Robust Nonlinear Estimation and Control of Clutch-to-Clutch Shifts

Thesis

Presented in Partial Fulfillment of the Requirements for the Degree Master of
Science in the Graduate School of The Ohio State University

By

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2016

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Abstract

Shift quality is an integral component of drivability. Thus, with the extension of feedback control to more powertrain functions, it is of paramount importance that systematic and robust control methods be developed for ensuring good shift quality. Increasingly, automatic transmissions employ a combination of two actively controlled clutches to implement shifts, which makes the gearshifts clutch-to-clutch shifts. In the current study, a model-based approach is presented for the development of a robust clutch-to-clutch shift controller. A detailed nonlinear model of the powertrain consisting of engine, torque converter, transmission system, and vehicle dynamics is developed. The transmission system consists of mechanical and hydraulic subsystems. While the mechanical subsystem is easier to model, the hydraulic subsystem, commonly referred to as shift hydraulic system, is usually challenging to model due to high dynamic order and significant nonlinearities. Starting from a high order nonlinear model of the shift hydraulic dynamics, developed from first principles in an earlier study, we apply systematic methods to arrive at a lower order control-oriented model, and use it to develop a closed loop clutch pressure controller. Specifically, state-space averaging and singular perturbation techniques were used to achieve this. The reduced order model was validated against experimental measurements.

Automotive systems in general, and transmission systems in particular, lack adequate sensing essential to successful operation of feedback controllers; thus there is a

need for observers for online estimation of important operating variables. In particular, the output shaft torque, turbine torque, reaction torque at the offgoing clutch, and online clutch pressures are critical to the performance of the nonlinear controller proposed in the current study. An observer scheme is proposed to estimate these variables, with the emphasis on robustness to modeling error and parametric variations. A combination of robust Luenberger and unknown input sliding mode observers (UI-SMO) is used to solve the estimation problem. The estimates show reasonable correspondence with the actual signals, even in the presence of appreciable parametric uncertainty.

A closed-loop torque phase controller is proposed to ensure good coordination between the offgoing and oncoming clutches, which is an open problem in controlling clutch-to-clutch shifts. The controller uses information on load transfer during the torque phase, represented by the estimated reaction torque at the offgoing clutch, to manipulate the offgoing clutch pressure so that it behaves functionally similar to a one-way clutch. A secondary objective of controlling the duration of the torque phase is also served by this controller. During the inertia phase, a reference trajectory for the oncoming clutch slip speed is specified, tracking which would satisfy the conflicting control objectives associated with this phase, namely, smooth synchronization of the input and output shaft speeds of the transmission system and a short phase duration. Specifically, for each phase, a combination of sliding mode and feedback linearization-based controllers are used. The sliding mode controller implements closed-loop clutch pressure control, where the reference clutch pressure trajectories are generated by the feedback linearization controller. Again, as was the case with estimator design, robustness of the proposed nonlinear controller against different sources of uncertainty

was deemed essential. Three strategies to achieve robustness were proposed and validated numerically. The third strategy is novel and can serve as a method for incorporating robustness into feedback linearization-based controllers in general. The validation results are encouraging and indicate feasibility of the proposed observer-based controller.

Dedicated to a beginning ...

Acknowledgments

First and foremost, I would like to thank my academic advisor, Professor Krishnaswamy Srinivasan, for his guidance and support. In particular, I got inspired by his perspective of control engineering. More importantly I learnt, how one should approach a research problem, which was essential to my growth as a researcher. I also wish to thank my committee member, Professor Levent Guvenc, for his time and useful suggestions.

I would like to thank Professor Vadim Utkin for teaching me essential concepts of sliding mode theory, which has been extensively used in the current study. I also want to acknowledge Professor Andrea Serrani for teaching me basics of system and control theory. Out of all the people who inspired me at OSU, I would like to mention Jubal's name, whose presence around me, during my initial days of grad school, made me see the beauty of mathematics and the profound impact it has on mankind.

Finally, I would like to thank the Department of Mechanical and Aerospace engineering for providing me with an opportunity to be a teaching assistant, due to which, I was introduced to a beautiful, yet challenging, world of teaching.

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Research Publications

Mishra, K., D., and Srinivasan, K., “Robust Nonlinear Control of Inertia Phase in Clutch-to-Clutch Shifts,” *4th IFAC Workshop on Engine and Powertrain Control, Simulation and Modeling*, vol.4, August 2015.

Fields of Study

Major Field: Mechanical Engineering

Nonlinear estimation and control, Model order reduction, and Automatic transmission

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Nomenclature

A	Cross-sectional area of the air-gap in the PWM solenoid valve [m ²]
A_a	Area of the accumulator piton [m ²]
A_{in}	Area of the supply orifice of the PCV [m ²]
A_{ex}	Area of the exhaust orifice of the PCV [m ²]
A_c	Effective pressurized clutch area [m ²]
$\Delta A_{pcv1}, \Delta A_{pcv2}$	Differences of the valve cross sectional areas for PCV [m ²]
a_0, a_1, a_2	Constants of the turbine torque model in the torque amplification mode [N-m/(rad/s) ²]
b_0, b_1, b_2	Constants of the pump torque model in the torque amplification mode [N-m/(rad/s) ²]
b_e	Linear damping constant of engine [N-m/rad/s]
C_1, C_2, C_3, C_4, C_5	Clutch friction coefficient model parameter
c_0, c_1, c_2	Constants of the torque converter model in the fluid coupling mode [N-m/(rad/s) ²]
d_{sp1}, d_{sp2}	Diameter of the valve lands at the supply and exhaust port of the PCV [m]
$d_{sol,in}$	Inlet orifice diameter of the PWM solenoid valve [m]
$d_{sol,ex}$	Exhaust orifice diameter of the PWM solenoid valve [m]
f_1, f_2	Load torque model parameters
f^+	Inflow vector field of the reduced order hydraulic model
f^-	Exhaust vector field of the reduced order hydraulic model
I_e	Lumped engine inertia [kg-m ²]
I_{sr}	Lumped inertia at the reaction-side sun gear [kg-m ²]
I_{si}	Lumped inertia at the input-side sun gear [kg-m ²]
I_{rr}	Lumped inertia at the reaction-side ring gear [kg-m ²]

I_{ri}	Lumped inertia at the input-side ring gear [kg-m ²]
I_{cr}	Lumped inertia at the reaction-side carrier gear [kg-m ²]
I_{ci}	Lumped inertia at the input-side carrier gear [kg-m ²]
I_t	Turbine-side inertia [kg-m ²]
K_a	Stiffness of the accumulator restoring spring [N/m]
K_{sol}	Stiffness of the restoring spring of the PWM solenoid valve [N/m]
K_s	Lumped driveline stiffness [N/(rad/s)]
k_1, k'_1	Torque phase turbine torque observer gains
k_2, k'_2	Torque phase oncoming clutch pressure observer gains
k_3, k'_3, k_4, k'_4	Inertia phase observer gains
k_d	Gain of the disturbance decoupling controller
L	Luenberger observer gain, Adaptive feedback linearization gain
LR	Low-reverse clutch
l_1, l_2	Elements of the Luenberger observer gain matrix
N	Number of turns of the PWM solenoid valve
N_c	Number of plates in a multi-plate clutch
2ND	Second clutch
OD	Over-drive clutch
P	Transformation matrix for kinematics of the planetary gearset
P_c	Clutch pressure [Pa]
$P_{c,2ND}$	Second clutch pressure [Pa]
$P_{c,LR}$	Low-reverse clutch pressure [Pa]
P_s	Supply pressure of the transmission hydraulic system [Pa]
P_{sol}	PWM solenoid valve pressure [Pa]
$p_{11}, p_{12}, p_{21}, p_{22}$	Elements of the transformation matrix P
Q_{in}	Inflow of the PWM solenoid valve [m ³ /s]
Q_{ex}	Exhaust flow of the PWM solenoid valve [m ³ /s]
Rev	Reverse clutch
R_d	Final drive ratio
R_c	Effective radius of the clutch [m]
RT_{2ND}	Reaction torque at the Second clutch [N-m]
RT_{LR}	Reaction torque at the Low-reverse clutch [N-m]
RT_{UD}	Reaction torque at the Under-drive clutch [N-m]
r	Wheel radius [m]
T_{sr}	Torque at the reaction-side sun gear [N-m]
T_i	Indicated torque of the engine [N-m]

T_{cr}	Torque at the reaction side carrier gear [N-m]
T_{si}	Torque at the input-side sun gear [N-m]
T_f	Constant component of friction torque on engine [N-m]
T_p	Torque converter pump torque [N-m]
T_t	Torque converter turbine torque [N-m]
T_s	Output shaft torque [N-m]
T_{rr}	Torque at the reaction-side ring gear [N-m]
T_L	Load torque on the vehicle [N-m]
T_c	Clutch torque capacity [N-m]
$T_{c,2ND}$	Clutch torque capacity of the Second clutch [N-m]
$T_{c,LR}$	Clutch torque capacity of the Low-reverse clutch [N-m]
ΔT_t	Desired time duration of the torque phase [s]
t_m	Time constant for step response of the accumulator piston position [s]
UD	Under-drive clutch
V_{in}	Input voltage of the PWM solenoid valve [V]
$V_{in,m}$	Maximum value of the input voltage of the PWM solenoid valve [V]
x_{sol}	Position of the PWM solenoid valve plunger [m]
x_{pcv}	Position of the pressure control valve (PCV) plunger position [m]
x_a	Position of the accumulator piston position [m]
x_{in}	Displacement of the PCV spool corresponding to the opening of the supply port
x_{out}	Displacement of the PCV spool corresponding to the closing of the exhaust port
$x_{sol,m}$	Maximum value of the PWM solenoid valve plunger position [m]
$x_{pcv,m}$	Maximum value of the PCV plunger position [m]
$x_{a,avg}$	Averaged value of the accumulator piston position [m]
$x_{a,m}$	Maximum value of the accumulator piston position [m]
α	Fractional throttle valve opening
μ	Coefficient of clutch friction
$\Delta\mu$	Additive uncertainty in clutch coefficient of friction
ϕ	Magnetic flux of the PWM solenoid valve [Wb]
ϕ_m	Maximum value of the magnetic flux in the PWM solenoid valve [Wb]
λ_I	Inertia phase controller gain
$\lambda_{I,p}, \lambda_{I,i}$	Loop-shaping based robustness strategy gains

λ_T	Torque phase controller gain
τ_1	Torque phase turbine torque observer low-pass filter time constant [s]
τ_3, τ_4	Inertia phase observer low-pass filter time constants [s]
τ_2	Torque phase oncoming clutch pressure observer low-pass filter time constant [s]
τ_d	Time constant of the low-pass filter of the disturbance decoupling controller [s]
δ_m	Maximum value of the additive uncertainty δ_2
δ_1	Lumped multiplicative uncertainty for robustness study
δ_2	Lumped additive uncertainty for robustness study
γ	Duty cycle of the PWM solenoid valve
ω_{ci}	Speed of the input-side carrier gear [rad/s]
ω_{rr}	Speed of the reaction-side ring gear [rad/s]
ω_{ri}	Speed of the input-side ring gear [rad/s]
ω_{cr}	Speed of the reaction-side carrier gear [rad/s]
ω_{sr}	Speed of the reaction-side sun gear [rad/s]
ω_{si}	Speed of the input-side sun gear [rad/s]
ω_v	Wheel speed [rad/s]
ω_e	Engine speed [rad/s]
ω_O	Speed of the output shaft of the final drive [rad/s]
ω_p	Torque converter pump-side speed [rad/s]
ω_t	Torque converter turbine-side speed [rad/s]
$\Delta\omega_c$	Clutch slip velocity [rad/s]

Chapter 1: Introduction

1.1 Background and motivation

A significant majority of production vehicles with automatic transmissions on the road today use either a planetary automatic transmission (AT) or a dual clutch transmission (DCT). Dual clutch transmissions, by design, employ two actively controlled plate friction clutches, capable of transmitting the load in both slip speed directions, to implement a gearshifts; making all shifts clutch-to-clutch shifts. On the other hand, planetary automatic transmissions usually employ a combination of friction clutches, which may be plate clutches or band clutches, and one-way clutches for activation during gearshifts. The number of clutches in such transmissions is usually high, and certainly greater than two. One-way clutches simplify the coordination of load transfer between clutches by allowing loads in only one direction of rotation. While shifts involving one-way clutches are easier to control because of the one-directional nature of load transfer, they are mechanically more complex. However, in the pursuit of compact mechanical packaging and cost reduction, newer generation ATs have replaced one-way clutches and band clutches with plate friction clutches, making all gearshifts in such transmissions clutch-to-clutch shifts, as is the case with DCTs.

In order to understand the fundamental challenges involved in controlling a clutch-to-clutch shift, we begin by describing the underlying mechanism. As was mentioned above, clutch to clutch shifts employ two plate friction clutches: an oncoming clutch and an offgoing clutch. The offgoing clutch represents the clutch to be released during a gearshift, and the oncoming clutch represents the clutch to be engaged during the shift. Figure 1.1 shows typical time evolution history of relevant system variables during a 1 – 2 upshift.

Figure 1.1: A representative diagram of system variables during a 1 – 2 upshift

It should be noted that the input torque - of the transmission system - is shown as being constant throughout the gearshift, implying that engine variables such as spark advance, throttle angle, etc. remain constant during the shift. If engine variables

are manipulated during the gear shift, this would not be the case. The field of integrated powertrain control uses manipulation of engine variables and coordinates such manipulation with transmission control, to further improve shift quality.

Clutch-to-clutch shifts consist of transfer of the load torque from the offgoing to the oncoming clutch (torque phase) and synchronization of speeds of the input and output shafts (inertia phase). On the commencement of the upshift, the transmission system goes into the torque phase, from its 1st gear configuration. During the torque phase, the oncoming clutch pressure is ramped-up, proportionately increasing the corresponding clutch torque capacity, which results in the transfer of load torque from the offgoing to the oncoming clutch. Consequently, during this transition, the output shaft torque reduces as well, since some of the load torque is transmitted at the lower torque amplification corresponding to the higher gear. In Figure 1.1, this is shown by an increasing oncoming clutch torque capacity, and a reduction in the output shaft torque and the reaction torque at the offgoing clutch. As the torque phase progresses, the reaction torque at the offgoing clutch approaches zero, implying no load on the offgoing clutch. At this time, ideal operation of the gearshift controller would ensure that the offgoing clutch torque capacity is reduced to zero, consequently causing the offgoing clutch to slip, and the oncoming clutch torque capacity be high enough to transmit all of the load torque and prevent any further reduction in the output shaft torque. Any inaccuracy in coordinating the offgoing and oncoming clutches would result in either an engine flare or tie-up. Engine flare represents a situation where, due to a reduction of load on engine crankshaft, the engine speed increases. On the other hand, an increment of load on engine crankshaft would result in a reduction of the engine speed, known as engine tie-up. This brings us to one of the fundamental

challenges involved in controlling a clutch-to-clutch shift. Transmission systems in production do not come equipped with torque sensors due to economic and durability reasons, implying that the controller has no real-time sensory information about the load transfer during the torque phase. Coordination of the offgoing and oncoming clutches, in the face of lack of torque sensing, is challenging, and is one of the issues we address in the current study.

Once the offgoing clutch starts to slip, the transmission system enters the inertia phase. During this phase, the oncoming clutch pressure is manipulated to ensure a *smooth* speed synchronization of the input and output shafts of the transmission system, which facilitates drivability. Also, the controller should ensure a small inertia phase duration, to enhance longevity of the friction elements and reduction of the frictional losses. The two objectives are conflicting in nature, which forms the basis of yet another fundamental problem involved in clutch-to-clutch shift control. This is addressed in the current study as well. Once the synchronization of the input and output shaft speeds is completed, the transmission system enters the 2nd gear configuration, which marks the end of the upshift. The second challenge described above is easier to address due to the availability of speed sensors in a transmission system.

Another fundamental problem, but one that we do not address in the current study, is that of clutch-fill detection. Before the commencement of the torque phase, the oncoming clutch needs to be completely filled so that any further stroking of the clutch piston, at the beginning of the torque phase, results in a spontaneous increase in the torque capacity of the oncoming clutch. So, there exists a preparatory phase which involves *judicious* filling of the oncoming clutch, known as clutch-fill

phase. Again, like the coordination of the offgoing and oncoming clutches, accuracy is of critical importance. An underfilling of the oncoming clutch will result in loss of spontaneity, further resulting in a sluggish gearshift. On the other hand, an overfilling would result in parasitic clutch drag losses, implying an increased drop in the output shaft torque during the torque phase. The key challenge for the clutch-fill phase controller is to detect the exact clutch-fill point and to achieve clutch fill at just the right time, so that further filling can be avoided along with low clutch drag losses. Specifically, the initial position of the clutch piston, i.e. position of the clutch piston at the start of the clutch-fill phase, is not known, as it is bound to change on a shift to shift basis. Thus, adaptive measures need to be taken, as is shown by [42] and [99]. However, the problem is still open, as a meaningful solution which is general enough for application to a wide spectrum of transmission systems is not currently available.

Clutch-fill detection is just one of the several challenges associated with transmission hydraulic systems which can be solved efficiently using model-based approaches. However, the fundamental requirement of any model-based controller is the availability of a *good* dynamic model representation of the plant, in this case the shift hydraulic system, which is generally underemphasized in the vast majority of reported literature, as will be shown in the next chapter. These studies employ some kind of system identified model for representing shift hydraulic system dynamics, which are only applicable to certain ranges of operating conditions, and fail to provide physical insights essential for control design. On the other hand, models developed from first principles are highly nonlinear and computationally expensive, which limits their applicability to controller design. We identify the tradeoff; and address this by applying model

reduction techniques to a model developed from first principles, in order to facilitate its validity for model-based control approaches.

1.2 Objectives of the research

There are three main objectives of the current study: development of a reduced order representation of the shift hydraulic system dynamics; ensuring accurate coordination of the offgoing and oncoming clutches during the torque phase of gearshifts; and satisfying conflicting objectives of the inertia phase, while ensuring the robustness of the observer-based controller proposed here to accommodate the sensor-poor nature of production transmissions. Starting from an experimentally validated nonlinear model of the shift hydraulic dynamics developed in an earlier work [110], we apply formal methods of model order reduction to reduce the original model, containing 13 states, into a first order nonlinear model. The reduced order model is subsequently used for designing a closed loop clutch pressure controller.

The second objective of ensuring good coordination between the offgoing and oncoming clutches during the torque phase is achieved by estimating the reaction torque at the offgoing clutch during the torque phase. This information of the load carried by the offgoing clutch serves as an indicator for the torque phase controller to help govern the release of the offgoing clutch. In order to estimate the reaction torque at the offgoing clutch, other variables such as the output shaft torque, turbine torque and oncoming clutch pressure, which are essential to robust performance of the controller, are estimated as well.

As was explained in the preceding section, the two control objectives during the inertia phase are conflicting in nature. The third objective of the current study is

to simultaneously resolve these conflicting objectives satisfactorily. This problem has, however, been well studied in the technical literature due to the availability of speed sensors in transmission systems, as will be shown in the next chapter. Thus, our objective here, in addition to satisfying the conflicting demand on the inertia phase controller, is to ensure robustness of the proposed observer-based controller to modeling errors such as error in the assumed clutch coefficient of friction.

1.3 Organization of the thesis

The thesis is organized as follows. Chapter 2 presents a comprehensive coverage of technical literature relevant to modeling, estimation and control of clutch-to-clutch shifts. We have tried to capture the paradigm shift from calibration-intensive and fragile open loop techniques to more systematic and inherently robust closed loop (feedback) techniques. The presented literature is segregated based on the relevance to modeling, estimation or control.

The model representation of the powertrain used in the current study is described in chapter 3. We mainly focus on the 1 – 2 upshift, which for the powertrain of interest is a clutch-to-clutch shift. In general, 1 – 2 gearshifts are the most difficult to control due to the higher gear ratio changes involved. Starting from a relatively high order nonlinear model of the shift hydraulic system dynamics developed in a previous work [110], formal techniques of model order reduction are applied, to arrive at a control-oriented shift hydraulics model. The reduced order model is validated against the full order nonlinear model, which was experimentally validated in the same study [110], and results are presented in chapter 3.

Chapter 4 presents the major contributions of this thesis. The algorithms for online estimation of the output shaft torque, turbine torque, reaction torque at the offgoing clutch, and oncoming clutch pressure are developed first. A controller is subsequently designed to meet the objectives of the thesis. In particular, we present a closed loop torque phase control law which ensures that the offgoing clutch mimics the operation of a one-way clutch, implying good coordination between the offgoing and oncoming clutches. A control law for smooth synchronization of input and output shaft speeds during the inertia phase is developed with the emphasis on robustness against modeling and estimation errors. In the pursuit of such robustness, novel techniques for ensuring robustness of feedback linearization based controllers are developed.

Validation results of the observer-based controller proposed in chapter 4, in the face of modeling and estimation errors, are presented in chapter 5. In particular, we test the proposed algorithm against an appreciable uncertainty in the coefficient of oncoming clutch friction. Shortcomings of the proposed observer-based controller are highlighted, wherever applicable. Based on these and other considerations, we conclude with the key findings and relevant implications, along with recommendations for future work, in chapter 6.

Chapter 2: Powertrain Modeling and Control: Literature Review

Digital computers have revolutionized the approach of control design for any dynamical system. This paradigm shift is very evident in the field of automotive controls, where there has been a gradual shift from fragile open loop to inherently robust feedback control techniques in an effort to improve the performance of powertrain or chassis functions as part of overall performance improvement of automobiles. Feedback control laws can either be generated using calibration-intensive adhoc methods or formal procedures based on the mathematical representation of the system, namely, its dynamic model. The two desirable characteristics of any model are its ability to capture the relevant dynamics of the system *sufficiently* accurately, and at the same time computationally inexpensively to facilitate its online implementation. The two characteristics are of conflicting nature and thus there exists a tradeoff that an engineer should resolve. The susceptibility to model errors is greater for open-loop control techniques as compared to its counterpart, closed-loop (feedback) control techniques, which are tolerant of a certain degree of modeling errors. An inherent requirement of any feedback control system is the online availability of different operating variables of the dynamical system, which in turn requires the presence of sensors and estimators. Model-based estimators, like controllers, also depend heavily on an accurate model of

the dynamical system. Here too, estimation may be performed open loop or closed loop, and the two approaches differ in their sensitivity to model error significantly, as for control. Thus, the requirement of a reasonably accurate model is of paramount importance.

This chapter makes an attempt to gather, summarize, and organize the *relevant* literature in the field of automotive powertrain control. Because of the underlying motivation being improved powertrain control with an emphasis on transmission control, the emphasis in modeling studies is on model fidelity at the low frequencies relevant for improved shift quality and fuel economy. The higher frequency phenomena relevant for noise, vibration, and harshness studies are of secondary importance for the current work and are not emphasized. The segregation of different scholarly articles is done here based on their relative relevance to automotive powertrain modeling, estimation and control. The literature review also makes the point that the model-based controls for transmission systems is not as advanced a field as the one for engines.

2.1 Powertrain Modeling

A conventional powertrain system (with a planetary stepped automatic transmission) typically consists of an internal combustion engine (ICE), torque converter, and transmission mechanical and hydraulic systems. The model representation, thus, should essentially have these components. The torque generated at the output shaft of the final drive (in the transmission) is used to provide propulsion power to the vehicle the dynamics of which are captured in the vehicle dynamics model. We review the modeling of these components, one by one.

Modeling, estimation and control of internal combustion engines is the most studied subject in systems-level studies of automobiles. The modeling techniques for engines have been around for over fifty years [10]. Some of the earlier works focused on steady-state and empirical data based models of engines. [111] and [9] created models for predicting fuel economy and vehicle emissions. Soon it was recognized that the applicability of steady state models for controller design is very limited. [98, 40, 85] are representative examples of attempts made at dynamical modeling of engines.

A nonlinear state-space model including the description of fuel dynamics and EGR valve was presented by [85]. A sampled data model and corresponding controller for constant speed operation of SI engines was presented by [40]. Various other control-oriented engine models were introduced in [22, 80, 12, 54, 58, 112]. Detailed experimental validation of an engine model was performed by [80]. Nonlinear control techniques were also introduced in this work, for improved engine control. [12] developed a three state nonlinear engine model and used bondgraph modeling techniques to graphically represent the dynamics of the transmission system. A mean value model of a turbocharged diesel engine was developed by [54]. It is worth mentioning that the mean value model of an engine has been extensively used in automotive control literature. A cylinder-by-cylinder model was developed by [58], where a time varying engine inertia was used in the cylinder pressure generation model to simulate the effect of discrete combustion events. [112] focused more on the simulation aspects of engine modeling. The literature for engine modeling is vast and can not be completely covered here. Interested readers are referred to survey articles by [86, 43] and books by [50, 36] for further reading.

Powertrains equipped with a planetary stepped automatic transmission system employ torque converters for the purposes of torque amplification at vehicle launch and driveline damping. Torque converters are the prime reason for the good launch characteristics and the comfortable ride of vehicles equipped with automatic transmissions. A torque converter may act as a fluid coupling or a torque amplification device depending on the operating conditions. Some of the earlier attempts at modeling torque converters include [46, 100, 65]. [46] developed a 4th order nonlinear model using bondgraph modeling concepts. Although only steady state validation was shown, the dynamic model was utilized to design a shift quality controller to demonstrate the application of the developed model. A linearization of the nonlinear model developed in [46] is presented by [19]. [100] focused on capturing the dynamics involved during the torque-converter clutch lock-up event. A nonlinear static model for the torque converter was developed by [65]. This model has been used often in the automotive control literature [88, 14, 121] due to its applicability under normal operating conditions of the powertrain [14]. However, one should be aware of the limitations of static models such as Kotwicki's. These models do not do a good job of representing the torque converter's filtering of engine torque pulsations resulting from discrete combustion events as they do not adequately capture fluid inertial effects involved in the torus flow. Nor do they do as good a job of capturing the torque amplification effects at vehicle launch.

The types of transmission systems that can be found in the literature include planetary automatic transmissions (AT), dual clutch transmissions (DCT), continuously variable transmissions (CVT), and manual and automated manual transmissions (MT

and AMT). Planetary automatic transmissions, increasingly, and dual clutch transmissions, invariably, employ clutch-to-clutch shifts and thus will be the focal point of discussion here. Clutch-to-clutch shifts involve active control and electronic coordination of the two clutches involved in a shift, as opposed to shifts that involve one actively controlled clutch and a one-way clutch. Earlier transmission designs with fewer speeds relied upon shifts of the latter type. As the number of speeds in transmissions has increased due to fuel economy considerations, cost considerations have dictated elimination of costly one-way clutches and greater reliance upon electronic control of clutch-to-clutch shifts.

Transmission systems can be broadly divided into mechanical and hydraulic subsystems. Transmission mechanical systems consist of different mechanical paths (for torque-speed modulation) and friction elements to alter power flow paths for proper matching of the available and demanded power at the wheels [60]. The modeling of the mechanical paths in DCTs is fairly straightforward due to the simplicity of the kinematics involved because of the layshaft configuration and the two parallel power flow paths, only one of which is effective in gear [105]. However, the same is not true for planetary automatic transmissions. Thus, various attempts have been made to model the kinematics of planetary transmissions [113, 6, 114, 93]. An energy approach was adopted in [113]. [6] used the lever analogy for modeling complex mechanical pathways in a transmission, an approach that has been widely used in engineering practice because of its graphical nature and appeal to intuition [101, 38]. Methods involving tables and signal flow graphs were used in [93] and [114] respectively, for performing the kinematic analysis of a planetary gear set.

Dynamic models of transmission mechanical systems are required for developing shift quality controllers. Bond graph methods have been used for the dynamic modeling of transmission mechanical paths as a prelude to shift controller development [14, 88, 47, 20]. Such methods enable convenient representation of power flow in a multi-domain system involving different forms of energy storage and transmission. Bond graph modeling techniques use flow and effort variables to describe the power-flow and the underlying dynamics of a system. [89, 79, 57] employed the Lagrange method for deriving dynamic equations of a transmission mechanical system. The Lagrange method is a generic energy based approach that is appropriate for the modeling of the dynamics of the mechanical components of transmission systems. One of the advantages of the Lagrange method is that there is no need to evaluate different reaction forces, which could be a daunting task for any complex mechanism for power transmission. An alternate approach to modeling the dynamics of a transmission system is by using static force analysis [84].

