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INITIALLY AT PLASMA TEMPERATURES.

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HEAT AND MOMENTUM TRANSFER IN LAMINAR FLOW:
HELIUM, INITIALLY AT PLASMA TEMPERATURES

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

by

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Approved by

Advisers

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SUMMARY

In the use of extremely high temperatures for chemical synthesis, it is necessary to rapidly cool gases. One method is to direct its flow through a small diameter heat exchange system. The design of such a system requires predicting the rates of heat and momentum transfer at temperature differences in the range of 10,000°R. Data for these predictions have not been available. The present investigation seeks to provide such data.

Extremely high temperatures may be achieved through the use of water-cooled plasma generators. Such a generator was built, and was novel in that cyclonic feed was employed. Using helium as the working gas, current levels were varied up to 500 amperes; the helium mass flow rates were varied from 0.213 to 1.263 lbm/hr. Corresponding energy inputs varied up to 12 kw or 40,000 Btu/hr. Average plasma temperatures in the generator anode section exceeded 50,000°R. Exit plasma temperatures, or more properly at these lower ionization levels, exit gas temperatures varied up to 15,000°R.

Rapid cooling of the generator exit gas occurred in a three-unit, water-cooled, heat exchange system. The

system was instrumented to provide heat transfer and pressure loss data. Each heat exchange unit was approximately one inch long. Having a one-eighth inch diameter inside flow tube, the assembled exchangers provided measurements at L/D values of 11, 19, and 27. Under isothermal flow conditions, prior to initiating the plasma arc, pressure losses varied up to 0.071 psig per inch. Reynolds numbers varied from 500 to 2300.

After initiation of the arc, pressure losses varied from a negative 0.115 to a positive 0.274 psig per inch, the decrease in velocity head at times more than offsetting the frictional pressure loss. Reynolds numbers varied from 100 to 700. Heat transfer rates in excess of 2,500,000 Btu/hr-ft² were obtained. The gas temperatures decreased by as much as 13,000°R through the exchangers. Gas bulk temperatures generally varied from 2000° to 11,000°R. Average temperature differences between the gas and heat exchange surfaces varied from low values to greater than 10,000°R. The average residence time of the gas in the exchange system was as low as 0.1 milliseconds. Mach numbers did not generally exceed 0.4.

With no heat input, pressure gradients were higher than predicted by the Langhaar methods for the entrance sections of circular ducts. This deviation was probably due

to the annular entrance geometry. At temperatures above 2760°R these entrance effects were less than 10 percent.

With heat input, the correlation of the heat and momentum transfer data required the estimation of viscosity, thermal conductivity, and heat capacity at temperatures far exceeding those where experimental data are available. Because of the exceptional properties of helium, this estimation was possible at temperatures up to 15,000°R. Normally the shifting of electrons within inner shells and the loss of electrons from outer shells must be taken into account. With helium, the high potential of its lowest electron orbit above ground state at 20.5 eV and its high ionization potential of 24.46 volts made this unnecessary. The electronic contribution to the heat capacity was negligible, the heat capacity being constant at 1.242 Btu/lbm-°R. The viscosity and thermal conductivity were calculated directly from kinetic theory.

Due to the great complexity of the problem, an over-all analytical solution was not attempted, the heat and momentum transfer data being calculated from heat balances and an over-all energy balance. The heat transfer data correlated favorably according to the Graetz solution for laminar flow. For Graetz numbers varying from 3 to 25, the Nusselt numbers varied from 1 to 8. Corresponding heat

transfer coefficients varied from 50 to 300 Btu/hr-ft²-°R. The momentum transfer data with heat input were correlated by plotting friction factor versus Reynolds number. Although the isothermal relationship, $f_a = 16/Re$, checked the data to within 50 percent, an L/D effect was apparent.

By making assumptions based on the conditions of this investigation, the energy balance can be simplified to yield an interesting relationship. These assumptions are:

- 1) The pressure gradient approaches zero.
- 2) The outlet gas temperature is small compared to the inlet gas temperature.

Under these conditions, the Reynolds analogy for laminar flow holds quite well but is improved if a temperature correction term is included:

$$\frac{f_a}{2} = \frac{2h_a}{\rho v c_p} \left(1 - \frac{T_w}{T_b}\right).$$

This relationship holds only for high temperature flow since the temperature correction reduces to zero under isothermal conditions.

Based on assumptions approached, but not met in this investigation, further relationships can be developed. The assumptions are then

- 1) The pressure gradient is zero.

- 2) The outlet gas temperature is negligible compared to the inlet gas temperature.

The relationships take the form of analogies and are:

$$\frac{f_a}{2} = \frac{2h_a}{\alpha \rho v c_p}$$

$$\frac{f_a}{2} = \frac{\rho v c_p}{2h_a (L/D)^2}$$

The equating of these results in two further relationships:

$$\frac{f_a}{2} = \frac{1}{\alpha (L/D)}$$

$$h_a = \frac{\rho v c_p}{2(L/D)}$$

These relationships account for the observed L/D effect and provide a simple means of approximating the friction factor and heat transfer coefficient under conditions approaching the above assumptions.

INTRODUCTION

High temperature research is currently receiving much attention. If we define high temperatures as being above $12,000^{\circ}\text{R}$, a temperature approximately equal to the surface of the sun, we are in an area where no known substance can exist as a solid or liquid unless under pressures greater than one atmosphere. If the substance is not easily ionizable, it will exist as a gas obeying the normal gas laws. Its viscosity, however, may be as much as seven times greater than it would be if the substance existed as a gas at room temperature.

If the substance is moderately or easily ionizable, it will exist in part or nearly wholly of ions and free electrons. Such a substance is termed a plasma. As opposed to the behavior of most solids, liquids, and gases, a plasma is easily affected by electric and magnetic fields. In addition, its properties change rapidly from that of the corresponding gas as the percentage of free electrons increases.

A tool for producing such materials is the plasma generator. In simplest form, it consists of a cathode and an anode supported in a gas. Once an arc has been struck between the two electrodes, the electrons emitted from

the cathode rush toward the anode, accelerated by the power source. In their travel, they collide with atoms of the gas, knocking off electrons and producing a fluid composed of ions, electrons, and neutral atoms. If now a gas is caused to flow continuously between the two electrodes, this same action will produce a continuous source of plasma.

Our initial interest in plasmas was directed toward analyzing the behavior of a plasma in a magnetic field. The exceedingly hot plasma would have to be contained in a flow tube and passed through a magnetic field. We were confident that we could somehow contain the plasma, but by virtue of the fact that the plasma is contained, infers extreme temperature gradients. The containing vessel must be much colder than the plasma. Thus any attempt at an analytical solution of the problem is confronted by extreme gradients in density, viscosity, thermal conductivity, and temperatures in both the radial and axial directions. Adding to this the problem of accounting for the effect of electrons in estimating physical properties, the problem takes on apparently insurmountable proportions. Nevertheless, an idea had occurred to us which we felt might aid sufficiently to allow an attack of the problem.

Previous experimental work in the field of magnetohydrodynamics had involved using liquid metals as the conducting medium. It occurred to us that electrolytes are also conductors and for our purposes offered some interesting possibilities. They, like plasmas, have negative, positive, and neutral species. But unlike plasmas, their physical properties are constant. Thus we could set up a magnetohydrodynamic experiment involving charged particles but without the disadvantage of property variation.

Such an experiment involving electrolytes was designed, as shown in the Appendix. Calculations in the design of this experiment showed it to be necessary to have magnetic field strengths on the order of 20,000 gauss and higher before a measurable magnetohydrodynamic effect could be observed. Such strong magnets were not available to us. Without this information, it appeared that even an analytical approximation of the plasma case would be near impossible. Only one solution of the equations of continuity, momentum, energy, and state for the magnetohydrodynamic plasma case was known and in addition to many simplifying assumptions, it involved the near impossible physical case of a radial magnetic field.

Our attention was thus directed to an experimental treatment of the flow of a plasma in a circular duct. To

our knowledge, this work had not been done and seemed a place where a significant contribution could be made. A number of plasma generators were built employing a straight-through flow in the hope of more easily maintaining laminar flow. In addition to the new idea of straight-through flow, we hoped to operate without cooling for periods sufficiently long to allow pressure loss measurements. To an extent our ideas were successful but arc instability caused us to abandon this approach.

Although numerous plasma generators have been built by others and operated successfully, the benefit of this experience is not readily available. Our own attempts at solving the arc instability problem led us to two new ideas. One involved the use of a cyclone type gas feed system. This type of feed system had the advantages of maintaining a cool gas film along the internal walls of the generator, promoting convective cooling of the cathode, and providing a constricted gas flow around the arc, thus tending to stabilize it.

The second idea, and the one which led to the work of this dissertation, lay in the fact that if both heat and momentum transfer were measured as a plasma flowed through a water-cooled circular duct, an over-all energy balance could be applied. The problem would not be solved

analytically but the approach would yield valuable insight into the over-all problem.

Some of the specific information obtained in this manner, that is, heat transfer coefficients and friction factors, would be valuable in the design of plasma cooling systems. Such systems might occur where chemicals are being synthesized by rapidly cooling plasmas to ambient temperature. As the plasma is cooled, products may "freeze out" which would normally not be formed in any other way. One means of achieving rapid cooling consists of allowing the plasma reactants to flow through a small diameter cooled tube. If the ionization of the plasma reactants is not too high, the viscosity will be at a high value because of the extreme temperature. The combined effect of high viscosities and small tube diameters causes the flow to lie in the laminar region over a far greater range of mass flow rates than is the usual case.

The final objectives of this dissertation were then

- 1) The building and operation of a novel plasma generator.
- 2) The measurement and correlation of heat and momentum transfer data in a gas quenching system under laminar flow conditions and at temperatures hitherto unreported.

LITERATURE SURVEY

As a fluid, starting from a reservoir at zero velocity, begins its flow through a duct, a certain distance and time are required for the steady state velocity profile to develop. In developing this profile, momentum is transferred to the walls at a greater rate than would be the case for a fully developed flow regime. If the fluid is flowing in a circular duct and the Reynolds number is less than 2100, this increase in momentum transfer can be calculated according to the methods of Langhaar.¹

Although little laminar flow data are reported on the effect of heat transfer on the friction factor, it must be inferred from the heat and momentum analogies that an interdependence exists. Looking at the Reynolds analogy in terms of dimensionless groups,

$$\frac{f}{4} \text{ Re} = \frac{\text{Nu}}{\text{Pr}} \quad (\text{laminar flow})$$

it would appear that the ratio of Nusselt to Prandtl numbers must be constant to prevent a scatter of points when friction factor is plotted against Reynolds number; that is, to ensure only one value of friction factor for each value of Reynolds number. Deissler² reports that this ratio will be constant for temperature changes not too

large if the fluid properties are evaluated at a film temperature defined by

$$T_{-0.27} = -0.27(T_w - T_b) + T_b \quad .$$

Although the relationships developed were based on a mathematical approach rather than laboratory data, the Nusselt number for laminar flow was shown to be constant at approximately 4.3 under the conditions assumed, and so independent of Reynolds number.

The heat conduction problem for a fluid flowing in a circular tube (assuming laminar flow or a parabolic velocity distribution, constant viscosity, and thermal conductivity, a uniform wall temperature, and heat conduction in the radial direction only) was solved by Graetz as reported by McAdams.³ Assuming rodlike flow for these otherwise identical conditions, again as reported by McAdams,³ the equations were solved by Drew. As reported by Deissler,² the cooling of a gas tends to alter the normal parabolic distribution of laminar flow into a flattened parabola, tending toward rodlike flow as the ratio of wall temperature to bulk temperature decreases. Conversely, if the gas is being heated, the parabola tends to become elongated. Cholette⁴ presents data for air flowing through steam heated tubes. Since the gas is being heated, its

velocity distribution should tend toward an elongated parabola, and the log-log plot of Nusselt versus Graetz numbers should fall below those for the Graetz solution. The fact that the data do not is probably due to the assumptions of the Graetz solution.

Taylor and Kirchgessner⁵ investigated heat and momentum transfer at heat fluxes up to 345,000 Btu/hr-ft² with helium as the fluid and surface temperatures up to 4533°R. Their work is almost entirely in the turbulent flow regime; however, some laminar data were reported which showed friction coefficients to be approximately four times as great as that predicted by the following formula:

$$f = \frac{16}{Re} \quad (\text{laminar isothermal flow}) \quad .$$

Fluid properties were evaluated at a temperature midway between the surface temperature and the fluid bulk temperature. Part of the fourfold deviation may have been due to the use of an α of 2 in the kinetic energy term,

$$v^2 / \alpha g_c \quad ,$$

rather than a value near 1 as would be expected in laminar flow.

A review of the analogies between heat and momentum transfer is covered by Knudsen and Katz.⁶ Among these is the Reynolds analogy,

$$\frac{f}{2} = \frac{h}{\rho v c_p} \quad .$$

This analogy is of prime interest in the work presented here, although the analogy applies only to turbulent flow.

EXPERIMENTAL METHODS

Equipment

The heart of the equipment is the plasma generator and associated heat exchangers shown in Figure 1. This particular plasma generator was the twelfth of those built, the twelve being shown consecutively from left to right, top to bottom in Figure 2. It was hoped initially that it would be possible to build an apparatus which could be operated without water cooling for a period sufficiently long to permit measuring pressure gradients. Graphite was selected as the material of construction, since it was easily machinable, did not melt or vaporize up to a temperature of 6330°F, and was relatively inexpensive. It was also thought that a straight-through flow would be preferable, since a laminar flow regime could be developed before the gas entered the arc region. The electrodes were placed at right angles to the direction of flow and were either of graphite or brass. The associated equipment initially consisted of a 650 ampere A.C. - D.C. (D.C. was used throughout this investigation) rectifier for power, a rotameter to measure flow rate of the gas, a 50 mv range voltmeter to indicate the voltage generated by the current flowing through an 800 ampere shunt to the electrodes, a

Figure 1. Plasma generator. heat exchangers, and associated equipment.

Figure 2. The eleven preliminary and final plasma generators.

0-150 volt voltmeter to measure the voltage between the electrodes, and a tank of pressurized nitrogen.

In a typical early plasma generator, the electrodes were not insulated from their graphite half-cylinder supports, although the two half cylinders were insulated from each other with either vycor or asbestos. The arc was started either by advancing one of the electrodes until it touched the other and then retracting it, or by placing steel wool between the two electrodes, the steel wool vaporizing and leaving a current path through the gas. The operation of these generators tended to be quite erratic. This behavior was definitely due to the fact that the plasma acted as a near short circuit between the two half cylinders of graphite and allowed the arc to move to the end of the flow tube and then blow out. Attempts were made to remedy this situation, first by recessing a space around the electrodes and then by insulating the electrodes from the two graphite half cylinders. This allowed some slight improvement, but the tendency was still to operate for a short period of time, ending with blowout of the arc. The other problems associated with this type of generator included the almost immediate melt-out of the electrodes if brass were used, and the rapid attrition of electrodes if graphite were used. Then, too, the entire assembly would become a glowing red color after a

short period of operation. The loss of brass electrode material was due simply to the temperature of operation being higher than the melting point of brass. The loss of graphite electrode material was due both to being near or above the sublimation point of graphite and to a reaction with the nitrogen carrier gas to produce cyanogen.⁷ The heating of the entire graphite assembly was due both to the relatively high electrical resistance of graphite and the lack of adequate heat removal.

Because of the difficulties encountered in nitrogen reacting with the electrode materials, it was decided to use a noble gas in further work. Noble gases, being monatomic, offer for ease of calculation the additional advantages of (1) having no infrared radiation, at least until ionization becomes significant; (2) having no energy of dissociation since they are monatomic to begin with; and (3) having a constant heat capacity over all temperature ranges, again up to the point where ionization becomes significant. Argon was chosen originally because of its relatively low cost, but it was later found advantageous to use helium since its higher ionization potential allowed attaining markedly higher temperatures.

Based on a suggestion by Dr. C. E. Dryden of the Ohio State University Chemical Engineering staff, it was next decided to build a generator wherein the gas feed

followed a path, from its entry to its exit through the anode, similar to that followed by a gas in flowing through a cyclone separator. It was also decided to use tungsten metal as electrode material, since although it melts at a temperature (6100°F) slightly below the sublimation point of graphite (about 6330°F), it does not boil until $10,700^{\circ}\text{F}$, significantly higher than graphite at 7600°F . Then, too, tungsten has a much higher electrical and thermal conductivity than graphite, resulting in less internal heating of the electrodes due to resistance and a more rapid flow of heat away from the electrodes. The gas to the generator entered tangentially around the anode; flowed back in a cyclic motion, washing the pyrex glass wall; and traveled forward along the cathode through the arc and out the center three-sixteenth inch hole in the anode. At low current densities, the device worked quite successfully, but at higher current densities, the one-eighth inch diameter tungsten cathode softened and fell away from the anode. Also at high current densities, the velocity of the vortex motion of the cool gas around the arc was too low to prevent flarebacks of the plasma, the flarebacks causing failure of the pyrex glass. Then, too, even at these relatively low current levels of 75 to 150 amperes, vaporization (but not reaction) of the graphite anode continued to be a problem. The general lack of

success with equipment uncooled except for radiation and natural convection led us to conclude that forced cooling would be required both in preventing undue vaporization of electrode material and in allowing runs of sufficient length to permit pressure gradient measurements.

The first water-cooled apparatus again allowed for straight-through flow of gas and employed one-eighth inch diameter pieces of tungsten rod imbedded in brass as the cathode and anode materials. The two tungsten pieces in each electrode were pointed diagonally toward each other across a one-half inch thick asbestos insulating section. The apparatus worked successfully for a short time, but the two pieces of tungsten on the anode side rapidly melted away, allowing the arc to strike the brass, the brass then melting through to the water side.

Again using straight-through flow, an apparatus was built with the cathode now constructed of one-eighth inch diameter tungsten pieces brazed together and given an outside thread to allow feeding toward the anode through the cathode water-cooled section. The anode itself was made by imbedding numerous tungsten pieces of one-eighth inch diameter by one-half inch length around a three-sixteenth inch hole in the water-cooled brass anode assembly. This apparatus was more successful, but the brass portion of

the anode surrounding the tungsten rods again soon melted and eroded through to the water side.

