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PHYSICS

MEASUREMENT OF THE NUCLEAR DEPENDENCE

OF F_2 AND $R = \sigma_L/\sigma_T$ IN THE NUCLEON RESONANCE REGION (223 pp.)

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Experiment E04-001 was carried out in Hall C at Jefferson Lab to measure the L/T separated structure functions from nuclear targets. The experiment ran in the nucleon resonance region ($1 < W^2 < 4.5 \text{ GeV}^2$) spanning the squared four-momentum transfer range $0.5 < Q^2 < 2.5 \text{ (GeV/c)}^2$. An important goal of this experiment was to provide high precision cross section results for nuclear targets in the resonance region. The Rosenbluth separation technique was used to extract the separated structure functions F_L, F_1, F_2 and the ratio $R = \sigma_L/\sigma_T$. The measurements of this experiment were used to study the nuclear dependence on the separated structure functions and R .

MEASUREMENT OF THE NUCLEAR DEPENDENCE
OF F_2 AND $R = \sigma_L/\sigma_T$ IN THE NUCLEON RESONANCE REGION

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by
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Chapter1: Introduction

In late 1967, the famous SLAC-MIT experiment [1, 2], which studied electron scattering from protons revealed strong evidence for their substructure in terms of partonic constituents. Since then, deep inelastic scattering (DIS) has been a primary tool for mapping out the nucleon substructure in term of the (more) recognized constituent quark. The nucleon substructure can be revealed by determining the longitudinal F_L , transverse F_1 and $F_2(\propto F_L + 2xF_1)$ structure functions, where x is a scaling variable known as the "Bjorken variable".

Initially, the nuclear effects of the structure functions were thought to be negligible in the DIS region, however, this was proven to be wrong by the European Muon Collaboration experiment [3]. The deviation of the iron to deuterium cross sections ratio from unity, called later the EMC effect, revealed a clear hint of medium modifications of the σ^A/σ^D ratio (A stands for nucleus and D stands for deuterium). The EMC effect seemed to increase with the mass number, A , of the nucleus, exhibiting an A -dependence that was consistent with both $\log A$ and average nuclear density. However, data of a later experiment in 2009 [4], which studied the effect on light nuclei, disagreed with the simple $\log A$ and average density dependence. Instead, the EMC effect seemed to depend on the local density of the nucleus, which suggests that the nuclear environment influences the internal structure of the nucleon. The arising nuclear dependence of the cross section ratios triggered considerable activity in the theoretical study of deep inelastic lepton scattering from nuclear targets.

In the DIS region, the structure function F_2 is proportional to the double differential cross section, $d^2\sigma/d\Omega dE'$, and many experiments ran resulting in precise measurements

of the structure function F_2 in the DIS region. One can extract the ratio F_2^A/F_2^D from the measured cross sections using:

$$\begin{aligned} \frac{\sigma^A}{\sigma^D} &= \frac{F_2^A(x, Q^2)(1 + R_D)(1 + \epsilon R_A)}{F_2^D(x, Q^2)(1 + R_A)(1 + \epsilon R_D)} \\ &\approx \frac{F_2^A(x, Q^2)}{F_2^D(x, Q^2)} \left[1 - \frac{(R_A - R_D)(1 - \epsilon)}{(1 + R_D)(1 + \epsilon R_A)} \right], \end{aligned} \quad (1-1)$$

where σ and F_2 refer here to the cross section per nucleon and the structure function per nucleon respectively, and R is the ratio of the longitudinal, σ_L , to transverse, σ_T , photonuclear cross sections: $R = \sigma_L/\sigma_T = F_L/2xF_1$ (for the definition of ϵ please see Eq. 2-17). Typically, $\sigma^A/\sigma^D = F_2^A/F_2^D$ only in the limit of $\epsilon = 1$ or $\Delta R = R_A - R_D = 0$, and $\sigma^A/\sigma^D = F_1^A/F_1^D$ only if $\epsilon = 0$. The assumption $\Delta R = 0$ implies that there is no (or negligible) nuclear dependence on R , and therefore F_L .

Measuring R is experimentally challenging; it requires separating the structure function F_2 into F_1 and F_L using the Rosenbluth separation technique, i.e. measuring the cross section (with high precision) at different energies at fixed x and Q^2 . Therefore, there are not enough data available for R and the separated structure functions F_1 and F_L as F_2 . So far, the world's published data on R exist mainly in the DIS region, and are associated with large uncertainties. In the DIS region, R is generally small which makes it difficult to detect any nuclear dependence in that region. Therefore, precise measurements of R in the resonance region, where Q^2 values are relatively low to moderate, will give a better understanding of the medium modification of R . Several experimental attempts were made to measure R [5-10]. While some experiments [7, 8] concluded that ΔR is compatible with zero, other experiments [9, 10] observed only hints of some nuclear dependence of R at low Q^2 .

Theoretical predictions of the nuclear dependence in R , and therefore F_L , have been

introduced, such as the theory of pion enhancement by Miller [11]. Miller predicted a significant nuclear dependence of R at low Q^2 values due to the exchange of nuclear pions between nucleons, but there has been no confirmation of Miller's prediction so far.

Experiment E04-001 [12] was carried at Jefferson Lab in 2005 to measure the longitudinal-transverse ($L - T$) separated structure functions F_2 and R in the resonance region and to study the medium modification of F_1 , F_L and R . Measurements were made in the nucleon resonance region ($1 < W^2 < 4.5 \text{ GeV}^2$) spanning the four-momentum transfer squared range $0.5 < Q^2 < 2.5 \text{ (GeV/c)}^2$. Data were taken with beam energies from 1.2 to 4.6 GeV, using the Hall C High Momentum Spectrometer (HMS) at various scattering angles ($10.65^\circ - 70^\circ$). Targets included Carbon, Iron and Aluminum. The decomposition of the inclusive electroproduction cross sections into longitudinal and transverse components was accomplished by performing Rosenbluth separations to extract the longitudinal F_L and transverse F_1 structure functions, and the ratio R . Since E04-001 ran at same kinematic points covered by Miller's predictions, the experiment results will help check the pion excess in the low Q^2 region.

The focus of this thesis is to analyze E04-001 data collected from Carbon and Iron targets. First, a "Theoretical Overview" will be presented, starting from the history of electron-nucleon scattering to the general physics aspects that E04-001 is studying. Then, the "Experimental Setup" of E04-001 will be summarized, describing how Hall C looked like in 2005 (the 6 GeV era). Later, the transition from detected raw data into useful physics quantities will be described in the "Analysis" chapter. The analysis chapter will describe in detail the process of extracting the cross sections and structure functions, and study the medium modifications. Finally, the "Results" of this work will be presented followed by the "Conclusions".

Chapter2: Theoretical Overview

2.1 A Bit of History

In the early years of the twentieth century, Rutherford taught us one of the most valuable lessons in nuclear physics: we can learn about the structure of matter by using the scattering process. Knowing the momentum of the incident particle and measuring the angle and energy of the particle scattered by an object, one can study the features of that object. The angular distribution of the scattered particles will help understand the force producing the scattering and, then, predict the "charge distribution" inside the object.

Rutherford's famous α -gold elastic scattering experiment [13] suggested that α particles are being scattered by a tiny, dense, positively charged nucleus. The cross section formula that Rutherford established was confirmed in the non-relativistic case:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 Z^2}{4(p\beta) \sin^4(\theta/2)} , \quad (2-1)$$

where $\alpha \approx 1/137$ is the fine structure constant, Z is the nucleus charge, β and p denote the velocity and momentum of the incident particle respectively and θ is the scattering angle in the laboratory frame.

One can say that a scattering process will "photograph" the object. So, in order to take a better picture with high resolution, one should use structureless, fast point-like projectiles such as electrons instead of the low energy α -particles. When using electrons, the Rutherford formula Eq. 2-1 needs to be modified, and that was calculated

by Mott [14]. Mott's cross section for a point-like (structureless) target is given by:

$$\left(\frac{d\sigma}{d\Omega}\right)_{Mott} = \left(\frac{d\sigma}{d\Omega}\right) \cdot \frac{\cos^2(\theta/2)}{1 + \frac{2E}{M} \sin^2(\theta/2)} , \quad (2-2)$$

where E and M are the electron beam energy and the proton mass, respectively.

For many years, the available electron energies were too small for the electrons to penetrate beyond the electron cloud of the atoms. However, when high-energy electrons became available, theorists and experimentalists showed great interest in using high-energy electrons as probes of the nucleus, and study the proton structure. Electron scattering became one of the most powerful tools to investigate the structure of the proton for many reasons. First, the electromagnetic interaction is well understood. Moreover, the electrons have no substructure and are easy to produce and easy to accelerate. In 1950, Rosenbluth [15] proposed an expression for the elastic differential cross section for the proton, involving an electric, G_E , and a magnetic, G_M , form factor. The form factors describe the deviation from a point-like behavior of the proton:

$$\left(\frac{d\sigma}{d\Omega}\right)_{ep \rightarrow ep} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \left[\frac{G_E^2 + \tau G_M^2}{1 + \tau} \cos^2(\theta/2) + 2\tau G_M^2 \sin^2(\theta/2) \right], \quad (2-3)$$

where $\tau \equiv -q^2/4M^2$, with $q^2 \equiv Q^2 = 4EE' \sin^2(\theta/2)$ being the squared four momentum transfer.

In 1950's, Hofstadter and his colleagues [16, 17] performed a series of elastic ep scattering experiments to investigate the substructure of the proton. Those experiments discovered that as the momentum transfer in the scattering process increased, the elastic scattering cross section dropped sharply relative to that from point charge. In other words, the proton has an extended structure, and it is not a point. This result confirmed

Rosenbulth's formula Eq. 2-3 and became a fundamental discovery. For the next several years, the experiments were limited to elastic scattering. Those elastic scattering experiments provided clues about the proton structure, but still were not able to see "what is inside the proton". In order to do so, one need to probe the nucleus deeper with even higher electron energies, which the inelastic ep scattering would provide.

"... the electron-scattering method appears to offer great promise in unraveling the problems of nuclear size and shape and the internal dynamics of nuclei. But it must be emphasized that we do not have complete information on such subjects at the present time. The experiments are just "scratching the surface". Many nuclei still need to be investigated and greater accuracy is needed; absolute cross sections must be obtained. Better resolution between elastic and inelastic scattering should be a fundamental objective and is at present a serious lack. Good electron counters must be made which distinguish high-energy electrons from all other particles having the same momenta. We may summarize these remarks in a familiar way: More and better data are needed." — R. Hofstadter (1956)

The results of those early elastic ep scattering experiments led to the proposal to a build two-mile linear accelerator at Stanford, the Stanford Linear Accelerator Center (SLAC). The major objective of SLAC was to investigate the structure of the nucleon (proton and neutron) by providing high energy electrons that would make both elastic and inelastic scattering possible. On the other hand, theorists were searching for a model that would describe the nucleon structure, and in 1964 both Gell-Mann [18] and Zweig [19] invented "quarks". In their approach, the nucleons are composed of point-like objects called "quarks", and these objects had to have electrical charges equal to $1/3$ or $2/3$ that of an electron or proton. Even though the quark model was useful as a mathematical construct in particle physics, the idea of fractional charges was not attractive to the physicists back then. However, surprising results came out in late 1967, when a collaboration from Massachusetts Institute of Technology (MIT) and SLAC performed the first series of experiments on highly deep inelastic scattering using beam energies up to 20 GeV. Those experiments led to the discovery of the presence of point-like

constituents (quarks) inside the nucleon [1, 2]. The pioneering work of the SLAC-MIT experiments for the development of the quark model was recognized with the 1990 Nobel in Physics award [20–22].

2.2 Deep Inelastic Scattering (DIS)

2.2.1 Inclusive Process $ep \rightarrow eX$

In deep inelastic scattering, a high energy electron e is scattered off the proton where the proton p is excited to a hadronic final state X with higher mass.

$$e(k) + p(P) \rightarrow e(k') + X$$

Considering the lowest order of perturbation theory, the electromagnetic interaction between the electron and the proton occurs via the exchange of a so-called virtual photon γ^* , where its energy and momentum are transferred to the proton (see Fig. 1).