Friction elements such as band brakes and clutches play a crucial role in the control of clutch-to-clutch shifts and thus their modeling should be discussed in detail. Modeling of friction has always been a central topic of research in many fields [83]. In particular, the phenomenon of friction presents a challenge for controller design because of the discontinuity in the friction effect accompanying a reversal of the direction of motion, as well as the phenomenon of clutch lock up which results in a change in the system description [1]. Many dynamic models of friction has been proposed in literature [29, 16]. [29] compared two dynamic models of friction; frictional phenomenon captured, behavior at zero velocity, and associated computational issues

being the metric for comparison. [16] proposed a new dynamic model of friction, and discussed properties of the model relevant to control and observation of friction.

In the context of automotive transmissions, [7], [115], [27], and [28] studied the clutch engagement dynamics. [7] presented a dynamic model for the clutch friction, fluid film thickness being the state of the proposed lumped parameter model. [115] (and [21]) proposed an extension to the dynamic model proposed by [7]. [27, 28] proposed and experimentally validated a static model capable of representing the effect of surface roughness and asperity contact on clutch friction, making the proposed model a higher dimensional static map. Such higher dimensional static models require more calibration; and dynamic models are too *rich* for controller design since these models increase the dimension of an already complex powertrain model. A static model relating the clutch friction linearly to the clutch pressure through a clutch slip-velocity dependent coefficient of friction is ubiquitous in field of automotive controls. The usage of look-up tables for modeling the coefficient of friction is pervasive in the control of systems with friction. However, such an approach presents few numerical challenges for the simulation of system response due to the discontinuity of friction at zero velocity [61]. In addition to this, the static model of the clutch friction described above is only valid for shifts involving high clutch pressure and engagement (mating) speed, i.e., a high-power shift. Due to its inability to capture the squeeze dynamics of the fluid film at the mating plane of the clutch, the static model, in general, is inadequate for accurately representing the clutch-engagement dynamics at low clutch pressures or slip speeds, as at these low-power shifts the fluid film dynamics is comparable to the overall engagement dynamics [21], [18]. Also, a static model with clutch coefficient of friction expressed solely as a function of the clutch

slip speed does not include effects of lubrication aging [8] and transmission fluid temperature variations [53].

Models for transmission hydraulic systems or shift hydraulic models are inherently nonlinear in nature and thus their use for controllable design is challenging. There is also a noticeable lack of open technical literature for detailed shift hydraulics modeling, [59, 121, 106, 110] being a few exception. This lack of literature may be attributed to the fact that the electronic integration of the control of shift hydraulic systems with engine control is relatively recent. In practice, therefore, control of shift quality often depended on mechanical coordination such as one-way clutches and passive control mechanisms such as accumulators. [59] developed a detailed nonlinear transmission hydraulic model from first principles for simulating both in-gear and gear shift dynamics of transmissions. However, no experimental validation of the model was presented. [118] developed and validated experimentally a second order model to capture the shift dynamics of a hydraulic system with solenoid valve. A physics based model of a shift hydraulic system was proposed by [106] for dual clutch transmissions. [110] carried out modeling of shift hydraulic system (with a PWM solenoid valve) using Newtonian dynamics. The nonlinear model so generated was of 13^{th} order which was subsequently reduced to 5^{th} order in the same study using energy analysis methods [74]. Subsequently, in the same study, this 5^{th} order model was reduced to 1^{st} order nonlinear model for clutch-pressure estimation, albeit in an adhoc way. In order to avoid dealing with nonlinearities involved in a hydraulic system, [4, 78] linearized the nonlinear model for control purposes. Empirical models have been the primary choice for a number of authors, particularly in the context of

transmission controller development [107, 48, 37, 13, 91, 30]. These black-box models lack physical insight and are valid over very limited ranges of operating conditions.

The study of vehicle dynamics is a research field in itself as was the case for IC engines. A lot of research has been carried out in this field resulting in high fidelity, nonlinear vehicle dynamic models. However, the usual practice for modeling the driveline and vehicle longitudinal dynamics for any gear shift control studies has been to treat the driveline as a torsional spring, the vehicle as a mass lumped at the center of gravity, and wheel inertias lumped at the wheels [14, 56, 121, 110]. The forces and torques at the wheel-road interface are determined based upon empirical models of the longitudinal slip. The same approach will be adopted in the current study as well.

2.2 Powertrain control and estimation

Powertrain control for a clutch-to-clutch shift can be achieved by manipulating the clutch pressures alone or by coordinating control of clutch pressures with that of other engine variables such as throttle valve opening, spark advance, fuel rate etc. Simultaneous control of engine and transmission variables is known as integrated powertrain control [2], whereas the former approach is one of transmission control. In the current study, clutch pressures alone are manipulated to achieve the desired shift quality as the focus of the work is on transmissions, such an approach being more limited in terms of the resulting shift quality as compared to integrated powertrain control. The approaches developed here can easily be used, in conjunction with engine control, to achieve further improvements in shift quality.

All shifts involve two distinct phases because of their reliance upon friction devices for power transfer. One phase is the torque phase where the primary phenomenon is that of load transfer from the off-going clutch to the on-coming clutch, with little shift-related speed adjustment of the transmission elements or the engine. The other phase is the inertia phase or speed adjustment phase and, as indicated by the name of the phase, the primary phenomenon that occurs during this phase is shift-related speed adjustment of the transmission elements and the engine. Due to the fact that the powertrain model is described by different sets of differential equations for the two phases of the shifts, and due to the fact that the primary phenomena manifest themselves so differently during the phases, the opportunities for on-line sensing are significantly different and the controller are designed separately for the two phases. For example, the feedback from on-line measurements of the transmission elements is much richer for inertia phase control than for torque phase control.

Torque phase control has been mostly accomplished using open-loop techniques [66, 39, 117], given the relative paucity of sensor information during this phase. Consequently, articles dedicated to torque phase control are far fewer than for inertia phase control where the technical literature is comparatively rich. In [66], a model describing the shift dynamics of clutch-to-clutch shifts in dual clutch transmissions is used for offline optimization of clutch pressure trajectories for achieving desired torque response. A similar approach was adopted in [39] where, in addition to offline optimization of clutch pressure, engine torque was also modulated during the inertia phase for better shift quality. However, it should be noticed that the shift studied in this study does not involve two actively controlled clutches. An open loop pressure trajectory control, determined based on offline studies, is proposed in [117]. A key

aspect of this work is that it included robustness studies for the proposed open loop technique. Due to the appealing nature of integrated powertrain control, many researchers have explored ways for achieving desired shift quality by manipulating both engine and transmission variables [106, 13]. The engine throttle valve opening was manipulated, in addition to clutch pressures, in [106] for controlling clutch-to-clutch shifts in dual clutch transmissions. A highlight of this work was the detailed modeling of shift hydraulic dynamics. [13] employed sliding mode control techniques to achieve integrated powertrain control of the clutch-to-clutch shifts. The control methodology focused on robustness against actuator modeling error to accommodate a second order system identified model of the shift hydraulic dynamics. [11, 63, 17, 34, 42] and [99] employed adaptive techniques to overcome different uncertainties present in the system. A model reference adaptive controller is proposed in [11] to prevent stick-slip transition during clutch engagement. [63] proposed and validated an iterative scheme where desired clutch pressure trajectories are updated on the shift-to-shift basis. [17] employed a two level iterative learning scheme where the top level controller periodically updates the trajectory that is to be tracked by the lower level controller. [34] focused on uncertainty due to coefficient of friction where-as [42] and [99], primarily adapted against the uncertainty in the clutch-fill time duration. In order to achieve good coordination between the offgoing and oncoming clutches, it is of critical importance to have good control over the clutch-fill duration. [96, 97] used dynamic programming to achieve optimal oncoming clutch pressure profile for the filling phase. Clutch-fill control is not include in the current study, but appreciation for its importance is noted here by highlighting relevant work in this field, as indicated above. Other work that has focused on control of clutch position or clutch engagement are

[78, 4, 67, 45, 120, 119] and [95]. Closed loop feedback control of the torque phase has been a rarity in the literature. In [35], clutch slip control of the offgoing clutch during the torque phase is proposed, to simulate the operation of a one-way clutch. The methodology is adequate for preventing clutch tie-up. However, this introduces additional risk of stick-slip oscillations for the offgoing clutch [15], which is not good for both shift quality and the durability of the friction elements. The fundamental reason for the lack of literature on closed-loop torque phase control is the unavailability of pressure and/or torque sensors in production automatic transmissions. Thus, in order to close the control loop during the torque phase, observers are needed.

Torque/pressure sensors are unavailable in production automatic transmissions due to their high cost and low durability. However, online information on output shaft torque and clutch pressures is required for the generation of closed loop feedback control laws for effective control of transmission output torque during the torque phase. [77] use torque converter characteristics and estimated acceleration signals (from turbine speed sensors) to calculate the output shaft torque. The method suffers from two problems. First, torque converter characteristics are only valid under low frequency conditions and are not robust against variations in different environmental conditions. Second, acceleration is estimated by taking the derivative of speed signals which, in general, is corrupted by noise. However, to some extent, the second problem can be addressed by using Kalman filters for acceleration estimation [41]). [49] rely up on a combination of torque converter characteristics and engine maps for torque estimation, the resulting estimate and suffering from lack of robustness against various uncertainties. [32] developed an observer based controller for the torque phase. A Luenberger [75] type observer was presented for the estimation of

the driveshaft torque. Again, this study has two weaknesses. First, the hydraulic model considered is highly simplified and unrealistic. Second, turbine torque is calculated using the torque converter characteristics which again poses robustness issues. [76] performed simulations to show the effectiveness of their proposed sliding mode observer for driveshaft torque estimation. [55] performed simultaneous estimation of drive torque and coefficient of friction using sliding mode observers. In this study, availability of clutch pressure sensors is assumed, which makes the application of this work limited since such sensors are usually not available in production transmissions. [110] designed and validated a sliding mode observer for clutch pressure estimation. The observer gains were selected to ensure robustness against different uncertainties, which makes this work noteworthy. It was also recognized that any uncertainty in the coefficient of friction introduces a proportional uncertainty in clutch pressure estimation. [33] performed rigorous robustness analysis for their proposed clutch pressure observer. However, a first order linear model for shift hydraulic dynamics was used, which raises questions on the range of validity of the proposed observer. [116] proposed nonlinear algorithm for turbine torque estimation. An adaptive sliding mode observer was proposed, with parameters of the torque converter model [65] being adapted for achieving convergence of the estimation error. [71] used the lumped stiffness of the driveline and the speed sensors at the transmission output shaft and the wheel to calculate the relative change in the driveshaft torque. Hardware modifications were employed in [70] to estimate the absolute value of the driveshaft torque, such hardware modification making the proposed solution less feasible for implementation in production vehicles. Another real problem lies in the fact that the lumped stiffness of the driveline is a source of uncertainty and should be accounted for. In the

current study, the observer proposed for the driveshaft torque estimation is described in a later chapter and shown to be robust against this uncertainty. A dynamic model [47] of the torque converter is used in [69] for the estimation of turbine and pump torques. However, dynamic models are computationally expensive and are seldom used for real-time control applications. Thus, the proposed observer can only be used for very few controllers.

As opposed to torque phase control, where lack of torque/pressure sensors have led to the lack of closed loop control implementations, inertia phase control is traditionally achieved using feedback control techniques. The primary reason for this is the availability of the speed sensors, or RPM sensors, in a production vehicle. In most reported work of this type, the primary approach used for achieving good inertia phase control is to ensure good oncoming clutch slip trajectory tracking. However, the generation of the desired reference oncoming clutch slip trajectory has not been investigated much. [56] make an attempt at analytically deriving a generic reference trajectory for inertia phase control. In order to make the analysis tractable, various assumptions were made, some of which might not hold for many production transmission systems. First, it was assumed that the offgoing clutch is disengaged at the initiation of the clutch-to-clutch shift, which was further considered as an instantaneous event as transmission hydraulic system dynamics is not modeled in the cited reference. Second, the turbine torque was assumed to be constant during the shift, which again might fail to hold in an integrated powertrain setting. Finally, the drive shaft is assumed to be rigid, which might not be valid for many powertrain configurations. Linear control theory has been used by [44, 33, 91, 90, 121] and [73] to achieve desired objectives during the inertia phase. [44] used state space based linear

control to achieve the desired shift quality. [33] proposed a two degree of freedom linear clutch slip controller for the inertia phase. [91] proposed a clutch-slip controller which was shown to be robust against modeling errors associated with shift hydraulic dynamics. [90] used μ -synthesis controller to achieve robustness against modeling errors and disturbance torque. Realizability of the proposed algorithm was demonstrated by simulating the closed loop system using an integration time step of 10 ms. A robust PID controller was proposed by [73] for clutch slip speed tracking and the controller was shown to be robust against uncertainty in friction characteristics. Linear control theory has been a prominent choice for researchers for inertia phase control due to the developed field of robust linear control theory. In recent years, nonlinear control theory has been applied to achieve inertia phase control [48, 30, 31]. [48] proposed a backstepping controller and studied the robustness of the closed loop system using the framework of input to state stability, for a dual clutch transmission. Similar analysis was performed by [30] for automatic transmissions. In both of these references, a highly simplified shift hydraulic dynamics model was used, which raises questions about the applicability of these works in practice. [31] proposed a two degree of freedom controller where the feedforward path was established using differential flatness [81] of the system, and the feedback path consisted of a linear controller to ensure robustness against different sources of uncertainty. However, a phenomenological (linear, first order) shift hydraulic model was used for this work as well. We will describe, as part of the current work, a combination of feedback linearization and sliding mode control techniques to achieve the inertia phase control. A formal treatment of the hydraulic system model was undertaken, and robustness

against uncertainty in the clutch coefficient of friction was ensured using adaptive means.

2.3 Conclusion

A thorough literature survey was conducted to summarize progress in the field of automotive powertrain modeling and control. The technical literature in this field is much broader than has been discussed here, and the selection of referenced articles here being based solely on relevance to the current study. The different studies discussed here were organized based on their relative relevance to powertrain modeling, estimation and control, especially as they relate to transmissions. The lack of formal treatment of shift hydraulic dynamics is a common theme observed in the different studies discussed here. The literature survey also highlights a few observations. First, there has been a gradual shift from calibration intensive control methods to much more systematic model based control design procedures. Second, the literature for the control of internal combustion engines is much richer than that of the control of any other vehicle subsystem. Third, model based control of transmission systems is still in its nascent stages, and offers considerable opportunity for improvement.

Chapter 3: Powertrain Modeling

The current study focuses on the control of clutch-to-clutch shifts with application to both planetary automatic and dual clutch transmissions. The first step in any controller design process is to develop a control-oriented model. The two desirable, albeit conflicting, properties of such a model are its ability to capture the relevant dynamics of the system and at the same time be computationally inexpensive enough to facilitate online implementation of the corresponding model-based controller. Due to the conflicting nature of the two modeling objectives, a trade-off exists between model fidelity and complexity. The development of a control-oriented powertrain model is presented in the current chapter, with explicit consideration of this tradeoff.

We adopt a modular approach for the exposition, i.e. models of different powertrain components are presented one-by-one. A powertrain can be broadly segregated into an engine, torque converter (for planetary automatic transmissions, and) transmission mechanical and hydraulic (more commonly known as shift hydraulic system) subsystems. A simplified map-based model is used for modeling the indicated torque generation in the engine. A widely used static model [65] is used for the representation of the torque converter. The transmission mechanical dynamics are linear in nature and are represented using Euler equations. Certain comments are also made regarding simulation aspects of the developed model. Shift hydraulic dynamics are

essentially nonlinear in nature and, in general, are unacceptably complex from the point of view of controller design. The current study uses a 4-speed Hyundai Motors automatic transmission, F4A42. [110] performed detailed modeling of the shift hydraulic dynamics of this automatic transmission using Newtonian dynamics, and validated the model against experimental data as part of a study which is one of the few of its kind in the open technical literature. The detailed validated simulation model developed in the cited reference is adopted in the current study as a proxy for an experimental transmission and further model order reduction is performed to increase its utility for model-based controller design. The detailed simulation model is thus the standard against which the reduced order models are evaluated. A simplified vehicle longitudinal dynamics model is used to represent interactions resulting from power transfer between the powertrain and the vehicle. The parameters for the transmission system are obtained from [110], to which readers are referred for the greater insight and further detail. Model representation of different powertrain components are presented next.

3.1 Engine Model

The previous chapter on literature review briefly described the evolution of the dynamic modeling of internal combustion engines. It is clear that different kinds of models with varying degree of fidelity exist in the literature, and the choice of the engine model to be used must be based on the intended application. A three degree of freedom mean value engine model [12], with the manifold air pressure, fuel flow rate and crankshaft speed representing the three energy storage elements, is a good choice for integrated powertrain control [121]. The use of the mean value model implies

that torque generation at the individual cylinders is averaged, with the expectation that low pass filtering phenomena in the powertrain perform the filtering in practice. The fluid coupling in the torque converter performs such a function. If the torque converter clutch is engaged, the damping mechanism included in the clutch performs the filtering function. Since engine variables are not manipulated in the current study for gear shift control, the engine is modeled as a one degree of freedom system here. If engine variables are to be manipulated as part of the shift control strategy, as would be the case in integrated powertrain control, the three degree of freedom model or an equivalent model would be needed to properly represent engine dynamics involved in the response to powertrain commands.

$$I_e \dot{\omega}_e = T_i(\omega_e, \alpha) - T_p - T_f - b_e \omega_e \quad (3.1)$$

where I_e , ω_e , α , T_f , T_p , b_e , and T_i denote the lumped engine inertia, engine speed, throttle angle, constant component of friction torque, pump torque, coefficient of viscous friction and indicated torque of the engine respectively. A static map [5] is used for modeling the indicated torque, which defines a nonlinear algebraic relationship between the output, T_i , and the inputs, α and ω_e , implying that the process of torque generation due to the combustion of the fuel in the cylinders is much faster than the other dynamic processes involved in powertrain rotational response. Figure 3.1 shows the engine map used for the current study.

Figure 3.1: Engine map

3.2 Torque Converter Model

The torque converter plays a critical role in the dynamic performance of automatic transmissions by providing damping and torque amplification at the vehicle's launch and lower gears [51]. Various models have been developed to capture the dynamics of the fluid flow within the torque converter [46, 19, 100]. However, these dynamic effects are seldom included in the models used for control law design in shift quality control applications because of the emphasis on low frequency aspects of torque converter dynamics, and also due to the additional computational burden posed by these models on the controller.

Kotwicki's model [65] has been widely used in the literature, due to its simplicity and applicability for low frequency operation of transmission shift control systems [12].

The same static model is used here and is represented by the following equations,

$$\begin{aligned} T_t &= a_0\omega_p^2 + a_1\omega_p\omega_t + a_2\omega_t^2 \\ T_p &= b_0\omega_p^2 + b_1\omega_p\omega_t + b_2\omega_t^2 \end{aligned} \tag{3.2}$$

for the torque amplification mode and by

$$T_t = T_p = c_0\omega_p^2 + c_1\omega_p\omega_t + c_2\omega_t^2 \tag{3.3}$$

for the fluid coupling mode. $T_t, T_p, \omega_t, \omega_p$ denotes the turbine torque, pump torque, turbine and pump speed respectively. The constants a_i, b_i, c_i ($i \in \{1, 2, 3\}$) are empirically determined model parameters and are identified by steady state testing of the torque converter. Physically, the torque amplification mode involves larger differences in the pump and turbine speeds, resulting in the flow geometry being such as to keep the stator element stationary and contribute to the torque amplification. The fluid coupling mode involves smaller differences in pump and turbine speeds, and a flow geometry that causes the stator element to freewheel. For the purpose of simulations, the torque amplification and fluid coupling modes are represented by the conditions, $\frac{\omega_t}{\omega_p} < 0.9$ and $\frac{\omega_t}{\omega_p} \geq 0.9$ respectively. As a general remark, for phenomenological torque converter models (such as the one used in the current study), identification of the model parameters and selection of the condition for switching between the torque amplification and fluid coupling phases must be performed carefully, to avoid any non-physical discontinuity appearing in the numerical simulation. In the current study, the model parameters are obtained from the manufacturer, and the switching condition based on turbine and pump speeds is the same as was used in [65].

3.3 Clutch Model

The clutches employed in the transmission system of interest are of the rotating wet-type, i.e. the rotating clutch disks are submerged in the transmission fluid. This kind of wet-type clutch has been usually modeled by a static relationship for clutch torque [88],

$$T_c = P_c \mu(\Delta\omega_c) A_c N_c R_c \text{sgn}(\Delta\omega_c) \quad (3.4)$$

where $T_c, P_c, A_c, N_c, R_c, \Delta\omega_c$ represent the clutch torque capacity, clutch pressure, effective pressurized area, number of clutch plates, effective radius and clutch slip speed respectively. μ is the coefficient of clutch friction and is modeled here by a fourth order polynomial in $\Delta\omega_c$

$$\mu = C_1 \Delta\omega_c^4 + C_2 \Delta\omega_c^3 + C_3 \Delta\omega_c^2 + C_4 \Delta\omega_c + C_5 \quad (3.5)$$

where the coefficients $C_1, \dots, C_5$ are identified for each clutch separately using the clutch friction coefficient data provided by the manufacturer. The current study focuses on 1-2 upshifts, which is of the clutch-to-clutch type, with $LR, 2ND$ being the offgoing and oncoming clutches respectively. Friction characteristics for these clutches are shown in Figure 3.2. The figure shows the data provided by the manufacturer and the corresponding polynomial fits. Note that the clutch friction characteristics for the two clutches are different, reflecting differences in the clutch geometry and clutch materials. Realistically, clutch friction characteristics will vary with the age of the clutch and the state of the transmission fluid. So, the models used here represent clutch friction characteristics at one point in time and usage only.

Figure 3.2: Clutch friction characteristics

Clearly, the clutch friction characteristics are discontinuous at zero clutch slip speed, and there is always a potential numerical problem during clutch lock-up. Thus, proper attention must be paid to the simulation of the clutch lock-up process [61]. For the current study, a simple approach was adopted where clutch lock-up was defined using a very small clutch slip speed threshold, i.e. the clutch was assumed to be locked if it had a very small velocity below the threshold. The method was found to work sufficiently well for the controller design evaluation process. However, it is emphasized that this approach would need to be further refined in more precise studies of shift quality, for the following reason. The clutch-lock up condition currently forces the clutch slip velocity to go to zero, if the condition on the threshold is satisfied. Necessarily, the acceleration of the rotating clutch discs will not be zero at clutch lock-up. Thus, an additional disturbance inertial torque is introduced artificially in the simulation, when the shift transfers from the inertia phase to the oncoming gear phase, in the case of the 1 - 2 shift, the second gear. It is emphasized that this disturbance torque is solely due to the limitations of the finite precision of the machine used for the

simulation. The deterioration in the simulated transmission performance is artificial and can be eliminated by using a better clutch lock-up condition. A condition based on both the clutch slip speed and acceleration would presumably work better, as it would allow for the disturbance inertial torque to be further minimized. However, this hypothesis is not tested in the current study.

3.4 Transmission Model

Transmission systems are used to match the effort (torque) supplied by the engine to the load torque demanded at the wheels, to best satisfy criteria related to driving performance and/or fuel economy. Functionally, transmission systems can be viewed as combination of alternative mechanical power flow paths and friction elements to switch between these paths, as dictated by the shift schedule. These friction elements are, in general, controlled through hydraulic actuators in conventional planetary automatic transmissions. The mechanical power flow paths typically consist of gear elements and interconnecting shafts.

For the transmission system of interest (Hundai Motors F4A42), the mechanical subsystem consists of a gear train composed of two planetary gear sets, three friction clutches (Rev, OD, and UD) and two band brakes where one of the mating clutch elements is stationary (2ND and LR), and a third gear set functioning as a final drive see Figure 3.3. The figure is a stick diagram of the transmission, a compact representation of transmissions widely used in practice that includes accepted symbols for plate clutches, band brakes, and the elements of planetary gear sets. A three-element torque converter is shown, along with the torque converter clutch TCC, at the gear box input. The planetary gearset that receives power from the engine (for

all forward gear operations) is known as the input-side planetary gearset, and the planetary gearset that supplies power to the wheels is known as the reaction side planetary gearset. These planetary gearsets are kinematically constrained as shown in the figure, as follows,

$$\omega_{ci} = \omega_{rr}, \omega_{ri} = \omega_{cr} \quad (3.6)$$

where ω_{kl} , $k \in \{s, c, r\}$, $l \in \{i, r\}$ represents the speed of the sun (s), carrier (c) or ring (r) gear elements of the input side (i) or the reaction side (r) planetary gearset of the gear train.

Figure 3.3: Stick diagram of transmission system

The shift hydraulic system in the transmission consists of a pump, supply pressure regulation system common to all the clutch-accumulator combinations, and for each clutch-accumulator combination, a PWM solenoid valve, pressure control valve (PCV), supply side orifice and clutch-accumulator combination, see Figure 3.4 on next page. We start by describing the model of the transmission mechanical subsystem.

Figure 3.4: Simplified schematic of the hydraulic system [110]

3.4.1 Transmisission Mechanical Model

As was mentioned before, the transmission system of interest consists of a gear train composed of two gear sets, five friction elements (three clutches and two band brakes) and a gear set acting as the final drive gear. The three clutches are the Under-drive (UD), Over-Drive (OD) and Reverse (REV) clutch and the two band brakes are the Low-Reverse (LR) and Second (2ND) brake. Different combinations of these clutches can be engaged to provide four forward speeds and one reverse speed. Table 3.1 shows the clutch engagement schedule and the overall transmission gear ratio for the different gears, including the final drive gear ratio of 2.84. The 1 – 2 gearshift involves the highest gear ratio change (a factor of 1.86, see Table 1) and thus

Gear	UD	OD	REV	LR	2ND	Gear Ratio
1 st	X			X		10.7165
2 nd	X				X	5.766
3 rd	X	X				3.7706
4 th		X			X	2.6871
Reverse			X	X		

Table 3.1: Clutch engagement schedule for the 4-speed transmission of interest

is expected to be the most difficult shift to control. Also, it should be noted that the 1 – 2 gearshift is a clutch-to-clutch shift, that is, it both the offgoing and oncoming clutches are actively controlled clutches. Therefore, in the current study, the 1 – 2 upshift was chosen to be the gearshift for evaluating the controller’s performance and thus it is sufficient to model this gearshift alone. It is emphasized that the controller proposed in the current study is model-based and, in principle, will work equally well for all gearshifts, once adapted for different gearshifts as dictated by the model parameters.

Planetary gear trains are complex mechanisms and in order to simplify their modeling, their kinematics and dynamics are modeled separately. In order to do so, the planetary gear train is modeled as the combination of two kinematically constrained ideal (with no rotational inertia or friction) planetary gearsets, with all the rotational inertia and frictional effects lumped appropriately at the four input/output *ports* of the transmission system (see Figure 3.5). The equations describing the kinematics of the compound gearset can, then, be represented as,

$$\begin{aligned}
\begin{bmatrix} \omega_{rr} \\ \omega_{si} \end{bmatrix} &= P \begin{bmatrix} \omega_{sr} \\ \omega_{cr} \end{bmatrix} \\
\begin{bmatrix} T_{sr} \\ T_{cr} \end{bmatrix} &= -P^T \begin{bmatrix} -T_0 \\ T_{cr} \end{bmatrix}
\end{aligned} \tag{3.7}$$

Figure 3.5: Stick diagram for 1-2 upshift

where,

$$\begin{aligned}
 P &= \frac{1}{R_{si}R_{rr}} \begin{bmatrix} -R_{si}R_{sr} & R_{si} \\ -R_{sr} & 1 - R_{ri}R_{rr} \end{bmatrix} =: \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} \\
 R_i &= \frac{N_i}{N_i + N_j}, \quad i, j \in \{si, ri\}, \quad i \neq j \\
 R_l &= \frac{N_l}{N_l + N_k}, \quad l, k \in \{sr, rr\}, \quad l \neq k
 \end{aligned} \tag{3.8}$$

where N_p ($p \in \{si, ri, sr, rr\}$) denotes the number of gear teeth on the p^{th} gear element and the subscripts si, ri, sr, rr denote the input-side sun, input-side ring, reaction-side sun and reaction-side ring gear respectively. $T_{si}, T_{cr}, T_{sr}, T_0$ denote the torques at the four ports of the transmission system, as can be seen in Figure 3.5. The generic dynamic equations for the planetary gear train can be derived by inspecting the free body diagrams for the different lumped inertias ($I_t, I_{si}, I_{rr}, I_{sr}, I_{cr}$).

$$\begin{aligned}
 I_t \dot{\omega}_t &= T_t - RT_{UD} \\
 I_{si} \dot{\omega}_{si} &= RT_{UD} - T_{si} \\
 I_{rr} \dot{\omega}_{rr} &= T_{rr} - R_d T_s \\
 I_{cr} \dot{\omega}_{cr} &= -RT_{LR} - T_{cr} \\
 I_{sr} \dot{\omega}_{sr} &= -RT_{2ND} - T_{sr}
 \end{aligned} \tag{3.9}$$

where I_i, ω_i, T_i ($i \in \{t, si, rr, cr, sr\}$) denote the lumped inertia, rotational speed and torque respectively. Subscripts t, si, rr, cr, sr denote the turbine side of torque converter, input-side sun gear, combined input-side carrier and reaction-side ring, combined input-side ring and reaction-side carrier, and reaction-side sun respectively. R_d, T_s represent final drive ratio and drive shaft torque (after the final drive) respectively. RT_i ($i \in \{UD, LR, 2ND\}$) denotes the reaction torque exerted on the clutch i , as described below. Subscripts $UD, LR, 2ND$ denote the clutches involved during the 1 – 2 upshift. If a particular clutch is not locked, then the corresponding reaction torque is equal to the torque capacity (4) of the clutch, i.e.

$$RT_i = T_{c,i} = P_{c,i} \mu(\Delta\omega_{c,i}) A_{c,i} N_{c,i} R_{c,i} \text{sgn}(\Delta\omega_{c,i}). \quad (3.10)$$

where $T_{c,i}$ is the torque capacity of the i^{th} clutch. For the case when a clutch is locked, the reaction torque is equal (and opposite) to the summation of all other torques acting on that clutch, as long as the reaction torque is less than the torque capacity. Thus, the reaction torque for a locked clutch depends on the motion as well as the mechanical parameters of the drivetrain elements coupled by the clutch.

