ON SPACING STATISTICS OF PLANT POPULATIONS

PRODUCED BY UNIFORM SEED-PLACEMENT DEVICES

ABSTRACT OF
DISSERTATION

Presented in Partial Fulfillment of the Requirements for the
Degree Doctor of Philosophy in the Graduate School of
The Ohio State University

By

Roger Phillip Rohrbach, B. Agr. E.

* * * * *

The Ohio State University
1968

Approved by

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Roger Phillip Rohrbach, Ph.D.

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Professors E. J. Barre
and Ross D. Brazee

Establishment of proper population densities in field crops has been a long standing problem. One apparent solution is to over-seed the field so that the resulting plant population could be thinned after emergence to the desired density and spacing. Coupled with the aforementioned idea is the speculation that there exists a physical plant characteristic which is correlated with the plant's ability to produce. Thus there is the possibility that a "selective" thinner could be employed to yield a highly productive and properly spaced plant population.

Before design criteria for a thinning device can be defined, it is necessary to focus attention on the plant populations emanating from uniform seed-placement devices since the "output" of the planter is in fact the "input" to the thinning device. The primary objective of this study was to develop a stochastic model for systems designed to produce uniform spacings of planted seeds and, as a consequence, to statistically characterize the plant spacings produced by these systems.

Most seed-placement systems designed to produce uniform plant spacings involve, in common, three basic physical processes. The first of these ideally is the selection and acquisition of a single seed from a large number of seeds *in* the hopper. The second process pertains to

the release or discharge of the seed and its subsequent placement in the soil. The third and final process is that of a seed's development into a plant, which usually has greater dependence upon events after, rather than during planting.

The hypothesis developed in this study can be stated as follows: A device designed to produce uniform seed spacing, germination, and early plant growth can be characterized in terms of three independent random variables. They are: the number of seeds that occupy a given seed cell at the time of discharge; the error in seed placement with respect to the desired uniform spacing; and the survival factor. Furthermore, the drop error random variable has a normal probability density function.

A digital computer-simulated planter was proposed and used as a Monte Carlo Model to predict field plant spacings. Specifically, corn, cucumber, and sugar beet populations were predicted and tested, one by one, against field observations.

The actual testing of the hypothesis consisted of observing field populations, deducing model parameters and predicting a comparable field population. Independent model parameter determination was unfeasible since too little is presently known about these parameters. The proposed Monte Carlo digital computer model generates valid predictions of field plant population spacing statistics. This conclusion is supported by a chi-square test for goodness of fit at a significance level of approximately 1 to 8 percent depending on the specific population being tested,

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LIST OF SYMBOLS

$C_{yy}(n)$	autocovariance function of y
E_i	expected number of observations in the i^{th} sample class
$E[y]$	expected value of y
$f(s_1)$	"mixed" normal probability density function of s_1
g	probability that a given seed survives to seedling
G_j^k	random survival variable for the k^{th} seed of the j^{th} seed cell discharge
J	Jacobian
N_j	number of seeds in the j^{th} seed cell discharge
$N(\mu, \sigma^2)$	shorthand notation for a normal probability density function with mean μ and standard deviation σ (this symbol is read "normally distributed, mean μ and variance σ^2 ")
O_i	observed number of observations in the i^{th} sample class
pdf	probability density function
P_i	probability that a seed cell contains i seeds at the time of discharge
$Pr(x)$	probability that x will occur
Pr_1	probability that a given spacing is defined by plants arising from i successive seed drops apart
Q_k	a statistic with an approximate chi-square probability density function with k degrees of freedom
R	theoretical drop or the average number of seeds discharged per seed drop
$R_{yy}(n)$	autocorrelation function of y

s_a	intended uniform spacing
s_1	length of the i^{th} spacing between the $(i+1)^{\text{th}}$ and i^{th} plants in a row
s_0	expected value of a normal pdf "folded" about its zero mean
s_0	artificial distance introduced to provide the necessary number of transform equations
$V(y)$	variance of y
W	probability that no plant survives from a given seed drop
x_1	random drop error
y_1^j	coordinate location of the i^{th} sequentially dropped seed in a row, the seed originating from the j^{th} seed drop
z_1	coordinate location of the i^{th} plant in a row
Z_1	probability that at least one seed survives from the i^{th} seed drop
σ	standard deviation of the drop error x_1
σ_0	standard deviation of a normal pdf "folded" about its zero mean
$\phi(x_1)$	normal probability density function

INTRODUCTION

Innumerable agricultural processes require some type of sorting operation. Harvesting operations can be visualized as sorting procedures which separate the harvestable product from the unwanted material. Weed control in its simplest form is a sorting problem where the sorting rule is to destroy the weeds without harming the crop.

Establishment of proper population densities in field crops has been a long standing problem. Present methods of obtaining desirable population densities vary among crops, but all methods seem to have at least one common characteristic, viz., inability to provide direct and reliable control over plant population. Over twenty years ago, Bainer (1947), Guelle (1947) and others were talking about precision planting and mechanical blocking of row crops. Nowhere in the literature has this author found a quantitative description of precision planting.

On the other hand, mechanical blocking is really a sorting operation. This fact is easily ascertained by considering the distribution of plant spacings, as obtained under field conditions and present seeding devices, to be somehow "altered" or "sorted" into a more desirable distribution by removing certain plants.

In any event, we must recognize that there is no quantitative method for characterizing plant populations and that design criteria for mechanical blockers require a complete understanding of the

existing population to be thinned as well as the thinning rule.

There also has been speculation, Mederski (1967) and Johnson (1967), that certain physical characteristics of a plant are correlated with the plant's ability to produce. If this were true, it would seem possible to overplant the number of seeds required with the idea that "selective" thinning could be employed to yield a highly productive stand as well as the exact desired population density and spacing.

INVESTIGATION

Objectives

The objectives of this investigation are four-fold: To develop a stochastic model for systems designed to produce uniform spacing of planted seeds; to statistically characterize the plant spacings produced by these systems; to provide an analytic basis for evaluating accuracy and precision of horizontal cell plate planters; and to provide an analytic basis for subsequent investigations into the selection of a desirable thinning rule.

Theoretical Analysis

Most seed-placement systems designed to produce uniform plant spacings involve, in common, three basic physical processes. The first of these ideally is the selection and acquisition of a single seed from a large number of seeds in the hopper. The second process pertains to the release or discharge of the seed and its subsequent placement in the soil. The third and final process is that of a seed's development into a plant, which usually has greater dependence upon events after, rather than during planting. In any event, the planting-system model will emerge from detailed considerations of these three factors.

Ideally the mechanism selects one seed at a time, but in actuality this is not true. A cell plate planter is so constructed that it is

possible to obtain zero, one, or two or more seeds per cell. The occurrence of zero, single, or multiple seed drops is, of course, random but may be influenced by the design of the seed plate and its cells, amount and gradation of seeds, and the peripheral velocity of the seed cell.

Upon recognition of the randomness of the number of seeds that may occupy a given seed cell at any time, we can introduce the following mathematical formalisms: Let p_i equal the probability that i seeds occupy a seed cell at the time of discharge. From the definition of p_i it follows that

$$\sum_{i=0}^{\infty} p_i = 1. \quad (1)$$

Furthermore, the theoretical drop R of the planter can be defined in terms of p_i as

$$R = \sum_{i=0}^{\infty} i p_i. \quad (2)$$

The number of seeds per cell at the j^{th} choice, N_j , is a random variable with a discrete probability density

$$\Pr(N_j = k) = p_k, \quad k = 0, 1, \dots. \quad (3)$$

It is conceivable that planter plate or seed cell irregularities might cause N_j to display some persistence or periodicity. For example, an enlarged cell may tend to contain, on the average, more

seeds than each of the remaining cells.

The autocorrelation $R_{NN}(n)$ is defined as

$$R_{NN}(n) = E[N_j N_{j+n}]. \quad (4)$$

The index j can be thought of as a running index which marks or counts each successive seed cell discharge.

The seed may be discharged from the seed cell at a foot or more above the seedbed. As it falls to the seedbed, the seed interacts with the tube which guides it to final placement in the soil. The seed, after undergoing such interactions and striking the soil, becomes randomly placed with respect to the desired uniform spacing. The amount of random dispersion may be influenced by seed tube design, soil conditions, and velocity of forward travel.

The a priori choice of a probability density function for the drop error is eased by the observation (Korn and Korn, 1961) that the continuous distribution having the largest entropy for a given variance σ^2 is the normal distribution. The formal definition of the drop error x_1 is completed by asserting its amplitude probability distribution function and its autocorrelation function. The normal probability density function of x_1 is

$$\phi(x_1) = \frac{1}{\sigma\sqrt{2\pi}} \exp \left[-\frac{x_1^2}{2\sigma^2} \right], \quad -\infty < x_1 < \infty. \quad (5)$$

The probability density function of equation (5) will be referred to using the shorthand notation $N(0, \sigma^2)$ which is read "normally

distributed, mean 0 and variance σ^2 ."

It is in general necessary to allow for the possibility of persistent or periodic behavior which could be introduced into the drop error random variable by nonuniformities in the planter mechanism. To this end the drop error autocorrelation function is introduced as

$$R_{xx}(n) = E \left[x_1 x_{1+n} \right] , \quad (6)$$

where

$$R_{xx}(0) = \sigma^2. \quad (7)$$

The correct placement of the seeds at a uniform spacing does not, unfortunately, guarantee uniform plant spacing. The third random variable in the stochastic planting system accounts for the fact that a given seed may or may not develop into a plant depending on seed viability, soil conditions, and the microenvironment surrounding the seed.