The next two models were further attempts to use the cyclone principle but employed smaller cone-shaped chambers. The smaller chambers allowed higher vortex velocities which tended to stabilize the arc without exceeding laminar flow conditions in the outlet tube. The models were both of uncooled graphite, the second being constructed with much thinner walls in the hope of dissipating the heat more rapidly. Although both operated successfully for short periods of time, vaporization of graphite remained a problem.

The tenth model was quite similar to the eighth but employed cyclone feed, was somewhat more sophisticated in its water cooling, and, what was felt to be most important, the tungsten rods were imbedded as closely together as possible in the anode section. The close imbedding, it was hoped, would allow the tungsten to melt together to form a continuous tungsten face on the anode and prevent melting through to the water side. The action was in part successful in that a half hour of operation was achieved before a melt-through occurred, the tungsten having in large part melted into a continuous surface. The problem seemed to be that the tungsten did not melt evenly enough to prevent

an irregular turbulence from forming which then preferentially eroded away one side. Observation of the arc was possible in this apparatus, since a vycor tube two and one-fourth inches in diameter by three-fourths inch long was used as the insulating material between the cathode and anode assemblies.

The problem of producing a continuous tungsten anode piece was solved on the basis of advice obtained from Battelle Memorial Institute of Columbus, Ohio. They stated that, contrary to available information, tungsten metal could be drilled with ordinary carbon steel high-speed twist drills. They also suggested that the anode be made removable, so that in case of anode failure it would be necessary to replace only the anode rather than the whole anode assembly. Based on this advice, one-fourth inch diameter thoriated tungsten rods as used in helium arc welding were purchased. After considerable trial and error in lathe rotational speeds, speed of lathe travel, and drill sharpening, a one-eighth inch diameter hole was drilled in the one-fourth inch tungsten rod, a one-half inch length being incorporated as the anode working surface of the eleventh plasma generator.

Feed gas in the eleventh generator could be injected both tangentially around the anode to give a vortex

motion or straight through. The vycor window was retained to allow viewing of the arc. The cathode was constructed of a short length of one-fourth inch diameter tungsten rod imbedded in a water-cooled holder, threaded to allow feeding toward the anode through the cathode support. This apparatus operated quite successfully, the single serious problem being a tendency for the current to arc across the tungsten to the water-cooled copper. Since the anode was made removable, this problem was solved by using a 1/2-inch, rather than a 1/4-inch, diameter tungsten rod 1/2-inch long drilled with a 1/8-inch center hole. By placing the cathode about 1/16 inch away from the anode, the resistance paths of the arc were now such that the arc struck to the surface inside the 1/8-inch diameter tube rather than outside across the tungsten to the copper. The less serious problems included the difficulties encountered in trying to maintain a tight seal along the threaded cathode and the generally unnecessary bulkiness of the unit. These factors lead to the decision to construct one final generator which incorporated as nearly as possible all the features we thought desirable in a small plasma generator.

Before an actual final design could be achieved, however, an unforeseen problem arose which required solution. The cooling water to the anode made a 360° turn around the central tube in going from the entrance to the

exit. In making this turn, the centrifugal force of the water tended to leave an area of low velocity water on the opposite side from the water tube inlet. With the plasma generator operating, incipient boiling occurred at this point, even though water outlet temperatures from the anode never exceeded 130°F, and in a short time led to a total melting away of the anode on that side. In building a new anode assembly, the problem was corrected by inserting a small copper baffle which forced the water to wash the central anode tube at fairly high velocities over the full 360°, completely eliminating incipient boiling.

The final plasma generator assembly is shown photographically in Figure 3 and schematically in Figure 4. A detailed schematic drawing of the anode assembly is shown in Figure 5 and it will be described first. The anode assembly is constructed of four major pieces, three of copper and one of tungsten, all copper walls being one-eighth inch thick except those around the tungsten, which are one-sixteenth inch thick. One copper piece, the one into which the tungsten is fitted, makes up the whole face looking toward the cathode assembly and extends back through the anode assembly to form a portion of the other face. After first turning out the general shape from copper bar stock, the face, looking toward the cathode assembly, was drilled to form a hole

Figure 3. Exploded views of plasma generator.

Figure 4. Plasma generator (to scale)

Figure 5. Anode assembly (to scale)

one-half inch deep and just slightly smaller in diameter than the one-half inch diameter tungsten. The one-half inch long tungsten piece was first polished with fine sandpaper, then electroplated with a thin plate of copper in a copper sulfate - sulfuric acid bath, and finally force fitted into the copper. To ensure as good electrical and thermal conductivity as possible, a combination of silver solder and brazing rod were used to further seal the tungsten to the copper piece. The best method, however, would be to produce a perfect force fit, since any solder used must form a eutectic with both copper and tungsten. Since the eutectic formed melts below the lowest melting component, the soldered joint will always melt out first.

The second copper piece is a cylinder $3/4$ -inch long by $1\frac{1}{4}$ -inches outside diameter, having a wall approximately $1/8$ -inch thick. The piece was cut from a length of copper pipe and silver soldered to the first copper piece. Two small copper pieces were now inserted, one being the small copper partition between what became later the cooling water inlet and outlet, and the other being the small copper baffle required at a point 180° from the cooling water inlet and outlet. After soldering these pieces in place and marking the outside to show their position, the third major copper piece was cut from bar stock, turned on a

lathe to proper size, and soldered into place. The assembly was now complete except for drilling the 3/16-inch diameter by 3/16-inch deep hole into which the heat exchanger assembly would fit, and drilling the water inlet and outlet holes.

In following the above method of construction, the copper acquired a fairly heavy oxide coating, due to its being severely heated in the soldering operations, and had to be cleaned with steel wool or sandpapered before it could be soldered into the anode support assembly. A more simple method was to form the parts so that the outside diameter of the anode assembly was just slightly larger than the hole into which it had to fit. The whole anode assembly was then turned down as a unit, so that it exactly fit the hole in the anode support assembly, the copper oxide being removed in the process.

The anode support assembly consists essentially of a cylindrical piece of brass turned to size on a lathe. A hole was drilled for the anode assembly and the side facing the cathode recessed on the lathe to allow for both the annular space in which the gas feed starts its vortex motion, and the small recess into which the vycor cylinder and rubber gasketing fit. Three other holes were then drilled, one tangentially to the annular space (the feed

inlet) and the other two for cooling water to and from the anode.

The anode assembly was fitted into the anode support assembly along with the cooling water tubes and the gas feed tube. These five parts were then soldered together using 50-50 tin-lead solder, the low melting solder being preferable in that it allowed easier disassembly of the parts.

The cathode assembly consists in part of a 1-3/4 inch length of tungsten, 1/4 inch in diameter, force fitted and brazed into a piece of copper 1/2 inch long by 1/2 inch in diameter, the copper piece in turn being silver soldered to the inside surface of one end of a brass tube with an inside diameter of 1/2 inch and an over-all length of 3-7/8 inches. The other end of the brass tube was closed with a 1/8-inch thick copper disc silver soldered into place. Two 1/4-inch diameter holes were drilled through the walls of the brass tube through which were fitted the cathode cooling water inlet and outlet tubes. The inlet tube was bent to a 90° turn and run to within 1/8 inch of the copper end plug, thus insuring that high velocity cooling water would be present at the maximum heat flux area. The copper tubes were silver soldered into place as was a 1/8-inch thick brass disc to which the negative power lead was connected.

The cathode support assembly, like the anode support assembly, was turned from a cylindrical piece of brass. After drilling the center hole for the cathode assembly, one end was recessed toward the center in a cone shape, this shape facilitating the flow of gas in its 180° turn back toward the anode. On this same face which looks toward the cathode, a small recess was turned down at the outside edge into which the vycor window and rubber gasketing fit. On the opposite face, four holes were drilled and tapped, two of these tapped holes being used to anchor the metal backup piece for the cathode sealing gasket. The other two holes were used to anchor the cathode screw feed mechanism.

The cathode screw feed mechanism consists of a two-inch diameter by one-fourth inch thick brass disc drilled and tapped at the center to allow a screw to be turned through it against one end of the cathode. A brass spring was included in this mechanism to aid in backing the cathode from the anode.

The cathode and anode support assemblies were held together by three bolts running through brass rings, one of the two rings fitting over the anode and the other over the cathode support assemblies. These rings were insulated from the support assemblies by teflon gasketing. The anode and cathode support assemblies were insulated from each other,

as stated before, by the vycor window and the rubber gasketing. The cathode assembly was slipped through the metal sleeve and rubber sealing gasket to within 1/16 inch of the anode. The metal sleeve was then screwed down to seal the gasket against the cathode cooling water jacket and the screw feed mechanism fitted over the cathode and screwed to the cathode support assembly. Silicon grease was used on the three pieces of rubber gasketing to provide a better seal.

The plasma generator was now assembled and required only the connecting of the negative power lead to the cathode binding post and the positive power lead to the anode cooling water lines.

In this final generator, a tangential gas feed, rather than a straight-through gas feed, was selected. Although little difference had been noted between the two, tangential feed was chosen for three reasons; (1) the vortex motion produced by this method should lead to a more stable arc since the cycling of cold gas around the arc tends to constrict the arc thermally to a path down the center of the vortex, (2) the cyclone action of the tangential feed causes the gas to flow first past the vycor window thus tending to protect it, and (3) the gas is preheated to a somewhat greater extent than the straight-through feed,

although the heat pickup is minor compared to energy input of the arc.

The heat exchangers shown photographically in Figure 6 and schematically in Figure 7, provide the test section for measuring heat and momentum transfer. The train consists of three exchangers in series, the first exchanger being 1-3/8 inches long and the other two each being one inch long. A pressure tap is included in the first heat exchanger (H-1) which allows measuring the pressure drop from that point to the atmosphere. As the second (H-2) and third (H-3) heat exchangers are added, the pressure drop can be measured over this increased length of tube to the atmosphere.

The heat exchangers are built of copper, the copper pieces being joined together with silver solder. All walls of the heat exchangers, except the lead wall of H-1, are approximately 1/8 inch thick. Each exchanger is built of four parts; the inside tube, the two end pieces, and the outside wall. The inside diameter of the inner tube is 1/8 inch. The lead end of each exchanger consists of a male part 3/16 inch in diameter by 3/16 inch long while the trailing end has a female part of corresponding dimensions. The anode also having a female part, the units may be force fitted together to give a continuous tube of three different lengths for measurements of heat and momentum transfer.

Figure 6. The three heat exchangers.

Figure 7. Schematic of heat exchangers (to scale)

A fourth heat exchanger was constructed to act as the final heat sink in bringing the temperature of the exit gas to within a few degrees of the reference temperature (75°F). Through this means it was possible to obtain a heat balance on the plasma generator and associated heat exchange equipment. The exchanger was constructed from 3 feet of 1/4-inch copper tubing coiled to a 1½-inch diameter by 5 inch length and soldered into a copper water jacket 5-3/4 inches long having a 2-inch inside diameter. The exchanger had a single inlet and outlet for cooling water and was designed with a male part to allow force fitting it to the trailing end of the third heat exchanger.

An over-all schematic of the plasma generator, heat exchangers, and associated equipment is shown in Figure 8. In this schematic, heat exchanger 4 is included but in normal operation it would be absent. As shown, the associated equipment includes (1) a tank of helium initially pressured to about 2600 psig; (2) a rotameter capable of measuring helium mass flow rates from 0.05 to 1.84 lbm/hr when the rotameter is under 14.8 psig pressure; (3) a pressure gauge to measure the pressure in the rotameter; (4) manually operated bypass and control valves; (5) a mercury manometer to measure the gas pressure entering the arc; (6) a water manometer to measure the pressure drop

Figure 8. Schematic of plasma generator, heat exchangers, and associated equipment

from the inlet of the heat exchange train to atmospheric pressure; (7) rubber tubing to serve as cooling water ducts; (8) six No. 30 B&S gauge copper-constantan thermocouples to measure cooling water and helium inlet temperatures; (9) a selenium rectifier to provide 75 to 650 ampere direct current; (10) an 800 ampere shunt; (11) a 0-50 millivoltmeter to indicate relative arc currents; and (12) a 0-150 volt-meter to measure arc voltage. Equipment not shown in the schematic includes (1) a 16-point automatic recorder for continuous temperature readings; (2) a wooden stand to support the plasma generator and heat exchangers; (3) a two-liter flask and stop watch to measure cooling water rates; (4) zinc strips which are inserted through the anode gas exit to short between the two electrodes and vaporize to form a current path for the arc; (5) asbestos gloves for handling the zinc strip in initiating the arc and in handling any equipment which is either electrically or thermally hot; (6) a wet test gas meter used in conjunction with the 2-liter flask and stop watch to calibrate the rotameter; (7) a potentiometer with standard cell to calibrate the millivoltmeter; (8) an accurate 0-200°C thermometer to check water temperatures; and (9) an assortment of tools to assemble and disassemble the equipment.

Procedure

The first step in starting the equipment is to turn on the cooling water. The recorder is then turned on, and after warm-up the temperature of the cooling water to the anode, cathode, and three heat exchangers is recorded. The rotameter is next pressured to 14.8 psig by adjusting the pressure control valve on the pressurized helium tank. This is done with the rotameter bypassed to avoid damaging the rotameter should the pressure be increased too rapidly. After the bypass is closed, the helium flow control valve is adjusted, by observing the rotameter float, to a previously decided helium flow rate. The first heat exchanger is then fitted into the anode and the rotameter reading, helium inlet pressure, and pressure drop across the first exchanger manually recorded. This done, the power supply is turned on and the anode and cathode shorted together with a zinc metal strip to initiate the arc. The current is adjusted to some low level, for instance 100 amperes, and the following manually recorded: relative amperes through arc in millivolts, voltage across the arc, helium inlet pressure, and the pressure drop across the exchanger. During this time, the heat transfer data in terms of cooling water outlet temperatures, are automatically recorded by the 16-point recorder. The raw data recorded for one particular set of conditions are given in Table 1.

Table 1
Raw Data for Sample Calculation

Rotameter Reading	10
Tube diameter, inch	1/8
Tube length, inch	3.37

No Heat Input Case

Helium temperature, °F	75
Ambient temperature, °F	75
Atmospheric pressure, mm Hg	746

Pressure Drop Data

Manometer reading with no flow (left leg), mm H ₂ O	521
Manometer reading with flow (left leg), mm H ₂ O	544

Heat Input Case

Helium temperature, °F	80.6
Ambient temperature, °F	80.6
Atmospheric pressure, mm Hg	746

Heat Transfer Data

Relative amperes to arc, mv	22.0
Voltage across arc, volts	27.0
Water flow and temperatures:	
Temperature(°F)	Time for 2 liters to flow, seconds
<u>into</u> <u>out</u>	
Anode 74.0 110.5	25.3
Cathode 66.0 73.0	77.8
H-1 71.5 102.0	77.0
H-2 71.0 76.0	85.4
H-3 70.8 74.0	74.7

Pressure Drop Data

Manometer reading with no flow (left leg), mm H ₂ O	521
Manometer reading with flow (left leg), mm H ₂ O	472

The arc amperage is then increased and the same process repeated until data have been obtained at from three to five current levels for the one exchanger at the one-flow rate. The second and third heat exchanger are in turn added and again the process is repeated. For each flow rate, then, there is a minimum of nine sets of data. (The exception is a mass flow rate of 1.263 lbm/hr where only two sets of data were taken. This occurred because one run was conducted where the power input was held approximately constant and only the flow rate and number of heat exchangers varied.)

Generally speaking, the helium mass flow rate was varied from 0.213 to 1.263 lbm/hr and the current through the arc from 75 to 550 amperes, although the current range was most often from 140 to 450 amperes.

Still photographs of the plasma generator in operation at 0.642 lbm/hr of helium and a 450-ampere current through the arc are shown in Figure 9. From left to right, top to bottom, the views show the generator in operation with none, one, two, and three heat exchangers connected. Power input to the arc was approximately 40,000 Btu/hr. It will be noted that with no heat exchangers connected to the anode, a laminar radiating gas or plasma extends away from the anode for a distance of three to four inches. With a one-inch length of exchanger, approximately one inch of

Figure 9. Plasma generator in operation with none, one, two, and three heat exchangers (0.642 lbm/hr of He at 450 amperes).

radiating gas is visible; with the two-inch length of exchanger, approximately a quarter inch of radiating gas, and with three inches of exchanger, the ions and electrons have all recombined to give an electrically neutral gas.

In Figure 10, a view is given of the tip of the cathode and a slight view of the inside of the anode when the arc was in operation. The brightest portion of the figure (the photograph was taken through a dark filter) is the arc itself as it travels from the cathode to the anode.

Figure 10. Arc in operation at 0.642 lbm/hr of He and 450 amperes. Photograph taken with dark filter.

EXPERIMENTAL RESULTS

Method of Calibration

Calibration of the rotameter involved two procedures, each intended to check on the other. The first involved pressuring the rotameter to 14.8 psig, adjusting the rotameter float to some particular level, via downstream control valve, and measuring the length of time required to displace the water from a two-liter flask. This method could be used up to a helium mass flow rate of 0.496 lbm/hr, but above which the rate of filling became too rapid to allow any reasonable accuracy. The other method involved using a wet test gas meter which was also capable of measuring mass flow rate to 0.496 lbm/hr. The wet test meter checked the calculated flow rates to an accuracy of five percent at 0.496 lbm/hr with the error approaching zero magnitude as the flow rate was decreased. Displacing water also checked the calculated values. This led us to accept the calculated flow rates over the whole rotameter range; that is, at 14.8 psig the mass flow rate was assumed to vary from 0.048 to 1.841 lbm/hr of helium gas as the rotameter reading varied from 2.2 to 25.3, the total useful range of the rotameter.

Of the 16 points available on the temperature recorder, 4 points checked the anode water outlet temperatures, 4 checked the cathode water outlets, 2 the H-1 water outlets, 2 the H-2 water outlets, 2 the H-3 water outlets, and 2 the helium inlet temperatures. Only one thermocouple from each group was used to obtain data, the others being merely check points. The combined accuracy of the copper-constantan thermocouples, recorder, and leads was checked by placing the thermocouple junctions in a Dewar flask of known water temperature. In the normal operating temperature range of 50° to 120°F the maximum errors are shown to be:

Anode thermocouple	-1.3°F at 94°F
Cathode thermocouple	-1.2°F at 94°F
H-1 thermocouple	-1.3°F at 94°F
H-2 thermocouple	-2.0°F at 94°F
H-3 thermocouple	-2.2°F at 94°F .

