An Energy-Based Experimental-Analytical Torsional Fatigue Life-Prediction Method

THESIS

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By

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Abstract

An energy-based cycle-dependent fatigue life prediction framework for the calculation of torsional fatigue life and remaining life has been developed. The framework for this fatigue prediction method is developed in accordance with previously developed energy-based axial and bending fatigue life prediction approaches, which state: the total strain energy density accumulated during both a monotonic fracture event and cyclic processes is the same material property, where each can be determined by measuring the area beneath the monotonic true stress-strain curve and the area within a hysteresis loop, respectively. The energy-based fatigue life prediction framework is composed of the following entities: (1) the development of a shear fatigue testing procedure capable of assessing cyclic plastic strain energy density accumulation in a pure shear stress state and (2) the incorporation of an energy-based fatigue life calculation scheme to determine the remaining fatigue life of in-service gas turbine materials subjected to pure shear fatigue. Validation of the improved theory was attempted but failed due to undesired axial loading occurring during testing. Future work was proposed to address the issues.

This is dedicated to my family.

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Chapter 1: Introduction and Review

The middle years of the 20th century witnessed the birth of the jet age, which provided an invigorating challenge to aircraft designers who for decades had focused their efforts on perfecting propeller-driven propulsion. Although initially of relatively simple design, jet-engine technology quickly grew in complexity. Today, these devices are constructed of thousands of complex moving parts assembled to exacting tolerances. Mounted to an airframe, they are exposed to both rigorous operating conditions and long, repeated duty cycles. Despite these harsh operating conditions, they must operate flawlessly yet with minimal maintenance to reduce costs. It is the intersection of these competing constraints that provide challenges to the technical designer and drive research into the damage mechanisms that assail jet-engines throughout their operational lifetime. Although these mechanisms are many and varied, one of the most prevalent and costly is high cycle fatigue (HCF).

High cycle fatigue, described as the accumulation of damage during a cyclic process wherein the cyclic stresses remain below the yield strength of the fatigued material, is especially insidious for two reasons^[17]. First, classical elastic theory is unable to account for the effect of HCF on a structure. Second, because fatigue damage is accrued over time, very few visual clues tend to develop warning of impending HCF failure. While all

structures within a jet-engine are susceptible to fatigue damage, it is those operating in high-frequency environments - such as turbine blades in an engine hot section - that are especially vulnerable to HCF damage. Within this hot-section, turbine blades rotate at high frequencies and are subjected to hot, fluctuating temperatures and multi-axial mechanical loads. Because of this complicated loading, determining the fatigue lifetime of turbine blades is a challenging problem requiring robust solution methods.

Classical methods for determining the fatigue lifetime of structures such as turbine blades include stress-versus-cycle (SN) plots, Goodman diagrams, and modified Goodman diagrams^[1]. Unfortunately, these techniques generally lack the ability to account for the aforementioned loading complexities and are relatively expensive to develop. Thus, even prior to the development of the jet-engine the need for a better approach to the prediction of fatigue lifetimes was recognized. One historical approach to attaining a more stringent life prediction method exploits the phenomenon of strain hardening. As described by Hanstock,

"Strain hardening is a result of plastic or irreversible straining of the metal and is characterized by an increase in the stress necessary to produce a defined amount of plastic strain, i.e. as an increase in the limit of proportionality or proof stress^[2]."

Due to strain hardening, a portion of the work done on a structure in a cyclic process is stored as strain energy density, an effect evidenced by the existence of finite area between the loading and unloading stress-strain lines of a specimen cycled between two

stress levels ^[7]. As strain energy density is a scalar quantity - and as such permits no vectorial distinction between loading axes - it would seem an ideal parameter to use in estimating fatigue lifetime for complex loading conditions.

Some of the initial work on the relationship between strain energy density accumulation and fatigue lifetime was conducted by Inglis in 1927 ^[3]. In his work, Inglis employed low-carbon steel fatigue specimens subjected to rotating bending (RB) stresses to compile data on the total strain energy density to fracture. Figures 1 and 2 depict Inglis' results and a typical SN plot for an equivalent material, respectively.

Figure 1: Maximum Stress vs. Work Done to Fracture, RB Case w/ Low-Carbon Steel

Adapted from Inglis ^[3]

Figure 2: Typical SN Curve - Low Carbon Steel

Adapted from *Atlas of Fatigue Curves* ^[4]

A brief comparison of the two figures indicates that similarities exist between the trend of a typical SN plot and the tendency of the total strain energies to fracture. These similarities indicated a possible relationship between the number of cycles to failure and the total accumulated strain energy, promoting the possibility of using strain energy density as a tool in estimating cyclic failure. In 1947, Hanstock attempted to quantify this relationship by developing the model found in Equation 1 as a metric for predicting fatigue failure ^[2].

$$\sum_0^N \Delta E = C + DN \quad (1)$$

Though tantalizingly simple, experimental efforts to validate this approach conducted by contemporary peers, including Forrest and Tapsell, ultimately met with little success ^[5].

With the validation of Hanstock's model unsuccessful, development of energy-based fatigue life prediction theory stagnated for eight years. The next advance occurred in 1955, when Enomoto proposed that:

"...failure occurs under cyclic loading when the energy absorbed in each cycle in excess of a certain nondamaging amount accumulates to a critical total value ^[6]."

This hypothesis was important, as it introduced the concept of a critical value at fatigue failure. Unfortunately, Enomoto failed to ascertain the nature of this critical total value, a shortfall rectified by Feltner and Morrow in 1959.

Following Enomoto's work, Feltner and Morrow crafted the uniaxial logarithmic life-prediction model found in Equation 2 ^[7].

$$\log(\sigma_a) = K - \left(\frac{n}{1+n}\right) \log(N_f) \quad (2)$$

Of greater importance were the four major assumptions that the model was founded on:

⁽¹⁾ plastic strain energy density is a measure of fatigue damage; ⁽²⁾ a logarithmic plot of static true stress versus true plastic strain is valid when extrapolated back into the fatigue stress region; ⁽³⁾ the damaging energy per cycle for a given stress amplitude is constant and is equal to the area under the static stress-plastic strain curve; and ⁽⁴⁾ the total damaging energy required to cause fatigue fracture is constant and as a first approximation is equal to the area under the static true stress-true strain curve. Both the third and fourth assumptions are especially important to the further development of energy-based theories. The third assumption relates the damaging energy in a cyclic

process to the area within the hysteresis loops formed due to cyclic strain hardening, while the fourth assumption relates Enomoto's critical total value at failure to the area under the monotonic stress-strain curve. Theoretical results obtained by Feltner and Morrow compared to experimental data can be found in Figure 3. The experimental data was captured for SAE 4340 steel fatigue specimens subjected to uniaxial fatigue.

Figure 3: Alternating Stress vs. Cycles to Failure, Uniaxial Case w/ SAE 4340 Steel

Adapted from Feltner & Morrow ^[7]

Feltner and Morrow's work was followed by that of Stowell, who in 1966 constructed a new energy-based fatigue prediction model by retaining the first, third, and fourth assumptions made by Feltner and Morrow while abandoning the logarithmic approach in favor of strain energy density integrals describing the energy accumulated both cyclically

and in a monotonic process ^[8]. In essence, Stowell equated the strain energy density accumulated in a monotonic fracture process with the strain energy density accumulated cyclically during a fatigue process,

$$N * W_m = W_f, \quad (3)$$

thereby necessitating knowledge of both the strain energies of the monotonic fracture process and the cyclic fatigue process to estimate fatigue failure. Stowell achieved this by assuming that the behavior of most engineering materials subjected to uniaxial loading was described by the relationship

$$\varepsilon = \frac{\sigma}{E} + \epsilon_o \sinh \left(\frac{\sigma}{\sigma_o} \right), \quad (4)$$

with the value of the constant σ_o defined as

$$\sigma_o = \frac{\sigma_f - \sigma_y}{\log (\epsilon_f / 0.002)}. \quad (5)$$

With these relationships in place, Stowell solved for W_f first by applying the definition of strain energy density, or

$$W = \int_0^\epsilon \sigma d\epsilon = \sigma\epsilon - \int_0^\sigma \epsilon d\sigma, \quad (6)$$

and then by replacing the aforementioned ε - σ relation (Equation 4) for ε at monotonic fracture,

$$W_f = \sigma_f \left(\epsilon_f - \frac{\sigma_f}{2E} \right) - \sigma_o \epsilon_o \left(\cosh \frac{\sigma}{\sigma_o} - 1 \right). \quad (7)$$

W_m was solved for in a similar fashion, first beginning with

$$W_l = \sigma\epsilon - 2 \int_0^\sigma \epsilon d\sigma, \quad (8)$$

and then again replacing the ϵ - σ relation for ϵ . An additional step - using the arithmetic mean of the cyclic strain energy density to account for fluctuations in the real cyclic material response - was also employed, producing the equation

$$W_m = \frac{1}{2}W_l = \frac{1}{2}\sigma_o\epsilon_o \left[\frac{\sigma}{\sigma_o} \sinh \frac{\sigma}{\sigma_o} - 2 \left(\cosh \frac{\sigma}{\sigma_o} - 1 \right) \right]. \quad (9)$$

Having defined W_f and W_m , Stowell's model for the number of cycles to failure was formulated by replacing the derived energy relationships into the initial equality (Equation 3), producing Equation 10.

$$N = 2 \frac{\frac{\sigma_f \epsilon_f - (\sigma_f/2E)}{\epsilon_o} - \cosh \frac{\sigma_f}{\sigma_o} + 1}{\frac{\sigma}{\sigma_o} \sinh \frac{\sigma}{\sigma_o} - 2 \left(\cosh \frac{\sigma}{\sigma_o} - 1 \right)} \quad (10)$$

Results for this model can be found in Figure 4, presented as a comparison of Stowell's experimental data and the predicted cycles to failure. Experimental data was generated for SAE 4130 steel fatigue specimens subjected to uniaxial fatigue.