A clutch-to-clutch shift consist of two phases, one involving the transfer of the load torque from the offgoing to the oncoming clutch (torque phase) and another involving the synchronization of speeds of the input and output shafts (inertia phase) of the transmission system. Thus, during the 1 – 2 clutch-to-clutch upshift, the transmission system goes from the 1st gear configuration to the torque phase, followed by the inertia phase. On the completion of the gearshift, the vehicle is in the 2nd gear configuration. Thus, it is essential to model all of these four configurations of the transmission system to completely describe the 1 – 2 upshift. During a downshift, the inertia phase occurs first, followed by the torque phase.

Equation (3.9) represents the general set of dynamic equations which is used to derive the equations for different configurations of the transmission system involved during a clutch-to-clutch gear shift, by applying the conditions corresponding to different phases of a gear shift (see Table 3.2).

Table 3.2: Conditions for different phases of a clutch-to-clutch shift

Phase	Conditions
1 st gear	$\omega_t = \omega_{si}, \omega_{cr} = 0, RT_{2ND} = 0$
Torque Phase	$\omega_t = \omega_{si}, \omega_{cr} = 0, RT_{2ND} = T_{c,2ND}$
Inertia Phase	$\omega_t = \omega_{si}, RT_{2ND} = T_{c,2ND}, RT_{LR} = T_{c,LR}$
2 nd gear	$\omega_t = \omega_{si}, \omega_{sr} = 0, RT_{LR} = 0$

First gear

For operation of the transmission system in first gear, the LR clutch is engaged (Table 3.1). Thus, the slip speed corresponding to the LR clutch (ω_{cr}) is identically zero. This further implies that the rate of change of this slip speed ($\dot{\omega}_{cr}$) is identically zero as well. Also, the $2ND$ clutch is disengaged when the vehicle is in the first gear, which means that the reaction torque acting on this clutch (RT_{2ND}) is zero. The condition $\omega_t = \omega_{si}$ simply denotes the rigid connection of turbine side of the torque converter and the input shaft of the transmission system. Using these conditions and the kinematic relationship of the compound planetary gearset (3.7), (3.8), the equations in (3.9) can be combined to give,

$$(I_{sr} + I_{rr}p_{11}^2 + p_{21}^2(I_t + I_{si}))\dot{\omega}_{sr} = -p_{11}R_dT_s + p_{21}T_t, \quad (3.11)$$

which serves as the model describing the response of the transmission system in the first gear. For the simulation, a simplified shift schedule is used, where the 1 - 2 upshift

is initiated based on the vehicle velocity exceeding a specified threshold. Once the upshift is initiated, the transmission is in the torque phase of the 1 - 2 upshift.

Torque Phase

It should be noted, from table 3.2, that the conditions corresponding to the 1 - 2 upshift torque phase are exactly the same as the conditions corresponding to the first gear configuration, except for the condition on the reaction torque for the $2ND$ clutch. A clutch-to-clutch upshift starts with the torque phase, where the load is transferred from the offgoing to the oncoming clutch. This is achieved by increasing the oncoming clutch pressure (and simultaneously decreasing the offgoing clutch pressure), and thus increasing the reaction torque on the oncoming ($2ND$) clutch. Since the oncoming clutch is still slipping, the reaction torque exerted on the clutch is equal to the clutch torque capacity (see equation (3.10)). Again, using the conditions corresponding to the torque phase and the kinematics of the transmission system, the equations in (3.9) can be combined to give,

$$(I_{sr} + I_{rr}p_{11}^2 + p_{21}^2(I_t + I_{si}))\dot{\omega}_{sr} = -p_{11}R_dT_s + p_{21}T_t - T_{c,2ND} \quad (3.12)$$

Simultaneously increasing the oncoming clutch pressure and decreasing the offgoing clutch pressure causes the torque capacity of the offgoing clutch to become less than the reaction torque exerted on the same. When this happens, the offgoing clutch starts slipping, which marks the end of the torque phase. Thus, in order to model the transition of the simulation from the torque to inertia phase, it is necessary to model the reaction torque on the offgoing clutch. The reaction torque on the offgoing clutch can be calculated from (3.9).

$$RT_{LR} = -p_{12}R_dT_s + p_{22}T_t - (p_{11}p_{12}I_{rr} + p_{22}p_{21}(I_t + I_{si}))\dot{\omega}_{sr} \quad (3.13)$$

Based on the above discussion, the criterion

$$RT_{LR} \geq T_{c,LR} \quad (3.14)$$

is used to indicate the transition from the torque phase to the inertia phase.

Inertia Phase

The inertia phase involves the synchronization of the input and output shaft speeds of the transmission system. During the inertia phase, both the offgoing and oncoming clutches are slipping, and thus the transmission system has two degrees of freedom. This further means that the reaction torques acting on both the clutches are equal to the respective clutch torque capacities, i.e.,

$$RT_{LR} = T_{c,LR}, \quad RT_{2ND} = T_{c,2ND}. \quad (3.15)$$

Based on the conditions corresponding to this phase, the differential equations describing the transmission system (during the inertia phase) can be derived from equation (3.9),

$$\begin{aligned} & (I_{sr} + p_{11}^2 I_{rr} + p_{21}^2 (I_t + I_{si})) \dot{\omega}_{sr} + (p_{11} p_{12} I_{rr} + p_{21} p_{22} (I_t + I_{si})) \dot{\omega}_{cr} \\ & \quad = -T_{c,2ND} - p_{11} R_d T_s + p_{21} T_t \\ & (I_{cr} + p_{12} p_{22} I_{rr} + p_{22}^2 (I_t + I_{si})) \dot{\omega}_{cr} + (p_{12} p_{11} I_{rr} + p_{22} p_{21} (I_t + I_{si})) \dot{\omega}_{sr} \\ & \quad = -T_{c,LR} - p_{12} R_d T_s + p_{22} T_t. \end{aligned} \quad (3.16)$$

It should be noted that the differential equations derived above contain an algebraic loop (Figure 3.6), which leads to difficulties in the simulation if the algebraic equation is nonlinear. However, the algebraic loop in (3.16) is linear (the loop in red in Figure 3.6), and was easily removed by direct substitution to facilitate the simulation of the transmission system during the inertia phase.

Figure 3.6: Algebraic loop in the inertia phase simulation

The resulting differential equations are,

$$\begin{aligned}
 \dot{\omega}_{sr} &= -(p_{12}\varphi_1 + p_{11}\varphi_2)R_d T_s + (p_{22}\varphi_1 + p_{21}\varphi_2)T_t - \varphi_2 T_{c,2ND} - \varphi_1 T_{c,LR} \\
 \dot{\omega}_{cr} &= -(p_{12}\varphi_3 + p_{11}\varphi_4)R_d T_s + (p_{22}\varphi_3 + p_{21}\varphi_4)T_t - \varphi_4 T_{c,2ND} - \varphi_3 T_{c,LR}
 \end{aligned} \tag{3.17}$$

where,

$$\begin{aligned}\varphi_1 &= \frac{(p_{11}p_{12}I_{rr} + p_{21}p_{22}(I_t + I_{si}))}{\Delta} \\ \varphi_2 &= -\frac{(I_{cr} + p_{12}p_{22}I_{rr} + p_{22}^2(I_t + I_{si}))}{\Delta} \\ \varphi_3 &= \frac{(I_{sr} + p_{11}^2I_{rr} + p_{21}^2(I_t + I_{si}))}{\Delta}\end{aligned}\tag{3.18}$$

$$\varphi_4 = \varphi_1$$

$$\begin{aligned}\Delta &= (p_{12}p_{11}I_{rr} + p_{22}p_{21}(I_t + I_{si}))^2 - \\ &\quad (I_{sr} + p_{11}^2I_{rr} + p_{21}^2(I_t + I_{si}))(I_{cr} + p_{12}p_{22}I_{rr} + p_{22}^2(I_t + I_{si})).\end{aligned}$$

The completion of the inertia phase marks the completion of the upshift. At the end of the upshift, the oncoming clutch must be locked. Thus, the criterion used to detect the end of the inertia phase must be $\omega_{sr} = 0$. Since it is difficult to simulate this condition due to the finite precision of digital computers (see the section on clutch model for further discussion of problems involved in the simulation of clutch lock-up), the following inequality was used as the criterion to mark the end of the inertia phase,

$$\omega_{sr} < \omega_{lock-up}\tag{3.19}$$

where $\omega_{lock-up}$ is a very small positive value which was adjusted to give reasonable results, i.e. to avoid any numerical instability. The value of $\omega_{lock-up}$ used in the current study is 0.01 *rad/s*.

Second gear

Once the oncoming clutch locks-up, the transmission system is in the second gear. Here, the *2ND* clutch is engaged and the *LR* clutch is free to rotate. Thus, the clutch slip speed and acceleration corresponding to the *2ND* clutch and the reaction torque corresponding to the *LR* clutch are identically zero. These conditions, along with

(3.7)(3.8)(3.9), leads to the following differential equation,

$$(I_{cr} + p_{12}^2 I_{rr} + p_{22}^2 (I_t + I_{si})) \dot{\omega}_{cr} = -p_{12} R_d T_s + p_{22} T_t, \quad (3.20)$$

which was used to simulate the response of the transmission system in second gear.

3.4.2 Transmission Hydraulic Model

The importance of modeling the dynamics of the shift hydraulic system may be better appreciated from Figure 3.7, where transmission hydraulic subsystem is viewed interpreted as the actuator in the closed loop system. It is a well-known fact in the controls technical literature that actuator dynamics could play a crucial role in the overall performance of the closed loop system, especially if actuation dynamics are comparable to the dynamics of the plant being controlled. Such is the case in the study of shift dynamics in automatic transmissions. Figure 3.7 shows a highly simplified functional block diagram for transmission control. The simplification rests in the fact that the nature of the control depends on the shift phase, being largely open-loop during the torque phase and involving feedback and closed loop control only during the inertia phase. At any rate, the importance of the representation in Figure 3.7 is that it highlights the importance of the dynamics of the transmission shift hydraulic system in determining the dynamics and quality of gear shifts. In view of its importance, it is surprising that model-based control of shifts is in its infancy, much more so than is model-based control of engine functions. Further, as noted in the previous chapter on literature review that many scholarly articles focusing on transmission control employ phenomenological models for representing the dynamics of the shift hydraulic system. The use of such models results in a few drawbacks. First,

development of such a model is a calibration-intensive process. Second, these models are only applicable for limited ranges of operating conditions and frequently overlook important model attributes such as nonlinearities. Third, these models lack physical insight and do not facilitate the process of controller design, resulting in a controller design process that is highly iterative. In order to overcome these problems, a detailed model, developed from the first principles, was employed in the current study. The model for the transmission hydraulic system is discussed next.

Figure 3.7: Functional block diagram representation of a transmission control system

A transmission shift hydraulic system consists of an electro-hydraulic actuation system for each clutch and a central line (supply) pressure regulation system (see Figure 3.4 for example). The line pressure regulation system consists of an oil sump, a pump and, in this case, a mechanical pressure regulation system. The regulation of the supply pressure is achieved through mechanical line pressure feedback, which is inherent in the construction of the regulation system. The clutch actuation system is composed of a PWM solenoid valve, a pressure control valve (PCV) and a clutch-accumulator assembly.

The detailed modeling and experimental validation of the hydraulic subsystem for the automatic transmission of interest was performed by [110]. In the cited work, a 13th order model for representing the dynamics of the shift hydraulic system was derived from Newtonian principles. The line pressure regulation system was modeled as a 4th order dynamical system, while the clutch electro-hydraulic actuation system was modeled (for each clutch) as a 9th order dynamical system. [110], in the same work, reduced the detailed model to 5th order, using energy analysis techniques introduced in [74]. The model order reduction method based on energy analysis involves the calculation of activity indices for different states of the system. The activity index for a state is the measure of the energy stored by the state, relative to the total energy contained in the system. In particular, the line pressure regulation and clutch electro-hydraulic actuation system models were reduced to second and third order respectively.

The fifth order model, so obtained, was chosen as the starting point for the current study, and needs to be simplified further to reduce the real-time computational requirements of model based controllers. In order to reduce the model order further, it was assumed that the line pressure of the hydraulic system is approximately constant. Simulation results of the full (13th) order model indicate that the line pressure fluctuates about an approximately constant mean value and at a frequency equal to that of the PWM solenoid valve. Since these fluctuations are attenuated by the accumulator dynamics, we consider it reasonable to neglect the line pressure dynamics. Thus, we have a third order model representation of the shift-hydraulic dynamics of each clutch. It is noted here that the assumption of constant line pressure offers

the additional advantage in that it decouples the shift hydraulic models for different clutches, leading to a simplified process of controller design.

A (simplified) schematic of the transmission hydraulic system is shown in Figure 3.8. In the current study, the clutch-filling phase is not modeled, i.e., it is assumed that the air cavity in the clutch-accumulator chamber has already been replaced by the transmission fluid, and any further stroking of the clutch (or accumulator) piston results in an increment of the clutch pressure. Due to the high bulk modulus of elasticity of the transmission fluid, and the dominance of accumulator restoring spring over the clutch restoring spring, the position of the clutch piston is not a dynamic variable, which is why clutch piston position is not represented as a state in Figure (3.8). It is recognized that the modeling and control of the clutch filling phase is of critical importance to shift quality. Modeling and control of the oncoming clutch filling phase, which precedes the torque phase in an upshift and the inertia phase in a downshift, remains a challenging task to date, with no formal procedures available. Thus, addition of the clutch filling phase dynamics to the existing model is left as a topic for recommended future work.

Figure 3.8: Simplified representation of the transmission hydraulic system

The three main energy storage elements in the reduced model are the magnetic flux (ϕ) in PWM solenoid valve, the pressure control valve (PCV) piston position (x_{pcv}), and the accumulator piston position (x_a). These three states were identified

as accounting for the bulk of the energy storage in the system, by the energy analysis technique cited earlier. In response to the duty cycle commanded by the controller, a voltage is generated by the PWM circuit. This voltage energizes the solenoid and leads to the production of magnetic flux. This flux results in a force on the plunger which causes the solenoid valve to open and close at the same frequency as the PWM voltage. The dynamic equation for the magnetic flux is,

$$\begin{aligned}\dot{\phi} &= \frac{V_{in}}{N} - \frac{\phi}{\mu_0 A N^2} (x_{sol,m} - x_{sol}) \\ x_{sol} &= \frac{1}{2} \frac{\phi^2}{\mu_0 A K_{sol}}\end{aligned}\tag{3.21}$$

where V_{in} is the voltage generated, N is the number of coils in the solenoid, μ_0 is the permeability of free space, A is cross-sectional area of the air-gap, K_{sol} is the solenoid valve return spring constant, $x_{sol,m}$ is the maximum solenoid plunger displacement, and x_{sol} is the solenoid plunger displacement, see Figure 3.8.

Corresponding to the opening and closing of the valve are the in-flow (Q_{in}) and exhaust-flow (Q_{ex}) rates given below,

$$\begin{aligned}Q_{in} &= C_d \pi d_{sol,in} (x_{sol,m} - x_{sol}) \sqrt{\frac{2(P_s - P_{sol})}{\rho}} \\ Q_{ex} &= C_d \pi d_{sol,ex} x_{sol} \sqrt{\frac{2P_{sol}}{\rho}} \\ P_{sol} &= \begin{cases} \frac{P_s}{a}, & \text{Inflow phase} \\ \frac{P_c}{a}, & \text{Exhaust phase} \end{cases} \\ a &= \frac{\Delta A_{pcv1}}{\Delta A_{pcv2}}\end{aligned}\tag{3.22}$$

where C_d is the discharge coefficient, ρ denotes the fluid density, $d_{sol,in}$, $d_{sol,ex}$ are inlet and exhaust orifice diameters respectively, P_s is the supply (line) pressure, P_{sol} is the controlled pressure at the outlet of solenoid valve, P_c is the clutch pressure, ΔA_{pcv1} is the difference in areas of face 1 and 2, and ΔA_{pcv2} is the difference in areas of face

4 and 3 (see Figure 3.8) of the pressure control valve. The flow from the solenoid valve exerts pressure on the spool of the PCV, the resulting motion being given by the following equation.

$$\dot{x}_{pcv} = \frac{1}{\Delta A_{pcv1}} (Q_{in} - Q_{ex}) \quad (3.23)$$

The opening and closing of the PCV gives rise to the flow to and from the clutch-accumulator chamber. This flow determines the motion of the accumulator piston (x_a), described by a first order nonlinear model, and given below.

$$\dot{x}_a = \begin{cases} \frac{1}{A_a} \left(C_d A_{incE} \sqrt{\frac{2(P_s - P_c)}{\rho}} \right), & \text{Inflow phase} \\ \frac{1}{A_a} \left(C_d A_{excE} \sqrt{\frac{2P_c}{\rho}} \right), & \text{Exhaust phase} \end{cases} \quad (3.24)$$

$$P_c A_a = K_a x_a$$

where A_a is the accumulator piston area, K_a represents the accumulator spring constant, A_{incE}, A_{excE} represent the effective supply orifice areas for the inflow and exhaust phases respectively.

$$\begin{aligned} A_{incE} &= \frac{A_{in}(x_{pcv}) A_{inc}}{\sqrt{A_{in}(x_{pcv})^2 + A_{inc}^2}} \\ A_{excE} &= \frac{A_{ex}(x_{pcv}) A_{inc}}{\sqrt{A_{ex}(x_{pcv})^2 + A_{inc}^2}}, \end{aligned} \quad (3.25)$$

where A_{inc} is the area of the supply-side orifice, and A_{in}, A_{ex} are the inflow and exhaust port areas of the PCV respectively. They are expressed by the following equations, where x_{in}, x_{out} are the displacements of PCV spool corresponding to the opening of the supply port and the closing of the exhaust port respectively. d_{sp1}, d_{sp2} are the diameters of valve lands at the supply and exhaust ports respectively, see Figure 3.8.

$$A_{in}(x_{pcv}) = (x_{pcv} - x_{in}) \pi d_{sp1} \quad (3.26)$$

$$A_{ex}(x_{pcv}) = (x_{out} - x_{pcv}) \pi d_{sp2}$$

For an in-depth treatment of the modeling of the transmission hydraulic system, readers are referred to [110]. It should be noted that, due to the nature of operation

of a PWM solenoid valve, the hydraulic model described above has a hybrid nature. This can be seen by the existence of two different phases of operation, inflow and exhaust. The model is continuous for each phase, however the transition from one phase to other is discontinuous. For the purpose of controller design, a hybrid systems framework would have been desirable. However, since the control theory for hybrid systems is relatively young, the more abundant tools from traditional control theory (which assume some degree of continuity of the system) are more attractive. If traditional nonlinear control techniques developed for continuous systems are pursued, one needs to be careful about accommodating the hybrid nature of the system in some form. This will be discussed in the section on model order reduction of the hydraulic system.

3.5 Vehicle Dynamics and Driveline Model

The vehicle dynamics model used here represents only the longitudinal dynamics of the vehicle, which has been found to be sufficiently rich for the evaluation of the transmission controller performance [12, 121]. The torque produced at the output shaft of the transmission system propagates through the driveline (final drive, shafts, couplings, etc.) to drive the vehicle. The driveline torsional characteristics are represented by a single compliant shaft. The resulting vehicle dynamics model is expressed as,

$$\begin{aligned} \dot{T}_s &= K_s(\omega_o - \omega_v) \\ \dot{\omega}_v &= \frac{1}{I_v}(T_s - T_L) \end{aligned} \tag{3.27}$$

where K_s , ω_o , ω_v , I_v denote the lumped driveline compliance, final drive output shaft speed, wheel speed, and effective inertia of the vehicle lumped at the wheel, respectively. T_L is the load torque on the wheel, which is a measure of vehicle road

load and is modeled as,

$$T_L = r(f_1 + f_2 r^2 \omega_v^2) \quad (3.28)$$

where r is the wheel's radius and f_1, f_2 are known constants. This model of vehicle longitudinal dynamics does not accommodate tire longitudinal slip, and is more appropriate for conditions involving low longitudinal slip. Inclusion of such longitudinal slip is straightforward [121]. Given the focus of the current work on modeling and control of transmission shift quality, it is of secondary importance.

3.6 Transmission Shift Hydraulic System Model Order Reduction

Electro-hydraulic actuators usually have fast dynamics which may enable further model order reduction by exploiting large differences in the time scales of the transients associated with the dynamic states of the systems. If such model order reduction can be done, the real time computational burden associated with controller implementation would be reduced. A non-linear first order model was, in fact, used for on-line clutch pressure estimation by [109], but the model order reduction was not formal, resulting in the range of validity of the approximation being unknown. It is the intent here to perform model order reduction using a formal procedure, thereby characterizing the range of validity of the reduced order model.

For the third order model presented above, it is reasonable to expect that the dominant dynamics would involve the accumulator piston position and the related accumulator/clutch pressure. This is because accumulators are designed to store and release energy in a controlled manner that would result in good shift quality. The above expectation is in fact borne out for the given transmission of interest.

The accumulator piston position (which is linearly related to the clutch pressure, see equation (3.24)) was found to be an order of magnitude slower than the other two states (magnetic flux and PCV piston position). This can be seen from Figure 3.9, which results from simulation of the open loop response of the hydraulic system model presented above, when the offgoing (LR) and oncoming (2ND) clutches were subjected to step-down and step-up inputs of voltage to the corresponding PWM solenoid valves, respectively. The step-down and step-up commands correspond to sudden release and engagement of the corresponding clutches. Clutch pressures in the figure are proportional to accumulator piston positions (see equation (3.24)). It can be noted from the figure that there is an order of magnitude difference in the time scales of the three states.

Figure 3.9: Open loop simulation of step response for the offgoing (LR) and oncoming (2ND) clutches

Formal methods of model order reduction use the difference in the time scales associated with different states to achieve a good reduced order approximation of a higher order model. One such popular method is model order reduction using the singular perturbation technique [64]. The singular perturbation technique utilizes the difference in the time scales of transients of different dynamics to scale the equations of motion for the system of interest. If the equations of motions are properly scaled, this difference in time scales results in term involving small parameters multiplying the derivative terms associated with faster dynamics. Such equations of motion are referred to as singularly perturbed. In order to see this, let us take a representative example [62]. Equation (3.29) describes the dynamics of an armature-controlled DC motor (with negligible damping), where $J_m, \omega_m, k_m, i_m, R_m, L_m, u_m$ denote the motor inertia, speed, armature constant (which is assumed to be the same as the torque constant), armature current, resistance, inductance and input voltage respectively.

$$\begin{aligned} J_m \frac{d\omega_m}{dt} &= k_m i_m, & \omega_m(0) &= \omega_{m,0} \\ L_m \frac{di_m}{dt} &= -k_m \omega_m - R_m i_m + u_m, & i_m(0) &= i_{m,0} \end{aligned} \quad (3.29)$$

Typically, motor current dynamics are considerably faster than speed dynamics, i.e. there exists a difference in the time scales of the two states of a DC motor. In order to see this more clearly, let us perform the following scaling of the dependent variables (ω_m, i_m, u_m) resulting in corresponding non-dimensional variables,

$$\omega_r = \frac{\omega_m}{\Omega_m}, i_r = \frac{i_m R_m}{k_m \Omega_m}, u_r = \frac{u_m}{k_m \Omega_m} \quad (3.30)$$

where Ω_m is a nominal speed of the motor. Equation (3.29) thus becomes,

$$\begin{aligned} T_m \frac{d\omega_r}{dt} &= i_r \\ T_e \frac{di_r}{dt} &= -\omega_r - i_r + u_r \end{aligned} \quad (3.31)$$

where $T_m := J_m R_m / k_m^2$, $T_e := L_m / R_m$ have dimensions of time and are usually referred to as the mechanical and electrical time constants respectively. For typical values of parameters, it is usually the case that $T_m \gg T_e$. Now, define a non-dimensional time $t_r = t / T_m$. Substituting in (31) we have,

$$\begin{aligned} \frac{d\omega_r}{dt_r} &= i_r \\ \epsilon_m \frac{di_r}{dt_r} &= -\omega_r - i_r + u_r \end{aligned} \tag{3.32}$$

where $\epsilon_m := T_e / T_m$. If $T_m \gg T_e$, i.e. if the mechanical time constant is significantly greater than the electrical time constant, then the existence of $\epsilon_m \ll 1$ as the coefficient of the rate of change of the faster state, the normalized and non-dimensional motor current, captures this difference in the time scales of the two states. Dynamical systems having a model representation similar to (3.32) are referred as singularly perturbed. Moreover, (3.32) is also said to be in *standard* form since, when $\epsilon \rightarrow 0$, the resulting equation

$$0 = -\omega_r - i_r + u_r \tag{3.33}$$

has an isolated root, i.e. i_r can be solved in terms of ω_r and u_r . $\square$

The above example highlights the importance of properly scaling the dependent and independent variables of the system. It should be noted that in the preceding example, appropriate scaling of dependent and independent variables led to the emergence of mechanical and electrical time constants, having *physical* insight. However, for more complicated nonlinear systems, this might not be the case. Also, in the preceding example, the system was found to be in standard form. A vast majority of the theory for singularly perturbed systems is only available for systems in standard form, as the existence of isolated roots makes the evaluation of the approximate reduced order

model possible. This is shown in the extension of the preceding example, presented next.

