SPECTRALLY SHAPED GENERALIZED MULTITONE DIRECT SEQUENCE
SPREAD SPECTRUM

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Abstract

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In the work of this dissertation, we propose a new modulation scheme called spectrally-shaped generalized multitone direct-sequence spread spectrum (SSG-MT-DS-SS) which is the generalized multitone/multicarrier-DS-SS (MC/MT-DS-SS) scheme. With SSG-MT-DS-SS, we can shape the spectrum of the transmitted signal without using any kind of pulse shape filter. The spectral shaping is achieved via allowing the data rate, bit energy, processing gain, and carrier frequency of each subcarrier to be adjustable. Specifically, the parameters of each subcarrier are not constrained to be identical as conventional MC/MT-DS-SS schemes. By varying these parameters, we alter the energy distribution of the SSG-MT-DS-SS signal, and, hence, the spectrum of the transmitted signal. In this dissertation, we investigate the spectral shaping mechanism and the shaping capabilities of SSG-MT-DS-SS. We provide a general form of the power spectrum density of SSG-MT-DS-SS with arbitrary parameters on each subcarrier, and develop a numerical method to select the parameters of SSG-MT-DS-SS according to some given design requirements. We derive the general form of the bit error probability of SSG-MT-DS-SS in additive white Gaussian noise channel and the dispersive channel. In addition, we also investigate the error performance of SSG-MT-DS-SS with sinusoidal chip waveforms on each subcarrier. It is shown that with the properly selected sinusoidal chip waveform and the dual sidebands combining scheme, we can improve the error

performance of SSG-MT-DS-SS in the dispersive channel. The use of error correction code, e.g., Hamming code, and the peak to average power ratio of the SSG-MT-DS-SS are also investigated in this dissertation.

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Chapter 1. Introduction

In this dissertation, we investigate a new modulation scheme called spectrally shaped generalized multitone direct sequence spread spectrum (SSG-MT-DS-SS). This includes a complete and precise mathematical definition of our modulation scheme, its spectral shaping capabilities, bit error performance in additive Gaussian white noise (AWGN) and dispersive fading channels for both single- and multi-user conditions. In addition to the basic SSG-MT-DS-SS modulation, we also investigate an extended form of SSG-MT-DS-SS that can yield increased diversity in dispersive fading wireless applications. In this chapter we provide background information on wireless communications and its recent growth, followed by the aims and outline of this dissertation.

1.1 Evolution of Cellular Networks

The growth in cellular radio has inspired much research and development in all aspects of wireless communications. The wireless cellular network, which is based on the idea of breaking a large service area into small “cells,” was first used in the analog first generation (1G) cellular network in the early 1980s. By breaking the large service area into small cells, the same carrier frequency can be shared by different, geographically separated cells, yielding an increased spectral efficiency for the whole cellular network, especially when the cell size is small. The commercially implemented 1G systems were the Advanced Mobile Phone Systems (AMPS) for North America, and the variety of Total Access Communication Systems (TACS) for Europe and the rest of

the world [1]. All these systems employed analog modulation and demodulation techniques. These systems were gradually replaced by the 2G systems where digital technology was used because of the tremendous advances in integrated circuit (IC) technology, and digital signal processing (DSP) technologies. The use of digital technologies brought the 2G systems many advantages over 1G systems, such as higher system capacity, better quality of the service (QoS), lower equipment cost, etc. [1].

In 1992 the first digital cellular system, the Global System for Mobile Communications (GSM) was implemented in Europe. In North America, although the GSM technology is used, most 2G systems are based on code division multiple access (CDMA) technology. These CDMA systems use a modulation technique called spread spectrum, described in detail in the sequel.

The 2G networks, either GSM or CDMA, offer both voice and (low rate) data services (9.6 kbps or 14.4 kbps). However, these 2G networks can not satisfy the increasing demands consumers have for high rate data services. The “2.5G” technologies, as an intermediate step towards 3G, were proposed to increase the data rate of these 2G systems. For example, the General Packet Radio Standard (GPRS), which increases the data rate of GSM up to 115 kbps, and the Enhanced Data rates for GSM Evolution (EDGE) standard, which is able to provide up to 384 kbps service to mobile applications, were both proposed to increase the data capacity of GSM systems [12]-[14]. For CDMA systems, the CDMA2000 1X standard was developed to enable delivery of data services at rates up to 144 kbps [12]-[14]

A current goal of the cellular network evolution is to upgrade the 2G networks into 3G networks, whose specifications were defined by the International

Telecommunication Union (ITU), under the International Mobile Telecommunications-2000 (IMT-2000) program. There are three international standards for 3G systems: CDMA2000, Wideband CDMA (WCDMA) and Time Division Synchronous CDMA (TDS-CDMA) [8]-[12]. All these 3G standards are based on CDMA technology, the subject of later section.

1.2 Other Wireless Systems

For completeness, we provide a brief discussion on other wireless systems in this section. In addition to terrestrial cellular networks, there are of course other wireless communication systems. One class of these systems is satellite communication systems which were introduced in the 1950s, and are now still in use. The satellite system can provide all the services provided by land based systems, e.g., voice, data, broadcasting (TV and radio), etc. [21]. For fixed point (non-mobile) telephone (voice) service, the telephone signal is first sent to an earth station where the signal is then sent to the satellite that forwards the signal to the designated earth station. The designated earth station then sends the received signal to the destination fixed telephone via land lines, or microwave point-to-point relay. For mobile telephone service, the earth stations at both sending and receiving sides may be omitted, and the telephones directly communicate with the satellite(s). In this scenario, the satellite is similar to the base station in terrestrial networks. An example of such system is the Iridium satellite system [10], which provides voice and data services for global coverage of the earth via 66 low-earth orbiting (LEO) satellites.

Wireless local area networks (WLANs) are another type of wireless systems that connect two or more computers without using wires. The WLAN systems utilize wireless technology such as spread spectrum or orthogonal frequency division multiplexing (OFDM) to enable two-way communication in a given area, usually indoor, and they are most commonly used to provide computers with high speed data capability. The presently popular WLAN systems are IEEE 802.11 (WiFi), IEEE 802.15 (Bluetooth), and IEEE 802.16 (WiMax) based systems. More information about WLANs can be found in [11].

1.3 Duplexing

Duplexing is the means by which resources are divided for two terminals to communicate to each other. Most modern communication systems can be categorized into either half duplex and/or full duplex systems. For half duplex systems, the communication between two terminals is allowed for both directions, but only in one direction at a time. For example, terminal A and B can communicate to each other, but at any given time only the transmission from A to B or from B to A is allowed. A typical example of half duplex is the “walkie-talkie” type of application, where communication is allowed in only one direction at any given time. The full duplex systems allow the two terminals to communicate to each other simultaneously. All telephone systems, wired and wireless, are full duplex systems.

Full duplex is achieved by establishing two separate links for terminals A and B at the same time, i.e., a link from terminal A to B and a link from terminal B to A . One way to achieve this is to assign two non-overlapped frequency channels to the two links,

resulting in frequency division duplex (FDD). One example of FDD in wireless communication is GSM-900, which uses the frequency band of 935-965 MHz for communications from base stations (BS) to mobile stations (MS), and uses the frequency band of 890-915 MHz for communications from MS to BS. Besides FDD, full duplex can also be achieved by assigning different time slots to the two links, resulting in time division duplex (TDD). Since the time slots in TDD systems are of very short duration, the communications between two terminals appear to take place simultaneously even though the communications from A to B and from B to A are not strictly simultaneous. In TD-SCDMA, one of the 3G standards, TDD is used to achieve full duplex and the time slots for downlink (BS to MS) and uplink (MS to BS) can also be adjusted dynamically, which brings the system the capability to easily support asymmetric rate services.

1.4 Code Division Multiple Access (CDMA)

As mentioned in previous sections, the 3G standards are all based on CDMA technology. In this section we give a brief introduction to the principles of CDMA, one of several possible multiple access techniques. The multiple access techniques control the user's access to the available radio frequency (RF) resources (time slots, frequency, code) such that all users can gain access to the RF resources for some period of time. By properly assigning the RF resources to each user, multiple users can be supported by the communication system. The primary multiple access techniques can be categorized as time division multiple access (TDMA), frequency division multiple access (FDMA), and code division multiple access (CDMA). The FDMA approach is perhaps the simplest and oldest multiple access technique, as it dates back to use in early mobile systems;

frequency division also has a long history of use in analog broadcast radio systems. For FDMA systems, multiple users are accommodated via assigning different non-overlapped frequency channels to each one of them. For TDMA systems, different users use different, non-overlapping time slots to transmit their signal without interfering with other users. This, of course, requires centralized timing control that precisely adjusts the transmission timing of each user signal. The CDMA systems, in contrast, utilize the whole allocated spectral band for the entire transmission time, e.g., for IS-95, a CDMA based 2G cellular standard, the whole downlink bandwidth of 824-849 MHz is used for BSs to send signals to active users. The way that commercial CDMA systems distinguish users is via the principles of direct sequence spread spectrum (DS-SS) technique, where each user's narrowband data signal is spread in frequency by a user-unique spreading code. The spreading results in the user signal occupying a much wider bandwidth than required for its information content. The spreading also yields a lower power spectral density level. The spread spectrum technique can also be achieved via frequency hopping (FH), where the carrier frequency of the signal is changed during transmission. The FH-SS technique has been mainly used for military applications.

The spread spectrum signaling approach was first proposed for military communication systems in World War II [7], where signal security was of great importance. The use of spread spectrum brings communication systems many advantages over non-spread systems. For example, the spread spectrum systems are robust to interference, intentional (jammers), and because of the noise-like signal structure, the DS-SS signal is difficult to be detected and intercepted by unauthorized users that do not have knowledge of the spreading code used at the transmitter [25].

Besides the increased security level of the DS-SS signaling scheme, its wide bandwidth helps it combat the multipath fading, which is often a dominant contributor to system performance degradation in wireless communications. Specifically, because of the wide bandwidth of the transmitted signal, the receiver is able to resolve different propagation paths, and when these resolvable paths are combined coherently, i.e., maximal ratio combined (MRC), the performance is greatly improved in such multipath environments. Due to these attractive properties of DS-SS, all the 3G standards are based on DS-CDMA technology.

For DS-CDMA systems, when transmitting, the user signals occupy the whole one-way (or two-way for TDD) bandwidth; thus, each user acts as interference to other users. As a result, the capacity of a CDMA system is limited by the number of active users, or by the multiple access interference (MAI).

To attack this limitation of CDMA, researchers looked at treating the MAI not as an unstructured interference, or “noise-like” signal, but as a signal with some known properties that could be exploited for improved performance. This led to so-called multi-user detection techniques [26]. The maximum likelihood sequence estimation (MLSE) multi user detection (MUD) technique was proposed in [27] to optimally remove all interference from all users. The algorithm proposed by [27], which uses the Viterbi algorithm to detect the transmitted data of all users jointly, is the (sequence) optimal MUD algorithm. Unfortunately, the complexity of this optimal MUD is an exponential function of the number of users, i.e., the computational complexity of MLSE increases exponentially as the number of user increases, which largely prohibits its implementation in real systems.

To reduce computational complexity, some sub-optimal MUD schemes have been proposed. This includes the “subtractive” schemes such as successive interference cancellation [28] and parallel interference cancellation [29]. In these schemes, the interfering users' signals are estimated, re-generated and subtracted from the received signal. The re-generation of all other users' signals requires the receiver to have accurate parameter estimates for all user signals, and, most importantly, is rather computationally expensive. Specifically, the computational complexity of SIC is a linear function of the number of users, and it is a linear function of the product of the number of users and the number of iterations for PIC. The computational complexity of both schemes, SIC and PIC, is not as intensive as that for the MLSE scheme.

The MAI from different users can also be treated analogously to inter-symbol-interference (ISI) from the same user [26]. Thus, ISI combating techniques can be used to eliminate the effect of MAI which results in two multi-user detectors: decorrelating detectors and minimum mean squared error (MMSE) detectors. The decorrelating detector, similar to zero forcing (ZF) equalizers, can completely eliminate the MAI. It was shown the decorrelating detector has many attractive properties [30], e.g., no amplitude information is needed, it is robust in near-far environments, etc. However, since it removes the MAI without considering the power of background noise and the correlations between the desired and interfering users, in some scenarios, it enhances the background noise, which greatly degrades its error performance. To circumvent the disadvantage of the noise enhancement of the decorrelating detector, the MMSE detector was proposed [31]. The MMSE detector, similar to a MMSE equalizer, balances the

background noise and signal power, and, as a result, has a better error probability performance than the decorrelating detector.

1.5 Multicarrier Modulation

Since DS-CDMA systems are widely used in commercial cellular networks, they are strong potential candidates for future wireless communication schemes, e.g., 3G and future networks. Despite its advantages, CDMA does have some drawbacks. One of its drawbacks is the potential susceptibility to ISI. For high data rate applications, i.e., multimedia and high speed internet, the symbol duration is much shorter than that of the channel dispersion. Although the RAKE receiver can help to combat the channel dispersion, a large number of RAKE fingers are needed to capture all or the majority of the energy dispersed in the channel for these high data rate applications. Thus the complexity and cost of the receiver is greatly increased in this scenario. Researchers are thus looking for methods to combat the ISI. One way is to use an equalizer, either in the time or frequency domain, to reduce the ISI, which, as with a RAKE, will inevitably increase the system complexity and cost.

Another way to overcome the ISI problem is to increase the symbol duration. This can be done via transmitting the signal through multiple carriers. One of the multicarrier transmission schemes recently studied is Orthogonal Frequency Division Multiplexing (OFDM) [35]. In the OFDM signaling scheme, the high rate data streams are first serial to parallel (S:P) converted to form a number of lower rate data streams, and these low rate streams are transmitted by orthogonal subcarriers, as shown in Figure 1-1. This S:P conversion will substantially increase the symbol duration, and when the

symbol duration is much longer than the channel dispersion, the effect of ISI introduced by the channel dispersion is greatly reduced. As a result, the equalizer at the receiver is simplified, or can even be avoided.

Figure 1-1: Block diagram of OFDM transmitter

The use of OFDM did not gain much attention until 1971 when Weinstein and Ebert presented their work on using the Fourier transform (FT) instead of a bank of sub-carrier oscillators to implement the OFDM signaling scheme. In their work they also introduced the idea of using a guard time between the transmitted symbols to largely avoid the effect of ISI.

With further advances in DSP techniques, OFDM can now be very efficiently implemented using the fast Fourier Transform (FFT). Recently, the OFDM scheme has been widely used for both wired and wireless applications. For instance, OFDM was selected for the European Digital Video Broadcasting standard [11], [18], several Wireless Local Area Network (WLAN) standards IEEE 802.11a/g/n [19] and future fixed wireless access standards IEEE 802.16a/e [20].

Recently, researchers have focused on combining the DS-CDMA and OFDM techniques to bring to communication systems the advantages of both. The combination results in (at least) three types of modulation schemes: MC-CDMA, MC-DS-CDMA, and MT-DS-CDMA [37]. Research shows that these multicarrier spread spectrum modulation schemes are robust to channel fading as well as narrow band interference [2], which makes them very promising for future wireless communication systems. Despite their performance advantages though, these multicarrier modulation schemes can suffer from a large peak to average power ratio (PAPR) and sensitivity to synchronization errors (frequency offsets). In the next chapter we give a detailed literature review on MC-SS system configurations, PAPR reduction methods, and synchronization algorithms for these multicarrier schemes.

Recent research also shows that MC/MT-DS-CDMA is very likely capable of supporting the requirements of next generation wireless networks. However, the MC/MT-DS-CDMA systems, even the “generalized” MC-DS-CDMA proposed by [38], [39], employ common parameter settings for all the subcarriers. Specifically, they employ identical data rates, chip rates, amplitudes, and frequency separations on each subcarrier. This will inherently limit the flexibility of the system design. In this dissertation we propose the SSG-MT-DS-SS modulation, which is a generalized MC/MT-DS-SS scheme that can bring to system design a new level of flexibility.

1.6 Dissertation Objective

In the work of this dissertation, we propose a new MC-SS scheme named spectrally shaped generalized multitone direct sequence spectrum (SSG-MT-DS-SS).

This scheme allows all the parameters of MC-SS to be adjustable, or selectable. By adjusting these parameters, we can freely shape the spectrum of the transmitted signal without using any kind of pulse shaping filter. This dissertation investigates this proposed scheme in terms of its spectral shaping capabilities, PAPR distribution, and error performance in AWGN and dispersive channels, with single and multiple users. All of this is done with simple rectangular pulses for all digital waveforms. To extend our schemes' capabilities, we also investigate the error performance of SSG-MT-DS-SS with sinusoidal chip waveforms. The sinusoidal chip waveform is used for each subcarrier of SSG-MT-DS-SS such that with a properly designed chip waveform and receiver, the error performance of SSG-MT-DS-SS with sinusoidal chip waveforms outperforms conventional MC-DS-CDMA in dispersive fading channels.

The remainder of this dissertation is organized as follows: Chapter 2 reviews related work on various types of MC-SS; Chapter 3 introduces the basic idea of SSG-MT-DS-SS, and describes its spectral shaping mechanism, error performance in an AWGN channel, its PAPR distribution, and possible error correction coding schemes that can complement and enhance its performance. Chapter 4 investigates the error performance of SSG-MT-DS-SS in dispersive fading channels and proposes design guidelines for such cases. In Chapter 5 we investigate the error probability performance of SSG-MT-DS-SS with our sinusoidal chip waveform in dispersive channels, and Chapter 6 summarizes the dissertation work, and outlines future work.

Chapter 2. Literature Review

In this chapter we review the related research on the topic of MC-SS, the modulation scheme that combines OFDM and DS-SS. As mentioned in Chapter 1, DS-SS can be used to support service for multiple users, via CDMA. This multiple access capability is accomplished via assigning a user-specific spreading code to each user. The OFDM technique, widely used for both wired and wireless applications, is a modulation scheme rather than a multiple access scheme. Thus, when multiuser service is needed, TDMA, FDMA or CDMA must be used to provide multiple access capability to the OFDM based systems. Example OFDM systems that use TDMA and FDMA can be found in the recent IEEE 802.16e standard [20]. In the quest for ever-better techniques, researchers have recently focused on combining the DS-CDMA and OFDM techniques to bring to communication systems the advantages of both. The combination of DS-CDMA and OFDM, often termed multicarrier CDMA, results in (at least) three types of modulation schemes: MC-CDMA, MC-DS-CDMA, and MT-DS-CDMA [37].

The three multicarrier CDMA schemes, MC-CDMA, MC-DS-CDMA, and MT-DS-CDMA, can be categorized into two types, depending on how the spreading is performed. For the MC-CDMA scheme, in a sense, the spectrum is spread in “frequency domain,” where the input data stream is spread by the spreading code, and each chip of the spread signal is transmitted via orthogonal subcarriers [37]. In contrast to spreading the spectrum in the “frequency domain,” the MC-DS-CDMA and MT-DS-CDMA schemes spread the spectrum in the “time domain,” where the input data stream is serial-to-parallel (S:P) converted to form the data stream of each subcarrier, and then these data streams are DS spread and transmitted through (possibly orthogonal) subcarriers. The

block diagrams of the MC-CDMA and MC/MT-DS-CDMA transmitters are shown in Figure 2-1. Note that in Figure 2-1 (a), $c(n)$ is the n^{th} chip of the spreading code, and the processing gain is M for MC-CDMA. The chip value $c(n)$ is a constant. In Figure 2-1 (b), $c(t)$ is the spreading code waveform, defined as $c(t) = \sum_{n=0}^{N-1} c(n)P_{T_c}(t - nT_c)$, where N is the processing gain of each subcarrier and $P_{T_c}(t)$ is the chip waveform. It is worth noting that the MC-DS-CDMA and MT-DS-CDMA schemes have asimilar transmitter structure (as shown in Figure 2-1 (b)), where the only difference between these two schemes is the intersubcarrier spacing. For MC-DS-CDMA, the intersubcarrier spacing for adjacent subcarriers is the chip rate of each subcarrier, whereas the intersubcarrier spacing is the data rate of each subcarrier for MT-DS-CDMA. The data rate for MT-DS-CDMA is the symbol rate, which equals the bit rate divided by the number of bits that represents one transmission symbol (or the bit rate divided by the number of subcarriers in high-data-rate, non-frequency-diversity forms). Usually, binary phase shift keying (BPSK) or quadrature phase shift keying (QPSK) is used for each subcarrier of MC/MT-DS-CDMA, and in this case the symbol rate is the bit rate for BPSK modulation, and half of the bit rate for QPSK.

(a) Transmitter diagram of MC-CDMA

(b) Transmitter diagram of MC/MT-DS-CDMA

Figure 2-1: Transmitter block diagram of MC-CDMA and MC/MT-DS-CDMA (a) MC-CDMA, (b) MC/MT-DS-CDMA

For a good MC-CDMA system design, it is essential to have each subcarrier's bandwidth less than the coherence bandwidth of the channel, i.e., each subcarrier should undergo frequency non-selective fading, as in [40], [41]. The coherence bandwidth, determined by the environment, is the bandwidth over which the fading is correlated, and

it is related to the root mean square delay spread (RMS-DS) of the environment, which is often obtained by field measurements (e.g., as [110]).

Thus in these MC-CDMA schemes, before copying, we may need to serial-to-parallel convert the input data to reduce the data rate of each subcarrier such that, after spreading, the chip duration of each subcarrier is long enough for each subcarrier to undergo flat fading. In [40], the performance of MC-CDMA was compared to that of single-carrier DS-CDMA in the frequency selective Rayleigh channel. In [40], the BER performance of synchronous MC-CDMA and DS-CDMA were compared, and the optimal parameter selection for MC-CDMA was suggested. It was shown in [40] that the optimal number of subcarriers for MC-CDMA and the guard interval increases as the channel RMS-DS increases. Also in this paper, the authors have shown that MC-CDMA has no major advantage over DS-CDMA in terms of bandwidth occupancy or BER, since both systems have similar error performance under the condition of equal bandwidth and data rate.

One thing we should note is that the comparison made in [40] was favorable for DS-CDMA since it assumed that the DS-CDMA receiver is capable of capturing all the (significant) multipath component energies scattered by the channel in time, i.e., all the multipath echoes. This assumption is not always true because it is often difficult to detect and coherently combine all the multipath echoes due to the limitation of receiver complexity and per-component signal-to-noise ratio (SNR). For MC-CDMA though, the receiver needs to combine the energies scattered in frequency, i.e., the same information transmitted via several frequency separated subcarriers. Since the subcarriers that transmit the same information are known by the receiver and all the subcarriers undergo

flat fading, the combining process can be easily performed. Thus MC-CDMA might outperform DS-CDMA in terms of BER performance for similar receiver complexity.

The users in these MC-CDMA systems e.g., in [40], are distinguished by assigning orthogonal codes to each; a common code set used is the Walsh Hadamard codes. This implies that the MC-CDMA system requires users to be synchronized in order not to destroy the orthogonality among users. Therefore, MC-CDMA is often used for the centralized downlink, e.g., BS to MS in cellular networks, where the BS transmits to all the users at the same time. For uplinks, i.e., communication from users to base station, user signals will no longer be orthogonal to each other if transmission in this direction is asynchronous, as it commonly is in practice. In [41], the authors investigated the performance of asynchronous MC-CDMA in the frequency selective Rayleigh fading channel. In this scenario, user signals are not orthogonal, and the multi user interference (MUI) was investigated by assuming the interference due to the MUI at the correlator output is a *Gaussian* random variable (RV). The bit error probability of asynchronous MC-CDMA was given in [41]. It was shown in [41] that for a frequency selective Rayleigh channel with exponentially decaying power delay profile (PDP), the asynchronous MC-CDMA outperforms DS-CDMA.

As mentioned before, the spectrum is spread in the “time domain” for the MC/MT-DS-CDMA schemes. The basic idea of MC/MT-DS-CDMA is that the signals on all subcarriers of these two schemes are DS-SS signals, and these DS-SS signals are transmitted through orthogonal subcarriers. The orthogonality between subcarriers can be achieved by setting the frequency separation between adjacent subcarriers to be either the chip rate on each subcarrier or the data rate on each subcarrier (this requires the

spreading codes be equal on all subcarriers for a given user in the MT case). When the frequency separation is the chip rate, the system is the so called MC-DS-CDMA, and it is MT-DS-CDMA when the frequency separation is the data rate. It is worth mentioning that as an optional format for the air-interface for CDMA2000, one of the international 3G standards, MC/MT-DS-CDMA can be used to upgrade the current single-carrier DS-SS CDMA systems to 3G systems, since each subcarrier of MC/MT-DS-CDMA is a DS-SS signal, and the existing signal processing techniques which are used for CDMA still apply for MC/MT-DS-CDMA.

The MC-DS-CDMA approach was first reported in early 1990s in [43], [44]. In [45], the authors did initiate research on the performance of MC-DS-CDMA schemes in the dispersive fading channel. For MC-DS-CDMA in [45], the input data stream is DS spread and the same DS spread signal is transmitted through orthogonal subcarriers, yielding frequency diversity if the channel on each subcarrier is not strongly correlated with that on other subcarriers. The frequency spacing between the adjacent subcarriers was selected to be *two* times the chip rate, yielding mostly non-overlapped spectra of each subcarrier. In addition, the chip rate was configured to be small enough such that each subcarrier experiences frequency non-selective fading. The receiver combines the output of each subcarrier's correlator output in a maximal ratio combining (MRC) fashion before detector hard decision. It was shown in [45] that this system was robust in the presence of channel fading and narrow-band-interference (NBI).

The MC-DS-CDMA system proposed in [45] can be efficiently implemented using Fast Fourier Transform (FFT) techniques as shown in [57], where the authors investigated the anti-jamming performance of a digitally implemented MC-DS-CDMA

scheme in a dispersive Rayleigh fading channel. Similar to common practice in OFDM schemes, a cyclic prefix is added to avoid the effects of ISI in [57]. As shown in [57] and [45], the MC-DS-CDMA scheme is more robust to multipath fading as well as narrow band interference than is single carrier DS-SS.

Systems similar to that in [45] were also proposed by researchers. For example, in [47], Sourour and Nakagawa proposed an MC-DS-CDMA system that uses a modification to the MC-DS-CDMA system of [45], where the input data stream is first serial-to-parallel converted before the DS spreading. After spreading, the same DS-SS signal is transmitted through *some* subcarriers, not all the subcarriers as in [45]. These subcarriers carrying the same information are interleaved in the frequency domain in such a manner that the frequency separation between any two of them is larger than the coherence bandwidth of the channel. It is worth noting that the authors assumed that the coherence bandwidth of the channel is known by the transmitter in order to properly interleave the subcarriers. In this case, the subcarriers that carry the same information undergo independent fading, so that this configuration brings full frequency diversity to the system.

In [56], the performance of MC-DS-CDMA with these two kinds of configurations, with and without serial-to-parallel conversion, and single carrier DS-CDMA, were compared in the downlink environment. In the down link scenario, the orthogonal codes are used for all three schemes. Thus, because of the diversity and the orthogonal codes, the MC-DS-CDMA scheme without serial-to-parallel conversion has the best error performance followed by the MC-DS-CDMA scheme with serial-to-parallel conversion, and finally by single carrier CDMA when the maximum number of users

supported by the system is small, i.e., orthogonal codes can be assigned to each user. As the number of users increases, the MC-DS-CDMA scheme without serial-to-parallel conversion faces the problem of running out of the orthogonal codes, and as a result, random codes are used in this case, whereas the MC-DS-CDMA system with serial-to-parallel conversion and single carrier CDMA can still employ the orthogonal codes due to their larger processing gain values. Therefore, when there are a large number of users, MC-DS-CDMA with serial-to-parallel conversion has the best error performance, followed by the single carrier CDMA and then by MC-DS-CDMA without serial-to-parallel conversion.

Because of the robustness to fading and interference and the flexibility multiple carriers provide, different variations of MC-DS-CDMA were proposed. For instance, instead of applying the same spreading sequence for each subcarrier of a given user signal, the MC-DS-CDMA system proposed in [51] used different spreading sequences on each of its subcarriers. The spreading sequences are constructed in a way such that they have zero cross correlation for all non-zero shifts in an “ideal phase-coherent” channel. The requirement of a phase-coherent channel means a channel without dispersion, or one in which channel dispersions have the same phase shift; this limits this scheme’s application to light-guide fiber or Rician channels with a strong line-of-sight (LOS) component.

In [52], You and Hong combined the MC-CDMA and MC-DS-CDMA approaches by spreading the spectrum in *both* time and frequency domains, to give the system even more flexibility in terms of the number of subcarriers and processing gain, while still keeping the advantages of both MC-CDMA and MC-DS-CDMA. In [55] the

authors borrowed the ideal of OFDMA to construct a nearly orthogonal MC-DS-CDMA set of signals called group orthogonal multicarrier CDMA. In the system proposed by [55], users share different orthogonal subcarriers, i.e., different users transmit their signal through different subcarriers. With ideal channel side information (CSI) at the receiver and a dynamic subcarrier assignment algorithm [58], [59], the BER performance of this group orthogonal multicarrier CDMA scheme with multiple users is close to that of a single user case.

The MC-DS-CDMA scheme that transmits the same data on all/some of its subcarriers [45]-[47] has the best performance because of frequency diversity, whereas the MC-DS-CDMA scheme with different data on each subcarrier has the maximum throughput [50] but with poorer error performance. Researchers are trying to maximize throughput of MC-DS-CDMA while still having acceptable BER performance. It was shown in [53] that a system that switches its configuration from split (different data on each subcarrier) to replication (same data on all subcarriers) according to channel gains on each subcarrier can maximize its instantaneous throughput.

The MT-DS-CDMA proposed by Vandendorpe [93] is also an attractive modulation scheme for wireless communication. Since the frequency separation between adjacent subcarriers is the data rate on each subcarrier, the subcarrier spectra overlap a great deal, almost entirely when processing gain is large. The large spectral overlapping among subcarriers allows the processing gain of MT-DS-CDMA systems to be larger than that of an equivalent MC-DS-CDMA system, which can be an asset for combating channel dispersion and the narrowband and multiuser interferences. In [94], the BER performance of MC-DS-CDMA and MT-DS-CDMA in the presence of partial band

jamming were investigated and compared. It is not surprising that with a larger processing gain on each subcarrier, the error performance of MT-DS-CDMA is often better than that of MC-DS-CDMA.

Inspired by the work of Vandendorpe, the downlink multiple access capacity of an MT-DS-CDMA system was investigated in the Additive White Gaussian Noise (AWGN) channel in [95], where the authors showed that the multiple access capacity of MT-DS-CDMA is essentially the same as that of a single carrier DS-CDMA system with the same data rate and the bandwidth. In other words, the large processing gain associated with MT-DS-CDMA does not enable it to increase its multiple access capacity.

A variation of conventional MT-DS-CDMA was proposed in [99], where the authors showed that the frequency spacing between two adjacent subcarriers can be reduced to half of the data rate on each subcarrier. The reduction of frequency spacing does improve the spectrum efficiency of the system, i.e., it reduces the overall bandwidth of MT-DS-SS. The authors in [99] also proposed new receiver designs for this reduced frequency spacing MT-DS-CDMA. One receiver proposed by [99] uses fewer local oscillators (LOs). It was also shown in [99] that the inter-subcarrier interference introduced by reducing the number of LOs can be properly cancelled with a moderate increase in baseband signal processing complexity.

The MC-DS-CDMA and MT-DS-CDMA approaches both spread the spectrum in the time domain; the only difference between these two schemes is the frequency separation between two adjacent subcarriers, and the resultant processing gain on each subcarrier. In MC-DS-CDMA the frequency separation is chosen to be the chip rate on

each subcarrier, whereas the frequency separation is the data rate for MT-DS-CDMA. It was shown in [38], [39] that two sinusoids are orthogonal within a symbol duration as long as the frequency separation between these two sinusoids is an integer multiple of the symbol rate. Thus, MC-DS-CDMA and MT-DS-CDMA are actually two special cases of the generalized MC-DS-CDMA system proposed in [38], [39], where the frequency separation between two adjacent subcarriers is an integer multiple of the symbol rate on each subcarrier. The error performance of this generalized MC-DS-CDMA in a Nakagami- m dispersive channel was investigated in [38]. The results shown in [38] indicate that MT-DS-CDMA outperforms MC-DS-CDMA only when the channel is severely faded (e.g., Nakagami- m with $m=1$, or Rayleigh fading), and as the channel condition improves, the MC-DS-CDMA technique outperforms MT-DS-CDMA. The authors also showed that both MC-DS-CDMA and MT-DS-CDMA systems are suboptimal, and an optimal frequency spacing that minimizes the BER does exist for a given channel condition, i.e., for a channel with certain coherence bandwidth and fading specified by the value of m , there exists an optimal frequency spacing that yields the best error performance.

In spread spectrum systems, the receiver must synchronize its locally generated spreading code with the spreading code of the incoming signal. This is a prerequisite for any data detection. This code synchronization is often done in two steps: coarse synchronization and fine synchronization. The coarse synchronization step aligns the two spreading codes--the locally generated spreading code and the spreading code of the incoming signal--to be within one-half of a chip interval. This gives an initial starting point to the code tracking loop, which performs the fine code synchronization. In [61],

the code acquisition scheme which exhaustively searches all possible code phases was proposed. In this acquisition scheme, all possible candidate code delays are tested and the one that generates the largest--maximum likelihood (ML)--value is selected to be the coarse acquisition output. The performance of this acquisition scheme for MC-DS-CDMA and single carrier DS-CDMA were compared in AWGN and flat Rayleigh fading environments in [61] . It was shown by the authors that with the aid of frequency diversity, i.e., with the same data transmitted on all subcarriers, the proposed code acquisition scheme yields better synchronization results for MC-DS-CMDA than it does for single carrier DS-CDMA in fading environments. This relationship also holds when MC-DS-CDMA and single carrier CDMA are received in the presence of narrow band interference. In this work, the channels over which the code acquisition performances of two systems were compared were non-fading and flat fading channels. These channel models cannot model the real channel between the transmitter and receiver of MC-DS-CDMA systems for all cases. In [62], Yang and Hanzo extended the work of [61] to include the Nakagami- m dispersive channel, a parametric distribution that provides good fits for experimental data in both indoor and outdoor urban channels. Results similar to those in [61] were obtained for MC-DS-CDMA in a dispersive Nakagami- m channel for different m values, or different channel fading scenarios. It was also shown by [61] that when compared with single carrier DS-CDMA, the MC-DS-CDMA systems have better code acquisition performance in terms of a smaller average acquisition time and smaller overall probabilities of missed detection. The authors of [62] showed that there exists an optimal acquisition threshold that minimizes the acquisition time and overall probability of missed detection.

The use of multicarrier CDMA systems, MC-CDMA and/or MC/MT-DS-CDMA, for wireless communication systems, was justified based upon their advantages, which include easy implementation via FFT techniques, robustness in fading environments, and robustness to interference. However, the large number of subcarriers associated with these multicarrier transmission schemes makes them sensitive to some receiver imperfections, e.g., frequency offset, timing jitter, and phase noise. The effects of these imperfections on multicarrier systems have also been extensively investigated by researchers. In [63]-[68], researchers investigated the effects of carrier frequency offsets on the error performance of various types of multicarrier transmission schemes. These researchers showed that the error performance of the multicarrier transmission schemes degrades significantly due to the frequency offsets between the locally generated carriers and those of the transmitted signal. The effects of timing jitter and timing offset between the transmitter and receiver clock, for MC-DS-CDMA, were studied by [69], where the authors showed that the effects of timing jitter are independent of the number of subcarriers, processing gain, and jitter noise spectrum, but only depend on the variance of the jitter noise, or the power of the jitter noise. The effects of the phase noise in a phase-locked loop (PLL) on the performance of MT-DS-SS were evaluated in [98]. These results showed that due to the loss of SNR and loss of orthogonality between subcarriers, phase noise degrades the performance of the MT-DS-SS system. These research results, [63]-[69] and [98], showed that compared with the timing and phase errors, frequency offsets are most detrimental to the error performance of the multicarrier transmission schemes. Thus, an accurate frequency synchronization algorithm, or frequency offset compensation algorithm, is very critical for good multicarrier system performance. The

study of synchronization techniques for multicarrier transmission scheme is outside the scope of this dissertation, and readers are referred to [63]-[69] and the references therein for discussion of synchronization techniques.

In addition to sensitivity to carrier frequency offsets, the multicarrier transmission schemes suffer from a large peak to average power ratio (PAPR). The high peak power values require the transmitter power amplifier to operate in a large linear range, which in turn increases the amplifier cost and possibly the complexity of any ancillary (e.g., “linearizing”) equipment. Since the PAPR of a multicarrier system is proportional to its number of subcarriers [72], and a large number of subcarriers are often used for OFDM systems, extensive research has been done to address the PAPR problem in OFDM systems, including characterizing the distribution of PAPR of OFDM signals [73]-[75], and more importantly, PAPR reduction methods for OFDM signals [76]-[84] (and references therein).

The simplest method to reduce the PAPR is to clip, or limit, the amplitude of the transmitted signal. When the amplitude of the OFDM signal is larger than the pre-set threshold, the signal is clipped [76]. This clipping of the OFDM signal though introduces both in-band distortion and out-of-band emissions, which destroy the orthogonality between subcarriers and hence degrades the error performance. Another method to reduce the PAPR is companding, which is a technique borrowed from analog communication systems, in which an analog signal with a large dynamic range is compressed before transmission, and then expanded after reception. Similarly, compressing can be performed before the OFDM signal is actually amplified, and expanding is performed before the signal is detected [77], [78]. Similar to clipping, the

companding is easy to implement in hardware, yet its use may also degrade error performance due to the consequent noise enhancement introduced when expanding the received signal.

In contrast to the clipping and companding techniques which directly modify the envelope of the transmitted signal, the PAPR can also be reduced via limiting the possible values in the transmitted data set; for example only signals with peak value less than a given threshold are *allowed* to be transmitted. This can be done by using block codes in which the signals are mapped to a higher dimensional signal space, and the signals with large peak levels are eliminated by this mapping [79], [80]. However, due to the computational complexity of encoding and decoding, these coding schemes are typically infeasible for multicarrier system with large number of subcarriers, e.g., $N \geq 32$.

Other than coding across subcarriers, another means to avoid the large peak values of the transmitted signal is the multiple signal representation (MSR) approach. In this technique, the amplitude of the transmitted data is kept unchanged but the phases are modified. Thus, the signals of all subcarriers are not coherently added, and the large peak values are avoided. Two methods of MSR were proposed by researchers [81]-[83]. One is selective mapping (SLM) [81], where instead of transmitting the original data vector $\mathbf{X}=[X_1, X_2, \dots, X_N]$, each element of $\mathbf{X}$ is phase rotated by the phase rotation vector $\mathbf{D}_i$ $i=1, 2, \dots, U$, and among all modified data vectors, the one with the lowest PAPR is transmitted. The side information regarding which phase rotation vector is used is transmitted to the receiver via some reliable channel such that the receiver can detect the received signal without distortion. The phase rotation vectors $\mathbf{D}_s$ are of length N , and each element of $\mathbf{D}$ is a random but fixed complex symbol with amplitude of 1. The

amount of the PAPR reduction of SLM depends on the number of possible phase rotation vectors U , and the design of the rotation vectors $\mathbf{D}_s$. Generally speaking, increasing U will increase the PAPR reduction capability. However, this will, of course, increase the computational complexity of the system since the system needs to perform U IFFTs in order to find which phase rotation vector to be used. In [81], the authors showed that with $U=4$ and each element of D_1 obtained from randomly selected QPSK symbols (with amplitude normalized to 1), SLM can greatly reduce the PAPR, e.g., for a 128 subcarrier OFDM system, the probability that PAPR value exceeds 8 dB is reduced from 5×10^{-1} to 5×10^{-3} .

Another method of MSR is partial transmit sequences (PTS) [82], which is also based on phase rotation of the transmitted signal. In PTS, the transmitted data vector is divided into V disjoint subgroups, and the signals within the subgroups are multiplied by the same phase rotation factor. The phase rotation factors of different subgroups are independent, and are selected through exhaustive search on all the possible phase rotation factors; as with MSR, this search must often be time limited by the computational complexity and delay requirements of the system [82]. Similar to SLM, in PTS the modified data vector with the lowest PAPR is selected and transmitted to the receiver. The PAPR reduction capability of different grouping strategies was investigated in [82]. It was shown that a random subcarrier grouping has a better PAPR reduction than adjacent and interleaved subcarrier grouping schemes. To take advantage of the better PAPR reductions of the random grouping scheme, additional information on the grouping must be transmitted along with the rotation factors of each subgroup via reliable links [83].

Some researchers proposed PAPR reduction schemes that employ the “unused” subcarriers of multicarrier schemes. These subcarriers do not carry any information due to their low SNR or for other reasons, and can be used to cancel the peak of the transmitted signal which results in the tone reservation (TR) method [84]. An advantage to this approach is that since these subcarriers are orthogonal to the actual information carrying subcarriers, they will not cause any performance degradation.

Other PAPR reduction techniques such as Tone Injection (TI) [85], Active Constellation Extension (ACE) [86], and interleaving [87] are also promising PAPR reduction schemes and are being extensively studied by researchers. These PAPR reduction schemes can be categorized into two types: distortionless and distortion-inducing. For the distortionless types, such as SLM, PTS, coding, TR, TI, and ACE, the orthogonality among subcarriers is kept and there is no BER degradation. However, the computational complexity of these distortionless methods is much greater than that of the distortion-inducing methods, the most common of which is simple clipping.

In the study of the PAPR of OFDM signaling schemes [76]-[87], researchers assume independent data on each subcarrier. This assumption is not always valid for multicarrier CDMA systems, where the same data are purposely transmitted through some or all subcarriers for the sake of frequency diversity. These subcarriers carrying the same data are called identical subcarriers. In [88] the PAPR of MC-DS-CDMA was analyzed, where the authors showed that as the number of identical subcarriers increases, the PAPR of the MC-DS-CDMA increases. The author also investigated an adaptive scheme [89], wherein instead of transmitting the data through all the identical subcarriers, the subcarrier with the highest signal-to-noise-and-interference is used to transmit the

data, and this scheme reduces the PAPR value.

In summary, researchers have proposed a number of variations of multicarrier CDMA systems. All of them, even the generalized multicarrier CDMA proposed by Yang and Hanzo [38], use identical data rates and chip rates on each subcarrier. In this dissertation, we propose a novel modulation scheme based on MT/MC-DS-CDMA in which we allow all the parameters of each subcarrier to be adjustable; that is, all subcarriers may have a different data rate, chip rate, bit energy, and carrier frequency separation from adjacent subcarriers. Since we allow all these parameters to be adjustable, and due to the consequent spectral shaping capabilities of our scheme, we call this new modulation scheme spectrally-shaped generalized multitone direct sequence spread spectrum (SSG-MT-DS-SS). In the following chapters we explore the properties of the SSG-MT-DS-SS systems. These properties include its spectral shaping capabilities, PAPR distribution, performance with new error correction coding schemes, and error performance in different types of multi-user channels.

