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DISSERTATION

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By
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* * * * *

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INTRODUCTION

The problem of ultrasonic drilling of rock is of practical significance and interest to the engineers and researchers involved in the development and application of sonic energy. The most important item of equipment for sonic drilling is the transducer which converts electrical energy into mechanical energy. It consists of piezoelectric ceramic rings sandwiched in a resonant horn assembly, vibrating at frequencies in the range of 10 to 40 kHz. The horn, which is an energy concentrator, is in turn connected to a transmission line. The free end of the transmission line is either shaped into different geometries to act as a drilling bit or drilling bits of various configurations can be attached to it. However, it is more common to transmit the vibration energy of the line to the rock through a bouncing drill bit. The bit is located between the free end of the transmission line and the rock.

Ultrasonic drilling has nothing in common with the usual rotary cutting action associated with conventional drilling, but may work on a principle similar to that of a pneumatic drill at a frequency in the ultrasonic range. Due to the reciprocating action the ultrasonic drill is not limited to drilling round holes. The bit can be made to the shape required and extremely complicated shapes can be drilled.

Ultrasonic vibrations are used extensively in machining hard and brittle materials that are difficult or impossible to cut by other methods. The method is particularly useful in producing accurate odd-shaped holes and recesses in hard materials such as carbides, ceramics, and semi-conductors.

Sonic drilling of rock results from many impacts. Therefore, if the energy transfer ratio, drill bit penetration, reflected stress waves, dynamic force-penetration characteristic of the rock, and other information of interest, corresponding to a single impact, are known, one may very conveniently use them to explain the entire process of sonic rock drilling.

It has to be noted that due to the complexities of rock properties and thus their behavior, it is impractical to make any assumption or develop a theory which can hold for all types of rock under all circumstances. However, it is understood that most rocks are generally brittle.

It has long been recognized that the mechanical behavior of materials under conditions of impact differs significantly from that under static load application. Numerous experiments have been conducted in an effort to determine the dynamic properties of solids.

The term "rapid loading" test is applied to those loads where the loading time, while relatively short is still large compared to the transit time of a wave over the specimen. Consequently, the test piece remains in a state of quasi-equilibrium, and wave propagation effects need not be considered.

If a very hard body is allowed to fall on the smooth surface of a much larger fragile mass the energy expended will be partly absorbed and the balance will appear as rebound. With extremely fragile material the energy will be completely absorbed, and there will be no rebound. With extremely tough material, such as rubber, no energy will be absorbed, and the rebound will substantially equal the drop. With material between these two limits, part of the

energy is absorbed and part appears as rebound. In brittle materials the absorbed energy is used to produce a zone of fracture.

In this paper a differential equation representing the penetration of a drill tip into rock is obtained. To solve this equation, one needs to establish the dynamic force-penetration relationship between the tip and rock. Using the results of a series of experiments on stress wave transmission into limestone, experimental force-penetration relations were obtained. It was found that in general these relations can be approximated by two lines. Using such a bilinear spring model, the above-mentioned differential equation is solved for the drill's penetration, velocity, contact force, and reflected stress waves.

To examine the effects of some major factors such as the shape, amplitude, and duration of the incident stress wave, tip geometry of the drill, number of impacts, static pre-stressing of the drill, and material characteristics of rock, a rather extensive series of tests were carried out in these regards. In these experiments a variety of rocks such as granite, limestone, quartz, sandstone, feather rock, cork rock, marble and concrete were impacted with flat as well as conical tips and with stress amplitudes as high as 39 ksi. and as low as 5 ksi. Either approximately rectangular or sinusoidal incident stress waves were reproduced. It was found that the general patterns of the reflected waves are independent of amplitude and duration of the incident wave. In general it can be concluded that the model fairly represents the actual dynamic force-penetration of the rocks.

The theoretical values obtained, using the model, are compared with those of the experiments. Considering the

experimental limitations and complexities of rock behavior, the theory is in good agreement with the experiments.

Attempts are also made towards finding the most desirable incident stress waves which give maximum efficiency in energy transmission into rock.

Another aim of the experiments was an investigation of the mechanism by which rocks fracture under impact. Each block of rock was impacted and the impacted surface was then examined under a microscope and photographed. This process was in some cases repeated for up to 50 impacts on the same area. No unique mechanism of fracture was observed, i.e., different types of rock may fracture in a different fashion, but it was noticed that medium hard rocks such as limestone and corkstone break through a number of rings, using flat tip. The first ring is the zone in the vicinity of the outer edge of the bit. The successive impacts widen the width of this ring both outward and inward towards the center of the drill. This process repeats itself till the central core is demolished. A theoretical explanation is given for this mechanism of fracture.

To find the stiffness constants of the bilinear spring model introduced, Griffith's, Murrell's, and McClintock-Walsh's fracture theories for polycrystalline materials such as rocks are used in conjunction with elasticity solutions to obtain force-penetration relations. These relations are linearized by assuming that the depth of penetration is very small with respect to the radius of the drill. This is actually the case in ultrasonic drilling. These equations are in terms of the material characteristics of rocks. One major difficulty encountered at this stage is that one is not able to find unique values for material properties of one type of rock. This is of course natural due to non-uniqueness of the structure of rock.

I- REVIEW OF LITERATURE

It is very difficult, but necessary, to put rocks into a specified category in terms of their general properties. Investigations have shown that many types of rock are relatively elastic when subjected to uniaxial compression tests, so long as the stresses are within the magnitudes usually encountered in drilling and mining [1]. However, only a few of these rocks exhibit anything approaching linear elasticity. Also, the elastic behavior of a given rock type from different geographical areas varies considerably. Many rocks usually fail by fracture, so their behavior fairly resembles a brittle material.

Generally the tensile strengths of rocks are very small compared to their compressive strengths. This is due to the presence of microscopic cracks in rocks. Nishihara [2] has suggested that the nonlinearities in the stress-strain relation are also due to these microcracks.

Of all the theories predicting failure of rocks, four are found to be most commonly used or referred to. These are the Coulomb-Navier, Griffith's, McClintock and Walsh [1] and Murrell's [3] failure theories. The last two theories are sometimes referred to as modified and generalized Griffith's theories, respectively. The maximum shear stress theory proposed by Coulomb postulates that failure occurs in a material when the maximum shear stress at a point reaches a specific value which is referred to as the shear strength. Navier modified this theory by assuming that the normal stress across the plane of failure increases the shear resistance of the material by an amount proportional to the magnitude of the normal stress.

It is known from observations that crystalline materials, such as rocks, contain microfractures. Griffith hypothesized that stress concentrations develop at the crack tips causing the cracks to propagate and ultimately contribute to macroscopic failure.

McClintock and Walsh modified Griffith's theory by assuming that in compression Griffith's cracks close and a frictional force develops across the crack surface. This modified Griffith's fracture theory is in better agreement with published results on fracture of rocks.

Fairhurst [4] has observed that a drill bit penetrates the rock surface as a result of crushing of the contact surface irregularities, elastic deformation of the rock, and pulverization of the rock in a zone directly beneath the point of contact.

Further investigations of Fairhurst [5] show that the efficiency of energy transfer from a drill bit to the rock is determined by the boundary conditions at the bit-rock interface, and that complete transmission can occur only if the effective penetration impedance, the ratio of bit contact force to penetration velocity, of the rock matches the characteristic impedance of the rod. Furthermore, he has observed that the amount of energy transmitted is not significantly affected by the included cone angle of the drill tip. Consideration of the penetration of a sharp wedge-shaped bit driven into the rock by a blow indicates immediately that it is impractical to completely match the rod impedance to the penetration impedance at all times during penetration, since the latter approaches zero at the start of penetration and becomes infinite at the end when the force is maximum and the rate of penetration is zero. Thus, the effective impedance of the rock varies widely while the rod impedance remains constant.

It is theoretically possible, however, to match impedances by control of the incident waveform, if the force-displacement characteristics of rock are known. Assuming that this relationship is linear, Fairhurst proves that if the incident wave has a rising exponential form, the criterion of maximum energy transfer will theoretically be achieved.

Simon [6] combining the force and velocity conditions at the bit-rock interface, has found a differential equation governing the penetration of the bit as a function of the incident stress wave. To solve this equation, one needs to know the dynamic force-penetration characteristic. Assuming that this characteristic is in the form of a right triangle with linear rise, and that the incident stress wave is in exponential form, he develops an expression for energy transfer and shows that 100% energy transfer, predicted by Fairhurst, is theoretically possible.

Since it seems impractical to develop the type of waveform needed to obtain 100% efficiency, (at least impractical to obtain by simple impacts involved in sonic drilling), Simon looks for the possibility of achieving 100% efficiency with some combination of the above-mentioned waveforms with non-linear force displacement characteristics. His analysis of this new version of the problem results in a decreasing interface force between the drill and the rock with increasing depth of penetration, which is virtually impossible except momentarily when fracture occurs.

Hawkes [7], after discussing some fundamentals of wave propagation, tries to find out whether rocks behave in a perfectly elastic manner when subjected to impact loads, and if so, whether their behavior can be predicted mathematically from the laws propounded for elastic materials. The author has attempted to solve this problem by subjecting cylindrical

specimens of rock to impact loads generated by falling weights, and measuring the behavior of the resulting stress waves in the specimens.

Using a vertical drop tester, Hartman [8] investigated the sequence of rock failure under the impact of a simple, chisel-shaped bit when changes were made, singly, in blow energy, striker geometry, blow velocity, indexing of the striker, blow transfer, etc. The author concludes that the most desirable bit shape is the wedge with included angle not greater than 90 degrees, because it promotes crater formation by chipping. Also, a high blow energy is desirable to penetrate the rock and break a larger crater, since the volume of the crater formed is approximately proportional to the energy, regardless of bit shape. Furthermore, a low-velocity blow is preferable to a high one, and faster drillings can be attained only by increasing the blow frequency. The fracture of rocks and the events occurring are believed to be: crushing of surface irregularities, elastic deformation of rock, radiation of subsurface cracks from lines of stress concentration at cutting edge boundaries, generation of a central crushed wedge, chipping out of large fragments along curved trajectories to the surface adjacent to the crushed zone, and crumbling away of the crushed zone. The entire sequence may be repeated if the blow energy is sufficient.

Evans and Murrel [10] have measured the force required to push steel wedges of various angles at a slow steady speed into small rectangular blocks of coal confined on all surfaces except one. The results are described in terms of penetration resistance. In the absence of friction between wedge and medium, this is theoretically equal to the normal stress exerted on the interface. It has been found that, to a first approximation, the penetration resistance is equal to the strength of the coal in uniaxial compression. Friction makes less contribution to penetration resistance than would

be deduced from available values of the coefficient of friction. This is attributed to the low shear strength of coal. Evidence is included suggesting that this approximation applies if the medium is of semi-infinite extent.

The authors [10] develop a theory which predicts that the force-penetration relationship is directly proportional to the tangent of the bit half angle, in the absence of friction, and directly proportional to the sum of the coefficient of friction and tangent of the bit half angle, otherwise.

Pariseau and Fairhurst [11] have been able to solve the equations of equilibrium for plane stress in conjunction with the Coulomb failure criterion for fracture of rock-like materials. The result is establishment of a force-penetration characteristic for a wedge drill bit. The force-penetration is a linear relationship with the slope depending on the angle of internal friction, the angle of the wedge, the coefficient of friction between rock and drill bit, the width of the bit, and the uniaxial compressive strength of the rock.

Of interest is the work done by Miller and Sikarskie [12] on the penetration of conical indentors into rock. Using the modified Coulomb-Mohr fracture criteria in conjunction with the equilibrium equation of a conical bit, a parabolic force-penetration relationship is found, as compared to a linear relation for a wedge drill bit. Expressions for the volume of rock removed and the associated energies are formulated. It is suggested that, prior to the formation of any chips, the force is proportional to the square of the penetration and that the constant of proportionality is independent of penetration depth.

Shepherd [13] looks into the problem of percussive drilling and urges the study of the means by which energy is transmitted

through steel bars and the mechanism by which this energy is converted into work. He suggests that the higher striking energies of larger drills do not materially increase the maximum value of the stress wave, but rather maintain this stress over a longer time period. These high striking energies usually break more rock because they provide sufficient force over a longer time so that rock continues to be broken until the stress level falls below the ultimate strength of the rock. The author further explains that there is not a unique mechanism by which rocks fracture, rather it depends on the rock type. However, rocks fail by being subjected to compression beyond their strength and thus, disintegrate into fine powder or comminute. The adjacent grains are also compressed and cause shearing stresses to develop. Since brittle substances are weak in shear, these shear stresses break the material exposed to them. In addition, the paper gives the relative drillability of 50 rocks.

Singh and Hartman [14] have pointed out that little is known about the mechanisms of rock fracture under dynamic loading. Thus, cracks caused by stress waves differ from those obtained under static loading conditions. They propagate slower than the stress waves and hence, cracks that may be formed do not have time to grow before the pulse has traveled past, removing the stress. Also, a stress pulse affects only a small portion of a solid at a time, and fractures may develop in a part of the material without being affected by what is occurring in the other parts.

Under dynamic loads, many materials which behave in a ductile manner under static loading are known to assume brittle characteristics.

The authors [14] have carried out impact experiments on

a special synthetic rock, and strain-time variations are recorded. Based on these experiments and a theory, a hypothesis of failure for rocks is proposed. It consists of crushing of interfacial irregularities, and a momentary stretching of the rock surface causing tension around the area of drill bit contact. The stresses induced in the rock are compressive in the vertical direction below the bit, but may be compressive or tensile in any horizontal plane a short distance below the surface, which may initiate a crack in a nearly vertical direction since the maximum shear stress trajectories are almost vertical in this region. The crack then tends to extend in the vertical plane, upward to the surface and downward until its energy is spent. Owing to the impact of the bit on the rock a stress pulse radiates outward with compressive effect and tensile at right angles to its direction of movement. This phenomenon has a tendency to cause radial tension cracks in the rock. As stress waves propagate through a medium, shear planes are induced. These waves bring about a shallow cratering of the rock by shear failure. One of the complications not predicted by the above hypothesis is the existence of flaws and microcracks in the material, which introduce points of stress concentration. Some other limitations of this hypothesis are that rock material characteristics, mode and confinement pressure, bit type, and manner of striking the blow are not accounted for.

Simon [15] considers the sources of energy dissipation for concentrated loadings on rocks in an attempt to account for the experimentally measured magnitude of the work required to break out a unit volume of rock from the free surface of an essentially semi-infinite medium. It is concluded that most of this work probably represents the elastic energy developed by the loading in a much larger volume of rock beneath the loaded region than the volume of the rock fragment

broken out to the side of the loaded region. This strain energy is largely dissipated in the form of the stress waves generated by the high rate of unloading produced by the propagating cracks. The energies associated with the new surfaces of cracks and with the stress wave generated directly by the loading process are computed to be negligibly small. Possibilities for improving the utilization of energy to drill rock are discussed. However, the improvement would probably be slight.

Teale [16] introduces the concept of "specific energy", defined as the energy required to excavate a unit volume of rock, which may also be interpreted as an index of mechanical efficiency in rock mechanics. Its minimum value is observed to be very roughly correlated with the crushing strength of the medium which is drilled. The author emphasizes that 'crushing strength' is in no sense an absolute measure of rock strength; it is, rather, dependent on many factors. Therefore, compressive strength is only one and not necessarily the best measure of rock strength. It is suggested that 'indentation' be used as the measure of rock strength.

Goldsmith, Austin, Wang, and Finnegan [17] have studied transmission and decay of stress waves in bars of igneous rocks. Ballistically suspended Hopkinson bars of these rocks were impacted longitudinally by steel spheres. The shape and velocity of propagation of the resultant wave in the rocks were measured. It was observed that these rocks were all brittle materials, that no plastic deformation was observed, and that neglecting dispersion is justified. Furthermore, it was noted that the dynamic results are generally more consistent than the static results. The experimental data of the authors indicate that when a transient

pulse with a stress level above a critical value traverses a rock bar, energy is absorbed by fractures, grain cleavages, and damage to grain-to-grain bonds resulting from general comminution. A simple macroscopic model applicable to rock must therefore incorporate the effects of these phenomena. A solid friction mechanism appears to be definitely more applicable for a description of such materials than a viscoelastic model. This is particularly true at the high rates of loading encountered in the propagation of stress waves induced by impact or explosive loading. Under these conditions the short duration of the pulses are insufficient to cause significant manifestations of time dependent viscous phenomena. The general absence of dispersion observed in the rocks tested offers strong support for this conclusion.

Thus, based on the papers briefly reviewed in the above, one may draw the following important and fundamental conclusions with regard to rock drilling.

1. Most rocks when subjected to impacts of moderate stress levels, as encountered in drilling of rocks, fairly behave as a brittle material. Having this property, there are failure theories which predict their fractures.
2. Mathematical expressions for the force and penetration velocity at the rock-drill bit boundary are available and can be combined to obtain a differential equation for wave transmission into rock.
3. The dynamic force-penetration relation for any rock is of prime necessity for solving the problem of the wave transmission into rocks.

4. Experimental methods and techniques are available to obtain the dynamic force-penetration relations for any rock, but no theoretical analysis exists to predict these relations in terms of the rock material properties and the drill tip geometry.
5. Stress waves of approximately square or sinusoidal shapes can be reproduced experimentally.
6. Although rock is considered to be a viscoelastic material, due to the high rate of loading encountered in the stress wave propagation induced by impact, the short duration of the pulses are insufficient to cause a significant manifestation of time dependent viscous phenomena.
7. When a rock specimen is subjected to an impulsive load, the following sequence of events are suggested to happen:
 - (a)-Crushing of irregularities at the contact surface.
 - (b)-Elastic deformation of the rock.
 - (c)-Pulverization of the rock in the zone directly beneath the point of contact.
 - (d)-Radiation of subsurface cracks from lines of stress concentration at the cutting edge and chipping out of large fragments.
8. The dynamic modulus of elasticity, and compressive and tensile strengths of rocks are generally higher than their corresponding static values.

higher than their corresponding static values.