The example presented above is extended below to illustrate the application of the singular perturbation technique to model order reduction of dynamical systems in the *standard* singularly perturbed form. For the following analysis, subscript r will be dropped from t_r , without loss of any generality. If the small parameter $\epsilon_m \rightarrow 0$, equation (3.32) becomes,

$$\dot{\bar{\omega}}_r = \bar{i}_r \quad (3.34)$$

$$0 = -\bar{\omega}_r - \bar{i}_r + u_r(t)$$

where $\bar{i}_r, \bar{\omega}_r$ denote the quasi-steady state value of the current (i_r) and speed (ω_r), respectively, in the limiting condition. Assume for the sake of discussion that the input is a ramp function, $u_r(t) = t$. From (3.34), it can be seen that the limiting quasi-steady state value ($\bar{i}_r$) of the non-dimensional current is given as,

$$\bar{i}_r = -\bar{\omega}_r + t \quad (3.35)$$

and the corresponding reduced order model can be evaluated to be,

$$\dot{\bar{\omega}}_r = -\bar{\omega}_r + t, \quad \bar{\omega}_r(0) = \frac{\omega_{m,0}}{\Omega_m} \quad (3.36)$$

Also, define $i_b := i_r - \bar{i}_r$, which is the difference between the instantaneous value of the non-dimensional current $i_r(t)$ and its quasi-steady state value. It is emphasized here that the variable $\bar{i}_r$ is fictitious and has been created for the sake of analysis and, there exists some difference between the i_r and $\bar{i}_r$, at least at the start of the process ($t = 0$). This can be seen by analyzing the initial value of the i_r and $\bar{i}_r$.

$$i_b(0) = i_r(0) - \bar{i}_r(0) = \frac{i_{m,0}R_m}{k_m\Omega_m} + \frac{\omega_{m,0}}{\Omega_m} =: i_{b,0} \neq 0 \quad (3.37)$$

We are interested in studying the evolution of the initial difference of the non-dimensional current (i_r) and its approximation (i.e. its quasi-steady state value, $\bar{i}_r$). If it can be guaranteed that this initial difference (which is the error in approximation) goes to zero sufficiently fast, then the system represented by (3.35),(3.36) would be considered a good reduced order approximation of (3.29). Thus, we have,

$$\frac{di_b}{dt} = \frac{di_r}{dt} - \frac{d\bar{i}_r}{dt} = \frac{1}{\epsilon_m}(-\omega_r - i_r + t) + \bar{i}_r - 1 \quad (3.38)$$

or, equivalently

$$\epsilon_m \frac{di_b}{dt} = -\omega_r - i_r + t + \epsilon_m(\bar{i}_r - 1) \quad (3.39)$$

Now let $\tau_m = t/\epsilon_m$. Thus, at the initial time $t = 0$, we have $\tau_m = 0$. It is useful to see τ_m as the *stretched* time, by rewriting $\tau_m = (t - 0)/\epsilon_m$. This is because when $\epsilon_m \rightarrow 0$, for any finite time t , $\tau_m \rightarrow \infty$. This in turn shows that τ_m is the *faster* time scale as compared to t , i.e. the time variable τ_m changes infinitely fast with respect to a small differential change (dt) in time variable t , which can also be seen from,

$$\frac{d\tau_m}{dt} = \frac{1}{\epsilon_m}. \quad (3.40)$$

Thus, it is reasonable to think that the time variable t stays at zero for any finite τ_m , if $\epsilon_m \rightarrow 0$. Consequently, the slower dynamics ω_r remains approximately equal to its initial value for any finite value of τ_m . Following this rationale, (3.39) can be written in τ_m time scale (and tending $\epsilon_m \rightarrow 0$),

$$\frac{di_b}{d\tau_m} = -i_b, \quad i_b(0) = i_{b,0}. \quad (3.41)$$

In the above we have used the fact that $\omega_r = \bar{\omega}_r$, when $\epsilon_m \rightarrow 0$. Since the equilibrium, $i_b = 0$, is exponentially stable in the large, $i_b(t) \rightarrow 0$ exponentially fast $\forall i_{b,0}$. Since the error in approximation goes to zero exponentially fast, the reduced order model

(3.36) is considered a good approximation of the model represented by (3.29). The system represented by (3.41) in the faster time scale τ_m is known as the boundary layer system. Thus, stability of the boundary layer system ensures a good reduced order approximation of the detailed model.

The two scales involved in the system can be understood in the following way. The motor electric current (i_r) evolves in the faster time scale τ_m , while the slowly varying motor speed (ω_m) remains approximately close to its initial value. The non-dimensional current (i_r) settles to its quasi-steady state value ($\bar{i}_r$) when $\tau_m \rightarrow \infty$, which in turn implies a finite time t (since $t = \tau_m \epsilon_m$). The motor speed (ω_m) evolves in this slower time scale t , while electric current remains approximately equal to its quasi-steady state value. $\square$

From the example presented above, model order reduction by singular perturbation technique can be divided into three steps,

1. Scaling the dependent and independent variables of the system, with the objective of transforming the system into the standard singularly perturbed form.
2. Deriving the reduced order model and boundary layer system in the limiting case ($\epsilon_m \rightarrow 0$).
3. Studying the stability of the boundary layer to ensure good reduced order approximation of the higher order model.

It should be noted that the method of singular perturbation technique as applied to model order reduction, presented by preceding examples, offers few advantage over an adhoc method, where a reduced model (such as (3.36)) can be obtained just by

observing the relative timescales of different states of the dynamic model from their respective step-responses, similar to those given by Figure 3.9, and assuming that the faster *dynamics* do not change when the slowest *dynamics* of interest is evolving in time. However, in such an adhoc method, the third step of evaluating the stability of the boundary layer system would be typically neglected, leading to an inaccurate reduced order model approximation. This situation usually arises for systems where multiple reduced order approximations exist corresponding to the full order model. A greater appreciation of this remark can be attained by observing the arguments presented in the following example [62]. Consider the following singularly perturbed system in *standard* form.

$$\begin{aligned}\dot{x} &= \frac{x^2(1+t)}{z}, \quad x(0) = 1 \\ \epsilon \dot{z} &= -[z + (1+t)x]z[z - (1+t)], \quad z(0) = \eta_0\end{aligned}\tag{3.42}$$

In the limiting case, i.e., when $\epsilon \rightarrow 0$, the quasi-steady state value of z , denoted by $\bar{z}$, could be any one of the three isolated roots of (3.42), i.e.,

$$\bar{z} = -(1+t)x, \quad \bar{z} = 0, \quad \text{and} \quad \bar{z} = 1+t\tag{3.43}$$

Due to the existence of three isolated roots, three reduced order approximations of (3.42) exist. One can easily show that the reduced order approximation corresponding to the isolated root $\bar{z} = 0$ is not a valid approximation, as the boundary layer system is unstable for this case. Similarly one can show that the isolated roots $\bar{z} = -(1+t)x$ and $\bar{z} = 1+t$ give rise to valid reduced order approximations of the full model (3.42) in the regions $\eta_0 < 0$ and $\eta_0 > 0$ respectively. It should be noted that the case $\eta = 0$ is not of interest, as the corresponding model represented by (3.42) is not well-defined. Thus in cases like these, one should be cautious while deriving

the correct reduced order approximation of the full model; and the formal method of singular perturbation technique serves as an appropriate tool for this purpose.

Another advantage offered by singular perturbation technique lies in the fact that the rate of convergence of the boundary layer system, to zero, can be analytically (numerically) evaluated for lower (higher) order systems, further implying the availability of the information about the rate of convergence of the reduced order model to the corresponding full model. In other words, a rough estimate of time after which the reduced order model represents a good approximation of the full model is available, which can be of critical importance to the success of model-based observers and controllers using the same reduced order model.

Thus, the first step in the model order reduction for the transmission shift hydraulic system of interest is to transform the model presented in the previous section into the standard singularly perturbed form by appropriately scaling the independent (time) and dependent (applied voltage, magnetic flux, PCV piston position, accumulator position) variables. The following scaling was adopted for this purpose,

$$\tilde{\phi} = \frac{\phi}{\phi_m}, \tilde{V}_{in} = \frac{V_{in}}{V_{in,m}}, \tilde{x}_{pcv} = \frac{x_{pcv}}{x_{pcv,m}}, \tilde{x}_a = \frac{x_a}{x_{a,m}}, \tilde{t} = \frac{t}{t_m}, \quad (3.44)$$

where $\tilde{\cdot}$ denotes the scaled, non-dimensional variables and the subscript m represents the maximum value of the corresponding variables. The maximum values of the magnetic flux (ϕ), input voltage (V_{in}), pressure control valve piston position (x_{pcv}) and accumulator piston position (x_a) were noted from the data provided by the manufacturer. The maximum value of the independent variable (t) was chosen to be 0.1 second, which was observed to be the slowest time constant (corresponding to the accumulator piston position dynamics, see Figure 3.9) present in the system. Substituting these in (3.21) through (3.26), the following set of scaled differential equations

was obtained.

$$\begin{aligned}\epsilon_1 \dot{\tilde{\phi}} &= f_1(\tilde{\phi}, \tilde{V}_{in}) \\ \epsilon_2 \dot{\tilde{x}_{pcv}} &= f_2(\tilde{\phi}, \tilde{x}_a) \\ \dot{\tilde{x}_a} &= f_3(\tilde{x}_a, \tilde{x}_{pcv})\end{aligned}\tag{3.45}$$

The functions on the right hand side (RHS) of (3.45) were found to be order of 1, and $\epsilon_1, \epsilon_2 \ll 1$. More precisely, $\epsilon_1 = 0.001$ and $\epsilon_2 = 0.005$, when rounded to three decimal places. Thus, the system is transformed in the singularly perturbed form. However, the differential equations presented in (3.45) are not in *standard* form. This can be seen by studying the limiting case, $\epsilon_1, \epsilon_2 \rightarrow 0$, which gives,

$$\begin{aligned}0 &= f_1(\tilde{\phi}, \tilde{V}_{in}) \\ 0 &= f_2(\tilde{\phi}, \tilde{x}_a).\end{aligned}\tag{3.46}$$

Clearly, $\tilde{x}_{pcv}$ cannot be solved in terms of $\tilde{\phi}, \tilde{V}_{in}$ from (3.46). This in turn means that a reduced order model (similar to (3.36) in the example of the DC motor) can not be derived.

Recently, there have been few developments in application of singular perturbation technique to singularly perturbed systems in *non-standard* form [82]. However, application of any such technique is not possible for the system of interest in the current study, as explained next. As can be easily seen, the normalized input $\tilde{V}_{in}$ in (3.45) is a discontinuous signal which renders the RHS of equation (3.45) discontinuous. Due to the fact that the system violates basic requirements on continuity, application of singular perturbation theory is not possible as it hinges upon the fundamental theorem of existence and uniqueness for nonlinear systems [62], which in turn requires that the dynamical system under investigation at least be locally lipshitz for the solutions to be uniquely defined over some finite interval of time. Local lipshitzness of a dynamical system is a stronger notion than the mere continuity of the vector field,

which, in addition to the continuity, requires boundedness of the vector field in some local subset of the state-space. Thus, further investigation into the stability of the boundary layer system is not possible in the current study.

The normalized equations of motion (3.45) show the existence of small coefficients of the derivative terms and thus corroborate the existence of fast dynamics which is also reflected by the simulation of the full order model (see Figure 3.9). Based on these observations, the dynamics of magnetic flux and spool position of PCV are neglected for controller design. Thus, the following first order nonlinear representation of the hydraulic system is obtained,

$$\begin{aligned} \dot{x}_a &= \begin{cases} \frac{1}{A_a} \left(C_d A_{incE} \sqrt{\frac{2(P_s - P_c)}{\rho}} \right), & \text{Inflow phase} \\ \frac{1}{A_a} \left(C_d A_{excE} \sqrt{\frac{2P_c}{\rho}} \right), & \text{Exhaust phase} \end{cases} \\ P_c A_a &= K_a x_a, \\ A_{incE} &= \frac{A_{in}(x_{pcv}) A_{inc}}{\sqrt{A_{in}(x_{pcv})^2 + A_{inc}^2}}, \\ A_{excE} &= \frac{A_{ex}(x_{pcv}) A_{inc}}{\sqrt{A_{ex}(x_{pcv})^2 + A_{inc}^2}}, \end{aligned} \tag{3.47}$$

where x_{pcv} is a fictitious input that needs to be expressed in terms of the actual input, V_{in} . The usual process for such an identification entails the generation of a look-up table, based on a set of steady state tests, that relates the input V_{in} to the variable x_{pcv} . It should be noted that due to the nature of PWM excitation, the PCV spool valve position, x_{pcv} never attains a steady state value but oscillates around a mean value which is determined by the commanded duty cycle. Thus, the process of look-up table identification is not straight forward. Also, the fictitious input, x_{pcv} enters the model in a non-affine manner which makes the process of nonlinear controller design very challenging in the sense that controller design procedures developed for affine systems cannot be used. Affine systems are systems where the control input

enters the system state equations linearly. In order to avoid having to deal with the oscillating nature of $\tilde{x}_{pcv}$ and simultaneously transform the model represented by (3.47) into the affine form, the method of state space averaging is used. This method has been used for pneumatic PWM control systems [92], but has not been applied to a hydraulic PWM system.

State-Space Averaging

Before introducing the technique of state-space averaging and its application to the hydraulic system of interest, some comments regarding PWM-controlled systems in general are in order. In such systems, the input power is delivered to the system in discrete packets. If the frequency of the PWM signal is much higher than the bandwidth of the system being controlled, the effect of the high frequency input signal is averaged and the controlled system response reflects only this average. It is worth noting that, unlike the proportional solenoid valve also used in transmission hydraulic systems, PWM solenoid valves do not have an inherent feedback from the output pressure and consequently these valves are either fully open or fully closed [3]. This results in the output pressure being equal to a fraction of the supply pressure, the fraction being determined by the duty cycle. In addition to simplifying the model for analysis, the state-space averaging technique converts a model which is non-affine in the control input to one which is affine in the control input, thus simplifying controller design. In considering state-space averaging for this application, we note a connection to Filippov's theorem [25], followed by application to the hydraulic system described above.

[25] considered solutions for the response of general nonlinear autonomous systems with discontinuous RHS such as that in equation (3.48).

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, u(\mathbf{x})), \quad \mathbf{x} \in \mathbb{R}^n, \quad u \in \mathbb{R} \quad (3.48)$$

It should be noted that u being a scalar is just a simplifying assumption that does not detract from the generality of the result, but it relates better to the application for our system. For the system shown above, u takes only discrete values (determined by the state vector, $\mathbf{x}$),

$$u \in \{u_1, u_2, u_3, \dots, u_p\}, \quad (3.49)$$

and for each of these values there exists a corresponding flow vector, i.e.

$$\mathbf{f} \in \{\mathbf{f}_1, \mathbf{f}_2, \mathbf{f}_3, \dots, \mathbf{f}_p\}. \quad (3.50)$$

For such a system, Filippov [25] proposed that the equation of motion is determined by a flow vector which lies in the convex hull of all the possible flow vectors. The convex hull of a set of flow vectors (3.50) is given by,

$$\mathcal{C}_{\mathbf{f}} = \left\{ \mathbf{x} : \mathbf{x} = \sum_{k=1}^p \gamma_{\mathbf{f},k} \mathbf{f}_k, \quad 0 \leq \gamma_{\mathbf{f},k} \leq 1, \quad \gamma_{\mathbf{f},1} + \gamma_{\mathbf{f},2} + \dots + \gamma_{\mathbf{f},p} = 1 \right\} \quad (3.51)$$

More precisely, $\forall \mathbf{x} \in \mathbb{R}^n$ and $\forall \delta$, if a ball, $\mathcal{B}(\mathbf{x}, \delta)$, of radius δ centered around $\mathbf{x}$ is constructed, then the flow vector, defining the motion of the system at that point, lies in the convex hull of all possible flow vectors at point $\mathbf{x}$ and passing through the ball $\mathcal{B}(\mathbf{x}, \delta)$. Loosely speaking, Filippov's theorem defines an effective flow vector corresponding to different possible flow vectors at a point in the state space.

Filippov's theorem directly gives the result that we require for state-space averaging. In order to see this, let's analyze the system represented by (3.48) for a certain

time period ΔT . During this time, assume $u(\mathbf{x})$ takes the values u_k ($k = 1 \dots p$) for the time periods Δt_k . Thus, necessarily, we have,

$$\Delta T = \sum_{k=1}^p \Delta t_k. \quad (3.52)$$

If the input, $u(\mathbf{x})$, switches sufficiently fast, the technique of state space averaging can be used to derive the averaged behavior of the system during the time period, ΔT . The basic idea involved here uses an averaged flow vector for describing the system's behavior ($\forall t \in \Delta T$), the average being given by

$$\mathbf{f}_{avg} = \frac{\Delta t_1 \mathbf{f}_1 + \Delta t_2 \mathbf{f}_2 + \dots + \Delta t_p \mathbf{f}_p}{\Delta T} \quad (3.53)$$

Thus, the averaged approximation ($\mathbf{x}_{avg}$) of (3.48) $\forall t \in \Delta T$ can be derived to be,

$$\dot{\mathbf{x}}_{avg} = \mathbf{f}_{avg} \quad (3.54)$$

which is the same result as the derived from Filippov's theorem by considering,

$$\gamma_{\mathbf{f},k} = \frac{\Delta t_k}{\Delta T}, \quad k \in \{1, \dots, p\} \quad (3.55)$$

For the transmission shift hydraulic system of interest, let u take values from the discrete set of 0 and 1, where 1 denotes the inflow phase and 0 denotes the exhaust phase as noted in equation (3.56). Inflow and exhaust phases denote a fully open and a fully closed PWM solenoid valve, respectively. A fully open and fully closed solenoid valve, in turn, implies maximum and minimum values of the position of the pressure control valve piston, respectively. Thus, the two corresponding flow vectors for the system are,

$$\dot{\mathbf{x}}_a = \begin{cases} f^+ = \frac{1}{A_a} \left(C_d A_{incE}(x_{pcv,max}) \sqrt{\frac{2(P_s - P_c)}{\rho}} \right), & u = 1 \\ f^- = \frac{1}{A_a} \left(C_d A_{excE}(x_{pcv,min}) \sqrt{\frac{2P_c}{\rho}} \right), & u = 0. \end{cases} \quad (3.56)$$

Let γ denote the duty cycle of the PWM solenoid valve, which means that for any given time period ΔT , the valve remains open for $\gamma\Delta T$ time units and remains closed for $(1 - \gamma)\Delta T$ time units. Now, going through the same exercise as above (or, alternatively, using Filippov's theorem), we can see that,

$$\dot{\mathbf{x}}_{a,avg} = \gamma f^+ + (1 - \gamma)f^-, \quad 0 \leq \gamma \leq 1 \quad (3.57)$$

It should be noted that γ , the commanded duty cycle, is calculated by the controller as a function of output feedback. Clearly, this system is affine in the input γ and does not depend on the oscillating value of x_{pcv} , which was the primary motivation for performing the above analysis. A similar presentation of the application of Filippov's theorem to state-space averaging (though, without any explicit mention of the phrase *state-space averaging*), and in a different context can be found in [102].

The reduced order model was validated against experimental measurements; Figures 3.10 through 3.13 show the validation results. In the current study, the 1 – 2 upshift is used to validate the observer-based controller proposed later in chapter 5, which is a clutch-to-clutch shift for the transmission system of interest. The *LR* and *2ND* clutches serve the roles of offgoing and oncoming clutches for this clutch-to-clutch shift. We validate both of these clutches for two gearshift sequences: 1 – 2 – 3 and 3 – 2 – 1. For the first sequence, the 1 – 2 upshift is initiated at 3 s, and the 2 – 3 upshift is initiated at 7 s. During the 2 – 3 upshift, the *2ND* clutch is released and the *LR* clutch does not participate. For the second gearshift sequence, the 3 – 2 downshift is initiated at 17.5 s, and the 2 – 1 downshift is initiated at 20.5 s. The duty cycle for the *2ND* clutch and the *LR* clutch corresponding to these gearshift sequences are shown in Figures 3.11 and Figure 3.13 respectively. Figure 3.10 shows the comparison of the reduced order model for the *2ND* clutch with the experimental

measurement of the $2ND$ clutch pressure corresponding to the gearshift sequences mentioned above. Clearly, the reduced order model is a good approximation of the physical system during a gearshift. However, when the transmission system is in fixed gear, one should not expect a similar performance from the reduced order model; the reason being the following. When the transmission system is in a fixed gear, the clutch pressures are determined by the supply pressure regulation system, which for the reduced order model, does not exist, as the supply pressure was assumed constant while deriving it. Figure 3.12 shows a good correspondence between the reduced order model and the experimental measurement for the LR clutch. Based on these, we conclude the validity of the reduced order models for the closed loop clutch pressure controller to be developed in chapter 5.

Figure 3.10: State space averaged model validation, $2ND$ Clutch, shifts: 1 – 2 – 3 and 3 – 2 – 1

Figure 3.11: Duty cycle, 2ND Clutch, shifts: 1 - 2 - 3 and 3 - 2 - 1

Figure 3.12: State space averaged model validation, LR Clutch, shifts: 1 - 2 - 3 and 3 - 2 - 1

Figure 3.13: Duty cycle, *LR* Clutch, shifts: 1 – 2 – 3 and 3 – 2 – 1

3.7 Conclusion

The powertrain model, which mainly comprises of an engine, torque converter, transmission system, driveline and vehicle longitudinal dynamics, was described in some detail. Various comments regarding simulation aspects of the model were also made. Starting from a high fidelity, 13th order nonlinear, experimentally validated model, a systematic procedure for model order reduction was carried out to reduce it to a first order nonlinear model. Time scale analysis and subsequent state space averaging were used to obtain the reduced order model. The technique of state space averaging offers the additional advantage of converting a model which is non-affine in the control input to the one which is affine in the control input. This greatly simplifies the process of controller design, described in Chapter 4. Also, we established a connection between the Filippov's theorem and the state space averaging technique

(we note that the similar arguments, albeit in a different context, was presented in [102]). The reduced order model was validated against experimental measurements under a wide range of operating conditions and for a number of clutches, establishing confidence in the modeling technique.

Chapter 4: Powertrain Estimation and Control

A strategy for model-based feedback control of clutch-to-clutch shifts is presented in this chapter. The success of a typical model-based feedback controller rests upon two factors: the first, adequate model fidelity; the second, availability of feedback signals to implement estimation of unmeasured signals and feedback control. The first condition dictates the development of a powertrain model capable of capturing the dynamics relevant to a gear shift, which was done in chapter 3. We begin this chapter by discussing algorithms for model-based estimation of unavailable signals, which are critical to the success of the feedback controller proposed in a subsequent section. The transmission system has different dynamical structures for the two phases of a clutch-to-clutch gearshift, which usually results in two controller strategies and architectures. A closed-loop torque phase control strategy is described first, followed by the description of the inertia phase controller. Three different strategies to ensure robustness against modeling and estimation errors, parametric uncertainties, and external disturbances are proposed. The third strategy culminates in a novel approach to decoupling the effect of modeling and estimation error, and external disturbances, on the performance of the closed loop controller.

4.1 Estimator design

The controller developed in section 4.2 requires information about the output shaft torque (T_s), turbine torque (T_t), oncoming clutch pressure ($P_{c,2ND}$), and reaction torque at the offgoing clutch (RT_{LR}) during the torque phase. Transmission systems, in general, are sensor-poor and the ones in production are not equipped with torque sensors due to their high cost and low durability. In addition to this, planetary automatic transmissions usually do not consist of any clutch pressure sensors because of the large number of clutches and the cost and reliability consequences of instrumenting all of the clutches. However, dual clutch transmissions come equipped with clutch pressure sensors as there are only two clutches, which eliminates the need for clutch pressure estimation. We present scenarios for estimating all of the required variables, including cases where clutch pressure sensors are unavailable. Shaft torque is estimated using a Luenberger observer, with gains selected to ensure robustness against parametric uncertainty. Reaction torque at the offgoing clutch need only be estimated for the torque phase alone, since it is zero by design during the inertia phase (as will be shown in section 4.2). The turbine torque and oncoming clutch pressures are estimated differently for the torque and inertia phases due to the differences in governing differential equations for the two phases.

As was emphasized in the literature survey in chapter 2, model-based estimation and control of engine and vehicle dynamics is more established than similar approaches for transmission systems. Also, the former vehicular subsystems are richer with respect to the level of sensing and on-line estimation available as compared to transmission systems. We note that sensory and estimation information from engines and chassis subsystems could be used to facilitate the development of algorithms for

estimating operating variables in the transmission subsystem. More specifically, load torque on the vehicle and the indicated torque of the engine will be assumed to be known quantities in the following discussion, due to the availability of well-developed and validated estimation algorithms for these operating variables.

4.1.1 Shaft torque observer design

A Luenberger observer was used to estimate the shaft torque (T_s), based on the vehicle dynamic and transmission driveline models of the powertrain. The model representation of the driveline and vehicle longitudinal dynamics, developed in chapter 3, is restated below,

$$\begin{aligned} \dot{T}_s &= K_s(\omega_o - \omega_v) \\ \dot{\omega}_v &= \frac{1}{I_v}(T_s - T_L) \end{aligned} \quad (4.1)$$

where K_s , ω_o , ω_v , I_v denote the lumped driveline compliance, final drive output shaft speed, wheel speed, and effective inertia of the vehicle lumped at the wheel, respectively. T_L is the load torque on the wheel, which is a measure of vehicle road load and is modeled as,

$$T_L = r(f_1 + f_2 r^2 \omega_v^2) \quad (4.2)$$

where r is the wheel's radius and f_1, f_2 are known constants. Equation (4.1) can be written in state space form,

$$\begin{aligned} \begin{bmatrix} \dot{T}_s \\ \dot{\omega}_v \end{bmatrix} &= \begin{bmatrix} 0 & -K_s \\ \frac{1}{I_v} & 0 \end{bmatrix} \begin{bmatrix} T_s \\ \omega_v \end{bmatrix} + \begin{bmatrix} K_s & 0 \\ 0 & -\frac{1}{I_v} \end{bmatrix} \begin{bmatrix} \omega_o \\ T_L \end{bmatrix} \\ y &= [0 \quad 1] \begin{bmatrix} T_s \\ \omega_v \end{bmatrix} \end{aligned} \quad (4.3)$$

where ω_o, T_L are the known inputs and y is the measured output, wheel speed. Due to the availability of speed sensors, ω_o is known by measurement. T_L is either known through (4.2) or estimated by some other means, since load torque on a vehicle is an

important quantity for vehicle stability controllers and thus is usually known. Define

$$\begin{aligned} \mathbf{x} &:= \begin{bmatrix} T_s \\ \omega_v \end{bmatrix}, \quad A := \begin{bmatrix} 0 & -K_s \\ \frac{1}{I_v} & 0 \end{bmatrix}, \quad B := \begin{bmatrix} K_s & 0 \\ 0 & -\frac{1}{I_v} \end{bmatrix} \\ C &= [0 \quad 1], \quad \mathbf{u} := \begin{bmatrix} \omega_o \\ T_L \end{bmatrix} \end{aligned} \quad (4.4)$$

The Luengerger observer is given by,

$$\dot{\hat{\mathbf{x}}} = A\hat{\mathbf{x}} + B\mathbf{u} + LC(y - \hat{y}), \quad L := [l_1 \quad l_2]^T \quad (4.5)$$

where $\hat{\mathbf{x}}$ denotes the estimate of $\mathbf{x}$. The column vector L denotes the observer gain. Observability of the model represented by (4.3) can be evaluated by calculating the rank of the observability matrix $\mathcal{O}$,

$$\mathcal{O} = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{1}{I_v} & 0 \end{bmatrix} \quad . \quad (4.6)$$

The rank of the matrix $\mathcal{O}$ is equal to the dimension of the state space, equal to 2, which implies that the system state is fully observable. Since the pair (C, A) is observable, eigenvalues of the closed loop system characterized by the matrix $A + LC$ can be placed arbitrarily to make it Hurwitz, leading to asymptotic convergence of the estimation error to zero.

There are several ways to choose the estimator gain matrix L . In the current study, L was chosen to ensure robustness against the uncertainty in the lumped driveline stiffness K_s , which usually is not known very accurately. Let the uncertainty in the lumped compliance be denoted by ΔK_s , then the following can be easily derived.

$$\tilde{T}_s(s) = \frac{s + l_2}{s^2 + l_2s + \frac{l_1 + K_s}{I_v}} \Delta K_s(\omega_o - \omega_v)(s) \quad , \quad (4.7)$$

where $\tilde{T}_s$ denotes the estimation error of the output shaft torque. Since $\Delta K_s, \omega_o$, and ω_v do not change substantially during the entire shift as its duration is short, it is

reasonable to assume $\Delta K_s(\omega_o - \omega_v)$ to be a low frequency signal. Thus l_1, l_2 were selected to shape the frequency response of the transfer function in equation (4.7) to reject low frequency disturbances as well as the high frequency sensor noise which might be present in the measured rotational speeds.

A few remarks regarding the above observer are in order. It can be shown that the effect of any magnitude of ΔK_s can be rejected by arbitrarily increasing the gains l_1 and l_2 , albeit at the cost of the potential risk of the *peaking* phenomenon [62]. *Peaking* phenomenon refers to a situation where non-zero initial tracking error (in case of controllers) or estimation error (in case of observers) results in an inappropriately large control or output injection action. This is one disadvantage of using high gains in any estimation or control scheme. Also, high observer gains might result in deterioration of the closed loop performance due to the presence of noise in sensor outputs being used for feedback. At any rate, if the estimation error of the shaft torque ($\tilde{T}_s$) due to the disturbance defined as $\Delta K_s(\omega_o - \omega_v)$ can be reduced sufficiently by selecting sufficiently high gains, the actual value of compliance can be calculated as,,

$$\hat{K}_s = \frac{\dot{\hat{T}}_s}{\omega_o - \omega_v} \quad (4.8)$$

where $\hat{K}_s$ represents the estimate of K_s and converges to it as $\hat{T}_s$ converges to T_s . It should be noted that there is no need to perform the differentiation for obtaining $\dot{\hat{T}}_s$ as it can be evaluated using (4.5). In the current study, the gains l_1, l_2 are tuned under the assumption that the maximum *constant* uncertainty in the lumped driveline compliance K_s is 30%.

