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NONLEPTONIC DECAYS OF HYPERONS
AND CURRENT ALGEBRA

DISSERTATION

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the Degree Doctor of Philosophy in the Graduate
School of the Ohio State University

By

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* * * * *

The Ohio State University
1968

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3. "Nonleptonic Decays of Hyperons and Current Algebra" (submitted to Physical Review for publication).

FIELDS OF STUDY

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INTRODUCTION

The subject of nonleptonic decays of hyperons¹ is an important, but the least understood aspect of weak interactions. The observed rates for these decays in general are quite high. Thus Λ decays into nucleon and pion with a branching ratio of almost a hundred percent. The characteristic hyperon mean lifetime is of the order of 10^{-10} second.² From angular momentum consideration there are two independent amplitudes for the decay: they are the S-wave amplitude which is parity violating (pv) and the P-wave amplitude which is parity conserving (pc).

The usual starting point for the discussion of non-leptonic decays is with the current-current interaction Hamiltonian.³ The part of the Hamiltonian which is responsible for our decays is,

$$H_W = \frac{G}{\sqrt{2}} \sin\theta \cos\theta \left[\{J_\lambda(\Delta S=0), J^{\lambda\dagger}(|\Delta S|=1)\}_+ + \text{h.c.} \right] \quad (\text{I-1})$$

¹Hyperons refer to Λ , Σ and Ξ particles.

²A. H. Rosenfeld et al., UCRL-8030 August 1966.

³M. Ademollo and R. Gatto, Phys. Rev. Letters 13, 264 (1964). For the development leading to Cabibbo Hamiltonian see Appendix B.

where the strangeness conserving current $J_\lambda (\Delta S=0)$ and the strangeness changing current $J_\lambda (|\Delta S|=1)$ both contain a vector and an axial vector part. There has been a good deal of study on these decays from symmetry considerations.⁴ It is evident from equation (I-1) and its transformation property in the unitary spin space that the only relevant contributions, in the symmetric product of two identical octets, come from the 8s and 27 irreducible representation of the group $SU(3)$.⁵ If one assumes that for some reason the 27 representation is suppressed compared to the 8s representation, then there are nine ways of making $SU(3)$ invariants from the four relevant octets, (the three physical particles plus H_W). Since there are more unknown $SU(3)$ amplitudes than the number of decays, no relations among the decay amplitudes can be obtained from the $SU(3)$ consideration alone without imposing other symmetry restrictions. Assuming octet dominance and the combined invariance of charge conjugation and parity (CP), the Lee-

⁴For a review of this, see for example, J. S. Bell, "Theory of Weak Interactions", High Energy Physics, Les Houches 1965, edited by C. DeWitt and M. Jacob (Gordon and Breach Science Publishers, 1965).

⁵For a collection of papers on the group $SU(3)$, see M. Gell-Mann and Y. Ne'eman, The Eightfold Way (W. A. Benjamin Inc., 1964).

Sugawara triangle relation⁶ for the parity violating amplitudes can be deduced. However the same relation for the parity conserving amplitudes could not be derived from similar consideration without imposing R or RP ⁷ invariance. Such R-invariance is not expected to be a valid symmetry for the strong interaction.

We now turn to a dynamical approach.⁸ This approach involves the use of current algebra and the hypothesis of partially conserved axial vector current (PCAC).⁹ The current algebra technique is well justified by its consequences, such

⁶B. W. Lee, Phys. Rev. Letters 12, 83 (1964); H. Sugawara, Prog. Theor. Phys. 31, 213 (1964); S. Coleman, S. Glashow and B. W. Lee, Ann. of Phys. 30, 348 (1964).

⁷R-invariance is a hypothetical symmetry in which the members of an octet are exchanged with their mirror images across the diagonal of the $SU(3)$ matrix.

⁸For a review of the bootstrap approach to nonleptonic hyperon decays see B. W. Lee, "Selected Topics in Unitarity Symmetry and Weak Interactions", High Energy Physics and Elementary Particles (International Atomic Energy Agency, Vienna, 1965); R. F. Dashen, S. C. Frautschi, and D. H. Sharp, Phys. Rev. Letters 13, 777 (1964); R. F. Dashen, Y. Dothan, S. C. Frautschi, and D. H. Sharp, Phys. Rev. 143, 1185 (1966); R. F. Dashen and S. C. Frautschi, Phys. Rev. 143, 1171 (1966); R. F. Dashen and S. C. Frautschi, Phys. Rev. 145, 1287 (1966). The results of these bootstrap calculations are remarkably similar to the current algebra results.

⁹M. Gell-Mann, Phys. Rev. 125, 1067 (1962); Physics 1, 63 (1964); Y. Nambu, Phys. Rev. Letters 4, 380 (1960); M. Gell-Mann and M. Levy, Nuovo Cimento 16, 705 (1960); J. Bernstein et al., Nuovo Cimento 16, 560 (1960); Nuovo Cimento 17, 757 (1960).

as the Goldberger-Treiman relation¹⁰ and the calculation of the renormalization effect of the axial vector coupling constant.¹¹ Gell-Mann made the suggestion of identifying the physical charges (the space integral of the time component of the physical hadronic current) with the generator of a symmetry group which is explicitly proposed to be $SU(3)$ for the vector current and $SU(3) \times SU(3)$ for the vector and axial vector currents. In so doing the equal time commutation relations between charge and currents are taken to generate the algebra of the corresponding group. The existence of the algebra is quite different from the hypothesis that the related group is a symmetry group. It is the commutation relations in the algebra which are assumed to be exact; they form the bases for the current algebra calculation.

The first papers written on the nonleptonic decays of hyperons using the technique of current algebra are only concerned with the S-wave amplitudes.¹² Following that there have been many attempts to extend the method to include the

¹⁰M. L. Goldberger and S. B. Treiman, Phys. Rev. 110, 1178 (1958); Phys. Rev. 111, 354 (1958).

¹¹S. L. Adler, Phys. Rev. Letters 14, 1051 (1965); W. I. Weisberger, Phys. Rev. Letters 14, 1047 (1965).

¹²M. Suzuki, Phys. Rev. Letters 15, 986 (1965); H. Sugawara, Phys. Rev. Letters 15, 870 (1965); 15, 997 (1965).

P-wave amplitudes.¹³ A typical calculation using current-current interaction was made by Brown and Sommerfield (this will be discussed in some detail in section III). It was found that within the framework of current-current interaction and current algebra the P-wave amplitudes were a factor of two to three smaller than the experimental values. One of the known problems has been that the extrapolation of the four momentum of the pion to zero for the P-wave amplitudes is not defined. Our work involves first developing a formalism which avoids such difficulty of extrapolation. The treatment is in analogy to that of the P-wave pion nucleon scattering lengths by Schnitzer.¹⁴ Based on this formalism two separate models are studied. The first model considers, in addition to the equal time commutator term and the Born term studied by Brown and Sommerfield, the contributions from the spin $3/2^+$ decuplet and the spin $1/2^-$ $Y^*(1405)$ intermediate states to the dispersion relation within the $SU(3)$ framework. The second model considers the additional contributions instead from the parity violating spurion matrix elements between two

¹³ L. S. Brown and C. M. Sommerfield, Phys. Rev. Letters 16, 751 (1966); Y. Hara, Y. Nambu, and J. Schechter, Phys. Rev. Letters 16, 380 (1966); S. Badier and C. Bouchiat, Phys. Letters 20, 529 (1966); Y. T. Chiu and J. Schechter, Phys. Rev. Letters 16, 1022 (1966); S. W. Biswas, Aditya Kumar and R. P. Saxena, Phys. Rev. Letters 17, 268 (1966); Y. T. Chiu, J. Schechter, and Y. Ueda, Phys. Rev. 150, 1201 (1967).

¹⁴ H. J. Schnitzer, Phys. Rev. 158, 1471 (1967).

baryon octets. These matrix elements would vanish in the exact $SU(3)$ limit.¹⁵ In this model they are included as unknown parameters. This approach is in the same spirit as Kumar and Pati¹⁶ and is one of $SU(3)$ breaking.

Our presentation follows the order given in the Table of Contents. In the Appendixes we try to supply most of the relevant information. The framework in which our theory develops pertains more to the formulation of L.S.Z.¹⁷ and the symmetry group $SU(3)$. In particular the vertices we are considering are always assumed to be $SU(3)$ symmetric.

It is found that within the framework of CP invariance, $SU(3)$ symmetry and dispersion relation, the first model can give a good fit to the experimental data with six $SU(3)$ reduced matrix elements. The additional contributions from the spin $3/2+$ decuplet and $Y_0^*(1405)$ contributes mainly to the S-wave amplitudes. The fit for the P-wave amplitudes is already very good with the octet baryon pole alone. With model 2, the result of the fit is within 20% which is roughly within the accuracy expected of current algebra calculation. The fit is not as good as model 1. The symmetry breaking

¹⁵M. Gell-Mann, Phys. Rev. Letters 12, 155 (1964).

¹⁶Arvind Kumar and J. G. Pati, Phys. Rev. Letters 18, 1230 (1967); G. S. Guralnik, V. S. Mathur and L. K. Pandit, University of Rochester preprint UR-875-216 (to be published in Physical Review); K. Nishijima and L. J. Swank, Phys. Rev. 146, 1161 (1966).

¹⁷H. Lehmann, K. Symanzik and W. Zimmermann, Nuovo Cimento 1, 1425 (1955); Nuovo Cimento 6, 319 (1957).

terms give about 10% correction. Admittedly the truth of the models would depend on the verification of the strength of the relevant spurion vertices.

GENERAL FORMULATION

Consider the fictitious scattering process denoted by the conservation equation $p_\alpha + k \rightarrow p_\beta + q$, where p_α , p_β , q and k are the four-momentum of the initial baryon, final baryon, pion and spurion respectively. The decay problem can be looked upon as the limiting case of such a scattering problem with possible parity violation, the weak Hamiltonian (H_w) being a spurion with two possible parities. We will work in the center of mass coordinate system. The Mandelstam variables are $s = (p_\alpha + k)^2$, $u = (p_\beta - k)^2$, $t = (p_\alpha - p_\beta)^2$ with $s + u + t = m_\alpha^2 + m_\beta^2 + q^2 + k^2$. Other useful kinematic relations are $s = W^2$,
 $|\vec{q}| = [(W + m_\beta)^2 - q^2]^{1/2} [(W - m_\beta)^2 - q^2]^{1/2} / 2W$ and
 $|\vec{k}| = [(W + m_\alpha)^2 - k^2]^{1/2} [(W - m_\alpha)^2 - k^2]^{1/2} / 2W$. We define the off mass-shell scattering amplitude for the process as,

$$T = (2\pi)^3 \sqrt{\frac{E_\alpha}{m_\alpha}} \sqrt{\frac{E_\beta}{m_\beta}} \frac{\mu^2 - q^2}{C_\pi} \int d^4x e^{iqx} \langle p_\beta | T \{ \gamma^\mu A_\mu^i(x), H_w(0) \} | p_\alpha \rangle \quad (\text{II-1})$$

where we have used the standard reduction technique and defined the pion field of isospin i by means of the

partially conserved axial current (PCAC) hypothesis,

$$\partial^\mu A_\mu^i(x) = C_\pi \phi_\pi^i(x) \quad (\text{II-2})$$

Equation (II-2) relates the pion field operator (ϕ_π^i) off the mass-shell to the divergence of the axial vector current with the constant C_π . From the basic identity,

$$T \left\{ \partial^\mu A_\mu^i(x), H_W(0) \right\} = \frac{\partial}{\partial x} T \left\{ A_\mu^i(x), H_W(0) \right\} - \delta(x_0) \left[A_0^i(x), H_W(0) \right]$$

we obtain

$$T = (2\pi)^3 \sqrt{\frac{E_\alpha}{m_\alpha}} \sqrt{\frac{E_\beta}{m_\beta}} \frac{\mu^2 - q^2}{C_\pi} \left\{ -iq^\mu R_\mu - \int d^4x e^{iqx} \delta(x_0) \langle p_\beta | [A_0^i(x), H_W(0)] | p_\alpha \rangle \right\} \quad (\text{II-3})$$

where

$$R_\mu = \int d^4x e^{iqx} \langle p_\beta | T \{ A_\mu^i(x), H_W(0) \} | p_\alpha \rangle \quad (\text{II-4})$$

The scattering amplitude T is now decomposed into invariant amplitudes,

$$T = \bar{u}(p_\beta) \left\{ A - \frac{1}{2} \gamma \cdot (q+k) B + i\gamma_5 C - \frac{1}{2} i\gamma \cdot (q+k) \gamma_5 D \right\} u(p_\alpha) \quad (\text{II-5})$$

(II-6)

In terms of Pauli spinor (χ) this can be expressed in the center of mass as

$$F = \chi^\dagger \left\{ f_1 + (\vec{\sigma} \cdot \hat{q})(\vec{\sigma} \cdot \hat{k}) f_2 + (\vec{\sigma} \cdot \hat{q}) f_3 + (\vec{\sigma} \cdot \hat{k}) f_4 \right\} \chi \quad (\text{II-7})$$

where

$$F = \frac{\sqrt{m_\alpha m_\beta}}{4\pi W} T \quad (\text{II-8})$$

The amplitudes f_1 , f_2 , f_3 and f_4 are given in terms of the invariant amplitudes by the relations,

$$f_1 = \frac{1}{16\pi W^2} \left[(W+m_\beta)^2 - q^2 \right]^{1/2} \left[(W+m_\alpha)^2 - k^2 \right]^{1/2} \left[A - \frac{2W-m_\alpha-m_\beta}{2} B \right] \quad (\text{II-9})$$

$$f_2 = \frac{1}{16\pi W^2} \left[(W-m_\beta)^2 - q^2 \right]^{1/2} \left[(W-m_\alpha)^2 - k^2 \right]^{1/2} \left[-A - \frac{2W+m_\alpha+m_\beta}{2} B \right] \quad (\text{II-10})$$

$$f_3 = \frac{1}{16\pi W^2} \left[(W-m_\beta)^2 - q^2 \right]^{1/2} \left[(W+m_\alpha)^2 - k^2 \right]^{1/2} \left[C + \frac{2W-m_\alpha+m_\beta}{2} D \right] \quad (\text{II-11})$$

$$f_4 = \frac{1}{16\pi W^2} \left[(W+m_\beta)^2 - q^2 \right]^{1/2} \left[(W-m_\alpha)^2 - k^2 \right]^{1/2} \left[-C + \frac{2W+m_\alpha-m_\beta}{2} D \right] \quad (\text{II-12})$$

Using the method of partial-wave decomposition¹⁸ for the parity conserving and parity violating amplitudes separately, we have obtained for the total partial-wave amplitude,

$$f_{\ell_{\pm}} = \frac{1}{2} \int_{-1}^{+1} dz \left[(f_1 + f_4) P_{\ell}(z) + (f_2 + f_3) P_{\ell \pm 1}(z) \right] \quad (\text{II-13})$$

where $\ell_{\pm}$ stands for the final states with orbital angular momentum ℓ and total angular momentum $j = \ell \pm 1$. For the parity conserving amplitude

$$\begin{aligned} f &= \sum (2\ell+1) (\vec{\sigma} \cdot \hat{q}) \left[f_{\ell+} \Lambda_{\ell+} + f_{\ell-} \Lambda_{\ell-} \right] P_{\ell}(\cos\theta) \\ &= (\vec{\sigma} \cdot \hat{q}) f_3 + (\vec{\sigma} \cdot \hat{k}) f_4 \end{aligned} \quad (\text{II-14})$$

while for parity violating amplitude

$$\begin{aligned} f &= \sum (2\ell+1) \left[f_{\ell+} \Lambda_{\ell+} + f_{\ell-} \Lambda_{\ell-} \right] P_{\ell}(\cos\theta) \\ &= f_1 + (\vec{\sigma} \cdot \hat{q})(\vec{\sigma} \cdot \hat{k}) f_2 \end{aligned} \quad (\text{II-15})$$

where $\Lambda_{\ell+}$ and $\Lambda_{\ell-}$ are the projection operators,

$$\frac{(\ell+1) + \vec{\sigma} \cdot \vec{L}}{2\ell+1} \quad \text{and} \quad \frac{\ell - \vec{\sigma} \cdot \vec{L}}{2\ell+1} . \quad \text{The above expressions can be}$$

inverted to give equation (II-13).