Figure 4: Strain Range vs. Cycles to Failure, Uniaxial Case w/ SAE 4130 Steel

Adapted from Stowell ^[8]

It is clear from the above figure that Stowell's use of strain energy density integrals - combined with Feltner and Morrow's first, third, and fourth assumptions - constitutes a powerful and reasonably accurate method of estimating uniaxial fatigue life.

With Stowell's work as a firm foundation, Scott-Emuakpor developed an improved energy-based failure criterion for determining the fatigue life of a structure subjected to uniaxial cyclic loading ^[9]. Scott-Emuakpor then extended Stowell's basic premise into uncharted territory, applying energy methods to the fatigue life prediction of structures subjected to diverse types of loading. Capabilities developed within this framework

included bending fatigue life prediction ^[10], uniaxial fatigue life prediction at various stress ratios ^[10], and transverse shear fatigue life prediction ^[11].

Of particular interest to this manuscript is the development of the fatigue life prediction model for transverse shear stress. Development of an energy-based fatigue life criterion for the transverse shear stress case was analogous to that of the uniaxial stress case established by Stowell and improved upon by Scott-Emuakpor ^[11]. In fact, the two models are nearly equivalent, with the few differences encapsulated in the characteristic parameters describing the material behavior. The material behavior for monotonic transverse shear stress was defined as

$$\gamma = \frac{\tau}{G} + \gamma_o \sinh \frac{\tau}{\tau_o}, \quad (11)$$

while the material behavior for cyclic transverse shear stress was defined as

$$\gamma_{cy} = \frac{\tau_{pp}}{G} + \frac{1}{c_s} \sinh \frac{\tau_{pp}}{\tau_c}. \quad (12)$$

The constant τ_o was approximated as

$$\tau_o = \frac{\tau_u - \tau_y}{\log(\gamma_u/0.002)}. \quad (13)$$

Fatigue life was determined in a consistent fashion by employing the same methodology established by Stowell for the uniaxial case and described above in Equations (3), (6)-(10). Thus, the number of cycles to failure for a structure subjected to cyclic transverse shear stresses was predicted as

$$N = C_s \frac{\tau_u \left(\gamma_u - \frac{\tau_u}{2G} \right) + \gamma_o \tau_o \left(\cosh \frac{\tau_u}{\tau_o} - 1 \right)}{2\tau_c \left[\frac{\tau}{\tau_c} \sinh \frac{2\tau}{\tau_c} - \left(\cosh \frac{2\tau}{\tau_c} - 1 \right) \right]}. \quad (14)$$

Results for this model compared to experimental data generated from Aluminum 6061-T6 fatigue specimens subjected to cyclic transverse shear stresses can be found in Figure 5. Primary results compare experimental data for a stress reversal ratio R of 0.1 with the model-based prediction, while the secondary results show the prediction for an R ratio of -1 (the fully-reversed case). Comparison with experimental data indicated the criterion provided reasonable estimates for transverse shear fatigue life.

Figure 5: Stress Amplitude vs. Cycles, Transverse Shear Case w/ AL 6061-T6

Adapted from Scott-Emuakpor^[11]

Indeed, validation for all of the models investigated by Scott-Emuakpor appeared to indicate that the energy method, in its current iteration, was thorough enough to initiate work on the larger goal attainable through energy-based fatigue prediction models.

Essentially, the transverse shear fatigue model - in addition to the models for uniaxial and bending fatigue - would serve as the building blocks of a truly multi-axial fatigue life prediction method through the superposition of loading axes. This model could then be applied to the finite element method (FEM), allowing for the prediction of fatigue life for complex structures under multi-axial mechanical loading. Thus, it would seem that the aim of modeling structures subjected to multi-axial stress states was nearly within reach.

However, the work of Scott-Emuakpor retained Feltner and Morrow's assertion that

"...the damaging energy per cycle for a given stress amplitude is constant... [7, 9-11] ."

This assumption is valid for a majority of cycles during an ideal fatigue process; nevertheless, actual in-service components experience measurable periods of material softening and hardening that affect the rate of strain energy density accumulation [12]. Scott-Emuakpor's model depends heavily on an accurate accounting of the plastic strain energy density accumulated during a cyclic process, thereby demanding incorporation of these cyclic hardening and softening characteristics in the fatigue life analysis.

Ozaltun was the first to quantify these effects for the uniaxial case. New investigations into the history of strain energy density accumulation experienced by a mechanically loaded structure both illustrated the expected variations in strain energy density accumulation rate and identified a drastic spike in the accumulation rate near the limit of the structure's fatigue life [12]. The strain energy density accumulation rate variations - in

addition to the rate spike near the limit of the fatigue lifetime - are visualized in Figure 6 for the uniaxial fatigue case.

Figure 6: Strain Energy Density Accumulation History, Uniaxial Case w/ AL 6061-T6
Adapted from Ozaltun^[12]

Besides simply accounting for the correct amount of plastic energy accumulation, it was suggested that this spike in strain energy density accumulation may itself indicate substantial changes occurring within the structure, serving as another metric of the progression structural damage.

It is through examination of the past work performed by Stowell, Scott-Emuakpor, and Ozaltun that the objectives of this research are identified. The first goal is to demonstrate

that the transverse shear analysis developed by Scott-Emuakpor is applicable to the torsion-based shear (torsional) stress state, thereby establishing it as a generalized shear life prediction model. The second goal is to examine the shear fatigue life prediction criterion with the intent of showing that the cyclic accumulation of plastic strain energy density for torsional fatigue follows a trend similar to that found in the uniaxial fatigue case. This reanalysis would support the development of advancements to the existing shear life prediction model, including both the ability to account for the changing rate of plastic strain energy accumulation over lifetime and the means to examine the effect of service on the remaining life of a structure subjected to a state of cyclic shear stress.

Chapter 2: Experimental Procedures

The stated objectives of this thesis are twofold; first, to show that the transverse shear life prediction method developed by Scott-Emuakpor is applicable to the torsional shear case; and second, to examine the plastic strain energy density accumulation trend for the shear fatigue case with the intent of using this analysis as a platform for enhancements to Scott-Emuakpor's energy-based shear fatigue criterion. Achievement of these goals required the development of three primary testing methods. First, the general fatigue response of the testing material had to be established for use as a benchmark in later verification. This material response was determined by pure torsion fatigue testing. Specimens used during this portion of testing were generally not instrumented, as the desired data was simply the correlation of operating stress to the number of cycles at failure for the construction of a classical SN curve. Next, it was necessary to develop a testing method to establish the frequency at which data would be collected. Denoted "frequency sweeps," this brand of testing investigated the evolution of the hysteresis loops formed during a cyclic process for increasing frequencies at constant fatigue amplitudes. Finally, a testing method was necessary to capture the changing plastic strain energy density accumulation rate over the lifetime of the fatigue specimens. The modified forward method tests (MFTs) were crafted to answer this need. These tests were separated into two unique tiers; the first tier involved a two-step procedure in which torsional fatigue was executed for a specified

number of cycles followed directly by monotonic torsion until failure, and the second tier was a continuous torsional fatigue process wherein data was captured at set intervals up to failure. It should be noted that the above testing methods are further discussed in Chapter 3.

Both sets of tests were performed using *ASTM E2207-08: Standard Practice for Strain-Controlled Axial-Torsional Fatigue Testing with Thin-Walled Tubular Specimens* as a guide ^[13]; however, it should be noted that load control was used in place of strain control. Load control was substituted for two reasons; first, load control was much simpler to implement than strain control; and second, previous energy-based predictive capabilities developed by Scott-Emuakpor were based on stress amplitude (load) control. Loss of fidelity due to the use of load control in place of strain control was assumed to be negligible.

The design of the specimen derived from the ASTM can be found in Figure 7. Each specimen was machined from the same batch of Aluminum 6061-T6 tube stock. Machining was performed with a computer numerically controlled (CNC) lathe. No surface treatment was applied, as the application of a delicate surface finish would have required a considerable addition to the total machining time for the volume of specimens desired.

