Smart Damage Prediction: a Distance to Bifurcation Based Approach

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ABSTRACT

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Every structure (civil, mechanical, aerospace, etc.) has an inherent lifespan over which it will be able to function properly. Consequently, health monitoring of these structures is necessary in order to ensure that they are operating under safe conditions. Damage detection and prognosis are essential components of a system's health monitoring. Many vibrations-based methods have been used for structural health monitoring; however, most have significant limitations and are based on an inaccurate assumption of linearity.

For this thesis, damage is proposed to be a nonlinear dynamical phenomenon that can be analyzed using bifurcation theory. A methodology for diagnosing a remaining useful service life of a structure is developed which utilizes the concepts of distance to stability boundary as estimated by bifurcation analysis. The proposed methodology is illustrated on both a two and a three degree of freedom non-linear mass-spring-damper system. These systems are tested using two different damage models in order to demonstrate the versatility of the methodology for estimating the evolution of various damage phenomena.

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1.0 INTRODUCTION

Every civil, mechanical and aerospace structure has an inherent lifespan over which it will be able to function properly. These complex systems are often modeled by engineers, and can range from an automobile engine to a suspension bridge. As parameters (mass, stiffness, damping) of a system change over time, qualitative changes in the long-term behavior and stability of a system can result. These qualitative changes are referred to as bifurcations [3]. The goal of this project is to develop a bifurcation-based methodology for health monitoring that can be compared to existing linear and non-linear health monitoring techniques and provide a more accurate and expansive capability of damage prognosis.

Health monitoring can be defined as using an autonomous system for the continuous monitoring, inspection and damage detection of a structure [11]. Health monitoring is becoming increasingly important in determining product safety. Vibrations-based health monitoring techniques are receiving increasing attention in research and literature [7]. The goals of vibrations-based health monitoring can be stated in four main steps [16]: damage detection, damage localization, damage quantification and prediction of a remaining useful life.

Current monitoring techniques are limited in that the majority of procedures rely on either visual or localized inspection methods [7]. Consequently, the damage in the system must be previously known, or be located at a visually accessible location. In addition, long time intervals can occur between inspection periods. Key damage to a

system can occur during these lapses of system monitoring, causing the system to go on operating under unsafe conditions that could lead to dramatic failure. Innovation in damage detection using vibrations measurements, seeks to eliminate these harmful barriers and supply a more reliable continuous monitoring technique. Continuous autonomous monitoring would allow for the extension of the remaining useful service life of a system, as is displayed in Figure 1.

Figure 1. Useful remaining life extension by continuous autonomous monitoring for civil and mechanical structures. [18]

Furthermore, development of a bifurcation-based approach to damage detection and prediction eliminates the inaccuracy of the linear-based detection techniques and proposes a methodology for predicting a remaining useful service life.

One of the most significant weaknesses surrounding most vibrations-based analysis methods developed to date is that they are based on linear assumptions. In reality, the behavior of all real systems is, to a some degree, nonlinear [20]. The concept of damage as a non-linear phenomenon is intuitive in that one would not expect a structure to fail at exactly twice the rate when twice a given force is applied to a structure. Material properties and initial conditions (for example, initial flaw distributions) are just two examples of factors that can influence how a system reacts to an applied force. One

of the strengths of a bifurcation-based analysis is that it is not based on linear assumptions. As a result, the accuracy of a bifurcation-based damage detection method should be better than those that are dependant on linear assumptions and more accurate for a wider range of system damage possibilities.

The limitation of bifurcation analysis is that, in order for an accurate analysis to result, the existing model must capture all damage phenomena of interest. For the bifurcation-based methodology developed in this paper, research by Adams and Nataraju [1], as well as that of Chelide, Cusumano and Chatterjee [4,5], is used as a basis for developing quantitative, non-linear models of damage to a linear system. These damage models demonstrate characteristics of sudden, catastrophic damage, as well as a more gradual damage to a system.

One of the leading challenges for current researchers of health monitoring and non-destructive evaluation is that of developing an all-inclusive methodology for damage detection, evaluation and prognosis that is both quantitative and qualitative in nature. Prognosis can indicate the rate at which the damage is accumulating, as well as the remaining useful service life of the structure. Prognosis in the methodology developed in this paper will focused on predicting a remaining useful service life for a structure, and will be assessed using a distance to bifurcation-based approach.

The concepts of predicting a remaining service life and performing ongoing health monitoring could have widespread affects on our maintenance of civil structures, automobiles, or even household appliances. If vibrations measurements could be used to accurately monitor system health conditions, anticipating problems ahead of time could

prevent many system failures. Continuous autonomous monitoring would allow for unnecessary routine maintenance to be replaced by conducting repairs on the system only when they are required. Furthermore, if the bifurcation theory can be used to analyze vibrations-based measurements taken on a system, this would provide non-linear methodology for health monitoring. This will lead to a smart damage detection and prediction methodology for civil and mechanical structures.