4.1.2 Torque phase observer design

Kotwicki's static model [65] for torque converters is usually considered to be sufficiently accurate for transmission shift controller design, for example, see [32]. However, it should be noted that Kotwicki's model does not take into account variations in temperature of the torque converter oil, fluid inertia, etc., and hence the open loop estimation of turbine torque based on Kotwicki's model is questionable for control and diagnosis purposes. This motivates the need for online closed loop estimation of the turbine torque.

The transmission system is a single degree of freedom system during the torque phase of a clutch-to-clutch shift. The differential equation corresponding to this phase was derived in chapter 3 to be,

$$(I_{sr} + I_{rr}p_{11}^2 + p_{21}^2(I_t + I_{si}))\dot{\omega}_{sr} = -p_{11}R_dT_s + p_{21}T_t - T_{c,2ND} \quad (4.9)$$

and the reaction torque at the offgoing clutch was shown to be,

$$RT_{LR} = -p_{12}R_dT_s + p_{22}T_t - (p_{11}p_{12}I_{rr} + p_{22}p_{21}(I_t + I_{si}))\dot{\omega}_{sr} \quad (4.10)$$

Due to the fact that the oncoming clutch slips during the torque phase, the following definition of the clutch torque capacity ($T_{c,2ND}$) applies.

$$T_{c,2ND} = P_{c,2ND}\mu(\omega_{sr})A_cR_cN_c\text{sgn}(\omega_{sr}) \quad (4.11)$$

The equations (4.9),(4.10) and (4.11) can be combined to give,

$$RT_{LR} = \alpha_1T_s + \alpha_2T_t + \alpha_3\mu(\omega_{sr})P_{c,2ND} \quad (4.12)$$

where,

$$\begin{aligned}
\alpha_1 &= -p_{12}R_d + \frac{p_{11}R_d(p_{11}p_{12}I_{rr} + p_{22}p_{21}(I_t + I_{si}))}{I_{sr} + I_{rr}p_{11}^2 + p_{21}^2(I_t + I_{si})} \\
\alpha_2 &= p_{22} - \frac{p_{21}(p_{11}p_{12}I_{rr} + p_{22}p_{21}(I_t + I_{si}))}{I_{sr} + I_{rr}p_{11}^2 + p_{21}^2(I_t + I_{si})} \\
\alpha_3 &= -A_c R_c N_c \frac{(p_{11}p_{12}I_{rr} + p_{22}p_{21}(I_t + I_{si}))}{I_{sr} + I_{rr}p_{11}^2 + p_{21}^2(I_t + I_{si})}
\end{aligned} \tag{4.13}$$

In the above, we have used the fact that the oncoming clutch slip speed (ω_{sr}) does not change sign during the torque phase, and $sgn(\omega_{sr})$ can be treated as a constant (equal to -1 for the transmission system of interest).

We begin by noting that, just before the start of the torque phase, the torque converter can either be locked by the torque converter clutch, or be in the fluid coupling or torque amplification mode. We assume here that the torque converter mode remains unchanged during the torque phase since speeds do not change substantially during this phase. For the torque-amplification mode, the turbine torque depends significantly upon the torque converter characteristics in addition to the pump torque, and hence its estimation requires significant information from the transmission side. However, during the torque phase, the transmission system just has one degree of freedom, further implying that simultaneous estimation of the turbine torque and oncoming clutch pressure is not possible without additional information. Thus, for this case, the controller is forced to use Kotwicki's model for the turbine torque information. For the remaining two cases, the transmission system is coupled to the engine and thus information from the engine side can be used to estimate the turbine torque.

In the current study, we assume that the torque converter is either locked or in the fluid coupling phase. If neither is the case, we would need to use Kotwicki's model of the torque converter to help in the estimation of turbine torque. It should

also be noted that the different modes of a torque converter can be determined from the velocities of the turbine and pump shafts. Since these velocities are measured variables, information regarding the operating mode of the torque converter is always known. The model representing the dynamics of the engine crankshaft was developed in chapter 3,

$$I_e \dot{\omega}_e = T_i(\omega_e, \alpha) - T_p - T_f - b_e \omega_e \quad (4.14)$$

where I_e , ω_e , α , T_f , T_p , b_e , T_i denote the lumped engine inertia, engine speed, throttle angle, constant component of friction torque, pump torque, coefficient of viscous friction in the engine and indicated torque of the engine. Under the assumption of locked torque converter clutch or fluid coupling mode, (4.14) is modified to,

$$\dot{\omega}_e = \frac{1}{I_e} [T_i(\omega_e, \alpha) - T_t - T_f - b_e \omega_e] \quad (4.15)$$

It is further assumed that T_i, T_f, b_e are known. We again emphasize that these variables are essential for any kind of model-based engine speed control. Assuming such a control environment, the assumption that these variables are known is justified. A sliding mode observer [102] is proposed for the estimation of turbine torque during the torque phase,

$$\dot{\hat{\omega}}_e = \frac{1}{I_e} [-b_e \hat{\omega}_e + v_1] \quad (4.16)$$

$$v_1 = k_1 \text{sign}(\omega_e - \hat{\omega}_e), \quad k_1 > 0$$

which gives the following error dynamics equation

$$\dot{\tilde{\omega}}_e = \frac{1}{I_e} [-b_e \tilde{\omega}_e + T_i - T_t - T_f - v_1] \quad (4.17)$$

Now if $k_1 > |T_i| + |T_t| + |T_f|$, then the estimation error $\tilde{\omega}_e$ converges asymptotically to zero. Upper bounds on T_i, T_f, T_t are usually known which can be used to select the gain k_1 . Note that finite time convergence of $\tilde{\omega}_e$ to zero can be achieved if

$k_1 > |T_i| + |T_t| + |T_f| + |b_e \tilde{\omega}_e|$ or $k_1 > |T_i| + |T_t| + |T_f| + |b_e \tilde{\omega}_e(0)|$, where $\tilde{\omega}_e(0)$ is the initial estimation error which can be made arbitrarily small by choosing the observer's initial condition in (4.16) to be close to the sensed value of ω_e at that initial time instant. This was the methodology adopted for the current study. Once convergence of the estimation error to zero takes place, the method of equivalent control [102] can be used to derive the following

$$v_{1,eq} = T_i - T_t - T_f \quad (4.18)$$

where $v_{1,eq}$ is the equivalent *control* corresponding to $k_1 \text{sign}(\tilde{\omega}_e)$ and physically represents its low frequency component. Thus, if $v_{1,eq}$ is known, and by using previous assumptions, the turbine torque can be calculated. The equivalent control ($v_{1,eq}$) can be extracted from its corresponding high frequency signal v_1 by low-pass filtering it [102]. Thus the following low pass filter was designed,

$$\tau_1 \dot{v}_{1,f} + v_{1,f} = v_1 \quad (4.19)$$

where τ_1 is the time constant of the filter and $v_{1,f}$ is the filter output. It was shown in [102] that $v_{1,f} \rightarrow v_{1,eq}$ if and only if $\tau_1 \rightarrow 0$ and $\Delta/\tau_1 \rightarrow 0$, where $\Delta \approx 1/f_s$ and f_s is the switching frequency for the term $k_1 \text{sign}(\tilde{\omega}_e)$, which in the case of an observer is limited by the time step of the fixed step solver. This necessary and sufficient condition says that the switching should be done at an infinite rate which consequently means an infinitely fast processor. Since this is not practical, there is a first order lag between the filter output and the equivalent control. This lag increases the estimation error and deteriorates the controller performance.

We introduce a novel approach for equivalent control extraction to overcome the above problem. The output injection term v_1 was redesigned as follows.

$$v_1 = k_1 \text{sign}(\tilde{\omega}_e) + \int_0^t k'_1 \text{sign}(\tilde{\omega}_e) dt, \quad k'_1 > 0 \quad (4.20)$$

The PI form of the output injection makes the effective switching signal a low frequency signal and thus the lag is highly minimized due to further filtering action. This was confirmed by the simulation results. We will therefore assume that the output of the low pass filter is approximately equal to the equivalent control.

We proceed by applying similar technique, as above, to (4.9) to estimate the term $\mu(\omega_{sr})P_{c,2ND}$, which, along with the estimated turbine and output shaft torques, will be used to evaluate the reaction torque at the offgoing clutch (see equation (4.12)). Towards this end, the following observer is proposed.

$$\begin{aligned} \dot{\hat{\omega}}_{sr} &= \frac{1}{I_{sr} + I_{rr}p_{11}^2 + p_{21}^2(I_t + I_{si})} (-p_{11}R_d\hat{T}_s + p_{21}\hat{T}_t + v_2) \\ v_2 &= k_2 \text{sign}(\tilde{\omega}_{sr}) + \int_0^t k'_2 \text{sign}(\tilde{\omega}_{sr}) dt, \quad k_2, k'_2 > 0 \\ \tau_2 \dot{v}_{2,f} + v_{2,f} &= v_2 \end{aligned} \quad (4.21)$$

where $\tilde{\omega}_{sr} := \omega_{sr} - \hat{\omega}_{sr}$, and $\hat{\omega}_{sr}$ denotes the estimated oncoming clutch slip speed. The observer gain k_2 was selected to ensure convergence of the estimation error $\tilde{\omega}_{sr}$ to zero, following a line of arguments similar to that presented for the selection of the gain k_1 . The observer gain k_2 was tuned to minimize the first order lag between the filter output $(v_{2,f})$ and the equivalent control $(v_{2,eq})$ corresponding to the signal v_2 . Once the estimation error converges to zero, we have the following.

$$v_{2,eq} = p_{11}R_d\tilde{T}_s - p_{21}\tilde{T}_t - A_c N_c R_c \mu(\omega_{sr})P_{c,2ND} \quad (4.22)$$

where the variables with $\sim$ denote the corresponding estimation errors. In arriving at (4.22), we have used the fact that $\text{sgn}(\omega_{sr}) = -1$ during the torque phase. Thus,

if the estimation errors in the turbine and output shaft torques converge to zero, equation (4.22) reduces to

$$v_{2,eq} = -A_c R_c N_c \mu(\omega_{sr}) P_{c,2ND} \quad (4.23)$$

Thus, the variable $v_{2,eq}$ contains the estimate of the term $\mu(\omega_{sr}) P_{c,2ND}$. Under the assumption of known coefficient of friction ($\mu(\omega_{sr})$), the oncoming clutch pressure can be evaluated from $\mu(\omega_{sr}) P_{c,2ND}$ as follows,

$$\hat{P}_{c,ND} = \frac{-1}{A_c R_c N_c \mu(\omega_{sr})} v_{2,eq} \quad (4.24)$$

where $\hat{P}_{c,ND}$ denotes the estimate of the oncoming clutch pressure. However, any uncertainty in the coefficient of friction will induce a proportional uncertainty in the clutch pressure estimate. Thus the controller will be made robust against this and other estimation errors in section 4.3. The reaction torque at the offgoing clutch can be estimated using equation (4.12)

$$\hat{R}T_{LR} = \alpha_1 \hat{T}_s + \alpha_2 \hat{T}_t + \alpha_3 \mu(\omega_{sr}) \hat{P}_{c,ND} \quad (4.25)$$

where the $\hat{R}T_{LR}$ denotes the estimate of the reaction torque at the offgoing clutch. The estimate of the reaction torque at the offgoing clutch is a good indicator of the amount of load torque transferred from the offgoing to the oncoming clutch during the torque phase. Consequently, it plays a critical role in the smooth integration of torque and inertia phase controllers.

4.1.3 Inertia phase observer design

The transmission system has two degrees of freedom during the inertia phase and the corresponding governing differential equations are

$$\begin{aligned}\dot{\omega}_{sr} &= -(p_{12}\varphi_1 + p_{11}\varphi_2)R_dT_s + (p_{22}\varphi_1 + p_{21}\varphi_2)T_t - \varphi_2T_{c,2ND} - \varphi_1T_{c,LR} \\ \dot{\omega}_{cr} &= -(p_{12}\varphi_3 + p_{11}\varphi_4)R_dT_s + (p_{22}\varphi_3 + p_{21}\varphi_4)T_t - \varphi_4T_{c,2ND} - \varphi_3T_{c,LR}\end{aligned}\quad (4.26)$$

where,

$$\begin{aligned}\varphi_1 &= \frac{(p_{11}p_{12}I_{rr} + p_{21}p_{22}(I_t + I_{si}))}{\Delta} \\ \varphi_2 &= -\frac{(I_{cr} + p_{12}p_{22}I_{rr} + p_{22}^2(I_t + I_{si}))}{\Delta} \\ \varphi_3 &= \frac{(I_{sr} + p_{11}^2I_{rr} + p_{21}^2(I_t + I_{si}))}{\Delta} \\ \varphi_4 &= \varphi_1\end{aligned}\quad (4.27)$$

$$\begin{aligned}\Delta &= (p_{12}p_{11}I_{rr} + p_{22}p_{21}(I_t + I_{si}))^2 - \\ &\quad (I_{sr} + p_{11}^2I_{rr} + p_{21}^2(I_t + I_{si}))(I_{cr} + p_{12}p_{22}I_{rr} + p_{22}^2(I_t + I_{si}))\end{aligned}$$

and,

$$\begin{aligned}T_{c,2ND} &= A_cN_cR_c\mu(\omega_{sr})P_{c,2ND} \\ T_{c,LR} &= A_cN_cR_c\mu(\omega_{cr})P_{c,LR}\end{aligned}\quad (4.28)$$

As will be shown in the next section on controller design, the offgoing clutch pressure is maintained at a sufficiently low level during the inertia phase, and hence is considered negligible. Thus equations (4.26) become

$$\begin{aligned}\dot{\omega}_{sr} &= -(p_{12}\varphi_1 + p_{11}\varphi_2)R_dT_s + (p_{22}\varphi_1 + p_{21}\varphi_2)T_t - \varphi_2A_cN_cR_c\mu(\omega_{sr})P_{c,2ND} \\ \dot{\omega}_{cr} &= -(p_{12}\varphi_3 + p_{11}\varphi_4)R_dT_s + (p_{22}\varphi_3 + p_{21}\varphi_4)T_t - \varphi_4A_cN_cR_c\mu(\omega_{sr})P_{c,2ND}\end{aligned}\quad (4.29)$$

The technique introduced for model based estimation of the turbine torque and the oncoming clutch pressure in the previous section needs to be modified for the inertia phase, since the transmission system has a different model representation in this phase. In addition to this, it is highly likely that the torque converter will cease

to operate in the fluid coupling mode and be in the torque amplification mode during the inertia phase, due to a substantial change in torque converter element speeds. Also, for the same reason, the torque converter clutch is not locked during this phase. Thus a new observer needs to be designed for the estimation of the turbine torque in this phase. The turbine torque and oncoming clutch pressure were simultaneously estimated. Towards this end, we begin by rewriting equations (4.29) for compact representation

$$\begin{aligned}\dot{\omega}_{sr} &= q_1 T_s + q_2 T_t + q_3 \mu(\omega_{sr}) P_{c,2ND} \\ \dot{\omega}_{cr} &= p_1 T_s + p_2 T_t + p_3 \mu(\omega_{sr}) P_{c,2ND}\end{aligned}\tag{4.30}$$

where,

$$\begin{aligned}q_1 &= -(p_{12}\varphi_1 + p_{11}\varphi_2)R_d, \quad q_2 = (p_{22}\varphi_1 + p_{21}\varphi_2), \quad q_3 = -\varphi_2 A_c R_c N_c \\ p_1 &= -(p_{12}\varphi_3 + p_{11}\varphi_4)R_d, \quad p_2 = (p_{22}\varphi_3 + p_{21}\varphi_4), \quad p_3 = -\varphi_4 A_c R_c N_c\end{aligned}\tag{4.31}$$

The following observer is designed using the same principles as that of the torque phase sliding mode observer.

$$\begin{aligned}\dot{\hat{\omega}}_{sr} &= q_1 \hat{T}_s + v_3 = q_1 \hat{T}_s + k_3 \text{sign}(\tilde{\omega}_{sr}) + \int_0^t k'_3 \text{sign}(\tilde{\omega}_{sr}) dt, \quad k_3, k'_3 > 0 \\ \dot{\hat{\omega}}_{cr} &= p_1 \hat{T}_s + v_4 = p_1 \hat{T}_s + k_4 \text{sign}(\tilde{\omega}_{cr}) + \int_0^t k'_4 \text{sign}(\tilde{\omega}_{cr}) dt, \quad k_4, k'_4 > 0 \\ \tau_3 \dot{v}_{3,f} + v_{3,f} &= v_3, \quad \tau_4 \dot{v}_{4,f} + v_{4,f} = v_4\end{aligned}\tag{4.32}$$

where the symbols have meanings similar to those defined for the torque phase observer. The selection of gains was performed so as to ensure convergence of the estimation error to zero, and minimize the lag between the low pass filter output and the equivalent control corresponding to the switching signal. Once the convergence error goes to zero, the following can be derived using the error dynamics of the observer presented above and the method of equivalent control.

$$\begin{aligned}0 &= q_1 \tilde{T}_s + q_2 T_t + q_3 \mu(\omega_{sr}) P_{c,2ND} - v_{3,eq} \\ 0 &= p_1 \tilde{T}_s + p_2 T_t + p_3 \mu(\omega_{sr}) P_{c,2ND} - v_{4,eq}\end{aligned}\tag{4.33}$$

which on rearranging gives,

$$\begin{bmatrix} T_t \\ \mu P_{c,2ND} \end{bmatrix} = \begin{bmatrix} q_2 & q_3 \\ p_2 & p_4 \end{bmatrix}^{-1} \left(\begin{bmatrix} v_{2,eq} \\ v_{3,eq} \end{bmatrix} - \begin{bmatrix} q_1 \\ p_1 \end{bmatrix} \tilde{T}_s \right) \quad (4.34)$$

where the inverse always exists. The terms $v_{3,eq}$ and $v_{4,eq}$ denote the equivalent control corresponding to the signals v_3 and v_4 respectively. Clearly, as the estimation error of the shaft torque goes to zero, i.e., $\tilde{T}_s \rightarrow 0$, information about the turbine torque and oncoming clutch pressure can be obtained from $v_{2,eq}, v_{3,eq}$. Again, under the assumption of known μ , the oncoming clutch pressure can be calculated. Based on the information provided by the observer, a controller was designed for the closed loop control of both the torque and inertia phases.

4.2 Controller design

Due to the differences in the governing differential equations of the transmission system, control design is implemented differently for the two phases. However, the basic control philosophy remains the same for both the phases. The structure of the controller is shown in Figure 4.1. The controller is segregated into two components having a cascaded connection. The feedback linearization controller calculates the clutch pressure trajectory required for desired speed tracking, which serves as a reference input for the sliding mode controller. The key thing to notice is the modularity of the controller structure which effectively reduces the problem of designing a controller for a larger state-space to the problem of designing two controllers for smaller state spaces with different robustness requirements, which simplifies the design problem in the sense that controller designs appropriate for the different levels of robustness can be used. In the field of sliding mode control, this is known as control design by

regular form [102]. Also, once a source of uncertainty is identified, it is enough to take measures for making the corresponding controller component robust against it.

Figure 4.1: Controller Architecture

4.2.1 Introduction to sliding mode control and feedback linearization

As can be inferred from the discussion above, the current study uses a combination of sliding mode control and feedback linearization for achieving the desired control objectives, for both the torque and inertia phases. Even though these nonlinear control techniques have been well studied and documented in the literature [52],[102], a short introduction to these techniques here would enhance appreciation of the controller design in the current study. We intend to keep the presentation fairly *informal*, and focus on the aspects of these control design tools which are relevant to the current study. For a rigorous treatment of these nonlinear control design techniques, readers are referred to [52], and [102].

Sliding mode control

Sliding mode control is a well-known variable structure control technique which changes the dynamical structure of the closed loop system depending on the available feedback, to achieve desired sliding motion of the closed loop system. These desired sliding modes, *surfaces* in the state space of the dynamical system, are pre-calculated based on physical specifications such as percentage overshoot, steady state accuracy, etc. We begin by defining the following autonomous (time-invariant) nonlinear system which is affine in the control input u .

$$\dot{x} = f(x) + g(x)u, \quad x \in \mathbb{R}^n, \quad u \in \mathbb{R}^n, \quad f : \mathbb{R}^n \rightarrow \mathbb{R}^n, \quad g : \mathbb{R}^n \rightarrow \mathbb{R}^n \times \mathbb{R}^n \quad (4.35)$$

where the output (measurement) equation has been omitted under the assumption of full state feedback. This assumption can be relaxed for cases with output feedback alone, under certain structural conditions. However, the assumption is justified for the system of interest in the current study. We further assume that the vector field f is bounded, i.e.,

$$|f(x)| < f_0 > 0, \quad \forall x \in \mathbb{R}^n \quad (4.36)$$

and the matrix valued function g is invertible, for the region of interest in the state space $\mathbb{R}^n$. Let $s(x) = 0$ denote the sliding surface which captures the specifications for the controller design. The choice of the sliding surface depends mainly on the class of control problem to be solved. For the class of tracking problems, a suitable choice of the sliding surface could be the tracking error, defined as $\tilde{x} := x - x^*$, where x^* denotes the reference trajectory for the state vector x . Thus,

$$s(x) = x - x^* \quad (4.37)$$

We make the well-justified assumption that the reference trajectory x^* and its first derivative $\dot{x}^*$ are bounded, as this would imply bounded closed loop system response and bounded rate of change with respect to time. In particular, let

$$|\dot{x}^*(t)| < \rho, \quad \forall t > 0 \quad (4.38)$$

Formally, the key idea involved in designing a sliding mode controller can be explained in the Lyapunov framework [23]; however, we present a more *intuitive* take on it. The tracking problem will be considered solved if $\tilde{x}(t) = 0$, $\forall t > T \geq 0$, where T is a positive scalar. This is equivalent to achieving sliding motion of the closed loop system on the surface defined by (4.37), i.e.,

$$s(x(t)) = 0 \quad \forall t > T \geq 0 \quad (4.39)$$

Equation (4.39) implies finite time convergence of the variable $s(x(t))$ to zero. The necessary and sufficient condition for achieving finite time convergence of the variable $s(x(t))$ to zero can be *intuitively* seen to be,

$$s(x(t))\dot{s}(x(t)) < 0, \quad \forall t > 0 \quad (4.40)$$

Informally, this means that the variable $s(x(t))$ and its time derivative, $\dot{s}(x(t))$, should always have opposite signs. The control input $u(x)$, can now be synthesized to satisfy equation (4.40). Towards this end, we begin by evaluating the time derivative of the variable $s(x(t))$.

$$\begin{aligned} \dot{s} &= \dot{x} - \dot{x}^* \\ &= f(x) + g(x)u - \dot{x}^* \end{aligned} \quad (4.41)$$

Thus, if the control input u is selected as,

$$u = g^{-1}(x)M \operatorname{sgn}(s), \quad M > f_0 + \rho \quad (4.42)$$

where $g^{-1}(\cdot)$ denotes the inverse of the matrix valued function $g(\cdot)$, then (4.40) is always satisfied. It is interesting to note that the assumption of a bounded vector field need not be true for a system in general. For such cases, the controller gain M needs to satisfy,

$$M > |f(x(t))| + \rho, \quad \forall t > 0 \quad (4.43)$$

in order to achieve the desired tracking performance. Thus, for certain initial conditions, it is not difficult to see that the required control becomes unbounded and the problem of actuator saturation arises, leading to a situation where the control objectives are not met.

A counter-intuitive aspect of sliding mode control systems is the evaluation of the dynamics of the closed loop system during the sliding motion, i.e., when $s = 0$. The counter-intuition arises due to the fact that the control input u is not defined during this phase, as the value of signum function is not defined when its argument is zero. Thus, the method of equivalent control was introduced in [102] to calculate the dynamics of the closed loop system during the sliding phase. In what follows, we assume the function $g(x)$ to be identity. The method requires replacing the actual switching control $M \operatorname{sgn}(s)$ by an *equivalent* control u_{eq} during the sliding phase, where the variable s and all its higher derivative are identically zero. Thus, equation (4.41) reduces to,

$$u_{eq} = \dot{x}^* - f(x) \quad (4.44)$$

Substituting this equivalent control in the equation of motion of the open loop plant (4.35), we obtain,

$$\dot{x} = \dot{x}^* \quad (4.45)$$

which describes the dynamics of the closed loop system during the sliding motion phase. Physically, equivalent control represents the low frequency component of the corresponding switching signal $M\text{sgn}(s)$, and can be obtained by processing the high frequency signal through a low pass filter of the following form.

$$\tau \dot{u}_f + u_f = M\text{sgn}(s) \quad (4.46)$$

where τ, u_f denote the filter time constant and filter output respectively. The time constant τ should be sufficiently small to filter the unwanted high frequency components but large enough to avoid significant attenuation of the equivalent control. A formal statement capturing these design guidelines was made earlier in the section on torque phase observer design.

We would like to conclude this short introduction to sliding mode control concepts by making a few final remarks regarding a phenomenon, which has proven to be a bottleneck for the implementation of sliding mode controllers historically. We emphasize that the point of the following discussion is not to detract from the practical utility of the sliding mode controllers, as the problem of chattering has been solved [68], [26], [87] for all practical purposes, but to highlight the precautions that need to be taken during its implementation.

The phenomenon of chattering can be defined as the appearance of undesired oscillations of finite magnitude and frequency in the closed loop system response. There are two well-known causes for chattering to appear in a closed loop system: the presence of fast unmodeled dynamics in the system, and the finite sampling rate of the digital microcontrollers used for the practical implementation of sliding mode controllers. The latter is critical to the physical realization of the controller, which is not pursued in the current study and has been left as a recommendation for future

work. We are more interested here in the former, as it serves as an indirect way to validate the reduced order model for the hydraulic system, in addition to the validation performed in chapter 3. [102] show that the presence of fast unmodeled dynamics lead to oscillations in the system response through numerous examples. In the current study, a sliding mode controller is designed for the transmission hydraulic system so that the oncoming clutch pressure tracks a specified reference trajectory. The model representation of the transmission hydraulic system used for the sliding mode controller design is a reduced order approximation of the full model. It was shown in chapter 3 that the full model consists of several fast dynamics, which were eliminated using arguments from singular perturbation and state-space averaging techniques to arrive at the reduced order model representation. These fast dynamics are equivalent to the unmodeled fast dynamics in the examples noted in [102] since the evaluation of the sliding mode controllers in the current work involves simulation of the full-order shift hydraulic system model validated in experimental work by [110], and constitute a potential source for chattering. Conversely, if the closed loop system response (clutch pressure) does not show any oscillations, signifying no chattering, then the reduced order model should indeed be considered a good representation of the transmission hydraulic system, a fact we will return to in chapter 5 when we present simulation results.

Feedback linearization

Feedback linearization, or more appropriately linearization by feedback, is a nonlinear control technique where a nonlinear system is linearized using the available feedback. We will restrict our attention to systems with full state feedback, as it relates to the current study better. The feedback is used to design a control law such

that the closed loop system has linear dynamics, which is only possible if the designed control law cancels the nonlinearities present in the plant. It should be noted that the technique of linearization by feedback is different than, and bears no connection to, the approximate linearization technique based on Taylor series. We further restrict our attention to single input single output (SISO) systems where the output is assumed to have a relative degree (to be defined) equal to the dimension of the system.

We begin by analyzing the following nonlinear autonomous system, which is affine (linear) in control input u .

$$\begin{aligned}\dot{x} &= f(x) + g(x)u, \quad x \in \mathbb{R}^n, \quad u \in \mathbb{R}, \quad f : \mathbb{R}^n \rightarrow \mathbb{R}^n, \quad g : \mathbb{R}^n \times \mathbb{R} \\ y &= h(x), \quad y \in \mathbb{R}, \quad h : \mathbb{R}^n \rightarrow \mathbb{R}\end{aligned}\tag{4.47}$$

It should be noted that the measurement equation stated above is not $y = x$, which is usually the case for systems with full state feedback. The possibility that a system can be linearized completely or partially using feedback depends primarily on the structure of the system (given by the state equation) and the output function $h(x)$. Thus $h(x)$ becomes a design variable, which can be selected to be some linear and/or nonlinear combination of the available states.

It is natural to determine conditions for the existence of a feedback law that would render the closed loop system linear. To answer this question, we need to define the *relative degree* of a dynamical system, which is a structural property of the system. Towards this end, let $L_f h(x)$ denote the Lie Derivative of h along the vector field f . Thus,

$$L_f h(x) = \frac{\partial h(x)}{\partial x} f(x)\tag{4.48}$$

The definition of the relative degree of a SISO system is given next [62]. The nonlinear system (4.47) is said to have relative degree ρ , $1 \leq \rho \leq n$, in a region $D_0 \subset \mathbb{R}^n$ if

$$L_g L_f^{i-1} h(x) = 0, \quad i = 1, 2, \dots, \rho - 1; \quad L_g L_f^{\rho-1} h(x) \neq 0, \forall x \in D_0 \quad (4.49)$$

The above definition, for all practical purposes, states that the relative degree of a SISO system is the minimum number of differentiations of the output function $h(x)$, that needs to be carried out for the control input u to appear in the Lie derivative. Also, it should be clear from the definition that the relative degree of a system might not be defined over some regions of the state space. These are usually points of singularities in state space.