Figure 7: ASTM Fatigue Specimen, [mm]

Loading values were calculated for the desired stress amplitude by the following equation:

$$T = \frac{\tau J}{r}, \quad (15)$$

where J , the polar moment of inertia, is given by the equation

$$J = \frac{\pi}{32}(d_o^4 - d_i^4), \quad (16)$$

and r , the radius at which the shear stress is evaluated, is given by

$$r = \frac{1}{4}(d_o + d_i) \quad (17)$$

such that r is evaluated at the mean radius of the gauge section. These equations were derived from the classical formulation of torsion in structural mechanics ^[14].

The experiments were conducted on an Instron model 1321 load frame equipped with MTS model 647 hydraulic grips, MTS model 647.02b 0.79-1.07 in. wedges, and an Instron model 6467-107 force transducer. The test setup is found in Figure 8.

Figure 8: Test Setup

Data was collected with multiple devices. VIC-3D, a visual strain gauge system produced by Correlated Solutions, obtained strain data during monotonic testing via the stereoscopic recording of the displacements of a paint-based speckle pattern applied to the surface of each specimen. The VIC-3D software then utilized these images - captured at a maximum of 19 frames per second - to calculate the strains on the surface of the specimen. Images were collected through an NI USB-6251 DAQ and processed on local hardware running the VIC-3D software suite. However, strain gauges were used to gather

data for the cyclic hysteresis loops, as the fidelity of the VIC-3D system for small displacements was poor. These Vishay CEA-13-062UV-350 gauges were placed in the center of the specimen gauge section and were connected to a Vishay model 2360 amplifier to condition the signal and provide the desired amplification of 1000μ strain per volt. As the unamplified signal was on the order of 10^{-3} , this amplification factor produced an output roughly on the order of magnitude 10^0 . Remaining data – such as cycle number, angle, and torque – were recorded by the MTS FlexTest data acquisition system. During fatigue testing, data was acquired two hundred times per cycle to produce a smooth hysteresis loop, regardless of the frequency of operation.

Testing conditions were controlled. Each test was performed at ambient temperature and humidity. Initial alignment of the Instron load frame was attempted utilizing a specimen gauged with two axial and two shear gauges separated circumferentially by 90 degrees. Results of this initial alignment test appeared to indicate the load frame was within the 5% alignment band permitted by *ASTM E606: Standard Practice for Strain Controlled Fatigue Testing*; however, it should be noted that this ASTM applies primarily to the uniaxial case, not the bi-axial loading case ^[18]. Thus, the procedure for determining alignment had to be modified to match bi-axial nature of the load frame.

Tests were set up following an established procedure in the effort to eliminate inconsistencies. This procedure detailed the gauging of each specimen, the placement of each specimen within the hydraulic grips, and the grip pressure utilized to secure each

specimen in place (1000 psi). Design and execution of the various tests was accomplished through Multi-Purpose Testware, a suite of MTS software coupled to the MTS controller. This software allowed the user to write reusable sets of commands to the MTS controller; thus, testing parameters remained consistent between tests of the same type. The proportional-integral-derivative (PID) values accessible through the MTS controller were optimized at the start of testing. Optimization entailed the tailoring of the PID values to reduce both the offset and time constant between the desired input signal and the actual machine input signal.

Chapter 3: Experimental Results and Discussion

3.1 Torsional Fatigue Test

Experimental results for torsional fatigue were developed in the form of an SN plot for validation purposes. Load control was utilized to maintain an R ratio of -1 to ensure consistency with later tests. Fatigue lifetimes were recorded at stress levels equivalent to 90%, 85%, 80%, and 70% of the shear yield strength, which was determined to be approximately 133MPa through preliminary torsional monotonic testing. Torsional fatigue failure was defined as the approximate cycle at which the maximum angle of rotation exceeded by 10% the averaged maximum or minimum angle derived from data well before failure. This definition of failure was chosen due to its use in the development of previous energy-based methods by Scott-Emuakpor, particularly for the transverse shear fatigue case ^[11]. A visual representation of this failure criterion can be found in Figure 9.

Figure 9: Torsional Fatigue Failure Criterion

The results of the torsional fatigue testing (Figure 10) show the general trend expected of an SN plot; however, note the cycles-to-failure evaluated at 85% of the shear yield stress do not conform to expectations. Data at 85% of the shear yield stress was taken several months prior to that at the other stress levels, and it is likely some inconsistency exists between the two sets of data. Despite this issue, the scatter of the data appears to be reasonable in relation to a "normal" SN plot; unfortunately, a torsional fatigue SN plot could not be found in existing literature to supply a direct comparison. Regardless, the results of this torsional fatigue testing - summarized by the SN plot - were deemed to be readily applicable to validation purposes.

Figure 10: SN Curve for AL 6061 T6 (R=-1)

3.2 Cyclic Stress-Strain Behavior

The basic tenant of energy methods applied to the task of estimating fatigue failure states that fatigue damage is the result of the accumulation of plastic strain energy density during a cyclic process. However, during HCF only a small portion of the applied mechanical loading results in actual fatigue damage. This may not be readily apparent in the examination of the total strain energy extracted from experimental stress-strain hysteresis loops, which generally tend to be a composite of both damaging (plastic) and anelastic effects. Anelastic effects - described by Feltner and Morrow to include magneto-elastic coupling, atomic diffusion, and other effects - are non-damaging mechanisms playing a negligible role in the mechanical damage scrutinized by energy-

based theories ^[7]. As such, the presence of these anelastic effects must be minimized to accurately capture only the cyclic plastic damage.

Previous work by Ozaltun on the uniaxial case exposed the relationship between total strain energy density derived from a hysteresis loop and cyclic frequency, showing that the total strain energy density increased monotonically with cyclic frequency ^[12]. Figure 11 depicts several experimental hysteresis loops generated for the same uniaxial stress amplitude at frequencies ranging from 1-50Hz.

Figure 11: Hysteresis Loop with Respect to Frequency, Uniaxial Fatigue Case

Adapted from Ozaltun ^[12]

From the results found in Figure 11, Ozaltun concluded that internal friction and damping in the test specimens became dominant at higher cyclic frequencies, and that those anelastic effects were the cause of the hysteresis loops becoming significantly larger as the cyclic frequency increased ^[12].

It was expected that a similar trend would be found for the relationship between the total strain energy density per cycle and the cyclic frequency in the torsional shear case. To that end, the frequency sweep method discussed in Chapter 2 was employed to determine the frequency at which the area within the fatigue hysteresis loops was minimized, thereby forcing said area to correspond more closely to the damaging plastic strain energy density. Hysteresis loops were acquired using an alternating torsional shear stress of 106MPa and operating frequencies ranging from 0.01-5Hz. Results from these frequency sweeps are displayed numerically in Table 1 and graphically in Figure 12. Strain was captured using the Vishay shear strain gauges discussed in Chapter 2, and testing was performed across increasing oscillation frequencies.

Frequency	Average Strain Energy Density
[Hz]	[J/m ³]
0.01	5875
0.05	4852
0.10	4572
0.25	3625
0.50	2307
1.00	543
2.50	8881
5.00	22542

Table 1: Average Strain Energy Density vs. Frequency

Figure 12: Average Strain Energy Density Accumulation vs. Frequency

Figure 12 demonstrates a significant reduction in the plastic strain energy density as the frequency increases from 0.01Hz to 1Hz, followed by an increase in plastic strain energy density from 1Hz to 5Hz. The source of increase in cyclic strain energy density with increasing frequency from 1Hz to 5Hz can be explained by anelastic effects; however, the cause for the initial decrease in cyclic strain energy density for increasing frequency between 0.01Hz and 1Hz is unknown. Fortunately, this peculiar trend simplifies the determination of the frequency at which the minimum cyclic strain energy density accumulation occurs, and it is quickly seen that this minimum appears at or around 1Hz. At this frequency, it must be inferred that most – if not all – of the strain energy density is due purely to plastic sources. Based on the findings of Figure 12, data acquisition during

the second tier of the modified forward testing was performed at an operating frequency of 1Hz.

It should be noted that an operating frequency of 1Hz represents the ideal operating case for this material and these testing conditions. Operation above this ideal frequency will result in the addition of anelastic effects to the energy accumulation, while operation below this ideal frequency will result in the addition of "other" effects to the energy accumulation. As it is assumed that only plastic strain energy density accumulation contributes to mechanical damage, the addition of these anelastic/"other" effects would not cause the part to fail at a different rate. This distinction confers two benefits to this method. First, although this energy-method is developed for the ideal frequency, it can account for a structure fatigued at any frequency. Second, although data must be acquired at 1Hz to minimize hysteretic non-damaging effects, other portions of testing during which data is not acquired can be accelerated to reduce overall testing time.

3.3 Failure Energy of In-Service Structures

In general, fatigue prediction techniques only estimate the cycles to failure of pristine structures; however, knowledge of the ability of an in-service structure to sustain fatigue loading after an arbitrary number of cycles would be valuable. Ozaltun showed that an investigation of the remaining lifetime of a structure subjected to uniaxial fatigue could be established by examining the progression of that structure's monotonic stress-strain

curves across increasing amounts of fatigue loading ^[12]. Application of this approach to torsional fatigue required the design of the first tier of the two-tiered testing procedure described in Chapter 2, wherein cyclic loading was applied to the fatigue specimen for a prescribed number of cycles followed by a monotonic torsion process at low strain rate until failure.

