Feedback Controllers as Financial Advisors for Low Income
Individuals

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Abstract

Feedback controllers are introduced to help manage an individual's or household's financial life and build savings. The controllers can be viewed as financial advisors for a household's resource allocation problem, which is modeled as a discrete time stochastic system with unknown parameters, income uncertainties, asset losses, and constraints on cash flow and credit. We first compare a model predictive controller (MPC), proportional-integral derivative (PID) controller, and adaptive controller (AC) with a benchmark method in used economics, stochastic dynamic programming (DP). We show that a higher percentage of households are able to achieve higher savings and income by following the spending and investment recommendations made by the feedback controllers compared to DP. Finally, we discuss prospective environment for the implementation of the feedback controllers.

This is dedicated to my beautiful wife Oliva and my son Hugo Rafael, my strength,
motivation and goal in this journey.

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Chapter 1: INTRODUCTION

Humanity is facing significant challenges to improve the quality of life of more than 1 billion people that live in extreme poverty. Engineers are increasingly committing themselves to developing and implementing technologies that improve the overall welfare of the poor [29], proposing practical applications based on the empirical and theoretical findings in the context of social, political, and economical systems. This paper introduces the use of “feedback control engineering” to help the poor manage their financial lives.

Current research aimed at alleviating poverty is clearly dominated by development economics, and most of the aid agencies and multi-lateral institutions’ efforts rely on findings in this area [41]. Concepts such as poverty traps [9, 11, 33] and economic growth [2] are mainstream, where the notion of utility-maximizing behavior of representative agents is central to these studies [1, 34]. On the other hand, studies in the field of finance take a more practical approach to acknowledge which are the best practices to overcome poverty, as noted in [30, 32]. The optimal allocation of investment and consumption has been studied in [24] with extensions to the household finance in [10] and some insights into households in developing economies can be found in [42]. Viewing economic systems as evolving dynamical systems where agents need to adapt themselves to a changing environment can lead to a holistic

view of the phenomena, and the design of better tools to help with the pressing need for managing the economic lives of the poor.

Various publications stress the importance of providing better tools for the financial lives of the poor [13, 18, 30, 43]. In [13], three fundamental financial needs are identified from a survey of 250 households from South Asia and South Africa. The first of these needs is to cope with irregular income to meet daily basic needs. The second is to deal with “shocks,” even with low savings. The third is to find strategies to accumulate larger sums of money via savings. This last need is considered of significant importance in [30], where, after analyzing individuals in 49 villages in Thailand, the authors concluded that households that could accumulate savings by any means often dealt better with uncertainties and shocks.

Treating an economic agent and the local market as a dynamical system opens the possibility of employing feedback controllers to manage budgets, overcome savings and credit constraints, become a tool to overcome low financial literacy of people in poverty [19], and achieve *reliability* by adapting to the changing environment when dealing with risk and uncertainties. It is our belief that a possible framework for implementation of these controllers could be a microfinance institution that deals with individuals (households), mainly small-scale entrepreneurs, seeking to improve their well-being and escape poverty. There has been a broad consensus among scholars on the role of microfinance as a tool to alleviate poverty [13, 18], and their close contact with people represents an attractive alternative to apply feedback controllers as financial advisors for their clients.

The interaction between control engineering and economics went through three different periods as noted by [26]. The first one began in early 1950’s with the works

by electrical engineer Arnold Tustin in [39], later followed with the work in [31]. These early attempts used linear feedback control tools and their analysis remained limited due to uncertainty in the parameters of economic models and lack of flexibility when it comes to modifying parameters in the controller. The second period corresponded to the application of stochastic optimal control theory in econometric models. Significant works of this period are [4, 12, 40]. In the third period, in the late 70's, control theory approaches in economics were not pursued because they failed to represent “rational expectations”, that is, the fact that the current state depends on future predictions (Lucas critique, [23]). The few publications in this third period have to do with the use linear quadratic regulators and dual control [20], and applications of sliding mode control, to follow a reference optimal control path under uncertainty [36]. A renewed interest in control theory tools applied to economics has been promoted by the use of Kalman filter [3] and adaptive control [14] to mitigate the rational expectations critique. For a more extensive review, see also [21].

Building on these facts, here we propose three controllers that can steer the economic dynamical model to meet the three needs mentioned above. Although it will be difficult to establish any analogy between controllers commonly found in industrial plants with humans, the idea remains valid for our goal in this work, which is to show the decisions a household must take to build the savings that will allow it to achieve a high income state and escape poverty.