Chapter 3. Spectrally Shaped Generalized Multitone Direct Sequence Spread Spectrum (SSG-MT-DS-SS)

In this chapter, we first provide the system model for our spectrally shaped generalized multitone direct sequence spread spectrum (SSG-MT-DS-SS) scheme, and illustrate the effects of different parameter settings on the spectral shaping capabilities of SSG-MT-DS-SS systems. To focus on these spectral shaping capabilities, we investigate the single user bit error ratio (BER) performance in the simple additive white Gaussian noise (AWGN) channel, and show the effects of different parameter sets on both spectrum and BER performance. This chapter concludes with a description of the method used to select SSG-MT-DS-SS parameter sets to meet a desired spectral mask.

3.1 System Model

In this section, we introduce the SSG-MT-DS-SS transmitter and receiver structure, followed by the definition of the parameters needed to identify and evaluate the BER performance of the SSG-MT-DS-SS system.

Transmitter Description

The SSG-MT-DS-SS system is a generalized multicarrier spread spectrum system. For simplicity of presentation, we use binary phase shift keying (BPSK) modulation on each subcarrier, but the results can be easily extended to other modulation schemes, e.g., M -PSK, or M-QAM.

Figure 3-1: Transmitter block diagram

A block diagram of the transmitter is shown in Figure 3-1. As shown in the block diagram, the input binary data stream $d(t)$, first enters the data transformation block **T**, which generates M parallel data streams from the input binary data stream $d(t)$. At the output of the data transformation block **T**, each subcarrier may have a unique data rate, i.e., the data rate of the i^{th} subcarrier may be different from that of the j^{th} subcarrier. The binary data streams of each subcarrier are then spread by the (generally subcarrier unique) spreading codes, $c_i(t)$, $i \in \{1, 2, \dots, M\}$. After spreading, the signals are upconverted by the carrier waveforms, specifically, the DS-SS signal of subcarrier i is upconverted to its radio frequency (RF) by the carrier waveform $s_i(t)$. Finally, the transmission power of each subcarrier is scaled by the subcarrier amplifier and the transmitted signal $v(t)$ is formed by adding the signals from all subcarriers. It is worth noting that the data transformation block **T** can be viewed as a generalized serial-to-

parallel (S:P) conversion block, i.e., when each subcarrier has the same data rate, the data transformation block $\mathbf{T}$ is the conventional serial-to-parallel converter, or demultiplexer.

The transmitted signal can be expressed as

$$v(t) = \sum_{i=1}^M A_i \sum_{n=-\infty}^{\infty} \left\{ d(n) [p_{T_{b,i}}(t - nT_{b,i}) \sum_{m=0}^{N_i-1} c_i(m) p_{T_{c,i}}(t - mT_{c,i})] \right\} s_i(t) \quad (3.1)$$

where the following definition apply:

- (1) M is the number of subcarriers;
- (2) $A_i = \sqrt{2E_{b,i}/T_{b,i}}$, where $E_{b,i}$ and $T_{b,i}$ are respectively bit energy and bit duration for the i^{th} subcarrier;
- (3) $d_i(n) \in \{-1, 1\}$ is the n^{th} transmitted data bit on the i^{th} subcarrier;
- (4) $c_i(m) \in \{-1, 1\}$ is the m^{th} chip of the spreading code corresponding to the i^{th} subcarrier, and $N_i = T_{b,i}/T_{c,i}$ is the processing gain of the i^{th} subcarrier;
- (5) $s_i(t) = \cos(2\pi f_{c,i}t)$ is the sinusoidal carrier for subcarrier i of user k and $f_{c,i}$ is the carrier frequency of the i^{th} subcarrier;
- (6) $p_T(t)$ is the rectangular pulse waveform, equal to one over $0 \leq t \leq T$, and zero elsewhere.

Based on the definition of the parameters of each subcarrier, we can see that the SSG-MT-DS-SS scheme is a completely *generalized* multicarrier DS-SS scheme, and MC/MT-DS-SS are two special case of SSG-MT-DS-SS, e.g., when each subcarrier has the same data rate, processing gain, and bit energy, SSG-MT-DS-SS is MC or MT depending on the frequency separation between adjacent subcarriers. In addition, the generalized MC-DS-SS [38], [39] can also be included in our SSG-MT-DS-SS framework. It is worth noting that we use BPSK modulation and rectangular chip

waveforms for all subcarriers of SSG-MT-DS-SS in this dissertation for simplicity of presentation; the SSG-MT-DS-SS technique could be further generalized by allowing different modulations and pulse shapes on each subcarrier, to enable shaping of the spectrum of SSG-MT-DS-SS with even more flexibility.

Receiver Description

In this chapter, we focus on the spectrum shaping capability of the SSG-DS-SS scheme. In this case, we assume the channel between the transmitter and receiver is a simple AWGN channel. This channel model, with its simplicity, often serves as the first “test bed” when a new modulation scheme is proposed and evaluated [101].

With the AWGN channel model, the receiver is a simple bank of conventional correlators. This receiver is optimal when the subcarriers of SSG-MT-DS-SS are orthogonal, or when SSG-MT-DS-SS is inter-subcarrier interference free. When the SSG-MT-DS-SS is not interference free, the outputs of the correlator bank are the inputs to the advanced types of detectors. The block diagram of our detector for BPSK modulated SSG-MT-DS-SS is shown in Figure 3-2, where the received signal is correlated with each subcarrier’s carrier waveform $s_i(t)$ and spreading code $c_i(t)$. After correlating, the i^{th} correlator’s output is sampled at the rate of $1/T_{b,i}$ to form the decision metric z_i of the i^{th} subcarrier. Then a hard decision is made based on the sign of the correlator outputs to form the detected bits of each subcarrier. The detected bits of each subcarrier are $\mathbf{T}^{-1}$ converted to form the detected binary data streams $\hat{d}$.

Figure 3-2: Receiver block diagram

Parameter Definitions

In the SSG-MT-DS-SS system, the data rate, processing gain, bit energy, and carrier frequency of each subcarrier are allowed to be selectable. Thus, it is convenient to represent the parameters in a matrix form as $\Psi = [\mathbf{R}_b^T, \mathbf{N}^T, \mathbf{E}_b^T, \Delta \mathbf{f}^T]$, where Ψ is the parameter set of a given system, and superscript T denotes transpose. In the parameter set (matrix) Ψ , $\mathbf{R}_b$, $\mathbf{N}$, and $\mathbf{E}_b$, are column vectors of length M with elements given by the data rate, processing gain, and bit energy of each subcarrier, respectively, and $\Delta \mathbf{f}$ is of length $M-1$ with its j^{th} element equal to the frequency difference between subcarriers j and $j+1$. To simplify presentation, we normalize the $\mathbf{R}_b$ and $\mathbf{E}_b$ vectors their 1^{st} element, i.e., $\mathbf{R}_b$ and $\mathbf{E}_b$ are normalized by the data rate and bit energy of the 1^{st} subcarrier, respectively. Similarly, we normalize the $\Delta \mathbf{f}$ vector by the data rate of the 1^{st} subcarrier.

An example of the use of this notation is now given. An $M=3$ subcarrier SSG-MT-DS-SS system with data rates of $R_{b1}=1$ Mbps, $R_{b2}=2$ Mbps, $R_{b3}=1$ Mbps, processing gains of $N_1=50$, $N_2=200$, $N_3=50$, equal bit energies on all subcarrier bits, and frequency separations between adjacent subcarriers all equal to R_{b1} Hz can hence be expressed in vector form as $\mathbf{R}_b=[1, 2, 1]$, $\mathbf{N}=[50, 200, 50]$, $\mathbf{E}_b=[1, 1, 1]$, and $\Delta\mathbf{f}=[1, 1]$. In this case, the parameter set matrix is

$$\Psi = \begin{bmatrix} 1 & 50 & 1 & 1 \\ 2 & 200 & 1 & 1 \\ 1 & 50 & 1 & - \end{bmatrix}$$

3.2 Spectral Shaping of SSG-MT-DS-SS

The SSG-MT-DS-SS modulation can shape the spectrum of the transmitted signal by adjusting the parameters of each subcarrier. By appropriately selecting these parameters, we can shape the spectrum of the SSG-MT-DS-SS signal to have reduced sidelobe levels without using conventional pulse shape filters. In this section we provide the general expression for the SSG-MT-DS-SS power spectral density (PSD) and illustrate the spectral shaping mechanism behind the SSG-MT-DS-SS approach.

For a binary modulation with uncorrelated data, the PSD is determined by the Fourier transform of its pulse waveform [3]. Therefore, the general PSD expression for a DS-CDMA signaling scheme is determined by the chip waveform:

$$S(f) = E_c |G(f)|^2 \quad (3.2)$$

where $E_c = E_b/N$ is the energy within one chip duration, or the chip energy, and $G(f)$ is the Fourier transform of the chip waveform. Thus, for the rectangular chip waveform, the PSD of CDMA signaling scheme is

$$S(f) = E_c \text{sinc}^2[T_c(f - f_i)] \quad (3.3)$$

where T_c is the chip duration and f_i is the carrier frequency of the DS-CDMA signal. In (3.3) $\text{sinc}(x)$ is defined as $\text{sinc}(x) = \sin(\pi x)/\pi x$. Since the data bits and the spreading codes for each SSG-MT-DS-SS subcarrier are independent, the PSD of the transmitted signal can be expressed by the sum of each individual subcarrier's PSD as

$$S_v(f) = \sum_{i=1}^M E_{c,i} \text{sinc}^2[T_{c,i}(f - f_{c,i})] \quad (3.4)$$

where, similar to the previous definition, $T_{c,i}$, $E_{c,i}$, and $f_{c,i}$ are respectively the chip duration, chip energy, and carrier frequency of subcarrier i . We note again that since SSG-MT-DS-SS is the most generalized multicarrier spread spectrum format, the PSD given by (3.4) can be used to model the PSD of MC/MT-DS-SS. To illustrate, the PSD of conventional MT-DS-SS with $M=5$ subcarriers and processing gain of $N=10$ in both linear scale and logarithmic scale are shown in Figure 3-3. From this figure we can see that the mainlobe bandwidth of a MC/MT-DS-SS system is given by

$$BW = (M - 1)\Delta f + 2/T_c \quad (3.5)$$

where M is the number of subcarriers, $2/T_c$ is the passband bandwidth of one subcarrier, and Δf is the frequency separation between adjacent subcarriers, which is the data rate of each subcarrier for MT-DS-SS and the chip rate for MC-DS-SS.

(a) Linear scale

(b) Logarithmic scale

Figure 3-3: PSD of conventional MT-DS-SS, $N=10$, $M=5$. (a) linear scale, (b) logarithmic scale.

From Figure 3-3, we can also observe that, when normalized by its peak value, the relative sidelobe level of each subcarrier of MT-DS-SS is approximately -13dB, and

because of the large spectral overlap among subcarriers, the sidelobe level of the complete MT-DS-SS signal, which is the summation of all subcarrier spectra, is likewise approximately -13dB. The sidelobes of the transmitted signal, often termed the out-of-band emission, can represent undesired radiated power which interferes with the systems operating in adjacent bands. Usually these out-of-band emissions are suppressed via filtering, e.g., a raised cosine filters, in order to satisfy the transmission requirements, namely, the spectral mask for wireless equipment. These filters, although efficiently implementable via digital signal processing (DSP) techniques, increase the complexity and the cost of equipment. In the design of SSG-MT-DS-SS system, we aim to reduce the sidelobe levels of the transmitted signal without using these pulse shape filters. To achieve this goal, as noted, we allow the parameters of all subcarriers to be arbitrary, which in turn can change the way the energy is distributed in frequency. Thus, by concentrating more energy in the mainlobe of the SSG-MT-DS-SS signal we can reduce the sidelobe levels of the transmitted signal. This mechanism is shown in Figure 3-4. In this example, the component and total PSD of a $M=3$ subcarrier SSG-MT-DS-SS signal with $\mathbf{R}_b=[1, 2, 1]$, $\mathbf{N}=[128, 32, 128]$, $\mathbf{E}_b=[1, 2, 1]$, and frequency separation of $\Delta\mathbf{f}=[1, 1]$ are compared with the PSD of conventional MT-DS-SS. To compare two systems fairly, the total transmitted power and total data rates of the two systems are set to be equal, and the processing gain of the MT-DS-SS is set to be $N=128$. The PSD plots shown in this figure are all normalized by the peak value of the SSG-MT-DS-SS spectrum in order to show the relative peak power of two systems. It can be seen from this figure that the peak value of the conventional MT-DS-SS spectrum is approximately 4 dB lower than that of the central subcarrier of the SSG-MT-DS-SS spectrum (its 2nd subcarrier). In

addition, the mainlobe bandwidths of all the subcarriers of SSG-MT-DS-SS are narrower than that of the conventional MT-DS-SS spectrum. The fact that the SSG-MT-DS-SS spectrum has narrower mainlobe bandwidths and higher peak values for some of its subcarriers means that we concentrate more energy in the mainlobe of the SSG-MT-DS-SS spectrum. As a result, when respectively normalized by their maximum values, the sidelobe levels of the SSG-MT-DS-SS spectrum are lower than those of conventional MT-DS-SS.

The PSD plots of the SSG-MT-DS-SS and conventional MT-DS-SS signals are shown again in Figure 3-5. In this plot, the PSDs of both signaling schemes are normalized by their spectral peaks. It can be seen that both spectral shaping and reduction of the spectral sidelobe peak level can be obtained for the SSG-MT-DS-SS system. Specifically, the spectrum of SSG-MT-DS-SS signal is *not* the $\text{sinc}^2(f)$ shape, and the sidelobe levels of SSG-MT-DS-SS are approximately 4 dB lower than those of the conventional MT-DS-SS scheme. This example also validates the generalized PSD expression for SSG-MT-DS-SS given by (3.4), corroborated by simulations.

Figure 3-4: Component and total spectra of SSG-MT-DS-SS with $M=3$ subcarrier case, $R_{b,s}=[1,2,1]$, $N_s=[128,32,128]$, $E_{b,s}=[1,2,1]$, and frequency separation $\Delta f_s=[1,1]$,

Figure 3-5: Simulated and analytical PSD for Figure 3-4

3.3 BER Performance Analysis

In this section, we study the BER performance of a single user SSG-MT-DS-SS system in the AWGN channel. First, we specify the condition under which any two subcarriers of the SSG-MT-DS-SS signal are orthogonal. Under this condition, the SSG-MT-DS-SS is inter-subcarrier interference free, and the BER performance is the same as that of BPSK in the AWGN channel. Since the SSG-MT-DS-SS is a generalized multicarrier DS-SS scheme, i.e., the parameters of each subcarrier are adjustable, the orthogonality condition may not always be satisfied, and when the orthogonality condition is violated, the SSG-MT-DS-SS suffers from inter-subcarrier same user interference (IS-SUI). In this section, we study the statistics of IS-SUI for all possible relationships among parameter combinations and derive the general form of the average bit error probability for single user SSG-MT-DS-SS in the AWGN channel.

Correlator Outputs

As shown in the previous section, the (basic) receiver is a bank of correlators, and the receiver is assumed to be able to perform coherent detection. Without loss of generality, we analyze detection of the zeroth symbol for each subcarrier. The decision statistic z_i for the i^{th} subcarrier is

$$z_i = \int_0^{T_{b,i}} r(t) \cos(2\pi f_{c,i} t) c_i(t) dt \quad (3.6)$$

where $r(t) = v(t) + n(t)$ is the received signal for the AWGN channel, and as defined previously, $f_{c,i}$ and $c_i(t)$ are respectively the carrier frequency and spreading waveform of subcarrier i . Upon substituting (3.1) into (3.6), the decision metric z_i given in (3.6) can

be separated into the desired signal term, an inter-subcarrier same-user-interference (IS-SUI) term, and an AWGN noise sample as

$$z_i = \int_0^{T_{b,i}} [v(t) + n(t)] \cos(2\pi f_{c,i} t) c_i(t) dt = \hat{d}_i(0) + \alpha_i + n_i \quad (3.7)$$

where $\hat{d}_i(0) = A_i T_{b,i} d_i(0)/2$ is the desired signal term, and n_i is the noise sample term, which is a Gaussian random variable with zero mean and variance $N_0 T_{b,i}/4$. The IS-SUI term α_i in (3.7) can be expressed as

$$\alpha_i = \sum_{\substack{j=1 \\ j \neq i}}^M \alpha_{i,j} \quad (3.8)$$

where $\alpha_{i,j}$ is the j^{th} subcarrier's interference seen by subcarrier i , given by

$$\begin{aligned} \alpha_{i,j} &= \int_0^{T_{b,i}} \sum_{n=0}^{\lceil T_{b,i}/T_{b,j} \rceil - 1} \frac{1}{2} A_j d_j(n) [p_{T_{b,j}}(t - nT_{b,j}) c_j(t - nT_{b,j})] c_i(t) \cos(2\pi \Delta f_{i,j} t) dt \\ &= \frac{A_j}{2} \int_0^{T_{b,i}} \cos(2\pi \Delta f_{i,j} t) c_i(t) \sum_{n=0}^{\lceil T_{b,i}/T_{b,j} \rceil - 1} d_j(n) [p_{T_{b,j}}(t - nT_{b,j}) c_j(t - nT_{b,j})] dt \end{aligned} \quad (3.9)$$

where $\lceil x \rceil$ is the ceiling function of x , equal to the integer part of x , *plus one*, and $\Delta f_{i,j} = f_{c,i} - f_{c,j}$ is the frequency separation between carrier frequencies of subcarriers i and j . Note that when deriving (3.9) we discard the high frequency component, and the expression for the IS-SUI terms applies to the case when $T_{b,i}/T_{b,j}$ is rational. In our performance analysis, the IS-SUI from the j^{th} subcarrier $\alpha_{i,j}$ is assumed to be a Gaussian distributed random variable for all subcarriers, and under some mild conditions, we show that this assumption is corroborated by our simulation.

Orthogonality Conditions

As shown in the previous analysis, the SSG-MT-DS-SS system can suffer from IS-SUI. In this section, we derive the conditions under which the SSG-MT-DS-SS signal is IS-SUI free. Without loss of generality, we study the interference from the j^{th} subcarrier seen by subcarrier i . For the zeroth chip, the interference from the j^{th} subcarrier is a function of the spreading codes and carrier waveforms of these two subcarriers

$$\rho_{i,j} = \int_0^{T_{c,i}} c_i(0)c_j(t) \cos(2\pi f_i t) \cos(2\pi f_j t) dt \quad (3.10)$$

This expression can be viewed as a generalized per-chip cross correlation.

Depending on the chip durations on the i^{th} and the j^{th} subcarriers, we can separate the cross correlation term into two cases as

$$\rho_{i,j} = \begin{cases} \frac{c_i(0)c_j(0)}{2\pi\Delta f_{i,j}} \sin(2\pi\Delta f_{i,j}T_{c,i}), & T_{c,i} \leq T_{c,j} \\ \frac{c_i(0) \sin(\pi\Delta f_{i,j}T_{c,j})}{\pi\Delta f_{i,j}} \sum_{\ell=0}^{\mu_{i,j}-1} c_j(\ell) \cos[\pi\Delta f_{i,j}T_{c,j}(2\ell+1)] & T_{c,i} > T_{c,j} \end{cases} \quad (3.11)$$

where $\mu_{i,j}$ is the chip duration ratio between the i^{th} and the j^{th} subcarriers, i.e.,

$\mu_{i,j} = T_{c,i}/T_{c,j}$, assumed an integer. In the SSG-MT-DS-SS system we can denote the

lowest data rate among all the subcarriers by $R_{b,basic}$, and express the data rate of all

subcarriers by an integer multiple of the basic data $R_{b,basic}$. Thus, the data rates of

subcarrier i, j can be expressed respectively by $R_{b,i} = \lambda_i R_{b,basic}$ and $R_{b,j} = \lambda_j R_{b,basic}$ where

λ_i, λ_j are integers. Similarly, the frequency separation between the i^{th} and the j^{th}

subcarrier can be expressed by $\Delta f_{i,j} = \kappa_{i,j} R_{b,basic}$, where $\kappa_{i,j}$ is of arbitrary value. The

cross correlation given by (3.11) can then be expressed as

$$\rho_{i,j} = \begin{cases} \frac{c_i(0)c_j(0)}{2\pi\Delta f_{i,j}} \sin[2\pi\kappa_{i,j}/(N_i\lambda_i)] & T_{c,i} \leq T_{c,j} \\ \frac{c_i(0)\sin[\pi\kappa_{i,j}/(N_j\lambda_j)]}{\pi\Delta f_{i,j}} \sum_{\ell=0}^{\mu_{i,j}-1} c_j(\ell) \cos[\pi\kappa_{i,j}(2\ell+1)/(N_j\lambda_j)] & T_{c,i} > T_{c,j} \end{cases} \quad (3.12)$$

Since the chip duration ratio between the i^{th} subcarrier and the j^{th} subcarrier, $\mu_{i,j}$, is an integer, the cross correlation given in (3.12) can be extended to represent the cross correlation of any chips of both subcarriers i and j by simply changing the chip index of both subcarriers. A zero value of both $\rho_{i,j}$ and $\rho_{j,i}$ means the cross correlations between any chips of the i^{th} subcarrier and the j^{th} subcarrier are of zero value, or, the two subcarriers are orthogonal to each other. It can be shown that by properly choosing the value of the frequency separation, i.e., the value of $\kappa_{i,j}$, the cross correlation is zero, i.e., $\rho_{i,j}=\rho_{j,i}=0$. Observing (3.12), we can find the value of $\kappa_{i,j}$ given by

$$\kappa_{i,j} = \Lambda LCM(N_i\lambda_i/2, N_j\lambda_j) \quad (3.13)$$

where $LCM(x,y)$ is the least common multiple function and Λ is a positive integer of any value. An example of an SSG-MT-DS-SS system with orthogonal subcarriers can be given as follows: let the data rate and processing gain vectors be respectively given by $\mathbf{R}_b=[1, 2, 1]$, and $\mathbf{N}=[128, 32, 128]$. With a frequency separation vector selected to be $\Delta\mathbf{f}=[64, 64]$, the subcarriers are orthogonal to each other. This BER performance for this example is shown in a subsequent section (Figure 3-18).

ISI-SUI Expression

The SSG-MT-DS-SS system can be configured without IS-SUI when the orthogonality condition given previously is satisfied. Otherwise, the SSG-MT-DS-SS will suffer from IS-SUI. To evaluate the BER performance of the SSG-MT-DS-SS in the

presence of IS-SUI, we should evaluate the statistics of the IS-SUI terms, $\alpha_{i,j}$. In this section we derive the expressions for the IS-SUI terms for all possible relative parameter combinations.

First, we separate the IS-SUI term between any two subcarriers into 9 distinct cases according to the combinations of their data rates and chip rates. If we leave the bit energy vector $\mathbf{E}_b$, and frequency separation vector $\Delta \mathbf{f}$ unconstrained, we can divide the combination of different data rate, and chip rate values between subcarrier i , and j into 9 distinct cases (see (3.9)). Specifically, three possible data rate relations between subcarrier i and subcarrier j , ($>, =, <$), and three possible chip rate relations between these subcarriers, yield nine total possible combinations. To describe the relationship between two different subcarriers, i.e., the i^{th} and the j^{th} subcarrier, we define the following additional parameters

- Data rate ratio: $\lambda_{i,j} = R_{b,i}/R_{b,j}$; we assume that $\lambda_{i,j}$ is an integer when $R_{b,i} \geq R_{b,j}$
- Bit energy ratio $\eta_{i,j} = E_{b,i}/E_{b,j}$; there is no restriction on this non-negative parameter;
- Chip duration ratio: $\mu_{i,j} = T_{c,i}/T_{c,j} = N_j/(\lambda_{i,j}N_i)$; similar to $\lambda_{i,j}$, we assume $\mu_{i,j}$ is an integer when $T_{c,i} \geq T_{c,j}$

Note that in order to ensure a closed form BER expression of SSG-MT-DS-SS, we place some restrictions on the parameter selection. Specifically, the data rate ratio $\lambda_{i,j}$ must be an integer when $R_{b,i} > R_{b,j}$ and similarly restriction applies on chip rate ratio $\mu_{i,j}$. This is a very minor restriction in practice. With these additional parameters defined, the IS-SUI from subcarrier j given by (3.9) can be evaluated for the nine combinations of different data rate and chip rate relations. In this section, three example IS-SUI terms for

different bit rate and chip rate relations from these nine cases are given ($\lambda_{i,j} \geq 1$, and $\mu_{i,j} \geq 1$), and the expressions for the rest of the cases are listed in Table 3-1.

Case 1. $\lambda_{i,j}=1$ and $\mu_{i,j}=1$

This case is the simplest case among all *nine* cases, where the data rates and chip rates of the i^{th} and the j^{th} subcarriers are identical. The chips from the i^{th} and the j^{th} subcarriers are aligned as shown in the Figure 3-6.

Figure 3-6: Timing diagram for $\lambda_{i,j}=1$, and $\mu_{i,j}=1$

For this case, the IS-SUI term from subcarrier j can be simplified as

$$\begin{aligned} \alpha_{i,j} &= \frac{1}{2} \sqrt{\frac{2E_{b,j}}{T_{b,j}}} d_j(0) \sum_{m=0}^{N_i-1} c_i(m) c_j(m) \int_{mT_{c,i}}^{(m+1)T_{c,i}} \cos(2\pi\Delta_{i,j}t) dt \\ &= \frac{1}{2} \sqrt{\frac{2E_{b,j}}{T_{b,j}}} d_j(0) T_{c,i} \text{sinc}(\Delta f_{i,j} T_{c,i}) \sum_{m=0}^{N_i-1} c_i(m) c_j(m) \cos[\pi\Delta f_{i,j} (2m+1) T_{c,i}] \end{aligned} \quad (3.14)$$

Case 5: $\lambda_{i,j}<1$ and $\mu_{i,j}>1$

In this case one symbol of subcarrier i is interfered with by $1/\lambda_{i,j}$ symbols from subcarrier j ; and one chip from the i^{th} subcarrier is interfered with by $\mu_{i,j}$ chips from the j^{th} subcarrier. For a specific example, one symbol of subcarrier i is interfered with by 4 symbols from subcarrier j , and $c_i(0)$ is interfered by $c_j(0)$, $c_j(1)$ as shown in Figure 3-7. In order to express the IS-SUI, we define the *cross processing gain* between the i^{th} and the

j^{th} subcarrier as $S_{i,j}=R_{c,i}/R_{b,j}$, which is the number of chips from the i^{th} subcarrier within one symbol duration of the j^{th} subcarrier.

Figure 3-7: Timing diagram for $\lambda_{i,j}=1/4$ and $\mu_{i,j}=2$ case

In this case, the integral in the IS-SUI expression in (3.9) can be evaluated via dividing it into a sum of integrals over the smallest chip duration as

$$\begin{aligned}
 \alpha_{i,j} &= \frac{1}{2} \sqrt{\frac{2E_{b,j}}{T_{b,j}}} \sum_{l=0}^{1/\lambda_{i,j}-1} d_j(l) \sum_{m=0}^{S_{i,j}-1} c_i(lS_{i,j} + m) \sum_{q=mk}^{(m+1)\gamma_{i,j}-1} c_j(q) \int_{qT_{c,j}}^{(q+1)T_{c,j}} \cos(2\pi\Delta f_{i,j}t) dt \\
 &= \frac{1}{2} \sqrt{\frac{2E_{b,j}}{T_{b,j}}} T_{c,j} \text{sinc}(\Delta f_{i,j}T_{c,j}) \sum_{l=0}^{1/\lambda_{i,j}-1} d_j(l) \sum_{m=0}^{S_{i,j}-1} c_i(lS_{i,j} + m) \\
 &\quad \times \sum_{q=mk}^{(m+1)\gamma_{i,j}-1} c_j(q) \cos[\pi\Delta f(2q+1)T_{c,j}]
 \end{aligned} \tag{3.15}$$

Case 9: $\lambda_{i,j} > 1$, and $\mu_{i,j} < 1$

In this case, the data rate on subcarrier i is larger than that of subcarrier j , i.e., $\lambda_{i,j} > 1$, so one symbol from the i^{th} subcarrier is interfered with by some portion of the symbol from the j^{th} subcarrier, and one chip from the j^{th} subcarrier interferes with $1/\mu_{i,j}$ chips from the i^{th} subcarrier, (e.g., chip $c_j(0)$ interferes with $c_i(0)$ and $c_i(1)$ as shown in the example of Figure 3-8)

Figure 3-8: Timing diagram for $\lambda_{i,j}=4$ and $\mu_{i,j}=1/2$ case

Following the method of separating the integral into a sum of integrals over the smallest time duration, the IS-SUI expression for this case is given by

$$\begin{aligned}
 \alpha_{i,j} &= \frac{1}{2} \sqrt{\frac{2E_{b,j}}{T_{b,j}}} d_j(0) \sum_{m=0}^{N_j/\lambda_{i,j}-1} c_j(m) \sum_{n=m\gamma_{i,j}}^{(m+1)/\mu_{i,j}-1} c_i(n) \int_{nT_{c,i}}^{(n+1)T_{c,i}} \cos(2\pi\Delta f_{i,j}t) dt \\
 &= \frac{1}{2} \sqrt{\frac{2E_{b,j}}{T_{b,j}}} d_j(0) T_{c,i} \text{sinc}(\Delta f_{i,j} T_{c,i}) \sum_{m=0}^{N_j/\lambda_{i,j}-1} c_j(m) \sum_{n=m\gamma_{i,j}}^{(m+1)/\mu_{i,j}-1} c_i(n) \cos[\pi\Delta f_{i,j}(2n+1)T_{c,i}]
 \end{aligned} \tag{3.16}$$

The IS-SUI expression of other cases can be evaluated following methods similar to those given in the previous examples. The IS-SUI expressions for all the nine distinct cases are listed in Table 3-1. In Table 3-1, for compactness, we also define the additional parameters $\psi_{i,i} = T_{c,i} \text{sinc}(\Delta f_{i,j} T_{c,i}) \sqrt{E_{b,j}/(2T_{b,j})}$, and $\psi_{i,j} = T_{c,j} \text{sinc}(\Delta f_{i,j} T_{c,j}) \sqrt{E_{b,j}/(2T_{b,j})}$, with $\text{sinc}(x) = \sin(\pi x)/(\pi x)$.

Table 3-1: IS-SUI equations for nine cases

Case	IS-SUI Expression $\alpha_{i,j}$ for Subcarrier i
1. $\lambda_{i,j}=1$ $\mu_{i,j}=1$	$\psi_{i,i} d_j(0) \sum_{m=0}^{N_i-1} c_i(m) c_j(m) \cos[\pi \Delta f_{i,j} (2m+1) T_{c,i}]$
2. $\lambda_{i,j}=1$ $\mu_{i,j}>1$	$\psi_{i,j} d_j(0) \sum_{m=0}^{N_i-1} c_i(m) \sum_{q=\mu_{i,j}m}^{(m+1)\mu_{i,j}-1} c_j(q) \cos[\pi \Delta f_{i,j} (2q+1) T_{c,j}]$
3. $\lambda_{i,j}=1$ $\mu_{i,j}<1$	$\psi_{i,i} d_j(0) \sum_{m=0}^{N_i-1} c_j(m) \sum_{q=m/\mu_{i,j}}^{(m+1)/\mu_{i,j}-1} c_i(q) \cos[\pi \Delta f_{i,j} (2q+1) T_{c,i}]$
4. $\lambda_{i,j}<1$ $\mu_{i,j}=1$	$\psi_{i,j} \sum_{s=0}^{1/\lambda_{i,j}-1} d_j(s) \sum_{m=0}^{N_j-1} c_i(sN_j+m) c_j(m) \cos[\pi \Delta f_{i,j} (2m+1) T_{c,j}]$
5. $\lambda_{i,j}<1$ $\mu_{i,j}>1$	$\psi_{i,j} \sum_{s=0}^{1/\lambda_{i,j}-1} d_j(s) \sum_{m=0}^{H_{i,j}-1} c_i(sH_{i,j}+m) \sum_{q=m\mu_{i,j}}^{(m+1)\mu_{i,j}-1} c_j(q) \cos[\pi \Delta f_{i,j} (2q+1) T_{c,j}]$
6. $\lambda_{i,j}<1$ $\mu_{i,j}<1$	$\psi_{i,i} \sum_{s=0}^{1/\lambda_{i,j}-1} d_j(s) \sum_{m=0}^{N_j-1} c_j(m) \sum_{n=0}^{1/\mu_{i,j}-1} c_i(s/\mu_{i,j}+n) \cos[\pi \Delta f (2n+1) T_{c,i}]$
7. $\lambda_{i,j}>1$ $\mu_{i,j}=1$	$\psi_{i,i} d_j(0) \sum_{m=0}^{N_i-1} c_i(m) c_j(m) \cos[\pi \Delta f_{i,j} (2m+1) T_{c,i}]$
8. $\lambda_{i,j}>1$ $\mu_{i,j}>1$	$\psi_{i,j} d_j(0) \sum_{m=0}^{N_i-1} c_i(m) \sum_{n=m\gamma_{i,j}}^{(m+1)\mu_{i,j}-1} c_j(n) \cos[\pi \Delta f_{i,j} (2n+1) T_{c,j}]$
9. $\lambda_{i,j}<1$ $\mu_{i,j}<1$	$\psi_{i,i} d_j(0) \sum_{m=0}^{N_j/\lambda_{i,j}-1} c_j(m) \sum_{n=m\mu_{i,j}}^{(m+1)/\mu_{i,j}-1} c_i(n) \cos[\pi \Delta f_{i,j} (2n+1) T_{c,i}]$

ISI-SUI Variance

With the expression of the IS-SUI term for all parameter combinations listed in Table 3-1, we can evaluate their variances. Recall that we model the IS-SUI as a Gaussian random variable. Since we assume the use of zero mean random binary data and random long codes on each subcarrier, the individual IS-SUI term $\alpha_{i,j}$ from the j^{th} subcarrier, is of zero mean, thus all we need to complete the specification of the Gaussians is their variances. We denote the total IS-SUI for the i^{th} subcarrier, i.e., the

sum of the IS-SUI terms from all other subcarriers, by α_i (3.8), and its variance by $\sigma_{\alpha,i}^2$.

The variance of the total IS-SUI α_i is its second moment i.e.,

$$\sigma_{\alpha,i}^2 = E \left(\sum_{\substack{j=1 \\ j \neq i}}^M \alpha_{i,j} \right)^2 = \sum_{\substack{j=1 \\ j \neq i}}^M E(\alpha_{i,j}^2) + \sum_{\substack{j=1 \\ j \neq i}}^M \sum_{\substack{k=1 \\ k \neq i, k \neq j}}^M E(\alpha_{i,j} \alpha_{i,k}) \quad (3.17)$$

where the first term in (3.17) is the sum of the variances of the IS-SUI from all M-1 subcarriers, the j^{th} term of which is $\sigma_{\alpha,ij}^2 = E(\alpha_{i,j}^2)$, and the 2^{nd} term is the cross correlation between the IS-SUI terms from subcarrier j and subcarrier k . Since different data bits are transmitted on different subcarriers, the cross correlation between any two IS-SUI terms is zero. Therefore, the variance of IS-SUI can be expressed as

$$\sigma_{\alpha,i}^2 = \sum_{\substack{j=1 \\ j \neq i}}^M \sigma_{\alpha,ij}^2 \quad (3.18)$$

With the assumption of random data and random spreading codes on each subcarrier, it is easy to obtain the expressions for all the $\sigma_{\alpha,ij}^2$. An example derivation of the IS-SUI variance is provided for case nine, where the data rate ratio, $\lambda_{i,j}$, and the chip duration ratio, $\mu_{i,j}$, between the i^{th} and the j^{th} subcarriers, are both less than 1.

Case 9. $\lambda_{i,j} < 1$, and $\mu_{i,j} < 1$

As listed in Table 3-1, the IS-SUI term for this case is

$$\alpha_{i,j} = \psi_{i,i} d_j(0) \sum_{m=0}^{N_j/\lambda_{i,j}-1} c_j(m) \sum_{n=m\mu_{i,j}}^{(m+1)/\mu_{i,j}-1} c_i(n) \cos[\pi \Delta f_{i,j} (2n+1) T_{c,i}] \quad (3.19)$$

For random binary data symbols and random long spreading codes the $\alpha_{i,j}$ term given by (3.19) is of zero mean. Thus, the variance of $\alpha_{i,j}$ is its second moment, which can be

evaluated as the expectation of the square of the terms in the summation of (3.19) as follows, where the independence of spreading code chips is employed:

$$\begin{aligned}
 \sigma_{\alpha,ij}^2 &= \psi_{i,i}^2 d_j^2(0) \mathbb{E} \left\{ \sum_{m=0}^{N_j/\lambda_{i,j}-1} c_j^2(m) \sum_{n=m/\mu_{i,j}}^{(m+1)/\mu_{i,j}-1} c_i^2(n) \cos^2[\pi \Delta f_{i,j} (2n+1) T_{c,i}] \right\} \\
 &= \psi_{i,i}^2 \sum_{m=0}^{N_j/\lambda_{i,j}-1} \sum_{n=m/\mu_{i,j}}^{(m+1)/\mu_{i,j}-1} \cos^2[\pi \Delta f_{i,j} (2n+1) T_{c,i}] \\
 &= \psi_{i,i}^2 \sum_{m=0}^{N_i-1} \cos^2[\pi \Delta f_{i,j} (2m+1) T_{c,i}]
 \end{aligned} \tag{3.20}$$

Note, we use the definition of $\mu_{i,j}$ to derive the last line of (3.20)

$$\mu_{i,j} = T_{c,i} / T_{c,j} = N_j / (\lambda_{i,j} N_i) \Rightarrow N_i = N_j / (\lambda_{i,j} \mu_{i,j}) \tag{3.21}$$

Now using the trig identity $\cos^2(x) = [\cos(2x) + 1]/2$, and then the following identity [5],

$$\sum_{m=0}^{N-1} \cos(my + x) = \frac{\cos\{x + [(N-1)/2]y\} \sin(Ny/2)}{\sin(y/2)} \tag{3.22}$$

(3.20) reduces to the last entry in Table 3-2

$$\sigma_{\alpha,ij}^2 = \psi_{i,i}^2 \left\{ \frac{N_i}{2} + \frac{\sin[4\pi \Delta f_{i,j} T_{c,i} N_i]}{4 \sin[2\pi \Delta f_{i,j} T_{c,i}]} \right\} \tag{3.23}$$

The variance of the IS-SUI terms for the other cases can be evaluated via similar methods, and these results are listed in Table 3-2.

Table 3-2: Variance Term in IS-SUI

Case	Variance of IS-SUI Term ($\sigma_{\alpha,ij}^2$)
1. $\lambda_{i,j}=1$ $\mu_{i,j}=1$	$\psi_{i,i}^2 \left\{ \frac{N_i}{2} + \frac{\sin[4\pi\Delta f_{i,j}T_{c,i}N_i]}{4\sin[2\pi\Delta f_{i,j}T_{c,i}]} \right\}$
2. $\lambda_{i,j}=1$ $\mu_{i,j}>1$	$\psi_{i,j}^2 \left\{ \frac{N_j}{2} + \frac{\sin[4\pi\Delta f_{i,j}T_{c,j}N_j]}{4\sin[2\pi\Delta f_{i,j}T_{c,j}]} \right\}$
3. $\lambda_{i,j}=1$ $\mu_{i,j}<1$	$\psi_{i,i}^2 \left\{ \frac{N_i}{2} + \frac{\sin[4\pi\Delta f_{i,j}T_{c,i}N_i]}{4\sin[2\pi\Delta f_{i,j}T_{c,i}]} \right\}$
4. $\lambda_{i,j}<1$ $\mu_{i,j}=1$	$\frac{1}{\lambda_{i,j}} \psi_{i,j}^2 \left\{ \frac{N_j}{2} + \frac{\sin[4\pi\Delta f_{i,j}T_{c,j}N_j]}{4\sin[2\pi\Delta f_{i,j}T_{c,j}]} \right\}$
5. $\lambda_{i,j}<1$ $\mu_{i,j}>1$	$\frac{1}{\lambda_{i,j}} \psi_{i,j}^2 \left\{ \frac{N_j}{2} + \frac{\sin[4\pi\Delta f_{i,j}T_{c,j}N_j]}{4\sin[2\pi\Delta f_{i,j}T_{c,j}]} \right\}$
6. $\lambda_{i,j}<1$ $\mu_{i,j}<1$	$\psi_{i,j}^2 \left\{ \frac{N_i}{2} + \frac{\sin[4\pi\Delta f_{i,j}T_{c,i}N_i]}{4\sin[2\pi\Delta f_{i,j}T_{c,i}]} \right\}$
7. $\lambda_{i,j}>1$ $\mu_{i,j}=1$	$\psi_{i,i}^2 \left\{ \frac{N_i}{2} + \frac{\sin[4\pi\Delta f_{i,j}T_{c,i}N_i]}{4\sin[2\pi\Delta f_{i,j}T_{c,i}]} \right\}$
8. $\lambda_{i,j}>1$ $\mu_{i,j}>1$	$\psi_{i,j}^2 \left\{ \frac{N_j}{2k_{i,j}} + \frac{\sin[4\pi\Delta f_{i,j}T_{c,j}N_j/\lambda_{i,j}]}{4\sin[2\pi\Delta f_{i,j}T_{c,j}]} \right\}$
9. $\lambda_{i,j}<1$ $\mu_{i,j}<1$	$\psi_{i,i}^2 \left\{ \frac{N_i}{2} + \frac{\sin[4\pi\Delta f_{i,j}T_{c,i}N_i]}{4\sin[2\pi\Delta f_{i,j}T_{c,i}]} \right\}$

BER Expression

The IS-SUI is modeled as a Gaussian random variable (RV), thus the variance of the decision statistic for subcarrier i , z_i , conditioned upon $d_i(0)=+1$, is the sum of the

AWGN and IS-SUI variances, or $(N_0 T_{b,i} / 4) + \sigma_{\alpha,i}^2$. It is well known [3] that the error probability for BPSK modulation in this case is

$$P_{b,i} = Q\left(\sqrt{\frac{E[z_i | d_i(0) = 1]^2}{\text{var}[z_i | d_i(0) = 1]}}\right) = Q\left\{\sqrt{\frac{1}{N_0 / 2E_{b,i} + 2\sigma_{\alpha,i}^2 / (E_{b,i} T_{b,i})}}\right\} \quad (3.24)$$

where $Q(x) = \int_x^\infty e^{-t^2/2} dt / \sqrt{2\pi}$, and some algebraic simplification has been performed to obtain the final expression of (3.24). Note that the bit error probability given in (3.24) is for subcarrier i , and the average error probability of the SSG-MT-DS-SS system can be obtained by averaging the error probability of all individual subcarriers by their data rates:

$$\bar{P}_b = \frac{\sum_{i=1}^M R_{b,i} P_{b,i}}{\sum_{i=1}^M R_{b,i}} \quad (3.25)$$

3.4 Numerical Examples

In this section, we show the effects of different parameter sets on the spectral shaping capability and the BER performance of various SSG-MT-DS-SS systems. The PSD and BER performance are compared with those of the conventional MT-DS-SS under the same transmission power, total data rate condition. For example the SSG-MT-DS-SS system with parameters $\mathbf{R}_b=[1, 4, 1]$ Mbps, $\mathbf{N}=[128, 32, 128]$, $\mathbf{E}_b=[1, 1, 1]$ μJ , and $\Delta\mathbf{f}=[1, 1]$ MHz are compared with a 3-subcarrier conventional MT-DS-SS system, in which each subcarrier has identical data rates of $R_b=2$ Mbps, processing gains of $N=128$, bit energies $E_b=1$ μJ , and frequency separation between adjacent subcarriers of $\Delta f=2$ MHz. In this case, both systems have the same data rate and transmit power of 6 MHz and 6 Watts respectively. For presentation purposes, we use the subscripts s and c to distinguish between the SSG-MT-DS-SS and conventional MT-DS-SS systems if necessary, and the parameter sets are given in relative vector form as described in section 3.2.