Let g equal the probability that a given seed becomes a seedling plant; and hence, $(1-g)$ equals the probability that any given seed will not survive to seedling. The random plant development variable G_j^k is defined to take on values of 0 and +1 with probabilities g and $(1-g)$ respectively. Thus the first order statistics are characterized by

$$\Pr(G_j^k = 0) = g,$$

$$\Pr(G_j^k = +1) = 1-g . \quad (8)$$

There would seem no reason to expect that the germination and early growth of one seed should affect any other seeds in any way. On the other hand, mature plants do compete with one another, but the intended concern of G_j^k is the probability of survival from seed to seedling.

In an effort to derive the probability density function (pdf) of the spacings produced by the planter model proposed above, we suppose that a sample row has been planted which might look like that indicated in figure 1. In the figure, the +'s represent the desired uniform spacing of s_a while the o's indicate the location of the established plants. The single-headed arrows labeled x_1 thru x_{12} are random samples from the drop error pdf. Notice that in general some of the x_1 's will be missing and that others will "overlap" causing the order of plants in the row to differ from the order of planting. The missing x_1 's are evidence that $g \neq 1$, and the s_1 's are plant spacings whose pdf we ultimately wish to predict.

Consider the row in figure 1 with a number scale superimposed, the origin being at the extreme left +. The desired uniform plant locations would be given on the number scale by

$$js_a, j = 0, 1, 2, \dots . \quad (9)$$

Let the actual locations of the plants be denoted by

+ ... Desired Drop Points
 O ... Actual Plant Points

Figure 1. Identification of the drop errors, x_i , and the resulting spacings, s_i , for a typical row.

$$z_i, i = 1, 2, \dots \quad (10)$$

In every case, the actual plant location can be thought of as a random selection from a $N(k\sigma_a, \sigma^2)$ population, where k is itself a random integer. The statistics of k are intimately related to the statistics of N_j and G_j^1 , as we shall see.

The distance between plants is defined as

$$s_i = z_{i+1} - z_i, i = 1, 2, \dots \quad (11)$$

We can obtain the pdf of s_i by considering the following model planter experiment.

Suppose m seeds,

$$m = \sum_{j=0}^n N_j, \quad (12)$$

are dropped at $n+1$ desired uniform spacing points from which l plants develop,

$$l = m - \sum_{j=0}^n \left[\sum_{k=1}^{N_j} G_j^k \right]. \quad (13)$$

According to the model there would be m random selections of x_i from a $N(0, \sigma^2)$ population. Furthermore, at the j^{th} desired drop point there are h_j choices of the x_i random variable associated with the surviving plants,

$$h_j = N_j - \sum_{k=1}^{N_j} G_j^k. \quad (14)$$

Under the assumption that the x_i 's are independent and uncorrelated, we see that the planter model is really equivalent to h_j random samples from a $N(js_a, \sigma^2)$ population, $j = 0, 1, \dots, n$. Let this sequence be called $\{y_i^j; i = 1, 2, \dots, l\}$ where the subscript denotes the sequential selection of the sample, one sample for every surviving plant, and the superscript indicates that the i^{th} sample came from a normally distributed population with mean js_a , the variance always being σ^2 . In general there may be repeated or missing values, or both, of the superscript.

Now the first plant to appear in the row is clearly the minimum of the set $\{y_i^j\}$ while the last plant is maximum of $\{y_i^j\}$. Let

$$\begin{aligned} z_1 &= \min \{y_i^j\}, \\ z_2 &= \min \{y_i^j\} \text{ with } z_1 \text{ removed,} \\ &\vdots \\ &\vdots \\ z_n &= \max \{y_i^j\}; \end{aligned} \quad (15)$$

then s_1 is given by equation (11).

The joint pdf of the set $\{y_i^j\}$ is

$$g(y_1^0, y_2^1, \dots, y_1^j, \dots, y_1^n) = \prod_{i=1}^l \phi_j(y_1^j), \quad -\infty < y_1^j < \infty,$$

$$i = 1, 2, \dots, l, \quad (16)$$

where $\phi_j(y_1^j)$ is the $N(js_a, \sigma^2)$ pdf of y_1^j .

The joint pdf of the set $\{z_1\}$ can be obtained from joint pdf given in equation (16) by the transformation equations (15). There are $l!$ different sets of one to one transforms required to arrange the y_1^j 's. If, for example,

$$y_1^0 < y_2^1 < \dots < y_1^j < \dots < y_1^n, \quad (17)$$

then

$$z_1 = y_1^j, \quad i = 1, 2, \dots, l. \quad (18)$$

A simple computation will confirm that the absolute value of the Jacobian of the transformation (18) is unity. Similarly, all $l!$ one to one transforms that map the set $\{y_1^j; i = 1, 2, \dots, l\}$ onto the set $\{z_1; i = 1, 2, \dots, l\}$ have Jacobians whose absolute value is unity.

The joint pdf of the z_1 's is given by

$$h(z_1, z_2, \dots, z_l) = \sum \prod_{i=1}^l \phi_j(z_1) |J_k|,$$

$$-\infty < z_1 < z_2 < \dots < z_l < \infty, \quad (19)$$

the summation being carried out over the $l!$ permutations of the set $\{z_i; i = 1, 2, \dots, l\}$ and J_k is the Jacobian of the transform equations of the k^{th} permutation.

Equation (11) defines a second and final transformation, viz.,

$$s_1 = z_2 - z_1,$$

$$s_2 = z_3 - z_2,$$

$$\begin{array}{ccc} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{array}$$

$$s_{l-1} = z_l - z_{l-1}.$$

(20)

The inverse transformation equations are

$$z_1 = s_0,$$

$$z_2 = s_0 + s_1,$$

$$z_3 = s_0 + s_1 + s_2,$$

$$\begin{array}{ccc} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{array}$$

$$z_l = s_0 + s_1 + \dots + s_{l-1},$$

(21)

where $z_1 = s_0$ is introduced to provide l rather than $(l-1)$ equations.

The absolute value of the Jacobian of the transformation equations (21) is again unity.

It is now possible to symbolically express the joint probability density function of the set $\{s_i; i = 1, 2, \dots, l\}$, that pdf being

$$w(s_0, s_1, \dots, s_{l-1}) = h(s_0, s_0 + s_1, \dots, s_0 + s_1 + \dots + s_{l-1}) |J|, \quad (22)$$

where J is the Jacobian of equations (21).

The major complication in the above theoretical development is associated with the interchange or ordering procedure of equations (15). If it were possible to say with a high degree of certainty that the ordering of seed drops was in fact the same as the ordering of plants in the row, then equations (15) could be replaced by equations (18) and the analysis would proceed more simply. Unfortunately, as in the case of sugar beet planters, this assumption would be totally unjustified. However, in the case of cucumber and corn planters, there would be ample justification as we shall see.

In an effort to evaluate the quantitative relationship between the intended uniform spacing s_a and the variance σ^2 we propose the following experiment. Suppose m seeds are singularly dropped at m desired drop points from which m plants develop. The coordinate of the i^{th} plant is y_i from a $N(is_a, \sigma^2)$ pdf. If we define the plant spacing to be

$$s_i = y_{i+1} - y_i, \quad (23)$$

then the pdf of s_1 can be shown to be $N(s_a, 2\sigma^2)$. If $s_1 < 0$ then an interchange has occurred. The probability that $s_1 < 0$ can be written as

$$\Pr(s_1 < 0) = \frac{1}{2\sigma\sqrt{\pi}} \int_{-\infty}^0 \exp\left[-\frac{1}{4}\left(\frac{s_1 - s_a}{\sigma}\right)^2\right] ds_1, \quad (24)$$

which is easily evaluated. In fact, if $s_a > 2\sqrt{2}\sigma$ then

$$\Pr(s_1 < 0) < 0.0227. \quad (25)$$

Equation (24) gives a quantitative relationship between the planter model parameters σ and s_a and the error involved by using equations (18) instead of equations (15). In other words, if $s_a > 2\sqrt{2}\sigma$ then less than 3(2.27) percent of the s_1 's will be incorrectly represented by equation (23). The factor of 3 occurs since every interchange affects three interval lengths. If, for example, $y_{i+1} < y_i$ then s_{i-1} , s_i and s_{i+1} intervals would be correctly represented by $y_{i+1} - y_{i-1}$, $y_i - y_{i+1}$, and $y_{i+2} - y_i$ respectively.

Since we wish to predict statistically the distance from the i^{th} to the $(i+1)^{\text{th}}$ plant in a row, it would be necessary for the mathematical model to always predict a positive number. Under the conditions described above, we could represent the i^{th} positive distance between the $(i+1)^{\text{th}}$ and the i^{th} plants by

$$s_i = |y_{i+1} - y_i|, \quad (26)$$

with a maximum 6.81 percent error.

Equation (26) is equation (23) with the addition of the absolute value. It can also be shown that the pdf of s_1 as defined by equation (26) is a $N(s_a, 2\sigma^2)$ pdf that has been "folded" about the origin to give the pdf shown in figure 2.

We now take the point of view that an interval s_1 is randomly observed in a row planted with the modeled planter under the most general conditions. The question is asked: What is the pdf of this length?

In the following analysis we shall assume $\sigma \ll s_a$ so that the interchange phenomenon discussed above is practically non-existent. Furthermore, the order of seeds and plants arising from the same seed drop is unimportant since all coordinate values are random samples from the same pdf. The pdf of $s_1 = |y_{i+1} - y_i|$, for y_{i+1} and y_i both from a $N(js_a, \sigma^2)$ pdf, is a $N(0, 2\sigma^2)$ pdf "folded" about the origin. For the remainder of this analysis we shall use

$$s_1 = y_{i+1} - y_i. \quad (27)$$

understanding that the absolute value function merely "folds" the pdf of s_1 about the origin.

The pdf of the observed spacing length s_1 is clearly a $N(ks_a, 2\sigma^2)$ pdf; k is zero if the second plant defining s_1 is from the same seed drop as the first plant and k is unity if the second plant is from the first successive seed drop after the first plant seed drop, etc. There remains only to determine what the pdf of k is in relation to the model random variables N_i and G_i^k .