Since it was the differences in temperature which were important, the error in the temperature level could be neglected. The maximum percentage error on temperature differences was five percent.

An error in temperature measurement, the magnitude of which we could not definitely determine, seemed to stem from the over-all power load on the power lines feeding the recorder. At times when the selenium rectifier,

feeding direct current to the arc, was turned on, there would be a marked change in the value of the temperatures read from the recorder, amounting to as much as 3°F. At other times there would be no change. It was decided that the differences in temperature would remain correct even though the temperature level was somewhat in error.

The current passing through the arc was determined by measuring the voltage drop across an 800 ampere shunt. The passage of 800 amperes through this shunt supposedly produced a 50-millivolt potential. By measuring the potential across the shunt, the current could be calculated; that is, each millivolt of potential would equal 16 amperes. The millivoltmeter used in measuring this potential was calibrated by checking the value of its readings for a given current flow against a standard potentiometer. Through a misunderstanding of where the millivoltmeter was to be connected, the calibration for Runs 5, 6, 7, 8, 9, and 10 was

$$1 \text{ mv} = 17.86 \text{ amperes} \quad .$$

The millivoltmeter had been at this time connected across two bolts on the shunt and was in parallel with a recording millivoltmeter. After Run 10, the recording millivoltmeter was disconnected because its response was extremely slow and so was of little value. At this time,

too, the millivoltmeter was connected properly across two wing nuts on the shunt. The calibration for Runs 11, 12, and 13 was then

$$1 \text{ mv} = 17.09 \text{ amperes}$$

with a constant value of 15.9 amperes to be subtracted in each calculation for the correct amperage.

The voltmeter measuring the arc voltage was calibrated with a vacuum tube voltmeter. This showed the error in reading the voltmeter to be ± 0.5 volt or a maximum error of about 2.5 percent.

Since it would be possible for part of the power supposedly feeding the arc to be lost in power inlet and outlet cables, high resistance connections, and in such things as the one-fourth inch diameter tungsten cathode of the plasma generator, the anode and cathode were shorted together and the current-voltage relationship determined. Within the accuracy of the voltmeter this showed a zero voltage drop across the equipment, up to a current level of 600 amperes. It could thus be concluded that all energy measured as going to the arc actually did so.

At one point in the investigation we had cause to doubt that the helium gas being used was exhibiting the same viscosity as reported in the literature. Using a capillary tube 0.001414 ft in diameter and 3.624 ft in

length, the viscosity of the gas at 75°F was determined to be 0.0488 lbm/ft-hr compared to the literature value of 0.0483 lbm/ft-hr, an excellent check. It became evident later in the work that the apparent discrepancy was caused by entrance effects associated with the geometry of our equipment.

The entrance effects mentioned above can be calculated by the methods of Langhaar¹ for the case where (1) the Reynolds numbers are less than 2100, (2) the fluid is entering a circular duct, and (3) the fluid starts from zero velocity. These assumptions fit the case described here, except for the fact of the cathode being so placed as to produce an incoming velocity profile different from that assumed by Langhaar. In the present case, the gas enters through an annular space, resulting in an initial velocity profile having zero velocity at the center and at the walls. In converting such a velocity profile to the characteristic parabolic profile of laminar flow, more momentum must be transferred to the walls than would be the case for the Langhaar assumptions; that is, greater pressure drops would be expected than those predicted.

Using the Langhaar method, knowing the length of tube over which the pressure drop is to be measured (three-inch length of one-eighth inch diameter or L/D equal to 24),

and assuming an L/D of 5 for the approach section of tube prior to the pressure tap (a value measured after the plasma generator had been operating for some hours), the friction factors were calculated for a range of Reynolds numbers. A plot of this calculation is shown as curve "A" in Figure 11. In Table 2 are shown the actual average pressure drops observed in this investigation for the case of no heat input. The H-123 data correspond to an L/D of 24, the value used in the Langhaar calculation. The results of calculating friction factor and Reynolds numbers for these data are plotted as curve "B" in Figure 11. The relationship for isothermal laminar flow,

$$f = 16/Re \quad ,$$

is plotted as curve "C". From this plot, it is seen that the data follow the same trend as that predicted by the Langhaar method. The chief deviation occurs in the observed data having friction factors higher than those calculated by the Langhaar method. This would appear to be due to the greater energy required in establishing a laminar profile when the gas enters through an annular space. A summary of these calculations is given in the Appendix.

Calculations similar to those described in the above paragraph can be carried out for any temperature. The calculations may also be made in terms of pressure drop rather

Figure 11. Friction factor versus Reynolds number for case of no heat input.

Table 2
Pressure Drop Data
(Averages--no heat input)

<u>w - lbm/hr</u>	<u>L/D</u>	<u>Δp</u>		<u>f_a</u>	<u>Re</u>
		<u>mm H₂O</u>	<u>lbf/ft²</u>		
0.213	11	3	0.62	0.0194	536
	19	6	1.23	0.0222	
	27	10	2.05	0.0260	
0.348	11	7	1.43	0.0167	874
	19	16	3.28	0.0221	
	27	22	4.51	0.0215	
0.496	11	13	2.66	0.0153	1240
	19	26	5.33	0.0177	
	27	38	7.79	0.0183	
0.642	11	21	4.30	0.0148	1610
	19	39	7.99	0.0158	
	27	54	11.1	0.0156	
0.785	11	29	5.94	0.0136	1970
	19	50	10.2	0.0135	
	27	76	15.6	0.0146	
0.925	11	40	8.20	0.0138	2320
	19	70	14.3	0.0138	
	27	105	21.5	0.0147	
1.429	11	75	15.4	0.0106	3600
	19	121	24.6	0.0098	
	27	195	40.0	0.0112	
1.842	11	108	22.1	0.0092	4640
	19	170	34.8	0.0083	
	27	273	55.9	0.0095	

than friction factor. The results of three of these calculations are shown in Table 3, the temperatures being 535°, 2760°, and 15,060°R. These calculations show that for mass flow rates less than 0.925 lbm/hr, the greatest error predicted by the Langhaar data is 30 percent; that is, the Langhaar data predict that the actual pressure drop will be a maximum of 30 percent higher than for true laminar flow. This is at a temperature of 535°R (75°F). Most of the work in this investigation was at a temperature of 2760°R or above. Here the maximum error is less than 10 percent with most error being less than 5 percent. Because the error in assuming laminar flow becomes so relatively small at these high temperatures, it was further assumed in this investigation that although the geometry used leads to higher pressure drops than would be predicted at low temperatures, the effect of the geometry would be very small in the high temperature work.

A heat balance was made on the equipment for three different flow rates. This balance was made possible by adding the fourth heat exchanger, as described under Experimental Methods. This last exchanger brought the temperature of the exit gas very near to that of the cooling water and to within a few degrees of 75°F, the reference temperature. The average loss of heat, as shown in Table 4, was 1700 Btu/hr.

Table 3
Percent Deviation of Actual Pressure Drop from that for True Laminar
Flow Due to Entrance Effects (based on Langhaar data)

Temperature, °R :	535				2760				15060			
Viscosity, lbm/ft hr :	0.0483				0.139				0.404			
Helium mass flow, lbm/hr	<u>Re</u>	<u>H-1</u>	<u>H-2</u>	<u>H-3</u>	<u>Re</u>	<u>H-1</u>	<u>H-2</u>	<u>H-3</u>	<u>Re</u>	<u>H-1</u>	<u>H-2</u>	<u>H-3</u>
0.213	540	4	2	1	190	0.4	0.0	0.0	64	0.0	0.0	0.0
0.348	880	9	4	3	310	1.6	0.4	0.1	105	0.0	0.0	0.0
0.496	1250	16	8	4	430	3	1.0	0.3	150	0.0	0.0	0.0
0.642	1610	21	12	7	560	5	1.7	0.7	190	0.3	0.0	0.0
0.785	1980	25	14	10	690	7	2.5	1.0	240	0.7	0.0	0.0
0.925	2340	29	19	13	810	9	3.5	1.5	280	1.5	0.0	0.0

Table 4
Heat Balances
(Calculated Data--Run 13)

<u>W,</u> <u>lbm/hr</u>	<u>Arc Input</u> <u>Btu/hr</u>	<u>Heat Transfer, Btu/hr</u>							<u>Heat Loss</u> <u>Btu/hr</u>
		<u>Anode</u>	<u>Cathode</u>	<u>H-1</u>	<u>H-2</u>	<u>H-3</u>	<u>H-4</u>	<u>Total</u>	
0.348	32,800	23,200	400	5400	400	400	600	30,400	2400
0.642	34,500	18,900	1300	8700	1600	1100	1900	33,500	1000
0.925	34,400	15,100	1200	8500	2400	1800	3800	32,800	1600

This figure was accepted as the true loss of heat for all runs having an arc energy input of 25,000 Btu/hr or higher. (Rough calculations indicated that the loss is entirely due to radiation from the cathode and anode.) For runs having an energy input of less than 25,000 Btu/hr, the loss was assumed to be 2500 Btu/hr. This second value was determined by a study of the heat lost and the heat leaving with the exit gas. These figures showed appreciably more heat going to exit gas and loss at low heat inputs than would reasonably be estimated from the heat transferred in the exchangers. A search of the literature⁸ revealed that in the use of carbon electrodes to produce an arc in air, there are at least three definite arc forms: (1) the low current arc below 130 amperes where the magnetic field produced by the flowing electrons is not sufficient to contract the arc; (2) the contracted arc existed from 130 to 400 amperes where the magnetic field produced is sufficiently strong to fully contract the arc; and (3) a winding arc above 400 amperes where the self-induced magnetic field is sufficiently strong to cause the arc to become unstable and begin winding around the anode. These changes do not occur at such precise current values as listed above; that is, the transitions may be gradual and in some cases may not occur at all, but the evidence is definite that different

arc forms do exist. It also appears reasonable that with the low current arc (where the arc originates over a much larger area of the cathode), the loss to radiation would be higher. The temperature of individual spots might be much higher at the higher current levels, but this radiation would be seen mainly by the cathode and anode themselves, since the high current arc tends to leave the cathode from its tip and to arc to some place inside the anode. This tendency to higher losses at low currents could be borne out qualitatively by observing the arc visually. Not always, but very often at low current levels, an exceedingly bright glow would appear, extending back for one-half inch along the cathode. An investigation of the data also revealed what appeared to be arc transitions. Three sets of these data are shown in Table 5. The data led us to conclude that with the geometry, gas, and materials of construction in our generator, an arc transition occurred at about 270 amperes and again at about 400 amperes. No change in heat loss figures appeared justified for currents above the second transition.

Method of Calculation

In the following discussion on the method of

Table 5
Possible Arc Transitions

Run No. :	5	12	10			
w, lbm/hr :	0.642	0.496	0.642			
Transition Current, amps :	200 - 267	267 - 392	392 - 455			
Transition Voltage, volts :	27.5 - 27.5	28.0 - 22.0	23.0 - 27.0			
Arc Input, Btu/hr :	more than 18,800	less than 25,100	more than 25,530	less than 29,420	more than 36,100	less than 35,700
Heat Transfer, Btu/hr						
Anode	10,750	15,000	15,230	20,380	22,690	25,130
Cathode	1,160	1,180	1,250	1,170	820	970
H-1	3,080	4,100	3,800	2,490	5,590	4,780
H-2	540	700	520	220	1,220	1,070
H-3	490	550	-	-	790	770
Gas and Loss	2,700	3,600	4,730	5,160	5,000	2,900
Pressure Drop, mm H ₂ O :	-62	-66	-2	-8	-94	-216
Helium Pressure entering Arc Region, mm Hg :	-	-	110	78	268	202

calculation, these assumptions are made:

- 1) Ionization of helium gas in the test section is small enough not to affect the transport properties of the gas.
- 2) Radiation from the helium gas in the test section is sufficiently small for its contribution to heat transfer to be negligible.
- 3) The Mach number is sufficiently less than 1 for compressibility effects not to predominate.

The reasoning that led to these assumptions is given under Discussion and Analysis.

As an illustration of the method of calculation to be discussed here, a sample calculation is given in the Appendix, the raw data for this calculation being given in Table 1.

Viscosity. Having made the assumption of negligible ionization, the viscosity of a monatomic gas may be estimated according to the theory of Chapman-Enskog and the use of the Leonard-Jones⁹ potentials. The equation for this calculation is

$$\mu = (4.81 \times 10^{-3}) \frac{(MT)^{\frac{1}{2}}}{\Omega} , \quad (1)$$

which shows the viscosity to be directly proportional to the square root of the product of the molecular weight and temperature and to be inversely proportional to the square of a force constant typical of the gas and to the temperature-dependent Leonard-Jones function for first approximation. The units of the equation are consistent with those of this work except for the Leonard-Jones function, which remains as reported by Condon and Odishaw.⁹ The equation checks well with actual data showing a maximum deviation at a temperature of 1960°R where the actual viscosity is 0.1141 lbm/ft-hr, approximately 4 percent higher than predicted by the above equation. No viscosity data were found in the literature at higher temperatures, making it necessary to depend on theory in estimating higher temperature viscosities. The fairly excellent check of the Chapman-Enskog theory with data at lower temperatures led us to assume its validity at higher temperatures. Viscosities could be calculated from the data reported by Condon and Odishaw,⁹ for temperatures up to 7400°R. In order to extend the results of this theory to temperatures higher than 7400°R (to the general upper temperature limit of 15,000°R encountered in the test section), an empirical relation was developed which fit the temperature-viscosity data calculated from the Chapman-Enskog theory to an

accuracy of better than 1 percent over the temperature range 368 to 7400°R. This relation is as follows:

$$\mu = (8.50 \times 10^{-4}) T^{0.643} \quad (2)$$

and was assumed to hold from 500° to 15,000°R.

Thermal Conductivity. Again according to the theory of Chapman-Enskog, the thermal conductivity of a monatomic gas is related to the viscosity in the following manner:

$$k = \frac{15}{4} \frac{R}{M} \mu \quad (3)$$

For helium, this equation becomes

$$k = 1.859 \mu \quad (4)$$

This equation was assumed to hold over the same temperature range as the equation for viscosity, from 500° to 15,000°R.

Prandtl Number. The Prandtl number is equal to $c_p \mu / k$. Since the specific heat of helium is constant at 1.242 Btu/°R-lbm for all temperatures up to the point where ionization becomes significant, and the ratio of viscosity to thermal conductivity is constant, the Prandtl number will be constant and equal to 0.668. Experimentally, the Prandtl number of helium at room

temperature is approximately 0.69. The calculations of Taylor and Kirchgessner⁵ show that the Prandtl number of helium varies from 0.68 to 0.93 as the temperature varies from 400° to 6000°R. Since the specific heat is constant, it must be concluded from their work that the ratio of viscosity to thermal conductivity varies with temperature; however, not actually knowing the extent of this variation at temperatures up to 15,000°R, it was decided that the theory of Chapman-Enskog would have to suffice.

Friction Factors for no Heat Input Case. For the isothermal flow with no heat input, the Fanning friction factor was calculated from the equation

$$f_a = \Delta p_f \frac{2 g_c}{4(L/D)\rho v^2} \quad (5)$$

where Δp_f is taken as $(p_1 - p_2)$. Actually, this equation has no restrictions and applies to nonisothermal as well as isothermal flow. The subscript "a" on the friction factor refers to the fact that it is an average over some distance of pipe. For incompressible flow, this distinction is unnecessary but for compressible flow the distinction is helpful since the value of the friction factor will change depending on the point in the duct at which it is measured. The effect is very minor for the no heat input

case; however, the subscript will be retained as being more descriptive.

Reynolds Numbers for no Heat Input Case. The Reynolds numbers (for the no heat input case) are simply calculated from

$$Re = \frac{Dv\rho}{\mu} \quad (6)$$

Energy Input to the Arc. The energy to the arc in Btu/hr is found by multiplying the product of the current and voltage inputs by 3.413.

Heat Transfer from Gas to Cooling Water. The heat picked up by the water in the anode, cathode, H-1, H-2, and H-3 is calculated from

$$Q = wc_p(T_2 - T_1) \quad (7)$$

where the quantities refer to water; T_1 and T_2 are the inlet and outlet temperatures of the water, respectively, for each unit.

Helium Temperatures. Since the heat picked up by the water must have come from the gas, an equation can be written,

$$Q = wc_p(T - T_R) \quad (8)$$

where Q refers to the sum of heats given up by the helium in cooling or being cooled to the reference temperature. T is the temperature of the helium at some point in the test section. Solving for T ,

$$T = \frac{Q}{wc_p} + T_R \quad (9)$$

The reference temperature was assumed to be 535°R throughout this work. This is an arbitrary base temperature which, for these calculations, was selected as room temperature for convenience.

Gas Side Heat Transfer Coefficients. The following equation holds for convective heat transfer

$$Q = h_a A_w \Delta T \quad (10)$$

where Q is the heat transferred through the wall of a heat exchanger, h_a is the average heat transfer coefficient, A_w is the inside area of the wall, and ΔT is the temperature difference between the gas and the wall. In this work, the temperature of the gas was taken to be the bulk temperature as defined by

$$T_b = \frac{T_1 + T_2}{2} \quad (11)$$

where T_1 and T_2 are the temperature of the gas entering and leaving a section, respectively.

Rough calculations of the inside wall temperature of the copper, starting from the water side, led us to the conclusion that this temperature never exceeded 240°F. This conclusion is based on the fact that incipient boiling did not occur. Thus, the water side of this copper wall was always below 212°F. Since the inside wall temperature is normally appreciably less than the bulk temperature of the gas, it was assumed to be constant at 240°F (700°R).