The organization of the paper is as follows: Section 2 begins with a literature review describing some of the most common vibrations-based techniques researched in the areas of health monitoring and non-destructive evaluation using vibrations-based methods. Section 3 follows by providing a comparison, based on research conducted by the author, of some of the existing damage detection techniques. Section 4 provides a mathematical background in some of the key components to this methodology, including a description of damage phenomena (specifically the models that are utilized in this methodology) and a background on bifurcation-based detection and prognosis. Section 5 outlines the key components of the methodology developed. Section 6 provides an illustrative example using the proposed methodology, and section 7 concludes with a discussion of conclusions and suggestions for possible future work.

2.0 LITERATURE REVIEW

2.1 LINEAR METHODS

One of the most commonly used linear-based methods of vibrations-based damage detection is done through monitoring the changes in natural frequencies of the structure. Changes in the structural properties of the system (mass, stiffness, damping) result in changes in the resonant frequencies of the structure. Although there has been a good deal of research showing that changes in natural frequencies can be used to detect the presence of damage, the low sensitivity of these shifts causes the method to be limited in its ability to provide localization and quantification information concerning the damage. [7] Some research has been done monitoring the shifts that occur in certain pre-determined states of damage and comparing them to the undamaged model to find the highest degree of similarity. However, extremely precise measurement capabilities or very large levels of damage are necessary for accurate results. [16] In addition, the method has difficulties distinguishing the location of damage occurring in symmetrical structures. [16]

Another one of the most largely researched vibrations damage detection methods monitors changes in the mode shapes of a structure. Mode shapes describe the geometry of a system's response to excitation. Mode shape changes can be tracked using MAC (Modal Assurance Criteria, which detects the degree of correlation between two different modes, or, in the case of COMAC (Co-ordinate Modal Assurance Criteria), the same mode before and after the presence of damage) values. If the two modes are identical, the

MAC/COMAC value will be equal to 1. If there is no correlation, the MAC/COMAC value will be equal to zero. [16] Tracking changes in eigenvalues and/or eigenvectors is another way to monitor changes in a system's modal properties. Eigenvalues and eigenvectors track the displacement and direction of a system's deformation. Some studies have shown that mode shape-based approaches are more successful indicators of damage than frequency-based techniques [7]. In addition, mode shape damage detection methods are able to indicate which modes are most affected by the damage [7]. Due to their nature as spatially distributed quantities, mode shape vectors can provide damage localization information; unfortunately, a large number of different measurement locations are necessary to develop the most accurate mode-shape vectors best able to determine damage location. [9].

Some research has suggested that observing changes in mode shape curvature can provide a more sensitive method for damage analysis, based on the fact that the second derivative of the mode shape is more responsive to changes than the original mode shape itself. [7] The mode shape curvature can be calculated using either the modal displacements or direct measurement of the curvature/strain. There is, however, a good deal of difficulty in obtaining accurate data for either of these two analysis techniques, and a larger amount of statistical uncertainty in the results exists for this technique in comparison to some of the other methods discussed [9].

Sensitivity functions are another tool used by researches for vibration-based damage detection. Sensitivity functions monitor the input and output frequency response functions of a system in a way which calculates the how reactive a certain vibration

property is to a change in one of its structural properties (mass, stiffness, damping). [23] For example, in a single degree of freedom system, taking the partial derivative of the forcing function with respect to an individual parameter (for example, stiffness) and plotting the results can indicate the frequencies at which the parameter is most sensitive to change. This principle can be expanded to include multi degree-of-freedom systems as well. One of the strengths of the sensitivity function method is that it does not require specific knowledge about the M , C K properties of the system [23].

There are several other methods used by researchers as vibration damage detection tools. One group of these are the matrix update methods, where the M , C , and K matrices are updated to produce a response (either static or dynamic) as close to the measured response as possible [7]. Another technique uses a Flexibility Matrix to measure changes in the structural stiffness as they relate to dynamically measured flexibility [9]. An additional category of analysis tools is the Neural Network-based methods. These methods use a computer-programmed algorithm to adjust outputs so that they minimize the error between the predicted and measured output responses [7].

Much research has been done in determining the capabilities of these damage-detection techniques. Each has their strengths and limitations in terms of the capabilities and accuracy of their analysis; still, it is important to note that most fail to go beyond the step of damage detection to a more comprehensive methodology that can determine damage localization, quantification or prognosis of remaining life.

Even the most effective and expansive of these methods is subject to an important limitation. All of the methods described above are all contingent upon the assumption

that a linear model is capable of accurately representing a system before and after damage has occurred. As a result of this assumption, the methods are inadequate in that their calculations can only be valid for small motion [20]. In most realistic modeling cases, damage has been shown to be a non-linear phenomenon, causing the structure to display non-linear behavior.

2.2 NON-LINEAR METHODS

Despite the inaccuracy and inefficiency of linear-based health monitoring techniques, sufficiently less research has been conducted in methods that are not dependant on a linear assumption [16]. Adding to their potential for benefit, non-linear damage detection techniques have been found to be less sensitive to environmental changes and interstructure variability [21]. Most non-linear damage detection techniques are based on the assumption that the damage scenarios give rise to non-linear vibration behavior that is sufficiently larger than the nonlinearities that are present in the undamaged structure.

