Hierarchical Framework for Demand-Side Frequency Control

Thesis

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By

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Abstract

This thesis develops a hierarchical framework for demand-side frequency control. First, we study the implementation of the hierarchical decentralized controller based on a reduced nonlinear multi-machine power system model in order to stabilize the system. The framework involves two decision layers. The supervisory layer determines a power control command for the aggregated power output response on each load bus using robust decentralized control theory. The device layer involves a large number of controllable loads, which switch probabilistically during contingencies based on a Markov-chain model, local frequency and angle measurements, so that the aggregated power output deviation from the controllable loads matches the desired droop amount according to the power control command determined in the supervisory layer. The proposed framework can deal with time-varying system operating conditions while respecting the physical constraints of individual devices. Realistic simulation results based on a 68-bus system are provided to demonstrate the effectiveness of the proposed strategy. Then, we extend the hierarchical implementation to a structure preserving multi-machine power system model with only frequency measurements. The idea of implementing primary frequency control through end-use devices is investigated. The hierarchical decentralized framework is implemented in order to provide the system with a droop-like response. The proposed framework enables the systematic design of practically implementable demand-side frequency controllers that can

effectively increase the system damping. At the supervisory layer, the power control command characterized by a feedback gain is determined. This gain specifies the desired aggregate power output response at each load if frequency deviates. This gain is periodically updated by taking into account the time-varying availability of end-use devices and the system oscillatory modes under different operating conditions. At the device layer, each end-use device changes its operating mode independently with certain probability that are calculated based on a Markov-chain model, the received control gain, and local frequency measurement. Simulation results based on a 13-bus system demonstrate the efficacy of the proposed demand-side primary frequency control strategy.

This thesis is dedicated to my parents, Bolivar Moya and Monica Calderon and my little sister Erika Moya, for all their support throughout my entire academic career.

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CHAPTER 1: INTRODUCTION

1.1 Background and Motivation

In the conventional power grid paradigm, the system generation is actively controlled to match the system load that is passive for most of the time. End-use loads are generally not controlled unless the system is experiencing severe stability issues which may cause under-frequency load shedding. The transient stability and small signal stability of the power grid are primarily maintained by the droop control of synchronous generators through speed governors. However, the conventional load-following control strategy becomes less effective in maintaining the system stability under the high penetration of intermittent renewable energy such as wind energy and solar energy. This is because the system inertia and damping will be greatly reduced with growing penetration of renewable generation that has very small or no rotating mass. Recently, the idea of making end-use loads controllable, that is, generation-following [3], is attracting considerable attention in order to compensate the inertia loss in the presence of renewable integration. Those smart end-use appliances in today's consumer market are capable of two-way communications, which can enable grid operators to influence the energy consumption. Although the benefits and possibilities of implementing demand-side control for the future power grid paradigm

become more obvious, a systematic design of practical load control strategies is still under investigation.

In the literature, several demand-side frequency control strategies have been proposed in [4–7]. In [4], decentralized load controllers were proposed to stabilize the system frequency after disturbances. In [5], local frequency based load control strategies were developed in order to balance the demand with supply, and regulate system frequency. Both approaches are based on a lumped linear power system model, which cannot accurately represent the transient dynamics of the complex power system. One of the first attempts to design demand-side frequency control based on the nonlinear multi-machine power system model was presented in [6]. This approach seeks to match the real-time power balance and regulate the frequency while minimizing the dis-utility of individual end-use loads. The effect of stochastic frequency measurement noise was studied in [7]. However, the proposed load control in [6] and [7] still suffers from the overly simplified power system model considered in the controller design. Moreover, none of the above load control strategies has taken into account end-use load constraints and end-user behaviors such as compressor time delays for thermostatically controlled loads, and stochastic load arrival and departure. These constraints and behaviors can significantly influence the aggregate response of end-use loads and thus deteriorate the overall load control performance.

Different from most existing works in the literature, we propose a hierarchical decentralized framework to enable a systematic design of demand-side frequency control. The proposed framework will be first based on a network reduced nonlinear multi-machine power system model. The hierarchical framework involves two decision layers. At the supervisory layer, it computes the feedback gains required from

the aggregated load response on each bus using a robust decentralized control approach [8]. Such an approach can achieve robust stability of the system without linearization of the power system model, and can be efficiently solved through Linear Matrix Inequalities (LMI). The robust decentralized control method has been used to design generation-side controllers [1, 9]. Its application to the demand-side frequency control is novel. At the device layer, a Markov-chain model is proposed to represent the aggregated load dynamics under demand-side frequency control. These aggregated dynamics have been considered on slow time scale in [10, 11]. However, properly incorporating them into frequency control has not been considered. The proposed Markov-chain model suffices to capture the aggregated response during the short frequency control period. Mode switching probabilities of individual devices are computed based on the Markov-chain model and the measured frequency and angle so that the total population response matches the desired one determined by the supervisory layer. Although our discussion is mainly based on Thermostatically Controlled Loads (TCLs), the method extends easily to deferrable loads. Compressor time delay constraints of the TCLs [10] are also considered in the design of the device layer. Simulations based on a 68-bus power system are performed in order to validate the hierarchical framework.

Building over our first results, we extend the hierarchical framework implementation to a more general power system model, the structure-preserving power model with only frequency measurements. This extension is the response to the following limitations of the first implementation. First, the controller design is based on the reduced multi-machine power system model, which does not represent system dynamics accurately during transient. Second, the controller design requires not only the local

frequency but also local angle measurement, which makes it difficult for practical implementation. Third, the effectiveness of our first load control implementation is guaranteed only if the population of end-use devices is sufficiently large. The extension of the proposed demand-side frequency control strategy therein can also greatly improve the system damping and restore the system frequency back to an stable value. Furthermore, it systematically takes into account the physical constraints of end-use loads at the same time.

The proposed extension of the framework for demand-side frequency control has two decision layers. At the supervisory layer the control design is based on structure preserving power system model [12]. Moreover, the resulting load controller only requires the local frequency measurement that is practically available. Furthermore, in order to respect the time-varying availability of end-use devices, the control gains are carefully designed by analyzing the system oscillatory behaviors based on the power system modal analysis [13–15]. In this way, we can avoid prematurely exhausting the capabilities of end-use loads in response to system frequency deviation. At the device layer switching probabilities are selected using the Markov-chain approach with only local frequency measurements. Simulation results confirm the efficacy of the newly proposed hierarchical design for demand-side primary frequency control.

1.2 Outline of the Thesis

This thesis is structured as follows:

CHAPTER 2: Hierarchical Framework for Demand Side Frequency Control. We design a hierarchical demand-side frequency controller that robust stabilizes a power system represented by a reduced network power system model. The

hierarchical implementation involves two layers. A supervisory layer efficiently computes a power control command using linear matrix inequalities (LMI). At the device layer switching probabilities are computed using local frequency deviation measurements and its integral (angle), such that the power consumption of controllable loads matches the desired response required at the supervisory layer. Simulation results confirm the efficacy of the proposed framework.

CHAPTER 3: Hierarchical Design for Demand-Side Primary Frequency

Control. We implement a hierarchical demand-side frequency controller that provides a droop-like response to a power system represented by a structure preserving power system model. A supervisory layer computes the power control command characterized by a feedback gain using a linearized model. At the device layer switching probabilities are computed using only local frequency measurements, such that the aggregate consumption of controllable loads matches the desired response required at the supervisory layer. Simulations are provided in order to validate the proposed strategy.

CHAPTER 4: Conclusion. Some key conclusions on the results of the present thesis are stated. Finally, some future research directions are suggested.

CHAPTER 2: HIERARCHICAL FRAMEWORK FOR DEMAND-SIDE FREQUENCY CONTROL

In this chapter, the demand-side frequency control problem is presented over a reduced power system model. We propose a hierarchical framework to implement demand-side frequency control. The framework consists of two interacting layers. At the supervisory layer a power control command is designed using Robust Control methods and efficiently computed through LMI. At the device layer individual switching probabilities are computed from local frequency measurements and its integral (angle deviation), such that the power output deviation of the total population of controllable loads matches the desired response required at the supervisory layer. Realistic simulation results based on a 68-bus power system are presented to demonstrate the effectiveness of the proposed strategy.

2.1 Reduced Multi-machine Power System

We consider a multi-machine power system represented by a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ is the set of n buses, and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of transmission lines that connect the buses. We assume that at each bus $i \in \mathcal{V}$ generation and loads coincide. Furthermore, each bus is considered to be a PV bus with constant voltage magnitude, i.e., $V_i = |V_i|$ for all $i \in \mathcal{V}$. Reactive power dynamics are neglected and loads at each

bus $i \in \mathcal{V}$ are considered to be frequency sensitive [12, 16]. Each transmission line $(i, j) \in \mathcal{E}$, is associated with a complex line admittance value $y[ij] = y[ji]$, where $\Re(y[ij]) = G_{ij} > 0$ is the conductance, and $\Im(y[ij]) = B_{ij} > 0$ is the suceptance of the line. Different from previous works, losses across the transmission network are considered, i.e., the line conductance is such that $\Re(y[ij]) = G_{ij} > 0$.

The dynamics of each bus $i \in \mathcal{V}$ are given by the classical swing-dynamics [17] as follows

$$\dot{\theta}_i = \omega_i, \quad \dot{\omega}_i = \frac{1}{M_i}(-D_{gi}\omega_i + P_{mi} - P_{ei}). \quad (2.1)$$

Here, θ_i is the rotor or electrical/bus angle measured with respect to a rotating frame, ω_i is the angular velocity (frequency deviation), $P_{mi} > 0$ is the power generated or mechanical power input from the generator, $M_i > 0$ is the inertia moment and $D_{gi} > 0$ is the mechanical damping constant. The electrical power P_{ei} at each bus $i \in \mathcal{V}$ is given by

$$P_{ei} = D_{li}\omega_i + f_i(\boldsymbol{\theta}), \quad (2.2)$$

where, $\boldsymbol{\theta} \triangleq [\theta_1, \dots, \theta_n]^T$, $D_{li} > 0$ is the load damping constant [12, 16], and $f_i(\boldsymbol{\theta})$ represents the active power flow from bus $i \in \mathcal{V}$ given in the following form

$$f_i(\boldsymbol{\theta}) = \sum_{j=1}^n (a_{ij} \cos \theta_{ij} + b_{ij} \sin \theta_{ij}).$$

In the above, $\theta_{ij} \triangleq \theta_i - \theta_j$, $a_{ij} \triangleq |V_i||V_j|G_{ij}$ and $b_{ij} \triangleq |V_i||V_j|B_{ij}$. In this thesis, generation-side controllers (such as generation governor droop control and automatic generation control (AGC)) will not be considered in the design. Without generation governor droop and AGC, the mechanical power set-point is constant, i.e., $P_{mi} = P_{mi}^o$. Furthermore, the electrical power (2.2) at each bus $i \in \mathcal{V}$ can be expressed as

$$P_{ei} = D_{li}\omega_i + |V_i|^2 G_{ii} + \sum_{j \neq i, j \in \mathcal{V}} (a_{ij} \cos \theta_{ij} + b_{ij} \sin \theta_{ij}),$$

where $|V_i|^2 G_{ii}$ is the steady state power consumption of the loads under bus $i \in \mathcal{V}$.

