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TOWARD AN OPTIMAL METHOD OF EQUATING
SUBGROUPS COMPOSED OF DIFFERENT SUBJECTS

DISSERTATION

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the Degree Doctor of Philosophy in the Graduate
School of the Ohio State University

By

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The Ohio State University
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I. INTRODUCTION

Whenever we wish to compare the merit of two or more individuals for assignment to a given task, we are faced first with the problem of determining what traits, abilities, skills, characteristics, etc. are relevant to success on the task. Secondly, we must either collect data on the individuals to be evaluated or use data already collected on them. If the number of individuals to be evaluated is relatively small, then a single evaluator might possibly collect all the data deemed necessary for each trait being studied. I shall define "evaluator" as any instrument which purports to possess the ability to give the relative standings of two or more individuals on a given "observable" (i.e., trait, ability, skill, etc.). An evaluator may, by this definition, be a rater, a paper-and-pencil test, a performance test, an electrode imbedded in the skin, etc.

If we have a large number of individuals to be evaluated we may have to employ several evaluators to accomplish the evaluation of each observable. This would be especially true in areas where ratings are to be used. It is quite possible that we might be able to find one person who we felt knew one group of the individuals well enough in a given area to rate them, a second rater for a second group, etc. For another observable we might use one form of a paper-and-pencil test on part of the group and a different form or forms on the remainder of the group. For yet another observable we might use English grades from high school A for some individuals, English grades from high school B for others, etc.

The moment we begin using more than one evaluator for the same observable we introduce a host of problems which may be subsumed under the general title of "equivalence of scale" problems. This title includes such problems as "leniency," the tendency of one rater to consistently rate everyone higher than would the average rater, and "exaggeration," the tendency of one rater to spread the scores of the ratees over a wider range than would an average rater. These are very common problems in the field of rating (22, 25, 34, 44). "Floor" and "ceiling" effects, the failure of an evaluator to adequately discriminate individuals at one end or the other of the continuum, are also "equivalence of scale" problems, but are most frequently discussed by those in the field of test construction (8, 23). "Composition" effects, concerned with the relative emphasis of different factors on the total evaluation score, have been discussed by both those who construct alternate forms of tests (3, 12) and those in the field of rating (13, 21, 27). The "halo" effect (5, 9, 10, 15, 32, 33, 44, 46, 47) perhaps the most widely known type of rating bias, is a "composition" effect. "Reliability" which broadly considered is concerned with the contribution of an error factor to the total evaluation score, and "validity" which is concerned with the overlap of factors in the evaluation score with those in a criterion score are also considered here to be "composition" effects. These and other "equivalence of scale" problems and their solutions constitute the general topic of this dissertation.

Because of differences in the magnitude of the above-mentioned effects from one "evaluator" to the next, we cannot validly compare the scores of two or

more individuals on a given observable when these have been obtained from different evaluators. For example, a rating of "85" is not necessarily indicative that a ratee is better than another ratee who received a rating of only "78" from a different evaluator. A cumulative point-hour ratio of 3.00 from one university may not indicate the same calibre of student as a 3.00 from another university. Even within the same university the "meaning" of marks may differ from department to department. In fact, the "meaning" of grades frequently differs among instructors within the same department. It is thus not surprising that even the best college entrance test can explain only about 50 per cent of the variance of actual college grades.

METHODS FOR PREVENTING THE OCCURRENCE OF "EQUIVALENCE OF SCALE" ERRORS

There are ways to avoid certain types of "equivalence of scale" errors. Among the "preventives" in the field of test construction are recommendations to include items from all levels of difficulty (18, 43). This is clearly an attempt to prevent a floor or ceiling effect from occurring. The entire history of the development of rating scales is really one attempt after another to develop scales which would be equivalent from one rater to another (anchor points, graphic rating scales, forced distributions, scaling techniques, etc.). In modern rating forms we usually find very careful instructions (sometimes even short training courses are given (14)) on how to avoid various types of bias in rating. This is done in the hope that "a word to the wise will be sufficient." The extent to which such instructions really prevent rating bias is rarely reported, and, if

reported, is based on letting the different raters rate the same group of individuals. What would happen if two or more raters are asked to rate separate groups which really drastically differ in both average ability and spread of ability on the trait to be rated has to my knowledge, either not been done or, if done, the degree of presence of the various types of bias has not been established.

While I believe such preventives should be applied, there is no guarantee that they will always work as designed, and methods of removing the effects of non-equivalence are badly needed.

METHODS FOR THE REMOVAL OF "EQUIVALENCE OF SCALE" ERRORS

Two major approaches to the problem of removing "equivalence of scale" errors are found in the literature. The first approach is based on the assumption that the group of individuals who were evaluated by evaluator A (rater A, form A, etc.) is similar (in all important aspects concerning possession of the trait or ability being measured) to the group of individuals who were evaluated by evaluator B. The second approach does not make this "equivalence of possession of trait rated" assumption. In the case where both evaluators have evaluated the same group of individuals, the assumption is usually made without question (although it is entirely possible that the two raters or two forms weren't measuring the same traits, and the assumption is not warranted). Until recently, the methods of equating alternate forms of a test have uniformly made this assumption. Some people have suggested that if the assumption is true, all that would be necessary to adequately equate the scales of the two evaluators would be a linear transformation to one or both groups of test scores which would make the means

and standard deviations of the two scales equal. Others (22), who have been bothered by the fact that the simple linear transformations would not remove differences between the two scales with regard to skewness and kurtosis, have recommended normalizing both sets of scores. Still others have felt that normalizing the scores might conceivably damage the interval nature of scale values in their more trusted form. For example, if form A of the test has been developed by carefully observing all the rules for obtaining an interval scale for that form, and the distribution of scores obtained is not normal, the developers may decide that the assumption of a normal distribution of the trait being measured is not warranted. These people have suggested that the percentile scores of the two forms be obtained and the raw score of form A be assigned to the equivalent percentile score of form B. These and other methods of equating scales, when the "equivalence of possession of trait rated" in the two groups is assumed, are discussed by Flanagan (19) and Gulliksen (24). While the assumption of equivalence of possession of trait rated" is always questionable, methods using the assumption have been successfully employed by many investigators. For example, Berkshire (4) demonstrated that using standard scores (within classes) rather than raw scores of successive classes of students in the U.S. Naval School of Pre-Flight significantly raised the average intercorrelation among the courses being taught. While this is not conclusive evidence that the assumption was warranted, it does indicate that grading procedures from class to class were varying more drastically than were the ranges and averages of ability. More recently, Lord (28) has

reported success at equating alternate forms of tests administered to different groups which had been constituted by random assignment of the entire group to the two forms.

Several methods using partial overlap among groups to be equated have been used. Thurstone's absolute scaling method (35) uses this procedure. In industrial psychology a majority of the methods for determining wage rates (39) may be considered to be variations of the partial overlap method. Usually a group of experts (from management and/or labor) select a number of "key" jobs which they rate and discuss until general agreement is reached on these "key" jobs concerning overall wage and/or amounts of various abilities required for each job. Independent raters may then evaluate different departments within the company and their ratings can be "equated" to the "key" job wage scale. Some of the key jobs should come from each department.

Several methods have been devised for equating the scales of two or more evaluators when there is no overlap among the groups evaluated by the different evaluators, and when one is unwilling to assume that the distribution of the trait being rated is the same from one subgroup to the next. The earliest statistical method for equating subgroups composed of different subjects was a method described by Toops (37) in 1933. It involved using one variable which was a "standardized" test (an intelligence test) as the variable against which the subgroup scores (grades from various high schools) could be "equated." Basically the method required finding the slope and intercept of each subgroup's scores with the scores of that subgroup on the standardization variable. Each score was

"transmuted" (or "equated") by subtracting that subgroup's intercept and dividing the remainder by that subgroup's slope. Because of the nature of Toops' method I shall refer to it as the "slope and intercept" method of equating subgroups. Toops in his article does not give the rationale behind his method other than to say that the "transmuted" scores would take into account the intellectual differences of different schools. Unfortunately, Toops' paper was published only in an Ohio College Association Bulletin and is not a well known article. No real data were "transmuted" in his paper and, to my knowledge, the efficiency of the method was never demonstrated empirically.

Gulliksen (24) describes a procedure similar to Toops' "slope and intercept" method for equating alternate forms of a test which have been administered to different groups of individuals. Gulliksen credits Tucker with developing the theory on which the procedure is based. Tucker's procedure was to estimate what the mean and standard deviation of each form would have been if both groups had taken both forms. This was accomplished by finding what the slope and intercept of the regression line of each form was to an "equating" variable (a variable on which the entire group has been tested), and then assuming that the slopes and intercepts would have been the same if the entire group had taken both forms. Using the statistics for correction for curtailment of range, Tucker could find the theoretical mean and standard deviation of each form if the entire group had taken both forms. A simple linear transformation to either form was then accomplished in order to make the theoretical means and standard deviations of the two forms

equal. The results of Tucker's procedure, if one has a single "equating" variable, would be identical to Toops' method.

Tucker also included a modified procedure to use when one has more than a single "equating" variable. This procedure was identical to the single "equating" variable method except that the slope and intercept of one form to the best composite of the "equating" variables for the group which took that form are used instead of the regression line of that form to a single "equating" variable. (This procedure seems somewhat questionable, however, since one equates each group, i.e., each alternate form, to a different composite variable. This point will be discussed more fully in the rationale section.) Tucker's method, like Toops', was not published in a standard journal. It was published (38) in 1953 as one of the papers delivered at an invitational conference on testing problems. Gulliksen's mention of Tucker's method appeared in 1950. Its use was reported by Englehart (16) in 1959.

While techniques for equating subgroups composed of different people are in existence, they are rarely discussed and generally unknown. There is, nevertheless, an obvious need for understanding the causes of "equivalence of scale" errors, and for techniques designed to accomplish the removal of such errors. Two areas of need already mentioned are better prediction of college success (this was the point of Toops' method) and alternate forms construction. I feel certain that the availability of a proven technique for handling these problems would be welcomed by the "selection" and "placement" fields of psychology. One example of an evaluation situation requiring a method of equating scores from subgroups

composed of different individuals is found in naval aviation training and will serve in this dissertation as the vehicle for an empirical demonstration of a possible solution to such problems.

BACKGROUND OF THE PROBLEM TO BE USED FOR EMPIRICAL DEMONSTRATION

The U.S. Navy trains several types of aviators. While part of the training of all naval aviators is virtually identical (i.e., standardized for all students), the advanced phases of training are quite different, depending on what type of aircraft the student will be expected to fly when he graduates from training and joins the fleet. The cost of training all types of aviators is tremendously high; so high, in fact, that the assignment of individuals to the various advanced training pipelines in an optimal fashion could effect substantial training economies. Such assignment can be accomplished by computers using linear programming (20), if a "figure of merit" can be established for every individual for each of the advanced training types. The "figure of merit" is a composite of such elements as "cost to train" (training for different types of aircraft is differentially costly), "worth to the Navy of this type of aviator," and "predicted differential success as each type of aviator" (fighter, patrol, helicopter, etc.). It is the need for this latter element which requires an ability to equate training success scores. Every student completing training does have an "Advanced Flight Grade" which indicates fairly accurately the relative success of a given aviator from that subgroup of aviators to other aviators from that same subgroup. The validity of these "Advanced Flight Grades" has been established by several investigations (30, 31).

Within a given type of advanced training these grades can be predicted fairly accurately by using data collected on the aviators prior to their assignment to the advanced training. However, a predicted grade, or an actual grade, of 302 in one subgroup does not necessarily indicate the same degree of absolute "success as a pilot" as a score (predicted or actual) of 302 from a different subgroup. The problem is complicated by the fact that there are seven different types of advanced training with different numbers of students in each type.

The practical problem then is to find a method for equating the "Advanced Flight Grades" among the different subgroups of students in advanced training. Those individuals in the past who have had the administrative responsibility for assigning students to the different types of advanced training have ignored the lack of "equivalence of scale" among these seven subgroups. This was due in part, I believe, to a lack of a comprehensive rationale which would embrace all of the "equivalence of scale" problems, and in part to the lack of proven method for equating subgroups composed of different individuals.

It is hoped that the rationale and the techniques to be set forth in this dissertation will, in some measure, help fill this need. It is perhaps fitting that we quote from Toops' original article on the "slope and intercept" method. He said, "This method is developed below, in the hope that others may use it, discover and report its faults, and, perchance, devise a better system, or one allowing adequately for more factors."

II. RATIONALE

REAL DIFFERENCES AMONG INDIVIDUALS

In the hypersphere concept (45) of correlation it is assumed that there are a finite number (m) of independent dimensions which account for all the real differences among all individuals. Each individual is represented (i.e., completely described) by a single point in this m -dimensional space, and all individuals could be enclosed by a m -dimensional envelope called the "hypersphere." A given individual's location may be specified by the perpendicular projections of the point representing that individual on the m independent dimensions. Further, we may express these projections in terms of standard scores for each dimension. Thus, following accepted factor analytic theory (36),

$$\text{Location of individual } i \rightarrow z_{1i}, z_{2i}, z_{3i}, \dots, z_{fi}, \dots, z_{mi}. \quad (1)$$

There are an infinite number of vectors running through this m dimension hypersphere and these vectors may be described in one of several ways. We may use either some trigonometric function (such as the cosine) of the angle between the specified vector and each independent dimension, or we could use the relationship (determined by the Pearson product-moment correlation coefficient based on all the cases available) between the specified vector and each independent dimension. Thus,

$$\text{Vector } X \rightarrow \cos \angle X01, \cos \angle X02, \dots, \cos \angle X0f, \dots, \cos \angle X0m. \quad (2)$$

$$\text{Vector } X \rightarrow r_{x_1}, r_{x_2}, r_{x_3}, \dots, r_{xf}, \dots, r_{xm}. \quad (3)$$

Wherry, Jr. (45) has shown that under certain conditions (when $\sigma_1 = \sigma_2 = \sigma_3 = \dots = \sigma_f = \dots \sigma_k$) that the two expressions above are identical. In the following discussion we shall assume that these conditions are true.

When a given observer (a rater, a test, or some other type of measuring instrument) observes the locations of a group of individuals we assume that what he is really doing is observing the locations of the individuals along some vector through the hypersphere. Usually when a set of observations is being made some random error, unrelated to any of the m dimensions, creeps in. We may think of the error as being another independent dimension. Thus, a given individual is imperfectly observed to be at

$$z_{x_i} = r_{x_1} z_{1_i} + r_{x_2} z_{2_i} + r_{x_3} z_{3_i} + \dots + r_{x_f} z_{f_i} + \dots + r_{x_m} z_{m_i} + r_{x_e} z_{e_i} \quad (4)$$

For simplification purposes, we shall simply call these r_{x_i} 's weights and symbolize them by "w's". Thus,

$$z_{x_i} = w_1 z_{1_i} + w_2 z_{2_i} + \dots + w_f z_{f_i} + \dots + w_m z_{m_i} + w_e z_{e_i} \quad (5)$$

Where z_{x_i} = the standard score expression of individual i 's supposed location on vector X ,

z_{f_i} = the standard score expression of the location of individual i on factor f ,

z_{e_i} = the standard score expression of error for individual i ,

m = the total number of independent real factors,

f = an unspecified factor, and

w_f = the weight given factor f ($= r_{xf}$).

DIFFERENCES AMONG EVALUATORS

When a given rater is asked to rate a group of individuals on some given trait, or variable, the w 's are usually unspecified for the most part. A second rater will undoubtedly have a different set of weights for the m factors than did the first rater. Further, the relative contribution of error (w_e) and the error score itself (z_{e_i}) will probably be different among different raters.

The entire population of people might be broken down into k subgroups and a different rater might be assigned to rate each subgroup. If we were able to discover what weights were used for each subgroup for each factor, we might table the values as seen in Table I. Notice that each subgroup has a different error factor. This is to say that the error in one subgroup is unrelated to the error in a second subgroup.