Figure 2. Probability density function of $s_i = |y_{i+1} - y_i|$.

We can now ask: What is the probability that the observed spacing has a pdf that is $N(0, 2 \sigma^2)$, i.e. what is the probability that the two plants defining s_1 came from the same seed drop? Let Pr_0 be the probability in question.

To derive an expression for Pr_0 we must identify all the mutually exclusive and independent events which add up to the desired outcome. Clearly if the two plants at y_1 and y_{i+1} are from the same seed drop then there must have been two or more seeds dropped and the first plant can at worst be the next to last surviving seed.

The total fraction of surviving plants from double drops is $\frac{2p_2}{R}$. The probability the first plant is from the first seed of a double seed drop is $\frac{1}{2} \frac{2p_2}{R}$ and the probability that the second seed survived is g , hence the probability that the observed interval, s_1 , is from a double seed drop is $\frac{p_2}{R} g$.

The fraction of surviving plants from triple drops is $\frac{3p_3}{R}$. However, there are three ways in which the two plants can be observed to occur. The first plant can be from either the first or second seed while the second plant can be from either the second or third seed. The probability that the observed interval is from a triple drop is $\frac{p_3}{R} [2 + (1-g)] g$.

The general expression for Pr_0 is

$$Pr_0 = \frac{g}{R} \left\{ p_2 + p_3 [2 + (1-g)] + p_4 [3 + 2(1-g) + (1-g)^2] + \dots \right. \\ \left. + p_k [(k-1) + (k-2)(1-g) + \dots] \right\}$$

$$+ 2 (1-g)^{k-3} + (1-g)^{k-2} \Big] + \dots \Big\}, \quad (28)$$

$$\begin{aligned} \text{Pr}_0 = \frac{g}{R} \Big\{ p_2 + p_3 \Big[2 + (1-g) \Big] + \dots \\ + \frac{p_k}{g} \Big[(k-1) - \frac{(1-g) - (1-g)^k}{g} \Big] + \dots \Big\}, \end{aligned} \quad (29)$$

which finally takes the form

$$\text{Pr}_0 = \frac{1}{R} \sum_{k=2}^{\infty} p_k \Big[(k-1) - \frac{(1-g) - (1-g)^k}{g} \Big]. \quad (30)$$

Let Pr_1 be the probability of observing s_1 from a pdf that is $N(s_a, 2 \sigma^2)$. Now the two plants in question must come from successive seed drops. The probability that the first plant is from a single drop is $\frac{p_1}{R}$. Reasoning as above we have

$$\begin{aligned} \text{Pr}_1 = \frac{Z_1}{R} \Big\{ p_1 + \Big[1 + (1-g) \Big] p_2 + \Big[1 + (1-g) + (1-g)^2 \Big] p_3 + \dots \\ + \Big[1 + (1-g) + \dots + (1-g)^{k-1} \Big] p_k + \dots \Big\}, \end{aligned} \quad (31)$$

where Z_1 is defined as the probability that at least one plant from the next successive seed drop survives.

The probability that at least one seed survives from a given seed drop is

$$Z_1 = gp_1 + \Big[g + g(1-g) \Big] p_2 + \Big[g + (1-g)g + g(1-g)^2 \Big] p_3 + \dots$$

$$+ \left[g + g(1-g) + \dots + g(1-g)^{k-1} \right] p_k + \dots, \quad (32)$$

$$Z_1 = \sum_{k=1}^{\infty} p_k \left[1 - (1-g)^k \right]. \quad (33)$$

Hence,

$$Pr_1 = \frac{1}{Rg} \left\{ \sum_{k=1}^{\infty} p_k \left[1 - (1-g)^k \right] \right\}^2. \quad (34)$$

Let Pr_n be the probability that the second plant is from the n^{th} successive seed drop after the seed drop that established the first plant. Reasoning as above we obtain

$$\begin{aligned} Pr_n = \frac{W^{n-1} Z_n}{R} & \left\{ p_1 + \left[1 + (1-g) \right] p_2 + \dots \right. \\ & \left. + \left[1 + (1-g) + \dots + (1-g)^{k-1} \right] p_k + \dots \right\}, \end{aligned} \quad (35)$$

where Z_n is defined as the probability that at least one plant from the n^{th} successive seed drop survives. We define W as the probability that no plants survive from any of the $(n-1)$ intervening seed drops.

Interestingly enough, Z_n is identical to Z_1 as given by equation (33). On the other hand, W is given by

$$W = p_0 + p_1 (1-g) + p_2 (1-g)^2 + \dots + p_k (1-g)^k + \dots, \quad (36)$$

$$W = \sum_{m=0}^{\infty} p_m (1-g)^m. \quad (37)$$

After some work, we can write equation (35) in the form

$$\Pr_n = \frac{1}{Rg} \left\{ \sum_{k=1}^{\infty} p_k [1 - (1-g)^k] \right\}^2 \left\{ \sum_{m=0}^{\infty} p_m (1-g)^m \right\}^{n-1}. \quad (38)$$

What is of ultimate interest is the pdf of the observed spacing.

This pdf is

$$f(s_1) = \sum_{n=0}^{\infty} \Pr_n N(ns_a, 2\sigma^2), \quad s_1 > 0, \quad (39)$$

where we must keep in mind the necessary "folding" about the origin as discussed earlier. Equation (39) is more conveniently written as

$$f(s_1) = \Pr_0 N(0, 2\sigma^2) + \sum_{n=1}^{\infty} \Pr_n N(ns_a, 2\sigma^2), \quad s_1 > 0, \quad (40)$$

which expands to

$$\begin{aligned} f(s_1) = & \frac{1}{R} \sum_{k=2}^{\infty} p_k \left[(k-1) - \frac{(1-g) - (1-g)^k}{g} \right] N(0, 2\sigma^2) \\ & + \frac{1}{Rg} \sum_{n=1}^{\infty} \left\{ \sum_{k=1}^{\infty} p_k [1 - (1-g)^k] \right\}^2 \left\{ \sum_{m=0}^{\infty} p_m (1-g)^m \right\}^{n-1} N(ns_a, 2\sigma^2), \\ & s_1 > 0, \quad (41) \end{aligned}$$

upon substitution.

Equation (41) represents the probability density of spacings produced by a planter which drops i seeds, with probability p_i , at each desired plant point every s_a feet down the row. The variance of the

drop error is σ^2 and 100×g percent of the seeds ultimately develop into plants.

We now turn our attention to a suitable expression for the autocorrelation function of the spacings. Had it been possible to develop equation (22) into a more usable form, we could have easily obtained both the amplitude pdf and the autocorrelation directly.

The autocorrelation function can be defined as

$$R_{ss}(n) = E \left[s_i s_{i+n} \right]. \quad (42)$$

From the definition of expected values, equation (42) becomes

$$R_{ss}(n) = \int_0^\infty \int_0^\infty s_i s_{i+n} g(s_i, s_{i+n}) ds_i ds_{i+n}, \quad (43)$$

where $g(s_i, s_{i+n})$ is the joint pdf of s_i and s_{i+n} . As discussed above, this joint pdf can be obtained by considering it as a "marginal" pdf of the joint distribution given in equation (22). It becomes apparent that it is not possible to evaluate equation (43) under the present problem definition. Physically, the reason for this is that the seed order from multiple seed drops is identifiable sequentially; but, the interchange rate is always significant, regardless of the magnitudes of σ and s_a .

The autocovariance function $C_{ss}(n)$ has a significantly non-zero value for $n = 0$ and $n = 1$, as evidenced by Table III. It appears possible that a pulse coded autocorrelation or cross-correlation might be employed to gain some insight into the structure of the

autocovariance function $C_{ss}(n)$. A pulse coded field signal $u(x)$ could be defined as

$$u(x) = \begin{cases} \frac{1}{a} & \text{when } (z_1 - \frac{1}{2}a) < x < (z_1 + \frac{1}{2}a), i = 1, 2, \dots \\ 0 & \text{otherwise} \end{cases}, \quad (44)$$

where a is a small positive number, while a pulse coded uniform spacing signal $v(x)$ could be similarly defined as

$$v(x) = \begin{cases} \frac{1}{a} & \text{when } (js_a - \frac{1}{2}a) < x < (js_a + \frac{1}{2}a), j = 0, 1, \dots \\ 0 & \text{otherwise} \end{cases}, \quad (45)$$

The coding results in a sequence of unit strength rectangular pulses which are centered about the plant location. Preliminary studies have indicated that $R_{uu}(x)$ or possibly $R_{uv}(x)$ might be of value in determining the complete nature of $R_{ss}(n)$.