Figure 1: Deep inelastic lepton nucleon scattering

where $\mathbf{k} = (E, \vec{k})$ and $\mathbf{k}' = (E', \vec{k}')$ are the four-momenta of the incident and scattered electron respectively, and $\mathbf{q} = (\nu, \vec{q})$ and $\mathbf{P} = (E_p, \vec{p})$ are those of the exchanged virtual

photon and the target nucleon, respectively. The DIS process can be described using Lorentz-invariant quantities:

$$q^2 = -Q^2 = (\mathbf{k} - \mathbf{k}')^2 \quad \text{squared four momentum transfer} \quad (2-4)$$

$$W^2 = (\mathbf{P} + \mathbf{q})^2 \quad \text{squared invariant mass of the final hadronic state} \quad (2-5)$$

In the laboratory reference frame, the target is usually fixed, $\mathbf{P} = (M, 0)$, and:

$$Q^2 \simeq 4EE' \sin^2 \frac{\theta}{2} \quad (2-6)$$

$$W^2 = M^2 + 2M\nu - Q^2 \quad , \quad (2-7)$$

where M is the proton mass, θ is the scattering angle in the laboratory frame, $\nu = E - E'$ is the energy of the virtual photon transferred from the incident electron to the proton.

In deep inelastic ep scattering, only the electron is observed and the composition of the hadronic final state is unknown except for its invariant mass W . The unpolarized differential cross section for ep scattering -obtained by scattering unpolarized electrons from unpolarized target- can be written as:

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{\alpha^2}{Q^4} \frac{E'}{E} L^{\mu\nu} W_{\mu\nu} \quad , \quad (2-8)$$

where α is the fine structure function, and $L^{\mu\nu}$ and $W_{\mu\nu}$ are the leptonic and hadronic tensors, respectively. The former is given by:

$$L^{\mu\nu} = 2(k_\mu k'_\nu + k'_\mu k_\nu - g_{\mu\nu} k \cdot k') \quad , \quad (2-9)$$

where k and k' are initial and final electron momentum respectively, and $g_{\mu\nu}$ is the metric

tensor. On the other hand, the hadronic tensor can be expressed as:

$$W^{\mu\nu} = W_1(\nu, Q^2) \left(\frac{q^\mu q^\nu}{q^2} - g^{\mu\nu} \right) + \frac{W_2(\nu, Q^2)}{M^2} \left(p^\mu + \frac{p \cdot q}{q^2} q^\mu \right) \left(p^\nu + \frac{p \cdot q}{q^2} q^\nu \right) . \quad (2-10)$$

The structure functions W_1 and W_2 are functions of the two independent variables ν and Q^2 . Substituting Eq. 2-9 and Eq. 2-10 into Eq. 2-8, we obtain the total differential cross section for inclusive ep scattering:

$$\left(\frac{d^2\sigma}{dE'd\Omega} \right)_{ep \rightarrow eX} = \frac{\alpha^2}{4E^2 \sin^4(\theta/2)} \left(W_2(\nu, Q^2) \cos^2(\theta/2) + 2W_1(\nu, Q^2) \sin^2(\theta/2) \right) . \quad (2-11)$$

The expression Eq. 2-11 is the analog of the Rosenbluth cross section of Eq. 2-3 for the elastic scattering case and was first introduced by Drell and Walecka [23].

2.2.2 DIS Cross Section as γ^*p Scattering

In deep inelastic scattering, as seen in Fig. 1, the interaction occurs between the virtual photon γ^* and the proton. The electron beam has a simple role, that is to provide the virtual photon for the interaction. Therefore, in order to have a better understanding of the cross section, it is useful to find an expression of the total cross section for $\gamma^*p \rightarrow X$ scattering. One can write the total cross section for a polarized photon interacting with an unpolarized proton as:

$$\sigma_\lambda^{tot} = \frac{4\pi^2\alpha}{K} \epsilon_\lambda^{\mu*} \epsilon_\lambda^\nu W_{\mu\nu} , \quad (2-12)$$

where $\epsilon_\lambda^{\mu(\nu)}$ is the polarization vector of the virtual photon, λ and K are the virtual photon helicity and flux respectively, with:

$$K = \frac{W^2 - M^2}{2M} , \quad (2-13)$$

and

$$\begin{aligned}\lambda = \pm 1 : \epsilon_{\pm} &= \frac{1}{\sqrt{2}} \mp (0; 1, \pm i, 0) \ , \\ \lambda = 0 : \epsilon_0 &= \frac{1}{\sqrt{Q^2}} (\sqrt{\nu^2 + Q^2}; 0, 0, \nu) \ .\end{aligned}\tag{2-14}$$

Using Eq. 2-14 along with the structure function $W_{\mu\nu}$ formula Eq. 2-10, the longitudinal and the transverse photonuclear cross sections can be expressed as:

$$\begin{aligned}\sigma_T &\equiv \frac{1}{2}(\sigma_+^{tot} + \sigma_-^{tot}) = \frac{4\pi^2\alpha}{K} W_1(\nu, Q^2) \ , \\ \sigma_L &\equiv \sigma_0^{tot} = \frac{4\pi^2\alpha}{K} \left[\left(1 + \frac{\nu^2}{Q^2} \right) W_2(\nu, Q^2) - W_1(\nu, Q^2) \right] \ .\end{aligned}\tag{2-15}$$

Substituting $W_1(\nu, Q^2)$ and $W_2(\nu, Q^2)$ from Eq. 2-15 into 2-11, one can express the total differential cross section in terms of longitudinal and transverse photonuclear cross sections:

$$\left(\frac{d^2\sigma}{dE'd\Omega} \right) = \Gamma (\sigma_T + \epsilon\sigma_L) \ ,\tag{2-16}$$

where ϵ is the relative flux of longitudinal polarized virtual photons and is given by:

$$\epsilon = [1 + 2(1 + \nu^2/Q^2) \tan^2(\theta/2)]^{-1} \ ,\tag{2-17}$$

and Γ is the flux of transverse virtual photons,

$$\Gamma = \frac{\alpha K E'}{2\pi^2 Q^2 E (1 - \epsilon)} \ .\tag{2-18}$$

Equation 2-16 is of great important to extract separated transverse and longitudinal cross sections from the measured differential cross section. When taking the ratio of the

differential cross section to the flux Γ , we obtain:

$$\sigma_R = \frac{1}{\Gamma} \frac{d^2\sigma}{dE'd\Omega} = \sigma_T + \epsilon\sigma_L, \quad (2-19)$$

where σ_R is the so called reduced cross section. It is clear from Eq. 2-19 that σ_R is linear with ϵ . Therefore, when plotting the reduced cross section versus ϵ (see Fig. 2), the intercept of the linear fit ($\epsilon = 0$) gives the value of σ_T , and the slope gives that of σ_L . The above technique of extracting the transverse and longitudinal cross sections from the measured cross section is called *Rosenbluth separation technique* [15].

Figure 2: An example for implementing the Rosenbluth separation technique to extract the longitudinal and transverse structure functions from the measured cross section.

2.2.3 Bjorken Scaling and The Quark Parton Model (QPM)

As mentioned above, in DIS the interaction occurs between the virtual photon γ^* and the hadron. If the Q^2 of the virtual photon is large enough so that the wavelength is smaller than the size of the scattering center, i.e. $\lambda \approx 1/\sqrt{Q^2} \ll 1$ fm, the proton constituents (quarks) will appear as point-like, spin- $\frac{1}{2}$ particles. Those quarks will start behaving as *free* Dirac particles. Hence, the proton structure functions can be written as:

$$\begin{aligned} 2MW_1(\nu, Q^2) &= \frac{Q^2}{2M\nu} \delta(1 - Q^2/2M\nu) , \\ \nu W_2(\nu, Q^2) &= \delta(1 - Q^2/2M\nu) . \end{aligned} \tag{2-20}$$

From Eq. 2-20, it is clear that the "*point-like*" structure functions, depend only on the ratio $Q^2/2M\nu$ and not on the kinematics variables Q^2 and ν independently. Therefore, it is safe to state that: "at high enough Q^2 , the virtual photon will resolve *point-like* constituents inside the proton, and the structure functions will depend only on one dimensionless variable, $Q^2/2M\nu$ ".

It was Bjorken [24] who predicted, in 1969, that in the limit of Q^2 and ν approaching infinity, the structure functions W_1 and W_2 will become functions of a single dimensionless variable, the so called Bjorken variable which is given by:

$$x = \frac{Q^2}{2M\nu} . \tag{2-21}$$

in a reference frame where the proton moves with infinite momentum ("*infinite momentum frame*"). That is, in the so called "Bjorken limit", where $\nu \rightarrow \infty$ and $Q^2 \rightarrow \infty$, with x being fixed:

$$\begin{aligned} MW_1(\nu, Q^2) &= F_1(x) , \\ \nu W_2(\nu, Q^2) &= F_2(x) . \end{aligned} \tag{2-22}$$

This property is called "scaling" in the variable x . F_1 and F_2 are structure functions that depend only on one variable, namely x , and said to satisfy the "Bjorken Scaling". Having the structure functions Q^2 -independent suggests strongly that the target proton behaves as a collection of point-like constituents in the infinite momentum frame.

The point-like structure of the proton proposed by Bjorken, is in agreement with the concept of **partons** proposed by Fynman [25] and the *Quark Parton Model* (QPM) [26, 27]. According to the QPM, the nucleon is composed of pointlike constituents "partons" with spin 1/2, named "quarks". The inelastic electron-nucleon scattering is hypothesized to arise from the sum of incoherent elastic scattering of electrons on the quarks in the nucleon. The fractional momentum of the nucleon carried by the struck quark is given by the Bjorken scaling variable, x . Neglecting any quark momentum transverse to the nucleon so that

$$P_{quark} = xP_{nucleon} \ ,$$

It is easy to obtain the elastic cross section for a quark with momentum $xP_{nucleon}$. In the QPM, and in the limit $\nu \rightarrow \infty$, $Q^2 \rightarrow \infty$ with fixed x (Bjorken limit), the structure functions F_1 and F_2 can be expressed in terms of quark distribution functions:

$$F_1(x) = \frac{1}{2} \sum_i e_i^2 q_i(x), \quad F_2(x) = x \sum_i e_i^2 q_i(x), \quad (2-23)$$

where e_i is the charge of quark type i , and $q_i(x)$ is the momentum distribution of the quark i inside the proton. In other words, $q_i(x)$ gives the probability to strike a quark of flavor i inside the nucleon carrying a momentum in the range between x and $x + dx$, and the sum runs over all quarks whose charges are e_i . The relation of Eq. 2-23 is known as the Callan-Gross relation.

The proton is assumed to consist of three "valence" quarks, that form the proton

charge and quantum number, and "sea quarks". The latter are basically a collection of many quark-antiquark pairs. The proton and neutron structure functions can be written as:

$$\begin{aligned}\frac{1}{x}F_2^p &= \frac{1}{9}[4u_v + d_v] + \frac{4}{3}S, \\ \frac{1}{x}F_2^n &= \frac{1}{9}[u_v + 4d_v] + \frac{4}{3}S\end{aligned}\tag{2-24}$$

where $u_v(d_v)$ is the probability distribution of the valence up(down) quark inside the proton, S is the distribution function of the sea quarks of all quark flavors, which is assumed to be the same for proton and neutron. For more details see Ref. [26].

Experimentally, SLAC [2,28] uncovered the first evidence that the nucleon consists of structure-less particles, and that the cross cross section is an incoherent sum of individual elastic scattering cross sections from these constituents. When plotting the differential cross section divided by the Mott cross section $[(d^2\sigma/d\Omega dE')/(d\sigma/d\Omega)_{Mott}]$ versus the invariant momentum transfer Q^2 for constant values of the invariant mass W^2 (Fig. 3), it can be seen that the probability of the deep inelastic scattering decreases with Q^2 much more slowly than for elastic scattering as compared for inelastic scattering [2]. Such a result indicates that the electrons were hitting "hard cores" inside the nucleon. When the x variable is kept fixed for $Q^2 > 1 \text{ GeV}^2$ and $W^2 > 4 \text{ GeV}^2$, the outstanding phenomenon of Bjorken scaling is observed (Fig. 4).

Figure 3: $(d^2\sigma/d\Omega dE')/(\sigma/\sigma_{\text{Mott}})$ versus Q^2 as introduced in Ref. [2].

Figure 4: World data on the structure function $F_2(x)$ of the proton from the Particle Data Group [29].