¹⁸See Appendix C.

From equations (II-9) to (II-13) the S-wave and P-wave amplitudes can be expressed in terms of the invariant amplitudes. The decay amplitude is supposed to be an analytic continuation of the scattering amplitude in its variables. It is obtained from the scattering amplitude by setting $k = 0$, $s = m_\alpha^2$, $u = m_\beta^2$ and $q^2 = \mu^2$ corresponding to the physical point of decay. The assumption of PCAC says that the invariant amplitudes will extrapolate smoothly to $q^2 = 0$ as justified by the Goldberger-Treiman relation. Thus the decay amplitude can be approximated by the fictitious scattering amplitude at $k = 0$, $s = m_\alpha^2$, $u = m_\beta^2$ and $q^2 = 0$ and hence $t = 0$,

$$f_{s\frac{1}{2}} = \frac{1}{8\pi m_\alpha} \left[(m_\alpha + m_\beta)^2 - \mu^2 \right]^{\frac{1}{2}} \left[A(s=m_\alpha^2, q^2=0, t=0) - \frac{m_\alpha - m_\beta}{2} B(s=m_\alpha^2, q^2=0, t=0) \right] \quad (\text{II-16})$$

$$f_{p\frac{1}{2}} = \frac{1}{8\pi m_\alpha} \left[(m_\alpha - m_\beta)^2 - \mu^2 \right]^{\frac{1}{2}} \left[C(s=m_\alpha^2, q^2=0, t=0) + \frac{m_\alpha + m_\beta}{2} D(s=m_\alpha^2, q^2=0, t=0) \right] \quad (\text{II-17})$$

The conventional S-wave and P-wave nonleptonic decay

amplitudes denoted by S and P as defined by the decay width formula,¹⁹

$$\Gamma = \frac{|\vec{q}| (E_\beta + m_\beta)}{4\pi m_\alpha} (|S|^2 + |P|^2) \quad (\text{II-18})$$

are related to $f_{s1/2}$ and $f_{p1/2}$ by a common factor

$$\left[(m_\alpha + m_\beta)^2 - \mu^2 \right]^{1/2} / 8\pi m_\alpha. \text{ They can be worked out easily}$$

from equations (II-8) and (II-18). With this correction we have

$$S = A(s=m_\alpha^2, q^2=0, t=0) - \frac{m_\alpha - m_\beta}{2} B(s=m_\alpha^2, q^2=0, t=0) \quad (\text{II-19})$$

$$P = K \left[C(s=m_\alpha^2, q^2=0, t=0) + \frac{m_\alpha + m_\beta}{2} D(s=m_\alpha^2, q^2=0, t=0) \right] \quad (\text{II-20})$$

where

$$K = \left[\frac{(m_\alpha - m_\beta)^2 - \mu^2}{(m_\alpha + m_\beta)^2 - \mu^2} \right]^{1/2} \quad (\text{II-21})$$

¹⁹See Appendix D.

THE CONVENTIONAL APPROACH RE-EXAMINED

III.1 Born Approximation

To recapitulate earlier work, the calculation of the P-wave amplitudes of Brown and Sommerfield was given by the Born approximation to the first term of equation (II-4). Charge conjugation invariance in the exact SU(3) limit implies that only the scalar part of H_W couples to the baryon octet, so that the Born approximation contributes only to the P-wave decays. For the process, $\alpha \rightarrow \beta \pi^1$ we are considering, the Born term is given by

$$\begin{aligned}
 B_{\beta 1 \alpha}(q) = & \sum_s \bar{u}(p_\beta) i\gamma_5 (\not{p}_\beta + \not{q} - m_s)^{-1} u(p_\alpha) g_{\beta s} S_{s\alpha} \\
 & + \sum_s \bar{u}(p_\beta) (\not{p}_\alpha - \not{q} - m_s)^{-1} i\gamma_5 u(p_\alpha) S_{\beta s} g_{s\alpha}
 \end{aligned}
 \tag{III-1}$$

where the structure of the scalar spurion vertex $S_{s\alpha}$ and the SU(3) octet Yukawa coupling constants $g_{\beta s}$ will be specified later. What is to be noticed from equation (III-1) is that the extrapolation of the pion momentum q to zero is not well defined. The contribution of the

intermediate state δ for $m_\delta \neq m_\beta$ is of the order $(m_\delta + m_\beta)^{-1}$ in the limit $q \rightarrow 0$, whereas for q at its physical value the contribution is of the order $(m_\delta - m_\beta)^{-1}$. Hence in extrapolating the pion momentum to its unphysical value $q=0$ there is a variation of the order $m/\Delta m$. The extrapolation of the pion momentum to zero for an intermediate state degenerate in mass with either the initial or the final baryon is also not permissible since there contains in the limit $q \rightarrow 0$ an undefined term $1/(p_\beta \cdot q)$.

What is usually done is to write the first term of equation (II-4), here denoted by $I(q)$, as

$$I(q) = \lim_{q \rightarrow 0} \left[I(q) - B_{\beta i \alpha}(q) \right] + B_{\beta i \alpha}(q) \quad (\text{III-2})$$

and then it can be shown that the combination of terms in bracket not only has a well defined limit for $q \rightarrow 0$ but also such contribution is negligible so that $I(q)$ is approximated by $B_{\beta i \alpha}(q)$. Such has been the conventional approach and the P-wave amplitudes were assumed to be given solely by the Born term, $B_{\beta i \alpha}(q)$.

Before turning to our method let us examine, for completeness, the combination of terms in equation (III-2). The degenerate Born term is given from equation (III-1) by

$$\begin{aligned}
B_{\beta 1\alpha}(q) = & \sum_{\delta} \bar{u}(p_{\beta}) \frac{i\gamma_5(-m_{\beta}+m_{\delta}+1)}{q^2+2p_{\beta}\cdot q+m_{\beta}^2-m_{\delta}^2} u(p_{\alpha}) g_{\beta\delta} S_{\delta\alpha} \\
& + \sum_{\gamma} \bar{u}(p_{\beta}) \frac{(-m_{\alpha}+m_{\gamma}-1)i\gamma_5}{q^2-2p_{\alpha}\cdot q+m_{\alpha}^2-m_{\gamma}^2} u(p_{\alpha}) S_{\beta\gamma} g_{\gamma\alpha} \quad (\text{III-3})
\end{aligned}$$

The same degenerate baryon pole terms lead to singularities in $I(q)$.

$$\begin{aligned}
\lim_{q \rightarrow 0} I(q) = & \bar{u}(p_{\beta}) \left\{ -\frac{1}{2m_{\beta}} i\gamma_5 \frac{\gamma \cdot p_{\beta} + m_{\beta}}{2p_{\beta} \cdot q} g_{\beta\delta} S_{\delta\alpha} \right. \\
& \left. + \frac{\gamma \cdot p_{\alpha} + m_{\alpha}}{2p_{\alpha} \cdot q} i\gamma_5 \frac{1}{2m_{\alpha}} S_{\beta\gamma} g_{\gamma\alpha} \right\} u(p_{\alpha}) \quad (\text{III-4})
\end{aligned}$$

Combining equations (III-3) and (III-4) we have

$$\lim_{q \rightarrow 0} [I(q) - B_{\beta 1\alpha}(q)] = \bar{u}(p_{\beta}) \left\{ \frac{1}{2m_{\beta}} i\gamma_5 g_{\beta\delta} S_{\delta\alpha} + \frac{1}{2m_{\alpha}} i\gamma_5 S_{\beta\gamma} g_{\gamma\alpha} \right\} u(p_{\alpha}) \quad (\text{III-5})$$

This contribution is indeed small compared with the Born term at its physical pion momentum which is of the order $1/\Delta m$ as mentioned. For this reason this combination of the terms has been neglected in most of the previous treatments.

Our approach depends on finding an accurate determination of the various contributions to the invariant amplitudes defined in equation (II-6) in the decay limit. To study the octet baryon pole contribution to the P-wave amplitudes by our method, the spurion matrix element between two baryon octets can be defined as follow,

$$\langle \delta, p_\delta | H_W^{(\nu_4, \nu_5)}(0) | \alpha, p_\alpha \rangle = \frac{1}{(2\pi)^3} \sqrt{\frac{m_\alpha}{E_\alpha}} \sqrt{\frac{m_\delta}{E_\delta}} \bar{u}(p_\delta) S_{\delta\alpha}^{(\nu_4, \nu_5)} u(p_\alpha) \quad (\text{III-6})$$

For our purpose it is sufficient to know that $S_{\delta\alpha}$ is a scalar spurion and in the unitary spin space it is given by

$$S_{\delta\alpha}^{(\nu_4, \nu_5)} = \sum_{\nu} \begin{pmatrix} 8 & 8 & 8_s \\ \nu_4 & \nu_5 & \nu \end{pmatrix} \left[\begin{pmatrix} 8 & 8 & 8_s \\ \alpha & \nu & \delta \end{pmatrix} a_{8_s} + \begin{pmatrix} 8 & 8 & 8_a \\ \alpha & \nu & \delta \end{pmatrix} a_{8_a} \right] \\ + \sum_{\nu} \begin{pmatrix} 8 & 8 & 27 \\ \nu_4 & \nu_5 & \nu \end{pmatrix} \begin{pmatrix} 8 & 27 & 8 \\ \alpha & \nu & \delta \end{pmatrix} a_{27} \quad (\text{III-7})$$

with the reduced matrix elements a_{8_s} , a_{8_a} and a_{27} ; $\nu_4 = (0, 1, -1)$ and $\nu_5 = (1, \frac{1}{2}, \frac{1}{2})$ representing the quantum numbers (Y, I, I_z) .²⁰ The subscripts α, β etc. serve both as particle and SU(3) indices. The structure of the weak Hamiltonian has the transformation property corresponding

²⁰The notation follows that of Sugawara in reference 12.

to the symmetric product of two identical octets as expressed in equation (III-7); for nonleptonic decay only the 8 and the 27 representations are relevant. The axial vector current matrix element between two octet baryons is given by,

$$\begin{aligned}
 & \langle \beta, p_\beta | A_\mu^i(0) | \delta, p_\delta \rangle \\
 &= \frac{1}{(2\pi)^3} \sqrt{\frac{m_\beta}{E_\beta}} \sqrt{\frac{m_\delta}{E_\delta}} \bar{u}(p_\beta) \left[\left(g_A(q^2) \right)_{\beta\delta} i\gamma_\mu \gamma_5 + \left(h_A(q^2) \right)_{\beta\delta} i q_\mu \gamma_5 \right] u(p_\delta)
 \end{aligned}
 \tag{III-8}$$

where the axial vector coupling constant $(g_A)_{\beta\delta}$ is related to the Yukawa coupling constant $g_{\beta\delta}$ by the Goldberger-Treiman relation

$$\frac{f_\pi^2}{C_\pi} \approx \frac{g_{\beta\delta}}{(m_\beta + m_\delta)(g_A)_{\beta\delta}}
 \tag{III-9}$$

The induced pseudoscalar form factor $(h_A)_{\beta\delta}$ does not come into the discussion when $q^2=0$. In the spherical basis $g_{\beta\delta}$ is related to the pion-nucleon coupling constant g as follows,

$$g_{\beta\delta} = (-1)^{Q_{V_1}} \left[\sqrt{3} \begin{pmatrix} 8 & 8 & 8_s \\ \beta & \nu_1 & \delta \end{pmatrix} f + \sqrt{\frac{5}{3}} \begin{pmatrix} 8 & 8 & 8_s \\ \beta & \nu_1 & \delta \end{pmatrix} d \right] g \quad (\text{III-10})$$

The symmetric and antisymmetric coupling constants have the normalization $f + d = 1$.

With the help of equations (III-6), (III-8) and (III-9) the octet baryon pole contribution to the invariant amplitudes in the decay limit is given by

$$C = - \left[\frac{1}{m_\beta + m_\delta} + \frac{m_\beta - m_\alpha - 2m_\delta}{2(m_\delta^2 - s)} \right] g_{\beta\delta} S_{\delta\alpha}^{c(\nu_4, \nu_5)} - \left[\frac{1}{m_\alpha + m_\gamma} + \frac{m_\alpha - m_\beta - 2m_\gamma}{2(m_\gamma^2 - u)} \right] S_{\beta\delta}^{c(\nu_4, \nu_5)} g_{\gamma\alpha} \quad (\text{III-11})$$

$$D = \frac{1}{m_\delta^2 - s} g_{\beta\delta} S_{\delta\alpha}^{c(\nu_4, \nu_5)} + \frac{1}{m_\gamma^2 - u} S_{\beta\delta}^{c(\nu_4, \nu_5)} g_{\gamma\alpha} \quad (\text{III-12})$$

Substituting into equation (II-20) we have the octet baryon pole contribution to the P-wave amplitudes as,

$$\begin{aligned}
P(\alpha \rightarrow \beta \pi^{\frac{1}{2}}) = K(\alpha \rightarrow \beta \pi^{\frac{1}{2}}) \left\{ - \frac{m_{\alpha} + m_{\beta}}{(m_{\beta} + m_{\delta})(m_{\alpha} - m_{\delta})} g_{\beta\delta} S_{\delta\alpha}^{c(\nu_4, \nu_5)} \right. \\
\left. + \frac{m_{\alpha} + m_{\beta}}{(m_{\gamma} + m_{\alpha})(m_{\gamma} - m_{\beta})} S_{\beta\delta}^{c(\nu_4, \nu_5)} g_{\delta\alpha} \right\} \quad (\text{III-13})
\end{aligned}$$

The previous ambiguity lies in extrapolating the pion four-momentum to zero. Here extrapolation is made from $q^2 = \mu^2$ to $q^2 = 0$ which is in accordance with the spirit of PCAC. The P-wave formula derived above presumably contains the small correction term expressed in equation (III-5) which has been left out in the previous treatments. The formula is free from the ambiguity of the baryon masses used; the intermediate state that can contribute is dictated by SU(3) symmetry for the vertex.

III.2 Equal Time Commutator Term

This is the second term in equation (II-4). As stated previously the Cabibbo Hamiltonian is proportional to the symmetric product of the strangeness changing and strangeness conserving currents. The strangeness changing and strangeness conserving currents transform in the unitary spin space as K^+ and π^- respectively each containing vector and axial vector parts. In the following we take the viewpoint that possible Schwinger terms are paired off with possible noncovariant contributions so that they are irrelevant for our consideration.²¹ The equal time commutator term can then be written as the commutator of axial charge with the weak Hamiltonian. Using the chiral invariance of the Hamiltonian which can be derived from the equal time commutation relations between charge (Q) and current:

$$[Q_5^j(0), A^k(\vec{x}, 0)] = [Q^j(0), V^k(\vec{x}, 0)] \quad (\text{III-14})$$

$$[Q_5^j(0), V^k(\vec{x}, 0)] = [Q^j(0), A^k(\vec{x}, 0)] \quad (\text{III-15})$$

(where the axial charge is distinguished from the ordinary charge by a subscript 5), we can write the equal time

²¹J. D. Bjorken, Phys. Rev. 148, 1467 (1966); Lowell S. Brown, Phys. Rev. 150, 1338 (1966).

commutator term as

$$= - \frac{g}{2m_N g_A(0)} (2\pi)^3 \sqrt{\frac{E_\alpha}{m_\alpha}} \sqrt{\frac{E_\beta}{m_\beta}} \langle p_\beta | [Q^i, H_w^{c(\nu_4, \nu_5)}(0)] | p_\alpha \rangle$$

In the spherical basis²² the equal time commutator term contribution to the invariant amplitudes is,

$$A = - \frac{g}{2m_N g_A(0)} \sqrt{3} \left\{ \begin{pmatrix} 8 & 8 & 8_A \\ \nu_1 & \nu_4 & \nu_6 \end{pmatrix} S_{\beta\alpha}^{(\nu_5, \nu_6)} + (\nu_4 \leftrightarrow \nu_5) \right\} \quad (\text{III-16})$$

Thus the contribution of the non-Born term to the S-wave amplitudes gives the following results,²³

$$S(\Lambda_-^0) = \sqrt{2} S(\Lambda_\bullet^0) \quad (\text{III-17})$$

$$S(\Xi_-^-) = -\sqrt{2} S(\Xi_0^0) \quad (\text{III-18})$$

$$-\sqrt{2} S(\Sigma_0^+) = S(\Sigma_+^+) + S(\Sigma_-^-) \quad (\text{III-19})$$

and

$$2 S(\Xi_-^-) = S(\Lambda_-^0) + \sqrt{\frac{3}{2}} S(\Sigma_-^-) \quad (\text{III-20})$$

²²J. J. de Swart, Rev. Mod. Phys. 35, 916 (1963); B. W. Lee, "Selected Topics in Unitarity Symmetry and Weak Interactions", High Energy Physics and Elementary Particles (International Atomic Energy Agency, Vienna, 1965).