Table I

Theoretical Weights Used on m Independent Factors and Error Factors for Determining an Individual's X score

		Real Factors					Error Factors						
Subgroup	1	2	...	f	...	m	E_1	E_2	...	E_1	...	E_k	E_T
1	w_{1_1}	w_{2_1}	...	w_{f_1}	...	w_{m_1}	w_{E_1}	.00	...	.00	...	.00	.00
2	w_{1_2}	w_{2_2}	...	w_{f_2}	...	w_{m_2}	.00	w_{E_2}	...	.00	...	.00	.00
...	...	...	...	...	...	...	...	...	...	...	...	...	...
g	w_{1_g}	w_{2_g}	...	w_{f_g}	...	w_{m_g}	.00	.00	...	w_{E_g}	...	.00	.00
...	...	...	...	...	...	...	...	...	...	...	...	...	...
k	w_{1_k}	w_{2_k}	...	w_{f_k}	...	w_{m_k}	.00	.00	...	.00	...	w_{E_k}	.00
Standard Y	$\bar{w}_1$	$\bar{w}_2$	...	$\bar{w}_f$	...	$\bar{w}_m$	.00	.00	...	.00	...	.00	$\bar{w}_E$

The "Reliability" Effect

Since each w is a correlation coefficient, it represents the square root of the proportion of variance explainable by its factor, and because the factors are independent, the sum of the w_g^2 's for any subgroup will equal 1.00. In this way we may specify that the "reliability" (λ_g) effect in subgroup g is equal to the square root of one minus the $w_{E_g}^2$, i.e.,

$$\begin{array}{l} \text{Index of} \\ \text{reliability} \\ \text{for subgroup} \\ g \end{array} = \lambda_g = \sqrt{1 - w_{E_g}^2} = \sqrt{\frac{m}{\sum w_g^2}} . \quad (6)$$

We do not assume that the index of reliability would be the same among subgroups. (This would depend on the various w_E 's.)

$$\lambda_1 \neq \lambda_2 \neq \dots \neq \lambda_g \neq \lambda_k . \quad (7)$$

Further, we do not assume that the weight in one subgroup for any given factor will be the same as the weight for that factor in any other subgroup.

$$w_{f_1} \neq w_{f_2} \neq \dots \neq w_{f_g} \neq \dots \neq w_{f_k} . \quad (8)$$

There must be an average weight ($\bar{w}$) assigned to each of the m independent factors which would represent a "standard" (Y) on which our subgroups might be equated. However, it is not particularly easy to say how these average weights should be defined. For example, one way might be

$$\bar{w}_f = \frac{n_1 w_{f_1} + n_2 w_{f_2} + \dots + n_g w_{f_g} + \dots + n_k w_{f_k}}{N} . \quad (9)$$

In one sense this would give the "average" w_f , but this is differentially determined by the size of each subgroup. A second method would be to give equal weight to each subgroup's opinion about what magnitude w_f should be. Thus,

$$\bar{w}_f'' = \frac{w_{f_1} + w_{f_2} + \dots + w_{f_g} + \dots + w_{f_k}}{k} \quad (10)$$

Still another method might be to consider the proportion of variance being explained, thus

$$\bar{w}_f''' = \frac{n_1 w_{f_1}^2 + n_2 w_{f_2}^2 + \dots + n_g w_{f_g}^2 + \dots + n_k w_{f_k}^2}{N} \quad (11)$$

Actually there are many other justifiable ways to define the "average" w_f . We show eight of these in Table II.

Table II

Various Ways of Defining the "Average" Weight ($\bar{w}_f$) for Factor f

Subgroup Weighting	w_{fg}	w_{fg}^2	Usable Functions of " w_{fg} "	
			$\sqrt{w_{fg}^2/\lambda_g^2}$	w_{fg}^2/λ_g^2
Weight each subgroup equally	$\frac{\sum w_{fg}}{k}$	$\frac{\sum w_{fg}^2}{k}$	$\frac{\sum \sqrt{w_{fg}^2/\lambda_g^2}}{k}$	$\sqrt{\frac{\sum (w_{fg}^2/\lambda_g^2)}{k^2}}$
Weight each subgroup by sample size	$\frac{\sum n_g w_{fg}}{N}$	$\frac{\sum n_g w_{fg}^2}{N}$	$\frac{\sum n_g \sqrt{w_{fg}^2/\lambda_g^2}}{N}$	$\sqrt{\frac{\sum (n_g w_{fg}^2/\lambda_g^2)}{N^2}}$

There would seem to be plausible reasons for using any one of these ways to define $\bar{w}_f$. Each would yield a slightly different vector (the Y vector) against

which subgroups could be equated. The relative merits of the different methods are academic considerations since we would rarely hope to be able to define the independent factors (usually we only have one score per individual) much less accurately determine the weights that were actually being used on each factor in each of the k subgroups. Dingman and Guilford (13) and Jones (27) have discussed the problem of finding the optimal vector to maximize composition effects among different evaluators.

Let us merely say that we shall attempt to find a set of scores (Y scores) which contain an "average" composite of the factor weights being used in the subgroups. These scores will be the "standard" against which we will attempt to equate the subgroups. An individual's score (expressed in standard score form) on the Y vector would be

$$z_{Y_i} = \bar{w}_1 z_{F_{1_i}} + \bar{w}_2 z_{F_{2_i}} + \dots + \bar{w}_f z_{F_{f_i}} + \dots + \bar{w}_m z_{F_{m_i}} + \bar{w}_E z_{E_i} \quad (12)$$

In turn we may combine the true portion of the z_Y score as follows

$$Y_i = \bar{w}_1 z_{F_{1_i}} + \bar{w}_2 z_{F_{2_i}} + \dots + \bar{w}_f z_{F_{f_i}} + \dots + \bar{w}_m z_{F_{m_i}} \quad (13)$$

Equation (12) really becomes a definition of the Y vector. We may say that his entire score is

$$z_{Y_i} = \lambda_y \cdot Y_i + \sqrt{1 - \lambda_y^2} \cdot e_{y_i} \quad (14)$$

$$\text{where } \lambda_y = \sqrt{\frac{\sum \bar{w}_f^2}{m}} \quad .$$

The "Composition" Effect

The correlation between the scores the entire sample would have received if they had all been assigned to subgroup g and the standard would be

$$r_{X_{N_g} Y_N} = \dot{\theta}_g = \lambda_g \lambda_Y \sum w_{fg} \bar{w}_f, \text{ and} \quad (15)$$

The correlation between the "reliable" portion of X and the reliable portion of Y would be

$$\theta_g = \frac{\dot{\theta}_g}{\lambda_g \lambda_Y} \quad (16)$$

This "unattenuated" correlation (θ_g) will be called the "composition" effect.

Now we may consider a person's subgroup score to be composed in part of the "true" part of the standard and that part of the subgroup score which is independent of the "true" part of the standard. We may call this second part the "uniqueness" (U_{g_1}) of subgroup g 's score for individual i . Thus,

$$X_{g_1} = \theta_g Y_i + \sqrt{1 - \theta_g^2} \cdot U_{g_1} \quad (17)$$

The uniqueness in turn can be subdivided into a non-error part (the "true" variance in subgroup i 's scores unrelated to the Y vector), and an error part. The former we shall denote by the letter " S_g " (the "specific" portion of a subgroup g scores) while the latter is the previously discussed e_{g_1} . Since we already know that the weight that e_{g_1} gets is w_{E_g} ($= \sqrt{1 - \lambda_g^2}$) then we can show

$$\sqrt{1 - \theta_g^2} \cdot U_{g_1} = \sqrt{\lambda_g^2 - \theta_g^2} \cdot S_{g_1} + \sqrt{1 - \lambda_g^2} \cdot e_{g_1} \quad (18)$$

The "Exaggeration" Effect

Any given subgroup evaluator may tend to spread the observed scores (X_{g_i}) out or to compress them too much. This tendency is known as the "exaggeration" effect. We shall assume that this works in a simple multiplicative fashion on the X_g scores. Thus,

$$\dot{X}_{g_i} = \dot{\beta}_{gt} X_{g_i} . \quad (19)$$

Where $\dot{\beta}_{gt}$ = the "exaggeration" effect in subgroup g at time t .

The "Leniency" Effect

The "leniency" effect, the tendency to give each person in a given subgroup a higher or lower score than he deserves, is assumed to work in an additive fashion. We may now write the equation for the i^{th} individual in subgroup g when both an "exaggeration" and "leniency" effect are operating.

$$\dot{X}_{g_i} = \dot{\alpha}_{gt} + \dot{X}_{g_i} = \dot{\alpha}_{gt} + \dot{\beta}_{gt} X_{g_i} , \quad (20)$$

where $\dot{\alpha}_{gt}$ = the "leniency" effect at time t in subgroup g .

FLUCTUATIONS WITHIN EVALUATORS

It will be helpful in understanding "leniency" and "exaggeration" to think of each evaluator as if he had been given a "rubber yardstick" to measure the individuals in his group. "Exaggeration" effects result because different evaluators stretch their yardsticks differentially taut. To assume that $\dot{\beta}_{gt}$ is constant for all subjects rated by evaluator g is to assume that evaluator g can maintain the same degree of tautness in his yardstick at all times. Such an assumption is probably not warranted if the "evaluator" is a rater. It is entirely possible that the degree of tautness in one rater's yardstick may undergo gradual

changes as a function of time. Similar comments may be true of "leniency" effects as well. A rater or instructor may conceivably become progressively more lenient as a function of time, or raters may be more lenient on Mondays than they are on Fridays. Coombs' (11) rationale recognizes such fluctuations.

The "composition" effect will probably also show fluctuation from day to day as well as gradual shift over time. It is certainly not difficult to imagine a rater's definition of what constitutes "leadership" changing in the course of time. We can also hypothesize fluctuations in the reliability of a rater's ratings over time. As a rater learns more and more about the individuals he is rating, his ratings should be more free of error.

While within-evaluator changes in non-equivalence effects is a fascinating problem, especially those of a gradual shift nature, it is beyond the scope of this dissertation. We shall assume that over the period of time that the ratings we have were obtained, the various fluctuations mentioned above have occurred in a random fashion. Our concern will be with the average "leniency," the average "exaggeration," etc. We shall let

$$\dot{\alpha}_{gt} = \alpha_g + \Delta\alpha_{gt}, \text{ and} \quad (20.1)$$

$$\dot{\beta}_{gt} = \beta_g + \Delta\beta_{gt}, \quad (20.2)$$

where α_g = the average "leniency" effect, in subgroup g ,

β_g = the average "exaggeration" effect in subgroup g ,

$\Delta\alpha_{gt}$ = the difference between the actual and the average "leniency" effects, and

$\Delta\beta_{gt}$ = the difference between the actual and the average "exaggeration" effects.

Then, substituting Eqs. (20.1) and (20.2) into Eq. (20)

$$\ddot{X}_{g_i} = \alpha_g + \beta_g X_{g_i} + (\Delta\alpha_{g_i} + \Delta\beta_{g_i} \cdot X_{g_i}). \quad (20.3)$$

Since the fluctuations (Δ 's) are random, they will not correlate with any other terms in our equations. Since the fluctuations are also deviation scores, they will sum to zero for the entire group. We shall make similar assumptions regarding fluctuations in "composition" and "reliability" effects. With these assumptions made we can consider all fluctuation terms (Δ 's) to have already been accounted for in our error term for individual i (e_{g_i}). Dropping the Δ terms from Eq. (20.3) and substituting Eqs. (17) and (18) we obtain

$$\ddot{X}_{g_i} = \alpha_g + \beta_g \theta_g Y_i + \beta_g \sqrt{\lambda_g^2 - \theta_g^2} \cdot S_{g_i} + \beta_g \sqrt{1 - \lambda_g^2} \cdot e_{g_i}, \quad (21)$$

where $\ddot{X}_{g_i}$ = the i^{th} individual's raw score in subgroup g ,

α_g = the "leniency" effect in subgroup g ,

β_g = the "exaggeration" effect in subgroup g ,

θ_g = the "composition" effect in subgroup g ,

λ_g = the "reliability" effect in subgroup g ,

Y_i = individual i 's location on the Y vector, the vector being used to equate the subgroups,

S_{g_i} = individual i 's location on the "specific" dimension in subgroup g .

e_{g_i} = individual i 's location on the error dimension in subgroup g .

The "standardizing" variable is assumed to also have undergone an "exaggeration" and "leniency" effect. Thus from Eq. (14) we may write the final¹ equation for a "standardizing" variable (the variable to be used to equate the subgroups) score as

$$\dot{Y}_i = \alpha_Y + \beta_Y \lambda_Y Y_i + \beta_Y \sqrt{1 - \lambda_Y^2} \cdot e_{Y_i} \quad (22)$$

Where α_Y = the "leniency" effect on the standardizing variable,

β_Y = the "exaggeration" effect on the standardizing variable,

λ_Y = the "reliability" effect on the standardizing variable,

Y_i = i's location of the vector being used to equate the subgroups,

$\dot{Y}_i$ = i's raw score on the standardizing variable, and

e_{Y_i} = i's location on an error dimension in standard score form.

Now the sum of the raw scores in subgroup g would be

$$\sum_{i=1}^{n_g} \dot{X}_g = n_g \alpha_g + \beta_g \theta_g \sum_{i=1}^{n_g} Y_g + \beta_g \sqrt{\lambda_g^2 - \theta_g^2} \cdot \sum_{i=1}^{n_g} S_g + \beta_g \sqrt{1 - \lambda_g^2} \cdot \sum_{i=1}^{n_g} e_g .$$

Dividing the above equation by n_g , we could find the mean of the raw scores in subgroup g.

$$\bar{\dot{X}}_g = \frac{\sum_{i=1}^{n_g} \dot{X}_g}{n_g} = \alpha_g + \beta_g \theta_g \bar{Y}_g + \beta_g \sqrt{\lambda_g^2 - \theta_g^2} \cdot \bar{S}_g + \beta_g \sqrt{1 - \lambda_g^2} \cdot \bar{e}_g . \quad (23)$$

A deviation score in subgroup g would be found by subtracting Eq. (23) from (21).

$$\ddot{x}_{g_i} = \dot{X}_{g_i} - \bar{\dot{X}}_g = \beta_g \theta_g Y_g + \beta_g \sqrt{\lambda_g^2 - \theta_g^2} \cdot s_{g_i} + \beta_g \sqrt{1 - \lambda_g^2} \cdot e_{g_i} \quad (24)$$

¹ Inclusion of "floor" or "ceiling" and "differential treatment of extremes" effects will be considered later.