First, we show that our Gaussian approximation for the IS-SUI is a valid one. Simulated and analytical BER curves, obtained from (3.25), are plotted in Figure 3-9, where the SSG-MT-DS-SS signal has the same parameter sets as the SSG-MT-DS-SS system shown in Figure 3-5. Again, the relative bit energy vector, data rate vector, processing gain vector, and the frequency separation vector are respectively $\mathbf{R}_b=[1, 2, 1]$, $\mathbf{N}=[128, 32, 128]$, $\mathbf{E}_b=[1, 2, 1]$, and $\Delta\mathbf{f}=[1, 1]$. It can be seen that the simulation result matches the analytical result, which corroborates our Gaussian assumption for the IS-SUI. For this parameter set, the performance of SSG-MT-DS-SS is worse than that of the

conventional MT-DS-CDMA with the same total data rate, same transmission power, and processing gain of $N=128$ on each subcarrier. From this figure we can see that we lose about 1.6 dB compared with conventional MT-DS-SS in terms of the required E_b/N_0 for $P_b \cong 10^{-3}$. The performance loss is due to the lower processing gain on the 2nd subcarrier of SSG-MT-DS-SS, and the lower average signal to noise plus interference ratio (SNIR) of this 2nd subcarrier.

Figure 3-9: Simulated and analytical average P_b vs E_b/N_0 for an $M=3$ subcarrier, with $\mathbf{R}_b=[1, 2, 1]$, $\mathbf{N}=[128, 32, 128]$, $\mathbf{E}_b=[1, 2, 1]$, and frequency separation $\Delta\mathbf{f}=[1, 1]$

The Gaussian approximation for IS-SUI is valid for the SSG-MT-DS-SS with minimum processing gain value greater than 2, e.g., with $\mathbf{N}=[8, 2, 8]$, the Gaussian approximation is no longer valid. The simulation results for SSG-MT-DS-SS with $\mathbf{N}=[8, 2, 8]$ and all other parameters held constant are shown in Figure 3-10, where, clearly, we can observe the discrepancy between the simulation and the analytical results. Because of the lower processing gain value—2 for the 2nd subcarrier—the central limit theorem is

no longer valid and we cannot approximate the IS-SUI to be a Gaussian RV. In the following examples we investigate the effects of different parameter sets on the BER and PSD of our SSG-MT-DS-SS scheme with processing gains on each subcarrier greater than 8 in order to use the analytical bit error probability expression given by (3.25).

Figure 3-10: simulated and analytical average P_b vs E_b/N_0 for an $M=3$ subcarrier, with $\mathbf{R}_b=[1, 2, 1]$, $\mathbf{N}=[8, 2, 8]$, $\mathbf{E}_b=[1, 2, 1]$, and frequency separation $\Delta\mathbf{f}=[1, 1]$

Note that in the previous examples, the E_b/N_0 used on the plot abscissa is the average energy per bit to noise ratio of both the SSG-MT-DS-SS and conventional MT-DS-SS systems. We will often refer to this bit energy to noise density ratio as the signal-to-noise ratio (SNR). The bit energy here is determined by the division of the total transmit power by the total data rate of the system, which are identical for both SSG-MT-DS-SS and conventional MT-DS-SS under our same total data rate and transmission

power condition. In the following examples, the SNR is the average SNR unless stated explicitly.

In Figure 3-9, the BER performance of SSG-MT-DS-SS is worse than that of the conventional MT-DS-SS; however, as shown in Figure 3-5, the SSG-MT-DS-SS signal does have a shaped spectrum and reduced spectral sidelobes. In SSG-MT-DS-SS systems, we allow all parameters of each subcarrier to be adjustable, i.e., the elements of the $\mathbf{E}_b$, $\mathbf{R}_b$, $\mathbf{N}$, and $\Delta\mathbf{f}$ vectors can be of any values. Thus each given parameter set, i.e., a given, $\Psi = \{\mathbf{E}_b, \mathbf{R}_b, \mathbf{N}, \Delta\mathbf{f}\}$, is a sample of a multiple dimensional space. As a result, due to the large number of values the parameter set matrix can take, the effects of different parameters on the spectral shaping capability and BER performance are difficult to explore if we allow all the parameters to vary simultaneously. To circumvent this difficulty, we allow only *one* parameter of interest to vary, within a limited range, and keep the rest of the parameters fixed. We use this method to explore the effects of different parameter sets on the spectral shaping and BER performance of SSG-MT-DS-SS throughout this section, and in particular, to see the effects caused by varying each of the parameter types individually.

In the first example, the effects of different bit energies are shown. In this example, the SSG-MT-DS-SS system is a three subcarrier system with the same bit energies on the 1st and 3rd subcarriers, i.e., $E_b(1) = E_b(3)$, and the bit energy of the central, i.e., the 2nd subcarrier, is varied. The average P_b curves for different bit energy distributions among subcarriers are shown in Figure 3-11, where the bit energy ratio between the 1st and 2nd subcarriers is varied from 0.1 to 3. The three SSG-MT-DS-SS systems shown in this figure are of the same processing gain vector and frequency

separation vector i.e., $\mathbf{N}=[128, 32, 128]$ $\Delta\mathbf{f}=[1, 1]$, but they have different data rate vectors, specifically, $\mathbf{R}_b=[1, 2, 1]$, $\mathbf{R}_b=[1, 1, 1]$, and $\mathbf{R}_b=[1, 1/2, 1]$. Note also that in this figure, we keep the average SNR fixed at $E_b/N_0=7$ dB, and, as noted previously, the E_b here is not the bit energy of any individual subcarrier but the average E_b of the whole system, that is, $E_b=P_T/R_b$ where P_T is the total transmission power and R_b is the total data rate.

Figure 3-11: P_b vs. E_{b2}/E_{b1} at $E_b/N_0=7$ dB, $\mathbf{N}=[128, 32, 128]$ $\Delta\mathbf{f}=[1, 1]$.

From this figure, we can observe that the average P_b of SSG-MT-DS-SS is a function of the bit energy distribution, and that the best performance is reached when the bit energies of all subcarriers are equal, i.e., $\mathbf{E}_b=[1, 1, 1]$.

The PSD plots of the SSG-MT-DS-SS signals with these different bit energy vectors are also given in Figure 3-12. From this figure, it is clear that the mainlobe bandwidth of SSG-MT-DS-SS is narrower than that of the conventional MT-DS-CDMA for all $\mathbf{E}_b$ vectors, and the sidelobe levels are approximately 3-5dB lower than that of the

conventional MT-DS-SS spectrum, depending on the bit energy distribution among subcarriers. In addition, the bit energy distribution does not affect the mainlobe bandwidth but it *does* affect the shape of spectrum and the BER performance of the SSG-MT-DS-SS.

Figure 3-12: PSD for $\mathbf{R}_b=[1, 2, 1]$ with different $\mathbf{E}_b$ vectors.

In the previous example, the data rates and the processing gain values of the 1^{st} and the 3^{rd} subcarriers are the same, and their frequency spacings from the center subcarrier are also the same. Thus the spectrum of SSG-MT-DS-SS is symmetric in this scenario. The spectrum of SSG-MT-DS-SS can also be shaped to be asymmetric. An asymmetric spectrum is obtained by letting any one of the parameters, i.e., the data rates, processing gains, and the frequency spacings from the center frequency of the 1^{st} and 3^{rd} subcarriers be different. In Figure 3-13 we show the effect of different bit energies on the error performance of three SSG-MT-DS-SS systems, each with asymmetric spectra. The parameters of the three SSG-MT-DS-SS systems are $\mathbf{R}_b=[2, 2, 1]$, $\mathbf{N}=[128, 32, 128]$,

$\Delta\mathbf{f}=[1, 1]$ for SSG-MT-DS-SS1; $\mathbf{R}_b=[1, 1, 2]$, $\mathbf{N}=[128, 128, 32]$, $\Delta\mathbf{f}=[1, 1]$ for SSG-MT-DS-SS2; and $\mathbf{R}_b=[2, 2, 1]$, $\mathbf{N}=[128, 128, 32]$, $\Delta\mathbf{f}=[64, 1]$ for SSG-MT-DS-SS3.

Figure 3-13: P_b vs. E_{b2}/E_{b1} at $E_b/N_0=7$ dB for three SSG-MT-DS-SS systems with asymmetric spectra.

From Figure 3-13 we can also observe that SSG-MT-DS-SS has the best BER performance when the bit energy vector is an all ones vector. This fact also holds for larger number of subcarriers. Through numerous investigations, we have found that the SSG-MT-DS-SS system reaches its best error probability performance when the bit energies of all subcarriers are equal, i.e., $\mathbf{E}_b$ is an all ones vector, for any arbitrary number of subcarriers and arbitrary values of the $\mathbf{R}_b$, $\mathbf{N}$, and $\Delta\mathbf{f}$ vectors. Therefore, to obtain the best BER performance, we keep the bit energies on all subcarriers equal, and shape the

spectrum of transmitted signal via adjusting the other parameters in the following examples.

One of the design goals for our SSG-MT-DS-SS schemes is to reduce the sidelobe levels while still having BER performance comparable to that of the equivalent conventional MT-DS-SS scheme. As we have shown previously, the sidelobe levels can be reduced via concentrating more energy in the mainlobe of the transmitted signal. One way to concentrate the energy is to choose the parameter set such that the subcarriers with high chip energy are set near the center frequency, or spectrum center. The chip energy, which is given by $E_c = E_b/N$, can be increased by either increasing the bit energy or decreasing the processing gain of a particular subcarrier. Since we have shown that the BER performance of SSG-MT-DS-SS reaches its best performance when the bit energies of all subcarriers are equal, we keep the bit energy fixed to be equal for all subcarriers and change the chip energy by varying the processing gain of each subcarrier. As expected, this has significant impact upon the resulting power spectra. In Figure 3-14 and Figure 3-15, the BER performance and the spectrum of SSG-MT-DS-SS with different processing gain vectors $\mathbf{N}$ are compared with those of conventional MT-DS-SS. The SSG-MT-DS-SS systems are configured to have parameter vectors $\mathbf{R}_b = [1, 2, 1]$, $\mathbf{E}_b = [1, 1, 1]$, $\Delta\mathbf{f} = [1, 1]$, and different $\mathbf{N}$. Note that the processing gain vectors in Figure 3-14 and Figure 3-15 are normalized by that of the conventional MT-DS-SS, which is 128 for both examples.

Figure 3-14: P_b vs. E_b/N_0 for different processing gain, $\mathbf{R}_b=[1, 2, 1]$, $\mathbf{E}_b=[1, 1, 1]$, $\Delta f=[1, 1]$.

Figure 3-15: PSD of different processing gain vectors, $\mathbf{R}_b=[1, 2, 1]$, $\mathbf{E}_b=[1, 1, 1]$, $\Delta f=[1, 1]$.

From this example we can observe that the values of processing gain are of little consequence on the BER performance of SSG-MT-DS-SS, whereas the PSD of the transmitted signal is greatly affected by the selection of the processing gain vectors. As

we increase the processing gain of the central subcarrier, while keeping the other parameters fixed, the chip energy of the central subcarrier is reduced. Therefore, the energy is not concentrated in the mainlobe of the transmitted signal, and the sidelobe levels are higher. This can also be explained by examining the PSD of the individual subcarriers. As the processing gain of the 2^{nd} subcarrier increases, its mainlobe bandwidth increases accordingly. As a result, the mainlobe of the transmitted signal, the summation of the PSD of all subcarriers, increases as well.

Similar to the effects of varying processing gains, the effects of different data rate vectors are demonstrated in Figure 3-16 and Figure 3-17. Clearly, it can be seen that the data rate distribution on each subcarrier has little effect on the BER performance of the SSG-MT-DS-SS when each subcarrier has the same bit energy and processing gain is held constant. However, the data rate distribution does have great effect on the PSD of the SSG-MT-DS-SS systems. To keep the total data rate of SSG-MT-DS-SS identical to that of the conventional MT-DS-SS system, the data rate of the non-central subcarriers decreases as we increase the data rate of the central subcarrier, i.e., subcarrier 2. Since we are keeping the processing gain of SSG-MT-DS-SS fixed in this example, the chip rates of these non-central subcarriers decreases, and thus the mainlobe bandwidth of SSG-MT-DS-SS decreases.

Figure 3-16: P_b vs. E_b/N_0 for different data rate vectors, $\mathbf{E}_b=[1, 1, 1]$ $\mathbf{N}=[1, 1/4, 1]$, $\Delta\mathbf{f}=[1, 1]$.

Figure 3-17: PSD of different data rate vectors, $\mathbf{E}_b=[1, 1, 1]$ $\mathbf{N}=[1, 1/4, 1]$, $\Delta\mathbf{f}=[1, 1]$.

Having shown the effects of bit energy, processing gain, and data rate vectors on the BER performance and spectral shaping of SSG-MT-DS-SS, we next demonstrate the effects of different frequency separations. Figure 3-18 shows the BER curves of SSG-MT-DS-SS with different frequency separations with all other parameters held constant.

It is interesting to see that as the frequency separation between the adjacent subcarriers of SSG-MT-DS-SS increases, the BER performance of SSG-DS-SS improves (average P_b decreases). For this example, with $\mathbf{N}=[128, 32, 128]$, $\mathbf{R}_b=[1, 2, 1]$, and $\mathbf{E}_b=[1, 1, 1]$, when the frequency separation between adjacent subcarriers is greater than $\Delta f = 27 R_{b,1}^s$, i.e., $\Delta f = 40 R_{b,1}^s$, the SSG-MT-DS-SS system outperforms the conventional MT-DS-SS with different codes on each subcarrier, and it attains the AWGN channel, orthogonal subcarrier performance when the frequency separation is $\Delta f = 64 R_{b,1}^s = 0.5 R_{c,1}^s$, which is in agreement with the analytical IS-SUI-free conditions between subcarriers provided in the previous section. The PSDs of SSG-MT-DS-SS with different frequency separations are shown in Figure 3-19, for which we can see that the mainlobe bandwidth of SSG-MT-SS increases as the frequency separation increases, and that the SSG-MT-SS signal that meets the IS-SUI free frequency separation configuration, i.e., $\Delta f = 64 R_{b,1}^s = 0.5 R_{c,1}^s$, has a slightly larger mainlobe bandwidth than that of conventional MT-DS-SS. Thus, if the BER performance is the main concern in system design, we should select the frequency separation of SSG-MT-DS-SS to be IS-SUI free.

Figure 3-18: P_b vs. E_b/N_0 for different frequency separation vectors, $\mathbf{R}_b=[1, 2, 1]$, $\mathbf{E}_b=[1, 1, 1]$, $\mathbf{N}=[128, 32, 128]$.

Figure 3-19: PSD for different frequency separation vectors, $\mathbf{R}_b=[1, 2, 1]$, $\mathbf{E}_b=[1, 1, 1]$, $\mathbf{N}=[128, 32, 128]$.

Having shown results for $M=3$ subcarriers, we next show examples using a larger number of subcarriers to shape the spectrum. The advantage of increasing the number of subcarriers in a constant bandwidth system is that we can increase the processing gain of

each subcarrier and thus improve the BER performance [12]. Figure 3-20 compares BER performance and PSD of SSG-MT-DS-SS with those of conventional MT-DS-SS schemes with $M=3$ and 5, in which all systems have the same total data rate and transmit power but a different number of subcarriers.

With the total data rate and the processing gains on each subcarrier held constant, increasing the number of subcarriers from $M=3$ to $M=5$ greatly reduces the mainlobe bandwidth of conventional MT-DS-SS. Thus, in this example, we double the processing gain of conventional MT-DS-SS when we increase the number of subcarriers from $M=3$ to $M=5$, i.e., the processing gains of the two conventional MT-DS-SS signals are respectively $N_1^c = 128$ for $M=3$ and $N_2^c = 256$ for $M=5$. From this example we can observe that by increasing the number of the subcarriers and the processing gain of each subcarrier at the same time, we increase the mainlobe bandwidth of the conventional MT-DS-SS signal. In contrast, even with increased processing gain, the SSG-MT-DS-SS with an $M=5$ system can have the same mainlobe bandwidth as the $M=3$ conventional MT-DS-SS system, and achieve additional 1 dB sidelobe reduction over that of the $M=3$ SSG-MT-DS-SS case by properly choosing the parameters of the $\mathbf{R}_b$ and $\mathbf{N}$ vectors.

(a) P_b vs. E_b/N_0 for different numbers of subcarriers.

(b) PSD for different numbers of subcarriers.

Figure 3-20: P_b vs. E_b/N_0 and PSD for SSG-MT-DS-SS with different numbers of subcarriers; for $M=3$, $\mathbf{R}_b=[1, 2, 1]$, $\mathbf{E}_b=[1, 1, 1]$, $\mathbf{N}=[128, 32, 128]$, $\Delta\mathbf{f}=[1, 1]$; for $M=5$, $\mathbf{R}_b=[1, 4, 2, 4, 1]$, $\mathbf{E}_b=[1, 1, 1, 1, 1]$, $\mathbf{N}=[256, 64, 256, 64, 256]$, $\Delta\mathbf{f}=[1, 1, 1, 1]$,

Similar to the previous example, we can use even larger numbers of subcarriers for SSG-MT-DS-SS to shape the spectrum. In Figure 3-21 the BER and PSD of the S-MT-DS-SS system with a $M=15$ subcarriers are shown. We make the processing gain of

the conventional MT-DS-SS with $M=15$ to be $N_2^c = 6N_1^c$ in order to have mainlobe bandwidth similar to that of the $M=3$ conventional system. From this example we can see that with the increased number of subcarriers, the SSG-MT-DS-SS can reduce sidelobe levels to be lower than -20 dB. As illustrated in Figure 3-20(a) and Figure 3-21(a), the price paid for shaping the SSG-MT-DS-SS spectrum with a larger number of subcarriers is the performance degradation. Specifically, compared with the conventional MT-DS-SS scheme, we required 0.5 dB more E_b/N_0 to obtain the bit error probability of $P_b=10^{-3}$ for SSG-MT-DS-SS with $M=5$, and approximately another 0.5 dB more E_b/N_0 is required to obtain the same error performance for $M=15$ SSG-MT-DS-SS.

a. P_b vs. E_b/N_0 for different numbers of subcarriers.

(b) PSD for different numbers of subcarriers.

Figure 3-21: P_b vs. E_b/N_0 and PSD for SSG-MT-DS-SS with different numbers of subcarriers, with $N_1^c = 128$, $N_2^c = 6N_1^c$; $M=3$ subcarriers has $\mathbf{R}_b=[1, 2, 1]$, $\mathbf{E}_b=[1, 1, 1]$, $\mathbf{N}=[1, 1/4, 1] N_1^c$; $\Delta\mathbf{f}=[1, 1]$; $M=15$ subcarriers has $\mathbf{R}_b=[1, 2, 2, 2, 4, 2, 4, 1, 4, 2, 4, 2, 2, 1]$, $\mathbf{E}_b=[1_{1 \times 15}]$, $\mathbf{N}=[1, 1/4, 1/4, 1/4, 1/4, 1/4, 1/2, 1/4, 1/4, 1/4, 1/2, 1/4, 1/4, 1/4, 1] N_2^c$, $\Delta\mathbf{f}=[1_{1 \times 15}]$;

(a) P_b vs. E_b/N_0 for different numbers of subcarriers, and (b) PSD for different numbers of subcarriers.

In addition to the typical symmetric spectrum, we can also vary the parameter sets to shape the spectrum asymmetrically for certain applications, such as accounting for a channel that has a known asymmetric amplitude response. For example, in an $M=3$ case we can set the data rate, processing gain, and frequency separations of the 1st and 3rd subcarriers to be different, instead of the same, to obtain an asymmetric spectrum. Analogous asymmetric parameter settings can be employed for other numbers of subcarriers.

In Figure 3-13 we showed that the SSG-MT-DS-SS with equal bit energies on all subcarriers has the best BER performance even when it has an asymmetric spectrum. The PSDs of those three SSG-MT-DS-SS systems are shown in Figure 3-22. It can be observed that the SSG-MT-DS-SS3 system has the most obvious asymmetric spectrum while the spectra of other two systems are less asymmetric. To be specific, the PSDs of the positive frequency and negative frequencies are of similar shape for both SSG-MT-DS-SS1 and SSG-MT-DS-SS2, and the difference between the 1st sidelobe levels for the positive and negative frequencies are very small, e.g., respectively 0.18 dB for SSG-MT-DS-SS1 and 0.16 dB for SSG-DS-SS2. This is because the inter-subcarrier frequency separations for SSG-MT-DS-SS1 and SSG-MT-DS-SS2 equal the data rate of the 1st subcarrier, and due to the largely overlapped spectra of each subcarrier, the asymmetry of the overall spectrum is less obvious for these two systems.

Figure 3-22: PSDs of SSG-MT-DS-SS with $\mathbf{R}_b=[2, 2, 1]$, $\mathbf{N}=[128, 32, 128]$, $\Delta\mathbf{f}=[1, 1]$ for SSG-MT-DS-SS1, $\mathbf{R}_b=[1, 1, 2]$, $\mathbf{N}=[128, 128, 32]$, $\Delta\mathbf{f}=[1, 1]$ for SSG-MT-DS-SS2, and $\mathbf{R}_b=[2, 2, 1]$, $\mathbf{N}=[128, 128, 32]$, $\Delta\mathbf{f}=[64, 1]$ for SSG-MT-DS-SS3.

In Figure 3-23 (b) we show another example of SSG-MT-DS-SS with an asymmetric spectrum. The parameters of the SSG-MT-DS-SS system are selected to be $\mathbf{R}_b=[1, 1, 2]$, $\mathbf{E}_b=[1, 1, 1]$, $\mathbf{N}=[1, 1, 1/4]$, and $\Delta\mathbf{f}=[1, 9]$. We can observe from Figure 3-23 (b) that, for the baseband PSD, the spectra of the positive and negative frequencies are not the same. Specifically, the first sidelobe levels for positive frequency and negative frequency are respectively -17 dB and -16 dB, and the spectra of the positive and the negative frequencies are of different shapes. From Figure 3-23 (a) we see that we lose less than 0.1 dB at $P_b=10^{-3}$ compared to conventional MT-DS-SS, which is a small price paid for spectral shaping.

(a) P_b vs. E_b/N_0 for an asymmetric spectrum

(b) PSD for asymmetric spectrum

Figure 3-23: P_b vs. E_b/N_0 and PSD for asymmetric spectrum of SSG-MT-DS-SS with $M=3$, $\mathbf{R}_b=[1, 1, 2]$, $\mathbf{E}_b=[1, 1, 1]$, $\mathbf{N}_s=[1, 1, 1/4]$, $\Delta\mathbf{f}=[1, 9]$. (a) P_b vs. E_b/N_0 and (b) PSD.

As we have shown in a previous example, we can further shape the spectrum of SSG-MT-DS-SS with a larger number of subcarrier. In Figure 3-24, we show the BER performance and PSD of SSG-MT-DS-SS with $M=5$ subcarriers. To shape the spectrum asymmetrically, we select the parameters to be $\mathbf{R}_b=[1, 1, 1, 1, 2]$, $\mathbf{N}=[128, 128, 32, 32, 32]$, $\mathbf{E}_b=[1, 1, 1, 1, 1]$, and $\Delta\mathbf{f}=[1, 1, 15, 20]$. With this set of parameters the sidelobe levels of both positive and negative frequencies are further suppressed (compared with the $M=3$ system), and in addition, the spectra of the positive and negative frequencies differ more. The difference between the first sidelobe levels of the positive and negative frequencies is larger for the $M=5$ system than that for the $M=3$ system shown in the previous example, e.g., it is approximately 2.2 dB when $M=5$, and 1.0 dB when $M=3$. The price paid for the spectral shaping is again the BER performance degradation. Specifically, as shown in Figure 3-24(a), we lose approximately 0.3 dB at $P_b=10^{-3}$ compared to conventional MT-DS-SS, while for the $M=3$ case we only have 0.1 dB of SNR loss at the same bit error probability.

(a) P_b vs. E_b/N_0 for an asymmetric spectrum with $M=5$

(b) PSD for asymmetric spectrum with $M=5$

Figure 3-24: P_b vs. E_b/N_0 and PSD for asymmetric spectrum of SSG-MT-DS-SS with $M=5$, $\mathbf{R}_b=[1, 1, 1, 1, 2]$, $\mathbf{N}=[128, 128, 32, 32, 32]$, $\mathbf{E}_b=[1, 1, 1, 1, 1]$, and $\Delta\mathbf{f}=[1, 1, 15, 20]$. (a) P_b vs. E_b/N_0 and (b) PSD

3.5 SSG-MT-DS-SS with Error Correction Codes

In Section 3.4, we showed examples of the spectral shaping capability of the SSG-MT-DS-SS modulation for different parameter sets. It was shown that with properly selected parameters for each subcarrier (data rate, processing gain, bit energy, and carrier frequency), the SSG-MT-DS-SS signal can be made to have a narrower mainlobe bandwidth and lower sidelobe levels than those of conventional MT-DS-SS. In addition, the SSG-MT-DS-SS reaches its best BER performance--which is comparable to that of conventional MT-DS-SS--when the bit energies are equal for all subcarriers. Although this equal bit energy configuration of SSG-MT-DS-SS results in the best BER performance, it also produces higher sidelobe levels than appropriately designed schemes with un-equal bit energies.

An example of this sidelobe level comparison is shown in Figure 3-12, where we can observe that the sidelobe level of the equal bit energy system is approximately 1 dB higher than that of the un-equal bit energy systems in the figure. Moreover, the equal bit energy constraint restricts our parameter selection space for the SSG-MT-DS-SS system, reducing flexibility. As a result, the spectral shaping capability of SSG-MT-DS-SS is limited by the equal bit energy constraint. To enhance the spectral shaping capability of SSG-MT-DS-SS, we would like to remove the equal bit energy constraint. As we have illustrated, this essentially always results in an increase of the BER of the system. A traditional means to reduce BER (generally at the expense of bandwidth), is the use of error correction codes [3].

In this section, we provide a generalized method to analyze the average bit error probability of a simple block code, the Hamming code, with different code symbol energies. This method is then extended to the SSG-MT-DS-SS system where Hamming codes are used to encode the data bits transmitted through subcarriers with different bit energies.

Bit Error Probability of Hamming code with Different Code Symbol Energies

The Hamming code is a simple block code that can detect at most two errors and correct one error within a codeword. The (n,k) Hamming encoder encodes a block of k information bits into a block of length n —an n bit codeword. The decoder we study decodes the received symbols in a maximum likelihood (ML) fashion, i.e., for a received vector of $\mathbf{r}$, the decoder outputs estimated information vector $\mathbf{U}_i$ if the following equation is satisfied:

$$P(\mathbf{r} | \mathbf{U}_i) = \max_{\text{over all } \mathbf{U}_j} P(\mathbf{r} | \mathbf{U}_j) \quad (3.26)$$

where in (3.26) $\mathbf{r}$ is the vector of length n with each element being the receiver output, and $\mathbf{U}_j$ is the vector of length k with each element being the possible binary information symbol. Note that the elements of $\mathbf{r}$ are generally a real number until quantization. The ML detection rule given by (3.26) is computationally extensive when the length of $\mathbf{U}$ is large, or the number of possible $\mathbf{U}$ vectors is large. Thanks to the properties of the Hamming codes, they can be decoded efficiently using the algebraic methods, defined in the binary field. The detection algebra can be found in many textbooks such as [3] and is

omitted in this dissertation. Algebraic decoding is not ML decoding, and hence the performance is sacrificed for lower complexity.

For a binary symmetric channel (BSC) with error probability of p , the block (codeword) and bit error probability of an (n,k) Hamming code are respectively given by

$$P_{block} = \sum_{j=2}^n \binom{n}{j} p^j (1-p)^{(n-j)} \quad (3.27)$$

$$P_b = \frac{1}{n} \sum_{j=2}^n \beta_j \binom{n}{j} p^j (1-p)^{(n-j)} \quad (3.28)$$

Note since the Hamming code is capable of correcting one error, the block error probability is the probability that the channel introduces two or more errors in the received codeword, and the bit error probability is the weighted version of the block error probability [23]. The value of the weighting factor β_j is approximately given by $\beta_j = j$ for $j > 1$ [23] which is for the average number of symbol errors in the codeword after decoding.

The block and bit error probabilities given by (3.27) and (3.28) assume the received symbol error probabilities, p , are the same within one block transmission (bit energies and channel statistics remain constant during the transmission of the n -bit block). This assumption, however, is not applicable when the code symbols within the codeword have different energies, that is, when the code symbols are transmitted through a multicarrier system with different bit energies on each subcarrier. An example of such a system is shown in Figure 3-25, where a block of k information bits, $\mathbf{b}$, is first encoded by the (n,k) Hamming encoder. After encoding, the codeword $\mathbf{c}$, a block of n symbols, is serial to parallel converted and transmitted through n subcarriers where the transmission

power of each subcarrier is adjusted by each subcarrier's amplifier individually. The received signal is passed through the bank of n correlators, where the received signal is correlated with each subcarrier carrier waveform, $s_i(t)$, and a hard decision, often used for block codes, is made based on the correlator output. These hard decisions are parallel to serial converted to form the Hamming decoder input vector $\mathbf{r}$, and finally the decoder estimates the transmitted data vector $\hat{\mathbf{b}}$. In the block diagram, the signals of form $s_i(t) = \cos(2\pi f_i t)$ are the carrier waveforms of subcarrier i , and vectors $\mathbf{b}$, $\mathbf{c}$, $\mathbf{r}$, and $\hat{\mathbf{b}}$ are binary vectors of length k , n , n , k , respectively.

Figure 3-25: Block diagram of Hamming code with different code symbol energies

The average block error probability can be bounded by the following expression [3], which is the sum of the conditional probabilities of all possible received code symbol vectors, 2^n to be exact, given the codeword $\mathbf{c}$ is transmitted, i.e.,

$$P_{block} \leq \sum_{j=0}^{2^n-1} P(\mathbf{r}_j | \mathbf{c}) \quad (3.29)$$

where, again as defined previously, $\mathbf{c}$ is the transmitted codeword, and $\mathbf{r}_j$ is the j^{th} possible codeword out of 2^n possible received codewords, and $P(\mathbf{r}_j|\mathbf{c})$ is the conditional probability of receiving $\mathbf{r}_j$ given $\mathbf{c}$ is transmitted. For a linear block code, the block and bit error probabilities are the same for all the possible codewords, i.e., P_{block} and P_b are the same for all possible $\mathbf{c}$, or in other words, these probabilities are invariant to the choice of transmitted codeword. Therefore, we can assume $\mathbf{c}$ is an all-zero codeword for simplicity. For the equal symbol error probability case, i.e., when all the symbol error probabilities within the codeword are identical, the conditional probability of receiving $\mathbf{r}_j$ when $\mathbf{c}$ is transmitted is

$$P(\mathbf{r}_j | \mathbf{c}) = (p)^{d_j} (1-p)^{(n-d_j)} \quad (3.30)$$

where n is again the length of the codeword, p is the symbol error probability of each symbol in the codeword, and d_j is the Hamming distance between the codeword $\mathbf{c}$ and $\mathbf{r}_j$. The conditional probability given by (3.30) is valid only when the symbol error probabilities of the codeword are the same. When the code symbols are of different code symbol energies, (3.30) is no longer valid. To illustrate our method of analyzing the unequal bit energy case, and for simplicity of presentation, we assume there are two different symbol error probabilities, p_1 and p_2 , within a codeword. In this case of two symbol energies, the conditional probability of receiving $\mathbf{r}_j$ given that $\mathbf{c}$ is transmitted is

$$P(\mathbf{r}_j | \mathbf{c}) = (p_1)^{d_m} (1-p_1)^{(n_1-d_m)} (p_2)^{d_n} (1-p_2)^{(n_2-d_n)} \quad (3.31)$$

where the code symbols within the codeword are grouped by their error probabilities, i.e., the “sub-codeword” of n_1 code symbols has symbol error probability p_1 , and the “sub-codeword” of n_2 code symbols has symbol error probability p_2 . The Hamming distances

between the partial transmitted and received messages for the two sub-codewords are denoted by d_m and d_n .

As noted previously, any single error, or error pattern with Hamming weight of 1, can be corrected by a Hamming code and the codewords with error patterns of Hamming weight greater than 1 cause the decoder to wrongly decode the received message. In Table 3-3, the decoder output of a Hamming (7,4) code for different error patterns are listed when the message 0000 is transmitted. Note that in Table 3-3, D_j is the number of different bits between the original message vector $\mathbf{b}$ and the Hamming decoder output $\hat{\mathbf{b}}_j$ for the j^{th} received message. Since the transmitted message is a all-zero message, the j^{th} received message is actually the j^{th} error pattern, and D_j is the number of decoding (bit) errors for the j^{th} error pattern. The number of decoding errors, D_j , is a discrete random variable with allowable values listed in Table 3-3. The probability of a given value of D_j is simply the conditional probability of receiving $\mathbf{r}_j$ when $\mathbf{c}$ is transmitted, $P(\mathbf{r}_j|\mathbf{c})$. Therefore, the expectation of D_j , or the average number of decoded bit errors is given by

$$\begin{aligned}\overline{D} &= \sum_{j=0}^{2^n-1} P(\mathbf{r}_j | \mathbf{c}) D_j \\ &= \sum_{0 \leq d_m + d_n \leq n} \sum \left[D_j (p_1)^{d_m} (1-p_1)^{(n_1-d_m)} (p_2)^{d_n} (1-p_2)^{(n_2-d_n)} \right]\end{aligned}\tag{3.32}$$

where we assume two code symbol error probabilities exist in a code word of length n , and as defined previously, d_m and d_n are the Hamming distances between the received and transmitted sub-codewords of fixed lengths n_1 and n_2 respectively. Since k information bits are encoded by the (n,k) Hamming encoder, the average bit error probability for each information bit is thus given by

$$P_b = \frac{\overline{D}}{k} = \frac{1}{k} \sum_{0 \leq d_m + d_n \leq n} \sum [D_j (p_1)^{d_m} (1 - p_1)^{(n_1 - d_m)} (p_2)^{d_n} (1 - p_2)^{(n_2 - d_n)}] \quad (3.33)$$

Table 3-3: (7,4) Hamming decoder output for different received codewords

b	c	r_j	($\hat{\mathbf{b}}_j$)	D_j
0000	0000000	0000001	0000	0
0000	0000000	0000010	0000	0
0000	0000000	0000011	0011	2
0000	0000000	0000101	1101	3
...	...	...	...	...
0000	0000000	1111111	1111	4

For simplicity, we have only shown the method for finding the average bit error probability of a Hamming (n,k) code with two distinct symbol energies within a codeword. This method can be extended to the case of multiple symbol error probabilities. To extend the method, we can simply group the code symbols into different sub-codewords according to their symbol error probabilities and find the conditional probability of received message $\mathbf{r}_j$ when the codeword $\mathbf{c}$ is transmitted accordingly. We then apply the first line of (3.32) to find the average number of errors, $\overline{D}$, and finally find the average bit error probability by averaging $\overline{D}$ over the number of information bits k .

This method can also be extended to include uncoded bits. An example of including uncoded bits is now described. We have an SSG-MT-DS-SS system in which the relative data rate vector and bit energy vector are respectively $\mathbf{R}_b=[1, 2, 1]$, $\mathbf{E}_b=[1, 2, 1]$. The code symbols and the uncoded bits of a Hamming (7,4) code are distributed according to the diagram shown in Figure 3-26, where the “U” denotes the uncoded input

bit, and the numerals 1, 2,...7 denote the 1st, 2nd ... 7th output code symbols of a (7,4) Hamming encoder

1	2	Subcarrier 1
3	4	Subcarrier 2
5	U	Subcarrier 2
6	7	Subcarrier 3

Figure 3-26: Bit distribution in a 3 subcarrier SSG-MT-DS-SS

As derived in the previous section, the symbol error probabilities of subcarriers 1, 2, 3 are respectively p_1 , p_2 , and p_3 , and again, the error probability of the i^{th} subcarrier is given by

$$p_i = Q \left\{ \sqrt{\frac{1}{N_0 / 2E_{s,i} + 2\sigma_{\alpha,i}^2 / (E_{s,i} T_{s,i})}} \right\} \quad (3.34)$$

where $N_0/2$ is the power spectral density of the AWGN, $E_{s,i}$, $T_{s,i}$ are respectively the code symbol energy and code symbol duration of the i^{th} subcarrier, and $\sigma_{\alpha,i}^2$ is the variance of the IS-SUI term seen by the i^{th} subcarrier, which is given in Table 3-2. The code symbol energy is related to the input bit energy E_b by $E_s = kE_b/n$ where as defined previously n , k are respectively the number of input information bits and the number of code symbols in a codeword. Note that the uncoded bits are transmitted though the 2nd subcarrier because of its higher bit energy and lower bit error probability.

Since we have the uncoded bit transmitted with the coded symbols simultaneously, the code symbols and the uncoded bit should be considered jointly. Thus, Table 3-3 is extended by adding the uncoded bits as shown in Table 3-4:

Table 3-4: SSG-MT-DS-SS receiver output for different received codewords

b	c	r_j	$\hat{(\mathbf{b}_j)}$	D_j
00000	00000000	0000001	00001	1
00000	00000000	0000010	00000	0
00000	00000000	0000011	00001	1
00000	00000000	0000100	00000	0
00000	00000000	0000101	00001	1
...	...	...	...	...
0000	0000000	1111111	11111	5

Based on Table 3-4 and the fact that the three subcarriers have three different symbol error probabilities, we extend (3.33) as follows

$$\begin{aligned}
 P_b &= \frac{\overline{D}}{k + N_u} \\
 &= \frac{1}{k + N_u} \sum_{0 \leq d_m + d_n + d_q \leq n} \sum \sum \left[D_j (p_1)^{d_m} (1 - p_1)^{(n_1 - d_m)} (p_2)^{d_n} (1 - p_2)^{(n_2 - d_n)} (p_3)^{d_q} (1 - p_3)^{(n_3 - d_q)} \right]
 \end{aligned} \tag{3.35}$$

where N_u is the number uncoded bits, which is l in this case, p_1 , p_2 , and p_3 are the symbol error probabilities of three subcarriers, and d_m , d_n , and d_q are the Hamming distances between the received and sent messages. It is worth noting that the average bit error probability given by (3.35) can be applied to 3-subcarrier SSG-MT-DS-SS with three distinct bit error probabilities, even though the SSG-MT-DS-SS system in this example, with $\mathbf{E}_b = [1, 2, 1]$, has only two distinct error probabilities for its subcarriers, e.g., $p_1 = p_3$. In Figure 3-27 we show that our analytical results based upon (3.35) match the simulation results. Note that the E_b/N_0 axis in the figure is the average SNR for all un-coded bits. From this figure, we can see that with a simple $(7,4)$ Hamming code, the BER performance of the SSG-MT-DS-SS is greatly improved. This then indicates that the use

of such codes may allow us to relax the constraint that requires all subcarriers to have identical bit energies in order to obtain good error probability performance.

Figure 3-27: P_b vs. E_b/N_0 for SSG-MT-DSSS with and without the use of a (7,4) Hamming code, $M=3$, $\mathbf{R}_b=[1, 2, 1]$, $\mathbf{E}_b=[1, 2, 1]$, $\mathbf{N}=[1, 1/4, 1]$, and $\Delta\mathbf{f}=[1, 1]$.

3.6 Peak to Average Power Ratio (PAPR) of SSG-MT-DS-SS

A large peak to average power ratio (PAPR) is a major drawback for multicarrier modulation schemes. Large PAPR values require the amplifier of the transmitter to have a large dynamic range, and this increases the cost and complexity of equipment. A large amount of work has been done to reduce the PAPR of multicarrier schemes, and some techniques include for example clipping, selected mapping (SLM), partial transmit sequence (PTS), tone reservation (TR), tone injection (TI), and constellation extension (ACE) [73]-[87]. Although these methods can reduce the PAPR value of multicarrier modulation schemes, they also either degrade the BER performance of the multicarrier modulation schemes or introduce additional complexity to the transmitter. The SSG-MT-DS-SS, however, can be configured to have a reduced PAPR naturally, without introducing any complexity-increasing schemes, as many of the explicit PAPR reduction methods do. In this section, we prove that with properly selected parameters, the PAPR of SSG-MT-DS-SS is lower than that of the conventional multicarrier scheme.

The PAPR of a signal $v(t)$ is defined as

$$PAPR = \frac{\max \{ [v(t)]^2 \}}{P_{avg}} \quad (3.36)$$

where P_{avg} is the average power of the transmitted signal $v(t)$. For a signal with symbol duration of T , the average power is given by

$$P_{avg} = \frac{1}{T} \int_0^T v^2(t) dt \quad (3.37)$$

The transmitted signal of a multicarrier modulation scheme is the summation of all subcarrier signals. Thus, when all the signals from different subcarriers are co-phased,

or add coherently, the maximum value of $v(t)$ is proportional to the number of the subcarriers. Since the amplitude of the transmitted signal is the major concern when analyzing the PAPR of the SSG-MT-DS-SS system, the transmitted signal given by (3.1) can be expressed in the alternative form

$$v(t) = \sum_{i=1}^M A_i X_i(t) \cos[2\pi f_{c,i} t] \quad (3.38)$$

where, as defined previously in (3.1), $A_i = \sqrt{2E_{b,i}/T_{b,i}}$, and $f_{c,i}$ are respectively the amplitude and carrier frequency of the i^{th} subcarrier, and $X_i(t)$ is the product of the data bits and spreading waveform chips of the i^{th} subcarrier. The waveform $X_i(t)$ is a sequence of equiprobable binary chips.