The equation for the calculation of the average heat transfer coefficient is thus

$$h_a = \frac{Q}{A_w(T_b - T_w)} = \frac{Q}{A_w\left(\frac{T_1 + T_2}{2} - 700\right)} \quad (12)$$

Nusselt Numbers. Nusselt numbers were evaluated using average heat transfer coefficients and thermal conductivities based on bulk temperatures,

$$Nu_b = \frac{h_a D}{k_b} \quad (13)$$

Graetz Numbers. Graetz numbers were calculated using thermal conductivities evaluated at the bulk temperature of the gas

$$Gz_b = \frac{w c_p D}{k_b L} \quad (14)$$

Friction Factors for Heat Input Case. In order to calculate an average friction factor for the case of heat input, it is necessary to sum the energy changes over a section of the heat exchanger. This equation in differential form is

$$w \left(\frac{g}{g_c} dZ + JdE + p dV_s + V_s dp + \frac{v_p dv_p}{g_c} \right) = J\delta Q + \delta W_o . \quad (15)$$

Since

$$JdE = JTdS - p dV_s \quad (16)$$

and

$$wJTdS = J\delta Q - w\delta F_f , \quad (17)$$

equation (15) becomes

$$w \left(\frac{g}{g_c} dZ + V_s dp + \frac{v_p dv_p}{g_c} \right) = W_o + w\delta F_f . \quad (18)$$

Then, since there is no change in elevation and no outside work involved, equation (18) becomes

$$V_s dp + \frac{v_p dv_p}{g_c} = \delta F_f . \quad (19)$$

Using the relationships

$$V_s = \frac{1}{\rho}$$

and

$$F_f = \frac{dp_f}{\rho} ,$$

equation (19) becomes

$$\frac{dp}{\rho} + \frac{v_p dv_p}{g_c} = \frac{dp_f}{\rho} \quad (20)$$

In order to integrate the first terms on both sides of the equation, it is necessary to know the manner in which the density varies with the two types of pressure. Since this relationship is only known for the cases of isothermal and adiabatic flow, the present case must be estimated. Using data from this experiment and making the isothermal and adiabatic calculations, led to the conclusion that an average density could be assumed according to the following relationship

$$\rho_a = \frac{2\rho_1\rho_2}{\rho_1 + \rho_2} \quad (21)$$

The second term of equation (20) must be integrated point by point over the actual velocity profile. The usual convention in this integration is to use average velocities and place an α in the denominator of the integrated term. The term, α , varies with the velocity profile, being one for laminar flow and nearly two for turbulent flow. The final integrated equation is

$$\frac{p_2 - p_1}{\rho_a} = \frac{(p_2 - p_1)_f}{\rho_a} - \frac{(v_2^2 - v_1^2)}{\alpha g_c} \quad (22)$$

Transposing signs and letting $\Delta p = p_1 - p_2$

$$\Delta p = \Delta p_f + \frac{f_a}{ag_c} (v_2^2 - v_1^2) \quad . \quad (23)$$

Using equation (21) and the relationships

$$\rho v = G \quad (24)$$

$$\rho = \frac{pM}{JRT} \quad (25)$$

$$R' = \frac{JR}{M} \quad (26)$$

we obtain from equation (23)

$$\Delta p_f = \Delta p - \left(\frac{2G^2 R'}{ag_c} \right) \left(\frac{T_2}{p_2} - \frac{T_1}{p_1} \right) \quad . \quad (27)$$

In the data of this investigation the frictional pressure drop and the velocity head change nearly cancel each other so that the assumption, $p_1 = p_2$, can be made with almost no error. Using this fact together with equations (5) and (21), and rearranging to solve for f_a , we obtain

$$f_a = \left(\frac{pg_c}{G^2 R'} \right) \left(\frac{1}{L/D} \right) \left(\frac{\Delta p}{T_1 + T_2} \right) + \left(\frac{2}{gL/D} \right) \left(\frac{T_1 - T_2}{T_1 + T_2} \right) \quad (28)$$

which is the equation used to calculate friction factors.

Reynolds Numbers for Heat Input Case. The Reynolds numbers were calculated based on viscosities evaluated at the bulk temperature

$$Re_b = \frac{Dv\rho}{\mu_b} \quad . \quad (29)$$

Method of Correlation

Heat Transfer Correlation. The heat transfer data were correlated in accordance with the dimensionless groups Graetz found to be important for laminar heat transfer. Evaluated at bulk temperatures, these groups are

$$\text{Nusselt number} = Nu_b = \frac{h_a D}{k_b} \quad (30)$$

$$\text{Graetz number} = Gz_b = \frac{w c_p}{k_b L} \quad (31)$$

$$= \frac{\pi D}{4L} \text{RePr} \quad (32)$$

The numerical values of these groups are shown in Table 6 of the Appendix and plotted in Figure 12. The lines shown on this plot are

- (A) For Graetz numbers under 4, the fluid outlet temperature is equal to the wall temperature and the exact Graetz solution is

$$\frac{h_a D}{k} = \frac{2}{\pi} \left(\frac{w c_p}{k L} \right) \quad (33)$$

- (B) For Graetz numbers between 4 and 10 there is a transition. For Graetz numbers above 10 there are a number of possibilities, three of which are given below. The first is the Graetz solution for parabolic flow,

Figure 12. Heat transfer correlations.

approximated by

$$\frac{h_a D}{k} = 1.75 \left(\frac{w c_p}{k L} \right)^{0.33} . \quad (34)$$

(C) The solution of Drew for rodlike flow is approximated by

$$\frac{h_a D}{k} = 1.75 \left(\frac{w c_p}{k L} \right)^{0.46} . \quad (35)$$

(D) The data of Cholette and Kroll as reported in McAdams³ for the steam heating of air is approximated by

$$\frac{h_a D}{k_b} = 1.5 \left(\frac{w c_p}{k_b L} \right)^{0.4} . \quad (36)$$

Friction Factor Correlation. For the case of no heat input, the usual plot of friction factor vs Reynolds number is made, the average data for this investigation having been shown in Figure 11. Because of entrance effects, the data do not fall on the usual

$$f_a = \frac{16}{Re} \quad (37)$$

line for laminar flow.

For the case of heat input, the data are plotted similarly except for the Reynolds number being based on the viscosity calculated at the bulk temperature. The data of Run 5 (0.642 lbm/hr) have been plotted in Figure 13

Figure 13. Friction factor versus Reynolds number (Helium mass flow rate equal to 0.642 lb_m/hr).

for an α of 1, corresponding to laminar flow and an α of 2 corresponding to turbulent flow. All of the data have been plotted for an α of 1, laminar flow, in Figure 14.

Analogies. Multiplying the numerator and denominator of the right side of equation (5) by ρ_a , remembering ρv is constant, and solving for Δp_f , we have

$$\Delta p_f = 4f_a \frac{LG^2}{D\rho_a^2 g_c} \quad (38)$$

Substituting this expression into equation (23), using the definition of ρ_a from equation (21), and solving for f_a we obtain

$$f_a = \left(\frac{2}{\alpha L/D}\right) \left(\frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}\right) + \frac{2g_c \rho_a \Delta p}{4(L/D)G^2} \quad (39)$$

Again making use of the perfect gas law in eliminating density from this equation and subtracting and adding $p_2 T_2$ to the numerator and denominator of the first term on the right, we have

$$f_a = \left(\frac{2}{\alpha L/D}\right) \left[\frac{p_2(T_1 - T_2) + T_2(p_2 - p_1)}{p_2(T_1 + T_2) + T_2(p_1 - p_2)} \right] + \frac{2g_c \rho_a \Delta p}{4(L/D)G^2} \quad (40)$$

If now we substitute

$$T_1 - T_2 = \frac{Q}{wc_p}$$

$$T_1 + T_2 = 2T_b$$

$$Q = h_a A_w (T_b - T_w) = h_a \pi DL (T_b - T_w)$$

Figure 14. Momentum transfer correlation.

$$w = A \rho v = \frac{\pi D^2}{4} \rho v$$

$$\Delta p = p_1 - p_2$$

and rearrange the equation, we obtain, for the cooling of a gas

$$f_a = \frac{4h_a}{\alpha \rho v c_p} \left(1 - \frac{T_w}{T_b}\right) \left[\frac{1 - \left(\frac{T_2}{T_1 - T_2}\right) \left(\frac{\Delta p}{p_2}\right)}{1 + \left(\frac{T_2}{T_1 + T_2}\right) \left(\frac{\Delta p}{p_2}\right)} \right] + \frac{2g_c \rho_a \Delta p}{4(L/D)G^2} \quad (41)$$

In the present investigation where Δp approaches zero for nearly all runs, the equation reduces to

$$f_a = \frac{4h_a}{\alpha \rho v c_p} \left(1 - \frac{T_w}{T_b}\right) \quad (42)$$

For turbulent flow with α equal to 2 and where $T_b \gg T_w$, this equation reduces to the familiar Reynolds analogy

$$\frac{f_a}{2} = \frac{h_a}{\rho v c_p} \quad (\text{turbulent flow}) \quad (43)$$

For laminar flow with α equal to 1 and where $T_b \gg T_w$, equation (42) becomes

$$\frac{f_a}{2} = \frac{2h_a}{\rho v c_p} \quad (\text{laminar flow}) \quad (44)$$

This then would be the correct form of the Reynolds analogy for laminar flow. Data for this analogy are tabulated in Table 7 (Appendix) and plotted in Figure 15.

Figure 15. Reynolds analogy (laminar flow) for heat and momentum transfer.

When α equals 1, equation (42) becomes

$$\frac{f_a}{2} = \frac{2h_a}{\rho v c_p} \left(1 - \frac{T_w}{T_b}\right) \quad (45)$$

which is a corrected form of the Reynolds analogy. The data for this analogy are tabulated in Table 7 (Appendix) and plotted in Figure 16.

For the case where a gas is being heated, equation (41) becomes

$$f_a = \frac{4h_a}{\alpha \rho v c_p} \left(\frac{T_w}{T_b} - 1\right) \left[\frac{1 + \left(\frac{T_2}{T_1 - T_2}\right) \left(\frac{\Delta p}{p_2}\right)}{1 + \left(\frac{T_2}{T_1 + T_2}\right) \left(\frac{\Delta p}{p_2}\right)} \right] + \frac{2g_c \rho_a \Delta p}{4(L/D)G^2} \quad (46)$$

If the terms in equation (40) are arranged to give a $(T_1 + T_2)$ term in the numerator and a $(T_1 - T_2)$ in the denominator rather than the other way around, the following equation can be developed in the same manner as equation (41),

$$f_a = \frac{\rho v c_p}{\alpha h_a (L/D)^2 \left(1 - \frac{T_w}{T_b}\right)} \left[\frac{1 - \left(\frac{T_2}{T_1 + T_2}\right) \left(\frac{p_1 + p_2}{p_2}\right)}{1 + \left(\frac{T_2}{T_1 - T_2}\right) \left(\frac{p_1 + p_2}{p_2}\right)} \right] + \frac{2g_c \rho_a \Delta p}{4(L/D)G^2} \quad (47)$$

Figure 16. Proposed analogy for heat and momentum transfer in laminar flow with gas cooling at high temperature differences.

When p_1 is approximately equal to p_2 , with T_2 approaching T_w , with T_w small compared to T_1 , and with α equal to 1, the following analogy is formed

$$\frac{f_a}{2} = \frac{(\rho v c_p)}{2h_a (L/D)^2} \quad (\text{laminar flow}) \quad . \quad (48)$$

If the two analogies of equations (44) and (48) are equated in their general form, that is, with α included, the following relationship is developed

$$\alpha f_a = \frac{4h_a}{\rho v c_p} = \frac{2}{L/D} \quad (49)$$

which states that within the limits of the assumptions, that is

- either 1) Δp approaches zero
- 2) T_1 is large compared to T_2 (reversed if gas is heated)
- 3) the perfect gas law holds

- or 1) T_1 is large enough compared to T_2 that the resulting ρ_a in the second term on the right side of the equation causes this second term to be small compared to the first (temperatures reversed if gas is heated)

2) the perfect gas law holds ,

f_a becomes dependent only on α , indicative of the flow regime, and the geometric term L/D . The relationship derived for predicting the heat transfer coefficient is

$$h_a = \frac{\rho v c_p}{2(L/D)} \quad . \quad (50)$$

This relationship is interesting for two reasons. First, it states that the heat transfer coefficient has become independent of the flow regime. Secondly, if the right side of the equation is multiplied by k/k , D/D , π/π , and $2/2$, and the equation rearranged, we have derived equation (33), the Graetz solution for the case where the outlet temperature of the fluid is the same as the wall temperature,

$$\frac{h_a D}{k} = \frac{2}{\pi} \left(\frac{w c_p}{k L} \right) \quad . \quad (33)$$

DISCUSSION AND ANALYSIS

Three topics will be discussed in this section: (1) the validity of the assumptions, (2) the sources of error, and (3) the experimental results.

It has been assumed that ionization in the test section is sufficiently low that it will not affect the transport properties of the fluid. In order to estimate the degree of ionization, the temperature must first be estimated. A plot of the temperature entering the test section is shown in Figure 17. There it is seen that the approximate maximum temperature entering the test section is 15,000°R. The resulting degree of ionization can be calculated from the Saha¹⁰ equation,

$$\frac{x^2}{1-x^2} p = (1.55)(10^{-4}) T^{2.5} e^{-\frac{uI}{KT}} \quad (51)$$

For a maximum temperature of 15,000°R, this calculation shows 0.0031 percent of the gas is ionized at equilibrium conditions. This value is sufficiently small that ionization need not be considered in the calculation of transport properties. This low value of ionization is directly due to helium's high ionization potential of 24.46 eV. This unusual behavior of helium was previously exhibited in the negligible contribution of higher electronic states to

Figure 17. Temperature at inlet of test section versus energy into the arc.

the heat capacity. At these temperatures, the electronic contribution would be of considerable importance with most other substances.

The error that might be incurred in the above assumption would be due to nonequilibrium conditions. Unless thermal equilibrium had been reached, the ionization level would be higher than that predicted by the calculation. This is because ionization in the anode section is much higher than at the entrance to the test section. Unless the recombination time of electrons and ions was instantaneous, they would be carried some distance downstream before recombination occurred. This error due to nonequilibrium conditions was assumed to be negligible.

This assumption was based on the results obtained with a 1,000 gauss magnetic field placed across the test section. During this test, recombination of ions and electrons was occurring throughout the test section as evidenced by visible radiation from the exit gas of the third heat exchanger. The magnetic field did affect both the rate of heat transfer and the pressure loss, heat transfer increasing and the pressure loss becoming more negative. It at first appeared that the magnetic field had directly affected the individual path of the charged particles in the flow tube. A more rigorous inspection

of all the data revealed, however, that heat transfer to the cathode and anode was equally affected. Our conclusion was that the magnetic field had affected the arc and in this way the heat and momentum transfer. This qualitative test did not prove the ionization level to be as low as predicted but indicated that it was not markedly higher than predicted.

Another factor to be considered is the extent to which radiative heat transfer may or may not play a part in total heat transfer. Since helium is a monatomic gas there are no rotational or vibrational energies involved, and the gas will not emit or absorb infrared radiation until ionization has become significant. The radiation emitted by the gas should then be principally from the electrons falling back into their orbitals, this energy being released in the visible and ultraviolet regions. For the purposes of this calculation, the radiation energy emitted was assumed to be that of the most prominent ultraviolet spectral line, $584.4 \text{ \AA}$. The equation

$$E_1 = h\nu \quad (52)$$

gives the energy released in a single recombination and

$$q_1 = \frac{1}{2} N x h \nu \quad (53)$$

gives the total release of energy from all recombinations per unit time. Assuming the maximum operating temperature of the test section to be 15,000°R, the total energy release from ion recombination appears to be less than 10 Btu/hr, an insignificant amount compared to the total energy transferred.

Prior to accepting the validity of the normal convective heat transfer relations, it was necessary to estimate the magnitude of compressibility effects in terms of the Mach numbers encountered. The general relationship for determining the velocity of sound in a gas is

$$c = 27.9 \left(\frac{g \gamma R T}{M} \right)^{\frac{1}{2}} \quad (54)$$

which reduces to

$$c = 5.17 \times 10^5 T^{\frac{1}{2}} \quad (55)$$

for helium, provided the ratio of the heat capacities is constant (constant at 1.66 for helium), an assumption made in this study. The reasoning behind this assumption is as follows: the critical temperature and pressure of helium are respectively 9.5°R and 4,800 lbf/ft² (2.26 atm). At an operating temperature of 15,000°R and an operating pressure of 2,124 lbf/ft², the reduced temperature will be 1,580 and the reduced pressure 0.442. Under these

conditions, the gas should closely follow the behavior of an ideal gas, the compressibility factor being near 1.

For an ideal gas

$$C_p = C_v + R \quad (56)$$

and since both C_p and R are constant, C_v must be constant and thus the ratio must be constant.

For a given mass flow rate, the local average velocity is

$$v = v_R \left(\frac{T}{T_R} \right) \quad (57)$$

this expression being based on the assumption that the ideal gas law holds and the pressure is constant at 2,124 lbf/ft². Remembering that the Mach number is the ratio of the local velocity to the velocity of sound, we obtain by combining equations (55) and (57):

$$T = 7.67 \times 10^{16} \left(\frac{M_c}{v_R} \right)^2 \quad (58)$$

which for a Mach number of 1 becomes

$$T = \frac{7.67 \times 10^{16}}{v_R^2} \quad (59)$$

Thus a temperature can be calculated based on the local average velocity of the gas at the reference temperature (535°R). This velocity is of course constant for each mass

flow rate, the temperature calculated being the lower limit of temperature required to produce Mach 1. The results of the calculation are as follows:

<u>w - lbm/hr</u>	<u>T(°R) for Mach 1</u>
0.213	1,290,000
0.348	484,000
0.496	239,000
0.642	142,000
0.785	95,100
0.925	68,200
1.263	36,500

These data illustrate that the operating temperature must be at least twice the operating temperature of the present study (15,000°R) before sonic velocities are reached. The closest approach is at 1.263 lbm/hr where a 15,000°R operating temperature produces a Mach number of 0.41.

