**SUSPENSION PARAMETER MEASUREMENT USING SIDE-PULL TEST TO
ENHANCE MODELING OF VEHICLE ROLL**

A Thesis

Presented in Partial Fulfillment of the Requirements for

the Degree of Master of Science in the

Graduate School of The Ohio State University

By

Levent Erdogan, B.S.

* * * * *

The Ohio State University

1999

Master's Examination Committee:

Prof. D. A. Guenther, Adviser

Dr. G. J. Heydinger

Approved by

Adviser

Department of Mechanical Engineering

ABSTRACT

A new laboratory test facility for measuring suspension parameters that affect rollover is described. The Side-Pull mechanism rolls the test vehicle through a cable attached rigidly at its center of gravity (CG). Changes in wheel camber and wheel steer angles are measured as a function of body roll angle. The roll test simulates a steady-state cornering. Thus, both compliance and kinematic forces are fed simultaneously to the vehicle as they would be applied in a real cornering situation. The lateral load transfer, and roll angle as a function of simulated lateral acceleration is determined.

The Side-Pull Roll Measurement has advantages over the conventional roll tests where the rolling force couple is applied vertically. The Side-Pull mechanism rolls the vehicle in a natural way with horizontal forces applied at the tire / pad contact and the CG location. Thus, the measurements take into account coupling of compliance with roll. The Side-Pull method eliminates the need to run separate tests for suspension kinematics and compliance properties.

Description of the hardware and testing method is given. Results from the side-pull measurement for the 1996 Jeep Cherokee are compared with field test results and with conventional SPMD results. It is found that slow continuous side-pull test simulates closely the steady-state cornering. The results from superposition of kinematics and

compliance are compared with the side-pull results. Recommendations are given for development of the Side-Pull Suspension Measurement Facility to make use of it in roll modeling .

Anne ve Babama

ACKNOWLEDGMENTS

I would like to express my gratitude to my adviser, Dennis Guenther, for giving me the opportunity to work towards an advanced degree and do research in one the most exciting fields in engineering. He gave me intellectual freedom and provided all necessary means to carry out my research.

I would like to thank Gary Heydinger who has been an invaluable mentor and support throughout the project. Without his input, this study would not have been the same.

I would also like to thank Ron Bixel for his help with data acquisition; Jim Nowjack for his help with experimental setup, and everyone else at SEA Inc., who helped along the way.

Finally, I would like to thank Dr. W. Riley Garrott and VRTC for sharing their data and allowing me to use their vehicle for testing that was essential for this experiment.

VITA

August 3, 1974Born - Istanbul, Turkey

1997B.S.M.E., The Ohio State
University, Columbus, Ohio

1997 to presentGraduate Research Associate,
The Ohio State University,
Columbus, Ohio

Field of Study

Major Field: Mechanical Engineering

Studies in Vehicle Dynamics - Dr. Dennis A. Guenther

TABLE OF CONTENTS

ABSTRACT	ii
DEDICATION.....	iv
ACKNOWLEDGMENTS.....	v
VITA	vi
LIST OF FIGURES.....	ix
LIST OF TABLES.....	xi
CHAPTER 1 - INTRODUCTION	
1.1 General	1
1.2 Objectives	2
1.3 Thesis Overview	4
CHAPTER 2 - BACKGROUND	
2.1 Definitions.....	6
2.1.1 Roll	6
2.1.2 Lateral Load Transfer	7
2.1.3 Suspension and Suspension Parameters	7
2.2 Roll Center	10
2.3 Past Efforts in Suspension Parameter Measurement.....	14
CHAPTER 3 - SIDE-PULL SUSPENSION PARAMETER MEASUREMENT	
3.1 Introduction.....	18
3.2 Concept and Design.....	19
3.3 Description of the Side-Pull SPMF	23
3.3.1 Actuator and Cable Attachment	24
3.3.2 Force Measurement	25
3.3.3 Wheel Displacement Measurement	25
3.3.4 Roll Angle Measurement	25
3.3.5 Data Acquisition	26
3.3.6 Test Procedure	28

CHAPTER 4 - MODELING OF VEHICLE ROLL	
4.1 Introduction.....	29
4.2 Light Vehicle Dynamics Simulation.....	29
4.3 Vehicle Dynamics Analysis Non-Linear Simulation	32
4.4 Verification of Lateral Load Transfer Theory.....	33
CHAPTER 5 -RESULTS	
5.1 Dynamic vs. Quasi-Static Side-Pull Test	41
5.2 Results of Field Test Compared to Side-Pull Test	49
5.3 Verification of Kinematics and Compliance Superposition Theory	51
5.4 Verification of Load Transfer	57
CHAPTER 6 - CONCLUSIONS	
6.1 Summary of Results.....	64
6.2 Recommendations	65
LIST OF REFERENCES	67
BIBLIOGRAPHY	69
APPENDIX A - Figures and Tables	70
APPENDIX B - Matlab Code for Data Analysis	77

LIST OF FIGURES

Figure 1- Vehicle Body Roll in Cornering	3
Figure 2- Camber Angle Change with Roll	9
Figure 3- Roll Steer of a Hotchkiss Axle	9
Figure 4- Roll Center Measurement	10
Figure 5- Roll Center Analysis for Hotchkiss Suspension	11
Figure 6- Roll Center Analysis for 4-Link Suspension	12
Figure 7- Roll Center Determination for the Front Suspension of Test Vehicle	13
Figure 8- Standard vs. Side-Pull Suspension Measurement	21
Figure 9- Justification of Not Using Winch Height Adjustment	23
Figure 10- Representative Drawing of Side-Pull Setup for Measurement	23
Figure 11- Chain Arrangement to Fix the Cable Attachment at CG Location	24
Figure 12- Linear Pot Arrangement to Determine Camber and Steer Angles	27
Figure 13- Modeling of Roll in LVDS Simulation	30
Figure 14- Discrepancy between LVDS and Field Test Results	31
Figure 15- Lateral Load Transfer Model	34
Figure 16- Vehicle Load Transfer in 3-D	35
Figure 17- Roll Center Shift with Bump Stop Contact	39
Figure 18- Virtual Roll Center Computation from Side-Pull Test Results	40

Figure 19- Side-Pull Rolling Speed and Verification of Roll Angle Measurement	42
Figure 20- Dynamic vs. Quasi-Static Side-Pull for Rear Roll Stiffness Measurement ..	44
Figure 21- Dynamic vs. Quasi-Static Side-Pull for Front Roll Stiffness Measurement ..	45
Figure 22- Dynamic vs. Quasi-Static Side-Pull Test for Front Roll Camber and Front Roll Steer Measurement.....	47
Figure 23- Dynamic vs. Quasi-Static Side-Pull Test for Rear Roll Camber and Rear Roll Steer Measurement.....	48
Figure 24- Validation of Simulated Lateral Acceleration via Side-Pull Test	50
Figure 25- Determination of Front Roll Camber Coefficients.....	52
Figure 26- Determination of Front Roll Steer Coefficients	51
Figure 27- Determination of Rear Roll Camber Coefficients.....	53
Figure 28- Determination of Rear Roll Steer Coefficients.....	54
Figure 29- Theoretical Rear Tire Lift-Off.....	59
Figure 30- Theoretical Front Tire Lift-Off	60
Figure 31- Axle Roll Moment Variation and Determination of Bump Stop Contact	62
Figure 32- Lateral Load Transfer Plot	63
Figure 33- View of Front Axle From Underneath	72
Figure 34- View of Rear Axle From Underneath	73
Figure 35- Drawing of Support Pole for the Wheel Pads	74
Figure 36- Drawing of Wheel Pad Base	75
Figure 37- Jeep Cherokee during Side-Pull Test	76

LIST OF TABLES

Table 1 - Choosing a Design Concept for Suspension Parameter Measurement	17
Table 2 - Test Conditions of the 1997 Jeep Cherokee Sport	26
Table 3 - Comparison of Dynamic and Quasi-Static Side-Pull Test Results	41
Table 4 - Coefficients of Roll Camber and Roll Steer from Side-Pull Test.....	49
Table 5 - Coefficients of Roll Camber and Roll Steer from Conventional S.P.M.D	49
Table 6 - Verification of Superposition Theory for K&C Measurement.....	54
Table 7 - Failure Analysis of the Designed Components	75

CHAPTER 1

INTRODUCTION

1.1 General

The increased computing speeds during 1990's made computer simulations of road vehicle dynamics become more popular. Today computer simulations are widely used to predict vehicle response in real world scenarios for design, research and development, for accident reconstruction and rule-making. However, the predictions of these simulations become questionable when dealing with severe maneuvers where the vehicle body is rolled to its limits by lateral acceleration forces. Without having information on how the suspension behaves at high degree of roll angle, it is not possible to model vehicles accurately. Consequently with an inaccurate model, the simulation results may be misleading at the limits of vehicle performance, especially when dealing with discrete events such as spin out or rollover. Therefore, this research aims to develop a measurement system that will determine the suspension parameters of vehicles for severe roll cases. Modeling of the bump stops and other non-linear suspension characteristics will enable the dynamic simulations to predict discrete events

more precisely at least for smooth road surfaces. Although the simulations cannot replace the full scale field tests which are costly and potentially dangerous, they can provide enough information to reduce their number.