We can, however, obtain the expected value of s_1 and an approximate expression for the variance of s_1 . The pdf of s_1 was obtained in equation (41) and is given explicitly by

$$f(s_1) = \frac{\text{Pr}_0}{\sqrt{\pi} \sigma} \exp \left[-\frac{1}{4} \left(\frac{s_1}{\sigma} \right)^2 \right] + \frac{1}{2\sqrt{\pi} \sigma} \sum_{k=1}^{\infty} \text{Pr}_k \exp \left[-\frac{1}{4} \left(\frac{s_1 - ks_a}{\sigma} \right)^2 \right], \quad s_1 > 0. \quad (46)$$

We have, by definition,

$$E(s_1) = \int_0^{\infty} s_1 f(s_1) ds_1, \quad (47)$$

and provided $\sigma \ll s_a$

$$E(s_1) \approx s_0 Pr_0 + s_a \sum_{k=1}^{\infty} k Pr_k, \quad (48)$$

$$\approx s_0 Pr_0 + \frac{1 - Pr_0}{1 - W} s_a, \quad (49)$$

where

$$s_0 = \frac{1}{\sqrt{\pi} \sigma} \int_0^{\infty} s_1 \exp \left[-\frac{1}{4} \left(\frac{s_1}{\sigma} \right)^2 \right] ds_1, \quad (50)$$

$$= \frac{2 \sigma}{\sqrt{\pi}}. \quad (51)$$

Similarly, the variance of s_1 which is related to $R_{ss}(0)$, is defined as

$$V(s_1) = E \left[\left(s_1 - E(s_1) \right)^2 \right], \quad (52)$$

$$= E(s_1^2) - \{ E(s_1) \}^2. \quad (53)$$

The $E(s_1^2)$ can be evaluated approximately by

$$E(s_1^2) = \int_0^{\infty} s_1^2 f(s_1) ds_1, \quad (54)$$

$$\approx \text{Pr}_0(\sigma_o^2 + s_o^2) + \sum_{k=1}^{\infty} \text{Pr}_k(2\sigma^2 + k^2 s_a^2). \quad (55)$$

The variance is given in its final form by

$$\begin{aligned} V(s_1) \approx & \text{Pr}_0(\sigma_o^2 + s_o^2) + \frac{2\text{Pr}_1}{1-W} \sigma^2 \\ & + \text{Pr}_1 s_a^2 \sum_{k=0}^{\infty} (k+1)^2 W^k - \left\{ E(s_1) \right\}^2, \end{aligned} \quad (56)$$

where σ_o^2 is the variance of the distribution from which s_o was also computed. Specifically

$$\sigma_o = \sqrt{2 - 4/\pi} \sigma. \quad (57)$$

We have developed the theory to the point that given any set of model parameters, subject to $\sigma \ll s_a$, we can predict the histogram. We now wish to predict the model parameters, given the histogram.

There are four independent equations of which two can be obtained with the aid of a histogram. They are

$$\sum_{i=0}^{\infty} p_i = 1, \quad (1)$$

$$R = \sum_{i=0}^{\infty} i p_i, \quad (2)$$

$$Pr_0 = \frac{1}{R} \sum_{k=2}^{\infty} p_k \left[(k-1) - \frac{(1-g) - (1-g)^k}{g} \right], \quad (30)$$

and

$$W = \sum_{m=0}^{\infty} p_m (1-g)^m. \quad (37)$$

To be sure, numerical values for Pr_n , $n = 1, 2, \dots$ could be obtained from the histogram, but these values are in fact related to W and Pr_0 , viz.,

$$Pr_n = (1-Pr_0) (1-W) W^{n-1}, \quad n = 1, 2, \dots \quad (58)$$

If we consider p_0, p_1, p_2 , and R as unknowns and g as a parameter in the solution, we can solve the vector equation

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & (1-g) & (1-g)^2 & 0 \\ 0 & 0 & g & -Pr_0 \\ 0 & 1 & 2 & -1 \end{bmatrix} \begin{bmatrix} p_0 \\ p_1 \\ p_2 \\ R \end{bmatrix} = \begin{bmatrix} 1 \\ W \\ 0 \\ 0 \end{bmatrix}, \quad (59)$$

resulting from equations (1), (2), (30), and (37), to yield the vector solution

$$\begin{bmatrix} p_0 \\ p_1 \\ p_2 \\ R \end{bmatrix} = \frac{1}{\Delta} \times \begin{bmatrix} (1-Pr_0)g^2 + (W-1)g + Pr_0(1-W) \\ (1-W)(g-2Pr_0) \\ (1-W)Pr_0 \\ (1-W)g \end{bmatrix}, \quad (60)$$

where $\Delta = (1-Pr_0)g^2$.

The physical reason that only four independent equations can be obtained is that only four planter parameters are unique. No seed being planted is equivalent to one seed being planted with negative germination or development; in other words, we cannot tell in general if a seed was planted and did not grow or if no seed was planted.

Digital Computer Analysis

In view of the restrictive limitations placed upon the theoretical analysis by the necessity of keeping $\sigma \ll s_a$, a computer simulation was developed based on equations (15) rather than equations (18) used in the theoretical analysis.

The actual computer simulated planter operates in the following manner (see also appendix B). The input consists of specifying the intended uniform spacing s_a , the standard deviation of the drop error σ , the percent theoretical drop, and the germination and emergence percentage. The theoretical drop random variable is simulated by filling 100 subscripted locations in the computer memory with the desired percentages of zeros, ones, twos, etc. For example, a distribution of 4 percent of the cells with zero seeds, 82 percent of the cells with one seed, and 14 percent of the cells with two seeds

would be obtained by filling the 100 subscripted locations with 4 zeros, 82 ones, and 14 twos. The drop error x_i has a $N(0, \sigma^2)$ pdf.

The simplified steps the computer-simulated planter takes are;

1. Sets up 1,001 intended seed drop points at the average spacing s_a .
2. At each seed drop point the computer determines the number of seeds to be dropped by addressing one of the 100 memory locations with a random and uniformly distributed subscript.
3. For each seed dropped, a random selection from a normally distributed population of mean zero and variance σ^2 is made and thus placing the seed in the row with its associated drop error.
4. Each seed is randomly determined to have developed or not by a scheme similar to the one used for determining the seeds per cell. This procedure produces a series of established plant coordinates that are arranged in ascending order; the differences of which are the desired interval lengths.

One of the factors affecting the validity of the predictions of the computer model is the computer's ability to make random selections from the desired populations. Appendix A offers evidence that the computer-produced random variables have the proper first order statistics and are uncorrelated.

To illustrate the computer model, some typical values for the planter parameters were assumed. Corn in 40 inch rows at an average

spacing of 8 inches would yield 19,600 plants per acre. Andrews (1959) obtained the seeds per cell distribution for cell plate corn planters under a wide variety of conditions. A reasonable and realistic set of values for the seeds per cell random variable would be

$$\begin{aligned} \Pr(N_i = 0) &= 0.009, \\ \Pr(N_i = 1) &= 0.981, \\ \Pr(N_i = 2) &= 0.005, \\ \Pr(N_i = 3) &= 0.005, \end{aligned} \tag{61}$$

while the germination and emergence might be 94 percent. Under these conditions the intended uniform spacing would need to be 0.627 feet to obtain an average spacing of 8 inches. The standard deviation of the drop error was assumed to be 0.125 feet. Figure 3 shows the spacing characteristics of the computer-produced corn population while figures 4 and 5 show similar results for planter parameters associated with cucumbers and sugar beets, respectively.

The autocovariance functions, $C_{ss}(n)$, $n = 0, 1, \dots$, of the spacings produced by the three simulated planters are given in Table I. The autocovariance computer program (see Appendix B) calculates $C_{ss}(n)$ according to

$$C_{ss}(n) = E \left[\left(s_i - E(s_i) \right) \left(s_{i+n} - E(s_{i+n}) \right) \right]. \tag{62}$$

The theory predicts that the frequency histogram of plant spacings will be a summation of "folded" normal pdf's provided $\sigma \ll s_a$.

Figure 3. Frequency histogram of plant spacings produced by a computer-simulated corn planter.

Figure 4. Frequency histogram of plant spacings produced by a computer-simulated cucumber planter.

Figure 5. Frequency histogram of plant spacings produced by a computer-simulated sugar beet planter.

TABLE I

AUTO-COVARIANCE $C_{ss}(n)$ OF THE SIMULATED-
PLANTER-PRODUCED PLANT SPACINGS

Lag n	$C_{ss}(n)$ for Corn	$C_{ss}(n)$ for Cucumbers	$C_{ss}(n)$ for Sugar Beets
0	0.066	0.441	0.0163
1	-0.012	0.016	-0.0004
2	0.000	0.003	-0.0003
3	-0.001	-0.006	-0.0005
4	0.000	-0.004	-0.0004
5	0.004	0.027	-0.0014
6	0.001	-0.008	0.0005
7	-0.001	0.012	-0.0011
8	-0.003	-0.018	0.0009
9	0.000	0.020	-0.0001
10	-0.003	-0.004	0.0000

The normal distribution separator program (see Appendix B) will extract the appropriate parameters by reflecting or "unfolding" the histogram about the origin. We proceed with two parallel analyses, the results of which are found in Table II. Firstly we use the theoretical equations to obtain the parameter based estimates and secondly we use the normal distribution separator to obtain another estimate.

Table II contains planter parameter based estimates and histogram based estimates for various histogram quantities. The table does not show results for sugar beets since the basic premise, $\sigma \ll s_a$, is false. It should be noted that

$$C_{ss}(n) = R_{ss}(n) - [E(s_i)]^2. \quad (63)$$

The purpose of Table II is to give the reader a numerical basis for comparing the theoretical equations with the computer model and normal distribution separator programs. For a given choice of s_a , σ^2 , g and the p_i 's, the equations listed in the table can be used to compute the independent histogram parameters s_a , σ^2 , Pr_0 and W . For the same choice of s_a , σ^2 , g and the p_i 's, the computer-simulated planter will produce a histogram of plant spacings. This histogram can then be analyzed with the normal distribution separator to yield the histogram parameters s_a , σ^2 , Pr_0 , Pr_1 and Pr_2 . The $E(s_i)$ and the $V(s_i)$ can be computed from theoretical equations and also from the computer-produced spacings.

In the case of the corn histogram, the distribution separator program has difficulty in determining the small area percentages

TABLE II

COMPARISON OF HISTOGRAM PARAMETER ESTIMATES
 BASED ON THE THEORETICAL EQUATIONS AND A
 STATISTICAL ANALYSIS OF THE HISTOGRAM

Parameter	Theoretical estimate based on figure 3	Normal distribution separator estimate from figure 3	Theoretical estimate based on figure 4	Normal distribution separator estimate from figure 4	Number of equation used
Pr_0	0.009	0.048	0.235	0.234	(30)
Pr_1	0.922	0.872	0.531	0.534	(58)
Pr_2	0.063	0.072	0.162	0.166	(58)
Pr_3	0.004	0.006	0.049	0.052	(58)
W	0.064	0.082	0.305	0.310	(37)
σ	0.125	0.120	0.125	0.125	--
s_a	0.627	0.628	0.800	0.746	--
$E(s_1)$	0.665	0.666	0.914	0.843	(49)
$V(s_1)$	0.059	0.066	0.515	0.441	(56)

associated with the "peaks" at zero and $2s_g$. Another problem arises in the computer-simulated-planter due to the small values of p_0 and p_2 . There were actually five zero drops and thirteen double drops where there should have been ten of each. This difference is not statistically significant but certainly affects the theoretically predicted histogram parameters. The actual values of g and the p_i 's that emerged from the simulated planter were used in the theoretical equations. They were $g = 0.94$, $p_0 = 0.005$, $p_1 = 0.982$, and $p_2 = 0.013$.