For the purposes of this testing, monotonic torsional failure was defined as the true shear strain at which the shear stress attained a maximum value. This point represented the ultimate shear stress of the fatigue specimen; thus, it both characterized the material and was simple to identify in the stress-strain curve.

Several tests were conducted with this procedure. Each test was designed to accumulate the same strain energy density as a fully-reversible fatigue test at an alternating stress of 106MPa. As specimens fatigued at this shear stress generally experienced failure at 22k cycles, the first-tier modified forward method was conducted by first performing a cyclic operation on specimens up to a specified cycle count; specifically 6k, 18k, and 21k cycles. Each cyclic operation was immediately followed by a monotonic torsion process to failure using displacement control with a user-specified strain rate of 1.372E-4/s. The results of these tests are compiled in Figure 13.

Figure 13: Comparison of Tier 1 MFTs

The progression of stress-strain curves found in Figure 13 indicates that the accumulation rate of plastic strain energy density remains slow until in the vicinity of fatigue failure. This result can be directly correlated to the toughness of the material, as toughness is defined as the area under the stress-strain curve. Thus, it can be said that the toughness of the material degrades slowly with increasing cyclic damage. It will be seen that this trend will be repeated in the examination of the energy accumulation history of the continuous fatigue process, as discussed in Section 3.4.

3.4 Energy Accumulation History

The energy-based fatigue models developed by Scott-Emuakpor assumed the accumulation of plastic strain energy density to be a constant cyclic process ^[9-11]. However, the development of internal micro-cracks near the point of structural failure challenges the veracity of this assumption ^[12]. Thus, the second tier of the modified forward method was designed to investigate the variation in plastic strain energy density accumulation over the duration of the torsional fatigue operation.

During testing, loading was applied to produce a fully-reversible alternating torsional shear stress of 106MPa for consistency with previous tests. Specimens were cycled at 1Hz during data acquisition; otherwise, all other portions of testing used a cyclic frequency of 5Hz to reduce total testing time. Shear fatigue failure was defined previously in Section 3.1. Because of the low number of cycles to failure, strain gauges survived throughout testing with minimal zero-drift, thereby allowing for data to be recorded continuously directly to failure.

Results of the second tier modified forward method can be found in Figures 14 and 15. Figure 14 displays the plastic strain energy density accumulation from pristine condition to a spike in the accumulation rate, while Figure 15 depicts the plastic strain energy density accumulation from pristine condition to failure.

Figure 14: Plastic Strain Energy Density Accumulation, 0 to Pre-Spike, 106MPa

Figure 15: Plastic Strain Energy Density Accumulation, 0 to Failure, 106MPa

Several features of the plastic strain energy density accumulation should be noted from Figures 14 and 15. First, the constant nature of the accumulation rate during the first 3k cycles is an artifact of the lack of data during this portion of testing. Thus, the behavior of the material during this portion of its lifetime is unknown. Second, both Figures 13 and 14 show softening of the material from 3k cycles to 12k cycles, followed by hardening of the material between 12k cycles and 21k cycles. This slow hardening of the material accelerates near failure, transitioning directly to a large spike in the plastic strain energy density accumulation rate at approximately 22k cycles. This rapid, intense material softening continues until fatigue failure. The spike in plastic strain energy density accumulation may denote the existence of swiftly growing internal micro-cracks as of yet not visible on the surface of the specimen.

The total plastic strain energy density accumulated by the specimen in Figures 14 and 15 is represented by the area under each curve. These energies are tabulated below in Table 2.

Test Type	Failure Energy
[Type]	[J/m ³]
MFT, Tier 2 0 Cycles to Pre-Spike	14827124
MFT, Tier 2 0 Cycles to Failure	14864428

Table 2: Strain Energy Density Accumulation Totals

Comparison of these energies indicates a portion of the total energy was accrued in the final few cycles of the specimen's lifetime; however, this comparison is highly dependent

on the definition of the point of fatigue failure as discussed below in Section 3.5. Additionally, by comparing the plastic strain energy density of the continuous run specimen loaded at 106MPa from 6k cycles to the accumulation rate spike (Figure 16) with the plastic strain energy density accumulated by a similar specimen fatigued for 6k cycles at 106MPa and then monotonically sheared to failure (Figure 17), it can be shown that a sizeable amount of plastic strain energy density is unaccounted for between these two equivalent processes. Table 3 tabulates the cumulative strain energies of the two processes.

Figure 16: Plastic Strain Energy Density Accumulation, 6k to Pre-Spike, 106MPa

Figure 17: Plastic Strain Energy Density Accumulation, 6k Fatigue/Monotonic

Test Type	Failure Energy
[Type]	[J/m ³]
MFT, Tier 2 6k Cycles to Pre-Spike Fatigue/106MPa	11319581
MFT, Tier 1 6k Fatigue (106MPa)/Monotonic Torsion	14071725

Table 3: Failure Energy Comparison

The amount of plastic strain energy density accumulated from 6k cycles to the accumulation rate spike represents roughly 80% of the monotonic strain energy density from 6k cycles to monotonic failure. Past the accumulation rate spike of the fatigue process, it is assumed that substantial internal changes have begun that will shortly lead to failure. Thus, when the sum of the accrued plastic strain energy density equals 80% of the total monotonic strain energy density, the in-service part should be replaced.

3.5 Experimental Challenges

Successful completion of this research faced two significant hurdles. The first stemmed from the definition of failure as stated in Section 3.1. Figure 18 shows an image of two specimens post fatigue failure.

Figure 18: Failed Shear Fatigue Specimens

Note that these specimens exhibit primarily vertical cracking, a feature common to all of the specimens tested. Because of the orientation of the crack with respect to loading, each specimen was generally able to sustain modest loading even after the surface crack had formed. Indeed, catastrophic failure of the specimen - defined as the separation of the specimen into two distinct pieces - would require considerable cycling beyond the

defined point of fatigue failure. The discrepancy between the defined point of failure and the actual catastrophic failure translates to a large amount of additional cyclic strain energy accumulation that could be considered. Thus, the definition of fatigue failure has a direct impact on the accounting of the cyclic strain energy accumulation.

The second hurdle was the lack of a device to ensure the alignment of the load frame. During testing, most specimens consistently exhibited cracking at the same location. Statistically, the random orientation of each sample within the wedge grips argues against any sort of failure consistency. Misalignment of the load frame would explain this regularity, as each specimen would experience undesired axial loading producing the preferential cracking location. With this axial load in place - and possibly slightly changing in magnitude as samples were replaced - the lifetime of both the fatigue and monotonic specimens may have been affected due to multi-axial loading.

This theory is further supported by crack propagation data from research conducted on fatigue specimens subjected to a 3-to-2 shear-to-axial biaxial stress state and a pure-shear stress state, depicted in Figures 19A and 19B respectively. ^[15].

Figure 19: Fatigued Dog-Bone Specimens

Adapted from Wu and Yang ^[15]

Marked similarities are observable when Figures 18 and 19A are compared. This supports the idea that axial loading was present during testing. As the cyclic axial load was held to zero during each test using the control software, the only source of axial load would be that generated by a misalignment of the load frame.

Chapter 4: An Energy-Based Cycle-Dependent Fatigue Criterion

4.1 Theoretical Process

Each of the elements detailed in the preceding chapter played important roles in the development of the modified energy-based shear fatigue life prediction model. The process of enhancing the model is based on that proposed by Ozaltun for the uniaxial case, wherein the equation estimating the number of cycles to failure was amended by a representative constant stemming from the analysis of the energy accumulation history^[12]. This constant is intended to capture the acceleration of the plastic strain energy density accumulation rate near the point of failure due to the substantial internal structural changes assumed to be occurring, essentially incorporating a safety factor directly into the estimation of fatigue life. Because the constant is developed from the energy accumulation history, the modified fatigue life prediction model may be described as being "cycle dependent."

A stronger method of incorporating this cycle dependency would be to model the energy accumulation history from empirical data; however, this method requires considerably more data than is presently available and thus represents an opportunity for future expansion of the work in this manuscript.