In Chapter 2 we develop a multiple input, multiple output (MIMO) stochastic model of household's financial dynamics based on models found in the finance and economics literature. This model features uncertain income, sudden losses of wealth

(shocks), and the inherent liquidity and subsistence constraints that head of households have to take into account when deciding where to allocate current wealth to ensure successful adaptation to the its uncertain environment. The model also provides the possibility to implement feedback controllers to help manage these decisions and achieve higher savings and consumption via a quadratic objective function.

In Chapter 3 we present various feedback controllers that could make the necessary recommendations on spending and investment to achieve the desired goal. We start with our base case, the stochastic dynamic programming (DP), common in economics and finance studies. The model predictive controller (MPC) is also a frequent candidate to be considered when dealing with a system with constraints on the input as in our case, and hence is also explored in this work. The prevalent proportional, integral and derivative (PID) controller is also shown in the form of parallel controllers for each input. Finally, the MIMO indirect adaptive controller (AC) is also implemented to deal with the seasonal variations of income.

In Chapter 4, we display the results of single run and Monte Carlo simulations of the model for each of the controllers. The advantages of PID and AC in relation to the DP controller are shown via several plots. A graphical representation of a “poverty trap” is also discussed in the final section of the chapter.

In Chapter 5, concluding remarks are given and some implications for a practical implementation of the controllers are discussed.

Chapter 2: A MODEL OF ECONOMIC AGENT'S DYNAMICS

“Households” in development economics are often portrayed as both consumption and production units, either wage earners or small entrepreneurs [38]. Building on the model in [17], we propose a discrete time stochastic model with constraints that reflect income uncertainties, investments, risks, and liquidity constraints for underprivileged households. The dynamical model presented here intends to fit some of the empirical findings in [13] and [30]

2.1 Household Dynamics

The state variable at time k is $x(k) = [x_1(k), x_2(k)]^T$ where $x_1(k) \in \mathbb{R}$ corresponds to cash in hand, or household savings, and $x_2(k) \in \mathbb{R}$ is the capital invested on a risky asset. The control or decision variable is $u(k) = [u_1(k), u_2(k)]^T$ where $u_1(k) \in \mathbb{R}$ is household consumption and $u_2(k) \in \mathbb{R}$ is household investment in period k . Here, we will represent an entrepreneur who, at each time step, must decide how much to consume for household's needs, how much to invest into the working capital $x_2(k)$ to secure future income and how much to save, i.e. increase cash in hand x_1 in order to overcome the uncertain income and possible shocks. The model is

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 1 & v(k) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} -1 & -(1-v(k)) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1(k) \\ u_2(k) \end{bmatrix} + \begin{bmatrix} g(k) - \delta(k) \\ -\gamma(k) \end{bmatrix} \quad (2.1)$$

where $v(k)$ is a random return on the capital invested $x_2(k)$. As the markets in developing countries evolve in a different way than in the developed world and there is a gap in the literature in representing those markets, we are choosing to model this variable as $v(k) = \sigma + \omega(k)$ where σ is the expected return on the investment and $\omega(k)$ is a zero mean Gaussian random variable with variance κ . Furthermore, we consider a deterministic income $g(x) \geq 0$ that could be thought of as wages or cash transfers and random variables $\delta(k) \geq 0$ and $\gamma(k) \geq 0$ that represent negative shocks to cash in hand and working capital respectively, and will be modeled as independent and uncorrelated discrete time events with a specific probability distribution.

2.2 Constraints

The constraints on the control space (u_1, u_2) can be modeled as

$$u_1(k) \geq c_{min} \quad (2.2)$$

$$u_2(k) \geq -x_2(k) \quad (2.3)$$

$$u_1(k) + u_2(k) \leq x_1(k) \quad (2.4)$$

where the lower bound on $u_1(k)$ is the subsistence constraint or minimum consumption needed to survive. The lower bound on $u_2(k)$ is the available amount for disinvestment, which represents the fact that the agent cannot borrow and must overcome any possible deficit at cash in hand $x_1(k)$ by transferring up to all of his/her own

working capital $x_2(k)$. Next, the upper bound on the sum $u_1(k) + u_2(k)$ is the liquidity constraint, and it simply implies that at the beginning of each period k , when the income from the investment $v(k)(x_2(k) + u_2(k))$ has not been realized yet, the agent cannot create a budget that will deplete all of his cash in hand, and if the desired consumption $u_1(k)$ is greater than the available cash, i.e., if $u_1(k) > x_1(k)$, then the deficit must be covered by the working capital $x_2(k)$ via negative investment $u_2(k) < 0$ as explained above. Note that this last statement does not imply that we are assuming that $x_1(k) \geq 0$ as we are allowing it to go negative after the realization of the shock $\delta(k)$. Figure 2.1 shows the constraints on the control space at some k , where in the left plot we see the existence of a feasible set Ω_k (shaded), whereas the constraints in the right plot do not allow the existence of a feasible solution.