In the case of the cucumber histogram, there is good agreement between the histogram parameter estimates.

Hypothesis

A device designed to produce uniform seed spacing, germination, and early growth can be characterized in terms of three independent random variables. They are: the number of seeds that occupy a given seed cell at the time of discharge; the error in seed placement with respect to the desired uniform spacing; and the survival factor. Furthermore, the drop error random variable has a normal probability distribution function.

The computer-simulated planter is proposed as a Monte Carlo Model to predict field plant spacings. Specifically, corn, cucumber, and sugar beet populations will be predicted and tested, one by one, against field observations.

Testing the Hypothesis

It is the author's hope to produce evidence in support of the hypothesis and to provide a plausible and reasonable basis for

acceptance. The actual test will consist of observing field populations, deducing model parameters and predicting a comparable field population based on these model parameters.

The information contained in figures 3, 4, and 5 and in Table I is identical to the type information we can obtain from a real world planter. Figures 6, 7, and 8 show field data from corn, cucumber, and sugar beet stands respectively. The corn and cucumber stands were established using a standard horizontal cell plate corn planter with appropriate plates while the sugar beets were planted with a horizontal cell plate "precision" vegetable planter. Table III shows the autocovariance functions for the different field populations.

By analyzing the field histograms and autocovariance functions, the planter model parameters can be deduced and used to predict the histograms shown in figures 9, 10, and 11. Table IV shows the autocovariance functions for the different computer-predicted field populations.

Figure 6. Frequency histogram of plant spacings from a row of corn.

Figure 7. Frequency histogram of plant spacings from a row of cucumbers.

Figure 8. Frequency histogram of plant spacings from a row of sugar beets.

TABLE III

AUTO-COVARIANCE $C_{ss}(n)$ OF THE
FIELD DATA PLANT SPACINGS

Lag n	$C_{ss}(n)$ for Corn	$C_{ss}(n)$ for Cucumbers	$C_{ss}(n)$ for Sugar Beets
0	0.250	0.914	0.0076
1	-0.085	0.053	-0.0001
2	-0.001	-0.112	-0.0002
3	0.020	-0.107	-0.0001
4	-0.011	0.013	-0.0002
5	0.013	0.072	0.0000
6	-0.006	-0.044	0.0001
7	-0.004	-0.051	-0.0005
8	0.001	0.074	-0.0002
9	-0.025	-0.022	0.0003
10	0.038	0.020	-0.0001

Figure 9. Frequency histogram of plant spacings predicted by computer-simulated planter for a row of corn. Compare with figure 6.

Figure 10. Frequency histogram of plant spacings predicted by computer-simulated planter for a row of cucumbers. Compare with figure 7.

Figure 11. Frequency histogram of plant spacings predicted by computer-simulated planter for a row of sugar beets. Compare with figure 8.

TABLE IV
 AUTOCOVARIANCE $C_{ss}(n)$ OF THE COMPUTER-
 PREDICTED FIELD PLANT SPACINGS

Lag n	$C_{ss}(n)$ for Corn*	$C_{ss}(n)$ for Cucumbers*	$C_{ss}(n)$ for Sugar Beets*
0	0.362	0.895	0.0062
1	-0.044	-0.122	-0.0015
2	0.003	0.078	-0.0007
3	-0.007	-0.004	-0.0003
4	0.023	-0.018	0.0002
5	-0.016	0.019	-0.0004
6	-0.009	-0.038	0.0001
7	-0.003	0.000	0.0001
8	0.010	-0.013	-0.0001
9	-0.001	-0.039	0.0002
10	0.004	-0.007	0.0001

* Compare with TABLE III.

CONCLUSIONS AND SUMMARY

The chi-square test for goodness of fit was employed to provide a statistical basis for accepting or rejecting the hypothesis that the computer-predicted and field histograms were statistically identical. The field histogram was considered the expected histogram, where E_i was the expected number of samples in the i^{th} class, while the predicted histogram was considered the observed histogram, where O_i was the observed number of samples in the i^{th} class. Cramer (1946) suggests that $E_i \geq 10$ in each class; therefore, it was necessary to group the tails of the histograms.

The ten classes chosen for corn were bounded by 0, 0.2, ..., 1.6, 1.8 and ∞ . Eleven cucumber classes were chosen and were bounded by 0, 0.2, ..., 1.2, 1.4, 1.8, 2.2, 2.8 and ∞ while the sixteen sugar beet classes were bounded by 0, 0.02, 0.04, ..., 0.24, 0.26, 0.30, 0.36 and ∞ .

In every case, if the field spacing pdf is equal to the predicted spacing pdf then the quantity Q_{k-1} has an approximate chi-square distribution with $k-1$ degrees of freedom where k is the number of classes and

$$Q_{k-1} = \sum_{i=1}^k \frac{(O_i - E_i)^2}{E_i} . \quad (64)$$

The computational results of equation (64) and conclusions are as follows:

1. Corn ... $Q_9 = 20.84$

The hypothesis cannot be rejected at a significance level of approximately 1.7 percent. It is also interesting to note that of the 20.84 points of Q_9 , 11.87 points are due to the last class, i.e., 1.8 to ∞ . This indicates that the field data are insufficient to accurately determine the tail.

2. Cucumbers ... $Q_{10} = 16.63$

The hypothesis cannot be rejected at a significance level of approximately 8.5 percent.

3. Sugar Beets ... $Q_{15} = 31.26$

The hypothesis cannot be rejected at a significance level of approximately 0.8 percent. As in the case of corn, a significant number of points of Q_{15} is due to the last class. In fact, the 7.36 points attributable to the last class again indicate there are some problems in obtaining a large enough sample to completely define the tail.

In every case, it may be concluded that there is not sufficient evidence to reject any of the hypotheses.

Tables III and IV can be compared directly. $C_{ss}(0)$ is the variance, and beyond one lag there would seem to be no correlations.

Little has been said to this point about the sugar beet data. The computer-simulated planter model will produce sugar beet plantings, but parameter extraction from the field histogram is impossible since σ is of the order of s_a . The parameters used in the computer model to

generate figure 11 were obtained by trial and error. There is no reason to believe that one could not find another set of planter parameters which would lower the chi-square test value between the pdf's of figures 8 and 11, thus increasing the significance at which the test can be reported.

In summary we can see that the four basic objectives of this study have been fulfilled. A stochastic model has been obtained and the model parameters have been related to the resulting plant populations. The plant populations, on the other hand, have been characterized by certain histogram parameters. The variance of the drop error would be the logical choice for specifying the seed placement "precision" of a planter mechanism.

RECOMMENDATIONS FOR FUTURE STUDY

While this study has concentrated on a stochastic planter model, it must be recognized that this objective was not an end in itself. The most obvious extension of this work would be to use the resulting analytic characterizations of plant populations as the basis for investigations into various methods or rules for thinning. One might also investigate the possibility of using a selective thinning mechanism for weed control since the statistical location of the plants can be obtained.

The speculation that a correlation exists between a plant parameter and the plants ability to produce is another subject for future work. If this correlation should be found and a theoretical basis for measuring the plant parameter, a "selective" thinning rule could be chosen to produce a highly productive and properly spaced plant population.

APPENDIXES

Appendix A
Digital Computer Normal and Uniform
Probability Density Function Generators

The computer-simulated planter requires random and uncorrelated samples from both normally and uniformly distributed populations. This appendix serves as justification that these requirements were met.

A computer-produced random sample of 1,000 observations was obtained from each of the two populations in question. Each population sample was divided into two sub-samples of 500 observations by splitting the population sample between the 500th and 501st random observation. For each of the two populations, the first 500 observation sample was analyzed maintaining the sampling order, while the second 500 observation sample was "shuffled" and then analyzed. The "shuffling" was accomplished by sorting, in ascending order, the 6th column to the right of the decimal of the random observation and was intended to destroy any computer-produced periodicities.

Figures 12 and 14 show very close agreement between the observed cumulative pdf's and the theoretical pdf's. Figures 13 and 15 indicate that there were virtually no computer-induced correlations.

Figure 12. Cumulative probability density function for the computer uniform probability density function generator.

Figure 13. Autocovariance of the samples from the computer uniform probability density function generator.

Figure 14. Cumulative probability density function for the computer normal probability function generator.

Figure 15. Autocovariance of the samples from the computer normal probability function generator.

A chi-square test for goodness of fit in figure 12 was formulated to test the hypothesis H_{01} against all alternatives.

$$H_{01} : \Pr [\xi \in A_i] = p_i = \frac{1}{20}, i = 1, 2, \dots, 20,$$

where ξ is a single observation and

$$A_i = \left\{ \xi ; 0.05(i-1) \leq \xi < 0.05i \right\}, i = 1, 2, \dots, 20.$$

Let n observations be made and let q_i be the number of observations falling in A_i , $i = 1, 2, \dots, 20$.

If H_{01} is true the random variable

$$Q_{19} = \sum_{i=1}^{20} \frac{(q_i - np_i)^2}{np_i} \quad (65)$$

has an approximate chi-square distribution with 19 degrees of freedom.

For the total sample of 1,000 observations $Q_{19} = 18.72$ indicating that the data are consistent with H_{01} .

A chi-square test for goodness of fit in figure 14 was also constructed to test the hypothesis H_{02} against all alternatives.