Traditionally the measurement devices used in the past take separate measurements of suspension, kinematic, and compliance characteristics assuming superposition of these. The Side-Pull Suspension Parameter Measurement Facility used in this project has the advantage of combining all the inputs, giving a more realistic test and thus eliminates all possible errors due to the assumption of superposition. More importantly, this test device gives the possibility of determining the instantaneous roll centers of the vehicle that can be utilized in replacing the fixed roll center models widely used. The test apparatus designed and built for this project is capable of running a quasi-static or slow dynamic roll test by pulling laterally on the overall center of gravity (CG) of the vehicle. The pulling force is applied to the sprung mass by an electric winch cable attached to a fixture anchored inside the vehicle at the CG location.

1.2 Objectives

The objective of this research is further understanding of vehicle suspension and chassis behavior when operating around high roll angles. High roll angles typically occur during severe maneuvers where the body of the vehicle is leaned and contacts the bump stops (Figure 1). Both experimental and theoretical work are carried out in order to achieve this goal. The experimental part of the project will require designing and

building a new Side-Pull Suspension Parameter Measurement Facility, to measure the non-linear characteristics of the suspension at the higher-than-normal roll angles including the instantaneous roll center. New techniques developed enables one to determine how the bump stops behave, and how the forces and moments are transmitted to the chassis around “wheel lift-off” angles.

The test vehicle chosen for this experiment is a 1997 Jeep Cherokee Sport. This vehicle has been previously tested on track by Vehicle Research and Test Center (VRTC) for National Highway Traffic and Safety Administration’s (NHTSA) rollover research. Its suspension and inertia parameters have been measured by SEA Inc. [1]. Thus, for comparison of the results, the availability of field test results and the suspension characteristics makes this vehicle an ideal candidate.

Figure 1-Vehicle Body Roll in Cornering

The parameters measured include: roll steer, roll camber, roll center height, and bump stop contact angle as well as the lateral load transfer as a function of roll angle in the linear and non-linear range.

Theoretical part compares the previous test results from the in-house Suspension Parameter Measurement Device (SPMD) and the Side-Pull test results in order to verify the superposition theory of the suspension kinematics and compliance characteristics. In addition, the Side-Pull Test results are compared with actual steady state cornering data. The results will show the level of accuracy for the simulation of the steady state turn. Suggestions on how to use the Side-Pull Test results to improve modeling will be made.

1.3 Thesis Overview

The first part of Chapter 2 gives definitions of important concepts and terms related to roll mode. The effects of roll motion, lateral load transfer during cornering, camber and steer change with roll are explained. This section is useful for readers who are not familiar with suspension measurement. The second part of Chapter 2 talks about roll center measurement. Conventional experimental and theoretical methods are described. A new method for roll center height determination using the Side-Pull Test is explained in detail. The third part covers the history of suspension measurement devices. The capabilities and limitations of actual suspension measurement machines will be outlined.

Chapter 3 is on Side-Pull Suspension Measurement Facility. Its design philosophy and its functioning will be described. Details on its hardware and software as well as its characteristics, capabilities and limitations are given here.

Chapter 4 focuses on modeling of vehicle roll. Two different methods to model roll behavior are reviewed and their limitations explored. An improved method is proposed.

Chapter 5 discusses the results of the side-pull suspension parameter measurement. The results are compared to field test results and to the previously performed SPMD. test results.

Chapter 6 summarizes the results of the experiment and suggests improvements for future applications.

CHAPTER 2

BACKGROUND

2.1 Definitions

In the following paragraphs important concepts and terms of suspension parameters concerning the roll mode are described. Definitions of roll, lateral load transfer, suspension, camber change, and steer change with roll are given [2, 3, 4, 5].

2.1.1 Roll

Suspension Roll is formally defined by Society of Automotive Engineers (S.A.E.) as rotation of the vehicle sprung mass about a fore-aft axis with respect to a transverse line joining a pair of wheel centers. It is positive for clockwise rotation viewed from rear. In general, during roll the spring on the more compressed side becomes stiffer causing some heave of the body. This phenomenon, known as spring jacking is important to recognize since roll motion cannot always be decoupled from bounce (Refer to Figure 1). The effects of roll motion on the handling characteristics are generated by the following as described by Ellis [4]:

1. Change in normal force at tire/road interface
2. Roll Steer induced forces and moments
3. Lateral velocity change at each tire/road interface due to tire scrub
4. Roll Camber induced forces and moments

2.1.2 Lateral Load Transfer

During cornering of a vehicle, the d'Alembert force generated at the CG creates a moment couple along with the tire lateral forces. This couple in return shifts some of the vertical force under the tires along with the load transfer induced by body roll. Its effects on handling depend on the roll dynamics and the front/rear roll moment distribution [2,3,5].

2.1.3 Suspension and Suspension Parameters

Suspension of a vehicle is defined as the system of springs, dampers (shock absorbers) and locating linkages that support a vehicle body and powertrain on its wheels. The suspension has two important functions. First, it controls the motion of the road wheels and keeps them in contact with the road surface under the most demanding conditions. Second, it is responsible for the ride comfort. Suspension parameters define the changes that occur at the tire/road interface due to vehicle body movement and

forces required to control the vehicle. Hence they determine the kinematic and compliance characteristics of a suspension.

The kinematic characteristics describe the wheel motion due to roll, bounce and pitch movement of the vehicle body without including the lateral or longitudinal forces at the tire/ road interface. The compliance characteristics are road wheel displacements resulting from the application of forces at the contact surface of the tire and road. For the determination of suspension characteristics, camber, steer and toe changes with respect to roll motion and force inputs are typically measured. Positive camber angle is defined as the inclination of the top of the wheel outward from the body (Figure 2).

Gillespie [2] defines Roll Steer as the steering motion of the front or rear wheels with respect to the sprung mass that is due to rolling motion of the sprung mass. Positive roll steer coefficient causes the wheels to steer in the direction of body roll. The solid axle supported with leaf springs for the configuration shown in Figure 3, steers when there is a rolling motion of the body. In other words, the roll steer coefficient determined as $d\delta/d\phi$ can significantly modify the understeering characteristics of the vehicle. This is because the point at which the axle is attached to the springs (A), does not follow a perfect vertical motion but rather follows the trajectory as shown in Figure 3.

γ_g = Camber angle with respect to the ground
 γ_b = Camber angle of the wheel with respect to the body
 ϕ = Roll angle of the vehicle

Figure 2 - Camber Angle Change with Roll

Figure 3 - Roll Steer of a Hotchkiss Axle

2.2 Roll Center

The roll center can be defined as the point in the vertical transverse plane through the wheel centers at which lateral forces applied to the sprung mass causes no suspension roll [5]. Roll Center Location can be determined both experimentally and theoretically using graphical or test methods. Bergman, in a paper published in 1969 [6], describes the effects of compliance on roll characteristics. He introduces a new concept of roll axis and separation of “kinematic roll axis” from the “dynamic roll axis”. The compliance in suspension and tires are responsible for the differences between measured and calculated values. Even in the case of a vehicle with stiff suspension bushings the roll center deflection can exceed 3 inches.

Measured and calculated positions of the kinematic roll center of a typical passenger car at various degrees of roll

Figure 4 - Roll Center Measurement [6]

The graph in Figure 4 shows the typical downward shift of the kinematic roll center height as a result of deflection. Because of this, it would be incorrect to use a constant roll center height in modeling.

Theoretical Roll Center Determination

For the leaf spring solid rear axle, the design roll center is determined simply from its geometry as shown in the Figure 5. The roll axis is established by the front spring eye and the shackle attachment point on the frame. The roll center lies at the intersection of the roll axis with the wheel centerline. For the test vehicle, the rear roll center height is found to be 19 inches.

Figure 5 - Roll Center Analysis of Hotchkiss Suspension

The roll center of 4-link solid axle suspension in the front of our test vehicle is also determined geometrically. The pictures of the suspension layout is provided in the Appendix. Figure 6 explains the principle while Figure 7 shows the actual determination of roll center location with data obtained from [1]. The procedure followed is described in [2] : First, determine the reaction points A and B on the centerline of the vehicle for forces in the link. Second, locate the points A and B in the side view to identify the roll axis. Finally, the roll center is the point in the side view where the roll axis crosses the vertical centerline of the wheels. With this method, the front roll center height for the test vehicle is found to be 10 inches.

Figure 6- Roll Center Analysis of a four-link suspension [2]

Figure 7 - Roll Center Determination for the Front Suspension of the Test Vehicle

There are several ways of experimentally determining the roll center. One way, is to pull the vehicle at different heights through a belt wrapped around the vehicle until finding the height where there is no body roll. However, this method only gives the roll center height at zero roll angle.

The conventional roll test [7] can be used to determine the roll center height as well. No lateral force is applied on the tires during the test. Arcs of relative displacements between sprung mass and the tire contact patch are recorded. Intersection of perpendicular lines to two different tangent lines gives the roll center.

However this method does not take into account the roll center height shift due to compliance in the suspension. Another similar technique is to take superimposed pictures of the vehicle while it is being rolled in a constant radius turn. The arcs of displacement of a fixed point on the sprung mass can be obtained from the picture. This method is tedious and requires a field test. In order to accurately measure the instantaneous roll center height at each roll angle the side-pull test can be used in combination with the roll stiffness obtained from in-house SPMD machine. The roll center height deflection due to suspension and tire compliance during cornering is determined according to theory explained below.