Figure 2.1: Existence of a feasible set Ω_k .

2.3 Objective Function

For the sake of simplicity, we kept the model two dimensional, although it can be easily modified to consider additional investment options or the possibility of

borrowing by augmenting the dimension of the system via the addition of integrators. With this model, we will try to develop controllers that help the entrepreneur to build savings that will make him/her more resilient to shocks and uncertainty. Our goal is not to model the agent's behavior by employing cost functions such as the CRRA (constant relative risk aversion) utility function or other standard isoelastic cost function [1]; instead, we seek to minimize of a running quadratic cost over a finite horizon with target states $x^* \in \mathbb{R}^2$ and controls $u^* \in \mathcal{U}$,

$$g(x(k), u(k)) = \min_{u(k)} (\|x(k) - x^*\|^T Q \|x(k) - x^*\| + \|u(k) - u^*\|^T R \|u(k) - u^*\|) \quad (2.5)$$

Throughout the paper, we make the assumption that agents have perfect knowledge of the states, i.e., the agent has information about his/her cash in hand and working capital, which is usually the case for individuals or households with a basic level of financial literacy. This avoids the use of estimators and observers for the state.

Chapter 3: HOUSEHOLD ECONOMIC DECISION MAKING USING FEEDBACK

3.1 Dynamic Programming

A very common method employed to solve dynamic economic models such as the one above is dynamic programming (DP). Using the terminology found [8], the basic problem in DP uses

$$x(k+1) = f_k(x(k), u(k), w(k)), \quad k = 0, 1, \dots, N-1$$

where the state $x(k)$ is an element of $S(k)$ and control $u(k) \in U_k(x(k))$. The random disturbance $w(k)$ is assumed to be independent on past values of the disturbance. A policy is a sequence of functions

$$\pi = \{\mu_0, \dots, \mu_{N-1}\}$$

where μ_k maps states $x(k)$ to controls $u(k) = \mu_k(x(k))$ and it is “admissible” when $\mu_k(x(k)) \in U_k(x(k))$ for all $x(k) \in S(k)$. For given functions $g(k)$, $k = 0, 1, \dots, N$, also called utility functions, given initial state x_0 and an admissible policy π the expected cost is

$$J_\pi(x_0) = \mathbb{E}_{w(k)} \left\{ g_N(x_N) + \sum_{k=0}^N g_k(x(k), \mu_k(x(k)), w(k)) \right\}$$

An optimal policy π^* is one that minimizes this cost, that is

$$J_{\pi^*}(x_0) = J^*(x_0) = \min_{\pi \in \Pi} J_{\pi}(x_0)$$

where Π is the set of all admissible policies.

The principle of optimality implies that the optimal cost $J^*(x_0)$ is equal to $J_0(x_0)$, where the function J_0 is given by the last step of the following routine which proceeds backward in time from period $N - 1$ to period 0 by

$$J_N(x_N) = g_N(x_N)$$

$$J_k(x(k)) = \min_{u(k) \in U_k(x(k))} \mathbb{E}_{w(k)} \{g_k(x(k), u(k), w(k)) + J_{k+1}(f_k(x(k), u(k), w(k)))\}$$

for $k = 0, 1, \dots, N - 1$.

In general, a closed-form solution for the DP routine is not possible except for some specific cases. One typically has to resort to a discretization of the state space and proceed with a numerical computation. The computational effort, that increases with the number discretization points, leads to the “curse of dimensionality,” a term coined by Bellman which implies that realistic models with a large number of variables can become quickly intractable.

For linear systems with quadratic cost, an analytical solution is available for both the finite and infinite horizon via the Riccati equation and the algebraic Riccati equation respectively. Also, for LQR with constraints on states and control, a closed form solution is available [7]. However, as our model presents state-dependent saturation and a random state matrix besides the additive disturbances, our goal is to find the optimal solution to a non-linear time-varying stochastic model. For this reason, we resort to the finite horizon minimization of the quadratic cost for the system.