The mean score of the standardizing variable scores in subgroup g would be from equation (22), following the same logic.

$$\bar{\dot{Y}}_g = \frac{\sum_{n_g} \dot{Y}_g}{n_g} = \alpha_y + \beta_y \lambda_y \bar{Y}_g + \beta_y \sqrt{1 - \lambda_y^2} \cdot \bar{e}_{y_g} . \quad (25)$$

A deviation score on the standardizing variable would be found by subtracting Eq. (25) from Eq. (22). Thus,

$$\dot{y}_{g_i} = \beta_y \lambda_y y_{g_i} + \beta_y \sqrt{1 - \lambda_y^2} \cdot e_{y_i} . \quad (26)$$

Squaring these deviation scores and summing through the n_g cases in subgroup g

$$\sum_{n_g} \dot{y}_g^2 = \beta_y^2 \left[\lambda_y^2 \sum_{n_g} y_g^2 + 2\lambda_y \sqrt{1 - \lambda_y^2} \cdot \sum_{n_g} y e + (1 - \lambda_y^2) \cdot \sum_{n_g} e_y^2 \right] ,$$

but $\sum_{n_g} y e$ should equal 0.0 and $\sum_{n_g} e_y^2$ will equal n_g , therefore,

$$\sum_{n_g} \dot{y}^2 = \beta_y^2 \left[\lambda_y^2 \sum_{n_g} y_g^2 + (1 - \lambda_y^2) n_g \right] . \quad (27)$$

The sum of the cross products of a $\dot{y}$ score and a $\ddot{x}$ score will be

$$\begin{aligned} \sum_{n_g} \ddot{x} \dot{y}_g &= \beta_y \lambda_y \beta_g \theta_g \sum_{n_g} y_g^2 + \beta_y \lambda_y \beta_g \sqrt{\lambda_g^2 - \theta_g^2} \sum_{n_g} s_g y \\ &\quad + \beta_y \lambda_y \beta_g \sqrt{1 - \lambda_y^2} \sum_{n_g} y e_g + \beta_y \sqrt{1 - \lambda_y^2} \beta_g \theta_g \sum_{n_g} y_g e_y \\ &\quad + \beta_y \sqrt{1 - \lambda_y^2} \beta_g \sqrt{\lambda_g^2 - \theta_g^2} \sum_{n_g} s_g e_y \\ &\quad + \beta_y \sqrt{1 - \lambda_y^2} \beta_g \sqrt{1 - \lambda_g^2} \sum_{n_g} e_g e_y . \end{aligned} \quad (28)$$

However, there should be no correlation among S , Y , e_g , or e_y ; therefore,

$$\begin{aligned} \sum_{n_g} s_g y &= \sum_{n_g} y e_g = \sum_{n_g} y_g e_g = \sum_{n_g} s_g e_y = \sum_{n_g} e_g e_y = 0.0, \text{ and} \\ \sum_{n_g} \ddot{x}_g \dot{y}_g &= \beta_y \lambda_y \beta_g \theta_g \sum_{n_g} y_g^2. \end{aligned} \quad (29)$$

Dividing $\sum_{n_g} x y_g$ by $\sum_{n_g} y_g^2$, we may find the slope (b_g) of $\ddot{X}_g$ on $\dot{Y}_g$.

$$\begin{aligned} b_g &= \frac{\sum_{n_g} \ddot{x}_g \dot{y}_g}{\sum_{n_g} \dot{y}_g^2} = \frac{\beta_y \lambda_y \beta_g \theta_g \sum_{n_g} y_g^2}{\beta_y^2 \left[\lambda_y^2 \sum_{n_g} y_g^2 + (1 - \lambda_y^2) n_g \right]}, \\ &= \frac{\beta_g \theta_g}{\beta_y \left[\lambda_y + \frac{(1 - \lambda_y^2)}{\lambda_y} \cdot \frac{n_g}{\sum_{n_g} y_g^2} \right]}. \end{aligned} \quad (30)$$

The intercept (a_g) of the $\ddot{X}$ scores on the $\dot{Y}$ scores for subgroup g is found by

$$a_g = \bar{\ddot{X}}_g - b_g \bar{\dot{Y}}_g.$$

Substituting Eqs. (23), (25), and (30) into the above and assuming $\bar{e}_g = \bar{e}_{y_g} = 0.0$,

we obtain

$$\begin{aligned}
a_g &= \alpha_g + \beta_g \theta_g \bar{Y}_g + \beta_g \sqrt{\lambda_g^2 - \theta_g^2} \cdot \bar{S}_g - \frac{\alpha_y \beta_g \theta_g}{\beta_y \left[\lambda_y + \frac{(1 - \lambda_y^2)}{\lambda_y} \cdot \frac{n_g}{\Sigma y_g^2} \right]} \\
&\quad - \frac{\beta_g \theta_g \beta_y \lambda_y \bar{Y}_g}{\beta_y \lambda_y \left[1 + \frac{1 - \lambda_y^2}{\lambda_y^2} \cdot \frac{n_g}{\Sigma y_g^2} \right]}, \\
&= \alpha_g + \beta_g \theta_g \bar{Y}_g + \beta_g \sqrt{\lambda_g^2 - \theta_g^2} \bar{S}_g \\
&\quad - \frac{\alpha_y \beta_g \theta_g}{\beta_y \lambda_y \left[1 + \frac{(1 + \lambda_y^2)}{\lambda_y^2} \cdot \frac{n_g}{\Sigma y_g^2} \right]} - \frac{\beta_g \theta_g \bar{Y}_g}{1 + \frac{(1 + \lambda_y^2)}{\lambda_y^2} \cdot \frac{n_g}{\Sigma y_g^2}} \quad (31)
\end{aligned}$$

TRANSFORMING A SET OF SCORES TO HAVE A SLOPE OF 1.00 AND AN INTERCEPT OF 0.0 WITH A SECOND SET OF SCORES

In order to transform a set of scores so they will have a slope of 1.00 and an intercept of 0.0 with a second set of scores all we must do is subtract the present intercept and divide the remainder by the present slope. This is, of course, Toop's "slope and intercept" method. Let us now transform our $\ddot{X}$ scores in subgroup g so that they have a slope of 1.00 and an intercept of 0.0 with the $\dot{Y}$ scores. These new transformed scores will be symbolized as $\ddot{\ddot{X}}$, thus

$$\ddot{\ddot{X}}_{g_1} = \frac{\ddot{X}_{g_1} - a_g}{b_g} \quad (32)$$

Substituting Eq. (21) for $\ddot{X}_{g_1}$ and Eqs. (30) and (31) for the values of a_g and b_g , we obtain

$$\ddot{X}_{g_1} = \frac{\left\{ \begin{aligned} &\alpha_g + \beta_g \theta_g Y_1 + \beta_g \sqrt{\lambda_g^2 - \theta_g^2} \cdot s_{g_1} + \beta_g \sqrt{1 - \lambda_g^2} \cdot e_{g_1} \\ &- \alpha_g - \beta_g \theta_g \bar{Y}_g - \beta_g \sqrt{\lambda_g^2 - \theta_g^2} \cdot \bar{s}_g + \frac{\alpha_y \beta_g \theta_g}{\beta_y \lambda_y \left[1 - \frac{(1 - \lambda_y^2)}{\lambda_y^2} \cdot \frac{n_g}{\Sigma y_g^2} \right]} \\ &+ \bar{Y}_g \cdot \frac{\beta_g \theta_g}{1 - \frac{(1 - \lambda_y^2)}{\lambda_y^2} \cdot \frac{n_g}{\Sigma y_g^2}} \end{aligned} \right\}}{\frac{\beta_g \theta_g}{\beta_y \lambda_y \left[1 - \frac{(1 - \lambda_y^2)}{\lambda_y^2} \cdot \frac{n_g}{\Sigma y_g^2} \right]}} .$$

Collecting like terms and dividing the last two terms in the numerator by the denominator

$$\ddot{X}_{g_1} = \frac{\beta_g \theta_g Y_1 - \beta_g \sqrt{\lambda_g^2 - \theta_g^2} \cdot s_{g_1} + \beta_g \sqrt{1 - \lambda_g^2} \cdot e_{g_1}}{\beta_g \theta_g} + \alpha_y + \beta_y \lambda_y \bar{Y}_g \cdot \frac{1}{\beta_y \lambda_y \left[1 - \frac{(1 - \lambda_y^2)}{\lambda_y^2} \cdot \frac{n_g}{\Sigma y_g^2} \right]}$$

Rearranging the terms and reducing

$$\begin{aligned}
 \ddot{X}_{g_1} = & \alpha_Y + \beta_Y \lambda_Y Y_1 + \beta_Y \lambda_Y \left[1 - \frac{(1 - \lambda_Y^2)}{\lambda_Y^2} \cdot \frac{n_g}{\Sigma Y_g^2} \right] \cdot Y_{g_1} \\
 & + \beta_Y \lambda_Y \cdot \frac{\sqrt{\lambda_g^2 - \theta_g^2}}{\theta_g} \cdot s_{g_1} \cdot \left[1 - \frac{(1 - \lambda_Y^2)}{\lambda_Y^2} \cdot \frac{n_g}{\Sigma Y_g^2} \right] \\
 & + \beta_Y \lambda_Y \cdot \frac{\sqrt{1 - \lambda_g^2}}{\theta_g} \cdot \left[1 - \frac{(1 - \lambda_Y^2)}{\lambda_Y^2} \cdot \frac{n_g}{\Sigma Y_g^2} \right] \cdot e_{g_1}
 \end{aligned} \tag{33}$$

DIFFERENCES AMONG GROUPS EVALUATED

The "Curtailment" Effect

In several parts of the above equation we see the term $n_g / \Sigma Y_g^2$ which is equal to $1 / \sigma_{Y_g}^2$. The reader will recall that the Y 's referred to here are standard scores (see Eq. (13)), and that the total variance of Y 's would be 1.00 if we include all persons in the entire group rather than only those in subgroup g . Thus, $1 / \sigma_{Y_g}^2$ is the ratio of the original variance of the Y 's to the curtailed variance of the Y 's in the g^{th} subgroup. We may symbolize this "curtailment" effect as Φ_g , thus

$$\Phi_g = \frac{\sigma_Y^2 \text{ total}}{\sigma_{Y_{n_g}}^2} = \frac{\text{original variance of the } y \text{ vector scores for entire group}}{\text{curtailed variance of the } y \text{ vector scores for subgroup } g} \tag{34}$$

Substituting Eq. (34) into Eq. (33) and regrouping our terms, we obtain

$$\begin{aligned}\ddot{X}_{g_1} = & \alpha_Y + \beta_Y \lambda_Y (\bar{Y}_g + y_{g_1}) - \left[\beta_Y \lambda_Y \cdot \frac{(1 - \lambda_Y^2)}{\lambda_Y^2} \cdot \Phi_g \right] \cdot y_{g_1} \\ & + \beta_Y \lambda_Y \frac{\sqrt{\lambda_g^2 - \theta_g^2}}{\theta_g} \cdot \left[1 - \frac{(1 - \lambda_Y^2)}{\lambda_Y^2} \Phi_g \right] \cdot s_{g_1} \\ & + \beta_Y \lambda_Y \frac{\sqrt{1 - \lambda_g^2}}{\theta_g} \cdot \left[1 - \frac{(1 - \lambda_Y^2) \Phi_g}{\lambda_Y^2} \right] \cdot e_{g_1} .\end{aligned}$$

However, $\bar{Y}_g + y_{g_1} = Y_{g_1}$ and substituting this and collecting all terms with $\beta_Y \lambda_Y$ and all terms with $(1 - \lambda_Y^2) \theta_g / \lambda_Y^2$ together, we obtain

$$\begin{aligned}\ddot{X}_{g_1} = & \alpha_Y + \beta_Y \lambda_Y \left[Y_{g_1} + \frac{\sqrt{\lambda_g^2 - \theta_g^2}}{\theta_g} \cdot s_{g_1} + \frac{\sqrt{1 - \lambda_g^2}}{\theta_g} \cdot e_{g_1} \right. \\ & - \frac{(1 - \lambda_Y^2) \Phi_g}{\lambda_Y^2} \cdot \left(y_{g_1} + \frac{\sqrt{\lambda_g^2 - \theta_g^2}}{\theta_g} \cdot s_{g_1} \right. \\ & \left. \left. + \frac{\sqrt{1 - \lambda_g^2}}{\theta_g} \cdot e_{g_1} \right) \right] \quad (35)\end{aligned}$$

where α_Y = the "leniency" effect in the "standardization" variable,

β_Y = the "exaggeration" effect in the "standardization" variable,

λ_Y = the "reliability" effect in the "standardization" variable,

λ_g = the "reliability" effect in the "subgroup" scores,

θ_g = the "composition" effect in the "subgroup" scores,

Φ_g = the "curtailment" effect in the subgroup,

Y_{g_i} = location of individual i on the "common" vector in standard score form,

s_{g_i} = the deviation of individual i in standard score units on the "specific" (true but unrelated to the "common" Y factor) dimension in subgroup g ,

e_{g_i} = the deviation of individual i in standard score units on an "error" dimension, and

y_{g_i} = the deviation of individual i from the average score mean ($\bar{z}_{Y_g}$) Y score in subgroup g , and

$\ddot{X}_{g_i}$ = individual i 's "transmuted" subgroup score.

These $\ddot{X}$ scores are called "transmuted" scores following Toop's notation (28).

Since α_Y and β_Y will be the same for a transmuted score from any subgroup, and amount only to a linear transformation of all the transmuted scores we need not concern ourselves with these terms.

If we sum the transmuted scores for subgroup g and divide by n_g , we may obtain the mean transmuted score for subgroup g , thus

$$\bar{\ddot{X}}_g = \frac{\sum_{i=1}^{n_g} \ddot{X}_{g_i}}{n_g} = \alpha_Y + \beta_Y \lambda_Y \bar{Y}_g \quad (36)$$

This is true because the deviation scores in a given subgroup will sum to zero. From Eq. (35) we see that the transmuted scores of the subgroups will definitely equate the means of the k subgroups regardless of the value of λ_Y as long as λ_Y does not equal zero. That is, if we wish to find the relative standings of various subgroups with respect to the "true" portion of a "standardizing" variable, Eq. (36) will accomplish this goal.

However, we usually wish to not only equate the subgroups but also to equate individuals from different subgroups as well. Let us now examine how well the "transmutation" equation accomplishes this goal.

The "Transmutation Regression" Effect

From Eq. (35) we may examine the relative contribution of the various "effects." One of the more interesting terms in Eq. (35) is the y_{g_i} term. The reader may see that this term is always working against fully equating individuals from different subgroups as it is a function of where an individual was on the Y vector with reference to his own subgroup. Further, the sign of this effect is always negative. Thus a person who was high on the Y vector within his own group gets penalized while one who is low on the Y vector in his own group gets a bonus. There is thus a regressing toward their own mean, $\bar{Y}_g$, in the transmutation equation. In fact, we shall call this the "transmutation regression" effect.

The extent of the "transmutation regression" effect is in turn a function of "reliability" effect of the standardization variable and the "curtailment" effect in that subgroup. That is, the narrower the range of the Y vector ability in a given subgroup, the larger the "transmutation regression" effect.

The reliability of the standardizing variable has an interesting influence on the "transmutation regression" effect in that the higher the reliability of the "standardization" variable the smaller the "transmutation regression" effect, regardless of the magnitude of the "curtailment" effect.

In fact, as the "reliability" effect of the standardization variable approaches 1.00 then

$$\ddot{X}_{g_1} \rightarrow \alpha_y + \beta_y \left[Y_{g_1} + \frac{\sqrt{\lambda_g^2 - \theta_g^2}}{\theta_g} \cdot s_{g_1} + \frac{\sqrt{1 - \lambda_g^2}}{\theta_g} \cdot e_{g_1} \right] \quad (37)$$

One might say that he is not concerned over what the various subgroups are measuring in "common" but he has a set of scores on some variable that he would like to equate the subgroups on. He may even maintain that he has a perfect set of scores to which he would like to equate various subgroups. In such a case he may assume $\lambda_y = 1.00$ and Eq. (37) becomes applicable. The extent to which he has equated the subgroups then becomes merely a function of the "reliability" and "composition" effects within the various subgroups. The proportion of the $\ddot{X}$ variance that is attributable to the "standardization" variable, in such a case, turns out to be θ_g^2 .

TOWARD AN OPTIMAL SOLUTION

The point may be raised that while the preceding rationale is satisfactory from the standpoint that one may see the relative contributions of such influences as "leniency," "exaggeration," "reliability," "composition," etc., we do not know if the "transmutation" equations we obtain for the subgroups are, in fact, the optimal ones.

The "transmutation" equation for subgroup g , as shown in Eq. (32), will be of the form

$$\ddot{X}_{g_1} = \frac{\ddot{X}_{g_1} - a_g}{b_g} ,$$

where a_g = the intercept of the $\ddot{X}$ scores on the $\dot{Y}$ scores for subgroup g ,
and

b_g = the slope of the $\ddot{X}$ scores on the $\dot{Y}$ scores for subgroup g .

We may rewrite the above equation as

$$\ddot{X}_{g_1} = \frac{\ddot{X}_{g_1}}{b_g} - \frac{a_g}{b_g} \quad . \quad (38)$$

Letting

$$\begin{aligned} \dot{b}_g &= \frac{1}{b_g} \quad \text{and} \\ \dot{a}_g &= \frac{a_g}{b_g} \quad , \end{aligned}$$

we can put the "transmutation" equation in the more familiar form of a linear transformation equation where

$$\ddot{X}_{g_1} = \dot{a}_g + \dot{b}_g \ddot{X}_{g_1} \quad . \quad (39)$$

The question of whether there possibly may be better sets of $\dot{a}$'s and $\dot{b}$'s for transmuting the various subgroups' raw scores demands that we define what the criterion for "better" would be. It seems reasonable to assume that the higher the correlation between the $\ddot{X}$ (transmuted) scores and the $\dot{Y}$ (standardizing) scores, for the entire group, the better the solution.

