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MAGNETIC COMPRESSION OF AXIALLY SYMMETRIC
BRILLOUIN-FOCUSED ELECTRON BEAMS

DISSERTATION

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The Ohio State University

By

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* * * * *

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CHAPTER I

INTRODUCTION

General

Many types of electron devices require an axially symmetric electron beam of high current density. High current density beams have also found application in electron beam welding and in plasma-beam interaction experiments. Because of cathode emission limitations, it is usually necessary to compress the beam, that is, to reduce the beam diameter from its initial diameter at the cathode. Besides reducing cathode loading, beam compression makes possible larger electron gun sizes, an advantage for high frequency tubes in particular. Beam compression is often obtained by using a convergent Pierce type electron gun.¹ Although electrostatic methods have been employed, usually the electron beam formed by the gun is maintained at a constant diameter by an axial magnetic field which prevents space-charge spreading. Beam compression can also be achieved through a suitable build-up in the axial magnetic field. This has been referred to in the literature as "magnetic compression."

Several methods of magnetic focusing of axially symmetric electron beams have been employed. For "Brillouin flow"² the cathode is completely shielded from the magnetic field. Electrons in the beam move with a constant angular velocity such that the inward force due to the magnetic field exactly cancels the outward space charge and centrifugal forces. This method requires the least magnetic field for a given beam, but the necessary conditions for a well-focused beam are difficult to

achieve. Another method commonly used is to immerse the device completely in a strong magnetic field. This is referred to as "confined flow."³ Intermediate between the extremes of confined flow and Brillouin flow are focusing methods in which a portion of the magnetic field is allowed to penetrate into the electron gun region and link the cathode. The general case has been called "space-charge-balanced flow."⁴ Periodic magnetic focusing has also been used.⁵ Magnetic compression of an electron beam can be achieved in regions of magnetic field build-up for any one of types of magnetic field focusing.

Review of previous work

Pierce was perhaps the first to suggest that a Brillouin-focused beam could be compressed by a gradual increase in the magnetic field without upsetting the focusing conditions.⁶ Hines⁷ determined the requirements for maintaining a uniformly convergent, conical electron beam with Brillouin focusing. He found that a parabolic magnetic field distribution was necessary and could be obtained by shaping the pole pieces of the solenoid. Moster and Molnar,⁸ using an analog computer to calculate beam trajectories, were the first to make a careful study of the magnetic build-up region for Brillouin focusing. Their design curves indicate a reduction in beam diameter for larger pole pieces, that is, for a more gradual field build-up. Brück⁹ showed that Brillouin flow is possible for a magnetic build-up of the form $B_z = B_0 \sin^2 \pi z/2L$ where L is the length of the build-up region. Müller¹⁰ in a related paper obtained similar results for magnetic field distributions of the forms $\cos z$ and $\exp(-z^2)$. The results of Moster and Molnar were confirmed by Bevc, Palmer, and Susskind who extended the analysis to include space-charge-balanced flow and periodic focusing.¹¹ They also presented a method of matching the magnetic transition region to Pierce-type electron guns. Experimental results for a high perveance electron gun using magnetic compression were reported by Geppert.¹² The electron gun and focusing system gave an overall area convergence of 130 to 1 which included a magnetic compression of about 16 to 1. In his analysis, Geppert assumed an exponential build-up of the magnetic field. An approximation to this field distribution was obtained by shaping the magnetic pole pieces. Ash¹³ developed equations describing trajectories as functions

of assumed magnetic and electric field distributions. Using an analog computer, he synthesized magnetic field distributions required to achieve desired beam compressions. A numerical example resulted in an area convergence of 100 to 1 for a Brillouin focused beam of perveance 0.45×10^{-6} . Kikushima and Johnson,¹⁴ using a high convergence electron gun of perveance 2.2×10^{-6} , obtained an overall convergence of 1350 to 1 by additional magnetic compression. In their analysis a field distribution $B_z = B_0/2(1 - \cos \pi z/L)$ was assumed, where B_0 is the Brillouin field, and L is the build-up length. They obtained design curves giving radii and slope requirements at the entrance to the magnetic field, that is, where B_z is zero. A more detailed account of this work was reported later by Talbot and Johnson.¹⁵ A theoretical analysis of magnetic compression for the cases of Brillouin flow and space-charge-balanced flow has been presented by Ghandi and Vaidya.¹⁶ They assumed a magnetic build-up of a form $B_z = B_0 e^{\alpha z}$. Their results show that Brillouin-focused beams experience a much stronger compression than space-charge-balanced beams. In a later communication, Ghandi treated magnetic compression for completely immersed beams.¹⁷ The results are similar to those for the space-charge-balanced case. Vaidya and Ghandi^{18,19} have reported experimental results which seem to confirm the theory for Brillouin flow and space-charge-balanced flow.

Outline of the problem

Previous work has shown that magnetic compression can appreciably increase the convergence of high perveance beams. Further, it has been shown that Brillouin flow is possible with magnetic compression. However, analyses of the problem have been limited by simplifying assumptions, such as a constant current density in the beam, constant axial velocity of electrons, and a simple analytic form of magnetic field distribution. In most treatments, anode aperture effects and the effect of the drift tube or interaction structure on the focused beam have been neglected. In this paper the analysis of the problem is based on accurate calculation of electron trajectories. The electrostatic fields for given systems of electrodes are found by using a resistance network analog. In this way, aperture and drift tube effects are included. The effect of non-uniform current can also be included with the network analog. Electron trajectories are calculated from the electrostatic field data and the magnetic field distribution. Equations for calculating electron paths in combined electrostatic and magnetic fields are developed in the next chapter. By this method, general design data for the case of Brillouin flow are obtained. This method is also used as a design procedure for a particular system having moderately high beam compression and at the same time approaching the conditions for ideal Brillouin flow. Experimental results for this system are presented.

CHAPTER II

THEORY

Geometry of the problem

The geometry of the system is axially symmetric and is described in cylindrical coordinates. It is convenient to distinguish three regions as shown in Figure 1. In the electron gun region, electrons emitted from the cathode are formed into a beam and accelerated by electrostatic fields. Through the transition region, the converging electron beam experiences the increasing magnetic field and continues to converge. This is the region of magnetic convergence. Axial distances in the transition region will be given by z' -coordinates, as shown in Figure 1. At $z' = 0$, the beam radius is r_m ; at $z' = L$, the beam radius is r_0 . In the focused beam region, the electron beam maintains a constant radius r_0 under conditions of Brillouin flow.

Basic equations

The equations of radial and transverse motion of electrons in a cylindrical beam are given by²⁰

$$\frac{d^2 r}{dt^2} - \eta \frac{\partial V}{\partial r} - r \left(\frac{d\theta}{dt} \right)^2 + \eta B_z r \frac{d\theta}{dt} = 0 \quad (1)$$

$$\frac{d^2 z}{dt^2} - \eta B_r r \frac{d\theta}{dt} - \eta \frac{\partial V}{\partial z} = 0 \quad (2)$$

where $\eta = e/m$, V is the electric potential, and B_z and B_r are the magnetic field components.

Fig. 1. Electron gun and beam focusing system

Busch's theorem²¹ relates the angular velocity of the electrons to the magnetic flux.

$$\frac{d\theta}{dt} = \frac{n}{2\pi r^2} (\phi - \phi_0) \quad (3)$$

For Brillouin flow, the flux $\phi_0 = 0$. The flux ϕ through a circular cross section is found by integrating B_z times the element of area,

$$\phi = \int_0^r B_z 2\pi r dr \quad (4)$$

Assuming B_z is not a function of r ,

$$\phi = \pi r^2 B_z \quad (5)$$

Substituting (5) into Eq. (3) gives

$$\frac{d\theta}{dt} = \frac{n}{2} B_z \quad (6)$$

the Larmor frequency.²² Now substituting (6) into Eq. (1) yields

$$\frac{d^2 r}{dt^2} - n \frac{\partial V}{\partial r} + \frac{n^2 B_z^2 r}{4} = 0 \quad (7)$$

and substituting (6) into Eq. (2) results in

$$\frac{d^2 z}{dt^2} - n \frac{\partial V}{\partial z} - \frac{n^2 B_r B_z}{2} r = 0 \quad (8)$$

Solutions of Eqs. (7) and (8) give the r, z coordinates of electrons in the cylindrical beam. The electrons may also have motion in the θ -direction, but, because of the symmetry of the problem, the θ position is not needed in determining the shape and characteristics of the beam. Plotting the r, z coordinates of the outermost electron of the beam gives the beam envelope.

For Brillouin flow, the magnetic field components are zero in the electron gun region and Eqs. (7) and (8) reduce to

$$\frac{d^2 r}{dt^2} - \eta \frac{\partial V}{\partial r} = 0 \quad (9)$$

$$\frac{d^2 z}{dt^2} - \eta \frac{\partial V}{\partial z} = 0 \quad (10)$$

In the transition region, the magnetic field components are functions of the coordinates r, z .