In spite of the significant number of papers written on the field of stress wave transmission in rock, or in areas related to this problem, this investigator has not been able to find a paper or a combination of papers which has developed the theory which can predict the experimental outcomes.

II-EXPERIMENTS ON WAVE TRANSMISSION INTO ROCK

Sonic drilling of rock is the result of fracture of rock under impact of a drill bit. The source of energy is the vibration of the tip of the transmission line of a sonic transducer. The end of the transmission line vibrates at a high frequency but the amplitude of vibration is extremely small. In the process of rock drilling the following sequence of events occur: (1) the vibrating line impacts the drill bit, (2) the drill bit rebounds from the line, (3) the drill bit impacts the rock, (4) the rock fractures and deforms elastically, (5) the rock relaxes elastically, and the drill bit rebounds from the rock. The above events occur extremely rapidly in an actual drilling process.

Although ultrasonic drilling of rock results from very many impacts, the most basic problem is the analysis corresponding to only one of these impacts. To solve this problem, a differential equation, which is the result of combining the force and the velocity at the surface of contact of the rock and the drill bit, is obtained. The solution of this equation requires that the dynamic force-penetration characteristics of the impacted rock be known. To find such a relationship, a series of experiments on stress wave transmission into rock were carried out.

When two elastic-homogeneous rods of the same cross section and material impact, a stress wave is generated. If the contact surfaces are plane, the stress wave is theoretically of rectangular shape, propagating at 20×10^4 inches per second if the rods are of steel. The length of this wave is twice the length of the impacting rod. If the contact end of the impacting rod is hemi-spherical,

the wave is approximately of parabolic or sinusoidal shape, i.e. if the contact surface is reduced to a point.

When a stress wave encounters a boundary it reflects. The shape of this reflected wave depends on the physical condition of the boundary, the shape of the incident wave, and the properties of the material at the boundary. If the boundary is free or unconstrained, the reflected wave has the same shape as the incident wave but opposite in sign, i.e., a compressive stress wave will be reflected as a tensile wave, and vice versa. When a wave encounters a fixed end, it reflects as a pulse of the same shape and sign. Thus, tension reflects as tension, and compression as compression.

When a block of rock is laid against the end of a rod, then the boundary is neither free nor fixed. Thus, the shape of the reflected waves are different from the incident wave. If these reflected waves can be determined theoretically or experimentally, then the problem of energy transmission to the rock is the difference between the energy corresponding to the incident wave and that corresponding to the reflected wave. The reflected stress can be easily determined experimentally by mounting two strain gages diametrically on a rod which is laid against a block of rock. The gages are connected to an oscilloscope. Proper arrangement can be made to avoid overlapping of the incident and the reflected waves.

2.1-Experimental Determination of Dynamic Force -Penetration Characteristics of Rock.

2.1.1-Experimental Techniques and Arrangements.

A steel rod of about 40 inches in length and $\frac{3}{8}$ inch in diameter was laid against a block of rock. Two strain gages were mounted diametrically on the rod at about 13.5 inches from the end in contact with the rock. The gages were connected to a bridge in such a way that bending effects were cancelled. An oscilloscope was used to observe and record both the incident and reflected waves. Precautions were made to avoid overlapping of the incident and the reflected waves. A spring gun was used to shoot bars of different lengths, (three, six, nine, and twelve inches), against the stationary transmission rod. Different springs were used in the gun to obtain stresses of different amplitudes. After each series of experiments, the geometry of the end of the transmission line which was in contact with the rock was changed to ascertain the effect of the tip geometry. The tip geometries used were flat, and cones of 135° , 90° , and 60° apex angles. Figure (1) shows the experimental arrangements, schematically.

Fig.1- Schematic arrangement of the experiment. (Oscilloscope, Wheatstone bridge, and Camera are not shown.)

As already mentioned, when two elastic bars of the same cross section and material impact against each other, a stress wave is initiated at the interface. Considering the experimental limitations and imperfections, the pulses obtained were in very good agreement with the theoretical predictions, from the point of view of amplitude, shape, and duration; however, it had a gradual rise time and more gradual fall. The discrepancy between the theoretical and actual results might be attributed to the insufficiency of one dimensional theory of wave propagation in rods, departure of the experimental conditions from the mathematically ideal situation analyzed, and imperfections of the contact surfaces. The latter increases the rise and the fall times of the pulse. See Figures 2 and 3 for pulse shapes obtained using the apparatus described. The incident pulse is the first pulse shown, (on left), and is compressive. It is immediately followed by the reflected pulse from the end; if free, the pulse should be the same as the incident, but reversed in sign (thus tension). As is seen this is the case. The delay time is due to the placement of the gages on the rod.

2.1.2- Theoretical Basis for the Test and Data Reduction

If $\sigma_i(t)$ is the incident wave in a bar and $\sigma_r(t)$ is the corresponding reflected wave from an end of the bar, then the force at the boundary of the rod is given by

$$F = A \left[\sigma_i(t) + \sigma_r(t) \right] \quad (1)$$

where A is the cross sectional area of the bar. Similarly

Fig.2- Incident and reflected stress waves from a free end of a rod.

Time base: 50 μ seconds per major division.

Vertical: 12.5 ksi. per major division.

(These calibrations apply to all subsequent records unless otherwise stated).

Fig.3- (left), a 4"x4"x1" plate is welded to the tip of the transmission line. (Right), the corresponding stress wave reflection which has a clamped end reflection phenomenon.

the velocity of the rod at the boundary is given by

$$\dot{y} = \left(\frac{g}{\rho c}\right) [\sigma_i(t) - \sigma_r(t)] \quad (2)$$

where A , ρ , and c , are the displacement of the boundary, the weight density, and the elastic wave velocity of the rod, respectively, and g is the gravitational acceleration.

During the contact of the rod with the rock, one has conditions of continuity of penetration, y , velocity, $\dot{y}$, and the force, F . Thus if $\sigma_i(t)$ and $\sigma_r(t)$ are known, Equations (1) and (2) supply a basis for very conveniently calculating the contact force, F , and the rod penetration into the rock.

Figures (4), (5), (6) are the actual-photographically-recorded - traces on the oscilloscope corresponding to flat and conical tips of 135° and 90° apex angles. These recording are used, in conjunction with Equations (1) and (2) to find the contact force and penetration velocity. Then graphically integrating the velocity by the simple process of calculating the area under the curve, one obtains the time variation of the penetration.

A typical calculation and the resulting force-penetration relation are given below. Note that for a steel bar

$$\begin{aligned} c &= 20 \times 10^4 \text{ (in/sec),} \\ \rho &= 490 \text{ \#/ft}^3, \\ g &= 32.2 \text{ ft/sec}^2. \end{aligned}$$

This yields

$$g/\rho c = (32.2 \times 12^4) / (490 \times 20 \times 10^4) = .006738 \text{ (in}^3\text{/\#-sec.)}$$

Also, having two gages of gage factor 2.04 and voltage supply of 12 volts to the bridge, one finds that:

$$1 \text{ millivolt} = 2.449 \text{ K\#/in}^2$$

Fig.4- Incident and reflected waves due to a flat tip.

Fig.5- Incident and reflected waves due to a 135° tip.

Fig.6- Incident and reflected waves due to a 90° tip.

In an oscilloscope recording, such as the one shown in Figure 4, the first rectangular shape is the incident wave. The following irregular wave is the first reflection from the tip of the rod, which is in contact with the rock. In the first column of the following table, after the column which contains time, one finds the ordinates of the incident wave corresponding to different times. In the second column, one sees the values of the reflected wave as taken from the picture. Note that the zero time in the latter case, is where the reflected waves starts to rise. Columns 3 and 4 are the values of the differences and the sums of the incident and reflected waves. Note that in these calculations, compressive stresses are taken to be positive, and vice versa. The next four columns in Table 1 are the same as the first ones except that they belong to a drill bit of conical tip with an apex angle of 135° . Similarly, the last four columns give the values of the incident stress, reflected stress, their differences, and their sums for a flat tip.

Having the difference and the sum of the incident and the reflected waves for different times, now one can substitute them in Equations (1) and (2) to find the penetration velocity and the contact force. Table 2 contains the results of such calculations. The second and third columns from the left contain the penetration velocity and the contact force, respectively. Similary Table 2 also contains time variations of velocity and contact force for the conical tip geometries. Columns 4, 7, and 10 are the drill penetration into the rock. These values were obtained by integrating the time-velocity curve, by approximating the area under this curve by the sum of the areas of a series of trapezoidals elements. Figures (7) and (8) are graphical representations of penetration and contact forces corresponding to the results tabulated in Table 2 . In Figure (9)

Table 1-Experimental values of the incident and reflected waves, as measured from typical oscilloscope recordings.

Time (μ -sec)	Stresses (in Millivolts) For 90° Tip				Stresses (in Millivolts) For 135° Tip				Stresses (in Millivolts) For Flat Tip			
	σ_i	σ_r	$\sigma_i - \sigma_r$	$\sigma_i + \sigma_r$	σ_i	σ_r	$\sigma_i - \sigma_r$	$\sigma_i + \sigma_r$	σ_i	σ_r	$\sigma_i - \sigma_r$	$\sigma_i + \sigma_r$
0	0	0	0	0	0	0	0	0	0	0	0	0
10	16	-15.	31.	1	14.	-14.	28.	0	15.5	-15.	30.5	.5
20	15.9	-14.	29.9	1.9	13.9	-11.	24.9	2.9	16.	-11.	27	5.
30	15.9	-13.	28.9	2.9	13.9	-9.	22.9	4.9	16.	-6.5	22.5	9.5
40	15.8	-11.5	27.3	4.3	13.8	-6.5	20.3	7.3	16.	0	16.	16
50	15.8	-10.5	26.3	5.3	13.6	-5.	18.6	8.6	15.9	3.5	12.4	19.4
60	15.7	-9.	24.7	6.7	13.4	-3.	16.4	10.4	15.9	6.	9.9	21.9
70	15.6	-8.	23.6	7.6	13.2	-2.	15.2	11.2	15.8	8.	7.8	23.8
80	15.6	-6.5	22.1	9.1	13.0	-1.	14.	12.	15.4	9.	6.4	24.9
90	15.5	0	15.5	15.5	8.	0	8	8	10.	11.	-1.	21.
100	10.5	5.	+5.5	15.5	1.	9.	-8	10	1.	15.	-14.	16.
110	0	4.5	-4.5	4.5	0.	7.5	-7.5	7.5	0	8.5	-8.5	8.5
120	-	4.	-4.	4.	-	4.5	-4.5	4.5	-	6.	-6.	6.
130	-	2.5	-2.5	2.5	-	3.5	-3.5	3.5	-	4.	-4.	4.
140	-	2.	-2.	2.	-	2.5	-2.5	2.5	-	2.5	-2.5	2.5
150	-	1.8	-1.8	1.8	-	2.	-2.	2.	-	2.	-2.	2.
160	-	1.6	-1.6	1.6	-	1.5	-1.5	1.5	-	1.5	-1.5	1.5
170	-	1.2	-1.2	1.2	-	1.	-1.	1.	-	1.	-1.	1.
180	-	1.0	-1.0	1.0	-	.8	-.8	.8	-	.8	-.8	.8
190	-	.8	-.8	.8	-	.5	-.5	.5	-	.5	-.5	.5
200	-	.6	-.6	.6	-	.3	-.3	.3	-	.4	-.4	.4
220	-	.4	-.4	.4	-	.1	-.1	.1	-	.2	-.2	.2
250	-	0	0	0	-	0	0	0	-	0	0	0

Compression (+)

Table 2-Results for force, velocity, and penetration from data in Table 1.

	90° TIP			135° TIP			PLANE TIP		
Time	Contact Velocity	Contact Force	Penetration	Contact Velocity	Contact Force	Penetration	Contact Velocity	Contact Force	Penetration
μ -sec.	(ft/sec)	(K#/in ²)	(ft) $\times 10^{-6}$	(ft/sec)	(K#/in ²)	(ft) 10^{-6}	(ft/sec)	(K#/in ²)	(ft) $\times 10^{-6}$
0	0	0	0	0	0	0	0	0	0
10	42.63	2.449	213.15	38.50	0	192.50	41.94	1.224	209.70
20	41.12	4.653	631.90	34.24	7.102	556.20	37.13	12.245	605.05
30	39.74	7.102	1036.20	31.49	12.000	879.85	30.94	23.265	945.40
40	37.54	10.531	1422.60	27.91	17.878	1176.85	22.00	39.184	1210.10
50	36.17	12.980	1791.15	25.58	21.061	1444.30	17.05	47.511	1405.35
60	33.96	16.408	2147.65	22.55	25.470	1684.95	13.61	53.633	1558.65
70	32.45	18.612	2479.70	20.90	27.429	1902.20	10.73	58.286	1680.35
80	30.39	22.286	2793.90	19.25	29.388	2102.95	8.80	60.980	1778.00
90	21.31	37.959	3052.40	11.00	19.592	2254.20	-1.37	51.429	1815.15
100	+7.56	37.959	3196.75	-11.00	24.490	2254.20	-19.25	39.184	1712.00
110	-6.19	11.021	3164.55	-10.31	18.367	2147.65	-11.69	20.817	1557.30
120	-5.50	9.796	3106.10	-6.19	11.020	2065.15	-8.25	14.694	1457.60
130	-3.44	6.122	3061.40	-4.81	8.571	2010.15	-5.50	9.796	1388.85
140	-2.75	4.898	3031.40	-3.44	6.123	1968.90	-3.44	6.123	1344.15
150	-2.48	4.408	3005.25	-2.75	4.898	1937.95	-2.75	4.898	1313.20
160	-2.20	3.918	2981.85	-2.06	3.674	1915.40	-2.06	3.674	1289.15
170	-1.65	2.939	2962.60	-1.37	2.449	1898.25	-1.37	2.449	1272.00
180	-1.37	2.449	2947.50	-1.10	1.960	1885.90	-1.10	1.960	1259.65
190	-1.10	1.959	2935.15	-.68	1.224	1877.00	-.68	1.224	1250.75
200	-.82	1.469	2925.55	-.41	0.735	1871.55	-.55	0.980	1244.60
220	-.55	.980	2918.70	-.14	0.245	1868.80	-.28	0.490	1240.45
250	0	0	2915.95	0	0	1861.80	0	0	1239.05

Fig. 7-Experimental time variation of penetration due to a rectangular incident stress wave of about 39 ksi.

Fig. 8-Experimental time variation of the contact force.
($\sigma = 39$ ksi)

Fig.9-Experimental force-penetration for various tips.
($\sigma_c = 39$ ksi)

these two graphs are combined and an experimental force-penetration is obtained.

2.1.3 - Effects of Parameter Variations on the Reflected Wave

To observe the significance of some parameters such as the tip geometry, amplitude of the incident waves, number of impacts, static prestressing of the rod against the rock, length of the impacting bars, the material characteristics of different types of rock, and shape of the incident wave on the reflected stress wave, a rather extensive series of experiments were carried out. These parameters were varied one at a time to examine their effects. The results are discussed briefly in below.

(a) Tip Geometry:

It was observed that the pattern of the reflected wave was greatly affected by changing the geometry of the rod end. Figures 4, 5, and 6, which are oscilloscope recordings, show the patterns of reflected waves corresponding to rods of flat, 135° , and 90° tips.

In general the reflected stress wave from a flat end is more of compressive nature and thus has more resemblance with the reflected wave from a fixed end. On the other hand, for sharp conical tips the reflections are more tensile and thus exhibit more similarity to the reflections from a free end.

All the reflected waves have a compressive tail which is of damping-out character. This part corresponds to the elastic re-expansion of the rock. Although all are compressive in nature for all tip geometries considered, their amplitudes are noticeably larger for the flat tip and diminish as the tip angle of the conical drill decreases. In the

following figure, three traces corresponding to conical tips of 90° and 135° and a flat tip are superimposed for better exhibition of the aforementioned phenomena. Note that in all these cases the incident wave is the same.

Fig.10- Effects of tip geometry on reflected waves.(Actual)

(b) The spring gun was modified to produce rectangular incident waves of approximately 5.9, 7.8, 24.5, and 39.0

Ksi. Again different tip geometries were used. Within the experimental range of the stresses considered there were no noticeable variations in the reflected stress patterns. Figures 11, 12, & 13 are the dynamic force penetrations obtained from these experiments.

(c) Number of Impacts:

During the course of the experiments, it was noted that even for the same incident stress wave and conical tip, the reflected stress wave pattern changed quite significantly. Starting with a new tip of 60° apex angle and rock of fresh surface, a series of impacts were conducted. Superimposing the reflected waves on an oscilloscope, the picture shown in Figure 14 was obtained.

Fig. 11-Experimental force-penetration for various tips.
($\sigma_c = 24.5$ ksi).

Fig.12-Experimental force-penetration for various tips. ($\sigma_c = 7.8$ Ksi.)

Fig. 13-Experimental dynamic force-penetration for various tips. ($\sigma_c = 5.9$ ksi)

Fig.14- Effects of number of impacts on the reflected wave.
(60° tip used)

Fig.15- Effects of static force on the pattern of stress wave. (The upper trace corresponds to application of static force).

Fig.16- Phenomenon of proportionality of incident wave length to the impact bar's length. (Each major vert. div.=2mv., and each horiz. major div.=20 μ -sec.)
Note: The wave having first vert. line, from left, & the last is that of a 12" bar. The wave having first & fourth is that of a 9" bar. The one having 2nd and 3rd lines is that of a 6" rod.

Table 3- Effects of number of impacts on contact force, velocity, and penetration when a conical tip of 60 degree was used.