²³For a discussion of these results see reference 12.

MODEL 1

Thus far we have developed a method of extrapolation in section II and have made use of it to obtain the octet baryon pole contribution to the P-wave decay amplitude as given in equation (III-13). With this contribution alone it is generally recognized that there does not exist a consistent fit of the experimental decay data for both the S- and P-wave amplitudes. Our formalism provides us with a natural way of looking into other intermediate states that may be important. Thus within the framework of SU(3), the decay amplitude will depend on a correct model for the term R_μ which is assumed to obey unsubtracted dispersion relation.

In this model the spin $3/2^+$ decuplet and $Y_0^*(1405)$ states are favoured. A typical member of the spin $3/2^+$ decuplet is the well-known $3-3$ resonance $N^*(1236)$. We will assume that it can be approximated by the Rarita-Schwinger field.²⁴ The spin $3/2$ wave function (u_μ) obeys

²⁴W. Rarita and J. Schwinger, Phys. Rev. 60, 61 (1941);
H. Umezawa, Quantum Field Theory (North Holland Publishing Co., 1956).

$(\not{p} - m)u_\mu(p) = 0$ and $\gamma_\mu u_\mu(p) = 0$. It can contribute to R_μ when it is off mass-shell since it then has a spin 1/2-component. The spurion matrix element between baryon octet and decuplet can be defined as follow:

$$\begin{aligned} & \langle s^*, p_s^* \mid H_w^{(\nu_4, \nu_5)}(0) \mid \alpha, p_\alpha \rangle \\ &= \frac{1}{(2\pi)^3} \sqrt{\frac{m_s^*}{E_s^*}} \sqrt{\frac{m_\alpha}{E_\alpha}} \bar{u}_\nu(p_s^*) p_\alpha^\nu (S_{s\alpha}^v + i\gamma_5 S_{s\alpha}^c) u(p_\alpha) \end{aligned} \quad (\text{IV-1})$$

where the parity violating spurion $S_{s\alpha}^v$ (also the parity conserving spurion $S_{s\alpha}^c$) in the unitary spin space is given by

$$\begin{aligned} S_{s\alpha}^v (S_{s\alpha}^c) &= \frac{1}{\sqrt{2}} \begin{pmatrix} 8 & 8 & 8_s \\ \nu_4 & \nu_5 & \nu \end{pmatrix} \begin{pmatrix} 8 & 8 & 10 \\ \alpha & \nu & s^* \end{pmatrix} b_8^v (b_8^c) \\ &+ \frac{1}{\sqrt{2}} \begin{pmatrix} 8 & 8 & 27 \\ \nu_4 & \nu_5 & \nu \end{pmatrix} \begin{pmatrix} 8 & 27 & 10 \\ \alpha & \nu & s^* \end{pmatrix} b_{27}^v (b_{27}^c) \end{aligned} \quad (\text{IV-2})$$

(For the value of the SU(3) Clebsch-Gordan coefficients Appendix E is referred). There are four independent form factors for the decuplet-octet axial vector vertex.²⁵ It

²⁵J. D. Bjorken and J. D. Walecka, *Annals of Physics* 38, 35 (1966).

can be written such that three of them are transverse to the pion four-momentum and hence do not enter into the calculation. The relevant term is,

$$\langle \gamma^*, p_\delta^* | A_\mu^i(0) | \alpha, p_\alpha \rangle = \frac{1}{(2\pi)^3} \sqrt{\frac{m_\gamma^*}{E_\gamma^*}} \sqrt{\frac{m_\alpha}{E_\alpha}} \bar{u}_\mu(p_\gamma^*) \left(g_A^*(q^2) \right)_{\delta\alpha} u(p_\alpha) \quad (\text{IV-3})$$

where the form factor $g_A^*(q^2)$ at $q^2=0$ is given by

$$(g_A^*)_{\beta\delta} = (-1)^{q_{\beta\delta}} \begin{pmatrix} 8 & 8 & 10 \\ \beta & \nu & \delta \end{pmatrix} g^* \quad (\text{IV-4})$$

The appropriate spin $3/2^+$ positive energy projection operator is

$$\sum_{\text{spin}} u_\mu(p_\delta^*) \bar{u}_\nu(p_\delta^*) = \left[g_{\mu\nu} - \frac{2}{3m_\delta^{*2}} (p_\delta^*)_\mu (p_\delta^*)_\nu - \frac{1}{3} \gamma_\mu \gamma_\nu - \frac{1}{3m_\delta^*} (p_\delta^*)_\mu \gamma_\nu + \frac{1}{3m_\delta^*} \gamma_\mu (p_\delta^*)_\nu \right] \left[\frac{\gamma \cdot p_\delta^* + m_\delta^*}{2m_\delta^*} \right] \quad (\text{IV-5})$$

With equations (IV-1), (IV-3) and (IV-5) the computation of the contribution to R_μ from the decuplet intermediate state is rather lengthy but straight forward. In terms of the invariant amplitudes in the decay limit the results are,

$$A = - \frac{g}{2m_N g_A(0)} \left\{ \left(c_1 + \frac{m_\alpha - m_\beta}{2} c_2 \right) (g_A^*)_{\beta\delta} s_{\delta\alpha}^v \right. \\ \left. + \left(c_5 + \frac{m_\alpha - m_\beta}{2} c_6 \right) s_{\beta\delta}^v (g_A^*)_{\delta\alpha} \right\} \quad (\text{IV-6})$$

$$B = \frac{g}{2m_N g_A(0)} \left\{ c_2 (g_A^*)_{\beta\delta} s_{\delta\alpha}^v + c_6 s_{\beta\delta}^v (g_A^*)_{\delta\alpha} \right\} \quad (\text{IV-7})$$

$$C = - \frac{g}{2m_N g_A(0)} \left\{ \left(c_3 - \frac{m_\alpha + m_\beta}{2} c_4 \right) (g_A^*)_{\beta\delta} s_{\delta\alpha}^c \right. \\ \left. - \left(c_7 + \frac{m_\alpha + m_\beta}{2} c_8 \right) s_{\beta\delta}^c (g_A^*)_{\delta\alpha} \right\} \quad (\text{IV-8})$$

$$D = \frac{g}{2m_N g_A(0)} \left\{ c_4 (g_A^*)_{\beta\delta} s_{\delta\alpha}^c + c_8 s_{\beta\delta}^c (g_A^*)_{\delta\alpha} \right\} \quad (\text{IV-9})$$

where the constants C_i 's are functions of the masses:

$$C_1 = \frac{1}{6m_\delta^*{}^2(m_\delta^* - m_\alpha)} (m_\alpha^2 - m_\beta^2)(m_\delta^* + m_\beta)(3m_\delta^* - 2m_\alpha) \quad (\text{IV-10})$$

$$C_2 = \frac{1}{6m_\delta^*{}^2(m_\delta^* - m_\alpha)} \left[(m_\delta^* - m_\alpha)(2m_\alpha^2 - 3m_\beta^2 - m_\alpha m_\delta^*) - m_\alpha(m_\delta^* + m_\beta)^2 \right] \quad (\text{IV-11})$$

$$c_3 = \frac{1}{6m_\delta^{*2}(m_\delta^* + m_\alpha)} (m_\alpha^2 - m_\beta^2)(m_\delta^* + m_\beta)(3m_\delta^* + 2m_\alpha) \quad (\text{IV-12})$$

$$c_4 = \frac{1}{6m_\delta^{*2}(m_\delta^* + m_\alpha)} \left[(m_\delta^* + m_\alpha)(2m_\alpha^2 - 3m_\beta^2 + m_\alpha m_\delta^*) + m_\alpha(m_\delta^* + m_\beta)^2 \right] \quad (\text{IV-13})$$

$$c_5 = \frac{1}{6m_\gamma^{*2}(m_\gamma^* - m_\beta)} (m_\alpha^2 - m_\beta^2)(m_\gamma^* + m_\alpha)(3m_\gamma^* - 2m_\beta) \quad (\text{IV-14})$$

$$c_6 = \frac{1}{6m_\gamma^{*2}(m_\gamma^* - m_\beta)} \left[(m_\gamma^* - m_\beta)(2m_\beta^2 - 3m_\alpha^2 - m_\beta m_\gamma^*) - m_\beta(m_\gamma^* + m_\alpha)^2 \right] \quad (\text{IV-15})$$

$$c_7 = \frac{1}{6m_\gamma^{*2}(m_\gamma^* + m_\beta)} (m_\alpha^2 - m_\beta^2)(m_\gamma^* + m_\alpha)(3m_\gamma^* + 2m_\beta) \quad (\text{IV-16})$$

$$c_8 = \frac{1}{6m_\gamma^{*2}(m_\gamma^* + m_\beta)} \left[(m_\gamma^* + m_\beta)(2m_\beta^2 - 3m_\alpha^2 + m_\beta m_\gamma^*) + m_\beta(m_\gamma^* + m_\alpha)^2 \right] \quad (\text{IV-17})$$

The $Y_0^*(1405)$ contributes to the Σ_+^+ and Σ_-^- decays only and in equal magnitude through the u-channel. A model computation is given in Appendix F. However for our purpose it is appropriate to introduce two parameters y_s and y_p to represent its contribution to the S- and P-wave amplitudes respectively.

Combining with the results given in equations (III-13) and (III-16) we have the following formulae for the decay amplitudes,

$$\begin{aligned}
 S(\alpha \rightarrow \beta \pi^1) = & - \frac{g}{2m_N g_A(0)} \left\{ \sqrt{3} \left[\begin{pmatrix} 8 & 8 & 8_a \\ \nu_1 & \nu_4 & \nu_6 \end{pmatrix} s_{\beta\alpha}^{(\nu_5, \nu_6)} + (\nu_4 \leftrightarrow \nu_5) \right] \right. \\
 & + \frac{m_\alpha - m_\beta}{6m_\delta^*} \left[(m_\alpha + m_\beta)(2m_\alpha + m_\delta^*) + 2m_\beta m_\delta^* \right] (g_A^*)_{\beta\delta^*} s_{\delta\alpha}^{v(\nu_4, \nu_5)} \\
 & + \frac{m_\alpha - m_\beta}{6m_\gamma^*} \left[(m_\alpha + m_\beta)(2m_\beta + m_\gamma^*) + 2m_\alpha m_\gamma^* \right] s_{\beta\gamma^*}^{v(\nu_4, \nu_5)} (g_A^*)_{\gamma\alpha^*} \left. \right\} \\
 & + y_\pi \delta_{\beta N} \delta_{\alpha \Sigma} \quad (IV-18)
 \end{aligned}$$

$$\begin{aligned}
 P(\alpha \rightarrow \beta \pi^1) = & K(\alpha \rightarrow \beta \pi^1) \left\{ - \frac{m_\alpha + m_\beta}{(m_\beta + m_\delta)(m_\alpha - m_\delta)} g_{\beta\delta} s_{\delta\alpha}^{(\nu_4, \nu_5)} \right. \\
 & + \frac{m_\alpha + m_\beta}{(m_\gamma + m_\alpha)(m_\gamma - m_\beta)} s_{\beta\gamma}^{(\nu_4, \nu_5)} g_{\gamma\alpha} + \frac{g}{2m_N g_A(0)} \\
 & \left[\frac{m_\alpha + m_\beta}{6m_\delta^*} \left[(m_\alpha - m_\beta)(2m_\alpha - m_\delta^*) + 2m_\beta m_\delta^* \right] (g_A^*)_{\beta\delta^*} s_{\delta\alpha}^{c(\nu_4, \nu_5)} \right. \\
 & + \frac{m_\alpha + m_\beta}{6m_\gamma^*} \left[-(m_\alpha - m_\beta)(2m_\beta - m_\gamma^*) + 2m_\alpha m_\gamma^* \right] s_{\beta\gamma^*}^{c(\nu_4, \nu_5)} (g_A^*)_{\gamma\alpha^*} \left. \right] \left. \right\} \\
 & + y_\pi \delta_{\beta N} \delta_{\alpha \Sigma} \quad (IV-19)
 \end{aligned}$$

where the kronecker delta $\delta_{\beta N} \delta_{\alpha \Sigma}$ is introduced to mean that the Y_0^* resonance contributes only to the Σ_+^+ and Σ_-^- decays.

In the following discussion it is more appropriate to assume octet dominance since the 27 reduced matrix element of the baryon, a_{27} violates $\Delta I = \frac{1}{2}$ rule for the P-wave and the 27 reduced matrix element of the decuplet, b_{27} violates $\Delta I = \frac{1}{2}$ rule for the S-wave. With this assumption there are six unknown parameters in the decay formula. If we define

$$A_{8s} = \frac{1}{2m_N g_A(0)} \frac{1}{20} a_{8s} g \quad (\text{IV-20})$$

$$A_{8a} = \frac{1}{2m_N g_A(0)} \frac{1}{20} a_{8a} g \quad (\text{IV-21})$$

$$B_s = - \frac{1}{2m_N g_A(0)} \frac{1}{4\sqrt{30}} \mu b_8^v g^* g \quad (\text{IV-22})$$

$$B_p = - \frac{1}{2m_N g_A(0)} \frac{1}{4\sqrt{30}} \mu b_8^c g^* g \quad (\text{IV-23})$$

the S- and P-wave amplitudes can be written out from equations (IV-18) and (IV-19),

$$S(\Lambda_-^0) = \sqrt{3} A_{8s} + \sqrt{15} A_{8a} + \sqrt{6} s_2^*(\Lambda) B_s \quad (\text{IV-24})$$

$$S(\Sigma_+^+) = 0 + 0 + [2s_1^*(\Sigma) - s_2^*(\Sigma)] B_s + y_s \quad (\text{IV-25})$$

$$S(\Sigma_0^+) = 3 A_{8s} - \sqrt{5} A_{8a} - \sqrt{2} [2s_1^*(\Sigma) + s_2^*(\Sigma)] B_s \quad (\text{IV-26})$$

$$S(\Sigma_-^-) = -3\sqrt{2} A_{8s} + \sqrt{10} A_{8a} + [6s_1^*(\Sigma) + s_2^*(\Sigma)] B_s + y_s \quad (\text{IV-27})$$

$$S(\Xi_-^-) = -\sqrt{3} A_{8s} + \sqrt{15} A_{8a} + \sqrt{6} [s_1^*(\Xi) + s_2^*(\Xi)] B_s \quad (\text{IV-28})$$

and

$$P(\Lambda_-^0) = K(\Lambda_-^0) \left\{ \sqrt{3} [p_1(\Lambda) - 2p_2(\Lambda)] A_{8s} + \frac{\sqrt{15}}{3} [3p_1(\Lambda) + 2p_2(\Lambda)] A_{8a} + \sqrt{6} p_2^*(\Lambda) B_p \right\} \quad (\text{IV-29})$$

$$P(\Sigma_+^+) = K(\Sigma_+^+) \left\{ \sqrt{2} [-3p_1(\Sigma) + p_2(\Sigma) - 3p_3(\Sigma)] A_{8s} + \sqrt{10} [p_1(\Sigma) + p_2(\Sigma) + p_3(\Sigma)] A_{8a} + [2p_1^*(\Sigma) - p_2^*(\Sigma)] B_p \right\} + y_p \quad (\text{IV-30})$$

$$P(\Sigma_0^+) = K(\Sigma_0^+) \left\{ 3 [-p_1(\Sigma) - 2p_3(\Sigma)] A_{8s} + \sqrt{5} [p_1(\Sigma) + 2p_3(\Sigma)] A_{8a} - \sqrt{2} [2p_1^*(\Sigma) + p_2^*(\Sigma)] B_p \right\} \quad (\text{IV-31})$$

$$\begin{aligned}
P(\Sigma^-) = K(\Sigma^-) \left\{ \sqrt{2} [p_2(\Sigma) + 3p_3(\Sigma)] A_{8S} + \sqrt{10} [p_2(\Sigma) - p_3(\Sigma)] A_{8A} \right. \\
\left. + [6p_1^*(\Sigma) + p_2^*(\Sigma)] B_P \right\} + Y_P
\end{aligned} \quad (IV-32)$$