One factor which would cause confusion, unless explained, concerns the use of lower L/D values for the no heat input case than for the heat input case. For the three exchangers connected in series, the distance from the pressure tap to the atmosphere is equivalent to an L/D of 24. The total length of the exchangers is equivalent to an L/D of 27. Thus with no heat transferred, an L/D of 24 is used and when heat is transferred, an L/D of 27 is used, since heat is transferred over the whole length and not just over that length downstream from the pressure tap. The magnitude of this error should be quite small since

the frictional pressure loss and the velocity head change tend to cancel, resulting in measured pressure losses of nearly zero value.

The most serious source of error resulted from our inability to determine the temperatures accurately. The temperatures, as stated, were not measured directly but calculated from heat balances. The chief factor contributing to the uncertainty in the temperature values was the error involved in obtaining the correct voltage. If the arc was in a period of stable operation, the voltage could be read to an accuracy of ± 0.5 volt. Depending on the current level, the following error could be expected in the calculated temperature levels

	$\pm$ °F			
w - lbm/hr:	<u>0.213</u>	<u>0.496</u>	<u>0.785</u>	<u>1.263</u>
<u>Amperes</u>				
100	645	277	176	109
200	1290	522	351	217
300	1930	829	527	326
400	2570	1110	703	435
500	3220	1380	879	543

The error in temperature level cancels when a straightforward temperature difference is involved ($T_1 - T_2$), but causes appreciable error if the sum of temperatures is involved, for instance in ($T_b - T_w$) where T_b involves the sum of T_1 and T_2 . Since both the heat transfer coefficient and the friction factor are inversely proportional to this

error, they will both change in the same direction as this error changes, their relation to each other remaining in the same proportion. Their absolute values may be considerably in error, particularly at low mass flow rates and high current levels, where the error may have reached 50 percent. At high mass flow rates and current levels, this error becomes less than 10 percent. The negative extremes of this error (where it would imply negative temperatures) have not been included in the correlations of this study. These amounted to 14 out of a total of 94 data points. If it were assumed that an equal number of points showed less energy taken out of the gas, than would be expected for the arc energy input, there would be 14 points calculated as having markedly higher temperatures than would actually be the case. Thus we might expect to find at least 14 heat transfer coefficients and friction factors having values on the order of 50 percent less than would be the actual case. The lesser errors in temperature would simply tend to cause scatter of the points.

An error that exists in reading both the voltage and current values stems from the sometimes unsteady nature of the arc. Although the voltage and current may have originally been set at some definite values, the arc may alter its characteristics while temperatures were being read

and return to its original values by the time the values were again checked. The temperatures would then be recorded for some energy input other than that recorded. Fortunately, the current and voltage, for any single setting of the power supply, tend to vary in such a way that the power input varies only very slightly. The error due to this cause should be small.

Another source of error due to the sometimes uncertain nature of the arc stems from the radiation loss. As previously stated, the radiation loss for the low current arc was assumed to be 2,500 Btu/hr and for the contracted or winding arc operation, 1,700 Btu/hr. These values are, we hope, approximately correct, but uncertainty does arise about when the arc is actually operating under a particular regime. The error incurred in this choice should be no more than 10 percent, however.

Small errors were also incurred by assuming that--

- 1) The wall temperature was constant throughout its length at 240°F. This temperature exists probably only at the inlet to the test section under the higher heat input rates. The outlet wall temperature was undoubtedly quite near to the water temperature, a value near 70°F. This discrepancy of 170° is quite minor, however, compared to

the over-all temperature differences which always exceeded 2000°.

2) The inlet helium temperature was 75°F. At times there was at least a 10° variation in this temperature, but here again, the error is minor.

3) The pressure of the helium gas in the rotameter was always at 14.8 psig. With an initial setting of 14.8 psig, the pressure would drop as low as 13.8 psig as the helium flow rate was increased. This error amounts to less than 2 percent.

A source of error, the exact magnitude of which we could not estimate, was due to the condensation of zinc and tungsten vapor on the inside walls of the flow tube. This buildup of powder, although quite thin, must have tended to decrease heat transfer rates and increase pressure losses. A similar effect was at times produced by small pieces of molten zinc or tungsten solidifying in the flow tube. This could usually be detected by abrupt changes in pressure loss or heat transfer rates. When these sudden changes occurred, the equipment was shut down and the tube cleaned. For the most part, however, these effects need not be considered since the powder film accumulated quite slowly and the molten metal

had little chance to lodge in the flow tube at the usual velocities encountered.

A problem concerned not with error but with reproducibility stems from where the arc maintains its path onto the anode. Since the cathode was fed toward the anode to maintain a one-sixteenth inch spacing, it soon developed that the cathode extended into the anode because the anode eroded away gradually. With the cathode extending into the anode, the arc could travel either a radial path from the cathode to the anode, or could arc to the inside of the anode outlet tube. If the arc struck radially to the anode, the gas would be less effectively heated on a bulk basis, since much of the gas would bypass the arc, and more heat would be lost to the anode cooling water. If the arc struck to the inside of the anode outlet tube, the gas would be very effectively heated and less heat would be lost to the anode cooling water. Thus, for a constant arc input energy, the temperature of the gas entering the test section might vary considerably. As stated, this does not produce error in the results but makes it quite difficult to reproduce results. Also leading to nonreproducibility is the changing geometry due to gradual erosion of the anode; that is, as the anode walls get thinner, more heat is transferred through the walls to the cooling water. It might in fact be stated

that no two runs will ever be exactly alike since even though the rate of erosion varies, the geometry is never again exactly the same once an arc has been struck.

In analyzing the results of this study, the most striking generality would be the apparently good correlations achieved in the face of so much potential error. Starting with Figure 11, it is seen that although the isothermal pressure loss data, as reflected in a friction factor vs Reynolds number plot, do not follow the laminar ($f_a = 16/Re$) line because of entrance effects, it does follow the trends indicated by the Langhaar method. It differs from data calculated by the Langhaar method in that the friction factors are generally higher as would be expected for the geometry of the present study; i.e., for an annular entrance area. The data, at the expected Reynolds number of approximately 2,100, indicate a transition from laminar development of velocity profile to a turbulent development. The transition is not great because entrance effects tend to mask the change.

Figure 12 is the correlation of Nusselt and Graetz numbers. For a given Graetz number, the data would indicate that the Nusselt numbers are 50 percent lower than would be predicted by the asymptotic Graetz solution of equation (33). The more likely conclusion is that, with

temperatures known more accurately, the data would fall well within what might be expected as normal scatter around the Graetz solution for both the asymptotic and parabolic cases. Except for three points which are above the equation for rodlike flow, the data appear to be taking the same turn as indicated by the work of Cholette; that is, undergoing a transition at Graetz numbers of about 4.5 (more nearly 9 in the present work) to the solution for a parabolic or rodlike velocity profile. This might indicate that the calculated thermal conductivities are too low. The reason the data points at the lower left are lower than would be predicted by the asymptotic solution of Graetz is in all probability due to the temperature levels being calculated as higher than is actually the case.

The plot of friction factor vs Reynolds numbers for heat input at the single mass flow rate of 0.642 lbm/hr (Figure 13) indicates two points:

- 1) Assuming laminar flow, $\alpha = 1$, the data with heat input coincides significantly better with the corresponding isothermal data than if turbulent flow is assumed, $\alpha = 2$. This tends to verify our assumptions based on calculations from the Langhaar data, that the flow regime at high temperatures is essentially laminar after an approach section of 5 L/D.

2) The value of L/D has a definite effect on the slope of friction factor vs Reynolds number line. Stated another way, the greater the L/D , the lower the friction factor. This is theoretically confirmed by equation (49) which states that under the limits of the assumptions, the friction factor is inversely proportional to (L/D) . A point to remember in interpreting both Figure 13 (being discussed here) and Figure 14 is that although the flow appears to be much more laminar than turbulent, an α of one is not exactly correct since heat transfer causes deformation of the laminar velocity profile. It would appear that the effect is not too great.

The momentum transfer correlation of Figure 14 (friction factor vs Reynolds number plot) shows a rather wide scatter of points around the isothermal laminar line of $f_a = 16/Re$. This scatter appears to be due to three effects:

1) The effect of L/D , as demonstrated in Figure 13, which produces a different slope of friction factor vs Reynolds number line for each L/D value. This effect would be completely masked when comparing high L/D values, but when comparing small L/D values, the effect is quite discernible.

2) The entrance effects in the case of the no heat input causing the points to lie on a curve which runs at an angle to the isothermal laminar line.

3) The effect produced by the error in calculated temperatures. The three exceptionally low values are undoubtedly caused by this error.

The plot in Figure 14 shows that (with no better information available) the use of the isothermal laminar line is adequate for engineering estimates of pressure losses at high heat transfer rates and small L/D values.

Some remarks are in order concerning the raw data from which the friction factors were calculated. As stated previously, many of the pressure loss data points were negative for the heat input cases. This appears contrary to reason, that is, that a gas will flow from a lower to a higher pressure. One possible explanation would be that a vena contracta was formed as the gas flowed from the converging anode section. This explanation does not appear entirely acceptable since a vena contracta normally occurs at a point less than one pipe diameter downstream from the orifice. Pressure recovery is generally complete at some point prior to eight pipe diameters downstream. With the pressure tap in this study

at a point five pipe diameters downstream from the convergent section outlet, the effects of a vena contracta would be expected to be minor if present at all.

The explanation which we have found most reasonable is that the momentum of the gas, with heat input, is large enough that the gas will flow through the tube despite the opposing pressure gradients which sometimes occurred. This explanation is supported by the fact that with the fourth heat exchanger connected, negative pressure losses did not occur. Thus negative pressure losses would appear to be phenomena which tend to occur only in short lengths of duct at extreme heat transfer rates. Although no explanation was given, similar negative pressure gradients were recently reported by Talbot.¹¹

Data for the laminar flow Reynolds analogy are plotted in Figure 15. Essentially 100 percent of the points fall down and to the right of the 45° line which represents this analogy. In Figure 16, the same data are plotted with the inclusion of the temperature correction factor developed in equation (45). It is seen that a marked improvement has been made in the correlation by using this factor.

That such corrections may be required to obtain rigorous analogies can be demonstrated by deriving the

heat and momentum analogy for molecular transport.

For momentum diffusivity

$$\frac{\mu}{\rho} = \frac{Df_a v \gamma_m}{8} \quad (60)$$

For heat diffusivity

$$\frac{k}{\rho c_p} = \frac{Dh_a \gamma_h}{4 \rho c_p} \quad (61)$$

where the γ terms refer to the ratio of the difference between the average concentration of the transport property and its value at the wall, to the difference between the concentration of the property at the center and at the wall.¹² A ratio of γ_m to γ_h would provide a measure of the difference in velocity and temperature profiles. Such differences have been discussed by Deissler.²

The ratio of momentum diffusivity to heat diffusivity is the Prandtl number. Taking this ratio and rearranging, we obtain

$$\frac{f_a}{2} = \frac{h_a}{\rho v c_p} \left(\frac{\gamma_h}{\gamma_m} \text{Pr} \right) \quad (62)$$

Since this equation refers only to laminar flow where molecular transport is controlling, it corresponds to equation (45), the inference being

$$\frac{\gamma_h}{\gamma_m} \text{Pr} = 2 \left(1 - \frac{T_w}{T_b} \right) \quad (63)$$

For a Prandtl number of one, this would indicate that

$$\gamma_m = 1/2 \quad (64)$$

$$\gamma_h = \left(1 - \frac{T_w}{T_b}\right) \quad (65)$$

which is not an unreasonable interpretation of the γ terms. Actually, for the system of interest in this investigation, it is not possible to establish exactly the proper form of γ_m and γ_h from their definitions. This occurs because of the extreme variations in density in the radial direction.

CONCLUSIONS

Equipment

Graphite plasma generators, without forced cooling, can be built which have operating times on the order of minutes. Failure of these generators is due to excessive heating and vaporization of the electrodes.

In order to construct plasma generators capable of a long operating life it is necessary to have

- 1) Forced cooling of cathode and anode,
- 2) Electrodes of high refractoriness, thermal conductivity, and electrical conductivity, such as tungsten,
- 3) Heat transfer surfaces of high thermal conductivity such as copper, and
- 4) A working gas which does not react chemically to any appreciable extent with the electrode material.

A plasma generator, using these principles, can be built for less than two hundred dollars, including labor, capable of delivering 12 kw (40,000 Btu/hr) to the working gas. Bulk temperatures of over 50,000°R are achieved in the anode section. Operating times of 40 hours can be obtained before anode renewal is required.

Single tube, water-cooled, heat exchangers of one inch length and one-eighth inch inside diameter are capable of transferring over 10,000 Btu/hr at average gas inlet temperatures in excess of 15,000°R. At inlet water temperatures of 70°F and water rates of 0.5 GPM, the outlet water temperatures do not exceed 120°F. Heat fluxes in these exchangers exceed 2,500,000 Btu/hr-ft² at arc input energies of 40,000 Btu/hr. Residence times of the gas in the three heat exchange units are as low as 0.1 milliseconds.

Experimental

The experimental conclusions are based on the cooling of helium gas from high temperatures under near laminar conditions. The single exception is the friction factor vs Reynolds number curve for no heat input conditions. Here it was established that the Langhaar method of predicting pressure losses for the inlet sections of circular ducts, was adequate for predicting trends when the gas entered through an annular space but did not predict the magnitude. This is to be expected because of the greater energy required in establishing this laminar flow.

At temperatures above 2760°R, an approach section of five L/D is adequate to insure a less than ten percent deviation from true laminar flow in the subsequent section of eleven L/D.

Negative pressure losses (pressure increases) are experienced in the rapid cooling of a gas in short sections of pipe. The increase in pressure occurs because the decrease in velocity head is greater than the frictional pressure losses.

The Graetz solution for heat conduction in laminar flow appears quite adequate for the estimation of heat transfer coefficients at bulk temperatures up to 10,000°R. This temperature corresponds to an inlet temperature of 15,000°R and a ΔT approaching 10,000°R.

The isothermal laminar line, $f_g = 16/Re$, also appears adequate under the conditions of short pipe sections and high heat transfer rates, but is not exact. The deviation is due to an L/D effect, this effect being the result of the fact that in laminar flow, as the outlet gas temperature becomes negligible compared to the inlet gas temperature, and the pressure loss approaches zero, the friction factor becomes identically equal to $(L/D)^{-1}$.

Again under the conditions of short pipe sections and high heat transfer rates, the Reynolds analogy for laminar flow holds quite well but can be improved by the inclusion of a temperature correction term,

$$\left(1 - \frac{T_w}{T_b}\right) .$$

Although the data of this investigation do not prove the following, they do indicate that ionization of helium is not an important factor in the calculation of transport properties up to a temperature of 15,000°R. The data also indicate that the theory of Chapman and Enskog in conjunction with the use of the Leonard-Jones potentials to the first approximation, is adequate for the prediction of viscosity and thermal conductivity to bulk temperatures of 10,000°R.

Theoretical

A theoretical analogy can be derived for the case where the decrease in velocity head, by virtue of cooling, exactly balances the frictional pressure drop. This equation is

$$\frac{f_a}{2} = \frac{2h_a}{\alpha \rho v c_p} \left(1 - \frac{T_w}{T_b}\right) .$$

With the additional restriction of the outlet gas temperature being negligibly small compared to the inlet gas temperature (T_w then of necessity being negligible compared to T_b), the equation reduces to

$$\frac{f_a}{2} = \frac{2h_a}{\alpha \rho v c_p} .$$

This equation represents a family of analogies depending on the value of α , reducing further to the Reynolds analogy

for turbulent flow when α is equal to two. It should be emphasized that α may, theoretically, assume a value anywhere from zero to two depending on the deformation of the velocity profile through heating, cooling or other means. For the case of laminar flow, α is one, which yields

$$\frac{f_a}{2} = \frac{2h_a}{\rho v c_p} \quad .$$

The above analogies hold quite well even though the outlet gas temperature may not be negligibly small compared to the inlet gas temperature. If the outlet gas temperature is truly negligibly small compared to the inlet temperature and the pressure loss again approaches zero, the following analogy holds

$$\frac{f_a}{2} = \frac{\rho v c_p}{2H_a(L/D)^2} \quad .$$

Equating the two analogies leads to the relation

$$\alpha f_a = \frac{4h_a}{\rho v c_p} = \frac{2}{L/D} \quad .$$

Thus for extreme temperature differences in short sections of line when cooling a gas, a simple means is available for approximation of both the friction factor and the heat transfer coefficient

$$f_a = \frac{2}{\alpha(L/D)}$$

$$h_a = \frac{\rho v c_p}{2(L/D)} \quad .$$

RECOMMENDATIONS

1) Experimental work should be conducted at high temperatures to verify the theory of Chapman-Enskog in the prediction of transport properties.

2) An investigation should be conducted to determine the conditions of current, working gas, geometry, and electrode composition under which arc transitions take place. This investigation could include the calculation and experimental verification of radiative losses from the cathode and anode. In respect to the cathode, this would be a particularly interesting problem in that the geometry is simple, heat originates at the tip of the cathode at the presumably known temperature of 6560°R (the melting point of tungsten), and in addition to the radiative loss, there is the simultaneous transfer of heat by conduction and convection, convection being small compared to the other two.

3) Although the relationship

$$\alpha f_a = \frac{4h_a}{\rho v c_p} = \frac{2}{L/D}$$

appears to approach theoretical exactness for the conditions of the pressure loss being zero and the gas outlet

temperature being negligible compared to the gas inlet temperature, it would be very interesting to check this experimentally. This could be done through use of liquid nitrogen as the cooling medium (boiling point equal to 321°R), although water would probably be adequate. Assuming a wall temperature of 700°R (approximately correct for water), an inlet gas temperature of $27,700^{\circ}\text{R}$ is required to reduce the temperature correction term to 0.95, this provided the outlet temperature is essentially equal to the wall temperature. The data of this investigation would be used to determine the L/D and flow rate required to achieve the proper gas inlet and outlet temperatures and to approach the zero pressure loss required.

APPENDIX A

Effect of Magnetic Field on Heat Transfer

EFFECT OF MAGNETIC FIELD ON HEAT TRANSFER

The following data were inadvertently recorded while a 1,000-gauss magnetic field was in position across the three heat exchangers. The magnet had $1\frac{1}{2}$ -inch diameter pole faces and a 3-inch gap. The magnet had been positioned across the flow tube to determine its effect on the pressure loss. The data reported here are thus not complete since the object had not been to collect heat transfer data but the partial results seem important enough that they should still be reported.