3.2 Model Predictive Control

In the recent decades, a variety of approaches for MPC have emerged to operate large scale industrial control systems with uncertainties and constraints [16]. MPC features the explicit use of a model to predict the system's behavior, the calculation of a control sequence minimizing a cost function with a receding horizon strategy. The process is repeated at each time step with the use of the output measurements. Note that MPC resembles human decision making with the use of a model (mental model of the process) [28]. Several applications of MPC can be found in economics and finance, including portfolio optimization [15, 17], rational expectations [25], and also in sustainability studies [22]. It should be noted that our goal is not to cast doubt on the standard approach in economics of using dynamic programming, but to offer a connection with feedback control aiming at more practical implementations.

To present the basic formulation of the MPC, suppose

$$x(k+1) = f_k(x(k), u(k), w(k)), \quad k = 0, 1, \dots, N-1$$

with the same assumptions as DP with respect to $x(k) \in S_k$ and $u(k) \in U_k(x(k))$.

Then, the MPC algorithm proceeds as follows at each time k

1. Given a time horizon $k+m$, find a controller sequence $\{\bar{u}(k), \bar{u}(k+1), \dots, \bar{u}(k+m-1)\}$

that minimizes

$$\sum_{i=k}^{k+m-1} g_i(x(i), u(i))$$

subject to the constraints

$$x(i+1) = f_i(x(i), u(i)), \quad u(i) \in U_i, \quad i = k, k+1, \dots, k+m-1$$

2. Apply the control input

$$\bar{\mu}(x(k)) = \bar{u}(k)$$

3. Return to step 1 until the final time N .

Solving an open loop problem lets us avoid the complications of the “curse of dimensionality” while maintaining closed loop performance.

3.3 Proportional Integral Derivative (PID) Control

A widely successful controller for many industrial applications is the proportional integral derivative (PID) controller. An advantage of this controller is that no prior knowledge of the plant parameters is required and yet, via appropriate tuning, good performance is often obtained even for nonlinear models with random variables as is the case here.

3.3.1 Formulation

As the PID controller does not explicitly considers the minimization of a cost function in its definition, in order to compare the performance of this controller with the previous one, we are defining the vector $\bar{y}(k)$ as

$$\bar{y}(k) = Cx(k) + Du(k) \tag{3.1}$$

where $\bar{y}(k) = [\bar{y}_1(k), \bar{y}_2(k)]^T$ and C and D are matrices with appropriate dimensions. Here, we study a tracking problem where we will design a controller to make $\bar{y}(k)$ track reference input $r(k)$ which is defined as

$$r(k) = Cx^* + Du^*$$

where the target vectors x^* and u^* are used in Equation (2.5). We choose the matrices C and D to be

$$C = Q, \quad D = R$$

where R and Q are the matrices from the cost function in Equation (2.5). This implies that goal of the controller will be to drive the error $e(k) = r(k) - \bar{y}(k) = C(x^* - x) + D(u^* - u)$ to zero, which is also the minimum in the Equation (2.5).

We define a PID controller as the feedback law corresponding to each input $u_i(k)$ for $k \in \{1, 2\}$. Each controller produces an input based on the error value defined as $e_i(k) = r_i(k) - \bar{y}_i(k)$, then

$$u_i(k) = P_i(k) + I_i(k) + D_i(k) \quad \text{for } k \in \{1, 2\}$$

where

$$P_i(k) = K_{p,i}e_i(k), \quad I_i(k) = I_i(k-1) + K_{t,i}e_i(k)$$

and

$$D_i(k) = \frac{K_{d,i}}{K_{d,i} + K_{n,i}}D_i(k-1) + \frac{K_{d,i}K_{n,i}}{K_{d,i} + K_{n,i}}(e_i(k) - e_i(k-1))$$

where $P_i(k)$, $I_i(k)$, and $D_i(k)$ are the proportional, integral and derivative terms and in the same way, the constants $K_{p,i}$, $K_{t,i}$, and $K_{d,i}$ are the proportional, integral, and derivative gains, and $K_{n,i}$ is the “filter coefficient.” A diagram for the PID controller is shown in Figure 3.1.