$$\begin{aligned}
P(\Xi^-) = K(\Xi^-) \left\{ \sqrt{3} [2p_1(\Xi) - p_2(\Xi)] A_{8S} + \frac{\sqrt{15}}{3} [2p_1(\Xi) + 3p_2(\Xi)] A_{8A} \right. \\
\left. + \sqrt{6} [p_1^*(\Xi) + p_2^*(\Xi)] B_P \right\}
\end{aligned} \quad (IV-33)$$

where

$$s_1^*(\Lambda) = \frac{m_\Lambda - m_N}{6m_\Delta^* \mu} \left[(m_\Lambda + m_N)(2m_\Lambda + m_\Delta^*) + 2m_N m_\Delta^* \right] \quad (IV-34)$$

$$s_2^*(\Lambda) = \frac{m_\Lambda - m_N}{6m_Y^* \mu} \left[(m_\Lambda + m_N)(2m_N + m_Y^*) + 2m_\Lambda m_Y^* \right] \quad (IV-35)$$

$$s_1^*(\Sigma) = \frac{m_\Sigma - m_N}{6m_\Delta^* \mu} \left[(m_\Sigma + m_N)(2m_\Sigma + m_\Delta^*) + 2m_N m_\Delta^* \right] \quad (IV-36)$$

$$s_2^*(\Sigma) = \frac{m_\Sigma - m_N}{6m_Y^* \mu} \left[(m_\Sigma + m_N)(2m_N + m_Y^*) + 2m_\Sigma m_Y^* \right] \quad (IV-37)$$

$$s_1^*(\Xi) = \frac{m_\Xi - m_\Lambda}{6m_Y^* \mu} \left[(m_\Xi + m_\Lambda)(2m_\Xi + m_Y^*) + 2m_\Lambda m_Y^* \right] \quad (IV-38)$$

$$s_2^*(\Xi) = \frac{m_\Xi - m_\Lambda}{6m_\Xi^* \mu} \left[(m_\Xi + m_\Lambda)(2m_\Lambda + m_\Xi^*) + 2m_\Xi m_\Xi^* \right] \quad (IV-39)$$

and

$$p_1^*(\Lambda) = - \frac{m_\Lambda + m_N}{6m_\Delta^*} \mu \left[(m_\Lambda - m_N)(2m_\Lambda - m_\Delta^*) + 2m_N m_\Delta^* \right] \quad (\text{IV-40})$$

$$p_2^*(\Lambda) = - \frac{m_\Lambda + m_N}{6m_Y^*} \mu \left[-(m_\Lambda - m_N)(2m_N - m_Y^*) + 2m_\Lambda m_Y^* \right] \quad (\text{IV-41})$$

$$p_1^*(\Sigma) = - \frac{m_\Sigma + m_N}{6m_\Delta^*} \mu \left[(m_\Sigma - m_N)(2m_\Sigma - m_\Delta^*) + 2m_N m_\Delta^* \right] \quad (\text{IV-42})$$

$$p_2^*(\Sigma) = - \frac{m_\Sigma + m_N}{6m_Y^*} \mu \left[-(m_\Sigma - m_N)(2m_N - m_Y^*) + 2m_\Sigma m_Y^* \right] \quad (\text{IV-43})$$

$$p_1^*(\Xi) = - \frac{m_\Xi + m_\Lambda}{6m_Y^*} \mu \left[(m_\Xi - m_\Lambda)(2m_\Xi - m_Y^*) + 2m_\Lambda m_Y^* \right] \quad (\text{IV-44})$$

$$p_2^*(\Xi) = - \frac{m_\Xi + m_\Lambda}{6m_\Xi^*} \mu \left[-(m_\Xi - m_\Lambda)(2m_\Lambda - m_\Xi^*) + 2m_\Xi m_\Xi^* \right] \quad (\text{IV-45})$$

and

$$p_1^*(\Lambda) = - \frac{m_\Lambda + m_N}{m_\Lambda - m_N} g_A(0) \quad (\text{IV-46})$$

$$p_2^*(\Lambda) = \frac{2m_N(m_\Lambda + m_N)}{(m_\Sigma + m_\Lambda)(m_\Sigma - m_N)} g_A(0) \quad (\text{IV-47})$$

$$p_1(\Sigma) = - \frac{m_\Sigma + m_N}{m_\Sigma - m_N} g_A(0) \quad (\text{IV-48})$$

$$p_2(\Sigma) = \frac{2m_N(m_\Sigma + m_N)}{(m_\Lambda + m_\Sigma)(m_\Lambda - m_N)} g_A(0) d \quad (\text{IV-49})$$

$$p_3(\Sigma) = \frac{m_N(m_\Sigma + m_N)}{m_\Sigma(m_\Sigma - m_N)} g_A(0) f \quad (\text{IV-50})$$

$$p_1(\Xi) = - \frac{2m_N(m_\Xi + m_\Lambda)}{(m_\Lambda + m_\Xi)(m_\Xi - m_\Lambda)} g_A(0) d \quad (\text{IV-51})$$

$$p_2(\Xi) = \frac{m_N(m_\Xi + m_\Lambda)}{m_\Xi(m_\Xi - m_\Lambda)} g_A(0) (d-f) \quad (\text{IV-52})$$

Taking the SU(3) baryon-meson octet coupling ratio $d/f = 1.75^{26}$ and $g_A(0) = 1.3$, we have

$$S(\Lambda_-^0) = 1.732 A_{8_S} + 3.873 A_{8_A} + 2.619 B_S \quad (\text{IV-53})$$

$$S(\Sigma_+^+) = 0 + 0 + 2.357 B_S + y_S \quad (\text{IV-54})$$

$$S(\Sigma_0^+) = 3.000 A_{8_S} - 2.236 A_{8_A} - 7.907 B_S \quad (\text{IV-55})$$

²⁶W. Willis et al., Phys. Rev. Letters 13, 291 (1964); N. Brene, L. Veje, M. Roos and C. Cronstrom, Phys. Rev. 149, 1288 (1966); C. F. Carlson, Phys. Rev. 152, 1433 (1966).

$$S(\Sigma_-^-) = -4.243 A_{8s} + 3.162 A_{8a} + 13.54 B_s + y_s \quad (\text{IV-56})$$

$$S(\Xi_-^-) = -1.732 A_{8s} + 3.873 A_{8a} + 7.305 B_s \quad (\text{IV-57})$$

and

$$P(\Lambda_-^0) = -2.387 A_{8s} - 2.363 A_{8a} - 0.4957 B_p \quad (\text{IV-58})$$

$$P(\Sigma_+^+) = 4.320 A_{8s} + 0.1119 A_{8a} - 0.4337 B_p + y_p \quad (\text{IV-59})$$

$$P(\Sigma_0^+) = 1.395 A_{8s} - 1.040 A_{8a} + 1.817 B_p \quad (\text{IV-60})$$

$$P(\Sigma_-^-) = 2.505 A_{8s} + 1.608 A_{8a} - 3.058 B_p + y_p \quad (\text{IV-61})$$

$$P(\Xi_-^-) = -3.146 A_{8s} - 1.373 A_{8a} - 1.501 B_p \quad (\text{IV-62})$$

With the above equations a consistent fit of the experimental data can proceed as follow. The contribution of the decuplet and Y_0^* states to the P-wave decays is expected to be of the order $\Delta m/m$ smaller compared with that to the S-wave decays. As a first approximation it can be assumed that such contribution is insignificant. Then looking at the P-wave amplitudes alone, with $P(\Sigma_-^-) \cong 0$, we would have a fixed ratio of A_{8s} and A_{8a} ; the remaining P-wave amplitudes can be expressed in terms of one unknown, and hence are related. For the S-wave

amplitudes, with $S(\Sigma_0^+)$ as input, the contribution of the decuplet is determined automatically. The contribution of Y_0^* is then fixed by the condition, $S(\Sigma_+^+) = 0$. The results of the fit as given in Tables 1 and 2 have been obtained without prejudice with the help of computer.²⁷ They correspond to the solutions: $A_{8_S} = 9.323 \times 10^{-8}$, $A_{8_a} = -1.462 \times 10^{-7}$, $B_S = 3.6 \times 10^{-8}$, $B_P = 0.5 \times 10^{-8}$, $y_S = -8.485 \times 10^{-8}$ and $y_P = 1.400 \times 10^{-8}$. The Lee-Sugawara relations are reasonably well obeyed for the decay amplitudes. One thing to note is that the P-wave fit can be given by the octet baryon pole alone.

A few comments may be of interest. First this is a good SU(3) model in the sense that the usual argument of CP invariance and SU(3) symmetry does not apply to the parity violating matrix elements between the decuplet and the octet baryon. Secondly, since the four-momentum of the pion need not vanish in our approach, it is natural to examine other possible intermediate states that may contribute to R_{π} in equation (II-4). The t-channel pole, K and K* mesons have been considered and found to be insignificant.²⁸ In principle there are also those spin

²⁷Experimental data taken from S. A. Bludman, Cargèse Lectures (1966); see Appendix D.

²⁸Lowell S. Brown and H. Crater (private communication to H. J. Schnitzer).

$3/2^-$ states such as $Y_0^*(1520)$, $N^*(1518)$ and $\Xi^*(1816)$, and perhaps the existence of a spin $1/2^-$ octet.²⁹ In view of the experimental uncertainty we have not included them. Thirdly, the 27 amplitude is expected to be small because of the empirical $\Delta I = 1/2$ rule which appears quite good for Λ and Ξ decays. However, the 27 amplitude between octet baryons gives comparatively the largest contribution to the Σ_+^+ and Σ_-^- decays so that the possibility of replacing the Y_0^* terms appear to exist. This can be done except that the $\Delta I = 1/2$ rule for the P-wave Ξ decays would be violated rather badly. Lastly this model favours a S-wave rather than a P-wave correction to the conventional approach. It is not clear from physical arguments how important the decuplet contribution should be compared with the equal time commutator term (E.T.C.). With the parity violating Hamiltonian proportional to the divergence of axial vector current as proposed by Nishijima, for example one would have the S-wave amplitudes given entirely by terms proportional to the baryon mass difference.

²⁹S. Okubo, "New Approach to Algebra of Current and Weinberg Sum Rules", University of Rochester preprint, August 1967.

Table 1 The contributions of various terms to the S-wave amplitudes and comparison with experiment for model 1 with Yukawa coupling $d/f = 1.75$, and spurion reduced matrix elements as in text.

Amplitude	E.T.C.	Decuplet	Y_0^*	Total	Experiment
$S(\Lambda_-^0) \times 10^6$	-0.40	0.09	0	-0.31	-0.33
$S(\Sigma_+^+) \times 10^6$	0	0.08	-0.08	0.00	0
$S(\Sigma_0^+) \times 10^6$	0.61	-0.29	0	0.32	0.33
$S(\Sigma_-^-) \times 10^6$	-0.86	0.49	-0.08	-0.45	-0.40
$S(\Xi_-^-) \times 10^6$	-0.73	0.26	0	-0.47	-0.44

Table 2 The contributions of various terms to the P-wave amplitudes and comparison with experiment for model 1 with Yukawa coupling $d/f = 1.75$, and spurion reduced matrix elements as in text.

Amplitude	Baryon	Decuplet	Y_0^*	Total	Experiment
$P(\Lambda_-^0) \times 10^7$	1.23	-0.02	0	1.21	1.25
$P(\Sigma_+^+) \times 10^7$	3.86	-0.02	0.14	3.98	4.02
$P(\Sigma_0^+) \times 10^7$	2.82	0.09	0	2.91	2.54
$P(\Sigma_-^-) \times 10^7$	-0.02	-0.15	0.14	-0.03	-0.03
$P(\Xi_-^-) \times 10^7$	-0.93	-0.07	0	-1.00	-0.90

MODEL 2

In the previous section a very reasonable model has been obtained which gives good agreement with experiment. Here other possibilities are investigated. This model to be discussed is very much in the spirit of Kumar and Pati.³⁰ It involves the assumption that the parity violating spurion matrix element between two baryon octets which would vanish in exact SU(3) limit is not zero. The non-vanishing of similar physical matrix elements occurs in other example such as K meson decaying into two pions. Here we define,

$$\langle b, p_b | H_w^{v(\nu_4, \nu_5)}(0) | \alpha, p_\alpha \rangle = \frac{1}{(2\pi)^3} \sqrt{\frac{m_b}{E_b}} \sqrt{\frac{m_\alpha}{E_\alpha}} \bar{u}(p_b) i\gamma_5 S_{b\alpha}^{v(\nu_4, \nu_5)} u(p_\alpha) \quad (V-1)$$

where the parity violating spurion $S_{b\alpha}^v$ in the unitary spin space has the structure,

³⁰See reference 16.

$$\begin{aligned}
S_{\delta\alpha}^{v(\nu_4, \nu_5)} &= \frac{1}{\sqrt{6}} \begin{pmatrix} 8 & 8 & 8_s \\ \nu_4 & \nu_5 & \nu \end{pmatrix} \left[\begin{pmatrix} 8 & 8 & 8_s \\ \alpha & \nu & \delta \end{pmatrix} a_{8s}^v + \begin{pmatrix} 8 & 8 & 8_a \\ \alpha & \nu & \delta \end{pmatrix} a_{8a}^v \right] \\
&+ \frac{1}{\sqrt{6}} \begin{pmatrix} 8 & 8 & 27 \\ \nu_4 & \nu_5 & \nu \end{pmatrix} \begin{pmatrix} 8 & 27 & 8 \\ \alpha & \nu & \delta \end{pmatrix} a_{27}^v \quad (V-2)
\end{aligned}$$

The SU(3) notation is as before. With the axial vector current matrix elements as equation (III-8), the baryon pole contributions from the parity violating part of the Hamiltonian to the invariant amplitudes in the decay limit are,

$$\begin{aligned}
A &= - \left[\frac{1}{m_\beta + m_\delta} + \frac{m_\alpha + m_\beta - 2m_\delta}{2(m_\delta^2 - s)} \right] g_{\beta\delta} S_{\delta\alpha}^{v(\nu_4, \nu_5)} \\
&- \left[\frac{1}{m_\alpha + m_\gamma} + \frac{m_\alpha + m_\beta - 2m_\gamma}{2(m_\gamma^2 - u)} \right] S_{\beta\gamma}^{v(\nu_4, \nu_5)} g_{\gamma\alpha} \quad (V-3)
\end{aligned}$$

$$\begin{aligned}
B &= \frac{1}{m_\delta^2 - s} g_{\beta\delta} S_{\delta\alpha}^{v(\nu_4, \nu_5)} - \frac{1}{m_\gamma^2 - u} S_{\beta\gamma}^{v(\nu_4, \nu_5)} g_{\gamma\alpha} \\
&\quad (V-4)
\end{aligned}$$

The amplitudes A and B give to the S-wave decays a combined contribution of the order of baryon mass difference.