To comprehend this remark better, consider the following example [52]. The differential equations in (4.50) model a *ball* of point mass m on a frictionless beam of infinite length and moment of inertia J . The position of the ball with respect to the origin is given by r , while φ denotes the angle by which the beam is rotated with respect to the horizontal line (Figure 4.2). The constant g denotes the acceleration due to gravity.

Figure 4.2: *A ball on a beam* problem schematic

$$\begin{aligned}
\dot{r} &= v \\
\dot{v} &= -g \sin \varphi + r \omega^2 \\
\dot{\varphi} &= \omega \\
\dot{\omega} &= -\frac{2mr v \omega}{J + mr^2} - \frac{mrg \cos \varphi}{J + mr^2} + u
\end{aligned} \tag{4.50}$$

Denote by $x_0 = (r_0, v_0, \varphi_0, \omega_0)$ any operating point of the four dimensional state space. For the system described above, with r selected as output, i.e., $y = r$, the relative degree of the system is not defined for any point of the form $(r_0, 0, 0, 0)$. This can be seen from the successive differentiations of the output,

$$\begin{aligned}
y &= r \\
\dot{y} &= v \\
\ddot{y} &= -g \sin \varphi + r \omega^2 \\
\ddot{\ddot{y}} &= -g \cos \varphi \dot{\varphi} + 2r \omega \dot{\omega} \\
&= -g \cos \varphi \dot{\varphi} + 2r \omega \left(-\frac{2mr v \omega}{J + mr^2} - \frac{mrg \cos \varphi}{J + mr^2} + u \right)
\end{aligned} \tag{4.51}$$

Clearly, the control input does not appear in the equation for $\ddot{\ddot{y}}$ if $\omega = 0$, and the relative degree of the system is not defined. However, if rotation angle of the beam is selected as the output, i.e., $y = \varphi$, then the system has a globally defined relative degree. This shows the importance of selecting an appropriate output function $h(x)$ in (4.47), and corroborates the remark made earlier about the output function being a design variable.

We now define the necessary and sufficient condition for existence of a feedback law which would render the closed loop system corresponding to a nonlinear plant linear. A nonlinear system, of the form (4.47), can be *completely* linearized by feedback if the relative degree of the system is equal to the dimension of the state space. In

this case, the system is said to have full relative degree. Partial linearization of a nonlinear system is possible if relative degree of the system is less than its dimension, in which case the dynamics appearing in the input-output map are linear. However, some of the dynamics use nonlinear laws, which are rendered unobservable by the feedback used for partial linearization, and constitute the internal dynamics of the system. We are interested in systems with full relative degree. Thus we assume that the nonlinear system defined by (4.47) has full relative degree.

In order to derive the linearized form of the closed loop system, a transformation of the original system given by (4.47) is required. Towards this end, define the following nonlinear transformation.

$$\eta = T(x) \quad (4.52)$$

where $T(x)$ is a diffeomorphism (analogous to its linear counter-part, change of basis) given by,

$$T(x) = \begin{bmatrix} h(x) \\ L_f h(x) \\ \vdots \\ L_f^{(n-1)} h(x) \end{bmatrix} \quad (4.53)$$

The transformed system has the following form.

$$\begin{bmatrix} \dot{\eta}_1 \\ \dot{\eta}_2 \\ \vdots \\ \dot{\eta}_{n-1} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ & & & \ddots & \\ 0 & 0 & \dots & 0 & 1 \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \\ \vdots \\ \eta_{n-1} \\ \eta_n \end{bmatrix} \quad (4.54)$$

$$\dot{\eta}_n = \alpha(\eta) + \beta(\eta)u$$

$$y = \eta_1$$

where,

$$\alpha(\eta) = L_f^n h(x)|_{x=T^{-1}(\eta)}, \quad \beta(\eta) = L_g L_f^{n-1} h(x)|_{x=T^{-1}(\eta)} \quad (4.55)$$

The transformation lumps distributed nonlinearities present in the system, in the original coordinates, into the dynamics of the *directly controllable* state in the new coordinates, where the nonlinearities can be canceled by appropriately selecting the control input u . The feedback law which linearizes the given nonlinear system can be easily seen, in the new coordinates (η) , to be,

$$u = \frac{1}{\beta(\eta)}(-\alpha(\eta) + v) \quad (4.56)$$

where v is the input to the linearized system, which is designed to achieve the specified control objective. By the very definition of relative degree, the term $\beta(\eta) \neq 0$. In the η -coordinates, the input-output map of the closed loop system is given by,

$$\frac{y(s)}{v(s)} = \frac{1}{s^n} \quad (4.57)$$

Thus, the input-output map is a chain of n integrators. A linear controller can now be designed for the linearized plant to achieve certain performance goals. Figure 4.3 summarizes the technique of controller design by feedback linearization. In the Figure, $C(s)$ is a linear controller designed to achieve some performance objective, such as making the output y track the reference r . The closed loop system consists of two loops: the inner loop linearizes the nonlinear system into a chain of integrators, and the outer loop controls this chain of integrators to solve the control problem.

Figure 4.3: Controller design by feedback linearization

A few general remarks regarding the feedback linearization technique are in order. The method relies heavily on exact knowledge of the model, which further implies that any modeling error or parametric uncertainty will lead to deterioration of the closed loop performance. Thus, robustness of the closed loop system should be ensured by the appropriate design of the linear controller $C(s)$. Also, it should be obvious that part of the control effort is consumed just to linearize the system by feedback. Moreover, the method is blind to the type of nonlinearities, i.e., it does not differentiate *good* nonlinearities from *bad* ones. Consider the following scalar system.

$$\dot{x} = -x^3 + u + d \quad (4.58)$$

where x is the state variable, u is the control input and d is a constant disturbance to be rejected. A controller can be designed by using the technique of feedback linearization to reject the effect of the disturbance d on the output x . One possible

feedback control law which achieves this is,

$$\begin{aligned} u &= x^3 + v \\ v &= -K_p x - K_I \int x dt \end{aligned} \tag{4.59}$$

It is trivial to see that the linear controller $C(s)$, in this case, is a PI controller with K_p, K_I as the controller gains. The point to note is that a part of the control effort is wasted in canceling the *useful* nonlinearity x^3 . The nonlinearity is useful in the sense that it helps in stabilizing the origin of the system since

$$\dot{x} = -x^3 \tag{4.60}$$

is asymptotically stable in the sense of Lyapunov. In summary, the feedback linearization technique is not robust against modeling errors and might lead to an (unnecessarily) *expensive* control law. Thus, the technique must not be blindly used for any feedback linearizable nonlinear system.

We now apply the concepts introduced for sliding mode control and feedback linearization techniques to the transmission system of interest. As was briefly discussed earlier, the controller designed for the current study consists of a combination of feedback linearization and sliding mode controllers. The former computes the desired clutch pressure trajectory in order for the oncoming clutch slip speed to track a specified reference. The computed clutch pressure trajectory serves as the reference input for the oncoming clutch pressure, which is controlled by the latter. Thus, the sliding mode controller is designed to solve the trajectory tracking problem for the oncoming clutch pressure, the reference trajectory being computed by the feedback linearization controller.

4.2.2 Sliding mode control of the transmission shift hydraulic system

A reduced order model for the transmission shift hydraulic system was derived in chapter 3, and is given below.

$$\begin{aligned} \dot{\mathbf{x}}_{a,avg} &= \gamma f^+ + (1 - \gamma) f^-, \quad 0 \leq \gamma \leq 1 \\ P_c &= \frac{K_a}{A_a} \mathbf{x}_{a,avg} \end{aligned} \quad (4.61)$$

where,

$$\begin{aligned} f^+ &= \frac{1}{A_a} \left(C_d A_{incE}(x_{pcv,max}) \sqrt{\frac{2(P_s - P_c)}{\rho}} \right) \\ f^- &= \frac{1}{A_a} \left(C_d A_{excE}(x_{pcv,min}) \sqrt{\frac{2P_c}{\rho}} \right) \end{aligned} \quad (4.62)$$

where $\mathbf{x}_{a,avg}$ denotes the averaged trajectory of the accumulator piston position, γ is the commanded duty cycle of the PWM solenoid valve, P_c denotes the clutch pressure, K_a, A_a denote the accumulator spring constant and piston area respectively, $A_{incE}(x_{pcv,max})$ denotes the effective area during the inflow phase corresponding to the maximum displacement of the pressure control valve (PCV) piston position ($x_{pcv,max}$), $A_{excE}(x_{pcv,min})$ denotes the effective area during the exhaust phase corresponding to the minimum displacement of the PCV piston position ($x_{pcv,min}$), P_s denotes the supply (line) pressure and ρ denotes the density of the transmission fluid. The commanded duty cycle γ and the clutch pressure P_c serve the role of the control input and the output for the dynamical system described by (4.61) respectively. To see this better, (4.61) can be rearranged as,

$$\begin{aligned} \dot{\mathbf{x}}_{a,avg} &= f^- + (f^+ - f^-) \gamma \\ P_c &= \left(\frac{K_a}{A_a} \right) \mathbf{x}_{a,avg} \end{aligned} \quad (4.63)$$

It should be noted that the control input γ takes values in the closed set $[0, 1]$, which is different from what is usually the case for control inputs in sliding mode control

systems, where the control input assumes values symmetric about zero. We design the following linear transformation for the control input γ in order to adhere to the convention.

$$u = 2\gamma - 1 \quad (4.64)$$

The equations in (4.63) can be combined and rewritten to incorporate this transformation as,

$$\dot{P}_c = \frac{K_a}{A_a} \left(\frac{f^+ + f^-}{2} + \frac{f^+ - f^-}{2} u \right), \quad -1 < u < 1 \quad (4.65)$$

The problem of trajectory tracking can now be formally stated: for a given reference clutch pressure trajectory P_c^* , design a sliding mode controller such that the trajectory tracking error, $\tilde{P}_c$, converges to zero asymptotically. Sliding mode controllers possess the property of finite time convergence, i.e., the tracking error goes to zero after some finite time, if feedback is available from sensor measurements. However, if the signals involved in the feedback are estimated with the estimates converging asymptotically to exact values, then this attractive property of finite time convergence is lost. In the current study, availability of the oncoming clutch pressure sensor is not assumed, and the required clutch pressure is estimated asymptotically. Consequently, the control problem statement requires only asymptotic convergence of the tracking error to zero.

The following sliding surface (manifold) was designed to solve the stated control problem,

$$s = P_c - P_c^* \quad (4.66)$$

Following the design methodology presented previously, the rate of change of this sliding variable was calculated as,

$$\dot{s} = \frac{K_a}{A_a} \left(\frac{f^+ + f^-}{2} \right) + \frac{K_a}{A_a} \left(\frac{f^+ - f^-}{2} \right) u - \dot{P}_c^* \quad (4.67)$$

In order to ensure the condition

$$s\dot{s} < 0 \quad (4.68)$$

the control input u was selected as,

$$\begin{aligned} u &= u_{eq} - \epsilon \operatorname{sgn}(s) \\ u_{eq} &= \frac{2A_a}{K_a(f^+ - f^-)} \left(-\frac{K_a}{A_a} \left(\frac{f^+ + f^-}{2} \right) + \dot{P}_c^* \right) \end{aligned} \quad (4.69)$$

The control input has two component: u_{eq} denotes the equivalent control while the second term, $\epsilon \operatorname{sgn}(s)$, is added for robustness. The quantity ϵ can be a constant parameter that needs to be calibrated, or it can be calculated as a function of states using Lyapunov arguments [92]. In the current study, a constant value of ϵ was used. Also, $f^+ > 0$, $f^- < 0$ during a gearshift, which implies that the factor $(f^+ - f^-) \neq 0$; further implying that the control input is well defined during the gearshift.

Some additional comments regarding the controller implementation are in order. First, the control input should switch between -1 and 1 , which implies that a saturation block must be employed for the simulation purposes. Second, the control input should be synthesized using a zero-order hold, the sampling frequency being chosen to be the same as the PWM valve, 64 Hz for the valve used in the production transmission studied here.

4.2.3 Torque phase controller design

The primary goal in torque phase controller design is to prevent any mis-coordination of the offgoing and oncoming clutches resulting in engine tie-up or flare, as both of

these situations result in a larger torque hole in the output shaft torque response. A secondary goal is to control the duration of the torque phase. The two objectives were met by the following controller. The primary goal of coordinating the two clutches can be achieved by monitoring the estimated reaction torque at the offgoing clutch and designing a controller such that the offgoing clutch behaves like a one-way clutch, i.e. the offgoing clutch transfers the load only in one direction. This further means that the offgoing clutch pressure must be manipulated to maintain a clutch torque capacity which is higher than the offgoing clutch reaction torque for all times during the torque phase and converges to zero with it. Zero clutch torque capacity implies non-zero slip velocity of the offgoing clutch and thus denotes the end of the torque phase. This can be done in a closed-loop fashion for transmission systems equipped with a pressure sensor for the offgoing clutch, as it is not possible to estimate the offgoing clutch pressure using just the speed measurements during the torque phase. For the current study, availability of the offgoing clutch pressure sensor is assumed so that we may demonstrate one possible way to do this.

The desired torque capacity of the offgoing clutch ($T_{c,LR}^*$) is defined to be a linear function of the estimated reaction torque, i.e.

$$T_{c,LR}^* = \varepsilon \hat{R}T_{LR}, \quad \varepsilon > 1 \quad (4.70)$$

where ε is a constant greater than one. The pressure at the offgoing clutch will be manipulated to achieve this desired torque capacity. The constant ε is a safety factor (cushion) in the face of uncertainty in the estimate of the reaction torque at the offgoing clutch. If some maximum estimate of the estimation error of the reaction torque ($\tilde{R}T_{LR}$) at the offgoing clutch, denoted by $\tilde{R}T_{LR,max}$, is known, then ε can be

selected to ensure the following at the start of the torque phase,

$$T_{c,LR}^*(t_o) > \hat{R}T_{LR}(t_o) + \tilde{R}T_{LR,max} \quad (4.71)$$

where t_o denotes the start of the torque phase. This torque capacity can be converted to the desired clutch pressure of the offgoing clutch using (4.28).

$$P_{c,LR}^* = \frac{1}{\mu(\Delta\omega_c)A_cR_cN_csgn(\Delta\omega_c)}T_{c,LR}^* \quad (4.72)$$

The desired clutch pressure trajectory so obtained is sent to the sliding mode controller where, by using feedback from the offgoing clutch pressure sensor, the problem of clutch pressure trajectory tracking is solved. For transmission systems without any clutch pressure sensors, this needs to be done in an open loop fashion. The only piece of information that is needed then is the delay between the command given to the PWM solenoid valve to release the clutch, and the actual release of the clutch, which is usually known to a sufficient degree of accuracy. In case of uncertainty in this information, adaptive measures can be taken on a shift-to-shift basis. This approach, however, has not been pursued in the reported study, but is intended as future work.

In order to control the duration of the torque phase, we start by analyzing (10), which is rewritten below for compact representation.

$$RT_{LR} = \beta_1 T_s + \beta_2 T_t + \beta_3 \dot{\omega}_{sr} \quad (4.73)$$

where,

$$\beta_1 = -p_{12}R_d, \quad \beta_2 = p_{22}, \quad \beta_3 = I_{sr} + I_{rr}p_{11}^2 + p_{21}^2(I_t + I_{si}) \quad (4.74)$$

Since there are only small speed changes during the torque phase, we assume that $\dot{\omega}_{sr} \approx 0$. Thus (4.38) becomes,

$$RT_{LR} \approx \beta_1 T_s + \beta_2 T_t \quad (4.75)$$

Due to the fact that the engine indicated torque is not manipulated in the current study, the turbine torque roughly remains constant,

$$\dot{RT}_{LR} \approx \beta_1 \dot{T}_s \quad (4.76)$$

Define,

$$\xi := -\frac{\hat{RT}_{LR}(t_o)}{\Delta t_T} \quad (4.77)$$

where $\hat{RT}_{LR}(t_o)$ denotes the estimated value of the reaction torque at the start of the torque phase and Δt_T is the desired duration of the torque phase. Now, the problem of controlling the duration of the torque phase can be converted into an equivalent, but more tractable, trajectory tracking problem for the speed of the final drive output shaft (ω_o). The reference trajectory (ω_o^*) for ω_o was defined as,

$$\omega_o^* = \omega_v + \frac{\xi}{K_s \beta_1} \quad (4.78)$$

Under the assumption that ω_o converges to ω_o^* , using equation (4.1),

$$\dot{T}_s = \frac{\xi}{\beta_1} \quad (4.79)$$

This gives, using equation (4.41),

$$\dot{RT}_{LR} \approx \xi \quad (4.80)$$

Integrating both sides, we have

$$RT_{LR}(t_o + \Delta t_T) - RT_{LR}(t_o) \approx \xi \Delta t_T \quad (4.81)$$

Now, using the definition of ξ and under the assumption that $\hat{RT}_{LR}(t_o) \rightarrow RT_{LR}(t_o)$,

$$RT_{LR}(t_o + \Delta t_T) \approx 0 \quad (4.82)$$

which shows that the duration of the torque phase is Δt_T . It should be noted that the quantity $\hat{R}T_{LR}(t_o)$ was not calculated using the torque phase observer described above, but by using a similar observer for the transmission system in the first gear. This is important to understand since the torque phase observer will take finite time to converge and thus the assumption of $\hat{R}T_{LR}(t_o) \rightarrow RT_{LR}(t_o)$ will not be valid, no matter how small that time duration is. The only additional result that remains to be shown is the convergence of ω_o to ω_o^* .

During the torque phase, ω_o is kinematically related to the oncoming clutch slip speed (ω_{sr}) as,

$$\omega_o = -R_d \frac{R_{sr}}{R_{rr}} \omega_{sr} \quad (4.83)$$

where,

$$R_{sr} = \frac{N_{sr}}{N_{sr} + N_{rr}}, \quad R_{rr} = \frac{N_{rr}}{N_{sr} + N_{rr}} \quad (4.84)$$

N_{sr}, N_{rr} denote the numbers of teeth on the reaction side sun and ring gear respectively. Thus, equation (4.9) can be rewritten as,

$$\dot{\omega}_o = d_1 T_s + d_2 T_t + d_3 \mu(\omega_{sr}) P_{c,2ND} \quad (4.85)$$

where,

$$\begin{aligned} d_1 &= \left(R_d \frac{R_{sr}}{R_{rr}} \right) \frac{p_{11} R_d}{I_{sr} + I_{rr} p_{11}^2 + p_{21}^2 (I_t + I_{si})} \\ d_2 &= - \left(R_d \frac{R_{sr}}{R_{rr}} \right) \frac{p_{21}}{I_{sr} + I_{rr} p_{11}^2 + p_{21}^2 (I_t + I_{si})} \\ d_3 &= \left(R_d \frac{R_{sr}}{R_{rr}} \right) \frac{A_c N_c R_c}{I_{sr} + I_{rr} p_{11}^2 + p_{21}^2 (I_t + I_{si})} \end{aligned} \quad (4.86)$$

Using the feedback linearization control technique, the following desired clutch pressure trajectory is proposed.

$$\begin{aligned} P_{c,2ND}^* &= \frac{1}{d_3 \mu(\omega_{sr})} (-d_1 \hat{T}_s - d_2 \hat{T}_t + \dot{\omega}_o^* + v) \\ v &= -\lambda_T (\omega_o - \omega_o^*) \end{aligned} \quad (4.87)$$

where λ_T is a positive constant which determines the rate of convergence of the tracking error ($\tilde{\omega}_o := \omega_o - \omega_o^*$) to zero. Under the assumption that the estimation errors for T_s, T_t converge to zero sufficiently fast, we have (from (4.85),(4.87))

$$\dot{\tilde{\omega}}_o = -\lambda_T \tilde{\omega}_o \quad (4.88)$$

which clearly shows that $\tilde{\omega}_o \rightarrow 0$ asymptotically. The clutch pressure trajectory tracking is achieved through the sliding mode controller which uses the estimated value of the oncoming clutch pressure, as for the transmission system of interest, the availability of the oncoming clutch pressure sensor is not assumed. This concludes the description of torque phase controller design.

4.2.4 Inertia phase controller design

The inertia phase controller should be designed to ensure a short inertia phase duration and smooth clutch slip speed change. A short inertia phase duration implies longevity of the friction elements (friction clutches and band brakes) of the transmission system, albeit at the potential compromise of shift *smoothness*. The two objectives are conflicting in nature and thus the design of the reference clutch slip speed is critically important. Due to the continuing lack of formal approaches for the selection of such a reference, the lack of a formal framework for resolving the conflicting demands of the inertia phase being the reason, it is of paramount importance to select the reference trajectory using simpler physical reasoning. The following reference clutch slip speed trajectory (ω_{sr}^*) is selected in the current study [32].

$$\omega_{sr}^*(t) = \frac{2\omega_{sr}(t_i)}{(t_f - t_i)^3}(t - t_i)^3 - \frac{3\omega_{sr}(t_i)}{(t_f - t_i)^2}(t - t_i)^2 + \omega_{sr}(t_i) \quad (4.89)$$

where t_i, t_f represent the start and end times of the inertia phase, $t \in [t_i, t_f]$, and $\omega_{sr}(t_i)$ denotes the sensed slip speed of the oncoming clutch at the start of the inertia phase.

Figure 4.4 shows one such trajectory with $t_i = 3.4$ s, $t_f = 4.2$ s, $\omega_{sr}(t_i) = -160$ rad/s. A few remarks regarding the selected reference trajectory are in order. The initial tracking error ($\tilde{\omega}_{sr}(t_i) := \omega_{sr}(t_i) - \omega_{sr}^*(t_i)$) and derivative of the reference trajectory are zero at the start of the inertia phase, which prevents controller saturation at this time instant. Also, the derivative of the desired reference trajectory is zero at the end of the inertia phase which ensures zero clutch lockup torque.

The inertia phase controller uses a combination of feedback linearization and sliding mode controllers to achieve the desired clutch slip trajectory tracking. First, note that, due to the fact that the offgoing clutch is completely released at the end of the torque phase, the differential equation for the oncoming clutch slip speed during the inertia phase becomes,

$$\dot{\omega}_{sr} = q_1 T_s + q_2 T_t + q_3 \mu(\omega_{sr}) P_{c,2ND} \quad (4.90)$$

Figure 4.4: Reference trajectory for the oncoming clutch slip speed

Adopting a procedure similar to the one described for the tracking control of final drive output shaft speed (ω_o), the following desired oncoming clutch pressure trajectory is proposed.

$$P_{c,2ND}^* = \frac{1}{q_3\mu(\omega_{sr})}(-q_1\hat{T}_s - q_2\hat{T}_t + \dot{\omega}_{sr}^* + v) \quad (4.91)$$

$$v = -\lambda_I(\omega_{sr} - \omega_{sr}^*)$$

where $\lambda_I > 0$ controls the rate of convergence of the tracking error $\tilde{\omega}_{sr}$ to zero, which can be seen from the resulting error dynamics equation,

$$\dot{\tilde{\omega}}_{sr} = -\lambda_I\tilde{\omega}_{sr} \quad (4.92)$$

In deriving the above error dynamics, we have assumed that the estimation errors of the turbine and shaft torques converge to zero sufficiently fast. Again, the desired clutch pressure trajectory is used by the sliding mode controller to make the oncoming clutch pressure track it. In the absence of any estimation error and modeling

uncertainty, the combination of torque and inertia phase controllers achieves good shift quality as will be shown in the next chapter. However, it is not reasonable to assume zero estimation error and/or zero modeling uncertainty. Thus, robustness of the controller to these errors needs to be evaluated, and appropriate measures taken to incorporate a sufficient level of robustness in the controller.

4.2.5 Robust controller design

For each phase of the shift, the controller uses a combination of feedback linearization and sliding mode control techniques. It is a widely accepted fact that the sliding mode controller is robust against parametric uncertainties and external disturbances which satisfy the matching condition [23]. The matching condition implies that the disturbance (and parametric uncertainties, which can be modeled as disturbances) enters the system dynamics through the same channels as the control input, i.e., the disturbance should share the same range space as the control input. On the other hand, as was previously discussed, the feedback linearization technique requires exact information about the model and its performance degrades with any kind of uncertainty. In the current study, estimation errors and the coefficient of friction were identified as the two major sources of uncertainty. It is assumed that the change of coefficient of friction over time is positive, which is to say that the friction can only increase through aging. Due to the absence of a reliable aging model for clutch friction, the assumption is difficult to justify formally. Uncertainties used in the current study are of the additive type and are denoted by the symbols $\Delta T_s, \Delta T_t, \Delta\mu(\omega_{sr})$, representing estimation errors for the shaft torque, turbine torque and uncertainty in the oncoming clutch friction coefficient respectively.

Three strategies will be discussed next to ensure robustness of the proposed controller against the sources of uncertainty mentioned above. The first strategy employs an adaptive technique to compensate for the uncertainty in the oncoming clutch friction coefficient, which is a major source of uncertainty for most production transmissions since credible online estimation techniques for the clutch coefficient of friction are not available, at least in the open literature. The second strategy incorporates robustness in the closed loop system against different sources of uncertainty by using loop shaping techniques. It should be noted that the linear controller used in conjunction with the feedback linearized system (see Figure (4.3)) for the proposed controller is just a proportional gain (λ_T, λ_I for the torque and inertia phases, respectively). The loop shaping technique relies upon modifying this linear controller to minimize the sensitivity of the closed loop system to the disturbances. The second strategy is appealing due to the availability of rich literature for robust linear controller design. However, the design usually turns out to be conservative for cases where any apriori knowledge of the frequency spectrum of the disturbance is not available. The third strategy employs an unknown input sliding mode observer (UI-SMO) [24] to reconstruct the disturbance and subsequently nullify its effect on the closed loop performance. The technique is novel and does not require any apriori knowledge of the disturbance, which is an artifact of the property that any sliding mode technique realizes infinite loop gain.

Adaptive feedback linearization

Clutch friction characteristics change due to various reasons such as deterioration of the automatic transmission fluid, clutch wear, temperature changes, etc. Reliable techniques for online estimation of the clutch coefficient of friction are not available,

implying that the assumption of a perfectly known model is not valid, which is key to the successful implementation of any feedback linearization based controller. Either the controller can be made robust against such a parametric uncertainty or adaptive measures can be taken to compensate for it. We rely upon the latter here in order to accommodate coefficient of friction uncertainty, as the former leads to conservative designs.

Any uncertainty in the friction characteristics of the oncoming clutch is reflected in the dynamics of its slip speed, which can be seen from the following equation.

$$\dot{\omega}_{sr} = q_1 T_s + q_2 T_t + q_3 \mu(\omega_{sr}) P_{c,2ND} \quad (4.93)$$

Thus, the oncoming clutch slip speed, which is available from measurement, will be used to derive an adaptation law for modifying the feedback linearization controller proposed earlier. Towards this end, an observer is designed to estimate oncoming clutch slip speed and consequently by using Barbalat's lemma [94], an adaptation law for the clutch friction coefficient is developed. We make the following two assumptions: the friction coefficient varies slowly with respect to time which can be adequately justified; and availability of the oncoming clutch pressure sensor, which is true for clutch-to-clutch shifts in dual clutch transmissions (DCTs).