		Water Temperatures (°F)		
		inlet	outlet	
			<u>magnet off</u>	<u>magnet on</u>
Anode		71.2	101.5	112.0
Cathode		70.5	76.6	87.5
H-1		70.2	109.5	121.8
H-2		71.5	80.5	93.2
H-3		71.2	77.7	89.5

		ΔT (°F)		Heat Transfer (Btu/hr)	
				<u>magnet off</u>	<u>magnet on</u>
	Water (lbm/hr)	<u>magnet off</u>	<u>magnet on</u>		
Anode	673	30.3	40.8	20,390	27,460
Cathode	219	6.1	17.0	1,340	3,720
H-1	215	39.3	51.6	8,450	11,090
H-2	196	9.0	22.0	1,760	4,310
H-3	231	6.5	18.0	1,500	4,160
Total				33,440	50,740

With the magnet off, the heat input to the arc was 35,050 Btu/hr. This figure corresponds well with the heat transferred with the magnet off but is considerably low for the heat transferred with the magnet on. Thus it would appear that the main effect of the magnetic field was to increase the resistance to the flow of electrons from the cathode to the anode resulting in much higher arc input energies for a given setting of the power supply.

APPENDIX B

Scaling Equipment to Measure the Effect of a Magnetic Field on Flowing Electrolytes

SCALING EQUIPMENT TO MEASURE THE EFFECT OF A MAGNETIC FIELD ON FLOWING ELECTROLYTES

Consider the flow of a conducting fluid between two parallel plates and through a perpendicular homogeneous magnetic field. With unidirectional laminar flow, the equation for predicting the pressure loss is

$$p = \frac{LV\mu}{a^2} \left(\frac{M^2 \tanh M}{M - \tanh M} \right) \quad (1)$$

where

- p = pressure loss in newtons/meter²
- L = length in meters
- V = velocity in meters/second
- μ = viscosity in kg/meter-second
- a = half-width of channel in meters
- M = Hartman number and is dimensionless .

The Hartman number is equal to

$$M = \mu_e H_0 a \left(\frac{\sigma}{\mu} \right)^{\frac{1}{2}} \quad (2)$$

where

- $\mu_e = 4\pi \times 10^{-7}$ webers/meter-ampere
- $H_0 = (H_0 \text{ in gauss}) / (4\pi \times 10^{-3} \text{ gauss/ampere/meter})$
 $= 79.6(H_0 \text{ in gauss}) \text{ ampere/meter}$
 $= \text{magnetic field in ampere/meter}$
- $\sigma = \text{electrical conductivity in (ohm-meter)}^{-1}$

The foregoing equations are found in:

Hartman, J., I. Theory of the laminar flow of an electrically conductive fluid in a homogeneous magnetic field, K. (Konlige) Danske Videnskab. Selskob. Mat.-fys. Medd 15, No. 6 (1937);

Hartmann, J., and Lazarus, F., II. Experimental investigations of the flow of mercury in a homogeneous magnetic field., *ibid.*, No. 7, 45 pp. (in English).

Whereas Hartman and Lazarus were concerned with the flow of mercury, we will concern ourselves with the flow of electrolytes. The calculations will be specifically for aqueous sodium chloride solutions since it represents a more conservative case in that it has an electrical conductivity somewhat less than aqueous solutions of hydrogen chloride for instance. Also, in these calculations, the depth of the channel will be 17.46 times the width, as recommended by Hartman and Lazarus for the neglecting of end effects.

Now, if we have a micromanometer which can measure a two-inch fluid differential to ± 0.0002 inch, this is equivalent to a ± 0.000006 psi change when the manometer fluid has a density of 0.810 gm/cc at 20°C. Allowing this variation to be one percent of the total variation

due to a magnetic field, the magnetic field must produce a 0.0006 psi change (4.14 newtons/meter²).

Assuming the channel to have a half-width of 1 mm and that the pressure loss will be measured over a length of 10 cm, we will first calculate the pressure loss for the case where there is no magnetic action. With no magnetic field, H_0 is equal to zero, M is equal to zero, and the last term in equation (1) is equal to 3. Thus, we have from equation (1)

$$\begin{aligned} p &= (10/100)(V)(\mu)(3) / (1/1000)^2 \\ &= 3 \times 10^5 V\mu. \end{aligned}$$

For the flow to be laminar, the Reynolds number must be less than about 2,000. Thus

$$\frac{DVe}{\mu} = \text{Re} = 2,000$$

where

$$\begin{aligned} D &= (4)(\text{hydraulic radius}) = (4)(\text{area}) / \\ &\hspace{15em} (\text{wetted perimeter}) \\ &= \frac{(4)(2a)(2b)}{2a + 2b}, \text{ } b \text{ being equal to } 17.46a \\ &= 7.57a = 0.00757 \text{ meter} \end{aligned}$$

$$\begin{aligned} \rho &= 1194 \text{ kg/meter}^3 \text{ for 26 weight percent} \\ &\text{NaCl at } 20^\circ\text{C.} \end{aligned}$$

$$\mu = 2.5 \times 10^{-3} \text{ kg/meter-second for } \text{NaCl at } 20^{\circ}\text{C.}$$

Thus, the velocity is calculated to be 0.553 meters/second and the pressure loss with no magnetic field is

$$p = 414 \text{ newtons/meter}^2 \text{ (no magnetic field) .}$$

Knowing that the last term of equation (1) is equal to three with no magnetic field, the ratio of equation (1) for no magnetic field can be ratioed to equation (1) for a magnetic field to yield

$$\left(\frac{M^2 \tanh M}{M - \tanh M} \right) = 3 \left(\frac{p}{p_m} \right) . \quad (3)$$

Since we had previously calculated that the magnetic field must cause a 4.14 newton/meter² pressure loss, we find

$$\begin{aligned} \left(\frac{M^2 \tanh M}{M - \tanh M} \right) &= 3(414) / (414 + 4.14) = 3.0300 \\ &= 3 + \frac{M^2}{5} \text{ for } M \text{ less than one .} \end{aligned}$$

Thus M is equal to 0.388.

Then, knowing that the electrical conductivity of 26 weight percent NaCl at 20°C is 22.59 (ohm-meter)⁻¹, the strength of the magnetic field can be calculated from equation (2). It was found that

$$H_0 = 40,800 \text{ gauss .}$$

To summarize the above, we have calculated that a 40,000 gauss magnet with a 10 cm pole face is required to produce a one percent increase in pressure loss. Although a more practical compromise might be reached by varying the geometry, the many calculations of the writer indicate that the above calculation offers the best chance of accurately measuring the pressure change. If some device more sensitive than a micromanometer were available, the required magnet size might be reduced to more reasonable levels. Such devices might include a capacitance meter to measure level changes or some current measuring device to sense the change in velocity as the pressure loss increases due to the magnetic field.

APPENDIX C

Friction Factors with no Heat Input

FRICTION FACTORS WITH NO HEAT INPUT

Langhaar Calculation

<u>Re</u>	<u>$\frac{L_1/D}{Re}$</u>	<u>$\frac{L_2/D}{Re}$</u>	<u>$f_a(L_1/D)$</u>	<u>$f_a(L_2/D)$</u>	<u>$f_a(\frac{L_2 - L_1}{D})$</u>	<u>f_a</u>
200	0.025	0.145	0.65	2.2	1.5	0.063
400	0.125	0.0725	0.40	1.3	0.9	0.038
800	0.00625	0.0363	0.29	0.86	0.57	0.024
1600	0.003125	0.0181	0.20	0.52	0.32	0.013

where

$$\begin{aligned} L_1/D &= 5 \\ L_2/D &= 29 \\ \frac{L_2 - L_1}{D} &= 24 \end{aligned}$$

Calculation on Observed Data

<u>w-lbm/hr</u>	<u>Δp</u>		<u>v-ft/hr</u>	<u>Re</u>	<u>f_a</u>
	<u>mm H₂O</u>	<u>lbf/ft²</u>			
0.213	10	2.05	2.44×10^5	536	0.0293
0.348	22	4.51	3.98×10^5	874	0.0242
0.496	38	7.79	5.76×10^5	1240	0.0206
0.642	54	11.1	7.34×10^5	1610	0.0176
0.785	76	15.6	8.98×10^5	1970	0.0164
0.925	105	21.5	1.05×10^6	2320	0.0165
1.428	195	40.0	1.64×10^6	3600	0.0126
1.841	273	55.9	2.11×10^6	4640	0.0107

where

$$f = \Delta p \left(\frac{2g_c}{4L/D \rho v^2} \right)$$

$$\begin{aligned} L/D &= 24 \\ &= 1.02 \times 10^{-2} \text{ lbm/ft}^3 \end{aligned}$$

APPENDIX D

Sample Calculation

SAMPLE CALCULATION

I. No Heat Input Case

A. Friction Factor vs Reynolds Number Calculation

1. Friction Factor

$$f_a = \Delta p_f \frac{2g_c}{4L/D \rho v^2}$$

Δp_f is assumed equal to Δp since there is no change in pressure drop due to position, and a negligibly small change due to velocity head changes. Entrance and exit effects tend to cancel each other and may be neglected.

$$g_c = 4.17 \times 10^8 \frac{\text{lbm-ft}}{\text{lbf-ft}^2}$$

$$L = 3.37 \text{ inches} = 0.2812 \text{ ft}$$

$$D = 1/8 \text{ inch} = 1.042 \times 10^{-4} \text{ ft}$$

$$L/D = 0.2812 / 1.042 \times 10^{-4} = 27$$

$$\rho = 1.02 \times 10^{-2} \text{ lbm/ft}^3 \text{ at } 75^\circ\text{F}$$

$$v = w/A\rho$$

$$w = 0.642 \text{ lbm/hr from Rotameter reading of 10 at } 75^\circ\text{F and } 14.7 \text{ psig}$$

$$A = \frac{\pi D^2}{4} = \left(\frac{3.1416}{4}\right)(1.042 \times 10^{-2})^2 = 8.52 \times 10^{-5} \text{ ft}^2$$

$$v = 0.642 / (8.52 \times 10^{-5})(1.02 \times 10^{-2}) = 7.34 \times 10^5 \text{ ft/hr}$$

$$\Delta p_f = 2(544 - 521) = 46 \text{ mm H}_2\text{O} = 9.39 \text{ lbf/ft}^2$$

$$f_a = (9.39) \frac{(2)(4.17 \times 10^8)}{(4)(27)(1.02 \times 10^{-2})(7.34 \times 10^5)^2} = 0.0132$$

2. Reynolds Number Calculation

$$Re = \frac{Dve}{\mu}$$

$$\mu = 0.0483 \text{ lbm/ft-hr at } 75^\circ\text{F}$$

$$Re = \frac{(1.042 \times 10^{-2})(7.34 \times 10^5)(1.02 \times 10^{-2})}{0.0483} = 1610$$

The values of friction factor and Reynolds number calculated here are for an actual data point and do not check exactly the averages plotted.

II. Heat Input Case

A. Heat Transfer Calculation

1. Energy to Arc

The equation for conversion of mv to amperes is

$$\text{amperes} = 17.8 (\text{mv reading})$$

$$\text{amperes} = (17.8)(22.0) = 392$$

$$\text{Power input} = (392)(27.0) = 10.58 \text{ kw} = 36,100 \text{ Btu/hr.}$$

2. Water Flow Rates

	<u>Time for 2 liters, seconds</u>	<u>lbm/hr</u>
Anode	25.3	625
Cathode	77.8	203
H-1	77.0	205
H-2	85.4	185
H-3	74.7	211

3. Heat Transfer to Water from each Unit

$$Q = wc_p (T_2' - T_1) = wc_p \Delta T$$

$$c_p \text{ for water} = 1 \text{ Btu/}^\circ\text{R-lbm}$$

	<u>$wc_p \left(\frac{\text{Btu}}{^\circ\text{R-hr}} \right)$</u>	<u>$\Delta T (^\circ\text{R})$</u>	<u>$Q (\text{Btu/hr})$</u>
Anode	625	36.5	22,810
Cathode	203	7.0	1,420
H-1	205	30.5	6,250
H-2	185	5.0	930
H-3	211	3.2	680
Total			32,090

4. Helium Average Gas Temperatures (T)

The heat transfer in (3) above shows

$$36,100 - 32,090 = 4,010 \text{ Btu/hr}$$

unaccounted for. From heat balance work, the loss of heat to radiation would be approximately

1700 Btu/hr at this current level. The residual heat to the exiting gas would then be

$$4010 - 1700 = 2,310 \text{ Btu/hr}$$

Again

$$Q = wc_p(T - T_R)$$

where the symbols now refer to helium. The reference temperature is taken to be 535°R (75°F) or

$$T_R = 535^\circ\text{R}$$

and T is the average of the radial temperatures.

$$T = \frac{Q}{wc_p} + T_R$$

$$w = 0.642 \text{ lbm/hr of helium}$$

$$c_p = 1.242 \text{ Btu/lbm-}^\circ\text{R} = \text{constant for helium}$$

	<u>Q (heat in gas leaving, Btu/hr)</u>	<u>Q/wc_p</u>	<u>T (temperature of exit gas - °R)</u>
Anode	10,170	12,810	13,340
H-1	3,920	4,930	5,460
H-2	2,990	3,760	4,290
H-3	2,310	2,900	3,430

5. Gas Side Heat Transfer Coefficients

$$Q = h_a A_w \Delta T$$

where

$$Q = \text{heat transferred in Btu/hr}$$

$$h_a = \text{average heat transfer coefficient in Btu/hr-ft}^2 \text{ }^\circ\text{R}$$

$$A_w = \text{wall area in ft}^2$$

ΔT = the difference between the bulk temperature of the gas and the temperature of the wall in $^{\circ}\text{R}$.

The bulk temperature of the gas is assumed to be

$$T_b = \frac{T_1 + T_2}{2}$$

where T_1 and T_2 are the temperatures entering and leaving a section. Calculations from the water side of the heat exchangers show the inside temperature of the copper wall to be 240°F or less. The wall temperature, being low compared to the gas temperature, is thus taken to be constant at 700°R .

$$h_a = Q/A_w \Delta T$$

	<u>Q</u>	<u>L</u>	<u>A_w</u>	<u>T₁</u>	<u>T₂</u>	<u>T_b</u>	<u>ΔT</u>	<u>h</u>
H-1	6250	0.1146	3.78×10^{-3}	13,340	5460	9400	8700	190
H-12	930	0.1979	6.53×10^{-3}	13,340	4290	8820	8120	135
H-123	680	0.2812	9.28×10^{-3}	13,340	3430	8380	7680	112

$$\text{where } A_w = \pi DL$$

6. Nusselt Numbers-- Nu_b

$$Nu_b = \frac{h_a D}{k_b}$$

$$k_b = 1.859 \mu_b$$

$$\mu_b = (8.5 \times 10^{-4}) T_b^{0.643}$$

	<u>T_b</u>	<u>μ_b</u>	<u>k_b</u>	<u>h_a</u>	<u>Nu_b</u>
H-1	9400	0.305	0.567	190	3.5
H-12	8820	0.292	0.543	135	2.6
H-123	8380	0.281	0.522	112	2.2

7. Graetz Numbers--Gz_b

$$Gz_b = \frac{wc_p}{k_b L}$$

$$wc_p = (0.642)(1.242) = 0.7974$$

	<u>k_b</u>	<u>L</u>	<u>Gz_b</u>
H-1	0.567	0.1146	12.3
H-12	0.543	0.1979	7.4
H-123	0.522	0.2812	5.4

The heat transfer data are then correlated by plotting Nusselt vs Graetz numbers.