Time μ sec	first impact					20th. impact			
	σ_c mv	σ_r mv	F k#/in. ²	v ft/sec.	Pen. 10 ⁻⁶ ft	σ_r mv	F k#/in. ²	v ft/sec.	Pen. 10 ⁻⁶ ft
0	0	0	0	0	0	0	0	0	0
10	7.	-7.	0	19.2	96.	-7.	0	19.3	86
20	7.	-6.8	.5	19.	287	-6.	2.4	17.9	272
30	7.	-6.6	1.	18.7	475	-5.	4.9	16.5	444
40	7.	-6.4	1.5	18.4	661	-4.	7.4	15.1	602
50	7.	-6.2	2.	18.1	844	-2.	12.3	12.4	739
60	7.	-6.	2.4	17.9	1024	-1.	14.7	11.	856
70	7.	-5.8	2.9	17.6	1201	-.5	15.9	10.3	962
80	7.5	-5.6	4.7	18.	1389	0	18.4	10.3	1065
90	7.5	-5.4	5.1	17.7	1567	0	18.4	10.3	1168
100	7.5	-5.2	5.6	17.5	1743	.5	17.2	9.6	1268
110	7.	-5.	4.9	16.5	1913	.5	18.4	8.4	1360
120	3.	-1.	4.9	5.5	2023	2.	12.2	1.4	1412
130	0	1.5	3.7	-2.1	2006	6.	14.7	-8.2	1378
140						4.5	11.	-6.2	1306
150		1.	2.5	-1.4	1971	3.5	8.6	-4.8	1251
160						2.5	6.1	-3.4	1210
170						2.	4.9	-2.8	1179
180		0	0	0	1950	1.	2.4	-1.4	1158
200						.5	1.2	-.7	1148
220						0	0	0	1144

Fig.17-Limits of force-penetration for 60° tip as number of impacts increased.

The stress variation seems to be due to the fact that, as the number of impacts increases, so does the area of contact between the rock and rod. Also, the geometry of the tip as well as the contact surface of the rock undergo noticeable changes.

It is noteworthy to mention that beyond a certain number of impacts, which depends on the geometry and material characteristic of the tip as well as the type of rock, the stress patterns remain unchanged. (See Fig. 17).

(d) Static Prestressing

To provide static pressure between the rod and the rock, a disk was screwed to the rod. Two springs were hooked to one of the supports of the rod. The other ends of the springs were attached to the disk by means of two screws. The position of the disk and the resulting amount of extension of the springs could be adjusted by the side screws on the disk. Observations support the idea that no significant distortion occurs in the stress patterns for static forces of relatively small magnitude with respect to the amplitude of the stress due to impact. However, the patterns were shifted as much as the corresponding static force. A typical result is shown in Fig. (15).

(e) Length of the Impacting Bar

Impacting bars of 3", 6", 9", and 12" length were used to ascertain the significance of this factor. It was observed, as predicted by the theory, that only the duration of the pulses change in proportion to the length of the impact bar. Figure (16) depicts the experimental results.

(f) Material Characteristics of Rock

Different types of rocks were impacted. Attempts were made to select rocks of varying hardness. They included quartz, Georgia marble, granite, limestone, Tennessee quartzite

Quartzite

Corkstone

Georgia marble

Sandstone

Fig. 18 - Incident and reflected stress waves for different rocks.

Limestone

Concrete

Feather rock

Tenn. quartzite

Granite

Fig.19 -Incident and reflected stress waves for various types of rock.

Fig. 20- Experimental time variations of contact force for sandstone, Tenn. quartz, concrete, Ga. marble, corkrock, and featherstone

Fig. 21-Experimental time-penetration for various rock.

Fig. 22-Experimental dynamic force-penetration for Ga. marble, limestone, concrete, Tenn. quartz, and sandstone.

Fig. 23- Experimental dynamic force-penetration for featherstone, corkrock , and quartz.

sandstone, corkstone, and featherstone. A block of concrete was also impacted. Because of its practical importance in sonic drilling only a flat-tip rod was used in this series of the experiments. Figures 18 and 19 show the reflected waves corresponding to these rocks and concrete. From these photographs one may recognize that reflected waves corresponding to quartz, granite, Georgia marble, concrete, limestone and Tennessee quartz exhibit similar trends in their stress patterns. The reflected wave for featherstone is unique in shape as though corresponding to a free end. The cork rock's reflected stress wave is mostly tensile for a major part of the first reflection whereas those of the first group of rocks were compressive. The reflected wave of sandstone appears to be between those of first group and the corkstones.

Using these pictures with equation (1) and (2), and going through the same mathematical process which was discussed earlier, time variation of the rod penetration in each rock and the contact forces were calculated. Having these results in hand, then, experimental dynamic force-penetration relations were plotted in each case. Results are shown in Figures (20) through (23).

(g) Shape of Incident Wave

In addition to rectangular stress waves, it was possible to produce waves of approximately parabolic or sinusoidal shapes. These pulses are generated at the contact point of a flat ended bar with a bar of hemispherical end. These kind of waves are of practical interest. To generate such waves, the spring gun was modified. A three-inch bar of hemispherical end which was made of hard steel to avoid excessive deformations, was used for shooting. The round end bar was put in front of a nine-inch long bar. Otherwise,

the gun could not be used to shoot the 3-inch rod directly. To avoid multi-impact resulting from shooting two bars at the same time, the longer bar was screwed to the disk which was located in front of the spring. The disk acted as a stopper for the longer bar. In the sketch below such an incident wave is compared and approximated by a sine wave. In Figures (24), (25), and (26) are the incident and refelected waves corresponding to a flat tip and two conical tips of 90° and 135° .

Fig.23b-Approximation of an actual incident wave by a sinusoidal wave.

(h) Rate of Loading

Within the range of stress amplitudes considered, the time required for an incident stress wave of rectangular shape to attain its maximum value was experimentally found to be about 8 microseconds. Furthermore it seems to be independent of the stress amplitude. Considering that the stress amplitudes ranged from 6 Ksi. to 40 Ksi., one obtains the corresponding loading rates which are $.75 \times 10^9$ and 5×10^9 psi./sec., respectively.

The slopes of the experimental force-penetration characteristics show no general and consistent pattern of change within the above range of loading range. One may conclude that loading rates play no significant role on the force -penetration relations, at least under impulsive loadings. However, one should not totally ignore the effects of loading rates for considerably wider ranges of loading rates.

Fig.24-Sinusoidal incident & the reflected waves due to flat tip.

Fig.25-Sinusoidal incident & the reflected waves due to 135° tip.

Fig.26- Sinusoidal incident & the reflected waves due to 90° tip.

2.2 Observations on Rock and Concrete Fracture Under Impact

A series of impact tests were carried out on various types of rocks. The aim was to observe how different rocks fracture under impact and see whether most rocks fracture in the same fashion. The stress level was kept at about 30 Ksi. and the incident pulses were approximately rectangular in shape. In a few cases the stress level was about 40 Ksi. A variety of rocks were tested ranging from very hard ones such as quartzite, granite, Georgia marble, Tennessee quartzite, to very soft rocks such as featherstone and corkstone. The featherstone was so soft and spongy that a series of 15 impacts bored a hole of $\frac{1}{2}$ " in depth. The result is shown in Fig. 31.

With hard rocks the observations are very inconsistent and inconclusive, realizing that in these cases as many as 30 to 40 impacts were applied to a point without the drill actually penetrating into the rock. Fig. 27 to 29 show granite and concrete before impact and after a number of impacts.

Limestone and corkstone showed similar features in fracture. Fig. 30 shows that limestone fails around the boundary of the drill. This seems to be due to high stress concentrations at the boundary when a flat drill is used. The successive impacts increase the width of this fracture both inwards and outwards. In Figures 32 and 33 the same features are observed in corkstone. They indicate that an island is developed after 7 impacts and that the contact area with the drill is appreciably reduced.

The experiments, although not exhibiting any definite common result, may suggest that rock fractures layer by layer, when subjected to impacts, and that each layer fails in a ring-like fashion. The first ring to fracture is the material both from inside as well as outside of the contact area. From then on an island forms which reduces in diameter in each successive impact and finally vanishes.

Fig. 27-Surface of the granite before any impact.

Fig. 28- Damage inflicted on the granite after 20 impacts.

Fig. 29- From top to bottom, respectively, are shown a block of concrete before and after 20 and 50 impacts.

Fig. 30- Fracture occurred on limestone after five,(top),
and thirty,(bottom), impacts:

Fig. 31- Cylindrical hole bored in feather rock by 15
impacts.

Fig. 32 - From top to bottom, respectively, are shown the cork-rock before any impact and after one and three impacts.

Fig. 33- Sequence of fracture inflicted on corkrock by five, (top), seven, (middle), and ten, (bottom), impacts.

2.3 Results of the Experimental Investigations.

The series of experiments carried out on impacting different types of rock with drills of various tip geometries and incident stress waves of different durations, amplitudes, and shapes were aimed at obtaining an insight into the force-penetration relations of rock under short duration dynamic loadings. Considering difficulties in achieving any concrete and conclusive results in this area, the outcome has been promising in founding the later works in this investigation on a realistic basis.

Although coming up with any mathematical model which can represent the complicated behaviour of rock, under dynamic loadings seems unfeasible and impractical, it is observed that the rocks considered, within the experimental range in this work, exhibit a bilinear spring phenomena in their dynamic force-penetration relation. Their stiffnesses depend on the rock material characteristics and tip geometry but are independent of amplitude, duration, and shape of the incident stress waves. Such an approximation of the dynamic force-penetration relations is rather crude in most cases. However, there are some cases where this is far from reality and there are some other instances in which agreement is surprisingly good. But during the process of calculating the contact force and the drill penetration by means of the oscilloscope traces, it is felt that these experimentally-calculated dynamic force penetration variations are subject to unavoidable errors. Because a very negligible inaccuracy in measuring stresses from the pictures, which cannot be avoided, a notable error appears in both the velocity and contact force. Furthermore, this inaccuracy is magnified in evaluating drill penetration by means of numerical integration. For this reason, some of the disagreement of this bilinear spring model from the experimental dynamic force-penetration can be expected.

III-THE THEORY

Ultrasonic drilling of rock is the result of fracture of rock under impact of a drill bit. The source of energy is the vibration of the tip of the transmission line of a sonic transducer. The end of the transmission line vibrates with high frequencies but the amplitude of vibration is extremely small. In the process of rock drilling the following sequence of events occur :

- (1) the vibrating line impacts the drill bit.
- (2) the drill bit rebounds from the line.
- (3) the drill bit impacts the rock.
- (4) the rock deforms elastically and fractures.
- (5) the rock relaxes elastically and the drill bit rebounds from the rock.

These events occur extremely rapidly in an actual drilling process. Although ultrasonic drilling of rock results from very many impacts, the most basic problem is an analysis corresponding to only one of these impacts. To solve this problem, a differential equation is obtained which is the result of combining the force and the velocity at the surface of contact of the rock and the drill bit. The solution of this differential equation requires that the dynamic force -penetration characteristics of the impacted rock be known.

When a block of rock is laid against the end of a rod, then the boundary is neither free nor fixed. Thus the shape of the reflected waves are different from the incident wave. If these reflected waves can be determined theoretically, then the problem of energy transmission into the rock is very easily answered since the energy transmitted to the rock is the difference between the energy corresponding to the incident wave and that of the reflected wave.

3.1 Development of the Differential Equation

Let $\sigma_i(t)$ represent the incident stress wave at the bit end of the drill steel as a function of time, t , with $t=0$ at the instant of arrival of the wave front of the stress wave at the interface between the bit and the rock. Likewise, let $\sigma_r(t)$ represent the stress amplitude of the wave reflected at the bit end of the drill steel. Then, as already mentioned, the instantaneous force between the bit and the rock, F , is given by

$$F = A [\sigma_i(t) + \sigma_r(t)] + F_0 \quad (3)$$

where A is the cross sectional area of the drill steel and F_0 is any force that may have existed between the bit and the rock just prior to arrival of the stress wave. The velocity of motion of the bit into the rock, dy/dt , where the algebraic sign of y is positive for displacement toward the rock, is given by

$$dy/dt = g [\sigma_i(t) - \sigma_r(t)] / (\rho c) + v_0 \quad (4)$$

where $\rho c/g$ is the specific acoustic impedance of the drill steel material for propagation of the longitudinal bar waves and v_0 is any velocity that the bit may have had just before $t=0$. It is to be noted that compressive stresses are taken to be positive and tensile stresses negative, for convenience.

Combining the above equations by eliminating $\sigma_r(t)$, one obtains

$$dy/dt + g/(\rho c A) F = g [2 \sigma_i(t) + F_0/A] / (\rho c) + v_0 \quad (5)$$

To solve this differential equation, one needs to know the dynamic-force-penetration relation for the rock and the time variation of the incident wave.

It has to be emphasized at this point that due to complexities of rock properties and to their great variation and thus uncertainties in their behavior, it is impossible to establish any theory which can rigorously predict the

behavior of rocks, or make any assumptions which can hold for all rocks under all circumstances. Therefore the aim of this theoretical work is an attempt towards investigating the feasibility of obtaining a solution which may fairly represent typical behavior of some rocks under short duration loadings, which is the case in sonic drilling.

Observations suggest that when a drill impacts a rock, it advances as the result of both fracture and elastic deformation. It is far from reality to give a unique mechanism for fracture of rocks, but it can be said that the process of fracture may include any or all of chipping, cracking, compaction, and pulverization of rock depending on the rock and the drill bit geometry. However, chipping and cracking are more present when the drill is flat-ended. Likewise, pulverization and compaction are major factors of fracture when conical drills are used.

Let us assume that the force-penetration relation for rock, when subjected to impacts, can be fairly approximated as shown in the following figure.

Fig. 34-Approximate dynamic force-penetration for rock.

This is mathematically equivalent to

$$F(y) = \begin{cases} F_1 = k_1 y_1 & ; y \geq 0, \dot{y} > 0 \text{ or } 0 \leq t \leq t_1 \\ F_2 = k_2 (y_2 - \alpha) & ; y > 0, \dot{y} < 0 \text{ or } t \geq t_1 \end{cases} \quad (6)$$

where t_1 = duration of the incident stress wave.

The values of k_1 and k_2 seem to depend on the material properties of the rock as well as the tip geometry of the drill bit. It is experimentally noticed that neither the stress wave duration nor its amplitude, within the range considered, have any appreciable effect on k_1 and k_2 . However, the actual drill penetration, α , depends on the rock characteristics, bit angle, stress wave duration, and stress amplitude.

To find the net drill penetration, let the maximum drill penetration be y_e . Then from the above figure one has

$$k_1 y_e = k_2 (y_e - \alpha)$$

from which one obtains

$$\alpha = \left(1 - \frac{k_1}{k_2}\right) y_e \quad (7)$$

where $y_e = y_1(t_1)$.

Substituting Equations (6) into Equation (5), one obtains

$$dy_1/dt + K_1 y_1 = g \left[2\sigma_i(t) + F_0/A \right] / (\rho c) + v_0 \quad ; 0 \leq t \leq t_1 \quad (8)$$

$$dy_2/dt + K_2 y_2 = 0 + K_2 \alpha + v_0 + (g F_0) / (\rho c A) \quad ; t \geq t_1 \quad (9)$$

where

$$K_1 = (g k_1) / (\rho c A) \quad (10)$$

$$K_2 = (g k_2) / (\rho c A) \quad (11)$$

Equations (8) and (9) are linear first order differential equations and their solutions can conveniently be obtained for any arbitrary incident stress wave, $\sigma_i(t)$.

3.2 General Solutions

Let

$$f(t) = g \left[2 \sigma_c(t) + F_0/A \right] / (\rho c) + v_0 \quad (12)$$

in Eqn. (8). Then one has

$$dy_1/dt + K_1 y_1 = f(t).$$

To solve this differential equation, multiply both sides by $e^{K_1 t}$. Thus one obtains

$$d(e^{K_1 t} y_1) = e^{K_1 t} f(t) dt$$

Integrating this equation yields

$$e^{K_1 t} y_1 = \int_0^t e^{K_1 x} f(x) dx$$

where x is a dummy variable chosen for convenience in integration. Hence

$$y_1 = e^{-K_1 t} \int_0^t e^{K_1 x} f(x) dx$$

This is the particular solution. Knowing that $G_1 e^{-K_1 t}$ and $G_2 e^{-K_2 t}$ are the homogeneous solutions for Equations (8) and (9), respectively, one can say that the general solutions are

$$y_1 = e^{-K_1 t} \left[G_1 + \int_0^t e^{K_1 x} f(x) dx \right] \quad ; 0 \leq t \leq t_1 \quad (13)$$

$$y_2 = G_2 e^{-K_2 t} + \alpha + F_0/k_2 + v_0/K_2 \quad ; t \geq t_1 \quad (14)$$

Imposing the condition of continuity of penetration at $t=t_1$, i.e., $y_1(t_1) = y_2(t_1)$, one can solve for one of the unknowns in terms of the other.

3.3 Special Cases

In most of the practical cases in rock impact, the static force, F_0 , is absent and the rod may start with no initial velocity.

3.3.1 No Static Load and No Initial Velocity

Substituting these conditions into Equations (12) through (14), one obtains

$$f(t) = \frac{2g}{\rho_c} \sigma_i(t),$$

$$y_1(t) = e^{-Kt} \left[G_1 + \frac{2g}{\rho_c} \int_0^t e^{Kx} \sigma_i(x) dx \right] \quad ; \quad 0 \leq t \leq t_1 \quad (15)$$

and

$$y_2(t) = G_2 e^{-K_2 t} + \alpha \quad ; \quad t \geq t_1 \quad (16)$$

Imposing the conditions of no penetration at $t=0$, and the continuity of penetration at $t=t_1$, one has

$$G_1 = 0 \quad (17)$$

$$G_2 = (k_1 y_e / k_2) e^{K_2 t_1} \quad (18)$$

where $y_e = y_1(t_1)$.