The cross section for inelastic scattering Eq. 2-11 can be re-written in terms of the structure functions F_1 and F_2 of the nucleon as:

$$\frac{d^2\sigma}{dE'd\Omega}(E, E', \theta) = \frac{4\alpha^2(E')^2}{Q^4} \cos^2\left(\frac{\theta}{2}\right) \left[\frac{F_2(x)}{\nu} + \frac{2F_1(x)}{M} \tan^2\left(\frac{\theta}{2}\right) \right] . \quad (2-25)$$

The structure functions F_1 and F_2 are related to the photoabsorption cross sections for transverse photons (with helicity ± 1), σ_T , and longitudinal scalar photons (with helicity 0), σ_L , given by Eq. 2-15, as follows:

$$\begin{aligned} F_1 &= \frac{MK}{4\pi^2\alpha} \sigma_T , \\ F_2 &= \frac{\nu K}{4\pi^2\alpha} \frac{Q^2}{Q^2 + \nu^2} (\sigma_T + \sigma_L) , \end{aligned} \quad (2-26)$$

where K is the effective momentum of the virtual photons. The structure function F_1 is only related to σ_T , therefore it is purely transverse. F_2 , however, is a combination of both σ_T and σ_L . A purely longitudinal structure function F_L can be introduced, and is defined as follows:

$$F_L = \left(1 + \frac{Q^2}{\nu^2}\right) F_2 - 2xF_1 . \quad (2-27)$$

One can also introduce the reduced cross section of Eq. 2-19 in terms of the structure functions F_1 and F_L :

$$\sigma_R = \frac{1}{\Gamma} \frac{d^2\sigma}{dE'd\Omega} = \frac{4\pi^2\alpha}{x(W^2 - M^2)} [2xF_1 + \epsilon F_L] . \quad (2-28)$$

The Rosenbluth separation technique can be implemented as mentioned earlier to extract the structure functions F_1 , F_L and F_2 . Introducing the quantity R , which is the ratio of

the longitudinal to transverse structure function (see Introduction/Chapter 1),

$$R = \frac{\sigma_L}{\sigma_T} = \frac{F_L}{2xF_1} \quad , \quad (2-29)$$

equation 2-16 can be re-written in terms of the ratio R as :

$$\frac{d^2\sigma}{dE'd\Omega} = \Gamma\sigma_T(1 + \epsilon R) \quad . \quad (2-30)$$

This form of the cross section (Eq. 2-30) is of a great significance in studying the distribution of quarks and gluons inside the nucleon, which will be discussed in detail later.

2.2.4 Scaling Violation

In the infinite momentum frame, and using the scaling invariance property, the structure function F_2 is well understood in the perturbative QCD (pQCD) framework. F_2 is interpreted as an incoherent sum of quark distribution functions (Eq. 2-23). Therefore, it is assumed that the number of partons does not change with Q^2 and the scaling should not be violated at all. However, data (Fig. 4) show that the structure function F_2 has some Q^2 dependence. This clearly indicates a scaling violation when the four-momentum transfer is reduced. Therefore, the Bjorken scaling is not expected to hold for low Q^2 . Data show that for low x values, F_2 increases with increasing Q^2 , while for high x , F_2 slightly decreases with increasing Q^2 . This scaling violation can be explained in terms of gluon splitting and gluon radiation. At low x , the sea quarks dominate and the gluon splits into a quark-antiquark pair, which leads into higher F_2 with increasing Q^2 , while at high x , the valence quarks dominate and some may radiate a gluon. This gluon radiation pushes the quark into lower x causing F_2 to fall with increasing Q^2 . Additional processes may cause the scaling violation within the pQCD, such as gluon self-couplings, or gluon

annihilations (see Ref. [30]).

Perturbative QCD predicts a logarithmic scaling violation at high Q^2 . The structure functions cannot be calculated within the framework of QCD. However, knowing the structure function value at a certain Q_0^2 , it is possible to calculate the Q^2 evolution of the structure function in pQCD. The Q^2 evolution of the structure functions is described by the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equations [31–33], that predict a $\log(Q^2)$ dependence of the structure functions. This dependence is well verified and is given by processes such as gluon radiation shown in parts b, c and d of Fig. 5. The corrections arising from such processes, are described by the DGLAP equations, which are applicable for high Q^2 values. However, for small values of Q^2 , the Q^2 evolution of the structure functions is not fully understood.

Figure 5: Some of the gluon radiation corrections (b, c and d) to the QPM (a)

For the low Q^2 regime, the Q^2 -dependence of the structure functions contains additional power corrections in the form of $1/(Q^2)^n$. These power corrections to pQCD are due to the non-vanishing target nucleon mass and to higher twist effects. In the QPM, and for large Q^2 , the target mass and higher twist effects are negligible. However, for low Q^2 , these effects can be significant and correction terms have to be applied [34].

A useful variable to study the QCD scaling violation in the structure functions is

Nachtmann's variable ξ that was introduced by O. Nachtmann [35], and is given by:

$$\xi = \frac{2x}{1 + \sqrt{1 + \frac{4M^2x^2}{Q^2}}} . \quad (2-31)$$

From the expression in Eq. 2-31, it is clear that as $Q^2 \rightarrow \infty$, $\xi \rightarrow x$. For low Q^2 values, it is useful to use the Nachtmann variable for scaling instead of x .

2.3 Structure Functions in the Nuclear Resonance Region

At low Q^2 , the kinematic region in W^2 that is dominated by the excitation of nucleon resonant states, is called the nucleon resonance region. This region is typically valid when $W^2 < 4 \text{ GeV}^2$ and $0.5 \leq Q^2 \leq 4.5 \text{ GeV}^2$. Resonances are excited bound states of quarks and gluons. There are almost twenty well-established nucleon resonances with masses below 2 GeV which give rise to distinct enhancements in the measured inclusive cross section (see Fig. 6). The first resonance region is due to single state, the $P_{33}(1232)$ Δ isobar, while the others are a composite of overlapping states. The second resonance region, that is comprised of primarily the $S_{11}(1535)$ and $D_{13}(1520)$ states, is generally referred to as the S_{11} region due to the dominance of the $S_{11}(1535)$ state at higher Q^2 .

Figure 6: The $ep \rightarrow eX$ cross section as a function of the missing mass W . Data are from SLAC (see [30], pg. 180).

In DIS, both of the energy transfer ν and the four-momentum transfer Q^2 are sufficiently large. Therefore, as mentioned in Section 2.2.3, the nucleon will be seen by the virtual photon as a collection of *asymptotically free* quarks and gluons. The structure function is interpreted, then, as an incoherent sum of the *point-like* quark distribution functions (Eq. 2-23). In addition, for $W^2 > 4 \text{ GeV}^2$ the perturbative QCD (pQCD)

provides good prediction for the Q^2 -dependence of the structure function. In addition, R is assumed to be very small in the QPM, $\sigma_L/\sigma_T \rightarrow 0$, which implies that the virtual photon is being scattered by a spin- $\frac{1}{2}$ parton. However, for small W^2 and Q^2 , i.e. in the resonance region, a scaling violation along with target mass corrections and higher twist effects will become significant, moreover, R may not be small due to strong gluon binding effects [36]. Therefore, the structure function of the resonance region is not expected to be trivial as the one in DIS region.

2.4 Quark-Hadron Duality

As mentioned earlier, at high energies, pQCD is well defined and can fully describe the experimental results in terms of asymptotically free quarks and gluons, while the chiral perturbation theory (ChPT) is effective to the study of the dynamics of QCD at low energies. *Duality* is an experimental technique, to connect the free partons at high energies with the confined partons at low energies.

By examining early inclusive electron-proton scattering data from SLAC, Bloom and Gilman [37–39] discovered a remarkable connection between the structure function in the resonance region and the deep inelastic region. Defining a new dimensionless variable w' :

$$w' = \frac{2M\nu + M^2}{Q^2} = w + \frac{M^2}{Q^2} = 1 + \frac{W^2}{Q^2} \quad (2-32)$$

where M is the nucleon mass, $w = 1/x$, Q^2 and W^2 are defined in Eq. 2-6 and Eq. 2-7 respectively. The goal was to find a variable w' such that $w' \rightarrow w$ as $Q^2 \rightarrow \infty$, yet suitable to study scaling when ν and Q^2 are finite. Bloom and Gilman, using w' , were the first to make a quantitative observation of quark-hadron duality in electron scattering.

The structure function for the resonance region was extracted from the measured cross section using a fixed value of the longitudinal and transverse cross section ratio

$R = \sigma_L/\sigma_T = 0.18$. The scaling curve in Fig. 7 is a parametrization of high Q^2 data [40].

Bloom and Gilman made the following observations:

- The resonance data clearly oscillate about, and average to the scaling curve;
- The agreement of the resonance data with the scaling curve gets better with increasing Q^2 .

The above observations, led Bloom and Gilman to conclude that: *" the resonances are not a separate entity but are an intrinsic part of the scaling behavior of νW_2 "*. This remarkable connection is called Quark-Hadron Duality. In order to make a quantitative

Figure 7: The function νW_2 plotted versus w' compared to a smooth fit to the data in the scaling-limit region. The data were taken for $Q^2 = 0.75 - 3.0 \text{ GeV}^2$ and $R = 0.18$ (see text).

connection between the low- W^2 region and the deep inelastic region, Bloom and Gilman derived a finite energy sum rule, given by

$$\frac{2M}{Q^2} \int_0^{\nu_m} \nu W_2(\nu, Q^2) d\nu = \int_1^{1+W_m^2/Q^2} \nu W_2(w') dw' . \quad (2-33)$$

The upper limit, $\nu_m = (W_m^2 - M^2 + Q^2)/2M$, corresponds to the maximum value of $w' = 1 + W_m^2/Q^2$, where W_m is the hadron mass and $\simeq 2$ GeV. W_m is a cutoff value so that the integral of the scaling function covers the same range in w' as the resonance region data.

The area under the resonance region (Fig. 7) was compared to the area under the smooth scaling curve, for values of Q^2 from 1.0 GeV² up to 3.0 GeV² (table I in Ref. [38]). A comparison of both sides of Eq. 2-33 showed that the relative differences ranged from $\sim 10\%$ at $Q^2 = 1$ GeV², to $\sim 2\%$ for energies beyond $Q^2 = 2$ GeV². Increasing the value of Q^2 led to changing both sides of the equation by about an order of magnitude. Using Bloom-Gilman duality along with the above sum rule, one can describe qualitatively the data in the range $1 \leq Q^2 \leq 10$ GeV².

2.5 Nuclear Dependence of R

Figure 8: The ratio of the nucleon structure functions per nucleon F_2^N measured on iron and deuterium as a function of Bjorken x [41] .

The difference in the cross section ratio for iron and deuterium targets (see Fig. 8), known as the EMC effect, has inspired a series of theoretical studies that try to explain the change of quark distributions in nuclei compared to those in free nucleons. In order to compare the theoretical predictions for the structure function ratio with the experimentally extracted cross section ratio, it is important to measure the differences in R for a variety of nuclear targets.

The extraction of R is experimentally challenging; it requires separating the F_2 structure function into F_1 and F_L using the Rosenbluth separation technique, i.e. measuring the cross section with high precision at different ϵ values (energies) at fixed x and Q^2 . A few experiments were able to obtain model-independent data, such as NMC [7] and SLAC [8], to extract R for several nuclear targets. These experiments concluded that ΔR was consistent with zero (see Fig. 9). However, they were obtained at high Q^2 , where R itself is nearly zero and so any difference between nuclei would be quite small. Because

it is both a small and challenging to measure quantity, these experiments were not very sensitive to any nuclear dependence.

However, some model-independent experiments [9,10] provided a first hint of a nuclear dependence of R for deuterium ($\sim 30\%$) at low Q^2 (Fig. 10)

Figure 9: The SLAC [8] results for $R_A - R_D$ (Left), and the NMC [7] measurements for $R_C - R_{Ca}$ (Right) as a function of x .

The transition from the cross section ratio, σ^A/σ^D , to the structure function ratio, F_2^A/F_2^D is valid with the assumption $\Delta R = 0$ or $\epsilon = 1$ (see Eq. 1-1). The extraction of the structure function ratio F_1^A/F_1^D , on the other hand, from the measured cross section ratio is done using the following formula:

$$\begin{aligned} \frac{\sigma^A}{\sigma^D} &= \frac{F_1^A(x, Q^2)(1 + \epsilon R_A)}{F_1^D(x, Q^2)(1 + \epsilon R_D)} \\ &\approx \frac{F_1^A(x, Q^2)}{F_1^D(x, Q^2)} \left[1 + \frac{\epsilon \Delta R}{1 + \epsilon R_D} \right]. \end{aligned} \quad (2-34)$$

It can be seen from Eq. 2-34 that at $\epsilon = 0$, $\sigma^A/\sigma^D = F_1^A/F_1^D$.