B. Friction Factor vs Reynolds Number Calculation

1. Friction Factor

The equation for calculating friction factors as developed in Method of Calculation is

$$f_a = \left(\frac{Pg_c}{G^2 R^1} \right) \left(\frac{1}{L/D} \right) \left(\frac{\Delta p}{T_1 + T_2} \right) + \left(\frac{2}{L/D} \right) \left(\frac{T_1 - T_2}{T_1 + T_2} \right)$$

$$p = (746/760)(14.75)(144) = 2085 \text{ lbf/ft}^2$$

$$g_c = 4.17 \times 10^8 \frac{\text{lbm ft}}{\text{lbf hr}^2}$$

$$G^2 = (7.49 \times 10^3)^2 = 5.61 \times 10^7$$

$$R' = 386 \frac{\text{ft lbf}}{\text{lbm}^\circ\text{R}}$$

$$L/D = 27$$

$$\Delta p = 2(472 - 521) = -98 \text{ mm H}_2\text{O} = -20.1 \text{ lbf/ft}^2$$

$$T_1 = 13,340^\circ\text{R}$$

$$T_2 = 3430^\circ\text{R}$$

$$f_a = \frac{(2085)(4.17 \times 10^8)}{(5.61 \times 10^7)(386)} \left(\frac{1}{27}\right) \left(\frac{-20.1}{13340 + 3430}\right) +$$

$$\left(\frac{2}{27}\right) \left(\frac{13340 - 3430}{13340 + 3430}\right)$$

$$= -0.0018 + 0.0438$$

$$= 0.0420 .$$

2. Reynolds Number

$$\text{Re}_b = \frac{D v_e}{\mu_b}$$

$$= \frac{(1.04 \times 10^{-2})(7.49 \times 10^3)}{0.281} = 277$$

$$\text{since } v_e = \text{constant} = 7.49 \times 10^3 .$$

III. Analogies

1. Reynolds Analogy--laminar flow

$$\frac{f_a}{2} = \frac{2h_a}{\rho v c_p}$$

$$\frac{f_a}{2} = \frac{0.0420}{2} = 0.0210$$

$$\frac{2h_a}{\rho v c_p} = \frac{2(112)}{(7.49 \times 10^3)(1.242)} = 0.0241$$

$$\% \text{ error} = \left(\frac{0.0241 - 0.0210}{0.0210} \right) (100) = 14.8\%$$

2. Analogy with Temperature Correction

$$\frac{f_a}{2} = \frac{2h_a}{\rho v c_p} \left(1 - \frac{T_w}{T_b} \right)$$

$$\frac{f_a}{2} = 0.0210$$

$$\frac{2h_a}{\rho v c_p} \left(1 - \frac{T_w}{T_b} \right) = (0.0241) \left(1 - \frac{700}{8380} \right) = (0.0241)(0.916)$$

$$= 0.0221$$

$$\% \text{ error} = \frac{(0.0221 - 0.0210)}{0.0210} (100) = 5.2\%$$

APPENDIX E

Table 6. Heat and Momentum Transfer

Table 6

**Heat and Momentum Transfer
(Calculated Data--Run 9)**

Helium mass flow rate, lbm/hr 0.213

<u>Arc Input</u> <u>Btu/hr</u>	<u>L/D</u>	<u>T_b</u>	<u>μ_b</u>	<u>Re_b</u>	<u>f_a</u>	<u>k_b</u>	<u>h_a</u>	<u>Nu_b</u>	<u>Gz_b</u>
0	11	535	0.0483	536	0.0194	-	-	-	-
8,100	11	-	-	-	-	-	-	-	-
20,000	11	2820	0.140	185	0.126	0.260	126	5.0	8.9
30,800	11	6460	0.238	109	0.103	0.442	89	2.1	5.2
0	19	535	0.0483	536	0.0222	-	-	-	-
12,400	11	2820	0.140	185	-	0.260	126	5.0	8.9
	19	2820	0.140	185	0.0694	0.260	73	2.9	5.1
18,200	11	-	-	-	-	-	-	-	-
	19	-	-	-	-	-	-	-	-
26,800	11	4610	0.193	134	-	0.359	79	2.3	6.4
	19	4610	0.193	134	0.0487	0.359	45	1.3	3.7
0	27	535	0.0483	536	0.0260	-	-	-	-
12,400	11	3680	0.167	155	-	0.310	84	2.8	7.5
	19	3420	0.159	163	-	0.296	61	2.2	4.5
	27	3180	0.151	171	0.0555	0.281	53	2.0	3.4
19,100	11	4290	0.184	141	-	0.342	73	2.2	6.8
	19	3990	0.176	147	-	0.327	54	1.7	4.1
	27	3720	0.167	155	0.0506	0.310	46	1.5	3.0
26,800	11	-	-	-	-	-	-	-	-
	19	-	-	-	-	-	-	-	-
	27	-	-	-	-	-	-	-	-

(Table continued)

Table 6 (continued)

Heat and Momentum Transfer
(Calculated Data--Run 8)

Helium mass flow rate, lbm/hr 0.348

Arc Input Btu/hr	L/D	T_b	μ_b	Re_b	f_a	k_b	h_a	Nu_b	Gz_b
0	11	535	0.0483	867	0.0167	-	-	-	-
14,300	11	5020	0.204	207	0.0703	0.379	103	2.8	10.0
24,800	11	6720	0.246	172	0.0860	0.457	121	2.8	8.3
33,400	11	9160	0.298	142	0.0899	0.554	121	2.3	6.8
0	19	535	0.0483	867	0.0221	-	-	-	-
12,400	11	5070	0.204	207	-	0.379	65	1.8	10.0
	19	5040	0.204	207	0.0264	0.379	39	1.1	5.8
22,800	11	6960	0.252	167	-	0.468	95	2.1	8.1
	19	6890	0.248	170	0.0381	0.461	57	1.3	4.7
34,800	11	13470	0.383	110	-	0.712	88	1.3	5.3
	19	13160	0.377	112	0.0423	0.701	56	0.8	3.1
0	27	535	0.0483	867	0.0215	-	-	-	-
11,000	11	2300	0.123	343	-	0.229	127	5.8	16.5
	19	2140	0.116	364	-	0.216	97	4.7	10.1
	27	1980	0.112	377	0.0479	0.208	88	4.4	7.4
23,700	11	8740	0.289	146	-	0.537	92	1.8	7.0
	19	8220	0.279	151	-	0.519	66	1.3	4.2
	27	7870	0.272	155	0.0374	0.506	53	1.1	3.0
34,100	11	-	-	-	-	-	-	-	-
	19	-	-	-	-	-	-	-	-
	27	-	-	-	-	-	-	-	-

(Table continued)

Table 6 (continued)

Heat and Momentum Transfer
(Calculated Data--Runs 11 and 12)

Helium mass flow rate, lbm/hr 0.496

Arc Input Btu/hr	L/D	T_b	μ_b	Re_b	f_a	k_b	h_a	Nu_b	Gz_b
<u>Run 11</u>									
8,460	11	-	-	-	-	-	-	-	-
20,480	11	3430	0.159	378	0.0802	0.296	193	6.8	18.2
39,900	11	16340	0.434	138	0.0578	0.807	108	1.4	6.7
40,140	11	13250	0.378	159	0.0634	0.703	119	1.8	7.7
<u>Run 12</u>									
0	11	535	0.0483	1240	0.0153	-	-	-	-
9,960	11	-	-	-	-	-	-	-	-
19,660	11	-	-	-	-	-	-	-	-
31,950	11	9110	0.298	202	0.105	0.554	193	3.6	9.7
34,400	11	9660	0.308	195	0.100	0.573	182	3.3	9.4
35,600	11	4310	0.184	327	0.135	0.342	287	8.7	15.7
0	19	535	0.0483	1240	0.0177	-	-	-	-
13,690	11	-	-	-	-	-	-	-	-
19	19	-	-	-	-	-	-	-	-
22,940	11	5170	0.196	307	-	0.364	225	6.4	14.8
	19	4750	0.197	305	0.0772	0.366	163	4.6	8.5
27,030	11	4640	0.187	321	-	0.348	167	5.0	15.5
	19	4470	0.189	318	0.0510	0.351	110	3.3	8.9
30,100	11	4630	0.187	321	-	0.348	204	6.1	15.4
	19	4490	0.189	318	0.0601	0.351	129	3.8	8.9
35,630	11	-	-	-	-	-	-	-	-
	19	-	-	-	-	-	-	-	-

(Table continued)

Table 6 (continued)

Heat and Momentum Transfer
(Calculated Data--Runs 11 and 12)

Helium mass flow rate, lbm/hr 0.496

Arc Input Btu/hr	L/D	T_b	μ_b	Re_b	f_a	k_b	h_a	Nu_b	Gz_b
<u>Run 12 (continued)</u>									
0	27	535	0.0483	1240	0.0183	-	-	-	-
10,070	11	2600	0.132	455	-	0.245	129	5.5	21.9
	19	2600	0.132	455	-	0.245	75	3.2	12.7
	27	2570	0.132	455	0.0085	0.245	56	2.4	8.9
17,200	11	2520	0.130	462	-	0.242	214	9.2	22.2
	19	2430	0.128	470	-	0.238	140	6.1	13.1
	27	2300	0.123	489	0.0298	0.229	119	5.4	9.6
27,030	11	6240	0.232	259	-	0.431	101	2.4	12.5
	19	6130	0.231	260	-	0.429	64	1.5	7.3
	27	5970	0.225	267	0.0177	0.418	50	1.3	5.2
30,100	11	3540	0.162	371	-	0.301	225	7.8	17.9
	19	3330	0.156	385	-	0.290	155	5.6	10.7
	27	3100	0.150	401	0.0440	0.279	133	5.0	7.9
35,630	11	5390	0.213	282	-	0.363	188	5.4	14.8
	19	5040	0.204	295	-	0.379	133	3.7	8.2
	27	4690	0.196	307	0.0466	0.364	114	3.3	6.0

(Table continued)

Table 6 (continued)

133

Heat and Momentum Transfer
(Calculated Data--Run 5)

Helium mass flow rate, lbm/hr 0.642

Arc Input Btu/hr	L/D	T_b	μ_b	Re_b	f_a	k_b	h_a	Nu_b	Gz_b
0	11	535	0.0483	1610	0.0148	-	-	-	-
8,400	11	2500	0.130	599	0.0373	0.242	135	5.8	28.8
13,900	11	3880	0.173	450	0.0450	0.322	141	4.6	21.6
19,800	11	6470	0.221	352	0.0406	0.411	157	4.0	16.9
26,400	11	8180	0.283	275	0.0585	0.526	160	3.2	13.2
34,400	11	7690	0.272	286	0.0865	0.506	233	4.8	13.8
40,100	11	8690	0.293	266	0.101	0.545	263	5.0	12.8
0	19	535	0.0483	1610	0.0158	-	-	-	-
9,100	11	3200	0.153	509	-	0.284	152	5.6	24.5
	19	3070	0.149	523	0.0322	0.277	105	3.9	14.5
15,800	11	5550	0.215	362	-	0.400	150	3.9	17.4
	19	5110	0.203	384	0.0415	0.377	120	3.3	10.7
19,800	11	6120	0.230	339	-	0.428	178	4.3	16.3
	19	5610	0.215	362	0.0502	0.400	139	3.6	10.1
24,600	11	7010	0.252	309	-	0.468	175	3.9	14.9
	19	6480	0.238	327	0.0491	0.442	133	3.1	9.1
29,900	11	7890	0.272	286	-	0.506	181	3.7	13.8
	19	7150	0.255	305	0.0508	0.474	140	3.1	8.5
37,500	11	10260	0.321	243	-	0.597	162	2.8	11.7
	19	9510	0.307	254	0.0485	0.571	122	2.2	7.1

(Table continued)

Table 6 (continued)
Heat and Momentum Transfer
(Calculated Data--Run 5)

Helium mass flow rate, lbm/hr 0.642

Arc Input Btu/hr	L/D	T_b	μ_b	Re_b	f_a	k_b	h_a	Nu_b	Gz_b
<u>Run 5(continued)</u>									
0	27	535	0.0483	1610	0.0156	-	-	-	-
8,400	11	2980	0.146	534	-	0.271	126	4.8	25.7
	19	2740	0.139	560	-	0.258	110	4.4	15.6
	27	2500	0.130	599	0.0302	0.242	107	4.6	11.7
13,600	11	3810	0.170	458	-	0.316	166	5.5	22.0
	19	3520	0.162	481	-	0.301	131	4.5	13.4
	27	3290	0.156	499	0.0362	0.290	115	4.1	9.8
18,800	11	5030	0.201	388	-	0.374	188	5.2	18.6
	19	4690	0.193	404	-	0.359	139	4.0	11.2
	27	4390	0.185	421	0.0448	0.344	120	3.6	8.2
25,100	11	7090	0.253	308	-	0.470	168	3.7	14.8
	19	6650	0.243	321	-	0.452	124	2.9	8.9
	27	6300	0.235	331	0.0371	0.437	103	2.5	6.5
28,800	11	7140	0.255	305	-	0.474	168	3.7	14.7
	19	6670	0.243	321	-	0.452	124	2.9	8.9
	27	6260	0.234	333	0.0385	0.435	106	2.5	6.5
36,100	11	9400	0.305	255	-	0.567	190	3.5	12.3
	19	8820	0.292	267	-	0.543	135	2.6	7.4
	27	8380	0.281	277	0.0420	0.522	112	2.2	5.4

(Table continued)

Table 6 (continued)

Heat and Momentum Transfer
(Calculated Data--Run 10)

Helium mass flow rate, lbm/hr 0.642

Arc Input Btu/hr	L/D	T_b	μ_b	Re_b	f_a	k_b	h_a	Nu_b	Gz_b
13,400	11	3020	0.147	530	-	0.273	211	8.0	25.5
	19	2800	0.139	560	0.0339	0.258	160	6.5	15.6
27,300	11	6910	0.249	313	-	0.463	210	4.7	15.0
	19	6410	0.238	327	0.0497	0.442	153	3.6	9.1
40,100	11	10950	0.334	233	-	0.621	169	2.8	11.2
	19	10320	0.323	241	0.0449	0.600	120	2.1	6.7
43,500	11	11010	0.336	232	-	0.625	185	3.1	11.1
	19	8790	0.292	267	0.0399	0.543	156	3.0	7.4
15,800	11	5790	0.222	351	-	0.413	158	4.0	16.8
	19	5390	0.213	366	-	0.396	120	3.2	10.2
	27	5000	0.203	384	0.0371	0.377	108	3.0	7.5
24,600	11	7470	0.262	297	-	0.487	161	3.4	14.3
	19	6960	0.251	310	-	0.467	122	2.7	8.6
	27	6530	0.241	323	0.0366	0.448	105	2.4	6.3
36,100	11	10750	0.332	235	-	0.617	147	2.5	11.3
	19	9980	0.316	247	-	0.587	112	2.0	6.9
	27	9480	0.306	255	0.0359	0.569	93	1.7	5.0
35,700	11	7380	0.259	301	-	0.481	189	4.1	14.5
	19	6710	0.246	317	-	0.457	149	3.4	8.8
	27	6220	0.233	334	0.0451	0.433	129	3.1	6.6

(Table continued)

Table 6 (continued)

Heat and Momentum Transfer
(Calculated Data--Run 6)

Helium mass flow rate, lbm/hr 0.785

Arc Input Btu/hr	L/D	T_b	μ_b	Re_b	f_a	k_b	h_a	Nu_b	Gz_b
0	11	535	0.0483	1973	0.0136	-	-	-	-
15,500	11	2850	0.140	681	0.0753	0.260	306	12.2	32.7
27,300	11	6140	0.230	414	0.0817	0.428	267	6.5	19.9
41,500	11	10000	0.316	302	0.0801	0.587	238	4.2	14.5
0	19	535	0.0483	1973	0.0135	-	-	-	-
16,200	11	4970	0.202	472	-	0.376	168	4.7	22.6
	19	4690	0.196	486	0.0337	0.364	125	3.6	13.5
28,300	11	6750	0.247	386	-	0.459	247	5.6	18.5
	19	6130	0.230	414	0.0580	0.428	194	4.7	11.5
40,100	11	7870	0.272	350	-	0.506	310	6.4	16.8
	19	7140	0.255	374	0.0748	0.474	234	5.1	10.4
0	27	535	0.0483	1973	0.0146	-	-	-	-
14,300	11	4470	0.190	502	-	0.353	151	4.5	24.1
	19	4070	0.177	538	-	0.329	132	4.2	15.0
	27	3780	0.168	567	0.0333	0.312	122	4.1	11.1
30,100	11	-	-	-	-	-	-	-	-
	19	-	-	-	-	-	-	-	-
	27	-	-	-	-	-	-	-	-
42,800	11	9310	0.300	318	-	0.558	284	5.3	15.2
	19	8330	0.281	339	-	0.522	224	4.5	9.4
	27	7770	0.269	354	0.0602	0.500	187	3.9	6.9

(Table continued)

Table 6 (continued)

Heat and Momentum Transfer
(Calculated Data--Run 7)

Helium mass flow rate, lbm/hr 0.925

Arc Input Btu/hr	L/D	T_b	μ_b	Re_b	f_a	k_b	h_a	Nu_b	Gz_b
0	11	535	0.0483	2320	0.0138	-	-	-	-
16,200	11	4870	0.199	564	0.0542	0.370	199	5.6	27.1
30,100	11	9520	0.306	367	0.0533	0.569	180	3.3	17.6
42,800	11	11560	0.347	323	0.0589	0.645	188	3.0	15.5
0	19	535	0.0483	2320	0.0138	-	-	-	-
14,800	11	4570	0.190	591	-	0.353	159	4.7	28.4
	19	4370	0.185	606	0.0300	0.344	116	3.5	16.9
27,300	11	6740	0.247	454	-	0.459	246	5.6	21.8
	19	6250	0.234	479	0.0547	0.435	186	4.5	13.3
41,500	11	-	-	-	-	-	-	-	-
	19	-	-	-	-	-	-	-	-
0	27	535	0.0483	2320	0.0147	-	-	-	-
14,300	11	4550	0.192	584	-	0.357	135	3.9	28.1
	19	4220	0.183	613	-	0.340	118	3.6	17.1
	27	3970	0.174	645	0.0273	0.323	108	3.5	12.7
28,700	11	8250	0.281	399	-	0.522	185	3.7	19.2
	19	7590	0.265	423	-	0.493	151	3.2	11.8
	27	7200	0.255	440	0.0363	0.474	128	2.8	8.6
40,100	11	-	-	-	-	-	-	-	-
	19	-	-	-	-	-	-	-	-
	27	-	-	-	-	-	-	-	-

(Table continued)

Table 6 (continued)

Heat and Momentum Transfer
(Calculated Data--Run 13)

Helium mass flow rate, lbm/hr:
variable

Flow Rate lbm/hr	Arc Input Btu/hr	L/D	T_b	μ_b	Re_b	f_a	k_b	h_a	Nu_b	Gz_b
0.213	30,100	11	-	-	-	-	-	-	-	-
0.348	31,570	11	-	-	-	-	-	-	-	-
0.496	32,700	11	10850	0.332	181	0.105	0.617	196	3.3	8.7
0.348	32,420	11	-	-	-	-	-	-	-	-
		19	-	-	-	-	-	-	-	-
0.496	31,980	11	8340	0.283	212	-	0.526	241	4.8	10.2
		19	7580	0.266	226	0.0894	0.494	175	3.7	6.3
0.642	33,350	11	9980	0.314	248	-	0.584	240	4.3	11.9
		19	8930	0.294	265	0.0768	0.547	188	3.6	7.4
0.785	32,940	11	9130	0.298	320	-	0.554	278	5.2	15.4
		19	8040	0.275	347	0.0764	0.511	229	4.7	9.6
1.263	30,550	11	7640	0.266	576	-	0.494	278	5.9	27.7
		19	6840	0.247	621	0.0522	0.459	245	5.6	17.3
0.348	31,600	11	-	-	-	-	-	-	-	-
		19	-	-	-	-	-	-	-	-
		27	-	-	-	-	-	-	-	-
0.496	32,300	11	8320	0.283	212	-	0.526	238	4.7	10.2
		19	7670	0.267	225	-	0.496	168	3.5	6.3
		27	7210	0.255	236	0.0686	0.474	136	3.0	4.6
0.642	32,800	11	10470	0.326	239	-	0.606	214	3.7	11.5
		19	9570	0.309	252	-	0.574	161	2.9	7.0
		27	9000	0.296	263	0.0548	0.550	133	2.5	5.2
0.785	32,700	11	10230	0.321	297	-	0.597	225	3.9	14.3
		19	9240	0.301	317	-	0.560	179	3.3	8.8
		27	8600	0.287	332	0.0515	0.534	153	3.0	6.5
1.263	31,300	11	8210	0.279	549	-	0.519	247	5.0	26.4
		19	7420	0.262	585	-	0.487	217	4.6	16.3
		27	6760	0.247	621	0.0428	0.459	206	4.7	12.2