Velocity of Penetration:

From Equations (8) and (9), in this case, one obtains

$$dy_1/dt = \frac{2g}{\rho_c} \sigma_i(t) - K_1 y_1$$

$$dy_2/dt = K_2 (\alpha - y_2).$$

Substituting (17) and (18) into Equations (15) and (16) and the result into the last two equations, one obtains expressions for velocity of the drill bit penetration into the

rock. The results are

$$dy_1/dt = \frac{2g}{\rho_c} \left[\sigma_i(t) - K_1 e^{-K_1 t} \int_0^t e^{K_1 x} \sigma_i(x) dx \right] \quad ; \quad 0 \leq t \leq t_1 \quad (19)$$

$$dy_2/dt = -(K_1 y_e) e^{-K_2(t-t_1)} \quad ; \quad t \geq t_1 \quad (20).$$

Reflected Stress Waves

From Eqn. (4), for $v_0=0$, one gets

$$\sigma_r(t) = \sigma_i(t) - \left(\frac{\rho_c}{g} \right) \dot{y}$$

Substituting expressions for velocity into the above one obtains

$$\sigma_r(t) = \begin{cases} -\sigma_i(t) + 2K_1 e^{-K_1 t} \int_0^t e^{K_1 x} \sigma_i(x) dx & ; \quad 0 \leq t \leq t_1 \\ (k_1 y_e / A) e^{-K_2(t-t_1)} & ; \quad t \geq t_1 \end{cases} \quad (21)$$

3.3.2 Rectangular Incident Stress Wave

When two elastic-homogeneous rods of the same cross section and material impact, a stress wave is generated. If the contact surfaces are plane, as they usually are in sonic drilling, the stress wave is theoretically of rectangular shape, propagating at a velocity equal to $(E/\rho)^{1/2}$ where E is the modulus of elasticity and ρ is the mass density of the rod. The length of this wave is twice the time required for the wave to travel the length of the impacting bar with this velocity. Such a wave can be mathematically expressed by

$$\sigma_i(t) = \begin{cases} \sigma_0 & ; 0 \leq t \leq t_1 \\ 0 & ; t \geq t_1 \end{cases}$$

In the following drill penetration, velocity, reflected wave, and contact force corresponding to a rectangular incident pulse will be derived.

Penetration

Substituting Equations (17), (18), and (22) into Eqns. (15) and (16), one obtains

$$y_1(t) = 2A\sigma_0(1 - e^{-K_1 t})/k_1 \quad ; 0 \leq t \leq t_1 \quad (23)$$

$$y_2(t) = (k_{ye}/k_2) e^{-K_2(t-t_1)} + \alpha \quad ; t \geq t_1 \quad (24)$$

Penetration Velocity

Differentiation yields

$$dy_1/dt = \left(\frac{2g\sigma_0}{\rho c}\right) e^{-K_1 t} \quad ; 0 \leq t \leq t_1 \quad (25)$$

$$dy_2/dt = -(k_{ye}) e^{-K_2(t-t_1)} \quad ; t \geq t_1 \quad (26)$$

Reflected Stress Wave

From Equations (20) and (21) one obtains

$$\sigma_r(t) = \begin{cases} \sigma_0(1 - 2e^{-K_1 t}) & ; 0 \leq t \leq t_1 \end{cases} \quad (27)$$

$$\begin{cases} (k_{ye}/A) e^{-K_2(t-t_1)} & ; t \geq t_1 \end{cases} \quad (28)$$

Contact Force at Rod-Rock Interface

To find the contact force one can either substitute Equations (23) and (24) into Eqn. (16) or use Eqns. (27) and (28) in conjunction with the boundary condition as given by Eqn. (3). However, the result is the same and as follows.

$$F = \begin{cases} 2A\sigma_0(1-e^{-K_1 t}) & ; 0 \leq t \leq t_1 \quad (29) \\ (k_y e) e^{-K_2(t-t_1)} & ; t \geq t_1 \quad (30) \end{cases}$$

In general there is a discontinuity in the reflected wave and penetration velocity at $t=t_1$.

Fig.35-Some possible shapes of the reflected wave due to a rectangular incident wave depending on k_1 and k_2 .

3.3.3 Sinusoidal Incident Stress Wave

When a bar of hemispherical end impacts a bar of flat end the contact surface is very small and the impact generates a stress wave of parabolic shape which can be approximated by a sine function. Let

$$\sigma_i(t) = \begin{cases} \sigma_0 \sin wt & ; 0 \leq t \leq t_1 \\ 0 & ; t \geq t_1 \end{cases}$$

where $wt_1 = \pi$.

Again assuming no initial velocity and no static force, from Equation (9) one gets

$$f(t) = \left(\frac{2g\sigma_0}{\rho c} \right) \sin wt$$

Substituting this into Equations (13) and (14) one obtains

$$y_1(t) = E_1 e^{-K_1 t} + \frac{2g\sigma_0}{\rho c(K_1^2 + w^2)} (K_1 \sin wt - w \cos wt)$$

$$y_2(t) = E_2 e^{-K_2 t} + \alpha$$

where E_1 and E_2 are constants. Applying the conditions of zero penetration at $t=0$, one gets

$$E_1 = \frac{2g\sigma_0 w}{\rho c(K_1^2 + w^2)} \quad (31)$$

Another condition is continuity of penetration at $t=t_1$, i.e., $y_1(t_1) = y_2(t_1)$, which yields

$$E_2 = (k_1 y_e / k_2) e^{K_2 t_1} \quad (32)$$

where in this case

$$y_e = \frac{2g\sigma_0 w}{\rho c(K_1^2 + w^2)} (1 + e^{-K_1 t_1})$$

Hence one obtains the drill penetration into the rock as follows

$$y_1(t) = \frac{2g\sigma_0}{\rho c(K_1^2 + w^2)} (w e^{-K_1 t} + K_1 \sin wt - w \cos wt); 0 \leq t \leq t_1 \quad (33)$$

and

$$y_2(t) = (k_1 y_e / k_2) e^{-K_2 (t - t_1)} + \alpha; t \geq t_1 \quad (34)$$

Differentiating these equations one gets the velocities;

$$\dot{y}_1(t) = \frac{2g\sigma_0 w}{\rho c(K_1^2 + w^2)} [K_1 (\cos wt - e^{-K_1 t}) + w \sin wt] \quad (35)$$

and

$$\dot{y}_2(t) = -(K_2 y_e) e^{-K_2 (t - t_1)} \quad (36)$$

Substituting these expressions into Eqn. (4), one obtains reflected stress waves corresponding to a sinusoidal stress wave. The results are given below:

$$\sigma_r(t) = \begin{cases} \left(\frac{\sigma_0}{K_1^2 + w^2} \right) \left[(K_1^2 + w^2) \sin wt - 2K_1 w (\cos wt - e^{-K_1 t}) \right] & ; 0 \leq t \leq t_1 \quad (37) \\ (k_1 y_e / A) e^{-K_2 (t - t_1)} & ; t \geq t_1 \quad (38) \end{cases}$$

where $wt_1 = \pi$.

3.3.4 Trapezoidal Incident Stress Wave

It is very difficult, if not impossible, to obtain perfectly rectangular stress waves experimentally. The pulses experimentally obtained are more trapezoidal due to necessity of a short time for the stress wave to reach it's maximum value and requirement of a rather longer time for the pulse to die out and attain zero value.

In the present analysis an attempt will be made to obtain a more realistic solution of the drill penetration, contact force, and reflected stress waves by assuming a trapezoidal incident stress wave instead of a rectangular one.

Let

$$\sigma_i(t) = \begin{cases} (\sigma_0 / t_0) t & ; 0 \leq t \leq t_0 \\ \sigma_0 & ; t_0 \leq t \leq (t_0 + t_1) \\ (\sigma_0 / t_f) (T - t) & ; (t_0 + t_1) \leq t \leq T \end{cases} \quad (39)$$

where

$$T = t_0 + t_1 + t_f$$

is the total duration of the pulse. This is the mathematical representation of the pulse shown in Fig. 36,

Fig.36 A trapezoidal incident stress wave with notations used.

where t_0 and t_f are the rising and falling times respectively. Substituting Eqn. (39) into Eqn. (15), one obtains

$$y_1(t) = \begin{cases} y_{11} = A_1 e^{-K_1 t + (\frac{2A\sigma_0}{k_1 t_0})(t-1/K_1)} & ; 0 \leq t \leq t_0 \\ y_{12} = A_2 e^{-K_1 t + (2A\sigma_0/k_1)} & ; t_0 \leq t \leq (t_0 + t_1) \\ y_{13} = A_3 e^{-K_1 t + (\frac{2A\sigma_0}{k_1 t_f})(T-t+1/K_1)} & ; (t_0 + t_1) \leq t \leq T \end{cases}$$

and

$$y_2(t) = (k_1 y_e / k_2) e^{-K_2(t-T)} + (1 - k_1/k_2) y_e$$

where

$$y_e = y_{13}(T).$$

To find A_1, A_2 , and A_3 , one has the following conditions:

$$\begin{aligned} y_{11}(0) &= 0, \\ y_{11}(t_0) &= y_{12}(t_0), \end{aligned}$$

and

$$y_{12}(t_0 + t_1) = y_{13}(t_0 + t_1).$$

These conditions, when imposed, yield the following equations for penetration corresponding to a trapezoidal incident stress wave:

$$y_1(t) = \begin{cases} \left(\frac{2A\sigma_0}{k_1 t_0} \right) \left[\frac{e^{-K_1 t}}{K_1} + (t-1/K_1) \right] & ; 0 \leq t \leq t_0 \\ \left(\frac{2A\sigma_0}{k_1 t_0} \right) \left[\frac{1}{K_1} (1 - e^{K_1 t_0}) e^{-K_1 t + t_0} \right] & ; t_0 \leq t \leq (t_0 + t_1), (40) \\ \left(\frac{2A\sigma_0}{k_1 t_0} \right) \left\{ \frac{1}{K_1} \left[1 - e^{K_1 t_0} \left(1 + \frac{t_0}{t_f} e^{K_1 t_1} \right) \right] e^{-K_1 t} + \frac{t_0}{t_f} \left[(T-t) + 1/K_1 \right] \right\} & ; t_0 + t_1 \leq t \leq T \end{cases}$$

and

$$y_2(t) = (k_1 y_e / k_2) e^{-K_2(t-T)} + (1 - k_1/k_2) y_e. \quad ; t \geq T.$$

Proceeding in exactly the same manner as already described, i.e., by differentiation and use of Eqn. (4), one obtains

the following expressions for the velocity and reflected wave;

$$y_1 = \begin{cases} \left(\frac{2A\sigma_0}{k_1 t_0}\right) (1 - e^{-K_1 t}) & ; 0 \leq t \leq t_0 \\ \left(\frac{2A\sigma_0}{k_1 t_0}\right) (e^{K_1 t_0} - 1) e^{-K_1 t} & ; t_0 \leq t \leq t_0 + t_1 \\ \left(\frac{2A\sigma_0}{k_1 t_0}\right) \left\{ \left[e^{K_1 t_0} \left(1 + \frac{t_0}{t_f}\right) e^{K_1 t_1} - 1 \right] e^{-K_1 t - t_0/t_f} \right\} & ; t_0 + t_1 \leq t \leq T \end{cases} \quad (41)$$

and

$$y_2 = -(K_1 y_e) e^{-K_2(t-T)} \quad ; t \geq T$$

$$\sigma_r(t) = \begin{cases} (\sigma_r)_{11} = \sigma_0(t/t_0) - 2\sigma_0(1 - e^{-K_1 t})/(K_1 t_0) & ; 0 \leq t \leq t_0 \\ (\sigma_r)_{12} = \sigma_0 - 2\sigma_0[e^{K_1 t_0} - 1]e^{-K_1 t}/(K_1 t_0) & ; t_0 \leq t \leq t_0 + t_1 \\ (\sigma_r)_{13} = \sigma_0(T-t)/t_f - \frac{2\sigma_0}{K_1 t_0} \left\{ \left[\left(1 + \frac{t_0}{t_f}\right) e^{K_1 t_1} \right] e^{K_1 t_0} - 1 \right\} e^{-K_1 t - t_0/t_f} & \\ (\sigma_r)_{14} = (k_1 y_e / A) e^{-K_2(t-T)} & ; t \geq T. \end{cases} \quad (42)$$

An examination of the reflected wave reveals that $(\sigma_r)_{11}$ and $(\sigma_r)_{12}$ are the same at $t=t_0$. Similarly $(\sigma_r)_{12}$ and $(\sigma_r)_{13}$ are equal at $t=t_0+t_1$. This is contrary to the case of a rectangular incident stress wave.

To avoid mathematical complications resulting from the above analysis, one may at least think of a logical way to approximate the imperfect experimentally-obtained rectangular pulses. One logical way seems to be the finding of a rectangular pulse of equivalent energy to that of an approximately trapezoidal stress wave.

Later on in this paper it will be shown that the energy of a rectangular wave of amplitude σ_0 and duration τ is

$$E_i = \left(\frac{2A\sigma_0^2}{\rho c} \right) \tau \quad (43)$$

To obtain the energy of a trapezoidal pulse one may make use

of

$$E_i = \int_0^{t_1} \left(\frac{EA}{\rho c} \right) [\sigma_i(t)]^2 dt$$

where t_1 is its duration. Substituting Eqn. (35) in the above and performing the integration, one gets

$$E_i = gA \sigma_0^2 [t_0/3 + t_1 + (T - t_0 - t_1)^3 / (3t_f^2)] / (\rho c) \quad (44)$$

Equating Eqns. (43) and (44), and making use of $t_f = T - t_0 - t_1$, one obtains

$$\tau = t_1 + (t_0 + t_f)/3 \quad (45)$$

which implies that on the basis of equality of energy, a trapezoid of $(t_0 + t_1 + t_f)$ duration should be approximated by a rectangular pulse of $t_1 + (t_0 + t_f)/3$. This approximation is schematically shown in the following figure.

Fig.37 Approximation of a trapezoidal wave by an equivalent rectangular pulse.

3.4 Energy Transmission Efficiency

The energy corresponding to an incident wave $\sigma_i(t)$, of duration t_1 is given by

$$E_i = (gA/\rho c) \int_0^{t_1} [\sigma_i(t)]^2 dt \quad (46)$$

Similarly, the energy of a reflected wave is given by

$$E_r = (gA/\rho c) \int_0^{t_2} [\sigma_r(t)]^2 dt \quad (47)$$

where t_1 is the duration of the reflected wave. Usually t_1 and t_2 are not equal, unless the transmission end is either free or fixed.

The energy transmitted to the rock, during a single rock-drill contact, is the difference between the incident energy and the reflected energy. Thus using equations (39) and (40), one obtains

$$E_t = (gA/\rho c) \left(\int_0^{t_1} [\sigma_i(t)]^2 dt - \int_0^{t_2} [\sigma_r(t)]^2 dt \right) \quad (48)$$

Hence, defining the efficiency as being the ratio of the transmitted energy, E_t , to the incident energy, E_i , and denoting this ratio by $\mathcal{E}$ one has

$$\mathcal{E} = 1 - \int_0^{t_2} [\sigma_r(t)]^2 dt / \int_0^{t_1} [\sigma_i(t)]^2 dt \quad (49)$$

There are two possible situations in calculating the efficiency of energy transmission into the rock, depending on whether the impact is a single contact process or a multi-contact process.

3.4.1 Single Contact Case

If the rod contacts the rock only once during each impact, Equation (49) can be very conveniently used to calculate

the transmission efficiency. Because after the first reflections, the reflected wave travels back and forth in the rod without any further energy being transferred to the rock.

3.4.2 Multiple Contact Case

It is possible that corresponding to each impact, the rod may contact the rock more than one time. Then computation of the efficiency of transmission is not as simple as already discussed, although Eqn. (49) still holds. This is because the reflected wave, after the first contact, travels in the rod, and upon reaching the rock boundary, acts as a new incident wave and its shape changes as it reflects for the second time. This process may be repeated as many as the number of contacts between the rock and the rod for a single impact. To calculate the transmission energy, one may assume that the ratio of the energy of the reflected wave to the energy of the incident wave remains unchanged. Call this ratio e . After the first contact

$$(E_r)_1 = eE_i \quad \text{i.e.} \quad (E_t)_1 = (1 - e)E_i$$

where the subscript 1 indicates the first contact. Now the energy reflected in the bar acts as a new incident wave for the second contact.

Thus

$$(E_r)_2 = e(E_i)_2 = e(E_r)_1 = e^2(E_i)_1$$

and

$$(E_t)_2 = (E_i)_2 - (E_r)_2 = e(1 - e)E_i$$

Obviously the total energy transmitted to the rock so far will be

$$(E_t)_1 + (E_t)_2 = (1 - e)(1 + e)E_i = (1 - e^2)E_i$$

To find the most general case of n contacts between the rock and rod, one realizes that this process is a geometric series, i.e.

$$E_t = (E_t)_1 + (E_t)_2 + \text{----} + (E_t)_n$$

or

$$E_t = [(1 - e) + e(1 - e) + e^2(1 - e) + \text{-----} + e^{(n-1)}(1 - e)] E_i$$

or

$$E_t = (1 - e) [1 + e + e^2 + \text{----} + e^{(n-1)}] E_i$$

Multiply this by e and subtract, the process yields

$$(1 - e)E_t = (1 - e)(1 - e^n)E_i$$

Hence, the expression for transmission efficiency will be

$$\mathcal{E} = \frac{E_t}{E_i} = (1 - e^n) \quad (50)$$

where n is the number of impacts.

3.4.3 - Optimization of Energy Transmission into Rock

It is of theoretical interest and practical importance to find the conditions for maximum transfer of energy into rock. One major factor seems to be duration of the incident stress wave. To ascertain significance of this factor and to avoid mathematical difficulties which may arise in calculating the energy transfer for any arbitrary stress pulses, only two cases of greater interest, rectangular and sinusoidal incident stress waves, will be considered. However, in case

of any arbitrary pulse the basic process involved in obtaining the pulse length, which makes maximum energy to be transferred, is exactly the same as given below.