We have,

$$\begin{aligned} \text{Roll Moment} &= \text{Side Pull Force} \times \text{Roll Center Height} \\ &= \text{Roll Stiffness} \times \text{Roll Angle} \end{aligned}$$

Rearranging,

$$\text{Roll Center Height} = \frac{\text{Roll Stiffness} \times \text{Roll Angle}}{\text{Side Pull Force}} \quad (1)$$

2.3 Past Efforts in Suspension Parameter Measurement

Several devices have already been developed and used to measure kinematic and compliance characteristics of suspension systems. One of the earliest designs

which belongs to Citroen and Peugeot in collaboration with Michelin dates from the 1940's [8]. The developments in the U.K. and the U.S.A. followed it in 1960's [10]. The first rigs traditionally moved the body of the vehicle while the wheels were supported by air bearings for unrestricted motion. In 1967, a Suspension Parameter Measurement Device (SPMD) was developed at Cranfield Institute of Technology under the direction of Ellis and Sharp [10]. The work of Basso followed the same principles [11, 12]. In 1985, VRTC built a SPMD for the National Highway Transportation Safety Administration (NHTSA) to measure both kinematic and compliance properties [13]. The problem of the device was the extra work and time required to remove the suspension unit from the vehicle and mount it on fixtures in order to run the test. Other existing apparatus that could have both kinematic and compliance inputs at that time was the Chevrolet VHF facility described in a paper published by Nedley and Wilson [9]. The particularity of the latter was that it held the chassis down by clamps and moved the wheels. The maneuvers simulated remained in the low range of acceleration which is below 0.3 g. This was the first machine to apply roll, lateral force and aligning torque in any ratios desired simultaneously while camber and steer angles are measured. A maximum of 10 degrees of platform angle was achievable without raising the CG too much. The latest generation of SPMD was designed and built by SEA, Inc. for Goodyear in 1990 [14, 7]. It had significant improvements compared to the NHTSA SPMD. The entire suspension system could be tested on the vehicle without any dismounting. The upgraded computer and electronic

hardware allowed the automatic testing cycles with ease. In 1991, MTS Systems Corporation designed a vehicle suspension kinematics and compliance (K&C) testing facility that moves all wheels independently [15]. In 1996, another similar measurement facility was designed by Anthony Best [16, 17]. The device can test two axles simultaneously. In addition to the described devices, several laboratories and universities have their simple mechanisms to measure the suspension parameters needed for their simulations.

Although the current suspension measurement devices are fully automated and can combine both the kinematic and compliance inputs, the problem is to decide which combination of roll and lateral tire force will act simultaneously. Furthermore, for suspension systems where there is a coupling of bounce with roll motion for example, this testing method becomes more questionable.

Another problem is that even the most recent suspension measurement devices are not capable of taking measurements beyond approximately 10 degrees of body roll. Even though this level of roll is enough to lift both tires off the ground for most vehicles, it is interesting to analyze what happens to bump-stops and suspension at higher roll angles.

The Side-Pull Test has never been used before to measure suspension characteristics. However, J. P. Chrstos had used it for the first time to simulate a steady state turn and analyze the lateral load transfer thus comparing it with simulation results [18, 19]. His experiment used a cable pull mechanism with a feedback control for the

pull height. The sophistication of the device caused a high hysteresis level in the measurements. The results, although comparable with the real test data, lacked the desired level of accuracy. As he suggested in his thesis [18], the side pull test device of our experiment eliminates the trip rail by replacing it with a high friction surface wheel pad, and reduces the hysteresis by a simpler open loop control design.

CHAPTER 3

SIDE-PULL SUSPENSION PARAMETER MEASUREMENT FACILITY

3.1 Introduction

The suspension characteristics of the 1997 Cherokee Sport was measured by S.E.A Inc. using an in-house SPMD [1]. This machine is manually operated and can run separate bounce, roll and steering compliance tests as well as lateral force compliance tests using hydraulic jacks as actuators.

The side-pull test uses the same measurement apparatus to measure additional roll characteristics of the suspension system. The difference is the method with which roll moment is applied to the vehicle. For the Side-Pull Suspension Measurement, we will only deal with the measurement of the suspension parameters for the roll mode, excluding the pitch and bounce body motions. However, it should be noted that measurement techniques chosen here will automatically take into account the suspension jacking resulting from the roll motion. The standard suspension parameter

measurement uses separate tests for determining both the compliance and kinematics characteristics assuming superposition of these. Suspension parameter measurement is performed quasi-statically in the laboratories, while the dynamic effects of the shock absorbers and tires are measured separately.

3.2 Concept and Design

The application of roll forces to the test vehicle is one of the most important design criteria. For accurate modeling, these forces need to be applied as closely as they would be in a real world scenario. Since our test is not dynamic, we chose to simulate a steady state turn. The best way to simulate cornering in a laboratory would consist of a turn-table like device where the car is positioned on a circular platform in the direction of the rotation. When the platform rotates around its center, the vehicle is submitted to a lateral acceleration proportional to its distance from the center. By increasing the speed of rotation, the vehicle is rolled to its limit. Table 1 summarizes the different measurement concepts for the simulation of steady state turn.

	DESCRIPTION	PROS (+)	CONS (-)
In-House Suspension Measurement Device	Rolls vehicle body through clamps from the chassis spot welds	<ul style="list-style-type: none"> • Already in use • Only simple modification to reach high roll angles 	<ul style="list-style-type: none"> • Moment couple composed of a pair of vertical forces yielding an unrealistic test
Side-Pull Test Device	Rolls the vehicle through a cable attached to a fixture at CG location	<ul style="list-style-type: none"> • Moment couple composed of horizontal forces yielding a realistic test 	<ul style="list-style-type: none"> • Device not readily available • Cable attachment to CG is difficult
Turn-Table Suspension Measurement Device	Vehicle is positioned on a rotating platform to simulate cornering	<ul style="list-style-type: none"> • Most realistic test since lateral forces are distributed on each component 	<ul style="list-style-type: none"> • Design from scratch • High Cost

Table 1 - Choosing a Design Concept for Suspension Parameter Measurement

STANDARD MEASUREMENT :

AND

SIDE-PULL MEASUREMENT:

Figure 8 - Standard vs. Side-Pull Suspension Measurement

The influence of tire forces generated is included in the kinematic parameters. The measurement follows the modeling principle. Thus both tire forces and roll motion inputs are included simultaneously in the measurement of wheel displacement outputs. The vehicle roll occurs generally in cornering where the centripetal force is applied to the center of gravity of the vehicle and lateral forces in the plane of the road are applied to the tires. The Side Pull Test simulates closely these inputs unlike other test methods.

During the concept stage the guiding requirement was to build a device that would measure the forces transmitted to the chassis at tip-up roll angles. This meant that the pull cable would not remain parallel to the horizontal line throughout the roll range [Figure 10]. So the question asked was: Do we need to adjust the pull cable height? The answer is “No”. Because the error of simulated lateral acceleration force due to deviation of the pull cable from the horizontal plane is minimal within the range of roll measurement. Up to bump stop contact the CG height downward shift is less than 1/12 of an inch. Once the rigid body roll initiates, the roll center is moved to the inner tire/ground contact and the CG displaces upward creating an angle between the horizontal and the pull cable (Figure 17). Assuming bump-stop contact at 5.7 degree roll the following graph shows that at 15 degree roll angle the error in the applied roll moment is less than 0.1%. Note that the error is near zero for the spring range where the body rolls about its design roll center.

Figure 9 - Justification of Not Using Winch Height Adjustment

3.3 Description of the Side-Pull SPMD Hardware

The Side-Pull SPMD is composed of actuators and sensors as described below. A representative drawing of the experimental set-up is given in Figure 10. In the actual test the winch is mounted on the front of a 7000 lb fork lift which serves as an anchor.

Figure 10 - Representative Drawing of Side-Pull Suspension Measurement

3.3.1 Actuator and Cable Attachment

The actuator is an electric winch operating at constant height, the initial CG height of the vehicle. It is fed by two 12 Volt Batteries connected in parallel. During the roll test, it is controlled manually by an on/off switch.

The front passenger door and the front seats are removed for the test setup. The cable is attached to a hook, fixed exactly at the CG location by chains that are bolted to the front seat anchors. A pulley-cable arrangement is used to reduce the roll speed and the load on the winch motor (Figure 8).

Figure 11 - Chain arrangement for fixing the attachment of the cable at CG location

3.3.2 Force Measurement

The side-pull force is measured directly through a load cell placed on the winch-vehicle attachment cable. The vertical forces under each tire is recorded to a computer through an electronic scale. The load cell can be seen on Figure 11.

3.3.3 Wheel Displacement Measurement

Three LVDT's touching the "T" shaped plate mounted to the rim, collect necessary information to determine the rotational movements of the wheel (See Figure 12). Wheel displacement measurements are made simultaneously on both the right and left sides of the vehicle. Separate tests are run for the front and rear axle. The SAE vehicle dynamics coordinate system is used to define the directions of the measured quantities. All quantities are measurements made relative to the 'normal' vehicle state prior to testing.

3.3.4 Roll Angle Measurement

Roll angle of the vehicle body is measured with two different methods simultaneously. One method is an inclinometer mounted on the front bumper replacement collects roll data. The other uses two string pots attached perpendicularly on each end of the bumper replacement take measurements of the vertical and lateral motion of the vehicle body allowing the computation of corresponding roll angle.

3.3.5 Data Acquisition

The software used for data acquisition is Lab Notebook. The circuit board is able to collect data from 20 channels simultaneously with a sampling rate of 50 Hz for dynamic testing. For the quasi-static test, at each manual triggering, data are collected for a window of 0.1 second at 50 Hz. and the average value is saved.