4.2 Development of a Cycle-Dependent Fatigue Criterion

With the results of Section 3.4, the original shear fatigue life prediction model (Equation 14) is modified to determine the critical number of in-service cycles. As it was shown in Section 3.4 that the energy accumulated during second-tier modified forward method testing represented 80% of the total monotonic strain energy density, the applied constant will provide the basis for the improvements to the model. To achieve this modification, Equations 11, 12, and 13 remain unchanged, while Equation 14 becomes Equation 18 as shown below.

$$N = 0.8C_s \frac{\tau_u \left(\gamma_u - \frac{\tau_u}{2G} \right) + \gamma_o \tau_o \left(\cosh \frac{\tau_u}{\tau_o} - 1 \right)}{2\tau_c \left[\frac{\tau}{\tau_c} \sinh \frac{2\tau}{\tau_c} - \left(\cosh \frac{2\tau}{\tau_c} - 1 \right) \right]} \quad (18)$$

4.3 Model Verification

Two components are necessary to verify the model set forth in Equation 18. First, the amount of plastic strain energy density accumulated cyclically must be extracted from a representative hysteresis loop for the denominator of Equation 18. This representative hysteresis loop was derived from results generated during the frequency sweep tests and can be found in Figure 20. The polarity of this plot is the result of the process which produced the fatigue data: as the program was load-controlled, the generating sine function first approached the minimum operating torque and then continued to the

maximum operating torque. Data for the chosen hysteresis loop is presented below in Table 4.

Figure 20: Representative Hysteresis Loop, 106MPa

τ_{pp}	γ_{pp}
[Pa]	[m/m * 10^{-3}]
0	0
681	2.45
2882	2.80
6277	3.34
10428	4.00
15189	4.75
19711	5.46
23817	6.11
26845	6.59
28684	6.89
29187	6.96

Table 4: Representative Hysteresis Loop Data

Data presented in Table 4 has been adjusted to range from zero to the peak-to-peak value. This step is necessary in the calculation of curve-fit constants used by Equation 18, as it allows for a single simple equation (Equation 12) to model the experimentally derived cyclic stress-strain data.

The second necessary component is the total strain energy density accumulated in a monotonic torsion test, which comprises the numerator of Equation 18. Curve-fit constants essential to Equation 18 were developed from consideration of Equation 11, which utilizes those same constants to represent the experimental monotonic torsion test data. Characteristic material properties (such as the ultimate shear stress, shear yield stress, ultimate shear strain, and shear modulus) were also determined from the monotonic torsion data. The curve-fit parameters and characteristic material properties generated from these two data sets can be found below in Table 5. Additionally, the resulting life prediction for fully-reversed torsional fatigue over a range of torsional stress amplitudes can be seen in Figure 21, where it is presented with respect to the experimental SN curve detailed in Section 3.1.

C_s	29.41
τ_f [MPa]	191.9
γ_f [m/m]	0.1138
G [GPa]	23.48
τ_o [MPa]	10.09
γ_o [m/m * 10^{-9}]	1.17
τ_c [MPa]	786

Table 5: Curve-fit parameters for Equation 18

Figure 21: S-N Curve (R=-1) Experimental vs. Theoretical

Several issues with the validation are readily apparent. While the general trend of the theoretical lifetime estimate appears correct, their actual number of cycles to failure for a given stress level does not correspond to the experimental cycles to failure for that same stress level.

The source of the discrepancy between the theoretical and experimental results is attributable to the experimental challenges discussed in Section 3.5. Both the localized region of crack initiation with respect to the orientation of the specimen in the load frame and the similarity in crack propagation path to that observed in the 3-to-2 shear-to-axial fatigue load path of Figure 19A indicate significant axial loads were being applied to each specimen in addition to the prescribed shear load. Thus, the state of stress acting on

each specimen resembled that depicted below in Figure 22, where the actual state of stress has been made multi-axial due to the addition of uniaxial misalignment stress.

Figure 22: Comparison of Expected/Actual Stress State

Adapted from Scott-Emuakpor ^[16]

Scott-Emuakpor verified the presence of this multi-axial loading by comparing the experimental fatigue data with a new multi-axial life prediction model based on an assumed user-specified 3-to-2 torsional-to-axial stress input ^[16]. The results of this comparison can be found below in Figure 23.

Figure 23: Experimental Results vs. Multi-axial Life Prediction Model

Adapted from Scott-Emuakpor ^[16]

Figure 23 indicates that application of the suspected axial loading to the prediction method results in a strong correlation between the prediction and the experimental results. Thus, the poor correlation between the un-modified theoretical prediction and the experimental results can be summarized by the following: each step used in the procedure for developing the energy-based shear fatigue life-prediction model required pure shear loading during experimentation. This requirement impacts not only the user-specified shear stress amplitude τ , but also the curve fit components fundamental to the energy-based method. Therefore, the life-prediction results suffered because the method was designed only to measure the fatigue life of a specimen experiencing a pure shear stress

state and was unable to account for the energy accumulated due to the undesired axial loading.

It should be noted that the theoretical results erroneously under-predict the fatigue lifetime compared to the experimental results: that is, the application of undesired axial loading should cause the experimental lifetimes to be *shorter* than the predicted lifetimes, not longer. However, this can be accounted for by the fact that the presence of axial misalignment would have caused the monotonic torsion tests to fail sooner than if the misalignment had not been present. This translates into a reduction in the plastic strain energy density accumulated during the monotonic torsion process. Because this monotonic plastic strain energy density accumulation formed the numerator of the fatigue life prediction model, the total number of cycles to failure predicted by the shear life prediction method was lower than the experimental results.

Chapter 5: Conclusion

Scott-Emuakpor's transverse shear fatigue prediction method was shown to be applicable to torsional shear fatigue life prediction. A technique was developed to account for the plastic strain energy density accumulation of a cyclic process at an operating frequency of 1Hz. This development allowed for the extension of the ideas put forth by Ozaltun in the uniaxial case, which stated that the plastic strain energy density accumulation rate should change over the lifetime of the specimen, especially near the end of the specimen's lifetime ^[12]. Using the energy accumulation as a guide, improvements were made to the energy-based shear life prediction model for the consideration of the remaining life of in-service parts. The model was shown to have the correct trend but incorrect magnitudes caused by an unwanted axial load present in the experimental data.

Future work has been proposed to resolve the issues arising from the undesired axial loading. First, the acquisition of an alignment fixture for the tension-torsion load frame would guarantee alignment, removing any axial loading component that could modify the data set. Second, the development of a well-constructed alignment specimen would both ensure consistency between testing and provide warning as to when recalibration of the alignment fixture was necessary.

Despite the issues encountered in validating the modified shear fatigue life prediction method, energy methods remain a powerful tool for the estimation of fatigue life. Indeed, re-evaluation of the shear method under the improvements outlined for future work would increase its predictive accuracy and move strain energy density-based fatigue life prediction methods closer to solving truly multi-axial fatigue problems.

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Appendix A: Nomenclature

C = curve fit constant

C_s = MATLAB curve fit parameter

d_o = outer diameter of gauge section

d_i = inner diameter of gauge section

D = curve fit constant

ΔE = hysteretic energy

G = shear modulus

J = polar moment of inertia

K = intercept at one cycle

n = slope of the logarithmic stress-plastic strain curve

N = number of cycles to failure

N_f = number of cycles to failure

N_c = number of cycles to erosion of 80% of material toughness

r = mean radius of gauge section

R = stress reversal ratio, minimum stress to maximum stress

T = torque

W_f = strain energy density accumulated in a monotonic fracture process

W_m = strain energy density accumulated in a cyclic process

Greek Letters

γ = engineering shear strain

γ_u = ultimate engineering shear strain

γ_{cy} = cyclic shear strain

γ_o = curve fit constant

σ_a = peak stress

τ = shear stress at mean radius of gauge section

τ_u = ultimate shear stress

τ_y = 0.2% yielding shear stress

τ_{pp} = twice the alternating shear stress

τ_o = constant defined by Equation 2

τ_c = MATLAB curve fit parameter_[7]

Appendix B: Matlab Code

The following code represents the various programs utilized in the data reduction phase of the work in this thesis. Data accumulated by testing was output as a text file, which was directly imported into MATLAB using a user-designed read function.