2.2 Problem Formulation

At each bus $i \in \mathcal{V}$ of the power system, we can distinguish the following two types of loads:

- $\mathcal{L}_{\text{cont},i}$: set of controllable loads.
- $\mathcal{L}_{\text{no-cont},i}$: set of non-controllable loads.

To simplify our discussion, we assume that the set of controllable loads $\mathcal{L}_{\text{cont},i}$ is composed only of thermostatically controlled loads (TCLs). Some examples of TCLs are refrigerators, HVACs, etc. Such loads are characterized by the fact that they can be switched ON/OFF within a short period of time without affecting the end-use performance [10].

Denote the constant power consumption of the set of non-controllable loads as $P_{\text{no-cont},i}$. Let the individual consumption of each TCL load $j \in \mathcal{L}_{\text{cont},i}$ be given by

$$q_j^i = \begin{cases} p_{\max,j}^i & \text{when load } j \text{ is ON} \\ 0 & \text{when load } j \text{ is OFF.} \end{cases}$$

Let $P_{\text{cont},i}$ be the power consumption of the set of controllable loads under bus $i \in \mathcal{V}$ computed as

$$P_{\text{cont},i} = \sum_{j \in \mathcal{L}_{\text{cont},i}} q_j^i.$$

Then, to account for the power consumption from controllable loads at each bus $i \in \mathcal{V}$, it is necessary to decompose $P_{\text{cont},i}$ as follows

$$P_{\text{cont},i} = \sum_{j \in \mathcal{L}_{\text{cont},i}} q_j^i = P_{\text{cont},i}^0 + u_i. \quad (2.3)$$

In other words, the power consumed by the set of TCL loads is given by the steady state consumption of the loads denoted by $P_{\text{cont},i}^0$, and the controllable power under bus $i \in \mathcal{V}$ denoted by u_i . Let P_{l_i} be the active power consumption at each bus $i \in \mathcal{V}$ given in the following form

$$P_{l_i} = P_{\text{no-cont},i} + P_{\text{cont},i} = |V_i|^2 G_{ii} + u_i.$$

Then, it can be easily verified that the electrical power at bus $i \in \mathcal{V}$ considering the controllable power u_i is given by

$$P_{e_i} = D_{l_i} \omega_i + u_i + f_i(\boldsymbol{\theta}). \quad (2.4)$$

For convenience, and using (2.4) we collect the bus dynamics (2.1) and the controllable power from loads u_i at each bus $i \in \mathcal{V}$, in the following form

$$\begin{aligned} \dot{\theta}_i &= \omega_i, \\ \dot{\omega}_i &= -\frac{D_i}{M_i} \omega_i - \frac{1}{M_i} u_i - \frac{1}{M_i} ((f_i(\boldsymbol{\theta}) - P_{m_i})), \\ u_i &= P_{\text{cont},i} - P_{\text{cont},i}^0, \end{aligned} \quad (2.5)$$

where $D_i \triangleq D_{l_i} + D_{g_i}$.

Define the bus state vector as $\mathbf{x}_i \triangleq [\theta_i, \omega_i]^T$ and the overall interconnected state vector as $\mathbf{x} \triangleq [x_1, \dots, x_n]^T$. Then (2.5) can be written in a state-space form as

$$\dot{\mathbf{x}}_i = \mathbf{A}_i \mathbf{x}_i + \mathbf{B}_i u_i + \mathbf{G}_i F_i(\mathbf{x}), \quad (2.6)$$

where, matrices $\mathbf{A}_i \in \mathbb{R}^{2 \times 2}$, $\mathbf{B}_i \in \mathbb{R}^{2 \times 1}$ and $\mathbf{G}_i \in \mathbb{R}^{2 \times 1}$ are given by

$$\mathbf{A}_i \triangleq \begin{bmatrix} 0 & 1 \\ 0 & -\frac{D_i}{M_i} \end{bmatrix}, \quad \mathbf{B}_i \triangleq \begin{bmatrix} 0 \\ -\frac{1}{M_i} \end{bmatrix}, \quad \mathbf{G}_i \triangleq \begin{bmatrix} 0 \\ -\frac{1}{M_i} \end{bmatrix}, \quad (2.7)$$

and the nonlinear function $F_i : \mathbb{R}^n \rightarrow \mathbb{R}^2$ is given by: $F_i(\mathbf{x}) = f_i(\boldsymbol{\theta}) - P_{m_i}$. We are particularly interested in the power system dynamics during a contingency event.

Thus, it is convenient to work in relative coordinates with respect to the pre-fault equilibrium,

$$\Delta \dot{\mathbf{x}}_i = \mathbf{A}_i \Delta \mathbf{x}_i + \mathbf{B}_i u_i + \mathbf{G}_i h_i(\Delta \mathbf{x}), \quad \forall i \in \mathcal{V}, \quad (2.8)$$

where, $\mathbf{A}_i$, $\mathbf{B}_i$, and $\mathbf{G}_i$ are defined as in (2.7), $\Delta \mathbf{x}_i \triangleq \mathbf{x}_i - \mathbf{x}_i^0$, where $\mathbf{x}_i^0 \triangleq [\theta_i^0, \omega_i^0 = 0]^T$ is the pre-fault equilibrium, $h_i : \mathbb{R}^n \rightarrow \mathbb{R}^2$ is the nonlinear interconnection function, and u_i is the controllable power at bus $i \in \mathcal{V}$.

More generally, the overall dynamics of the interconnected power system can be written in the following form

$$\Delta \dot{\mathbf{x}} = \mathbf{A}_D \Delta \mathbf{x} + \mathbf{B}_D \mathbf{u} + \mathbf{G}_D \mathbf{h}(\Delta \mathbf{x}). \quad (2.9)$$

Here, $\Delta \mathbf{x} \triangleq [\Delta \mathbf{x}_1, \dots, \Delta \mathbf{x}_n]^T \in \mathbb{R}^{2n}$ denotes the overall state vector of the interconnected power system, $\mathbf{A}_D \triangleq \text{diag}\{\mathbf{A}_1, \dots, \mathbf{A}_n\}$, $\mathbf{B}_D \triangleq \text{diag}\{\mathbf{B}_1, \dots, \mathbf{B}_n\}$, $\mathbf{G}_D \triangleq \text{diag}\{\mathbf{G}_1, \dots, \mathbf{G}_n\}$, denote the system matrices, $\mathbf{h} : \mathbb{R}^n \rightarrow \mathbb{R}^n$ represents the overall interconnection function $\mathbf{h} \triangleq [h_1, \dots, h_n]^T$, and $\mathbf{u} \triangleq [u_1, \dots, u_n]^T$ is the controllable power from all buses $i \in \mathcal{V}$.

With the above notations, we can now give a precise statement of the demand-side frequency control problem.

Problem 1 *For each participating controllable load $j \in \mathcal{L}_{cont,i}$, find the probability μ of switching its current state (e.g. turning ON or OFF for a TCL load), such that the controllable power u_i under each bus $i \in \mathcal{V}$, stabilizes the interconnected power system (2.9).*

2.3 Hierarchical Decentralized Controller Design

We present a formal solution to the *demand-side frequency control* problem using a hierarchical framework approach as shown in Fig. 2.1. The hierarchical controller involves two decision layers. The top layer referred to as *supervisory layer* computes the desired power control command u_i characterized by a feedback gain. The gain determines the required power output deviation necessary to stabilize the system under the current system conditions. At the lower layer under each bus referred to as *device layer*, decentralized controllers are designed for each device based on a Markov-chain model. The controllers compute individual switching probabilities μ , so that the power output deviation of the set of controllable loads ($P_{\text{cont},i} - P_{\text{cont},i}^0$) matches the desired power control command u_i determined at the supervisory layer.

Figure 2.1: Hierarchical implementation for demand-side frequency control problem

2.3.1 Supervisory Layer Control Design

At the supervisory layer, we are interested in designing the power control command u_i , that guarantees stability of (2.9) under a given disturbance. To enable a decentralized analysis and implementation of the proposed strategy, we set the power

control command u_i as a state feedback control input, i.e., it is required that the local control input follows a decentralized information pattern, where each bus is controlled using only its locally available states. Therefore, the state feedback power control command at the i th bus is given by

$$u_i(\Delta \mathbf{x}_i) = \mathbf{K}_i \Delta \mathbf{x}_i = K_{i1} \Delta \theta_i + K_{i2} \Delta \omega_i, \quad (2.10)$$

where $\mathbf{K}_i \in \mathbb{R}^{1 \times 2}$ is the feedback control gain matrix, that will be **transmitted** to the device layer under bus $i \in \mathcal{V}$. Let the overall interconnected power control command be defined as

$$\mathbf{u}(\Delta \mathbf{x}) = \mathbf{K}_D \Delta \mathbf{x}. \quad (2.11)$$

Here, the overall feedback matrix denoted by $\mathbf{K}_D \in \mathbb{R}^{n \times 2n}$ is structured as follows $\mathbf{K}_D \triangleq \text{diag}\{\mathbf{K}_1, \dots, \mathbf{K}_n\}$. From the dynamics of the interconnected power system (2.9) and the overall state feedback power control command (2.11), the interconnected power system in closed loop is given by

$$\Delta \dot{\mathbf{x}} = \hat{\mathbf{A}}_D \Delta \mathbf{x} + \mathbf{G}_D \mathbf{h}(\Delta \mathbf{x}), \quad (2.12)$$

where $\hat{\mathbf{A}}_D \triangleq \mathbf{A}_D + \mathbf{B}_D \mathbf{K}_D$. Our goal is to design $\mathbf{K}_D$, such that the equilibrium of the closed loop system is asymptotically stable.

First, note that the nonlinear interconnection function in relative coordinates with respect to the pre-fault equilibrium is such that $h_i(\Delta \mathbf{x}) \triangleq f_i(\boldsymbol{\theta}) - f_i(\boldsymbol{\theta}^0)$, i.e, the power flow in relative coordinates,

$$h_i(\Delta \mathbf{x}) = \sum_{j=1}^n a_{ij} [\cos \theta_{ij} - \cos \theta_{ij}^o] + \sum_{j=1}^n b_{ij} [\sin \theta_{ij} - \sin \theta_{ij}^o].$$

Moreover, observe that h_i is a nonlinear function of the state $\Delta \mathbf{x}$, and it is of an uncertain nature during a contingency event; however, it satisfies the following Lemma.

Lemma 1 *The nonlinear function $h_i(\Delta \mathbf{x})$ for each bus $i \in \mathcal{V}$, satisfies the following quadratic bound*

$$h_i^T(\Delta \mathbf{x})h_i(\Delta \mathbf{x}) \leq \alpha_i^2 \Delta \mathbf{x}^T \mathbf{H}_i^T \mathbf{H}_i \Delta \mathbf{x}, \quad (2.13)$$

for some constant matrix $\mathbf{H}_i$ and scalar value $\alpha_i > 0$.