Similarly for the equal time commutator term, the contribution from the parity violating spurion, in analogy to equation (III-16), is

$$C = - \frac{g}{2m_N g_A(0)} \sqrt{3} \left\{ \begin{pmatrix} 8 & 8 & 8_a \\ \nu_1 & \nu_4 & \nu_6 \end{pmatrix} S_{\beta\alpha}^{v(\nu_5, \nu_6)} + (\nu_4 \leftrightarrow \nu_5) \right\} \quad (V-5)$$

Combining with the results from the parity conserving spurion as given in equations (III-13) and (III-16), we have for this model,

$$\begin{aligned} S(\alpha \rightarrow \beta \pi^1) = & \left[- \frac{1}{m_\beta + m_\pi} + \frac{1}{m_\delta + m_\alpha} \right] g_{\beta\delta} S_{\delta\alpha}^{v(\nu_4, \nu_5)} \\ & + \left[- \frac{1}{m_\gamma + m_\alpha} + \frac{1}{m_\gamma + m_\beta} \right] S_{\beta\gamma}^{v(\nu_4, \nu_5)} g_{\gamma\alpha} \\ & - \frac{g}{2m_N g_A(0)} \sqrt{3} \left[\begin{pmatrix} 8 & 8 & 8_a \\ \nu_1 & \nu_4 & \nu_6 \end{pmatrix} S_{\beta\alpha}^{c(\nu_5, \nu_6)} + (\nu_4 \leftrightarrow \nu_5) \right] \end{aligned} \quad (V-6)$$

$$\begin{aligned}
P(\alpha \rightarrow \beta \pi^1) = K(\alpha \rightarrow \beta \pi^1) \left\{ \left[-\frac{1}{m_\beta + m_\delta} - \frac{1}{m_\alpha - m_\delta} \right] s_{\beta\delta} s_{\delta\alpha}^{c(\nu_4, \nu_5)} \right. \\
+ \left[-\frac{1}{m_\gamma + m_\alpha} + \frac{1}{m_\gamma - m_\beta} \right] s_{\beta\gamma}^{c(\nu_4, \nu_5)} s_{\gamma\alpha} \\
\left. - \frac{g}{2m_N g_A(0)} \sqrt{3} \left[\begin{pmatrix} 8 & 8 & 8_a \\ \nu_1 & \nu_4 & \nu_6 \end{pmatrix} s_{\beta\alpha}^{v(\nu_5, \nu_6)} + (\nu_4 \leftrightarrow \nu_5) \right] \right\}
\end{aligned}
\tag{V-7}$$

What has gone into the formulae above is essentially the complete octet baryon pole approximation to R_{μ} . The parity violating spurion matrix elements between two baryon octets are usually neglected because of the extended charge conjugation invariance to a $SU(3)$ multiplet have been included as unknown parameters. The expressions for the decay amplitudes are:

$$\begin{aligned}
S(\Lambda_-^0) = \sqrt{3} A_{8s}^c + \sqrt{15} A_{8a}^c + \frac{2\sqrt{2}}{3} A_{27}^c + \sqrt{3} [s_1(\wedge) - 2s_2(\wedge)] A_{8s}^v \\
+ \frac{\sqrt{15}}{3} [3s_1(\wedge) + 2s_2(\wedge)] A_{8a}^v + \frac{2\sqrt{2}}{9} [3s_1(\wedge) + 4s_2(\wedge)] A_{27}^v
\end{aligned}
\tag{V-8}$$

$$\begin{aligned}
S(\Lambda_o^0) = & \frac{\sqrt{6}}{2} A_{8s}^c + \frac{\sqrt{30}}{2} A_{8a}^c + \frac{2}{3} A_{27}^c + \frac{\sqrt{6}}{2} [s_1(\Lambda) - 2s_2(\Lambda)] A_{8s}^v \\
& + \frac{\sqrt{30}}{6} [3s_1(\Lambda) + 2s_2(\Lambda)] A_{8a}^v + \frac{2}{3} [s_1(\Lambda) - 2s_2(\Lambda)] A_{27}^v
\end{aligned}
\tag{V-9}$$

$$\begin{aligned}
S(\Sigma_+^+) = & \frac{20\sqrt{3}}{9} A_{27}^c + \sqrt{2} [-3s_1(\Sigma) + s_2(\Sigma) - 3s_3(\Sigma)] A_{8s}^v + \sqrt{10} [s_1(\Sigma) \\
& + s_2(\Sigma) + s_3(\Sigma)] A_{8a}^v + \frac{4\sqrt{3}}{9} [2s_1(\Sigma) + s_2(\Sigma) - 3s_3(\Sigma)] A_{27}^v
\end{aligned}
\tag{V-10}$$

$$\begin{aligned}
S(\Sigma_o^+) = & 3 A_{8s}^c - \sqrt{5} A_{8a}^c - \frac{4\sqrt{6}}{9} A_{27}^c - 3 [s_1(\Sigma) + 2s_3(\Sigma)] A_{8s}^v \\
& + \sqrt{5} [s_1(\Sigma) + 2s_3(\Sigma)] A_{8a}^v + \frac{4\sqrt{6}}{9} [s_1(\Sigma) + 2s_3(\Sigma)] A_{27}^v
\end{aligned}
\tag{V-11}$$

$$\begin{aligned}
S(\Sigma_-^-) = & -3\sqrt{2} A_{8s}^c + \sqrt{10} A_{8a}^c - \frac{4\sqrt{3}}{3} A_{27}^c + \sqrt{2} [s_2(\Sigma) + 3s_3(\Sigma)] A_{8s}^v \\
& + \sqrt{10} [s_2(\Sigma) - s_3(\Sigma)] A_{8a}^v + \frac{4\sqrt{3}}{9} [s_2(\Sigma) + 3s_3(\Sigma)] A_{27}^v
\end{aligned}
\tag{V-12}$$

$$\begin{aligned}
S(\Xi_-^-) = & -3 A_{8s}^c + \sqrt{15} A_{8a}^c - \frac{2\sqrt{2}}{3} A_{27}^c + \sqrt{3} [2s_1(\Xi) - s_2(\Xi)] A_{8s}^v \\
& + \frac{\sqrt{15}}{3} [2s_1(\Xi) + 3s_2(\Xi)] A_{8a}^v + \frac{2\sqrt{2}}{9} [-4s_1(\Xi) - 3s_2(\Xi)] A_{27}^v
\end{aligned}
\tag{V-13}$$

$$\begin{aligned}
s(\Xi^0) = & \frac{\sqrt{6}}{2} A_{8\pi}^c - \frac{\sqrt{30}}{2} A_{8\pi}^c + \frac{2}{3} A_{27}^c + \frac{\sqrt{6}}{2} [-2s_1(\Xi) + s_2(\Xi)] A_{8\pi}^v \\
& + \frac{\sqrt{30}}{6} [-2s_1(\Xi) - 3s_2(\Xi)] A_{8\pi}^v + \frac{2}{3} [-2s_1(\Xi) + s_2(\Xi)] A_{27}^v
\end{aligned}
\tag{V-14}$$

where

$$s_1(\Lambda) = - \frac{m_\Lambda - m_N}{m_\Lambda + m_N} g_A(0) \tag{V-15}$$

$$s_2(\Lambda) = \frac{2m_N(m_\Lambda - m_N)}{(m_\Lambda + m_\Sigma)(m_N + m_\Sigma)} g_A(0) \quad d \tag{V-16}$$

$$s_1(\Sigma) = - \frac{m_\Sigma - m_N}{m_\Sigma + m_N} g_A(0) \tag{V-17}$$

$$s_2(\Sigma) = \frac{2m_N(m_\Sigma - m_N)}{(m_\Sigma + m_\Lambda)(m_N + m_\Lambda)} g_A(0) \quad d \tag{V-18}$$

$$s_3(\Sigma) = \frac{m_N(m_\Sigma - m_N)}{m_\Sigma(m_\Sigma + m_N)} g_A(0) \quad f \tag{V-19}$$

$$s_1(\Xi) = - \frac{2m_N(m_\Xi - m_\Lambda)}{(m_\Xi + m_\Sigma)(m_\Lambda + m_\Sigma)} g_A(0) \quad d \tag{V-20}$$

$$s_2(\Xi) = \frac{m_N(m_\Xi - m_\Lambda)}{m_\Xi(m_\Xi + m_\Lambda)} g_A(0) \quad (d-f) \tag{V-21}$$

and

$$\begin{aligned}
P(\Lambda_0^-) = K(\Lambda_0^-) \left\{ \sqrt{3} [p_1(\wedge) - 2p_2(\wedge)] A_{8a}^c + \frac{\sqrt{15}}{3} [3p_1(\wedge) + 2p_2(\wedge)] A_{8a}^c \right. \\
\left. + \frac{2\sqrt{2}}{9} [3p_1(\wedge) + 4p_2(\wedge)] A_{27}^c + \sqrt{3} A_{8a}^v + \sqrt{15} A_{8a}^v + \frac{2\sqrt{2}}{3} A_{27}^v \right\}
\end{aligned}
\quad (V-22)$$

$$\begin{aligned}
P(\Lambda_0^0) = K(\Lambda_0^0) \left\{ \frac{\sqrt{6}}{2} [p_1(\wedge) - 2p_2(\wedge)] A_{8a}^c + \frac{\sqrt{30}}{6} [3p_1(\wedge) + 2p_2(\wedge)] A_{8a}^c \right. \\
\left. + \frac{2}{3} [p_1(\wedge) - 2p_2(\wedge)] A_{27}^c + \frac{\sqrt{6}}{2} A_{8a}^v + \frac{\sqrt{30}}{2} A_{8a}^v + \frac{2}{3} A_{27}^v \right\}
\end{aligned}
\quad (V-23)$$

$$\begin{aligned}
P(\Sigma_+^+) = K(\Sigma_+^+) \left\{ \sqrt{2} [-3p_1(\Sigma) + p_2(\Sigma) - 3p_3(\Sigma)] A_{8a}^c + \sqrt{10} [p_1(\Sigma) + p_2(\Sigma) \right. \\
\left. + p_3(\Sigma)] A_{8a}^c + \frac{4\sqrt{3}}{9} [2p_1(\Sigma) + p_2(\Sigma) - 3p_3(\Sigma)] A_{27}^c + \frac{20\sqrt{3}}{9} A_{27}^v \right\}
\end{aligned}
\quad (V-24)$$

$$\begin{aligned}
P(\Sigma_0^+) = K(\Sigma_0^+) \left\{ -3 [p_1(\Sigma) + 2p_3(\Sigma)] A_{8a}^c + \sqrt{5} [p_1(\Sigma) + 2p_3(\Sigma)] A_{8a}^c \right. \\
\left. + \frac{4\sqrt{6}}{9} [p_1(\Sigma) + 2p_3(\Sigma)] A_{27}^c + 3 A_{8a}^v - \sqrt{5} A_{8a}^v - \frac{4\sqrt{6}}{9} A_{27}^v \right\}
\end{aligned}
\quad (V-25)$$

$$\begin{aligned}
P(\Sigma_-^-) = K(\Sigma_-^-) \left\{ \sqrt{2} [p_2(\Sigma) + 3p_3(\Sigma)] A_{8a}^c + \sqrt{10} [p_2(\Sigma) - p_3(\Sigma)] A_{8a}^c \right. \\
\left. + \frac{4\sqrt{3}}{9} [p_2(\Sigma) + 3p_3(\Sigma)] A_{27}^c - 3\sqrt{2} A_{8a}^v + \sqrt{10} A_{8a}^v - \frac{4\sqrt{3}}{3} A_{27}^v \right\}
\end{aligned}
\quad (V-26)$$

$$\begin{aligned}
P(\Xi_-^-) = K(\Xi_-^-) \left\{ \sqrt{3} [2p_1(\Xi) - p_2(\Xi)] A_{8a}^c + \frac{\sqrt{15}}{3} [2p_1(\Xi) + 3p_2(\Xi)] A_{8a}^c \right. \\
\left. - \frac{2\sqrt{2}}{9} [4p_1(\Xi) + 3p_2(\Xi)] A_{27}^c - \sqrt{3} A_{8a}^v + \sqrt{15} A_{8a}^v - \frac{2\sqrt{2}}{3} A_{27}^v \right\}
\end{aligned}
\quad (V-27)$$

$$\begin{aligned}
P(\Xi^0_0) = K(\Xi^0_0) \left\{ -\frac{\sqrt{6}}{2} [2p_1(\Xi) - p_2(\Xi)] A_{8s}^c - \frac{\sqrt{30}}{6} [2p_1(\Xi) + 3p_2(\Xi)] A_{8a}^c \right. \\
\left. + \frac{2}{3} [-2p_1(\Xi) + p_2(\Xi)] A_{27}^c + \frac{\sqrt{6}}{2} A_{8s}^v - \frac{\sqrt{30}}{2} A_{8a}^v + \frac{2}{3} A_{27}^v \right\}
\end{aligned}
\tag{V-28}$$

where the constants p_i are defined as in equations (IV-46) to (IV-52). The amplitudes A_{8s}^c , A_{27}^c , A_{8s}^v etc. are related to a_{8s}^c , a_{27}^c , a_{8s}^v etc. by the same constant factor as shown in equations (IV-20) and (IV-21).

The experimental data can be fitted with six unknown SU(3) reduced matrix elements allowing for small $\Delta I = \frac{1}{2}$ rule violation. The results of a fit with the Yukawa coupling ratio $d/f = 2$ are given in Tables 3 and 4. They correspond to $A_{8s}^c = 6.612 \times 10^{-8}$, $A_{8a}^c = -9.936 \times 10^{-8}$, $A_{27}^c \approx 0$, $A_{8s}^v = 5.112 \times 10^{-8}$, $A_{8a}^v = 1.237 \times 10^{-7}$ and $A_{27}^v = 1.312 \times 10^{-7}$. On the whole the results are good to 10-20%, which is within the accuracy expected of PCAC.

Table 3 The contributions of various terms to the S-wave amplitudes and comparison with experiment for model 2 with Yukawa coupling $d/f = 2$, and spurion reduced matrix elements as in text.

Amplitude	E.T.C.	Baryon	Total	Experiment
$S(\Lambda_{-}^0) \times 10^6$	-0.27	-0.06	-0.33	-0.33
$S(\Lambda_0^0) \times 10^6$	-0.19	-0.06	-0.25	-0.23
$S(\Sigma_{+}^{+}) \times 10^6$	0	-0.01	-0.01	0
$S(\Sigma_0^{+}) \times 10^6$	0.42	-0.02	0.40	0.33
$S(\Sigma_{-}^{-}) \times 10^6$	-0.60	0.06	-0.54	-0.40
$S(\Xi_{-}^{-}) \times 10^6$	-0.50	-0.01	-0.51	-0.44
$S(\Xi_0^0) \times 10^6$	0.35	0.03	0.38	0.34

Table 4 The contributions of various terms to the P-wave amplitudes and comparison with experiment for model 2 with Yukawa coupling $d/f = 2$, and spurion reduced matrix elements as in text.

Amplitude	E.T.C.	Baryon	Total	Experiment
$P(\Lambda_-^0) \times 10^7$	0.37	0.71	1.08	1.25
$P(\Lambda_0^0) \times 10^7$	0.26	0.50	0.76	0.87
$P(\Sigma_+^+) \times 10^7$	0.51	2.91	3.42	4.02
$P(\Sigma_0^+) \times 10^7$	-0.26	2.19	1.93	2.54
$P(\Sigma_-^-) \times 10^7$	-0.12	-0.18	-0.30	-0.03(-0.21)*
$P(\Xi_-^-) \times 10^7$	-0.16	0.90	-0.74	-0.90
$P(\Xi_0^0) \times 10^7$	-0.11	0.64	0.53	0.57

*For the Σ_-^- decay the data in the parenthesis is from D. Berley et al., Phys. Rev. Letters 19, 979 (1967).

CONCLUSION

To sum up briefly. We have developed a formalism based on which decay amplitudes can be expressed in terms of the invariant amplitudes for the fictitious scattering, spurion + baryon $\rightarrow$ pion + baryon. With the momentum of spurion going to zero, the S and P partial-wave amplitudes are indeed connected with the parity violating and parity conserving decay amplitudes. The decay amplitudes are obtained by assuming a model for the term R_{μ} . We have considered the baryon pole in detail and have arrived at a formula for the decay amplitudes avoiding the usual ambiguity of the extrapolation for the P-wave decays. Two models are studied which give good agreement with experiment. Model 1 is entirely within the symmetry framework. The decuplet and Y^* intermediate states are assumed to contribute appreciably to S-wave amplitudes. The use of these intermediate states is not new in the

non-current-algebra approach.³¹ It was suggested by Hara,³² for example, to give asymmetry to the pole model of Feldman, Mathews and Salam.³³ In our model it gives an important correction to the equal time commutator terms which is all that we would have for the S-wave decays in the degenerate mass limit. The second model involves the conjecture that the parity violating matrix element between baryons is important. The basic difficulty in current-current Hamiltonian lies in the evaluation of the spurion matrix element which consists of a product of two form factors.³⁴ We have assumed that the parity violating spurion matrix elements can be written in the form of equation (V-1). There may be some mass dependence of these spurion reduced matrix elements. However since the correction coming from the parity violating matrix

³¹J. Schwinger, Phys. Rev. Letters 13, 348 (1964); 13, 500 (1964); Riasuddin, Fayyazuddin, and A. H. Zimmerman, Phys. Rev. 137, B1556 (1965).

³²Y. Hara, Phys. Rev. Letters 12, 378 (1964).

³³G. Feldman, P. T. Mathews and A. Salam, Phys. Rev. 121, 302 (1961).