The "Floor or Ceiling" and "Differential Treatment of Extremes" Effects

Beyond the problem of whether $\dot{a}_g$ and $\dot{b}_g$ are the optimal constants for a linear transformation is the additional problem of whether a simple linear transformation is adequate. After all, we really have no way of knowing if a more realistic equation might not be

$$\ddot{X}_{g_1} = \alpha_g + \beta_g \ddot{X}_{g_1} + \gamma_g \ddot{X}_{g_1}^2 + \delta_g \ddot{X}_{g_1}^3 + \dots \quad (40)$$

Certainly Eq. (40) would give us a far more general model in which to work, and there are some reasons to believe that the higher order terms would be psychologically as meaningful as the "leniency" (α) and "exaggeration" (β) terms. For example, the quadratic effect (γ) may be indicative of a "floor or ceiling" effect in a given subgroup as seen in Figures 1 and 2. The cubic effect (δ) may be indicative of a "differential treatment of extremes" effect as seen in Figures 3 and 4.

Fig. 1. - The Result of an Evaluator
Letting $\ddot{X} = \alpha + \beta X + \gamma X^2$.

Fig. 2. - The Result of an Evaluator
Letting $\ddot{X} = \alpha + \beta X - \gamma X^2$.

Fig. 3. - The Result of an Evaluator
Letting $\ddot{X} = \alpha + \beta X + \delta X^3$.

Fig. 4. - The Result of an Evaluator
Letting $\ddot{X} = \alpha + \beta X - \delta X^3$.

The "slope and intercept" method is obviously not capable of handling these higher order terms. The desire to find a method capable of handling the higher order terms and one which would simultaneously solve the equations for all groups resulted in a different approach to the problem of equating subgroups.

A Technique for Handling Nonlinear "Non-Equivalence of Scale" Errors

The technique which will be described below was developed as an outgrowth of a proof that the correlation ratio (η) is actually a multiple correlation coefficient (R) and should be considered as part of the Pearson family of correlation coefficients. In 1944 Wherry, Sr. (42) had proven that the optimal score values, from a least squares standpoint, to assign to categorical data for use in multiple correlational studies would be the mean of the criterion scores, each subgroup (or class) of the categorical variable had obtained. It seemed rather obvious that the traditional method of coding categorical data, by the use of nominal scaling, was not the only way of recording categorical information; each class could be treated as a separate dichotomous variable. Thus we could have one dichotomous variable on which members of the second class received a code of "1" and members of all other classes received a code of "0," etc. If there were k classes in the categorical variable, one could create k dichotomous variables. If the k dichotomous variables are then used as predictor variables in a multiple correlation analysis to predict a non-categorical variable, the multiple correlation coefficient (R) will be equal to the correlation ratio (η). Several other interesting findings are worth noting. First, any $k-1$ of the k dichotomous

variables will yield identical multiple correlations, because after selecting any set of $k-1$ dichotomous variables, no variance will remain for the unselected dichotomous variable. This gives the proof that the appropriate degrees of freedom for the correlation ratio is $k-1$. Secondly, the raw score beta weights and the constant term of the prediction equation derived do, in fact, yield Wherry, Sr.'s "Multi-serial" weights. The third finding is that the shrunken multiple correlation coefficient ($\bar{R}$) is equal to Kelly's epsilon (ϵ), the unbiased estimate of the correlation ratio. The prediction equation obtained is of the form (assuming the k^{th} dichotomous variable was the one not selected)

$$\hat{Y}_1 = b_1 X_{1_1} + b_2 X_{2_1} + b_3 X_{3_1} + \dots + b_{k-1} X_{k-1_1} + C.$$

For an individual who "belonged" to class 1, $X_1 (=1.00)$ is the only score in the prediction equation that will be non-zero, and the above equation will reduce to

$$\hat{Y}_{1_1} = b_1 (1) + C = b_1 + C.$$

For an individual who belonged to the k^{th} class, the equation would reduce to

$$\hat{Y}_{k_1} = C.$$

Another interesting finding concerned the matrix of intercorrelations among these created dichotomous variables. It can be shown that a single factor can account for all the off-diagonal entries if the loadings on this factor have the values $\sqrt{(p_g/q_g)} i$, where p_g = the proportion of people in class g , q_g = the proportion of people not in class g , and "i" is the imaginary mathematical number ($=\sqrt{-1}$). Because of the "imaginary" nature of these dichotomous variables,

I have referred to them as "pseudo" variables to indicate their "unreal" or "imaginary" qualities. The k dichotomous pseudo variables may be symbolized

$$P_{D_1} \rightarrow P_{D_k}.$$

Other interesting findings included showing that if the variable being predicted was a set of ranks rather than an "interval" type variable, the squared multiple correlation coefficient (R^2) was equal to the Kruskal-Wallis "H" divided by $(N-1)$. If the variable to be predicted was itself a dichotomous variable, the squared multiple correlation was equal to the Chi Square test for unmatched samples divided by N . It was also established that Kendall's coefficient of concordance (W), rather than being a measure of relationship, is a measure of the proportion of variance of the ranked data that is explainable by knowing which individuals the various judges were ranking, i.e., $R^2 = W$.

Prior to my own work in this area, and unknown to me at the time, Bottenberg and Christal (6) had developed an "iterative technique for clustering criteria which would retain optimum predictive efficiency." In their work, they had made use of what I referred to above as "dichotomous pseudo variables." In addition, they had used what I will refer to as linear pseudo variables. Linear pseudo variables are used in their technique for non-categorical data. Given a variable "X," we can create k linear pseudo variables ($P_{L_1} \rightarrow P_{L_k}$). A person who was in the first subgroup will receive a score of "X" on P_{L_1} and a score of "0" on all the other P_L variables. A person from the second subgroup will receive a score of "X" on P_{L_2} and a score of "0" on the other P_L variables, etc.

Bottenberg and Christal showed that by using dichotomous and linear pseudo variables as predictor variables, it was possible to simultaneously solve different multiple linear regression equations for different subgroups of individuals. Further, they stated that the weights derived for each subgroup by their method would be identical to the weights for that subgroup if that subgroup's data were considered separately.

I should point out that Bottenberg and Christal did not call their predictor variables by any special name as I have. I think, however, referring to these "created" variables as "pseudo" variables is a good method for keeping them from being confused with regular interval or dichotomous variables. It is perhaps fitting to point out that the pseudo linear variables are products of the pseudo dichotomous variables and the "X" variable. Therefore, referring to the Bottenberg and Christal technique as a method of "Producing Simultaneous Equations Using Dichotomous Observation," or simply, the "pseudo" technique, seems, to me, to be a good way of capturing the essence of the technique.

In order to extend the Bottenberg and Christal technique to handle simultaneous optimal solutions of non linear regression equations for different subgroups, all that was necessary was to create higher order polynomial pseudo variables; k quadratic pseudo variables ($P_{Q_1} \rightarrow P_{Q_k}$) and k cubic pseudo variables ($P_{C_1} \rightarrow P_{C_k}$).

During the accomplishment of this dissertation, Bottenberg and Ward (7) published an excellent and comprehensive paper on multiple regression techniques which includes a section on the kind of model extension which I am proposing as

a method for nonlinear equating of non-overlapping subgroups. The decision to choose a polynomial approach rather than some other type of nonlinear system (40) was governed by considerations mentioned in the rationale section.

Creating the "Pseudo" Variables for Equating Subgroups

There will be one dichotomous pseudo variable (P_{D_1} through P_{D_k}) for each subgroup. If a person is a member of subgroup 1, he will receive a score of "1" on P_{D_1} and a score of "0" on the other P_D variables. If he is a member of subgroup g , he would receive a score of "1" on P_{D_g} and a score of "0" on all the other P_D variables. In the same fashion there will be one linear pseudo variable (P_{L_1} through P_{L_k}) for each subgroup. A person will receive a score of X_{g_1} (his non-transmuted or original score) on the linear pseudo variable (P_{L_g}) representing his subgroup and a score of "0" on all the other P_L variables. There will also be one quadratic pseudo variable (P_{Q_1} through P_{Q_k}) and one cubic pseudo variable (P_{C_1} through P_{C_k}) for each subgroup. A person from subgroup g will receive a score of $X_{g_1}^2$ on P_{Q_g} , a score of $X_{g_1}^3$ on P_{C_g} and scores of "0" on all other P_Q and P_C variables. Table III shows how the scores for the various pseudo variables are derived.

Table III

Procedure for Creating Dichotomous, Linear, Quadratic, and Cubic Pseudo Variables

Subgroup	Score to be Equated	Pseudo Variables																	
		Dichotomous						Linear						Quadratic					
		P_{D_1}	P_{D_2}	...	P_{D_g}	...	P_{D_k}	P_{L_1}	P_{L_2}	...	P_{L_g}	...	P_{L_k}	P_{Q_1}	P_{Q_2}	...	P_{Q_g}	...	P_{Q_k}
	X_{1_1}	1	0	...	0	...	0	X_{1_1}	0	...	0	...	0	$X_{1_1}^2$	0	...	0	...	0
1	X_{1_2}	1	0	...	0	...	0	X_{1_2}	0	...	0	...	0	$X_{1_2}^2$	0	...	0	...	0
	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	$X_{1_{n_1}}$	1	0	...	0	...	0	$X_{1_{n_1}}$	0	...	0	...	0	$X_{1_{n_1}}^2$	0	...	0	...	0
	X_{2_1}	0	1	...	0	...	0	0	X_{2_1}	...	0	...	0	0	$X_{2_1}^2$	...	0	...	0
2	X_{2_2}	0	1	...	0	...	0	0	X_{2_2}	...	0	...	0	0	$X_{2_2}^2$	...	0	...	0
	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	$X_{2_{n_2}}$	0	1	...	0	...	0	0	$X_{2_{n_2}}$	...	0	...	0	0	$X_{2_{n_2}}^2$	...	0	...	0
	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
g	X_{g_1}	0	0	...	1	...	0	0	0	...	X_{g_1}	...	0	0	0	...	$X_{g_1}^2$	...	0
	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
k	$X_{k_{n_k}}$	0	0	...	0	...	1	0	0	...	0	...	$X_{k_{n_k}}$	0	0	...	0	...	$X_{k_{n_k}}^2$
																			$X_{k_{n_k}}^3$

If all of the pseudo variables are used in a multiple correlational method to predict the standardization variable (Y) we may obtain a prediction equation of the form

$$\begin{aligned}\hat{Y}_i = & \alpha_{PD_1} \cdot X_{PD_{1i}} + \alpha_{PD_2} \cdot X_{PD_{2i}} + \dots + \alpha_{PD_g} \cdot X_{PD_{gi}} + \dots + \alpha_{PD_k} \cdot X_{PD_{ki}} \\ & + \beta_{PL_1} \cdot X_{PL_{1i}} + \beta_{PL_2} \cdot X_{PL_{2i}} + \dots + \beta_{PL_g} \cdot X_{PL_{gi}} + \dots + \beta_{PL_k} \cdot X_{PL_{ki}} \\ & + \gamma_{PQ_1} \cdot X_{PQ_{1i}} + \gamma_{PQ_2} \cdot X_{PQ_{2i}} + \dots + \gamma_{PQ_g} \cdot X_{PQ_{gi}} + \dots + \gamma_{PQ_k} \cdot X_{PQ_{ki}} \\ & + \delta_{PC_1} \cdot X_{PC_{1i}} + \delta_{PC_2} \cdot X_{PC_{2i}} + \dots + \delta_{PC_g} \cdot X_{PC_{gi}} + \dots + \delta_{PC_k} \cdot X_{PC_{ki}} + C.\end{aligned}\quad (41)$$

Where α_{PD} = a raw score prediction weight for a dichotomous pseudo variable,

X_{PD_i} = the score individual i received on a dichotomous pseudo variable,

β_{PL} = a raw score prediction weight for a linear pseudo variable,

X_{PL_i} = the score individual i received on a linear pseudo variable,

γ_{PQ} = a raw score prediction weight for a quadratic pseudo variable,

X_{PQ_i} = the score individual i received on a quadratic pseudo variable,

δ_{PC} = the raw score prediction weight for a cubic pseudo variable,

X_{PC_i} = the score individual i received on a cubic pseudo variable, and

C = a constant to be added.

For an individual in the g^{th} subgroup the only scores in the entire prediction equation which will not necessarily equal zero will be $X_{P_{D_g}} (=1)$, $X_{P_{L_g}} (= \ddot{X}_{g_1})$, $X_{P_{Q_g}} (= \ddot{X}_{g_1}^2)$, and $X_{P_{C_g}} (= \ddot{X}_{g_1}^3)$.

Thus for all the persons in the g^{th} subgroup Eq. (41) will simplify to

$$\hat{Y}_{1(g)} = (\alpha_{P_{D_g}} + C) + \alpha_{P_{L_g}} \cdot \ddot{X}_{g_1} + \gamma_{P_{Q_g}} \cdot \ddot{X}_{g_1}^2 + \delta_{P_{C_g}} \cdot \ddot{X}_{g_1}^3. \quad (42)$$

Now letting $\hat{a}_g = (\alpha_{P_{D_g}} + C)$,

$$\hat{b}_g = \alpha_{P_{L_g}},$$

$$\hat{c}_g = \gamma_{P_{C_g}}, \text{ and}$$

$$\hat{d}_g = \delta_{P_{C_g}}, \text{ we obtain}$$

$$\hat{Y}_{1(g)} = \hat{a}_g + \hat{b}_g \ddot{X}_{g_1} + \hat{c}_g \ddot{X}_{g_1}^2 + \hat{d}_g \ddot{X}_{g_1}^3. \quad (43)$$

Similarly for each subgroup we may obtain an equation such as Eq. (43) above.

We know that $\hat{a}$, $\hat{b}$, $\hat{c}$, and $\hat{d}$ are the optimal weights to use for transmuting the $\ddot{X}$ scores for each subgroup. If only the dichotomous and linear pseudo variables were used in arriving at the prediction equation, we could find out if $\hat{a}_g$ and $\hat{b}_g$ (derived by the "slope and intercept" method) are equivalent to $\hat{a}_g$ and $\hat{b}_g$ (derived from the "pseudo variable" method). This will be done in the results section. According to Bottenberg and Christal (6) they should be.

Since the solution has optimized the correlation between the "standardization" variable $\dot{Y}$ and the pseudo variables we know that the "transformation" equations are the "best" that can be arrived at using only the

first three powers of the scores to be equated. Of course, it is possible that we may have fit too much error by using all the terms, but this shortcoming could be resolved by using a test-selection technique instead of selecting all but one of the pseudo dichotomous variables. Thus, if for a given subgroup, the cubic effect does not add significantly to the amount of variance explained, there would then be no "d" term in that subgroup's final "transformation" equation.

Finding the "Best Possible" Standardization Variable (Y)

Toops (37), in his original article on "The Transmutation of Marks," which set forth the "slope and intercept" method, had used an intelligence test as the "standardization" variable against which he equated grades from various high schools. He had the following comments regarding the "standardization" variable used to equate subgroups.

...it will be clear that the transmuting equation is affected by the validity of the measuring rod used to equate the marks. It follows, then, that one should employ the best possible measuring rod. This means ideally--we take it--the best cross-section of the 'content' of the curricula of the several schools of the geographical area over which the marks are being rendered comparable. The average of four annual intelligence tests of high school pupils would be better than one... (37)

Since the "slope and intercept" method of transmuting scores requires a single "standardization" variable it is obvious that if one has but a single variable to use as the "standardization" variable that it is the "best available" one, even if it might not be the "best possible" one. While I am in agreement with Toops that an average of four annual intelligence tests would undoubtedly be better than a single intelligence test, I do not necessarily believe it to be the "best available" one. If, for example, differentially weighting the four annual intelligence tests (in his case)

would ultimately yield a correlation between the entire group's transmuted marks and the weighted composite of the four annual intelligence tests which was higher than the correlation between the transmuted marks and a simple average of the intelligence tests, then the weighted composite would be a better "standardization" variable than the simple average. This is true because it would have raised the overall "composition" effects (θ 's) for the various subgroups.