Proof: Using the arguments introduced in [9], it can be easily verified that the nonlinear function h_i satisfies the following bound

$$h_i^T(\Delta \mathbf{x})h_i(\Delta \mathbf{x}) \leq \Delta \boldsymbol{\theta}^T \mathbf{T}_i^T \mathbf{D}_i \mathbf{T}_i \Delta \boldsymbol{\theta}, \quad (2.14)$$

where $\Delta \boldsymbol{\theta} \triangleq [\Delta \theta_1, \dots, \Delta \theta_n]^T$, $\mathbf{T}_i$ is a constant matrix with entries,

$$[\mathbf{T}_i]_{l_1, l_2} = \begin{cases} -1 & \text{if } l_2 = i, \ l_2 \neq l_1 \\ 1 & \text{if } l_1 = l_2 \neq i \\ 0 & \text{otherwise,} \end{cases}$$

and $\mathbf{D}_i$ is a diagonal matrix with entries $d_{kk}^{(i)} \triangleq 4P_{ik} \sum_{j=1}^n P_{ij} > 0$, where $P_{ij} \triangleq |V_i||V_j||y_{ij}|$ is the maximum power transferred from bus i to bus j . Clearly, (2.14) is of the form (2.13) since $\Delta \boldsymbol{\theta}$ is part of the state $\Delta \mathbf{x}$. ■

Therefore, whenever the nonlinear interconnection function h_i satisfies Lemma 1 for all $i \in \mathcal{V}$, a sufficient condition for the robust stability of the multi-machine interconnected system (2.9) can be established through the following theorem [8].

Theorem 1 *The multi-machine power system (2.9) is robustly stabilized by decentralized linear state feedback power control command (2.10), if the following LMI optimization problem over $\gamma_i, \mathbf{Y}_D$ and $\mathbf{L}_D$ is feasible.*

$$\begin{aligned} & \text{minimize} \quad \sum_{i=1}^n \gamma_i \\ & \text{subject to} \quad \mathbf{Y}_D > 0 \end{aligned}$$

$$\begin{aligned}
& \begin{bmatrix} \mathbf{W}_D & \mathbf{G}_D & \mathbf{Y}_D \mathbf{H}_1^T & \dots & \mathbf{Y}_D \mathbf{H}_n^T \\ \mathbf{G}_D^T & -\mathbf{I} & 0 & \dots & 0 \\ \mathbf{H}_1 \mathbf{Y}_D & 0 & -\gamma_i \mathbf{I} & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{H}_n \mathbf{Y}_D & 0 & 0 & \dots & -\gamma_n \mathbf{I} \end{bmatrix} < 0 \\
& \begin{bmatrix} -\kappa_{L_i} \mathbf{I} & \mathbf{L}_i^T \\ \mathbf{L}_i & -\mathbf{I} \end{bmatrix} < 0, \begin{bmatrix} \mathbf{Y}_i & \mathbf{I} \\ \mathbf{I} & \kappa_{Y_i} \mathbf{I} \end{bmatrix} > 0,
\end{aligned} \tag{2.15}$$

where $i = 1, \dots, n$, $\gamma_i \triangleq \frac{1}{\alpha_i^2}$, $\mathbf{W}_D \triangleq \mathbf{A}_D \mathbf{Y}_D + \mathbf{Y}_D \mathbf{A}_D^T + \mathbf{B}_D \mathbf{L}_D + \mathbf{L}_D^T \mathbf{B}_D^T$ with $\mathbf{Y}_D \triangleq \text{diag}\{\mathbf{Y}_1, \dots, \mathbf{Y}_n\}$ and $\mathbf{L}_D \triangleq \text{diag}\{\mathbf{L}_1, \dots, \mathbf{L}_n\}$. Furthermore, the scalars κ_{L_i} and κ_{Y_i} are prescribed positive limits selected to bound the norm of the decentralized gain matrices in the following form

$$\|\mathbf{K}_i\| \leq \|\mathbf{L}_i\| \|\mathbf{Y}_i^{-1}\| < \sqrt{\kappa_{L_i} \kappa_{Y_i}}. \tag{2.16}$$

Proof: We follow similar arguments as in [8, 9]. Whenever the nonlinear interconnection function h_i satisfies Lemma 1, the overall interconnection function $\mathbf{h}(\Delta \mathbf{x})$ satisfies the following quadratic bound

$$\mathbf{h}^T(\Delta \mathbf{x}) \mathbf{h}(\Delta \mathbf{x}) \leq \Delta \mathbf{x}^T \left(\sum_{i=1}^n \alpha_i^2 \mathbf{H}_i^T \mathbf{H}_i \right) \Delta \mathbf{x},$$

which can be expressed as a matrix inequality as

$$\begin{bmatrix} \Delta \mathbf{x} \\ \mathbf{h} \end{bmatrix}^T \begin{bmatrix} -(\sum_{i=1}^n \alpha_i^2 \mathbf{H}_i^T \mathbf{H}_i) & 0 \\ 0 & \mathbf{I} \end{bmatrix} \begin{bmatrix} \Delta \mathbf{x} \\ \mathbf{h} \end{bmatrix} \leq 0. \tag{2.17}$$

Then, in order to establish sufficient conditions for asymptotic stability, we define the following quadratic Lyapunov function candidate,

$$\mathbf{V}_D(\Delta \mathbf{x}) = \Delta \mathbf{x}^T \mathbf{P}_D \Delta \mathbf{x},$$

where $\mathbf{P}_D = \mathbf{P}_D^T \succ 0$ and $\mathbf{P}_D \triangleq \text{diag}\{\mathbf{P}_1, \dots, \mathbf{P}_n\}$. Then, a sufficient condition for asymptotic stability is given by

$$\mathbf{P}_D \succ 0, \text{ and } \dot{\mathbf{V}}(\Delta \mathbf{x}) < 0.$$

The conditions can be expressed as a pair of matrix inequalities,

$$\mathbf{P}_D \succ 0, \quad \begin{bmatrix} \Delta \mathbf{x} \\ \mathbf{h} \end{bmatrix}^T \begin{bmatrix} \hat{\mathbf{A}}_D^T \mathbf{P}_D + \mathbf{P}_D \hat{\mathbf{A}}_D & \mathbf{P}_D \mathbf{G}_D \\ \mathbf{G}_D^T \mathbf{P}_D & 0 \end{bmatrix} \begin{bmatrix} \Delta \mathbf{x} \\ \mathbf{h} \end{bmatrix} < 0.$$

Using S-procedure [18] we conclude that whenever $\mathbf{h}(\Delta \mathbf{x})$ satisfies (2.17), sufficient conditions for asymptotic stability are equivalent to the existence of $\beta > 0$ such that,

$$\mathbf{P}_D \succ 0, \quad \begin{bmatrix} \hat{\mathbf{A}}_D^T \mathbf{P}_D + \mathbf{P}_D \hat{\mathbf{A}}_D + \beta \sum_{i=1}^n \alpha_i^2 \mathbf{H}_i^T \mathbf{H}_i & \mathbf{P}_D \mathbf{G}_D \\ \mathbf{G}_D^T \mathbf{P}_D & -\beta \mathbf{I} \end{bmatrix} \prec 0. \quad (2.18)$$

Define matrix variable $\mathbf{Y}_D \triangleq \beta \mathbf{P}_D^{-1}$, then an equivalent expression for (2.18) is given as

$$\mathbf{Y}_D \succ 0, \quad \begin{bmatrix} \hat{\mathbf{A}}_D \mathbf{Y}_D + \mathbf{Y}_D \hat{\mathbf{A}}_D^T + \mathbf{Y}_D (\sum_{i=1}^n \alpha_i^2 \mathbf{H}_i^T \mathbf{H}_i) \mathbf{Y}_D & \mathbf{G}_D \\ \mathbf{G}_D^T & -\mathbf{I} \end{bmatrix} \prec 0. \quad (2.19)$$

Applying Schur-complement lemma [18] n times, and defining $\gamma_i \triangleq \frac{1}{\alpha_i^2}$ and $\mathbf{L}_D \triangleq \mathbf{K}_D \mathbf{Y}_D$ we can obtain the LMI optimization problem (2.15) over variables $\gamma_i, \mathbf{Y}_D$ and $\mathbf{L}_D$. Therefore, we conclude that the equilibrium of the overall interconnected power system is robustly asymptotically stabilized if the LMI optimization problem is feasible. ■

Once the LMI optimization problem (2.15) is solved we can compute the decentralized feedback gain matrices of the local controllers as

$$\mathbf{K}_D = \mathbf{L}_D \mathbf{Y}_D^{-1}. \quad (2.20)$$

Therefore, whenever the LMI optimization problem (2.15) is feasible, we can obtain a decentralized power control command u_i characterized by feedback gain matrix $\mathbf{K}_i$ for each bus $i \in \mathcal{V}$, that will be transmitted to the device layer. This power control command u_i renders the equilibrium of the interconnected system (2.9) asymptotically stable.

Finally, it can be observed that at the supervisory layer there exists limits over the maximum power available from the set of controllable loads. This implies that control gains have to respect the power available from controllable loads. Hence, we use the prescribed limits over the control gains (2.16), such that $\|\mathbf{K}_i\| < \sqrt{\kappa_{L_i}\kappa_{Y_i}} \leq \|\mathbf{K}_{\text{desired}}\|$, where $\|\mathbf{K}_{\text{desired}}\|$ is selected such that the power available from controllable loads can always support the power control command at each bus. It should be noted that the available power from controllable loads is time-varying. Thus, a systematic design of the gain bounds is needed and will be introduced in Chapter 3.

2.3.2 Device Layer Control Design

During a given contingency a non-zero frequency deviation from the equilibrium point ($\omega_i \neq \omega_i^0 = 0$) occurs at every bus $i \in \mathcal{V}$ of the system. Therefore, the set of available controllable loads (TCL loads) at bus $i \in \mathcal{V}$ will change its power output consumption according to the derived power control command $u_i = \mathbf{K}_i \Delta \mathbf{x} = K_{i1}(\Delta \theta_i) + K_{i2}(\Delta \omega_i)$. Hence, the objective for each individual participating load $j \in \mathcal{L}_{\text{cont},i}$ is to calculate its individual switching probability μ to turn ON (or OFF), such that $P_{\text{cont},i} - P_{\text{cont},i}^0 = u_i$ holds for all buses.

For simplicity, the temperature dynamics of TCL loads are neglected since their time scale is significantly greater than the frequency control time scale (sub-minute time scale). In addition, and in order to incorporate a more realistic behavior of the TCLs, compressor time delay constraints are incorporated in the device layer controller design. This compressor time delay relay is typically installed to each device in order to ensure that the compressor remains OFF for a minimum amount of

time, which is often referred to as the lockout period. During this period a switching ON control signal will be ignored [10].