$H_{02} : \zeta$ has a $N(0, 1)$ pdf, ζ being a single computer sample.

If H_{02} is true the random variable

$$Q_{k-1} = \sum_{i=1}^k \frac{(O_i - E_i)^2}{E_i} \quad (66)$$

has an approximate chi-square distribution with $(k-1)$ degrees of

freedom. The expected number of observations in the i^{th} class is E_i while O_i observations made are in the i^{th} class and the total number of classes is k . Cramer (1946) suggests that to correctly apply the chi-square test $E_i \geq 10$ for all i . In other words the tails of the $N(0, 1)$ pdf are gathered so that the expected number of observations under either tail is at least ten. The thirteen classes used were bounded by $-\infty, -2.2, -1.8, -1.4, \dots, 1.4, 1.8, 2.2, \text{ and } \infty$.

For the total sample of 1,000 observations $Q_{12} = 5.26$ again indicating that the data are consistent with H_{02} .

Appendix B
Digital Computer Programs

Program I

The autocovariance and hanning power spectral density program is based on numerical calculations from Bendat and Piersol (1966) and the reader is referred there for a complete discussion of the equations in this program.

```

STATEMENT LISTING FOR PROGRAM I

C   AUTOVARIANCE AND HANNING POWER SPECTRUM
C   THIS PROGRAM WAS ADAPTED FROM EQUATIONS APPEARING IN BENDAT AND
C   PERSOL, MEASUREMENT AND ANALYSIS OF RANDOM DATA, 1966, WILEY AND
C   SONS, INC.
C   DIMENSIONU(1100),X(1100),R(1000),G(1000),GS(1000)
C   READ(5,12)NSETS
C   12 FORMAT(I10)
C   DO11 JJ=1,NSETS
C   READ(5,1)H,L,NPLUS1,C,(X(I),I=1,NPLUS1)
C   THE INPUT PARAMETERS ARE THE SAMPLING INTERVAL H, THE NUMBER OF
C   LAGS FOR WHICH THE AUTOVARIANCE FUNCTION IS TO BE CALCULATED L,
C   THE NUMBER OF SAMPLES IN THE RECORD NPLUS1, A SCALE FACTOR FOR THE
C   SAMPLE VALUE C, AND THE SAMPLE VALUE X(I).
C   1 FORMAT(F10.5,2I10,F15.7/(10F7.3))
C   N=NPLUS1-1
C   SUMU=C.
C   DO2 I=1,N
C   U(I)=X(I+1)-X(I)
C   2 SUMU=SUMU+U(I)
C   FN=N
C   AVGU=SUMU/FN
C   DO3 I=1,N
C   3 X(I)=C*(U(I)-AVGU)
C   SUMS=0.
C   DO4 I=1,N
C   4 SUMS=SUMS+X(I)*X(I)
C   FN1=N-1
C   S=SQRT(SUMS/FN1)
C   WRITE(6,13)AVGU,S
C   13 FORMAT(1H1///7X,7HMEAN = ,F10.5//7X,21HSTANDARD DEVIATION = ,F10.5
C   B/// 12X,42H AUTOVARIANCE AND HANNING POWER SPECTRUM/ 12X,58H
C   B      I      R(LAG)      LAG      GS(F)      F)
C   DO6 J=0,L
C   M=J+1

```

STATEMENT LISTING FOR PROGRAM I, CONTINUED

```

SUMR=0.
R(M)=0.
NL=N-J
DO5 I=1,NL
IJ=I+J
5 SUMR=SUMR+X(I)*X(IJ)
FNJ=N-J
6 R(M)=SUMR/FNJ
FL=L
LI=L-1
L2=L+1
DO7 J=0,L
M=J+1
G(M)=0.
SUMG=0.
FJ=J
DO8 I=2,L
FI=I-1
8 SUMG=SUMG+R(I)*COS((3.1416*FI*FJ)/FL)
7 G(M)=2.*H*(R(1)+2.*SUMG+COS(3.1416*FJ)*R(L2))
GS(1)=0.5*G(1)+0.5*G(2)
DO9 I=2,L
9 GS(I)=0.25*G(I-1)+0.5*G(I)+0.25*G(I+1)
GS(L2)=0.5*G(L)+0.5*G(L2)
DO15 I=1,L2
K=I-1
FK=K
FLAG=FK*H
FR=FK/(2.*H*FL)
15 WRITE(6,10)K,R(I),FLAG,GS(I),FR
10 FORMAT(/12X,I10,4F12.5)
11 CONTINUE
STOP
END

```

Program II

The normal distribution separator program in its present form will fit up to five normal distributions to an observed frequency histogram with maximum likelihood.

The histograms of plant spacings must be reflected about the origin and then translated to the right so that all coordinate values are positive.

The program input requires the number of classes in the histogram, the number of normal distributions to be fitted to the data, the cut-off points, the left hand limits of the classes, and the cumulative frequency to each class limit. The cut-off points are the classes where the subpopulations appear to meet.

The original program was developed by Dr. Victor Hasselblad of the University of Washington and was subsequently modified by Dr. Robert D. Fox of the Fine Particle Physics Pioneering Laboratory, U. S. Department of Agriculture, Wooster, Ohio, to run in Fortran II. Basically the program is still in Fortran II, but it has been reconverted to Fortran IV. Many of the statements could be eliminated by the efficient use of Fortran IV.

STATEMENT LISTING FOR PROGRAM II

```

C      NORMAL DISTRIBUTION SEPARATOR USING STEEPEST DESCENT METHOD
C      THIS PROGRAM WAS ADAPTED FROM HASSELBLAD, TECHNOMETRICS, VOL. 8,
C      AUG. 1966, PP 431 TO 444.
C      DIMENSION Q(100,5), QQ(100), P(5), TU(5), SIG(5), F(101), FS(100), NS(5)
C      DIMENSION PP(5), TUP(5), SIGP(5), NX(5), XB(101)
C      2 READ(5,102) TIT1, TIT2, N, K, (NX(I), I=1,5), LOGNOR
C      N IS THE NO. OF FREQUENCIES, K IS THE NO. OF DISTRIBUTIONS, AND
C      NX(J) ARE THE CUT-OFF POINTS.
C      LOGNOR = 1 IF THE DATA ARE TO BE FITTED WITH A LOG NORMAL CURVE OR
C      LOGNOR = 0 IF THE DATA ARE TO BE FITTED WITH A NORMAL CURVE.
C      102 FORMAT(2A5,8I5)
C      NX(K)=N
C      N1=N+1
C      XB(I) IS THE LOWER LIMIT OF THE I-TH INTERVAL
C      READ(5,4)(XB(I), F(I), I=1, N1)
C      4 FORMAT(14F5.1)
C      IF(LOGNOR) 105, 105, 103
C      103 DO 104 I=1, N1
C      IF(XB(I)) 722, 722, 723
C      722 WRITE(6,3)
C      3 FORMAT(43H TAKING A LOG OF A NEGATIVE NUMBER OR ZERO )
C      GO TO 2
C      723 XB(I)=ALOG10(XB(I))
C      104 CONTINUE
C      105 FSUM=F(N1)
C      DO 107 I=2, N1
C      I1=I-1
C      F(I1)=F(I1)-F(I1)
C      IF(F(I1)) 453, 454, 454
C      454 IF(XB(I1)-XB(I1)) 453, 107, 107
C      453 WRITE(6,455)
C      455 FORMAT(19HNEGATIVE DIFFERENCE)
C      GO TO 2
C      107 CONTINUE

```


STATEMENT LISTING FOR PROGRAM II, CONTINUED

```

      DO 14 I=1,N
14    FS(I)=F(I)
      NB=1
      ASUM =0.
      DO 16 J=1,K
      NE=NX(J)
      A=0.
      B=0.
      C=0.
      I=0
C      COMPUTE INITIAL ESTIMATES USING HALD PROCEDURE
      DO 15 II=NB,NE
      I=I+1
      N1 =NE+1-I
      I1 =NE+2-I
      KK =NB+I
      LLL=NB+I-1
      CI=(XB(I1)+XB(N1))/2.-XB(NB)
      CF=FS(II)*(XB(I1)-XB(N1))/(XB(KK)-XB(LLL))
      A=A+CF
      B=B+CF*CI
      C=C+CF*CI*CI
15    CONTINUE
      X=A*C/(2.*B*B)
      IF(X-0.55) 500,40,40
500   AVE=B/A
      SIG(J)=SQRT ((C-B*B/A)/A)
      X=AVE/SIG(J)
      GO TO 39
40    X1=ALOG (X-0.5)
      Z=SQRT (-2.*X1)
      X=((-5.381936*Z+38.338991)*Z -69.16544)*Z+34.758138)
      Z=((1.6336409*Z -7.0780578)*Z+5.01975036)*Z+1.)
      X = X/Z

```

STATEMENT LISTING FOR PROGRAM II, CONTINUED

```

CALL PROTWO(X,PORM,PHI)
PORM=1.-PORM
X1=1./(PHI/PORM-X)
SIG(J)=B*X1/A
AVE=-X*SIG(J)
X = -X
39 CALL PROTWO(X,PORM,PHI)
PKQ=PORM
P(J)=A/PKQ
ASUM=ASUM+P(J)
N1=N+1
DO 17 KK=2,N1
II=KK+NE -1
IF(II-N)501,501,18
501 CI =(AVE+XB(KK))/SIG(J)
II=KK-1
CF= (AVE+XB(II))/SIG(J)
X=CI
CALL PROTWO(X,PORM,PHI)
CI = PORM
X=CF
CALL PROTWO(X,PORM,PHI)
PKK=CI-PORM
PKK=A*PKK/PKQ
SUBTRACT ESTIMATED JTH SAMPLE FROM REMAINING ONES
IF(FS(II)-PKK)7,7,8
7 FS(II)=0.
GO TO 9
8 FS(II)=FS(II)-PKK
9 IF(PKK-.05)18,17,17
17 CONTINUE
18 NB=NE+1
II=NX(J)
II=II+1