In Toops' case, where the four possible equating variables are highly homogeneous with regard to content, the difference between a weighted average and a simple average would probably be minimal. If, however, the possible equating variables were markedly different in content then, the weighted composite should be markedly superior to a simple average. This presents a second problem of deciding how to weight possible equating variables to obtain the optimum "standardization" variable.

One method which suggested itself was to use the equating variables to optimally predict the "variable-to-be-equated" for the entire sample. This would yield a set of weights for the equating variables which could be used to obtain a predicted "variable-to-be-equated" score ($\hat{X}$) for each person which would then serve as a "standardization" variable score (i.e., $\dot{Y} = \hat{X}$). Since these predicted scores are the result of a single equation which is the same for each subgroup there should be little, if any, differential "leniency," "exaggeration," "floor," "ceiling," or "differential treatment of the extremes" effects from one subgroup to another in this standardization variable. Further, because we have found the optimal set of weights for the equating variables we should have also maximized the total contribution of the "composition" effects for all subgroups. In order to keep the "reliability" effect of the "standardization" variable maximal a test-

selection method, such as Wherry, Sr.'s (41), could be used. This will guard against overfitting of error in the prediction equation.

III. EMPIRICAL DEMONSTRATION AND CROSS VALIDATION

SAMPLE

The data analyzed in this study were gathered on all graduates of the naval aviation training program who had entered training between 1 January 1960 and 31 December 1960. As students enter training they are assigned a class number and an alphabetical sequence number.

Experimental Sample

Alternately numbered students were assigned to the experimental sample. The composition of the experimental sample with regard to subgroup classification is given in Table IV.

Table IV

The Composition of the Experimental and Cross Validation Sample
With Regard to Subgroup Membership

Subgroup (Advanced Flight Training Pipeline)	N of Experimental Sample	N of Cross Validation Sample
1	65	70
2	109	103
3	128	121
4	41	47
5	71	66
6	38	34
7	160	159
<u>TOTAL</u>	<u>612</u>	<u>605</u>

Cross Validation Sample

The remainder of the graduating students ($n = 605$) were assigned to the cross validation sample. The composition of the cross validation sample with regard to subgroup classification is given in Table IV.

VARIABLES

Equating Variables

A. Selection Test Variables. Prior to being designated as a flight student, all applicants for naval flight training must successfully pass the following tests.

1. Aviation Qualification Test. This is a sixty-minute omnibus intelligence test designed to reject applicants incapable of performing adequately in the ground school portion of their training. High correlations with commercial tests of intelligence have been demonstrated (2). Administrations of the test at receiving stations (field) and during the first week at the U.S. Naval School, Pre-Flight (laboratory) correlated .71. This coefficient is attenuated somewhat since only those applicants passing the field administration were accepted into the program. While only the total AQT raw score was used in this study the AQT is composed of five sub-scales.

1a) Mathematics: This sub-scale is composed of 45 items emphasizing basic mathematical reasoning and is concerned primarily with solving verbally stated problems algebraically and with the handling of geometric relationships.

1b) Vocabulary: This scale is composed of 24 five alternate multiple choice items covering word meaning.

1c) Practical Judgment: This scale is composed of 10 items in which conflict situations are presented; one of five alternative courses of action must be chosen.

1d) Dial Reading: In this 21-item scale the cadet is instructed to perform a specified series of operations if dial A has such-and-such a reading, to perform a different series of operations if dial B has a reading of such-and-such, et cetera. In each item, the cadet is presented with 3 to 5 dials which he must first read and then indicate the proper operations to be performed.

1e) Matching: The 15 items in this scale present two sets of letter-number combinations to which the subject is required to indicate whether the sets are identical or different.

2. Mechanical Comprehension Test. This is a forty-five minute test dealing with ability to perceive physical relationships and handle familiar concepts of everyday mechanics. Each of the 76 items is pictorial in nature. The subject responds to a question about the pictured relation with one of three alternatives. A reliability of .87 has been reported for the test (17).

3. Biographical Inventory. This is an untimed questionnaire containing elements of personal history, expressions of interest and attitudes, and selected informational items.

4. Spatial Apperception Test. This is a ten-minute spatial orientation test. Specifically, it requires the subject to visualize the relationship between the presented attitude of a plane and the territory over which it is flying. Alternate-form reliability of .70 was found (1) on a rather restricted sample.

B. Pre-Flight School Variables. Prior to receiving in-flight training, all students must complete the academic courses listed below.

5. Mathematics. This is a twenty-four hour course on basic mathematical principles. The content ranges from elementary algebra to a brief introduction to calculus. The course grade is derived from phase quizzes and a final examination.

6. Physiology. This is a nine-hour course covering selected topics from elementary physiology and the effect of flight on physiological systems. The course grade is determined by a final examination.

7. Principles of Flight. This is a thirty-hour course on basic aerodynamics. The course grade is derived in the same manner as the mathematical grade.

8. Engineering. The content of this course is centered around a thirty-one hour study of reciprocating engines and aircraft systems. Three hours at the end of the course are devoted to jet engines. The course grade is derived in the same manner as the mathematical grade.

9. Navigation. This thirty-three hour course is designed to give the student an understanding of the basic concepts of navigation and proficiency in the use of the E-10 computer. Course content is limited to dead reckoning navigation, except for a brief introduction to celestial navigation at the end of the course. The course grade is derived in the same manner as the mathematical grade.

While the above courses do not exhaust the curriculum of the U.S. Naval

School, Pre-Flight, they do represent courses which all students going through naval aviation training are required to take.

C. Primary Flight Training Variables. Upon successful completion of Pre-Flight Training, all students received their primary flight training which is composed of two in-flight stages of training.

10. Pre-Solo. This stage, flown in the T-34, is devoted to dual instruction for the purpose of qualifying students to fly solo. It consists of ten dual instructional hops, a check hop by a member of the unit's check board, and a solo flight by the student. Major instructional areas in the pre-solo stage of training include achieving correct plane attitude in flight, effective use of trim tabs, coordinating of controls, and pre-flight plane checks, emergency procedures, take-offs and landings.

11. Precision (T-34). This stage is devoted to instruction and practice in precision maneuvers in the T-34 aircraft. Emergency procedures, full-flap, crosswind, and touch-and-go landings, chandelles, wing-overs, spins, stalls, steep turns, and small field procedures were covered. The student has the same instructor that he had in pre-solo. This stage includes seven dual hops, ten solo hops, and a final dual check hop.

D. Basic Flight Training Variables. All students in this study followed the same basic flight training curriculum which is composed in part, of the following stages of training.

12. Transition (T-28). This stage provides dual instruction in the T-28 aircraft for the purpose of qualifying students to fly solo in the T-28. Material

covered in this stage corresponds closely to that covered in the pre-solo (T-34) stage. It includes seven dual instruction hops, a dual check hop, and a solo flight by the student.

13. Precision-Acrobatcs (T-28). This variable is the student's average grade from the following two stages.

13a) Precision. This stage is devoted to instruction and practice in precision maneuvers in the T-28. Material covered is identical to that found in the precision (T-34) stage. The student has the same instructor as in the transition stage. This stage includes two dual instructional hops, four solo hops, and a dual check hop.

13b) Acrobatics. This stage is flown in the T-28 and is devoted to teaching the student acrobatics and further developing his precision flying techniques. A portion of each dual flight is devoted to practice of precision stage maneuvers, with major emphasis on small field procedures and wing-overs. Barrel rolls, slow rolls, loops, Immelmann turns, 1/2 Cuban 8's, and other instructor approved acrobatics are introduced. The stage includes three dual hops, six solo hops, and a check hop. The student has the same instructor for acrobatics as he had in the two preceding stages of transition and precision.

14. Basic Instruments (T-28). This stage, flown in the T-28, consists of eleven instructional hops and a final check hop. Almost all of the maneuvers flown in this stage are performed by the student under the instrument flying hood. Major emphasis, from the standpoint of grades, is on such fundamentals as climb to

altitude, nose and wing position, cockpit procedures, and standard and half standard rate turns. More intricate maneuvers and patterns occur later in this stage.

15. Night Familiarization (T-28). This stage, flown in the T-28, introduces the student to flying at night. Emphasis is primarily on take-offs, landings, and flying to various checkpoints.

16. Radio Instruments (T-28). This stage, flown in the T-28, consists of various radio instruments for bearing and radial interception, tracking, station passage, orientation, penetration, and GCA. Two two-hour hops or a single four-hour hop is devoted to flying to various check points by radio navigation. A final check hop covers all material given in this stage.

17. Formation (T-28). This stage consists of eight dual and seven solo hops. Major emphasis is on rendezvous procedures, formation turns, and altering positions in formation. This stage is also flown in the T-28.

18. Gunnery (T-28). This stage, flown in the T-28, introduces the student to weapon systems in naval aircraft. The student receives practice at firing at targets from the aircraft.

E. Variable to be Equated

19. Advanced Flight Grade. This grade is a composite of students' advanced in-flight training grades. The number and nature of the grades in the composite vary according to which advanced pipeline the student was in. It is this grade which is to be equated.

RESULTS

Experimental Study Group Analysis (Slope and Intercept Method)

The analysis will be broken into eight procedural steps for discussion purposes.

1. Matrix of Correlation Coefficients Among "Equating" Variables and "Variable-to-be-Equated" Obtained

Pearson product-moment correlation coefficients were computed among the 18 "equating" variables and the "variable-to-be-equated" (Advanced Flight Grade) for the entire experimental sample ($N = 612$). These correlation coefficients are shown in Appendix I.

2. Wherry Test-Selection Technique Used to Find "Optimal Standardization" Variable

Using the variable-to-be-equated (variable 19) as the criterion variable and the 18 "equating" variables as potential selector variables, the Wherry Test-Selection technique (41) was accomplished. Only eight variables were selected for the prediction equation. The results of step 2 are shown in Table V.

3. Prediction of Scores of the Variable-to-be-Equated; "Standardization" Variable Scores Obtained

Using the prediction equation weights shown in Table V a prediction of what score each of 612 subjects in the experimental sample should have received in Advanced Flight Grade was computed. These predicted grades were then used as the "standardization" variable scores against which the subgroups were equated. The standardization variable appears as variable 20 in Appendix I.

Table V

Results of Wherry Test-Selection Technique Using Variable 19 as
The Criterion and the "Common" Variables as Potential Selector Variables

Order of Variables Selected	Number of Selected Variable	Name of Variable	Shrunk Multiple R $\bar{R}$	Prediction Equation Raw Score Betas
01	17	Formation	.505	.191532
02	09	Navigation	.583	.169281
03	12	Transition	.618	.100652
04	16	Radio Instruments	.634	.051899
05	14	Basic Instruments	.640	.047576
06	11	Precision	.641	.038367
07	07	Aerodynamics	.642	-.036411
08	04	S.A.T.	.642	.033569
Constant to be added to sum of products of raw score betas times raw scores = 165.15				

4. Subgroup Data Sums on the "Standardization" Variable ($\bar{Y}$) and the
"Variable-to-be-Equated"

The 612 cases were separated into their seven subgroups and the n's, sums, sums of squares, and sums of cross products were computed. These data are shown in Table VI.

Table VI

Summary Raw Score Data for "Standardization" Variable ($\bar{Y}$) and "Variable-to-be-Equated" on the Seven Subgroups Comprising the Experimental Sample

Subgroup	n_g	Variable-to-be-Equated		$\Sigma \bar{Y}$	Standardization Variable	
		$\Sigma \bar{X}$	$\Sigma \bar{X}^2$		$\Sigma \bar{Y}^2$	$\Sigma \bar{X}\bar{Y}$
1	65	19 599	5 887 257	19 505	5 854 071	5 870 212
2	109	32 772	9 854 896	33 023	10 005 633	9 929 320
3	128	38 468	11 564 928	38 355	11 494 819	11 528 856
4	41	12 495	3 808 967	12 391	3 745 333	3 776 636
5	71	21 471	6 494 971	21 356	6 424 742	6 459 189
6	38	11 471	3 463 931	11 399	3 419 885	3 441 537
7	160	47 656	14 201 688	47 851	14 312 561	14 254 914
Total	612	183 892	55 276 638	183 880	55 257 044	55 260 664

5. Computation of Means, Standard Deviations, Correlations, and Transmutation Constants for the Subgroups

The subgroup statistical data shown in Table VII were computed. The transmutation constants are functions of the slopes (b_g 's) and intercepts (a_g 's) of the various subgroups X scores on the Y scores.

$$\dot{a}_g = -a_g/b_g, \text{ and}$$

$$\dot{b}_g = 1/b_g.$$

This allows the computation of a "transmuted" score to be in the form $\ddot{X}_{g_1} = \dot{a}_g + \dot{b}_g \ddot{X}_{g_1}$, rather than $\ddot{X}_{g_1} = (\ddot{X}_{g_1} - a_g)/b_g$.

Table VII

The Means and Standard Deviations of the "Standardization" Variable and of the "Variable-to-be-Equated" and the Correlation Coefficients and the Transmutation Constants for the Subgroups

Subgroup	Variables to be Equated		Standardization Variable			Transmutation Constant	
	$\bar{X}_g$	σX_g	$\bar{Y}_g$	σY	$r_{\ddot{X}Y}$	$\dot{a}_g$	$\dot{b}_g$
1	300.907	5.267	300.076	4.058	.7251	131.961	.558
2	300.660	3.889	302.963	2.833	.5048	192.369	.367
3	300.531	5.653	299.648	3.753	.7288	154.231	.483
4	304.756	5.035	302.219	3.598	.5426	184.021	.387
5	302.408	5.252	300.788	3.932	.6505	153.519	.486
6	301.868	5.615	299.973	3.572	.7069	164.189	.449
7	297.850	6.776	299.068	3.374	.6815	197.986	.339
Total	300.477	5.899	300.457		.6481		

6. Transformation of $\ddot{X}$ scores so they will have a slope of 1.00 and an intercept of zero with the $\ddot{Y}$ variable by-passed

One could at this point multiply each $\ddot{X}_1$ score by $\dot{b}_g$ and add $\dot{a}_g$ to the product. These would give transmuted scores for each subgroup that would have a

slope of 1.00 and an intercept of zero with the standardized variable. The total group mean and standard deviation of the "standardization" variable were themselves derived from the total group mean and standard deviation of the "variable-to-be-equated." This was inherent during the accomplishment of the Wherry Test-Selection technique. The standard deviation of the transmuted scores will, in turn, be much smaller than the standard deviation of the "standardization" variable. The net result of this would be that the total group standard deviation of the transmuted scores would probably be much smaller than the original total group standard deviation of the "variable-to-be-equated." It was decided, therefore, to do one total group linear transformation to these "first" transmuted scores to make their mean and standard deviation equal to the mean and standard deviation of the original total group mean and standard deviation of the "variable-to-be-equated." In order to find the linear transformation capable of doing this we needed to know what the mean and standard deviation of these "first" transmuted scores would be.

The over-all linear transformation to the transmuted scores is of the form

$$\ddot{X}'_{g_1} = c + d \ddot{X}_{g_1}$$

$$\text{where } d = \sigma_{\ddot{X}_t} / \sigma_{\ddot{X}_t} ,$$

$$c = \bar{\ddot{X}}_t - d \bar{\ddot{X}}_t ,$$

$$\ddot{X}_{g_1} = \dot{a}_g + \dot{b}_g \ddot{X}_{g_1} , \text{ and}$$

$$\ddot{X}'_{g_1} = \text{a "new" transmuted score, which will have a total group mean equal to } \bar{\ddot{X}}_t , \text{ and a total group standard deviation equal to } \sigma_{\ddot{X}_t} .$$

The intermediate summary data necessary to compute the values of c and d are shown in Table VIII.

Because, however, $\ddot{X}_{g_1} = \dot{a}_g + \dot{b}_g \ddot{X}_{g_1}$ and $\ddot{X}'_{g_1} = c + d\ddot{X}_{g_1}$, then

$$\begin{aligned}\ddot{X}'_{g_1} &= c + d (\dot{a}_g + \dot{b}_g \ddot{X}_{g_1}), \\ &= c + d\dot{a}_g + d\dot{b}_g \ddot{X}_{g_1}.\end{aligned}$$

Letting $\dot{a}'_g = c + d\dot{a}_g$, and $\dot{b}'_g = d\dot{b}_g$, we may compute "new" transmutation constants for each subgroup. These values are also shown in Table VIII.