In the focused beam region, the beam radius is constant and the space charge and centrifugal forces are exactly canceled by the magnetic field force arising from the cross-product of the angular velocity and magnetic field. Setting $d^2 r/dt^2 = 0$ in Eq. (7) gives

$$\frac{\partial V}{\partial r} = \frac{\eta B_z^2 r}{4} \quad (11)$$

Integrating (11), with B_z constant, results in an expression for the potential in a Brillouin beam

$$V(r) = \frac{\eta B_z^2 r^2}{8} + V_0 \quad (12)$$

where V_0 is the potential at $r = 0$, that is, on the axis of the beam.

Within the beam the potential must satisfy Poisson's equation:

$$\nabla^2 V = -\rho/\epsilon_0 \quad (13)$$

In the focused beam region the electron beam is within a drift tube or unipotential interaction structure. The only electric field is due to the space charge in the beam. This field is in the radial direction.

Thus Poisson's equation becomes

$$\frac{\partial^2 V}{\partial r^2} + \frac{1}{r} \frac{\partial V}{\partial r} = - \frac{\rho}{\epsilon_0} \quad (14)$$

then

$$\rho = - \epsilon_0 \left(\frac{\partial^2 V}{\partial r^2} + \frac{1}{r} \frac{\partial V}{\partial r} \right) \quad (15)$$

By (11), Eq. (15) becomes

$$\rho = - \frac{\epsilon_0 \eta B_z^2}{2} \quad (16)$$

The electrons in the beam are moving only in the z and θ directions.

The total kinetic energy is then given by

$$\frac{1}{2} m \left(\frac{dz}{dt} \right)^2 + \frac{1}{2} m \left(r \frac{d\theta}{dt} \right)^2 = eV \quad (17)$$

from which

$$\frac{dz}{dt} = \sqrt{2\eta V - \left(r \frac{d\theta}{dt} \right)^2} \quad (18)$$

Substituting (6) and (12) into Eq. (18) gives

$$\frac{dz}{dt} = \sqrt{2\eta V_0} \quad (19)$$

The electrons in the Brillouin beam have constant z-velocity. The current in a beam of radius r_0 is then

$$I = - \pi r_0^2 \rho \frac{dz}{dt} \quad (20)$$

Substituting (16) and the constants in mks units gives

$$I = 1.45 \times 10^6 B_z^2 V_0^{1/2} r_0^2 \quad (21)$$

Equation (21) is the relation between beam current I and the quantities V_0 , B_z , and the beam radius r_0 for a Brillouin-focused beam.

The beam shape and electron trajectories are defined in the focused beam region for Brillouin flow. In the electron gun and transition regions, an analytic solution for the equations of motion is difficult, if not impossible, for cases of interest. In analyses of magnetic compression appearing in the literature, the usual approach has been to use Eq. (7) in obtaining maximum and minimum radii of excursion for various magnetic field distribution functions. By using the resistance network analog and trajectory tracing equations it is possible to obtain actual electron trajectories in both the electron gun and transition regions.

Resistance network analog

Electrostatic field data needed for plotting electron trajectories is obtained by use of a resistance network analog. The network analog for axially symmetric problems has been described in several papers.^{23,24,25,26,27,28} The particular network used was designed by Anand²⁹ and consists of 30 meshes in the radial direction and 45 meshes in the axial direction. The arrangement of resistors is shown in Figure 2.

The resistance network essentially solves Poisson's equation with boundaries consisting of a given configuration of electrodes. For an axially symmetric field, Poisson's equation becomes

$$\frac{\partial^2 V}{\partial r^2} + \frac{1}{r} \frac{\partial V}{\partial r} + \frac{\partial^2 V}{\partial z^2} = - \frac{\rho}{\epsilon_0} \quad (22)$$

Consider any set of five adjacent nodes in the network as shown in Figure 3. The potentials V_1 , V_2 , V_3 , and V_4 can be expressed as a power series in h , the distance between nodes.

$$V_1 = V_0 - \left(\frac{\partial V}{\partial r}\right)_0 h + \frac{1}{2} \left(\frac{\partial^2 V}{\partial r^2}\right)_0 h^2 \quad (23)$$

$$V_2 = V_0 - \left(\frac{\partial V}{\partial z}\right)_0 h + \frac{1}{2} \left(\frac{\partial^2 V}{\partial z^2}\right)_0 h^2 \quad (24)$$

$$V_3 = V_0 + \left(\frac{\partial V}{\partial r}\right)_0 h + \frac{1}{2} \left(\frac{\partial^2 V}{\partial r^2}\right)_0 h^2 \quad (25)$$

$$V_4 = V_0 + \left(\frac{\partial V}{\partial z}\right)_0 h + \frac{1}{2} \left(\frac{\partial^2 V}{\partial z^2}\right)_0 h^2 \quad (26)$$

where third and higher order terms are neglected. Combining Eqs. (23) to (26) and using Eq. (22) with $(V_3 - V_1)$ from Eqs. (25) and (23) gives

Fig. 2. Arrangement of resistances in resistance network.

Fig. 3. Set of network nodes used in analysis of network.

$$V_0 = \frac{1}{4}(V_1 + V_2 + V_3 + V_4) + (V_3 - V_1)\frac{h}{8r} + \frac{\rho}{4\epsilon_0} h^2 \quad (27)$$

A relation between potentials can be found by applying Kirchoff's Current Law. For the current I_f fed into the center node, Figure 3,

$$I_f = \frac{V_0 - V_1}{R_1} + \frac{V_0 - V_2}{R_2} + \frac{V_0 - V_3}{R_3} + \frac{V_0 - V_4}{R_4} \quad (28)$$

Let the resistances be given by

$$R_1 = \frac{R_0}{\frac{r}{h} - \frac{1}{2}} \quad (29)$$

$$R_2 = R_4 = \frac{R_0}{\frac{r}{h}} \quad (30)$$

$$R_3 = \frac{R_0}{\frac{r}{h} - \frac{1}{2}} \quad (31)$$

where r/h has integral values and R_0 is an arbitrary resistance characteristic of the network. Now combining (28) - (31) one obtains

$$V_0 = \frac{1}{4}(V_1 + V_2 + V_3 + V_4) + (V_3 - V_1)\frac{h}{8r} + \frac{I_f R_0 h}{4r} \quad (32)$$

Comparing Eqs. (27) and (32) it is seen that for an analogy

$$\frac{\rho h^2}{4\epsilon_0} = \frac{I_f R_0 h}{4r} \quad (33)$$

or

$$I_f = \frac{\rho h r}{\epsilon_0 R_0} \quad (34)$$

Therefore, the potential distribution on the resistance network will

satisfy Poisson's equation if the resistances are chosen according to Eqs. (29) through (31) and if the space charge density is simulated according to Eq. (34) at each node in the network.

Electrodes are simulated by connecting together nodes at positions corresponding to the actual electrodes. Electrode positions which do not coincide with the nodes of the network can be simulated by using shunt resistors. The shunt resistors in effect shift the node point to the desired position.

Consider the simulation of an electrode whose position with respect to five nodes of the network is shown in Figure 4. From the fact that the field is continuous at V_0 , the following relations can be written:

$$\frac{V_1 - V_0}{h} = \frac{V_0 - V_3}{h} \quad (35)$$

$$\frac{V_e - V_0}{ah} = \frac{V_0 - V_2}{h} \quad (36)$$

where V_0, V_1, V_2, V_3, V_4 are the potentials of the nodes, V_e is the potential of the electrode, and 'a' the fractional distance between node 0 and node 4.

As an approximation, let the resistors connecting the five nodes have the same value R as shown in Figure 4. By Kirchoff's Law,

$$\frac{V_1 - V_0}{R} + \frac{V_2 - V_0}{R} + \frac{V_3 - V_0}{R} + \frac{V_e - V_0}{xR} = 0 \quad (37)$$

where xR is the resistance needed to effectively shift the position of node 4 to coincide with the position of the electrode.

Fig. 4. Simulation of electrode lying between nodes.

Combining Eqs. (35) - (37) it is found that

$$x = a. \quad (38)$$

Then to shift node 4 to coincide with the physical position of the electrode which is ah from the center node, the resistance between node 4 and node 0 should have a value aR . To reduce a resistance of value R to aR , a shunt resistance

$$R_{sh} = \frac{a}{1-a} R \quad (39)$$

is added.