Rectangular Incident Stress Wave

From Eqn. (46) the total energy of a rectangular pulse of σ_0 amplitude and t_1 duration is

$$E_i = \left(\frac{EA\sigma_0^2}{\rho c} \right) t_1 \quad (51)$$

To find the energy transmitted to the rock, one may calculate the area under the force-penetration, using Fig. (34), instead of using Eqn. (47). Thus the approximate value of the energy is

$$E_t = \frac{1}{2} k_1 y_e^2 - \frac{1}{2} k_2 (y_e - \alpha)^2 \quad (52)$$

where the first term represents the total energy initially transferred to the rock and the second term is the portion of the energy which is reflected back, due to elastic re-expansion of the rock. Also y_e , as already defined, is the maximum penetration and is given by

$$y_e = y_1(t_1).$$

Substituting for α , one obtains

$$E_t = \frac{1}{2} k_1 (1 - k_1/k_2) y_e^2 \quad (53)$$

The efficiency of transmission, in case of single contact, will be

$$\mathcal{E} = E_t/E_i = \frac{\rho c k_1 (1 - k_1/k_2)}{2gA\sigma_0^2} y_e^2 t^{-1}$$

Substituting Eqn. (23) into the above yields

$$\mathcal{E} = 2\rho c A (1/k_1 - 1/k_2) (1 - e^{-K_1 t_1})^2 / (gt). \quad (54)$$

Differentiation gives

$$\partial \mathcal{E} / \partial t_1 = \frac{2\rho c A (1/k_1 - 1/k_2) (1 - e^{-K_1 t_1})}{g} [2K_1 e^{-K_1 t_1} \cdot t_1 - (1 - e^{-K_1 t_1})] / t_1^2$$

To find the value of t_1 for which $\mathcal{E}$ is maximum, this derivative should vanish. Note that $t_1 \neq 0$ and $K_1 \neq 0$ and therefore $(1 - e^{-K_1 t_1}) \neq 0$. Thus, one obtains

$$2K_1 e^{-K_1 t_1} - (1 - e^{-K_1 t_1}) / t_1 = 0$$

which yields

$$1 + 2K_1 t_1 = e^{K_1 t_1} \quad (55)$$

This transcendental equation can be solved for t_1 . Note that $t_1 = 0$ is a trivial solution. The smallest non-zero root of this equation is found to be

$$t_1^* = 1.26 / K_1 \quad (56)$$

Hence, by direct substitution, one obtains the optimum efficiency

$$\mathcal{E}^* = .814(1 - K_1/K_2).$$

Sinusoidal Incident Stress Wave

Substituting $\sigma_i(t) = \sigma_0 \sin wt$ into Eqn. (53) and performing the integration yields

$$E_i = \frac{1}{2} \left(\frac{g \sigma_0^2}{\rho c} \right) t_1 \quad (57)$$

This is one half the energy of a rectangular pulse of the same duration and amplitude.

Substituting for y_e from Eqn. (33) into Eqn. (53), one obtains

$$E_t = 2 \left(\frac{g \sigma_0 w}{\rho c} \right)^2 k_1 (1 - k_1/k_2) (1 + e^{-K_1 t_1})^2 / (K_1^2 + w^2)^2 \quad (58)$$

Substituting for $w = \pi/t_1$ and then using Eqns. (57) and (58) yields,

$$\mathcal{E} = 4\pi^2 K_1 (1 - k_1/k_2) \cdot (1 + e^{-K_1 t_1})^2 t_1 / [(K_1 t_1)^2 + \pi^2]^2$$

By differentiation, one finds that the t_1 , which maximizes efficiency, should satisfy

$$(\pi^2 - 3K_1^2 t_1^2) e^{K_1 t_1} - (K_1 t_1)^2 (3 + 2K_1 t_1) + \pi^2 (1 - 2K_1 t_1) = 0.$$

This equation is solved by trial and error and the approximate root is found to be

$$t_1^* = 1.12/K_1 \quad (59)$$

which suggests that optimum sinusoidal incident pulses are relatively shorter than the rectangular pulses. Hence, one obtains the optimum efficiency

$$\mathcal{E}^* = .63(1 - k_1/k_2) \quad (60)$$

Comparison of the optimum efficiency corresponding to a rectangular incident pulse, with the above, proves that rectangular pulses are more efficient in a ratio of

$$(\mathcal{E}^*)_{\text{rect.}} / (\mathcal{E}^*)_{\text{sin.}} = .814 / .630 = 1.29.$$

3.5 Effects of Static Loads

Suppose that the transmission line or the drill bit is pressed against the surface of the rock by a force F_0 prior to the arrival of the incident wave and that this force remains unchanged during the process.

From Eqn. (9) one has

$$f(t) = \left(\frac{g}{\rho c}\right) [2\sigma_i(t) + F_0/A]$$

Substituting this into Eons. (11) and (12) one obtains the corresponding penetration. Thus,

$$y_1(t) = e^{-K_1 t} \left[G_1 + \frac{2g}{\rho c} \int_0^t e^{K_1 x} \sigma_i(x) dx + F_0 (e^{K_1 t} - 1)/k_1 \right]$$

and

$$y_2(t) = G_2 e^{-K_2 t} + \alpha + F_0/k_2$$

From continuity of penetration at $t=t_1$ and that $y_1(0) = F_0/k_1$, one gets

$$G_1 = F_0 / k_1$$

and

$$G_2 = \left[\frac{2g}{\rho c} e^{-K_1 t_1} \int_0^t e^{K_1 x} \sigma_i(x) dx + F_0 (1/k_1 - 1/k_2) - \alpha \right] e^{K_2 t_1} \quad (61)$$

3.5.1 Penetration

From the above analysis one obtains

$$y_1(t) = \left(\frac{2g}{\rho c} \right) e^{-K_1 t} \int_0^t e^{K_1 x} \sigma_i(x) dx + F_0 / k_1 \quad ; 0 \leq t \leq t_1 \quad (62)$$

$$y_2(t) = G_2 e^{-K_2 t} - F_0 / k_2 \quad ; t \geq t_1 \quad (63)$$

3.5.2 Penetration Velocity

Differentiation of the above equations yields,

$$\dot{y}_1 = \frac{2g}{\rho c} \left[\sigma_i(t) - K_1 e^{-K_1 t} \int_0^t e^{K_1 x} \sigma_i(x) dx \right] \quad ; 0 \leq t \leq t_1 \quad (64)$$

$$\dot{y}_2 = -K_2 G_2 e^{-K_2 t} \quad ; t \geq t_1 \quad (65)$$

3.5.3 Reflected Stress Waves

Substituting the above expressions into Eqn. (19), one gets

$$\sigma_r(t) = \begin{cases} -\sigma_i(t) + 2K_1 e^{-K_1 t} \int_0^t e^{K_1 x} \sigma_i(x) dx & ; 0 \leq t \leq t_1 \\ (k_2/A) G_2 e^{-K_2 t} & ; t \geq t_1 \end{cases} \quad (66)$$

$$(67)$$

IV-COMPARISON OF THEORY AND EXPERIMENT

To ascertain validity of the bilinear spring model adopted to represent dynamic behavior of rock under impact and theory founded on this approximation, one needs to compare a few things of interest such as penetration of the drill, contact force, the penetration rate (velocity), and energy transmission as predicted by the theory and as evaluated from experimental results. To be able to make such a comparison one needs to know the equivalent spring constants k_1 and k_2 for different rocks subjected to drills of various tip geometries. Since the relationships between rock material characteristics and tip geometries of drills and k_1 and k_2 are not yet established, their average -approximate values as calculated by approximating the experimentally found dynamic force-penetration relations by two lines, will be used. Thus in the following, Figures 10 through 14 will be reproduced and the foregoing approximations used. The approximate values of k_1 and k_2 are shown on the graphs as well as in the following Table. As mentioned earlier, they correspond to drills of flat and conical tips and rectangular incident pulses of amplitudes 39.0, 24.5, 7.8, and 5.9 ksi, referred to as high, medium-high, medium, and low stresses, respectively.

Table 4-Experimental values of k_1 and k_2 for limestone.

TEST	FLAT TIP		135° TIP		90° TIP	
	(k_1/A)	(k_2/A)	(k_1/A)	(k_2/A)	(k_1/A)	(k_2/A)
High stress	2.74	9.10	1.18	6.64	0.67	5.85
Med.-high "	2.89	7.21	1.20	5.56	0.45	4.74
Medium "	3.69	6.82	1.80	4.74	0.75	3.79
Low "	3.17	6.67	1.42	5.89	0.65	4.26
Ave. Values	3.12×10^6	7.45×10^6	1.40×10^6	5.71×10^6	0.63×10^6	4.66×10^6

Fig. 38- Approximate dynamic force-penetration relations for limestone subjected to various tips.

Fig.39 -Approximate dynamic force-penetration for limestone.

Using the notations $K_{1,2} = (gk_{1,2}) / (\rho c A)$, $\alpha = (1 - k_1/k_2)y_e$ and the above average values, one obtains

$$\begin{aligned}
 \text{(a) Flat tip: } & K_1 = .6738 \times 3.12 \times 10^4 = 2.10 \times 10^4 & 1/\text{sec.} \\
 & K_2 = .6738 \times 7.45 \times 10^4 = 5.02 \times 10^4 & 1/\text{sec.} \\
 & \alpha = .582y_e \\
 \text{(b) } 135^\circ \text{ tip: } & K_1 = .6738 \times 1.40 \times 10^4 = 0.94 \times 10^4 & 1/\text{sec.} \\
 & K_2 = .6738 \times 5.71 \times 10^4 = 3.85 \times 10^4 & 1/\text{sec.} \\
 & \alpha = .756y_e \\
 \text{(c) } 90^\circ \text{ tip: } & K_1 = .6738 \times 0.63 \times 10^4 = 0.42 \times 10^4 & 1/\text{sec.} \\
 & K_2 = .6738 \times 4.66 \times 10^4 = 3.14 \times 10^4 & 1/\text{sec.} \\
 & \alpha = .866y_e
 \end{aligned}$$

Now using these values in conjunction with the expressions developed already for drill penetration, drill velocity, and reflected stress waves corresponding to different incident pulses one will be able to calculate their theoretical values. In the following these functions are calculated and the results are tabulated. Also a comparison between theoretical and experimental results are presented graphically.

Considering a rectangular incident stress wave of amplitude σ_0 and duration of 100 microseconds, Equations (23) through (30) were used to calculate drill penetration, drill rate, reflected waves, and the contact force. The results are tabulated in Tables 5 and 6 and are plotted in Figures 40 through 43. These values are all normalized with respect to σ_0 for convenience and then are compared with their corresponding experimental average values as obtained in the series of experiments already referred to as low, medium, high-medium and high tests.

Table 5-Theoretical Values of Penetration and Velocity.*

time	Penetration Velocity/ σ_0			(Penetration)/ σ_0		
$\mu\text{sec.}$	$10^{-2}\text{in.}^3/(\text{\#-second})$			$(10^{-6}\text{ in.}^3/\text{\#})$		
	Flat	135°	90°	Flat	135°	90°
0	1.348	1.348	1.348	0	0	0
10	1.090	1.230	1.290	.122	.129	.130
20	.885	1.120	1.240	.220	.246	.257
30	.715	1.030	1.190	.300	.352	.374
40	.582	.924	1.140	.364	.448	.481
50	.471	.842	1.090	.416	.536	.602
60	.368	.765	1.050	.458	.616	.708
70	.310	.697	1.000	.493	.690	.810
80	.250	.636	.955	.521	.755	.908
90	.205	.576	.992	.554	.815	1.000
100	.163, -1.180	.525, -.820	.885, -.460	.562	.872	1.090
110	-.714	-.558	-.336	.470	.804	1.050
120	-.432	-.380	-.246	.411	.757	1.021
130	-.262	-.258	-.180	.379	.725	1.000
140	-.158	-.176	-.131	.358	.704	.984
150	-.096	-.120	-.096	.346	.690	.973
160	-.058	-.081	-.068	.340	.680	.966
170	-.035	-.056	-.051	.334	.673	.958
180	-.021	-.038	-.037	.331	.668	.954
190	-.011	-.025	-.027	.330	.665	.951
200	-.008	-.017	-.020	.329	.662	.958

* Rectangular incident stress wave.

Table 6-Theoretical values of reflected wave and contact F.*

time	(Reflected wave)/ σ_0			Contact force/($\Lambda\sigma_0$)		
μsec	non-dimensional			non-dimensional		
0	-1.000	-1.000	-1.000	0	0	0
10	-.662	-.820	-.918	.380	.181	.082
20	-.314	-.656	-.838	.686	.344	.162
30	-.064	-.508	-.764	.936	.492	.236
40	.136	-.372	-.690	1.135	.626	.302
50	.300	-.250	-.620	1.300	.750	.380
60	.432	-.138	-.554	1.430	.861	.446
70	.540	-.036	-.490	1.540	.965	.510
80	.628	-.056	-.428	1.628	1.055	.572
90	.698	.140	-.370	1.730	1.140	.630
100	.754, 1.754	.220, 1.220	-.314, .686	1.752	1.221	.686
110	1.061	.830	.501	1.061	.830	.501
120	.641	.565	.386	.641	.565	.386
130	.389	.384	.268	.389	.384	.268
140	.235	.261	.195	.235	.261	.195
150	.142	.178	.143	.142	.178	.143
160	.086	.121	.104	.086	.121	.104
170	.053	.083	.076	.053	.083	.076
180	.032	.056	.056	.032	.056	.056
190	.019	.038	.040	.019	.038	.040
200	.012	.026	.030	.026	.026	.030

* Rectangular incident stress wave.

Fig. 40- Comparison of average-normalized experimental and theoretical penetration due to a rectangular incident stress wave and various tips.

Fig. 41-Comparison of experimental and theoretical time variation of normalized velocity for various drills.

Fig.42-Comparison of experimental and theoretical time variation of the contact force for various tips.

Fig.43 -Comparison of experimental and theoretical normalized reflected waves, σ_r/σ_0 , corresponding to a rectangular pulse and various tip geometries.

Similarly, considering a sinusoidal incident stress wave of amplitude σ_0 and duration of 50 microseconds, which yields $w=\pi \times 10^6/50=2\pi \times 10^4$, Equations (37) and (38) were used to obtain the theoretical values of the reflected waves. Table 7 shows the results and Figure 44 exhibits their forms as compared to the average experimental results.

Table 7-Theoretical reflected waves corresponding to sinusoidal incident waves.

Time (μ -seconds)	Normalized reflected stress wave.*		
0	0	0	0
5	-.276	-.291	-.305
10	-.419	-.526	-.563
15	-.554	-.683	-.763
20	-.546	-.746	-.862
25	-.440	-.714	-.879
30	-.250	-.587	-.791
35		-.383	-.616
40	.277	-.119	-.367
50	.812	.476	.242
60	.491	.324	.178
70	.296	.230	.130
80	.180	.150	.095
90	.109	.102	.070
100	.066	.070	.051
120	.024	.032	.027

*Note that from left to right the first, second, and third columns correspond to flat, 135°, and 90° tips, respectively.

Fig.44-Comparison of experimental and theoretical reflected wave corresponding to sinusoidal incident stress wave.

4.1-Reasons for Discrepancies

The agreement between the theory and the average experimental results obtained seems to be satisfactory considering the simplicity of the model used in predicting such a complex material as rock. However, one can mention the following items as the major factors for discrepancies:

- (a) Under experimental conditions it is impossible to obtain perfectly rectangular or sinusoidal stress pulses, because it takes some time for the stress to build up from the condition of rest to some value σ_0 in the material. This is known as rise time. Similarly, some time is required for the stress σ_0 to decay and finally vanish in the material.
- (b) Another major source of error is in reading stresses from the pictures taken from the oscilloscope traces. This becomes more serious when magnified in calculating the drill velocity and reflected stress waves. Of the same category, one has to mention approximate graphical integration of velocity in calculating the experimental penetration.
- (c) The bilinear spring model introduced in this paper is a mathematical simplification for behavior of rock. It is noticed that although such a model may represent rocks for the major portion of the contact time, it differs during the time of transition i.e., when both the contact force and penetration are at their maximum values and on the verge of falling off. The following sketch represents this period of departure of the model from the real rock.
- (d) The rock fractures as the result of impacts. During this process the geometry of the contact surface certainly changes after each impact. This is partly due to fracture and removal of rock material and partly because of distortion which occurs in the tip geometry. The latter becomes more critical for very sharp tips. As a result the force

penetration constants K_1 and K_2 change which in turn affects all theoretical values.

Fig.45 Departure of the model from actual behavior of rock.

4.2-Trapezoidal vs. Rectangular Approximation of the Incident Stress Wave.

As mentioned earlier one of the sources of discrepancy of the theory from the experimental outcome seems to be imperfection of the incident pulse actually produced from the shapes assumed in calculations. To lessen this difference and at the same time to see its significance, one may approximate the actual wave by a trapezoid instead of a rectangle.

Observations indicate that the rise and fall times remain unchanged for incident pulses of different durations, and for the ranges considered in these experiments they are about 8 and 12 microseconds, respectively. These values may rather depend on the stress amplitude. Having these values let us find a trapezoidal pulse whose rectangular counterpart pulse of equivalent energy has a duration of 100 microseconds. Using Eqn. (45), one will have

$$100 = t_1 - (8+12)/3$$

Hence $t_1 = 93$ microsec.

and.

$$T = 93 + 8 + 12 = 113 \text{ microseconds.}$$

Fig.4b-Equivalent-Energy Waves.

Now let us assume that $k_1 = 2.10 \times 10^4$ and $k_2 = 5.02 \times 10^4$, which correspond to flat-tip drill and limestone. Substituting these and the above values into Eqn. (40) and (42) one obtains drill penetration and a reflected stress wave corresponding to a trapezoidal incident stress wave. In the following these values are plotted and compared with experimental results and those corresponding to a rectangular incident wave of 100 micro-seconds.