In many non-linear based experiments on damage detection, a bilinear spring is often used to model the opening and closing behavior of a cracked beam. A comparison between this predefined state of damage and the steady state response of a cracked beam show that this model is capable of accurately demonstrating the behavior of such damage. The prominence of the effect can then be used to relate the crack depth and position, providing a method for damage localization and quantification [16].

A different non-linear technique combines the concepts of Instantaneous Frequency and Empirical Mode Decomposition. In this technique, empirical mode decomposition is used to decompose a signal into several monocomponent signals. The Instantaneous frequency, which can be loosely described as a frequency graph of a sine curve, is then fit to a signal. The basis behind this approach is the assumption that inelastic behavior and other types of damage will affect the frequency composition of the signal. Currently, this technique has been proved to be capable of identifying and quantifying damage in an idealized setting of noiseless conditions; however, the methodology has yet to be tested under more realistic circumstances [2].

Phase Plots are another tool used in vibrations damage detection that are not dependent on a linear assumption. Phase plots are charts created in phase space which graph a system's response of linear or angular position (x or θ) against a response of linear or angular velocity ($\dot{x}$ or $\dot{\theta}$). Changes in these phase plots with respect to time can be correlated with changes in a system's parameters (stiffness, damping, etc.) that can be characteristic of system damage. Little research has been done to shown the capacity for this damage detection methodology; however, it provides potential for tracking system damage, and possibly even predicting a remaining life.

2.3 HOW THIS WORK IS DIFFERENT

This methodology's status as a non-linear damage prognosis technique makes it unique among much of the current research. In addition, the methodology does not focus on the area of damage detection and assessment, which has already been heavily covered

in vibrations-based damage detection literature. Instead, it centers on creating a methodology for predicting a remaining useful service life of a structure. This approach, using a distance to bifurcation-based technique, is revolutionary in the area of vibrations health monitoring.

3.0 COMPARATIVE ANALYSIS OF EXISITING METHODS

In order to verify results obtained in previous literature, as well as obtain a better idea of each technique's strengths and weaknesses, some quantitative and qualitative experiments analyzing existing methodologies were performed in Matlab on a three degree-of-freedom mass-spring-damper system. More detailed results from some of these experiments can be found in the appendices; however, a table summarizing the advantages and disadvantages of each is given in Table 1.

<u>Method</u>	<u>Advantages</u>	<u>Disadvantages</u>
1. Changes in Natural Frequencies		
<i>linear</i>	effective at showing existence of damage	dependent on linear assumption stops short of showing localization, quantification or prognosis
<i>nonlinear</i>	not dependent on linear assumption	significant damage amount needed in order to detect change
2. Changes in Eigen Values		
<i>linear</i>	imaginary eigen data shows existence of damage quickly	dependent on linear assumption non-imaginary eigen data inconsistent stops short of showing localization, quantification or prognosis
<i>nonlinear</i>	not dependent on linear assumption	significant damage amount needed in order to detect change
3. Phase Plots	not dependent on linear assumption has capability of going beyond detection of damage to relationship with remaining useful life	busy nature of plots makes it difficult to track changes
4. Empirical Mode Decomposition	not dependent on linear assumption	data inconclusive; unable to accurately isolate damage signal

Table 1: Comparative Analysis of Existing Methods

4.0 MATHEMATICAL BACKGROUND

4.1 DAMAGE: A NONLINEAR PHENOMENON

Damage is a complex, nonlinear phenomenon which is difficult to model and/or extract from experimental data. Damage dynamics are dependent upon many variables, including initial damage states, loading history, and other environmental factors. There are several techniques for extracting damage models from a nonlinear system. A review of such approaches can be found in [7]. This paper does not focus on the methodology for developing such damage models; instead, it assumes that the models used are representative of all of the possible damage states of the system.

For this work, two low-order models of damage dynamics are used to demonstrate the proposed methodology. These models represent the normal forms of bifurcation phenomenon exhibited by non-linear systems. The first, which is similar to a model developed by [1] is a first-order single-degree-of-freedom damage model which represents sudden damage events with arresting plasticity. The model has been studied by various researchers, including Orrigner [17], and can represent the dynamics of crack propagation in joined sheet metal structures. The model can be represented by equation 1, where d is the damage state and $\dot{d}$ is the derivative with respect to time. γ_1 and γ_2 are variable parameters.

$$\dot{d} = \gamma_1 + \gamma_2 d - d^3 \quad (1)$$

Depending on the value of its parameters, $(\gamma_1 \text{ and } \gamma_2)$ this damage model has either one or three equilibrium points (Figure 4). It also represents a normal form for an asymmetric pitchfork bifurcation [19]. Saddle node bifurcation occurs all along the boundary in parameter space which divides the region into the one with one fixed point and the other having three fixed points (Figure 2). At the cusp point $(\gamma_1, \gamma_2 = 0, 0)$ a co-dimension two bifurcation is exhibited. When $\gamma_1 = 0$ this model exhibits a symmetric pitchfork bifurcation, but for $\gamma_1 \neq 0$ the pitchfork is asymmetric with two pieces; the upper piece consists entirely of stable fixed points whereas the lower piece has both stable and unstable branches.