B.1: Frequency Sweep

```
%%%%%%%%PROGRAM TO PROCESS DATA%%%%%%%%
%%%%%%%%FATIGUE%%%%%%%%

%%%%%%%%%-USER SPECIFIED-%%%%%%%%%
do_gage = 0.65*25.4; % mm
di_gage = 0.53*25.4; % mm
% The outer and inner diameters of the gage section. These will
% be used to calculate the polar moment of area (J0) and the
% average radius (r0).
%%%%%%%%%-END-%%%%%%%%%

Jo = pi/32*(do_gage^4-di_gage^4);
ro = (do_gage+di_gage)/4;

% Call function and read data from file including headers
[header,loop_data,j] = read_data('input/Frequency_Sweep_2_80.dat');
matrix_size           = size(loop_data);
number_of_cycles      = matrix_size(3);

% Retrieving cycle numbers
n = zeros(j-1,1);
n(1,1) = str2num(header(4,13,1));
for i = 2:j-1
    if i <= 9
        n(i) = str2num(header(2,13,i));
    end
    if i > 9 & i <= 99
        n(i) =
str2num(strcat(header(2,13,i),header(2,14,i),header(2,15,i)));
    end
    if i > 99 & i <= 999
        n(i) =
str2num(strcat(header(2,13,i),header(2,14,i),header(2,15,i),header(2,16
,i)));
    end
end
```

```

        if i > 999
            n(i) =
str2num(strcat(header(2,13,i),header(2,14,i),header(2,15,i),header(2,16
,i),header(2,17,i)));
        end
    end
    % Retrieving Frequencies
    m = zeros(j-1,1);
    m(1,1) =
str2num(strcat(header(3,22,1),header(3,23,1),header(3,24,1),header(3,25
,1)));
    for i = 2:j-1
        if i <= 10
            m(i) =
str2num(strcat(header(1,22,i),header(1,23,i),header(1,24,i),header(1,25
,i)));
        end
        if i > 10 & i <= 36
            m(i) =
str2num(strcat(header(1,22,i),header(1,23,i),header(1,24,i)));
        end
        if i > 36 & i <= 62
            m(i) =
str2num(strcat(header(1,22,i),header(1,23,i),header(1,24,i),header(1,25
,i)));
        end
        if i > 62 & i <= 113
            m(i) =
str2num(strcat(header(1,22,i),header(1,23,i),header(1,24,i)));
        end
        if i > 113 & i <= 164
            m(i) = str2num(header(1,22,i));
        end
        if i > 164 & i <= 240
            m(i) =
str2num(strcat(header(1,22,i),header(1,23,i),header(1,24,i)));
        end
        if i > 240
            m(i) = str2num(header(1,22,i));
        end
    end
end

% Setting error count
errors = 0;

for cycle_number = 1:number_of_cycles
    angle          = loop_data(:, 3, cycle_number);
    torque          = loop_data(:, 4, cycle_number);
    gam_strain      = loop_data(:, 5, cycle_number)/1000;
    eps_strain      = gam_strain/2;
    % gam_strain is the engineering shear strain, where gamma =
2*epsilon
    % eps_strain is the shear strain, where epsilon = (1/2)*gamma

```



```

% The strain gage reads gamma; however, we need shear strain
% to calculate strain energy

% Removing zeros in data file
[angle,torque,gam_strain,eps_strain] =
zero_check(angle,torque,gam_strain,eps_strain);

% Checking for loop completion/completing the loop if necessary
data_count = length(torque);
if abs(torque(data_count)) < abs(0.95*torque(1)) |
abs(torque(data_count)) > abs(1.05*torque(1))
    fprintf('\nThe values are in error for cycle count
%i\n',n(cycle_number));
    n(cycle_number) = 0;
    m(cycle_number) = 0;
    errors = errors+1;
elseif abs(eps_strain(data_count)) < abs(0.95*eps_strain(1)) |
abs(eps_strain(data_count)) > abs(1.05*eps_strain(1))
    fprintf('\nThe values are in error for cycle count
%i\n',n(cycle_number));
    n(cycle_number) = 0;
    m(cycle_number) = 0;
    errors = errors+1;
else
    % Setting the last data-point equal to the initial data-point
to
    % close the curve
    torque(data_count+1) = torque(1);
    eps_strain(data_count+1) = eps_strain(1);
    shear_stress = torque*ro/Jo*1000^2;

    % Plotting hysteresis loop and energy vs. cycle
    cycnum = num2str(n(cycle_number,1));
    freqnum = num2str(m(cycle_number,1));
    figure(1)
    plot(eps_strain,shear_stress,'+-
','Color','BLACK','LineWidth',1); grid on;
    energy(cycle_number) = polyarea(shear_stress,eps_strain);
    energy = energy';
    energy2 = num2str(energy(cycle_number));
    f = ['Stress - Strain Cycle      ',cycnum,'/',freqnum,'
Energy: ',energy2]
    title([f],'FontName','Tahoma','FontSize',12)
    xlabel('\epsilon [m/m]','FontName','Tahoma','FontSize',12);
    ylabel('\tau [Pa]','FontName','Tahoma','FontSize',12);
    filename = cycnum;
    filepath = 'output\';
    SSC = [filepath,filename, '.jpeg'];
    saveas(gcf,SSC, 'jpeg')
    pause
end
end

```

```

% Processing only valid cycles
for i = 1:number_of_cycles-errors
    if n(i) == 0
        n(i) = [];
    end
    if m(i) == 0
        m(i) = [];
    end
    if energy(i) == 0
        energy(i) = [];
    end
end

energy3(:,1) = m;
energy3(:,2) = energy;

z1 = 0;
z2 = 0;
z3 = 0;
z4 = 0;
z5 = 0;
z6 = 0;
z7 = 0;
for i = 1:length(m)
    if energy3(i,1) == 0.010
        freq1(i,1) = energy3(i,2);
    end
    if energy3(i,1) == 0.050
        freq2(i-z1,1) = energy3(i,2);
    else
        z1 = z1+1;
    end
    if energy3(i,1) == 0.100
        freq3(i-z2,1) = energy3(i,2);
    else
        z2 = z2+1;
    end
    if energy3(i,1) == 0.250
        freq4(i-z3,1) = energy3(i,2);
    else
        z3 = z3+1;
    end
    if energy3(i,1) == 0.500
        freq5(i-z4,1) = energy3(i,2);
    else
        z4 = z4+1;
    end
    if energy3(i,1) == 1.000
        freq6(i-z5,1) = energy3(i,2);
    else
        z5 = z5+1;
    end
    if energy3(i,1) == 2.500

```

```

        freq7(i-z6,1) = energy3(i,2);
    else
        z6 = z6+1;
    end
    if energy3(i,1) == 5.000
        freq8(i-z7,1) = energy3(i,2);
    else
        z7 = z7+1;
    end
end

for i = 1:length(freq1)-1
    freq1_mod(i,1) = freq1(i);
end
clear freq1;
freq1 = freq1_mod;
for i = 1:length(freq2)-1
    freq2_mod(i,1) = freq2(i);
end
clear freq2;
freq2 = freq2_mod;
for i = 1:length(freq3)-1
    freq3_mod(i,1) = freq3(i);
end
clear freq3;
freq3 = freq3_mod;
for i = 1:length(freq4)-1
    freq4_mod(i,1) = freq4(i);
end
clear freq4;
freq4 = freq4_mod;
for i = 1:length(freq5)-1
    freq5_mod(i,1) = freq5(i);
end
clear freq5;
freq5 = freq5_mod;
for i = 1:length(freq6)-1
    freq6_mod(i,1) = freq6(i);
end
clear freq6;
freq6 = freq6_mod;
for i = 1:length(freq7)-1
    freq7_mod(i,1) = freq7(i);
end
clear freq7;
freq7 = freq7_mod;
for i = 1:length(freq8)-1
    freq8_mod(i,1) = freq8(i);
end
clear freq8;
freq8 = freq8_mod;

freq1_mean = mean(freq1);

```

```

freq2_mean = mean(freq2);
freq3_mean = mean(freq3);
freq4_mean = mean(freq4);
freq5_mean = mean(freq5);
freq6_mean = mean(freq6);
freq7_mean = mean(freq7);
freq8_mean = mean(freq8);

energy_mean =
[freq1_mean;freq2_mean;freq3_mean;freq4_mean;freq5_mean;freq6_mean;freq
7_mean;freq8_mean];
m_mean = [0.01;0.05;0.1;0.25;0.5;1;2.5;5];

% Plotting loop energy vs. freq
figure(2)
plot(m_mean,energy_mean,'o','Color','BLACK','LineWidth',1); grid on;
hold on;
%title('Loop Energy vs. Freq','FontName','Tahoma','FontSize',12)
xlabel('Freq [Hz]','FontName','Tahoma','FontSize',12);
ylabel('Loop Energy [J/m^3]','FontName','Tahoma','FontSize',12);
filename = 'Energy vs. Frequency';
filepath = 'output\';
SSC = [filepath,filename, '.jpeg'];
% saveas(gcf,SSC, 'jpeg')

%Displaying % unusable
unusable_cycles = errors;
percent_unusable = errors/number_of_cycles*100;
disp(sprintf('\n%i percent of the cycles were deemed
unusable.',percent_unusable))
disp(blanks(2))

```

B.2 Remaining Lifetime

B.2.1 Monotonic

```

%%%%%%%%PROGRAM TO PROCESS DATA%%%%%%%%
%%%%%%%%          MONOTONIC          %%%%%%%%%

% Call function and read data from file including headers
[header,run_data] =
read_data('input/Remaining_Lifetime_Fatigue_1_6k_MP.dat');

eps_strain      = run_data(:,1);
shear_stress    = run_data(:,2);

% Decide on failure point
[a,b] = max(shear_stress);

```

```

for i = 1:b
    eps_strain_mod(i,1) = eps_strain(i,1);
    shear_stress_mod(i,1) = shear_stress(i,1);
end

% Determine axis bounds
c = max(eps_strain);

% Output failure point
fprintf('Ultimate failure point occurs at %g.\n',a)

% Plot of stress-strain curve
figure(1)
plot(eps_strain_mod,shear_stress_mod/1E6,'-
','Color','BLACK','LineWidth',2); grid on;
% Total monotonic strain energy
energy = trapz(eps_strain,shear_stress);
f = ['Stress - Strain']%, '          Total Energy:
',num2str(round(energy))];
%title(f,'FontName','Tahoma','FontSize',12)
axis tight
xlabel('Strain [m/m]' , 'FontName','TimesNewRoman','FontSize',12);
ylabel('Stress [MPa]' , 'FontName','TimesNewRoman','FontSize',12);
filename = 'Stress-Strain';
filepath = 'output\';
SSC = [filepath, filename, '.jpeg'];
saveas(gcf,SSC, 'jpeg')

fprintf('Total energy to failure: %g\n',energy)