3.3.2 Constraint Management

To tune the controller gains for the two controllers, simulations were performed to find the parameters that minimize both the error $e(k) = [e_1(k), e_2(k)]^T$ and at the

Figure 3.1: PID controller diagram.

same time confer reliability to the process, defined in this paper as the ability of the household to find a control input $u(k)$ within the feasible set Ω_k at each time k . In order to simultaneously deal with the constraint $u_1(k) + u_2(k) \leq x_1(k)$, we develop the following “projection algorithm” to ensure the control input remains in the feasible set Ω_k . A graphical representation of the algorithm is shown in Figure 3.2.

Note above that the term “management failure” refers here to the case where, at some time period k , the controllers are unable to find a solution within the feasible set Ω_k , and represents the situation where there is not enough liquidity to ensure the acquisition of basic needs to ensure subsistence.

3.4 Adaptive Controller

The model we are using represents a household’s finance dynamics with income generation, capital investments, and consumption. Due to the heterogeneity in the

Algorithm 1 Projection Algorithm

```

1: if  $u_{c1}(k) + u_{c2}(k) > x_1(k)$  then
2:   project the pair  $u_{c1}(k) + u_{c2}(k)$  to the closest point  $u_{p1}(k) + u_{p2}(k)$  on the line
    $u_1 + u_2 = x_1$ 
3:   if  $u_{p1}(k) \leq u_{min}$  and  $u_{p2}(k) > -x_2(k)$  then
4:      $u_1(k) \leftarrow u_{min}$ 
5:      $u_2(k) \leftarrow x_1(k) - u_1(k)$ 
6:   else if  $u_{p2}(k) \leq -x_2(k)$  and  $u_{p1}(k) > u_{min}$  then
7:      $u_2(k) \leftarrow -x_2(k)$ 
8:      $u_1(k) \leftarrow x_1(k) - u_2(k)$ 
9:   else
10:    No feasible solution. Terminate simulation as a “management failure”.
11:   end if
12: else
13:    $u_1(k) \leftarrow u_{c1}(k)$ 
14:    $u_2(k) \leftarrow u_{c2}(k)$ 
15: end if

```

Figure 3.2: Projection algorithm, graphical representation.

structure of households and their financial life, there is no precise deterministic model that can portray all possible behaviors; hence we have to deal with a model with uncertain parameters and also random variables for uncertain income and shocks. Adaptive controllers are well-suited for such highly uncertain systems [37].

There are two commonly used approaches to adaptive control. The first one is called “direct adaptive control,” which has an adaptation mechanism that observes the inputs and outputs of the process and adapts the parameters of the controller to maintain performance despite variations in the process. “Indirect adaptive control” makes use of an online identifier to estimate the plant input-output mapping and uses this to adjust a controller for the process. If the process parameters change, the identifier will take into account these changes and the controller designer will subsequently tune the parameters to maintain performance. The structure of the identifier model for both cases could be either linear with adjustable coefficients or alternatively it could be neural or fuzzy system with tunable parameters. After some simulations, we chose to implement the indirect adaptive control for the greater reliability (as defined above) showed and from now on it is going to be referred simply as the adaptive controller (AC). We will consider the discrete-time non-linear input-output difference equations of the form

$$y(k+1) = \alpha(x(k)) + \beta(x(k))u(k) \quad (3.2)$$

where the output is $y(k+1) = [y_1(k+1), y_2(k+1), \dots, y_n(k+1)]^T$, the input is $u(k) = [u_1(k), u_2(k), \dots, u_n(k)]^T$ and the functions $\alpha(x(k)) \in \mathbb{R}^n$ and $\beta(x(k)) \in \mathbb{R}^{n \times n}$ are the ones we are interested in estimating. For a full set of assumptions of the form of Equation (3.2) the reader is referred to [27]. Using a “certainty equivalence” approach, the feedback linearizing control law is

$$u^*(k) = \hat{\beta}^{-1}(x(k))(-\hat{\alpha}(x(k)) + r(k+1) + \nu(k))$$

where $\hat{\alpha}(x(k))$ and $\hat{\beta}(x(k))$ are estimates of $\alpha(x(k))$ and $\beta(x(k))$ respectively and $\nu(k)$ is an input to the linearized plant dynamics. The estimates can be written as

$$\hat{\alpha}(x(k)) = [\hat{\alpha}_1^T(x(k)), \hat{\alpha}_2^T(x(k)), \dots, \hat{\alpha}_n^T(x(k))]^T$$

$$\hat{\beta}(x(k)) = \begin{bmatrix} \hat{\beta}_{11}(x(k)) & \cdots & \hat{\beta}_{1n}(x(k)) \\ \vdots & \ddots & \vdots \\ \hat{\beta}_{n1}(x(k)) & \cdots & \hat{\beta}_{nn}(x(k)) \end{bmatrix}$$