```

STATEMENT LISTING FOR PROGRAM II, CONTINUED

```

      TU(J)=(XB(I1)+XB(I1))/2. -AVE
16  CONTINUE
      DO 19 J=1,K
19  P(J)=P(J)/ASUM
      TL1=0.
C    COMPUTE LIKELIHOOD FUNCTION TL1
      DO 21 I=1,N
      QQ(I)=0.
      I1=I+1
      CI=(XB(I1)+XB(I))/2.
      DO 20 J=1,K
      X1=(XB(I1)-XB(I))/SIG(J)
      Q(I,J)=0.39894228*EXP (-((CI-TU(J))**2)/(2.*SIG(J)**2))*X1
20  QQ(I)=QQ(I)+Q(I,J)*P(J)
      TL1=TL1+F(I)*ALOG10(QQ(I))
21  CONTINUE
C    START METHOD OF STEEPEST DESCENT
      DO 99 II=1,51
      LLL=II
      PSUM=0.
      DO 23 J=1,K
      ASUM=0.
      B=0.
      C=0.
      DO 24 I=1,N
      A=F(I)*Q(I,J)/QQ(I)
      ASUM=ASUM+A
      I1=I+1
      CI=(XB(I1)+XB(I))/2.
      CF=CI-TU(J)
      B=B+A*CI
      C=C+A*CF*CF
24  C=C+A*CF*CF
      TUP(J)=B/ASUM
      SIGP(J)=SQRT(C/ASUM)

```

STATEMENT LISTING FOR PROGRAM II, CONTINUED

```

      PP(J)=ASUM*P(J)
23  PSUM=PSUM+PP(J)
      DO 26 J=1,K
26  PP(J)=PP(J)/PSUM
      TL=0.
      DO 31 I=1,N
      QQ(I)=0.
      I1=I+1
      CI=(XB(I1)+XB(I))/2.
      DO 22 J=1,K
      X1=(XB(I1)-XB(I))/SIGP(J)
      Q(I,J)=0.39894228*EXP (-((CI-TUP(J))**2)/(2.*SIGP(J)**2))*X1.
22  QQ(I)=QQ(I)+Q(I,J)*PP(J)
      TL=TL+F(I)*ALOG10(QQ(I))
31  CONTINUE
C   TEST FOR IMPROVED LIKELIHOOD FUNCTION TL
      IF(TL-TL1)30,502,502
502  DO 25 J=1,K
      P(J)=PP(J)
      TU(J)=TUP(J)
25  SIG(J)=SIGP(J)
      TL1=TL
99  CONTINUE
30  LLL=LLL-1
      DO 37 I=1,N
      QQ(I)=0.
      DO 38 J=1,K
      I1=I+1
      X=(XB(I1)-TU(J))/SIG(J)
      CALL PROTWO(X,PCRM,PHI)
      Z=PORM
      X=(XB(I)-TU(J))/SIG(J)
      CALL PROTWO(X,PORM,PHI)
      C(I,J)=Z-PORM

```

STATEMENT LISTING FOR PROGRAM II, CONTINUED

```

38 QQ(I)=QQ(I)+Q(I,J)*P(J)
   FS(I)=QQ(I)*FSUM
37 CONTINUE
   X=(-TU(1)/SIG(1))
   CALL PROTWO(X,PORM,PHI)
   QQ(1)=QQ(1)+PORM*P(1)
   N1=N+1
   X=(TU(K)-XB(N1))/SIG(K)
   CALL PROTWO(X,PORM,PHI)
   QQ(N)=QQ(N)+PORM*P(K)
   FS(1)=QQ(1)*FSUM
   FS(N)=QQ(N)*FSUM
   KK=TU(1)/(XB(2)-XB(1))+1.
   X1=0.05*FSUM
   IF(X1-10.)114,114,115
115 X1=10.
114 PPR=0.
C   COLLAPSE CHI-SQUARE TABLE
   DO 200 I=1,KK
   II=I
   PPR=PPR+FS(I)
   IF(PPR-X1)200,201,201
200 CONTINUE
201 NLOW=II
   N1=N+1
   KK=(XB(N1)-TU(K))/(XB(N1)-XB(N))+1.
   PPS=0.
   DO 203 I=1,KK
   II=N-I+1
   PPS=PPS+FS(II)
   IF(PPS-X1)203,204,204
203 CONTINUE
204 NTOP=II
   AFR=0.

```

STATEMENT LISTING FOR PROGRAM II, CONTINUED

```

DO 210 I=1,NLOW
210 AFR=AFR+F(I)
   CHIS=(AFR-PPR)**2/PPR
   N1 =NTOP-1
   I1 =NLOW+1
DO 211 I=I1,N1
211 CHIS=CHIS+(F(I)-FS(I))**2/FS(I)
   AFR=0.
DO 213 I=NTOP,N
213 AFR=AFR+F(I)
   CHIS=CHIS+(AFR-PPS)**2/PPS
   ND=NTOP-NLOW-3*K+1
   WRITE OUT RESULTS
   N1 =FSUM+0.5
   IF(LOGNOR)112,112,111
111 WRITE(6,410)TIT1,TIT2
410 FORMAT(28HRESULTS FOR DATA IDENT. NO. ,2A5,
132H USING LOG-NORMAL DISTRIBUTIONS )
   GO TO 113
112 WRITE(6,408)TIT1,TIT2
408 FORMAT(29H1RESULTS FOR DATA IDENT. NO. ,2A5,
128H USING NORMAL DISTRIBUTIONS )
113 WRITE(6,401)LLL,N1
401 FORMAT(13H VALUES AFTER,15,11H ITERATIONS,10X,
1 19HTOTAL SAMPLE SIZE =, I5)
   WRITE(6,416)
416 FORMAT(6H GROUP,5X,4HMEAN,8X,8HVARIANCE,5X,7HPERCENT,4X,4HSIZE)
DO 402 J=1,K
IF(LOGNOR)109,109,108
108 A=EXP (SIG(J)*2.30259)
   A=A*A
   TU(J)=EXP (TU(J)*2.30259)
   GO TO 110
109 A=SIG(J)*SIG(J)

```

STATEMENT LISTING FOR PROGRAM II, CONTINUED

```

110 B=P(J)*FSUM
    C=P(J)*100.
    WRITE(6,94)J,TU(J),A,C,B
94  FORMAT (I3,F13.5,F14.6,F11.2,F9.1)
402 CONTINUE
    WRITE(6,405)
405 FORMAT(47H COMPARISON OF CALCULATED FREQUENCIES WITH DATA)
    WRITE(6,406)
406 FORMAT(19H0INTERVAL X-VALUE ,5X,20HCALCULATED FREQUENCY,
    15X,14HDATA FREQUENCY,5X,11HCAL. - DATA)
    DO 106 I=1,N
    X1=FS(I)-F(I)
    IF(LOGNOR)450,450,451
451 WRITE(6,452)I,XB(I),FS(I),F(I),X1
452 FORMAT(I4,3X,F8.4,15X,F8.1,14X,F8.1,9X,F8.1)
    GO TO 106
450 WRITE(6,407)I,XB(I),FS(I),F(I),X1
407 FORMAT(I4,3X,F8.1,15X,F8.1,14X,F8.1,9X,F8.1)
106 CONTINUE
    WRITE(6,404)
404 FORMAT(1CX,14HCUT-OFF POINTS)
    WRITE(6,81)(NX(I),I=1,5)
81  FORMAT(5I5)
    WRITE(6,403)NLOW,NTOP,ND
403 FORMAT(25H LOWER COLLAPSING POINT =,I3, 16HUPPER COLLAPSING,
    18H POINT =,I3,3X,16HDEG OF FREEDOM =,I3)
C    FROM ABRAMOWITZ P941.
    X=CHIS
    ASUM=0.
    IF(X)430,430,431
430 CHISQ=1.
    GO TO 432
431 IF (ND)433,433,434
433 CHISQ=1.

```

STATEMENT LISTING FOR PROGRAM II, CONTINUED

```

      GO TO 432
434 IF (ND/2*2-ND)435,436,435
436 IF (ND-2)437,437,438
438 KK=ND/2-1
      TL=1.
      DO 439 I=1, KK
      CI=I
      CI=2.*CI
      TL=TL*X/CI
439 ASUM=ASUM+TL
437 CHISQ=EXP (-X/2.)*(1.+ASUM)
      GO TO 432
435 TL' =1./X
      IF (ND-1)440,440,441
441 KK=(ND-1)/2
      DO 442 I=1, KK
      CI=I
      CI=2.*CI-1.
      TL=TL*X/CI
442 ASUM=ASUM+TL
440 X=SGRT (X)
      CALL PROTWO(X,PORM,PHI)
      CHISQ=2.-2.*PORM+X*EXP (-X*X/2.)*0.7978846*ASUM
432 WRITE(6,409)CHIS,CHISQ,TL1
409 FORMAT(19H CHI SQUARE VALUE =,F8.3,5X,7HPROB. =,F6.3,3X,
119HLOG OF LIKELIHOOD =,E12.4,1H-)
      GO TO 2
      END
      SUBROUTINE PROTWO(X,PORM,PHI)
      C      HANDBOOK OF MATHEMATICAL FUNCTIONS BERAMOWITZ P932
      X1=X
      IF (X1)223,224,224
223 X1=-X1
224 X1=1./(1.+X1*0.7978846)

```


STATEMENT LISTING FOR PROGRAM II, CONTINUED

```
PHI=0.39894228*EXP (-X*X/2.)  
PORM=1.781477937+X1*(-1.821255978+X1*1.330274429)  
PORM=X1*(0.319381530+X1*(-.356563782+X1*PORM))*PHI  
IF (X)220,220,221  
221 PORM=1.-PORM  
220 CONTINUE  
RETURN  
END
```

Program III

The basic operation of the computer-simulated planter was outlined in the body of this dissertation, and the reader is referred to that section. The program itself is written in SCATLAN and is included in the interest of completeness since the program cannot be used any place but at the Ohio State University Computer Center. However, anyone with a basic knowledge of computer programming can easily convert the program to another language.