We propose a Markov-chain model to capture the aggregated power consumption of the set of controllable loads $\mathcal{L}_{\text{cont},i}$ at bus $i \in \mathcal{V}$, incorporating the locked state. Each load $j \in \mathcal{L}_{\text{cont},i}$ is associated with a discrete operating state defined by $\sigma_j \triangleq \{1(\text{ON}), 0(\text{OFF}), 2(\text{LCK})\}$, and the possible state switching transitions illustrated in Fig. 2.2.

Figure 2.2: TCL state transition diagram

The time course evolution of σ_j is governed by a Markov-chain with transition probabilities denoted by

$$P_j\{\sigma_j(t_k) = s \mid \sigma_j(t_{k-1}) = r\} = p_{rs}.$$

In addition, the overall transition probability matrix is structured as

$$\mathbf{A}_i(t_k) = \begin{bmatrix} p_{11} & p_{01} & p_{21} \\ p_{10} & p_{00} & p_{20} \\ p_{12} & p_{02} & p_{22} \end{bmatrix} = \begin{bmatrix} 1 - \mu_{12}(t_k) & \mu_{01}(t_k) & 0 \\ 0 & 1 - \mu_{01}(t_k) & 0 \\ \mu_{12}(t_k) & 0 & 1 \end{bmatrix}, \quad (2.21)$$

where μ_{01} and μ_{12} denote the switching probabilities that can be modified by the decentralized controller to achieve the desired power output deviation that matches the power control command u_i . Since the frequency control period is much shorter

than the typical lockout time (3 min to 5 min [10]), we assume that each device, once get **LOCKED**, it cannot be switched ON again during the frequency control period. This is reflected in the model as the zero transition probabilities from the LOCKED state to ON and OFF states. Then, from the state transition matrix (2.21) the following discrete time system can be defined,

$$\begin{aligned}\mathbf{p}_i(t_{k+1}) &= \mathbf{A}_i(t_k)\mathbf{p}_i(t_k) \\ y_i(t_k) &= \mathbf{C}[\mathbf{p}_i(t_k) - \mathbf{p}_i(0)]P_{\text{tot},i}.\end{aligned}\tag{2.22}$$

In the above, $\mathbf{p}_i(t_k) \triangleq [p_{\text{on},i}(t_k), p_{\text{off},i}(t_k), p_{\text{lock},i}(t_k)]^T \in \mathbb{R}^{3 \times 1}$ represents the distribution of controllable loads at bus $i \in \mathcal{V}$, $y_i(t_k) \in \mathbb{R}$ is the power output deviation from controllable loads, $\mathbf{C} = [1, 0, 0]$, $\mathbf{p}_i(0) \in \mathbb{R}^{3 \times 1}$ is the initial distribution of controllable loads, and $P_{\text{tot},i}$ is the maximum power from the set $\mathcal{L}_{\text{cont},i}$ at bus $i \in \mathcal{V}$, computed as $P_{\text{tot},i} \triangleq \sum_{j \in \mathcal{L}_{\text{cont},i}} p_{\text{max},j}^i$.

From the Markov-chain implementation, the probability $\mathbf{p}_i(t_k)$ can be computed as $\mathbf{p}_i(t_k) = \prod_{n=1}^{t_k} \mathbf{A}_i(n)\mathbf{p}_i(0)$. Then, the power output deviation $y_i(t_k)$ can be expressed in the following alternative form

$$y_i(t_k) = \mathbf{C} \left[\prod_{n=1}^{t_k} \mathbf{A}_i(n)\mathbf{p}_i(0) - \mathbf{p}_i(0) \right] P_{\text{tot},i}.\tag{2.23}$$

Our goal is to approximate the power output deviation $y_i(t_k)$ of the set $\mathcal{L}_{\text{cont},i}$ to the power control command u_i obtained using LMI at the supervisory layer. This aggregate power output deviation is achieved whenever the following expression holds

$$y_i(t_k) = \mathbf{K}_i(\Delta \mathbf{x}_i(t_k)) = K_{i1}(\Delta \theta_i) + K_{i2}(\Delta \omega_i).$$

Note that this implies that the change of power consumption by the TCL loads $(P_{\text{cont},i} - P_{\text{cont},i}^0)$ has to match the power control command $u_i \forall i \in \mathcal{V}$, with steady state consumption given by $P_{\text{cont},i}^0 = p_{\text{on},i}(0)P_{\text{tot},i}$.

Moreover, it can be easily verified that the transition matrix (2.21) can be decomposed in the following form

$$\mathbf{A}_i(t_k) = \mathbf{I} + \mathbf{B}_1\mu_{12}(t_k) + \mathbf{B}_0\mu_{01}(t_k), \quad (2.24)$$

where, $\mathbf{B}_1$ and $\mathbf{B}_0$ are some constant matrices. Define $\mathbf{A}_i(0) = \mathbf{I}$, then the expression $y(t_k) = u_i$, can be represented as

$$y_i(t_k) = \mathbf{C} \left[\prod_{n=1}^{t_k} \mathbf{A}_i(n) \mathbf{p}_i(0) - \mathbf{p}_i(0) \right] P_{\text{tot},i} = u_i. \quad (2.25)$$

Furthermore, using the decomposed version of the transition matrix (2.24), we can re-write (2.25) as

$$a(t_k)\mu_{12}(t_k) + b(t_k)\mu_{01}(t_k) = c(t_k),$$

where

$$\begin{aligned} a(t_k) &\triangleq \mathbf{C} \left(\prod_{n=0}^{t_k-1} \mathbf{A}_i(n) \right) \mathbf{B}_1 \mathbf{p}_i(0), \\ b(t_k) &\triangleq \mathbf{C} \left(\prod_{n=0}^{t_k-1} \mathbf{A}_i(n) \right) \mathbf{B}_0 \mathbf{p}_i(0), \\ c(t_k) &\triangleq \frac{1}{P_{\text{tot},i}} [\mathbf{K}_i(\Delta \mathbf{x}_i(t_k))] + \left[\mathbf{I} - \prod_{n=0}^{t_k-1} \mathbf{A}_i(n) \right] \mathbf{p}_i(0). \end{aligned}$$

In other words, each controllable load $j \in \mathcal{L}_{\text{cont},i}$ has to compute μ_{12} and μ_{01} which are the device-level switching probabilities such that (2.25) holds at every decision time period denoted as $\Delta t_d \triangleq t_k - t_{k-1}$ (sub-second time scale) in the following form

$$u_{12}(t_k) = \begin{cases} 0 & \text{if } \Delta \omega_i(t_k) \geq \Delta \omega_i(t_{k-1}) \\ \frac{c(t_k)}{a(t_k)} & \text{otherwise,} \end{cases} \quad (2.26)$$

$$u_{01}(t_k) = \begin{cases} 0 & \text{if } \Delta \omega_i(t_k) \leq \Delta \omega_i(t_{k-1}) \\ \frac{c(t_k)}{b(t_k)} & \text{otherwise.} \end{cases} \quad (2.27)$$

Notice that $\mathbf{p}_i(0)$, $P_{\text{tot},i}$ and $\prod_{n=0}^{t_k-1} \mathbf{A}_i(n)$ are given since $\mathbf{A}_i(t_{k-1})$ was already computed in the previous iteration with $\mathbf{A}_i(0) = \mathbf{I}$.

2.4 Overall Hierarchical Implementation

The overall hierarchical framework is shown in Fig. 2.3. The decision layers operate in the following control time periods.

- $\mathcal{I}$: control period for supervisory layer (e.g. 15-30 minutes).
- Δt_d : control period for device layer (sampling: sub-second time scale).

Figure 2.3: Hierarchical framework for demand-side frequency control

We now describe the operation of the two layers and their information exchange.

Supervisory Layer Information Exchange

During the control period $\mathcal{I}$, the supervisory layer obtains the following system information:

- Relevant statistical information related to the amount of loads participating in demand-side frequency control at each bus $\{\mathbf{p}_i(0), P_{\text{tot},i}\}$.

- System information such as nominal power flows, renewable generation, etc.

All the information obtained is updated for the current period $\mathcal{I}$. It should be noted that the design of the feedback gains $\mathbf{K}_i$ will be based on this information, which allows for adaptation to time-varying conditions. On the other hand, at the beginning of each control period $\mathcal{I}$, the supervisory layer will transmit the following information to the device layer under each bus $i \in \mathcal{V}$.

- Current feedback gains $\mathbf{K}_i$.
- Relevant statistical information of the controllable load population $\{\mathbf{p}_i(0), P_{\text{tot},i}\}$ to each participating end-use device.

Device Layer Information Exchange

During the supervisory layer control period $\mathcal{I}$, each controllable device will exchange relevant information about its individual status to the supervisory layer. Moreover, at the beginning of the each control period $\mathcal{I}$, each individual controllable load receives the following information:

- Current feedback gains $\mathbf{K}_i$.
- Relevant statistical information of the controllable load population $\{\mathbf{p}_i(0), P_{\text{tot},i}\}$.

This information is necessary for the implementation of the Markov-chain model. On the other hand, if a contingency event occurs during the current control period $\mathcal{I}$, then each participating device will compute the switching probabilities μ_{01} and μ_{12} based on local measurements, namely, frequency deviation and its integral (angle deviation). This computation is done every device-decision period Δt_d , and then the device will switch according to the computed transition probabilities. It should be

noted that after each Δt_d , the current frequency measurement has to be stored and the Markov-chain distribution will be updated. This information will be used as the initial condition to compute the transition probabilities at the next device-decision control period Δt_d .

2.5 Simulation Results

The proposed hierarchical decentralized control strategy is validated using the IEEE 16-machine, 68-bus, 5-area power system as shown in Fig. 2.4 (adapted from [1]). It was implemented using the Power System Toolbox (PST) from MATLAB [19]. The source code for transient stability studies in PST was modified in order to incorporate load frequency response to local frequency measurement.

Figure 2.4: IEEE 68-bus power system (adopted from [1])

The model contain 24 load buses. We assume that all the load buses are considered to participate with controllable loads (TCLs). In particular, we consider that 40% of the load at each bus is controllable. The distribution of controllable loads at each load bus at the beginning of control period $\mathcal{I}$ is simulated as follows:

- $p_{\text{on},i}(0) = 0.3$, 30 % of the controllable loads are ON.
- $p_{\text{off},i}(0) = 0.6$, 60 % of the controllable loads are OFF.
- $p_{\text{lock},i}(0) = 0.1$, 10 % of the controllable loads are LOCKED.

For simulating a disturbance, a symmetrical three phase fault on line (36,64) followed by tripping of generator 12 is considered. The fault sequence was simulated in the following form:

- In the time interval $T_{ss} = [0, 5.1)[s]$, the system is at pre-fault steady state.
- At $t = 5.1[s]$ the fault occurs.
- At $t = 5.15[s]$ the near end of the line is cleared.
- At $t = 5.2[s]$ the far end of the line is cleared.
- Finally at $t > 5.2[s]$, the system is in post-fault state.

Our design requires that at each bus generation and loads coincide. Therefore, in our simulation a reduced power model is obtained by grouping load and generator buses that are strongly connected (e.g. area level). This process is called coherency [14] and is used to identify power systems at the area level. We identify 5 areas in the model; therefore, we reduce the network to a 5-generator system as

Figure 2.5: 5-bus (areas) reduced power system model

shown in Fig. 2.5. The dynamics of each reduced bus are governed by the center of angle (COA) of each area.