It can be shown that the signal expressed in (3.38) reaches its peak value when the signals of all subcarriers add coherently, i.e., $\max[v(t)] = \sum_{i=1}^M A_i$. Thus, the signal's peak power, or $\{\max[v(t)]\}^2$, is

$$\{\max[v(t)]\}^2 = \left(\sum_{i=1}^M A_i \right)^2 = \sum_{i=1}^M \sum_{j=1}^M A_i A_j \quad (3.39)$$

Based on the inequality that $x^2 + y^2 \geq 2xy$, with equality obtained when $x=y$, the maximum power given by (3.39) can be upper bound by

$$\begin{aligned} [\max(v(t))]^2 &= \sum_{i=1}^M \sum_{j=1}^M A_i A_j \leq \frac{1}{2} \sum_{i=1}^M \sum_{j=1}^M (A_i^2 + A_j^2) \\ &= \frac{M}{2} \left(\sum_{i=1}^M A_i^2 + \sum_{j=1}^M A_j^2 \right) = M \sum_{i=1}^M A_i^2 \end{aligned} \quad (3.40)$$

The peak value of the transmitted signal $v(t)$ reaches its upper bound when the amplitudes of all subcarriers are equal, i.e., when $A_i=A_j$ for $i=1..M$ and $j=1..M$. This is

exactly the case for conventional MC/MT-DS-SS schemes. In SSG-MT-DS-SS, the data rates, processing gains, and bit energies on each subcarrier are adjustable, which means we can configure the SSG-MT-DS-SS system to have different amplitudes on each subcarrier. This, as indicated by (3.40), reduces its maximum peak value. Then in our comparisons, since both SSG-MT-DS-SS and MT-DS-SS have the same total data rate input to the transmitter, the same transmitting power, and the same number of subcarriers, the average power, P_{avg} , of the two systems are the same. Therefore, the SSG-MT-DS-SS signal, with properly selected parameters, i.e., with different amplitudes on each subcarrier, has lower PAPR than that of conventional MT-DS-SS.

In Table 3-5, example PAPR reduction capabilities of SSG-MT-DS-SS are shown for two parameter sets, where the PAPR reduction in this table is defined as the relative PAPR difference between the conventional MT-DS-SS and SSG-MT-DS-SS as

$$p = \frac{PAPR^c - PAPR^s}{PAPR^c} \times 100\% \quad (3.41)$$

where $PAPR^c$ and $PAPR^s$ are respectively the PAPR of conventional MT-DS-SS and SSG-MT-DS-SS.

Table 3-5: Example PAPR reductions for two different parameter sets

R_b	E_b	N	PAPR Reduction
[1,2,1]	[1,2,1]	[1,1/4,1]	11.11
[1,4,2,4,1]	[1,2,1,2,1]	[1,1/4,1/2,1/4,1]	17.72

Note: PAPR reduction is defined in (3.41)

In practice, the distribution of the PAPR value is of more interest to the system designer, since this distribution gives the system designer an idea on how to select the

PAPR reduction scheme. For example, if the large PAPR values that extend beyond the amplifier dynamic range occur with very low probability, the simple clipping algorithm can be implemented without much performance penalty; otherwise more advanced PAPR reduction algorithms, such as SLM, PTS, TR, TI and ACE might need to be implemented. The distribution of the PAPR is often given by the complementary cumulative distribution function (CCDF), defined as $Pr(PAPR > PAPR_0)$, or the probability that the PAPR value is greater than a given PAPR value, $PAPR_0$. The CCDFs of conventional MT-DS-SS and the SSG-MT-DS-SS schemes with parameters listed in Table 3-5 are plotted in Figure 3-28. As shown in this figure, the SSG-MT-DS-SS signals have a lower PAPR than those of conventional MT-DS-SS, i.e., the SSG-MT-DS-SS signals have a smaller probability of reaching all these PAPR values than do those of the corresponding MT-DS-SS signals.

Figure 3-28: CCDF of SSG-MT-DS-SS and conventional MT-DS-SS with SSG-MT-DS-SS parameters given in Table 3-5.

3.7 SSG-MT-DS-SS Parameter Selection

As shown in the previous sections, the advantages of a properly configured SSG-MT-DS-SS system include reduced sidelobe levels, narrower mainlobe bandwidth, shaped spectra, and lower PAPR. In addition, the BER performance of the SSG-MT-DS-SS system is very close to that of the conventional MT-DS-SS system in AWGN when all subcarriers have equal bit energies. In this section, we describe the numerical method we employ to find the desired parameter set by “quasi-exhaustively” searching over the possible parameter sets according to the design goals of the SSG-MT-DS-SS. These goals may include desired sidelobe levels, mainlobe bandwidths, and even a specific spectral mask.

First, we repeat the general expression for the PSD of the SSG-MT-DS-SS signal, which is determined by the data rate vector $\mathbf{R}_b$, bit energy vector $\mathbf{E}_b$, processing gain vector $\mathbf{N}$, and carrier frequency vector $\mathbf{f}_c$. In this section, the absolute carrier frequency vector instead of the frequency separation vector is used to illustrate the PSD of SSG-MT-DS-SS. The PSD of the SSG-MT-DS-SS signal in a general form is given by (3.4), and it is given again here for reference

$$S_v(f, \mathbf{E}_b, \mathbf{R}_b, \mathbf{N}, \mathbf{f}_c) = \sum_{i=1}^M \frac{E_{b,i}}{N_i} \text{sinc}^2 \left(\frac{f - f_{c,i}}{R_{b,i} N_i} \right) \quad (3.42)$$

where $\mathbf{E}_b$, $\mathbf{R}_b$, $\mathbf{N}$, $\mathbf{f}_c$ are respectively the vector of bit energies, data rates, processing gains, and carrier frequencies, with the i^{th} element of these vectors being the bit energy, data rate, processing gain, and absolute carrier frequency of the i^{th} subcarrier. The effects of varying different parameters on the spectral shaping capabilities of the SSG-MT-DS-SS

signaling were shown in previous section. We show here some example PSD plots of the SSG-MT-DS-SS scheme with different parameter sets in Figure 3-29 and Figure 3-30.

Figure 3-29: SSG PSD for $M=3$, $\mathbf{E}_b=[1, 2, 1]$, $\mathbf{R}_b=[1, 2, 1]$, $\mathbf{N}=[1, 1/4, 1]$, $\mathbf{f}_c=[-1, 0, 1]$

Figure 3-30: SSG PSD for $M=3$, $\mathbf{E}_b=[1, 2, 1]$, $\mathbf{R}_b=[1, 1/2, 1]$, $\mathbf{N}=[128, 32, 128]$, $\mathbf{f}_c=[-1, 0, 1]$.

In these examples, all the parameters except the data rate vectors of the two SSG-MT-DS-SS signals are the same. With only one parameter different, the spectra of the two SSG-MT-DS-SS signals are quite different. Since we allow all the parameters to be adjustable, it is very difficult and time-consuming (and impossible in the case of parameters that take on real number values) to manually examine all the possible parameter sets and find the particular parameter set that results in the desired PSD shape. Therefore, we have developed a numerical method that can find the desired parameters according to the user specified design goal. Worth noting is there may be more than one parameter set that satisfies a given set of spectral constraints.

The design goals, or parameter selection criteria, of the SSG-MT-DS-SS scheme include reduced sidelobe levels and approximately the same, or a slightly changed mainlobe bandwidth, relative to the equivalent conventional MT-DS-SS signal. The mainlobe bandwidth of the PSD is defined as

$$BW_{main} = f_{R,Null} - f_{L,Null} \quad (3.43)$$

where $f_{L,Null}$, $f_{R,Null}$ are the frequencies of the first null to the left of the spectral peak and to the right of the spectral peak, respectively (first nulls of the positive and negative frequencies of the PSD of the baseband signal). For example, the scheme in Figure 3-29 has $BW_{main} \cong 1.4R_c$, and the scheme in Figure 3-30 has $BW_{main} \cong 2.4R_c$. The quantities $f_{R,Null}$ and $L_{R,side}$ are jointly defined as follows:

$$L_{R,side} = \max[S_v(f)] \quad \text{when } f > f_{R,Null} \quad (3.44)$$

$$S_v(f_{R,Null} \pm \delta_f) > S_v(f_{R,Null}) \quad (3.45)$$

$$L_{R,side} - S_v(f_{R,Null}) > \delta_{m,R} \quad (3.46)$$

where δ_f is the frequency resolution of the analysis, and $\delta_{m,R}$ is the desired minimal difference between the sidelobe level and null level for positive frequency. Note that (3.44) guarantees that $L_{R,side}$ is the level of the first sidelobe to the right of the spectral peak, and (3.45), (3.46) together guarantee that the PSD has a null at $f_{R,Null}$. For example, for (3.46), the PSD of Figure 3-29 would allow us to use $\delta_{m,R} \approx 17 - (-49) = 32$ dB. We define $f_{L,Null}$ and $L_{L,side}$ analogously:

$$L_{L,side} = \max[S_v(f)] \quad \text{when } f < f_{L,Null} \quad (3.47)$$

$$S_v(f_{L,Null} \pm \delta_f) > S_v(f_{L,Null}) \quad (3.48)$$

$$L_{L,side} - S_v(f_{L,Null}) > \delta_{m,L} \quad (3.49)$$

As we have shown in section 3.4, the PSD of SSG-MT-DS-SS can be shaped either symmetrically or asymmetrically. For a symmetric PSD, only one side of the spectrum needs to be examined, and we have $-f_{L,Null} = f_{R,Null}$, $L_{L,side} = L_{R,side}$, and $\delta_{m,R} = \delta_{m,L}$. Here, we define $f_{L,side}$ as the frequency at which $L_{L,side}$ occurs, and similarly for $f_{R,side}$. When the spectrum is shaped asymmetrically, both sides of the PSD should be examined and all the values $f_{L,Null}$, $L_{L,side}$, $L_{R,side}$, and $f_{R,Null}$ may no longer be symmetric with respect to the spectrum center.

Given the definition of the PSD nulls, sidelobe levels, and the specification of analysis parameters, i.e., frequency resolution δ_f and the desired minimal differences between the first sidelobe level and first null level for both positive and negative frequencies ($\delta_{m,R}$, $\delta_{m,L}$), we can find the $f_{R,Null}$ and $f_{R,side}$ of a given SSG-MT-DS-SS system by the standard calculus method of finding local minima and maxima, e.g., find f for which

$$\frac{d[S_v(f, \mathbf{E}_b, \mathbf{R}_b, \mathbf{N}, \mathbf{f}_c)]}{df} = 0 \quad (3.50)$$

Then for the frequency at which (3.50) is satisfied, we check the sign of the second derivative. We can represent the solutions of (3.50) by a vector $\mathbf{F}_{\text{Null}} = [f_{\text{Null},1}, f_{\text{Null},2}, \dots, f_{\text{Null},k}]$, for the first k nulls and $\mathbf{F}_{\text{Peak}} = [f_{\text{Peak},1}, f_{\text{Peak},2}, \dots, f_{\text{Peak},k}]$ for the first k peaks to the right of the first null. For positive frequencies (baseband), $\mathbf{F}_{\text{Null}}$ and $\mathbf{F}_{\text{Peak}}$ are vectors of positive numbers. Then $f_{R,\text{Null}}$ is the first element in $\mathbf{F}_{\text{Null}}$ that satisfies (3.44)-(3.46), and we can find $f_{L,\text{Null}}$ similarly (if needed, for asymmetric spectra). Likewise, we can describe the sidelobe peak levels by a vector $\mathbf{L} = [L_{L,\text{side}}, L_{R,\text{side}}]$.

Since the closed form solution of (3.50) is difficult to obtain in most cases, we generally have to solve (3.50) numerically, i.e., to find the values for $f_{L,\text{Null}}$ and $f_{R,\text{Null}}$ (and corresponding values for $L_{L,\text{side}}$ and $L_{R,\text{side}}$) for a given parameter set we use a numerical method. To find the values of $\mathbf{F}_{\text{Null}}$, $\mathbf{F}_{\text{Peak}}$, and $\mathbf{L}$ for all possible parameter sets of interest, we substitute one set of parameters, say, $\Psi_1 = [\mathbf{E}_{b1}, \mathbf{R}_{b1}, \mathbf{N}_1, \mathbf{f}_{c1}]$, solve (3.50) numerically to find $\mathbf{F}$ and $\mathbf{L}$, and repeat with Ψ_2 , etc. The pseudo-code that describes how we find all values for $\mathbf{F}$ and $\mathbf{L}$ associated with all parameter sets (all Ψ_i for a given number of subcarriers M) is listed below:

- Initialization: specify the analysis parameters: δ_f , $\delta_{m,R}$, $\delta_{m,L}$ and index $i=1$,
- 1: select parameter set Ψ_i ,
 - 2: solve (3.50) numerically to obtain vectors $\mathbf{F}$ and $\mathbf{L}$,
 - 3: employ (3.44)-(3.46) on $\mathbf{F}$ and $\mathbf{L}$ to find the $f_{L,\text{Null}}$, $f_{R,\text{Null}}$, $L_{L,\text{side}}$ and $L_{R,\text{side}}$ for Ψ_i ,
 - 4: $i=i+1$, and go back to step 1 for all i .

With the values of $\mathbf{F}$ and $\mathbf{L}$ found by this numerical method we can easily find the mainlobe bandwidth, BW_{main} , and the sidelobe levels of the SSG-MT-DS-SS for all parameter sets Ψ , and select the parameter set of the SSG-MT-DS-SS according to the desired sidelobe levels and the mainlobe bandwidth. It is worth noting that since the elements of $\mathbf{E}_b$ and $\mathbf{f}_c$ are real numbers, we can only test some subset of discrete values for $\mathbf{E}_b$ and $\mathbf{f}_c$. This is analogous to representing a continuous signal via its discrete samples. In the work of this dissertation, we *can* use any arbitrary value (limited of course by the computer's numerical precision) between two samples. Specifically, the difference between the values of each element in $\mathbf{E}_b$ from the parameter set Ψ_i and the next parameter set Ψ_{i+1} can be of any arbitrary value, and a similar comment applies for the $\mathbf{f}_c$ vectors. In this work, the step size for the bit energy change is chosen to be 1% of $E_{b,0}$, which is the bit energy of the conventional MT-DS-SS scheme with the same total transmission power and data rate of our SSG-MT-DS-SS scheme. In addition, when normalized by parameters of the 1st subcarrier, the bit energy, data rate, and processing gain of each subcarrier, excluding the 1st subcarrier, are restricted to vary in the range of $[1/4, 4]$; similarly, we restrict the frequency separation between two adjacent subcarriers to be less than the chip rate of the 1st subcarrier. Note that the restrictions on the parameter selections are somewhat arbitrary. We can, of course, relax the restrictions to enlarge the possible number of parameter sets Ψ , e.g., we could allow the inter-subcarrier frequency separation to be greater than the chip rate of the 1st subcarrier and also allow the other parameters to vary in a range outside $[1/4, 4]$; however this will increase the computational burden of our parameter selection method. This could be an area for future work.

The method of selecting an optimal parameter set according to sidelobe levels and mainlobe bandwidth can be used to select the parameters of an SSG-MT-DS-SS signal to meet a specific spectral mask. An example base band spectral mask is shown in, where f_0 is the center frequency of the transmitted signal, and $A, B, \dots, F$ are PSD levels of at frequency $f_A, f_B, \dots, f_F$.

Figure 3-31: Example spectral mask

Given the spectral mask, the bandwidth of the signal occupied is determined by the values of f_B , and f_E , and the out-band radiation is determined by the values of B and E . In practice, spectral masks are specified by the regulatory authorities or international standards, e.g., the spectral mask of wireless LAN transmitters is given by the FCC [22]. Often times, the spectral mask is symmetric about the spectral center or center frequency f_0 ($f_0=0$ for baseband spectral mask), implying $A=D$, $B=E$, $C=F$, $|f_A-f_0|=|f_D-f_0|$, $|f_B-f_0|=|f_E-f_0|$, and $|f_C-f_0|=|f_F-f_0|$. With a slight modification, the numerical method that finds all

$f_{L,Null}$, $f_{R,Null}$, $L_{L,side}$, and $L_{R,side}$ associated with all parameter sets (all Ψ_i) can be used to select the parameter set of SSG-MT-DS-SS that satisfies a given spectral mask. The modified method is given as follows:

Initialization: specify the analysis parameters: δ_f , $\delta_{m,R}$, $\delta_{m,L}$, f_A , $f_B, \dots, f_F$, A , $B, \dots, F$

and set index $i=1$,

1: select parameter set Ψ_i ;

2: solve (3.50) numerically to obtain vector $\mathbf{F}_{Null}$, $\mathbf{F}_{Peak}$ and $\mathbf{L}$;

3: employ (3.44)-(3.46) on $\mathbf{F}_{Null}$, $\mathbf{F}_{Peak}$ to find the $f_{L,Null}$, $f_{R,Null}$, $L_{L,side}$ and $L_{R,side}$ for

Ψ_i ,

4: if the $f_{L,Null}$, $f_{R,Null}$, $L_{L,side}$, and $L_{R,side}$ values and the PSD levels at f_C and f_F satisfy

the relation between $f_A f_B \dots f_F$, A , $B, \dots, F$, record the parameter sets Ψ_i , and

stop the loop;

5: $i=i+1$, and go back to step 1 for all i ;

Output: output the parameter sets which are recorded in 4.

Note that in step 4, the $f_{L,Null}$, $f_{R,Null}$, $L_{L,side}$, and $L_{R,side}$ are compared with the desired $f_A, f_B, \dots, f_F$, A , $B, \dots, F$ values. If the following relationships are satisfied, the parameter set is considered to be the parameter set that satisfies the given desired spectral mask:

$$|f_A - f_{R,Null}| \leq \delta_{m,R} \quad (3.51)$$

$$|f_B - f_{R,Null}| \leq \delta_{m,R} \quad (3.52)$$

$$L_{R,side} \leq B \quad (3.53)$$

$$S_v(f_C) \leq C \quad (3.54)$$

$$|f_D - f_{L,Null}| \leq \delta_{m,R} \quad (3.55)$$

$$|f_E - f_{L,Null}| \leq \delta_{m,R} \quad (3.56)$$

$$L_{L,side} \leq E \quad (3.57)$$

$$S_v(f_F) \leq F \quad (3.58)$$

Here we show an example of using the modified method to design the SSG-MT-DS-SS according to the given spectral mask. The parameters of the spectral mask are given by Table 3-6, where the frequencies are normalized by the chip rate of the conventional MT-DS-SS system as in the previous examples. In this example, we design the SSG-MT-DS-SS to have a symmetric spectrum, i.e., the frequency regions are symmetric with the center frequency f_0 , and the values A and B are the same.

Table 3-6: Spectral mask parameters

	A	B	C	D	E	F
Frequency	1	1.1	4	-1	-1.1	-4
Normalized Amplitude (dB)	0	-17	-25	0	-17	-25

With this modified numerical search method, we can design the SSG-MT-DS-SS system whose PSD satisfies the spectral mask parameters given in Table 3-6. The PSD of the designed SSG-MT-DS-SS and the spectral mask are plotted in Figure 3-32. In this figure the parameters of the 3-subcarrier SSG-MT-DS-SS are $\mathbf{E}_b=[2,1,2]$, $\mathbf{R}_b=[1,2,1]$, $\mathbf{N}=[128, 32, 128]$, and $\mathbf{f}_c=[-36, 0, 36]$ (or $\Delta\mathbf{f}=[36, 36]$). With these parameters the PSD is symmetric and has a spectral null at $f=\pm 1.171$ (normalized by the chip rate of the conventional MT-DS-SS system) and the first sidelobe level is approximately -17.51 dB. From this figure we can clearly see that the PSD of the SSG-MT-DS-SS satisfies the

spectral mask; however since the $\mathbf{E}_b$ vector is not an all-ones vector, the BER performance of the SSG-MT-DS-SS of this configuration is degraded. As shown in Figure 3-33, we have approximately 1.4 dB of SNR loss at the bit error probability of $P_b=10^{-3}$.

Figure 3-32: Designed SSG-MT-DS-SS to satisfy spectral mask.

Figure 3-33: BER performance of the designed SSG.

3.8 SSG-MT-DS-SS with Line Codes

In telephony systems, line codes are often used to encode the digital information that is about to be transmitted through the physical channel (often copper telephone lines). After line encoding, the encoded digital signals are directly put into the AC coupled transmission line. Thus, a good line code is DC free. Other considerations for choosing a line coding scheme are its power spectrum; transparency, i.e., the line code should represent (allow) all binary 1 and 0 patterns; and timing information, etc. [113]. There are two common types of line code: return to zero (RZ) and non-return to zero (NRZ). For a detailed description of line codes please refer to [113]. It is also worth to note that the rectangular chip waveform is actually a special case of a line code – the polar NRZ code.

In this section we use the Manchester and Miller codes to show the spectral shaping results of SSG-MT-DS-SS with line codes. The Manchester code, also called digital biphas, is a NRZ line coding scheme, and it is used in Ethernet LAN systems. For the Manchester code a binary 1 is represented by a pulse that takes a positive voltage level for the 1^{st} half of the bit duration and a negative voltage level for the 2^{nd} half. Similarly, the binary 0 is represented by a negative voltage level for the 1^{st} half and a positive voltage level in the 2^{nd} half. With Manchester encoding for each chip, the PSD of the i^{th} subcarrier of SSG-MT-DS-SS is given by [113].

$$S_i(f) = E_{c,i} \text{sinc}^2[(f - f_{c,i})T_c / 2] \sin^2[(f - f_{c,i})T_{c,i} / 2] \quad (3.59)$$

where similar to their definitions in (3.3) and (3.4) $E_{c,i}$, $T_{c,i}$, and $f_{c,i}$ are respectively the chip energy, chip duration, and the carrier frequency of subcarrier i . Besides the

Manchester code, the Miller code [113] is also one of the widely used line codes. With a Miller code, the binary one is represented by a pulse with a transition at the middle of bit duration, and binary zero is represented by a pulse without a transition. In addition, for two consecutive binary zeros, a transition occurs at the bit boundary. When the Miller code is used to encode the DS-SS chips, the PSD of subcarrier i is [113].

$$S_i(f) = \frac{E_{c,i}}{2[\pi(f - f_{c,i})T_{c,i}]^2(17 + 8[\pi(f - f_{c,i})T_{c,i}]_c)} \left\{ 23 - 2\cos[\pi(f - f_{c,i})T_{c,i}] \right. \\ - 22\cos[\pi(f - f_{c,i})T_{c,i}] - 12\cos 3[\pi(f - f_{c,i})T_{c,i}] + 5\cos 4[\pi(f - f_{c,i})T_{c,i}] \\ + 12\cos 5[\pi(f - f_{c,i})T_{c,i}] + 2\cos 6[\pi(f - f_{c,i})T_{c,i}] - 8\cos 7[\pi(f - f_{c,i})T_{c,i}] \\ \left. + 2\cos 8[\pi(f - f_{c,i})T_{c,i}] \right\} \quad (3.60)$$

The PSDs of Manchester encoded and Miller encoded DS-SS signals (single carrier) are shown in Figure 3-34. From this figure, we can see that, compared with the polar NRZ code (rectangular chip waveform), the energy of the Manchester encoded and Miller encoded DS-SS signals are reduced at DC frequency. When the Manchester code is used, the spectrum has nulls at DC, and approximately $2R_c$. This indicates the signal is DC-free, and the (baseband) transmission bandwidth is approximately $2R_c$. In addition, we find that the I^{st} sidelobe level of Manchester encoded DS-SS signal is approximately -10 dB, about 3 dB higher than that of the polar NRZ scheme. We should note that the PSD of the Miller code given by (3.60) is a function composed of a number of cosine terms, and no “*sinc*” term. Thus, when observing its PSD shown in Figure 3-34, we can see that it is of a totally different shape from the “*sinc*” function based spectra (polar NRZ, and Manchester code).

Figure 3-34: PSD of Manchester code and Miller code.

In (3.59) and (3.60) we provide the PSD expression of one subcarrier. For SSG-MT-DS-SS, since different data and spreading codes are used for each subcarrier, the spectrum of the transmitted signal is the summation of all subcarrier spectra as

$$S(f) = \sum_{i=1}^M S_i(f) \quad (3.61)$$

where the spectrum of individual subcarrier $S_i(f)$ is given by (3.59) or (3.60) depending on which line code scheme is used.

In Figure 3-35, the PSD of SSG-MT-DS-SS system with $\mathbf{R}_b=[1, 2, 1]$, $\mathbf{N}=[128, 32, 128]$, $\mathbf{E}_b=[1, 1, 1]$, and $\Delta\mathbf{f}=[1, 1]$ are shown with and without Manchester encoding. From this figure, we can observe that with the Manchester coding used on each subcarrier, the spectrum of SSG-MT-DS-SS is shaped to have a spectral null at DC. In

addition, since the Manchester code widens the mainlobe bandwidth, the mainlobe bandwidth of SSG-MT-DS-SS is larger than that of SSG-MT-DS-SS without the Manchester code. We can also observe the sidelobe levels of SSG-MT-DS-SS are larger when Manchester code is used. Nonetheless, the SSG-MT-DS-SS approach reduces the Manchester sidelobes, just as it does for rectangular NRZ chips (from -10 dB to near -14 dB).

Figure 3-35: PSDs of SSG-MT-DS-SS with and without Manchester encoding, $\mathbf{R}_b=[1, 2, 1]$, $\mathbf{N}=[128, 32, 128]$, $\mathbf{E}_b=[1, 1, 1]$, and $\Delta\mathbf{f}=[1, 1]$.

The PSD of SSG-MT-DS-SS with the same parameters but with the Miller code on each subcarrier is shown in Figure 3-36. As shown in this figure, with the Miller code used on each subcarrier, the spectra of SSG-MT-DS-SS has a null at the frequency of $1.5R_c$. If we take this frequency as the cut-off frequency of the mainlobe, the 1st sidelobe

level of this example is approximately -24 dB, which is approximately 8dB lower than that of SSG-MT-DS-SS without the Miller code. For Manchester codes, since the binary one is represented by 1,-1 and the binary zero is represented by -1, 1, and these two waveforms are “antipodal”, there is no BER performance degradation when it is applied to the SSG-MT-DS-SS system. The Miller code, however, represents the binary one and zero by orthogonal waveforms (transition at the mid-bit point for binary one, and no transition for binary zero), and as a result, we have a 3 dB performance loss when it is applied to SSG-MT-DS-SS.

Figure 3-36: PSDs of SSG-MT-DS-SS with and without Manchester encoding, $\mathbf{R}_b=[1, 2, 1]$, $\mathbf{N}=[128, 32, 128]$, $\mathbf{E}_b=[1, 1, 1]$, and $\Delta\mathbf{f}=[1, 1]$.

3.9 Conclusions

In this chapter we introduced the SSG-MT-DS-SS signaling scheme and discussed the mechanism by which our SSG-MT-DS-SS modulation shapes the spectrum of the transmitted signal. For the initial discussion of SSG-MT-DS-SS, we investigated the effects of different parameters on the spectral shaping and on the single user BER performance in the AWGN channel. We also derived a general method to evaluate the average bit error probability of a block code that uses different code symbol energies. This method was extended to evaluate the BER performance of a coded SSG-MT-DS-SS system. Finally, numerical methods were described for enabling selection of the parameters of SSG-MT-DS-SS signals according to some design goals, including sidelobe levels, mainlobe bandwidth, and a spectral mask.

We show in this chapter the effects of different parameter sets on the spectral shaping and BER performance of SSG-MT-DS-SS. The effects of the parameters, when we vary just one parameter and keep the rest constant, are as follows:

- i. Bit energy vector: the distribution of the bit energies on each subcarrier does not affect the mainlobe bandwidth but does affect the sidelobe levels of SSG-MT-DS-SS. Specifically, with higher bit energy values on the central subcarriers, we can concentrate more energy in the mainlobe of SSG-MT-DS-SS, and, as a result, reduce its sidelobe levels. In addition, with the total transmission power fixed, the larger the bit energy on some of the subcarriers the lower the PAPR value. One extreme example is that when the bit energy of one subcarrier is much higher than other

subcarriers, the PAPR of SSG-MT-DS-SS system approaches to 1, as in single carrier case.

- ii. Processing gain vector: The processing gains of each subcarrier have little effect on the BER performance for the AWGN single user setting, but they do affect the spectra of SSG-MT-DS-SS. With different processing gain vectors, the BER performance of SSG-MT-DS-SS is approximately the same. However, with larger processing gain values, the SSG-MT-DS-SS signal has a larger mainlobe bandwidth and higher sidelobe levels. The processing gain vector also affects the PAPR of the SSG-MT-DS-SS system. The chip amplitude is reduced when we increase the processing gain value on a given subcarrier. Thus, with larger processing gains on some subcarriers of SSG-MT-DS-SS, we can reduce its PAPR.
- iii. Data rate vector: similar to the processing gain vector, the distribution of data rates on each subcarrier has little effect on the BER performance. The data rates on each subcarrier do affect the spectra of the SSG-MT-DS-SS signal. Specifically, with a lower data rate on the subcarrier whose processing gain value is small, for a constant total data rate, the data rates of the other subcarriers must increase, and with constant N , this implies increased chip rates on those subcarriers; this yields a wider mainlobe bandwidth but lower sidelobe levels. Under the constant total data rate requirement, the chip amplitude of each subcarrier is affected by the data rate vector. Thus, with different data rate distributions, we have different sets of chip amplitudes for each subcarrier, and, as a result, the PAPR varies as we vary the data rate vector. The effect of the data rate vector on the PAPR is complex and system dependent, i.e., depends upon the bit energy vector and processing gain vector.

- iv. Frequency separation vector: The frequency separation vector affects both the spectra and BER performance of SSG-MT-DS-SS. With larger inter-subcarrier spacing, the SSG-MT-DS-SS has a larger mainlobe bandwidth and approximately the same sidelobe levels. In addition, for some given inter-subcarrier spacing values the SSG-MT-DS-SS is IS-SUI free in the AWGN (or a flat fading) setting. Since the chip amplitude is not a function of inter-subcarrier spacing, the frequency separation vector does not affect the PAPR.

In the next chapter, we analyze the BER performance of SSG-MT-DS-SS in more realistic channels—dispersive fading channels—and also analyze its multiple access capacity.

Chapter 4. SSG-MT-DS-SS in Dispersive Channels

In Chapter 3 we introduced the SSG-MT-DS-SS system and derived expressions for its BER performance in the AWGN channel for the single user setting. In this chapter we investigate the BER performance of SSG-MT-DS-SS in a more realistic model for wireless channels, in the multiuser case. As for any DS-SS system, users are distinguished by the user-specific spreading codes, i.e., CDMA is used to support multiple users. In this scenario, we term the multiuser SSG-MT-DS-SS system an SSG-MT-DS-CDMA system, since CDMA is employed to support multiple users. In this chapter, we provide an overview of the primary features of the wireless channel, followed by our augmented system models for the transmitter, receivers, and the fading channel itself. We then derive the bit error probability of SSG-MT-DS-CDMA in these dispersive channels, and finally we conclude this chapter by providing some numerical examples to corroborate and extend our analysis.

4.1 Wireless Channel

The SSG-MT-DS-CDMA system, as with other multicarrier spread spectrum techniques, is a modulation scheme based on the combination of DS-CDMA and multicarrier modulation schemes. Thus, the SSG-MT-DS-CDMA scheme can be made to have the advantages of both schemes, e.g., multiple access capabilities and robustness in fading channels. These advantages make the SSG-MT-DS-CDMA suitable for use in a wireless environment. In this section we give a brief overview of the wireless channel.

A wireless channel is the medium through which the radio wave propagates and over which information is conveyed from the transmitter to the receiver. The radio waves that propagate through the wireless channel are reflected, absorbed, scattered, and diffracted by different objects such as walls, buildings, trees, moving vehicles, etc[3] [4]. If the radio-frequency electromagnetic characteristics of the obstructing objects and all the detailed geometric details are known, one could apply the electromagnetic theory, i.e., Maxwell's equations with boundary conditions, to estimate the wireless channel's effect upon the transmitted signal. However, the direct application of electromagnetic theory is very computationally extensive, and in practice, some times the results are inaccurate because the detailed environment data is often hard to obtain. Great efforts have been devoted to modeling wireless channels using statistical methods, which provide much simpler and "statistically accurate" models. To model the wireless channel, we often separate the effects of the channel fading into two parts: large scale fading and small scale fading.

Large Scale Fading

Large scale fading is used to model the average received signal power and its fluctuation as a function of distance, over a local area, where this local area is typically defined as being of many (e.g., tens) of wavelengths in extent. The average received signal power decreases as a function of distance between transmitter and receiver, and the functional form of this decrease can be modeled deterministically. In addition to this attenuation, also known as path loss, spreading loss, or transmission loss, there are fluctuations in received power that are conveniently modeled as being random. These

fluctuations are mainly due to the obstruction of the radio signal, known as shadowing. The path loss, which is a function of the distance between the transmitter and receiver, is typically characterized by an attenuation factor relating the transmitted signal power and received signal power. When there are no objects between the transmitter and receiver and the medium is perfectly uniform in energy transmission, the medium, or channel, is called the *free space* channel, and the path loss for the free space environment is given by [3]

$$L(d) = \left(\frac{4\pi d}{\lambda} \right)^2 \quad (4.1)$$

where d is the distance between the transmitter and receiver, and λ is the wavelength of the radio wave. The free space path loss model is a path loss model for an *ideal* environment, and is not accurate for most real terrestrial applications. Hence modifications are made to model the path loss for different environments.

A commonly used model for the average path loss is given by the following [4]:

$$\bar{L}(d)_{dB} = L_s(d_0)_{dB} + 10n \log(d / d_0) \quad (4.2)$$

In (4.2) d_0 is a reference distance for path loss modeling, and the first term $L_s(d_0)_{dB}$, is the path loss measured at the reference distance d_0 . The reference distance is selected differently for different settings, e.g., 1 m for indoor environments and 100 m for outdoor environments. The value of n in (4.2) is environment specific, and is called the path loss exponent, via analogy to (4.1) which has an exponent of two. The value of n can be greater than two when there are obstructions between the transmitter and receiver.

In addition to basic path loss, the received signal strength is also affected by the presence of objects in the signal transmission path. The objects that absorb or scatter the

radio energy in a random manner make the received signal strength attenuate randomly.

This random attenuation caused by the objects in the transmission paths is termed shadowing, and it is often modeled by the log-normal distribution [4] as

$$p(x) = \frac{10/\ln 10}{x\sigma_{dB}\sqrt{2\pi}} \exp\left[-\frac{10\log x - \mu_{dB}}{2\sigma_{dB}^2}\right] \quad (4.3)$$

where μ_{dB} and σ_{dB}^2 are respectively the mean and variance of the attenuation variable x in dB. Note in (4.3) that x is on a linear scale. When x is expressed in a logarithmic scale, the pdf of (4.3) becomes the normal, or Gaussian, distribution

$$p(x_{dB}) = \frac{1}{\sigma_{dB}\sqrt{2\pi}} \exp\left[-\frac{x_{dB} - \mu_{dB}}{2\sigma_{dB}^2}\right] \quad (4.4)$$

Since path loss and shadowing both occur over a large spatial scale, we can combine the average path loss and shadowing models together to model the large scale fading as [4]

$$L(d)_{dB} = L(d_0)_{dB} + 10n\log(d/d_0) + X \quad (4.5)$$

where X is a Gaussian random variable with zero mean and variance of σ_{dB}^2 . Note that since the value of n and the variance of X in (4.5) are usually obtained via curve fitting empirical data, the average shadowing effect, or the mean of log-normal distribution, μ_{dB} , is incorporated into the average path loss, and thus X is of zero mean.

Small Scale Fading

Because of the reflectors/scatterers in the wireless channel and the possible movement of the transmitter/receiver, the transmitted signal reaches the receiver antenna via multiple transmission paths. This phenomenon is termed as multipath, and the fading

caused by multipath is called small scale fading, because its effects can be seen over spatial scales on the order of one-half wavelength.

When the signal is transmitting through a multipath channel, multiple replicas of the transmitted signal reach the receiver antenna at slightly different times. In addition, the amplitudes and phases of these transmitted signal replicas exhibit a random nature because of the scattering and reflection. When these signals are added together at the receiver, the resultant received signal is modeled as having a random amplitude and phase [3], [4]. Next, we introduce the main characteristics of the multipath fading channel.

- Frequency selective and flat fading [3], [4]: frequency selectivity is used to describe the variation of the channel in the frequency domain. This can be translated, by Fourier relations, into the time spreading (or dispersive) effects the multipath channel has on the signal. For the multipath channel, the transmitted signal reaches the receiver antenna at different times, which causes inter-symbol-interference (ISI). If the multipath delay spread is greater than (or even a substantial fraction of) a symbol duration, the ISI can be non-negligible. The effect of the ISI induced by the channel dispersion can also be observed in the frequency domain, where the spectral components of the transmitted signal are affected by different channel attenuations and phases. In this scenario, the coherence bandwidth, the bandwidth over which the fading is correlated by some specified amount, is less than the signal bandwidth. In summary, when the coherence bandwidth is less than the signal bandwidth, the channel is frequency selective, otherwise it is non-frequency selective, or the fading is called flat fading.

- Fast and slow fading [3], [4]: The time varying nature of the multipath channel is described by fast or slow fading. The slow and fast fading is related to the time correlation of the channel. If the coherence time, the time period over which the fading process is correlated by some specified amount, is less than the symbol duration, the channel is fast fading channel, otherwise it is called a slow fading channel.

The wireless channel can be categorized into flat or frequency selective fading and fast or slow fading according to the relations between the coherence bandwidth and coherence time to the symbol rate and symbol time of the signaling scheme. In this dissertation, we assume the channel between the transmitter and receiver is a slow fading channel, i.e., the channel is quasi-static over a block of transmitted symbols. This assumption is widely used by researchers, i.e., [38] -[57], and is valid for high speed data transmission over terrestrial channels where mobile velocities are typically below 120 kph and carrier frequencies are below 10 GHz.

When a wideband signal passes through the frequency selective channel, its different spectral components are affected differently by the channel. The frequency selectivity results in channel dispersion in the time domain, which is often modeled by a tapped delay line model (TDL) as

$$h(t) = \sum_{l=0}^{L-1} \alpha_l \exp(-j\phi_l) \delta(t - \tau_l) \quad (4.6)$$

where L is the number of channel taps which is given by

$$L = \lfloor R_s / B_c \rfloor + 1 \quad (4.7)$$

In (4.7), $\lfloor x \rfloor$ is the integer part of x , R_s is the symbol rate of the signaling scheme, and B_c is coherence bandwidth of the channel. Equation (4.6) is the complex baseband, or

lowpass equivalent, of the channel's impulse response (IR). Note that this TDL model can also be used to model the flat fading channel by setting the number of channel taps to be one. In (4.6) α_l , φ_l , τ_l , are respectively the tap amplitude, phase shift, and delay for the l^{th} tap, and this model can be thought of as discretizing the received multipath components. The tap amplitudes are commonly modeled by one of the Rayleigh, Rician, or Nakagami- m *statistical* distributions introduced later. These tap amplitudes are often modeled to be statistically independent RVs under the assumption of uncorrelated scattering. The average powers of the channel taps, e.g., $\Omega_l = E[\alpha_l^2]$ for the l^{th} tap, are related to the power delay profile (PDP) or multipath intensity profile (MIP), which is often approximated as a decaying exponential function,

$$\Omega_l = \Omega_1 e^{-\eta l} \quad (4.8)$$

where l is the tap index and η is the decay factor of the channel. Generally, η is of value greater than zero, which means that the average power of the channel response decreases as the delay increases. For a channel with a decay factor of zero, or $\eta=0$, the channel has a uniform PDP, and the average power of all taps are equal.

In (4.6), the phases of different taps are assumed to be independent identical distributed (i.i.d.) RVs, which are uniformly distributed within the interval $[0, 2\pi)$. The delays are also assumed to be i.i.d. RVs uniformly distributed in $[0, T_s)$, where T_s is the symbol duration of the signal. The decays could also be assumed to be uniformly distributed in $[0, T_c)$ (or some other interval) depending upon the actual physical environment and link distance.

We often use Rayleigh, Rician, or Nakagami- m distributions to model the amplitude of each of the taps of the TDL. Among these distributions, the Rayleigh distribution is often used to model the amplitude fading in non-line of sight (NLOS) environments. In the Rayleigh distribution, the amplitude of the channel, r is given by [3]

$$p(r) = \frac{2r}{\Omega} \exp\left(-\frac{r^2}{\Omega}\right) \quad (4.9)$$

where $\Omega = E[r^2]$ is the average channel power. Due to the NLOS nature, Rayleigh fading is often considered to be a worst case of the fading channel (e.g, [47], [49]). The Rician distribution is used to model the propagation environment with a strong LOS component and some weak NLOS components. The pdf of the Rician channel amplitude is given as [3]

$$f(r) = \frac{2r(k+1)}{\Omega} \exp\left(-k - \frac{(k+1)r^2}{\Omega}\right) I_0\left(2r\sqrt{\frac{k(k+1)}{\Omega}}\right) \quad (4.10)$$

where the k is the Rician “ k factor,” which is the power ratio between the LOS component and the NLOS components, and $I_0(\cdot)$ is the 0^{th} -order Bessel function of the first kind. When $k=0$ there is no energy transmitted through LOS path, and the Rician distribution reduces to Rayleigh distribution.

The Nakagami- m distribution is the distribution named after Nakagami for his work on measuring and modeling the urban wireless channel of Tokyo in 1960s [102]. Although a physical justification of the Nakagami- m distribution does not exist, this empirical distribution provides good fits to the data collected for both indoor and outdoor

environments [103], and is widely used to model the wireless channel with different channel conditions. The pdf for the Nakagmi- m distribution is given by [38], [39], [102].

$$p_R(r) = \frac{2}{\Gamma(m)} \left(\frac{m}{\Omega} \right)^m r^{2m-1} e^{-mr^2/\Omega} \quad (4.11)$$

where $\Gamma(\cdot)$ is the gamma function, and m is defined as $m = \Omega^2 / E[(R^2 - \Omega)^2]$. The m factor ranges from $1/2$ to ∞ , and is called the fading parameter; it can be used to represent the severity of the channel fading. Specifically, when $m=1/2$ the Nakagami- m distribution becomes the one sided *Gaussian* distribution; for $m=1$, it becomes the well known Rayleigh distribution, and when $m \rightarrow \infty$, it becomes an impulse, or a non-fading distribution. The Rician distribution can also be approximated by the Nakgami- m distribution with $m \cong (k+1)^2 / (2k+1)$ where k is the Rician k factor.

4.2 System Model

In this section, we provide the system model for the SSG-MT-DS-CDMA system. This system model includes the transmitter and receiver structures, and the channel model between the transmitter and receiver. Note again that we term the multi-user SSG-MT-DS-SS system as SSG-MT-DS-CDMA since CDMA is the multiple access technique used to support multiple users.

Transmitter Structure

The transmitter structure of the SSG-MT-DS-CDMA scheme is the same as that of the single user transmitter shown in Chapter 3 (Figure 3-1). We repeat the block diagram of the transmitter here for reference. Note that in this block diagram the superscript of (k) is used to identify the k^{th} user.