Figure 12 - Linear Pot Arrangement for Determination of Camber and Steer Angles

3.3.6 Test Procedure

Each tire is positioned on a high friction surface wheel-pad which transmits the vertical forces to the scale underneath. The steering wheel is locked. No brakes are applied. The automatic transmission is however left in the “Park” position. The vehicle is loaded to match the VRTC field test loading with outriggers and data acquisition system. The loading conditions are in Table 2.

VIN #	IJ4FJ6853VL579212	
Tire Load / Pressure	Left	Right
Front	1125 lb / 32 psi	1046 lb / 32 psi
Rear	975 lb / 32 psi	966 lb / 32 psi

Table 2 - Test Conditions of the 1997 Jeep Cherokee Sport

Representative characteristics determined from the side-pull roll test are front lateral load transfer, rear lateral load transfer, instantaneous roll center, front roll steer, rear roll steer, front roll camber, rear roll camber, individual bump stop contact, roll angle, and instantaneous CG height.

CHAPTER 4

MODELING OF VEHICLE BODY ROLL

4.1 Introduction

In this chapter, two representative simulations developed for NHTSA Crash Avoidance Research are reviewed for their roll modeling. The simulation of these models do not always match precisely with actual field test results. Especially, when dealing with high roll angles for discrete events such as spin out or rollover, modeling of the suspension with high fidelity becomes critical.

4.2 Light Vehicle Dynamics Simulation (LVDS)

The first simulation model analyzed is the Light Vehicle Dynamics Simulation (LVDS). The details of the simulation are described in a paper published by Nalecz [20]. This paper contains the description of a rollover model used in the simulation of limit maneuvers. The authors demonstrated that the components of lateral weight transfer strongly depend upon suspension types and configuration, and that their front and rear distribution can significantly change the wheel normal loads and tire forces generated.

Nomenclature

$\phi_{s/u}$ = Roll angle of the unsprung mass

ϕ_u = Roll angle of the sprung mass relative to the unsprung mass

Y_u = Lateral displacement of the unsprung mass center

Figure 13 - Modeling of Roll in LVDS Simulation

The positions of roll centers and vehicle roll axis are determined by the LVDS program. The suspension analysis provided by the LVDS is based on the assumption that suspension systems are mechanisms consisting of rigid links and that suspension deflections result from the compliance of the spring and shock absorbers. The rollover model is a planar model where the front and rear wheels are combined. The model consists of two masses, sprung and unsprung, which are connected through the elements of the suspension system (Figure 13). Bump stops are modeled as non-linear hardening springs. They limit the suspension motion and become infinitely stiff if they are fully compressed. The external forces applied to the planar model are lateral forces generated by the tires, inertia forces acting on the sprung and unsprung masses, and gravity forces.

Figure 14- Discrepancy between Simulation and Dynamic Test Results

The authors claim that LVDS's capabilities in accurately predicting severe spin out and roll-over were validated using experimental results obtained from several vehicles. But the Roll Angle versus Time plot provided (Figure 14) for the non-rollover case of the Suzuki Samurai, show that there is a 90% overshoot in the LVDS results while the actual vehicle exhibits no overshoot. This discrepancy is enough to question the validity of the roll model used in the simulation. First, a planar model that combines

the front and rear suspensions cannot alone predict a roll-over event. In the real world, the front and rear suspension deflections are not identical, neither do the bump stops hit at the same time. Secondly, LVDS planar model ignores front/rear weight shift which occurs at limit maneuvers.

4.3 Vehicle Dynamics Analysis, Non-Linear (VDANL)

The second simulation software examined is the VDANL. This simulation was originally developed under a NHTSA contract to analyze vehicle handling and driver/vehicle interaction in a variety of severe maneuvering scenarios.

The VDANL simulation has been extensively evaluated by Systems Technology Incorporated via comparison with measured results from full scale tests [15, 22, 23]. The simulation was found to represent vehicle response quite well for most maneuvers but not rollover and spin outs.

A paper by Garrott and Heydinger [21] was an attempt to relate the dynamic vehicle responses to rollover accident rates. Even the most significant response metric identified (low g lateral acceleration gain as determined from the 25 mph slowly increasing maneuver) is not a good predictor of rollover propensity. One reason could be that the model did not represent well the discrete events. The vehicle parameters required for the simulation included roll steer coefficient (front/rear), tire camber angle coefficient (front/rear) as a function of vehicle roll angle and suspension torsional stiffness (front/rear). However, suspension springs and bump stops were not modeled with high fidelity beyond the linear range in the simulation.

A report by G. J. Heydinger submitted to Isuzu Motors America, Inc. for the Evaluation of VDANL for Predicting Limit Performance of a 1996 Isuzu Trooper [24]. The report also evaluated VDANL's ability to predict rollover events. It states that wheel lift-off is predicted in many of the simulation runs of the experimental test maneuvers, when in fact no wheel lift was observed during any of the actual field tests. This is partly due to the fact that, the modeling of bump stops and non-linearity of the suspension was not modeled with enough precision to simulate high lateral acceleration maneuvers.

From the comparison of previous side-pull test results with VDANL simulation performed by Chrstos [18, 19] it is found that there is disagreement in the roll angle prediction. The VDANL has a tendency to underestimate the roll angle predictions for steady state maneuvering.

Since VDNL predicts a lower-than-actual roll for the steady state cornering, and higher-than-actual roll for the dynamic maneuver, there is reason to explore making improvements to the accuracy of modeling in the roll mode.

4.4 Modeling of Lateral Load Transfer

Determination of Lateral Load Transfer is important because the roll moment distribution contributes to understeer mechanisms. More roll moment will increase understeering if applied to the front axle and oversteering if applied to the rear axle [2]. Anti-roll bars are used to trim vehicle handling characteristics following this principle. It should be noted that the largest part of the lateral load transfer in cornering comes from lateral acceleration force independent of the roll angle and roll moment distribution. A smaller yet important portion of the lateral load transfer comes from the vehicle roll and

its effect depends on the roll dynamics. Accurate calculation of the roll moment distribution is an important step towards the prediction of roll dynamics.

Figure 15 - Lateral Load Transfer Model

Figure 16- Vehicle Load Transfer in 3-D

Modeling of Lateral Load Transfer For Front Axle

Vertical Force on the Front Inner Tire

$$F_{ZFI} = 1/2 \times m_{SF} \times g - F_{TF} \quad (2)$$

The total front lateral load transfer

$$F_{TF} = F_{UF} + F_{LF} + F_{KF} \quad (3)$$

where the individual contributors are;

Unsprung Mass Front Load Transfer

$$F_{UF} = \frac{m_{UF} \times A \times h_{UF}}{T_F} \quad (4)$$

Sprung Mass Lateral Load Transfer Acting Through Dynamic Roll Center

$$F_{LF} = \frac{m_{SF} \times A \times h_{RCF}}{T_F} \quad (5)$$

Front Roll Stiffness Load Transfer

$$F_{KF} = \frac{K_F (\phi_S - \phi_{UF})}{T_F} \quad (6)$$

Where ,

K_F = Front Roll Stiffness

ϕ_{UF} = Front Front Axle Roll

A = Lateral Acceleration

h_{UF} = Front Unsprung Mass Height

m_{UF} = Front Unsprung Mass

ϕ_S = Sprung Mass Roll

T_F = Front Track Width

h_{RCF} = Front Roll Center Height

m_{SF} = Front Sprung Mass

g = Gravitational Acceleration

Modeling of Lateral Load Transfer For Rear Axle

Vertical Force on the Rear Inner Tire

$$F_{ZRI} = 1/2 \times m_{SR} \times g - F_{TR} \quad (7)$$

The total rear lateral load transfer

$$F_{TR} = F_{UR} + F_{LR} + F_{KR} \quad (8)$$

where the individual contributors are;

Unsprung Mass Rear Load Transfer

$$F_{UR} = \frac{m_{UR} \times A \times h_{UR}}{T_R} \quad (9)$$

Sprung Mass Lateral Load Transfer Acting Through Dynamic Roll Center

$$F_{LR} = \frac{m_{SR} \times A \times h_{RCR}}{T_R} \quad (10)$$

Rear Roll Stiffness Load Transfer

$$F_{KR} = \frac{K_R(\phi_S - \phi_{UR})}{T_R} \quad (11)$$

Where ,

K_R = Rear Roll Stiffness

ϕ_{UR} = Rear Axle Roll

A = Lateral Acceleration

h_{UR} = Rear Unsprung Mass Height

m_{UR} = Rear Unsprung Mass

ϕ_S = Sprung Mass Roll

T_R = Rear Track Width

h_{RCR} = Rear Roll Center Height

m_{SR} = Rear Sprung Mass

g = Gravitational Acceleration

The roll center heights used in the numerical computation are found using the graphical methods as explained in Chapter 2, Section 2 .

The sprung mass is computed by subtracting the sum of all components constituting the unsprung mass, from the total mass. The unsprung mass heights are computed from the measured locations of each component.

The front and rear axle load transfers are computed using the constants described above. The inputs are the simulated lateral acceleration and the corresponding roll angles. Note that the application of the theory requires that roll angle is known as a function of lateral acceleration.

The idea of modeling the roll mode with high fidelity as a function of lateral acceleration requires all parameters to be a function of lateral acceleration. The Side-Pull Test Suspension Parameter Measurement Facility is capable of providing this information.

Spring Roll Motion upto Bump Stops

Roll-Over begins with tire lift-off

Figure 17 - Roll Center Shift with Bump Stop Contact

Figure 18 - Virtual Roll Center Computation from Side-Pull Test Results

CHAPTER 5

RESULTS

In this chapter we first validate the Dynamic Side-Pull Test by comparing its results to the Quasi-Static Test. Secondly we compare the results of the Side-Pull Test with Field Test Results obtained by VRTC. Thirdly, we check the superposition theory of compliance and kinematics by comparing the new results with the results previously obtained using SEA Inc.'s SPMD. Lastly, the theoretical load transfer is compared with experimental results.