³⁴See last of reference 13.

element is only 10% or so, the chance is that the overall result would not be much affected. With these remarks we wish to end this discussion. Hopefully the subject has been exposed from the point of view of current algebra and current-current interaction.

APPENDIX A

NOTATIONS AND CONVENTIONS

We have followed closely the notation used in An Introduction to Relativistic Quantum Field Theory by S. S. Schweber.³⁵ For completeness the free field equations as well as the projection operators for the spin 1/2 and 3/2 fields are presented.

A1. Metric and Units

A real metric tensor $g_{\mu\nu}$ is defined such that

$$\begin{aligned} g_{\mu\nu} = & +1 & \mu = \nu = 0 \\ & -1 & \mu = \nu = 1, 2, 3 \\ & 0 & \text{otherwise} \end{aligned} \quad (\text{A-1})$$

$$\text{and } g_{\mu\nu} = g^{\mu\nu} \quad (\text{A-2})$$

The relation between a covariant vector (p_μ) and a contravariant vector (p^μ) is given by

³⁵ S. S. Schweber, An Introduction to Relativistic Quantum Theory (Row, Peterson and Company, 1961).

$$p_\mu = g_{\mu\nu} p^\nu \quad (\text{A-3})$$

$$p^\mu = g^{\mu\nu} p_\nu \quad (\text{A-4})$$

The scalar product $x \cdot p$, for example, can be written as

$$x \cdot p = x_\mu p^\mu = g_{\mu\nu} x^\mu p^\nu = x^0 p^0 - \vec{x} \cdot \vec{p} \quad (\text{A-5})$$

We use units with $\hbar = c = 1$ so that mass = energy = time⁻¹ = length⁻¹. Thus for example, 1 MeV = 1.520 x 10²¹ sec.⁻¹.

A2. Free Field Equations

The spin 1/2 equation in the momentum representation is,

$$(\not{p} - m) u(p) = 0 \quad (\text{A-6})$$

where u is a four component spinor and $\not{p} = p_\mu \gamma^\mu$. With $\bar{u} = u^\dagger \gamma_0$ the normalization is

$$\sum \bar{u} u = \frac{p^0}{|p^0|} \quad (\text{A-7})$$

The positive energy projection operator Λ_+ is

$$\Lambda_+ = \frac{1}{2m}(\not{p} + m) \quad (\text{A-8})$$

The spin 3/2 field obeys

$$(\not{p} - m) u^\mu = 0 \quad \mu = 0, 1, 2, 3 \quad (\text{A-9})$$

$$p_\mu \cdot u^\mu = 0 \quad (\text{A-10})$$

$$\gamma_\mu \cdot u^\mu = 0 \quad (\text{A-11})$$

where the spinor indices have been suppressed. The u^μ is a sixteen component object. In the non-relativistic limit under the above conditions it reduces to four components as expected of a spin 3/2 field. As seen from equation (A-10) $u^0 = 0$ in the rest frame. Therefore the spin projection operator Θ must have the form,

$$\Theta_\mu^\nu = \delta_\mu^\nu + c \gamma^\nu \gamma_\mu \quad (\mu, \nu = 1, 2, 3) \quad (\text{A-12})$$

Using equation (A-11) $c = -1/3$. The spin projection operator Θ is to project out from an s -degree tensor the components transforming as irreducible representation of spin s ($D(s)$). Following Fronsdal³⁶ the positive

³⁶C. Fronsdal, Nuovo Cimento 9, 416 (1958).

energy projection operator is

$$\sum_{\text{spin}} u_{\mu_1, \dots, \mu_s} \bar{u}^{\nu_1, \dots, \nu_s} = \Theta_{\mu_1, \dots, \mu_s}^{\nu_1, \dots, \nu_s} \Lambda^+ \quad (\text{A-13})$$

Thus to construct such an operator all we need is to find the Lorentz invariant expression for equation (A-12).

Such an expression is

$$\Theta_{\mu}^{\nu} = \delta_{\mu}^{\nu} - \frac{1}{m^2} p^{\nu} p_{\mu} - \frac{1}{3} \gamma^{\rho} \left(\delta_{\rho}^{\nu} - \frac{1}{m^2} p^{\nu} p_{\rho} \right) \gamma_{\lambda} \left(\delta_{\mu}^{\lambda} - \frac{1}{m^2} p^{\lambda} p_{\mu} \right) \quad (\text{A-14})$$

Equation (A-13) together with equation (A-14) gives the projection operators for the decuplet we need.

APPENDIX B

WEAK INTERACTION PHENOMENOLOGY

The weak interaction can be characterized by the neutrino emission with a weak coupling constant, $G \cong 10^{-5} m_p^{-2}$, where m_p is the mass of proton. It violates many of the symmetries respected by strong interaction, such as isotopic spin (I), hypercharge (Y), charge conjugation (C) invariance and parity (P). In the following we attempt to review some aspects of leptonic and semileptonic decays with emphasis on the concept of universality.

B1. Strangeness Conserving ($\Delta S=0$) Decays

Experimental evidence has shown that the leptonic decay can be successfully described by an effective Hamiltonian of the form³⁷

$$H = \frac{G}{\sqrt{2}} j^\lambda j_\lambda^\dagger \quad (B-1)$$

³⁷ See G. Källén, Elementary Particle Physics (Addison-Wesley Publishing Co., Inc., 1964); R. E. Marshak, University of Rochester preprint, UR-875-187 (1967).

where j^λ is a sum of muon and electron currents,

$$j^\lambda = \bar{\mu} \gamma^\lambda (1 - i\gamma_5) \nu_\mu + \bar{e} \gamma^\lambda (1 - i\gamma_5) \nu_e + \text{h.c.} \quad (\text{B-2})$$

The particle symbols stand for the free field operator. Since the leptons do not interact strongly, there is no problem of renormalization and the conventional perturbation theory is adequate.

We can extend the current-current picture to study semileptonic decays. Thus for neutron β -decay, ($n \rightarrow p + e + \bar{\nu}$) the effective Hamiltonian is taken to be

$$H = \frac{G_\beta}{\sqrt{2}} j^\lambda j_\lambda^\dagger + \text{h.c.} \quad (\text{B-3})$$

where

$$j^\lambda = \bar{p} \gamma^\lambda (1 - i g_A \gamma_5) n + \text{h.c.} \quad (\text{B-4})$$

Experimentally G_β turns out to be approximately equal to G .³⁸ This suggests the possibility of universality. In fact the conserved vector current hypothesis (CVC)³⁹ is

³⁸C. S. Wu, Rev. Mod. Phys. 36, 618 (1964).

³⁹R. Feynman and M. Gell-Mann, Phys. Rev. 109, 193 (1958);
S. S. Gerstein and J. B. Zeldovitch, Soviet Physics (JETP) 2, 576 (1957).

prompted to explain the absence of renormalization for the vector coupling constant. It is postulated that J^λ belongs to the isotopic spin current corresponding to the isotopic spin raising operator I^+ . If one assumes that the weak current is a component of the same isovector which appears in the electromagnetic current and hence conserved, then one would expect G_β to be equal to G at zero momentum transfer.

Some of the direct consequences of the CVC hypothesis have been verified experimentally. For example the decay rate of $\pi^+ \rightarrow \pi^0 + e + \nu$ with a coupling constant fixed by β -decay has been compared very well with experiment. Also the weak magnetism term is correctly predicted.⁴⁰

For the axial vector current there is no reason to believe that it is divergenceless. In fact the partially conserved axial vector (PCAC) hypothesis says that it is proportional to the pion field operator. One of the great successes of current algebra is that the renormalization effect of strong interaction for the axial vector current coupling constant g_A is successfully calculated,⁴¹ so that it appears that we do have a universal theory for the $\Delta S = 0$ decays at least.

⁴⁰See reference 38.

⁴¹See reference 11.

B2. Strangeness Changing ($|\Delta S|=1$) Decays

A typical strangeness changing $|\Delta S|=1$ decay is the $\Lambda - \beta$ decay ($\Lambda \rightarrow p + e + \bar{\nu}$). The decay rate calculated on the current-current picture as given in equations (B-3) and (B-4) has come out to be an order of magnitude larger than experimental observed values.⁴² Similar calculations on other $|\Delta S|=1$ processes fail to the same extent. Cabibbo proposed the weak current for strongly interacting particles should be given by⁴³

$$J^\lambda = \cos\theta J^\lambda(\Delta S=0) + \sin\theta J^\lambda(|\Delta S|=1) \quad (\text{B-5})$$

He assumed that for the bare coupling constants the vector angle θ^V and the axial vector angle θ^A are equal. A detailed analysis of the semileptonic hyperon decays carried out by Willis et al. and recently by N. Brene and C. F. Carlson,⁴⁴ shows that the Cabibbo theory is essentially correct. The angle $\theta \approx 0.26$. Thus we still have a universal theory for both leptonic and semileptonic

⁴²See reference 4.

⁴³N. Cabibbo, Phys. Rev. Letters 10, 531 (1963).

⁴⁴See reference 26.

decays but with an extra parameter, the Cabibbo angle θ . The success of the above discussion can be summed up in the form of a weak Hamiltonian given by

$$H_w = \frac{G}{\sqrt{2}} \{ J_\mu + J_\mu^\dagger \} \quad (B-6)$$

where

$$J_\mu = j_\mu + \cos\theta J_\mu(\Delta S=0) + \sin\theta J_\mu(|\Delta S|=1) \quad (B-7)$$

B3. Nonleptonic Decays and CP Invariance

With the success of current-current interaction in leptonic and semileptonic decays, it is natural to assume that the nonleptonic decays can also be described with the Hamiltonian given in equation (B-6), that is,

$$H_w = \frac{G}{\sqrt{2}} \left\{ \cos\theta J_\mu(\Delta S=0) + \sin\theta J_\mu(|\Delta S|=1), \cos\theta J_\mu^\dagger(\Delta S=0) + \sin\theta J_\mu^\dagger(|\Delta S|=1) \right\} \quad (B-8)$$

The strangeness changing part is given by,

$$H_w^{\text{nonleptonic}}(|\Delta S|=1) = \frac{G}{\sqrt{2}} \sin\theta \cos\theta \left[\{ J_\mu(\Delta S=0), J_\mu^\dagger(|\Delta S|=1) \} + \{ J_\mu(|\Delta S|=1), J_\mu^\dagger(\Delta S=0) \} \right] \quad (B-9)$$

If we assume that these currents belong to the same octet, then the symmetric product of two identical octets $\underline{8} \times \underline{8}$ is given by the $\underline{8}$ and $\underline{27}$ irreducible representations of $SU(3)$. That is, our weak Hamiltonian for nonleptonic decay of the hyperon has the transformation property of an $\underline{8}$ and a $\underline{27}$ representations in the unitary spin space. It is the $\underline{27}$ piece which can give rise to $\Delta I = 3/2$. Hence the assumption of a pure $\Delta I = 1/2$ rule corresponds to assuming that only the octet part of the Hamiltonian contributes.

To complete this section the restriction of CP invariance and $SU(3)$ symmetry as applied to nonleptonic decays is discussed briefly. The charged mesons and baryons are not eigenstates of the charge conjugation operator C . However the notion of C parity can be generalized to an entire octet. Then a baryon octet transforms under C as $B \rightarrow CBC^{-1} = \gamma_c \bar{B}^T$, where

$$B = \begin{pmatrix} \frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda^0 & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda^0 & n \\ -\Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}} \Lambda^0 \end{pmatrix}$$

(B-10)

$$\bar{B} = \begin{pmatrix} \frac{1}{\sqrt{2}} \bar{\Sigma}^0 + \frac{1}{\sqrt{6}} \bar{\Lambda}^0 & \bar{\Sigma}^- & -\frac{1}{\sqrt{6}} \bar{\Xi}^- \\ \bar{\Sigma}^+ & -\frac{1}{\sqrt{2}} \bar{\Sigma}^0 + \frac{1}{\sqrt{6}} \bar{\Lambda}^0 & \bar{\Xi}^0 \\ \bar{p} & \bar{n} & -\frac{2}{\sqrt{6}} \bar{\Lambda}^0 \end{pmatrix} \quad (\text{B-11})$$

and

$$\gamma_c(O^-) = \gamma_c(A) = 1 \quad (\text{B-12})$$

$$\gamma_c(1^-) = \gamma_c(V) = -1 \quad (\text{B-13})$$

The product of two octets G and H under the extended charge conjugation operation transforms as

$$G H \rightarrow \gamma_c(G) \gamma_c(H) G^T H^T \quad (\text{B-14})$$

so that

$$[G, H]_+ \rightarrow \gamma_c(G) \gamma_c(H) [G, H]_+^T \quad (\text{B-15})$$

$$[G, H]_- \rightarrow -\gamma_c(G) \gamma_c(H) [G, H]_-^T \quad (\text{B-16})$$

Hence, for a symmetric product of G and H the C parity is $\eta_c(G)\eta_c(H)$. The nonleptonic current-current Hamiltonian can be separated into parity conserving and parity violating parts. Symbolically $H_W = H_W^C + H_W^V$ with,

$$H_W^C = \frac{G}{\sqrt{2}} \sin\theta \cos\theta \left[\{V, V^\dagger\} + \{A, A^\dagger\} \right] \quad (B-17)$$

$$H_W^V = \frac{G}{\sqrt{2}} \sin\theta \cos\theta \left[\{V, A^\dagger\} + \{A, V^\dagger\} \right] \quad (B-18)$$

where V and A are the appropriate vector and axial vector currents. From our previous argument, it follows,

$$\eta_c(H_W^C) = 1 \quad (B-19)$$

$$\eta_c(H_W^V) = -1 \quad (B-20)$$

and H_W is CP invariant. The above discussion on charge conjugation properties of the nonleptonic weak Hamiltonian has two interesting consequences:

$$(1) \quad \langle 2\pi | H_W^V | K \rangle = 0 \quad (B-21)$$

$$(2) \quad \langle B \bar{B} | H_W^V | 0 \rangle = 0 \quad (B-22)$$

Since the symmetric (8s, 27) baryon-antibaryon combination has $\eta_c = 1$, the second relation means that the baryon pole contributes only to P-wave hyperon decays. At this point it should perhaps be emphasized that such relations hold only in the exact SU(3) limit when all mesons and baryons appear symmetrically. When mass splitting among baryons and mesons are considered effective coupling such as $\bar{B} \gamma_\mu (A + i\gamma_5 B) B \gamma^\mu M$ contributes proportionally to the baryon mass differences.

As an illustration of equation (B-22), one can define $\lambda_i^T = \epsilon_i \lambda_i$. Then under charge conjugation,

$$\begin{aligned}
 C(d_{ijk} \bar{B}^j B^k) C^{-1} &= - d_{ijk} \epsilon_j B^j \epsilon_k \bar{B}^k \\
 &= - \epsilon_i d_{ijk} B^j \bar{B}^k \\
 &= \epsilon_i d_{ijk} \bar{B}^k B^j \\
 &= \epsilon_i (d_{ijk} \bar{B}^j B^k)
 \end{aligned} \tag{B-23}$$

where use has been made of the identity

$$\epsilon_i \epsilon_j \epsilon_k d_{ijk} = d_{ijk} \tag{B-24}$$

Equation (B-23) simply says η_c for the symmetric product 8s is 1 as asserted.