Figure 4-1: Block diagram of SSG-MT-DS-CDMA transmitter for k^{th} user

Channel Model

As we have introduced in the previous section, the wireless channel, flat or frequency selective, can be modeled by the TDL model. For DS-CDMA systems, the

transmitted bandwidth is much wider than the required information bandwidth, and often times this bandwidth is larger than the coherence bandwidth of the channel. Therefore, the channel between the transmitter and receiver is a frequency selective or dispersive channel, with the complex baseband channel model given by (4.6) and the number of channel taps determined by

$$L = \lfloor R_c / B_c \rfloor + 1 \quad (4.12)$$

In (4.12), instead of the symbol rate, the chip rate, R_c , is used to find the number of channel taps. In the multi-user multi-carrier case, we model the baseband channel between the transmitter and receiver for user k 's subcarrier i by

$$h_i^{(k)}(t) = \sum_{l=0}^{L_i^{(k)}-1} \alpha_{i,l}^{(k)} \exp(-j\varphi_{i,l}^{(k)}) \delta(t - \tau_{i,l}^{(k)}) \quad (4.13)$$

where $\alpha_{i,l}^{(k)}$, $\varphi_{i,l}^{(k)}$, $\tau_{i,l}^{(k)}$, are respectively the channel attenuation, phase shift, and delay for the l^{th} path of the channel between the transmitter and receiver of the i^{th} subcarrier of user k . Similar to any TDL model, $L_i^{(k)}$ is the number of resolvable paths for user k 's i^{th} subcarrier, and it can be calculated by substituting the chip rate of the i^{th} subcarrier of user k and the coherence bandwidth of the channel into (4.12).

The channel attenuation factors, $\alpha_{i,l}^{(k)}$'s, are modeled as Nakagami- m distributed random variables with pdf given by (4.11). We also assume the channels for all subcarriers have an exponentially decaying PDP with the same decay factor of η . In the work of this dissertation, we investigate the effects of multipath fading on the performance of SSG-MT-DS-CDMA. Thus, the large scale fading, including path loss and shadowing, is assumed to be compensated by a power control scheme. In addition,

under the uncorrelated scatterer assumption, the tap amplitudes are assumed to be independent RVs¹. This means the fading parameter $\alpha_{i,l}^{(k)}$ in (4.13) are independent Nakagami- m RVs with different average powers. The phase shifts of different paths, different subcarriers, and different users are assumed to be independent RVs uniformly distributed within the interval $[0, 2\pi)$, and the delays are also assumed to be independent RVs, uniformly distributed in $[0, T_b)$, where T_b is the bit duration for a single carrier (SC) transmission scheme with the same total data rate and bandwidth as the SSG-MT-DS-SS system.

Receiver Structure

The k^{th} user's receiver block diagram for SSG-MT-DS-CDMA in a dispersive channel is given in Figure 4-2, where, as shown in this figure, a RAKE receiver is used for each subcarrier [39]. For subcarrier i , coherent detection is performed by $L_i^{(k)}$ correlators to capture the energy dispersed in the channel, and the correlator outputs are maximal ratio combined (MRC) to form the decision statistic for the subcarrier i of user k . Then a hard decision is made based on the sign of the decision metric of subcarrier i . Finally, the detected bit stream is generated by $\mathbf{T}^{-1}$ transforming the bits detected for all M subcarriers. To perform the coherent detection and the MRC, we assume the receiver is capable of synchronizing to each multipath echo of the desired user's signal, and perfectly estimating its attenuation factor and phase, and we also assume the receiver has knowledge of the transmitted signal parameters, i.e., data rates, bit energies, processing gains, and frequency separations, of the desired user.

¹ The assumption of independence is common, and facilitates analysis. It is accurate if scattering is uncorrelated and the taps are well modeled as Gaussian.

Figure 4-2: Receiver block diagram for user k

4.3 Performance Analysis

In this section, we investigate the bit error probability of the SSG-MT-DS-CDMA system in the channel model given in (4.13). The bit error probability can be derived via the statistics of the decision metric. If we have a total of K asynchronous users (e.g., a cellular uplink), and the channel model is as given in (4.13), the received signal is

$$r(t) = \sum_{k=1}^K \sum_{i=1}^M \sum_{l=0}^{L_i^{(k)}-1} \alpha_{i,l}^{(k)} A_i^{(k)} d_i^{(k)}(t - \tau_{i,l}^{(k)}) c_i^{(k)}(t - \tau_{i,l}^{(k)}) \times \cos(2\pi f_i^{(k)} t - \theta_{i,l}^{(k)}) + n(t) \quad (4.14)$$

Where, as defined in the previous chapter, $d_i^{(k)}(t)$ and $c_i^{(k)}(t)$ are respectively the data and the spreading waveforms of user k 's subcarrier i . The composite phases $\theta_{i,l}^{(k)} = 2\pi f_i^{(k)} \tau_{i,l}^{(k)} + \phi_{i,l}^{(k)}$ are uniform RVs on $[0, 2\pi)$, and $n(t)$ is AWGN with double sided PSD of $N_0/2$ W/Hz. The received signal $r(t)$ is fed into the receiver shown in Figure 4-2 with perfect estimation of the amplitudes, $\alpha_{i,l}^{(k)}$, delays $\tau_{i,l}^{(k)}$, and composite phases $\theta_{i,l}^{(k)}$, of all taps of user k 's subcarrier i . The decision metric for subcarrier i of user k is thus given by

$$Z_i^{(k)} = \sum_{q=0}^{L_i^{(k)}-1} Z_{i,q}^{(k)} \quad (4.15)$$

where the correlator output for the q^{th} tap of subcarrier i for user k is

$$Z_{i,q}^{(k)} = \int_{\tau_{i,q}^{(k)}}^{\tau_{i,q}^{(k)} + T_{b,i}^{(k)}} r(t) \alpha_{i,q}^{(k)} c_i^{(k)}(t - \tau_{i,q}^{(k)}) \cos(2\pi f_i^{(k)} t - \theta_{i,q}^{(k)}) dt \quad (4.16)$$

Correlator Outputs

In this section, we analyze the correlator output by separating it into the desired

signal term and interference terms. Without loss of generality, we analyze the 0^{th} bit of subcarrier i . Upon substituting (4.14) into (4.16), and some straightforward algebra, the decision metric of the l^{th} correlator output of user k 's subcarrier i , can be expressed as

$$Z_{i,q}^{(k)} = D + N + \sum_{\substack{l=0 \\ l \neq q}}^{L_i^{(k)}-1} I_{1,l} + \sum_{\substack{j=1 \\ j \neq i}}^M \sum_{l=0}^{L_j^{(k)}-1} I_{2,j,l} + \sum_{\substack{u=1 \\ u \neq k}}^K \sum_{l=0}^{L_i^{(u)}-1} I_{3,l}^{(u)} + \sum_{\substack{u=1 \\ u \neq k}}^K \sum_{\substack{j=1 \\ j \neq i}}^M \sum_{l=0}^{L_j^{(u)}-1} I_{4,j,l}^{(u)} \quad (4.17)$$

where D is the desired signal given by $D = A_i^{(k)} T_{b,i}^{(k)} d_i^{(k)}(0) [\alpha_{i,q}^{(k)}]^2 / 2$, and N is the AWGN noise sample, which is of zero mean and variance of $N_0 T_{b,i}^{(k)} [\alpha_{i,q}^{(k)}]^2 / 4$. The rest of the terms in (4.17) are interference terms with definitions given as follows. First, $I_{1,l}$ is the interference from different paths (l) of the same subcarrier (same i) of the same user (user k), which can be expressed by

$$I_{1,l} = \frac{\alpha_{i,l}^{(k)} \alpha_{i,q}^{(k)} A_i^{(k)}}{2} \int_0^{T_{b,i}^{(k)}} d_i^{(k)}(t - \tau_{i,l}^{(k)}) c_i^{(k)}(t - \tau_{i,l}^{(k)}) c_i^{(k)}(t) \cos[\phi_{(i,q;i,l)}^{(k,k)}] dt \quad (4.18)$$

Note that to express the interference terms of $Z_{i,q}^{(k)}$, we let the delay of the q^{th} tap, $\tau_{i,q}^{(k)}$, equal zero and let the delay of the rest of the channel taps be the delays relative to this. The interference from the same user (user k), but different subcarriers (j) and all paths is denoted by $I_{2,j,l}$, and is given by

$$I_{2,j,l} = \frac{\alpha_{j,l}^{(k)} \alpha_{i,q}^{(k)} A_j^{(k)}}{2} \int_0^{T_{b,i}^{(k)}} d_j^{(k)}(t - \tau_{j,l}^{(k)}) c_j^{(k)}(t - \tau_{j,l}^{(k)}) c_i^{(k)}(t) \cos(2\pi \Delta f_{i,j}^{(k,k)} t - \phi_{(i,q;j,l)}^{(k,k)}) dt \quad (4.19)$$

The interference from different paths (l) of the same subcarrier (i), but different users (u) is $I_{3,l}^{(u)}$, given by

$$I_{3,l}^{(u)} = \frac{\alpha_{i,l}^{(u)} \alpha_{i,q}^{(k)} A_i^{(u)}}{2} \int_0^{T_{b,i}^{(k)}} d_i^{(u)}(t - \tau_{i,l}^{(u)}) c_i^{(u)}(t - \tau_{i,l}^{(u)}) c_i^{(k)}(t) \cos[\phi_{(i,q;l,l)}^{(k,u)}] dt \quad (4.20)$$

Finally, the interference from all paths (l) of different users' (u) different subcarriers (j) can be expressed by

$$I_{4,j,l}^{(u)} = \frac{\alpha_{j,l}^{(u)} \alpha_{i,q}^{(k)} A_j^{(u)}}{2} \int_0^{T_{b,i}^{(k)}} d_j^{(u)}(t - \tau_{j,l}^{(u)}) c_j^{(u)}(t - \tau_{j,l}^{(u)}) c_i^{(k)}(t) \cos(2\pi \Delta f_{i,j}^{(k,u)} t - \phi_{(i,q;j,l)}^{(k,u)}) dt \quad (4.21)$$

In (4.18)-(4.21), $\Delta f_{i,j}^{(k,u)} = f_j^{(u)} - f_i^{(k)}$ is the frequency difference between the carrier frequency of the i^{th} subcarrier of user k and that of the j^{th} subcarriers of user u , and $\phi_{(i,q;j,l)}^{(k,u)} = \theta_{i,q}^{(k)} - \theta_{j,l}^{(u)}$ is the phase difference between the q^{th} path of the i^{th} subcarrier of user k and the l^{th} path of the j^{th} subcarrier of user u . As with the θ 's, these phases are also i.i.d. RVs uniformly distributed within $[0, 2\pi)$. Via the assumption of independent random data bits and spreading codes, both with zero mean, all the interference terms in (4.18)-(4.21) are zero mean. In next sub-section, we investigate the statistics of the decision metrics and give the general form for the bit error probability of SSG-MT-DS-CDMA in the Nakagami- m dispersive channel.

Statistics of Decision Metrics

Observing the interference given in (4.18)-(4.21) we can see that the integrals of all the interference terms (4.18)-(4.21) are of the following form

$$R_{i,j}^{(k,u)} = \int_0^{T_{b,i}^{(k)}} d_j^{(u)}(t - \tau_{j,l}^{(u)}) c_j^{(u)}(t - \tau_{j,l}^{(u)}) c_i^{(k)}(t) g(t) dt \quad (4.22)$$

where $g(t) = \cos[2\pi \Delta f_{i,j}^{(k,u)} t - \phi_{(i,q;j,l)}^{(k,u)}]$. When $i=j$ and $u=k$, $R_{i,j}^{(k,u)}$ is the integral in (4.18);

when $i \neq j$ and $u=k$, it is the integral in (4.19); and when $u \neq k$ it becomes the integral in

either (4.20) or (4.21), depending on the values of the $\Delta f_{i,j}^{(k,u)}$. Specifically, when $\Delta f_{i,j}^{(k,u)} = 0$, it is the integral in (4.20); otherwise, it is the integral in (4.21). Since we assume zero data bits and random spreading codes on all subcarriers, we can approximate $R_{i,j}^{(k,u)}$ as a zero mean Gaussian RV, which implies that the interference terms in (4.17) are also all zero mean Gaussian RVs. The BER performance can hence be obtained with a knowledge of the variances of the interference terms, which requires the variance of the integral $R_{i,j}^{(k,u)}$.

Here we provide an example analysis of the variance of $R_{i,j}^{(k,u)}$ for “case 2” where the data rate ratio and chip duration ratio between the i^{th} and the j^{th} subcarrier of user k and u are respectively equal and greater than one, i.e., $\lambda_{i,i}^{(k,u)} = 1$ and $\mu_{i,j}^{(k,u)} > 1$. Note that the superscripts and subscripts of the data rate ratio and chip duration ratio are used to distinguish the subcarriers and users. In this case, user k ’s subcarrier i and user u ’s subcarrier j have identical bit durations, and one chip from user k ’s subcarrier i is interfered with by $\mu_{i,j}^{(k,u)}$ chips from user u ’s subcarrier j . An example timing diagram for $\lambda_{i,i}^{(k,u)} = 1$ and $\mu_{i,j}^{(k,u)} = 4$ is shown in Figure 4-3.

Figure 4-3: Timing diagram of $\lambda_{i,i}^{(k,u)} = 1$ and $\mu_{i,j}^{(k,u)} > 1$

To analyze the variance of $R_{i,j}^{(k,u)}$, it is convenient to quantize the delay, $\tau_{j,l}^{(u)}$, by the chip durations of both subcarrier i and subcarrier j of user k and u as

$$\tau_{j,l}^{(u)} = L_i T_{c,i}^{(k)} + \varepsilon_i = L_j T_{c,j}^{(u)} + \varepsilon_j \quad (4.23)$$

where L_i is an integer number given by $L_i = \lfloor \tau_{j,l}^{(u)} / T_{c,i}^{(k)} \rfloor$ and $\varepsilon_i = \tau_{j,l}^{(u)} - L_i T_{c,i}^{(k)}$ is the fractional-chip part of $\tau_{j,l}^{(u)}$. Similar definitions apply for L_j and ε_j . With this quantization of delay $\tau_{j,l}^{(u)}$, we can express the integral of (4.22) by the summation of two partial correlation functions as:

$$R_{i,j}^{(k,u)} = d_j^{(u)}(-1)R_{i,j}^{(k,u)}(\tau_{i,l}^{(u)}) + d_j^{(u)}(0)\hat{R}_{i,j}^{(k,u)}(\tau_{i,l}^{(u)}) \quad (4.24)$$

The partial correlation functions in (4.24) are defined as

$$\begin{aligned} R_{i,j}^{(k,u)}(\tau_{i,l}^{(u)}) = & \sum_{m=0}^{L_i} c_i^{(k)}(m) c_j^{(u)}(m \mu_{i,l}^{(k,u)} - L_j - 1) \int_{m T_{c,i}^{(k)}}^{m T_{c,i}^{(k)} + \varepsilon_j} g(t) dt \\ & + \sum_{m=0}^{L_i-1} c_i^{(k)}(m) \left[\sum_{n=0}^{\mu_{i,l}^{(k,u)}-2} c_j^{(u)}(m \mu_{i,l}^{(u,k)} - L_j + n) \int_{A(m,n)}^{A(m,n)+T_{c,j}^{(u)}} g(t) dt \right. \\ & \left. + c_j^{(u)}(m \mu_{i,l}^{(k,u)} + \mu_{i,l}^{(k,u)} - L_j - 1) \int_{B(m)}^{B(m)+T_{c,j}^{(u)}-\varepsilon_j} g(t) dt \right] \\ & + c_i^{(k)}(L_i) \sum_{n=0}^{\lfloor \varepsilon_i / T_{c,j}^{(u)} \rfloor - 1} c_j^{(u)}(-1-n) \int_{C(n)}^{C(n)+T_{c,j}^{(u)}} g(t) dt \end{aligned} \quad (4.25)$$

and

$$\begin{aligned}
\hat{R}_{i,j}^{(k,u)}(\tau_{i,l}^{(u)}) &= c_i^{(k)}(L_i) \sum_{n=0}^{\mu_{i,l}^{(k,u)} - \lfloor \varepsilon_j / T_{c,j}^{(u)} \rfloor - 2} c_j^{(u)}(n) \int_{C(n)}^{C(n)+T_{c,j}^{(u)}} g(t) dt \\
&+ \sum_{m=L_i}^{N_i^{(k)}-1} c_i^{(k)}(m) c_j^{(u)}(m\mu_{i,j} + \mu_{i,j}^{(k,u)} - L_j - 1) \int_{B(m)}^{B(m)+T_{c,j}^{(u)}-\varepsilon_j} g(t) dt \\
&+ \sum_{m=L_i+1}^{N_i^{(k)}-1} c_i^{(k)}(m) \left[\sum_{n=0}^{\mu_{i,j}^{(k,u)}-2} c_j^{(u)}(m\mu_{i,j}^{(k,u)} - L_j + n) \int_{A(m,n)}^{A(m,n)+T_{c,j}^{(u)}} g(t) dt \right. \\
&\left. + c_j^{(u)}(m\mu_{i,j}^{(k,u)} - L_j - 1) \int_{mT_{c,i}^{(k)}}^{mT_{c,i}^{(k)}+\varepsilon_j} g(t) dt \right]
\end{aligned} \tag{4.26}$$

where the terms in the integration limits of (4.25) and (4.26) are given by

$$A(m, n) = mT_{c,i}^{(k)} + \varepsilon_j + (m\mu_{i,j}^{(k,u)} - L_j + n)T_{c,j}^{(u)} \tag{4.27}$$

$$B(m) = (m+1)T_{c,i}^{(k)} - T_{c,j}^{(u)} + \varepsilon_j \tag{4.28}$$

$$C(n) = L_i T_{c,i}^{(k)} + nT_{c,j}^{(u)} + \varepsilon_j \tag{4.29}$$

Since random long spreading codes are used, these two partial correlation functions are of zero mean, and their variances can be evaluated by the expectation of the squares of the terms in the summation of (4.25) and (4.26). Via some straightforward algebra, the variance of $R_{i,j}^{(k,u)}$ for $\Delta f_{i,j}^{(k,u)} = 0$, in terms of the ε_j , can be expressed as

$$\text{var}[R_{i,j}^{(k,u)}] = E \left\{ \frac{N_i^{(k)}}{2} [\varepsilon_j^2 + [(T_{c,j}^{(u)} - \varepsilon_j)]^2 + (\mu_{i,j}^{(k,u)} - 1) [T_{c,j}^{(u)}]^2] \right\} \tag{4.30}$$

Note that to derive the expression given in (4.30), we used the fact that $E[\cos^2(\theta)] = 1/2$ when θ is uniformly distributed within $[0, 2\pi)$, and the expectation in (4.30) is taken with respect to the RV ε_j which is by assumption uniformly distributed within the interval of $[0, T_{c,j}^{(u)})$. Therefore, the variance of $R_{i,j}^{(k,u)}$ is then given by

$$\text{var}[R_{i,j}^{(k,u)}] = \frac{N_i^{(k)}}{2} \left\{ \frac{2}{3} + (\mu_{i,j}^{(k,u)} - 1) \right\} [T_{c,j}^{(u)}]^2 \tag{4.31}$$

Following the same method, we can express the variance of $R_{i,j}^{(k,u)}$ in terms of ε_j

for the $\Delta f_{i,j}^{(k,u)} \neq 0$ case as

$$\text{var}[R_{i,j}^{(k,u)}] = \frac{N_i^{(k)}}{2} \frac{\sin^2[\pi \Delta f_{i,j}^{(k,u)} \varepsilon_j] + \sin^2[\pi \Delta f_{i,j}^{(k,u)} (T_{c,j}^{(u)} - \varepsilon_j)] + (\mu_{i,j}^{(k,u)} - 1) \sin^2[\pi \Delta f_{i,j}^{(k,u)} T_{c,j}^{(u)}]}{(\pi \Delta f_{i,j}^{(k,u)})^2} \quad (4.32)$$

Here, we use the fact that $E[\sin^2(x + \theta)] = 1/2$ when x takes on a deterministic value and

θ is uniformly distributed within $[0, 2\pi)$. Upon averaging (4.32) with respect to ε_j , the

variance of $R_{i,j}^{(k,u)}$ for $\Delta f_{i,j}^{(k,u)} \neq 0$ is

$$\begin{aligned} \text{var}[R_{i,j}^{(k,u)}] = & \frac{N_i^{(k)}}{2(\pi \Delta f_{i,j}^{(k,u)})^2} \left\{ 1 - \sin^2 \left[2 \Delta f_{i,j}^{(k,u)} T_{c,j}^{(u)} \right] \right. \\ & \left. + (\mu_{i,j}^{(k,u)} - 1) \sin^2 \left[\pi \Delta f_{i,j}^{(k,u)} T_{c,j}^{(u)} \right] \right\} \end{aligned} \quad (4.33)$$

The variance of $R_{i,j}^{(k,u)}$ for other eight cases can be evaluated via the same method.

In Table 4-1 we list the variances of $R_{i,j}^{(k,u)}$ for all nine cases.

Table 4-1: Variance of integral $R_{i,j}^{(k,u)}$

Case	Variance of $R_{i,j}^{(k,u)}$	
1 $\lambda_{i,j}^{(k,u)} = 1$ $\mu_{i,j}^{(k,u)} = 1$	$\Delta f_{i,j}^{(k,u)} = 0$	$N_i^{(k)} \left[T_{c,i}^{(k)} \right]^2 / 3$
	$\Delta f_{i,j}^{(k,u)} \neq 0$	$N_i^{(k)} \left\{ 1 - \sin c \left[2\Delta f_{i,j}^{(k,u)} T_{c,i}^{(k)} \right] \right\} / 2(\pi\Delta f_{i,j}^{(k,u)})^2$
2 $\lambda_{i,j}^{(k,u)} = 1$ $\mu_{i,j}^{(k,u)} > 1$	$\Delta f_{i,j}^{(k,u)} = 0$	$N_i^{(k)} \left\{ 2/3 + (\mu_{i,j}^{(k,u)} - 1) \right\} \left[T_{c,j}^{(u)} \right]^2 / 2$
	$\Delta f_{i,j}^{(k,u)} \neq 0$	$N_i^{(k)} \left\{ 1 - \text{sinc} \left[2\Delta f_{i,j}^{(k,u)} T_{c,j}^{(u)} \right] + (\mu_{i,j}^{(k,u)} - 1) \sin^2 \left[\pi\Delta f_{i,j}^{(k,u)} T_{c,j}^{(u)} \right] \right\} / 2(\pi\Delta f_{i,j}^{(k,u)})^2$
3 $\lambda_{i,j}^{(k,u)} = 1$ $\mu_{i,j}^{(k,u)} < 1$	$\Delta f_{i,j}^{(k,u)} = 0$	$N_j^{(u)} \left\{ 2/3 + (1/\mu_{i,j}^{(k,u)} - 1) \right\} \left(T_{c,i}^{(k)} \right)^2 / 2$
	$\Delta f_{i,j}^{(k,u)} \neq 0$	$N_j^{(u)} \left\{ 1 - \text{sinc} \left[2\Delta f_{i,j}^{(k,u)} T_{c,i}^{(k)} \right] + (1/\mu_{i,j}^{(k,u)} - 1) \sin^2 \left[\pi\Delta f_{i,j}^{(k,u)} T_{c,i}^{(k)} \right] \right\} / 2(\pi\Delta f_{i,j}^{(k,u)})^2$
4 $\lambda_{i,j}^{(k,u)} < 1$ $\mu_{i,j}^{(k,u)} = 1$	$\Delta f_{i,j}^{(k,u)} = 0$	$N_i^{(k)} \left[T_{c,i}^{(k)} \right]^2 / 3$
	$\Delta f_{i,j}^{(k,u)} \neq 0$	$N_i^{(k)} \left\{ 1 - \sin c \left[2\Delta f_{i,j}^{(k,u)} T_{c,i}^{(k)} \right] \right\} / 2(\pi\Delta f_{i,j}^{(k,u)})^2$
5 $\lambda_{i,j}^{(k,u)} < 1$ $\mu_{i,j}^{(k,u)} > 1$	$\Delta f_{i,j}^{(k,u)} = 0$	$N_i^{(k)} \left\{ 2/3 + (\mu_{i,j}^{(k,u)} - 1) \right\} \left[T_{c,j}^{(u)} \right]^2 / 2$
	$\Delta f_{i,j}^{(k,u)} \neq 0$	$N_i^{(k)} \left\{ 1 - \text{sinc} \left[2\Delta f_{i,j}^{(k,u)} T_{c,j}^{(u)} \right] + (\mu_{i,j}^{(k,u)} - 1) \sin^2 \left[\pi\Delta f_{i,j}^{(k,u)} T_{c,j}^{(u)} \right] \right\} / 2(\pi\Delta f_{i,j}^{(k,u)})^2$
6 $\lambda_{i,j}^{(k,u)} < 1$ $\mu_{i,j}^{(k,u)} < 1$	$\Delta f_{i,j}^{(k,u)} = 0$	$N_i^{(k)} \mu_{i,j}^{(k,u)} \left\{ 2/3 + (1/\mu_{i,j}^{(k,u)} - 1) \right\} \left(T_{c,i}^{(k)} \right)^2 / 2$
	$\Delta f_{i,j}^{(k,u)} \neq 0$	$N_i^{(k)} \mu_{i,j}^{(k,u)} \left\{ 1 - \text{sinc} \left[2\Delta f_{i,j}^{(k,u)} T_{c,i}^{(k)} \right] + (1/\mu_{i,j}^{(k,u)} - 1) \sin^2 \left[\pi\Delta f_{i,j}^{(k,u)} T_{c,i}^{(k)} \right] \right\} / 2(\pi\Delta f_{i,j}^{(k,u)})^2$
7 $\lambda_{i,j}^{(k,u)} > 1$ $\mu_{i,j}^{(k,u)} = 1$	$\Delta f_{i,j}^{(k,u)} = 0$	$\lambda_{i,j}^{(k,u)} N_j^{(u)} \left[T_{c,i}^{(k)} \right]^2 / 3$
	$\Delta f_{i,j}^{(k,u)} \neq 0$	$\lambda_{i,j}^{(k,u)} N_j^{(u)} \left\{ 1 - \sin c \left[2\Delta f_{i,j}^{(k,u)} T_{c,i}^{(k)} \right] \right\} / 2(\pi\Delta f_{i,j}^{(k,u)})^2$
8 $\lambda_{i,j}^{(k,u)} > 1$ $\mu_{i,j}^{(k,u)} > 1$	$\Delta f_{i,j}^{(k,u)} = 0$	$N_i^{(k)} \left\{ 2/3 + (\mu_{i,j}^{(k,u)} - 1) \right\} \left[T_{c,j}^{(v)} \right]^2 / 2$
	$\Delta f_{i,j}^{(k,u)} \neq 0$	$N_i^{(k)} \left\{ 1 - \text{sinc} \left[2\Delta f_{i,j}^{(k,u)} T_{c,j}^{(u)} \right] + (\mu_{i,j}^{(k,u)} - 1) \sin^2 \left[\pi\Delta f_{i,j}^{(k,u)} T_{c,j}^{(u)} \right] \right\} / 2(\pi\Delta f_{i,j}^{(k,u)})^2$
9 $\lambda_{i,j}^{(k,u)} > 1$ $\mu_{i,j}^{(k,u)} < 1$	$\Delta f_{i,j}^{(k,u)} = 0$	$\lambda_{i,j}^{(k,u)} N_j^{(u)} \left\{ 2/3 + (1/\mu_{i,j}^{(k,u)} - 1) \right\} \left[T_{c,i}^{(k)} \right]^2 / 2$
	$\Delta f_{i,j}^{(k,u)} \neq 0$	$\lambda_{i,j}^{(k,u)} N_j^{(u)} \left\{ 1 - \text{sinc} \left[2\Delta f_{i,j}^{(k,u)} T_{c,i}^{(k)} \right] + (1/\mu_{i,j}^{(k,u)} - 1) \sin^2 \left[\pi\Delta f_{i,j}^{(k,u)} T_{c,i}^{(k)} \right] \right\} / 2(\pi\Delta f_{i,j}^{(k,u)})^2$

Similar to our method for the AWGN channel, the interference terms in (4.22) are modeled as independent additive *Gaussian* RVs. Therefore, with the variances of the $R_{i,j}^{(k,u)}$ known, the variance of the q^{th} RAKE finger output of the i^{th} subcarrier of user k is

$$\begin{aligned}
\text{var}[Z_{i,q}^{(k)}] &= \frac{[\alpha_{i,q}^{(k)} A_i^{(k)}]^2 \text{var}[R_{i,i}^{(k,k)}]}{4} \sum_{\substack{l=0 \\ l \neq q}}^{L_i^{(k)}-1} \Omega_{i,l}^{(k)} + \frac{[\alpha_{i,q}^{(k)}]^2}{4} \sum_{\substack{j=1 \\ j \neq i}}^M \sum_{l=0}^{L_j^{(k)}-1} \Omega_{j,l}^{(k)} [A_j^{(k)}]^2 \text{var}[R_{i,j}^{(k,k)}] \\
&+ \frac{[\alpha_{i,q}^{(k)} A_i^{(u)}]^2 \text{var}[R_{i,i}^{(k,u)}]}{4} \sum_{\substack{u=1 \\ u \neq k}}^K \sum_{l=0}^{L_i^{(u)}-1} \Omega_{i,l}^{(u)} + \frac{[\alpha_{i,q}^{(k)}]^2}{4} \sum_{\substack{u=1 \\ u \neq k}}^K \sum_{\substack{j=1 \\ j \neq i}}^M \sum_{l=0}^{L_j^{(u)}-1} \Omega_{j,l}^{(u)} [A_j^{(u)}]^2 \text{var}[R_{i,j}^{(k,u)}] \\
&+ \frac{[\alpha_{i,q}^{(k)}]^2 N_0 T_{b,i}^{(k)}}{4}
\end{aligned} \tag{4.34}$$

Applying the independent additive Gaussian assumption of the interference terms to (4.15), we can show the variance of the decision metric is the summation of the variances of all RAKE finger outputs:

$$\text{var}[Z_i^{(k)}] = \sum_{q=0}^{L_i^{(k)}-1} \text{var}[Z_{i,q}^{(k)}] \tag{4.35}$$

Similarly the mean of the decision metric is

$$E[Z_i^{(k)}] = \sum_{q=0}^{L_i^{(k)}-1} E[Z_{i,q}^{(k)}] = \frac{1}{2} A_i^{(k)} T_{b,i}^{(k)} d_i^{(k)}(0) \sum_{q=0}^{L_i^{(k)}-1} [\alpha_{i,q}^{(k)}]^2 \tag{4.36}$$

Average Bit Error Probability

The error probability for the BPSK modulation is a function of the ratio of the square of the mean and the variance of the decision metric. Thus, the bit error probability of the i^{th} subcarrier of user k can then be expressed as

$$P_{b,i}^{(k)}(\gamma_i^{(k)}) = Q\left(\sqrt{\frac{(E[Z_i^{(k)}])^2}{\text{var}[Z_i^{(k)}]}}\right) = Q\left(\sqrt{2\gamma_i^{(k)}}\right) \tag{4.37}$$

where $Q(x) = \int_x^\infty e^{-t^2/2} dt / \sqrt{2\pi}$. We further assume that channels of different subcarriers of different users have the same PDP distribution and using the exact same approach as in

[38], [39], we replace the average power of the j^{th} subcarrier of user k 's l^{th} tap, $\Omega_{j,l}^{(k)}$, by $1/L_i^{(k)}$. (This approximation is very good for small values of decay factor η .) The instantaneous signal to noise plus interference ratio (SNIR) can then be expressed as

$$\gamma_i^{(k)} = \left\{ \frac{N_0}{E_{b,i}^{(k)}} + \frac{2(L_i^{(k)} - 1)}{3N_i^{(k)} L_i^{(k)}} + SUI_i^{(k)} + MUI_i^{(k)} \right\}^{-1} \sum_{q=0}^{L_i^{(k)}-1} [\alpha_{i,q}^{(k)}]^2 = \sum_{q=0}^{L_i^{(k)}-1} \bar{\gamma}_i^{(k)} [\alpha_{i,q}^{(k)}]^2 \quad (4.38)$$

where the average SNIR is given by

$$\bar{\gamma}_i^{(k)} = \left\{ \frac{N_0}{E_{b,i}^{(k)}} + \frac{2(L_i^{(k)} - 1)}{3N_i^{(k)} L_i^{(k)}} + SUI_i^{(k)} + MUI_i^{(k)} \right\}^{-1} \quad (4.39)$$

In (4.39), the 2nd term is the “same user inter-chip interference,” and the same user (inter-subcarrier) interference (SUI) and multiple user interference (MUI) seen by the i^{th} subcarrier of user k are defined as follows:

$$SUI_i^{(k)} = 2 \sum_{\substack{j=1 \\ j \neq i}}^M \sum_{l=0}^{L_j^{(k)}-1} \frac{\Omega_{j,l}^{(k)} E_{b,j}^{(k)} \text{var}[R_{i,j}^{(k,k)}]}{E_{b,i}^{(k)} T_{b,i}^{(k)} T_{b,j}^{(k)}} \quad (4.40)$$

$$MUI_i^{(k)} = 2 \sum_{\substack{u=1 \\ u \neq k}}^K \sum_{j=1}^M \sum_{l=0}^{L_j^{(u)}-1} \frac{\Omega_{j,l}^{(u)} E_{b,j}^{(u)} \text{var}[R_{i,j}^{(k,u)}]}{E_{b,i}^{(k)} T_{b,i}^{(k)} T_{b,j}^{(u)}} \quad (4.41)$$

where the values of $R_{i,j}^{(k,u)}$ can be found in Table 4-1 for different combinations of i, j, k , and u . Note that the bit error probability of user k 's subcarrier i given by (4.37) is the instantaneous bit error probability, and we need to average this over the joint pdf of the tap amplitudes, i.e., $\alpha_{i,0}^{(k)}, \alpha_{i,1}^{(k)} \dots \alpha_{i,L_i^{(k)}-1}^{(k)}$, in order to obtain the average bit error probability.

Once again, we assume the environment yields uncorrelated scattering. Thus, we assume the channel attenuation factors are independent RVs (as is commonly done by other

researchers, e.g., [38], [39]), and the joint pdf can be expressed by the product of the individual pdfs of the $\alpha_{i,q}^{(k)}$, or

$$p_{\alpha_{i,0}^{(k)}, \alpha_{i,1}^{(k)}, \dots, \alpha_{i, L_i^{(k)}-1}^{(k)}}(\alpha_{i,0}^{(k)}, \alpha_{i,1}^{(k)}, \dots, \alpha_{i, L_i^{(k)}-1}^{(k)}) = \prod_{q=0}^{L_i^{(k)}-1} p_{\alpha_{i,q}^{(k)}}(\alpha_{i,q}^{(k)}) \quad (4.42)$$

The average bit error probability, which can be obtained via averaging (4.37) over the joint pdf of the channel taps given by (4.42), is given by

$$\overline{P}_{b,i}^{(k)}(\gamma_i^{(k)}) = \underbrace{\int_0^\infty \dots \int_0^\infty}_{L_i^{(k)} \text{ fold}} Q\left(\sqrt{2\gamma_i^{(k)}}\right) \prod_{q=0}^{L_i^{(k)}-1} p_{\alpha_{i,q}^{(k)}}(\alpha_{i,q}^{(k)}) d\theta d\alpha_{i,0}^{(k)} d\alpha_{i,1}^{(k)} \dots d\alpha_{i, L_i^{(k)}-1}^{(k)} \quad (4.43)$$

As we can see, (4.43) is an $L_i^{(k)}$ fold integration, which is very difficult to calculate since the usual integral formula for the Q -function has its argument in the lower limit of the integral. To bypass this difficulty, we rely on an alternative form of the Q -function whose integration limits are independent of the argument and are of finite values. The alternative form of Q -function is given by [104],

$$Q(x) = \frac{1}{\pi} \int_0^{\pi/2} \exp\left(-\frac{x^2}{2\sin^2 \chi}\right) d\chi \quad (4.44)$$

Upon applying (4.44), and interchanging the order of integration, the average bit error probability given in (4.43) is

$$\begin{aligned} P_{b,i}^{(k)} &= \frac{1}{\pi} \int_0^{\pi/2} \underbrace{\int_0^\infty \dots \int_0^\infty}_{L_i^{(k)} \text{ fold}} \exp\left(-\frac{\sum_{q=0}^{L_i^{(k)}-1} \gamma_i^{(k)} [\alpha_{i,q}^{(k)}]^2}{\sin^2 \chi}\right) \prod_{q=0}^{L_i^{(k)}-1} p_{\alpha_{i,q}^{(k)}}(\alpha_{i,q}^{(k)}) d\chi d\alpha_{i,0}^{(k)} d\alpha_{i,1}^{(k)} \dots d\alpha_{i, L_i^{(k)}-1}^{(k)} \\ &= \frac{1}{\pi} \int_0^{\pi/2} \prod_{q=0}^{L_i^{(k)}-1} G_{i,q}^{(k)}(\chi) d\chi \end{aligned} \quad (4.45)$$

where $G_{i,q}^{(k)}(\chi)$ is defined as

$$G_{i,q}^{(k)}(\chi) = \int_0^\infty \exp\left(-\frac{\gamma_i^{-(k)} [\alpha_{i,q}^{(k)}]^2}{\sin^2 \chi}\right) p_{\alpha_{i,q}^{(k)}}(\alpha_{i,q}^{(k)}) d\alpha_{i,q}^{(k)} \quad (4.46)$$

Observing (4.46), we can see it is actually the moment generating function of the Nakagami- m distribution, which is given by

$$G_{i,q}^{(k)}(\theta) = \left(\frac{m \sin^2 \chi}{\gamma_{i,q}^{-(k)} + \sin^2 \chi} \right)^m \quad (4.47)$$

Note in (4.47), $\gamma_{i,q}^{-(k)}$ is the average signal-to-noise-plus-interference ratio (SNIR) on the q^{th} tap of the i^{th} subcarrier of the desired k^{th} user, and this can be expressed in terms of the average SNIR of the 0^{th} tap of the same subcarrier by

$$\gamma_{i,q}^{-(k)} = \gamma_i^{-(k)} \exp(-\eta q) / \Omega_{i,0}^{(k)} \quad (4.48)$$

Now substituting (4.47) into (4.45), we have the average bit error probability for the i^{th} subcarrier of user k given by

$$P_{b,i}^{(k)} = \frac{1}{\pi} \int_0^{\pi/2} \prod_{q=0}^{L_i^{(k)}-1} \left(\frac{m \sin^2 \chi}{\gamma_{i,q}^{-(k)} + \sin^2 \chi} \right)^m d\chi \quad (4.49)$$

This expression must be evaluated numerically, and the total average error probability for the k^{th} user is obtained by averaging the error probabilities of all M subcarriers.

4.4 Numerical Examples

In this section, we illustrate the BER performance of SSG-MT-DS-CDMA with different parameter sets in the presence of dispersive fading channels. Similar to the results presented in Chapter 3, the performance of SSG-MT-DS-CDMA is compared with that of conventional MT-DS-CDMA under the conditions that the two systems have the same data rate, transmission power, and number of subcarriers. In addition, for the multiple user case, the two systems support the same number of users.

As we have shown in Chapter 3, with larger numbers of subcarriers, the SSG-MT-DS-CDMA system has a greater capability to shape the spectrum. However, the increased spectral shaping is generally at the price of BER performance degradation. In this section, we use the 3-subcarrier SSG-MT-DS-CDMA system to illustrate its BER performance in the dispersive channel. We configure the reference conventional MT-DS-CDMA system to have $M=3$ subcarriers, with a processing gain of $N=128$ on each subcarrier, and assume the chip rate of each subcarrier of MT-DS-CDMA is larger than that of the channel coherence bandwidth, which results in a dispersive channel for each subcarrier. We assume the number of resolvable channel taps for MT-DS-CDMA is $L=5$, and the number of resolvable channel taps for SSG-MT-DS-CDMA can be calculated using (4.12).

First we show simulated and analytical BER curves of SSG-MT-DS-CDMA in Figure 4-4. In this example, the BER performance of the SSG-MT-DS-CDMA system with parameter set given by $\mathbf{R}_b=[1, 2, 1]$, $\mathbf{E}_b=[1, 1, 1]$, $\mathbf{N}=[128, 32, 128]$, and $\Delta\mathbf{f}=[1, 1]$ were simulated for both single and 5-user cases. The channel for SSG-MT-DS-CDMA is

a Nakagami- m dispersive channel with $m=1$ (Rayleigh fading) and 4, 2, 4 channel taps exist for the three subcarriers. We assume the receiver is capable of capturing all the energies dispersed in the channel, and hence the receiver has 4, 2, 4 RAKE fingers for the respective subcarriers. These simulated results can be compared with the analytical results obtained via numerically evaluating (4.49). This example shows that the simulated results match the analytical results, which validates our *Gaussian* assumption for the interferences (SUI, MUI). As in Chapter 3, the Gaussian assumption for interference is not always a valid assumption when the central limit theorem is not applicable. For example, for single user SSG-MT-DS-CDMA with $\mathbf{N}=[8, 2, 8]$, and the remaining parameters the same as in the previous example, we can observe the difference between the simulated and analytical results. The BER curves are plotted in Figure 4-5. In the rest of this section, we use the analytical P_b to gain insight into how the parameters and channel conditions affect the BER performance of SSG-MT-DS-CDMA.