APPENDIX F

Table 7. Heat and Momentum Analogies

Table 7

Heat and Momentum Analogies
(Calculated Data--Run 9)

Helium mass flow rate, lbm/hr 0.213

Arc Input Btu/hr	L/D	T_b	$(1 - \frac{T_w}{T_b})$	$\frac{f_a}{2}$	$\frac{2h_a}{e^{vc}_p}$	$\frac{2h_a}{e^{vc}_p} (1 - \frac{T_w}{T_b})$
0	11	535	0	0.0097	-	0
8,100	11	-	-	-	-	-
20,000	11	2820	0.752	0.0630	0.0819	0.0616
30,800	11	6460	0.892	0.0515	0.0579	0.0516
0	19	535	0	0.0111	-	0
12,400	19	2820	0.752	0.0347	0.0475	0.0357
18,200	19	-	-	-	-	-
26,800	19	4610	0.848	0.0244	0.0295	0.0250
0	27	535	0	0.0130	-	0
12,400	27	3180	0.780	0.0278	0.0342	0.0267
19,100	27	3720	0.812	0.0253	0.0299	0.0243
26,800	27	-	-	-	-	-

(Table continued)

Table 7 (continued)

Heat and Momentum Analogies
(Calculated Data--Run 8)

Helium mass flow rate, lbm/hr 0.348

Arc Input Btu/hr	L/D	T_b	$(1 - \frac{T_w}{T_b})$	$\frac{f_a}{2}$	$\frac{2h_a}{e v c_p}$	$\frac{2h_a}{e v c_p} (1 - \frac{T_w}{T_b})$
0	11	535	0	0.0084	-	0
14,300	11	5020	0.861	0.0352	0.0408	0.0351
24,600	11	6720	0.896	0.0430	0.0479	0.0429
33,400	11	9160	0.924	0.0450	0.0479	0.0433
0	19	535	0	0.0111	-	0
12,400	19	5040	0.861	0.0132	0.0154	0.0133
22,800	19	6890	0.898	0.0191	0.0225	0.0202
34,800	19	13160	0.947	0.0212	0.0221	0.0209
0	27	535	0	0.0108	-	0
11,000	27	1980	0.646	0.0240	0.0350	0.0226
23,700	27	7870	0.911	0.0187	0.0211	0.0192
34,100	27	-	-	-	-	-

(Table continued)

Table 7 (continued)
Heat and Momentum Analogies
(Calculated Data--Runs 11 and 12)

Helium mass flow rate, lbm/hr 0.496

Arc Input Btu/hr	L/D	T_b	$(1 - \frac{T_w}{T_b})$	$\frac{f_a}{2}$	$\frac{2h_a}{\rho v c_p}$	$\frac{2h_a}{\rho v c_p} (1 - \frac{T_w}{T_b})$
<u>Run 11</u>						
8,460	11	-	-	-	-	-
20,480	11	3430	0.796	0.0401	0.0537	0.0427
39,900	11	16340	0.957	0.0289	0.0300	0.0287
40,140	11	13250	0.947	0.0317	0.0331	0.0313
<u>Run 12</u>						
0	11	535	0	0.0077	-	0
9,960	11	-	-	-	-	-
19,660	11	-	-	-	-	-
31,950	11	9110	0.923	0.0525	0.0537	0.0495
34,400	11	9660	0.928	0.0500	0.0506	0.0470
35,600	11	4310	0.838	0.0675	0.0798	0.0669
0	19	535	0	0.0079	-	0
13,690	19	-	-	-	-	-
22,940	19	4750	0.853	0.0386	0.0453	0.0386
27,030	19	4470	0.843	0.0255	0.0306	0.0258
30,100	19	4490	0.844	0.0301	0.0359	0.0303
35,630	19	-	-	-	-	-
0	27	535	0	0.0092	-	0
10,070	27	2570	0.728	0.0043	0.0155	0.0113
17,200	27	2300	0.696	0.0149	0.0331	0.0230
27,030	27	5970	0.883	0.0089	0.0139	0.0122
30,100	27	3100	0.774	0.0220	0.0370	0.0286
35,630	27	4690	0.787	0.0233	0.0317	0.0249

(Table continued)

Table 7 (continued)
Heat and Momentum Analogies
(Calculated Data--Run 5)

Helium mass flow rate, lbm/hr 0.642

Arc Input Btu/hr	L/D	T_b	$(1 - \frac{T_w}{T_b})$	$\frac{f_a}{2}$	$\frac{2h_a}{e_{vc} p}$	$\frac{2h_a}{e_{vc} p} (1 - \frac{T_w}{T_b})$
0	11	535	0	0.0074	-	0
8,400	11	2500	0.720	0.0187	0.0292	0.0210
13,900	11	3880	0.820	0.0225	0.0305	0.0250
19,800	11	6470	0.885	0.0203	0.0339	0.0300
26,400	11	8180	0.917	0.0293	0.0346	0.0317
34,400	11	7690	0.912	0.0433	0.0503	0.0459
40,100	11	8690	0.922	0.0505	0.0568	0.0524
0	19	535	0	0.0079	-	0
9,100	19	3070	0.772	0.0161	0.0227	0.0175
15,800	19	5110	0.863	0.0208	0.0259	0.0224
19,800	19	5610	0.875	0.0251	0.0300	0.0263
24,600	19	6480	0.892	0.0246	0.0287	0.0256
29,900	19	7150	0.902	0.0254	0.0302	0.0272
37,500	19	9510	0.926	0.0243	0.0264	0.0244
0	27	535	0	0.0078	-	0
8,400	27	2500	0.720	0.0151	0.0231	0.0166
13,600	27	3290	0.787	0.0181	0.0248	0.0195
18,800	27	4390	0.841	0.0224	0.0259	0.0218
25,100	27	6300	0.889	0.0186	0.0222	0.0197
28,800	27	6260	0.888	0.0193	0.0229	0.0203
36,100	27	8380	0.916	0.0210	0.0242	0.0222

(Table continued)

Table 7 (continued)
Heat and Momentum Analogies
(Calculated Data--Run 10)

Helium mass flow rate, lbm/hr 0.642

Arc Input Btu/hr	L/D	T_b	$(1 - \frac{T_w}{T_b})$	$\frac{f_a}{2}$	$\frac{2h_a}{e v c_p}$	$\frac{2h_a}{e v c_p} (1 - \frac{T_w}{T_b})$
13,400	19	2800	0.750	0.0170	0.0346	0.0260
27,300	19	6410	0.891	0.0249	0.0330	0.0294
40,100	19	10320	0.932	0.0250	0.0259	0.0241
43,500	19	8790	0.920	0.0200	0.0337	0.0310
15,800	27	5000	0.860	0.0186	0.0233	0.0200
24,600	27	6530	0.893	0.0183	0.0227	0.0203
36,100	27	9480	0.926	0.0180	0.0202	0.0187
35,700	27	6220	0.887	0.0226	0.0279	0.0247

(Table continued)

Table 7 (continued)

Heat and Momentum Analogies
(Calculated Data--Run 6)

Helium mass flow rate, lbm/hr 0.785

Arc Input Btu/hr	L/D	T_b	$(1 - \frac{T_w}{T_b})$	$\frac{f_a}{2}$	$\frac{2h_a}{\rho v c_p}$	$\frac{2h_a}{\rho v c_p} (1 - \frac{T_w}{T_b})$
0	11	535	0	0.0068	-	0
15,500	11	2850	0.754	0.0377	0.0539	0.0406
27,300	11	6140	0.886	0.0409	0.0470	0.0416
41,500	11	10000	0.930	0.0401	0.0419	0.0390
0	19	535	0	0.0068	-	0
16,200	19	4690	0.851	0.0169	0.0220	0.0187
28,300	19	6130	0.886	0.0290	0.0341	0.0302
40,100	19	7140	0.902	0.0374	0.0412	0.0372
0	27	535	0	0.0073	-	0
14,300	27	3780	0.815	0.0167	0.0215	0.0175
30,100	27	-	-	-	-	-
42,800	27	7770	0.910	0.0301	0.0329	0.0299

(Table continued)

Table 7 (continued)

Heat and Momentum Analogies
(Calculated Data--Run 7)

Helium mass flow rate, lbm/hr 0.925

Arc Input Btu/hr	L/D	T_b	$(1 - \frac{T_w}{T_b})$	$\frac{f_a}{2}$	$\frac{2h_a}{\rho v c_p}$	$\frac{2h_a}{\rho v c_p} (1 - \frac{T_w}{T_b})$
0	11	535	0	0.0069	-	-
16,200	11	4870	0.856	0.0271	0.0297	0.0254
30,100	11	9520	0.926	0.0267	0.0268	0.0248
42,800	11	11560	0.939	0.0295	0.0280	0.0263
0	19	535	0	0.0069	-	-
14,800	19	4370	0.840	0.0150	0.0173	0.0145
27,300	19	6250	0.888	0.0274	0.0277	0.0246
41,500	19	-	-	-	-	-
0	27	535	0	0.0074	-	-
14,300	27	3970	0.824	0.0137	0.0161	0.0133
28,700	27	7200	0.903	0.0182	0.0191	0.0172
40,100	27	-	-	-	-	-

(Table continued)

Table 7 (continued)
Heat and Momentum Analogies
(Calculated Data--Run 13)

Helium mass flow rate, lbm/hr variable

Flow Rate lbm/hr	Arc Input Btu/hr	L/D	T_b	$(1 - \frac{T_w}{T_b})$	$\frac{f_a}{2}$	$\frac{2h_a}{\rho v c_p}$	$\frac{2h_a}{\rho v c_p} (1 - \frac{T_w}{T_b})$
0.213	30,100	11	-	-	-	-	-
0.348	31,570	11	-	-	-	-	-
0.496	32,700	11	10850	0.935	0.0525	0.0545	0.0510
0.348	32,420	19	-	-	-	-	-
0.496	31,980	19	7850	0.908	0.0447	0.0487	0.0442
0.642	33,350	19	8930	0.922	0.0384	0.0406	0.0374
0.785	32,940	19	8040	0.913	0.0382	0.0403	0.0368
1.263	30,550	19	6840	0.898	0.0261	0.0267	0.0240
0.348	31,600	27	-	-	-	-	-
0.496	32,300	27	7210	0.903	0.0343	0.0378	0.0341
0.642	32,800	27	9000	0.922	0.0274	0.0287	0.0265
0.785	32,700	27	8600	0.919	0.0258	0.0269	0.0247
1.263	31,300	27	6760	0.896	0.0214	0.0225	0.0202

APPENDIX G

Table 8. Contents of Summary Tables

Table 8
Contents of Summary Tables of Raw Data
and Preliminary Calculated Data Filed
in Laboratory Notebook

<u>Table No.</u>	<u>Title</u>
1.	Modified Rotameter Readings Compared to True Rotameter Readings.
2.	Rotameter Calibration Data.
3.	Modified Rotameter Readings vs. Helium Volume and Mass Flow Rates.
4.	Helium Viscosity Data taken in Capillary Tubing.
5.	Pressure Drop Data (No heat input).
6.	Calibration Data of Millivoltmeter to Read Arc Amperage.
7.	Heat Balances (Raw data).
8.	Heat Balances (Calculated data).
9.	Heat Transfer and Pressure Drop (Raw data).
10.	Water Flow Rates.
11.	Heat Transfer and Pressure Drop (Calculated data).
12.	Equation Coefficients for Calculation of Helium Temperatures.
13.	Operating Temperatures.

NOMENCLATURE

		<u>Units</u>
A	inside cross-sectional area of pipe	ft^2
A_w	inside area of pipe wall	ft^2
c	velocity of sound	ft/hr
c_p	specific heat (1.242 for He)	$\text{Btu}/^\circ\text{R}\text{-lbm}$
C_p	heat capacity at constant pressure	$\text{Btu}/^\circ\text{R}\text{-lb mole}$
C_v	heat capacity at constant volume	$\text{Btu}/^\circ\text{R}\text{-lb mole}$
D	inside diameter of pipe	ft
e	natural logarithm base (2.718)	none
eV	electron volts	Btu
E	internal energy	Btu/lbm
E_1	recombination energy for single ion	Btu
f	friction factor	none
f_a	average friction factor	none
F	volume flow	ft^3/hr
F_f	frictional pressure loss	$\frac{\text{ft lbf}}{\text{lbm}}$
g	acceleration of gravity (4.17×10^8)	ft/hr^2
g_c	dimensional constant (4.17×10^8)	lbm ft/lbf hr^2
G	mass velocity	$\text{lbm/ft}^2 \text{ hr}$
Gz	Graetz number (wc_p/kL)	none
Gz_b	Graetz number with k evaluated at T_b	none
h	heat transfer coefficient	$\text{Btu/hr ft}^2 ^\circ\text{R}$
h_a	average heat transfer coefficient	$\text{Btu/hr ft}^2 ^\circ\text{R}$

h_c	Planck's constant (1.77×10^{40})	Btu hr
H-1	heat exchanger 1 (L/D of 11)	none
H-2	heat exchanger 2 (L/D of 8)	none
H-3	heat exchanger 3 (L/D of 8)	none
H-4	heat exchanger 4 (coiled)	none
H-12	heat exchangers 1 and 2 in series (L/D of 19)	none
H-123	heat exchangers 1, 2, and 3 in series (L/D of 27)	none
I	ionization potential of gas (22.4 for He)	volts
J	mechanical equivalent of heat (778)	ft lbf/Btu
k	thermal conductivity	Btu/hr ft ² °R/ft
k_b	thermal conductivity based on T_b	Btu/hr ft ² °R/ft
K	Boltzman's constant (7.267×10^{-24})	volt coulomb/°R
L	length of pipe	ft
m	mass	lbm
mm	millimeters	mm
M	molecular weight	lb mole
Mc	Mach number	none
MRR	modified rotameter reading	none
n	$n_n + n_1$ (original concentration of atoms)	number/ft ³
n_e	electron concentration	number/ft ³
n_1	ion concentration	number/ft ³
n_n	neutral atom concentration	number/ft ³

N	Avogadro's number (2.73×10^{26})	atoms/lb mole
Nu	Nusselt number ($h_a D/k$)	none
Nu _b	Nusselt number based on k evaluated at T_b	none
p	pressure	lbf/ft ²
p ₁	pressure at entrance to exchanger	lbf/ft ²
p ₂	pressure at outlet of exchanger	lbf/ft ²
Δp	net pressure drop	lbf/ft ²
Δp _f	pressure drop due to friction	lbf/ft ²
Pr	Prandtl number ($c_p k/\mu$)	none
Pr _b	Prandtl number based on k and μ evaluated at T_b	none
q	heat flux	Btu/hr ft ²
q ₁	rate of energy release in ion recombination	Btu/hr
Q	heat transfer rate	Btu/hr
R	gas constant (1.985)	Btu/lb mole °R
R'	gas constant for He (386)	ft lbf/lb mass °R
Re	Reynolds number ($Dv\rho/\mu$)	none
Re _b	Reynolds number based on μ evaluated at T_b	none
RTP	reference temperature and pressure	°R and lbf/ft ²
S	entropy	Btu/lbm °R
T	temperature	°R
T _b	bulk temperature = $(T_1 + T_2) / 2$	°R
T _R	reference temperature (535)	°R

T_w	wall temperature	$^{\circ}\text{R}$
T_1	temperature at inlet to exchanger	$^{\circ}\text{R}$
T_2	temperature at outlet of exchanger	$^{\circ}\text{R}$
u	unit electric charge (15.9×10^{-24})	coulombs
v	average velocity	ft/hr
v_p	point velocity	ft/hr
v_r	average reference velocity referred to 535°R	ft/hr
v_1	average velocity at entrance to exchanger	ft/hr
v_2	average velocity at outlet of exchanger	ft/hr
V	volume	ft^3
V_s	specific volume	ft^3/lbm
w	mass flow rate	lbm/hr
x	$n_1/n = n_e/n =$ fractional ionization	none
α	function varying between 0 and 2 depending on velocity profile (approximately 1 for isothermal laminar flow and 2 for isothermal turbulent flow)	none
W_o	work delivered from outside to gas	$\frac{\text{ft lbf}}{\text{hr}}$
Z	distance above reference plane	ft
γ	ratio of heat capacities	none
γ_m	ratio parameter for momentum transfer	none
γ_h	ratio parameter for heat transfer	none
μ	viscosity	lbm/ft hr

μ_b	viscosity evaluated at T_b	lbm/ft hr
ν	frequency of photon oscillation	hr ⁻¹
π	constant (3.1416)	none
ρ	density	lbm/ft ³
ρ_1	density at entrance to exchanger	lbm/ft ³
ρ_2	density at outlet of exchanger	lbm/ft ³
σ	force constant for Leonard-Jones model (2.576 for He)	angstroms
Ω	Leonard-Jones function for first approximation of viscosity and thermal conductivity	none

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AUTOBIOGRAPHY

I was born in Milwaukee, Wisconsin, August 6, 1923. After elementary schooling at Lutsen, Minnesota, and secondary schooling at Grand Marais, Minnesota, I entered the University of Minnesota in the fall of 1941 to pursue a degree in Chemical Engineering. In August of 1948, after an interim period of two years in the U. S. Navy as an electronic technician, I graduated with a B.Chem.E. degree.

My industrial experience began with five years as a process engineer for Texaco, Inc. This was followed by graduate study at the University of Washington where I obtained an M.S. degree in Chemical Engineering in December of 1954. My thesis was concerned with the development of a new process for producing titanium metal electrolytically from TiCl_4 in a fused salt system.

Further industrial experience included two years as planning engineer for the M. W. Kellogg Company of New York. This was followed by two additional years of experience with the Industrial Nucleonics Corporation of Columbus, Ohio, as a petroleum and chemicals engineer, product manager, and chief applications engineer.

In March of 1958 I entered the Ohio State University to begin work toward a Ph.D. degree in Chemical Engineering.