APPENDIX C

PARTIAL WAVE DECOMPOSITION

The scattering amplitude for the process $p_\alpha + k \rightarrow p_\beta + q$ can be written as

$$T(s, t, q^2, k^2) = \bar{u}(p_\beta) \left[A - \frac{1}{2} \gamma \cdot (q+k) B + i \gamma_5 C - \frac{1}{2} i \gamma \cdot (q+k) \gamma_5 D \right] u(p_\alpha) \quad (C-1)$$

With the definition

$$F = \frac{\sqrt{m_\alpha m_\beta}}{4\pi W} T \quad (C-2)$$

the differential cross-section in the center of mass coordinate is

$$\frac{d\sigma}{dr} = \frac{|\vec{q}|}{|\vec{k}|} |F|^2 \quad (C-3)$$

Some of the useful kinematics in this coordinate system are,

$$s = (p_\alpha + k)^2 = W^2 \quad (C-4)$$

$$u = (p_\beta - k)^2 \quad (C-5)$$

$$t = (p_\alpha - p_\beta)^2 = q^2 + k^2 - 2q_0 k_0 + 2|\vec{q}||\vec{k}| \cos\theta \quad (C-6)$$

$$|\vec{q}|^2 = \frac{[(W+m_\beta)^2 - q^2][(W-m_\beta)^2 - q^2]}{4W^2} \quad (C-7)$$

$$|\vec{k}|^2 = \frac{[(W+m_\alpha)^2 - k^2][(W-m_\alpha)^2 - k^2]}{4W^2} \quad (C-8)$$

$$E_\alpha = \frac{W^2 + m_\alpha^2 - k^2}{2W} \quad (C-9)$$

$$E_\beta = \frac{W^2 + m_\beta^2 - q^2}{2W} \quad (C-10)$$

In terms of the Pauli spinor

$$F = \chi^\dagger \left[f_1 + (\vec{\sigma} \cdot \hat{q})(\vec{\sigma} \cdot \hat{k}) f_2 + (\vec{\sigma} \cdot \hat{q}) f_3 + (\vec{\sigma} \cdot \hat{k}) f_4 \right] \chi \quad (C-11)$$

The relations between f and the invariant amplitudes can be work out easily as given in equations (II-9) to (II-12). A partial-wave decomposition is presented below for both

the parity conserving and parity violating amplitudes.⁴⁵

For the parity violating amplitude,

$$f(\cos\theta) = \sum_{\ell} (2\ell+1) \left[f_{\ell+} \wedge_{J=\ell+\frac{1}{2}} + f_{\ell-} \wedge_{J=\ell-\frac{1}{2}} \right] P_{\ell}(\cos\theta) \quad (C-12)$$

where the projection operators,

$$\wedge_{J=\ell+\frac{1}{2}} = \frac{(\ell+1) + \vec{\sigma} \cdot \vec{L}}{2\ell+1} \quad (C-13)$$

$$\wedge_{J=\ell-\frac{1}{2}} = \frac{\ell - \vec{\sigma} \cdot \vec{L}}{2\ell+1} \quad (C-14)$$

are constructed with the help of the identities

$$J(J+1) = \ell(\ell+1) + \frac{3}{4} + \vec{\sigma} \cdot \vec{L} \quad (C-15)$$

Using

$$\begin{aligned} (\vec{\sigma} \cdot \vec{L}) P_{\ell}(\cos\theta) &= \vec{\sigma} \cdot (\vec{r} \times \vec{q}) P_{\ell}(\hat{q} \cdot \hat{k}) \\ &= -i \vec{\sigma} \cdot (\hat{q} \times \hat{k}) P'_{\ell}(\cos\theta) \\ &= - \left[(\vec{\sigma} \cdot \hat{q})(\vec{\sigma} \cdot \hat{k}) - \hat{q} \cdot \hat{k} \right] P'_{\ell}(\cos\theta) \end{aligned} \quad (C-16)$$

⁴⁵M. L. Goldberger and K. M. Watson, Collision Theory (John Wiley and Sons Inc., 1964).

$$(\ell+1)P_\ell(\cos\theta) + \cos\theta P'_\ell(\cos\theta) = P'_{\ell+1}(\cos\theta) \quad (C-17)$$

$$\ell P_\ell(\cos\theta) - \cos\theta P'_\ell(\cos\theta) = -P'_{\ell-1}(\cos\theta) \quad (C-18)$$

We have

$$f(\cos\theta) = \sum_{\ell} \left\{ \left[f_{\ell+} P'_{\ell+1}(\cos\theta) - f_{\ell-} P'_{\ell-1}(\cos\theta) \right] + (\vec{\sigma} \cdot \hat{q})(\vec{\sigma} \cdot \hat{k})(f_{\ell-} - f_{\ell+}) P'_\ell(\cos\theta) \right\} \quad (C-19)$$

Relating this to the parity violating amplitude

$$f(\cos\theta) = f_1 + (\vec{\sigma} \cdot \hat{q})(\vec{\sigma} \cdot \hat{k}) f_2 \quad (C-20)$$

gives

$$f_1 = \sum_{\ell} \left[f_{\ell+} P'_{\ell+1}(\cos\theta) - f_{\ell-} P'_{\ell-1}(\cos\theta) \right] \quad (C-21)$$

$$f_2 = \sum_{\ell} \left[f_{\ell-} - f_{\ell+} \right] P'_\ell(\cos\theta) \quad (C-22)$$

The above equations can be inverted with the help of the following identities:

$$\frac{1}{2} \int_{-1}^1 dz P'_{\ell+1}(z) P_\ell(z) = 1 \quad (C-23)$$

$$\frac{1}{2} \int_{-1}^1 dz P'_{\ell+2}(z) P_\ell(z) = 0 \quad (C-24)$$

$$\int_{-1}^1 dz P_n'(z) P_\ell(z) = 0 \quad \text{for } n < \ell+1 \quad (\text{C-25})$$

etc. to give

$$f_{\ell\pm} = \frac{1}{2} \int_{-1}^1 dz \left[f_1 P_\ell(z) + f_2 P_{\ell\pm 1}(z) \right] \quad (\text{C-26})$$

Similarly for the parity conserving amplitudes we have

$$\begin{aligned} f &= \sum_{\ell} (2\ell+1) (\vec{\sigma} \cdot \hat{q}) \left[f_{\ell+} \wedge_{J=\ell+\frac{1}{2}} + f_{\ell-} \wedge_{J=\ell-\frac{1}{2}} \right] P_\ell(\cos\theta) \\ &= (\vec{\sigma} \cdot \hat{q}) f_3 + (\vec{\sigma} \cdot \hat{k}) f_4 \end{aligned} \quad (\text{C-27})$$

Repeating the same procedure we arrive at the following result:

$$f_{\ell\pm} = \frac{1}{2} \int f_3 P_\ell(z) dz + \frac{1}{2} \int f_4 P_{\ell\pm 1}(z) dz \quad (\text{C-28})$$

Thus the total amplitude can be given by,

$$f_{\ell\pm} = \frac{1}{2} \int_{-1}^1 dz \left[(f_1+f_4) P_\ell(z) + (f_2+f_3) P_{\ell\pm 1}(z) \right] \quad (\text{C-29})$$

To obtain the S-wave and P-wave amplitudes we use the

connection between f_1, f_2, f_3, f_4 and A, B, C, D ; and the formulae

$$A = \frac{1}{2} \int_{-1}^1 dz P_\ell(z) A(s, t) \quad (C-30)$$

$$A_0(s) = \frac{1}{2} \int_{-1}^1 dz A(s, t) = A(s, 0) \quad (C-31)$$

$$A_1(s) = \frac{1}{2} \int_{-1}^1 dz z A(s, t) = \frac{2}{3} |\vec{q}| |\vec{k}| \left(\frac{\partial A}{\partial t} \right)_{\substack{q^2=0 \\ k=0}} \quad (C-32)$$

At the point of decay corresponding to $q^2=0$, $k=0$, and $s=m_\alpha^2$, the relevant partial-wave amplitudes are given by

$$\begin{aligned} f_{s/2} \xrightarrow{\text{decay}} & \frac{1}{16\pi m_\alpha^2} \left\{ 2m_\alpha \left[(m_\alpha + m_\beta)^2 - q^2 \right]^{1/2} \left[A(0) - \frac{m_\alpha - m_\beta}{2} B(0) \right] \right. \\ & - \frac{4m_\alpha |\vec{q}|^2 |\vec{k}|^2}{3 \left[(m_\alpha + m_\beta)^2 - q^2 \right]^{1/2}} \left[\frac{\partial A(0)}{\partial t} + \frac{3m_\alpha + m_\beta}{2} \frac{\partial B(0)}{\partial t} \right] \\ & - \left[(m_\alpha + m_\beta)^2 - q^2 \right]^{1/2} |\vec{k}| \left[C(0) - \frac{3m_\alpha - m_\beta}{2} D(0) \right] \\ & \left. + \frac{8m_\alpha^2 |\vec{q}|^2 |\vec{k}|}{3 \left[(m_\alpha + m_\beta)^2 - q^2 \right]^{1/2}} \left[\frac{\partial C(0)}{\partial t} + \frac{m_\alpha + m_\beta}{2} \frac{\partial D(0)}{\partial t} \right] \right\} \quad (C-33) \end{aligned}$$

$$\begin{aligned}
f_{p/2} \xrightarrow{\text{decay}} & \frac{1}{16\pi m_\alpha^2} \left\{ \frac{4}{3} m_\alpha \left[(m_\alpha + m_\beta)^2 - q^2 \right]^{1/2} |\vec{q}| |\vec{k}| \left[\frac{\partial A(0)}{\partial t} - \frac{m_\alpha - m_\beta}{2} \frac{\partial B(0)}{\partial t} \right] \right. \\
& - \frac{2 m_\alpha |\vec{q}| |\vec{k}|}{\left[(m_\alpha + m_\beta)^2 - q^2 \right]^{1/2}} \left[A(0) + \frac{3 m_\alpha + m_\beta}{2} B(0) \right] \\
& - \frac{2}{3} |\vec{q}| |\vec{k}|^2 \left[(m_\alpha + m_\beta)^2 - q^2 \right]^{1/2} \left[\frac{\partial C(0)}{\partial t} - \frac{3 m_\alpha - m_\beta}{2} \frac{\partial D(0)}{\partial t} \right] \\
& \left. + \frac{4 m_\alpha^2 |\vec{q}|}{\left[(m_\alpha + m_\beta)^2 - q^2 \right]^{1/2}} \left[C(0) + \frac{m_\alpha + m_\beta}{2} D(0) \right] \right\} \\
& \qquad \qquad \qquad (C-34)
\end{aligned}$$

with $\vec{k} = 0$ the surviving terms are as given in equations (II-16) and (II-17).

APPENDIX D

CALCULATION OF EXPERIMENTAL DATA

D1. The Decay Rate

The decay rate is given by

$$\Gamma = \prod_{\text{initial}} \frac{\epsilon_i}{2E_i} \int \prod_{\text{final}} \frac{\epsilon_f}{(2\pi)^3 2E_f} d^3 p_f (2\pi)^4 \delta^4(p_f - p_i) \cdot (\langle f | T | i \rangle)^2 \quad (\text{D-1})$$

where $\epsilon = 1$ for bosons and $\epsilon = 2m$ for fermions. The covariant matrix element T_{if} is a Lorentz invariant function of its argument. In our case $\alpha \rightarrow \beta \pi^1$, the formula reduces to

$$\Gamma = \frac{m_\alpha m_\beta}{2E_\alpha} \frac{1}{(2\pi)^2} \int \frac{d^3 p_\beta d^3 q}{E_\beta q_0} \delta^4(p_\alpha - p_\beta - q) |T_{if}|^2 \quad (\text{D-2})$$

The evaluation of the phase space integral is particularly simple for two body final state. Using

$$\frac{d^3 p_\beta}{E_\beta} = 2 \delta(p_\beta^2 - m_\beta^2) d^4 p_\beta \quad (D-3)$$

we have in the rest frame of the initial state,

$$\begin{aligned} \Gamma &= \frac{m_\beta}{(2\pi)^2} \int \frac{d^3 q}{q_0} \delta(m_\alpha^2 + m_\pi^2 - m_\beta^2 - 2m_\alpha q_0) |T_{if}|^2 \\ &= \frac{m_\beta}{(2\pi)^2} \int d\Omega dq_0 |\vec{q}| \delta(m_\alpha^2 + m_\pi^2 - m_\beta^2 - 2m_\alpha q_0) |T_{if}|^2 \\ &= \frac{m_\beta}{2\pi m_\alpha} |\vec{q}| |T_{if}|^2 \end{aligned} \quad (D-4)$$

where

$$|\vec{q}| = \frac{[(m_\alpha + m_\pi)^2 - m_\beta^2]^{\frac{1}{2}} [(m_\alpha - m_\pi)^2 - m_\beta^2]^{\frac{1}{2}}}{2m_\alpha} \quad (D-5)$$

Our covariant T matrix can be put into the form of

$\bar{u}(p_\beta) [A' + i\gamma_5 B'] u(p_\alpha)$. Thus averaging over spin

$$\begin{aligned} \frac{1}{2} \sum |T_{if}|^2 &= \frac{1}{2} \text{Tr} \left\{ (A' + i\gamma_5 B') \left(\frac{\not{p}_\alpha + m_\alpha}{2m_\alpha} \right) (A'^* - i\gamma_5 B'^*) \left(\frac{\not{p}_\beta + m_\beta}{2m_\beta} \right) \right\} \\ &= \frac{1}{2m_\alpha m_\beta} \left\{ A' A'^* (p_\alpha \cdot p_\beta + m_\alpha m_\beta) + B' B'^* (p_\alpha \cdot p_\beta - m_\alpha m_\beta) \right\} \end{aligned} \quad (D-6)$$

Substituting into equation (D-4) gives the decay rate:

$$\Gamma(\alpha \rightarrow \beta \pi^1) = \frac{E_{\beta+m\beta}}{4\pi m_\alpha} \left\{ A'A'^* + B'B'^* \left(\frac{E_{\beta-m\beta}}{E_{\beta+m\beta}} \right) \right\} |\vec{q}| \quad (D-7)$$

In terms of Pauli spinor the covariant T matrix element can be written in the form of

$$T(\alpha \rightarrow \beta \pi^1) \sim \chi^\dagger \{ S + P (\vec{\sigma} \cdot \hat{q}) \} \chi \quad (D-8)$$

$$\text{where} \quad S = A' \quad (D-9)$$

$$P = \frac{|\vec{q}|}{E_{\beta+m\beta}} B' \quad (D-10)$$

If we average over the initial states allowing for the possibility of polarization, then

$$|T_{if}|^2 \sim \chi_f^\dagger \int \chi_f \quad (D-11)$$

where the final state density matrix $\int$ is given by,

$$\int = (S + P \vec{\sigma} \cdot \hat{q}) \frac{1}{2} (1 + \vec{\sigma} \cdot \vec{\Sigma}) (S^* + P^* \vec{\sigma} \cdot \hat{q}) \quad (D-12)$$

$\vec{\Sigma}$ being the polarization of the initial state. Simplifying equation (D-12)

$$\rho \propto \frac{1}{2} \left\{ (1 - \alpha \vec{\Sigma} \cdot \hat{q}) + \vec{\sigma} \cdot \left[(\vec{\Sigma} \cdot \hat{q} - \alpha) \hat{q} + \beta \vec{\Sigma} \times \hat{q} + \gamma (\vec{\Sigma} - \hat{q} \hat{q} \cdot \vec{\Sigma}) \right] \right\} \quad (D-13)$$

where the symmetry parameters are defined as,

$$\alpha = - \frac{2 \operatorname{Re} S^* P}{|S|^2 + |P|^2} \quad (D-14)$$

$$\gamma = \frac{|S|^2 - |P|^2}{|S|^2 + |P|^2} \quad (D-15)$$

$$\beta = \frac{2 \operatorname{Im} S^* P}{|S|^2 + |P|^2} \quad (D-16)$$

$$\text{where } \alpha^2 + \beta^2 + \gamma^2 = 1 \quad (D-17)$$

The parameter α can be measured as the longitudinal polarization of the decay baryons from unpolarized parents. This can be done in effect by averaging over $\vec{q}$ of the decay. A practical way of measuring α is from the angular distribution of the decay products if the polarization of the parent is known. The parameter γ can be obtained by considering decays in which $\vec{q}$ is at right angle to $\vec{\Sigma}$ and measuring the final baryon polarization in the direction

of $\vec{\Sigma}$ as indicated by the last term in equation (D-13). Similarly the parameter β can be obtained by measuring the final polarization perpendicular to the original polarization and $\vec{q}$.

The experimental data as shown in Table 5 below are taken from Bludman's Cargèse Lectures (1966). The S-wave and P-wave amplitudes have been computed with the help of equations (D-7) and (D-14). The signs of the amplitudes are not without ambiguity.