Figure 4-4: Simulated and analytical average P_b vs. E_b/N_0 for SSG-MT-DS-CDMA with $\mathbf{R}_b=[1, 2, 1]$, $\mathbf{E}_b=[1, 1, 1]$, $\mathbf{N}=[128, 32, 128]$, and $\Delta\mathbf{f}=[1, 1]$

Figure 4-5: Simulated and analytical average P_b vs. E_b/N_0 for SSG-MT-DS-CDMA with $\mathbf{R}_b=[1,2,1]$, $\mathbf{E}_b=[1, 1, 1]$, $\mathbf{N}=[8, 2, 8]$, and $\Delta\mathbf{f}=[1, 1]$, $K=1$

Similar to our approach in Chapter 3, we show the effects of varying one parameter at a time on the BER performance of SSG-MT-DS-CDMA. In Figure 4-6, we keep the data rates, processing gains, and frequency separations vectors of the 3-subcarrier SSG-MT-DS-SS systems fixed, and show the average P_b of the single user SSG-MT-DS-CDMA system at the average SNR of $E_b/N_0=7$ dB for different bit energy vectors. Similar to the example in Chapter 3, the 1st and 3rd subcarriers have the same bit energies, i.e., $\mathbf{E}_b(1)=\mathbf{E}_b(3)$, and the ratio between the bit energy of the 2nd subcarrier and that of the 1st subcarrier is varied from 0.1 to 3. The three SSG-MT-DS-CDMA systems in this example are of the same processing gain vector, $\mathbf{N}=[128, 32, 128]$, and frequency separation vector, $\Delta\mathbf{f}=[1, 1]$, but different data rate vectors i.e., $\mathbf{R}_b=[1, 2, 1]$, $\mathbf{R}_b=[1, 1, 1]$, and $\mathbf{R}_b=[1, 1/2, 1]$. The channel between the transmitter and receiver of the SSG-MT-

DS-CDMA signal is a dispersive Nakagami- m fading channel with $m=1$, decay factor of $\eta=2$. The number of channel taps for the respective subcarriers are respectively 4, 2, 4 when $\mathbf{R}_b=[1, 2, 1]$; 5, 2, 5 when $\mathbf{R}_b=[1, 1, 1]$; and 5, 1, 5 when $\mathbf{R}_b=[1, 1/2, 1]$,

Figure 4-6: Average P_b vs. E_{b2}/E_{b1} at $E_b/N_0=7$ dB, for $\mathbf{N}=[128, 32, 128]$ $\Delta\mathbf{f}=[1, 1]$ and different data rate

From this figure, we can observe the same phenomenon as observed in Chapter 3, where the SSG-MT-DS-CDMA had its best performance when the bit energies of all subcarriers are equal, i.e., $\mathbf{E}_b$ is an all-ones vector. In this figure, we can also observe that even with the same bit energy vectors, the average P_b of the SSG-MT-DS-SS systems varies as the data rate vector varies. For instance, when $E_b(2)/E_b(1)=2$, the SSG-MT-DS-SS system with $\mathbf{R}_b=[1, 1/2, 1]$ has the lowest average P_b , followed by the SSG-MT-DS-SS systems with $\mathbf{R}_b=[1, 1, 1]$, and $\mathbf{R}_b=[1, 2, 1]$. To show the effects of the data rate vector on the performance of the SSG-MT-DS-CDMA system, the BER curves of SSG-

MT-DS-CDMA with different data rate vectors and the same channel conditions are shown in Figure 4-7, where 5 active users are supported by the SSG-MT-DS-CDMA system, and the bit energy vectors of the 3-subcarrier SSG-MT-DS-CDMA systems are chosen to be $\mathbf{E}_b=[1, 1, 1]$ in order to obtain the best BER performance.

From this example, we can see that the SSG-MT-DS-CDMA systems with $\mathbf{R}_b=[1, 1/2, 1]$ and $\mathbf{R}_b=[1, 1, 1]$ have similar BER performance, and the system with $\mathbf{R}_b=[1, 2, 1]$ has the worst among all these systems; the conventional MT-DS-CDMA has the best BER performance. In addition, all systems--SSG-MT-DS-CDMA and conventional MT-DS-CDMA--have an error floor, i.e., at some value of E_b/N_0 , an increase in the value of E_b/N_0 does not lower the average P_b . This error floor is because of the channel dispersion and the MUI. Specifically, when the E_b/N_0 is large, the ISI caused by the channel dispersion and the MUI caused by multiple active users dominates the performance degradation.

The performance difference between these systems can be explained by reviewing their PSDs, shown in Figure 3-17, where the PSDs of the SSG-MT-DS-SS signals with different data rate vectors are shown. From Figure 3-17, we can see that the SSG-MT-DS-SS with data rate vector $\mathbf{R}_b=[1, 1/2, 1]$ has the largest mainlobe bandwidth followed by the schemes with $\mathbf{R}_b=[1, 1, 1]$ and $\mathbf{R}_b=[1, 2, 1]$. This ordering matches the BER performance order shown in Figure 4-6 and Figure 4-7.

Figure 4-7: Average P_b vs. E_b/N_0 for different data rate vector vectors, $\mathbf{E}_b=[1, 1, 1]$, $\Delta f=[1,1]$, $\mathbf{N}=[128, 32, 128]$

We also note that even with a larger mainlobe bandwidth, the SSG-MT-DS-CDMA scheme with $\mathbf{R}_b=[1, 1/2, 1]$ has a BER performance worse than that of conventional MT-DS-CDMA. This is because for this configuration, the number of resolvable channel taps for SSG-MT-DS-CDMA is 5, 1, 5 for the respective subcarriers. For the 2nd subcarrier, the number of tap is 1, which means the 2nd subcarrier is undergoing flat fading, thus, there is no multipath diversity for this subcarrier. In addition, since the processing gain vector is $\mathbf{N}=[128, 32, 128]$, the 2nd subcarrier has the lowest processing gain, which reduces its ability to suppress the interferences (SUI and MUI). These two factors, flat fading and low processing gain value, cause the BER performance degradation of the 2nd subcarrier and consequently of the whole system. The 1st and the 3rd subcarrier, however, have approximately the same BER performance

as the conventional MT-DS-CDMA scheme because they have the same processing gain and the same number of channel taps. Thus, the overall BER, averaged over all three subcarriers, is worse for SSG-MT-DS-CDMA than for the conventional MT-DS-CDMA.

In Chapter 3, we showed that the subcarriers of SSG-MT-DS-SS can be orthogonal to each other when the frequency separation orthogonality conditions are satisfied. In this case, all subcarriers are orthogonal to each other, the SSG-MT-DS-SS system is inter-subcarrier-interference-free, and the single user SSG-MT-DS-SS scheme in the AWGN channel has the same performance as that of single-user single-carrier BPSK signaling in the AWGN channel. However, this is not true for the multiuser SSG-MT-DS-CDMA system we are considering. As in any asynchronous CDMA system, the users of SSG-MT-DS-CDMA transmit asynchronously, or each user starts its data transmission at any arbitrary time. The asynchronous nature of the SSG-MT-DS-CDMA system, and the imperfectness of the spreading codes (the cross correlations of the spreading codes with non-zero offsets are of non-zero values), make user signals of SSG-MT-DS-CDMA interfere with each other. Moreover, the dispersive channel, which smears the signal in the time domain, introduces interference from both the same and different subcarriers of both the same and different users.

The interference between users of SSG-MT-DS-CDMA can be suppressed via either increase the processing gain of some or all the subcarriers, or by increasing the frequency spacing between adjacent subcarriers. Increasing the processing gain value of the SSG-MT-DS-CDMA system can improve its BER performance. The BER curves of an SSG-MT-DS-CDMA system with $\mathbf{E}_b=[1, 1, 1]$, $\mathbf{R}_b=[1, 2, 1]$, $\Delta\mathbf{f}=[1, 1]$ but different processing gain values are shown in Figure 4-8. The dispersive channel is the same as in

the previous example, i.e., $m=1$ and $\eta=2$, and the numbers of channel taps are respectively 4, 2, 4 for the three subcarriers when $N=[128, 32, 128]$, and 8, 4, 8 when $N=[256, 64, 256]$. As can be observed in this figure, by increasing the processing gains of each subcarrier, we can improve the BER performance of SSG-MT-DS-CDMA. However, as we have shown in Chapter 3 (Figure 3-15), increasing the processing gain increases the bandwidth and (possibly) the sidelobe levels of the SSG-MT-DS-SS signal. Since we want to keep the mainlobe bandwidth of the SSG-MT-DS-CDMA comparable to that of conventional MT-DS-CDMA, we will not address increasing the processing gain of some/all subcarriers of SSG-MT-DS-CDMA.

Figure 4-8: P_b vs E_b/N_0 for different processing gain values, $\mathbf{R}_b=[1, 2, 1]$, $\mathbf{E}_b=[1, 1, 1]$, $\mathbf{N}=[1, 1/4, 1]$, $\Delta\mathbf{f}=[1, 1]$

In Figure 4-9, we show the average P_b of a 5 user SSG-MT-DS-CDMA system with different frequency separation vectors. The parameters of SSG-MT-DS-CDMA in

this example are $\mathbf{E}_b=[1, 1, 1]$, $\mathbf{R}_b=[1, 2, 1]$, $\mathbf{N}=[128, 32, 128]$, with different Δf shown in the figure. The dispersive channel is the same as in the previous examples, i.e., $m=1$ and $\eta=2$, and the numbers of channel taps for this system configuration are 4, 2, 4 respectively for three subcarriers. As shown in Figure 4-9, the BER performance of SSG-MT-DS-CDMA improves as the frequency separation between the adjacent subcarriers increases. However, as mentioned previously, because of the interference from both the same and other users, even when the system is configured to satisfy the orthogonality conditions, $\Delta f=[64, 64]$ in this case, it is not interference free. In addition, with the lower processing gain value on the 2nd subcarrier, the SSG-MT-DS-CDMA is less capable of suppressing the interference, and, therefore, its BER performance is worse than that of conventional MT-DS-CDMA.

Figure 4-9: Average P_b vs E_b/N_0 for different frequency separations, $\mathbf{R}_b=[1, 2, 1]$, $\mathbf{E}_b=[1, 1, 1]$, $\mathbf{N}=[128, 32, 128]$

Having shown the effects of different parameters sets on the BER performance of SSG-MT-DS-CDMA, we continue to illustrate the effects of different channel conditions on the BER performance of SSG-MT-DS-CDMA systems. In Figure 4-10, the average P_b at average SNR of $E_b/N_0=7$ dB is shown for single user SSG-MT-DS-CDMA with $M=3$ subcarriers, $\mathbf{E}_b=[1, 1, 1]$, $\mathbf{R}_b=[1, 2, 1]$, $\mathbf{N}=[128, 32, 128]$, and $\Delta\mathbf{f}=[1, 1]$ with different PDP decay factors. The channel between the transmitter and the receiver of the SSG-MT-DS-CDMA system is a dispersive channel with $m=1$, $L=4, 2, 4$ taps for three subcarriers but the decay factor η varies from $\eta=0$ to $\eta=2.5$.

Figure 4-10: Average P_b vs. η at $E_b/N_0=7$ dB, $\mathbf{E}_b=[1, 1, 1]$, $\mathbf{R}_b=[1, 2, 1]$, $\mathbf{N}=[128, 32, 128]$, and $\Delta\mathbf{f}=[1, 1]$.

As we can observe from this example, the performance of conventional MT-DS-CDMA is better than that of SSG-MT-DS-CDMA at all η values; but the difference between the average P_b of the two systems decreases as the value of η increases. This is because the RAKE receiver reaches its best performance when the PDP is of the channel

is uniform, or $\eta=0$. As the value of decay factor η increases, the tap energy decreases as the delay increases, and the low energy taps are less helpful to combat fading when the major channel taps, those channel taps with relatively high channel energy and small delay, are deeply faded.

In Figure 4-11 and Figure 4-12 we consider the BER performance of both single user and multiuser SSG-MT-DS-CDMA in a dispersive channel with different fading parameters. In these figures, the parameters of SSG-MT-DS-CDMA are the same as in the previous example, and the channel is a dispersive Nakagami- m fading channel with $\eta=2$, $L=4,2,4$ taps for the three subcarriers of SSG-MT-DS-CDMA. The fading parameter m is selected to be $m=1$ and $m=20$ to represent both channels with severe fading ($m=1$ or Rayleigh), and with good channel conditions ($m \gg 1$). From the results of Figure 4-11, we can observe that SSG-MT-DS-SS has approximately the same BER performance as conventional MT-DS-SS in the single user case for a wide range of channel conditions. Figure 4-12 shows that in a multiuser setting, when the channel condition is severe ($m=1$), SSG-MT-DS-CDMA has BER performance again comparable to that of conventional MT-DS-CDMA, but the conventional MT-DS-SS scheme outperforms SSG-MT-DS-CDMA when the channel quality is good ($m \gg 1$).

Figure 4-11: Average P_b vs. E_b/N_0 for single user SSG-MT-DS-SS for different m , $\mathbf{E}_b=[1, 1, 1]$, $\mathbf{R}_b=[1, 2, 1]$, $\mathbf{N}=[128, 32, 128]$, and $\Delta\mathbf{f}=[1, 1]$, $L=4,2,4$ taps for three subcarriers, $\eta=2$.

Figure 4-12: Average P_b vs. E_b/N_0 for 5-user SSG-MT-DS-SS for different m , $E_b=[1, 1, 1]$, $R_b=[1, 2, 1]$, $N=[128, 32, 128]$, and $\Delta f=[1, 1]$, $L=4, 2, 4$ taps for three subcarriers, $\eta=2$.

The average bit error probability vs. the number of active users is shown in Figure 4-13. The parameters of the SSG-MT-DS-CDMA system are the same as in the previous example, and the dispersive channel is the Nakagami- m fading channel with $\eta=2$, $L=4$, 2, 4 taps for the respective subcarriers of SSG-MT-DS-CDMA. We show the average bit error probabilities for two fading parameters, $m=1$, and $m=20$, as we did in the previous example. From this figure, we can observe that the error performance of both SSG-MT-DS-CDMA and conventional MT-DS-CDMA degrades as the number of users increases. In addition, the performance difference between different channel conditions ($m=1$ and $m=20$) is decreasing as the number of users keep increasing, which indicates that the dominate effect on the performance degradation is the MUI instead of the channel fading when the number of users is large, e.g., $K>10$. Also, as noted before, the SSG-MT-DS-

CDMA performance is worse than that of conventional primarily because of its lower processing gain on some subcarriers.

Figure 4-13: Average P_b vs. number of users K at $E_b/N_0=7$ dB, $\mathbf{E}_b=[1, 1, 1]$, $\mathbf{R}_b=[1, 2, 1]$, $\mathbf{N}=[128, 32, 128]$, and $\Delta\mathbf{f}=[1, 1]$.

4.5 Conclusions

In this chapter, we investigated the BER performance of SSG-MT-DS-CDMA in dispersive fading channels. The average BER performance expressions for SSG-MT-DS-CDMA with different parameter sets were derived, and these results were compared with those of the conventional MT-DS-CDMA scheme under conditions of the same number of users, same number of subcarriers, same total data rate, and same transmission power. It was shown in this chapter that the SSG-MT-DS-CDMA scheme has approximately the same BER performance as conventional MT-DS-CDMA when only one user is supported by the system, and for multiple user systems the conventional MT-DS-CDMA has better performance than that of SSG-MT-DS-CDMA. The error performance degradation of SSG-MT-DS-CDMA is due to the lower processing gains for some of the SSG-MT-DS-CDMA subcarriers, which lower its capability to combat fading and MUI. In the next chapter, we introduce the SSG-MT-DS-CDMA signaling scheme with spectra shaped by using sinusoidal chip waveforms. The use of these sinusoidal chip waveforms helps combat the channel fading, and thus improves the BER performance.

Chapter 5. SSG-MT-DS-SS with Increased Diversity

In Chapter 4, we investigated the BER performance of the SSG-MT-DS-CDMA scheme in dispersive channels, and compared it with that of conventional MT-DS-CDMA. It was shown that the BER performance of SSG-MT-DS-CDMA is generally slightly worse than that of conventional MT-DS-CDMA. The performance degradation is mainly due to the low processing gain on some of its subcarriers, and is the price paid for spectral shaping and reduced PAPR. As in conventional modulation (spread and unspread) spectral shaping can also be achieved using chip waveforms. In [105] the spectrum of a single carrier DS-SS signal was shaped to be flat within a specified band by using a band limited chip waveform. The purpose of this band limited chip waveform was to minimize multiple access interference (MAI) in a CDMA application. Following this work, authors in [106], [107] used numerical methods to find the optimal time-limited chip waveform that minimizes both the MAI and the out-of-band energy of the signal.

These time-limited waveforms (obtained via numerical methods) shape the spectra of the baseband equivalent of the transmitted signals to have their energy concentrated around DC. Therefore, the spectra have only one primary lobe, and the lower sideband (LSB) and upper sideband (USB) of the passband signal are in a sense concatenated in the frequency domain. As a result, the two “sideband” signals often undergo correlated fading when this passband signal is passed through a dispersive fading channel. The BER performance expressions for DS-CDMA with different time limited chip waveforms were also provided in [108]. In addition to analyzing the BER

performance of these known chip waveforms, the numerical methods used to search for the optimal time limited chip waveforms were conducted under some predetermined conditions. For example, in [106], the family of prolate spheroidal wave functions (PSWFs) was used to find the optimized symmetric chip waveform that minimizes the MAI under the out-of-band energy and other constraints. The work of [106] was extended by [107] to include the both symmetric and asymmetric chip waveforms.

In this chapter we introduce the multicarrier SS modulation scheme that uses sinusoidal chip waveforms to shape the spectra of each individual subcarrier. For a given subcarrier, the chip waveform of our proposed scheme shapes the spectrum of the transmitted signal to have two distinct lobes, a lower sideband (LSB) and an upper sideband (USB). With a properly selected chip waveform parameter, the separation between these two lobes can be wider than the channel's coherence bandwidth, resulting in the two sidebands undergoing uncorrelated fading. This uncorrelated fading of the LSB and USB can be utilized to greatly improve performance by virtue of diversity.

5.1 System Model

In this section we introduce the transmitter block diagram, channel models and receiver model of the DS-SS multicarrier modulation scheme that uses sinusoidal chip waveforms.

Transmitted Signal

The transmitter block diagram of this new multicarrier modulation scheme is shown in Figure 5-1. As shown in this figure, the transmitter block diagram is similar to that of the MC/MT-DS-CDMA where the input data stream is first serial to parallel converted to form M lower rate data streams, and the data streams of each subcarrier are spread by the spreading waveforms and upconverted to their respective carrier frequencies. The transmitted signal is formed by summing the signal of all subcarriers.

Figure 5-1: Transmitter block diagram.

The new multicarrier modulation scheme assumes each subcarrier has the same data rate, processing gain, and the same spreading waveform, i.e., the same spreading code and chip waveform for all subcarriers of a given user. With the same data rate, bit

energy, processing gain, and the same sinusoidal chip waveform on each subcarrier, this new modulation scheme can be viewed as the extension of the SSG-MT-DS-SS. Thus, we can term this scheme as sinusoidal chip waveform SSG-MT-DS-CDMA (SW-SSG-MT-DS-CDMA). With BPSK modulation used on each subcarrier, the transmitted signal of user k can be expressed as:

$$s^{(k)}(t) = \sqrt{\frac{2E_b}{T_b}} \sum_{i=1}^M \sum_{n=-\infty}^{\infty} d_i^{(k)}(\lfloor n/N \rfloor) c^{(k)}(n) p(t - nT_c) \cos(2\pi f_i t) \quad (5.1)$$

where, similar to the SSG-MT-DS-SS scheme, E_b , T_b are respectively the bit energy and bit duration of each subcarrier; $d_i^{(k)}(n) \in \{-1, 1\}$ is the n^{th} transmitted data bit of the i^{th} subcarrier; $\lfloor x \rfloor$ is the integer part of x ; $N = T_b/T_c$ is the spreading gain or processing gain of one subcarrier; $c^{(k)}(m) \in \{-1, 1\}$ is the m^{th} chip of the spreading code corresponding to the user k ; $p(t) = \sqrt{2} \sin(J\pi t/T_c)$ over the interval $0 \leq t \leq T_c$ is the chip waveform with the integer $J \in \{2, 3, \dots\}$ the selectable parameter; and f_i is the carrier frequency of the i^{th} subcarrier.

Note that the use of sinusoidal chip waveforms is different from a simple upconversion of the baseband subcarrier signal to some carrier frequency. Specifically, when J is odd, phase discontinuities are enforced at chip boundaries where they otherwise would not be for an upconversion (when two consecutive chip pulses $c^{(k)}(m)$, $c^{(k)}(m+1)$ are equal). Most of our results employ odd values of J . In addition, we use the same spreading waveform for all subcarriers, i.e., the spreading sequence and the chip waveform (specified by integer J) are the same for all the subcarriers of all the users, or $c^{(k)}(t) = \sum c^{(k)}(n) p(t - nT_c)$ is the same for all subcarriers of all users. The frequency

separation between adjacent subcarriers is properly selected such that any two subcarriers are orthogonal. Specifically, when $i \neq j$ this requires

$$\int_0^{T_b} \sum_{n=0}^{N-1} p^2(t - nT_c) \cos(2\pi f_i t) \cos(2\pi f_j t) dt = 0 \quad (5.2)$$

With some straightforward algebra, we can show that (5.2) is satisfied when the frequency separation between the i^{th} and the j^{th} subcarrier is an integer multiple of the data rate $1/T_b$ of each subcarrier, i.e., $\Delta f_{i,j} = |f_i - f_j| = \mathcal{G}/T_b$ where $\mathcal{G}$ is an integer. In the work of this dissertation, we allow λ to vary from 1 to N corresponding to configuring the system to vary from MT-DS-CDMA to MC-DS-CDMA (as in [38], [39]).

The transmitted signal expressed in (5.1) can also be expressed by a summation of the LSB and USB of each subcarrier as

$$s^{(k)}(t) = \sum_{i=1}^M \left[s_{i,L}^{(k)}(t) + s_{i,U}^{(k)}(t) \right] \quad (5.3)$$

where $s_{i,L}^{(k)}(t)$ is the LSB signal on user k 's subcarrier i , and a similar definition applies for $s_{i,U}^{(k)}(t)$. With the help of the Hilbert transform, we can express the sideband signal of subcarrier i as

$$s_{i,X}^{(k)}(t) = \sqrt{\frac{E_b}{2T_b}} \sum_{n=-\infty}^{\infty} d_i^{(k)}(\lfloor n/N \rfloor) c^{(k)}(n) [p(t - nT_c) \cos(2\pi f_i t) \pm \hat{p}(t - nT_c) \sin(2\pi f_i t)] \quad (5.4)$$

where $\hat{p}(t)$ is the Hilbert transform of the infinite-duration extension of $p(t)$, truncated to one chip duration, and $X=L$ or U represents the LSB or USB of the transmitted signal. With the definition of chip waveform $p(t)$, we can easily find the Hilbert transform of

the infinite duration sinusoid to be $\hat{p}(t) = -\sqrt{2} \cos(J\pi t/T_c)$, which we truncate to be non-zero only in the range $0 \leq t \leq T_c$. Upon substituting $p(t)$ and $\hat{p}(t)$ into (5.4), we can write the sideband signal of subcarrier i as

$$s_{i,X}^{(k)}(t) = \sqrt{\frac{E_b}{T_b}} \sum_{n=-\infty}^{\infty} d^{(k)}(\lfloor n/N \rfloor) c^{(k)}(n) E_{i,X}(t - nT_c, 0) \quad (5.5)$$

In (5.5), $E_{i,X}(t, \varphi)$ is the passband chip waveform for subcarrier i , which is

$$E_{i,X}(t, \varphi) = \begin{cases} \sin\left[\frac{J\pi}{T_c}(t) \mp 2\pi f_i t + \varphi\right] & 0 \leq t \leq T_c \\ 0 & \text{otherwise} \end{cases} \quad (5.6)$$

In (5.6), the minus sign represents the passband chip waveform for the LSB signal, and similarly, the plus sign represents the chip waveform for the USB signal

Channel Model and Receiver Structure

As we have shown in Chapter 3, the PSD of each subcarrier is determined by the Fourier transform of its chip waveform [3]. With the sinusoidal chip waveform $p(t)$, the baseband PSD of one subcarrier is

$$\psi(f) = \frac{E_b}{N} \left(\frac{2J}{4\pi(T_c f)^2 - \pi I^2} \right)^2 \left[1 - (-1)^J \cos(2\pi T_c f) \right] \quad (5.7)$$

The PSD of the chip waveform for $J=5$ is shown in Figure 5-2. As can be observed in this figure, the PSD of the sinusoidal chip waveform for one subcarrier is indeed different from that of the subcarrier signal that is up-converted by the carrier waveform $\sin(5\pi t/T_c)$. Specifically, despite the same mainlobe shapes, the sidelobe levels of the two schemes—using our chip waveform and conventional up-conversion—are different. The frequency separation between the peaks of two sidelobes is

approximately J/T_c , and the null-null bandwidth of each sidelobes is $2/T_c$. Therefore, two sideband signals incur independent fading when the chip waveform parameter is properly chosen, i.e., the frequency separation between the two mainlobes of the LSB and USB is greater than the coherence bandwidth of the channel.

Figure 5-2: PSD plots of one subcarrier signal with $J=5$

Similar to the investigation in Chapter 4, the channel is represented by a TDL model, and the complex baseband channel for the i^{th} subcarrier of user k is given by

$$h_{i,X}^{(k)}(t) = \sum_{l=0}^{H-1} \alpha_{i,l}^{(k,X)} \exp(-j\theta_{i,l}^{(k,X)}) \delta(t - \tau_{i,l}^{(k,X)}) \quad (5.8)$$

where $X=L$ or U stands for the channel for LSB or USB. In (5.8) H is the number of resolvable channel paths with its value given by

$$H = \lfloor BW / B_c \rfloor + 1 \quad (5.9)$$

where $BW = 2/T_c$ is the baseband null-to-null bandwidth of either sideband signal. In (5.8), $\alpha_{i,l}^{(k,X)}$, $\theta_{i,l}^{(k,X)}$, $\tau_{i,l}^{(k,X)}$ are respectively the amplitude, phase shift, and delay of the l^{th} tap for user k 's LSB or USB signal on subcarrier i . Under the assumption of uncorrelated scattering (US) and independent fading for the LSB and USB signals, the phase shifts of different user's different paths on different subcarriers are independent (i.i.d.) RVs uniformly distributed within the range $[0, 2\pi)$. Similarly, the delay $\tau_{i,l}^{(k,X)}$ for different i , l , k and X are i.i.d RVs uniformly distributed within the interval $[0, T_b)$. The tap amplitudes for different sidebands, different taps and different users are assumed to be independent Nakagami- m RVs with the pdf given in Chapter 4 by (4.11). The average power on each channel tap is determined by the PDP which is assumed to be identical for both sideband signals of all users' subcarriers, i.e., $\Omega_{i,l}^{(k,X)} = \Omega_l$ for $X=L, U$, $i=1, 2, \dots, M$, and $k=1, 2, \dots, K$. In addition, the PDP is again assumed to be exponentially decaying with decay factor of η i.e., the average power of the l^{th} tap is given by $\Omega_l = \Omega_0 e^{-\eta l}$, and the total power of the channel is normalized to be unity, i.e., $\sum_{l=0}^{H-1} \Omega_l = 1$.

Assuming we have K asynchronous users, and independent channels for LSB and USB signals for all subcarriers of all users, the received signal is

$$r(t) = \sum_{k=1}^K \sum_{i=1}^M \left[r_{i,L}^{(k)}(t) + r_{i,U}^{(k)}(t) \right] + n(t) \quad (5.10)$$

where $n(t)$ is the AWGN with double sided PSD of $N_0/2$ W/Hz, and the received LSB or USB signal of the i^{th} subcarrier is given by

$$r_{i,X}^{(k)} = \sqrt{\frac{E_b}{T_b}} \sum_{l=0}^{H-1} \sum_{n=-\infty}^{\infty} \alpha_{i,l}^{(k,X)} d_i^{(k)}(\lfloor n/N \rfloor) c^{(k)}(n) E_{i,X}(t - nT_c - \tau_{i,l}^{(k,X)}, \phi_{i,l}^{(k,X)}) \quad (5.11)$$

In (5.11), $\phi_{i,l}^{(k,X)} = 2\pi f_i \tau_{i,l}^{(k,X)} - \theta_{i,l}^{(k,X)}$ is the composite phase shift due the delay and phase shift of tap l , which, by assumption, is also uniformly distributed within the range of $[0, 2\pi)$. For coherent detection, we assume that the receiver is capable of estimating the channel state information (CSI), i.e., the amplitude, phase, and delay of each of the channel taps. With this assumption, the receiver block diagram for user k is shown in Figure 5-3, where two RAKEs, one for the LSB and one for the USB, are used for each subcarrier. After MR combining the RAKE fingers, the output of the two sideband RAKEs are combined to form the decision metric for each subcarrier. Finally, a hard decision is made based on the sign of the decision metric, and these detected bits are parallel to serial converted to form the detected bit stream.

Figure 5-3: Receiver block diagram for user k .

The decision metric for the j^{th} subcarrier of user k is given by

$$Z_j^{(k)} = \sum_X \sum_{q=0}^{H-1} Z_{j,q}^{(k,X)} \quad (5.12)$$

where $Z_{j,q}^{(k,X)}$ is the output of the q^{th} RAKE finger of the j^{th} subcarrier of user k .

5.2 Performance Analysis

In this section, we analyze the BER performance of the SW-SSG-MT-DS-CDMA. Similar to the methods used in previous chapters, we analyze the correlator output and its statistics. With coherent detection on each RAKE finger, we can separate the output of the q^{th} RAKE finger of the j^{th} subcarrier of user k as follows:

$$\begin{aligned} Z_{j,q}^{(k,X)} &= \int_0^{T_b} r(t) \alpha_{j,q}^{(k,X)} \sum_{n=-\infty}^{\infty} c^{(k)}(n) E_{j,X}(t - nT_c) dt \\ &= D + n + I_1 + I_2 + I_3 \end{aligned} \quad (5.13)$$

where $D = \sqrt{E_b T_b} d_j^{(k)}(0)/2$ is the desired signal term, n is the AWGN sample with zero mean and variance $N_0 T_b/4$. The I_1 term in (5.13) is the interference from the same subcarrier of the same user; I_2 is the interference term from the same subcarrier of different users; I_3 is the interference term from other subcarriers. The interference from the same subcarrier of the same user, I_1 , is given by

$$\begin{aligned} I_1 &= \sqrt{\frac{E_b}{T_b}} \sum_{l=0, l \neq q}^{P-1} \alpha_{j,q}^{(k,X)} \alpha_{j,l}^{(k,X)} \beta_{j,j}^{(k,k)}(\tau_{j,l}^{(k,X)}, \phi_{j,l}^{(k,X)}) \\ &\quad + \sqrt{\frac{E_b}{T_b}} \sum_{l=0}^{P-1} \alpha_{j,q}^{(k,X)} \alpha_{j,l}^{(k,Y)} \hat{\beta}_{j,j}^{(k,k)}(\tau_{j,l}^{(k,Y)}, \phi_{j,l}^{(k,Y)}) \end{aligned} \quad (5.14)$$

where we denote the correlation function between the same sideband waveform from user v 's subcarrier i and that of user k 's subcarrier j by $\beta_{i,j}^{(v,k)}$, and similarly the $\hat{\beta}_{i,j}^{(v,k)}$ function is defined as the correlation function between corresponding different sideband signals. Note that since the LSB and USB signal are undergoing independent fading, the interference from the q^{th} path of the other sideband signal (indicated as Y) is not excluded from the summation of the second term of (5.14).

Similarly, the interference from the same subcarriers of different users, I_2 , and the interference from other subcarriers of all users, I_3 , are respectively given by

$$I_2 = \sqrt{\frac{E_b}{T_b}} \sum_{v=1, v \neq k}^K \sum_{l=0}^{H=1} \alpha_{j,q}^{(k,X)} \alpha_{j,l}^{(v,X)} \beta_{j,j}^{(v,k)}(\tau_{j,l}^{(v,X)}, \phi_{j,l}^{(v,X)}) + \sqrt{\frac{E_b}{T_b}} \sum_{v=1, v \neq k}^K \sum_{l=0}^{H=1} \alpha_{j,q}^{(k,X)} \alpha_{j,l}^{(v,Y)} \hat{\beta}_{j,j}^{(v,k)}(\tau_{j,l}^{(v,Y)}, \phi_{j,l}^{(v,Y)}) \quad (5.15)$$

$$I_3 = \sqrt{\frac{E_b}{T_b}} \sum_{v=1}^K \sum_{i=1, i \neq j}^M \sum_{l=0}^{H=1} \alpha_{j,q}^{(k,X)} \alpha_{i,l}^{(v,X)} \beta_{i,j}^{(v,k)}(\tau_{i,l}^{(v,X)}, \phi_{i,l}^{(v,X)}) + \sqrt{\frac{E_b}{T_b}} \sum_{v=1}^K \sum_{i=1, i \neq j}^M \sum_{l=0}^{H=1} \alpha_{j,q}^{(k,X)} \alpha_{i,l}^{(v,Y)} \hat{\beta}_{i,j}^{(v,k)}(\tau_{i,l}^{(v,Y)}, \phi_{i,l}^{(v,Y)}) \quad (5.16)$$

The cross correlation functions, $\beta_{i,j}^{(v,k)}$ and $\hat{\beta}_{i,j}^{(v,k)}$, in (5.14)-(5.16) are defined as

$$\beta_{i,j}^{(v,k)}(\tau, \phi) = \int_0^{T_b} \sum_{n=-\infty}^{\infty} d_i^{(v)}(\lfloor n/N \rfloor) c_i^{(v)}(n) E_{i,X}(t - nT_c - \tau, \phi) \sum_{m=0}^{N-1} c_j^{(k)}(m) E_{j,X}(t - nT_c, 0) dt \quad (5.17)$$

and

$$\hat{\beta}_{i,j}^{(v,k)}(\tau, \phi) = \int_0^{T_b} \sum_{n=-\infty}^{\infty} d_i^{(v)}(\lfloor n/N \rfloor) c_i^{(v)}(n) E_{i,Y}(t - nT_c - \tau, \phi) \sum_{m=0}^{N-1} c_j^{(k)}(m) E_{j,X}(t - nT_c) dt \quad (5.18)$$

We again assume the interference terms given in (5.14)-(5.16) are additive Gaussian RVs and, similar to our technique in previous chapters, we derive the statistics of these interference terms and then derive the bit error probability of the SW-SSG-MT-DS-CDMA scheme.

Observing the definitions of these two cross correlation functions ($\beta_{i,j}^{(k,v)}$ and $\hat{\beta}_{i,j}^{(k,v)}$), we can see that the cross correlation functions between the same subcarriers of

different users can be obtained by equating the user indices k and v ; similarly, the cross correlation between the different multipath echoes of the given subcarrier from the same user can be found by letting $i=j$ and $v=k$.

Following the method given in Chapter 4, we quantize the delay, τ , as

$$\tau = ST_c + \varepsilon \quad (5.19)$$

where S is an integer given by $S = \lfloor \tau / T_c \rfloor$ and $\varepsilon = \tau - ST_c$ is the fractional chip part of τ , which is uniformly distributed in $[0, T_c)$. After the quantization, $\beta_{i,j}^{(v,k)}$ and $\hat{\beta}_{i,j}^{(v,k)}$ can be express as

$$\beta_{i,j}^{(v,k)}(\tau, \phi) = d_i^{(v)}(-1)R_{i,X;j,X}^{(v,k)}(\tau, \phi) + d_i^{(v)}(0)\hat{R}_{i,X;j,X}^{(v,k)}(\tau, \phi) \quad (5.20)$$

$$\hat{\beta}_{i,j}^{(v,k)}(\tau, \phi) = d_i^{(v)}(-1)R_{i,Y;j,X}^{(v,k)}(\tau, \phi) + d_i^{(v)}(0)\hat{R}_{i,Y;j,X}^{(v,k)}(\tau, \phi) \quad (5.21)$$

where $R_{i,Y;j,X}^{(v,k)}$ and $\hat{R}_{i,Y;j,X}^{(v,k)}$ are two partial correlation functions with definitions given by

$$\begin{aligned} R_{i,Y;j,X}^{(v,k)}(\tau, \phi) = & \sum_{m=0}^S c^{(k)}(m)c^{(v)}(m-S-1) \int_0^\varepsilon E_{i,Y}(t-\varepsilon, \phi) E_{j,X}(t, 0) dt \\ & + \sum_{m=0}^{S-1} c^{(k)}(m)c^{(v)}(m-S) \int_\varepsilon^{T_c} E_{i,Y}(t-\varepsilon, \phi) E_{j,X}(t, 0) dt \end{aligned} \quad (5.22)$$

$$\begin{aligned} \hat{R}_{i,Y;j,X}^{(v,k)}(\tau, \phi) = & \sum_{m=S+1}^{N-1} c^{(k)}(m)c^{(v)}(m-S-1) \int_0^\varepsilon E_{i,Y}(t-\varepsilon, \phi) E_{j,X}(t, 0) dt \\ & + \sum_{m=S}^{N-1} c^{(k)}(m)c^{(v)}(m-S) \int_\varepsilon^{T_c} E_{i,Y}(t-\varepsilon, \phi) E_{j,X}(t, 0) dt \end{aligned} \quad (5.23)$$

Since we assume the data symbols and the spreading codes are random and binary, the two partial correlation functions given by (5.22) and (5.23) are of zero mean, and their variance is easy to evaluate. It is easy to show the variances of these two cross correlation function are

$$\text{var}[\beta_{i,j}^{(v,k)}(\tau, \phi)] = 2N\text{var}\left[\int_0^\varepsilon E_{i,X}(t-\varepsilon, \phi)E_{j,X}(t, 0)dt\right] \quad (5.24)$$

$$\text{var}[\hat{\beta}_{i,j}^{(v,k)}(\tau, \phi)] = 2N\text{var}\left[\int_0^\varepsilon E_{i,Y}(t-\varepsilon, \phi)E_{j,X}(t, 0)dt\right] \quad (5.25)$$

Let's denote the integral in (5.24) as I_X :

$$I_X = \int_0^\varepsilon E_{i,X}(t-\varepsilon, 0)E_{j,X}(t, 0)dt \quad (5.26)$$

With the definition of $E_{i,X}$ given in (5.6), it is easy to show that I_X is

$$I_X = \frac{1}{2} \int_0^\varepsilon \cos\left(2\pi\Delta f_{i,j}t + \frac{J\pi}{T_c}\varepsilon + \theta\right)dt \quad (5.27)$$

where $\Delta f_{i,j} = f_i - f_j$ is the frequency separation between the i^{th} and the j^{th} subcarrier, and $\theta = \phi + 2\pi f_i \varepsilon - I\pi\varepsilon/T_c$ is the aggregate phase due to the delay and the channel, which is uniformly distributed in $[0, 2\pi)$. Note that to obtain (5.27) we discard the high frequency component, i.e., that at $f_i + f_j$. Upon averaging I_X given by (5.27) over the aggregate phase θ , and the fractional chip part of the delay, ε , the variance of I_X is

$$\text{var}[I_X] = \frac{T_c^2}{32} \frac{2\pi\Delta f_{i,j}T_c - \sin(2\pi\Delta f_{i,j}T_c)}{\pi\Delta f_{i,j}T_c} \quad (5.28)$$

When $\Delta f_{i,j}=0$, I_X is used to represent the same-subcarrier same-sideband interference, and the variance of I_X in this case is

$$\text{var}[I_X] = \frac{T_c^2}{24} \quad (5.29)$$

Similarly, the integral in the cross correlation functions between different sideband signals can be denoted by I_Y , and it can be expressed as

$$\begin{aligned}
I_Y &= \int_0^\varepsilon E_{i,Y}(t-\varepsilon)E_{j,X}(t)dt \\
&= \frac{1}{2} \int_0^\varepsilon \cos\left(\frac{J\pi}{T_c}(2t-\varepsilon) - 2\pi\Delta f_{i,j}t + \theta\right)dt
\end{aligned} \tag{5.30}$$

Following the same method, the variance of I_Y is

$$\text{var}[I_Y] = \begin{cases} \frac{T_c^2}{32} \frac{2\pi J - 2\pi\Delta f_{i,j}T_c + \sin(2\pi\Delta f_{i,j}T_c)}{(\pi J - \pi\Delta f_{i,j}T_c)^3} & \Delta f_{i,j}T_c \neq J \\ \frac{T_c^2}{24} & \Delta f_{i,j}T_c = J \end{cases} \tag{5.31}$$

To simplify the presentation of the SNIR expression we define the following functions

$$V_X(\Delta f_{i,j}T_c) = \frac{2\pi\Delta f_{i,j}T_c - \sin(2\pi\Delta f_{i,j}T_c)}{\pi\Delta f_{i,j}T_c} \quad \Delta f \neq 0 \tag{5.32}$$

$$V_Y(\Delta f_{i,j}T_c) = \frac{2\pi J - 2\pi\Delta f_{i,j}T_c + \sin(2\pi\Delta f_{i,j}T_c)}{(\pi J - \pi\Delta f_{i,j}T_c)^3} \quad \Delta f_{i,j}T_c \neq J \tag{5.33}$$

Then, the variance of I_X and I_Y as a function of $\Delta f_{i,j}T_c$ can be expressed by

$$\text{var}[I_X(\Delta f_{i,j}T_c)] = \begin{cases} \frac{T_c^2}{32} V_X(\Delta f_{i,j}T_c) & \Delta f_{i,j} \neq 0 \\ \frac{T_c^2}{24} & \Delta f_{i,j} = 0 \end{cases} \tag{5.34}$$

and

$$\text{var}[I_Y(\Delta f_{i,j}T_c)] = \begin{cases} \frac{T_c^2}{32} V_Y(\Delta f_{i,j}T_c) & \Delta f_{i,j}T_c \neq J \\ \frac{T_c^2}{24} & \Delta f_{i,j}T_c = J \end{cases} \tag{5.35}$$

With the variance of I_X and I_Y given in (5.34) and (5.35), the variances of the interference terms are

$$\text{var}[I_1] = \frac{E_b T_b}{N} \left[\frac{M-1}{12M} + \frac{1}{8I^2 \pi^2} \right] (\alpha_{j,q}^{(k,X)})^2 \quad (5.36)$$

$$\text{var}[I_2] = (K-1) \frac{E_b T_b}{N} \left[\frac{1}{12} + \frac{1}{8J^2 \pi^2} \right] (\alpha_{j,q}^{(k,X)})^2 \quad (5.37)$$

$$\text{var}[I_3] = \frac{KE_b T_b}{N} \sum_{\substack{j=1 \\ j \neq i}}^M \left(\text{Var}[I_X(\Delta f_{i,j} T_c)] + \text{Var}[I_Y(\Delta f_{i,j} T_c)] \right) (\alpha_{j,q}^{(k,X)})^2 \quad (5.38)$$

Thus, the mean and variance of the j^{th} subcarrier decision metric are

$$\mathbb{E}[Z_j^{(v)}] = \frac{1}{2} \sqrt{E_b T_b} d(0) \sum_X \sum_{q=0}^{M-1} \left[(\alpha_{j,q}^{(k,X)})^2 \right] \quad (5.39)$$

$$\begin{aligned} \text{var}[Z_j^{(v)}] = & \left\{ \frac{N_0 T_b}{4} + \frac{1}{24J^2 \pi^2 N} \left[K(3 + 2J^2 \pi^2 - 2J^2 \pi^2 / M) \right] \right. \\ & \left. + \frac{K}{N} \sum_{i=1, i \neq j}^M \left(\text{Var}[I_X(\Delta f_{i,j} T_c)] + \text{Var}[I_Y(\Delta f_{i,j} T_c)] \right) \right\} \sum_X \sum_{q=0}^{H-1} \left[(\alpha_{j,q}^{(X)})^2 \right] \end{aligned} \quad (5.40)$$

The statistics of the decision metric of any given subcarrier from all users are the same since each user has the same parameter set, i.e., same number of subcarriers, same data rate, processing gain on each subcarrier, and the same chip waveform. Thus the superscript which indicates the user is removed without ambiguity. It is well known that the instantaneous bit error probability for BPSK modulation is

$$P_b = Q \left(\sqrt{\frac{\mathbb{E}^2[Z_j]}{\text{var}[Z_j]}} \right) = Q(\sqrt{2\gamma}) \quad (5.41)$$

where the instantaneous SNIR, γ , is

$$\gamma = \sum_X \sum_{q=0}^{P-1} \gamma_j^{(X)} (\alpha_{j,q}^{(X)})^2 \quad (5.42)$$

with the average SNIR of the j^{th} subcarrier's LSB or USB RAKE output given by

$$\begin{aligned} \bar{\gamma}_j^{(X)} = & \left\{ \frac{2N_0}{E_b} + \frac{1}{3J^2\pi^2N} \left[K(3 + 2J^2\pi^2 - 2J^2\pi^2/M) \right. \right. \\ & \left. \left. + \frac{8K}{N} \sum_{i=1, i \neq j}^M \left[V_X(\Delta f_{i,j} T_c) + V_Y(\Delta f_{i,j} T_c) \right] \right] \right\}^{-1} \end{aligned} \quad (5.43)$$

To obtain the average P_b , the instantaneous P_b given by (5.41) should be averaged over the joint pdf of channel tap amplitudes as was done in Chapter 4. With the uncorrelated scattering assumption, we assume the tap amplitudes are independent RVs and thus the joint pdf of the tap amplitudes can be expressed as the product of the pdfs of all the tap amplitudes

$$p_{\alpha_{0,X}, \alpha_{1,X}, \dots, \alpha_{P-1,X}}(\alpha_{0,X}, \alpha_{1,X}, \dots, \alpha_{P-1,X}) = \prod_X \prod_{q=0}^{P-1} p_{\alpha_{q,X}}(\alpha_{q,X}) \quad (5.44)$$

Following the same method as in Chapter 4, the average P_b can thus be expressed as

$$\begin{aligned} \bar{P}_b = & \frac{1}{\pi} \underbrace{\int_0^\infty \int_0^\infty \dots \int_0^\infty}_{2M \text{ fold}} \int_0^{\pi/2} \exp \left(- \frac{\sum_X \sum_{q=0}^{P-1} \bar{\gamma}_j^{(X)} (\alpha_{j,q}^{(X)})^2}{\sin^2 \chi} \right) d\chi \\ & \cdot \prod_X \prod_{q=0}^{P-1} p_{\alpha_{q,X}}(\alpha_{q,X}) d\alpha_{0,L}, d\alpha_{1,L}, \dots, d\alpha_{M-1,U} \\ = & \frac{1}{\pi} \int_0^{\pi/2} \prod_X \prod_{q=0}^{P-1} G(\bar{\gamma}_{j,q}^{(X)}, \chi) d\chi \end{aligned} \quad (5.45)$$

where $\bar{\gamma}_{j,q}^{(X)} = \bar{\gamma}_j^{(X)} e^{-\eta q}$ is the average SNIR of the q^{th} RAKE finger output for the LSB or USB RAKE of the j^{th} subcarrier, and $G(x)$ is the moment generating function of Nakagami- m distribution, which is given by

$$G(\chi) = \left(\frac{m \sin^2 \chi}{\gamma + \sin^2 \chi} \right)^m \quad (5.46)$$

5.3 Numerical Examples

In this section, we compare the performance of the SW-SSG-MT-DS-CDMA scheme with that of conventional MC-DS-CDMA. The performance is compared using the same number of subcarriers, same total data rate, same transmission power, and the same bandwidths for both schemes. First we show the spectrum of the proposed multicarrier modulation scheme with sinusoidal chip waveforms. Recall that the bandwidth of one sideband signal of the SW-SSG-MT-DS-CDMA scheme is $2/T_c$, and the frequency separation of the peaks of the two sidelobe signals is approximately J/T_c . Thus, the null-null bandwidth of our scheme is

$$BW = (J + 2)/T_c + (M - 1)\Delta f \quad (5.47)$$

where Δf is the frequency separation between the adjacent subcarriers, i.e., $\Delta f = \lambda R_b$. The null-null bandwidth of the MC-DS-CDMA scheme with frequency separation equal to the chip rate is

$$BW = 2/T_{c,MC} + (M - 1)/T_{c,MC} \quad (5.48)$$

Thus under the equal bandwidth and data rate condition, the processing gain relation between the MC-DS-CDMA scheme and the SW-SSG-MT-DS-CDMA scheme is

$$N_{MC} = \left\lfloor \frac{(J + 2)N + \lambda(M - 1)}{M + 1} \right\rfloor \quad (5.49)$$

In Figure 5-4 we compare the baseband PSD of the SW-SSG-MT-DS-CDMA scheme with that of the conventional MC-DS-CDMA with $M=5$ subcarriers and processing gain of 64 on each subcarrier. The two examples of the SW-SSG-MT-DS-CDMA scheme

have chip waveform parameter $J=3$, and frequency separations of I and N respectively.