Figure 2: Equilibrium points as a function of the two parameters (γ_1, γ_2) for damage model (Equation 1)

The second damage model (Equation 2) used in this work represents the phenomenon of Hopf bifurcation [19]. This represents a damage (fault) which results in

an oscillatory response with increasing magnitude of oscillations as the associated parameter μ is varied.

$$\begin{aligned}\dot{d}_1 &= \mu d_1 - d_2 + d_1 d_2^2 \\ \dot{d}_2 &= d_1 + \mu d_2 + d_2^3\end{aligned}\tag{2}$$

For this model (Equation 2) as the parameter μ increases through zero, the fixed point (origin) changes from a stable spiral to an unstable spiral and a sub critical Hopf bifurcation takes place at $\mu = 0$. This model and its bifurcations are presented in various texts including [10,12,15,19,20]. These oscillations are present in systems ranging from electromechanical actuation and positioning systems to biological and chemical systems. It should be noted that the size of limit cycle oscillations grows continuously from zero as the parameter μ is varied beyond the bifurcation point (Figure 3).

Figure 3: Numerical estimate of limit cycle amplitude: Hopf bifurcation (bifurcation occurs at $\mu = 0$) due to change in parameter for damage model as shown in Equation 2

4.2 DAMAGE DETECTION/ PROGNOSIS: DISTANCE TO BIFURCATION

For analysis, most physical systems can be represented as lumped parameter approximation by deterministic nonlinear models using ordinary differential equations in a state vector format. A general damaged nonlinear dynamic system in state vector format is given in Equation 3

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}, \mathbf{d}, \mathbf{u}, \mathbf{p}) \\ \dot{\mathbf{d}} &= \mathbf{g}(\mathbf{d}, \mathbf{p})\end{aligned}\tag{3}$$

where $\mathbf{x} \in \mathbf{R}^n$ are the state variables and $\mathbf{d} \in \mathbf{R}^m$ are the damage states. Further, $\mathbf{p} \in \mathbf{R}^p$ and $\mathbf{u} \in \mathbf{R}^u$ are the system parameters and inputs respectively. A similar model was proposed in [1]. This model assumes that the system states are coupled with the damage states. The primary interest is to track and explore the initiation and growth of damage states to *evaluate and predict the nature and extent of damage*. The output ($\mathbf{y}$) of the system dynamics (Equation 4) is considered independent of the damage states and thus ensuring that diagnosis is an inverse problem.

$$\mathbf{y} = \mathbf{h}(\mathbf{x}, \mathbf{p}, \mathbf{u})\tag{4}$$

The parameter space related to this model is a p -dimensional space. Thus the above state space model describes a $\mathbf{p}$ -parameter family of systems with a nominal parameter value represented by $\mathbf{p}_{\text{nom}}$. The equilibrium state ($\mathbf{x}_o$ and $\mathbf{d}_o$) for the system described in Equation 3 is the solution of an algebraic system (Equation 5).

$$\begin{aligned}\mathbf{f}(\mathbf{x}_o, \mathbf{d}_o, \mathbf{p}_o) &= 0 \\ \mathbf{g}(\mathbf{d}_o, \mathbf{p}_o) &= 0\end{aligned}\tag{5}$$

where $\mathbf{x}_o$ and $\mathbf{d}_o$ represent the equilibrium state vector for a given parameter vector $\mathbf{p}_o$.

The first order linearization of this system (Equation 3) about the equilibrium point ($\mathbf{x}_o$ and $\mathbf{d}_o$) can be used to study the stability of this system by analyzing the associated Jacobian matrix as given in Equation 6.

$$\mathbf{J} = \begin{bmatrix} \partial \mathbf{f} / \partial \mathbf{x} & \partial \mathbf{g} / \partial \mathbf{x} \\ \partial \mathbf{f} / \partial \mathbf{d} & \partial \mathbf{g} / \partial \mathbf{d} \end{bmatrix}_{(\mathbf{x}_o, \mathbf{d}_o)} \quad (6)$$

The nonlinear system (Equation 3) can be extended into a system as given in Equation 7 and then the center manifold theorem can be applied. The details of center manifold theorem are illustrated in various texts on bifurcation theory including [8,12,15,19,20,22]. The center manifold is tangent to the parameter plane as well as the eigenspace [20].

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}, \mathbf{d}, \mathbf{u}, \mathbf{p}) \\ \dot{\mathbf{d}} &= \mathbf{g}(\mathbf{d}, \mathbf{p}) \\ \dot{\mathbf{p}} &= \mathbf{0} \end{aligned} \quad (7)$$

All essential events near the bifurcation parameter value occur on the invariant manifold and are captured by the projection of the system on the center manifold. Essential dynamics include any and all qualitative changes associated around the equilibrium point. Standard bifurcation theory deals with the study of the stability of systems that move from one equilibrium condition to another equilibrium condition as the parameters slowly change. In a multi-parameter system, there is a bifurcation curve/plane in the parameter space along which the system has an equilibrium point exhibiting the

same bifurcation [20]. A smooth scalar function $\psi = \psi(\mathbf{x}, \mathbf{d}, \mathbf{p})$ can be constructed in terms of Jacobian matrix (Equation 6). This results in a system as shown in Equation (8).