Although Eq. (39) is based on an approximation, it is sufficiently accurate for most problems. More precise formulas, which take into account the curvature of electrodes, have been derived.³⁰

Space-charge simulation

Equation (34) gives the value of current to be injected at each node within a space-charge region. Using the relation between current density, charge density, and electron velocity

$$J = \rho u \quad (40)$$

and the equation for velocity in terms of potential

$$u = \sqrt{2} \eta V \quad (41)$$

Eq. (34) can be written as

$$I_f = \frac{J h r}{\sqrt{2} \eta V \epsilon_0 R_0} \quad (42)$$

In this expression, V is the potential at the node where current is injected. For mks units

$$I_f = 1.9 \times 10^5 \frac{J r h}{R_0 \sqrt{V}} \quad (43)$$

For currents injected at nodes along the axis, another formula is used:³¹

$$I_f = \frac{\rho h^2}{8 \epsilon_0 R_0} \quad (44)$$

Using (40) and (41) and constants in the mks system of units, Eq. (44) becomes

$$I_f = 2.375 \times 10^4 \frac{J h^2}{R_0 \sqrt{V}} \quad (45)$$

An alternate formula for injected currents can be derived for use in beam problems. Consider a typical electron gun geometry and beam profile as shown in Figure 5. Equations (43) and (45) apply when currents

Fig. 5. Division of beam region on resistance network for space-charge simulation.

are injected at all nodes within the space-charge region. For many problems sufficient accuracy in space-charge simulation is achieved without feeding currents to every node. As shown in Figure 5, the space-charge region can be divided into areas A in the r - z plane. The current to be fed into each area at a single node is simply Eq. (43) multiplied by A .

$$I_f = 1.9 \times 10^5 \frac{J r h A}{R_0 \sqrt{V}} \quad (46)$$

$$\text{where } A = a b. \quad (47)$$

The distances a and b are measured in unit lengths between the nodes of the network. In Figure 5 the portion of the beam current ΔI in the cylindrical stream of radius r is given by

$$\Delta I = 2\pi r a h J \quad (48)$$

and the current density is then

$$J = \frac{\Delta I}{2\pi r a h} \quad (49)$$

Substituting (47) and (49) into Eq. (46) gives

$$I_f = 3.03 \times 10^4 \frac{\Delta I b}{R_0 \sqrt{V}}. \quad (50)$$

Since the value of R_0 for the network used is 6000 ohms,

$$I_f = 5.04 \frac{\Delta I b}{\sqrt{V}} \quad (51)$$

Equation (51) is convenient to use in beam problems. Equation (44), the on-axis formula, is not required if the space-charge region is divided in a manner similar to that shown in Figure 5.

Recalculation of current density
at the cathode

An approximation often made in analyses of electron beam systems is that of a constant current density across the beam. However, it is known that in actual systems this is seldom true. In a high perveance Pierce-type electron gun, the electron current density is highest at the edge of the beam. Nonuniformity of current density is one of the factors which prevent achievement of Brillouin flow in some systems.³² Use of the resistance network analog allows recalculation of current densities at the cathode.³³ Thus in designing a beam focusing system with the network, it is possible to take into consideration the current density distribution and make corrections necessary to achieve desired distributions.

Again consider a typical electron gun configuration as shown in Figure 6. The point P is at a node of the network at a distance d from the cathode. The potential at point P is given by

$$V = V_0 - V_p \quad (52)$$

where V_0 is the potential at P without space charge and V_p is the potential at P due to space charge. The current density for space-charge limited flow between concentric spheres is given as³⁴

$$J_c = 2.335 \times 10^{-6} \frac{v^{3/2}}{\alpha^2 r_c^2} \quad (53)$$

where α has the value

$$\alpha = \ln \frac{r_a}{r_c} - 0.3 \left(\ln \frac{r_a}{r_c} \right)^2 + 0.075 \left(\ln \frac{r_a}{r_c} \right) + \dots \quad (54)$$

Fig. 6. Recalculation of current density.

In the above expressions r_c is the radius of the outer sphere and r_a is the radius of the inner sphere. Now substituting $r_a = r_c - d$ and the expression for α , Eq. (50) can be written as

$$J_c = \frac{2.335 \times 10^{-6} v^{3/2}}{d^2 [1 + 1.6 \frac{d}{r_c} + 2.06 (\frac{d}{r_c})^2 + \dots]} \quad (55)$$

For points very near the cathode $d/r_c \ll 1$ and Eq. (55) can be approximated by

$$J_c = \frac{2.335 \times 10^{-6} v^{3/2}}{d^2 (1 + 1.6 \frac{d}{r_c})} \quad (56)$$

Substituting (52)

$$J_c = \frac{2.335 \times 10^{-6} (V_o - V_p)^{3/2}}{d^2 (1 + 1.6 \frac{d}{r_c})} \quad (57)$$

Now consider two different cathode current densities J_{c1} , an approximate value, and J_c the recalculated value. If V_{p1} is the potential due to the space charge for the approximation, and V_p is the potential due to the space charge in the recalculated case, then

$$V_p = \frac{V_{p1}}{J_{c1}} J_c \quad (58)$$

Substituting (58) into Eq. (57)

$$J_c = \frac{2.335 \times 10^{-6} v_o^{3/2}}{d^2 (1 + 1.6 \frac{d}{r_c})} (1 - \frac{V_{p1}}{V_o J_{c1}} J_c)^{3/2} \quad (59)$$

From this expression the recalculated current density can be found graphically by plotting

$$F_1(J_c) = J_c \quad (60)$$

$$F_2(J_c) = \frac{2.335 \times 10^{-6} V_o^{3/2}}{d^2(1 + 1.6 \frac{d}{r_c})} (1 - \frac{V_{o1}}{V_o J_{c1}} J_c)^{3/2} \quad (61)$$

This procedure is lengthy since several points are needed to accurately determine the current density across the electron beam. A cubic equation in J_c results from squaring both sides of Eq. (59).

$$\begin{aligned} F_3(J_c) = J_c^3 + \left[\frac{d^2(1 + 1.6 \frac{d}{r_c})^2 J_{c1}^3}{5.4 \times 10^{-12} V_{o1}} - 3 \frac{V_o J_{c1}}{V_{o1}} \right] J_c^2 + \\ + 3 \frac{V_o^2 J_{c1}^2}{V_{o1}^2} J_c - \frac{V_o^3 J_{c1}^3}{V_{o1}^3} = 0 \quad (62) \end{aligned}$$

A standard computer subroutine for finding real and complex roots of a polynomial using Muller's method is available. Using the computer, Eq. (62) could conveniently be solved for any number of node points.

Electron trajectory tracing

The resistance network provides field data in a form suitable for numerical calculation of trajectories. The calculation is based on methods described by Haine and Vine³⁵ and Hechtel.³⁶ The method has been extended to include magnetic fields and has been programmed for use with a digital computer.

The calculation proceeds as follows: The field data is a set of voltages $V_{r,z}$ as shown in Figure 7. The trajectory calculation begins with an electron at a known position $r = r_b$, $z = b$, as shown in Figure 8. The unknown r_{b+1} coordinate can be written in a Taylor's series in terms of the interval $(z - z_0)$ as

$$r_{b+1} = r_b + \left(\frac{dr}{dz}\right)_{z=b} (z - z_0) + \frac{1}{2} \left(\frac{d^2r}{dz^2}\right)_{z=b} (z - z_0)^2 \quad (63)$$

where third and higher order terms are neglected. An expression for the first derivative is obtained directly by differentiating Eq. (63):

$$\left(\frac{dr}{dz}\right)_{z=b} = \left(\frac{dr}{dz}\right)_{z=b-1} + \left(\frac{d^2r}{dz^2}\right)_{z=b-1} (z - z_0) \quad (64)$$

The second derivative, which appears in Eqs. (63) and (64), can be found from the general ray equation³⁷

$$\frac{d^2r}{dz^2} \bigg|_{z=b} = \frac{1 + \left(\frac{dr}{dz}\right)_{z=b}}{2V_{r_b,b}} \left[\left(\frac{\partial V}{\partial r}\right)_{r_b,b} - \left(\frac{dr}{dz}\right)_{r_b,b} \left(\frac{\partial V}{\partial z}\right)_{r_b,b} \right] \quad (65)$$

where $V_{r_b,b}$, $(\partial V/\partial r)_{r_b,b}$, and $(\partial V/\partial z)_{r_b,b}$ are calculated from the nine potentials surrounding the electron at $r = r_b$ as shown in Figure 8. Expressions for these quantities are derived at the end of this section.

Fig. 7. Field data points in the r - z plane.

Fig. 8 Trajectory within potential mesh

In the calculation of a trajectory, the interval $(z - z_0)$ in Eqs. (14) and (15) is taken as one unit. Having found the r_{b+1} coordinate, the procedure is repeated for the r_{b+2} coordinate, and is continued until the complete trajectory is found as a series of r -coordinates at unit intervals of z .