and it is assumed that a projection algorithm is used to ensure the invertibility of matrix $\hat{\beta}(x(k))$. The estimates are given by

$$\hat{\alpha}(x(k)) = \theta_\alpha^T(k) \phi_\alpha(x(k))$$

$$\hat{\beta}(x(k)) = \theta_\beta^T(k) \phi_\beta(x(k))$$

where $\theta_\alpha^T(k)$ and $\theta_\beta^T(k)$ are adjusted in order to improve the approximation. $\phi_\alpha(x(k))$ and $\phi_\beta(x(k))$ are the approximators vectors and their structure depends on the approximator used.

The normalized gradient method is chosen for the adaptation mechanism [37].

The optimization parameters are

$$\theta_\alpha(k) = [\theta_{\alpha_1}(k), \dots, \theta_{\alpha_n}(k)]$$

$$\theta_\beta(k) = \begin{bmatrix} \theta_{\beta_{11}}(x(k)) & \cdots & \theta_{\beta_{1n}}(x(k)) \\ \vdots & \ddots & \vdots \\ \theta_{\beta_{n1}}(x(k)) & \cdots & \theta_{\beta_{nn}}(x(k)) \end{bmatrix}$$

and the adaptation law is

$$\theta_{\alpha_i}(k) = \theta_{\alpha_i}(k-1) - \frac{\eta_{\alpha_i} \phi_{\alpha_i}(k-1)}{1 + \gamma_{\alpha_i} |\phi_{\alpha_i}(k-1)|^2} e_{\epsilon_i}(k) \quad \forall i \in \{1, 2, \dots, n\}$$

$$\theta_{\beta_{ij}}(k) = \theta_{\beta_{ij}}(k-1) - \frac{\eta_{\beta_{ij}} \phi_{\beta_{ij}}(k-1)}{1 + \gamma_{\beta_{ij}} |\phi_{\beta_{ij}}(k-1)|^2} e_{\epsilon_i}(k) \quad \forall i \in \{1, 2, \dots, n\}, \quad \forall j \in \{1, 2, \dots, n\}$$

where η_{α_i} and $\eta_{\beta_{ij}}$ are adaptation gains and γ_{α_i} , $\gamma_{\beta_{ij}}$ are “robustification parameters” for $\theta_{\alpha_i}(k)$ and $\theta_{\beta_{ij}}(k)$ respectively. The term $e_{\epsilon_i}(k)$ is a “dead zone modification” of the tracking error corresponding to the i^{th} state and is useful to achieve stability with finite size approximators.

In our case, for the approximator vectors we choose

$$\phi_\alpha(x(k)) = \phi_h(y(k))$$

$$\phi_\beta(x(k)) = \phi_h(y(k))u^T(k)$$

A Takagi-Sugeno fuzzy approximator with p rules was used to build the vector $\phi_h(y(k))$. In this case, considering that the approximator takes as an input the vector $y(k)$

$$\phi_h(y(k)) = [\zeta_1, \dots, \zeta_p, y_1(k)\zeta_1, \dots, y_1(k)\zeta_p, \dots, y_n(k)\zeta_1, \dots, y_n(k)\zeta_p]^T$$

where

$$\zeta_j = \frac{\mu_j}{\sum_{i=1}^p \mu_i}$$

with μ_j representing the value that the membership function for the j^{th} rule takes on at input $y(k)$. For more details on Takagi-Sugeno fuzzy systems, see [37].

Chapter 4: SIMULATION RESULTS

4.1 Model Parameters

The time horizon of the study is $T = 100$ weeks, with the sample interval equal to one week. The random return on investment is

$$v(k) = \sigma(k) + \omega(k)$$

where the mean value σ fluctuates to account for seasonal variations (within a year) on income noted in [13] according to

$$\sigma(k) = \bar{\sigma} + 0.18 \sin \left(2\pi k \frac{1}{26} - \xi\pi \right)$$

with $\bar{\sigma} = 0.35$ chosen in accordance to the results in [30], and ξ is a random number that modifies the phase of the sinusoid to consider different seasonal variations in the Monte Carlo simulations. The variable $w(k)$ is the volatility of the return on the investment and it follows a discrete normal distribution with zero mean and covariance $\kappa = 0.2$. Furthermore, the variables $\gamma(k)$ and $\delta(k)$, corresponding to discrete shocks, were generated in the following way [5] and both follow the same probability mass function

$$f_{\gamma}(\gamma(k)) = \begin{cases} 0.98 & \text{if } \gamma(k) = 0 \\ 0.01 & \text{if } \gamma(k) = 1 \\ 0.005 & \text{if } \gamma(k) = 2 \\ 0.005 & \text{if } \gamma(k) = 4 \end{cases} \quad (4.1)$$