When the incident stress wave is approximated by a rectangular pulse, then the penetration suffers from an abrupt change in its slope at $t=t_1$. By approximateing the incident wave by a trapezoid this change is greatly reduced. Similarly the reflected stress wave improves by becoming a continuously increasing function at $t=t_1$. However, considering the increased mathematical complications, it does not seem to be worthwhile to use a trapezoidal approximation instead of a simple rectangular one. However, one conclusion from this analysis may be the fact that discontinuity of penetration and reflected stress wave at $t=t_1$, is not because of the bilinear-spring model for rocks but rather because of the

Fig.47-Comparison of the penetration due to rectangular and trapezoidal approximation of an incident stress wave and the average of the experimental results.

Fig.48-Reflected stress waves corresponding to rectangular and trapezoidal incident pulses of equivalent energy.

shape with which we approximate the incident stress wave.

4.3- Optimum Incident Stress Waves and Optimum Efficiency

Equation (56) is of much interest and significance. To make its importance clear again let's resort to numerical evaluations. In the following, the most favorable rectangular incident stress waves for drills of various geometries will be calculated.

(a) Flat Tip: ($k_1 = 2.10 \times 10^4 \text{ sec}^{-1}$)

Having $K_1 t_1 = 1.26$, one obtains

$$t_1^* = \frac{1.26}{2.10 \times 10^4} = 60 \times 10^{-6} \text{ sec.}$$

(b) 135° Tip: ($k_1 = .94 \times 10^4 \text{ sec}^{-1}$)

Hence

$$t_1^* = \frac{1.26}{.94 \times 10^4} = 134 \times 10^{-6} \text{ sec.}$$

(c) 90° Tip: ($k_1 = .42 \times 10^4 \text{ sec}^{-1}$)

Thus

$$t_1^* = \frac{1.26}{.42 \times 10^4} = 300 \times 10^{-6} \text{ sec.}$$

Note that these results imply that pulses of the same duration can not be effective for all tip geometries, furthermore, the length of the most favorable pulses are significantly longer for drills of sharper tip angles.

To find the optimum efficiency of energy transfer, $\mathcal{E}^*$, one needs to substitute $k_1 t_1$ into Equation (54) which yields

$$\mathcal{E}^* = .814(1 - k_1/k_2)$$

Substituting for k_1 and k_2 one obtains $\mathcal{E}^* = .474, .616$ and

0.703 for flat and conical drills of 135° and 90° , respectively, for the rock tested and rectangular incident stress waves of duration $t_1 = 1.26/K_1$. From this analysis one concludes that conical drills are more effective in transmitting the energy, and transmission efficiency further increases as the cone angle becomes smaller.

Substituting for y_e from Eqn. (23) into Eqn. (53) one obtains

$$E_t = 2(A\sigma_e)^2 (1/k_1 - 1/k_2) (1 - e^{-K_1 t_1})^2$$

which yields

$$E_t = .372 A \sigma_e^2 (1 - e^{-2.10 \times 10^4 t_1})^2 \times 10^{-6},$$

$$E_t = 1.080 A \sigma_e^2 (1 - e^{-.94 \times 10^4 t_1})^2 \times 10^{-6},$$

and

$$E_t = 2.750 A \sigma_e^2 (1 - e^{-.42 \times 10^4 t_1})^2 \times 10^{-6}$$

for flat, and conical drills of 135° and 90° cone angles, respectively. Here one can examine how the amount of energy transmitted and the required input energy vary as the length of the incident wave changes. Note that in this case, from Equation (51)

$$E_i = .006738 A \sigma_e^2 t_1.$$

Fig.49- Energy transmitted for various tips as the incident stress pulse duration varies. The input energy and the optimum pulses are also shown.

Fig. 50- Efficiency of energy transmission as the incident stress wave duration varies.

V - AN APPROXIMATE METHOD FOR DETERMINING THE DYNAMIC FORCE-PENETRATION SLOPES IN TERMS OF MATERIAL CHARACTERISTICS OF ROCK

Calculation of the penetration of a drill into rock, the reflected stress wave, and the efficiency of energy transmission can be very conveniently done by means of the expressions already developed in the last Section. However one needs the slopes of the dynamic force-penetration characteristic as well as incident wave data to be able to perform such calculations.

Unfortunately, due to uncertainties in the behavior of rocks and complexities of their structure, it is impossible to come up with a mathematical model or formulation which can represent rock behavior under all circumstances. With this realization, in this section the author attempts to find a basis on which generally and roughly most rocks are expected to behave.

Realizing that k_1 and k_2 represent two noticeably different phenomena of rock, one feels the necessity of resorting to two different methods for finding them. Since k_1 represents the slope of force-penetration, and because the penetration is mainly caused by fracture, fracture theories for rocks will be utilized in this regard. On the other hand k_2 characterizes the elastic behavior of rock, namely the fact that some portion of the total advancement of the drill will be compensated by elastic relaxation of the rock after the contact force is reduced. Thus for finding k_2 a logical and natural means would seem to be utilization of elasticity principles.

Ultrasonic drilling is generally done by flat tip drills. For this reason and for avoiding mathematical complexity in this section, all the developments will be based on flat tip

drills. However the method is applicable, with some modifications for sake of reducing complications in the algebra to other tips, such as conical drills.

5.1 Determining the Penetration Slope, k_1 , in terms of Rock Material Properties.

To achieve the objective of determining the penetration slope, k_1 , one needs to specify how the strengths of rocks are affected by the state of stress. Generally, an unconfined brittle rock will fracture when subjected to a certain stress, but under confinement the same rock will withstand a much greater stress before it fractures or deforms excessively. Of the various theories of failure that have been proposed for complex polycrystalline substances such as rock, only those that have been found on the basis of experiment to be reasonably valid for rock are considered.

5.1.1-Fracture Theories for Brittle Materials

(a) Griffith's Theory

Crystalline materials, such as most rocks, contain microfractures, and Griffith hypothesized that when such materials are subjected to loads, stress concentrations develop at the end of these cracks causing the crack to propagate and ultimately contribute to macroscopic failure. For a thin elastic strip of unit thickness containing an elliptical hole oriented with its long axis perpendicular to an applied tensile stress, the maximum stress, $\sigma_{\max}$, at the apex of the ellipse, depends on $\mathcal{R}$, the radius of curvature at the apex, and $2C$, the length of the crack. Inglis [9] determined this stress to be

$$\sigma_{\max} = 2\sigma_0 (C/\mathcal{R})^{\frac{1}{2}}$$

where σ_0 is the applied mean stress.

Griffith [1] computed the difference of energy in the strip with and without the hole to be

$$W_e = \pi \sigma_o^2 C^2 / E$$

The surface energy resulting from the formation of the crack is

$$W_s = 4CT$$

where T is the surface tension. Hence the elliptical hole has decreased the total energy by

$$W = W_e - W_s = \pi \sigma_o^2 \frac{C^2}{E} - 4CT$$

Instability will result and the crack will propagate if $\partial W / \partial C = 0$, that is, if the total energy becomes the maximum. Hence

$$\sigma_o = \left(\frac{2ET}{\pi C} \right)^{\frac{1}{2}} = T_o$$

where T_o is the tensile strength of the material. If in the two dimensional case, randomly distributed and oriented cracks occur throughout the body, the criterion for fracture is as follows:

If $\sigma_1 > \sigma_3$ and $\sigma_1 + 3\sigma_3 \leq 0$, i.e., compressive state of stress, fracture will occur when

$$(\sigma_1 - \sigma_3)^2 + 8T_o(\sigma_1 + \sigma_3) = 0 \quad (68)$$

where σ_1 is the greatest of the three principal stresses at a point and σ_3 is the smallest of the three. This is an approximation of the three dimensional state of stress by two dimensional fracture criterion, and the fact that the intermediate stress, σ_2 , plays no significant role in the fracture.

If $\sigma_1 > \sigma_3$ and $\sigma_1 + 3\sigma_3 > 0$, i.e. in tensile state of

stress, fracture will occur when

$$\sigma_1 = T_0$$

(b) McClintock-Walsh's Theory

McClintock and Walsh [23] modified Griffith's theory by assuming that in compression Griffith's cracks close and a frictional force develops across the crack surface. Thus their theory predicts that fracture initiates when the state of stress reaches

$$\mu(\sigma_1 + \sigma_3) + (1 + \mu^2)^{\frac{1}{2}}(\sigma_1 - \sigma_3) = 4T_0 \quad (69)$$

where μ is the coefficient of friction for the cracks. If the $\sigma_0 = -\sigma_3$ and $\sigma_1 = 0$ are the conditions for simple compression, the ratio of the compression to tensile strength is

$$\sigma_0/T_0 = 4 / [(1 + \mu^2)^{\frac{1}{2}} - \mu]$$

(c) Murrell's Theory

Murrell*, by analogy, generalized Griffith's theory to triaxial stress conditions, and proved that fracture occurs in a compressive region when the state of stress satisfies

$$(\sigma_1 - \sigma_3)^2 + (\sigma_3 - \sigma_2)^2 + (\sigma_2 - \sigma_1)^2 + \alpha(\sigma_1 + \sigma_2 + \sigma_3) = 0 \quad (70)$$

where α is a constant.

5.1.2 Stresses in a Half-Space Subjected to a Load Uniformly Distributed over a Circular Area

So far, we have established the criteria or the state of stresses by which a brittle material, such as rock, fractures. The next step would be to determine the stresses in the rock when subjected to a flat punch.

Sneddon [18] has solved the problem of a half-space subjected to a flat-circular-rigid punch. However, the stresses become unbounded at the boundary of the punch. This phenomena makes it difficult to use these stresses in any of

* See [3].

the criteria. To avoid this problem, one may assume that the load is distributed uniformly on the rock-drill interface. This^{is} equivalent to averaging the stresses due a flat punch over the area.

Experiments by Goldman [19] on glass indicate that holes drilled ultrasonically by flat drills are approximately cylindrical in shape. These holes are usually oversized at the top with respect to the diameter of the drill and the over-size depends on the penetration depth.

Experiments, by the author, on rock indicate noticeable differences in the shapes of the holes produced by impacts. The shape also varies with the type of rock. However, considering that the depth of such holes is very small, usually less than a milli-inch for a stress of about 40,000 psi, one may assume that such holes are approximately cylindrical.

We further assume that the ratio of punch radius to the smallest dimension of the rock is very small so that the variation of the stresses in the vicinity of the centerline of the punch, at any depth, but within the radius of the punch, is negligible.

Let r, θ , and y be the coordinates with y being an axis through the drill and running into the media in the positive direction.

Fig.51-System of coordinates.

Let $q=P/A$ where P is the force exerted on the rock and A is the area of the load.

This problem is solved by Timoshenko [20], and the stresses on the y -axis and at any depth are given as follows

$$\sigma_y = q \left[-1 + y^3 / (a^2 + y^2)^{3/2} \right] \quad (71)$$

$$\sigma_r = \sigma_\theta = q \left[-\frac{1}{2} (1+2\nu) + (1+\nu)y / (a^2 + y^2)^{\frac{1}{2}} - \frac{1}{2} y^3 / (a^2 + y^2)^{3/2} \right] \quad (72)$$

$$\tau_{ry} = 0, \quad \tau_{r\theta} = 0, \quad \text{and} \quad \tau_{y\theta} = 0 \quad (73)$$

One notices that all these stresses are principal stresses. Let

$$\begin{aligned} \sigma_1 &= \sigma_r, \\ \sigma_2 &= \sigma_\theta, \\ \text{and} \quad \sigma_3 &= \sigma_y. \end{aligned}$$

Hence

$$(\sigma_1 + \sigma_3) = q \left[-\frac{1}{2} (3+2\nu) + (1+\nu)y / (a^2 + y^2)^{\frac{1}{2}} + \frac{1}{2} y^3 / (a^2 + y^2)^{3/2} \right] \quad (74)$$

and

$$(\sigma_1 - \sigma_3) = q \left[\frac{1}{2} (1-2\nu) + (1+\nu)y / (a^2 + y^2)^{\frac{1}{2}} - \frac{3}{2} y^3 / (a^2 + y^2)^{3/2} \right] \quad (75)$$

5.1.3-Development of Linearized Force-Fracture Depth Relations.

Now that the approximate expressions for stresses are known, we substitute these relations into any of the fracture criteria and, after linearizing them, find a relation between the load and the depth at which fracture occurs. In addition, we will be able to find the minimum stress for which fracture initiates and the corresponding minimum load which causes fracture initiation.

(a) Griffith's Theory

Substituting Equations (74) and (75) into Griffith criterion for fracture one obtains

$$q \left[\frac{1}{2}(1-2v) + (1+v)y/(a^2+y^2)^{\frac{1}{2}} - \frac{3}{2} y^3/(a^2+y^2)^{3/2} \right]^2 + 8T_0 \left[-\frac{1}{2}(3+2v) + (1+v)y/(a^2+y^2)^{\frac{1}{2}} + \frac{1}{2}y^3/(a^2+y^2)^{3/2} \right] = 0 \quad (76)$$

where since $q \neq 0$ the expression is divided by q .

Since the depth of penetration is very small with respect to the radius of the drill, $(y/a)^2$ and higher order terms will be neglected. Thus solving for q one obtains

$$q(y) = \frac{32T_0(1+v)}{(1-2v)^2} \left[\frac{(3+2v)}{(2+2v)} - y/a \right] / \left[1 + \frac{4(1+v)}{(1-2v)} y/a \right]$$

Multiplying the denominator and numerator by the bracket in the denominator and then ignoring the terms containing $(y/a)^2$, one gets

$$q(y) = \frac{16T_0(3+2v)}{(1-2v)^2} - \frac{32T_0(7+2v)}{(1-2v)^3} (y/a) \quad (77)$$

This Equation now gives the depth of fracture for any given load, provided that the fracture depth is small relative to radius of the punch.

Since $q > 0$, one obtains, from the above Equation,

$$y < \frac{(3+2v)(1-2v)a}{2(7+2v)} \quad (78)$$

which is about 0.019 inches for $v=0.3$ and $a=3/16$ inches.

To find the minimum stress, q^* , which causes fracture to initiate, one can differentiate Eqn. (76) with respect to y and equate the result to zero. The outcome will be

$$\left[\frac{1}{2}(1-2v) + (1+v)z - \frac{3}{2} z^3 \right] \left[-(1+v) - \frac{1}{2}(1-2v)z^2 + \frac{3}{2}z^4 \right] = 2 \left[(1+v) - \frac{1}{2}(1+2v)z^2 + \frac{9}{2}z^4 \right] \left[\frac{1}{2}(3+2v) - (1+v)z - \frac{1}{2}z^3 \right]$$

where

$$z = (y/a)/(1+y^2/a^2)^{\frac{1}{2}} \quad (79)$$

Since (y/a) is very small, then certainly $z < 1$ and z^3 and higher order terms can be ignored. Thus, one obtains

$$\frac{(65+54v)}{4(1+v)} z^2 + (1+v)z - \frac{1}{2}(7+2v) = 0 \quad (80)$$

which can be easily solved for any rock with a given value of v . Discarding the negative root of this equation, because it corresponds to negative penetration, one obtains the following sample results,

v	z	(y/a)
0	.434	.481
$\frac{1}{4}$	.451	.505
$\frac{1}{2}$	.462	.521

Substituting these values into Eqn. (76), one can find the minimum value of q for which fracture occurs. The sample results are as follow,

v	(y/a)	q^*
0	.481	12.42 T_0
$\frac{1}{4}$	.505	19.18 T_0
$\frac{1}{2}$	.521	34.70 T_0

where T_0 is the uniaxial tensile strength of the rock considered.

(b) Murrell's Fracture Theory

Substituting $\sigma_2 = \sigma_3$ into the Murrell's fracture equation, one obtains

$$2(\sigma_1 - \sigma_3)^2 + \alpha(\sigma_1 + 2\sigma_3) = 0 \quad (81)$$

Substituting the expressions for σ_1 and σ_3 , one obtains

$$2q \left[\frac{1}{2}(1-2v) + (1+v)y/(a^2+y^2)^{\frac{1}{2}} - 1.5y^3/(a^2+y^2)^{3/2} \right]^2 + \alpha \left[-\frac{1}{2}(5+2v) + (1+v)y/(a^2+y^2)^{\frac{1}{2}} + 1.5y^3/(a^2+y^2)^{3/2} \right] = 0 \quad (82)$$

Now we proceed in exactly the same manner to obtain the linearized force-depth of fracture relation, the depth at which fracture initiates, and finally the minimum stress, q^* , which can cause fracture. The results are as follow,

$$q(y) = \frac{\alpha(5+2v)}{(1-2v)^2} - \frac{2\alpha(1+v)(11+2v)}{(1-2v)^3 a} y \quad (83)$$

Since $q > 0$ and $y > 0$, from the above one has

$$y < \frac{(5+2v)(1-2v)a}{2(1+v)(11+2v)} \quad (84)$$

which is about 0.014 inches for $v = .3$ and $a = 3/16$ inch as its maximum value.

From Equation (82), the y corresponding to minimum stress, q^* , will be obtained from

$$\frac{(103+20v+v^2)}{2(1+v)} z^2 + 2(1+v)z - (11+2v) = 0 \quad (85)$$

where z was previously defined.