The measurements of R provided by NMC [7] were taken with $\epsilon \rightarrow 1$, which implies that $\sigma^A/\sigma^D = F_2^A/F_2^D$ even if $\Delta R \neq 0$. On the other hand, for SLAC experiment E140 [8], $\epsilon \neq 0$, which means that the extraction of F_1^A/F_1^D depends explicitly on the

Figure 10: $R_D - R_H$ obtained from JLab E00-002 [9] and E99-118 [10] experiments as a function of Q^2 .

assumption adopted for ΔR . The impact of the nonzero ΔR on extracting the ratios F_2^A/F_2^D and F_1^A/F_1^D was introduced by V. Guzey *et al.* [42]. It was found that the nuclear enhancement in the interval $0.1 < x < 0.3$ (so called antishadowing region) becomes significantly reduced (or even disappears in some cases) for the ratio of the transverse structure functions F_1^A/F_1^D (see Fig. 11). This observation indicates that antishadowing is dominated by the longitudinal contribution rather than the transverse one (i.e., the antishadowing is dominated by gluons rather than quarks).

A nonzero ΔR might be also an indication of the existence of the nuclear pions. According to the nuclear pionic model, pions are believed to be the carrier of the nuclear force and hence the dominant agent responsible for the binding of nuclei. Yukawa's original theory proposed that the strong nuclear force is transmitted between nucleons by the exchange of pions. Several attempts were made to explain the nuclear dependence

Figure 11: The impact of the nontrivial nuclear dependence of R on the structure function ratios around the antishadowing region for NMC Ca/D [7] (Left) and for SLAC E140 [8] Fe/D (Right).

of the structure function in terms of nuclear pions, however, they failed to explain the experimental data at all kinematics. The question "where are the nuclear pions?" became an outstanding problem in nuclear physics.

Measurements of $R = \sigma_L/\sigma_T$ could verify the pion existence in the nucleus. A small value of R is a signature of spin- $\frac{1}{2}$ partons, while a large value of R is an indication of bosonic constituents. Calculations made by Miller [11] predict a significant pion enhancement in the σ_L^A/σ_L^D ratio at low Q^2 and moderate x as shown in Fig. 12.

Figure 12: Enhancement of longitudinal cross sections, as a function of Q^2 and x .

To study the role of pionic effects on σ_L for nuclear and nucleon targets, one may use the pionic contribution to the hadronic tensor:

$$\delta^\pi W^{\mu\nu} = \frac{1}{4\pi M} \int d^4\xi e^{iq \cdot \xi} \langle P | J_\pi^\mu(\xi) J_\pi^\nu(0) | P \rangle , \quad (2-35)$$

where $|P\rangle$ represents the nucleon or nuclear target ground state of total momentum P , $J_\pi^\mu(\xi)$ is the current operator that accounts for the effects arising from the nuclear pions:

$$J_\pi^\mu = i(\phi^* \partial^\mu \phi - \phi \partial^\mu \phi^*) , \quad (2-36)$$

with ϕ being the complex pion field operator. Using the standard formula for σ_L from Eq. 2-12 [30], we obtain:

$$\sigma_L = \frac{4\pi^2 \alpha Q^2}{(Q^2 + \nu^2)^{3/2}} W^{00} . \quad (2-37)$$

Now, using Eq. 2–35, one can evaluate $\delta^\pi W^{00}$ by using J_π^0 . Ignoring the interactions between the struck pion with the residual nucleus, taking the energy of the outgoing pion to be equal to ν , and ignoring the effects of nonzero values of nuclear excitation energies appearing in the energy-momentum conserving delta function in Eq. 2–35, we obtain:

$$\delta^\pi W^{00}(N, A) = \frac{\nu^2}{2Q^2} \frac{2}{3} \frac{f_{\pi/(N,A)}(\xi)}{M} F_\pi^2(Q^2) \ , \quad (2-38)$$

where $F_\pi(Q^2)$ is the pion form factor, $f_{\pi/(N,A)}(y)$ is the probability to find an excess pion in the nucleon (N) or the nucleus (A), with a momentum given by ym_N . The function $f_{\pi/N}(y)$ is constrained by the data on the nucleon sea which restricts the $\bar{u}$ and $\bar{d}$ distributions as similar [43, 44]. Equations 2–37 and 2–38 allow us to connect the nuclear longitudinal cross section per nucleon $\sigma_L(A)$ to the corresponding pionic contribution (per nucleon) $\delta^\pi \sigma_L$, so that $\sigma_L(A) = \sigma_L(D) + \delta^\pi \sigma_L$. It is desirable to present the pionic enhancement using the ratio $\sigma_L(A)/\sigma_L(D)$ (Fig. 12), which can be evaluated using Eq. 2–38 to be:

$$\frac{\sigma_L(A)}{\sigma_L(D)} = 1 + x \frac{2}{3} f_\pi(\xi) \frac{\nu^2}{(Q^2 + \nu^2)} \frac{F_\pi^2(Q^2)}{F_2^D R_D} (1 + R_D) \ , \quad (2-39)$$

where F_2^D and R_D are parametrized using Whitlow's algorithms [5].

Fig. 12 shows Miller's results of σ_L on ^{56}Fe [11]. The largest effect is expected for the lowest value of $Q^2 = 0.3 \text{ GeV}^2$, corresponding to a very low value of $\nu \sim 0.4 \text{ GeV}$.

2.6 E04-100 Experiment

Jefferson's lab experiment 04-001, "Measurement of Longitudinal-Transverse (L-T) Separated Structure Functions F_2 and $R = \sigma_L/\sigma_T$ from Nuclear Targets in the Resonance Region" [12], was carried out in Hall C in January 2005. The experiment, *Jan05*, is considered to be the first phase of two experiments. The second experiment 06-009 (*Rosen07*) was carried out in the summer of 2007 which covers a higher Q^2 region. Fig.13 shows the kinematics of both experiments.

E04-001 experiment was part of a full program to investigate the quark-hadron duality in electron scattering on nuclear targets for the separated structure functions. The targets included Carbon, Iron, and Aluminum to complement the existing data on Hydrogen and Deuterium. The experiment ran in the nucleon resonance region ($1 < W^2 < 4.5$ GeV²) spanning the four-momentum transfer range $0.5 < Q^2 < 2.5$ (GeV/c)² at identical kinematic points to those that ran during JLab experiment 02-109 [6]. The targets used during the experiment closely resemble nuclear targets commonly used in neutrino experiments such as the low statistics bubble chamber experiments on Hydrogen, Deuterium, Neon, and Argon and high statistics neutrino experiments on Scintillators (MINERvA, MiniBONNE, K2K and JPARC), Water (SuperK, JPARC), Steel (MINOS) and Argon (CNGS).

The data from E04-001 experiment will be used as input for vector form factors in a future analysis of neutrino data in order to investigate the quark-hadron duality in the axial structure function of nucleons and nuclei, which will be done on collaboration with the MINERvA neutrino experiment [45] at Fermilab. The measurements of *Jan05* should help in reducing the uncertainties in neutrino oscillation parameters for neutrino oscillation experiments such as JPARC and NUMI Off-axis.

In addition, a large unknown is the value of R in the resonance region (see Section 2.5).

The nuclear dependence of R in the resonance region at low Q^2 and W^2 is not known. A measurement of R in the region covered by this experiment provides information about the dynamics of the internucleon force inside nuclei. For example, as mentioned before, the calculations made by Miller (see Section 2.5), predict a significant pion excess enhancement in the σ_L^A/σ_T^D ratio at low Q^2 and moderate x . The kinematics of Miller's maximum pionic enhancement effects fall within the measurements of this experiment.

Figure 13: Kinematic coverage range of Phase I (*Jan05*) (Top) and Phase II (*Rosen07*) (Bottom) for JLab experiments E04-001, E02-109 and E06-009.

Chapter3: Experimental Setup

Overview

Measuring the inclusive cross section, $d^2\sigma/dE'd\Omega$ (Eq. 2–11), requires a precise measurement of the electron's incident beam energy, scattering energy and angle. The beam was generated by the Continuous Electron Beam Accelerator Facility (CEBAF) of JLab. In order to have a successful implementation of the physics program, the beam had to be transported with specific requirements onto the reaction target. The scattered particles were detected with the Hall C High Momentum Spectrometer (HMS), while the background events were detected using the Short Orbit Spectrometer (SOS). The HMS provided information about the energy and angular distribution of the scattered events. The unscattered beam was transported to a high power water cooled beam dump. Data from the detectors were digitized by front-end hardware, and the events were recorded and selected using a data acquisition system (DAQ) and a designed trigger system.

The experimental setup of experiment E04-001 will be described in this chapter, and will be organized as follows:

- The Beamline:
 - The Accelerator
 - Beam position measurement
 - Beam energy measurement
 - Beam current moneters
 - Beam raster

- Experimental Hall C
- Target System
- High Momentum Spectrometer
- Data Acquisition System DAQ and the Trigger Design

Experiment E04-001 was carried out at Jefferson Lab in January 2005. Since then, JLab went through a 12 GeV upgrade [46], and Hall C has replaced one of the spectrometers that were used during this experiment [47]. Therefore, this chapter will describe the beam properties and Hall C apparatus as they were during the time of the data taking of the E04-001 experiment.

3.1 Beamline

3.1.1 The Accelerator

The Continuous Electron Beam Accelerator Facility (CEBAF), a five-pass recirculating machine (Ref. [48]) provided a high power electron multi-GeV beam simultaneously to three end Halls A, B and C ¹. The accelerator produced short term beam pulses ≈ 1.67 ps, which provided a continuous wave (CW) electron beam. The basic design of the accelerator, in 2005, is shown in Fig. 14. The injector, which operated as an electron gun, produced electrons up to 67 MeV with polarization $\sim 85\%$. The electrons were then accelerated by the North Linac which added 600 MeV to the kinetic energy of the electrons. The beam was steered through the East Arc and then was accelerated again in the South Linac by another 600 MeV. When the electron beam passes by the North Linac and South Linac once, we call it *one pass*, and the electron beam can pass up to five passes through the linacs reaching energies up to 6 GeV. After the beam reached

¹In 2005, only three halls were running. Nowadays, there is a fourth hall (Hall D).

the desired energy, it was extracted at the end of the South Linac to be delivered to the experimental Hall.

Figure 14: Schematic view of Jefferson Lab Accelerator.

Figure 15: Hall C Arc and measurement devices.

3.1.2 Beam Position Measurement

The beam profile position was monitored by superharps installed along the beamline of the Hall C Arc. The superharp consists of a wooden fork holding three tungsten perpendicular wires. When the beam passes by, the electrons will interact with the wires. The horizontal and vertical position of the beam are determined by digitizing the amplitude of the charge collected from the electrons using an Analog to Digital Converter (ADC) connected to each wire. The fork with the wires can move across the beam by means of a stepper motor system, which is connected to a position encoder to measure the position of the wires. Combining the information collected from both ADC's and position encoders, the beam position profile can be determined (see Fig. 16). Using the superharps to measure the beam position is very precise, yet destructive. In other words, the superharp scans can not be performed during the data taking, and must be performed before or after. The beam position was adjusted so that the beam axis intersected with the HMS axis at the pivot.

Figure 16: Schematic view of a superharp monitor.

To be able to measure the beam position while taking data, one can use the Beam Position Monitors (BPM's) which are less destructive. The accuracy of the BPM's is

checked by calibrating them using the superharps information.

The BPM is a cavity with four antennae rotated at 45° with respect to the horizontal and vertical axes (Fig. 17). This angle is needed in order to prevent any damage from synchrotron radiation .

Figure 17: Design of the Beam Profile Monitor (BPM).

When the beam passes through the BPM cavity, each antennae will pick up the fundamental frequency of the beam. The amplitude of the signal picked up is proportional to the distance of the beam from the antennae. This signal is then digitized to determine the beam's center of gravity (X' and Y'). By projecting the X' and Y' axis on the X and Y axis, both absolute and relative beam positions can be determined. The relative beam position measurements must be compared to the superharp measurements in order to get an accurate absolute beam position during data taking.

3.1.3 Beam Energy Measurement

When an electron moves within a magnetic field, the Lorentz force will cause it to move in a circular trajectory. Knowing the magnet's magnetic field and the electron's trajectory, one can determine the momentum of the electron.

During the beam energy measurement in the Hall C Arc, the focusing elements were

turned off and only dipole magnets were used to bend the beam into the hall (Fig. 18). The currents of the dipole magnets were varied to steer the beam to the central path of the magnets. Knowing the dipole magnetic field and the position of the electrons from the superharps, the electron momentum can be determined using the formula

$$P = \frac{e}{\theta_{arc}} \int B dl , \quad (3-1)$$

where e is the electron charge, θ_{arc} is the bending angle of the arc and dl is the beam path determined by superharps. The uncertainty of the beam energy using this method is $\approx 4 \times 10^{-4}$ [49].