Table VIII

Computed Sums of $\ddot{X}$, $\ddot{X}^2$, and Final Transmutation Constants ($\dot{a}'_g$ and $\dot{b}'_g$) for Each Subgroup

Subgroup	$\Sigma \ddot{X}$	$\Sigma \ddot{X}^2 = \Sigma \ddot{X} \ddot{Y}$	$\dot{a}'_g$	$\dot{b}'_g$
1	19505	5 853 563	-58.3002	1.1896
2	33023	10 004 977	70.3261	.7832
3	38354	11 493 973	-10.8806	1.0302
4	12391	3 744 109	52.5492	.8258
5	21355	6 424 109	-12.3976	1.0369
6	11399	3 419 642	10.3214	.9577
7	47850	14 311 585	82.2862	.7226
Total	183880	55 252 808		

$$\sigma_{\ddot{X}_t} = \sqrt{\frac{(612)(\Sigma \ddot{X})^2 - (\Sigma \ddot{X})^2}{612}} = 2.77; d = \frac{\sigma_{\ddot{X}_t}}{\sigma_{\ddot{X}_t}} = \frac{5.889}{2.77} = 2.1292$$

$$\bar{\ddot{X}}_t = \frac{183880}{612} = 300.457; c = \bar{\ddot{X}} - d\bar{\ddot{X}} = 300.477 - (2.1292)(300.457) = -339.2865$$

7. Computation of Total Group's Correlation Between the "Transmuted" Scores and the "Standardization" Variable Scores

Supposedly we have now equated the seven subgroups according to the equations in the "rationale" section of this paper. We now ask the question of whether these $\dot{a}'_g$'s and $\dot{b}'_g$'s have significantly changed the relative locations of the individuals for intra-subgroup comparisons.

Since the transmutation scores within a subgroup are merely linear transformations of the original scores then the transmuted scores will not be able to increase in the proportion of the standardization variable's variance within a subgroup that is explainable. However, since different subgroups have received different linear transformations the total proportion of the variance of the standardization variable explainable should have increased for the group as a whole. The correlation between the transmuted scores and the standardization variable for the entire group was found to have increased from .6481 to .7246. Table IX shows that the F-ratio for the transmutation equations was found to be 10.886 with 12 and 591 degrees of freedom. An F-ratio of this magnitude is significant beyond the .001 level and tells us that the subgroups did have significantly different "leniency" and "exaggeration" effects. (Only 12 degrees of freedom are lost because we may consider one subgroup as the "standard" and equate the other subgroups to it.)

Table IX

Table for Testing the Significance of Transmutation Equations
Derived by the Slope and Intercept Method

Source	D.F.	R	R ²	Proportion of Variance Attributable to	MS	F-ratio
Multiple Correlation Equation	8	.6481	.4200	.4200	.0525	65.321*
Transmutation Equation	12	.7246	.5250	.1050	.00875	10.886*
Residual	<u>591</u>			<u>.4750</u>	.0008037	
Total	611			1.0000		

* Significant beyond the .001 level.

8. Need for Iterating the Solution Established and Transmuted Scores Obtained

We know from the preceding step that the subgroups did originally have significantly different "leniency" and "exaggeration" effects. Transmuted "Advanced Flight Grades" were computed for each subject using the transmutation constants ($\hat{a}_g$ and $\hat{b}_g$) found in Step 6 for the subgroup of which he was a member.

Since these grades have less bias in them the total group's Advanced Flight Grade may now correlate somewhat differently with the "predictor" variables. This, in turn, might allow a better determination of a "standardization" variable on which to equate the groups, i.e., the subgroups may have better "composition" with a new composite variable. For these reasons it was decided that the procedure should be iterated.

FIRST ITERATION ACCOMPLISHED

The correlation matrix was obtained and the Wherry Test-Selection technique accomplished in the same manner as before. The multiple correlation coefficient ($R_{\hat{X}\hat{Y}}$) was found to be .7339 which indicated the presence of an obtainable vector ($\hat{Y}'$) that would be slightly more related to the transmuted scores than the original vector (original $R_{\hat{X}\hat{Y}} = .7246$). (It is difficult to say whether this increase is significant or not. $\hat{Y}'$ was a composite of fewer variables than was $\hat{Y}$. In one sense, then, it has fewer degrees of freedom than $\hat{Y}$.) Using the new prediction equation, new "standardization" scores were computed for all subjects. Steps 3 through 7 were accomplished and yielded the transmutation constants seen in Table X.

It can readily be seen that the $\hat{b}_g$'s are approaching 1.00 and $\hat{a}_g$'s are approaching 0.0, and we should expect very little change in the total group

correlation between Advanced Flight Grade and the "standardization" variable. This was found to be true; the correlation coefficient increased only from .7339 to .7343. The F-ratio was actually less than 1.00 ($F = .8$) which indicated "over-fitting" of random error was probably present in the "transmutation constants" in Table X. We should, in the face of a non-significant F-ratio, not use this second set of constants.

Table X

Table of Transmutation Constants and r'_{XY} 's Found in the First Iteration

Subgroup	r'_{XY}	Transmutation Constants	
		a'_g	b'_g
1	.7606	-16.1094	1.0533
2	.5249	- 6.6847	1.0229
3	.7200	8.7624	.9706
4	.5031	20.1103	.9340
5	.6525	7.9457	.9723
6	.7011	7.7030	.9732
7	.6889	9.3158	.9688

SECOND ITERATION ACCOMPLISHED

To find if this was true a second iteration was accomplished. As anticipated the multiple correlation dropped from .7343 to .7329, and the final overall correlation after transmuting the scores a third time showed no increase at all.

COMPARISON OF THE "SLOPE AND INTERCEPT" METHOD WITH THE "PSEUDO" VARIABLE METHOD

In addition to the variables previously mentioned dichotomous, linear, quadratic, and cubic pseudo variables were manufactured for each subgroup. The intercorrelations among these and the other variables are also shown in Appendix I. In order to compare the results of the "slope and intercept" method with the "pseudo

variable" method only the dichotomous and linear pseudo variables were used to predict the same standardization variable (variable 20) used during the "slope and intercept" method.

Using all 14 of these pseudo variables (variables 21 - 34), the multiple correlation was found to be equal to .7247. The raw score regression weights are shown in Table XI. Comparison of these weights ($\hat{a}$'s and $\hat{b}$'s) and those shown in Table VII (the $\dot{a}$'s and $\dot{b}$'s) shows that the two techniques are in fact identical (within rounding error) as far as results obtained are concerned. This is what was expected from the Bottenberg and Christal paper(6). Thus, if one is concerned only with removing "leniency" and "exaggeration" bias, the "slope and intercept" method will yield a solution identical with that given by the "pseudo variable" method using only dichotomous and linear variables.

Table XI

Comparison of "Transmutation" Constants Derived from the "Slope and Intercept" Method and the "Pseudo Variable" Method
(Using Only Dichotomous and Linear Pseudo Variables)

Subgroup	Slope and Intercept Constants		Pseudo Variable Constants	
	$\hat{a}$	$\hat{b}$	$\hat{a}$	$\hat{b}$
1	131.961	.558	131.968	.558
2	192.369	.367	192.346	.367
3	154.231	.483	154.236	.483
4	184.021	.387	184.004	.387
5	153.519	.486	153.486	.487
6	164.189	.449	164.225	.449
7	197.986	.339	198.007	.339
Multiple R	.7246		.7247	

The Pseudo Variable Approach using Quadratic and Cubic Terms

A multiple correlation using all of the 28 pseudo variables as predictors was computed. The raw score weights to be used in the equation

$$\ddot{X}_{g_1} = \overset{\circ}{a}_g + \overset{\circ}{b}_g \ddot{X}_{g_1} + \overset{\circ}{c}_g \ddot{X}_{g_1}^2 + \overset{\circ}{d}_g \ddot{X}_{g_1}^3$$

are shown in Table XII.

Table XII

Transmutation Constants Derived from the "Pseudo Variable" Approach
Using Dichotomous, Linear, Quadratic, and Cubic Pseudo Variables

Subgroup	$\overset{\circ}{a}$	$\overset{\circ}{b}$	$\overset{\circ}{c}$	$\overset{\circ}{d}$
1	249.39576	- .22001	.001303	-.00000004026
2	244.57538	.04108	.000451	.00000019183
3	251.66059	- .07556	.000484	.00000099245
4	-439.63646	4.11595	-.004012	-.00000495186
5	332.74979	- .51062	.000730	.00000201406
6	503.45862	-1.56635	.002103	.00000281772
7	309.06867	- .30635	.000605	.00000104215

The multiple correlation using the 28 pseudo variables as predictors was found to be .7385. The question of whether this solution, using the cubic and quadratic terms, is better than the solution obtained when only dichotomous and linear pseudo variables were used can be answered by finding if the increase in the multiple correlation is a significant one. Table XIII presents the F-table information which shows that this solution is better beyond the .05 level.

Table XIII

F-Table Showing the Level of Significance of the Increase in the Proportion of Variance Explainable Using All Pseudo Variables Over Using Only Dichotomous and Linear Pseudo Variables

Source	d.f.	R	R ²	% of Variance	MS	F	Level
Original Multiple R	8	.6481	.4200	42.00	.052500	66.6209	***
Dichotomous and Linear	12	.7247	.5250	10.50	.008750	11.1035	***
Quadratic and Cubic	14	.7385	.5453	2.03	.001450	1.84	*
Unexplained Variance	<u>577</u>			<u>45.47</u>	.000788		
Total	611			100.00			

***Significant beyond the .001 level.

* Significant beyond the .05 level.

While the proportion of variance explained by the addition of the quadratic and cubic term did increase a significant amount, the absolute increase was quite small and is of little practical significance. A plot of the non linear equations over the range of data in their respective subgroups is shown in figure 5. From inspection of figure 5, it appears that the cubic term was relatively unimportant in all but the fourth subgroup.

THE CROSS VALIDATION STUDY

In addition to the question of whether the linear and nonlinear transmutation equations allow us to explain significantly more of the "standardization" variable's variance than without transmutation, there is also the question of how

Figure 5. - A Plot of the Nonlinear Equations Shown in Table XII Over the Range of Data in Each Subgroup

well the transmutation equations derived from one sampling of a population will perform in a second sampling of the population.

To determine this the linear and non linear equations derived from the experimental sample were used to transmute the Advanced Flight Grades of the cross validation sample. This procedure yields three different sets of Advanced Flight Grades: (1) the "raw" or non-transmuted, (2) the linearly transmuted, and (3) the non linearly transmuted. The question of which of these sets of scores is the most "error-free" can be, I believe, answered by determining which set has the most predictable variance. Since no degrees of freedom are lost in doing a priori transformations on the cross validation data, the set of scores having the most predictable variance would be the best set to use (i.e., it would be the most "error-free" set).

Pearson product-moment correlation coefficients were obtained among all the equating variables and these three sets of Advanced Flight Grades. These correlation coefficients are given in Appendix II. Three multiple correlations were accomplished using the equating variables as predictors and each of the sets of Advanced Flight Grades as criteria. The results of this procedure are shown in Table XIV.

From inspection of Table XIV it is quite obvious that either of the transmuted sets of grades would be "better" (i.e., more "error-free") than the non-transmuted grades. The increase of 9.30 in the per cent of variance explainable between the non-transmuted and linearly transmuted scores is comparable to that found in the experimental sample (i.e., 10.50 per cent). The increase of only 0.13 per cent

from the linearly transmuted to the nonlinearly transmuted scores was somewhat disappointing compared with the increase found in the experimental sample (i.e., 2.03). However, this does detract from the fact that the nonlinearly transformed set of scores was, indeed, the most error-free set.

Table XIV

Comparison of Non-Transmuted, Linearly Transmuted, and Nonlinearly Transmuted Advanced Flight Grades in the Cross Validation Sample Using the Transmutation Constants Derived from the Experimental Sample

Type of Advanced Flight Grade	Multiple Correlation	% of Variance Predictable	Differences in % of Variance
Non-Transmuted	.7029	49.41	09.30
Linearly Transmuted	.7663	58.71	00.13
Nonlinearly Transmuted	.7671	58.84	

The question of whether transmutation equations derived from one sample of data will work on a different sample of data was not asked with a view toward recommending the use, for example, of transmutation equations derived from one year's data to equate the next year's data. This would involve the problem of the stability of transmutation equations over time, a problem which, while interesting, is not investigated here. The cross validation procedure followed here was concerned instead with how much confidence we could have in using the transformation equations on the data they were derived from. Clearly, if they work (and they did) on a separate random sample of individuals spanning the same time period, then we should not hesitate to use the equations on the sample from which they were derived.

IV. DISCUSSION

In the foregoing chapters I have attempted to present a rationale which would be applicable to most of the major suspected causes of non-equivalence of scales among evaluators. The techniques which have been reported in this dissertation are intended for the removal of four types of these non-equivalence errors: "leniency," "exaggeration," "floor" or "ceiling" effects, and "differential treatment of extremes." The rationale section, to some extent, shows how other non-equivalence errors, such as the magnitude of the "reliability," and "composition" effects, and some confounding problems, such as the magnitude of the "curtailment" and "transmutation regression" effects enter into the general problem of equating subgroups composed of different individuals.

The removal of non-equivalence errors attributable to differential "composition" effects within subgroups was not attempted in this study. It seems, however, to be a combination of a factor analytic type of a problem and a criterion definition problem which would easily fit into the rationale presented.

COMMENTS REGARDING THE "OPTIMAL" STANDARDIZATION VARIABLE

The method used for obtaining the "standardization" variable, by using all available equating variables to predict the "variable-to-be-equated," and then using the predicted variable-to-be-equated scores as the "standardization" variable scores, seems, to me, to be superior to using either the single best available variable or a simple average of available variables. (I must confess, however, some disappointment that the solution showed no significant improvement after

iterating it.) Upon further consideration of the optimal standardization problem I have thought of one method which would, without question, find the optimal "standardization" vector for any sample of data regardless of the number of subgroups. Since the "optimal" standardization vector is that vector which will have the highest correlation with the "best" set of weights for all the pseudo variables, then the problem can be perfectly resolved by Hotelling's Canonical correlation technique (26), using the "equating" variables as the "criteria" and the pseudo variables as the "predictors." Whether this technique would yield a significantly better solution than the one used in this dissertation is a question that will remain unanswered here. There is the possibility, however, that the Hotelling procedure might do better in the experimental sample but worse in cross validation because it would have used all the equating variables and thus fit too much error.

COMMENTS REGARDING THE UTILITY OF THE PSEUDO VARIABLE APPROACH

The pseudo variable approach allowed us to prove that Toop's "slope and intercept" method does provide the optimal linear transformation equation for each subgroup. The pseudo variable approach accomplished this because it was a method which solved for all subgroups simultaneously, maximizing the proportion of variance explainable in the entire group; only with such a solution could this have been ultimately proven. (Bottenberg and Christal's paper had, of course, indicated that the results would turn out this way.) However, use of the pseudo variable approach to equate subgroups requires one pseudo variable for each term in the transmutation equation for each subgroup. If one had many subgroups to equate (e.g., fifty high schools) and wanted to include up to the quadratic term

in the transmutation equations, he might encounter some difficulty in finding a computer large enough to handle the problem (200 pseudo variables plus the predictor variables). Obviously, then, a solution which would solve one subgroup at a time is ultimately more desirable. Much as Toops found the best linear transformation equation for a given subgroup, one could find the best non linear transformation equation for a given subgroup. This could be done in several different ways. One method would be to simply create, for example, an $\ddot{X}^2$ and $\ddot{X}^3$ score in addition to the standardization variable score ($\dot{Y}$) and the "variable-to-be-equated" score ($\ddot{X}$) which one would already have. Using only the data in a given subgroup compute the correlations for the four variables and then use $\ddot{X}$, $\ddot{X}^2$, and $\ddot{X}^3$ to predict $\dot{Y}$. The raw score beta weights for $\ddot{X}$, $\ddot{X}^2$, and $\ddot{X}^3$ will correspond to $\hat{b}$, $\hat{c}$, and $\hat{d}$ respectively, and the constant to be added will be $\hat{a}$. The pseudo variable approach can be used to prove that "transmutation" equations derived in this manner are, in fact, optimal.