$$\begin{aligned} \mathbf{f}(\mathbf{x}, \mathbf{d}, \mathbf{p}, \mathbf{u}) &= 0 \\ \mathbf{g}(\mathbf{d}, \mathbf{p}) &= 0 \\ \psi(\mathbf{x}, \mathbf{p}) &= 0 \end{aligned} \tag{8}$$

which, generically defines a curve Γ passing through the equilibrium point $(\mathbf{x}_o, \mathbf{d}_o, \mathbf{p}_o)$ in $\Re^{n+m+p}$. Γ defines the equilibrium satisfying the defining bifurcation condition and the standard projection of Γ onto the parameter space $\mathbf{p}$ -plane results in the corresponding *bifurcation boundary*. This function $\psi = \psi(\mathbf{x}, \mathbf{d}, \mathbf{p})$ for fold bifurcations is, defining the curve of equilibria having at least one zero eigenvalue, given below (Equation 9):

$$\psi = \psi(\mathbf{x}, \mathbf{d}, \mathbf{p}) = \det \begin{bmatrix} \partial \mathbf{f} / \partial \mathbf{x} & \partial \mathbf{g} / \partial \mathbf{x} \\ \partial \mathbf{f} / \partial \mathbf{d} & \partial \mathbf{g} / \partial \mathbf{d} \end{bmatrix} \tag{9}$$

and for the Hopf bifurcation is (Equation 10)

$$\psi = \psi(\mathbf{x}, \mathbf{d}, \mathbf{p}) = \det \left(2 \begin{bmatrix} \partial \mathbf{f} / \partial \mathbf{x} & \partial \mathbf{g} / \partial \mathbf{x} \\ \partial \mathbf{f} / \partial \mathbf{d} & \partial \mathbf{g} / \partial \mathbf{d} \end{bmatrix} \Theta I \right) \tag{10}$$

where Θ denotes the bi-alternate product [10,15] of two matrices. If more than one parameter is varied simultaneously to track a bifurcation curve Γ , then the following events might occur to the monitored non-hyperbolic equilibrium at some parameter values: extra eigenvalues can approach the imaginary axis and thus change the dimension of the center manifold or some of the non-degeneracy conditions for the co-dimension

one bifurcation can be violated. This bifurcation analysis can be used to characterize the parametric space of nonlinear systems for stability behavior. A boundary can be developed which separates the stable and unstable regions of the parametric space as defined through the bifurcation analysis. This boundary in the parametric space can be defined as the *stability boundary*.

This notion of stability boundary can be utilized to compute the *distance to closest-bifurcation* in the p -dimensional parameter space as originally outlined by Dobson [6]. It is proposed in this work that this distance to closest bifurcation is an effective metric to track damage evolution and predict time to failure in the framework of bifurcation theory. The closest-bifurcation method [6] estimates how much the system parameters can be changed from their nominal value (a vector of size p) without causing a change in the stability of the equilibrium point. The parameter vector, starting at a nominal point, can be varied in infinitely many directions; however in this closest-bifurcation method this variation is constrained along a direction in the parameter space. This direction can be defined by $\mathbf{p} = \mathbf{p}_{nom} + \delta \mathbf{dir}_i$ starting at $\mathbf{p}_{nom}$, where δ is the scalar step size along the $\mathbf{dir}_i$ as specified by a set of unit direction vector of size p . Each direction in parameter space generates a set of systems with their corresponding Jacobians. Each of these systems is analyzed for bifurcation stability. For each direction in parameter space, starting at a stable nominal the first point which exhibits the loss of stability via bifurcations is the bifurcation value of the parameter set in that direction and is denoted by $\mathbf{p}_{bif}$. The difference between the $\mathbf{p}_{nom}$ and $\mathbf{p}_{bif}$ is a measure of robustness and stability of the system to the changes in system parameters. This difference is the *distance to*

bifurcation ($\Delta(\text{dir}_i) = \left| \mathbf{p}_{\text{bif}} - \mathbf{p}_{\text{nom}} \right|_{\text{dir}_i}$) along that direction in parameter space. For each direction in parameter space, starting from the nominal system, an estimate of distance to bifurcation can be developed. These changes in parameters are sometimes due to the onset and/or propagation of damage. So if the nominal parameter vector, $\mathbf{p}_{\text{nom}}$, represents the current system and $\mathbf{p}_{\text{bif}}$ represents the system parameter at which the system loses stability and hence has no useful life. The distance to bifurcation is thus a measure of useful remaining life as depicted in the parameter space and provides an effective tool for capturing and quantifying the extent of damage in the system. The closest-bifurcation method gives an estimate of the global *minimum* distance to bifurcation in any direction in the parameter space.

Closest distance to bifurcation method first determines a single distance to bifurcation along a specified (initial) search direction starting from a nominal parameter (Figure 4). Then, using an iterative distance minimization procedure, a search for minimum distance to bifurcation can be determined. This minimum distance to bifurcation is perpendicular to a tangent plane of the bifurcation surface. The iterative procedure updates the search direction to be perpendicular to the bifurcation boundary (surface) whenever the parameter vector crosses the bifurcation boundary. The search is started at the nominal point along this updated direction. The search direction converges to the perpendicular direction at the bifurcation boundary, and then the point on the bifurcation boundary has the minimum distance to bifurcation from that nominal point. A damage prediction methodology is presented in the Section 7, which utilizes the notion of bifurcation stability boundary and the closest distance to bifurcation as a metric of useful

remaining life. This parameter space investigation also suggests a measure of when and how this failure due to a possible damage might happen.