Now consider the addition of an axially-symmetric magnetic field, $\vec{B} = \vec{r}B_r + \vec{z}B_z$. If $B_r \neq 0$, electrons moving with a z -component of velocity will experience a force in the θ -direction. At any point r, z the kinetic energy of an electron in terms of its components of velocity would be

$$\frac{1}{2} m(u_r^2 + u_\theta^2 + u_z^2) = e V_{r,z} \quad (66)$$

where $u_\theta^2 = (r \frac{d\theta}{dt})^2$

If $1/2 m u_\theta^2$ is subtracted from both sides of (25), one obtains

$$\frac{1}{2} m(u_r^2 + u_z^2) = e V_{r,z} - \frac{1}{2} m(r \frac{d\theta}{dt})^2 \quad (67)$$

or $\frac{m}{2e} (u_r^2 + u_z^2) = V_{r,z} - \frac{m}{2e} (r \frac{d\theta}{dt})^2 \quad (68)$

The right-hand side of Eq. (68) can be considered as an expression for the "equivalent potential" for motion in r and z directions.

$$V_{eq,r,z} = V_{r,z} - \frac{m}{2e} (r \frac{d\theta}{dt})^2 \quad (69)$$

The array of these potentials can then be used to calculate trajectories in the r - z plane using Eqs. (63) through (65) where (69) is substituted

for $V_{r,z}$, The angular velocity at $z = b$ is given by Eq. (6):

$$\left(\frac{d\theta}{dt}\right)_b = \frac{n}{2} B_{z_b} \quad (6)$$

The equivalent potential at $r = a$, $z = b$ can now be written as

$$V_{eq_{a,b}} = V_{a,b} - \frac{a^2 e}{8m} B_{z_b}^2 \quad (70)$$

The above numerical method of calculating electron trajectories has been programed in SCATRAM for use with the IBM 7094 digital computer. The SCATRAM statements are given in the Appendix. Sets of trajectories describing a system can be calculated in a fraction of a minute. The necessary data consists of the array of electrostatic potentials obtained from the network, the corresponding array of z-components of B, and the initial positions, slopes, and velocities of the electrons.

Expressions for $V_{r_b,b}$, $(\partial V/\partial r)_{r_b,b}$, and $(\partial V/\partial z)_{r_b,b}$ which appear in Eq. (65) will now be derived. Again consider an electron in a position $r = r_b$, $z = b$ as shown in Figure 8. The potentials of the surrounding nodes can be expressed in Taylor's series:

$$V_{a+1,b} = V_{a,b} + \left(\frac{\partial V}{\partial z}\right)_{a,b} + \frac{1}{2} \left(\frac{\partial^2 V}{\partial z^2}\right)_{a,b} \quad (71)$$

$$\begin{aligned} V_{a+1,b+1} = V_{a,b} &+ \left(\frac{\partial V}{\partial z}\right)_{a,b} + \left(\frac{\partial V}{\partial r}\right)_{a,b} \\ &+ \frac{1}{2} \left(\frac{\partial^2 V}{\partial z^2}\right)_{a,b} + \frac{1}{2} \left(\frac{\partial^2 V}{\partial r^2}\right)_{a,b} + \left(\frac{\partial^2 V}{\partial z \partial r}\right)_{a,b} \end{aligned} \quad (72)$$

$$V_{a,b+1} = V_{a,b} + \left(\frac{\partial V}{\partial r}\right)_{a,b} + \left(\frac{\partial^2 V}{\partial r^2}\right)_{a,b} \quad (73)$$

$$\begin{aligned}
V_{a-1,n+1} = V_{a,b} - \left(\frac{\partial V}{\partial z}\right)_{a,b} + \left(\frac{\partial V}{\partial r}\right)_{a,b} \\
+ \frac{1}{2} \left(\frac{\partial^2 V}{\partial z^2}\right)_{a,b} + \frac{1}{2} \left(\frac{\partial^2 V}{\partial r^2}\right)_{a,b} - \left(\frac{\partial^2 V}{\partial r \partial z}\right)_{a,b} \quad (74)
\end{aligned}$$

$$V_{a-1,b} = V_{a,b} - \left(\frac{\partial V}{\partial z}\right)_{a,b} + \frac{1}{2} \left(\frac{\partial^2 V}{\partial z^2}\right)_{a,b} \quad (75)$$

$$\begin{aligned}
V_{a-1,b-1} = V_{a,b} - \left(\frac{\partial V}{\partial z}\right)_{a,b} - \left(\frac{\partial V}{\partial r}\right)_{a,b} \\
+ \frac{1}{2} \left(\frac{\partial^2 V}{\partial z^2}\right)_{a,b} + \frac{1}{2} \left(\frac{\partial^2 V}{\partial r^2}\right)_{a,b} + \left(\frac{\partial^2 V}{\partial r \partial z}\right)_{a,b} \quad (76)
\end{aligned}$$

$$V_{a,b-1} = V_{a,b} - \left(\frac{\partial V}{\partial r}\right)_{a,b} + \frac{1}{2} \left(\frac{\partial^2 V}{\partial r^2}\right)_{a,b} \quad (77)$$

$$\begin{aligned}
V_{a+1,b-1} = V_{a,b} + \left(\frac{\partial V}{\partial z}\right)_{a,b} - \left(\frac{\partial V}{\partial r}\right)_{a,b} \\
+ \frac{1}{2} \left(\frac{\partial^2 V}{\partial z^2}\right)_{a,b} + \frac{1}{2} \left(\frac{\partial^2 V}{\partial r^2}\right)_{a,b} - \left(\frac{\partial^2 V}{\partial r \partial z}\right)_{a,b} \quad (78)
\end{aligned}$$

By combining Eqs. (73) and (77) expressions for $(\partial V/\partial r)_{a,b}$ and $(\partial^2 V/\partial r^2)_{a,b}$ are obtained:

$$\left(\frac{\partial V}{\partial r}\right)_{a,b} = \frac{1}{2} (V_{a,b+1} - V_{a,b-1}) \quad (79)$$

$$\left(\frac{\partial^2 V}{\partial r^2}\right)_{a,b} = V_{a,b+1} - 2V_{a,b} + V_{a,b-1} \quad (80)$$

Combining (71) and (75) gives expressions for $(\partial V / \partial z)_{a,b}$ and $(\partial^2 V / \partial z^2)_{a,b}$

$$\left(\frac{\partial V}{\partial z}\right)_{a,b} = \frac{1}{2} (V_{a+1,b} - V_{a-1,b}) \quad (81)$$

$$\left(\frac{\partial^2 V}{\partial z^2}\right)_{a,b} = V_{a+1,b} - 2V_{a,b} + V_{a-1,b} \quad (82)$$

An expression for $(\partial^2 V / \partial r \partial z)_{a,b}$ is obtained by combining (72), (74), (76), and (78)

$$\left(\frac{\partial^2 V}{\partial r \partial z}\right)_{a,b} = \frac{1}{4} (V_{a+1,b+1} - V_{a-1,b+1} + V_{a-1,b-1} - V_{a+1,b-1}) \quad (83)$$

The potential at $r = r_b$, $z = b$ can be expanded in a Taylor's series in terms of the interval $r_b - a$:

$$V_{r_b,b} = V_{a,b} + \left(\frac{\partial V}{\partial r}\right)_{a,b} (r_b - a) + \frac{1}{2} \left(\frac{\partial^2 V}{\partial r^2}\right)_{a,b} (r_b - a)^2 \quad (84)$$

The derivatives of V at $r = r_b$, $z = b$ are then

$$\left(\frac{\partial V}{\partial r}\right)_{r_b,b} = \left(\frac{\partial V}{\partial r}\right)_{a,b} + \left(\frac{\partial^2 V}{\partial r^2}\right)_{a,b} (r_b - a) \quad (85)$$

$$\left(\frac{\partial V}{\partial z}\right)_{r_b,b} = \left(\frac{\partial V}{\partial z}\right)_{a,b} + \left(\frac{\partial^2 V}{\partial r \partial z}\right)_{a,b} (r_b - a) \quad (86)$$

where the derivatives at a,b are given by (79) - (83). Taking the second derivative of Eqs. (85) and (86) it is seen that

$$\left(\frac{\partial^2 V}{\partial r^2}\right)_{r_b, b} = \left(\frac{\partial^2 V}{\partial r^2}\right)_{a, b} \quad (87)$$

$$\left(\frac{\partial^2 V}{\partial z^2}\right)_{r_b, b} = \left(\frac{\partial^2 V}{\partial z^2}\right)_{a, b}$$

$$\left(\frac{\partial^2 V}{\partial r \partial z}\right)_{r_b, b} = \left(\frac{\partial^2 V}{\partial r \partial z}\right)_{a, b}$$

Equations (79) - (87) provide the necessary relations in terms of the potential data to evaluate Eq. (65) at each point of the trajectory calculation.

CHAPTER III

BEAM COMPRESSION IN TRANSITION REGION

Method of analysis

Data on magnetic compression of electron beams in the transition region for a range of beam perveances were obtained using the resistance network analog and trajectory tracing methods described in the previous chapter. The method of analysis used is similar to that suggested by Kikushima and Johnson.¹⁴ The magnetic field distribution in the transition region is assumed to be

$$B_z = \frac{1}{2} B_0 (1 - \cos \frac{\pi z'}{L}) \quad (88)$$

where z' and L are as shown in Figure 1. Trajectories are plotted backward through the transition region starting within the focused beam region where an ideal Brillouin beam is assumed. In this way the initial radii, slopes, and velocities of the electrons are exactly defined. The electron beam envelope diverges from its Brillouin radius r_0 to a radius of r_m at $z' = 0$. Because of the reciprocal nature of electron flow in a given field, the paths of electrons determined in this way will be the same as those of electrons entering the transition region at $z' = 0$ and forming a Brillouin beam at $z' = L$. By this analysis, one can determine the amount of area compression $A_m/A_0 = \pi r_m^2/\pi r_0^2$, and the entrance conditions at $z' = 0$ required to form a Brillouin beam.