STATEMENT LISTING FOR PROGRAM III

```

C      A COMPUTER-SIMULATED PLANTER -
C
C      THIS PROGRAM WAS DEVELOPED BY R. ROHRBACH, THE OHIO STATE -
C      UNIVERSITY, COLUMBUS, OHIO, 1967-1968. -
C      START      READINPUT,AIN, (AVGX,VARX, (NS(I),I=1,1,I.LE.100),(GER(I),I=
C                  1,1,I.LE.100)) -
C      AVGX IS THE INTENDED UNIFORM PLANT SPACING, VARX IS THE -
C      STANDARD DEVIATION OF THE NORMALLY DISTRIBUTED DROP ERROR, -
C      NS(I) IS THE NUMBER OF SEEDS PER CELL RANDOM VARIABLE, AND -
C      GER(I) IS THE RANDOM SURVIVAL VARIABLE. -
C      F AIN      (2F10.5,4(/5011))-
C                  DIMENSION(X(2501),NS(101),GER(101),Y(2501),XX(1501))-
C                  INTEGER(GER)-
C                  PCTNS=0.0-
C                  PCTGER=0.0-
C                  DOTHROUGH(LOOP0),I=1,1,I.LE.100-
C                  PCTNS=PCTNS+NS(I)-
C                  PCTGER=PCTGER+GER(I)-
C                  NPLTSR=1001-
C                  N=0-
C                  K0=0-
C                  K1=0-
C                  K2=0-
C                  K3=0-
C                  K4=0-
C                  DOTHROUGH(LOOP1),I=0,1,I.LE.1000-
C                  X(I)=AVGX*I-
C                  CALL FUNCTION(Z)=FRANUD.( ) -
C                  MSUB=100*Z+1.0-
C                  NSEEDS=NS(MSUB)-
C                  TRANSFERTO(TR1,T, TR3)PRINTED(NSEEDS-1)-
C                  X(I)=0.0-
C                  K0=K0+1-
C                  TRANSFERTO(LOOP1)-
C
C      TR1

```

STATEMENT LISTING FOR PROGRAM III, CONTINUED

```

TR2      CALL FUNCTION(DEIXI)=RANND.()-
          X(I)=X(I)+DELXI*VARX-
          K1=K1+1-
          TRANSFERTO(LOOP1)-
TR3      TRANSFERTO(TR4,TR5,TR6)PROVIDED(NSEEDS-3)-
TR4      CALL FUNCTION(DEIXI)=RANND.()-
          CALL FUNCTION(DELXJ)=RANND.()-
          X1=X(I)-
          X(I)=X(I)+DELXI*VARX-
          XX(N)=X1+DELXJ*VARX-
          N=N+1-
          K2=K2+1-
          TRANSFERTO(LOOP1)-
TR5      CALL FUNCTION(DEIXI)=RANND.()-
          CALL FUNCTION(DELXJ)=RANND.()-
          CALL FUNCTION(DELXK)=RANND.()-
          X1=X(I)-
          X2=X(I)-
          X(I)=X(I)+DELXI*VARX-
          XX(N)=X1+DELXJ*VARX-
          XX(N+1)=X2+DELXK*VARX-
          N=N+2-
          K3=K3+1-
          TRANSFERTO(LOOP1)-
TR6      CALL FUNCTION(DEIXI)=RANND.()-
          CALL FUNCTION(DELXJ)=RANND.()-
          CALL FUNCTION(DELXK)=RANND.()-
          CALL FUNCTION(DELXL)=RANND.()-
          X1=X(I)-
          X2=X(I)-
          X3=X(I)-
          X(I)=X(I)+DELXI*VARX-
          XX(N)=X1+DELXJ*VARX-
          XX(N+1)=X2+DELXK*VARX-

```

STATEMENT LISTING FOR PROGRAM III, CONTINUED

```

      XX(N+2)=X3+DELXL*VARX-
      N=N+3-
      K4=K4+1-
      CONTINUE-
      WRITEOUTPUT,AOUT,(K0,K1,K2,K3,K4)-
      (8X,Q*COMPUTER SIMULATED PLANTER STATISTICS*//      8X,I4,
      Q* = NUMBER OF CELLS THAT PLANTED 0 SEEDS. * //      8X,I4,
      Q* = NUMBER OF CELLS THAT PLANTED 1 SEED. * //      8X,I4,
      Q* = NUMBER OF CELLS THAT PLANTED 2 SEEDS. * //      8X,I4,
      Q* = NUMBER OF CELLS THAT PLANTED 3 SEEDS. * //      8X,I4,
      Q* = NUMBER OF CELLS THAT PLANTED 4 SEEDS. * //      8X,I4,
      DOTHROUGH(LOOP2),I=1001,1,I.LE.N+1001-
      X(I)=XX(I-1001)-
      L=0-
      DOTHROUGH(LOOP3),I=0,1,I.LE.N+1001-
      TRANSFERTO(LOOP3)PROVIDED(X(I).E.0.0)-
      CALL FUNCTION(Z)=FRANUD.()-
      MSUB=100*Z+1.0-
      GERNO=GER(MSUB)-
      TRANSFERTO(LOOP3)PROVIDED(GERNO.E.1.0)-
      X(I)=0.0-
      L=L+1-
      CONTINUE-
      M=0-
      DOTHROUGH(LOOP4),I=0,1,I.LE.1001+N-
      TRANSFERTO(LOOP4)PROVIDED(X(I).E.0.0)-
      Y(N)=X(I)-
      M=M+1-
      CONTINUE-
      DOTHROUGH(LOOP5),I=0,1,I.LE.M-1-
      DOTHROUGH(LOOP5),J=I+1,1,J.LE.M-
      TRANSFERTO(LOOP5)PROVIDED(Y(I).LE.Y(J))-
      TEMP=Y(I)-
      Y(I)=Y(J)-

```

LOOP1

F AOUT

LOOP2

LOOP3

LOOP4

STATEMENT LISTING FOR PROGRAM III, CONTINUED

```

      Y(J)=TEMP-
      CONTINUE-
      PUNCHCARDS,OUT,(M+1,(Y(I),I=0,1,I.LE.M))-
      (I5/(10F7.3))-
      DOTHROUGH(LOOP6),I=1,1,I.LE.M-
      X(I)=Y(I)-Y(I-1)-
      D1=1000*AVGX-
      D2=Y(M)-Y(C)-
      RAVGX=D2/(M-1)-
      MM1=K1+2*K2+3*K3+4*K4-
      WRITEOUTPUT,BOUT,(MM1,M)-
      (8X,Q*THE TOTAL NUMBER OF SEEDS PLANTED WAS *,I5/
      8X,Q*WHILE THE NUMBER OF SEEDS THAT GERMINATED WAS *,I4
      ,Q*.)-
      WRITEOUTPUT,COUT,(NPLTSR,D1,AVGX,M ,D2,RAVGX)-
      (8X,Q*THE COMPUTER SIMULATED PLANTER WAS ASKED TO PLANT*,I5
      /8X,Q*PLANTS IN A *,F6.1,Q* FOOT ROW AT AN AVERAGE SPACING
      OF *,F5.3/8X, Q*FEET. IT ACTUALLY PLANTED *,I4,Q* PLANTS IN
      A *,F6.1,Q* FOOT ROW*/8X,Q*AT AN AVERAGE SPACING OF *,F5.3,
      Q* FEET.*/)-
      WRITEOUTPUT,DOUT,(PCTNS,PCTGER,VARX)-
      (8X,Q*THE PERCENT THEORETICAL DROP WAS* ,F6.1/8X,Q*THE PER
      CENT GERMINATION WAS*,F6.1/8X,Q*THE VARIANCE WAS*,F6.3)-
      DOTHROUGH(LOOP7),I=1,1,I.LE.M-1-
      DOTHROUGH(LOOP7),J=I+1,1,J.LE.M-
      TRANSFERTO(LOOP7)PROVIDED(X(I).LE.X(J))-
      TEMP=X(I)-
      X(I)=X(J)-
      X(J)=TEMP-
      CONTINUE-
      WRITEOUTPUT,EOUT,((X(I),I=1,1,I.LE.M))-
      (1H1/7X,53HSORTED VALUES OF PLANT SPACING FROM SIMULATED PLA
      NTER/7X,40H(READ FROM LEFT TO RIGHT, TOP TO BOTTOM)/26(7X,6
      F10.2)/(1H1/7X,64HSORTED VALUES OF PLANT SPACING FROM SIMUL

```

STATEMENT LISTING FOR PROGRAM III, CONTINUED

```

      ATED PLANTER, CONT.      /7X,40H(READ FROM LEFT TO RIGHT, TOP
      TO BOTTOM)/26(/7X,6F10.2/)))-
      BSTOP=0.0-
      USTOP=0.1-
      LSTOP=0-
      WRITEOUTPUT,FOUT-
      (1H1,7X,Q* THERE WERE      INTERVALS BETWEEN      AND *)-
      X(M+1)=X(M)+0.1-
      DOTHROUGH(LOOP8),I=1,1,I.LE.M+1-
      TRANSFERTO(LOOP8)PROVIDED(X(I).L.USTOP)-
      I=I-1-
      NUMBER=I-LSTOP-
      WRITEOUTPUT,GOUT,(NUMBER,BSTOP,USTOP)-
      (19X,14,19X,F4.1,5X,F4.1)-
      BSTOP=BSTOP+0.1-
      USTOP=USTOP+0.1-
      LSTOP=I-
      CONTINUE-
      CALL SUBROUTINE( )=ENDJOB.( )-
      END PROGRAM (START)-

```

F FOUT

F GOUT

LOOP8

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