Figure 4: Distance to bifurcation boundary in parameter space: distance 1 and distance 3 are along the directions of first and second parameters respectively. Closest distance to bifurcation is obtained by the perpendicular distance to bifurcation boundary from the nominal point. Distance 2 is a measure of allowable parameter variation before the emergence of bifurcation as the parameters are varied along this direction.

5.0 SMART DAMAGE PREDICTION: A METHODOLOGY

The main goal of this research is to develop an overall methodology that can be used for smart damage prediction. Although this research does not detail the exact procedure for every step in the methodology, it presents an overall technique that can be used for the process.

The first step in the proposed methodology consists of developing a system model and selecting an appropriate representation of relevant damage phenomena. The goal of developing a mathematical model is to capture an accurate representation of a system which contains all of the essential characteristics of its components, but whose behavior can be described by mathematical equations. In order for the proposed methodology to be effective, the models developed for the system and related damage must truly be representative of all of the states of the system. The focus of this paper does not lie on constructing mathematical models for systems; however, the necessity of completing this step and validating the accuracy of the models should not be underestimated.

The next step is to locate the nominal system with respect to the parameters evaluated in parameter space. Knowledge of the location of the initial position of the system within parameter space is necessary to determine the shortest distance to bifurcation.

Any nonlinear system can have a large set of associated parameters and states. Investigating the full parameter space can be an overwhelming task, even for an efficient method such as bifurcation analysis. Consequently, is it helpful if some insight about the

influence of parameters on the system dynamics is known in advance. This knowledge can be gained either by numerical simulation or by experimental testing. In light of these insights it may be possible to reduce the dimensionality of the parameter space to include the parameters which are most influential in the damage propagation. Several methodologies for reduction of parameter space are known and as an example interested reader may consult [14]. Combining this knowledge with that of the initial state of the system, an initial 1-parameter investigation using bifurcation analysis can be conducted to capture the nature of possible damage via loss of bifurcation stability. This information can then be used to guide 2-parameter investigation for the distance to bifurcation. Together, this can provide an iterative process for selection of an appropriately reduced parameter space used to compute the bifurcation boundary.

Once the appropriate parameter space has been selected, a detailed bifurcation stability analysis can be conducted to compute the bifurcation boundary (and/or surface for a three-parameter investigation). This boundary will classify the states of the system into stable and unstable regions. Calculating the bifurcation boundaries and their respective projections can be done on a two-dimensional parameter space model for ease of graphical representation.

Next, starting at the nominal point and using the estimated bifurcation boundary, a distance to bifurcation along several key directions in parameter space are generated. This data will provide insight about the various paths in parameter space, and their associated effects on the remaining useful service life of a structure.

Using the iterative optimization process outlined by Dobson [6] and discussed in section 4.2, a closest distance to bifurcation is estimated for the nominal system. This distance to bifurcation provides a direct correlation to the remaining useful life of the structure, which, in turn, provides an accurate method for determining damage prognosis.

In practice, the methodology can be used as an iterative process, updating the system and damage models to reflect any changes in system parameters. This knowledge can best be obtained through experimental investigation and analysis. Continuous updating of the system will help in obtaining the most accurate estimate of the distance to bifurcation/remaining useful life. In addition, if specific knowledge about individual parameters is available, then this information can be used to compute the distance to bifurcation along the respective direction.

The proposed methodology is outlined in Figure 5.

Figure 5: Smart damage detection and prediction methodology: a distance to bifurcation-based approach

6.0 AN ILLUSTRATIVE EXAMPLE

To show its versatility, the proposed methodology has been illustrated on both a two and a three degree-of-freedom mass-spring-damper system. Both systems were tested twice, once using each of the different damage phenomena previously defined in equations 11 and 12.

The schematic of a two-degree-of-freedom system is shown in Figure 6. The mathematical models derived for the system as they are subjected to each of the damage phenomena are given in equations 1 and 2. The nominal system parameters are $m_1=1$, $m_2=10$, $c_{11}=5$, $c_{12}=10$, $c_{22}=4$, $k_{11}=100$, $k_{12}=k_{22}=800$, $\gamma_1=-1$, $\gamma_2=-1$, $u_1=-1$.