Table 5 Experimental Results

Decay	Rate $\times 10^{-10}$	α	$S \times 10^6$	$P \times 10^7$
$\Lambda \rightarrow p\pi^-$	0.2652	0.6633	-0.33	1.25
$\Sigma^+ \rightarrow n\pi^+$	0.5828	0.0085	0	4.02
$\Sigma^+ \rightarrow p\pi^0$	0.6512	-0.9600	0.33	2.54
$\Sigma^- \rightarrow n\pi^-$	0.6040	-0.0166	-0.40	-0.03
$\Xi^- \rightarrow \Lambda\pi^-$	0.5720	-0.3906	-0.44	-0.90

Some of the more recent data are the following:

$$\begin{aligned} \alpha(\Lambda \rightarrow p\pi^-) &= 0.65 \pm 0.02 \text{ and } \gamma(\Lambda \rightarrow p\pi^-) = 0.75 \pm 0.02;^{46} \\ \alpha(\Sigma^- \rightarrow n\pi^-) &= -0.017 \pm 0.042, \alpha(\Sigma^+ \rightarrow n\pi^+) = -0.006 \pm 0.043, \\ \text{and } \alpha(\Sigma^+ \rightarrow p\pi^0) &= -0.960 \pm 0.067;^{47} \alpha(\Sigma^- \rightarrow n\pi^-) = -0.104 \pm 0.04 \\ \text{and } \gamma(\Sigma^- \rightarrow n\pi^-) &\approx 1.^{48} \end{aligned}$$

⁴⁶O. E. Overseth and R. F. Roth, Phys. Rev. Letters 19, 391 (1967).

⁴⁷Roger O. Bangester, Angela Barbaro-Galtieri, J. Peter Berge, Joseph J. Murray, Frank T. Solmitz, M. Lynn Stevenson and Robert D. Tripp, Phys. Rev. Letters 17, 495 (1967).

⁴⁸D. Berley; S. Hertzbach, R. Kofler, G. Meisner, J. B. Shafer, and S. Yamamoto; W. Heintzelman, M. Schiff, J. Thompson, and W. Willis, Phys. Rev. Letters 19, 979 (1967).

APPENDIX E

SU(3) CLEBSCH-GORDAN COEFFICIENTS

The purpose of this appendix is to explain the notations used in connection with our problem. For the subject matter itself there are many good texts and literature.⁴⁹ We follow closely the notations and convention of de Swart's article.⁵⁰

An irreducible representation of SU(3) is denoted

⁴⁹F. E. Low, "Symmetries and Elementary Particles", and B. W. Lee, "SU(6) in Particle Physics", Particle Symmetries, Brandeis Lectures 1965, edited by M. Chrétien and S. Deser (Gordon and Breach Science Publishers, 1965); G. C. Wick, "Group Theory, Invariance Principle, Symmetries", and R. H. Dalitz, "Quark Models for the 'Elementary Particles'", High Energy Physics, Les Houches 1965, edited by C. DeWitt and M. Jacob (Gordon and Breach Science Publishers, 1965); M. Gourdin, Unitary Symmetries and their Application to High Energy Physics (North-Holland, Amsterdam, Interscience, New York, 1967); Harry J. Lipkin, Lie Groups for Pedestrians (John Wiley and Sons, Inc., 1965); C. Fronsdal, "Group Theory and Applications to Particle Physics", Elementary Particle Physics and Field Theory, Brandeis Lectures 1962, (W. A. Benjamin, Inc., 1963); S. Gasiorowicz, Elementary Particle Physics (John Wiley and Sons, Inc., 1966); L. C. Biedenharn, "Group Theory and the Classification of the Elementary Particles", CERN (1965).

⁵⁰See reference 22.

by $D(p,q)$ or simply by the dimension of the representation; for example, 8 stands for the irreducible representation $D(1,1)$, in which the dimension is eight. The integers p and q indicate the number of the upper and lower indices of an irreducible tensor representation of $SU(3)$. The dimension N of $D(p,q)$, that is, the number of basis vectors is⁵¹

$$N = (1 + p)(1 + q) \left[1 + \frac{1}{2} (p + q) \right] \quad (E-1)$$

The product representation of two irreducible representation can be symbolically written

$$D(p_1, q_1) \otimes D(p_2, q_2) = \sum_{P, Q} \oplus \sigma(P, Q) D(P, Q) \quad (E-2)$$

where $\sigma(P, Q)$ is a number of times which $D(P, Q)$ is contained in the direct product of $D(p_1, q_1)$ and $D(p_2, q_2)$. If we denote the eigenvalues of the Casimir operator collectively by μ and the eigenvalues of (Y, I, I_z) by β , we can write the eigenstate of the representation $D(p_1, q_1)$ as $\phi_{\beta_1}^{(\mu_1)}$.

⁵¹See F. E. Low, reference 49.

Then the eigenstate in the product representation (ϕ) will be given by

$$\psi \begin{pmatrix} \mu_1 \mu_2 \mu_\gamma \\ \beta \end{pmatrix} = \sum_{\beta_1, \beta_2} \begin{pmatrix} \mu_1 \mu_2 \mu_\gamma \\ \beta_1 \beta_2 \beta \end{pmatrix} \phi_{\beta_1}^{(\mu_1)} \phi_{\beta_2}^{(\mu_2)} \quad (\text{E-3})$$

The coefficients $\begin{pmatrix} \mu_1 \mu_2 \mu_\gamma \\ \beta_1 \beta_2 \beta \end{pmatrix}$ are the Clebsch-Gordan coefficients of SU(3).

A few of the useful properties of the Clebsch-Gordan coefficients are the orthogonal relations

$$\sum_{\beta_1, \beta_2} \begin{pmatrix} \mu_1 \mu_2 \mu_\gamma \\ \beta_1 \beta_2 \beta \end{pmatrix} \begin{pmatrix} \mu_1 \mu_2 \mu_{\gamma'} \\ \beta_1 \beta_2 \beta' \end{pmatrix} = \delta_{\mu\mu'} \delta_{\beta\beta'} \delta_{\gamma\gamma'} \quad (\text{E-4})$$

$$\sum_{\mu, \gamma, \beta} \begin{pmatrix} \mu_1 \mu_2 \mu_\gamma \\ \beta_1 \beta_2 \beta \end{pmatrix} \begin{pmatrix} \mu_1 \mu_2 \mu_\gamma \\ \beta'_1 \beta'_2 \beta \end{pmatrix} = \delta_{\beta_1 \beta'_1} \delta_{\beta_2 \beta'_2} \quad (\text{E-5})$$

and the symmetry properties such as

$$\begin{pmatrix} \mu_1 \mu_2 \mu_\gamma \\ \beta_1 \beta_2 \beta \end{pmatrix} = \sum_1 \begin{pmatrix} \mu_2 \mu_1 \mu_\gamma \\ \beta_2 \beta_1 \beta \end{pmatrix} \quad (\text{E-6})$$

$$\begin{pmatrix} \mu_1 & \mu_2 & \mu_3 \\ \beta_1 & \beta_2 & \beta \end{pmatrix} = \xi_3 \begin{pmatrix} \mu_1^* & \mu_2^* & \mu_3^* \\ -\beta_1 & -\beta_2 & -\beta \end{pmatrix} \quad (\text{E-7})$$

where the phase factors ξ_1 and ξ_3 are either +1 or -1 depending on μ_1, μ_2 and μ , and are given in de Swart's article.

Table 6 SU(3) Clebsch-Gordan Coefficients for the Baryon-Meson Coupling Constants.

Decay	Intermediate States		$g_{\beta\delta}$	$g_{\gamma\alpha}$
$\alpha \rightarrow \beta \pi^1$	δ	γ	g	g
$\Lambda \rightarrow p \pi^-$	n	Σ^+	$-\frac{1}{\sqrt{2}} (f+d)$	$\frac{1}{\sqrt{3}} d$
$\Lambda \rightarrow n \pi^0$	n	Σ^0	$-\frac{1}{2} (f+d)$	$\frac{1}{\sqrt{3}} d$
$\Sigma^+ \rightarrow p \pi^+$	p	Λ^0	$\frac{1}{\sqrt{2}} (f+d)$	$-\frac{1}{\sqrt{3}} d$
		Σ^0		f
$\Sigma^+ \rightarrow p \pi^0$	p	Σ^+	$\frac{1}{2} (f+d)$	f
$\Sigma^- \rightarrow n \pi^-$		Λ^0		$-\frac{1}{\sqrt{3}} d$
		Σ^0		- f
$\Xi^- \rightarrow \Lambda \pi^-$	Σ^-	Ξ^0	$-\frac{1}{\sqrt{3}} d$	$\frac{1}{\sqrt{2}} (d-f)$
$\Xi^0 \rightarrow \Lambda \pi^0$	Σ^0	Ξ^0	$\frac{1}{\sqrt{3}} d$	$-\frac{1}{2} (d-f)$

Table 7 SU(3) Clebsch-Gordan Coefficients for the Spurious Matrix Elements between Two Baryon Octets.

Decay	$S_{\delta\alpha}$			$S_{\beta\gamma}$		
	a_{8s}	a_{8a}	a_{27}	a_{8s}	a_{8a}	a_{27}
$\Lambda \rightarrow p \pi^-$	$-\frac{3}{10\sqrt{6}}$	$-\frac{3}{2\sqrt{30}}$	$-\frac{1}{15}$	$-\frac{3}{10}$	$\frac{1}{2\sqrt{5}}$	$\frac{4}{15\sqrt{6}}$
$\Lambda \rightarrow n \pi^0$	$-\frac{3}{10\sqrt{6}}$	$-\frac{3}{2\sqrt{30}}$	$-\frac{1}{15}$	$-\frac{3}{10\sqrt{2}}$	$\frac{1}{2\sqrt{10}}$	$-\frac{1}{5\sqrt{3}}$
$\Sigma^+ \rightarrow p \pi^+$	$-\frac{3}{10}$	$\frac{1}{2\sqrt{5}}$	$\frac{4}{15\sqrt{6}}$	$-\frac{3}{10\sqrt{6}}$	$-\frac{3}{2\sqrt{30}}$	$-\frac{1}{15}$
				$-\frac{3}{10\sqrt{2}}$	$\frac{1}{2\sqrt{10}}$	$-\frac{1}{5\sqrt{3}}$
$\Sigma^+ \rightarrow p \pi^0$	$-\frac{3}{10}$	$\frac{1}{2\sqrt{5}}$	$\frac{4}{15\sqrt{6}}$	$-\frac{3}{10}$	$\frac{1}{2\sqrt{5}}$	$\frac{4}{15\sqrt{6}}$
$\Sigma^- \rightarrow n \pi^-$				$-\frac{3}{10\sqrt{6}}$	$-\frac{3}{2\sqrt{30}}$	$-\frac{1}{15}$
				$-\frac{3}{10\sqrt{2}}$	$\frac{1}{2\sqrt{10}}$	$-\frac{1}{5\sqrt{3}}$
$\Xi^- \rightarrow \Lambda \pi^-$	$-\frac{3}{10}$	$-\frac{1}{2\sqrt{5}}$	$\frac{4}{15\sqrt{6}}$	$-\frac{3}{10\sqrt{6}}$	$\frac{3}{2\sqrt{30}}$	$-\frac{1}{15}$
$\Xi^0 \rightarrow \Lambda \pi^0$	$-\frac{3}{10\sqrt{2}}$	$-\frac{1}{2\sqrt{10}}$	$-\frac{1}{5\sqrt{3}}$	$-\frac{3}{10\sqrt{6}}$	$\frac{3}{2\sqrt{30}}$	$-\frac{1}{15}$

Table 8 SU(3) Clebsch-Gordan Coefficients for the Decuplet Pole.

Decay	Intermediate States		$g_{\beta\delta^*}$	$g_{\delta\alpha}^*$		$g_{\gamma\alpha}^*$	$s_{\beta\gamma^*}$	
	δ^*	γ^*	g^*	b_8	b_{27}	g^*	b_8	b_{27}
$\Lambda \rightarrow p \pi^-$	Δ^0	γ^{*+}	$\frac{1}{\sqrt{6}}$	0	$\frac{5}{12\sqrt{5}}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{5}}$	$-\frac{2}{3\sqrt{30}}$
$\Lambda \rightarrow n \pi^0$	Δ^0	γ^{*0}	$-\frac{1}{\sqrt{3}}$	0	$\frac{5}{12\sqrt{5}}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{10}}$	$\frac{1}{2\sqrt{15}}$
$\Sigma^+ \rightarrow p \pi^+$	Δ^+	γ^{*0}	$\frac{1}{\sqrt{6}}$	$\frac{1}{2\sqrt{5}}$	$\frac{2}{3\sqrt{30}}$	$\frac{1}{\sqrt{12}}$	$-\frac{1}{2\sqrt{10}}$	$\frac{1}{2\sqrt{15}}$
$\Sigma^+ \rightarrow n \pi^0$	Δ^+	γ^{*+}	$-\frac{1}{\sqrt{3}}$	$\frac{1}{2\sqrt{5}}$	$\frac{2}{3\sqrt{30}}$	$\frac{1}{\sqrt{12}}$	$-\frac{1}{2\sqrt{5}}$	$-\frac{2}{3\sqrt{30}}$
$\Sigma^- \rightarrow n \pi^-$	Δ^-	γ^{*0}	$\frac{1}{\sqrt{2}}$	$\frac{3}{2\sqrt{15}}$	$-\frac{1}{6\sqrt{10}}$	$\frac{1}{\sqrt{12}}$	$-\frac{1}{2\sqrt{10}}$	$\frac{1}{2\sqrt{15}}$
$\Xi^- \rightarrow \Lambda \pi^-$	γ^{*-}	Ξ^{*0}	$\frac{1}{2}$	$\frac{1}{2\sqrt{5}}$	$-\frac{1}{6\sqrt{30}}$	$-\frac{1}{\sqrt{6}}$	$-\frac{3}{2\sqrt{30}}$	$\frac{1}{12\sqrt{5}}$
$\Xi^0 \rightarrow \Lambda \pi^0$	γ^{*0}	Ξ^{*0}	$-\frac{1}{2}$	$\frac{1}{2\sqrt{10}}$	$-\frac{1}{2\sqrt{15}}$	$\frac{1}{\sqrt{12}}$	$-\frac{3}{2\sqrt{30}}$	$\frac{1}{12\sqrt{5}}$

APPENDIX F

CALCULATION OF $Y_0^*(1405)$ CONTRIBUTION

The computation of the intermediate state $Y_0^*(1405)$ contribution to the invariant amplitudes can be carried out if we define the relevant vertices as

$$\langle p_\delta | H_W(0) | p_\alpha \rangle = \frac{1}{(2\pi)^3} \sqrt{\frac{m_\delta}{E_\delta}} \sqrt{\frac{m_\alpha}{E_\alpha}} \bar{u}(p_\delta) (S_{\delta\alpha}^c i\gamma_5 + S_{\delta\alpha}^v) u(p_\alpha) \quad (F-1)$$

$$\langle p_\beta | A_\mu^i(0) | p_\delta \rangle = \frac{1}{(2\pi)^3} \sqrt{\frac{m_\beta}{E_\beta}} \sqrt{\frac{m_\delta}{E_\delta}} \bar{u}(p_\beta) \left(g_A^*(q^2) \right)_{\beta\delta} \gamma_\mu u(p_\delta) \quad (F-2)$$

Assuming R_μ (which is defined in equations (II-4) and (II-5)) obeys unsubtracted dispersion relation, we have

$$q_\mu R_\mu = \frac{1}{i(2\pi)^3} \sqrt{\frac{m_\alpha}{E_\alpha}} \sqrt{\frac{m_\beta}{E_\beta}} \bar{u}(p_\beta) \left\{ (S_{\beta\alpha}^c i\gamma_5 + S_{\beta\alpha}^v) (g_A^*)_{\gamma\alpha} \left(\frac{p_\alpha^2 - 4 + m_\delta^*}{m_\gamma^{*2} - u} \right) \right\} u(p_\alpha) \quad (F-3)$$

Y_0^* contributes only to the u-channel. The detail structure of the spurion is immaterial, since the SU(3) Clebsch-Gordan coefficients are the same for Σ_+^+ and Σ_-^- decays which are the only decays that Y_0^* can contribute. The results of its contribution to the S-wave and P-wave amplitudes, denoted by y_s and y_p respectively, are

$$y_s = -\frac{\mu^2}{C_\eta} \left(1 - \frac{m_\alpha - m_\gamma^*}{m_\beta - m_\gamma^*} \right) S_{\beta\gamma^*}^v (g_A^*)_{\gamma\alpha} \quad (F-4)$$

and

$$y_p = -\frac{\mu^2}{C_\eta} K \left(1 + \frac{m_\alpha - m_\gamma^*}{m_\beta + m_\gamma^*} \right) S_{\beta\gamma^*}^c (g_A^*)_{\gamma\alpha} \quad (F-5)$$

where K is defined in equation (II-21).

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