We obtain the feedback control gain matrices $\mathbf{K}_i$ for each load bus using the following procedure:

1. The 68-bus system is reduced to a 5-bus system by grouping generators and loads of the same area as identified in Fig. 2.4.
2. Compute the feedback control gain matrices $\mathbf{K}_i$ using LMI optimization problem (2.15) for the reduced 5-bus system.
3. Assign these feedback control gain matrices $\mathbf{K}_i$ to all load buses of the corresponding area.

Then, we implement a device-controller using the Markov-chain model for TCLs under each load bus, where the individual switching probabilities are computed according to the feedback control gains of each area. Simulation results are presented for load bus 4. They are developed for the following three cases:

- The power consumed at load bus 4 is set to be equal to the power control command u_i (without Markov-chain implementation).
- The power consumed at load bus 4 is the actual power output deviation of the TCLs loads $(P_{\text{cont},i} - P_{\text{cont},i}^0)$, where each load switches probabilistically based on the Markov-chain model.
- System without demand-side frequency control.

For all three cases, we decreased the generation control capacity (4 machines were modeled using the classical model), in order to explore the potential of demand-side frequency control as a complementary strategy to conventional generation control. The system frequency response is illustrated in Fig. 2.6. Active power consumption at load bus 4 is shown in Fig. 2.7. Several remarks about the simulations are in order.

- The proposed strategy can stabilize the frequency deviation under contingency and achieve a desired steady-state response. The response is similar to the generation-side droop control (all machines with droop control) and AGC.
- The response of the device-layer (power output deviation) matches well with the power control command computed at the supervisory layer. This shows a strong potential of using the proposed strategy for real demand-side frequency control programs.

Figure 2.6: Frequency response at load bus 4

Figure 2.7: Active power at load bus 4

- The design of the controller is done by solving the LMI problem with no heuristic tuning. This is of crucial importance because (i) it provides a systematic way to design load frequency controllers with stability guarantees, and (ii) it allows for automatic adaptation to time-varying operating conditions.
- An important parameter in the formulation is the gain bound. Note that a larger gain bound will lead to a better transient response. At the same time, a larger gain bound will result in larger transient load power output deviation. The actual achievable power output deviation of the population of controllable loads depends on the amount of controllable loads. Therefore, for simulation purposes, the gain bound was adjusted properly once a while to respect the actual power from participating controllable loads.

Comparison with conventional frequency control

We present in Fig. 2.8 the response of the system using demand side frequency control compared with conventional generation control. Conventional generation control is divided in two control actions performed in different time frames, identified as follows:

- Primary frequency control, also known as governor droop control. It is active immediately after the fault occurs. It is in charge of frequency synchronization among buses.
- Secondary control, also known as Automatic Generation Control (AGC). It is active after system has reached frequency synchronization. It is in charge of convergence to the nominal frequency (60 [Hz]).

Figure 2.8: Frequency response governor droop control with AGC vs demand-side frequency control

It can be observed that the response of the demand-side frequency controller has a better transient performance as shown in Fig. 2.8. However, the convergence to the nominal frequency is developed in a unique time frame.

2.6 Concluding Remarks

We conclude this chapter with some remarks about the implementation of the proposed strategy.

- In order to coincide generation and loads in the same bus, a reduction process was performed over the power network. Therefore, the structure and topology of the network were not preserved.

- Local controllers at the device level are required to measure frequency deviation and its integral (angle deviation). From a practical point of view the only available state to measure locally by each controllable load is frequency.
- Formulating and solving the LMI optimization problem requires a considerable amount of global information of the system. This particular feature makes the implementation of the proposed approach in a real large-scale power network difficult.
- As shown in the comparison example, generation-based frequency control is developed in two different time frames. This differs from our proposed approach where frequency converges to the nominal ($60[\text{Hz}]$) in a unique time frame.

In order to deal with the limitations on the implementation of the proposed demand-side frequency control approach described above, we extend our proposed solution of demand-side frequency control in the next chapter.

CHAPTER 3: HIERARCHICAL DESIGN FOR DEMAND-SIDE PRIMARY FREQUENCY CONTROL

In this chapter, we extend the hierarchical design for demand-side frequency control on a structure preserving power system model using only local frequency measurements from individual controllable loads. The hierarchical control implementation involves the two interacting layers. The supervisory layer design a power control command characterized by a feedback gain. The power control command is computed such that the population of controllable loads efficiently increase the system damping. Control gains are selected based on a linearized model and the power available from controllable loads. At the device level individual switching probabilities are computed using the Markov-chain approach based on local frequency measurements.

3.1 Motivating Example

We present the following example in order to show the influence of the oscillatory behavior of frequency deviation after a contingency event. The frequency response of the power network to a 3-phase fault over a transmission line is shown in Fig. 3.1. The power consumption at the load bus is presented in Fig. 3.2. Furthermore, we illustrate in Fig. 3.2 the concept of available power from controllable loads. In particular, this available power shows the bulk power that can be turned on (i.e. not locked).

For the present example, demand-side controller gains were obtained by solving the LMI optimization problem (2.15). Moreover, the gains were bounded using (2.16); however, this bound condition was not selected properly.

Figure 3.1: Frequency response of the power network example

The following conclusions are obtained by analyzing the example

- It can be observed that the available power from controllable loads reduces while responding to a decreasing frequency deviation. This reduction is due to the fact that individual loads get locked when they turn off (decrease load population power consumption).
- If the gains are not properly bounded the entire population of loads may get locked as shown in Fig. 3.2 after 15 seconds.

Figure 3.2: Maximum available power at a load bus with controllable loads

From the analysis of this example we conclude that the control gain has to be designed in order to respect the power that is available from the set of controllable loads. In particular, control gains design has to be related to the oscillatory response of system's frequency. We present a solution for this problem over a structure-preserving power network introduced next.

3.2 Structure-Preserving Power System

The power system under consideration in this chapter can be represented by a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ is the set of n buses, and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of transmission lines connecting different buses. In this graph, we have introduced fictitious buses that represent the internal generator voltages. We assume that there is a set of n_g internal generator buses denoted by $\mathcal{V}_g \subset \mathcal{V}$, and a set of n_l load buses denoted by $\mathcal{V}_l \subset \mathcal{V}$. Among n_l load buses, there is a set of n_c load buses denoted by $\mathcal{V}_c \subset \mathcal{V}_l$, which contains controllable loads.

We also assume that reactive power dynamics are neglected and the loads at each bus $i \in \mathcal{V}_l$ are frequency sensitive [12, 16]. On the other hand, each line $(i, j) \in \mathcal{E}$, is assumed to be lossless, that is, $\Re(y[ij]) = 0$, where $y[ij]$ is the associated line admittance value. We have $y[ij] = y[ji]$ and $\Im(y[ij]) = B_{ij} \geq 0$, which is the susceptance of the line. As pointed out in [12], the assumption of lossless lines is known to be accurate in the studies of transient stability using such a structure preserving power system model.

Let θ_i be the bus voltage phase angle for all $i \in \mathcal{V}$, and $\omega_i = \dot{\theta}_i$ be the angular velocity (frequency deviation). Then the active power flow, denoted as $f_i(\boldsymbol{\theta})$, at each bus $i \in \mathcal{V}$ is determined by

$$f_i(\boldsymbol{\theta}) = \sum_{j=1}^n (b_{ij} \sin \theta_{ij}),$$

where $\boldsymbol{\theta} \triangleq [\theta_1, \dots, \theta_n]^T$, $\theta_{ij} \triangleq \theta_i - \theta_j$ and $b_{ij} \triangleq |V_i||V_j|B_{ij}$. For each $i \in \mathcal{V}_g$, the dynamics of internal generator bus are given by the following differential equation [17],

$$M_i \ddot{\theta}_i + D_{g_i} \dot{\theta}_i + f_i(\boldsymbol{\theta}) = P_{m_i}^0, \quad (3.1)$$

where $M_i > 0$ is the generator inertia moment, $D_{g_i} > 0$ is the generator damping, and $P_{m_i}^0 > 0$ is the constant mechanical power input or power generated by the generator. To simplify our implementation, we do not explicitly account for generator-side controls, such as speed governor and automatic generation control (AGC). Nevertheless, our framework and the load control design method can also be applied when generation-side controls are considered.

For each bus $i \in \mathcal{V}_l$, and because we assume that loads are frequency sensitive, the load bus has the following dynamics

$$D_{li} \dot{\theta}_i = -(P_{li} + f_i(\boldsymbol{\theta})), \quad (3.2)$$

where $D_{li} > 0$ is the load damping constant as defined in [12, 16] and P_{li} is the total power consumption of the loads under load bus $i \in \mathcal{V}_l$.

3.3 Demand-Side Primary Frequency Control Problem

For each load bus with controllable loads $i \in \mathcal{V}_c$, we assume there are both uncontrollable and controllable loads, where the set of uncontrollable loads is denoted by $\mathcal{L}_{\text{no-cont},i}$ and the set of controllable loads is denoted as $\mathcal{L}_{\text{cont},i}$. In this chapter, we assume also that the controllable loads are only thermostatically controlled loads (TCLs). TCL loads are characterized by two operating modes ON and OFF, and can switch between them within a relatively short period of time [10].

Denote the total power consumption of uncontrollable loads as $P_{\text{no-cont},i}$, and the power consumption of controllable loads as $P_{\text{cont},i}$. The power consumption of each individual TCL load $j \in \mathcal{L}_{\text{cont},i}$ can be determined by

$$q_j^i = \begin{cases} p_{\max,j}^i & \text{if ON} \\ 0 & \text{if OFF.} \end{cases}$$

Then we can calculate $P_{\text{cont},i}$ as

$$P_{\text{cont},i} = \sum_{j \in \mathcal{L}_{\text{cont},i}} q_j^i.$$

To proceed, we represent the power consumption of controllable loads in the following form

$$P_{\text{cont},i} = P_{\text{cont},i}^0 + u_i, \tag{3.3}$$

where as usual $P_{\text{cont},i}^0$ represents the nominal power consumption of controllable loads without demand-side frequency control, and u_i is the power control command when demand-side frequency control is active. Thus, we can represent the total power

consumption of the loads at bus $i \in \mathcal{V}_c$ as

$$P_{l_i} = P_{\text{no-cont},i} + P_{\text{cont},i} = P_{l_i}^0 + u_i, \quad (3.4)$$

where $P_{l_i}^0 \triangleq P_{\text{no-cont},i} + P_{\text{cont},i}^0$ denotes the nominal load power at each bus $i \in \mathcal{V}_c$.