For $\lambda=I$ and $\lambda=N$ configurations, the processing gains are respectively 76 and 42.

Figure 5-4: PSD of SW-SSG-MT-DS-CDMA and conventional MC-DS-SS with $M=5$, $N_{MC}=64$, $J=3$, and $N=76$ for $\lambda=I$ and $N=42$ for $\lambda=N$.

It can be observed from this figure that $\lambda=I$ and $\lambda=N$ are two extreme examples of spectrum shaping. Specifically, when $\lambda=I$, the intersubcarrier frequency separation is the data rate on each subcarrier, and the PSD of the SW-SSG-MT-DS-CDMA scheme has spectral nulls around DC; conversely when $\lambda=N$, the PSD of the SW-SSG-MT-DS-CDMA scheme reaches its maximum value around DC². Thus, the SW-SSG-MT-DS-CDMA with different frequency separations can be used to adaptively compensate for channel fading when the wireless channel is known at the transmitter. For example, if the baseband equivalent channel has a spectral null around DC, we may want to shape the spectrum of the SW-SSG-MT-DS-CDMA scheme with $\lambda=I$ to “water-fill” the channel.

² This figure shows the PSD of the transmitted signal, i.e., the sum of the spectra of individual subcarriers. For each individual subcarrier the PSD is given by (5.7), and two distinct sidebands are separated by J/T_c .

It is also worth noting that the sidelobe levels of the SW-SSG-MT-DS-CDMA scheme are more than 17 dB down, whereas the conventional MC-DS-SS sidelobes are down by approximately 11 dB; hence our scheme suppresses the sidelobe levels naturally, by approximately 6 dB. With the spectral shaping capability, we can also classify this new multicarrier scheme as another form of SSG-MT-DS-CDMA.

Next, we compare the BER performance of SSG-MT-DS-CDMA with our sinusoidal chip waveform with that of conventional MC-DS-CDMA for different system configurations and channel conditions. As given in [38], the average P_b of the MC-DS-CDMA scheme in a dispersive Nakagami- m channel is given by

$$\bar{P}_b = \frac{1}{\pi} \int_0^{\pi/2} \prod_{q=0}^{H-1} G(\bar{\gamma}_q, \chi) d\chi \quad (5.50)$$

where H is the number of resolvable channel taps of one subcarrier, $\bar{\gamma}_q = \bar{\gamma} \Omega_q$ is the average SNIR of the q^{th} RAKE finger output, and $\bar{\gamma}$ is given by [38]:

$$\bar{\gamma} = \left[\left(\frac{E_b}{N_0} \right)^{-1} + \frac{2(KP-1)}{P} \left(\frac{1}{3N_{MC}} + (M-1)\bar{I}_M \right) \right]^{-1} \quad (5.51)$$

In (5.51), $\bar{I}_M$ is the average interference from other subcarriers, also given in [38]

$$\bar{I}_M = \frac{1}{M(M-1)} \sum_{i=1}^M \sum_{\substack{j=1 \\ j \neq i}}^M \frac{1}{2\pi^2(i-j)^2 N_{MC}} \quad (5.52)$$

where N_{MC} is the processing gain of MC-DS-CDMA. In Figure 5-5, we compare the analytical P_b derived in the previous section with simulation results. The SSG-MT-DS-CDMA results are for $K=10$ users, $M=5$ subcarriers, chip waveform parameter $J=3$, and processing gain of $N=32$ for both $\lambda=1$ and $\lambda=N$. The channel is a Nakagami- m

dispersive channel with $m=1$ (Rayleigh fading), PDP decay factor of $\eta=2$, and the number of resolvable paths for each sideband is $P=5$, for a 5-tap channel. From this figure, we can observe that the simulation results match the analytical results, corroborating our standard Gaussian assumption for the interference. The good agreement between the analytical results and simulation results hold for processing gain values greater than 8. Therefore, in the following examples we use analytical results to compare the BER performance of the sinusoidal chip waveform SSG-MT-DS-CDMA system with that of MC-DS-CDMA, in which we also consider 10 users, and 5 subcarriers, with processing gain and number of resolvable channel taps for each subcarrier set to be $N=64$ and $H=5$, respectively.

Note that in the example of Figure 5-5 we showed that analytical results match simulation results for the SW-SSG-MT-DS-CDMA scheme with two values of frequency separation without setting the bandwidths of these two configurations to be equal. For the examples in the rest of this section, we compare the BER performance of the sinusoidal chip waveform SSG-MT-DS-CDMA with that of the conventional MC-DS-CDMA scheme in which both systems have the same number of users, the same number of subcarriers, the same total data rate, the same bandwidth, and the same transmission power.

Figure 5-5: Analytical and simulated average P_b vs E_b/N_0 for $K=10$ users, $M=5$ subcarriers, $N=32$ for SW-SSG-MT-DS-CDMA with $J=3$, $\lambda=1$, $\lambda=N=32$, in Nakagami- m dispersive channel with $m=1$, $\eta=2$, $H=5$ channel taps.

The average P_b of the SW-SSG-MT-DS-CDMA scheme with $J=3$ and different frequency separations (λ 's), and for the MC-DS-CDMA system with parameters given previously are shown in Figure 5-6. The processing gains of the SW-SSG-MT-DS-CDMA schemes are 76 and 42 respectively for $\lambda=1$ and $\lambda=N$. In this example, the channel is a dispersive Nakagami- m channel with $m=1$, $\eta=2$, and the number of channel taps for the SW-SSG-MT-DS-CDMA scheme with $\lambda=1$ and $\lambda=N$ are respectively 6 and 4.

Figure 5-6: Average P_b vs E_b/N_0 for 10 user MC-DS-CDMA system with $M=5$ subcarriers and $N=64$, and SW-SSG-MT-DS-CDMA scheme with $J=3$, $\lambda=1$, $N=76$, and $J=3$, $\lambda=N=42$, in Nakagami- m dispersive channel with $m=1$, $\eta=2$, and $P=5$ channel taps for MC-DS-CDMA, $H=6$, and 4 for SW-SSG-MT-DS-CDMA with $\lambda=1$ and $\lambda=N$, respectively

From this example we can observe that the SW-SSG-MT-DS-CDMA system with $\lambda=N$ intersubcarrier frequency separation has better performance than that of the $\lambda=1$ case because of the smaller amount of intersubcarrier interference due to the lesser amount of spectral overlapping among subcarriers for the $\lambda=N$ configuration. The $\lambda=N$ system also has significantly better performance than conventional MC-DS-CDMA. In addition, in comparison to conventional MC-DS-CDMA, the SSG-MT-DS-CDMA with $\lambda=1$ has slightly better performance at SNRs less than 15 dB, and approximately the same performance for SNRs greater than 15 dB.

The effect of the chip waveform parameter J on the BER performance of the SW-SSG-MT-DS-CDMA with our sinusoidal chip waveform is shown in Figure 5-7, where

we fix the SNR at $E_b/N_0=15$ dB, and show the average P_b for our new SSG-MT-DS-CDMA scheme as a function of chip waveform parameter J . As with the previous example, we employ $K=10$ asynchronous users and the channel is a Nakagami- m dispersive channel with $m=1$ and $\eta=2$. The processing gains on each subcarrier of the SSG-MT-DS-CDMA systems are obtained via equating (5.47) and (5.48); the number of resolvable channel taps for each subcarrier of SW-SSG-MT-DS-CDMA is obtained via (5.9). From this figure, we can observe that as the value of the chip waveform parameter J increases, the BER performance of SW-SSG-MT-DS-CDMA degrades when the frequency separation is $\lambda=1$ whereas the performance is only slightly changed for the $\lambda=N$ configuration.

The reason for the performance degradation is mainly that as the value of value of J increases, the processing gain of SW-SSG-MT-DS-CDMA must decrease in order to keep the bandwidth unchanged. For example, the processing gains for the two frequency separations ($\lambda=1$ and $\lambda=N$) are reduced from 95 and 48, to 31 and 24, when the value of J increases from 2 to 10. For the $\lambda=1$ configuration, the spectra of all subcarriers overlap significantly, thus the loss in processing gain reduces its capability to suppress interference from other subcarriers. On the other hand, only the spectra of adjacent subcarriers overlap for the $\lambda=N$ configuration, yielding less intersubcarrier interference. As a result, the reduction of processing gain for the $\lambda=N$ scheme does not strongly affect the average P_b .

Figure 5-7: Average P_b vs. J for 10-user systems over Nakagami- m dispersive channel with $m=1$, and $\eta=2$, $M=5$ subcarriers, $N=64$ for MC-DS-CDMA, and $\lambda=1$ and $\lambda=N$ for SW-SSG-MT-DS-CDMA.

Having shown the effect of the chip waveform parameter on the BER performance of sinusoidal chip SW-SSG-MT-DS-CDMA, we next show the effects of different channel conditions on the BER performance of this SW-SSG-MT-DS-CDMA scheme. In Figure 5-8 the BER performance of 5-user SSG-MT-DS-CDMA systems with sinusoidal chip waveforms are compared with that of conventional MC-DS-CDMA for different PDP decay factors. In this figure, the SW-SSG-MT-DS-CDMA systems have $J=3$, processing gains of 76 and 42 for $\lambda=1$ and $\lambda=N$, respectively. The channel is a Nakagami- m fading channel with $m=1$, average SNR of $E_b/N_0=15$ dB, and the decay factor varies from 0 to 3. We assume the receiver is capable of capturing all the channel taps, i.e., we employ a 5-finger RAKE for one subcarrier of MC-DS-CDMA, and 4-finger

and 6-finger RAKEs for each sideband of the SW-SSG-MT-DS-CDMA schemes with $\lambda=1$ and $\lambda=N$ respectively.

Figure 5-8: Average P_b vs. channel power decay factor for 10 user system with, $N=64$, $M=5$ subcarriers for MC-DS-CDMA, and SW-SSG-MT-DS-CDMA with $J=3$, $\lambda=1$, $N=76$, and $J=3$, $\lambda=N=42$, in Nakagami- m dispersive channel with $m=1$, and $H=5$ for MC-DS-CDMA, $H=6$, and 4 for SW-SSG-MT-DS-CDMA with $\lambda=1$ and $\lambda=N$, respectively

The results shown in Figure 5-8 indicate that as the decay factor increases, the average P_b of all three system increases. This agrees with the fact that RAKE has the best performance for a uniform PDP, and the performance of the RAKE degrades as the value of the decay factor increases. The SW-SSG-MT-DS-CDMA with $\lambda=N$ outperforms the conventional MC-DS-CDMA system at any value of η , and when $\eta > 1.8$, our SW-SSG-MT-DS-CDMA scheme with $\lambda=1$ has a better performance than that of MC-DS-CDMA. As the decay factor increases, the majority of the energy is contained within the first few channel taps. Thus, when these taps incur deep fading, the

SNIR output of the MRC is low even if the low-energy channel taps are not faded. For the SW-SSG-MT-DS-CDMA scheme, however, the channels for LSB and USB are independent identical channels. Therefore, the probability of both sideband signals undergoing a deep fade is greatly reduced.

In Figure 5-9 we show the average P_b of three schemes for different Nakagami- m factors. The parameters of conventional MC-DS-CDMA and for our SW-SSG-MT-DS-CDMA schemes with $\lambda=1$ and $\lambda=N$ are the same as in the previous example, and the channel is similar to that of the previous example except that we keep the decay factor η , constant at $\eta=2$ and allow the fading parameter m to vary from 1/2 to 10. As we know, $m=1/2$ is the so called one-sided Gaussian distribution, or a very severe case of fading. As the value of m increases, the channel is less severely faded, and this example shows the effect of the diversity introduced via LSB and USB combining in the SW-SSG-MT-DS-CDMA scheme.

Figure 5-9: Average P_b vs. m for 10 user system with, $N=64$, $M=5$ subcarriers for MC-DS-CDMA and SW-SSG-MT-DS-CDMA with $J=3$, $\lambda=1$, $N=76$, and $J=3$, $\lambda=N=42$, in Nakgami- m dispersive channel with $\eta=2$, and $H=5$ for MC-DS-CDMA, $H=6$, and 4 for SW-SSG-MT-DS-CDMA with $\lambda=1$ and $\lambda=N$, respectively

As can be observed from Figure 5-9, the average P_b of all three schemes, MC-DS-CDMA and SW-SSG-MT-DS-CDMA with $\lambda=1$ and N , decreases as the value of m increases, as expected. Both SSG-MT-DS-CDMA with $\lambda=1$ and N outperform MC-DS-CDMA at $m=1/2$. As the value of m increases, the performance of the $\lambda=1$ scheme worsens relative to that of conventional MC-DS-CDMA, whereas the $\lambda=N$ scheme still has better performance than that of the MC-DS-CDMA scheme. The reason is that when the channel fading is milder (m increases), the dominant effect on performance becomes the MAI; in this case, since diversity is used to combat fading instead of suppress the interference, the diversity introduced by combining the LSB and USB signals in our scheme is less effective. As a result, the performance of the $\lambda=N$ scheme approaches that

of MC-DS-CDMA. This example, as well as the previous example, shows that the proposed scheme can be used in environments where the wireless channel is severely faded, i.e., a large PDP decay factor η and a small m value. For $\lambda=N$, our SW-SSG-MT-DS-CDMA scheme is superior to conventional for all values of η and m .

The average P_b of SW-SSG-MT-DS-CDMA versus number of subcarriers at $E_b/N_0 = 15$ dB, with $\lambda=1$ and $\lambda=N$, and that of conventional MC-DS-CDMA, are shown in Figure 5-10. We vary the number of subcarriers from 3 to 20, and adjust the processing gains of each subcarrier of our SW-SSG-MT-DS-CDMA schemes to make sure their bandwidths are the same as that of the conventional MC-DS-CDMA with a processing gain of $N=64$. The rest of the parameters are fixed; the channel is a Nakagami- m fading channel with $m=1$, $\eta=2$ and the number of resolvable channel taps for conventional MC-DS-CDMA is $H=5$, and the number of resolvable channel taps for our proposed schemes are given by (5.9). As demonstrated in this figure, the error performances of all three schemes are insensitive to the number of subcarriers, M , when M is greater than 6, i.e., the curves are flat as the number of subcarriers increases. In addition, the error performance of SW-SSG-MT-DS-CDMA with $\lambda=1$ is similar to that of conventional MC-DS-CDMA when the number of subcarriers is large because of the increased interference caused by strongly overlapped subcarriers; conversely, since the adjacent subcarriers overlap at their spectral nulls in SW-SSG-MT-DS-CDMA with $\lambda=N$, this scheme has lower bit error probabilities despite the increase in the number of subcarriers.

Figure 5-10: P_b vs. number of subcarriers for 10 user system with, $N=64$ for MC-DS-CDMA and SW-SSG-MT-DS-CDMA with $J=3$, $\lambda=1$, and $J=3$, $\lambda=N$, in Nakagami- m dispersive channel with $\eta=2$, and $H=5$ for MC-DS-CDMA

We also show the influence of different system load (number of users) on the average P_b of our SW-SSG-MT-DS-CDMA scheme. The parameters of conventional MC-DS-CDMA and for our proposed schemes with $\lambda=1$ and $\lambda=N$ and the channel are the same as in the example shown in Figure 5-6. We show the average P_b of three systems as a function of the number of active users. As expected, with more users added to the system, the error performance of all three schemes degrades, and the performance of SW-SSG-MT-DS-CDMA schemes with $\lambda=1$ and $\lambda=N$ approaches that of the conventional MC-DS-CDMA as the number of users increases. This is due to reasons similar to those of the example shown in Figure 5-9. Specifically, as the number of user increases, MAI

becomes the dominant factor in performance loss, and the diversity introduced by combining the LSB and USB signals in our scheme is less effective.

Figure 5-11: P_b vs. number of users, $N=64$, $M=5$ subcarriers for MC-DS-CDMA and SW-SSG-MT-DS-CDMA with $J=3$, $\lambda=1$, $N=76$, and $J=3$, $\lambda=N=42$, in Nakagami- m dispersive channel with $m=1$ $\eta=2$, and $H=5$ for MC-DS-CDMA, $H=6$, and 4 for proposed schemes with $\lambda=1$ and $\lambda=N$, respectively.

For yet another comparison, in Figure 5-12, we compare the error performance of SW-SSG-MT-DS-CDMA with the conventional MC-DS-CDMA with the half sine and raised cosine chip waveforms [39]. The parameters of conventional MC-DS-CDMA with the rectangular chip waveform, our SW-SSG-MT-DS-CDMA, and the channel are the same in previous example. Considering the spectral expansion of the half sine and raised cosine chip waveforms, the processing gains of MC-DS-CDMA with chip waveform other than rectangular are respectively 43 and 32 to ensure the same bandwidth of all schemes. It is shown in this figure that the SW-SSG-MT-DS-CDMA with $\lambda=N$ has the

best error performance followed by the proposed scheme with $\lambda=1$, conventional MC-DS-CDMA with rectangular chip waveform, MC-DS-CDMA with the half sine chip waveform, and MC-DS-CDMA with the raised cosine chip waveform. This again suggests that the SW-SSG-MT-DS-CDMA with $\lambda=N$ yields the maximum diversity gain. It is worth noting that the MC-DS-CDMA with rectangular, half sine, and raised cosine chip waveforms have similar error performance when their processing gain values are the same [39]. As we adjust the processing gain values of MC-DS-CDMA with chip waveforms other than rectangular, we can observe that, under the same bandwidth condition, the MC-DS-CDMA with the rectangular chip waveform has the best error performance followed by the half sine and raised cosine chip waveforms, which indicates that, other than shaping the spectra, the half sine and raised cosine chip waveforms do not improve the error performance of the system.

Figure 5-12: P_b vs. E_b/N_0 for 10 user MC-DS-CDMA system with $M=5$ subcarriers $N=64, 43$ and 32 respectively for rectangular, half-sine and raised-cosine chip waveforms, SW-SSG-DS-CDMA with $J=3$, $\lambda=1$, $N=76$, and $J=3$, $\lambda=N=42$, in Nakagami- m dispersive channel with $m=1$, $\eta=2$, and $H=5$ channel taps for MC-DS-CDMA, $H=6$, and 4 for proposed scheme with $\lambda=1$ and $\lambda=N$, respectively.

As a special case, the BER performance of “single carrier SW-SSG-MT-DS-CDMA,” or DS-SS-CDMA with our sinusoidal chip waveform, for different channel conditions, is now compared with that of DS-CDMA with a rectangular chip waveform and with the chip waveform proposed by [109], which is given by

$$p(t) = A_1 + A_2 \sin(2\pi t/T_c) + A_3 \cos(2\pi t/T_c) \quad (5.53)$$

where the coefficients A_1 , A_2 , and A_3 are chosen to flatten the spectrum, and should be normalized so that the energy is equal to T_c within one chip, i.e., so $\int_0^{T_c} p^2(t)dt = T_c$. The chip waveform given by (5.53) allows a wide variety of spreading waveforms, for

example, the conventional rectangular chip waveform is the special case when $A_1=I$ and $A_2=A_3=0$. In addition, it is relative easy to find the analytical BER performance of the system using this chip waveform.

Using the same system configuration as done in [105], where the same chip waveform is used for both transmitter and receiver, the average P_b of the system using the chip waveform given by (5.53) can be easily found as.

$$\overline{P_b} = \frac{1}{\pi} \int_0^{\pi/2} \prod_{l=0}^{P_{\text{Ref}}-1} \left(\frac{m \sin^2(\chi)}{m \sin^2(\chi) + \overline{\gamma}_l} \right)^m d\chi \quad (5.54)$$

where H_{Ref} is the number of resolvable channel taps for referenced systems, and $\overline{\gamma}_l = \overline{\gamma}_c \exp(-l\eta)$ is the average SNIR of the l^{th} tap. The average SNIR $\overline{\gamma}_c$ is given by

$$\overline{\gamma}_c = \left\{ \frac{N_0}{E_b} + \frac{2I(KH_{\text{Ref}} - 1)}{H_{\text{Ref}}} \right\}^{-1} \quad (5.55)$$

where I is

$$I = \frac{1}{192N\pi^2} \left[3A_3^4 - 72A_2^2A_1A_3 + 64A_1^4\pi^2 + 8A_3^4\pi^2 + 8A_2^4\pi^2 + 144A_3^2A_1^2 \right. \\ \left. - 192A_1^3A_3 - 6A_2^2A_3^2 + 144A_2^2A_1^2 + 15A_2^4 + 24A_3^2A_1 + 16A_2^2A_3^2\pi^2 \right] \quad (5.56)$$

Note for the conventional rectangular chip waveform with $A_1=I$, $A_1=A_2=0$, (5.56) reduces to well known result, $I/3N$. The coefficients of the chip waveform given by (5.53) are chosen to be $A_1=3.25$, $A_2=5.95$, and $A_3=1.55$ [109] to flatten the spectrum, and we term this chip waveform 1; similarly, the conventional rectangular chip waveform, or $A_1=I$, $A_2=A_3=0$, is termed chip waveform 2.

The average P_b for the DS-CDMA systems with these three chip waveforms are compared in the following examples. As in the previous examples, the comparison is

made under conditions of the same total data rate, same transmission power, and the same bandwidth for all systems. To ensure the same bandwidth of these single carrier systems, we need the processing gain of the two reference systems to be

$$\begin{aligned} N_{Chip1} &= (i + 2)N / 4 \\ N_{Chip2} &= (i + 2)N / 2 \end{aligned} \quad (5.57)$$

where N is the processing gain of our DS-CDMA signal that uses a sinusoidal chip waveform, and N_{chip1} , N_{chip2} are respectively the processing gains of the DS-CDMA systems with chip waveforms 1 and 2. Note that the null-to-null bandwidth of the DS-CDMA system using chip waveform 1 is approximately twice its chip rate as pointed out in [109].

The average P_b at $E_b/N_0 = 15$ dB and $K=10$ users for different PDP decay factors is shown in Figure 5-13. In this example, $J=3$ and $N=32$ for DS-CDMA with our sinusoidal chip waveform, and the processing gains of the DS-CDMA schemes with chip waveforms 1 and 2 are respectively, $N_{chip1}=40$, $N_{chip2}=80$. The channel is a dispersive channel with $m=1$, and $B_c=2R_c/5$, which implies the number of channel taps for the DS-CDMA with sinusoidal chip waveform, and the referenced chip waveforms are respectively, 5 and 12. The decay factor is varying from 0 to 2. We assume the receiver is capable of capturing all the channel taps. The results shown in Figure 5-13 indicate that as the decay factor increases, the average P_b of all three system increases. This agrees with the fact that RAKE has the best performance for a uniform PDP. The DS-CDMA scheme with our sinusoidal chip waveform outperforms the reference systems when $\eta > 0.5$, and the performance difference between the proposed system and the

referenced systems increases as η increases. This matches the observation for multicarrier case.

Figure 5-13: Average P_b vs. decay factor η for 5-user systems over Nakagami- m dispersive channel with $m=1$, and $N=32$, $J=3$, $H=5$ for the single carrier SW-SSG-MT-DS-CDMA and $N_{chip1}=40$, $N_{chip2}=80$, $H_{Ref}=25$ for the referenced schemes

Next, the BER performance as a function of Nakagami- m factor is shown in Figure 5-14, where the systems are the same as in the previous example. The decay factors are $\eta=0.5$ and $\eta=2$, and we allow the fading factor m to range from $\frac{1}{2}$ to 5. From this figure we can observe that the average P_b of the three systems decreases as the value of m increases, and that DS-CDMA with our sinusoidal chip waveform outperforms the reference schemes when $\eta=2$ and $m<4$. As we know, diversity is used to combat channel fading. However as the value of m increases, i.e., the channel is less severely faded, hence the MAI has a more significant effect on the performance degradation than fading

does (there is no inter-subcarrier interference for this single carrier case). Since the processing gain helps the DS-SS signal to suppress the MAI, the larger processing gains of the two referenced schemes are thus the reason for their better BER performance than the single carrier SW-SSG-MT-DS-CDMA scheme in better channel conditions. This example, as well as the previous example, shows that the proposed scheme can be used in the environments where the wireless channel is severely faded, i.e., the PDP decay factor η is larger than 0.5 and the m factor has a value less than 4.

Figure 5-14: Average P_b vs fading parameter m for 10-user systems over Nakagami- m dispersive channel, $E_b/N_0=15$ dB, $N=32$, $J=3$, $H=5$ for the single carrier SW-SSG-MT-DS-CDMA scheme and $N_{chip1}=40$, $N_{chip2}=80$, $H_{Ref}=15$ for the referenced schemes

5.4 Conclusion

In this chapter, we investigated the BER performance of SSG-MT-DS-CDMA with sinusoidal chip waveforms for increased diversity in dispersive fading environments. The sinusoidal chip waveform is able to shape the spectrum of each subcarrier of SSG-MT-DS-CDMA to have two distinct sidebands. With properly selected chip waveform parameter, the two sideband signals undergo independent fading, which provides diversity gain to the system to combat channel fading. It was shown in this chapter that with properly selected parameters, i.e., the chip waveform parameter and the intersubcarrier spacing, the SSG-MT-DS-CDMA signal sinusoidal chip waveforms outperforms the conventional MC-DS-CDMA scheme that uses rectangular chip waveforms for multiple subcarrier cases. For single subcarrier systems, a special case of our sinusoidal chip SSG-MT-DS-CDMA scheme has BER performance better than that of DS-CDMA schemes using either the rectangular chip waveform or chip waveforms proposed by [109] for the severely faded environments i.e., for channels with m factor less than 4 and PDP decay factor greater than 0.5.

Chapter 6. Conclusions and Future Work

In this chapter, we summarize the research work performed in this dissertation, and suggest several items of future work that could be pursued.

6.1 Summary of Research

In this dissertation, we proposed a new modulation scheme, which is a combination of multicarrier and direct sequence spread spectrum schemes. This new modulation scheme is called spectrally-shaped generalized multitone direct-sequence spread spectrum (SSG-MT-DS-SS) because it is a generalized multicarrier spread spectrum modulation scheme, and this generalization provides the spectral shaping capability. It was shown in this dissertation that the SSG-MT-DS-SS scheme can reduce the spectral sidelobe levels of the multicarrier spread spectrum signal's power spectrum without using pulse shape filters. This is achieved via allowing all the parameters of each subcarrier of SSG-MT-DS-SS to be adjustable. By adjusting the parameters of each subcarrier, we can concentrate more energy in the spectral mainlobe, and reduce the sidelobe levels. The "adjustment" specifically means that we do not constrain the parameters of each subcarrier to be identical, as in the conventional MT-DS-SS approach; rather, they can take almost arbitrary values. The parameters we can adjust for SSG-MT-DS-SS are the bit energy, data rate, chip rate (or processing gain), and frequency separation between adjacent subcarriers. We can also of course change the total number of subcarriers. In addition to sidelobe reduction, by varying the parameters of each subcarrier we can shape the power spectrum of transmitted signal.

In this dissertation we investigated the spectral shaping mechanism and capabilities of SSG-MT-DS-SS. We provided a general mathematical expression for the PSD of SSG-MT-DS-SS. With this general expression, we can find the PSD of SSG-MT-DS-SS with any arbitrary parameter set, i.e., with arbitrary values of data rate, processing gain, bit energy, and carrier frequency on each subcarrier. This PSD is a sum of “sinc” functions, whose actual form versus frequency must generally be obtained numerically. Based on this general expression, we developed a numerical method that is able to select the parameter set of SSG-MT-DS-SS by “quasi-exhaustive” search over the ranges of possible parameter sets, according to some given design requirements. These requirements may include mainlobe bandwidth, sidelobe levels, and a more complex set of spectral limits, such as a spectrum mask. We concluded that to substantially reduce the sidelobe levels of SSG-MT-DS-SS we need to concentrate energy in the mainlobe. This can be done by increasing the chip energy of the central subcarriers via either increasing the bit energies of these subcarriers or decreasing their processing gains.

We derived the general form of the bit error probability of single user SSG-MT-DS-SS in an AWGN channel. We first specified the condition under which any two subcarriers of SSG-MT-DS-SS are orthogonal to each other (in the AWGN channel). When the orthogonality condition is not satisfied, the SSG-MT-DS-SS suffers from inter-subcarrier same user interference (IS-SUI). We investigated the statistics of IS-SUI, and, by assuming it is a *Gaussian* disturbance, we were able to provide the general form of the bit error probability for the single user SSG-MT-DS-SS signal in the AWGN channel, with arbitrary parameters. The condition for the validity of the *Gaussian* approximation

was also provided. This mild condition is that the processing gain of each SSG-MT-DS-SS subcarrier must be greater than 2.

In addition to evaluating the single user SSG-MT-DS-SS performance in the AWGN channel, we also investigated its bit error probability in dispersive channels for multiple user settings. The channel we employed is a time dispersive channel, represented using the common tapped delay line model, with the amplitude of each channel tap modeled by the Nakagami- m distribution, and the phase of each tap uniformly distributed in $[0, 2\pi)$. With an alternative form of the Gaussian Q function, and the moment generating function of the Nakagami- m distribution, we were able to derive a closed form approximation for the BER of any arbitrary user in this dispersive Nakagami- m channel, as a function of the number of channel taps and their fading severity (m -factor), and the number of user signals present. The SSG-MT-DS-SS signal has BER performance comparable to that of conventional MT-DS-SS, except when significant spectrum shaping is attained by reducing the SSG-MT-DS-SS processing gains by more than a factor of two lower than that of conventional; in this case these lower SSG-MT processing gains translate to less immunity from all impairments—ISI, IS-SUI, MUI. The relative performance loss could be regained at the expense of a small expansion in bandwidth, while still retaining the spectral shaping features.

A large PAPR value is a potential major drawback of a multicarrier scheme. We investigated the PAPR of SSG-MT-DS-SS in this dissertation, and showed that the conventional multicarrier transmission schemes whose parameters are the same for all subcarriers have larger PAPR values than do properly designed SSG-MT-DS-SS schemes. In addition, with simulations, we showed that such properly designed SSG-

MT-DS-SS signals have smaller probabilities of reaching given PAPR values than do conventional MT-DS-SS signals. The SSG-MT-DS-SS signals attain lower PAPR values naturally, by virtue of unequal parameters on the subcarriers.

In this dissertation, we showed that SSG-MT-DS-SS has its best error performance when the bit energies of all subcarriers are identical. This, however, constrains our capability to shape the spectrum. Some times we do want the bit energies of each subcarrier to be different in order to gain more freedom to shape the spectrum. To improve the error performance in this (unequal bit energy) case we applied error correction codes. We derived the expressions for bit error probability of Hamming codes with different code symbol energies. This work can be used for SSG-MT-DS-SS when code symbols are transmitted through subcarriers with unequal bit energies. The BER expressions for such unequal symbol energy conditions are, we believe, new.

To shape the spectrum of SSG-MT-DS-SS, we allow its parameters to be adjustable. Thus, the energy distribution in the frequency domain is altered. We also showed that the spectrum of SSG-MT-DS-SS can be further shaped by using line codes to encode the chips of each subcarrier. Since the power spectra of the line codes are usually DC-free, we showed in this dissertation that with line codes used for each individual subcarrier, the power spectrum of the SSG-MT-DS-SS signal has a relatively small value around DC. The line codes also provided additional flexibility in spectral shapes.

Another way to alter the energy distribution in the frequency domain is to use chip waveforms other than rectangular chip waveforms. We investigated the use of sinusoidal chip waveforms to improve the error performance of SSG-MT-DS-SS. In this

portion of our work, all subcarriers employ the same data rate and chip rate, and the spectrum of each subcarrier is shaped by the sinusoidal chip waveform such that each subcarrier has two distinct spectral lobes from two sidebands. The sinusoidal chip waveform is properly selected such that the frequency spacing between these two lobes is wider than the coherence bandwidth of the channel to ensure that the two sideband signals undergo uncorrelated fading. The two uncorrelated faded sideband signals are then combined at the receiver to improve the error performance of SSG-MT-DS-SS in the dispersive fading channel.

6.2 Future Work

General Improvements and Extensions

In this dissertation, the same modulation, BPSK, is used for each subcarrier of SSG-MT-DS-SS. We could apply different modulation schemes on each subcarrier. For example, the use of higher-order PSK modulations, or QAM modulations would increase throughput, with the usual trade of degraded energy efficiency. This may give us additional flexibility to shape the spectra of the SSG-MT-DS-SS signals, by allowing unequal energies for different modulations on the different subcarriers.

In Chapter 3, we evaluated the BER performance of Hamming codes with different code symbol energies, and extended this method to be used in our SSG-MT-DS-SS schemes. The use of simple Hamming code does improve the BER performance of the SSG-MT-DS-SS systems, as expected. However, in practice we may need much more powerful FEC coding schemes to combat the performance degradation incurred because of more extreme spectral shaping, i.e., to compensate for the loss of SNR on

some of the subcarriers due to their lower bit energies. This is analogous to unequal error protection (UEP) coding schemes [111], [112]. We may want to provide “extra” protection on those subcarriers with larger bit error probabilities. Thus, the search for, or design of, new UEP codes for SSG-MT-DS-SS is another topic for future work.

In this dissertation, we placed some restrictions on the parameters of each SSG-MT-DS-SS subcarrier. For example, we restricted the data rate ratio between any two subcarriers to be an integer or a reciprocal of an integer, and a similar restriction was placed on the chip rate ratios between two subcarriers. With these restrictions, we can separate the combinations of data rates and chip rates of any two subcarriers into nine distinct cases, and derive the closed form BER expressions of SSG-MT-DS-SS. Further investigation on the spectral shaping capability of SSG-MT-DS-SS and its BER expression without these restrictions may also be of interest for future work. Although only modest changes from our results would be expected, this would further generalize the schemes.

Also in this dissertation, we used simple receiver techniques, i.e., a simple correlator bank for SSG-MT-DS-SS in the AWGN channel and a RAKE for each subcarrier in the dispersive channel. We could evaluate the performance of SSG-MT-DS-SS with some advanced receiver techniques for future work. Specifically, since the SSG-MT-DS-SS approach transmits different data on each subcarrier, when subcarriers interfere with each other, we can view the interference from different subcarriers analogously to that from different users; in other words, we can consider the interfering subcarriers as “virtual users.” Thus, the techniques of MUD—applied as multi-carrier detection—can be applied to SSG-MT-DS-SS even for the single user setting. For

instance, we can apply interference canceling (IC) schemes (e.g., [28], [29]) to cancel the effect of the interference from other subcarriers, and improve the BER performance of SSG-MT-DS-SS, or we can also jointly detect the data on all subcarriers in the ML way as in [27].

SSG-MT-DS-SS with Sinusoidal Chip Waveforms and Line Codes

In this dissertation, we used sinusoidal chip waveforms to improve the error performance of SSG-MT-DS-SS. The SSG-MT-DS-SS systems investigated in this portion of the work use the same parameters for each subcarrier. Specifically, the data rate, processing gain, bit energy, and frequency separation between adjacent subcarriers are the same for all subcarriers. Allowing different parameters, e.g., data rate and chip rate, on each subcarrier, is one topic for future work to further generalize the SSG-MT-DS-SS schemes with sinusoidal chip waveforms. This generalization, of course, requires us to again separate the combinations of data rates and chip rates for any two subcarriers into nine cases, and would consequently require that we investigate the statistics of the interference from different/same subcarriers for all these nine cases, as we have done in chapters 3 and 4. In addition to allowing different data rates and chip rates on each subcarrier, we could further generalize the SSG-MT-DS-SS signal to have different chip waveforms on each subcarrier.

These chip waveforms could include the half-sine, raised cosine, and Blackman [104] etc. We could use any one of these chip waveforms on any subcarrier of SSG-MT-DS-SS. This will naturally increase the complexity of the system, since we have to synthesize the chip waveform for each subcarrier individually. However, since these chip

waveforms all have some spectral shaping capabilities [104], their use could further extend the attainable SSG-MT-DS-SS spectral shaping. To analyze the BER performance of an SSG-MT-DS-SS system with different chip waveforms used on the subcarriers, we would follow the methods given in chapters 3 and 4, and analyze the statistics of the interference for all the nine (or more) cases with different chip waveform combinations.

In Chapter 3, we illustrated the spectral shaping obtainable in SSG-MT-DS-SS with the same line codes used on each subcarrier by studying the Manchester and Miller line codes as two examples. Future work for further exploration of the spectral shaping capability of SSG-MT-DS-SS could test other line coding schemes, such as the Alternate Mark Inversion (AMI) [109]. As with our suggestions on chip waveforms, we could also investigate the spectral shaping of SSG-MT-DS-SS with different line codes on each subcarrier.

Processing Extensions

In Chapter 3, we provided a numerical method to select the parameter set of SSG-MT-DS-SS from some possible parameter sets according to several criteria. The numerical method is based on “quasi-exhaustively” searching over the possible parameter sets, which are limited by some boundary conditions, i.e., the maximum data rate, processing gain, bit energy on a given subcarrier and the maximum frequency separation between two adjacent subcarriers. In addition, since the bit energies and intersubcarrier frequency spacing can take any real values, we have to sample these real values for our computer algorithm. One area for future work would be to produce some mathematical

guidance on how finely to sample these real values, analogous to how the sampling theorem does.

In Chapter 3 and Chapter 4, we showed that the SSG-MT-DS-SS system reaches its best BER performance when the bit energies of all subcarriers are identical for both AWGN and dispersive channels. We also examined the effects of the bit energy distribution by showing for a three-subcarrier system the average bit error probability vs. the bit energy ratio between the 2^{nd} and 1^{st} subcarriers (the bit energies of the 1^{st} and 3^{rd} subcarriers are identical for our example 3 subcarrier system). Figures 3-11, 3-13, and 4-5 all confirmed that the SSG-MT-DS-SS system has the best error performance when all subcarriers have the identical bit energies. Upon further examination of these examples, we can observe that the error performance of SSG-MT-DS-SS has smaller variation when the bit energy ratio between the 2^{nd} and 1^{st} subcarriers is in the range [0.75 1.5] than it does when the bit energy ratio lies outside this range. Further investigation on the bit error probability as a function of bit energy distribution for different bit energies on all subcarriers might be able to give us some guidance on additional parameter selections for SSG-MT-DS-SS. That is, instead of making the bit energies identical for all subcarriers, we could select the bit energy vector such that the bit energy ratios between any two subcarriers lie within the above range. In this case, we can enlarge the parameter selection space without sacrificing much in terms of error performance.