5.1 Dynamic Side-Pull Test vs Quasi-Static Test

The Side-Pull Test is carried out both dynamically and quasi-statically. Due to instrumentation limitations each test is performed twice to collect data from both front and rear wheels. The dynamic test is continuous without any interruption of the winch motor until one wheel starts to slide. The quasi-static test rolls the vehicle step by step with small increments controlled manually. The dynamic test rolls the body of the vehicle with an average rolling speed of 1 degree per second on the way up and 2 degrees per second on the way down. The plot in Figure 18 shows the history of a

typical dynamic test. The roll angle is measured with an inclinometer in addition to a linear string potentiometer arrangement which is used to double-check the measurement. Roll angles obtained with the two methods are very close giving confidence on the accuracy of roll angle measurement. The winch used during the test is not powerful enough to maintain a constant pulling speed throughout the test. This can be seen in the non-linearity of the curve for the first ten seconds (Figure 19).

Figure 19 - Side-Pull Rolling Speed and Verification for Roll Angle Measurement

From the shock absorber test data which was previously available, it was suspected that the dynamic test speed could be too high to ignore the dampers' effect. So the quasi-static tests are performed to check the validity of the continuous test. The axle roll moments are plotted against the roll angle for each axle and for each test type.

The axle roll moment is computed from the equation:

$$M_{xj} = (1/2) (\Delta F_{zi} - \Delta F_{zo}) T_j \quad j = \text{Front , Rear} \quad (11)$$

where ΔF_{zi} and ΔF_{zo} are the changes in the vertical inner and outer tire loads from their initial load and T_j is the average lateral distance between springs on each axle. The plots of Axle Roll Moment as a function of Roll Angle are given for both front and rear axles (Figure 19 and 20). The slopes of each curve obtained from the linear interpolation of data points represent the Axle Roll Stiffness and are summarized in Table 2. The results clearly show that the effect of the shock absorbers can be ignored for the dynamic test where the rolling speed is as low as 1 degree per second.

Roll Axle Stiffness	Dynamic Test	Quasi-Static Test	Error
Front Axle	7602 in-lb/deg	7523 in-lb/lb	1.0501 %
Rear Axle	6136 in-lb/deg	6135 in-lb/deg	0.0002 %

Table 2 - Comparison of Dynamic and Quasi-Static Side-Pull Test Results

Figure 20 - Dynamic vs. Quasi-Static Side-Pull Test for Front Roll Stiffness

Figure 21 - Dynamic vs. Quasi-Static Side-Pull Test for Rear Roll Stiffness Measurement

Validation of dynamic side-pull testing for suspension parameters measurement requires comparing camber and steer changes obtained with both methods. For this, we look at the Roll Steer vs. Roll Angle and Roll Camber vs. Roll Angle plots for the rear and front axle. The curves from dynamic and quasi-static test are plotted on the same graph for easy comparison (Figure 21 and 22). The juxtaposition of both curves in the first linear range show that the suspension characteristics measurement can be done with the dynamic test for the low roll angles. However the plots of the step-by-step test tend to separate and take higher values beyond 3 degrees of roll. This deviation may be due to the fact that the quasi-static test actually “shakes” the vehicle at the beginning and at the end of each pulling step. The disturbance overcomes the friction in the suspension system and the friction between the tire and wheel pad. This allows the wheel to move further than in the smooth, slow dynamic test. Another reason for the deviation between the dynamic and quasi-static test is that, the quasi-static pulling gives enough time to the wheel to fight the retarding forces and take its final position whereas the continuous pulling does not allow this.

As far as suspension measurement is concerned, the continuous pulling provides a quick and accurate results for the low roll range. For higher roll ranges (corresponding to lateral accelerations greater then 0.4 g based on Cherokee data) the quasi-static test is recommended. Because it is closer to a real maneuver where the body movements are not so smooth and where the wheel rolling ease the change wheel camber and steer angles.

Figure 22 - Dynamic vs. Quasi-Static Side-Pull for Front Camber and Steer Measurement

Figure 23 - Dynamic vs. Quasi-Static Side-Pull for Rear Camber and Steer Measurement

5.2 Field Test Results Compared to Side-Pull

In order to understand how well the laboratory Side-Pull Test simulates a real steady state cornering roll angle as a function of simulated acceleration from the side-pull test, needs to be compared with the results of the field test. The field test consists of a constant speed (25 mph) with increasing steer maneuver up to the limit understeering. This was performed by VRTC on the same vehicle two weeks before the Side-Pull Suspension Parameter Measurement Test. Lateral acceleration readings are corrected to take into account the inclination of the accelerometer. The roll angle output is passed through a low pass filter at 1 Hz cutoff frequency. Both results are plotted on the same graph for the ease of interpretation. As can be seen in Figure 23, the Roll Angle vs Lateral Acceleration curve obtained from the field test falls within the loop of the Side-Pull Test. Taking into account the hysteresis of the suspension system the results obtained are comparable, and shows that the Side-Pull Test simulates closely a steady state turn. Both filtered and unfiltered field test data are plotted versus corrected and uncorrected roll angles.

Figure 24- Validation of Simulated Cornering via Side-Pull Test

5.3 Verification of Kinematics and Compliance Superposition Theory

The Roll Camber and Roll Steer plots generated from the side-pull suspension measurement are shown in Figures 25, 26, 27, and 28. Although the curves are not linear, it is possible to divide them into linear portions in order to compare the results with the SPMD results. The roll camber and steer coefficients are summarized in Table 3.

	ROLL CAMBER COEFFICIENT (deg/deg)	ROLL STEER COEFFICIENT (deg/deg)
FRONT up to 3 deg. Roll	0.597	0.437
FRONT beyond 3 deg. roll	1.647	0.998
REAR up to 3 deg. Roll	0.631	0.027
REAR beyond 3 deg. Roll	1.543	0.199

Table 3 - Coefficients of Roll Camber and Roll Steer obtained from Side-Pull Test

	ROLL CAMBER COEFFICIENT	ROLL STEER COEFFICIENT
FRONT COMPLIANCE	0.00156 deg/lb	0.00181 deg/lb
FRONT KINEMATICS	0.347 deg/deg	0.0360 deg/deg
REAR COMPLIANCE	0.00199 deg/lb	0.000252 deg/lb
REAR KINEMATICS	0.0980 deg/deg	0.0160 deg/deg

Table 4 - Coefficients of Roll Camber and Roll Steer from conventional SPMD [1]

Figure 25 - Determination of Front Roll Camber Coefficients

Figure 26 - Determination of Front Roll Steer Coefficients

Figure 27 - Determination of Rear Roll Camber Coefficients

*Is the resolution
of sensor good enough?*

Figure 28 - Determination of Rear Roll Steer Coefficients

The roll camber and the roll steer changes are compared at the roll angle where the maximum friction is reached. At 5.56 degree of roll, the maximum friction of 1.01 is reached giving 1980 lb of vertical load for the rear outer tire and 2142 lb for the front outer tire. With this information, we extrapolate the camber and steer angles using the coefficients obtained from both methods. For the superposition of kinematics and compliance coefficients the equations used are :

$$\begin{aligned} \text{Camber Angle} = & (\text{Kinematic Camber Coefficient}) \times (\text{Roll Angle}) + \\ & (\text{Compliance Camber Coefficient}) \times (\text{Friction} \times \text{Vertical Tire Load}) \end{aligned}$$

(12)

and

$$\begin{aligned} \text{Steer Angle} = & (\text{Kinematic Steer Coefficient}) \times (\text{Roll Angle}) + \\ & (\text{Compliance Steer Coefficient}) \times (\text{Friction} \times \text{Vertical Tire Load}) \end{aligned}$$

(13)

	K & C COMBINED (deg)	SIDE-PULL (deg)
FRONT CAMBER ANGLE	5.3	6.0
REAR CAMBER ANGLE	4.5	5.8
FRONT STEER ANGLE	4.1	4.1
REAR STEER ANGLE	0.6	5.1

Table 5 - Verification of Superposition Theory for K&C Measurement

On the other hand, computation of the camber and steer angles at the same roll angle from the coefficients of the Side-Pull Test is straightforward. The results are displayed in Table 5.

The change in angles for both camber and steer are greater in the Side-Pull Test. The rear steer angle predicted by the superposition theory is 8.5 times smaller than the value predicted by the Side-Pull. This is partly due to the fact that the coefficients obtained from the compliance tests are for a lower lateral force range. Thus, extrapolation using the same coefficient for higher degrees of roll, results in underestimation of the roll steer change. Camber Angles found with both methods are comparable. However, it is not possible to check the superposition theory accurately with the available data. In order to accomplish the latter, camber and steer angles need to be compared throughout the entire range of roll and not just at the point of maximum acceleration which requires that lateral force at the individual wheel pads be measured. Also, both tests need to be run up to the same maximum value of roll and corresponding lateral force. For this project, the results from the kinematics and compliance measurement were obtained from a lower range test.

5.4 Verification of Lateral Load Transfer Theory

The Lateral Load Transfer Theory is explained in Section 4 of Chapter 4. All necessary constants to apply the theory for the test vehicle are found in a report prepared by SEA for NHTSA [1].