Analog model

Figure 9 shows the way the problem is simulated on the resistance network. The transition region is within a drift tube of constant diameter simulated by shorting together the $r = 15$ row of nodes on the network. The ratio of the drift tube radius to Brillouin radius was arbitrarily taken as 15 to 4 for convenience of representation and to allow a maximum area compression of at least 10 to 1. A beam profile is assumed and currents are injected at points within the area representing the beam. The actual subdivision of the beam area and nodes selected for current injection are shown in Figure 10.

The current, voltage, magnetic field, and linear dimensions for the network model were scaled from values which might be used in an actual microwave tube. For the actual system, one can start from a given beam voltage and desired beam diameter. The desired beam perveance, $K = I/V^{3/2}$, then gives the beam current. The magnetic field for a Brillouin-focused beam is then obtained through Eq. (21)

$$B_z = 8.3 \times 10^{-4} \frac{I^{1/2}}{v^{1/4} r_0} \quad (89)$$

Having found values of V , I , B , and a linear dimension ℓ in an actual system, the corresponding values of V' , I' , B' , and ℓ' for the analog model are obtained from the scaling relations

$$\frac{I'}{V'^{3/2}} = \frac{I}{V^{3/2}} = K \quad (90)$$

$$\frac{B'^2 \ell'^2}{V'} = \frac{B^2 \ell^2}{V} \quad (91)$$

Fig. 9. Setup of problem on resistance network.

Fig. 10. Division of beam region for space-charge simulation.

For the network model, the potential of the Brillouin beam voltage on the axis, V_0 , was picked to be 25 volts. Having established a suitable relation between the linear dimensions of the actual system and the network units of length, the values of I' and B' are calculated by Eqs. (90) and (91).

Results

The analysis was applied to a beam of perveance $K = 2.0 \times 10^{-6}$. For a beam voltage, $V' = V_0' = 25$ volts, the beam current $I' = KV_0'^{3/2} = 250 \times 10^{-6}$ amperes. An approximate beam profile was assumed and the area representing the beam was divided into sub-areas as in Figure 10. Figure 11 shows the cross section of the beam with A_1 and A_2 the cross sectional areas of the main divisions of the beam. The current through A_1 , $\Delta I_1 = 0.25 I' = 62.5 \times 10^{-6}$ amperes, and the current through A_2 , $\Delta I_2 = 0.75 I' = 187.5 \times 10^{-6}$ amperes. The length b of each sub-area is measured. The injected currents are now calculated by Eq. (51) using $V = V_0' = 25$ volts as a first approximation. The currents thus calculated are injected and the measured values of V at the injection nodes are then used to recalculate the injection currents. These currents are injected, the voltages at the injection nodes remeasured, the injection currents recalculated and injected. This procedure is repeated until self-consistency is achieved.

The accuracy of the network model of the Brillouin beam within the drift tube can be conveniently checked since the potential distribution across the beam and drift tube can be found analytically. The potential

Total area $A = 16 \text{ units}^2$

$$A_1 = 0.25A$$

$$A_2 = 0.75A$$

Fig. 11. Division of beam cross section.

distribution within the beam is given by Eq. (11)

$$V(r) = \frac{\eta B_z^2 r^2}{8} + V_0 \quad (11)$$

Outside the beam, a logarithmic function for the potential outside of a cylinder of charge is added to (11).³⁹ For $r > r_0$,

$$V(r) = \frac{\eta B_z^2 r_0^2}{8} + V_0 - \frac{\rho_0 r_0^2}{2 \epsilon_0} \ln \frac{r}{r_0} \quad (92)$$

The charge density in terms of B_z is given by Eq. (16)

$$\rho_0 = - \frac{\epsilon_0 \eta B_z^2}{2} \quad (16)$$

Substituting (16) into (92) gives

$$V(r) = V_0 + \frac{\eta B_z^2}{8} (r_0^2 + 2r_0^2 \ln \frac{r}{r_0}) \quad (93)$$

for the potential distribution outside the beam. A comparison between the potentials measured on the network and the analytical distribution calculated from Eqs. (11) and (93) is shown in Figure 12. The greatest error occurs at the axis where the analog is not exact, however, the percentage error is only 0.4%. In the vicinity of the beam boundary the accuracy is very good. At $r = 4$, the beam edge, the error is 0.15%.

Having set up the problem on the network and measured the potentials representing the electrostatic field data, the z-component of magnetic field is calculated at each z-coordinate along the transition region. This data is then used to calculate the equivalent potentials determining motion in the r,z plane by Eq. (70). Another check of the correctness of the analog model can be made with this data. Since the z-

Fig. 12. Potential distribution for a Brillouin-focused beam in a drift tube. The solid curve is the theoretical distribution and the circles are values measured on resistance network analog.

components of velocity in a Brillouin beam are the same and equal to velocity on the axis, the equivalent potentials within the beam must be the same and equal to the potential on the axis, $V'_0 = 25$ volts. The linear scale used was 1 network unit = 0.5×10^{-4} meters. The Brillouin magnetic field, $B'_0 = 0.0293$ webers/meter². Applying Eq. (70) in the focused beam region at $r = 4$, $z = 2$, the equivalent potential is

$$\begin{aligned} V_{eq4,2} &= 25.67 - \frac{e}{8m} (4 \times 0.5 \times 10^{-4})^2 (0.0293)^2 \\ &= 25.67 - 0.66 \\ &= 25.01 \\ &\approx 25.00 \end{aligned}$$

By varying L in Eq. (88), magnetic field data can be calculated for various transition lengths. In preparing data for use with the programmed trajectory equations, it was convenient to use transition lengths of $L = 36$, 18 , and 9 network units. The trajectories of two beam electrons were calculated for each transition length. The initial positions were taken within the focused beam region at $r = 4$, $z = 2$, and $r = 2$, $z = 2$ on the network. The electron initially at $r = 4$ traces the beam envelope. Since Brillouin flow is assumed in the focused beam region, the initial slopes $(dr/dz)_{2,2}$ and $(dr/dz)_{4,2}$ are zero. The initial z -velocities are also known from the assumed Brillouin beam, and are determined from $V_{eq2,2} = V_{eq4,2} = V'_0 = 25$ volts. The results of the trajectory calculations for the $K = 2.0 \times 10^{-6}$ beam are shown in Figures 13 through 16. Similar sets of trajectories were calculated for perveances of $K = 1.0 \times 10^{-6}$ and $k = 0.5 \times 10^{-6}$. In order to increase the relative transition length, the initial beam radii for these cases was taken to

be $r = 2.67$ units. The results of these calculations are summarized in the curves of Figures 17 through 19. It is noted in Figure 19 that the change in the required slope at $z' = 0$ decreases as the normalized transition length increases. This would indicate that matching of slope for Brillouin flow would be less critical for longer transition lengths.

Fig. 13. Beam trajectories. $K = 2.0 \times 10^{-6}$. Normalized transition length, $L/r_0 = 0$.

Fig. 14. Beam trajectories. $K = 2.0 \times 10^{-6}$. Normalized transition length, $L/r_0 = 2.25$.

Fig. 15. Beam trajectories. $K = 2.0 \times 10^{-6}$. Normalized transition length, $L/r_0 = 4.5$.

Fig. 16. Beam trajectories. $K = 2.0 \times 10^{-6}$. Normalized transition length, $L/r_o = 9.0$.

Fig. 17. Normalized radius at $z' = 0$ versus normalized transition length. $K = 1.0 \times 10^{-6}$ and $K = 0.5 \times 10^{-6}$.

Fig. 18. Area convergence versus normalized transition length. $K = 1.0 \times 10^{-6}$ and $K = 0.5 \times 10^{-6}$.

Fig. 19. Beam slope at $z' = 0$ versus normalized transition length. $K = 1.0 \times 10^{-6}$ and $K = 0.5 \times 10^{-6}$.

CHAPTER IV

MAGNETIC COMPRESSION EXPERIMENT

General objectives and considerations

The main objective of the experimental work was to achieve a high current density electron gun-focusing system having a high degree of magnetic compression. The design would be based on the results of Chapter III. Another objective was to approach ideal Brillouin flow in the focused beam region. In order to facilitate the study of the beam characteristics in the focused beam region, an extremely large overall convergence was not desirable. To satisfy these requirements, the electron gun should have a relatively high perveance, low convergence, and match the transition region entrance conditions. Also, the current density of the beam formed by the electron gun should be nearly uniform to satisfy that condition of Brillouin flow. The magnetic circuit must provide a desired magnetic field distribution and at the same time shield the cathode from the field.