Finally, in this dissertation, we spread the spectrum in the “time domain.” This is in analogy to the MC/MT-DS-CDMA schemes. Another area of future work would be to spread the spectrum in the “frequency domain” as does the MC-CDMA scheme. For one implementation of MC-CDMA, the same data bit is transmitted through orthogonal

subcarriers. For each subcarrier, the data bit is multiplied by one chip (of value either ± 1) from the spreading code (usually an orthogonal code) [37]. Thus, the chips transmitted through orthogonal subcarriers are essentially of infinite duration, and the number of orthogonal subcarriers is determined by the processing gain value of the spreading code. For some applications, the input data is first serial to parallel converted in order to ensure each subcarrier will undergo flat fading. These parallel data streams are then DS spread using constant chip values (different chips across subcarriers) and transmitted. Like SSG-MT-DS-SS, we could generalize the serial to parallel conversion such that after the generalized conversion, the parallel data streams are of different data rates. In addition, we could vary the number of orthogonal subcarriers for each parallel data stream. Thus, by adjusting the data rates and the number of subcarriers of each parallel data stream, we could obtain spectral shaping capabilities beyond that of the conventional MC-CDMA scheme.

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Appendix Matlab Programs

```

%%%-----
%%%simulation the BER of single user SSG in AWGN channel    %%%
%%%M: # of subcarriers  M=3                                %%%
%%%Ns: vector of the processing gain of each subcarrier    %%%
%%%df: vector of freq offset of each subcarrier            %%%
%%%k:  data rate ratio of each subcarrier                  %%%
%%%d:  bit energy ratio of each subcarrier                 %%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clear all; clc;
Ne=30;                                %%% # of errors expected
%%%parameters of conventional MT-DS-SS
M=3;
Nc=8;                                %%%processing gain
%%%parameters of S-MT-DS-SS
d =[1 ,2,1];                          %bit energy ratio
k =[1 ,2,1];                          %data rate ratio
rn=[1 ,1/4,1];                        %processing gain ratio
fs=[-1,0,1];                          %carrier frequency of each subcarrier
fc=[-1,0,1];
Ns=Nc*rn;
Nc=Nc(1);

Rbs=k/min(k);
Rcs=Rbs.*Ns;
Tcs=1./Rcs;
fsTc=Tcs/min(Tcs)*2;                  % 2 samples per chip

%%%parameters of  conventional MT-DSSS
Rbc=sum(Rbs)/M;                       %data rate of conventional MT-DS-SS
Tcc=1/Rbc/Nc;                         %chip duration
%%%IS-SUI of S-MT-DS-SS and conventional MT-DS-SS
beta=zeros(M,M-1);
betac=zeros(M,M-1);
sub=[1:M];                            %vector of subcarriers
for i=1:M
    residu=find(sub-i);               %sub other than i
    for ii=1:M-1
        j=residu(ii);
        beta(i,j) =betaij(Rbs(i),Rbs(j),Ns(i),Ns(j),d(i),d(j),
                           Rbs(1)*abs(fs(i)-fs(j)));
        betac(i,j)=betaij(Rbc,Rbc,Nc,Nc,1,1,Rbc*abs(fc(i)-fc(j)));
    end
end
I=sum(beta,2);
I=I';
%%%IS-SUI of conventional MT-DS-SS
Ic=sum(betac,2);
Ic=Ic';

EbN0=[0:10];    %in dB
len=length(EbN0);

```

```

pb_theo=zeros(M,len);
pb_awgn=zeros(M,len);
pbc_theo=zeros(M,len);
pb_theo_total=zeros(1,len);
pbc_theo_total=zeros(1,len);
pb_awgn_total=zeros(1,len);
pb_simu=zeros(M,len);
pb_simu_total=zeros(1,len);
N0=1;
for ii=1:length(EbN0)
    ii
    SNR=10^(EbN0(ii)/10);
    %%%
    Ebc=SNR*N0; %%%Eb of each subcarrier of conventional MT-DS-SS
    %%% SSG and conventional MT has the same tx power
    tEbs=M*Rbc*Ebc/sum(Rbs.*d);
    Ebs=d*tEbs; %%%Eb of SSG
    N0Esc=N0/Ebc/2;
    N0Ess=N0./Ebs/2;
    N0Es=1/SNR/2;
    %%% the bit error of each subcarrier
    for ch=1:M
        pb_theo(ch,ii)=qfunc(sqrt(1/(N0Ess(ch)+I(ch))));
        pb_awgn(ch,ii)=qfunc(sqrt(SNR));
        pbc_theo(ch,ii)=qfunc(sqrt(1/(N0Esc+Ic(ch))));
    end
    %%%the bit error of SSG
    pbc_theo_total(ii)=sum(pbc_theo(:,ii))/M;
    pb_theo_total(ii)=sum(pb_theo(:,ii)'.*Rbs)/sum(Rbs);
    pb_awgn_total(ii)=qfunc(sqrt(1/N0Esc));
    %%%find the minimal bits in simulation
    Nb=Ne./pb_theo(:,ii)';
    bpsimu=5e2; %%%bits per one simulaiton
    %bpsimu=1;
    Nb=ceil(Nb./k);
    minNb=max(Nb);
    Nb=ceil(minNb*Rbs);
    A=sqrt(Ebs./(fsTc.*Ns)); %amplitude of each chip
    Nsimu=ceil(minNb/bpsimu);
    error=zeros(1,M);
    for bit=1:Nsimu %simulation one bit a time
        sub=zeros(M,bpsimu*Ns(1)*fsTc(1));
        for ch=1:M
            %normalize the carrier frequency by the bit rate
            fcarr=fs-fs(ch);
            fcarrTb=fcarr./k;
            for jj=1:M
                b=BitGen(bpsimu*Rbs(jj)); %generate data bits
                b=b*A(jj); %amplitud of each chip
                bup(jj,:)=OverN(b,Ns(jj)*fsTc(jj)); %over sample
                c=BitGen(Ns(jj)); %generate spreading code
                cup=OverN(c,fsTc(jj));
                %generate carrier
                T=[0:Ns(jj)*fsTc(jj)-1]/Ns(jj)/fsTc(jj);
                carr=cos(2*pi*fcarrTb(jj)*

```

```

        [0:Ns(jj)*fsTc(jj)-1]/Ns(jj)/fsTc(jj));
    Sv=cup.*carr;
    Ccode(jj,:)=repmat(Sv,1,bpsimu*Rbs(jj));
    %DS spread data for subcarrier k
    sub(jj,:)=bup(jj,:).*Ccode(jj,:);
end
tx=sum(sub,1); %tx signal
%%demodulate subcarrier ch
VN=N0/2;
n=sqrt(VN)*randn(1,length(tx)); %AWGN
rx=n+tx; %recevied signal
shat=rx.*Ccode(ch,:); %demodulate
shat=reshape(shat,Ns(ch)*fsTc(ch),bpsimu*Rbs(ch));
integrate=sum(shat);
dec=integrate>0; % hard decison
bsent=reshape(bup(ch,:),fsTc(ch)*Ns(ch),bpsimu*Rbs(ch));
bsent=bsent(1,:)/A(ch);
bsent=(bsent+1)/2;
error(ch)=error(ch)+sum(abs(bsent-dec));
end

end %end simulation of every bits
pb_simu(:,ii)=(error./Nb)';
pb_simu_total(ii)=sum(error)/sum(Nb);
error=zeros(1,M);
end
figure; %plot the BER curve for SSG
semilogy(EbN0,pb_simu_total,'x',EbN0,pb_theo_total,'-
.',EbN0,pbc_theo_total,'g--',EbN0,pb_awgn_total,'r:', 'LineWidth',2);
str=strcat('Total Pb for Rb2/Rb1=',num2str(k), ' Eb2/Eb1=',num2str(d), '
delta f=',num2str(fs), 'Rb1');
%title(str);
xlabel('Eb/N0 (dB)'); ylabel('Pb');
legend('simulated Pb ', 'Theoretical Pb', 'conventional MT-DS-SS');
grid;

%%%-----
%%%simulate the Multi-user SSG in Dispersive Channel
%%%%%Wenhui Xiong
%%%%%last updated on 06/20/2005
clear all; clc;
close all;
K=1; %%%% # of users
M=3; %%%% # of subcarriers
%%Parameters of Desired user
Rb_ref=[1,2,1]; %%% data rate of referenced user
Eb_ref=[1,1,1]; %%% bit energy of reference user
N_ref=[1,1/4,1]*8; %%% processing gain of referenced user
df_ref=Rb_ref(1);
f_ref=[-1,0,1]*df_ref; %%% carrier frequency of each subcarrier

%Paramters of Interference user
Rb_int=[1,2,1]; %%% data rate of referenced user
Eb_int=[1,1,1]; %%% bit energy of reference user

```

```

N_int=N_ref;                %% processing gain of referenced user
df_int=Rb_ref(1);
f_int=[-1,0,1]*df_ref;    %% carrier frequency of each subcarrier
Lc=5;                      %% # of taps for conventional MT
eta=2;                     %% decay factor of PDP
m=1;                      %% m factor (m=1 Rayleigh)
%%%%conventional MT which has the same data rate and tx power as SSG
Rbc=sum(Rb_ref)/M;
Ebc=sum(Rb_ref.*Eb_ref)/Rbc/M;
Nc=N_ref(1);
%%find out the corresponding channel for SSG system
Bc=Rbc*Nc/(Lc-1);
Lv=TapCalcu(N_ref,Rb_ref,Bc);    %% # of taps for reference user
Lj=TapCalcu(N_int,Rb_int,Bc);    %% # of taps for interference users
Lmax=Lc;                      %% # of RAKE fingers for SSG
%%%%calculate the data duration, chip duration of SSG
Tb_ref=1./Rb_ref;
Tc_ref=Tb_ref./N_ref;
Tb_int=1./Rb_int;
Tc_int=Tb_int./N_int;
EbN0=[0:10];
SNR_av=10.^(EbN0/10);
Eb_av=1;
N0=Eb_av./SNR_av;
%%%%find out the analytical BER of SSG and conventional MT
for snr=1:length(EbN0)
    tSNR=SNR_av(snr)*sum(Rb_ref)/sum(Eb_ref.*Rb_ref);
    SNR_ref=tSNR*Eb_ref;        %%SNR on each subcarrier
    tSNR=SNR_av(snr)*sum(Rb_int)/sum(Eb_int.*Rb_int);
    SNR_int=tSNR*Eb_int;
    Pbs_sub_theo(:,snr)=SSG_BitError(Lc,Lmax,eta,m,Bc,SNR_ref,Rb_ref,
                                     N_ref,f_ref,K,SNR_int,Rb_int,N_int,f_int);
    SNRc=SNR_av(snr)*ones(1,M);
    Rbc_ref=Rbc*ones(1,M);
    Nc_ref=Nc*ones(1,M);
    fc_ref=[-Rbc,0,Rbc];
    Pbc_sub_theo(:,snr)=SSG_BitError(Lc,Lmax,eta,m,Bc,SNRc,Rbc_ref,
                                     Nc_ref,fc_ref,K,SNRc,Rbc_ref,Nc_ref,f_ref);
end
Pbs_theo_total=Rb_ref*Pbs_sub_theo/sum(Rb_ref);
Pbc_theo_total=sum(Pbc_sub_theo,1)/M;
Pb_AWGN=qfunc(sqrt(2*SNR_av));

%%%%start simulation
Pbs_simu_total=zeros(1,length(SNR_av));
Pbs_simu=zeros(M,length(SNR_av));
%%%%%%%%start simulation%%%%%%%%
%%find the Eb on each subcarrier of desired user and interference user
tEb_ref=sum(Rb_ref)*Eb_av/sum(Rb_ref.*Eb_ref);
Eb_ref=tEb_ref*Eb_ref;
tEb_int=sum(Rb_int)*Eb_av/sum(Rb_int.*Eb_int);
Eb_int=tEb_int*Eb_int;
Rc_ref=Rb_ref.*N_ref;
Rc_int=Rb_int.*N_int;
fs=max(max(Rc_ref),max(Rc_int))*4;  %% sample frequency, set to be 4

```



```

fsTc_int=fs./(Rc_int);
fsTc_ref=fs./(Rc_ref);

Rb_total=sum(Rb_ref);
Tb=1/Rb_total;
fsTb=Tb*fs;
blk=10;
bpsimu=blk*sum(Rb_ref);
Ne=20;
clear sub;
clear i;
clear j;
for ii=1:length(EbN0)
    EbN0_simu=EbN0(ii)
    error=zeros(1,M);
    Nb=zeros(1,M);
    tx_int=zeros(K-1,blk*Rb_int(1)*N_int(1)*fsTc_int(1));
    error=zeros(1,M);
    Nb=zeros(1,M);
    while (min(error)<Ne)
        %%generate the interference users' signal
        rand('state',sum(100*clock)); %%reset the random state
        for user=1:K-1
            sub=zeros(M,fsTc_int(1)*N_int(1)*blk*Rb_int(1));
            M_int=length(Eb_int);
            for ch=1:M_int
                %% the average power of each tap
                APower=AvgPower(eta,Lj(ch),'Exp');
                %% tap amplitudes
                Gainj=RdnGen(APower,'Rayleigh');
                phase=rand(1,Lj(ch))*pi*2; %% tap phases
                %% generate random bits for one subcarrier
                b=BitGen(blk*Rb_int(ch));
                bup=OverN(b,N_int(ch)*fsTc_int(ch)); %% over sample
                %% generate the spreading code (long) for one
                %%subcarrier
                c=BitGen(N_int(ch));
                cup=OverN(c,fsTc_int(ch));
                %% over sample the spreading code
                Ccode= repmat(cup,1,blk*Rb_int(ch));
                tx=Ccode.*bup; %% DS spread
                %%normalize the power
                P_tx=sum(tx.^2);
                P_desire=blk*Rb_int(ch)*Eb_int(ch);
                tx=sqrt(P_desire/P_tx)*tx;
                tmp=exp(-j*2*pi*f_int(ch)*[0:fsTc_int(ch)*N_int(ch)-
                    1]/fsTc_int(ch)/N_int(ch));
                %% generate the carrier
                carr=repmat(tmp,1,blk*Rb_int(ch));
                tx=tx.*carr; %%up-convert to its carrier ferg
                % generate random delays for each tap
                Delay=rand(1,Lj(ch));
            end
        end
        error=error+abs(tx_int(tx_int==0));
    end
    Nb=Nb+abs(tx_int(tx_int==0));
end

```

```

Delay=sort(Delay, 'ascend');
fsTao=floor(Delay*fsTb);
%%%generate the output of tap delay channel
for tap=1:Lj(ch)
    sub(ch,:)=sub(ch,:)+Gainj(tap)*exp(-
        j*phase(tap))*rshift(tx,fsTao(tap));
end
end
%%%one interfering user
tx_int(user,:)=sum(sub,1);
end
%%% sum all the interfering user signal
interfer=sum(tx_int,1);          %%%interference signal
Li=Lv;
%%%generate the reference user's signal
for ch=1:M                      %%%index for detection
    sub=zeros(M,fsTc_int(1)*N_int(1)*blk*Rb_int(1));
    for chl=1:M
        APower=AvgPower(eta,Li(chl), 'Exp');
        Gain_ref=RdnGen(APower, 'Rayleigh');
        phase=rand(1,Li(chl))*pi*2;
        b=BitGen(blk*Rb_ref(chl));
        bup=OverN(b,N_ref(chl)*fsTc_ref(chl));
        c=BitGen(N_ref(ch));
        cup=OverN(c,fsTc_ref(ch));
        Ccode=repmat(cup,1,blk*Rb_ref(ch));
        tx_sub=Ccode.*bup;
        %%%normalize the power
        P_tx=sum(tx_sub.^2);
        P_desire=blk*Rb_ref(chl)*Eb_ref(chl);
        tx_sub=sqrt(P_desire/P_tx)*tx_sub;
        %%%up-convert to its carrier frequency
        tmp=exp(-j*2*pi*f_ref(chl)*[0:fsTc_ref(chl)*N_ref(chl)-
            1]/fsTc_ref(chl)/N_ref(chl));
        carr=repmat(tmp,1,blk*Rb_ref(chl));
        tx_sub=tx_sub.*carr;
        %%%generate the output of tap delay channel
        Delay=rand(1,Li(chl));
        Delay=sort(Delay, 'ascend');
        Delay=Delay-min(Delay);
        %%%shift the delay with respect to the 1st tap
        fsTao=floor(Delay*fsTb);
        for tap=1:Li(chl)
            sub(chl,:)=sub(chl,:)+Gain_ref(tap)*exp(-
                j*phase(tap))*rshift(tx_sub,fsTao(tap));
        end
        if (chl==ch)
            %%% record the send out bits, spreading code, delay,
            %%% phase and amplitude of each channel tap
            bsent=(b+1)/2;
            dCcode=Ccode;
            dCarr=carr;
            dfsTao=fsTao;
            dphase=phase;
            dGain=Gain_ref;
        end
    end
end

```

```

        end
    end %%% generate the signal of reference user
    clear tmp;
    tx=sum(sub,1);
    VN=N0(ii)/2;
    %AWGN
    n=sqrt(VN)*randn(1,length(tx))
        +j*sqrt(VN)*randn(1,length(tx));
    rx=n+tx+interfer; %received signal
    %make decision on subcarrier "ch" of reference user
    for tap=1:min(Li(ch),Lmax) %MRC for each tap
        %demodulate
        shat=lshift(rx,dfsTao(tap)).*dCcode
            .*conj(dCarr)*exp(j*dphase(tap))*dGain(tap);
        shat=reshape(shat,N_ref(ch)*fsTc_ref(ch),blk*Rb_ref(ch));
        integrate(tap,:)=sum(shat);
    end
    MRC=sum(integrate,1); %MRC output
    dec=real(MRC)>0; %hard decision
    %record # of errors
    error(ch)=error(ch)+sum(abs(bsent(2:blk*Rb_ref(ch)-1)
        -dec(2:blk*Rb_ref(ch)-1)));
    Nb(ch)=Nb(ch)+blk*Rb_ref(ch)-2; %record # of bits send out
    clear integrate;
end
end
Pbs_simu(:,ii)=(error./Nb)';
Pbs_simu_total(ii)=sum(error)/sum(Nb);
end%%end of SNR loop
figure; %plot the BER curve for SSG
semilogy(EbN0,Pbs_simu_total,'-x',EbN0,Pbs_theo_total,
    'k-.',EbN0,Pbc_theo_total,'g-',EbN0,Pb_AWGN,'r:', 'LineWidth',2);
xlabel('Eb/N0 (dB)'); ylabel('Pb');
legend('simulated Pb ', 'Theoretical Pb ', 'Conventional MT-DS-
    SS', 'AWGN');
grid; axis([min(EbN0),max(EbN0),1e-6,1e0]);

%%-----
%%Simulate SW-SSG with sinusoidal chip wavefrom and LSB USB combining
clear all; clc; close all;
M=3; % # of subcarriers
N=32; % PG
K=10; % # of users
%%channel
Lc=5; % # of channel taps for each
sideband
eta=2; %decay factor
m=1; %m factor
Lmax=Lc; % # of RAKE fingers
Index=3; %sinusoidal frequency
lambda=1; %frequency separation, normalized by Rc
dfTb=N*lambda;
%%%%%%
EbN0=[0:10];
SNR=10.^(EbN0/10);

```

```

Eb=1;
N0=Eb./SNR;
%%%find the analytical Pb
for snr=1:length(EbN0)
    Pbn_sub_theo(:,snr) =BitErrorLSBUSB(Index,
                                         dfTb,Lc,Lmax,eta,m,SNR(snr),M,N,K);
end
PbDW_theo=sum(Pbn_sub_theo,1)/M;
Pb_simu=zeros(M,length(SNR));
Pb_simu_total=zeros(1,length(SNR));
Ne=50; % # of errors expected
blk=10; % # of bits in each simulation run
fsTc=16; % # of sample per chips
fsTb=fsTc*N;
t=[0:fsTc-1]/fsTc;
%%%generate the chip waveform, and find its Hilbert transform
for i=1:N
    ChipWave(i,:)=sqrt(2)*sin(Index*pi*t);
end
ChipWaveI=reshape(ChipWave',1,fsTc*N);
ChipWaveQ=imag(hilbert(ChipWaveI));

%%%carrier frequency of each subcarrier: freq normalized by the chip
rate
fc=[0:M-1]-(M-1)/2;
fc=fc*lambda;
for i=1:M
    tmp=exp(-j*2*pi*fc(i)*[0:fsTc-1]/fsTc);
    carr(i,:)=repmat(tmp,1,blk*N); %generate the carrier of each sub
end
%%%start simulation
for ii=1:length(EbN0)
    EbN0_simu=EbN0(ii)
    error=zeros(1,M);
    Nb=zeros(1,M);
    while (min(error)<Ne)
        %generate the interference users' signal
        rand('state',sum(100*clock)); %reset the random state
        for user=1:K-1
            sub=zeros(M,fsTc*N*blk);
            for ch=1:M
                subL=zeros(1,blk*fsTc*N);
                subU=subL;
                b=BitGen(blk); %generate the data bits
                bup=OverN(b,N*fsTc); %over sample
                c=BitGen(N); %generate the spreading code
                cup=OverN(c,fsTc); %over sample

                cI=cup.*ChipWaveI; %spreading waveform of I
                cQ=cup.*ChipWaveQ; %spreading waveform of Q
                cLSB=cI+j*cQ; %spreading waveform of LSB
                cUSB=cI-j*cQ; %spreading waveform of USB
                cLSB=repmat(cLSB,1,blk);
                cUSB=repmat(cUSB,1,blk);
            end
        end
    end
end

```

```

LSB=bup.*cLSB;          %%%spread the LSB and USB
USB=bup.*cUSB;
%%%normalize the power
P_tx=sum((LSB+USB).^2);
P_desire=blk*Eb;
LSB=sqrt(P_desire/P_tx)*LSB;
USB=sqrt(P_desire/P_tx)*USB;

USB=USB.*carr(ch,:);
LSB=LSB.*carr(ch,:);    %%%upconvert to carrier freq
%%%LSB and USB has independent fading
Delay=rand(1,Lc);        %%%delay of each channel tap
Delay=sort(Delay,'ascend');
fsTao=floor(Delay*fsTb);
phase=rand(1,Lc)*pi*2;   %%%phase of each channel tap
APower=AvgPower(eta,Lc,'Exp');
Gainj=RdnGen(APower,'Rayleigh');%%% tap amplitudes
for tap=1:Lc
    subU=subU+Gainj(tap)*
        exp(-j*phase(tap))*rshift(USB,fsTao(tap));
end
Delay=rand(1,Lc);
Delay=sort(Delay,'ascend');
fsTao=floor(Delay*fsTb);
phase=rand(1,Lc)*pi*2;
APower=AvgPower(eta,Lc,'Exp');
Gainj=RdnGen(APower,'Rayleigh');
for tap=1:Lc
    subL=subL+Gainj(tap)
        *exp(-j*phase(tap))*rshift(LSB,fsTao(tap));
end
sub_int(ch,:)=subL+subU;
end
%%%interference signal of one interfering user
tx_int(user,:)=sum(sub_int,1);
end
interfer=sum(tx_int,1);    %%%interference signal
%%%generate the reference user's signal
for ch=1:M                %%%index for detection
    for ch1=1:M
        subL=zeros(1,blk*fsTc*N);
        subU=subL;
        b=BitGen(blk);
        bup=OverN(b,N*fsTc);
        c=BitGen(N);
        cup=OverN(c,fsTc);
        cI=cup.*ChipWaveI;
        cQ=cup.*ChipWaveQ;
        cLSB=cI+j*cQ;
        cUSB=cI-j*cQ;
        cLSB=repmat(cLSB,1,blk);
        cUSB=repmat(cUSB,1,blk);
        LSB=bup.*cLSB;
        USB=bup.*cUSB;
        %%%normalize the power

```

```

P_tx=sum((LSB+USB).^2);
P_desire=blk*Eb;
LSB=sqrt(P_desire/P_tx)*LSB;
USB=sqrt(P_desire/P_tx)*USB;
USB=USB.*carr(ch1,:);
LSB=LSB.*carr(ch1,:);      %%%upconvert to carrier freq
Delay=rand(1,Lc);
Delay=sort(Delay,'ascend');
fsTaoU=floor(Delay*fsTb);
phaseU=rand(1,Lc)*pi*2;
APower=AvgPower(eta,Lc,'Exp');
GainjU=RdnGen(APower,'Rayleigh');
for tap=1:Lc
    subU=subU+GainjU(tap)
        *exp(-j*phaseU(tap))*rshift(USB,fsTaoU(tap));
end
%%%%%output of LSB
%%generate the output of tap delay channel
Delay=rand(1,Lc);
Delay=sort(Delay,'ascend');
fsTaoL=floor(Delay*fsTb);
phaseL=rand(1,Lc)*pi*2;
APower=AvgPower(eta,Lc,'Exp');
GainjL=RdnGen(APower,'Rayleigh');
for tap=1:Lc
    subL=subL+GainjL(tap)
        *exp(-j*phaseL(tap))*rshift(LSB,fsTaoL(tap));
end
sub(ch1,:)=subL+subU;      %%%tx signal
%%record the send out bits, spreading waveform, delay and
%%phases of all channel taps for both LSB and USB
if (ch1==ch)
    bsent=(b+1)/2;
    dcLSB=cLSB;
    dcUSB=cUSB;
    dcarr=carr(ch,:);
    dfsTaoL=fsTaoL;
    dfsTaoU=fsTaoU;
    dphaseL=phaseL;
    dphaseU=phaseU;
    dGainjU=GainjU;
    dGainjL=GainjL;
end
end
clear tmp;
tx=sum(sub,1);
VN=N0(ii)/2;
n=sqrt(VN)*randn(1,length(tx))
    +j*sqrt(VN)*randn(1,length(tx));
rx=n+tx+interfer;          %received signal
%%detect 'ch' subcarrier
%%Rake for LSB
for tap=1:min(Lc,Lmax)      %MRC for each tap
    shat=lshift(rx,dfsTaoL(tap)).*conj(dcLSB)
        .*conj(dcarr)*exp(j*dphaseL(tap))*dGainjL(tap);
end

```

```

        shat=reshape(shat,N*fsTc,blk);
        integrate(tap,:)=sum(shat);
    end
    MRCL=sum(integrate,1);
    %%%%RAKE for USB
    for tap=1:min(Lc,Lmax)        %%%MRC for each tap
        shat=lshift(rx,dfsTaoU(tap)).*conj(dcUSB)
            .*conj(dcarr)*exp(j*dphaseU(tap))*dGainjU(tap);
        shat=reshape(shat,N*fsTc,blk);
        integrate(tap,:)=sum(shat);
    end
    MRCU=sum(integrate,1);
    dec=real(MRCU+MRCL)>0;        %%% combine LSB and USB RAKEs
    error(ch)=error(ch)+sum(abs(bsent(2:blk-1)-dec(2:blk-1)));
    Nb(ch)=Nb(ch)+blk-2;
end
end %end of one SNR simulation
Pb_simu(:,ii)=(error./Nb)';
Pb_simu_total(ii)=sum(error)/sum(Nb);
end%%end of SNR loop

figure;        %%%plot BER curve
semilogy(EbN0,PbDW_theo,'r-',EbN0,Pb_simu_total,'g-','LineWidth',2);
xlabel('Eb/N0 (dB)'); ylabel('Pb');
legend('Analytical','Simulated');
grid;
title('new program')

%%%% -----%%%
%%%%compute the variance of decsion metrics
%%%%normalized with Tbi*Ebi
function bij=betaij(Rbi,Rbj,Ni,Nj,Ebi,Ebj,dfij)
Tbi=1/Rbi;        %%%bit duration of each subcarrier
Tbj=1/Rbj;
Tci=1/Rbi/Ni;        %%%chip duration of each subcarrier
Tcj=1/Rbj/Nj;
gama=Tci/Tcj;        %%%gamma i,j in the paper
kij=max(Rbi,Rbj)/min(Rbi,Rbj);
phii=Ebj*Tci^2*sinc(dfij*Tci)^2/Tbj/(Tbi*Ebi);
%%phi function in the paper
phij=Ebj*Tcj^2*sinc(dfij*Tcj)^2/Tbj/(Tbi*Ebi);

if (Rbi==Rbj)&(gama==1)        %%%case 1
    if dfij==0
        I=phii*Ni;
    else
        I=phii*(Ni/2+sin(4*pi*dfij*Tci*Ni)/(4*sin(2*pi*dfij*Tci)));
    end
elseif (Rbi==Rbj)&(gama>1)        %%%case 2
    if dfij==0
        I=phij*Nj;
    else
        I=phij*(Nj/2+sin(4*pi*dfij*Tcj*Nj)/sin(2*pi*dfij*Tcj)/4);
    end
end

```

```

elseif (Rbi==Rbj)&(gama<1)      %%case 3
    if dfij==0
        I=phii*Ni;
    else
        I=phii*(Ni/2+sin(4*pi*dfij*Tci*Ni)/sin(2*pi*dfij*Tci)/4);
    end
elseif (Rbi<Rbj)&(gama==1)      %%case 4
    if dfij==0
        I=kij*phij*Nj;
    else
        I=kij*phij*(Nj/2+sin(4*pi*dfij*Tcj*Nj)/sin(2*pi*dfij*Tcj)/4);
    end
elseif (Rbi<Rbj)&(gama>1)      %%case 5
    if dfij==0
        I=kij*phij*Nj;
    else
        I=kij*phij*(Nj/2+sin(4*pi*dfij*Tcj*Nj)/sin(2*pi*dfij*Tcj)/4);
    end
elseif (Rbi<Rbj)&(gama<1)      %%case 6
    if dfij==0
        I=phii*Ni;
    else
        I=phii*(Ni/2+sin(4*pi*dfij*Tci*Ni)/sin(2*pi*dfij*Tci)/4);
    end

elseif (Rbi>Rbj)&(gama==1)      %%case 7
    if dfij==0
        I=phii*Ni;
    else
        I=phii*(Ni/2+sin(4*pi*dfij*Tci*Ni)/sin(2*pi*dfij*Tci)/4);
    end

elseif (Rbi>Rbj)&(gama>1)      %%case 8
    if dfij==0
        I=phij*Nj/kij;
    else
        I=phij*(Nj/2/kij+sin(4*pi*dfij*Tcj*Nj/kij)/sin(2*pi*dfij*Tcj)/4);
    end
elseif (Rbi>Rbj)&(gama<1)      %%case 9
    if dfij==0
        I=phii*Ni;
    else
        I=phii*(Ni/2+sin(4*pi*dfij*Tci*Ni)/sin(2*pi*dfij*Tci)/4);
    end
end
bij=I;

%%% -----%%%
%%%calculates the BER of SSG in dispersive channel
%%% Lc: # of channel taps for conventional MT with same data rate and
%%% tx power
%%% Lmax: # of RAKE fingers for SSG
%%% eta: decay factor of the PDP
%%% m: m factors

```



```

%%% Bc: coherence bandwidht of the channel
%%% Rb_X, Eb_X, N_X, f_X, are data rate, bit energy, PG, carreir
%%% frequencies of reference user and interfering users.

Function Pbs_sub_theo=SSG_BitError(Lc,Lmax,eta,m,Bc,Eb_ref,Rb_ref,
                                N_ref,f_ref,K,Eb_int,Rb_int,N_int,f_int)
M=length(Eb_ref);
%%%find out the corresponding channel for SSG system
Nc=N_ref(1); %%% PG of conventional MT
Lv=TapCalcu(N_ref,Rb_ref,Bc); %%% #of taps for reference user
Lj=TapCalcu(N_int,Rb_int,Bc); %%% #of taps for interference user
for ch=1:M
    Li=Lv(ch);
    Pav=AvgPower(eta,Li,'Exp'); %%%tap average power
    %%%find the SUI and MUIs
    I1(ch)=SUI(ch,Lv,eta,Eb_ref,Rb_ref,N_ref,f_ref);
    I2(ch)=MUI(ch,eta,Lj,Eb_ref,Rb_ref,N_ref,f_ref,Eb_int,
                Rb_int,N_int,f_int);
    A(ch)=(Li-1)*2/3/Li/N_ref(ch);
    gamma=Eb_ref(ch)^(-1)+A(ch)+I1(ch)+I2(ch)*(K-1);
    gammai=1/gamma;
    %%%construct the integral expressions
    Integral='';
    for lp=1:min(Li,Lmax)
        gammaim=gammai*Pav(lp);
        m_s=num2str(m);
        gammaim_s=num2str(gammaim);
        str=strcat('(',' ',m_s,'*sin(theta).^2./(',' ,gammaim_s,'+',m_s,
            '*sin(theta).^2)',' ').^',' ,m_s,')');
        Integral=strcat(str, '.*',Integral);
    end
    Integral=strcat(Integral, '1');
    F=inline(Integral);
    p=quad(F,0,pi/2); %%%numerical integrate the expression
    p=p/pi;
    Pbs_sub_theo(ch)=p;
end

%%% -----%%%
%%% find the SUI the subcarrier i sees from all other subcarriers
%%% The expression of MUI is givne in SSG paper
function I=SUIij(i,L,yita,Eb,Rb,N,f)
M=length(Eb);
Tb=1./Rb;
tmp=0;
for j=1:M
    if (j~=i)
        Lj=L(j);
        df=abs(f(j)-f(i));
        Pav=AvgPower(yita,Lj,'Exp');
        for l=1:Lj

tmp=tmp+Pav(l)*Eb(j)/(Tb(i)*Eb(i)*Tb(j))*VarR(Rb(i),N(i),Rb(j),N(j),df);
        end
    end
end

```

```

end
I=tmp*2;

%%%-----
%%%find the MUI the subcarrier i sees from all the subcarriers of other
%%%users. The expression of MUI is givne in SSG paper
function I2=MUI(i,yita,L,Eb_ref,Rb_ref,N_ref,f_ref,Eb_int,
                Rb_int,N_int,f_int)
M=length(Eb_int);
Tb_ref=1./Rb_ref;
Tb_int=1./Rb_int;
tmp=0;
for j=1:M
    Lj=L(j);
    Pav=AvgPower(yita,Lj,'Exp');
    df=abs(f_ref(i)-f_int(j));
    for l=1:Lj
        tmp=tmp+Pav(l)*Eb_int(j)/(Eb_ref(i)*Tb_ref(i)*Tb_int(j))
            *VarR(Rb_ref(i),N_ref(i),Rb_int(j),N_int(j),df);
    end
end
I2=tmp*2;

%-----%
%%%calculate the variance of correlation between two subcarriers for 9
cases
%%%the formula for 9 cases are given in table
%%%      N_ref:  Processing gain of reference carrier
%%%      N_int:  Processing gain of interferenc carrier
%%%      Rb_ref: data rate of reference carrier
%%%      Rb_int: data rate of of interferenc carrier
%%%      df:     frequency separation between two carriers

function VarCorr=VarR(Rb_ref,N_ref,Rb_int,N_int,df)
Tb_ref=1/Rb_ref;
Tb_int=1/Rb_int;
Tc_ref=Tb_ref/N_ref;
Tc_int=Tb_int/N_int;

gamma=Tc_ref/Tc_int;
kij=1/(Rb_ref/Rb_int);

if (Rb_ref==Rb_int)&(gamma==1)           %%case 1
    if df==0
        I=Tb_ref^2/3/N_ref;
    else
        I=N_ref*(1-sinc(2*df*Tc_ref))/2/(pi*df)^2;
    end
elseif (Rb_ref==Rb_int)&(gamma>1)       %%case 2
    if df==0
        I=N_ref*(Tc_int)^2*(2/3+(gamma-1))/2;
    else

```

```

        I=N_ref*(1-sinc(2*df*Tc_int)+(gamma-
1)*(sin(pi*df*Tc_int))^2)/2/(pi*df)^2;
    end

elseif (Rb_ref==Rb_int)&(gamma<1)          %%case 3
    if df==0
        I=(N_int*(2/3+(1/gamma-1)))*Tc_ref^2/2;
    else
        I=N_int*(1-sinc(2*df*Tc_ref)+(1/gamma-
1)*(sin(pi*df*Tc_ref))^2)/2/(pi*df)^2;
    end

elseif (Rb_ref>Rb_int)&(gamma==1)          %%case 4
    if df==0
        I=Tc_ref^2/3/N_ref;
    else
        I=N_ref*(1-sinc(2*df*Tc_ref))/2/(pi*df)^2;
    end

elseif (Rb_ref>Rb_int)&(gamma>1)          %%case 5
    if df==0
        I=N_ref*(Tc_int)^2*(2/3+gamma-1)/2;
    else
        I=N_ref*(1-sinc(2*df*Tc_int)+(gamma-
1)*(sin(pi*df*Tc_int))^2)/2/(pi*df)^2;
    end

elseif (Rb_ref>Rb_int)&(gamma<1)          %%case 6
    if df==0
        I=N_ref*gamma*(Tc_ref)^2*(2/3+1/gamma-1)/2;
    else
        I=N_ref*gamma*(1-sinc(2*df*Tc_ref)+(1/gamma-
1)*(sin(pi*df*Tc_ref))^2)/2/(pi*df)^2;
    end

elseif (Rb_ref<Rb_int)&(gamma==1)          %%case 7
    if df==0
        I=kij*N_int*Tc_ref^2/3;
    else
        I=kij*N_int*(1-sinc(2*df*Tc_ref))/2/(pi*df)^2;
    end

elseif (Rb_ref<Rb_int)&(gamma>1)          %%case 8
    if df==0
        I=N_ref*Tc_int^2*(2/3+gamma-1)/2;
    else
        I=N_ref*(1-sinc(2*df*Tc_int)+(gamma-
1)*(sin(pi*df*Tc_int))^2)/2/(pi*df)^2;
    end

elseif (Rb_ref<Rb_int)&(gamma<1)          %%case 9
    if df==0
        I=kij*N_int*Tc_ref^2*(2/3+1/gamma-1)/2;
    else

```

```

        I=kij*N_int*(1-sinc(2*df*Tc_ref)+(1/gamma-
1)*(sin(pi*df*Tc_ref))^2)/2/(pi*df)^2;
    end
end
VarCorr=I;

%-----%
%%%calculate the BER of MC using sinusoidal chip waveform and LSB USB
%%%comibnors
%%Index:   chip waveform frequency
%%dfTb:    frequency separation between two subcarriers
%%Lc:      # of channel taps
%%Lmax:    # of RAKE fingers
%%eta:     decay factor of PDP
%%m:       m factor
%%M:       # of subcarriers
%%Nnew:    PG
%%K:       # of users
function Pb=BitErrorLSBUSB.m( Index,dfTb,Lc,Lmax,eta,m,SNR,M,Nnew,K)
for i=1:M
    L=Lc;
    I1=K*(3+2*pi^2*Index^2)-2*pi^2*Index^2/L; %%%same subcarrier ICI
    I1=I1/3/pi^2/Index^2/Nnew;

    I2=0; %%%other subcarrier interferencde
    for j=1:M
        if (j~=i)
            dfTc=abs(i-j)*dfTb/Nnew;
            if (dfTc==Index)
                tmp=1/3;
            else
                tmp=(2*pi*dfTc-sin(2*pi*dfTc))/(pi*dfTc)^3
                    +(2*pi*Index-2*pi*dfTc+sin(2*pi*dfTc))
                    /pi^3/(Index-dfTc)^3;
            end
            I2=I2+tmp;
        end
    end
    I2=I2*K/Nnew/2;
    I=I1+I2;
    gammai=(2*SNR.^(-1)+I).^(-1);
    Pav=AvgPower(eta,L,'Exp');
    %%%construct the integral expressions
    Integral='';
    for lp=1:L %min(Lc,Lmax)
        gammaim=gammai*Pav(lp);
        m_s=num2str(m);
        gammaim_s=num2str(gammaim);
        str=strcat('(',' ',m_s,'*sin(theta).^2./
            (' ',gammaim_s,'+',m_s,'*sin(theta).^2)',' ').^','m_s,')');
        Integral=strcat(str,'.*',Integral);
    end
    Integral=strcat(Integral,Integral);
    Integral=strcat(Integral,'1');

```

```

        F=inline(Integral);
        p=quad(F,0,pi/2);
        p=p/pi;
        Pb(i)=p;
    end

%-----%
%%%AvgPower calculates the average power of each tap in the
%%%dispersive channel
%%%The channel models we are using are: Exponential decay and Uniform
%%%Exponential decay:  $P_i=P_0 \cdot \exp(-n \cdot i)$  : $P_i$  is the average power of
%%%th tap and  $P_0$  is the power of 0th tap
%%%Uniform:           $P_1=P_2=\dots P_i$       : Equal power for all taps
%%%
%%%          n: decay factor
%%%          Ntap: # of taps
%%%          Mod: model used      Mod='Exp' or 'Uni'
function Pavg=AvgPower(n,Ntap,Mod)
Pavg=zeros(1,Ntap);
if(Mod=='Uni')          %%Uniform power
    for tap=1:Ntap
        Pavg(tap)=1/Ntap;
    end
elseif(Mod=='Exp')      %%exponential decay mean power
    for tap=1:Ntap
        Pavg(tap)=exp(-n*(tap-1));
    end
end
%%%normalize the total power to be 1
P_total=sum(Pavg);
Pavg=Pavg/P_total;

%-----%
%%%calculate the # of channel taps for SSG
%%%the reference system and SSG has same total data rate and # of
%%%subcarriers
%%% N_SSG:  processing gain of SSG for all subcarriers
%%% Rb_SSG: data rate of SSG for all subcarriers
%%% Np_ref: #of taps for reference system
%%% Np:      # of channel taps for M subcarriers of SSG system
function Np=TapCalcu(N_SSG,Rb_SSG,Bc)
M=length(N_SSG);          %%# of subcarriers of SSG system
Rc_SSG=Rb_SSG.*N_SSG;
Np=floor(Rc_SSG/Bc);      %%# of channel taps is integer
%%%if Np==0 then let Np=1
for ch=1:M
    if (Np(ch)==0)
        Np(ch)=1;
    else
        Np(ch)=Np(ch)+1;
    end
end
end

```

```

%-----%
%%%generate the random number of givne distribution the
%%%Dist: Rayleigh
function Gainij=RdnGen(AvgPower,Dist)
Gainij=zeros(1,length(AvgPower));
if (Dist=='Rayleigh')
    Gainij=raylrnd(sqrt(AvgPower/2));
end

%-----%
%%%generate N random bits
function x=BitGen(N)
x=rand(1,N);      %%generate N numbers uniformly distributed in (0,1)
x=x>0.5;          %%if x>0.5 then x=1 else x=-1
x=2*x-1;

%-----%
% Function OverN oversamples input vector x by N
function y=OverN(x,N)
lx=length(x);
x= repmat(x,N,1);
y=reshape(x,1,N*lx);

%-----%
%%left shift x by delay
%%x is a vector, delay is an integer, and the length of x is longer
than
%%the value of delay
function y=lshift(x,delay)
L=length(x);
y=zeros(1,L);
y(1:L-delay)=x(delay+1:L);
y(L-delay+1:L)=x(1:delay);

%-----%
%%right shift the vector x by delay
function y=rshift(x,delay)

L=length(x);
y=zeros(1,L);
y(1:delay)=x(L-delay+1:L);
y(delay+1:L)=x(1:L-delay);

```