The curves from the experiment and the theory for both front and rear axles are plotted (Figure 29, 30). The results match closely for the front axle with a maximum of approximately 4 percent error. For the rear axle, the theory predicts about 20 percent higher load transfer than in reality at its maximum value. Sources of error for this deviation are numerous. The measurements of mass and location of each component of the sprung mass, and changes in track width during roll motion are contributing factors. However, the most sensitive constants in the calculation are the Rear Roll Center Height and the Rear Roll Stiffness. The error is possibly the combination of these two.

The virtual roll center height could be used in modeling instead of a constant value. As shown in Figure 17, the roll center is subject to major shifts once the bump-stops are engaged. The shifting from the kinematic design roll center to the tire/ground contact needs to be modeled empirically using the information available from side-pull test. The virtual roll center height is computed following the equation (1) given in Chapter 2, Section 2. It varies between 12 and 17 inches. This result is in agreement with the design values calculated geometrically. The rear and front roll center heights were respectively 10 and 19 inches. The plotted roll center height values bracket the average value found geometrically (Figure 18).

Figure 29 - Theoretical Front Tire Lift-Off

Figure 30 - Theoretical Rear Tire Lift-Off

The high friction surface wheel pads enables one to simulate lateral acceleration levels exceeding 1 g. During our tests the front bump stop comes in to contact at around 5.2 degree of roll. This can be seen in the Axle Roll Moment vs. Roll Angle Plot (Figure 31). At the limit of rolling, first the rear axle starts to slide, the test is stopped at that point where the inner wheels are close to lift-off as can be seen in the load transfer plot shown in Figure 32.

Figure 31 - Axle Roll Moment Variation and Determination of Bump Stop Contact

Figure 32 - Lateral Load Transfer During Roll

CHAPTER 6

CONCLUSIONS

6.1 Summary of Results

The side-pull test is used for the first time to measure suspension parameters. The experiment gives promising results. The Side-Pull Suspension Parameter Measurement Facility provides means for deriving more complete combined kinematics and compliance models. Unrestrained roll into bump stops gives a more real-world-like measurement. The results compare well with field tests. And with some improvements, the facility has the potential to gather all suspension parameters necessary to model the vehicle limit roll.

The construction of the test facility is simple in nature and therefore not costly (See Appendix for design of the parts needed). The experiment time is much shorter than the kinematics and compliance tests run separately. The vehicle tested here did not pose any problems for setup. But, the ease of mounting the fixture for the cable, depend highly on inside geometry of the passenger cabin and the CG location of the vehicle

6.2 Recommendations

The Side-Pull Test could be used alone to measure suspension characteristics. It has the advantage of inputting the kinematics and compliance forces simultaneously. The measurement facility designed and tested here lacked the ability of measuring the tire lateral forces during the roll process. For future improvement, force sensors need to be mounted on the wheel pads to measure the lateral forces. This will enable researchers to gain knowledge of exact roll moment and hence the roll stiffness. Also, the wheel displacements could be easily compared with the kinematics and compliance superposition results at any given roll angle.

During the roll test, the measurement of wheel displacements could not be carried out to the maximum because of the extreme lateral wheel displacements. Replacement of existing LVDT's with ones for larger displacement range will enable one to measure camber and steer angles throughout the entire roll range.

Once these improvements are made, the Side-Pull Suspension Parameter Measurement Facility will provide enough information to model the roll mode of a vehicle for steady state cornering. It is possible to carry this analogy to predict the roll behavior for an emergency maneuver. When the roll angle, roll moment and tire forces are known as a function of lateral acceleration, it is straightforward to simulate the dynamic response of a vehicle for a given maneuver. The lateral acceleration required to perform a given emergency maneuver is approximated from the minimum radius of curvature of the path followed. This value is then used as an input along with the initial roll angle to predict whether a rollover takes place or not.

For dynamic modeling, the effects of dampers and inertia need to be included in the measurement. An improved version of the Side-Pull Facility could include a fast dynamic rolling mechanism where all the dynamic factors, except the rolling of wheels are included. The results obtained in laboratory, could for example be compared with hook maneuvers to get more insight on the roll-over tendency of road vehicles.

LIST OF REFERENCES

1. S.E.A. Inc., Vehicle Dynamics Division, "Vehicle Inertia Measurement Facility, Suspension Kinematics and Compliance, Shock Absorber, and Suspension Component Geometric and Inertia Measurement Results", report for NHTSA DTNH22-97-P-08122, 1997
2. Gillespie T., "*Fundamentals of Vehicle Dynamics*", SAE 1992
3. Dixon J. C., "*Tyres, Suspension and Handling*", Cambridge University Press, 1991
4. Ellis J. R., "*Vehicle Handling Dynamics*", Mechanical Engineering Publications, 1994
5. Genta G., "*Motor Vehicle Dynamics Modeling and Simulation*", Series on Advances in Mathematics for Applied Sciences -Vol. 43, 1997
6. Bergman Walter, "Effects of Compliance on Vehicle Handling Properties", SAE Paper 700369
7. Chen H. F., Coovert D. A., and Guenther D. A., "Suspension Testing on the New-Type Suspension Parameter Measurement Device", SAE Paper 920049
8. Dixon J. C., "Noise Reduction Design Features of the Peugeot 404", MIRA Report 1962
9. Nedley A. L. and Wilson W. J., "A New Laboratory Facility for Measuring Vehicle Parameters Affecting Understeer and Brake Steer", SAE Paper 720473
10. Ellis J. R., and Sharp R. S., "Measurement of Vehicle Characteristics for Ride and Handling", Symposium on Instrumentation and Test Techniques for Motor Vehicles, The Institution of Mechanical Engineers, Proceedings, 1967-68, Vol. 182, Part 3B, July 4-5 1967
11. Basso G. L., "Functional Derivation of Vehicle Parameters for Dynamic Studies", LTR-ST. 747, National Research Council of Canada, 1974
12. Basso G. L., "A Methodology for Measurement of Vehicle Parameters used in Dynamic Studies", National Aeronautical Establishment, NRC NO. 13497, National Research Council Canada, July, 1973

13. Ellis J. R., "The Design of a Suspension Parameter Measurement Device", SAE Paper 870567
14. Coovert D. A., Chen H. F., and Guenther D. A., "Design and Operation of a New Type Suspension Parameter Measurement Device", SAE Paper 920048
15. Chrstos J. P., and Heydinger G. J., "Suspension Parameter Measurement Needs Evaluation", VRTC-87-0086, 1991
16. Whitehead J. P., "The Essential Contribution of Rig Measurement to Suspension Design and Development", C498/7/061/95, IMechE Seminar Publication 1996-97
17. Best A., Neads S., Witehead J. P., and Willows I. R., "Design and Operation of a New Vehicle Suspension Kinematics and Compliance Facility", C524/133/97, IMechE Seminar Publication 1997-8
18. Chrstos J. P., "Simulation Evaluation Using the Side Pull Test", Thesis 1992, The Ohio State University
19. Chrstos J. P., and Guenther D. A., "Simulation Evaluation Using the Side-Pull Test", Institute of Mechanical Engineers (IMechE) C466/029/93, 1993
- ✓ 20. Nalecz A. G., "Development and Validation of Light Vehicle Dynamics Simulation (LVDS)", SAE Paper 920056
21. Garrott W. R., and Heydinger G. J., "An Investigation, Via Simulation, of Vehicle Characteristics that Contribute to Steering Maneuver Induced Rollover", SAE Paper 920585
22. Allen R. W., Rosenthal T. J., and Chrstos J. P., "Applying Vehicle Dynamics Analysis and Visualization to Roadway and Roadside Studies", System Technology Inc., Tehcnical Report 1301-1, FHWA Contract, Publication DTFH61-93-C-00209, 1998
23. Allen R. W., Szostak H. T., et al., "Vehicle Dynamic Stability and Rollover", NHTSA DTNH22-88-C-07384, June 1992

BIBLIOGRAPHY

1. Allen R. W., Rosenthal T. J., et al., "Requirements for Vehicle Dynamics Simulation Models", SAE Paper 940175
2. Lund Y. I., and James E. B., "Analysis of Simple Rollover Metrics", SAE Paper 950306
3. Bernard J. E., Clover C. L., "Validation of Computer Simulations of Vehicle Dynamics", SAE Paper 940231
4. Constantine C. J., Law E. H., "The Effect of Roll Control for Passenger Cars During Emergency Maneuvers", SAE Paper 940224
5. Kahrs J. J., Law E. H., "An Investigation of the Effects of Roll Control on Handling and Stability of Passenger Vehicles During Severe Maneuvers", SAE Paper 950305
6. Bernard J., Shannan J., Vanderploeg M., "Vehicle Rollover on Smooth Surfaces", SAE Paper 891991
7. Chen H. F., Guenther D. A., et al., "The Effects of Suspension Stiffness on Handling Responses", SAE Paper 911928
8. SAE Author, "Measurement of Vehicle And Suspension Parameters For Directional Control Studies", SAE J1574/ 1 May 1994, SAE Recommended Practice
9. Lund Y. I., and Bernard J. E., "The Relationship Between the Complexity of Linear Models and the Utility of the Computer Results", SAE Paper 920052
10. Heydinger G. J., Garrott W. R., Chrstos J. P., Guenther D. A., "A Methodology for Validating Vehicle Dynamics Simulations"
11. Ellis J. R., "A Study of Suspension Mechanisms, Cranfield Institute of Technology", ASAE Report No 5, 1969

APPENDIX A

Figures and Tables

Figure 33 - View of Front Axle from Underneath [Haynes Repair Manual]

1.1 Front suspension and steering components

1 Stabilizer bar	7 Coil spring
2 Upper suspension arms	8 Tie rod
3 Center link	9 Steering knuckle
4 Track bar	10 Lower suspension arms
5 Pitman arm	11 Front axle housing
6 Shock absorber (badly visible in this photo)	12 Steering damper

Figure 34 - View of Rear Axle from Underneath [Haynes Repair Manual]

Support Pole (x8)	Allowances: decimal +/- 0.010, angle +/- 0.5 degree unless otherwise specified
A: 1/4 inch thick carbon steel (1040) plate B: 1 inch diameter carbon steel (1040) rod	All dimensions are in inches

Figure 35 - Drawing of Support Pole for Wheel Pad

Skid Pad (x4)	Scale : 1/10 All dimensions in inches
Use 1/4 inch thick aluminum (2014) plate 1/8 steel 1040	Allowances: +/- 0.05 inches unless otherwise specified

Figure 36 - Drawing of Wheel Pad Base

Failure Analysis of the Support Poles

Material	1040 Carbon Steel Rod
Diameter	1.000 inch
Allowable Stress in Bending	30,600 psi
Design Stress (without gussets)	28,699 psi
Safety factor (without gussets)	1.02

Note: With the gussets welded on both sides of the rod, the safety factor is increased considerably.