We start with the following observer, where $L > 0$ is the observer gain, $\hat{\omega}_{sr}, \hat{\mu}, \hat{T}_s, \hat{T}_t$ are estimates of slip speed, friction coefficient, output shaft torque and turbine torque respectively, and $\sim$ denotes the corresponding error variable.

$$\dot{\hat{\omega}}_{sr} = q_1 \hat{T}_s + q_2 \hat{T}_t + q_3 \hat{\mu} P_{c,2ND} - L(\hat{\omega}_{sr} - \omega_{sr}) \quad (4.94)$$

Thus the following error dynamics are obtained,

$$\dot{\tilde{\omega}}_{sr} = q_1 \tilde{T}_s + q_2 \tilde{T}_t + q_3 \tilde{\mu} P_{c,2ND} - L\tilde{\omega}_{sr} \quad (4.95)$$

A globally radially unbounded positive definite Lyapunov function was selected as,

$$V = \frac{1}{2}(\tilde{\omega}_{sr}^2 + \tilde{\mu}^2) \quad (4.96)$$

which gives the following lie derivative,

$$\dot{V} = \tilde{\omega}_{sr}(q_1\tilde{T}_s + q_2\tilde{T}_t - L\tilde{\omega}_{sr}) + \tilde{\mu}(q_3\tilde{\omega}_{sr}P_{c,2ND} + \dot{\tilde{\mu}}) \quad (4.97)$$

Under the assumption that the estimates of the output shaft torque and turbine torque converges to their true values sufficiently fast, (4.97) reduces to,

$$\dot{V} = -L\tilde{\omega}_{sr}^2 + \tilde{\mu}(q_3\tilde{\omega}_{sr}P_{c,2ND} + \dot{\tilde{\mu}}) \quad (4.98)$$

Thus, if the error in the clutch friction coefficient is updated in the following manner, equation (4.99), then a negative semi-definite Lie derivative of Lyapunov function is obtained.

$$\dot{\tilde{\mu}} = -q_3\tilde{\omega}_{sr}P_{c,2ND} \quad (4.99)$$

$$\Rightarrow \dot{V} = -L\tilde{\omega}_{sr}^2 \quad (4.100)$$

Now, since the function $\dot{V}$ is uniformly continuous, by Barbalat's lemma the error in the slip speed and clutch friction estimates tend to zero, i.e.

$$\tilde{\omega}_{sr} \rightarrow 0, \tilde{\mu} \rightarrow 0 \quad (4.101)$$

Under the assumption of a slowly varying friction coefficient, equation (4.99) becomes,

$$\dot{\tilde{\mu}} = q_3\tilde{\omega}_{sr}P_{c,2ND} \quad (4.102)$$

which serves as an adaptation law. In order to make the feedback linearization controller adaptive, μ is replaced by its estimate, in equations (4.87) and (4.91). Thus,

the adaptive feedback linearization law for the torque phase becomes,

$$\begin{aligned} P_{c,2ND}^* &= \frac{1}{d_3 \hat{\mu}} (-d_1 \hat{T}_s - d_2 \hat{T}_t + \dot{\omega}_o^* + v) \\ v &= -\lambda_T (\omega_o - \omega_o^*) \end{aligned} \quad (4.103)$$

and for the inertia phase,

$$\begin{aligned} P_{c,2ND}^* &= \frac{1}{q_3 \hat{\mu}} (-q_1 \hat{T}_s - q_2 \hat{T}_t + \dot{\omega}_{sr}^* + v) \\ v &= -\lambda_I (\omega_{sr} - \omega_{sr}^*) \end{aligned} \quad (4.104)$$

Robustness through loop shaping technique

Loop shaping techniques have served as a primary tool for incorporating robustness in the closed loop configuration for linear systems [122]. In the current study, the nonlinear system of interest is rendered linear by using feedback, which is then controlled by a linear control law to solve the trajectory tracking problem for the oncoming clutch slip speed. In its current configuration, this linear control law (v) is just a proportional gain (see equations (4.87),(4.91) or equivalently equations (4.103),(4.104)). Modification of this simple linear controller will lead to increased levels of robustness for the closed loop configuration. Due to the similarities in the feedback linearization control law for the two phases, the following analysis is restricted to the inertia phase.

It can be easily shown that, under the influence of uncertainty in coefficient of friction and estimation errors, the error dynamics equation (4.92) now becomes,

$$\begin{aligned} \dot{\tilde{\omega}}_{sr} &= -\lambda_I \delta_1 \tilde{\omega}_{sr} + \delta_2 \\ \delta_1 &= 1 + \frac{\Delta\mu}{\mu} \\ \delta_2 &= \left(1 + \frac{\Delta\mu}{\mu}\right) (-q_1 T_s - q_2 T_t + \dot{\omega}_{sr}^*) \\ &\quad - q_3 \mu (\omega_{sr}) P_{c,2ND} + q_1 \Delta T_s + q_2 \Delta T_t \end{aligned} \quad (4.105)$$

which represents a combination of a multiplicative and an additive disturbance. Using the previously stated assumption regarding the coefficient of friction, we can see that $\lambda_I \delta_1 > 0 \forall t > 0$. Thus, the internal stability of the error dynamics is preserved under the influence of various sources of uncertainty. Moreover, the effect of the additive disturbance (δ_1) on the error dynamics can be arbitrarily reduced by selecting a sufficiently high gain (λ_I). However, since the closed loop system depends on a signal coming from a speed sensor which might be noisy, lower gains are preferred to keep the bandwidth of the system low. In order to achieve lower proportional gain, the linear controller (v), used in conjunction with the feedback linearization technique, was changed to the following,

$$\begin{aligned} P_{c,2ND}^* &= \frac{1}{q_3 \mu(\omega_{sr})} (-q_1 \hat{T}_s - q_2 \hat{T}_t + \dot{\omega}_{sr}^* + v) \\ v &= -\lambda_{Ip}(\omega_{sr} - \omega_{sr}^*) - \lambda_{Ii} \int_{t_i}^{t_f} (\omega_{sr} - \omega_{sr}^*) dt \end{aligned} \quad (4.106)$$

where $\lambda_{Ip}, \lambda_{Ii}$ represent proportional and integral gains respectively. Thus, a proportional control law was modified to include an integral term in order to reject low frequency disturbances. Here, the inherent assumption is that the disturbance δ_2 is primarily a low frequency signal. In general, if some information about the frequency spectrum of δ_2 is known apriori, through experimentation or application of the unknown input observer (UIO), an optimization problem can be solved [72] to generate better robust control laws. For the new linear controller, the effect of disturbance δ_2 on the tracking error $\tilde{\omega}_{sr}$ can be shown to be,

$$\tilde{\omega}_{sr}(s) = \frac{s}{s^2 + \delta_1 \lambda_{Ip} s + \delta_1 \lambda_{Ii}} \delta_2(s) \quad (4.107)$$

It is easy to see that the closed loop system is still internally stable (as $\delta_1 > 1$, by assumption). Again, as was mentioned earlier, the controller gains can be selected as

$|\lambda_{Ii}| \gg |\lambda_{Ip}| \gg 1$ to *arbitrarily* reduce the effect of the additive disturbance (δ_2) (of any kind) on the error dynamics, as is done in the design of high gain observers [62]. Moreover, there is no risk of a potential *peaking* phenomenon since the initial clutch slip speed tracking error ($\tilde{\omega}_{sr}(t_i)$) is zero by design, where t_i denotes the time corresponding to the start of the inertia phase. However, in order to avoid any deterioration of the controller performance that might be caused by the sensor noise, controller gains were selected to reject the low frequency disturbances and high frequency noise. Clearly, the process of gain selection is adhoc in absence of any apriori knowledge about the disturbance δ_2 .

We have seen that the method of adaptive feedback linearization is restricted to the case where uncertainty in the clutch coefficient of friction is the only form of uncertainty present in the model. Also, the assumption on availability of the on-coming clutch pressure sensor. On the other hand, incorporating robustness through loop shaping techniques is an attractive option due to the availability of robust linear control tools; however, a methodological design procedure requires information about the frequency spectrum of the disturbances affecting the closed loop error dynamics, which usually is not available. This motivates the need for a novel technique for incorporating robustness without any (or with minimal) information about disturbances and other uncertainties.

Disturbance decoupling controller

The main idea involved here is to estimate the disturbance δ_2 online, and decouple it from the tracking error $\tilde{\omega}_{sr}$. Sliding mode observers have been successfully used for the estimation of *bounded* disturbances (or input fault signals) [24]. We extend this idea to develop a disturbance decoupling controller.

We begin by analyzing the error dynamics equation (4.105), which includes various forms of uncertainty and corresponds to the case where a proportional gain is employed as the linear controller to be used in conjunction with the feedback linearization law. As we have already seen, the multiplicative disturbance δ_1 does not play an important role in determining the internal stability of the closed loop system. Thus, we make the simplifying assumption that $\delta_1 = 1$. Under this assumption, we have

$$\frac{\tilde{\omega}_{sr}}{\delta_2}(s) = \frac{1}{s + \lambda_I} \quad (4.108)$$

Thus, if $\lambda_I \rightarrow \infty$, the disturbance δ_2 can be decoupled from the tracking error $\tilde{\omega}_{sr}$. Hence, an infinite gain is required to solve the problem of disturbance decoupling, which cannot be realized in this form. However, infinite gains can be implemented by using sliding mode techniques [103].

Towards this end, the linear control law (v) was modified to include a disturbance decoupling term $u_d(\tilde{\omega}_{sr})$. This gives the following new error dynamics equation.

$$\dot{\tilde{\omega}}_{sr} = -\lambda_I \tilde{\omega}_{sr} + \delta_2 + u_d(\tilde{\omega}_{sr}) \quad (4.109)$$

where $u_d(\tilde{\omega}_{sr})$ is the output of the following observer based controller,

$$\sum : \begin{cases} \dot{\hat{\omega}}_{sr} = -\lambda_I \hat{\omega}_{sr} + k_d \text{sgn}(\tilde{\omega}_{sr} - \hat{\omega}_{sr}) \\ \tau_d \dot{v}_d + v_d = k_d \text{sgn}(\tilde{\omega}_{sr} - \hat{\omega}_{sr}) \\ u_d(\tilde{\omega}_{sr}) = v_d \end{cases} \quad (4.110)$$

In (4.110), $\hat{\omega}_{sr}$ is the estimate of the tracking error $\tilde{\omega}_{sr}$. The constant k_d denotes the observer gain and τ_d denotes the time constant of the low pass filter. The structure of the disturbance decoupling controller is shown in Figure 4.5.

Figure 4.5: Structure of the observer based disturbance decoupling controller

The controller Σ consists of two components: an observer and a low pass filter. The goal of the observer is not to estimate the tracking error $\tilde{\omega}_{sr}$, but the disturbance δ_2 , or at least some measure of it. In order to see this, the error dynamics equation corresponding to the observer part of the controller is calculated to be,

$$\dot{e}_d = -\lambda_I e_d + d - k_d \text{sgn}(e_d) \quad (4.111)$$

where $e_d := \tilde{\omega}_{sr} - \hat{\tilde{\omega}}_{sr}$, $d := \delta_2 - v_d$. The signal d is the effective disturbance acting on the plant (see Figure 4.5). If the estimation error e_d converges to zero, by the method of equivalent control explained earlier, we have

$$v_{d,eq} = d \quad (4.112)$$

where $v_{d,eq}$ denotes the equivalent control corresponding to the high frequency switching signal $k_d \text{sgn}(e_d)$. This can be rewritten as (using the definition of d),

$$v_{d,eq} = \delta_2 - v_d \quad (4.113)$$

Now, under the assumption that the low pass filter output v_d converges to the equivalent control $v_{d,eq}$, i.e., $v_d = v_{d,eq}$, the following holds.

$$d = \frac{\delta_2}{2} \quad (4.114)$$

Thus, it can be observed that the effect of the disturbance δ_2 on the oncoming clutch slip speed tracking error $\tilde{\omega}_{sr}$ has been reduced by one-half.

In order for this to happen, we need to justify the two assumptions that were made in arriving at (4.114). The second assumption that the low pass filter output v_d converges to the true value of equivalent control $v_{d,eq}$, can be justified by using the following theorem, which is stated without the proof. Interested readers are referred to [103] for a detailed proof.

Theorem 1. *The output of the low pass filter,*

$$\tau_d \dot{v}_d + v_d = k_d \text{sgn}(e_d) \quad (4.115)$$

tends to the equivalent control ($v_{d,eq}$) if and only if,

$$\tau_d \rightarrow 0, \frac{\Delta}{\tau_d} \rightarrow 0 \quad (4.116)$$

where $\Delta = \frac{1}{f_s}$, and f_s is the maximum possible switching frequency for the term $k_d \text{sgn}(e_d)$.

The theorem requires that the term $k_d \text{sgn}(e_d)$ switches with an infinite frequency, further implying the need for a digital microcontroller with infinite sampling frequency. Clearly, such a microcontroller does not exist. However, using simulation, it

was verified that the filter output does indeed tend to the equivalent control, with a fixed step solver of sampling frequency comparable to the sampling frequency of the microcontrollers employed in production vehicles. The theorem, however, indicates that the correspondence between the filter output and equivalent control deteriorates for microcontrollers with lower sampling frequencies. Thus, it is reasonable to assume that the filter output converges to the equivalent control.

The first assumption that was made in arriving at (4.114) was that the estimation error e_d converges to zero. This assumption can be satisfied by appropriate selection of the observer gain k_d . The next theorem proves the convergence of the estimation error e_d to zero, and develops guidelines for the selection of the gain k_d .

Theorem 2. *The necessary condition for the convergence of $e_d \rightarrow 0$ is*

$$k_d > \frac{\delta_m}{2} \quad (4.117)$$

and the sufficient condition for the same is

$$k_d > \delta_m \quad (4.118)$$

where k_d is the observer gain, and δ_m is the maximum value of the disturbance δ_2 .

Proof. The necessary and sufficient parts will be proved separately:

Necessity: Assume convergence of $e_d \rightarrow 0$. Thus, as was shown above, $v_{d,eq} = \frac{\delta_2}{2}$. In order for this to be true,

$$k_d > v_{d,eq} = \frac{\delta_2(t)}{2}, \quad \forall t > 0 \quad (4.119)$$

since $v_{d,eq}$ is the low frequency component of the switching signal, $k_d \text{sgn}(e_d)$. This is under the assumption that the filter output v_d converges to the equivalent control

$v_{d,eq}$, which was justified using the previous theorem. Equation (4.119) is always satisfied if

$$k_d > \frac{\delta_m}{2} \quad , \quad (4.120)$$

which proves the necessity.

Sufficiency: Assume (4.118) holds. Thus we can write k_d as,

$$k_d = \delta_m + \epsilon, \quad \epsilon > 0 \quad (4.121)$$

Substituting k_d in the error dynamics equation (4.111), the following is obtained.

$$\dot{e}_d = -\lambda_I e_d + \delta_2 - v_d - (\delta_m + \epsilon) \text{sgn}(e_d) \quad (4.122)$$

Before the estimation error e_d converges to zero, i.e., before the attainment of *sliding motion*, the system is in the reaching phase. The following two cases are possible during this phase: either $e_d > 0$ or $e_d < 0 \forall t_r > 0$, where t_r denotes the running time of the reaching phase. Let us assume, without the loss of generality, that $e_d > 0 \forall t_r > 0$.

$$\Rightarrow \text{sgn}(e_d(t_r)) = 1, \quad \forall t_r > 0 \quad (4.123)$$

Then, $v_d(t_r)$ can be evaluated using equations (4.115) and (4.121) as,

$$v_d(t_r) = (\delta_m + \epsilon) \left(1 - e_d^{-\frac{t_r}{\tau_d}} \right) \quad (4.124)$$

where we assumed that the initial value of the filter output $v_d(0) = 0$. Using equation (4.124) in (4.122), we obtain

$$\dot{e}_d = -\lambda_I e_d + \delta_2 - (\delta_m + \epsilon) \left(1 - e_d^{-\frac{t_r}{\tau_d}} \right) - (\delta_m + \epsilon) \quad (4.125)$$

Or,

$$\dot{e}_d = -\lambda_I e_d + u_f \quad (4.126)$$

where,

$$u_f := \delta_2 - (\delta_m + \epsilon) \left(2 - e_d^{-\frac{t_r}{\tau_d}} \right) \quad (4.127)$$

In (4.126), u_f is the forcing function. Since, by assumption, $e_d > 0$ during the reaching phase, we require a negative forcing function for the finite time convergence of the estimation error e_d to zero, i.e.,

$$u_f < 0 \quad (4.128)$$

Or,

$$\delta_2 - (\delta_m + \epsilon) \left(2 - e_d^{-\frac{t_r}{\tau_d}} \right) < 0 \quad (4.129)$$

Or,

$$(\delta_m + \epsilon) > \frac{\delta_2}{\left(2 - e_d^{-\frac{t_r}{\tau_d}} \right)} \quad (4.130)$$

which is always true. This concludes the proof of sufficiency. $\square$

So we have seen that application of the proposed disturbance decoupling controller reduced the effect of the disturbance δ_2 on the clutch slip tracking error $\tilde{\omega}_{sr}$, to one-half. It is a natural follow-on to implement one more identical disturbance decoupling controller and analyze if there is a further reduction in the effect of the disturbance on the closed loop performance. Figure 4.6 shows the architecture of the closed loop system corresponding to the modified controller.

Figure 4.6: Modified structure of the observer based controller

Formally, the modified observer-based disturbance decoupling controller is given by,

$$\sum_{i=1,2} : \begin{cases} \dot{\hat{\omega}}_{sr,i} = -\lambda_I \hat{\omega}_{sr,i} + k_d \text{sgn}(\tilde{\omega}_{sr} - \hat{\omega}_{sr,i}) \\ \tau \dot{v}_{d,i} + v_{d,i} = k_d \text{sgn}(\tilde{\omega}_{sr} - \hat{\omega}_{sr,i}) \\ u_{d,i}(\tilde{\omega}_{sr}) = v_{d,i} \end{cases} \quad (4.131)$$

The error dynamics equations corresponding to the two observers are given by,

$$\dot{e}_{d,i} = -\lambda_I e_{d,i} + \delta_2 - \sum_{i=1}^2 v_{d,i} - k_d \text{sgn}(e_{d,i}), \quad i \in \{1, 2\} \quad (4.132)$$

where $e_{d,i} := \tilde{\omega}_{sr} - \hat{\tilde{\omega}}_{sr,i}$. Under the same set of assumptions made for the previous case with one controller, we obtain

$$\begin{aligned} 0 &= \delta_2 - 2v_{d,1,eq} - v_{d,2,eq} \\ 0 &= \delta_2 - v_{d,2,eq} - 2v_{d,1,eq} \end{aligned} \quad (4.133)$$

Or,

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} v_{d,1,eq} \\ v_{d,2,eq} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \delta_2 \quad (4.134)$$

which gives

$$v_{d,i,eq} = \frac{\delta_2}{3}, \quad i \in \{1, 2\} \quad (4.135)$$

Thus, the effective disturbance d (see Figure 4.6) entering the plant is given by

$$d = \delta_2 - v_{d,1} - v_{d,2} = \delta_2 - v_{d,1,eq} - v_{d,2,eq} = \frac{\delta_2}{3} \quad (4.136)$$

The analysis performed above shows that the effect of the disturbance δ_2 on the output tracking error $\tilde{\omega}_{sr}$ is reduced further. Also, a key thing to note is that both the controller outputs or control efforts equal the effective disturbance d .

We can extend this idea to the case where the disturbance decoupling controller consists of n identical observer-controller units (Figure 4.7). As was shown for the case of two observer-controller pairs, it can be shown using a similar set of assumptions as were made previously, that the output of each observer-controller pair is identical and in turn is identical to the effective disturbance d , entering the plant. The effective disturbance d , in this case, can be evaluated from Figure 4.7 to be,

$$d = \frac{\delta_2}{n+1} \quad (4.137)$$

Thus, $n \rightarrow \infty$ implies $d \rightarrow 0$, which means that the effect of the disturbance δ_2 can be completely decoupled from the tracking error ω_{sr} . For the case where $n > 1$,

Theorem 1 is used to satisfy the assumption that the output of each lowpass filter converges to the corresponding equivalent control. The assumption that estimation error $e_{d,i}$ $i \in \{1, 2, \dots, n\}$ converges to zero is proved using a theorem which is identical to Theorem 2. The complete proof has been given in appendix A. It should be noted that the number of observer-controller pairs to be activated can be decided in real time by observing the tracking error $\tilde{\omega}_{sr}$.

Figure 4.7: Error dynamics with n disturbance decoupling controllers

Validation of the proposed disturbance decoupling controller, as implemented in the transmission control system, will be conducted in chapter 5. However, in order to

verify (4.137), a simulation identical to Figure 4.7 was created. The initial tracking error $\tilde{\omega}_{sr}(0)$ was assumed to be zero in order to clearly see the input-output response for the closed loop system. The proportional gain λ_I was chosen to be equal to 10. The additive disturbance δ_2 was modeled as a sine wave with amplitude and frequency equal to 2 Nm and 5 rad/s respectively (Figure 4.8). The simulation was performed with and without the proposed disturbance decoupling controller. For the former, the number of observer-controller pairs was varied. In Figure 4.9, the output signal op corresponds to the latter, while the output $op\#i$ $i \in \{1, 2, 3\}$ corresponds to the former. Clearly, the tracking error $\tilde{\omega}_{sr}$ is improved by increasing the number of the observe-controller pairs. Moreover, the relationship given by (4.137) is readily verified as well.

Figure 4.8: Additive disturbance δ_2

Figure 4.9: Tracking error $\tilde{\omega}_{sr}$: op denotes the tracking error without any disturbance decoupling controller; $op\#i$, $i \in \{1, 2, 3\}$, denotes the tracking error with i observer-controller pairs

4.3 Conclusion

The model developed in the preceding chapter was used to develop model-based approaches to estimation and control of clutch-to-clutch shifts. On the estimation side, algorithms were proposed to estimate the output shaft torque, turbine torque, reaction torque at the offgoing clutch, and oncoming clutch pressure. A Luenberger observer was proposed for the estimation of the output shaft torque. Robustness of the observer, to uncertainty in the lumped driveline compliance, was ensured by appropriate gain selection. An unknown input sliding mode observer (UI-SMO) was used to estimate the turbine torque and oncoming clutch pressure. The reaction torque

at the offgoing clutch is calculated as a linear combination of the other estimated variables.

A closed loop torque phase strategy was devised to ensure a smooth coordination between the offgoing and oncoming clutches, where the estimate of the reaction torque at the offgoing clutch played a pivotal role. A secondary objective of controlling the duration of the torque phase was also accomplished by the proposed torque phase control law. An inertia phase controller was proposed to ensure smooth synchronization of the input and output shaft speeds of the transmission system. The method of oncoming clutch slip speed tracking was used for the same. Measurements from speed sensors alone was used as the feedback for the proposed observer-based controller. Three strategies to ensure robustness of the proposed observer-based controller against modeling and estimation errors were proposed as well, the third being novel. The proposed observer-based controller was numerically validated on the powertrain model developed in chapter 2; results of which, are presented in the next chapter.

Chapter 5: Simulation Results and Discussions

The observer-based controller proposed in the preceding chapter has been validated on the MATLAB/SIMULINK powertrain model described in chapter 3, which represents a production stepped-automatic transmission system. We note that while the engine and transmission mechanical simulations are not particularly detailed, the simulation of the transmission hydraulic system is particularly detailed, including as it does a high order nonlinear model, and large portions of it have been validated in an earlier work [110]. A fixed step solver with an integration time step of 0.001 s was used for the simulation. The shift schedule is simplified and solely based on the vehicle speed. The 1 - 2 upshift is a clutch-to-clutch shift and is initiated when the vehicle speed is greater than a specified threshold. Clutch filling transients are not included in the simulation. Once the shift is commanded, the simulation goes into the torque phase. These trajectories are determined by the feedback linearization controller of the torque phase. Once the offgoing clutch starts slipping, the simulation goes into the inertia phase. In both phases, the feedback linearization controller corresponding to the phase is activated, and provides the reference pressure trajectory for the SMC controller. Towards the end of the inertia phase, when the slip speed of the oncoming clutch is very small, the feedback linearization controller is deactivated and the clutch

pressure is ramped up in an open loop fashion to ensure proper lock-up. Appendix B provides the details of the simulation.

We begin this chapter by presenting validation results for the estimation strategy proposed in the preceding chapter. We note first that the proposed controller uses a reduced first order nonlinear model of shift hydraulics, whereas the simulation of the shift hydraulics uses an experimentally validated high order nonlinear model. Thus, even if some parameter values such as driveline compliance and clutch friction coefficient are assumed to be known exactly for purposes of controller design, satisfactory performance of the controller on the simulation would validate the low order model of shift hydraulics as a basis for controller design. Since the observer-based controller proposed here employs the sliding mode technique, it is important to study the sensitivity of the controller performance with respect to sampling frequency of the fixed-step solver used for simulations. Robustness of the estimator and controller proposed in the preceding chapter to uncertainty in the lumped driveline compliance and to the clutch friction coefficient will be validated as well.

5.1 Estimator validation

The combination of nonlinear and linear estimators proposed in the current study is found to perform well, as hypothesized. Figures 5.1-5.3 correspond to the 1-2 upshift, commanded at 3 s. At this time, the simulation enters the torque phase which is followed by the transition to the inertia phase at 3.21 s. The upshift is completed at 3.69 s, at which point the transmission is in the 2nd gear. Figure 5.1 shows performance of the Luenberger observer for output shaft torque estimation, where the vertical dashed lines denote, from left to right, initiation of the upshift, transition

from the torque to the inertia phase, and end of the upshift respectively. Without any driveline compliance modeling uncertainty, the linear observer reproduces exactly the shaft torque signal T_s . In order to evaluate robustness of this observer, an uncertainty of 30% in the driveline compliance, i.e. $\Delta K_s = 0.3$, was introduced in the plant model. As one can see from the same figure, there is only marginal deterioration in the observer performance, which can be considered negligible for all practical purposes.

Sliding mode observers for estimation of the turbine torque and the oncoming clutch pressure produced accurate estimates (Figures 5.2, 5.3). In Figure 5.3, once the upshift is completed, the estimate of the clutch pressure is different than the actual value. This is due to the fact that there is no estimator designed for in-gear operation of the transmission system. It should be noted that the transition between the torque and inertia phase observers is *smoother* for the oncoming clutch pressure when compared to the same transition for the turbine torque. The reason for this lies in the difference in the output injection terms used in the observer designs for the two signals. The turbine torque is observed through engine speed in the torque phase and clutch slip speed in the inertia phase, implying that there is a change in the structure of the observer during the transition from the torque to the inertia phase. As opposed to this, the oncoming clutch pressure is observed through clutch slip speeds in both the torque and inertia phases, which means that the structure of the observer is preserved at the transition. The change in the structure of the observer during the transition from the torque to the inertia phase causes a deterioration in the accuracy of the turbine torque estimate. Finally, good estimates of the output shaft torque, turbine torque and oncoming clutch pressure result in a good estimate

of the reaction torque at the offgoing clutch (see Figure 5.4), as it is expressed as a linear combination of the three other quantities.

Figure 5.1: Robustness of the output shaft torque observer against uncertainty in the lumped driveline compliance, K_s

A time step of 1 *ms* was used for the simulation results shown in Figures 5.1 through 5.3. In order to check the validity of the observer-based controller for a control system limited in sampling rate of the microcontroller employed, same simulation was performed with a time step of 10 *ms*. Figures 5.5, 5.6 show comparison of the actual and estimated turbine torques and oncoming clutch pressures respectively. Clearly, the performance of the observer has deteriorated as compared to the simulation results corresponding to the 1 *ms* time step. Again, the degradation in turbine torque estimates is greater than that in clutch pressure estimates. However, the estimates are still acceptable for control purposes.

Figure 5.2: Estimated vs actual turbine torque

Figure 5.3: Estimated vs actual oncoming clutch pressure.

Figure 5.4: Estimated vs actual reaction torque at the offgoing clutch.

Figure 5.5: Estimated vs actual turbine torque

Figure 5.6: Estimated vs actual oncoming clutch pressure

A closer look at Figure 5.5 provides more insight into the *real* reason for this estimator performance deterioration. A higher sampling time for the fixed step solver results in chattering of the control input, subsequently causing the clutch pressure to chatter (compare Figure 5.6 with Figure 5.3). In the presence of this chattering, the observer actually does a decent job of averaging these signals out. This can be clearly seen from the estimate of the reaction torque at the offgoing clutch (Figure 5.7), where the actual reaction torque chatters and its estimate represents its approximate average value.

Figure 5.7: Estimated vs actual reaction torque at the offgoing clutch

The arguments presented above should not come as a surprise to readers since the proposed sliding mode controller was formulated in continuous-time domain, where we inherently assume that an infinitely fast processor is available. The above exercise - of checking the validity of the observer based controller for a higher sampling time

step - suggests that a reformulation of the sliding mode controller in discrete-time domain is required, where factors such as sampling time can be taken into account, if it were to be implemented on a production vehicle. This is left as one of the recommendations for future work.

5.2 Controller Validation

The results presented in the preceding section corresponded to the controlled closed-loop system simulation, where the controller did not employ any of the robustness schemes presented in the preceding chapter. The performance of this nominal observer-based controller was found to be robust to the uncertainty in the lumped driveline compliance (Figure 5.1), due to robust design of the output shaft torque observer. We continue here with evaluation of the nominal observer-based controller first, followed by evaluation of the robust controller that employs one of the robustness schemes referred to in chapter 4.

The nominal observer-based controller, consisting of the feedback linearization controller and sliding mode controller, performed well on a simulation of the nominal shift hydraulic system, as indicated in Figures 5.8 - 5.11. The linear control law for the feedback linearization controller in this case uses a proportional gain.