Electron gun design

The design of the electron gun was carried out on the resistance network. The final design evolved from a Pierce type gun.¹ The Pierce gun is based on electron flow in a spherical diode as analyzed by Langmuir and Blodgett.³⁴ In the Pierce gun the cathode surface consists of a portion of the outer sphere of the diode, and the anode of the gun consists

of a portion of the inner sphere. The beam of electrons consists ideally of a conical section of the space charge-limited flow of electrons in the diode. To compensate for the removal of the surrounding electrons, the electron gun has a beam-forming electrode at the edge of the cathode and at an angle of 67.5° with respect to the beam edge. With the beam forming electrode, the potential along the boundary of the beam is the same as in the diode. The anode cannot have the ideal spherical shape since it must have an aperture to allow the beam to pass through. Field distortions which are not compensated for by the beam forming electrode are caused by the aperture. One effect is to cause the electron beam to diverge, so that the beam has a minimum diameter further from the cathode than would be predicted for normal space charge spreading. Some beam divergence due to the field distortion occurs before the beam reaches the anode, causing beam current to be intercepted at the anode. In practice, the anode aperture is enlarged to reduce current interception; however, this further increases the distortion of the field. Another effect of the anode aperture is a reduced beam perveance since the distance between cathode and anode is effectively increased. Furthermore, the anode electrode will be closer to the edge of the cathode than to the center. This causes a higher current density at the edge of the beam. This last effect is an important consideration in designing a system for Brillouin flow since a uniform current density is required. Anode aperture effects are particularly significant for electron guns of high perveance and convergence where the aperture diameter approaches one-half or more the anode to cathode spacing.

Initially, a Pierce-type gun was designed for a nominal micro-

perveance of 2.0 and a half-angle of convergence of $\theta = 20^\circ$. It was thought that aperture effects would not be too serious for this relatively low convergence. The ratio of cathode to anode radii, r_c/r_a , is related to a parameter $(-\alpha)^2$ introduced by Langmuir and Blodgett.³⁴ For the desired perveance and half angle of convergence, the parameter $(-\alpha)^2$ can be obtained from the equation⁴⁰

$$I = 14.648 \times 10^{-6} (1 - \cos \theta) \frac{v^{3/2}}{(-\alpha)^2} \quad (94)$$

For a microperveance of 2.0 and $\theta = 20^\circ$, $(-\alpha)^2 = 0.44$. From a table,⁴¹ $r_c/r_a = 1.74$ for $(-\alpha)^2 = 0.44$. Since Eq. (94) is derived for a spherical diode, the actual perveance of the electron gun will be less due to the anode aperture. An approximation to the actual perveance can be obtained from the relation

$$K = K_{sp} \cos \theta \quad (95)$$

where K_{sp} is the perveance for the spherical diode and θ is the half angle of convergence. Applying this equation, $K = 2.0 \times 10^{-6} \cos 20^\circ = 1.88 \times 10^{-6}$. Measurements on the resistance network, described later, indicate this is closer to the actual value.

A Pierce type gun configuration having a ratio of $r_c/r_a = 1.74$ was set up on the resistance network using appropriate shunt resistances calculated by Eq. (39) to simulate electrodes. To determine the space charge simulation currents, a beam profile was assumed and subdivided as shown in Figure 20. The beam cross section was divided into six main areas and the currents, ΔI , for each area calculated as shown in Figure 21. Space charge simulation currents were then calculated by

Fig. 20. Division of beam region for space charge simulation currents.

$$I_{\text{total}} = 10.4 \text{ Amp}$$

$$\Delta I_1 = 0.29 \mu\text{A}$$

$$\Delta I_2 = 0.87 \mu\text{A}$$

$$\Delta I_3 = 1.44 \mu\text{A}$$

$$\Delta I_4 = 2.03 \mu\text{A}$$

$$\Delta I_5 = 2.60 \mu\text{A}$$

$$\Delta I_6 = 3.17 \mu\text{A}$$

Fig. 21. Currents in cross sectional areas of electron beam.

Eq. (51) using the ΔI , b , and the space charge free potential V for each sub-area. In the region next to the cathode, the analog should be as accurate as possible since electron trajectories are strongly affected by the fields in this part of the gun. For this reason, simulation currents are injected at every node as shown in Figure 20. Values of these currents can be calculated directly from Eq. (34). Having first calculated the simulation currents using the space charge free potentials and applied these to the network, the iterative procedure described in Chapter III was used to adjust the currents to their final values. The network analog of the gun configuration and space charge simulation is shown in Figure 22. A digital voltmeter was used to measure potentials.

The first trajectories calculated showed that a portion of the beam would be intercepted by the anode. The anode aperture was enlarged slightly and anode aperture effects were considered. Up to this point a uniform current density had been assumed. To determine the actual current density distribution across the cathode and the beam perveance, the current density was recalculated, using graphical solutions of Eq. (59), for several nodes near the cathode. The cathode surface area for the analog model was 870 square network units, so that the current density at the cathode, based on an assumed microperveance of 2.0, was $J_{c1} = 10.4 \times 10^{-6} / 870 = 1.2 \times 10^{-8}$ amperes/unit². The other values needed in Eq. (59) are V_0 , the space charge free potential; V_p , the potential due to space charge; and d , the distance of the node to the cathode. These were measured for seven nodes near the cathode and are listed in Table 1. The graphical solution for the node $r = 15$, $z = 6$ using Eqs. (60) and (61) is shown in Figure 23. The results of the recalculation of current

Fig. 22. Electron gun model on resistance network.

TABLE 1

Measured parameters for recalculation
of current densities. Pierce gun.

r	z	d	V_o	V	V_p
3	4	2.9	0.240	0.059	0.181
5		2.75	0.227	0.036	0.191
7		2.5	0.207	0.072	0.135
9	5	3.2	0.261	0.119	0.142
11		2.8	0.226	0.109	0.117
13	6	3.3	0.260	0.155	0.105
15		2.8	0.204	0.135	0.069

Fig. 23. Graphical solution of recalculated current density.
 $r = 15, z = 6$.

density across the beam, along with the assumed current density, are plotted in Figure 24 to show current density distribution. The recalculated current density is about 55 percent greater at the edge of the cathode than at the center. For the recalculated current, the beam perveance is approximately 1.8×10^{-6} , a reduction of about 10 percent. This agrees fairly well with the perveance calculated by Eq. (95).

The recalculated current density was used in determining the space charge simulation currents. Using the electrostatic field data thus obtained, beam trajectories were again calculated. The results are shown in Figure 25. The outer trajectory misses the anode indicating little possible beam interception. Although the beam trajectories for this gun are good, the nonuniform current distribution would fail to satisfy a condition of Brillouin flow.

The Müller electron gun,^{42,43} reported in the literature several years ago but not widely known, attempts to correct for anode aperture effects by means of an additional electrode placed in the vicinity of the anode. This electrode, which will be called the "concentrating electrode," is normally at cathode potential. Figure 26 shows the general Müller gun configuration. The concentrating electrode is set back of the anode aperture by a distance approximately one-third of the anode aperture. The aperture of the concentrating electrode is about twice the diameter of the anode aperture. This additional electrode near the anode has a somewhat similar effect near the anode as the beam forming electrode has near the cathode, in that it prevents beam spreading. Another feature of the Müller gun is the tubular, sharp edged anode.

Fig. 24. Current density versus distance on cathode surface for Pierce gun. Solid line is approximation and circles are recalculated values.

Fig. 25. Pierce gun beam trajectory.

Fig. 26. Müller gun.

This shape was found to improve the field in the immediate vicinity of the anode, in particular, to make the equipotential surfaces more nearly orthogonal to the assumed rectilinear beam edge.

A Müller-type gun was designed with the same relative dimensions and half-angle of convergence as the Pierce gun. Figure 27 shows the gun configuration as set up on the resistance network and the division of the area representing the beam for space charge simulation. Since the concentrating electrode is at cathode potential, the field at the cathode will be somewhat reduced, and therefore, a reduced perveance would be expected. A perveance of $K = 1.6 \times 10^{-6}$ was estimated and a uniform current density assumed. Simulation currents were again calculated and adjusted. Current density was recalculated at several nodes near the cathode. The values of the parameters used in the recalculation are given in Table 2. The recalculated current density distribution is shown in Figure 28. As is seen, the current density remains nearly uniform across the cathode. Perveance is reduced to $K = 1.3 \times 10^{-6}$. Space charge simulation currents were again adjusted and field data taken. Trajectories were calculated on the computer and are plotted in Figure 29. The outer trajectory misses the anode indicating little current interception. Also, there is good range in slope of the beam in the region beyond the anode, allowing some freedom in matching the entrance conditions at the transition region. This gun configuration was used in the experimental system. The actual gun was scaled by letting one network unit = 0.05 inch, so that the cathode diameter was 1.6 inches and the inner diameter of the anode was 1.0 inch. The rather large cathode diameter in the experimental system was chosen so that the focused beam

Fig. 27. Division of beam region for space charge simulation. Müller gun.