Figure 6: Schematic for two degree of freedom nonlinear mass-spring-damper system with damage (d)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{v}_1 \\ \dot{x}_2 \\ \dot{v}_2 \\ \dot{d} \end{bmatrix} = \begin{bmatrix} v_1 \\ -(k_{11} + d)x_1 - c_{11}v_1 - k_{12}(x_1 - x_2) - c_{12}(v_1 - v_2) / m_1 \\ v_2 \\ (-k_{12}(x_2 - x_1) - c_{12}(v_2 - v_1) - k_{22}x_2 - c_{22}v_2) / m_2 \\ \gamma_1 + \gamma_2 d - d^3 \end{bmatrix} \quad (11)$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{v}_1 \\ \dot{x}_2 \\ \dot{v}_2 \\ \dot{d}_1 \\ \dot{d}_2 \end{bmatrix} = \begin{bmatrix} v_1 \\ -(k_{11} + d_1)x_1 - c_{11}v_1 - k_{12}(x_1 - x_2) - c_{12}(v_1 - v_2) / m_1 \\ v_2 \\ (-k_{12}(x_2 - x_1) - c_{12}(v_2 - v_1) - k_{22}x_2 - c_{22}v_2) / m_2 \\ \mu d_1 - d_2 + d_1 d_2^2 \\ d_1 + \mu d_2 + d_2^3 \end{bmatrix} \quad (12)$$

For the first damage phenomena, γ_1 and γ_2 have been selected as the parameters of interest to be studied in the reduced parameter space investigation. According to the process proposed in the methodology, 1-D and 2-D direction searches in parameter space were evaluated in order to determine the appropriate range of values for the parameters of interest. The effect of varying the two parameters on the eigenvalues of the system is shown in Figure 7.

Figure 7: Effect of varying parameters (γ_1 and γ_2) on the eigenvalues of the two degree-of-freedom system. The system exhibits loss of stability via pitch fork bifurcation at (0,0).

Next, the stability boundary is calculated in the parameter space (Figure 8). From here, the shortest distance to bifurcation is determined. This distance is measured from the point designated by the values of the nominal parameters, $\gamma_1=-1$ and $\gamma_2=-1$. The

closest distance to bifurcation is found to be 1, and is along the direction dI , as labeled in the figure. This demonstrates that the system will fail when the parameter vary along the closest bifurcation direction and become equal to the value at the bifurcation point. The greater the closest distance to bifurcation, the larger is the remaining useful service life of the structure.

Figure 8: Bifurcation boundary as compared to nominal system parameters and closest distance to bifurcation ($d1$) for two degree-of-freedom system with damage model 1.

Several possible directions exist; however, most influential is the variation in the parameter γ_1

For the second damage phenomena, the parameter space of interest is one-dimensional since u is the only parameter to be directly involved in the damage model. In this case, the bifurcation boundary reduces to a point. The effect of varying μ on the bifurcation stability of the system is given in Figure 9. The bifurcation boundary reduces to a point for a one dimensional parameter space, and the possible direction in parameter space is restricted to the one changing parameter (μ). This bifurcation boundary

(bifurcation point) in the two dimensional parameter space of mass (m_1) and damage parameter μ is shown in Figure 10 along with the nominal system parameter. The closest distance to bifurcation is along the direction of μ , and is 1 for this case. This study suggests that the system will fail as $C=0$ by exhibiting increasing amplitudes of limit cycle oscillations, at which point the remaining useful service life of the structure would be zero.

Figure 9: Effect of varying parameter μ on the eigenvalues of the two degree-of-freedom system. The system exhibits loss of stability via Hopf bifurcation at ($\mu=0$)

Figure 10: Bifurcation point as compared to nominal system parameters and closest bifurcation for the two degree-of-freedom system with damage model 2.

The proposed methodology is next demonstrated on the three degree-of-freedom system shown in Figure 11. The mathematical models of the structure, as it is exposed to each damage model, are given in equations 13 and 14. In these equations, the variables are put into first order format, such that $y_1 = x_1, y_2 = \dot{x}_1, y_3 = x_2, y_4 = \dot{x}_2, y_5 = x_3, y_6 = \dot{x}_3, y_7 = d_1$, and $y_8 = d_2$. The nominal system parameters are $m_1=1, m_2=10, m_3=25, c_1=4, c_2=1, c_3=5, c_4=10, k_1=250, k_2=250, k_3=500, k_4=750, k_5=1000, \gamma_1=-1, \gamma_2=-1, u_1=-1$.

Figure 11: Schematic of a three degree of freedom nonlinear mass-spring-damper system with damage (d)

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \\ \dot{y}_4 \\ \dot{y}_5 \\ \dot{y}_6 \\ \dot{y}_7 \end{bmatrix} = \begin{bmatrix} y_2 \\ \left(k_2(y_3 - y_1) + k_5(y_5 - y_1) + c_2(y_4 - y_2) - k_1 y_1 - \gamma_1 y_1 - c_1 y_2 \right) / m_1 \\ y_4 \\ \left(k_3(y_5 - y_3) + c_2(y_6 - y_4) - k_2(y_3 - y_1) - c_2(y_4 - y_2) \right) / m_2 \\ y_6 \\ \left(-k_4 y_5 - c_4 y_6 - k_5(y_5 - y_1) - k_3(y_5 - y_3) - c_3(y_6 - y_4) \right) / m_3 \\ \gamma_1 + \gamma_2 d - d^3 \\ y_7 \end{bmatrix} \quad (13)$$