Failure Analysis of the Skid Pad

Material	1040 Steel Plate
Thickness	1/8 inch
Allowable Stress in Tension	19,350 psi
Design Stress (around holes)	10,000 psi
Allowable Load before buckling	5,582 lbf
Max. Design Load	2,500 lbf
Min. Safety Factor	1.9

Failure Analysis of the Anchors for the poles

Material	Rawl Steel Drop-in 1/4", FFS325, group 8
Maximum Design Load on the Anchors	1,612 lbf
Allowable Tension Load	3,400 lbf
Safety Factor	2.1

Failure Analysis of the Anchors for the winch

Material	Rawl Steel Drop-in 3/8", FFS325, group 8
Maximum Design Load on the Anchors	2,500 lbf @ 25 in c.g. height
Allowable Tension Load	6,400 lbf
Safety Factor	2.6

Table 7 - Failure Analysis of Designed Components

Figure 37 - Cherokee At The Side-Pull Test

APPENDIX B

Matlab Code For Data Analysis

%CHEROKEE DATA with the SIDE PULL TEST SETUP

```
m=4112;    %(lb) total mass
m_ur=376;  %(lb) rear unsprung mass
m_uf=422;  %(lb) front unsprung mass
m_sf=1750; %(lb) front sprung mass
m_sr=1565; %(kg) rear sprung mass
%I=653.02; %(kg-m^2) total vehicle roll moment inertia (stock)
%I_uf=100; %dummy values
%I_ur=100;
%I_s=453.02;

d=5.039;   %(in) distance btw RC and CG
h_cg=26.850; %(in) total vehicle CG height
h_ur=14.055; %(in) CG height of rear unsprung mass
h_uf=14.960; %(in) CG height of front unsprung mass
h_Rcf=10.000; %(in) Front Roll Center
%h_RCr=18.94; %(in) Rear Roll Center
h_RCr=19;
g=386.088;  %(lb-in/s^2) gravitational force

T_f=58.2;   %(in) front track width
T_r=57.3;   %(in) rear track width

%in-lbf/deg * (6.47) ==> Nm/rad

k_c=535091; %(N/m) pull cable spring rate
%K_f=7523;  %(in-lb/deg) front roll stiffness
K_f=7100;   %(SPMD)
%K_r=6140;  %(in-lb/deg) rear roll stiffness
K_r=4200;   %(SPMD)
K_a=37623;  %(in-lb/deg) Axle Roll Stiffness (from tires)
K_bsf=5*K_f; % front bump stop stiffness
K_bsr=5*K_r; % rear bump stop stiffness
%B_s=66.47; %(Nm-s/rad) Damping of 2 (left+right) shocks
```

```

%Lateral Load Transfer
% Assuming phi, A both positive
data_UK
A=F1/4112;
F_TF=((m_uf*A*h_uf)+(m_sf*A*h_RCf)+(K_f*phi))./T_f;      %Front LLT
F_ZFI=1125-F_TF;      %Front Inner Tire Load
F_TR=((m_ur*A*h_ur)+(m_sr*A*h_RCr)+(K_r*phi))./T_r;      %Rear LLT
F_ZRI=975-F_TR;      %Rear Inner Tire Load

```

```

figure(1)
clf
plot(A(j:k),F_ZFI(j:k),'o')
hold on
plot(A,1125+WLF,'b')
xlabel('Lateral Acceleration (g)')
ylabel('Vertical Force on Front Inner Tire (lb)')
legend('theoretical','side-pull')
title('Verification of Lateral Load Transfer Theory')
grid

```

```

figure(2)
h_RCf_computed=T_f*(-WLF(j:k)-(m_uf*A(j:k)*h_uf/T_f)-
(K_f*phi(j:k)/T_f))./(m_sf*A(j:k));
plot(phi(j:k),h_RCf_computed,'o')
grid
xlabel('Roll Angle (deg)')
ylabel('Virtual Front Roll Center Height (in)')

```

```

figure(3)
clf
plot(A,975+WLR)
hold on
plot(A(j:k),F_ZRI(j:k),'bo')
xlabel('Lateral Acceleration (g)')
ylabel('Vertical Force on Rear Inner Tire (lb)')
legend('side-pull','theoretical')
title('Verification of Lateral Load Transfer Theory')
grid