Now one can solve this equation for any value of v . Some values of z are given below,

v	z	(y/a)
0	.443	.494
$\frac{1}{4}$	.487	.557
$\frac{1}{2}$	.526	.618

Again substituting these values into Eqn. (82), one finds the following minimum stress as required for fracture initiation,

v	(y/a)	q^*
0	.494	1.46α
$\frac{1}{4}$	.557	1.88α
$\frac{1}{2}$	.618	3.92α

To find α , let $\sigma_3 = -C_0$ and $\sigma_1 = 0$, which are the conditions for uniaxial compressive test, in Eqn. (81). The result is

$$2C_0^2 - \alpha C_0 = 0$$

which yields

$$\alpha = 2C_0 \quad (86)$$

where C_0 is uniaxial strength of the rock. Another way to find α is using Murrell's fracture criterion for tensile principal stresses which gives

$$\alpha = 12T_0$$

(c) McClintock-Walsh's Fracture Theory

Equation (70) can be rewritten as

$$\left[(1+\mu^2)^{\frac{1}{2}} + \mu\right] \sigma_1 - \left[(1+\mu^2)^{\frac{1}{2}} - \mu\right] \sigma_3 = 4T_0$$

For convenience let

$$A_1 = \left[(1+\mu^2)^{\frac{1}{2}} + \mu\right] \quad (87)$$

and

$$B = \left[(1+\mu^2)^{\frac{1}{2}} - \mu\right] \quad (88)$$

Substituting Eqns. (71) and (72) into the above criterion for fracture and then solving for q , one obtains

$$q(y) = 4T_0 / \left\{ \left[B - \frac{1}{2}A_1(1+2v) \right] + (1+v)A_1y / (a^2+y^2)^{\frac{1}{2}} - \left(\frac{1}{2}A_1 + B \right) y^3 / (a^2+y^2)^{3/2} \right\} \quad (89)$$

Ignoring $(y/a)^2$ for smallness, the result will be,

$$q(y) = 4T_0 / \left\{ A_1(1+v) \left[\frac{B/A_1 - \frac{1}{2}(1+2v)}{(1+v)} + y/a \right] \right\}$$

Multiplying this by the bracketed terms in the denominator, and then ignoring $(y/a)^2$, it yields

$$q(y) = \frac{4T_0}{A_1\beta(1+v)} - \frac{4T_0}{A_1\beta^2(1+v)a}y$$

where

$$\beta = \left[B/A_1 - \frac{1}{2}(1+2v) \right] / (1+v) \quad (90)$$

From Equation (89), one notes that the minimum load for fracture corresponds to the depth which maximizes the denominator. Differentiating the denominator one obtains

$$\left[A_1(1+v) - \left[\frac{1}{2}(5+2v) + 3B \right] y^2 / (a^2 + y^2) + 3 \left(\frac{1}{2}A_1 + B \right) y^4 / (a^2 + y^2)^2 \right] / (a^2 + y^2)^{\frac{1}{2}}$$

Equating this expression to zero, as to satisfy the requirement of maximization, and then multiplying by $(a^2 + y^2)^{\frac{1}{2}}$ and finally introducing

$$\bar{z} = y^2 / (a^2 + y^2), \quad (91)$$

one gets

$$A_1(1+v) - \left[\frac{1}{2}(5+2v)A_1 + 3B \right] \bar{z} + 3 \left(\frac{1}{2}A_1 + B \right) \bar{z}^2 = 0 \quad (92)$$

This equation can be easily solved for any given value of v , A_1 , and B with the aid of Eqns. (87), (88), and the fact that

$$C_0/T_0 = 4 / \left[(1 + \mu^2)^{\frac{1}{2}} - \mu \right] \quad (93)$$

where C_0 and T_0 are the uniaxial compressive and tensile strengths respectively. Substituting the result into Eqn. (89), one will be able to find the minimum load for which the rock fractures

Equation (93) can be rewritten as

$$(1 + \mu^2)^{\frac{1}{2}} - \mu = 4T_0/C_0 = B$$

Solving for μ one obtains

$$\mu = (C_0/8T_0 - 2T_0/C_0).$$

Combining these two expressions one gets

$$(1 + \mu^2)^{\frac{1}{2}} + \mu = C_0/(4T_0) = A_1$$

and

$$\beta = \left[\left(\frac{4T_0}{C_0} \right)^2 - \frac{1}{2}(1+2v) \right] / (1+v)$$

Substitution of these results into $q(y)$ yields the following

$$q(y) = C_0 \left[1 - \frac{1}{2}(1+2v) \left(\frac{C_0}{4T_0} \right)^2 \right]^{-1} - \frac{16T_0^2(1+v)}{C_0 a} \left[\left(\frac{4T_0}{C_0} \right)^2 - \frac{1}{2}(1+2v) \right]^{-2} y \quad (94)$$

5.2-Determination of Elastic Relaxation Slope, k_2 , in Terms of Rock Properties.

In the earlier development of the theory it was postulated that a drill bit penetrates into the rock as the result of elastic deformation and fracture of the rock. This process continues till the level of the energy or the state of the stresses falls short of the minimum necessary for any further fracture. Then the drill bit regresses as the rock relaxes elastically.

The elastic deflection of a half-space subjected to a force P through a circular flat end punch is given by Sneddon, [18] and is

$$P = \left[\frac{8\mu(\lambda + \mu)a}{(\lambda + 2\mu)} \right] \epsilon \quad ; \quad r \leq a \quad (95)$$

where λ and μ are Lamé's constants for the media and a is the radius of the punch. Assuming that the deflection-force relation remains the same during the cycle of application and removal of the load, one obtains, from the above, the slope of the force-deflection relation:

$$k_2 \equiv P/\epsilon = \frac{8\mu(\lambda + \mu)a}{(\lambda + 2\mu)} \quad (96)$$

where ϵ is the deflection of the media at the interface.

5.3-Summary

Thus far we have established various equations relating the applied load to the depth of fracture. In the following, general expressions for k_1 and k_2 are given. Then, with the knowledge that the Poisson's ratio for most rocks is $\frac{1}{4}$, (see Table 9), these expressions are simplified. Furthermore, the ranges of k_1 and k_2 for various types of rock will be given. In getting these values a is assumed to be $3/16$ " and the mechanical properties given in Table 9 are used.

Table 8-General and specific values of k_1 and k_2 for various types of rock.

	Theory	General	If $v=\frac{1}{4}$	mar- ble	L.S.	S.S.	gra- nite	Q.
$\frac{k_1}{A}$	Griffith's	$\frac{32T(7+2v)}{a(1-2v)^3}$	$1920T_0/a$	10- 30	7- 37	6- 37	10- 37	14- 44
	Murrell's	$\frac{4C(1+v)(11+2v)}{a(1-2v)^3}$	$460C_0/a$	35- 88	10- 88	7- 60	35- 88	53- 105
	McClintock -Walsh	$\frac{16T^2(1+v)/C_0}{a[(4T/C_0)^2 - \frac{1}{2}(1+2v)]^2}$	$\frac{20T_0^2/C_0}{a[(4T/C_0)^2 - \frac{3}{4}]^2}$	17- 59			17- 138	19- 131
$k_2/A:$		$\frac{8u(\lambda+u)}{\pi(\lambda+2u)a}$	$.7E/a$	28- 41	5- 43	3- 43	11- 32	21- 31

Note: L.S., S.S., and Q. stand for limestone, sandstone, and quartzite, respectively.

Also, all values of k_1/A and k_2/A are in $10^6 \#/\text{in}^3$

The above tabulated values each have a rather wide range of variation for any specific type of rock. This is, of course due to the fact that the mechanical properties of rocks, as reported in literature, have lower and upper bounds with the

gap being considerably wide. These gaps depend on the geographical location and the structural composition of the rock.

Approximating the experimentally-obtained force penetration relations shown in Figures 22 and 23, the following values of k_1/A and k_2/A are calculated for a few types of rock.

Rock	Lime- stone*	Concrete	Tenn. quartz	Sand- stone	Corkrock	Feather- stone
k_1/A	9.0	6.5	4.4	3.9	1.1	.6
k_2/A	26	21	9	9	5	3

All the above values are in $10^6 \text{ \#/in.}^3$

* This limestone specimen was different from the type already mentioned in this work.

5.3-A Theoretical Explanation for Experimental Observations On Rock Fracture.

In the last Section it was assumed that the contact force is approximately uniformly distributed over the contact area. This seemed of necessity in order to avoid mathematical complications. In this part, to make the theory more realistic, it will be assumed that the distribution of the contact force will be that of a rigid punch pressed against a half-space. However, to make the solution simple to the extent which the theory can be of any use, a rather simple theory of failure such as the maximum shear strength theory will be applied. Such a mathematical formulation seems to be more realistic, because observations, as mentioned earlier, indicate that when a rock is impacted by a flat end drill, fracture initiates in the rock at the outer boundary of the drill and propagates outward as well as inward simultaneously. This phenomenon resembles that of a flat punch in regard to high stress concentration at the boundary. Thus the contact area reduces, and continues to do so after each impact, till it is completely demolished. This leaves an irregular new contact area. Then later impacts remove these irregularities and create a rather flat contact surface. The whole process, it seems logical to induce, repeats itself.

Fig. 52- A cycle of annular fracture of rock under impacts.

5.3.1-Stresses in an Elastic Half-Space Subjected to a Rigid Punch.

I.N. Sneddon [18] has solved for stresses produced by a flat punch on a half-space. The stresses are,

$$\sigma_z = -(P/2\pi a)(J_1^0 + \xi J_2^0) \quad (97)$$

$$\tau_{rz} = -(P/2\pi a)\xi J_2^1 \quad (98)$$

$$\sigma_\theta = -(P/2\pi a)\left[\lambda J_1^0 + \frac{\mu}{\rho}(J_0^1 - \frac{\lambda+\mu}{\mu}\xi J_1^1)\right]/(\lambda+\mu) \quad (99)$$

and

$$(\sigma_r + \sigma_\theta) = -(P/2\pi a)\left[(2\lambda+\mu)J_1^0 - (\lambda+\mu)\xi J_2^0\right]/(\lambda+\mu) \quad (100)$$

where P is the applied load, a is the radius of the punch, λ and μ are Lamé's constants of the elastic media, and the coordinates r, θ , and z are as already defined. Also,

$$\rho = r/a$$

$$\xi = z/a$$

and

$$\begin{aligned} J_n^m &= \int_0^\infty p^{n-1} \sin(p\xi) e^{-p\rho} J_m(p\rho) dp \\ &= \text{Im.} \int_0^\infty p^{n-1} e^{-p(\xi-i)} J_m(p\rho) dp \end{aligned} \quad (101)$$

Since the shearing stress τ_{rz} , is not, in general, zero at any point, the principal stresses are no longer given by σ_r, σ_θ , and σ_z . The principal stresses at any point are now determined by roots of the determinant of

$$\begin{bmatrix} (\sigma_r - \sigma) & 0 & \tau_{rz} \\ 0 & (\sigma_\theta - \sigma) & 0 \\ \tau_{rz} & 0 & (\sigma_z - \sigma) \end{bmatrix}$$

when equated to zero. The results are as follow,

$$\sigma_1 = \sigma_\theta \quad (102)$$

$$\sigma_2 = \frac{1}{2}(\sigma_r + \sigma_z) + \left[\frac{1}{4}(\sigma_r - \sigma_z)^2 + \tau_{rz}^2 \right]^{\frac{1}{2}} \quad (103)$$

and

$$\sigma_3 = \frac{1}{2}(\sigma_r + \sigma_z) - \left[\frac{1}{4}(\sigma_r - \sigma_z)^2 + \tau_{rz}^2 \right]^{\frac{1}{2}}. \quad (104)$$

The maximum shear is then half of the difference between the maximum and minimum principal stresses.

The most natural step at this stage seems to be substitution for stresses in terms of the load, P , material constants, λ and μ , and coordinates r , θ , and z into the fracture criterion. The result will be an equation which represents a family of surfaces. However, although this is true, this operation will be modified for two reasons; the extremely complicated algebra may not serve any practical purpose, second, singularities exist at $r=a$ because, according to linear elasticity, stresses attain unbounded values at the edge of the punch.

To simplify the problem, one may divide the medium into three regions :

- (a) Boundary of the medium but excluding the drill contact area, i.e., $z=0$ and $r \geq a$.
- (b) Boundary of the medium but within the drill contact area, i.e., $z=0$ and $r \leq a$.
- (c) Interior of the elastic medium, i.e., $z \neq 0$ and $0 \leq r \leq \infty$

Fig. 53-Zones of stresses.

Now we find stresses in each region. For convenience, let us designate these regions by I, II, and III, respectively.

Stresses in region I:

Substituting $\xi=0$ and $\rho \geq 1$ into the expressions for stresses already developed, one obtains

$$\sigma_z = -(P/2\pi a) \bar{J}_1^0$$

$$\tau_{rz} = 0$$

$$\sigma_\theta = -(P/2\pi a) (\lambda \bar{J}_1^0 + \frac{\mu}{\rho} \bar{J}_0^1) / (\lambda + \mu)$$

$$(\sigma_r + \sigma_\theta) = - \frac{P(2\lambda + \mu)}{2\pi a(\lambda + \mu)} \bar{J}_1^0$$

where barred $\bar{J}_m^n$ refers to its value at $\xi=0$ and $r \geq a$.

Stresses in region II:

Substituting $\xi=0$ and $\rho \leq 1$ into general equations of stresses one obtains

$$\sigma_z = -(P/2\pi a) J_1^{*0}$$

$$\tau_{rz} = 0$$

$$\sigma_\theta = -(P/2\pi a) (\lambda J_1^{*0} + \frac{\mu}{\rho} J_0^{*1}) / (\lambda + \mu)$$

and

$$(\sigma_r + \sigma_\theta) = - \frac{P(2\lambda + \mu)}{2\pi a(\lambda + \mu)} J_1^{*0}$$

where * indicates the value of J_m^n at $\xi=0$ and $\rho \leq 1$.

The stresses in the interior of the medium are as given in their most general form.

Introducing the following notations are helpful in calculating the stresses.

$$R^2 = (\rho^2 + \xi^2 - 1)^2 + 4\xi^2 \quad (105)$$

$$\theta = \tan^{-1} (1/\xi) \quad (106)$$

$$\phi = \tan^{-1} [2\xi / (\rho^2 + \xi^2 - 1)], \quad (107)$$

and

$$r^2 = 1 + \xi^2.$$

By these substitutions Sneddon has found that

$$J_1^0 = R^{-\frac{1}{2}} \sin \frac{1}{2}\phi \quad (108)$$

$$J_0^1 = (1 - R^{\frac{1}{2}} \sin \frac{1}{2}\phi) / \rho \quad (109)$$

$$J_1^1 = r\rho^{-1} \sin(\theta - \frac{1}{2}\phi) \quad (110)$$

$$J_2^0 = rR^{-3/2} \sin(3\phi/2 - \theta) \quad (111)$$

and

$$J_2^1 = \rho R^{-3/2} \sin 3\phi/2. \quad (112)$$

Now substituting $\xi=0$ these yield

$$R^2 = (\rho^2 - 1)^2 \quad \text{or} \quad R = \begin{cases} (\rho^2 - 1); & \rho \geq 1 \\ (1 - \rho^2); & \rho < 1 \end{cases}$$

$$\theta = \pi/2$$

$$\phi = \begin{cases} 0 & ; \rho \geq 1 \\ \pi & ; \rho < 1 \end{cases}$$

Hence

$$J_1^0 = \begin{cases} 0 & ; \rho \geq 1 \\ (1 - \rho^2)^{-\frac{1}{2}} & ; \rho < 1 \end{cases}$$

and

$$J_0^1 = \begin{cases} 1/\rho & ; \rho \geq 1 \\ (1/\rho) [1 - (1 - \rho^2)^{\frac{1}{2}}] & ; \rho < 1 \end{cases}$$

Now one has values of $\bar{J}_m^n$ and J_m^{n*} to be able to calculate the stresses in regions I and II. The results are as follow,

Stresses in region I:

$$\sigma_z = 0 \quad (113)$$

$$\tau_{rz} = 0 \quad (114)$$

$$\sigma_\theta = -\frac{P\mu}{2\pi a(\lambda+\mu)\rho^2} \quad (115)$$

and

$$\sigma_r = \frac{P\mu}{2\pi a(\lambda+\mu)\rho^2} \quad (116)$$

Stresses in region II:

$$\sigma_z = -P / [2\pi a(1-\rho^2)^{\frac{1}{2}}] \quad (117)$$

$$\tau_{rz} = 0 \quad (118)$$

$$\sigma_\theta = -\frac{P}{2\pi a(\lambda+\mu)} \left(\frac{\lambda}{(1-\rho^2)^{\frac{1}{2}}} + \frac{\mu}{\rho^2} [1-(1-\rho^2)^{\frac{1}{2}}] \right) \quad (119)$$

and

$$(\sigma_r + \sigma_\theta) = -\frac{P(2\lambda+\mu)}{2\pi a(\lambda+\mu)(1-\rho^2)^{\frac{1}{2}}} \quad (120)$$

Now assuming that this theory can predict what is experimentally observed, i.e., that fracture of rock happens through a series of rings, the first ring being in the vicinity of the punch's edge, we try to determine the extents of the fracture surfaces.

Fig. 54-Inner and outer extents of a shallow fractured zone.

5.3.2- Fracture Surface

The maximum shear stress in Region I will be

$$(\text{Shear max.})_{I=\frac{1}{2}}(\sigma_r - \sigma_\theta) = P\mu / [2\pi a (\lambda + \mu) \rho^2].$$

Let S_0 be the maximum shear strength of the rock. Then for fracture to occur one has

$$P\mu / [2\pi a (\lambda + \mu) \rho^2] \geq S_0$$

which implies that

$$\rho^2 \leq P\mu / [2\pi a (\lambda + \mu) S_0].$$

Hence the outer-extent of fracture, r_1 , will be

$$r_1^2 = P\mu / [2\pi a (\lambda + \mu) S_0] \quad (121)$$

Given the shear strength of the rock, as well as its elastic constants and the radius of the punch, one can very easily find r_1 for any value of P . However, since $r_1 \geq a$ in region I, one can also get the minimum value of load, P^* , which is required for fracture. Thus

$$P^* = 2\pi a^2 (\lambda + \mu) S_0 / \mu \quad (122).$$

To find the inner extent of fracture we proceed in exactly the same fashion, except that we use the stresses corresponding to region II.