Substituting 3.4 into 3.2, the dynamics of the load bus with controllable loads $i \in \mathcal{V}_c$ can be described as

$$D_{l_i} \dot{\theta}_i + u_i + f_i(\boldsymbol{\theta}) = -P_{l_i}^0. \quad (3.5)$$

In summary, and considering the dynamics of all type of buses from the network, the power system under consideration can be described by the following structure preserving model,

$$M_i \ddot{\theta}_i + D_i \dot{\theta}_i + u_i + f_i(\boldsymbol{\theta}) = P_{m_i}^0 - P_{l_i}^0, \quad (3.6)$$

where

$$\begin{cases} M_i > 0, D_i = D_{g_i}, P_{m_i}^0 > 0 & \text{if } i \in \mathcal{V}_g \\ D_i = D_{l_i}, P_{l_i}^0 > 0 & \text{if } i \in \mathcal{V}_l \\ M_i = D_i = P_{m_i}^0 = P_{l_i}^0 = 0 & \text{otherwise,} \end{cases}$$

and

$$u_i = \begin{cases} P_{\text{cont},i} - P_{\text{cont},i}^0 & \text{if } i \in \mathcal{V}_c \\ 0 & \text{otherwise.} \end{cases} \quad (3.7)$$

It is clear that the network topology has been preserved by the above model since no reduction was implemented. This model can more accurately represent the transient dynamics of the power system than the multi-machine power system model considered in Chapter 2.

Our goal in this chapter is to design practical demand-side frequency control strategies so that the controllable loads under each bus $i \in \mathcal{V}_c$ can effectively participate in the primary frequency control. More specifically, when the power system is subject to disturbances, each controllable load can make an independent decision to

change its current operating mode in such a way that the population of controllable loads under each bus can provide the droop-like aggregate response. In the following section, we will describe the hierarchical decentralized framework that can enable the systematic design of practical demand-side primary frequency control in order to effectively improve the system damping.

3.4 Hierarchical Decentralized Controller Design

In this section, we present a solution to the implementation of *demand-side primary frequency control* problem using the hierarchical framework approach. The hierarchical demand-side primary frequency controller involves two decision layers. The *supervisory layer* computes the desired power control command u_i characterized by a feedback gain k_i at each control period of time $\mathcal{I}$ (e.g 10 minutes). The control gain is selected based on the oscillatory behavior of system's frequency response and the available power from the set of controllable loads. Moreover, the control gain determines the required power output deviation necessary to effectively improve the system damping. At the *device layer*, controllers are designed based on the Markov-chain model so that the change of power consumption of controllable loads matches the desired power control command u_i using only local frequency measurements.

3.4.1 Supervisory Layer Implementation

At the supervisory layer, we are interested in designing the power control command u_i during each control period $\mathcal{I}$. The power control command u_i is designed as a feedback control input characterized by a control gain k_i . Different from the power control command designed in Chapter 2, we require that u_i depends only on local frequency measurements. Notice that in practice, the only available state to measure

locally by each load $j \in \mathcal{L}_{\text{cont},i}$ is the bus voltage frequency deviation defined as $\omega_i \triangleq \dot{\theta}_i$. Therefore, the feedback power control command can be stated as

$$u_i = k_i(\omega_i - \omega_i^0), \quad (3.8)$$

where $k_i \in \mathbb{R}$ is the control gain and $\omega_i^0 = \omega^0 = 0$ is the nominal frequency deviation for all $i \in \mathcal{V}_c$.

In Chapter 2 we showed that if the power control command is based on frequency deviation and angle measurements (2.10), then demand-side frequency control can achieve robust stability with respect to pre-fault equilibrium and the corresponding control gains can be computed using the linear matrix inequalities (LMI) optimization problem (2.15). However, formulating and solving the LMI optimization problem requires considerable global information about the the large-scale power system.

With only local frequency measurement, robust stability with respect to pre-fault equilibrium as developed in Chapter 2 can no longer be guaranteed. Nevertheless, the frequency synchronization and asymptotic stability around post-fault equilibriums can be satisfied under very mild conditions. The result is given in the following theorem.

Theorem 2 *If the post fault equilibrium of the power system **without** demand-side frequency control is asymptotically stable. Then the post-fault equilibrium of the power system coupled with demand-side frequency control is still asymptotically stable as long as the gains k_i are non-negative.*

Proof: The proof is structured as follows. First, we show that the equilibrium points for the uncontrolled system and the system coupled with demand-side primary frequency control coincide. Then, using an Energy like Lyapunov function approach

as in [20, 21], we will show that this equilibrium is locally asymptotically stable for both systems.

The dynamics of the structure-preserving power system are given in compact form as

$$\begin{aligned} (\mathbf{D}_l + \mathbf{K})\dot{\boldsymbol{\theta}}_l &= -\mathbf{F}_l(\boldsymbol{\theta}), \\ \dot{\boldsymbol{\theta}}_g &= \boldsymbol{\omega}_g, \\ \mathbf{M}_g\dot{\boldsymbol{\omega}}_g &= -\mathbf{F}_g(\boldsymbol{\theta}) - \mathbf{D}_g\boldsymbol{\omega}_g, \end{aligned} \tag{3.9}$$

where $\boldsymbol{\theta}_l \triangleq [\theta_1, \dots, \theta_{n_l}]^T$ denotes the load bus voltage angle vector, $\boldsymbol{\theta}_g \triangleq [\theta_1, \dots, \theta_{n_g}]^T$ denotes the generator rotor angle vector, $\boldsymbol{\omega}_g \triangleq [\dot{\theta}_1, \dots, \dot{\theta}_{n_g}]^T$ denotes the generator buses speed deviation vector, $\mathbf{D}_l \triangleq \text{diag}\{D_{l_i}\}$ for all $i \in \mathcal{V}_l$, $\mathbf{M}_g \triangleq \text{diag}\{M_i\}$ for all $i \in \mathcal{V}_g$, $\mathbf{D}_g \triangleq \text{diag}\{D_{g_i}\}$ for all $i \in \mathcal{V}_g$, $\mathbf{F}_l(\boldsymbol{\theta}) \triangleq [f_1(\boldsymbol{\theta}) + P_{l_i}^0, \dots, f_{n_l}(\boldsymbol{\theta}) + P_{l_{n_l}}^0]^T$ denotes the power flow from load buses, $\mathbf{F}_g(\boldsymbol{\theta}) \triangleq [f_1(\boldsymbol{\theta}) - P_{m_1}^0, \dots, f_{n_g}(\boldsymbol{\theta}) - P_{m_{n_g}}^0]^T$ denotes the power flow from generator buses and $\mathbf{K} = \text{diag}\{k_i\}$ for all $i \in \mathcal{V}_c$ denotes the overall control gain matrix. Note that the uncontrolled system dynamics are given by setting $k_i = 0$ for all $i \in \mathcal{V}_c$.

From the assumption of frequency sensitive loads we know that $\mathbf{D}_l \succ 0$ is positive definite. Then, it is clear that the equilibrium points of the uncontrolled systems are given by $\boldsymbol{\omega}_g = \mathbf{0}$, $\mathbf{F}_l(\boldsymbol{\theta}) = \mathbf{0}$ and $\mathbf{F}_g(\boldsymbol{\theta}) = \mathbf{0}$, which represents zero speed deviation from generators and the solution of the power flow equation for all buses. Then, if we select the control gains non-negative $k_i \geq 0$ for all $i \in \mathcal{V}_c$, matrix $\mathbf{D}_l + \mathbf{K}$ is positive definite and invertible. Therefore, the equilibrium points of the system with demand-side frequency control are also given by $\boldsymbol{\omega}_g = \mathbf{0}$, $\mathbf{F}_l(\boldsymbol{\theta}) = \mathbf{0}$ and $\mathbf{F}_g(\boldsymbol{\theta}) = \mathbf{0}$.

Assume $(\boldsymbol{\theta}^0, \mathbf{0})$ is a local asymptotically stable equilibrium of the uncontrolled system. We need to show that this equilibrium is also local asymptotically stable equilibrium of the system coupled with demand-side frequency control. Using an Energy like Lyapunov function approach as in [20, 21], we construct the following energy function $V : \mathbb{R}^{n_g} \times \mathbb{R}^n \rightarrow \mathbb{R}_{\geq 0}$,

$$V(\dot{\boldsymbol{\theta}}_g, \boldsymbol{\theta}) = \frac{1}{2} \dot{\boldsymbol{\theta}}_g^T \mathbf{M}_g \dot{\boldsymbol{\theta}}_g + U(\boldsymbol{\theta}, \boldsymbol{\theta}^0),$$

where $U : \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}$ is the potential energy function of the system given by [12, 21],

$$U(\boldsymbol{\theta}, \boldsymbol{\theta}^0) = \int_{\boldsymbol{\theta}^0}^{\boldsymbol{\theta}} [\mathbf{F}(\boldsymbol{\psi}) - \mathbf{F}(\boldsymbol{\theta}^0)]^T d\boldsymbol{\psi}.$$

Differentiating $V(\dot{\boldsymbol{\theta}}_g, \boldsymbol{\theta})$ along trajectories of the system gives, after some manipulation [12, 21],

$$\dot{V}(\dot{\boldsymbol{\theta}}_g, \boldsymbol{\theta}) = -\dot{\boldsymbol{\theta}}^T (\mathbf{D} + \hat{\mathbf{K}}) \dot{\boldsymbol{\theta}},$$

where $\mathbf{D} \triangleq \text{diag}\{\mathbf{D}_l, \mathbf{D}_g\}$ and $\hat{\mathbf{K}} \triangleq \text{diag}\{\mathbf{K}, \mathbf{0}_{n_g}\}$. Therefore, $\dot{V}$ is negative semi-definite if we choose $\mathbf{D} + \hat{\mathbf{K}} \succ 0$. Note, that $D_i > 0$. Therefore, a sufficient condition to ensure $\dot{V} \leq 0$ is that $k_i \geq 0$ for all $i \in \mathcal{V}_c$. Define $W = \{(\dot{\boldsymbol{\theta}}_g, \boldsymbol{\theta}) \in \mathbb{R}^{n_g+n} : \dot{V}(\dot{\boldsymbol{\theta}}_g, \boldsymbol{\theta}) = 0\}$. By LaSalle's invariance principle trajectories will converge to the largest invariant set contained in W . In particular, for power systems this largest invariant set is given by the equilibrium $(\boldsymbol{\theta}^0, \mathbf{0})$ [12, 21]. Therefore, we can conclude that the equilibrium $(\boldsymbol{\theta}^0, \mathbf{0})$ is also locally asymptotically stable of the system coupled with demand side frequency control as long as the control gains are selected non-negative. In particular, these control gains will provide a droop-like response and act similarly to account for dissipation of energy for the power system. ■

From the previous theorem we conclude that the control gains have to be chosen non-negative. However, from the motivating example we have shown that if the gain

is chosen too large, most of the loads can get locked during a contingency event. Also, we have shown in the motivating example that the maximum positive value that the gain can have is strictly related to the distribution of controllable loads and the oscillations of system's frequency response.

Moreover, notice that in practice the dimension of the sets of non-controllable loads $\mathcal{L}_{\text{no cont},i}$ and controllable loads $\mathcal{L}_{\text{cont},i}$ is time-varying, since customers can decide to participate or not in demand-side primary frequency control. However, if we use a transactive mechanism (e.g auction market), we can engage the end-use device in participating in demand-side frequency control for a fixed period of time. In this scenario, it is reasonable to assume that the amount of controllable and non-controllable loads remain approximately constant during each control period $\mathcal{I}$.