Figure 18: Beam energy measurement in Hall C Arc.

3.1.4 Beam Current Monitors

Since the particle yield to be determined is proportional to the charge delivered to the target, the quality of the beam current measurement is of great importance for scattering experiments. To measure the beam current continuously, four devices were installed along the Hall C beam line: three microwave cavity Beam Current Monitors (BCM's), and one parametric DC current transformer (Unser monitor). The Unser monitor, and BCM1

and BCM2 are installed on the beam line between the final Arc and Hall C target (see Fig. 15). BCM3 was located upstream of the target and was used by the accelerator division's beam accounting system.

The BCM's consisted of cylindrical wave guides that contain wire loop antennas. The dimensions of the cylinders were chosen so that their natural frequencies have harmonics at the pulse frequencies of the accelerator pulse rate (1497 MHz) and at the Hall C pulse rate (499 MHz). The antennae will pick up the signals from these cavities, then the picked-up signals will be integrated and will be proportional to the beam current. The BCM cavities have stable gain and offset and a high signal/noise ratio, however, they can not measure the beam current absolutely and need to be calibrated using the Unser several time throughout the run period.

The Unser can not be used for charge measurement because it is sensitive to thermal fluctuations which leads to a large drift in its zero-offset. However, the Unser monitor has a very stable gain which makes it a perfect tool to calibrate the gain of the BCM's. The BCM's, on the other hand, have a very stable offset, but the gain needs to be calibrated a few times during the experiment

The total beam charge delivered on the target, C_{tot} , is obtained from the current measurements using:

$$C_{tot} = I_{ave} \cdot t \quad (3-2)$$

where t is the time of the run, and I_{ave} is the average current measured using the beam charge monitors (BCM's) as discussed above. During the calibration run, the current was ramped up with alternating beam-on and beam-off intervals for about two minutes (Fig. 19).

Figure 19: Unser and BCM's frequency *vs.* time.

The BCM's calibration was achieved by comparison to the Unser monitor, and then was used to determine the charge of all runs. The beam-off intervals were used to determine the Unser zero offset, while the beam-on intervals were fitted to identify the gain. For each BCM, the gain and offset values for each calibration run were fitted linearly (see Fig. 20), and the fit value was used in the replay engine to calculate the beam current for each run.

The systematic uncertainty from on the current measurement due to the calibration procedure was of $0.2 \mu\text{A}$. This value was estimated by calculating the BCM fit residuals (Fig. 21).

Figure 20: BCM calibration results for E04-001 experiment.

Figure 21: BCM calibration residuals.

3.1.5 Beam Raster

CEBAF delivers a high current electron beam to Hall C, which has a small transverse size ($\leq 200\mu\text{A}$ FWHM). A beam with these characteristics can damage the target material, by causing a local boiling to the cryo-target or melting the solid target. Therefore,

the beam has to be rastered before striking the target using two fast-raster magnets located upstream of the target. One set rasters the beam in the vertical direction and the other in the horizontal direction. The raster size for this experiment was $2 \text{ mm} \times 2 \text{ mm}$.

3.2 Experimental Hall C

Hall C is the second largest hall, and is built underground in a circular shape with a diameter of about 32 m. Figures 22 and 23 show a top view of Hall C. In the 6 GeV era, the main components of Hall C were: the beam line, the target scattering chamber, the High Momentum Spectrometer (HMS) and the Short Orbit Spectrometer (SOS). Each of those components will be discussed in detail in the following sections.

Figure 22: Hall C top view. HMS on the right and SOS on the left. The beam line passes through the scattering chamber in the middle and ends up at the beam dump.

Figure 23: Schematic view of Hall C. Figure was taken from Ref. [50].

3.3 Target System

The Hall C targets were located in a cylindrical aluminum scattering chamber, which is centered at the spectrometer pivot. The inner radius of the scattering chamber was 61.6 cm and its height was 150 cm. The side facing the HMS had a 0.4 mm aluminum exit window. The targets were mounted on a target ladder (see Fig. 24) and can be remotely moved from the counting house in a period of few minutes. The solid targets ([51], [52]) consisted of Copper, Carbon, Gold and Iron (see Tables 1 and 2). The second and third loops of the cryotargets were filled with Hydrogen and Deuterium respectively, whereas the first loop was left as a spare. The solid targets that were used during the data taking are: C, Fe, and Al. The focus on this work is on the analysis of the Carbon and Iron data.

Figure 24: Hall C target ladder. Figure was taken from Ref. [53].

Target	Thickness (gm/cm ²)	Radiation Length (%)
Carbon	0.3494 ± 0.0007	0.819
Iron	0.1186 ± 0.0003	0.857

Table 1: Thickness and radiation lengths of the solid targets

Target	Thickness (gm/cm ²)	Radiation Length (%)
4 cm Dummy (²⁷ Al) upstream foil	0.2626 ± 0.0005	2.19
4 cm Dummy (²⁷ Al) downstream foil	0.2633 ± 0.0005	2.19

Table 2: Thickness and radiation lengths of the Dummy target

3.4 High Momentum Spectrometer (HMS)

As mentioned before, the High Momentum Spectrometer was used during this experiment to detect electrons. It has a moderately large acceptance, good angular resolution, an extended target acceptance, and a large angular range. The HMS can be rotated in

the horizontal plane to any angle between 10.5° and 80.0° relative to the beam axis. The optical properties of the HMS are shown in Table 3.

Max. central momentum	7.4 GeV/ c
Momentum acceptance	$\mp 10\%$
Momentum resolution	$< 0.1\%$
Solid angle	6.7 msr
Scattering angle acceptance	∓ 40.0 mr
Out-of-plane angle acceptance	∓ 80.0 mr
Extended target acceptance	10.0 cm
Scattering angle resolution	1.0 mr
Out-of-plane angle resolution	2.0 mr

Table 3: The optical properties of the HMS.

3.4.1 HMS Optics Design

The HMS consists of three superconducting quadrupoles (Q_1 , Q_2 and Q_3) and one superconducting dipole in a QQQD arrangement as shown in Fig. 25. The dipole magnet is a superconducting cryostable magnet with an effective length of 5.26 m, a bend radius of 12.06 m, and a gap width of 42 cm. When 7.4 GeV/ c momentum particles pass through the dipole, it provides 25° bending angle for them. The quadrupoles Q_1 , Q_3 and the dipole provide a vertical defocusing which results in a good momentum resolution. The quadrupole Q_2 focuses in the horizontal plane providing a large momentum bite, a large solid angle and an extended target acceptance as seen in Table 3.

Figure 25: Schematic lay-out of the HMS magnetic elements (three quadrupoles and one dipole magnet), and the detector hut where the detectors are located.

3.4.2 HMS Detector Package

Figure 26: A Schematic side view of the HMS detectors.

The HMS detector package consists of the following detectors (see Fig. 26):

- Two drift chambers DC1 and DC2, for tracking;
- Two pairs of scintillator hodoscopes S1 and S2, for triggering;

- One gas Čerenkov counter, for particle identification and;
- One lead-glass calorimeter, for particle identification.

3.4.2.1 Drift Chambers

The HMS drift chambers are ionization gas detectors that are filled with a 50/50 % argon-ethane gas mixture. They are used to determine the trajectory (position and direction) of the particle when passing through the spectrometer. This is done by measuring the drift time of the ionization particles in the gas and use it to calculate the coordinates of the ionizing incoming particle. Knowing the optical transfer properties of the HMS, one can use the drift chamber hit coordinates to reconstruct the particle trajectory, reaction vertex and momentum of the particle at the target. The HMS drift chamber has $140\ \mu\text{m}$ spatial resolution per plane and an efficiency better than 98%, even with high rates [54]. The layout of the drift chamber is shown in Fig. 27.

Figure 27: A Schematic view of the HMS drift chamber.

The distance between the centers of the two drift chambers is 81.2 cm, and each has an active area of about 113 cm (x) by 52 cm (y). Each drift chamber has six planes of sense wires (anodes) and field wires (cathodes). The sense wires are 25 μm diameter gold-plated tungsten wires, that are separated by 1 cm and kept in a ground potential. Whereas the cathodes are made of 150 μm gold-plated copper beryllium wires and the planes are separated by 1.8 cm. A field wire that is held at a negative potential (-1800 to -2500 V) is placed between each sense wire. On both sides of the sense planes is an additional plane of field wires held at the same negative potential. The order of the sense planes as seen by the incoming particles is X, Y, U, V, Y' and X'. The X and X' planes provide two measurements of the particle's position in the dispersive direction, whereas the Y and Y' planes provide two measurements of the position in the transverse direction. The U and V are rotated by $\pm 15^\circ$ from the X (X') planes, to prevent the right-left ambiguity.

When a charged particle passes through a drift chamber, it ionizes the gas atoms producing electrons and ions. Then, the ionization electrons drift with a constant velocity to the nearest sense wire, where the electric field is generated between the negative field wires and positive sense wires. As the electrons get closer to the wire, the electric field accelerates the electrons causing a secondary electron avalanche which produces a negative pulse on the wire. The signals from each individual sense wire are amplified and discriminated in the drift chamber cards, and then are sent to Time-to-Digital Converters (TDC's) located in the back of the detector hut. Knowing the electron drift velocity and the drift time provided by the TDC's, the perpendicular distance between the wire and the particle track can be calculated providing the particle position. Tracking software uses the position information of the particle from each drift chamber to determine the trajectory of the particle that went through the spectrometer.

Figure 28: A diagram of a drift chambers cell structure. The teal dots represent the field wires and the magenta dots the sense wires. As the charged particle ionizes the gas mixture in the drift chambers, the electrons are attracted to the sense wires by the electric field generated by the filed wires.

Drift Chamber Calibration

As mentioned before, drift chambers provided the tracking information of the particle. This information was crucial for a precise position and momentum reconstruction of the event at the target. Each wire in the drift chambers was connected to a TDC. The TDC information was, then, converted into a perpendicular distance from the wire. Since the wires were placed close together, the distribution of events about the wire should be flat. The drift chamber calibration consisted of adjusting the time-to-distance coefficients for each wire so that the drift distance distribution is flat.

3.4.2.2 Scintillator Hodoscopes

The HMS has 4 hodoscopes, that are used for triggering the detector read-out when a charged particle passes through the detector stack. The hodoscopes are paired in two horizontal-vertical X-Y sets (S1X-S1Y, and S2X-S2Y), that are separated by 220 cm. Each X hodoscope consists of 16 horizontal scintillator paddles as shown in Fig. 29, while the Y hodoscope consist of 10 vertical scintillator paddles. All paddles are 1 cm thick and 8 cm wide, and the paddles are placed with 0.5 cm overlap to avoid having

gaps. The X paddle is 75.5 cm long, whereas the Y paddle is 120.5 cm long. Each paddle is wrapped by one layer of light-tight Aluminum foil and two layers of Tedlar. Photomultiplier Tubes (PMT's) are attached through light guides at both ends of each scintillator paddle.

Figure 29: The HMS Hodoscope Structure.

When a charged particle passes by the paddle, it excites the paddle atoms. In order to return to the ground state, those excited atoms emit light that will be detected by the PMT's at the ends of the paddle. The signals from each PMT are sent to a splitter that gives two signals with $1/3$ and $2/3$ of the amplitude of the original signal. The signal with smaller amplitude is sent to Analog-to-Digital Converters (ADC's) to determine the signal integral. The second part of the signal is discriminated and one set of the outputs is sent to TDC's to provide timing information, and the other set is sent to logic

modules. The logic modules first generate the logical OR of all the signals on each side of the hodoscope planes. For example, for the right side of the hodoscope plane 1:

$$S1X+ \equiv (S1X+ \text{ OR } S1X2+ \text{ OR } \dots \text{ OR } S1X16+)$$

and similarly for the left side:

$S1X- \equiv (S1X- \text{ OR } S1X2- \text{ OR } \dots \text{ OR } S1X16-)$ These two logic signals from each side of a hodoscope are combined as $S1X \equiv (S1X+ \text{ AND } S1X-)$ which checks if the plane S1X was fired. The same procedure is followed for all other paddles S2X, S1Y and S2Y. The four output logic signals indicate which of the hodoscope planes were triggered. If three out of four planes have fired, a new logic signal is generated and called SCIN.

