.

3. For assurance that the complete hot-wire probe is within the potential core region of the nozzle (area of uniform velocity), the probe should be located at 0.25 inch from the nozzle exit and centered.

APPENDIX B
ADDITIONAL AMMONIA TRACE DATA

ADDITIONAL AMMONIA TRACE DATA

The ammonia trace data presented in Figures 57 through 59 were sufficient to describe the major results of the visualization portion of this study. However, additional trace data were obtained and are presented in Figures 76 through 81. For comparison, it should be recalled that the data presented in Figures 57 through 59 were obtained at $\theta=12^\circ$ and rotor speeds of 250, 700, and 990rpm. Again, the ammonia was released at chordwise locations of 5, 10, 15, 20, 25, 30, 35, 40, 50, 60, and 75% chord.

The data in Figures 76 through 78 were obtained at $\theta=13.5^\circ$ and rotor speeds of 250, 700, and 920rpm, respectively. The data in Figures 79 through 81 were obtained at $\theta=15^\circ$ and rotor speeds of 250, 700, and 855rpm respectively. Initially, it was hoped that data could be obtained at 1000rpm; however, the limited hover stand power reduced the maximum rotor speeds to 990, 920, and 855rpm at blade pitch angles of 12° , 13.5° , and 15° , respectively.

First, examining the 250rpm data, it is observed that the only distinct flow discontinuities occur at 50% radius. With a little bit of imagination, the 60%, 70%, and maybe even 80% radius traces show indications of

increased outflow in the vicinity of 10% chord. The most illustrative trace is at 50% radius in Figure 76. Although this trace is not as dark as the others in the vicinity of the discontinuity, the trace at 10% chord indicates that the flow is practically 100% spanwise outflow. This outflow pattern indicates the inconsistency of a small cross-flow assumption in the vicinity of laminar separation.

Next, a few comments must be made about the trends of the ammonia traces at 99% radius. The data near the blade tip illustrate the same patterns of flow for all rotor speeds. It seems that the effect of the tip vortex on the rotor boundary layer flow is at most a weak function of rotor speed. An interesting observation is that the last three chordwise traces near the tip indicate a gradual lessening in the spanwise inflow. Again, this trend is similar in every hover condition at which data was obtained. A possible explanation could be that the tip vortex has started to separate from the blade at this chord position. If so, the effect of the vortex would tend to create an outflow near the blade surface.

One final comment may be made concerning the ammonia data. As was mentioned in section 3.4.1, the absence of flow discontinuities in the higher Reynolds number data indicates that transition may have occurred prior to the bubble formation. Since the resulting turbulent flow will not support a laminar separation bubble, it

will separate in the more conventional and more gradual manner. By examining the ammonia data very near the leading edge, (at approximately the $1\frac{1}{2}$ chord position), the ammonia traces show a sudden expansion. This pattern of flow was discussed in reference 27 and was hypothesized as the location of transition. If this hypothesis were true, it could explain the absence of discontinuities throughout most of the data.

Figure 76. Ammonia Trace Data; $\theta=13.5^\circ$, 250rpm.

Figure 77. Ammonia Trace Data; $\theta=13.5^\circ$, 700rpm.

Figure 78. Ammonia Trace Data; $\theta \approx 13.5^\circ$, 920rpm.

Figure 79. Ammonia Trace Data; $\theta=15^\circ$, 250rpm.

Figure 80. Ammonia Trace Data; $\theta=15^\circ$, 700rpm.

Figure 81. Ammonia Trace Data; $\theta=15^\circ$, 855rpm.

APPENDIX C
ADDITIONAL HOT-WIRE VELOCITY DATA

ADDITIONAL HOT-WIRE VELOCITY DATA

In order to maintain continuity throughout the discussion of results, a large portion of the experimental hot-wire data are included in this appendix. These data are presented in Figures 82 through 91. Note that the graphical data in these figures are identical in form to Figures 62 and 63 which were discussed in section 3.4.3.