There are three possibilities for maximum shear stress in this case,

$$\begin{aligned} \text{(a)} \quad \tau_{\max} = \frac{1}{2} |(\sigma_z - \sigma_\theta)| &= \frac{P\mu}{4\pi a (\lambda + \mu)} \left(-(1-\rho^2)^{-\frac{1}{2}} + \rho^{-2} [1 - (1-\rho^2)^{\frac{1}{2}}] \right) \\ \text{(b)} \quad \tau_{\max} = \frac{1}{2} |(\sigma_r - \sigma_\theta)| &= \frac{P\mu}{4\pi a (\lambda + \mu)} \left((1-\rho^2)^{-\frac{1}{2}} - 2\rho^{-2} [1 - (1-\rho^2)^{\frac{1}{2}}] \right) \end{aligned}$$

$$(c) \tau_{\max} = \frac{1}{2} |(\sigma_r - \sigma_z)| = \frac{P\mu}{4\pi a(\lambda + \mu)\rho^2} [1 - (1 - \rho^2)^{\frac{1}{2}}]$$

The absolute value of one of these shear stresses must be the absolute maximum. To find it we see that (c) Δ (a), by inspection. To see whether (b) or (c) is the maximum, we use the binomial expansion, i.e.,

$$(1 - \rho^2)^{\frac{1}{2}} \approx 1 - \frac{1}{2}\rho^2$$

and

$$(1 - \rho^2)^{-\frac{1}{2}} \approx 1 + \frac{1}{2}\rho^2$$

where $\rho < 1$. Hence (b) gives

$$\left[(1 + \frac{1}{2}\rho^2) - 2\rho^{-2} [1 - (1 - \frac{1}{2}\rho^2)] \right] = \frac{1}{2}\rho^2$$

and

$$\rho^{-2} [1 - (1 - \frac{1}{2}\rho^2)] = -\frac{1}{2}$$

Hence, by comparison one finds that (c) is the absolute maximum. To satisfy the fracture criterion one will have,

$$P\mu [1 - (1 - \rho^2)^{\frac{1}{2}}] \geq 4\pi a(\lambda + \mu)\rho^2 S_0$$

Again using the binomial expansion, the above yields

$$\rho^2 \geq \frac{32\pi a(\lambda + \mu)S_0}{P\mu} - 4 \quad (123)$$

Note that increasing the force P , decreases ρ which is what is generally expected, i.e., movement of the fracture boundary towards the center of the punch as the load increases. Hence the "inner extent" of the fracture is given by

$$r_0^2 = \frac{32\pi a(\lambda + \mu)S_0}{P} - 4 \quad (124)$$

Thus, for any given applied load P and a rock of known material property constants, one will be easily able to determine the "inner and outer extents" of fracture surface.

To find the fracture surface for any value of the applied load and any type of rock is a difficult task. However there seems to be two ways, both approximate methods, to determine its general shape. One method is starting either from the outer or inner extents of fracture, at which $z=0$, and find $\sigma_r, \sigma_z, \tau_{rz}$, and σ_θ at

$$r_{00}=r_0+(r_1-r_0)/n$$

where n is any integer. Naturally the degree of accuracy depends on this number and increases as this number does so. Since r_{00} is a number, therefore $\sigma_z, \sigma_r, \tau_{rz}$, and σ_θ will be functions of the load, the rock's material constants, the punch's radius, and z . Using these expressions, find the corresponding principal stresses, σ_1, σ_2 , and σ_3 , for which general expressions are already given. Then find the maximum shear stress and use the maximum shear strength fracture theory, which will be

$$\begin{aligned} \tau_{\max} &= \tau_{\max}(z; \lambda; \mu; a; P) \geq S_0 \\ \tau_{\max} &= \tau_{\max}(z; \lambda; \mu; a; P) \geq S_0 \end{aligned} \quad (125)$$

where λ, μ, a , and P are parameters. Now solve this equation for its smallest root which is of interest in this type of problem(sonic drilling). Repeating this operation for each one of the following values,

$$r_{01}=r_{00}+(r_1-r_0)/n = r_0+2(r_1-r_0)/n$$

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$$r_{0(n-2)}=r_0+(n-1)(r_1-r_0)/n=r_1-(r_1-r_0)/n ,$$

one will be able to approximately plot the fracture surface.

One immediately realizes that such a task, the method already described, requires considerable amount of time and effort. To simplify this, one may resort to a more general and simpler method which can be of practical interest. Before doing so let us take a look at the material properties of some of the most often encountered rocks.

I.W. Farmer [21] quotes the following for a few rocks.

Table 9-Material properties of some rocks.

Rock Type	S_0 kg/cm ²	C_0 kg/cm ²	T_0 kg/cm ²	E 10 ⁵ kg/cm ²	ν
Granite	140-500	1000-2500	70-250	2-6	.25
Sandstone	80-400	200-1700	40-250	.5-8	.25
Limestone	100-500	300-2500	50-250	1-8	.25
Quartzite	200-600	1500-3000	100-300	4*-5.8*	
Marble	150-500	1000-2500	70-200	5.3-7.7	.25*

*See Krynine [22].

where S_0 , C_0 , T_0 , E , and ν are the shear, uniaxial compression, uniaxial tension, modulus of elasticity, and Poisson ratio, respectively.

From the above Table one sees that for most rocks Poisson's ratio is about .25. Using this value one gets

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)} = .4E$$

and

$$\mu = \frac{E}{2(1+\nu)} = .4E$$

Fortunately these values reduce the mathematical expressions of the stresses rather considerably. In the following table, the maximum shearing stresses corresponding to these values are evaluated for any load and punch radius. To do this the expressions already developed for stresses have been utilized.

Table 10- Variation of maximum shear stress with ρ and ξ .

$\rho \backslash \xi$	0	.2	.4	.6	.8	1.0	1.2	1.4	1.6	1.8	2.0	3.0	4.0	5.
0	.020	.028	.045	.061	.069	.070	.066	.060	.054	.048	.042	.023	.014	.010
.2	.020	.030	.048	.063	.071	.070	.066	.060	.054	.047	.042	.023	.014	.010
.4	.023	.036	.057	.070	.073	.071	.065	.059	.052	.046	.041	.025	.014	.010
.6	.030	.053	.074	.079	.077	.071	.064	.057	.050	.045	.040	.023	.014	.010
.8	.064	.095	.097	.097	.078	.070	.061	.054	.048	.042	.038	.022	.014	.009
1.0	∞	.116	.096	.076	.060	.053	.047	.041	.037	.033	.029	.017	.011	.007
1.2	.055	.070	.075	.066	.057	.049	.043	.039	.035	.031	.028	.016	.010	.007
1.4	.041	.025	.040	.043	.042	.041	.039	.036	.031	.029	.026	.016	.010	.006
1.6	.031	.014	.027	.035	.038	.039	.035	.031	.029	.026	.023	.015	.010	.007
1.8	.025	.011	.016	.024	.028	.029	.029	.027	.025	.023	.021	.014	.010	.007
2.0	.020	.009	.010	.017	.021	.023	.024	.023	.023	.021	.019	.013	.009	.006
3.0	.009	.006	.003	.002	.004	.005	.005	.006	.007	.007	.013	.007	.005	.004

*All values must be multiplied by (P/a^2) to obtain maximum shearing stress.

Fig. 55- Variation of maximum shearing stress, (normalized), with ρ and ξ .

The foregoing analysis enables one to construct the fracture envelopes in a rock subjected to a flat end punch and pressed together by a static force, P . However, one has to provide the necessary linkage in applying this analysis to the problem of rocks subjected to time-varying contact force, i.e., the existing problem is of dynamic nature whereas the developed analysis is of static character. Generally the contact force rapidly attains its maximum value and then decays with greater rapidity. Figure (20) is an example of such a force. Due to the impulsive nature of the loading, the whole contact time is a fraction of a second in most cases. It was observed experimentally that this time approximately varies between 2.2 to 2.5 times the duration of the incident stress waves. Due to the smallness of this contact time for short duration incident pulses, it may be assumed that the rock responds to the average of the applied load rather than responding to the instantaneous value of the force. This assumption is equivalent to the following mathematical approximation,

$$P = \frac{1}{T} \int_0^T F(t) dt, \quad (126)$$

where P is the average of the static contact force, $F(t)$ is the dynamic contact force, and T is the approximate contact time. The time variation of the dynamic contact time, $F(t)$, can be either calculated experimentally by means of the oscilloscope traces of the incident and reflected stress waves or by directly utilizing

various equation previously developed. The above equation is a linkage for applying the aforementioned static analysis to the problem of the approximate construction of the shallow fracture zones in rock.

5.3.3-Construction of fracture envelope.

Now one has all the necessary tools for constructing the approximate envelope of fracture for any rock for which the shear strength, S_0 , is known and its Poisson's ratio is .25. For example, take S_0 to be 300 kg/cm.² which is the average value given in Table 9 for limestone. This is equivalent to 4.3 k#/in.². Assume that this rock is subjected to a series of impacts with average contact stress, P/A , of 20,000 psi. Hence one obtains

$$S_0/(P/a^2) = (S_0/\pi)/(P/A) = (4.3/3.14)/20. = .0685$$

Corresponding to this value a horizontal line is drawn on Figure 55, which gives the following coordinates for fracture envelope.

ρ	.8	1.2	.6	1.2	.4	1.	0.	.8	.6	.4	0.
ξ	.038	.175	.350	.562	.575	.675	.775	1.	1.063	1.075	1.10

These coordinates represent four distinct zones of fracture which are plotted at the top of the following figure. These zones are designated by adc, cec', bdc, and bdfd'b'.

Fig. 56-Successive approximate zones of fracture as impacts are repeated.(Rock: limestone)

Of the four zones of fracture, it is experimentally observed that only the cross-hatched zones are actually separated from the rock. It is believed that the other two zones, although fractured, are pressed in their place due to the contact pressure and, since they are deeper, naturally fracture lines can not be observed, externally. Another reason may be the fact that only the cross-hatched zones have a free surface to escape.

After the first impact then, the contact area reduces, e.g., in this case the contact radius reduces from a to $0.8a$. To find the zones of fracture after the second impact, we base our calculation on the effective radius of the drill. The dotted lines with downward directions show the required scale modification after the first impact. Now starting as though with non-impacted rock, one can construct the new zones of fracture. This is shown in Figure 56. Repeating this process after each impact, one is able to construct the approximate fracture zone. This seems to be what actually happens in rock impacting.

One improvement in obtaining better results seems to be adjustment in the contact stress after each impact, because as the contact area reduces, the contact stress increases in the rock if the applied load is kept constant. For example, in the case mentioned above the contact stress increases by $1/(.8)^2 = 1.562$ and $1/(.8)^4 = 2.44$ for the second and fourth impacts, respectively.

A few general remarks are in order at this stage.

(1)- If the applied load is very small, then $S_0/(P/a^2)$ is large. Then the only zone of fracture is in the vicinity of the drill edge. This is a phenomenon exhibited in Figure (56).

(2)- As the load increases, $S_0/(P/a^2)$ decreases and

the fracture depth increases at the punch's edge. But the other fracture lines move upward towards the surface. This phenomenon seems to be in accord with the conclusions drawn in the last Section.

(3)- Figure 56 also exhibits another important feature. That is, as the load increases any further the shallow fracture envelopes move outward, $r > a$, and the deep fracture zones appear at $r = a$.

(4)- As mentioned previously, there are four fracture theories for brittle materials; Coulomb-Navier, Griffith's, McClintock-Walsh's, and Murrell's. The last three theories were used to establish relationships between the applied force and the resulting fracture depth. Their further use was avoided in this development because of mathematical complications, which evidently could result from the fact that stresses attain unbounded values at the punch edge. Thus, Coulomb-Navier failure theory was the only realistic theory to be applied in this development.

The maximum shear stress theory proposed by Coulomb postulates that failure occurs in a material when the shear stress at a point reaches the shear strength of the material. Navier modified this theory by assuming that the normal stress across the plane of failure increases the shear resistance by an amount proportional to the magnitude of the normal stress. However, realizing that the aim of the present analysis was to support the general pattern of failure of rock as observed experimentally, the Navier modification was also disregarded for the sake of generality rather than exactness.

CONCLUSIONS

- (1)- In general, a bilinear spring model seems to exhibit the rock behavior under impacts. Considering the complex behavior of rock, the experimental inaccuracies, and the mathematical errors, the agreement between the theory, founded on such a model, and the experiment is satisfactory.
- (2)- The major departure of the model from rock is at the transition time of the spring stiffness k_1 to k_2 . This phenomenon is evident in the theoretical values of penetration and its rate, the contact force, and reflected stress wave at the transition time, when compared with their experimental results at this time. However, due to the rapid decaying nature of these functions at $t > t_1$, the agreement between the theory and the experiment improves very rapidly.
- (3)- For the same rock, but drills of different tip geometry, it is experimentally verified that, in general, k_1 increases with increasing tip angle and this variation is quite noticeable. That is, the rock's resistance to penetration of a flat drill is much more than that for a sharp drill. On the other hand the value of k_2 seems to be rather insensitive towards tip angle variations.
- (4)- In order to transfer optimum energy to the rock, one significant factor is the incident pulse length. For rectangular incident stress waves this length is proved to be $t_1^* = 1.26/K_1$, and for sinusoidal incident pulses $t_1^* = 1.12/K_1$. Also it is proved that rectangular pulses are more effective in transmitting energy into rock. Furthermore, since k_1 is generally smaller for sharper tips, one can conclude that sharper tips

become even more efficient when subjected to longer duration pulses. The theory predicts that for the same rock an ideal pulse for a flat tip is 60 microseconds whereas for a 90° conical tip it must be 300 microseconds.

- (5)- It is also theoretically proved that sharper tip drills are more efficient and the efficiency increases as the apex angle decreases. But considering that such tips distort more rapidly and wear out faster to the point where their usefulness becomes questionable, there must be an economical limit to this angle.
- (6)- The analysis shows that approximating experimentally-obtained "rectangular" pulses by trapezoids does not improve the theoretical results considerably, provided that the rise and decay times of such a wave are small compared to its total length. When compared to simple approximation of such a pulse by a rectangular one, one realizes that the latter approximation is much simpler mathematically and agreement between the theory the experiments is good enough for practical purposes. However, solutions corresponding to trapezoidal approximation have continuity and smooth transition from one value to another at $t=t_1$.
- (7)- Observations on fracture of rock indicate that probably the mode or the mechanism of fracture depends on the structural composition and thus the strength of the rock. However, some similarities were observed between medium and low strength rocks. The resemblance being in annular fracture of rock when subjected to a flat-end drill. A theory based on linear elasticity solution and maximum shear strength failure criterion explains this phenomenon.

- (8)- The relations established between the force and the fracture depth are all indicative of the fact that fracture initiates within the media and propagates both upward and downward as the load increases. This is contrary to the common belief that fracture initiates on the surface of the media. Of course, this conclusion is based on the assumption that the applied load is uniformly distributed over the contact area. Removal of such a limitation or assumption may completely violate this finding. If the stresses corresponding to a rigid punch, which are unbounded at the punch edge, are used then fracture occurs for any value of the load, no matter how small it is. The fracture initiates at the surface of the media and in the vicinity of the punch edge. The latter result suffers from the shortcomings of linear elasticity; otherwise fracture can not happen for a very small applied force.
- (9)- In general the agreement between the theoretical and experimental values of K_1 and K_2 is not good. Also, the expressions relating rock material properties and spring stiffnesses are not claimed to be rigorous, realizing that such a task is impractical, if not impossible, for a material as complex and as unpredictable as rock. Nevertheless, the method used in obtaining these relations between the force and the fracture depth is new and may be improved in the future for better correlation between the theory and experiment.
- (10)- Of the three relations established between the force and the fracture depth, it is believed that the one based on McClintock-Walsh's theory of fracture gives much higher values of K_1 than experimentally evidenced. But those of Griffith's and Murrell's results

are more realistic values of K_1 .

- (11)- The theoretical values of K_2 seem to be more in agreement with the experimental results.
- (12)- Within the stress levels considered, the characteristics of the model, k_1 and k_2 , are independent of amplitude, shape, and duration of the incident stress wave. However, the maximum contact force and penetration vary directly with the amplitude of the incident pulse. Thus for such waves of different amplitudes one obtains a family of force-penetration relations shown below:

Fig.57-Effects of amplitude of the incident pulse on the force-penetration relations.

where f_1 and y_{e1} are the maximum contact force and maximum penetration corresponding to an incident stress wave of amplitude σ_0 , and f_2 , y_{e2} , f_3 , and y_{e3} are the maximum contact force and penetration corresponding to waves of $2\sigma_0$ and $3\sigma_0$ amplitude but the same duration, respectively.

It has to be mentioned that such a direct relationship does not exist between the incident pulse duration, t_1 , and the contact force or the penetration. Rather, such a relationship is of

exponential order. Thus in this case one will have

$$y_{e2} = y_{e1} (1 - e^{-2K_1 t_1}) / (1 - e^{-K_1 t_1}) = y_{e1} (1 + e^{-K_1 t_1})$$

and

$$f_2 = f_1 (1 + e^{-K_1 t_1})$$

where f_1 and y_{e1} are already defined and f_2 and y_{e2} are the maximum force and penetration corresponding to a wave of $2t_1$ duration but the same amplitude i.e., σ_0 and of rectangular shape. In the following an approximate family of force-penetration relations for incident stress waves of the same amplitude but different durations is shown:

Fig.58-Family of force-penetration relations for rectangular incident pulses of various lengths.

- (13)- As discussed earlier, the major sources of discrepancies between the theory and the experiments, other than that due to idealization of the rock behavior by the model, are imperfections of the experimentally-obtained incident stress waves from the mathematically-assumed shapes, human errors in measuring stresses from pictures of the oscilloscope traces, approximations in obtaining penetration from a time-velocity curve, and distortion of drill tip and rock contact surface during and after each impact.

- (14)- Within the experimental range considered, it must be emphasized that rate of loading effects or the visco-elastic properties of rock have not been included in the model, due to the fact the experimental results are presently inconclusive.

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