Another logic signal is generated if one plane at least from the X-Y pairs was fired, and called STOF. In another words, if the condition $S1 \equiv (S1X \text{ OR } S1Y) \text{ AND } S2 \equiv (S2X \text{ OR } S2Y)$ was achieved, an STOF logic signal is generated. All of the logic signal outputs are sent to the main trigger logic and to the scalers to be recorded. For more detailed information about the HMS hodoscopes see Ref. [55]

3.4.2.3 Gas Čerenkov Counter

The HMS gas Čerenkov counter is a threshold detector, that provides information about the particle identification. It consists of a large cylindrical tank of 150 cm diameter and 165 cm length, and is placed in the middle of the detector stack between the hodoscope pairs S1 and S2 (see Fig. 26). The tank is filled with C_4F_{10} gas. When a charged particle passes through the gas with velocity above the speed of light in that gas, an electromagnetic radiation (so-called Čerenkov radiation) is emitted. The light is emitted with an angle $\cos(\theta) = 1/n\beta$, where β is the velocity of the incoming particle relative to the speed of light, and n is the material's index of refraction. Light will be emitted only if $n\beta < 1$. A pair of front reflecting spherical mirrors are mounted vertically with 1 cm overlap at the end of detector, and at a 15° angle to focus the emitted Čerenkov

light to a pair of PMT's (see Fig. 30). The C_4F_{10} gas is chosen so that electrons at the spectrometer momentum will emit Čerenkov radiation while pions will not. However, when a pion produces a knock-on δ -electron, the Čerenkov detector is fired and the pion can be misidentified as an electron.

Figure 30: Čerenkov detector geometry.

The PMT's will generate signals proportional to the number of Čerenkov photons. Then, each PMT signal is sent to the ADC's and TDC's in the counting house in a similar fashion as the logic signals of the hodoscopes. For detailed information about the Čerenkov detector see Ref. [55].

Čerenkov Calibration

The Čerenkov detector was calibrated by determining the minimum response produced by each PMT. This response corresponded to a single photo-electron signal. Then, the position of the single photo-electron peak in the ADC spectra was determined for each PMT. The calibrated signals from each PMT are summed to evaluate the total number of photo-electrons produced by each event.

3.4.2.4 Calorimeter

Along with the Čerenkov detector, the HMS lead-glass calorimeter is used for particle identification. Each charged particle deposits different energy based on its type Bremsstrahlung radiation and pair production processes. The calorimeter consists of 52 TF1 type lead glass blocks of $10\text{ cm} \times 10\text{ cm} \times 70\text{ cm}$, and at each end of the block, a PMT is attached (see Fig. 31). A set of 13 lead-glass blocks are arranged in one layer. The calorimeter has four layers which gives a total thickness along the particle direction of 16 radiation lengths. The calorimeter is rotated by 5° with respect to the dispersive plane to prevent particles from passing in between the blocks.

Figure 31: The HMS calorimeter geometry.

Figure 32: Electromagnetic shower of particles in the HMS calorimeter block.

When a charged particle (electron or pion) passes through the lead-glass, it decelerates due to interaction with the medium nuclei. Consequently, the charged particle loses kinetic energy in the form of electromagnetic γ radiation (this process is called Bremsstrahlung). If the photon γ is energetic enough, it produces electron-positron pairs (pair production process). These pairs, in turn, radiate photons that will produce more $e^- - e^+$ pairs, which leads to a production of a shower of particles (e^-, e^+, γ) in the calorimeter (see Fig. 32). This process keeps going until the electron (or pion) loses all its energy inside the calorimeter. The radiation produced inside each block is detected by the PMT's attached at each end of it. The produced signals in the PMT's are proportional to the incident electron total energy. In order to distinguish electrons from pions, the ratio of the total deposited energy of the particles in the calorimeter to the track momentum is measured (see Fig. 33). The electron peak is located at 1 while the pion peak is located at ~ 0.3 . Even though this method is good enough to separate electrons from pions, it will not reject all pions. Pions can undergo charge-exchange reaction and produce neutral pions, that can in turn undergo the process $\pi \rightarrow 2\gamma \rightarrow e^- + e^+$. These

products can deposit all their energies and result to a high energy tail for pion peak which leads to a pion-electron misidentification.

Figure 33: The normalized calorimeter track energy spectrum of the HMS. The electrons peaked at 1, whereas pions peaked around $0.3/E'$. The central HMS momentum for the above run was 1.5 GeV, and the pions left a clear peak at 0.2.

All the signals from the calorimeter PMT's are sent to a read-out electronics in the counting house. Each signal is split into two copies, one goes to the ADC modules and the other is discriminated and sent to linear modules to be summed as a part of the trigger. The second copy of the signals is discriminated to give three logic signals: PRSUM, SHSUM and PRLO. PRSUM is the sum of all signals from the first layer of the calorimeter, and is used to form the high and low thresholds: PRHI and PRLO on the energy of the first layer. SHSUM is the sum of the entire calorimeter signals that represents the total energy deposited in the calorimeter. SHLO is a cut for the total energy in the calorimeter.

3.5 Data Acquisition System and the Trigger Design

The Hall C data acquisition (DAQ) is handled by the CEBAF Online Data Acquisition software package (CODA) that is developed by JLab CODA group. Data for each run are written directly to a memory disk, and contains three types of information:

1. Information from detectors ADC's and TDC's.
2. Information from scalers.
3. Information from EPICS.

The information collected from the ADC's and TDC's are recorded for each event, whereas the the information from scalers are recorded every two seconds. The EPICS database quantities are read out every 30 seconds and they provide information about the magnet sittings, beam position and target temperature and pressure. For more information about the data acquisition system see Ref. [56].

For most scattering experiments, only one process among many possible recorded events in the detectors is of interest. Therefore, the trigger design is crucial in data taking to make sure to select the desired events. However, there are some reasons for not recording all events of interest, such as the detector inefficiencies and deadtime (which will be discussed in detail in the next Chapter).

Figure 34: Schematic of the HMS trigger logic.

As described in the previous sections, the charged particles will trigger one or more detector when passing through the HMS. Ultimately, all triggers generated by the detectors PMT's are sent to DAQ. Certain combinations of these triggers are used to produce the pretrigger. An electron trigger [ELREAL] is produced in one of the following ways:

1. [ELLO] : the low - level electron trigger is fired when the event provides a Čerenkov signal [$\check{C}$], and at least two out of three of the following requirements are met: i) at least one of the two scintillator layers of both hodoscopes have fired [STOF] (STOF $\equiv$ S1 AND S2). ii) at least three out of four scintillator layers of both hodoscopes have fired [SCIN]. iii) The [PRLO] signal from the calorimeter has fired.
2. [ELHI] : the high - level electron trigger is fired when all of the following signals are fired: i) [SCIN] signal, ii) PRHI signal, and iii) [SHLO] signal from the calorimeter. The [ELHI] trigger does not require the Čerenkov detector to have fired.

If either [ELLO] or [ELHI] triggers fired, the electron trigger [ELREAL] is fired ($\text{ELREAL} \equiv \text{ELLO OR ELHI}$). Having two electron triggers, [ELHI] and [ELLO], helps to increase the trigger efficiency. The HMS trigger system has also a pion trigger, [PION], that required a [SCIN] signal and no Čerenkov signal [Č]. Then, the [PION] signal is combined with a prescaling circuit to form [PIPRE]. [PIPRE] ensures that a low-rate sample of pions is sent to the DAQ. Two copies for the [ELREAL] signal will be generated: one copy will be fanned out and sent to four logic units with dead-times set between 30 to 120 ns to measure the electronic dead-time. The second copy of [ELREAL] along with [PIPRE] will form the [PRETRIG] signal ($\text{PRETRIG} \equiv \text{ELREAL OR PIPRE}$), that will be sent to the Trigger Supervisor (TS).

When a run starts, the first 1000 pedestal triggers are generated by (PED PRETRIG) and the DAQ will record the ADCs read-outs. Then, the DAQ starts recording physics events. When a physics event forms a [PRETRIG] signal, the DAQ starts recording the detector ADC's and TDC's read-outs. During the time of the DAQ recording, the TS sets TS BUSY . After the DAQ is done recording the physics event, the TS BUSY is set to OFF to allow recording the next physics event.

Chapter4: Data Analysis

4.1 Overview

The goal of this work is to measure the nuclear dependence of F_2 and R in the nucleon resonance region. This requires first determining the yield and then the cross section for inclusively scattered electrons. Some analysis steps needed to be followed in order to extract physics variables from the raw data, and achieve the experiment goals. Those steps are as follows:

1. Event Reconstruction: vertex position, momentum, and angles (in-plane and out-of-plane) (see Sec. 4.2).
2. Event selection to identify inclusively scattered electrons (see Sec. 4.3).
3. Determination of inclusive yields (see Sec. 4.4).
4. Determination of inclusive cross section (see Sec. 4.5).
5. Extracting F_L , F_1 , F_2 and R (see Sec. 4.7).
6. Measure the nuclear dependence by extracting ΔR (see Sec. 4.8).

4.2 Tracking and Event Reconstruction

After recording the raw data, the Hall C analysis software [55] decodes the detector hits and the scalers output. In addition, the software reads the raw data for all events, generates the necessary information related to each particle such as the particle's track and deposited energy, resulting in three types of output files: 1) Paw Ntuple files that contain event-by-event information, such as the position, momentum, energy loss, etc for

each event. These Ntuples are converted later to ROOT files and used as the main output for data analysis. 2) Scaler files which contain all hardware and software settings during each run, also contain the scalers output and detector hits and calculated efficiencies, and beam information. 3) Paw HBOOK files which contain several histograms to check the detector and hardware performance.

The analyzer includes an event reconstruction code that reads the drift chambers TDC signals (see Sec. 3.4.2.1), and generates a trajectory of each particle at the focal plane. That defines the position and direction of the particle at the focal plane x_{fp}, y_{fp}, x'_{fp} and y'_{fp} . x_{fp} is the particle's position in the dispersive direction, where $+x_{fp}$ points vertically downwards when looking along the spectrometer central ray from the target. y_{fp} is the particle's position in the non-dispersive direction ($+y_{fp}$ points left toward the beamline relative the spectrometer central ray). x'_{fp} and y'_{fp} are the slopes of the track in x and y direction ($x'_{fp} = x_{fp}/dz$ and $y'_{fp} = y_{fp}/dz$), where z is the direction along the axis of the spectrometer. Using a mapping matrix, the four focal plane quantities can be transformed into the target quantities that define the initial position and momentum of the particle at the target. The target quantities reconstructed are $y_{tar}, x'_{tar}, y'_{tar}$ and $\delta p/p$ (or δ for short). y_{tar} is the horizontal position of the interaction point, x'_{tar}, y'_{tar} are the slopes at the interaction point relative to the spectrometer central ray in the dispersive and non-dispersive planes, respectively. $\delta p/p = \delta = (p - p_0)/p_0$ is the fractional deviation of the particle's momentum p from the central value p_0 . Since only four variables can be reconstructed from the four focal plane variables, the vertical position at the target, x_{tar} , was not given by the transformation and was assumed to be equal to zero (the vertical spread of the beam was neglected). This assumption is corrected for by studying the raster position.

The reconstruction matrix that transforms the focal plane quantities into target quantities is given by:

$$T_{tar} = \sum_{i,j,k,l} R_{i,j,k,l}^y (x_{fp})^i (y_{fp})^j (x'_{fp})^k (y'_{fp})^l . \quad (4-1)$$

The sum over i, j, k and l is constrained such as : $1 \leq i + j + k + l \leq N$, where N is the order of the transformation. $R_{i,j,k,l}^y$ is the row of the reconstruction matrix corresponding to the target variable T_{tar} . The matrix elements were determined in an iterative fitting procedure. Comparison to Monte Carlo simulation of the spectrometer is performed to check the accuracy of the event reconstruction (see Fig. 35).