First, let us examine the data in Figures 82 through 84. These data were obtained at $72\frac{1}{2}$ radius, a rotor speed of 100rpm, and pitch angles of 0° , 5° , and 15° respectively. At $\theta=0^\circ$, the chordwise profiles are "full"; however, they become less "full" as the $25\frac{1}{2}$ chord position is approached. Very little spanwise outflow is observed indicating that no separation bubble is present. A bubble could exist aft of the $25\frac{1}{2}$ chord position. At $\theta=5^\circ$, the chordwise profiles are more similar to separation type profiles at $25\frac{1}{2}$ chord than the $\theta=0^\circ$ data. Also, at $25\frac{1}{2}$ chord, the spanwise outflow is quite large indicating an action similar to the ammonia discontinuities. Next, recall that the data at $\theta=10^\circ$ was presented in Figure 62 and discussed at length. This data indicated large outflow at $20\frac{1}{2}$ chord. Now proceeding to the $\theta=15^\circ$ data of Figure 84, the trend of the hot-wire velocity profile data at 100rpm can be completed. At this pitch

angle, the variation of the chordwise profiles is reversed. That is, the profiles become fuller as the flow progresses from 10% to 25% chord. Very little outflow is observed in the spanwise profiles. This could indicate that a separation bubble exists in the flow just prior to the 10% chord position and that these data represent the development of reattached turbulent flow aft of the bubble.

The data at 100rpm and pitch angles of 0° , 5° , 10° , and 15° tend to confirm the trends of the location of the laminar separation bubbles that were observed with the ammonia trace technique. For example, ammonia data are presented in reference 27 for a four-inch chord, six-foot diameter, NACA 0015 airfoil at 72% radius and 400rpm. These data indicated that laminar separation occurred at chordwise positions of 46%, 23%, 15%, and 10% chord at the corresponding pitch angles of 0° , 5° , 10° , and 15° . A re-examination of Figures 82 through 84 and Figure 62 indicate velocity profile trends consistent with these locations of laminar separation.

Next, examine the data presented in Figures 85 through 87. These data were obtained at 72% radius, a rotor speed of 400rpm, and pitch angles of 0° , 5° , and 15° , respectively. An increase in the boundary layer thickness can be observed in chordwise profiles as the flow progresses from 10% chord to 25% chord. Note how

thin the boundary layer is at the 10% chord positions. This is a region of accelerated flow since the chordwise potential velocity is increasing. The $0-15^\circ$ data indicate a much increased boundary layer thickness; however, there are no indications of extremely large outflow in the 400rpm data. The data in Figures 85 through 87 and Figure 63 could be aft of the point of transition.

Figures 88 through 91 present data obtained at 92% radius. The data in Figures 88 and 89 were obtained at a rotor speed of 100rpm and pitch angles of 0° and 5° , respectively. The data in Figures 90 and 91 were obtained at 400rpm and pitch angles of 0° and 5° respectively. Due to the increased spanwise location, these data are similar to data taken at 72% radius, however, at slightly increased rotor speeds. These data were obtained only at chordwise positions of 15% and 25% chord. The data at 92% radius indicate trends similar to the data already discussed; however, with only two chordwise positions, these trends are not as obvious. One point should be noted in Figure 91. A large outflow seems to be evident in these data at 25% chord. This large outflow may be attributed to the very small value of potential spanwise velocity. Since all velocities were divided by their corresponding potential components, they become quite large as this potential component becomes small.

Figure 82. Velocity Data at 72% Radius, $\theta=0^\circ$, and 100rpm.

Figure 83. Velocity Data at 72% Radius, $\theta=5^\circ$, and 100rpm.

100 RPM
 $\theta = 5^\circ$ 187

Figure 84. Velocity Data at 72% Radius, $\theta=15^\circ$, and 100rpm.

100 RPM
 $\theta=15^\circ$
188

Figure 85. Velocity Data at 72% Radius, $\theta = 0^\circ$, and 400rpm.

Figure 86. Velocity Data at 72% Radius, $\theta=5^\circ$, and 400rpm.

400 RPM
 $\theta = 5^\circ$

Figure 87. Velocity Data at 72% Radius, $\theta = 15^\circ$, and 400rpm.

Figure 88. Velocity Data at 92% Radius, 100 RPM, $\theta = 0^\circ$, and 100rpm.

Figure 89. Velocity Data at 92% Radius, 100 RPM, $\theta = 5^\circ$, and 100rpm.

Figure 90. Velocity Data at 92% Radius, 400 RPM, $\theta = 0^\circ$, and 400rpm.

Figure 91. Velocity Data at 92% Radius, 400 RPM
 $\theta = 5^\circ$, and 400rpm.
 $\theta = 5^\circ$

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