```

B.2.2 Fatigue

```

%%%%%%%%PROGRAM TO PROCESS DATA%%%%%%%%
%%%%%%%%FATIGUE%%%%%%%%

%%%%%%%%%-USER SPECIFIED-%%%%%%%%%
do_gage = 0.65*25.4; % mm
di_gage = 0.53*25.4; % mm
% The outer and inner diameters of the gage section. These will
% be used to calculate the polar moment of area (J0) and the
% average radius (r0).
%%%%%%%%%-END-%%%%%%%%%

Jo = pi/32*(do_gage^4-di_gage^4);
ro = (do_gage+di_gage)/4;

% Call function and read data from file including headers
[header,loop_data,j] =
read_data('input/Remaining_Lifetime_Fatigue_1_6k_FP.dat');

```

```

matrix_size          = size(loop_data);
number_of_cycles     = matrix_size(3);

% Retrieving cycle numbers
n = zeros(j-1,1);
n(1,1) =
str2double(strcat(header(4,13,1),header(4,14,1),header(4,15,1),header(4
,16,1)));
for i = 2:j-1
    n(i) =
str2double(strcat(header(2,13,i),header(2,14,i),header(2,15,i),header(2
,16,i)));
end

% Setting energy matrix size
energy = zeros(j-1,1);

for cycle_number = 1:number_of_cycles
    angle          = loop_data(:, 3, cycle_number);
    torque         = loop_data(:, 4, cycle_number);
    gam_strain     = loop_data(:, 5, cycle_number)/1000;
    eps_strain     = gam_strain/2;
    % gam_strain is the engineering shear strain, where gamma =
2*epsilon
    % eps_strain is the shear strain, where epsilon = (1/2)*gamma
    % The strain gage reads gamma; however, we need shear strain
    % to calculate strain energy

    % Removing zeros in data file
    [angle,torque,gam_strain,eps_strain] =
zero_check(angle,torque,gam_strain,eps_strain);

    % Setting the last data-point equal to the initial data-point to
    % close the curve
    data_count = length(torque);
    torque(data_count+1) = torque(1);
    eps_strain(data_count+1) = eps_strain(1);
    shear_stress = torque*ro/Jo*1000^2;

    energy(cycle_number) = polyarea(shear_stress,eps_strain);
end

% Plotting loop energy vs. freq
figure(1)
plot(n,energy, '.', 'Color', 'BLUE', 'LineWidth', 1); grid on;
title('Loop Energy vs. Cycle', 'FontName', 'Tahoma', 'FontSize', 10)
xlabel('Cycle [#]', 'FontName', 'Tahoma', 'FontSize', 10);
ylabel('Loop Energy [J/m^3]', 'FontName', 'Tahoma', 'FontSize', 10);
filename = 'Energy vs. Cycle';
filepath = 'output\';
SSC = [filepath,filename, '.jpeg'];
saveas(gcf,SSC, 'jpeg')

```

B.3 Continuous Run

```
%%%%%%%%PROGRAM TO PROCESS DATA%%%%%%%%
%%%%%%%%          FATIGUE          %%%%%%%%%

%%%%%%%%%-USER SPECIFIED-%%%%%%%%%
do_gage = 0.65*25.4; % mm
di_gage = 0.53*25.4; % mm
% The outer and inner diameters of the gage section. These will
% be used to calculate the polar moment of area (J0) and the
% average radius (r0).
%%%%%%%%%-          END          -%%%%%%%%%

Jo = pi/32*(do_gage^4-di_gage^4);
ro = (do_gage+di_gage)/4;

% Call function and read data from file including headers
[header,loop_data,j] =
read_data('input/TorsionContinuous_Run_1_80.dat');
matrix_size          = size(loop_data);
number_of_cycles      = matrix_size(3);

% Retrieving cycle numbers
n = zeros(j-1,1);
n(1,1) =
str2double(strcat(header(4,13,1),header(4,14,1),header(4,15,1),header(4
,16,1)));
for i = 2:j-1
    if i <= 153
        n(i) =
str2double(strcat(header(2,13,i),header(2,14,i),header(2,15,i),header(2
,16,i)));
    end
    if i > 153
        n(i) =
str2double(strcat(header(2,13,i),header(2,14,i),header(2,15,i),header(2
,16,i),header(2,17,i)));
    end
end

% Setting error count
errors = 0;

% Setting energy matrix size
energy = zeros(j-1,1);
```

```

for cycle_number = 1:number_of_cycles
    angle          = loop_data(:, 3, cycle_number);
    torque         = loop_data(:, 4, cycle_number);
    gam_strain     = loop_data(:, 5, cycle_number)/1000;
    eps_strain     = (gam_strain)/2;
    % gam_strain is the engineering shear strain, where gamma =
    2*epsilon
    % eps_strain is the shear strain, where epsilon = (1/2)*gamma
    % The strain gage reads gamma; however, we need shear strain
    % to calculate strain energy

    % Removing zeros in data file
    [angle,torque,gam_strain,eps_strain] =
zero_check(angle,torque,gam_strain,eps_strain);
    % Setting the last data-point equal to the initial data-point to
    % close the curve
    data_count = length(torque);
    torque(data_count+1) = torque(1);
    eps_strain(data_count+1) = eps_strain(1);
    shear_stress = torque*ro/Jo*1000^2;
    energy(cycle_number) = polyarea(shear_stress,eps_strain);
end

% Averaging and displaying continuous curve
for i = 11:50
    energy_mod(i-10,1) = energy(i);
end
for i = 62:101
    energy_mod(i-61,2) = energy(i);
end
for i = 113:152
    energy_mod(i-112,3) = energy(i);
end
for i = 164:203
    energy_mod(i-163,4) = energy(i);
end
for i = 215:254
    energy_mod(i-214,5) = energy(i);
end
for i = 266:305
    energy_mod(i-265,6) = energy(i);
end
for i = 317:356
    energy_mod(i-316,7) = energy(i);
end
for i = 368:407
    energy_mod(i-367,8) = energy(i);
end
for i = 419:458
    energy_mod(i-418,9) = energy(i);
end

```



```

energy_mod = mean(energy_mod);
energy_mod = transpose(energy_mod);

for i = 1:length(energy_mod)
    energy_modmod(i+1,1) = energy_mod(i,1);
end
energy_mod = energy_modmod;
energy_mod(1,1) = energy_mod(2,1);

cycles = [0;3000;6000;9000;12000;15000;18000;20000;21000;22000];

for i = 450:length(energy)-59
    energy_mod(i-439) = energy(i);
    cycles(i-439,1) = n(i);
end

% Scaling factor determined by the ratio of average continuous loop
energy
% at 1 Hz to the average frequency sweep energy at 1 Hz
scalingfactor = 542.80209299/mean(energy_mod);

% Plotting the energy from 0 to the spike in dissipation
energy_mod_p = scalingfactor*energy_mod;

partial_energy = num2str(round(trapz(cycles,energy_mod_p)));

figure(1)
plot(cycles/1000,energy_mod_p,'-','Color','BLACK','LineWidth',2); grid
on;
axis([0 23 0 750])
f = ['Strain Energy Density vs. Cycles (0 to Pre-Spike)']%, '
Partial Energy: ',partial_energy];
title(f,'FontName','Tahoma','FontSize',12)
xlabel('Cycles [#'],'FontName','Tahoma','FontSize',12);
ylabel('Strain Energy Density
[J/m^3'],'FontName','Tahoma','FontSize',12);
filename = 'Energy vs. Cycles 0 to Pre-Spike';
filepath = 'output\';
SSC = [filepath,filename, '.jpeg'];
saveas(gcf,SSC, 'jpeg')

cycles1 = cycles;
cycles1(1) = [];
cycles1(1) = [];

energy_mod_p(1) = [];
energy_mod_p(1) = [];

partial_energy_6k = num2str(round(trapz(cycles1,energy_mod_p)));

figure(2)