TABLE 2

Measured parameters for recalculation
of current densities. Müller gun.

r	z	d	V_0	V	V_p
3	4	2.9	0.185	0.046	0.139
5		2.75	0.173	0.027	0.146
7		2.5	0.154	0.052	0.102
9	5	3.2	0.190	0.082	0.108
11		2.8	0.159	0.071	0.088
13	6	3.3	0.175	0.095	0.080
15		2.8	0.131	0.079	0.052

Fig. 28. Current density versus distance on cathode surface for Müller gun. Solid line is approximation and circles are recalculated values.

Fig. 29. Beam trajectory. Müller gun.

diameter would be large enough to easily observe in the beam tester to be described later. Area convergence for the gun is equal to $A_c/A_{\min}$, the area of the cathode divided by the area of the beam cross section at the minimum beam diameter. In network units, $A_c = 870 \text{ units}^2$, and from Figure 29, $A_{\min} = \pi r_{\min}^2 = \pi(6.6)^2 = 137 \text{ units}^2$. This gives an area convergence of $870/137 = 6.35$. It was desired to have a magnetic area compression of about 10. This would mean an overall area convergence of about 64, making the focused beam area $= 870/64 = 13.6 \text{ units}^2$. Then, in network units, $r_o = 2.1 \text{ units}$. For the scale chosen, $r_o = 0.1 \text{ inch}$, which would make the focused beam size large enough to be easily observed in the beam tester.

Magnetic circuit

By extrapolation of the results of Chapter III it was found that a normalized transition length, L/r_0 , of at least 25 would be needed to obtain a magnetic compression of about 10 to 1. For a focused beam having a radius $r_0 = 0.1$ inch, a transition length of at least 2.5 inches would be needed. To obtain this transition length, a magnetic pole piece was tapered from a minimum aperture diameter of 1.2 inches to a maximum aperture diameter of 2.6 inches. Allowing for certain fringing of the field beyond the pole piece aperture, it was estimated that this shape would provide the required transition length. From considerations in the design of the beam tester, the magnetic circuit was divided. The tapered portion was included in the vacuum envelope of the beam tester, and the remainder of the pole piece and solenoid were outside. Figure 30 shows the arrangement of pole pieces and solenoid. The tapered internal pole piece is fixed with respect to the electron gun. The external portion and solenoid can be moved axially, allowing some variation in the transition length. With the pole pieces and solenoid positioned as they would be in the beam tester, the magnetic field distribution along the axis of the system was measured using a gaussmeter and axial probe. The axial probe operates on the Hall Effect principle and measures only the z-component of magnetic field. It was mounted so that it could be moved along the axis of the system and its position could be accurately measured. The gaussmeter itself, a Bell Model 120, is a precision magnetic flux measuring instrument. For the field distribution measurements, it was calibrated against a standard magnet.

Fig. 30. Arrangement of pole pieces and solenoid.

Figure 31 shows the measured magnetic field distribution compared to the distribution assumed in the analysis of Chapter III. As is seen, the magnetic field does not go to zero. The $z' = 0$ position for the measured distribution is arbitrarily taken at a minimum value of B_z . This makes the transition length approximately 2.75 inches. At the position of the cathode, the measured magnetic field was not zero, but with additional external magnetic shielding it was reduced to less than 4 per cent of B_0 .

The position of the pole pieces and solenoid with respect to the electron gun could at best be approximated by extrapolating the results of Chapter III. Since the magnetic field does not actually go to zero, the $z' = 0$ boundary is indistinct. With these considerations, the electron gun and magnetic circuit were matched as carefully as possible to provide the correct beam entrance conditions at an approximate $z' = 0$ boundary. Some control over the beam entrance conditions at $z' = 0$ can be obtained by applying small positive or negative voltages to the concentrating electrode, making the matching of the gun and transition regions somewhat less critical.

Fig. 31. Magnetic field distribution along axis in experimental tube.
Solid line is the distribution assumed in the analysis.
Circles are the measured values.

Beam tester

A diagram of the beam tester, excluding the vacuum envelope, external pole piece and solenoid, is shown in Figure 32. The electron gun and magnetic circuit are as described in the previous sections. The cathode is of the oxide coated type heated with a bifilar heater. Other electrodes are made from nonmagnetic stainless steel. The tapered pole piece is of Armco iron. Electrodes and pole piece are accurately spaced with special ceramic washers having dimensional tolerances of ± 0.002 inch. The electron beam is collected on a thin graphite screen having a thickness of only 0.007 inch. It was found that for a beam current of about 20 milliamperes or greater, the impinging beam heats the screen sufficiently to cause the screen to glow, and the beam cross section can be viewed from the back side. Vaidya and Ghandi⁴⁴ considered radiation from such a collector and have calculated that radiation decreases rapidly outside the beam illuminated area, concluding that this method is quite accurate for beam diameter measurements. The graphite screen is fastened by nickel tabs to the end of a tube. This tube fits inside a drift tube and can be moved axially by means of a rod extending through a vacuum seal in the mounting plate. In this way, the graphite screen can be moved from the gun region through the transition region. Figure 33 shows the electrode system and movable screen mount suspended from the mounting plate. Figure 34 shows the complete beam tester including the vacuum envelope, external pole piece and solenoid. At the bottom of the

vacuum envelope is a mirror, set at an angle, for viewing the back side of the graphite screen. The system is continuously pumped while in operation. After outgassing and cathode activation, pressures of about 2×10^{-6} mm Hg were maintained through most of the experimental observations and measurements.

Fig. 32. Diagram of beam tester.

Fig. 33. Beam tester assembly.

Fig. 34. Beam tester.

Experimental results

Cathode activation was normal and full emission was obtained. The electron gun was operated space-charge-limited for beam voltages up to 800 volts. Beam current versus anode voltage is shown in Figure 35. For an anode voltage of 610 volts, the beam current was 21 milliamperes, giving a perveance of $K = 1.27 \times 10^{-6}$. This compares quite closely with the value of 1.30×10^{-6} obtained from the resistance network analog.

Figure 36 shows a diagram of the connections and metering of the electrodes. Because of the beam tester construction, anode and drift tube currents could not be separated. With V_a and V_b at 570 volts, the collector current, I_b , was 14.4 milliamperes and the current to the anode and drift tube combined was 4.8 milliamperes. Probably this current was due mainly to secondary electrons from the graphite screen collected by the drift tube. This was confirmed experimentally by applying a small negative voltage to the beam forming electrode. No decrease in current was observed, indicating little or no beam interception at the anode. Secondary emission could be reduced by using a Faraday cage type collector or by maintaining the collector at about 100 volts above the drift tube. Either of these methods would introduce changes in the field which would affect the beam trajectories.

At beam currents of about 20 milliamperes the beam cross section became distinctly visible as viewed from the back side of the collector. More distinct and brighter cross sections were obtained for a current of 24 milliamperes at 700 volts. A series of these cross sections were photographed at several positions along the transition region. These are shown, enlarged two times from actual size, in Figures 37 through 38.

Fig. 35. Beam current versus anode-drift tube voltage.

Fig. 36. Electrode connections.

$z' = -0.10$ in.

$r_b = 0.34$ in.

$z' = 0.28$ in.

$r_b = 0.26$ in.

$z' = 0.90$ in.

$r_b = 0.20$ in.

$z' = 1.40$ in.

$r_b = 0.18$ in.

Fig. 37. Beam cross section at various positions along axis. Enlarged 2x. Actual distance between reference spots is 0.55 inch.

$z' = 1.80$ in.

$r_b = 0.14$ in.

$z' = 2.15$ in.

$r_b = 0.11$ in.

$z' = 2.40$ in.

$r_b = 0.10$ in.

$z' = 2.53$ in.

$r_b = 0.08$ in.

Fig. 38. Beam cross section at various positions along axis. Enlarged 2x. Actual distance between reference spots is 0.55 inch.

Magnetic compression over the transition region is evident. The two small spots appearing in the pictures are due to holes in the graphite. The brightness is from the heated cathode surface seen through the holes. The holes have diameters of about 0.005 inch and the distance between them is 0.55 inch, providing a convenient means of measuring the beam diameters.

Figure 39 shows the beam profile as determined from the measured beam diameters. The profile is quite similar to those obtained by trajectory calculations in Chapter III. Slight irregularities may be due to beam scalloping. Some apparent beam scalloping was observed in the focused beam region, but no quantitative measurements were attempted.

From the beam cross section measurements, the area convergence over the transition length is about $(3.2)^2/(0.1)^2 = 10.2$. Overall area convergence is about 64.