During the experiment, the y'_{tar} reconstruction was observed to have odd structures and to not be modeled well by the simulation (see Fig. 36). The data y'_{tar} distribution seemed to have very sharp edges at $\sim \pm 25$ mrad. This discrepancy between data and simulation was observed to become more pronounced with decreasing E' runs below 1 GeV. This was found to be due to the installation of a thick (0.050 cm Ti) HMS vacuum window exit in the hut. At low E' , a large number of multiple scattering events populated the edges of the focal plane, which was outside the phase space covered by the fitted matrix elements. They, i.e. the matrix elements, are not well constrained and subject to overfitting at that region, and therefore are unreliable. To fix this issue, it was necessary to use two reconstruction matrices, the first is the standard (fitted) reconstruction matrix and the second is an alternate COSY reconstruction matrix (magneto-optics simulation package). The former is used to reconstruct events in the focal plane regions away from the edges, while the latter is used to reconstruct the events close to the edges of the phase space. This reconstruction issue was discussed in detail in Ref. [50]. Due to the use of the alternative matrix elements, an additional uncertainty to the cross section was assigned for the low E' runs. The uncertainty takes into account the fraction of data that had to be reconstructed using the alternative matrix elements, and was scaled to be

0% at $E' = 1.4$ GeV and 1% for the lowest E' value (0.4 GeV). The parametrization for the optics uncertainty is given by:

$$\Delta\sigma_{optics}(E') = 0.01(1.4 - E')^2 \quad . \quad (4-2)$$

Figure 35: Data to Monte Carlo simulation comparison of various target quantities for Carbon at $E = 2.347$ GeV, $E' = 1.091$ GeV and $\theta = 20.01^\circ$.

$$E = 2.3466 \text{ GeV}, E' = 0.514 \text{ GeV}, \theta = 30^\circ$$

Figure 36: Data to Monte Carlo comparison plots for y'_{tar} . Left plot is without the alternate COSY matrix elements, while the right plot is after applying the alternate COSY matrix elements. Yellow color represents Monte Carlo simulation while the blue markers represent data.

4.3 Event Selection

In scattering experiments, the primary measurement is the total number of inclusively scattered electrons. Determining this number requires selecting good events and rejecting the unwanted events that are coming from background sources. There are two types of cuts that need to be applied: the acceptance cuts and the Particle Identification (PID) cuts. The acceptance cuts are used to select a region where the spectrometer acceptance is well understood. The PID cuts help in selecting and identifying electrons based on their energy and interactions, and exclude the other particles.

4.3.0.1 Acceptance Cuts

Acceptance cuts are placed on the spectrometer reconstructed quantities δ , x'_{tar} and y'_{tar} taking into account the limitations of the HMS acceptance (see Table 3). These cuts (Table 4) are applied to make sure that only particles scattered from the target region are accepted, and eliminate the particles that originate from outside of the spectrometer acceptance.

HMS Acceptance Cuts
$-8 < \delta < 9$ (%)
$-0.1 < x'_{tar} < 0.1$ (rad)
$-0.05 < y'_{tar} < 0.05$ (rad)

Table 4: Acceptance cuts applied on the reconstructed HMS quantities

4.3.0.2 Particle Identification Cuts (PID)

In scattering experiments, particles are identified by the characteristic signatures they leave in the detector systems. Each component of the spectrometer tests a specific set of particle properties. The Čerenkov detector, for example, identifies the particle using the Čerenkov effect that was discussed in Sec. 3.4.2.3, and the ADCs give us the number of photo-electrons produced by that particle.

In this experiment, negative pions caused a significant source of background. Pions should not produce Čerenkov light, therefore, applying a cut to the Čerenkov ADC signal, to select only events that produce more than 2 photo-electrons ($hcer_npe > 2$), should give a clean electron sample.

Even though the Čerenkov cut will reject most of the pions, it is still possible for pions to produce a knock-on atomic electron of sufficient energy to emit Čerenkov light. The Čerenkov tank entrance window is made thin enough to reduce those knock-on electrons. However, it is still possible for pions to produce signals large enough to pass the Čerenkov cut. This will be discussed in detail in Sec. 4.4.3.2. The second PID

Figure 37: HMS calorimeter track energy E_{track}/E' vs. the number of Čerenkov photo-electrons. The green line represents the calorimeter cut, whereas the pink line represents the Čerenkov cut.

cut was applied on the normalized calorimeter energy spectrum. As discussed in Sec. 3.4.2.4, the calorimeter identifies electrons and pions based on the energy they deposit. When electrons pass through the calorimeter, they deposit all of their energy and will be completely absorbed. This is clearly observed in the normalized calorimeter track energy spectrum, E_{track}/E' that is also called *hsshtk*, where the electrons peaked at unity (see

Fig. 33). The pions, on the other hand, deposit around 300 MeV in the calorimeter causing a second peak in the spectrum around $0.3/E'$. The $E_{track}/E' > 0.7$ ($hsshtrk > 0.7$) cut is sufficient to remove most of the pions, however, it will remove some good electrons too. Electrons rejected by the calorimeter cut will be corrected for within the calorimeter cut efficiency which will be discussed in detail in Sec. 4.4.1.2. The summary of the PID cuts used for this analysis is listed in Table 5 (see Fig. 37).

PID Cuts
$hcer_npe > 2$
$hsshtrk > 0.7$

Table 5: List of the PID cuts used during the current analysis

4.4 Determination of Inclusive Yields

The data yield, Y_{Data} , is obtained from the raw number of electrons counted in the detectors, corrected for various detector effects (efficiencies, deadtimes) and the background events:

$$Y_{Data} = \frac{N^{e^-} - BG}{\epsilon_{tot}} , \quad (4-3)$$

where N^{e^-} and BG are the raw number of electrons and the background events, respectively, whereas ϵ_{tot} is the total detector efficiency. In the following Sections, the process of determining the data yield from the raw number of electrons will be discussed.

4.4.1 Efficiencies

4.4.1.1 Trigger Efficiency

Inefficiencies in the detectors can result in an inability to form a trigger when a good electron transverses the detector stack. The electron trigger efficiency for the HMS can be determined using the efficiencies for the individual component to the trigger using the

following formula:

$$\epsilon_{trig} = \epsilon_{PRLO} \times \epsilon_{STOF} + (1 - \epsilon_{PRLO}) \times \epsilon_{3/4} , \quad (4-4)$$

where ϵ_{PRLO} is the efficiency of the PRLO trigger (the signal with a low threshold on the energy in the first layer of the calorimeter), ϵ_{STOF} is the STOF trigger efficiency (the coincidence of two scintillator panels with one of the front pair of panels and one from the back pair of panels), and $\epsilon_{3/4}$ is the efficiency of the SCIN trigger (the coincidence 3 out of 4 scintillator panels). ϵ_{PRLO} was calculated as the ratio of the events that produced a coincidence of PRLO and Čerenkov to the events that produced from only Čerenkov:

$$\epsilon_{PRLO} = \frac{PRLO \ \& \ \check{C}er > 0.5}{\check{C}er > 0.5} , \quad (4-5)$$

ϵ_{STOF} is calculated as follows:

$$\begin{aligned} \epsilon_{STOF} &= \epsilon_{S1} \times \epsilon_{S2} \\ &= \left(1 - (1 - \epsilon_1) \cdot (1 - \epsilon_2) \right) \times \left(1 - (1 - \epsilon_3) \cdot (1 - \epsilon_4) \right) , \end{aligned} \quad (4-6)$$

$\epsilon_{3/4}$, on the other hand, is given by:

$$\epsilon_{3/4} = \prod_{i=1,4} \epsilon_i + \sum_{j=1,4} (1 - \epsilon_j) \prod_{i \neq j} \epsilon_i , \quad (4-7)$$

where $\epsilon_{1,2,3,4}$ are the efficiencies of the four scintillator panels S1X,S1Y,S2X and S2Y.

The efficiency of each panel is calculated as follows:

$$\text{panel efficiency} = \frac{\text{number of events that caused a hit and a track in the panel}}{\text{total number of events that left a track in the panel}} .$$

For experiment E04-001, a discriminator connected to the S1X panel went bad during

the data taking resulting in a drop in the total efficiency because of the drop in the 3/4 and STOF efficiency (see Fig. 38).

Figure 38: Efficiency for S1X panel (top) and 3/4 trigger (bottom) *vs.* run number for E04-001 experiment.

Due to the bad discriminator, an alternative method was implemented to calculate the trigger efficiency. The efficiency of each paddle of S1X was calculated individually before obtaining the total trigger efficiency. To determine the efficiency of each paddle, first we need to define an unbiased sample by selecting events which produced STOF trigger and a track ($N_{tracks} \geq 1$). Each event was, then, projected back to an S1X paddle and checked for whether the paddle fired. The efficiency of each paddle was taken to be the ratio events that left both a hit and a track ($N_{hits+tracks}$) to the total number of events that left a track (N_{tracks}),

$$\epsilon_{i=1,..16}^{paddle} = \frac{N_{hits+tracks}}{N_{tracks}} \quad , \quad (4-8)$$

where ϵ_i^{paddle} corresponds to the efficiency of paddle number i . In order to limit the effects of multiple scattering, the events that fired an adjacent paddle were excluded from the sample N_{tracks} .

After defining the paddle efficiency, it was straightforward to obtain the total trigger efficiency using Eq. 4-4. The trigger efficiency *vs.* S1X paddle number is shown in Fig. 39 (Top) for all Carbon and Iron runs, and the average trigger efficiency using this method was about 97.5%. The weighted trigger efficiency for each paddle, ϵ_i^w , was calculated over the total number of runs, n , using the formula [57]:

$$\epsilon_i^w = \frac{\sum_{k=1}^n (\epsilon_{ik}/\sigma_{ik}^2)}{\sum_{k=1}^n (1/\sigma_{ik}^2)} \quad , \quad (4-9)$$

where ϵ_{ik} is the paddle- i efficiency for the run k , and σ_{ik} is the associated statistical error. The general formula for the weighted average uncertainty, σ_i^w , is given by:

$$\sigma_i^w = \sqrt{\frac{1}{\sum_{k=1}^n (1/\sigma_{ik}^2)}} \quad . \quad (4-10)$$

The bottom plot of Fig. 39 shows the resulting weighted trigger efficiency for each paddle. The uncertainty due to the trigger efficiency was taken to be the spread of the points around the average value (Fig. 40).

Figure 39: (Top) The HMS trigger efficiency calculated *vs.* paddle number for all Carbon and Iron runs for E04-001 experiment. (Bottom) The weighted trigger efficiency per paddle number.

Figure 40: Electron trigger efficiency fit residuals.

4.4.1.2 Detector Efficiencies

Calorimeter Cut Efficiency: Even though applying the calorimeter cut ($E_{track}/E' > 0.7$) will reject most of the pions, some electrons will also be lost too due to this cut causing a source of an inefficiency. The total calorimeter cut efficiency is the fraction of electron events that passed the calorimeter cut (N^{cut}/N^{total}), where the electron sample, N^{total} , was chosen to be the number of events that produced more than 10 photo-electron pairs in the Čerenkov ADCs and passed the acceptance cuts listed in Table 4:

$$\epsilon_{cal}^{total} = \frac{N^{cut}(hsshtrk > 0.7 \ \& \ hcer_npe > 10 \ \& \ Acc_{cut})}{N^{total}(hcer_npe > 10 \ \& \ Acc_{cut})} . \quad (4-11)$$

Some electron events produce a momentum-normalized calorimeter energy less than 0.7 due to two factors: i) the calorimeter energy resolution reconstructs some electron calorimeter energies below the cut, and ii) the E_{track} sums energy deposited in blocks within some range of the projected track and, therefore, is sensitive to poorly tracked events. One source of poorly-tracked events is the selection of "ghost tracks" as the best

track for an event. At high rate runs, where there is a significant pileup of multiple particles within a given trigger, the tracking sometimes matches drift chamber hits from one particle in DC1 to the hit of a second particle in DC2 and builds a ghost track that doesn't match the trajectory of a real particle. Therefore, there will be little energy deposited around this track resulting in normalized calorimeter energy near zero.

In order to determine the calorimeter cut efficiency, we need to determine the inefficiency that is caused by each of the above components,

$$\epsilon_{cal}^{total} = \epsilon_{cal}(E') \times \epsilon_{cal}^{wrong}(rate) . \quad (4-12)$$

The calorimeter efficiency, ϵ_{cal} , arises from the calorimeter resolution and, therefore, is E' dependent, whereas the wrong-track efficiency, ϵ_{wrong} , is rate dependent, as it scales with the pileup probability.

When calculating ϵ_{cal} , and to avoid including events that are associated with multiple tracks, only events with a single track ($hntracks == 1$) were included. The fraction of one-track electron events that passed the calorimeter cut (N_1^{cut}/N_1^{total}) will give the calorimeter efficiency as:

$$\epsilon_{cal} = \frac{N_1^{cut}(hsshtrk > 0.7 \ \& \ hcer_npe > 10 \ \& \ hntracks == 1 \ \& \ Acc_{cut})}{N_1^{total}(hcer_npe > 10; \ \& \ hntracks == 1 \ \& \ Acc_{cut})} . \quad (4-13)$$

Fig. 41 shows the calorimeter efficiency *vs.* the scattering energy E' . The cut efficiency decreases as E' decreases for a fixed cut .