$$\begin{bmatrix} \bullet \\ y_1 \\ \bullet \\ y_2 \\ \bullet \\ y_3 \\ \bullet \\ y_4 \\ \bullet \\ y_5 \\ \bullet \\ y_6 \\ \bullet \\ y_7 \\ \bullet \\ y_8 \end{bmatrix} = \begin{bmatrix} y_2 \\ \left(k_2(y_3 - y_1) + k_5(y_5 - y_1) + c_2(y_4 - y_2) - k_1 y_1 - y_7 y_1 - c_1 y_2 \right) / m_1 \\ y_4 \\ \left(k_3(y_5 - y_3) + c_2(y_6 - y_4) - k_2(y_3 - y_1) - c_2(y_4 - y_2) \right) / m_2 \\ y_6 \\ \left(-k_4 y_5 - c_4 y_6 - k_5(y_5 - y_1) - k_3(y_5 - y_3) - c_3(y_6 - y_4) \right) / m_3 \\ \mu d_1 - d_2 - d_1 d_2^2 \\ d_1 + \mu d_2 + d_2^3 \end{bmatrix} \quad (14)$$

Just as in the two degree-of-freedom system, γ_1 and γ_2 are selected as the parameters of interest to be studied in the reduced parameter space investigation for the first damage model. The effect of varying the two parameters on the eigenvalues of the system is shown in Figure 12.

Figure 12: Effect of varying parameters γ_1 and γ_2 on the eigenvalues of the three degree-of-freedom system. The system exhibits loss of stability via pitch fork bifurcation at $(0,0)$.

Next, the stability boundary is calculated in the parameter space (Figure 13).

From here, the shortest distance to bifurcation is determined. This distance is measured

from the point designated by the values of the nominal parameters, $\gamma_1=-1$ and $\gamma_2=-1$. The closest distance to bifurcation is again found to be 1, and is along the direction dI , as labeled in the figure.

Figure 13: Bifurcation boundary as compared to nominal system parameters and closest distance to bifurcation ($d1$) for three degree-of-freedom system with damage model 1.

Several possible directions exist; however, most influential is the variation in the parameter γ_1

The significant difference between the amount of system noise shown in Figures 8 and 13 should be noted. Although the quality of Figure 8 was improved by running the simulation for a larger number of data points, (thus increasing the resolution of the plot) a considerably larger degree of noise can be seen around stability boundary for the three-degree-of-freedom system shown in Figure 13. The state of stability for the points around the bifurcation boundary can be better demonstrated in a three dimensional model of the damage surface, as shown in Figure 14, thus qualifying the determination of the closest distance to bifurcation demonstrated in Figure 13.

Figure 14: Three-dimensional model of damage surface for three degree-of-freedom system

Just as in the two degree of freedom example, the parameter space for the second damage model is one-dimensional since u is the only parameter to be directly involved in the damage phenomena. Again, the bifurcation boundary reduces to a point. The effect of varying μ on the bifurcation stability of the system is given in Figure 15. The bifurcation boundary reduces to a point for a one dimensional parameter space, and the possible direction in parameter space is restricted to the one changing parameter (μ). This bifurcation boundary (bifurcation point) in the two dimensional parameter space of mass (m_1) and damage parameter μ is shown in Figure 16 along with the nominal system parameter. The closest distance to bifurcation is along the direction of μ , and is again found to be 1.

Figure 15: Effect of varying parameter (u) on the eigenvalues of the three degree-of-freedom system. The system exhibits loss of stability via Hopf bifurcation at ($u=0$)

Figure 16: Bifurcation point as compared to nominal system parameters and closest bifurcation for the two degree-of-freedom system with damage model 2.

As demonstrated in these examples, the closest distance to bifurcation is not a computation in time domain, but instead is a measure of the distance in parameter domain (space). An equivalent time estimate can be developed if desired based on continuous tracking. However, since the damage is a nonlinear dynamical phenomenon any mapping between the time space to parameter space and *vice-versa* would be inherently nonlinear.

Thus, closest distance to bifurcation is an appropriate measure for remaining useful life under the framework of this methodology. Further, it should be noted that a continuous tracking and model update may be necessary for various systems to compute closest distance to bifurcation and evaluate remaining useful life, as is indicated in the iterative nature of the proposed methodology.

7.0 CONCLUSIONS AND FUTURE WORK

The primary purpose of this work is to propose a methodology for damage detection and predication, using a bifurcation-based approach. Since this approach is not constrained by the linear assumptions that limit the accuracy and scope of many other vibrations-based damage detection techniques, some of which have been identified and discussed in the background research for this paper, it has the potential to provide a much more accurate technique that can not only detect damage in a structure, but also provide a prognosis for the remaining useful life. This prognosis, determined by finding the closest distance to bifurcation, is given in terms of system parameters. For further validation of the accuracy of the results obtained through this methodology, continuous real-time tracking and model updates should be performed on the system. The concepts developed in this paper for predicting a remaining useful life have application in many different systems, and could have many positive effects in terms of cost savings and safety of civil structures.

The largest capacity for future work in this theory would come from providing this real-time verification of the accuracy of the results acquired by means of this methodology. Specifically, a two or three degree-of-freedom system could be tested being subjected to types of damage suggested in this paper. Appropriate values for the parameters that are characteristic of the system and damage models could be chosen, and the experimental and analytical results could be gauged against each other to provide quantitative proof of this methodology.

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