It is perhaps worth pointing out the dissimilarity of the application of the pseudo variable approach in the Bottenberg and Christal paper and in the problem which was studied here. Bottenberg and Christal were interested in obtaining simultaneous prediction equations of a criterion score for several subgroups (different schools), for which they linearly "pseudoized" the predictor variables and left the criterion variable as it was. The problem here was to equate the criterion scores from each subgroup for which the criterion score was non linearly "pseudoized" and the predictor variables were left as they were. Their concern seems to have been primarily with within-subgroup comparisons whereas mine has

been with between-subgroup comparisons. Now that the various subgroup scores have been equated we also would be concerned with getting different prediction equations for each subgroup. Our purpose in equating the subgroup scores was so that when we finally did obtain a predicted score of, for example, "302" in subgroup 1 for individual i and a predicted score of "304" in subgroup 2, we would know that the higher score indicated a higher degree of success as an aviator if the man is assigned to subgroup 2 than if he is assigned to subgroup 1.

Bottenberg and Christal used the pseudo variable approach to find out if the prediction equation for one school was significantly different from the prediction equation for a second school. Their stated purpose was to reduce the number of prediction equations needed or, more simply, to reduce the number of criterion groups. Of course, it is possible that two schools' prediction equations differed significantly not because of what was important for success at each school, but because the meaning of grades at one school was not equivalent to the meaning of grades at the second school. Thus, both in their application and the one studied here, the subgroup criterion scores should be equated first.

COMMENTS REGARDING THE RELATIVE EFFICACY OF LINEAR TO NON-LINEAR TRANSMUTATION EQUATIONS

While it is true that the data studied in this dissertation seemed to be relatively free of "floor" or "ceiling," and "differential treatment of extremes" effects (with the possible exception of subgroup 4), this should not be considered as a license to be satisfied with a simple linear transformation. The presence or absence of these kinds of effects will vary from study to study depending on the nature of the

observable being studied and the type of evaluator being used. For example, these effects may turn out to be highly important when dealing with ratings.

If the needed transformation to a given subgroup is really merely linear, then the constants for the quadratic and cubic terms of the non linear equation will turn out to be zero. It is, of course, impossible to explain less of a subgroup's variance by using the best non linear equation than by using the best linear equation. The question, then, is whether the non linear equation is significantly better than the linear equation. (This can be answered by accomplishing the F-test shown in Table XIII.) This brings up another possible improvement in the technique. One could use a test-selection technique to determine which variables($\bar{X}$, $\bar{X}^2$, and/or $\bar{X}^3$) should be used for a given subgroup. Thus, only when a term adds a significant amount to the predictable variance would it appear in the transmutation equation for that subgroup.

THE PRACTICAL IMPLICATIONS OF TRANSMUTING SCORES

We now have a proven technique for the removal of certain types of "non-equivalence" errors. The question of whether transmuting scores should be accomplished depends, first of all, on the expected use of the scores.

If the scores are to be used only to evaluate the relative merit of individuals from the same subgroup (i.e., within-subgroup comparisons), then there would be no gain from accomplishing linear transmutations and, perhaps, only a slight gain from use of nonlinear transmutations. If the scores are to be used to evaluate the relative merit of individuals from different subgroups (e.g., between-subgroup comparisons), then the technique is advantageous.

The cost today, unlike thirty years ago, of transmuting scores, is quite reasonable if computers are used. How much benefit one will realize from transmuting scores will have to be determined empirically for each new study. The ultimate worth of transmuting sets of scores is not only the gain in the proportion of variance that will be explainable, but also the degree of confidence we will be able to have in our data. That is, even if no gain derives from applying the transmutation technique, we would at least know that our scores were free of "leniency," "exaggeration," "floor or ceiling," and "differential treatment of extremes" effects. This alone may justify the use of the technique.

The ultimate worth of using the technique will be determined by whether savings resulting from the gain in operational validity of the transmuted scores offsets the cost of using the technique.

V. SUMMARY

Whenever two or more individuals are to be compared on the basis of scores obtained from different evaluators, certain differential non-equivalence errors should be removed from their scores. A rationale for understanding how these non-equivalence errors affect a set of scores is presented. Various non-equivalence errors such as "leniency," "exaggeration," "floor and ceiling," "differential treatment of extremes," and various "composition" effects such as differential emphasis of factors, reliability, and validity are included in the rationale. The confounding effect of differential spread of ability and differential average ability among subgroups is discussed.

The problem of optimally removing non-equivalence errors involves, first, finding the best variable or composite of variables to use as a "standardization" variable against which the data from different evaluators may be equated. One method for accomplishing this goal is described and used in the dissertation, and a second method, involving the use of canonical correlation, is discussed. Once the "standardization" variable has been determined, the problem of equating data from different evaluators may be accomplished in a variety of ways. Toops' "slope and intercept" method is here demonstrated to have the capability of removing linear non-equivalence (i.e., differential "leniency" and "exaggeration"). Another method, the "pseudo variable" approach, is proposed for the removal of linear and nonlinear ("floor or ceiling" and "differential

treatment of extremes") non-equivalence errors. Toops' method is shown to be a special case of the "pseudo variable" approach.

Data are analyzed and transmuted by both methods and some superiority of the nonlinear solution is demonstrated. Tests of significance for determining whether transmutation equations will significantly improve (i.e., make more "error-free") a set of scores are given. The transmutation equations derived from one sample ($N = 612$), seven subgroups of data, were shown to be able to remove 21.6 per cent of the non-transmuted unexplained variance. In a second sample ($N=605$) of data the same equations were able to remove 18.64 per cent of the unexplained variance.

It is recommended that whenever the relative merit of two or more individuals must be determined on the basis of scores derived from different evaluators use of the methods described here be considered.

APPENDIX I
Intercorrelations, Means, and Standard Deviations of
Variables Used In The Experimental Study

NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.
01	AVIA QUAL TEST	080.412	013.470
01 01	1.00000000	.347962520	-.001616198
01 07	.365719320	.274361270	.429739600
01 13	.098999772	.243713770	-.030897061
01 19	.147763650	.265565850	-.011719215
01 25	-.064107733	.026317257	-.034732249
01 31	-.004428722	-.061253559	.027279651
01 37	-.002339924	-.003592658	-.058571143
01 43	.100436340	.001057775	-.003149835

NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.
02	ECH COMP TEST	060.964	007.149
02 02	1.00000000	.066635884	.150195880
02 08	.432684300	.197815560	.127729810
02 14	.160096370	.019862510	.124880030
02 20	.203971930	-.045745959	.108681280
02 26	-.010072191	-.088558321	-.044828305
02 32	.000856223	-.009259638	-.087173246
02 38	.013578395	.002077860	-.008295243
02 44	.027945727	.014211418	.003055827

NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.
03	BIOGRAPH. INVEN.	032.141	008.363
03 03	1.00000000	-.013537920	-.165897960
03 09	-.058141344	.159343930	.124553380
03 15	.056418934	-.044854210	.115897570
03 21	.022742994	.070309999	-.046112474
03 27	.005120581	.023895087	.070302731
03 33	-.039295498	.006567955	.025142851
03 39	-.033577481	-.040243367	.008048798
03 45	.010910680	-.034783177	-.041140540

NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.
04	SPA. APPER. TEST	020.556	005.799
04 04	1.00000000	.058956894	.083443763
04 10	.144099110	.155529870	.076882193
04 16	.160249630	.163162130	.122966460
04 22	.015794325	.049117248	.027301564
04 28	.005138549	.015532674	.052051030
04 34	-.075728024	.006863252	.015355453
04 40	.007745409	-.074028040	.007975750
04 46	-.002019446	.008635331	-.071442504

NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.
05	MATHEMATICS	050.251	009.993
05 05	1.00000000	.238892420	.421481530
05 11	.029436060	.131045550	.128966650
05 17	.058268872	.065068743	.168591030
05 23	.031068931	.075875649	-.067155317
05 29	.108757800	.033599648	.077106556
05 35	.007382581	.108838900	.036036484
05 41	-.120619460	.007918390	.109003300
05 47	.003250841	-.120552700	.038838842

NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.
06	PHYSIOLOGY	052.371	008.270
06 06	1.00000000	.413483570	.410921670
06 12	.036843437	.061779830	.162563850
06 18	.102471530	.126086960	.165240680
06 24	.078864318	-.008844261	.094896016
06 30	-.028672471	.079438879	-.007339187
06 36	.051685266	-.027975907	.079979766
06 42	-.098712636	.051376904	-.027704138
06 48	-.031223344		.080171695

NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.
07	PRIN. OF FLIGHT	048.016	009.103
07 07	1.00000000	.585263690	.490584050
07 13	.234422500	.337235740	.053899496
07 19	.197703470	.304730870	.009868420
07 25	-.028117005	-.052531981	-.080320105
07 31	.054354064	-.026846385	-.051210658
07 37	-.002562163	.055964955	-.025665791
07 43	.112395930	-.001083143	.057712792

NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.
08	ENGINEERING	051.587	008.860
08 08	1.00000000	.391077190	.234776400
08 14	.253990210	.098000233	.202212460
08 20	.277783090	.023268313	.056911193
08 26	.029583962	-.086402284	.025347010
08 32	.052517729	.031087232	-.083909366
08 38	.028294280	.052915876	.032502339
08 44	-.045991717	.029165064	.054059989

NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.
09	NAVIGATION	051.072	006.446

09 09	1.00000000	.188478450	.218362570	.243324450	.203847290	.358828040
09 15	.059111803	.457900840	.149034250	.203209930	.364854420	.560186500
09 21	.047987738	.078951087	-.020694603	.063929801	.010999083	.031794213
09 27	-.145086460	.050004696	.079491946	-.016491712	.066448911	.014036261
09 33	.034249265	-.141544800	.052267647	.080193108	-.012319386	.068879565
09 39	.016946503	.036696761	-.137812110	.054255189	.080527911	-.008330882
09 45	.070787801	.020290197	.039043830	-.133998460		

NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.
10	PRE-SOLO T-34	295.908	008.343

10 10	1.00000000	.543685380	.404610810	.387773420	.337350840	.230902270
10 16	.299750480	.403963070	.194359520	.331597550	.507479520	-.019103997
10 22	.200658190	-.006399047	.043676182	.019875083	-.091325093	-.144575410
10 28	-.015250830	.201934490	-.003348418	.044373221	.021801537	-.090720258
10 34	-.140039670	-.011191991	.203267100	-.000067484	.044811225	.023813401
10 40	-.090124684	-.135288450	-.006339285	.204421650	.003030935	.045015785
10 46	.026029546	-.089262565	-.130433370			

NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.
11	PRECISION T-34	297.544	007.412

11 11	1.00000000	.333402850	.356628290	.293808940	.208851890	.287278650
11 17	.358063250	.176041600	.325891000	.503897220	.024779008	.185926840
11 23	-.029620434	.099364717	-.008695783	-.084659170	-.155548970	.028089893
11 29	.186947770	-.026874727	.098924723	-.005889504	-.083256256	-.151274120
11 35	.031695834	.187981830	-.024280807	.098412480	-.003053681	-.081993060
11 41	-.146997930	.035056783	.189127400	-.020765429	.097315177	-.000027415
11 47	-.079721665	-.142329570				

NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.
12	TRANSITION	295.809	010.406

12 12	1.00000000	.484305370	.374539510	.179109590	.351861060	.415522920
12 18	.267561050	.448402950	.688796870	-.049219371	.227718030	-.064683227
12 24	.067731558	.004203974	-.055787919	-.114857880	-.044996061	.228845840
12 30	-.059904768	.069331817	.007732644	-.053924689	-.109142040	-.040571965
12 36	.229832200	-.055089384	.070649742	.011055671	-.051914454	-.103228870
12 42	-.035411884	.230998460	-.049521861	.072125817	.015444693	-.050412405
12 48	-.096723588					

NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.
13	PRECISION-ACRO.	296.701	012.424

13 13	1.00000000	.367907360	.183976780	.329312870	.413985620	.218754140
13 19	.370679150	.635064030	.007442717	.223979040	-.114384410	.119547580
13 25	-.003190798	-.037950897	-.139218950	.011205364	.225393330	-.110638580
13 31	.121027150	-.001381889	-.036067860	-.135323900	.015064271	.226680320
13 37	-.106726290	.122235320	.000397312	-.034232760	-.131261270	.019801858
13 43	.228783470	-.102732740	.123454070	.002372340	-.031828541	-.127006800

NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.
14	BASIC INSTR.	301.180	013.944

14 14	1.00000000	.222910290	.509744900	.327697680	.157443850	.418107770
14 20	.641837260	-.023459209	.206243150	-.085572957	.124499180	-.028453433
14 26	-.062555811	-.099668901	-.020217132	.207819470	-.081496977	.125424480
14 32	-.026013857	-.059774958	-.093844952	-.016858977	.209511740	-.077720300
14 38	.126281640	-.023666949	-.056984170	-.088038707	-.012780460	.210460980
14 44	-.072203034	.126727040	-.020120746	-.053733265	-.081452618	

NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.
15	NIGHT FAMILIAR.	306.928	024.401

15 15	1.00000000	.176764050	.227232860	.105305050	.185033530	.267637000
15 21	.027313715	.099208542	-.006552625	.053622210	-.071491419	.011302557
15 27	-.084195510	.028258789	.100387460	-.004064405	.054050975	-.070308271
15 33	.012239749	-.082348094	.029342744	.101573570	-.001739482	.054309702
15 39	-.068990606	.013251978	-.080278940	.029906846	.102545830	.001155548
15 45	.054066946	-.067592351	.014555244	-.077865209		

NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.
16	RADIO INSTR.	300.157	015.326

16 16	1.00000000	.348668310	.205960790	.448285760	.687610750	-.002836090
16 22	.251040980	-.121130180	.065913704	.003949314	-.001308047	-.144142890
16 28	.000527648	.252798060	-.115941480	.067496573	.006954214	.001510993
16 34	-.139316800	.003851824	.254779910	-.110929260	.068905351	.009831529
16 40	.004449535	-.134159680	.008171711	.256135090	-.104960960	.069829024
16 46	.013428991	.006808869	-.128666700			

NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.
17	FORMATION	295.513	009.405

17 17	1.00000000	.324687140	.507379030	.781156570	-.073502941	.273400030
17 23	-.101103570	.105602930	.064870375	-.030595170	-.183493000	-.070123316
17 29	.275494540	-.095814512	.107291030	.067464260	-.028887049	-.176364250
17 35	-.066663999	.277393540	-.090550684	.108702280	.070195997	-.027166489
17 41	-.169114720	-.062387627	.280223910	-.084847868	.109882650	.072931450
17 47	-.025189945	-.160393100				

NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.
18	GUNNERY	307.013	013.584

18 18	1.00000000	.242487000	.357204550	-.036250083	.169013300	-.075915740
18 24	.034384018	-.014622257	.035143132	-.079685494	-.034122220	.170157720
18 30	-.073020296	.033813180	-.013419448	.035906556	-.075975528	-.031610513
18 36	.171193840	-.070062126	.033150866	-.012071139	.036815818	-.072166749
18 42	-.029301995	.172836720	-.066803631	.032917370	-.011049708	.036887617
18 48	-.069133851					

NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.			
19	ADVANCED FLIGHT	300.477	005.899	VARIABLE-TO-BE-EQUATED		
19 19	1.00000000	.648055720	.025159631	.014474053	.004718306	.194362660
19 25	.118600250	.060681323	-.264954160	.030542289	.018441340	.013987839
19 31	.198112770	.124182400	.065225620	-.249315550	.035998273	.022407693
19 37	.022957121	.201638110	.129606120	.069715875	-.233534590	.042562744
19 43	.027071656	.032839422	.205099580	.135433040	.073847762	-.216243080

NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.			
20	W-D PRED. ADV FL	300.458	003.817	STANDARDIZATION VARIABLE		
20 20	1.00000000	-.034372993	.305609280	-.109010010	.123700650	.031436571
20 26	-.032615482	-.216477710	-.029715918	.307835590	-.102052590	.125947640
20 32	.035648207	-.029443326	-.208248280	-.024854101	.310043180	-.095225554
20 38	.127909840	.039782197	-.026212701	-.199793760	-.019291195	.312456890
20 44	-.087428547	.129408740	.044790682	-.023006292	-.190194650	

NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.			
21	PSEUDO A GROUP 1	010.621	030.811	DICHOTOMOUS PSEUDO VARIABLE		
21 21	1.00000000	-.160469420	-.177274120	-.092371273	-.124880190	-.088694948
21 27	-.205094400	.999828620	-.160453090	-.177234460	-.092357759	-.124858890
21 33	-.088678590	-.205022560	.999297550	-.160403010	-.177119520	-.092319275
21 39	-.124796110	-.088629471	-.204805980	.998139850	-.160293530	-.176898690
21 45	-.092252728	-.124674810	-.088551645	-.204398160		

NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.	
22	PSEUDO A GROUP 2	017.810	038.260	DICHOTOMOUS PSEUDO VARIABLE
22 22	1.00000000	-.239393060	-.124739250	-.168639690 -.119774700 -.276961900
22 28	-.160441920	.999898270	-.239339510	-.124721010 -.168610920 -.119752610
22 34	-.276864890	-.160356680	.999586190	-.239184300 -.124669030 -.168526150
22 40	-.119686280	-.276572420	-.160170920	.998903900 -.238806080 -.124579170
22 46	-.168362350	-.119581180	-.276021700	

NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.	
23	PSEUDO A GROUP 3	020.915	040.670	DICHOTOMOUS PSEUDO VARIABLE
23 23	1.00000000	-.137802210	-.186300000	-.132317770 -.305965940 -.177243740
23 29	-.239368720	.999776340	-.137702060	-.186268220 -.132293360 -.305858780
23 35	-.177149580	-.239294000	.999127970	-.137724640 -.186174560 -.132220080
23 41	-.305535680	-.176944360	-.239130680	.997882280 -.137625380 -.185993610
23 47	-.132103980	-.304927280		

NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.	
24	PSEUDO A GROUP 4	006.699	025.001	DICHOTOMOUS PSEUDO VARIABLE
24 24	1.00000000	-.097074340	-.068946109	-.159428020 -.092355438 -.124726560
24 30	-.137771380	.999853800	-.097057774	-.068933391 -.159372180 -.092306378
24 36	-.124687630	-.137682040	.999437170	-.097008978 -.068895208 -.159203830
24 42	-.092199443	-.124602520	-.137510380	.998716760 -.096914686 -.068834712
24 48	-.158886820			

NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.				
25	PSEUDO A GROUP 5	011.601	032.024	DICHOTOMOUS PSEUDO VARIABLE			
25 25	1.00000000	-.093210837	-.215536750	-.124858800	-.166622540	-.186258330	
25 31	-.097060139	.999829480	-.093193646	-.215461250	-.124792460	-.168569910	
25 37	-.186137530	-.097019698	.999326750	-.093142025	-.215233650	-.124647900	
25 43	-.168454850	-.185905460	-.096949761	.998355410	-.093060238	-.214805060	

NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.				
26	PSEUDO A GROUP 6	006.209	024.132	DICHOTOMOUS PSEUDO VARIABLE			
26 26	1.00000000	-.153082880	-.088679747	-.119762520	-.132288180	-.068936022	
26 32	-.093194932	.999815590	-.153029260	-.088632638	-.119725140	-.132202370	
26 38	-.068907297	-.093148081	.999261800	-.152867610	-.088529959	-.119643420	
26 44	-.132037540	-.068857627	-.093057559	.998384340	-.152563710		

NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.				
27	PSEUDO A GROUP 7	026.144	043.942	DICHOTOMOUS PSEUDO VARIABLE			
27 27	1.00000000	-.205059250	-.276933720	-.305897510	-.159404710	-.215499960	
27 33	-.153054650	.999649710	-.204950310	-.276847290	-.305699120	-.159338280	
27 39	-.215391620	-.152969860	.998593750	-.204712890	-.276658320	-.305317960	
27 45	-.159223430	-.215182270	-.152835560	.996605330			

NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.			
28	PSEUDO B GROUP 1	031.959	092.727	LINEAR PSEUDO VARIABLE		
28 28	1.00000000	-.160425590	-.177204080	-.092341928	-.124837480	-.088663389
28 34	-.204987410	.999816940	-.160375530	-.177089160	-.092303452	-.124774720
28 40	-.188614277	-.204770890	.999078040	-.160266050	-.176868360	-.092236919
28 46	-.124653440	-.088536471	-.204363130			

NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.			
29	PSEUDO B GROUP 2	053.549	115.045	LINEAR PSEUDO VARIABLE		
29 29	1.00000000	-.239315140	-.124708310	-.168593760	-.119740430	-.276836700
29 35	-.160340360	.999891590	-.239159960	-.124656350	-.168509000	-.119674100
29 41	-.276544280	-.160154610	.999449030	-.238861760	-.124566500	-.168345210
29 47	-.119569020	-.275993600				

NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.			
30	PSEUDO B GROUP 3	062.856	122.254	LINEAR PSEUDO VARIABLE		
30 30	1.00000000	-.137751230	-.186226540	-.132263770	-.305790340	-.177109950
30 36	-.239240470	.999784760	-.137693840	-.186132920	-.132190500	-.305467340
30 42	-.176904770	-.239077170	.999016450	-.137594580	-.185952000	-.132074430
30 48	-.304859060					

NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.				
31	PSEUDO B GROUP 4	020.417	076.203	LINEAR PSEUDO VARIABLE			
31 31	1.00000000	-.097043576	-.068923307	-.159348860	-.092292875	-.124669390	
31 37	-.137661900	.999862660	-.096994788	-.068885128	-.159180550	-.092185957	
31 43	-.124584300	-.137490260	.999421100	-.096900510	-.068824645	-.158863570	

NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.				
32	PSEUDO B GROUP 5	035.083	096.860	LINEAR PSEUDO VARIABLE			
32 32	1.00000000	-.093177746	-.215424460	-.124771170	-.168541130	-.186105780	
32 38	-.097003140	.999831440	-.093126129	-.215196920	-.124626620	-.168426100	
32 44	-.185873730	-.090933217	.999222120	-.093044359	-.214768410		

NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.				
33	PSEUDO B GROUP 6	018.743	072.861	LINEAR PSEUDO VARIABLE			
33 33	1.00000000	-.153001030	-.088616291	-.119703050	-.132177990	-.068894588	
33 39	-.093130899	.999812590	-.152839420	-.088513632	-.119621350	-.132013190	
33 45	-.068844927	-.093040378	.999274210	-.152535070			

NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.				
34	PSEUDO B GROUP 7	077.869	130.926	LINEAR PSEUDO VARIABLE			
34 34	1.00000000	-.204878520	-.276750310	-.305592030	-.159282460	-.215316170	
34 40	-.152916280	.999642900	-.204641170	-.276561410	-.305211000	-.159167650	
34 46	-.215106890	-.152782010	.998410480				

NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.	
35	PSEUDO C GROUP 1	009.618	027.920	QUADRATIC PSEUDO VARIABLE
35 35	1.00000000	-.160290330	-.176995090	-.092254418 -.124708440 -.088567206
35 41	-.204662100	.999671910	-.160160920	-.176774410 -.092187919 -.124587220
35 47	-.088489436	-.204254570		

NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.	
36	PSEUDO C GROUP 2	016.108	034.617	QUADRATIC PSEUDO VARIABLE
36 36	1.00000000	-.239085300	-.124617440	-.168456410 -.119636750 -.276457960
36 42	-.160104630	.999769480	-.238787230	-.124527620 -.168292680 -.119531700
36 48	-.275907470			

NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.	
37	PSEUDO C GROUP 3	018.899	036.781	QUADRATIC PSEUDO VARIABLE
37 37	1.00000000	-.137604540	-.186012200	-.132104780 -.305269210 -.176790050
37 43	-.238922130	.999681380	-.137505350	-.185831420 -.131988780 -.304661370

NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.	
38	PSEUDO C GROUP 4	006.216	023.209	QUADRATIC PSEUDO VARIABLE
38 38	1.00000000	-.096954373	-.068856428	-.159114210 -.092147543 -.124532380
38 44	-.137432970	.999811710	-.096860133	-.068795964 -.158797380

NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.	
39	PSEUDO C GROUP 5	010.609	029.306	QUADRATIC PSEUDO VARIABLE
39 39	1.00000000	-.093079310	-.215088730	-.124563970 - .168341430 - .185780290
39 45	-.096884484	.999728140	-.092997580	-.214660430

NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.	
41	PSEUDO C GROUP 6	005.663	022.027	QUADRATIC PSEUDO VARIABLE
41 40	1.00000010	-.152754750	-.068464599	-.119555090 - .131940060 - .068806792
41 46	-.092988838	.999774400	-.152450570	

NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.	
41	PSEUDO C GROUP 7	023.208	039.062	QUADRATIC PSEUDO VARIABLE
41 41	1.00000000	-.204425020	-.276269280	-.304888610 - .158999520 - .214879670
41 47	-.152620620	.999504040		

NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.	
42	PSEUDO D GROUP 1	002.899	008.425	CUBIC PSEUDO VARIABLE
42 42	1.00000000	-.159995350	-.176569620	-.092081121 - .124442890 - .088386923
42 48	-.204017940			

NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.	
43	PSEUDO D GROUP 2	004.845	010.419	CUBIC PSEUDO VARIABLE
43 43	1.00000000	-.238624230	-.124442620	-.168177800 - .119450110 - .275719140

NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.	
44	PSEUDO D GROUP 3	005.688	011.084	CUBIC PSEUDO VARIABLE
44 44	1.00000000	-.137333900	-.185599700	-.131824210 - .304281500

NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.	
45	PSEUDO D GROUP 4	001.897	007.089	CUBIC PSEUDO VARIABLE
45 45	1.00000000	-.096790316	-.068746377	-.158682910

NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.	
46	PSEUDO D GROUP 5	003.208	008.869	CUBIC PSEUDO VARIABLE
46 46	1.00000000	-.092907191	-.214451790	

NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.	
47	PSEUDO D GROUP 6	001.709	006.653	CUBIC PSEUDO VARIABLE
47 47	1.00000000	-.152316720		

NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.	
48	PSEUDO D GROUP 7	006.908	011.651	CUBIC PSEUDO VARIABLE
48 48	1.00000000			

APPENDIX II
Intercorrelations, Means, and Standard Deviations of
Variables Used In The Validation Study

NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.
01	AVIA QUAL TEST	080.979	013.503
01 01	1.00000000	.371002950	-.011360781
01 07	.439941750	.346882470	.493165160
01 13	.053581011	.140902140	.016364385
01 19	.146797750	.156269600	.138533250

NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.
02	MECH COMP TEST	060.945	006.963
02 02	1.00000000	.108748490	.260982710
02 08	.468453970	.283183540	.153791530
02 14	.228335040	.017193659	.129580270
02 20	.266461000	.186134660	.187343540

NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.
03	BIOGRAPH. INVEN.	032.683	008.137
03 03	1.00000000	-.015034665	-.055634277
03 09	-.019908173	.200650310	.146472520
03 15	.006514781	-.006201482	.101279110
03 21	.079988316	.080167291	

NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.
04	SPA. APPER. TEST	020.441	005.827
04 04	1.00000000	.089315444	.084699904
04 10	.152365830	.134639650	.039361082
04 16	.135491800	.150814520	.110032950
04 22	.143707700		

NUMBER NAME OF VARIABLE MEAN ST.DEV.
05 MATHEMATICS 051.529 010.372

05 05 1.00000000 .191120070 .491532100 .291236950 .560931550 -.014631010
05 11 .062485199 -.002696570 .013975440 .126850940 .002803541 .183544040
05 17 -.013034899 .111652320 .120620120 .166182090 .119630920 .116849120

NUMBER NAME OF VARIABLE MEAN ST.DEV.
06 PHYSIOLOGY 051.671 009.085

06 06 1.00000000 .459806830 .476296510 .271215990 .071189152 .076247281
06 12 .059906250 .114634200 .160167220 .022799886 .162751100 .053725378
06 18 .091279271 .162211030 .234305850 .160656890 .160890360

NUMBER NAME OF VARIABLE MEAN ST.DEV.
07 PRIN. OF FLIGHT 048.117 009.751

07 07 1.00000000 .656285130 .537683680 .120252090 .112317070 .071840120
07 13 .131521010 .202399490 .014186081 .234346110 .008226526 .073684155
07 19 .127790000 .180075890 .142195660 .150840240

NUMBER NAME OF VARIABLE MEAN ST.DEV.
08 ENGINEERING 051.517 009.575

08 08 1.00000000 .441174720 .179910590 .178592220 .087083801 .094869475
08 14 .182936770 .051335556 .212065700 .130095500 .124467740 .190503290
08 20 .250083880 .188135790 .190291970

NUMBER NAME OF VARIABLE MEAN ST.DEV.
09 NAVIGATION 051.190 006.913

09 09 1.00000000 .118740540 .156880830 .131435350 .159913790 .234566420
09 15 .038950984 .325646100 .095085152 .157210900 .280263430 .395779460
09 21 .281980740 .283346210

NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.
10	PRE-SOLO T-34	296.355	010.094
10 10	1.00000000	.605258460	.381959080
10 16	.314712840	.384336340	.299014590
10 22	.482415770	.459955530	.655055850
10			.258391260
10			.484537900
NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.
11	PRECISION T-34	297.934	007.950
11 11	1.00000000	.344769660	.339405920
11 17	.370146770	.272690110	.365401960
11		.528342630	.234869710
11			.237207080
11			.402598820
NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.
12	TRANSITION	295.613	011.006
12 12	1.00000000	.504387910	.318763520
12 18	.232760040	.413741470	.566678270
12		.481211010	.296626860
12			.370177220
NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.
13	PRECISION-ACRO.	297.519	012.873
13 13	1.00000000	.393142040	.212373750
13 19	.445439290	.635301730	.502732550
13		.507549990	.437434380
13			.238773770
NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.
14	BASIC INSTR.	301.593	014.292
14 14	1.00000000	.192174120	.528207170
14 20	.673881270	.519521410	.406568180
14		.526672520	.198756090
14			.473484950

NUMBER		NAME OF VARIABLE	MEAN	ST.DEV.		
15		NIGHT FAMILIAR.	304.944	024.329		
15 15	1.00000000	.187906380	.275870070	.186111660	.237564230	.324177820
15 21	.270987600	.267024360				
NUMBER		NAME OF VARIABLE	MEAN	ST.DEV.		
16		RADIO INSTR.	300.843	015.172		
16 16	1.00000000	.362936930	.168503860	.513749390	.730524400	.546853850
16 22	.551009100					
NUMBER		NAME OF VARIABLE	MEAN	ST.DEV.		
17		FORMATION	295.874	009.106		
17 17	1.00000000	.342240720	.501038770	.715868910	.574770380	.576312340
NUMBER		NAME OF VARIABLE	MEAN	ST.DEV.		
18		GUNNERY	306.460	012.339		
18 18	1.00000000	.323730130	.462070350	.335333570	.327290450	
NUMBER		NAME OF VARIABLE	MEAN	ST.DEV.		
19		ADVANCED FLIGHT	300.952	005.962	NON-TRANSMUTED SCORES	
19 19	1.00000000	.700711870	.904463790	.889587180		
NUMBER		NAME OF VARIABLE	MEAN	ST.DEV.		
20		W-D PRED. ADV FL	300.932	004.202	ORIGINAL MULTIPLE VECTOR	
20 20	1.00000000	.760225250	.760600020			

NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.	
21	LINEAR EQUAT.	300.469	002.890	LINEARLY TRANSMUTED SCORES
21 21	1.00000000	.989012110		
NUMBER	NAME OF VARIABLE	MEAN	ST.DEV.	
22	NON-LINEAR EQUAT	300.621	002.935	NON-LINEARLY TRANSMUTED SCORES
22 22	1.00000000			

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