Figure 5.8 shows performance of the combined sliding mode and feedback linearization controllers for the torque phase for the nominal plant and controller. The desired duration of the torque phase was set to 0.2 s, which along with the estimated value of the reaction torque at the offgoing clutch, at the start of the torque phase, was used to generate the reference trajectory for the output shaft speed tracking problem, as was explained in chapter 4. On the other hand, the reference trajectory

shown in Figure 5.9 for oncoming clutch slip speed tracking was specified to be a cubic polynomial in time, as was discussed in the preceding chapter. Both the figures show good speed tracking performance for the proposed controller. It should also be noted that the convergence time for the speed tracking controller in the torque phase is very small (less than 0.035 s) and is practically equal to zero for the inertia phase. The convergence time of 0.035 s is due to the feedback linearization gain of the torque phase controller. Zero convergence time for the inertia phase controller is a consequence of the zero initial tracking error which, as was explained in the preceding chapter, is due to the intelligent design of the reference trajectory. In Figures 5.10, 5.11, the vertical dashed lines denote, from left to right, initiation of the upshift, transition from the torque to the inertia phase, and end of the upshift respectively. The two figures show the offgoing and oncoming clutch pressures, and the output shaft torque respectively, for the same simulation. The controlled clutch pressures in Figure 5.10 were found to very closely track the reference clutch pressures generated by the feedback-linearization controller. The output shaft torque response, shown in Figure 5.11, has a typical drop (torque hole) and a typical rise (torque hump) during the torque and inertia phases respectively, which is characteristic of clutch-to-clutch shifts being controlled through manipulation of clutch pressures alone. It is well-known that smoother shifts and smaller variations in the output shaft torque during shifts are possible by coordinated manipulation of transmission and engine variables in integrated powertrain control approaches, and we return to this topic when we discuss recommendations for future work. We note here that only minor oscillations are induced after the clutch lock-up, signifying a good upshift. During the torque phase, the controller is successful in satisfying both the control objectives:

mis-coordination of the offgoing and oncoming clutches, and the desired duration for the torque phase. The latter can be easily seen from Figure 5.11, where the duration of the torque phase is 0.21 s. In addition to this, Figure 5.12 shows that the duration of the torque phase can be easily controlled by the proposed closed loop torque phase controller. In order to support this claim, traces of the reaction torque at the offgoing clutch were recorded and are presented in Figure 5.13. It should also be noted that the magnitude of the negative load torque carried by the offgoing clutch at the end of the torque phase is small, representing good coordination between the offgoing and oncoming clutches.

Figure 5.8: Reference vs controlled output shaft speed during the torque phase

Figure 5.9: Reference vs controlled oncoming clutch slip speed during the inertia phase

Figure 5.10: Offgoing and oncoming clutch pressures response for the controlled closed-loop system

Figure 5.11: Output shaft torque response for the controlled closed-loop system

Figure 5.12: Output shaft torque response for simulations with different desired torque phase time durations Δt_T

Figure 5.13: Reaction torque at the offgoing clutch during the torque phase, $\Delta t_T = 0.2$ s

The controller validated above is the nominal controller, and does not incorporate any of the three features specifically designed to enhance control system robustness and discussed in chapter 4. In order to see the effectiveness of these controller features, a constant uncertainty of 10 % is introduced in the oncoming clutch coefficient of friction, i.e. $\Delta\mu = 0.1$. Figures 5.14, 5.15, 5.16, and 5.17 show the controller performance results corresponding to this simulated condition. The comparison of the output shaft torques, with and without the uncertainty in the clutch friction coefficient, is shown in Figure 5.14. Clearly, the comparison indicates significant degradation of the controller performance in the face of uncertainty in the oncoming clutch friction coefficient. The torque phase controller performance is only marginally affected, which can be seen in the form of a small increment in the magnitude of negative load torque carried by the offgoing clutch (Figure 5.15) resulting from a

small deterioration in the output shaft speed tracking performance (Figure 5.16). On the other hand, the inertia phase controller fails to track the reference trajectory for the oncoming clutch slip speed (Figure 5.17), resulting in premature termination of the inertia phase. Consequently, the clutch lock up torque increases (Figure 5.14), and oscillations are induced in the driveline. It is clear that the robustness of the control system does need to be enhanced significantly, as it is very reasonable to expect that the clutch friction coefficient would not be known exactly or, equivalently, that it changes with clutch usage or transmission fluid condition.

Figure 5.14: Effect of oncoming clutch friction coefficient uncertainty on output shaft torque response

Figure 5.15: Effect of oncoming clutch friction coefficient uncertainty on reaction torque at the offgoing clutch

Figure 5.16: Effect of oncoming clutch friction coefficient uncertainty on torque phase output shaft speed tracking

Figure 5.17: Effect of oncoming clutch friction coefficient uncertainty on inertia phase output shaft speed tracking

In order to test the three robustness schemes introduced in the preceding chapter, the assumed friction characteristic of the oncoming clutch, which corresponds to a new clutch, was replaced with a friction characteristic that is representative of old/worn-out clutches, and is believed to induce instability in the system response [108], see Figure 5.18. Figure 5.19 shows comparison of the performance of the three schemes designed to enhance controller robustness. All three schemes resulted in stable response, but differed considerably in performance. Scheme#1 denotes the adaptive feedback linearization strategy, which showed bad transients and higher clutch lock-up torque as compared to the other two schemes. It should be noted that a good transient performance is never guaranteed for an adaptive controller, unless explicitly ensured. The prime reason lies in the structure of the update law, which is just an integrator and lacks any kind of dissipation. Also, availability of the oncoming

clutch pressure signal had to be assumed for implementing the adaptive feedback linearization technique, which is a significant limitation for implementation. Scheme#2 employs loop-shaping techniques to incorporate robustness in the controller. The output shaft torque response corresponding to this scheme is very close to the one for the nominal plant case (without any uncertainty in the oncoming clutch coefficient of friction, see Figure 5.14). However, high gains had to be selected for this, signifying that this technique might not be a practical choice; the reason being that high gains in the control law tend to increase the bandwidth of the closed-loop system, making the performance prone to the measurement noise. The third scheme (scheme#3) employs the disturbance decoupling controller introduced in the previous chapter. The output shaft torque response corresponding to this scheme was found to be close to the nominal plant case as well. For the simulation, only one observer-controller pair was used.

Figure 5.18: Friction characteristics for the oncoming clutch: New vs worn-out clutch

Figure 5.19: Output shaft torque response corresponding to different robustness schemes

5.3 Conclusion

The proposed observer-based controller was validated on a high order nonlinear powertrain model, which included an experimentally validated shift hydraulic system dynamics model. The observer-based controller was found to perform well. It was shown that a discrete-time formulation of the proposed controller is required, if it were to be implemented on a vehicle. Robustness of the observer-based controller against the uncertainty in the lumped driveline compliance, and the coefficient of oncoming clutch friction was demonstrated as well. Based on various key findings, and other considerations, we proceed to conclude the current study, along with appropriate recommendations for future work, in the next chapter.

Chapter 6: Conclusions and Recommendations for Future Work

6.1 Conclusions

The thesis focused on the broad objective of developing systematic and robust methods for controlling clutch-to-clutch shifts, which occur with increasing frequency in gear shifts in production vehicles with automatic transmissions. The proposed observer-based controller was validated numerically on a powertrain model, which included an experimentally validated transmission hydraulic system model. Contributions were made in each of the three phases involved in a typical model-based approach: modeling, estimation, and control.

A control-oriented model of the powertrain was developed, and formal methods were presented to model the shift hydraulic system dynamics. Specifically, singular perturbation and state-space averaging techniques were employed to reduce a high order nonlinear model of the shift hydraulic system dynamics, developed from first principles, to a first order nonlinear model, which was validated against experimental measurements. The reduced order model so obtained is structurally similar to the model representation of turbulent flow through a sharp edged orifice, which is to say that the transmission hydraulic system behaves as a single restriction to the

transmission fluid flow. Under the assumption that this remains true for a wide majority of transmission hydraulic systems, the result gives a systematic procedure to model shift hydraulic dynamics for other transmission systems. However, we do not have any reason to believe that the assumption would hold for other transmission systems, especially systems that use different designs for the shift hydraulics. For example, transmission systems which rely upon high flow variable force solenoids instead of pilot operated pressure control valves may well have other dynamics.

Online estimation of critical operating variables such as the output shaft torque, turbine torque, reaction torque at the offgoing clutch, and oncoming clutch pressure was performed. The estimation algorithm employs an unknown input sliding mode observer (UI-SMO), which has been widely used for fault diagnosis and isolation (FDI) in dynamical systems. Accurate on-line information on the output shaft torque and turbine torque are critical to successful operation of the feedback linearization controller that generates the reference trajectory for the oncoming clutch pressure. The sliding mode controller, employed for closed loop control of the oncoming clutch pressure, requires feedback of the same, thus implying the necessity for online estimation of the oncoming clutch pressure. The online estimate of the reaction torque at the offgoing clutch provides the feedback needed for the closed loop torque phase controller that ensures good coordination between the offgoing and oncoming clutches. In developing the estimation algorithm, the lack of sensors, available in production transmission systems was compensated by *borrowing* available online information from the engine and vehicle dynamics subsystems. Specifically, it was assumed that the indicated torque of the engine and the load torque on the vehicle is known, as these are critical for successful operation of model-based engine and

vehicle stability controllers in a production vehicle. We would like to summarize two crucial assumptions of the estimation scheme: the first, during the torque phase, that the turbine torque estimation algorithm assumes that the torque converter is either locked by the torque converter clutch, or is in the fluid coupling mode; the second, that the coefficient of friction for the oncoming clutch is known accurately. The first assumption is a consequence of using information from the engine side to estimate the turbine torque. The second assumption is necessary, as any uncertainty in the clutch coefficient of friction will induce a proportional uncertainty in the oncoming clutch pressure estimate. This should be intuitively clear, as the information being used for the clutch pressure estimation is the clutch slip speed, which gets affected by any uncertainty in the friction coefficient.

On the controls side, a closed loop torque phase controller was proposed, to ensure good coordination between the offgoing and oncoming clutches. In order to demonstrate applicability of the proposed torque phase control law, the offgoing clutch pressure was assumed to be known. The assumption can be avoided if the delay between the commanded and the actual release of the offgoing clutch is known. An inertia phase controller was proposed for smooth synchronization of the input and output shaft speeds of the transmission system, while simultaneously controlling the phase duration. This was achieved through the well studied method of oncoming clutch slip speed tracking, where a reference trajectory is specified to meet the conflicting objectives of smooth shift quality and low clutch wear, for the inertia phase. For the controllers corresponding to both the phases, only measurements from speed sensors were utilized. Algorithmically, a combination of sliding mode and feedback linearization-based controllers was used, which provided modularity in the controller

structure, further simplifying the process of control design. Three schemes for ensuring robustness in the feedback linearization-based controller were presented, the third being novel. It was shown that the sliding mode controller, formulated in continuous-time domain, is not necessarily suitable for implementation on processors with low sampling frequencies, and a reformulation in discrete-time domain is required in such cases.

6.2 Recommendations for future work

We have segregated recommendations for future work in three subsections. In the first subsection, we summarize recommendations which are consequence of assumptions made in the current study, or are essential to vehicle implementation of the proposed control strategy. In the second subsection, we provide motivation for pursuing integrated powertrain control; and in the third subsection, we simply put forth a question regarding the possibility for improved gearshift control in an intelligent transportation system (ITS).

In the preceding section, various assumptions made in arriving at the proposed observer-based controller were summarized. Some of these, along with other considerations, lead us to our first group of recommendations for future work.

6.2.1 Filling the gaps

Here we summarize essential tasks that need to be addressed in order to make the observer-based controller, proposed in the current study, *vehicle-ready*.

1. The assumption made on the availability of the offgoing clutch pressure for the implementation of the torque phase control law is not acceptable for planetary

automatic transmissions. As was mentioned in that section, the implementation can be done in an open-loop fashion if the delay between the commanded and actual release of the offgoing clutch is known. However, if this were to be implemented, appropriate adaptive measures should be taken to accommodate variations from shift-to-shift.

2. Again from the preceding section, a discrete-time formulation of the sliding mode controller is in order, to facilitate wider implementation in vehicles.
3. In the current study, the clutch-fill phase was not modeled. The fundamental challenge involved in controlling the clutch-fill phase was explained in chapter 1. Thus, it is crucial to include a model representation of the clutch-fill phase, and study the problem of clutch-fill detection.
4. The proposed observer-based controller was formulated and validated (numerically) for a 1 – 2 upshift. [5] showed that a controller formulated for an upshift need not perform equally well for the corresponding downshift, albeit in the context of dual clutch transmissions. This motivates us to think about the validity of the proposed controller for a 2 – 1 downshift. However, it should be noted that the controller designed in the mentioned study was a PID-based controller, which may not operate well for different operating conditions of a nonlinear system, even if it is re-tuned for the corresponding operating condition. Our approach started with a nonlinear model and may not have the same problem. At any rate, this is an aspect that needs to be evaluated.
5. Estimation of the oncoming clutch pressure in the presence of uncertainty in the friction coefficient should be investigated. In a deterministic setting, this

could be difficult if speed sensors alone are used. Thus, stochastic approaches which are relatively more immune to modeling error and parametric uncertainties might be a potential solution to this problem. However, while going for such an approach, one should be careful of the fact that stochastic solutions are conservative in nature, i.e., performance is traded for robustness.

6.2.2 Integrated powertrain control

Integrated powertrain control approaches [2], for controlling a gearshift, involves manipulation of engine variables such as throttle angle, spark advance, etc., in addition to clutch pressures. We provide two motivations to pursue such a control philosophy. The first motivation is obvious: to eliminate the torque hole and hump during the torque and inertia phases respectively, see Figure 6.1. Figure 6.1 shows a typical output shaft torque response for an upshift with clutch pressures as the sole manipulated variables.

Figure 6.1: A typical output shaft torque response corresponding to an upshift, with clutch pressures as manipulated variables

The second motivation is less obvious. A combination of stringent emission rules and the pursuit for improved fuel economy and good drivability (in particular, passenger comfort and shift response time, as perceived by the driver) has led to the development of higher speed transmissions, with wider gear spread and smaller gear steps. However, the increased number of speeds increase the number and complexity of the shifts exponentially. Specifically, these transmissions frequently employ double transition shifts, which involve four actively controlled friction clutches, capable of transmitting the load in both the slip speed directions. Since the degree of the dynamical system to be controlled is higher in such a shift as compared to a clutch-to-clutch shift, a greater degree of freedom in exercising control would usually be required for a satisfactory performance. The engine variables such as the throttle angle and spark advance, which are available at ones' disposal, can play the role of these additional required control variables and degrees of control design freedom, to achieve good shift quality.

Integration of various control systems in a vehicle opens many avenues for the overall improved performance of the same. We present a possible benefit of integrating powertrain and chassis controllers. In situations where a vehicle is to slow down rather quickly on a road offering low traction, downshifting uses engine braking and minimizes brake application. Thus, the gear downshifting can be integrated with the brake control action to shorten braking distances while maintaining safety. Also, in the current study, by assuming the availability of load torque on the vehicle, we have already assumed some level of integration between the transmission and chassis stability controllers.

6.2.3 Looking ahead

Figure 6.2 shows a typical information exchange flow amongst different control systems in a vehicle, and with other vehicles and infrastructure in its environment. The latter is applicable in an intelligent transportation system (ITS) setting, where vehicle-to-vehicle (V2V) and vehicle-to-infrastructure (V2I) information exchange is allowed. The former however is true for any vehicle, as smooth integration of different control systems is ensured by some supervisory controller. In the preceding section on integrated powertrain control, we presented the existing and potential benefits of such an integration.

Figure 6.2: Information exchange in an intelligent transportation system (ITS)

We pose a question regarding the latter: can information from other vehicles on the road, and installed infrastructure such as GPS be integrated with gearshift controller performance in order to further improve the performance of a vehicle in isolation or in relationship to other vehicles? [104] shows that availability of information from its environment can play a critical role in deriving fuel efficient and *comfortable* shift schedules. However, no similar attempts have been made regarding gearshift controllers; the difference in time scales involved in a gearshift and those associated with events occurring in the environment of a vehicle is likely a reason. We present a case which motivates us to think a little more about the question posed. In a typical study concerning fuel consumption- and shift schedule-optimization, engine variables such as spark advance, throttle angle, etc. are calculated to solve the optimization problem. The following two assumptions are typical of such an optimization study. First, engine variables are assumed to be available, within their respective constraints, at all the times. Second, the gear shift is assumed to be instantaneous. The second assumption justifies the first. However, in an integrated powertrain control environment, these variables are not available for solving the optimization problem for finite duration of the gearshift. In order to understand a potential consequence of this, let's analyze a simulation result from [104]. Figure 6.3 shows the optimized gear-ratio time history for FTP75 drive cycle. The optimization problem was solved using dynamic programming (DP), where it was assumed that the shifts are instantaneous. Now, assuming a typical shift duration of 1 s for a clutch-to-clutch shift, one can note that for approximately 100 s, in a total trip time of 1800 s, manipulated engine variables are not available. Thus, if such a consideration is included in the optimization problem, the results are bound to change. One possible way to accommodate

this condition would be to include shift duration and clutch pressures as optimization variables. However, the *curse of dimensionality* associated with any optimization problem might be a hindrance. Clearly, if the potential benefits from incorporating gearshift controller performance are significant, that would motivate the development of appropriate solution procedures and is an area for future work.

Figure 6.3: Optimized gear-ratio time history for FTP75 drive cycle [104]

The discussion presented above also indicates a possibility of inclusion of transmission dynamic behavior in studies concerning platooning and automated driving. The question we have posed above becomes a necessity for cases in platooning and automated driving studies involving a short head-away distance between two consecutive vehicles, implying the comparability of time-scales of the events occurring in the environment of a vehicle and those occurring inside the transmission system. Thus, an improvement in the performance of gearshift controllers might offer a potential improvement in the performance of *string* of vehicles, and enhancement of safety in an automated driving setting.

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Appendix A: Convergence proof for the disturbance decoupling controller with n observer-controller pairs

In chapter 4, a novel technique to incorporate robustness into the feedback-linearization based controller was introduced. The proposed robustness strategy hinges upon estimation of uncertainty, disguised as a disturbance on the closed loop system, and its subsequent neutralization by appropriate manipulation of the clutch pressure actuator. It was shown that the effect of a disturbance δ_2 - which was manifestation of different uncertainties present in the system - can be reduced *arbitrarily* on the controller tracking performance. Precisely, the effective disturbance d affecting the error dynamics was shown to be,

$$d = \frac{\delta_2}{n+1} \tag{A.1}$$

where n is the number of observer-controller pairs used in the control law for the disturbance decoupling purpose. While deriving (A.1), it was assumed that the convergence error corresponding to all the observers go to zero. We state this assumption as a theorem, and provide a proof for the same. In what follows next, e_{di} denotes the convergence error corresponding to the observer of the i^{th} observer-controller pair.

Theorem 3. *The necessary condition for the convergence of $e_{di} \rightarrow 0$ is*

$$k_d > \frac{\delta_m}{n+1} \tag{A.2}$$

and the sufficient condition for the same is

$$k_d > \delta_m \quad (\text{A.3})$$

where k_d is the observer gain, identical for different observer-controller pairs, and δ_m is the maximum value of the disturbance δ_2 .

Proof. The necessary and sufficient parts will be proved separately:

Necessity: Assume convergence of $e_{di} \rightarrow 0$. Thus the error dynamics equation-set now becomes,

$$\begin{aligned} 0 &= \delta_2 - 2v_{1,eq} - v_{2,eq} - \dots - v_{n,eq} \\ 0 &= \delta_2 - v_{1,eq} - 2v_{2,eq} - \dots - v_{n,eq} \\ &\vdots \\ 0 &= \delta_2 - 2v_{1,eq} - v_{2,eq} - \dots - 2v_{n,eq} \end{aligned} \quad (\text{A.4})$$

or,

$$\begin{bmatrix} 2 & 1 & 1 & 1 & \dots & 1 \\ 1 & 2 & 1 & 1 & \dots & 1 \\ 1 & 1 & 2 & 1 & \dots & 1 \\ \vdots & & & \ddots & \dots & \vdots \\ 1 & 1 & 1 & \dots & 1 & 2 \end{bmatrix} \begin{bmatrix} v_{1,eq} \\ v_{2,eq} \\ \vdots \\ v_{n-1,eq} \\ v_{n,eq} \end{bmatrix} = \begin{bmatrix} \delta_2 \\ \vdots \\ \delta_2 \end{bmatrix} \quad (\text{A.5})$$

or,

$$v_{i,eq} = \frac{\delta_2}{n+1}, \quad i \in \{1, 2, \dots, n\} \quad (\text{A.6})$$

Now since $v_{i,eq}$ is the low frequency component of $v_i = k_d \operatorname{sgn}(e_{di})$, under the assumption that the output of the low pass filter converges to the equivalent control (by Theorem 1 presented in chapter 4), we have

$$k_d > \frac{\delta_2}{n+1} \quad (\text{A.7})$$

which is always true if

$$k_d > \frac{\delta_m}{n+1} \quad (\text{A.8})$$

This proves the necessity.

Sufficiency: Assume (A.3) holds. Thus we can write k_d as,

$$k_d = \delta_m + \epsilon, \quad \epsilon > 0 \quad (\text{A.9})$$

Substituting k_d in the i^{th} error dynamics equation, the following is obtained.

$$\dot{e}_{di} = -\lambda_I e_{di} + \delta_2 - \sum_{i=1}^n v_{di} - (\delta_m + \epsilon) \text{sgn}(e_d) \quad (\text{A.10})$$

where v_{di} is the output of the low pass filter of i^{th} observer-controller pair. Before the estimation error e_{di} converges to zero, i.e., before the attainment of *sliding motion*, the i^{th} observer loop is in the reaching phase. The following two cases are possible during this phase: either $e_{di} > 0$ or $e_{di} < 0 \forall t_{ri} > 0$, where t_{ri} denotes the running time of the reaching phase for the i^{th} observer-controller pair. Let us assume, without the loss of generality, that $e_{di} > 0 \forall t_{ri} > 0$.

$$\Rightarrow \text{sgn}(e_{di}(t_{ri})) = 1, \quad \forall t_{ri} > 0 \quad (\text{A.11})$$

Then, $v_{di}(t_{ri})$ can be evaluated using the solution of the low pass filter differential equation as,

$$v_{di}(t_{ri}) = (\delta_m + \epsilon) \left(1 - e^{-\frac{t_{ri}}{\tau_d}} \right) \quad (\text{A.12})$$

where we assumed that the initial value of the filter output $v_{di}(0) = 0$. In the above, τ_d denotes the time constant of first order low pass filters, which is identical for different observer-controller pairs. Also, without any loss in generality, assume for $j \neq i$, and $j \in \{1, 2, \dots, n\}$, $e_{dj}(t_{rj}) > 0$. In addition denote by $t_r = \min(t_{ri})$. Thus we have, from (A.12) and (A.10),

$$\dot{e}_{di} = -\lambda_I e_{di} + \delta_2 - n (\delta_m + \epsilon) \left(1 - e^{-\frac{t_r}{\tau_d}} \right) - (\delta_m + \epsilon) \quad (\text{A.13})$$

Or,

$$\dot{e}_{di} = -\lambda_I e_{di} + u_{fi} \quad (\text{A.14})$$

where,

$$u_{fi} := \delta_2 - (\delta_m + \epsilon) \left(n + 1 - n e_{di}^{-\frac{t_r}{\tau_d}} \right) \quad (\text{A.15})$$

In (A.14), u_{fi} is the forcing function. Since, by assumption, $e_{di} > 0$ during the reaching phase, we require a negative forcing function for the finite time convergence of the estimation error e_{di} to zero, i.e.,

$$u_{fi} < 0 \quad (\text{A.16})$$

Or,

$$\delta_2 - (\delta_m + \epsilon) \left(n + 1 - n e_{di}^{-\frac{t_r}{\tau_d}} \right) < 0 \quad (\text{A.17})$$

Or,

$$(\delta_m + \epsilon) > \frac{\delta_2}{\left(n + 1 - n e_{di}^{-\frac{t_r}{\tau_d}} \right)} \quad (\text{A.18})$$

which is always true. This concludes the proof of sufficiency. $\square$

One can verify that the necessary and sufficient conditions for the convergence of estimation error of a single observer-controller pair is a special case of the general result proved above.

Appendix B: Simulation paramaters

Engine Model

Parameter	Value [Unit]
I_e	0.4 [kg-m ²]
T_f	15.1 [N-m]
b_e	0.2 [N-m/rad/s]

Torque Converter Model

Parameter	Value [N-m/(rad/s) ²]
a_0	7.1315×10^{-3}
a_1	-3.0135×10^{-3}
a_2	-3.4140×10^{-3}
b_0	3.4276×10^{-3}
b_1	1.0719×10^{-3}
b_2	-2.9231×10^{-3}
c_0	4.4324×10^{-3}
c_1	6.9819×10^{-3}
c_2	-11.2612×10^{-3}

Clutch Model: $2ND$ (oncoming) clutch

Parameter	Value [Unit]
N_c	6 [NA]
R_c	0.1434 [m]
A_c	0.0055 [m ²]
K_a	44.86×10^3 [N/m]
x_{in}	0.0025 [m]
x_{out}	0.0013 [m]

Clutch Model: LR (offgoing) clutch

Parameter	Value [Unit]
N_c	12 [NA]
R_c	0.1495 [m]
A_c	0.0097 [m ²]
K_a	40.87×10^3 [N/m]
x_{in}	0.0025 [m]
x_{out}	0.0015 [m]

Parameters identical to both the offgoing and oncoming clutch are presented in the parameter table of transmission hydraulic system.

Transmission Mechanical Model

Parameter	Value [Unit]
p_{11}	-0.4032 [NA]
p_{12}	1.4032 [NA]
p_{21}	-1.1460 [NA]
p_{22}	2.1460 [NA]
I_t	0.05623 [kg-m ²]
I_{si}	0.0001389 [kg-m ²]
I_{rr}	0.0022299 [kg-m ²]
I_{cr}	0.0051057 [kg-m ²]
I_{sr}	0.0020026 [kg-m ²]
R_d	0.2652 [NA]
$\omega_{lock-up}$	0.01 [rad/s]

Transmission Hydraulic Model

Parameter	Value [Unit]
N	180 [NA]
A	1.8555×10^{-4} [m ²]
$x_{sol,m}$	0.0002 [m]
K_{sol}	400.9989×10^3 [N/m]
C_d	0.61 [NA]
P_s	8.35×10^5 [Pa]
$d_{sol,in}$	0.0014 [m ²]
$d_{sol,ex}$	0.002 [m ²]
ΔA_{pcv1}	4.95×10^{-5} [m ²]
ΔA_{pcv2}	3.89×10^{-5} [m ²]
A_a	9.0792×10^{-4} [m ²]
A_{inc}	3.1416×10^{-6} [m ²]
d_{sp1}	0.012 [m]
d_{sp2}	0.009721 [m]

Vehicle Dynamics Model

Parameter	Value [Unit]
K_s	7625 [N-m/rad/s]
I_v	169.82 [Kg-m ²]
r	0.3 [m]
f_1	0.2936 [N]
f_2	0.0025 [N/(m ² -rad ² /s ²)]

Output Shaft Observer

Parameter	Value [Unit]
l_1	9.35×10^3 [N-m/rad]
l_2	10 [s ⁻¹]

Torque Phase Observer

Parameter	Value [Unit]
k_1	30 [N-m]
k'_1	10000 [N-m/s]
τ_1	0.0011 [s]
k_2	500 [N-m]
k'_2	10000 [N-m/s]
τ_2	0.1592 [s]

Inertia Phase Observer

Parameter	Value [Unit]
k_3	350 [N-m]
k'_3	2000 [N-m/s]
τ_3	0.0011 [s]
k_4	110 [N-m]
k'_4	2000[N-m/s]
τ_4	0.0011 [s]

Sliding mode controller

Parameter	Value [Unit]
ϵ	0.1 [NA]

Torque phase controller

Parameter	Value [Unit]
ε	1.2 [NA]
λ_T	100 [s ⁻¹]

Inertia phase controller

Parameter	Value [Unit]
λ_I	10 [s ⁻¹]

Adaptive feedback linearization

Parameter	Value [Unit]
L	120 [s ⁻¹]

Robustness through loop-shaping techniques

Parameter	Value [Unit]
λ_{Ip}	10 [s ⁻¹]
λ_{Ii}	100 [s ⁻²]

Disturbance decoupling controller

Parameter	Value [Unit]
k_d	1000 [s ⁻²]
τ_d	0.01 [s]