```

```

plot(cycles1/1000,energy_mod_p,'-','Color','BLACK','LineWidth',2); grid
on;
axis([6 23 0 750])
g = ['Strain Energy Density vs. Cycles (6k to Pre-Spike)'],'
Partial Energy: ',partial_energy_6k];
title(g,'FontName','Tahoma','FontSize',12)
xlabel('Cycles [#'],'FontName','Tahoma','FontSize',12);
ylabel('Strain Energy Density
[J/m^3'],'FontName','Tahoma','FontSize',12);
filename = 'Energy vs. Cycles 6k to Pre-Spike';
filepath = 'output\';
SSC = [filepath,filename, '.jpeg'];
saveas(gcf,SSC, 'jpeg')

% Plotting the energy from 0 to failure

for i = 450:length(energy)-20
    energy_mod(i-439) = energy(i);
    cycles(i-439,1) = n(i);
end

energy_mod_t = scalingfactor*energy_mod;

total_energy = num2str(round(trapz(cycles,energy_mod_t)));

figure(3)
plot(cycles/1000,energy_mod_t,'-','Color','BLACK','LineWidth',2); grid
on;
axis([0 23 0 3500])
h = ['Strain Energy Density vs. Cycles (0 to Failure)'],'      Total
Energy: ',total_energy];
title(h,'FontName','Tahoma','FontSize',12)
xlabel('Cycles [#'],'FontName','Tahoma','FontSize',12);
ylabel('Strain Energy Density
[J/m^3'],'FontName','Tahoma','FontSize',12);
filename = 'Energy vs. Cycles 0 to Failure';
filepath = 'output\';
SSC = [filepath,filename, '.jpeg'];
saveas(gcf,SSC, 'jpeg')

% Plotting the energy from 6k cycles to failure

cycles2 = cycles;
cycles2(1) = [];
cycles2(1) = [];

energy_mod_t(1) = [];
energy_mod_t(1) = [];

total_energy_6k = num2str(round(trapz(cycles2,energy_mod_t)));

figure(4)

```

```

plot(cycles2,energy_mod_t,'-','Color','BLACK','LineWidth',2); grid on;
axis([6000 23000 0 3500])
l = ['Strain Energy Density vs. Cycles (6k to Failure)'],'      Total
Energy: ',total_energy_6k];
title(l,'FontName','Tahoma','FontSize',12)
xlabel('Cycles [#]','FontName','Tahoma','FontSize',12);
ylabel('Strain Energy Density
[J/m^3]','FontName','Tahoma','FontSize',12);
filename = 'Energy vs. Cycles 6k to Failure';
filepath = 'output\';
SSC = [filepath,filename, '.jpeg'];
saveas(gcf,SSC, 'jpeg')

```

B.4 Coefficient Generator

```

%%%%%%%%PROGRAM TO PROCESS DATA%%%%%%%%
%%%%%%%%      MODEL GENERATION      %%%%%%%%%

% Call function and read data from file including headers
[header,run_data] = read_data('input/FS_3_80_C_117_mod.dat');

shear_stress = run_data(:,1);
eps_strain   = run_data(:,2);
gam_strain   = 2*eps_strain;

% Specify material properties
G = 77.409E9; % Pa

% Specify required values
tau_c   = 1E8; % 1E8
eo_i    = 1E-2; % 1E-4
d_tau_c = 1E6; % 1E7
d_eo    = 1E-3; % 1E-5

% Specifying the number of iterations to be performed
total_iterations = 100;
std_dev_it       = 1000;

% Pre-allocating variables to increase speed
ep      = zeros(length(gam_strain),1);
e_diff  = zeros(length(gam_strain),1);
diff_sd = zeros(std_dev_it,1);
eo      = zeros(std_dev_it+1,1);
C       = zeros(total_iterations,1);
tau_c_v = zeros(total_iterations,1);
STD_diff = zeros(total_iterations,1);

% Beginning optimization loops

```

```

% Loop over total_iterations
for i = 1:total_iterations

    % Specifying initial eo and e_diff constant value
    eo(1,1) = eo_i;
    e_diff(1,1) = 0;

    % Loop over std_dev_it
    for j = 1:std_dev_it

        % Loop over vector gam_strain
        % Solving for the cyclic strain
        for k = 1:length(gam_strain)
            ep(k,1) =
(shear_stress(k,1)/G)+eo(j,1)*sinh(shear_stress(k,1)/tau_c);
        end
        % Determining the percent difference between
        % the experimental strain (gam_strain) and the calculated
        % cyclic strain (ep)
        for kk = 2:length(gam_strain)
            e_diff(kk,1) = 100*abs((gam_strain(kk,1)-
ep(kk,1))/gam_strain(kk,1));
        end

        % Determining the standard deviation of the difference
        diff_sd(j,1) = std(e_diff);

        % Increasing the eo constant value
        eo(j+1,1) = eo(j,1)+d_eo;

    end

    % Finding the minimum standard deviation
    diff_min = min(diff_sd);

    % Setting loop additive "a" and initial difference "del_diff"
    a = 0;
    del_diff = 1;

    while del_diff ~= 0
        a = a+1;
        del_diff = diff_sd(a,1)-diff_min;
    end

    C(i,1) = 1/eo(a,1);
    tau_c_v(i,1) = tau_c;
    STD_diff(i,1) = diff_min;
    tau_c = tau_c+d_tau_c;

end

```

```

% Determining global minimum standard deviation
STD_min = min(STD_diff);

% Setting loop additive "b" and initial difference "STD_del"
b = 0;
STD_del = 1;

% Finding the position corresponding to the global minimum standard
% deviation
while STD_del ~= 0
    b = b+1;
    STD_del = STD_diff(b,1)-STD_min;
end

% The optimum tau_c and C values occur at the global minimum standard
% deviation
tau_c_opt = tau_c_v(b,1);
C_opt = C(b,1);

% Calculated strain values
for l = 1:length(shear_stress)
    ep(l,1) =
    ((shear_stress(l,1)/G)+(1/C_opt)*sinh(shear_stress(l,1)/tau_c_opt));
end

% Plotting tau_c values versus the standard deviation
figure(1)
plot(tau_c_v,STD_diff); grid on;
title('Tau_C vs. STD','FontName','Tahoma','FontSize',10)
xlabel('\tau_c','FontName','Tahoma','FontSize',10);
ylabel('STD','FontName','Tahoma','FontSize',10);
filename = 'Tau_C vs. STD';
filepath = 'output\';
SSC = [filepath,filename, '.jpeg'];
saveas(gcf,SSC,'jpeg');

% Plotting the actual strain vs. stress and the theoretical strain vs.
% stress
figure(2)
plot(gam_strain,shear_stress,'+-','Color','BLACK','LineWidth',1); grid
on; hold on;
plot(ep,shear_stress,'+-','Color','RED','LineWidth',1);
title('Data Fit Comparison','FontName','Tahoma','FontSize',10)
xlabel('\gamma [mm/mm]','FontName','Tahoma','FontSize',10);
ylabel('\sigma [Pa]','FontName','Tahoma','FontSize',10);
filename = 'Data Fit Comparison';
filepath = 'output\';
SSC = [filepath,filename, '.jpeg'];
saveas(gcf,SSC,'jpeg');

C_opt

```

tau_c_opt

B.5 Read Data

```
function [header_mat,data_mat,j] = read_data(file)

if nargin < 1
    error('Function requires one input argument');
elseif ~ischar(file)
    error('Input argument must be a string representing a filename');
end

% Open the file.  If this returns a -1, we did not open the file
successfully.
fid = fopen(file);
if fid==-1
    error('File not found or permission denied.');
```

```
end

% Initialize loop variables
no_lines = 0;
max_line = 0;
ncols = 0;
data = [];
j=1;

% Start processing
line = fgetl(fid);
if ~ischar(line)
    disp('Warning: file contains no header and no data')
end;

while line~= -1
    [data, ncols, errmsg, nxtindex] = sscanf(line, '%f');

    while isempty(data) | nxtindex==1
        no_lines = no_lines+1;
        max_line = max([max_line, length(line)]);
        eval(['line', num2str(no_lines), '=line;'])
        line = fgetl(fid);
        if ~ischar(line)
            disp('Warning: file contains no data')
            break
        end;
        [data, ncols, errmsg, nxtindex] = sscanf(line, '%f');
    end

    header_mat(1:no_lines,1:max_line,j) = char(' '*ones(no_lines,
max_line));
```

```

for i = 1:no_lines
    if length(eval(['line' num2str(i)]))
        varname = eval(['line' num2str(i)]);
    else
        varname = ['line',num2str(i)];
    end

    header_mat(i,1:length(varname),j)=varname;
end

data = [data; fscanf(fid, '%f')];

% Resize output data
eval('data_mat(1:length(data)/ncols,1:ncols,j) = reshape(data,
ncols, length(data)/ncols)'';', '')
line = fgetl(fid);
j=j+1;
no_lines = 0;
end

fclose(fid);

```

B.6 Zero Check

```

function
[angle_modified,torque_modified,gam_strain_modified,eps_strain_modified
] = zero_check(angle,torque,gam_strain,eps_strain)

for i = 1:length(angle)
    if(angle(i) ~= 0)
        angle_modified(i) = angle(i);
    end
end

for i = 1:length(torque)
    if(torque(i) ~= 0)
        torque_modified(i) = torque(i);
    end
end

for i = 1:length(gam_strain)
    if(gam_strain(i,1) ~= 0)
        gam_strain_modified(i) = gam_strain(i);
    end
end

for i = 1:length(eps_strain)
    if(eps_strain(i) ~= 0)
        eps_strain_modified(i) = eps_strain(i);
    end
end

```

```
    end  
end  
  
return
```