The minimum consumption is $u_{min} = 0.5$ and the initial conditions were chosen to be $x(0) = x_0 = [2, 1]^T$. The reference state and input values are $x^* = [10, 6]^T$ and $u^* = [2.1, 0]^T$ respectively. Both matrices R and Q are considered equal to the identity matrix. Hence, the goal is to increase by five times the available cash in hand and by four times the minimum consumption.

4.2 Controller Parameters

With respect to controller parameters, for the DP approach we had to discretize both the state and control spaces and build a finite Markov chain to obtain a desired feedback law within our computational capabilities. For this, we chose $x_1(k) \in \{-1, -0.45, \dots, 12\}$, $x_2(k) \in \{0, 0.33, \dots, 8\}$, $u_1(k) \in \{0.5, 0.64, \dots, 6\}$, and $u_2(k) \in \{-2, -1.79, \dots, 3\}$. To model the random variable $w(k)$ we approximate the discrete Gaussian with $w(k) \in \{-0.13, -0.085, \dots, 0.13\}$, each with probability 0.2. The variables $\gamma(k)$ and $\delta(k)$ were modeled as in Equation (4.1). For the MPC, the prediction horizon was chosen as $m = 8$ simulating that the controller would have an estimation of the return on investment $v(k)$ of up to eight weeks. For the PID controller, we tuned the controller to obtain a good compromise between tracking and reliability performance. We chose, after tuning, for PID controller 1, $K_{p1} = -0.8$, $K_{i1} = -0.05$, $K_{d1} = 0$ and for PID controller 2, $K_{p2} = 0.17$, $K_{i2} = 0.24$, $K_{d2} = 0$. Finally, we also tuned the AC with the same objectives as the PID. We selected $\eta_1 = \eta_2 = 0.1$, $\eta_{11} = 0.005$, $\eta_{12} = 0.1$, $\eta_{21} = 0$, $\eta_{22} = 0.005$ and $\gamma_1 = .1$, $\gamma_2 = .05$, $\gamma_{11} = 0.005$, $\gamma_{12} = 0.05$, $\gamma_{21} = 0$, $\gamma_{22} = 0.0025$. A Takagi-Sugeno fuzzy approximator with 126 membership functions was used to build $\phi_h(y(k))$.

4.3 Single Run Simulation

The results for the four controllers are shown in Figure 4.1 for the same sequence of random variables, where it can be seen the time trajectories of cash in hand $x_1(k)$ (left column blue), working capital $x_2(k)$ (left column red), consumption $u_1(k)$ (right column blue) and investment $u_2(k)$ (left column red) for the various controllers. It can be seen that the MPC, PID and AC provide better regulation around the desired trajectory. On the other hand, the DP is affected by the seasonal variation of the income, which is noted in the oscillations of cash in hand $x_1(k)$. It is interesting to note that all controllers react to a sudden loss in working capital $x_2(k)$ by reducing consumption $u_1(k)$ and increasing investment $u_2(k)$. Also, notice the behavior at the beginning of the simulation. All controllers suggest large investments and reducing consumption to the minimum for a period of time. We can reduce this behavior by allowing the desired state trajectory to follow a ramp with a desired slope to the targeted values.

4.4 Tracking Performance Results

In Figure 4.1, when only one simulation run is considered, all controllers provide similar suggestions for consumption and investments. However, differences arise when we study the performance in a large number of simulations. For instance, the results of 2,500 Monte Carlo run are shown in Figure 4.2. Figure 4.2a corresponds to the mean of cash in hand $x_1(k)$ and consumption $u_1(k)$, denoted by the square and cross respectively and the standard distribution, represented by the vertical lines, for each controller. Figure 4.3b shows the mean and average final cost $J = \sum_{k=0}^N (g(x(k), u(k)))$, for the controllers, where $g(x(k), u(k))$ is defined in Equation (2.5). It can be seen

Figure 4.1: Time trajectories for the various controllers.

that the performance of the MPC and the two feedback controllers are superior to those of the DP controller in terms of tracking the desired states and input and in terms of cost, providing both a lower expected value and lower standard deviation on J .