Recall from Chapter 2 that each controllable load has three operating modes including ON(1), OFF(0) and LOCKED(2). The operating state of each controllable load evolves using the state transition diagram illustrated in Fig. 2.2. In particular, the locked state constraint makes the available controllable power from the set of controllable loads $\mathcal{L}_{\text{cont},i}$ time-varying, since loads get locked while responding to a decreasing system's frequency deviation as shown in Fig. 3.2.

Let the nominal consumption of the set of controllable loads be given by the consumption of participating controllable loads at the beginning of $\mathcal{I}$ ($P_{\text{cont},i}^0 = P_{\text{cont},i}(0)$ for all $t \in \mathcal{I}$). Recall that the maximum power from the set of controllable loads is given in the following way

$$P_{\text{tot},i} = \sum_{j \in \mathcal{L}_{\text{cont},i}} p_{\text{max},j}^i,$$

where $p_{\text{max},j}^i$ is the power consumed when $j \in \mathcal{L}_{\text{cont},i}$ is ON. Under the assumption of a large population of controllable loads, let $p_i(t)$ denote the distribution of controllable

loads at each bus $i \in \mathcal{V}_c$ as

$$p_i(t) = [p_{\text{on},i}(t), p_{\text{off},i}(t), p_{\text{lock},i}(t)]^T, \quad \forall t \in \mathcal{I},$$

where $p_{\text{on},i}(t)$ is the fraction of the loads that are ON, $p_{\text{off},i}(t)$ is the fraction that are OFF, and $p_{\text{lock},i}(t)$ is the fraction that are LOCKED, with initial distribution $p_i(0)$. Then, the nominal power can be defined in the following alternative form

$$P_{\text{cont},i}^0 = p_{\text{on},i}(0) \cdot P_{\text{tot},i}, \quad \forall t \in \mathcal{I}. \quad (3.10)$$

For purpose of control design, we assume that without control, the nominal power will be constant for all $t \in \mathcal{I}$. Indeed, this is a valid assumption because within a short period of time, the nominal power will not vary too much. Define the maximum (time-varying) available controllable power as

$$\bar{P}_i(t) = [p_{\text{off},i}(t) + p_{\text{on},i}(t)] \cdot P_{\text{tot},i}, \quad \forall t \in \mathcal{I},$$

which reflects the population of controllable loads that are not locked and can participate in demand-side primary frequency control during control period $\mathcal{I}$.

Finally, define the instantaneous active power consumption by the set of controllable loads by

$$P_i(t) = p_{\text{on},i}(t) \cdot P_{\text{tot},i}, \quad \forall t \in \mathcal{I}.$$

Using the above definitions, at the supervisor layer we identify that the design of the control gain $k_i \geq 0$ for all $i \in \mathcal{V}_c$, requires that the power consumption of the controllable loads matches the desired power control command u_i in the following form

$$u_i(t) = k_i(\omega_i(t) - \omega^0) = P_i(t) - P_{\text{cont},i}^0, \quad \forall t \in \mathcal{I}. \quad (3.11)$$

Notice that since the maximum available power from controllable loads $\bar{P}_i(t)$ is not increasing, k_i has to be selected such that (3.11) is always supported, i.e, $P_i(t) \leq \bar{P}_i(t)$ for all $i \in \mathcal{V}_c$ and for all $t \in \mathcal{I}$. This constraint clearly reflects the necessity to have enough power from the set of controllable loads to support the power control command u_i .

From generation-side frequency control strategies, we recognize that governor droop control design has a similar response characteristic as the one stated in (3.11). In particular, a generation droop constant is selected in order to respect the maximum power output of generation units. This droop constant denoted as R is normally chosen such that whenever the frequency deviation is less than 5 %, the desired droop power from machines can be achieved [13]. However, observe that this approach cannot be used to design the control gain k_i at load buses since the maximum available controllable power is time-varying due to oscillation of system's frequency during contingency.

Therefore, in order to study the influence from oscillations in the design of the control gain we present the following example. In Fig. 3.3 we illustrate an oscillatory waveform that we will assume is system's frequency deviation response to a given contingency event and characterizes the power control command u_i .

Our goal is to illustrate the time-varying evolution of the available power from controllable loads and the influence of the oscillatory behavior of system's frequency. Some relevant remarks about the example are in order

- During the time interval $[t_0, t_1]$, the first swing of the response occurs. This first swing of frequency deviation could be in both directions since faults can occur at the supply or demand side.

Figure 3.3: Frequency deviation example, and power consumed behavior

- During the time interval $[t_1, t_2]$, the aggregate power consumption of the set of controllable has to be reduced since the frequency deviation response $\Delta\omega_1$ is decreasing. This will cause that a certain amount of controllable loads will get locked.
- Notice that since a certain amount of controllable loads were locked during the interval of time $[t_1, t_2]$. The available power from controllable loads for the interval of time $[t_2, t_3]$ differs from the initial one.
- It is easy to verify that the consumption required from the set of controllable loads can be computed from the local maximum and minimum of system's frequency deviation response. For example, during time interval $[t_2, t_3]$ the aggregate power consumption of the controllable loads under demand-side primary frequency control has to match $u_i = k_i(|\Delta\omega_1(t_2)| + |\Delta\omega_1(t_3)|)$.

We conclude from the previous analysis that a systematic way to design control gains, that respects the actual power control command range is needed. In particular, the design has to be in concordance with the following features:

- Initial distribution of controllable loads.
- Amplitude and direction of the first swing.
- Oscillations that occur in system's frequency deviation.

Define, $t^{\max}$ as the maximum time the set of controllable loads performs demand-side primary frequency control (e.g 20 seconds). Then, in order to analyze the oscillations of system's frequency deviation after a disturbance and its influence over the population of controllable loads, we linearize the power system model (3.6) at the current operating point. Thus, we obtain the power system described by [13],

$$\dot{\mathbf{x}} = \mathbf{A}_{\text{lin}} \mathbf{x}, \quad (3.12)$$

where $\mathbf{x}$, is the system state vector including angles θ_i and angular velocities ω_i for all $i \in \mathcal{V}$.

Power system's oscillations and in particular oscillations of system's frequency are related to the modes of the system. Modes are the eigenvalues of a linear model [14]. They offer a considerable insight into the dynamic stability properties of a power system (small signal stability analysis [13]). The eigensolution of matrix $\mathbf{A}_{\text{lin}}$ provides all the eigenvalues/modes of the power system defined as $\lambda_l = \sigma_l \pm j\alpha_l$. Each complex eigenvalue will define a particular oscillatory response. From the set of eigenvalues of matrix $\mathbf{A}_{\text{lin}}$, we select the most unstable complex mode denoted as

$$\lambda^u \triangleq \sigma \pm j\alpha.$$

This most unstable eigenvalue λ^u features the worst oscillatory behavior of system's frequency deviation over the entire power network after a contingency event.

It is clear that a exact estimation of system's frequency response after disturbance cannot be achieved in real-time. However, the following waveform obtained from the selected most unstable eigenvalue λ^u , provides an insight of the worst case oscillations from system's frequency that the set of controllable loads needs to support,

$$\Delta\tilde{\omega}(t_k) = \omega^{\max} \exp(-\sigma t_k) \cos(\alpha t_k + \phi). \quad (3.13)$$

The amplitude $\omega^{\max}$ and phase shift ϕ characterize the first swing of the waveform of the corresponding selected mode λ^u . Their values are related to the specific disturbance that occurs in the network [14]. However, since faults can be of different types, they cannot be predicted accurately. Nevertheless, for our design and implementation we will select the amplitude of waveform (3.13) for the worst case scenario. In particular, we select $\omega^{\max}$ as the maximum value of frequency deviation for which it is required that controllable loads effectively participate in primary frequency control before tripping any security relay.

Then, as illustrated in the example the total power to support the corresponding power control command ($\tilde{u}_i = k_i \Delta\tilde{\omega}$) after the first swing depends on local maximum, minimum of waveform (3.13). Note that in particular $\Delta\tilde{\omega}$ achieves a local maximum/minimum at the time denoted by $t_m \triangleq \frac{m\pi - \phi}{\alpha} > 0$ and $m \in \mathbb{N}$.

Define the available power from controllable loads that is off (0) and can be turned on (1) as $\bar{P}_{01,i}$, and the available power that is on (1) and can be turned off (actually locked (2)) as $\bar{P}_{12,i}$. Furthermore, the initial values of the available power are defined as

$$\bar{P}_{01,i}^0 \triangleq \bar{P}_i(0) - P_{\text{cont},i}^0, \quad \bar{P}_{12,i}^0 \triangleq P_{\text{cont},i}^0.$$

Using the previous definitions and for any time interval of the form $[t_m, t_{m+1}]$, $t_m > 0$, $m \in \mathbb{N}$, the following conditions are required over $\bar{P}_{01,i}$ and $\bar{P}_{12,i}$ such that the power control command $\tilde{u}_i$ can be supported by the available power from controllable loads,

$$\begin{cases} \bar{P}_{01,i}(t_m) \geq k_i \sum_{j=0}^1 |\Delta\tilde{\omega}(t_{m+j})|, & \text{if } \Delta\tilde{\omega}(t_m) \text{ is min} \\ \bar{P}_{12,i}(t_m) \geq k_i \sum_{j=0}^1 |\Delta\tilde{\omega}(t_{m+j})|, & \text{if } \Delta\tilde{\omega}(t_m) \text{ is max.} \end{cases} \quad (3.14)$$

Then, working recursively and considering the first swing, it is easy to verify that a sufficient and necessary condition for (3.14) to hold is that the initial available power that can be turned ON and OFF denoted as $\bar{P}_{01,i}^0$ and $\bar{P}_{12,i}^0$, are respectively lower bounded by

$$\begin{aligned} \bar{P}_{01,i}^0 &\geq k_i \sum_{k=0}^N \Delta\tilde{\omega}\left(\frac{k\pi - \phi}{\alpha}\right), \\ \bar{P}_{12,i}^0 &\geq k_i \Delta\tilde{\omega}\left(\frac{-\phi}{\alpha}\right). \end{aligned} \quad (3.15)$$

In the above, N is selected such that $t_N \triangleq \frac{N\pi - \phi}{\alpha} < t^{\max}$. In particular, if we select $\phi = 0$ the solution suffices for all values of ϕ . Therefore, a bound condition for the control gain k_i for all $i \in \mathcal{V}_c$ can be formulated as follows

$$0 \leq k_i \leq \min \left\{ \frac{\bar{P}_{01,i}^0}{\sum_{k=0}^N \Delta\tilde{\omega}_i\left(\frac{k\pi}{\alpha}\right)}, \frac{\bar{P}_{12,i}^0}{\omega^{\max}} \right\}. \quad (3.16)$$

Therefore, at each load bus $i \in \mathcal{V}_c$ and for all $t \in \mathcal{I}$, the supervisory layer will select the corresponding control gain k_i using the bound condition stated in (3.16). Using the results described in this section we proceed to present a decision making procedure for the supervisory control layer.