```

```
%
% Rear Roll Test Analyses
%
clear
load r_cont.xlsx
full = r_cont;
```

```
G = 11.81; % Y Distance from Glass at LR1 to Wheel Center
H = 7.64; % Z Distance from LR1 to Wheel Center
SR4_0 = 124.0; % Initial Length from SR4 to Wheel Center
Tire_Y = 12.95; % Lateral Distance from Wheel Center to SR3
Height_0 = 11.0; % Initial Height (Length) of SR3
SR2_X = -35.83; % X Distance from A to Wheel Center
SR1_X = -35.83; % X Distance from SR1 to Wheel Center
LDT_X_Distance = 14.0; % X Distance Between LR2 and LR3
LDT_Z_Distance = 11.1; % Z Distance Between LR1 and LR's 2 & 3
```

```
L_G = 11.46; % Y Distance from Glass at LL1 to Wheel Center
L_H = 9.10; % Z Distance from LL1 to Wheel Center
SL4_0 = 124.6; % Initial Length from SL4 to Wheel Center
L_Tire_Y = 12.32; % Lateral Distance from Wheel Center to SL3
L_Height_0 = 9.29; % Initial Height (Length) of SL3
SL2_X = -37.05; % X Distance from SL2 to Wheel Center
SL1_X = -37.05; % X Distance from SL1 to Wheel Center
L_LDT_X_Distance = 14.0; % X Distance Between LL2 and LL3
L_LDT_Z_Distance = 11.0; % Z Distance Between LL1 and LL's 2 & 3
% N/A L_SB_0 = 63.2; % Initial Length (lateral distance) from SB
to vehicle body
```

```
y_distance_springs_f = 43.5; % Y-distance between front springs (in)
y_distance_springs_r = 37.7; % Y-distance between rear springs (in)
```

```
%
% Read in and Zero the Linear Displacement Transducer,
% String Potentiometer, Scale, Inclinometer and Load Cell Values
%
```

```
SL1 = full(:,1) - full(1,1); % Left Side - Vertical string potentiometer front
of wheel
SL2 = full(:,2) - full(1,2); % Left Side - Vertical string potentiometer rear of
wheel
SL3 = full(:,3) - full(1,3); % Left Side - Vertical string potentiometer wheel
center
```

```

SL4 = full(:,4) - full(1,4);           % Left Side - Longitudinal string potentiometer
wheel center

SR1 = full(:,5) - full(1,5);           % Right Side - Vertical string potentiometer front
of wheel
SR2 = full(:,6) - full(1,6);           % Right Side - Vertical string potentiometer rear
of wheel
SR3 = full(:,7) - full(1,7);           % Right Side - Vertical string potentiometer wheel
center
SR4 = full(:,8) - full(1,8);           % Right Side - Longitudinal string potentiometer
wheel center

LL1 = full(:,9) - full(1,9);           % Left Side - Top linear displacement transducer
LL2 = full(:,10) - full(1,10); % Left Side - Front linear displacement transducer
LL3 = full(:,11) - full(1,11); % Left Side - Rear linear displacement transducer
LR1 = full(:,12) - full(1,12); % Right Side - Top linear displacement transducer
LR2 = full(:,13) - full(1,13); % Right Side - Front linear displacement transducer
LR3 = full(:,14) - full(1,14); % Right Side - Rear linear displacement transducer

SA = full(:,15) - full(1,15); % String potentiometer between wheel pads
SB = full(:,16) - full(1,16); % String potentiometer - Lateral from body
SC = full(:,17) - full(1,17); % String potentiometer - Lateral from wheelpad

WLF = full(:,18) - full(1,18); % Left Front - Vertical load
WRF = full(:,19) - full(1,19); % Right Front - Vertical load
WLR = full(:,20) - full(1,20); % Left Rear - Vertical load
WRR = full(:,21) - full(1,21); % Right Rear - Vertical load

F1 = (full(:,22) - full(1,22)); % Load Cell

INC = (full(:,23) - full(1,23)); % Inclinator
INC=-INC;
%
% Determine the Change in Camber Angle
%

Camber_Angle_Rad = atan((LR1-(LR2+LR3)/2)/LDT_Z_Distance);
Camber_Angle_Deg = (180/pi)*Camber_Angle_Rad;

L_Camber_Angle_Rad = -(atan((LL1-(LL2+LL3)/2)/L_LDT_Z_Distance));
L_Camber_Angle_Deg = (180/pi)*L_Camber_Angle_Rad;

%
```

```
% Determine the Change in Steer Angle
%
```

```
Steer_Angle_Rad = atan((LR2-LR3)/LDT_X_Distance);
Steer_Angle_Deg = (180/pi)*Steer_Angle_Rad;
```

```
L_Steer_Angle_Rad = -(atan((LL2-LL3)/L_LDT_X_Distance));
L_Steer_Angle_Deg = (180/pi)*L_Steer_Angle_Rad;
```

```
%
% Determine the Chassis Vertical Deflection
%
```

```
Del_Z_Chassis = -SR2+((SR2-SR1)*SR2_X)/(SR2_X+SR1_X);

L_Del_Z_Chassis = -SL2+((SL2-SL1)*SL2_X)/(SL2_X+SL1_X);
```

```
%
% Determine the Wheel Center Lateral Deflection
%
```

```
J = -SR3;
K = G./cos(Camber_Angle_Rad);
L = (H+J).*sin(Camber_Angle_Rad);
Del_Y_Wheel_Prime = LR1+G-K-L;
```

```
J = -SL3;
K = L_G./cos(L_Camber_Angle_Rad);
L = (L_H+J).*sin(-L_Camber_Angle_Rad);
L_Del_Y_Wheel_Prime = -(LL1+L_G-K-L);
```

```
%
% For Roll Make No Correction for Chassis Lateral Movement
%
```

```
Lateral_Correction = 0;
```

```
Del_Y_Wheel = Del_Y_Wheel_Prime + Lateral_Correction;
```

```
L_Del_Y_Wheel = L_Del_Y_Wheel_Prime + Lateral_Correction;
```

```
%
% Determine the Tire Lateral Deflection
%
```

```
L_Del_Y_Tire = SC;
```

```
Del_Y_Tire = SC+SA;
```

```
%
```

```
% Determine the Wheel Vertical Deflection
```

```
%
```

```
A = Del_Y_Wheel - Tire_Y*(1-cos(Camber_Angle_Rad));
```

```
B = Height_0 + SR3;
```

```
Theta_String = asin(A./B);
```

```
Height = B.*cos(Theta_String)+Tire_Y*sin(Camber_Angle_Rad);
```

```
Del_Z_Wheel = Height_0-Height;
```

```
A = -L_Del_Y_Wheel - L_Tire_Y*(1-cos(-L_Camber_Angle_Rad));
```

```
B = L_Height_0 + SL3;
```

```
L_Theta_String = asin(A./B);
```

```
L_Height = B.*cos(L_Theta_String)+L_Tire_Y*sin(-L_Camber_Angle_Rad);
```

```
L_Del_Z_Wheel = L_Height_0-L_Height;
```

```
%
```

```
% Determine the Suspension Vertical Deflection
```

```
%
```

```
Del_Z_Susp = Del_Z_Chassis-Del_Z_Wheel;
```

```
L_Del_Z_Susp = L_Del_Z_Chassis-L_Del_Z_Wheel;
```

```
%
```

```
% Determine the Wheel Longitudinal Deflection
```

```
%
```

```
SR4_Angle =
```

```
asin((sqrt(Del_Y_Wheel.*Del_Y_Wheel+Del_Z_Chassis.*Del_Z_Chassis))./(SR4_0+SR4));
```

```
S = (SR4_0+SR4).*cos(SR4_Angle);
```

```
Del_X_Wheel = -(SR4_0-S-Tire_Y*sin(Steer_Angle_Rad));
```

```
SL4_Angle =
```

```
sin((sqrt(L_Del_Y_Wheel.*L_Del_Y_Wheel+L_Del_Z_Chassis.*L_Del_Z_Chassis))./(SL4_0+SL4));
```

```
L_S = (SL4_0+SL4).*cos(SL4_Angle);
```

```
L_Del_X_Wheel = -(SL4_0-L_S-L_Tire_Y*sin(-L_Steer_Angle_Rad));
```

```

%
%Correction of Roll Angle
%

phi_uncorrected=180/pi*asin((SR1-SL1)/73);
phi=180/pi*asin( ( ((25+SL1).^2-SL2.^2).^0.5 - ((25+SR1).^2-SL2.^2).^0.5 ) /73 );
Roll_Angle=phi;
time=linspace(0,length(INC)/50,length(INC))';
A_y=10*F1/4112;

%
% Determine the Front Axle Roll Stiffness
%

Roll_Moment_F = (WRF-WLF)*y_distance_springs_f/2;
Spring_Coefficients_F = polyfit(phi(10:460),Roll_Moment_F(10:460),1);
Roll_Stiffness_F = Spring_Coefficients_F(1)

%
% Determine the Rear Axle Roll Stiffness
%

Roll_Moment_R = (WRR-WLR)*y_distance_springs_r/2;
Spring_Coefficients_R = polyfit(phi(10:460),Roll_Moment_R(10:460),1);
Roll_Stiffness_R = Spring_Coefficients_R(1)

%
% Determine Roll Steer Coefficients
%

Roll_Steer_Coefficients = polyfit(Roll_Angle,Steer_Angle_Deg,1);
Rear_Roll_Steer = Roll_Steer_Coefficients(1)

%
% Compute the Linear Curve Fit for the Roll Camber
%

Rear_Roll_Camber_Coefficients = polyfit(Roll_Angle,Camber_Angle_Deg,1);
Rear_Roll_Camber = Rear_Roll_Camber_Coefficients(1)

%

```

```
% ***** BEGIN PLOTTING *****
%
% To Convert dimension of moment to 'kg-mm' in-lb*11.521 -> kg-mm
%
```

```
figure(1)
plot(Roll_Angle, Roll_Moment_R,'bo')
hold on
title('Rear Overall Roll Stiffness')
xlabel('Roll Angle (deg)')
ylabel('Roll Moment (in-lb)')
```

```
grid on
pause
```

```
figure(1)
plot(Roll_Angle, Roll_Moment_F,'ro')
hold on
title('Front Roll Stiffness')
xlabel('Roll Angle (deg)')
ylabel('Front Axle Roll Moment (in-lb)')
%plot(-phi,(Spring_Coefficients_F(2)+Spring_Coefficients_F(1)*phi),'r')
%text(4,3,'11904 lbf-in/deg')
grid on
pause
```

```
figure(2)
plot(Roll_Angle,Steer_Angle_Deg,'bo')
hold on
plot(Roll_Angle(1:350),Steer_Angle_Deg(1:350),'b-.')
title('Rear Roll Steer')
xlabel('Roll Angle (deg)')
ylabel('Steer Angle (deg)')
grid on
pause
```

```
figure(3)
plot(Roll_Angle(1:350),Steer_Angle_Deg(1:350),'bo')
hold on
title('Rear Roll Steer')
xlabel('Roll Angle (deg)')
ylabel('Steer Angle (deg)')
```



```

plot(phi(1:350),Roll_Steer_Coefficients(2)+Roll_Steer_Coefficients(1)*phi(1:350),'b')
grid on
pause

```

```

figure(4)
plot(Roll_Angle(1:350),Camber_Angle_Deg(1:350),'bo')
hold on
plot(Roll_Angle(1:350),Camber_Angle_Deg(1:350),'b-')
%plot(Roll_Angle,L_Camber_Angle_Deg,'go')
%plot(Roll_Angle,L_Camber_Angle_Deg,'g-')
title('Right Rear Roll Camber')
xlabel('Roll Angle (Deg)')
ylabel('Camber Angle (Deg)')
grid on
pause

```

```

figure(5)
plot(Roll_Angle(1:350),Camber_Angle_Deg(1:350),'bo')
hold on
%plot(Roll_Angle,L_Camber_Angle_Deg,'go')
plot(Roll_Angle(1:350),-Rear_Roll_Camber*Roll_Angle(1:350),'b')
%plot(Roll_Angle,L_Rear_Roll_Camber*Roll_Angle,'g')
title('Rear Roll Camber')
xlabel('Roll Angle (Deg)')
ylabel('Camber Angle (Deg)')
grid on
legend('b','Right')
pause

```

```

figure(6)
plot(Ay,INC,'o')
hold
plot(A_y,INC,'-')
title('SIDE-PULL ROLL')
xlabel('Simulated Lateral Acceleration (g)')
ylabel('Roll Angle (deg)')
pause

```

```

figure(7)
plot(Ay,WLF,'r',A_y,WRF,'b',Ay,WLR,'m',Ay,WRR,'g')
legend('Left Front','Right Front','Left Rear','Right Rear')
xlabel('Simulated Lateral Acceleration (g)')
ylabel('Vertical Tire Load (lbs)')
title('Lateral Load Transfer')

```

```
pause
```

```
figure(8)
```

```
hRC=Roll_Stiffness.*INC(90:800)./F1(90:800);
```

```
plot(INC(90:800),hRC,'o')
```

```
hold
```

```
plot(INC(90:800),hRC,'-')
```

```
hRCi=polyfit(INC(90:800),hRC,1);
```

```
hold on
```

```
plot(INC(90:800),hRCi(1)*INC(90:800)+hRCi(2),'b')
```

```
title('Instantaneous Roll Center')
```

```
ylabel('Roll Center Height (in)')
```

```
xlabel('Roll Angle (deg)')
```

```
grid
```