The magnetic field in focused beam region was 74 gauss for the 700 volt, 24 milliamperes beam. Using the measured focused beam radius, the Brillouin field calculated from Eq. (21) is 55 gauss. By this calculation the actual magnetic field is about 1.3 times the Brillouin field needed to focus a beam having the measured radius. The accuracy of the calculation is of course limited by the accuracy of the beam radius measurement. Another indication the Brillouin flow was achieved is the fact that the adjustment of the magnetic field was fairly critical. For magnetic fields slightly greater or less than the optimum value, the focused beam appeared to break up.

The experiment was repeated with the beam tester modified so that the screen could be moved through a longer interval in the focused-beam

Fig. 39. Beam profile. Circles are measured beam radii.

region. A micrometer attachment was also added which made it possible to accurately position the screen at small intervals. With this arrangement the focused beam was observed over a distance of approximately one inch at 0.01 inch intervals. For an optimally focused beam having a beam voltage of 540 volts and a beam current of 17.0 milliamperes, there was no measurable scalloping. Beam cross sections over a 0.40 inch interval, well within the focused beam region, are shown in Figure 40.

$z' = 3.80$ in.

$r_b = 0.08$ in.

$z' = 3.90$ in.

$r_b = 0.08$ in.

$z' = 4.00$ in.

$r_b = 0.08$ in.

$z' = 4.10$ in.

$r_b = 0.08$ in.

Fig. 40. Beam cross section over interval within focused-beam region. Enlarged 2x. Actual distance between reference spots is 0.25 inch.

CHAPTER V

SUMMARY AND CONCLUSIONS

An investigation of magnetic compression in high current density electron gun-focusing systems, employing Brillouin flow, was made. Analysis of such systems was by use of the resistance network analog and careful electron trajectory tracing. Equations for calculating electron trajectories in combined electrostatic and magnetic fields were developed and programmed for the IBM 7094 computer. Some general data on electron beam compression in the magnetic build-up region are presented in Chapter III.

An experimental system was designed and tested. The system employed a Muller-type electron gun which was found to have a nearly uniform current density, a requirement for Brillouin flow. A tapered magnetic pole piece provided a relatively long magnetic build-up region. Experimental results were in close agreement with design objectives. An area magnetic compression of about 10 to 1 and a focused beam approaching ideal Brillouin flow were achieved.

The feasibility of using a large degree of magnetic compression in systems of the type studied has been demonstrated. Besides the possibility of increasing overall convergence, certain other advantages are apparent. If a given amount of convergence is required, a smaller electron gun convergence is necessary, resulting in better beam formation. A relatively long transition region facilitates the formation of a

Brillouin focused beam, since entrance conditions of the beam into the magnetic field are less critical than for systems employing more abrupt field build-up.

In correcting for anode aperture effects, Müller-type electron guns provide a more uniform current density than the Pierce type. From the resistance network analysis and experimental results, it appears that the Müller type gun should be used in high perveance electron gun focusing systems attempting Brillouin flow. It is suggested that the nonuniform current density of the Pierce type gun has been one of the basic reasons for difficulty in achieving true Brillouin flow. A further investigation of the Müller gun designed for higher convergences and perveances would be of interest.

For a given gun-focusing system, a magnetic circuit similar to the type used in the beam tester could be used in adjusting the transition length and thus control the focused beam diameter. This would be useful in certain microwave tubes in maximizing beam coupling to the resonant circuit or in reducing beam interception.

APPENDIX

Computer Program for Electron Trajectory Tracing

The purpose of the program is to compute electron trajectories given a rectangular array of numbers representing a combination of electrostatic and magnetic fields. In the array, the elements in columns are designated by r , the elements in rows by z , corresponding to r, z in cylindrical coordinates. The symbols used in the program are as follows:

M = total number of z

N = maximum integral value of r

$N1(I)$ = minimum value of r for I th z -value

$N2(I)$ = maximum value of r for I th z -value (for each value of z there is a $N1(I)$ and a $N2(I)$)

$VO(I,J)$ = the values of the electrostatic potential at the point I,J in the array

$PHI(I,J)$ = magnetic field parameter at I,J

$M1$ = initial z -value of electron

$R(M1)$ = initial r -value of electron

$DR1$ = initial slope of electron

Any number of sets of initial conditions may be run at one time.

The output consists of the initial r, z coordinates and slope followed by the succeeding r -values of the trajectory at integral values of

z. If the calculation fails at any point in the calculation for lack of data, the statement, "The computation for $r = xx$ failed," is printed out. The program continues until trajectories for all sets of initial conditions have been calculated or otherwise terminated for lack of sufficient data.

SEEGER, JOHN A.

JOBN FGS010 001 050 0050

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*** SCATRAM

```

      DIMENSION (V(1250,M))-
      DIMENSION(V0(1250,N))-
      DIMENSION(A(1250,N))-
      DIMENSION(PHI(1250,N))-
      DIMENSION(N1(50),N2(50),V1(50),VA(50),VB(50),RC(50),RB(50),R
(50))-
S1    READINPUT,F1,(M,N)-
      DOTHROUGH(L3),I=1,1,I.LE.M-
      READINPUT,F2,(N1(I),N2(I))-
      J1=N1(I)-
      J2=N2(I)-
L3    READINPUT,F3,((VO(I,J),J=J1,1,J.LE.J2))-
      DOTHROUGH(L4),I=1,1,I.LE.M-
      READ INPUT,F2, (N1(I),N2(I))-
      J1=N1(I)-
      J2=N2(I)-
L4    READINPUT,F3,((PHI(I,J),J=J1,1,J.LE.J2))-
L8    DOTHROUGH(L9),I=2,1,I.LE.M-
      J1=N1(I)-
      J2=N2(I)-
      DOTHROUGH(L9),J=J1,1,J.LE.J2-
      A(I,J)=PHI(I,J)-
L9    V(I,J)=V0(I,J)-2.2*(A(I,J)).P.2-
L7    READINPUT,F4,(M1,R(M1),DR1)-
      PROVIDED(M1.E.0),CALL SUBROUTINE( )=END JOB.( )-
```

```

K=M1-
L=M1-
DR=DR1-
M4+M1-
L11    LA=R(K)-
        FL=LA-
        TRANSFERTO(L12,L12,L16)PROVIDED(FL+.5-R(K))-
L12    TRANSFERTO(L14,L13,L13)PROVIDED(N2(K)-LA-2)-
L13    L1=LA-
        L2=LA+1-
        L3=LA+2-
        TRANSFERTO(L35)-
L14    TRANSFERTO(L20,L15,L15)PROVIDED(N2(K)-LA-1)-
L15    L1=LA-1-
        L2=LA-
        L3=LA+1-
        TRANSFERTO(L35)-
L16    TRANSFERTO(L15,L15,L17)PROVIDED(N1(K)-LA+1)-
L17    TRANSFERTO(L13,L13,L20)PROVIDED(N1(K)-LA0-
L20    WRITEOUTPUT,F5,(R(M1))-
        DOTHROUGH(L21),I=M1,1,I.LE.K-
L21    WRITEOUTPUT,F7,(R(I))-
        TRANSFERTO(L7)-
L35    FL2=L2-
        V1(L)=V(M4,L2)+(V(M4,L3)-V(M4,L1))/2.0*(R(L)-FL2)+(V(M4,L3)-
        2.0*V(M4,L2)+V(M4,L1))/2. *(R(L)-FL2).P.2-

```

```

VA(L)=(V(M4,L3)-V(M4,L1))/2.0+(V(M4,L3)-2.0*V(M4,L2)+V(M4,L1
))* (R(L)-FL2)-
VB(L)=(V(M4+1,L2)-V(M4-1,L2))/2.0+(V(M4+1,L3)-V(M4+1,L1)+V(
M4-1,L1)-V(M4-1,L3))/4.0*(R(L)-FL2)+(V(M4+1,L3)-2.0*V(M4+1,L
2)+V(M4+1,L1)-V(M4-1,L3)+2.0*V(M4-1,L2)-V(M4-1,L1))* (R(L)-FL
2).P.2-
RC(M4)=((1.0+DR.P.2)*(VA(L)-DR*VB(L)))/(2.0*V1(L))-
RB(M4+1)=DR+RC(M4)-
R(M4+1)=R(M4)+DR+RC(M4)/2.0-
K=M4+1-
M4=M4+1-
DR=RB(M4)-
L=L+1-
TRANSFERTO(L11,L40,L40)PROVIDED(M4-M+1)-
L40      M7=M1+1-
          M8=M4-
          WRITEOUTPUT,F6,(M1,R(M1),DR1)-
          DOTHROUGH(L42),I=M7,1,I.LE.M8-
L42      WRITE OUTPUT,F7,(R(I))-
          TRANSFERTO(L7)-
F F1      (2I2)-
F F2      (2I2)-
F F3      (10F7.4)-

```

```
F F4      (I2,F5.2,F8.4)-  
F F5      (23H THE COMPUTATION FOR R=,3X,F5.2,3X,7HFAILED.)-  
F F6      (3H M=,I2,10X,2HR=,3X,F8.3,3HDR=,3X,F8.4)-  
F F7      (1H ,F8.4)-  
          ENDPROGRAM(S1)-
```


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