- *STEP 1:* At the beginning of each control period $\mathcal{I}$, the supervisory layer will collect the operating modes and the power consumption of all the controllable

loads participating in demand-side primary frequency control. Moreover, system information such as modal properties $\{\omega^{\max}, \sigma, \alpha\}$, power flows, renewable generation, will be obtained for the current period $\mathcal{I}$.

- *STEP 2:* Using this information the supervisory layer will compute feedback control gains based on bound (3.16). Notice that the design of the feedback control gains k_i are based on the set of parameters $\{\mathbf{p}_i(0), P_{\text{tot},i}, \omega^{\max}, \sigma, \alpha\}$ for the current control period $\mathcal{I}$, which allows for adaptation to time-varying system conditions.
- *STEP 3:* Current feedback control gains k_i and load population relevant statistics $\{\mathbf{p}_i(0), P_{\text{tot},i}\}$ will be transmitted to the device layer under each bus $i \in \mathcal{V}_c$.

3.4.2 Device Layer Implementation

After the control gain k_i , $i \in \mathcal{V}_c$, is determined at the supervisory layer, it is broadcasted to the device layer for each control period $\mathcal{I}$. With this control gain, each controllable load will have the knowledge of the desired power control command u_i based on the local bus frequency measurement, that is, $u_i = k_i \Delta \omega_i$ where $\Delta \omega_i \triangleq \omega_i - \omega^0$.

Now the remaining problem is how each controllable load can make an independent decision to change its current operating mode in such a way that the actual aggregate load response can reflect the desired power control command. To address this problem, we adopt the same Markov-Chain model (2.22) as previously described in Chapter 2, in order to capture the aggregate load response of controllable loads. Also, we identify $\mu_{01}(t_k)$ and $\mu_{12}(t_k)$ as the switching probabilities from OFF to ON and from ON to LOCKED, respectively for each controllable load.

We assume in this thesis that if one controllable load switches from ON to LOCKED, then it will not switch back to ON within each control period $\mathcal{I}$. With the Markov-Chain model (2.22), the power output deviation from the nominal power consumption of the set of controllable loads is given by (2.23). With these definitions, we summarize next the procedures for the decision making of each controllable load into the design process using the Markov-chain model (2.22).

- *STEP 1:* At the beginning of each control period $\mathcal{I}$, each controllable load reports its operating mode and power consumption to the supervisor. After the supervisor collects all the information, it sends $\mathbf{p}_i(0)$ and $P_{\text{tot},i}$ to each controllable load. In this way, each controllable load can initialize a local copy of the Markov-Chain model (2.22) for all the controllable loads under the same bus.
- *STEP 2:* At each time step $t_k \in \mathcal{I}$, each controllable load measures the local bus frequency to determine the desired power control command as $u_i(t_k) = k_i \Delta \omega_i(t_k)$. In order to calculate the switching probabilities $\mu_{01}(t_k)$ and $\mu_{12}(t_k)$, each controllable load forms the following equation adapted to only local frequency measurements by equalizing $y_i(t_k)$ defined in (2.23) with $u_i(t_k)$,

$$a(t_k)\mu_{12}(t_k) + b(t_k)\mu_{01}(t_k) = c(t_k),$$

where

$$\begin{aligned} a(t_k) &\triangleq \mathbf{C} \left(\prod_{i=0}^{t_k-1} \mathbf{A}(i) \right) \mathbf{B}_1 \mathbf{p}_i(0), \\ b(t_k) &\triangleq \mathbf{C} \left(\prod_{i=0}^{t_k-1} \mathbf{A}(i) \right) \mathbf{B}_0 \mathbf{p}_i(0), \end{aligned}$$

$$c(t_k) \triangleq \frac{1}{P_{\text{tot},i}} u_i(t_k) + \left[\mathbf{I} - \prod_{i=0}^{t_k-1} \mathbf{A}(i) \right] \mathbf{p}_i(0).$$

Note that we have only one equation and two unknowns. However, at each time step t_k one of the unknown variables is zero depending on the monotonicity of system's frequency response, i.e., controllable loads will only increase or decrease their consumption.

- *STEP 3:* Each controllable load solves the above equation to obtain the following switching probabilities,

$$\mu_{01}(t_k) = \begin{cases} 0 & \text{if } \Delta\omega_i(t_k) \leq \Delta\omega_i(t_{k-1}) \\ \frac{c(t_k)}{b(t_k)} & \text{otherwise,} \end{cases} \quad (3.17)$$

and

$$\mu_{12}(t_k) = \begin{cases} 0 & \text{if } \Delta\omega_i(t_k) \geq \Delta\omega_i(t_{k-1}) \\ \frac{c(t_k)}{a(t_k)} & \text{otherwise.} \end{cases} \quad (3.18)$$

Then each controllable load switches its operating mode according to calculated switching probabilities so that the actual deviation of aggregate load power from the nominal power will match the desired power control command for every decision making period of length $\Delta t_d = t_k - t_{k-1}$.

3.5 Simulations Results on a Structure-Preserving Model

The proposed hierarchical decentralized demand-side primary frequency control strategy is validated using the IEEE 4-machine, 13-bus power system as shown in Fig. 3.4. It was implemented using the Power System Toolbox (PST) [19] with modifications to incorporate load frequency response to local frequency measurement. The model contain 2 load buses. Both buses are considered to have controllable loads (TCLs). We assume that 40% of the load at each bus is controllable. The distribution

Figure 3.4: IEEE 4-machine, 13-bus power system (adapted from [2])

of controllable loads at each load bus at the beginning of control period $\mathcal{I}$ is simulated as follows:

- $p_{\text{on},i}(0) = 0.3$, 30 % of the controllable loads are ON.
- $p_{\text{off},i}(0) = 0.6$, 60 % of the controllable loads are OFF.
- $p_{\text{lock},i}(0) = 0.1$, 10 % of the controllable loads are LOCKED.

For simulating a disturbance, a symmetrical three phase fault on line (3,101) followed by the isolation of the line was implemented. The fault sequence was simulated as follows:

- In the time interval $T_{ss} = [0, 5.1)[s]$, the system is at pre-fault steady state.
- At $t = 5.1[s]$ the fault occurs.
- At $t = 5.15[s]$ the near end of the line is cleared.
- At $t = 5.2[s]$ the far end of the line is cleared.

- Finally at $t > 5.2[s]$, the system is in post-fault state.

The control gain for each one of the load buses with controllable loads is computed using the following design procedure

- Linearize the IEEE 13-bus 4-generators power system model around the steady state operating point.
- Perform modal analysis of the linearized model, i.e., obtain the eigenvalues of the matrix $\mathbf{A}_{\text{lin}}$.
- Select the most unstable eigenvalue λ^u for the entire network. This eigenvalue presents less damping and the worst oscillatory response in the system.
- Using the value of the most unstable eigenvalue λ^u and the statistics from the controllable loads $\bar{P}_{01,i}$ and $\bar{P}_{12,i}$, obtain the corresponding control gain using the bound condition (3.16).
- Implement device layer controllers using the Markov-chain model under each load bus.
- Individual switching probabilities are computed according to the control gain and local frequency measurement.

Simulation results are illustrated for load bus 4. The most relevant statistics from the population of controllable loads under bus 4 are given as follows:

- Maximum controllable power is $P_{\text{tot},4} = 543[\text{kW}]$ (5.43 in [p.u]).
- The number of participating loads is 136 with individual consumption of around 4[kW].

- The maximum initial available power to turn on is $\bar{P}_{01,4} = 326[\text{kW}]$ and the maximum available power to turn off is $\bar{P}_{12,4} = 161[\text{kW}]$.

For the control gain design $k_4 \geq 0$, the following procedure was implemented:

- After performing modal analysis the most unstable eigenvalue selected was $\lambda_4^u = -0.55 \pm j6.8$.
- The amplitude for the first swing support was selected as $\omega^{\max} = 0.5$.
- The computed value of the control gain is $k_4 = 1.82$.

Finally, we design each device layer controller such that each load has the same decision time Δt_d . Moreover, each load has their own individual copy of the Markov-chain model and do not necessary respond simultaneously.

System's frequency response at load bus 4 of the actual aggregated response of loads is shown in Fig. 3.5. Results are validated against the frequency response at load bus 4 without demand-side frequency control. For all cases, we decreased the generation control capacity (2 machines were simulated using the classical model), in order to explore the potential of demand-side control as a complementary strategy to conventional generation control. Additionally, the active power consumption at load bus 4 is illustrated in Fig. 3.6. Several remarks about the simulations are in order.

- The proposed strategy can improve the stability margin of frequency deviation under disturbances. The response is similar to primary frequency droop control.

Figure 3.5: Frequency response at load bus 4

Figure 3.6: Active power at load bus 4

- The controller design developed using only local frequency measurements shows that the frequency response converge to a stable frequency different than nominal. The convergence to the nominal value is developed practically through AGC(Automatic generation control), in a different time frame.

CHAPTER 4: CONCLUSION

In this thesis, a hierarchical framework has been developed in order to design demand-side frequency controllers. The framework consists of two decision interacting layers. A supervisory layer is responsible to gather system information, and determine the power control command for the responsive loads on each bus every few (e.g. 10-30) minutes. At the device layer, each decentralized controllable load will switch ON/OFF probabilistically based on local measurements so that the aggregated power output response from the population of controllable loads under each bus matches the desired power determined by the supervisory layer.

First, the design was studied over a reduced multi-machine power model with local frequency and angle measurements. It was shown that under these conditions the controller asymptotically stabilizes frequency in a unique time frame. The power control command designed in the supervisory layer was based on the decentralized robust control theory and efficiently solved through Linear Matrix Inequalities. On the other hand, the device switching probabilities were designed using a Markov-chain model with local frequency and angle measurements.

Then, building over the previous results, a hierarchical framework for demand-side primary frequency control was implemented on a structure preserving power model based only on local frequency measurements. The power control command at the

supervisory layer was characterized by a feedback control gain. The design of this gain was based on the time-varying available power of the controllable load population, and the oscillatory/transient behavior of system's frequency. The load switching probabilities were designed using a Markov-chain model with on-site frequency measurement. Under these conditions it was shown that the controllable loads provide a droop-like response and effectively increase system damping.

The proposed hierarchical framework strategy has several features. Firstly, it guarantees the closed-loop stability in the former case and effectively participates in primary frequency control in the later case. Secondly, the load response design can fully respect device constraints. Thirdly, the systematic computation of the gains every few minutes at the supervisory layer allows for adaptation to time-varying system conditions. The above mentioned features make the proposed control framework both theoretically sound and practically feasible.

Finally, future research will mainly focus on the following extensions. (i) Study the co-design with existing frequency controls. (ii) Extend the implementation considering other type of loads. (iii) Extend the design framework to provide AGC-like response. (iv) Study the feasibility of the proposed framework in a real large scale power system.

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