4.5 Reliability Performance Results

Figure 4.3a shows reliability results using 2,500 Monte Carlo run. The plot axes are the return on investment $\bar{\sigma}$ and initial wealth $w(0) = x_0(1) + x_0(2)$. For simplicity, we assumed in this particular simulation that $x_0(1) = x_0(2) = (1/2)w(0)$. Here, the curves corresponds to DP and adaptive controller. The curves divide the plane in two zones: the zone to the top-right of the curves corresponds to households with non-zero probability of reaching the desired states and control inputs, whereas that probability is zero for households with initial wealth and return on investment that

Figure 4.2: Results for 2,500 Monte Carlo simulations.

lies in bottom-left of the curves, lying in the called management failure zone. We only included AC because both MPC and PID controller show very similar results in terms of reliability. We can see that the superior reliability performance of the AC is shown by smaller failure zone is with respect to the DP. Figure 4.3b shows that this behavior is also observed in the deterministic case, that is considering all random variables equal to their expected values. These phenomenon can be interpreted as a poverty trap, using the terms defined in [11], where the low ability to obtain income and low initial endowment hinder the possibility to reach the desired state. However, here the poverty trap is a consequence of an inherent property of the system, controllability [6], rather than a phenomenon driven by multiple equilibria as in [11], as it can be argued that initial conditions in the management failure zone do not lie on the controllable subspace. Despite the existence of the poverty trap, we see that more households are likely to overcome the poverty trap by following the spending and investment recommendations by the adaptive feedback controller.

(a)

(b)

Figure 4.3: (a) Stochastic and (b) deterministic poverty trap.

Chapter 5: CONCLUSION

We have evaluated four feedback controllers that could help manage the financial lives of the poor living under \$2 per day. Based on the tracking performance and superior reliability shown by MPC, PID and AC with respect to DP, we conclude that these controllers are suitable tools to help manage the economic lives of the poor. The results also showed that wealth allocation to ensure present and future subsistence under limited resources requires sophisticated tools and in some cases it is not even possible as shown by the reliability plots. The controllers allowed households to be better prepared to deal with uncertain income and shocks via proper management of their financial life, building the savings that may grant them the ability to perform the necessary investments to improve their well-being.

5.1 Implications for Implementation

In this section we discuss the aspects to be considered when designing an automated financial advisor. We are assuming that the implementation of the feedback controller will be via a software that will receive as input the actual cash in hand, working capital, which could be of various accounts, and debt and will return the

daily, weekly, or monthly consumption, investments for each account and debt payment. The time step in either days, weeks, or month will be set by the user as well as the target wealth and consumption.

The following three options can be considered as implementation tools for the controller as identified in [29]:

1. Smart phone or cell phone: The UN estimates that of the 7.2 billion people in the world, over 6 billion persons have cell phone subscriptions. Two possibilities are identified: first, the program could use the computing power of the phone to run the PID or AC strategies. Second, the program can be running on a central computer and the communications can be made via short message service (SMS) using an SMS gateway or via voice messages using a similar platform. In the former case, involvement of the network carrier is needed in order to succeed.
2. Personal electronic device: A simple dedicated electronic device could be designed to implement the PID or AC strategy, and it could be made to be much less expensive than a smart or cell phone due to the fact that Microprocessors with the needed computing power are inexpensive nowadays.
3. Centralized computer: A computer at some central location (e.g., at micro-finance institution offices) could be used to manage the finances for poor persons in the area. Each week or month the person user inputs his/her data and the computer returns the spending, investment, and debt payment recommendations for the following time period. Of course, such a computer could be in a kiosk and be self-operated in either an urban or rural area. The controller can

run on “free-source” environments such as Scilab [35]. It can also be linked to free source management platforms for micro-finance institutions.

Additional analysis for the feedback controller strategy is needed to study the impact time delays in the feedback data, noise on the controller input, i.e. the person or households are not following the recommendations or other behavioral aspects not considered in this paper, the adequate graphic user interface (GUI) for various kind of users with different financial literacy, among others. All of these issues can be studied via field experiments and simulations.

5.2 Future Research Directions

The model can be expanded to add estimation of the dynamics of the return on the investment $v(k)$, noting that in the financial literature the markets are modeled as driven by Brownian motion processes. Future research directions could include also studying the effects of a kin-network on a household’s well-being via decentralized control approaches.

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