Using equations 4-12 and 4-13, the wrong track efficiency (ϵ_{wrong}) can be estimated. Fig. 42 shows the calorimeter wrong track efficiency *vs.* rate, providing an indication that the efficiency drops with increasing rate. The parametrization of both efficiencies, ϵ_{wrong} and ϵ_{cal} , were used to determine the calorimeter cut efficiency in a run-by-run

Figure 41: Calorimeter efficiency *vs.* the scattered electron energy E' .

basis. A systematic uncertainty of 0.1% was assigned to the calorimeter cut efficiency, which was obtained from the fit residuals (see Fig. 43).

Figure 42: Calorimeter wrong track efficiency *vs.* rate.

Figure 43: Calorimeter cut efficiency residuals.

Čerenkov Cut Efficiency: The choice of Čerenkov cut ($hcer_npe > 2$) was made to minimize the number of pions in the selected sample without sacrificing too many electron events. However, some good electrons produce less than 2 photo-electrons and, hence, they will be lost due to this cut. This leads to a Čerenkov cut inefficiency, that needs to be estimated and correct for.

In order to calculate the Čerenkov cut efficiency, we need to choose an unbiased sample of good electrons. This sample was taken to be the electrons that passed the acceptance cuts listed in Table 4, the calorimeter cut, and a Čerenkov cut of 1 ($hcer_npe \geq 1$). The latter cut was made to eliminate pions that can produce 0 photo-electrons in the Čerenkov ADC spectra. Then, the Čerenkov cut efficiency was taken to be the fraction of electrons that survived the Čerenkov cut,

$$\epsilon_{cer}^{cut} = \frac{N_{cut}^{e-}(hsshtrk > 0.7 \ \& \ hcer_npe > 2 \ \& \ Acc_{cut})}{N_{total}^{e-}(hsshtrk > 0.7 \ \& \ hcer_npe \geq 1 \ \& \ Acc_{cut})} . \quad (4-14)$$

Figure 44 shows the Čerenkov cut efficiency *versus* the HMS central momentum. It is clear that the runs with higher pion to electron ratios are affecting the efficiency. Fig. 45 shows how the Čerenkov cut efficiency drops with increasing π/e ratio. Therefore, it was best to exclude the runs that have high pion contamination when studying the Čerenkov cut efficiency (see Fig. 46). It was found that the Čerenkov cut efficiency is about 99.8% for lower HMS central momentum (< 1 GeV), and is about 99.9% for higher momentum runs.

The systematic uncertainty resulting from Čerenkov efficiency has been estimated from the spread of the fit residuals, and found to be ~ 0.04 % (see Fig. 47).

Figure 44: HMS Čerenkov cut efficiency *vs.* the HMS central momentum including the runs with high pion to electron ratio.

Figure 45: HMS Čerenkov cut efficiency *vs.* π/e ratio.

Figure 46: HMS Čerenkov cut efficiency *vs.* the HMS central momentum excluding the runs with high pion to electron ratio.

Figure 47: The residuals of Čerenkov efficiency fit.

4.4.2 Deadtimes

During data taking, some events can be lost due to two types of deadtime: the electronic deadtime and the computer deadtime. The word "deadtime" refers to the time interval that the device couldn't record an incoming event because it was busy processing an older event. For a precise calculation of the yields and cross sections, the lost events due to the deadtimes had to be corrected for.

4.4.2.1 Electronic Deadtime (EDT)

The electronic deadtime (EDT) comes from the trigger electronics, which are unable to record and can miss an incoming trigger while processing the previous trigger. This can happen when two signals from different events try to activate the logic gate module within a short period of time. If the second event reaches the module before the gate is closed, this event will be lost.

For a mean particle rate R , the probability of finding n events during a time t is:

$$P(n) = \frac{(Rt)^n e^{-Rt}}{n!} . \quad (4-15)$$

When an event arrives, the logic signal can not accept new events for some time τ , which is the gate width. Therefore, the probability of having zero events ($n = 0$) during the time τ , which is called the livetime ($= 1 - \text{deadtime}$), is given by:

$$LT = P(0) = e^{-R\tau} . \quad (4-16)$$

A 100% possibility of having zero events during τ (Eq. 4-16) leads to a 100% livetime, i.e. none of the events are lost. In reality, this is not true and the recorded events are usually less than the total events. For small $R\tau$, one can write Eq. 4-16:

$$LT = \frac{N_{\text{recorded}}}{N_{\text{total}}} \approx 1 - R\tau . \quad (4-17)$$

The electronic deadtime EDT can be written as follows:

$$EDT = 1 - LT = \frac{N_{\text{lost}}}{N_{\text{total}}} \approx R\tau \quad (4-18)$$

$$= \frac{N_1 - N_2}{N_1} \cdot \frac{\tau}{\tau_2 - \tau_1} , \quad (4-19)$$

where $N_{1,2}$ are the number of events recorded for two scalers with gate widths, τ_1 and τ_2 . For E04-001, scalers with four different gate widths ($t = 50, 100, 150, 200$ ns) have been used and the number of events recorded in each of them have been measured ($N_{50}, N_{100}, N_{150}, N_{200}$). The coincidence gate width τ for all components in the trigger was set to 60 ns. Therefore, using the trigger logic, units 100 ns and 150 ns scalers were

used to calculate the electronic deadtime :

$$EDT = \frac{N_{100} - N_{150}}{N_{100}} \cdot \frac{\tau}{50} , \quad (4-20)$$

with $\tau = 60$ ns. The electronic deadtime was calculated on a run-by-run basis. While effort was spent to keep the gate width of all logic modules to 60 ns, there are always variations when dealing with different many trigger components. Therefore, the systematic uncertainty due to the deadtime was assigned to be 20% to account for variations in the intrinsic limiting gate width.

4.4.2.2 Computer Deadtime (CDT)

Another source of deadtime is the computer deadtime (CDT), which depends on how fast the data acquisition computers can record the incoming events. If the DAQ system was busy processing an event, it can not process any new event and therefore, some events will not be recorded.

The computer livetime for each run was calculated as the ratio of triggers that have been successfully processed by the DAQ trigger supervisor ($N_{recorded}$) to the total number of triggers ($N_{trigger}$). Therefore, the computer livetime (LT) was calculated on a run-by-run basis using the following formula:

$$LT = \frac{N_{recorded}}{N_{trigger}} . \quad (4-21)$$

The systematic uncertainty for the computer deadtime was estimated to be equal to 0.2%. This number was obtained by comparing experimental yields at the same spectrometer kinematic setting but varying the prescale factors [58].

Figure 48 shows both electronic and computer deadtimes versus the SCIN rate and the trigger rate, respectively. The prescale factor ($PS = N_{trigger}/N_{pretrigger}$) was set to

try to keep the computer deadtime $\leq 15\%$. The electronic deadtime reaches 5% for the highest rate runs.

Figure 48: (Top) Electronic deadtime *vs.* rate, and computer deadtime *vs.* trigger rate (Bottom) for E04-001 experiment.

4.4.3 Background Events

Both the trigger system and the cuts were designed to select only good electrons from the data sample with a high efficiency. However, there are still some "false" events that were recorded with our sample that don't come from inclusive scattering, but rather other sources such as electrons from e^-e^+ pair production processes. Such events are considered unwanted "background events" and need to be corrected for.

4.4.3.1 Charge Symmetric Background (CSB)

During the experiment, electrons that did not come from inclusive scattering, but rather other processes, were also detected. These background electrons come mainly from high energy photons and neutral π^0 pions.

Photons can be produced, for instance, by the Bremsstrahlung process, which then annihilate into e^-e^+ pairs in the field of nuclei, $\gamma \rightarrow e^- + e^+$. Neutral pions, on the other hand, come mainly from photo-production processes, and then decay into two photons that eventually will produce electron-positron pairs ($\pi^0 \rightarrow 2\gamma \rightarrow e^- + e^+$).

In the pair production processes, the electrons and positrons are produced in equal numbers, and the background yield is charge symmetric. The electrons from charge symmetric pairs that scatter into the detector stack are indistinguishable from inclusive electrons and need to be subtracted. The total measured yield is then equal to the sum of the inclusive electrons, Y^{e^-} , and the CSB electrons, Y^{csb} :

$$Y^{tot} = Y^{e^-} + Y^{csb} . \quad (4-22)$$

For the E04-001 experiment, the most significant CSB was at $E = 2.35$ GeV and $\theta = 60^\circ$. where the CSB reached $\sim 20\%$ for the lowest E' runs (see Fig. 49 and Fig. 50). During the experiment, data were taken at $E = 2.35$ GeV and $\theta = 70^\circ$ to help better

determine the angular dependance of the CSB at this beam energy, where the background was the largest, even though the largest angle for this beam energy for inclusive data was 60° .

Figure 49: Data to Model cross section ratio for Carbon at $E = 2.35 \text{ GeV}$ and $\theta = 60^\circ$ prior to accounting for the CSB. The pink dashed lines indicate $\pm 2\%$ range.

Figure 50: Relative contribution of the charge symmetric cross section (determined from measuring e^+) to the total electron cross sections *vs.* E' for $E = 2.35 \text{ GeV}$ and $\theta = 20^\circ - 70^\circ$. The plots and results were taken from Ref. [59].

Since electrons and positrons are produced in equal numbers, measuring the positron yield gives a direct measurement of the CSB. Therefore, the spectrometer polarity was

reversed and the positron yield was determined for each target and kinematic settings where the background was thought to be significant. The positron cross section was then fitted (Fig. 51) using the function:

$$F(\theta, E') = e^{a(\theta)} \times \left(e^{b(\theta) \times (E - E')} - 1 \right), \quad (4-23)$$

where $a(\theta)$ and $b(\theta)$ are free parameters determined by the fit, and are interpolated in θ by a polynomial function of third degree.

Figure 51: Positron cross section per nucleon for Carbon target *vs.* E' at $E = 2.35$ GeV and $\theta = 20^\circ - 70^\circ$. The CSB plots and results for E04-001 experiment were taken from Ref. [59].

After determining the positron cross section, two methods were used to correct for the CSB: i) direct subtraction method, and ii) a multiplicative correction factor. In the subtraction method, we assume that the total radiated measured cross section for negative polarity is the sum of the inclusive electron cross section (including contributions

from radiative processes) and the CSB cross section, $\sigma^{total} = \sigma^{e^-} + \sigma^{CSB}$. Using the fit function given by Eq. 4-23, σ^{CSB} can be estimated for each kinematic. Therefore, the inclusive radiated electron cross section (prior to radiative corrections discussed in Section 4.4.4.5) can be determined by simply subtracting the positron cross section from the total measured cross section. The ratio of the measured cross section after subtraction and application of radiative corrections is shown in Fig. 52.

Figure 52: Data to Model cross section ratio after subtracting the CSB. The pink dashed lines indicate $\pm 2\%$ range.

In the second method (multiplicative), a scaling factor SF^{e+} was assigned to each simulated electron event in the Monte Carlo according to:

$$SF^{e+} = \frac{\sigma^{e^-} + \sigma^{e^+}}{\sigma^{e^-}} , \quad = 1 + \frac{\sigma^{e^+}}{\sigma^{e^-}} , \quad (4-24)$$

producing the same fractional enhancement in the MC yield as the data (See Eq. 4-45). The extraction of the inclusive cross section from the Monte Carlo yield will be discussed in detail in Sec. 4.5. Figure 53 shows the correction resulting of using the multiplicative method, which agrees with the subtraction method (Fig. 52) to better than 0.1%.

The uncertainty on the electron cross section coming from the CSB subtraction was

estimated to come from two sources [59]. The first uncertainty comes from the uncertainty in the positron cross section, this was estimated by multiplying the ratio $\sigma^{e^+}/\sigma^{e^-}$ by the positron cross section uncertainty. Another source of uncertainty was the sensitivity of the cross section extraction due to the pion contamination parametrization in the e^+ yield. The total systematic uncertainty assigned to the CSB was 0.05 - 2%.

Figure 53: Data to Model cross section ratio after subtracting the CSB using the multiplicative method. The pink dashed lines indicate $\pm 2\%$ range.

4.4.3.2 Pion Contamination

Pion contamination is another source of background, as a small number of pions can pass both Čerenkov and calorimeter cuts. Some pions can produce knock-on electrons of high enough energy to emit Čerenkov light, and will pass the Čerenkov cut. Figure 54 illustrates the rejection of pions by the Čerenkov and calorimeter cuts.

Looking at the normalized HMS calorimeter track energy spectrum, the pion contamination can be estimated. A Čerenkov cut ($\text{hcer_npe} \geq 2$) was applied to estimate the total number of events that pass the cut (top plot of Fig. 54). Then, a pion sample was selected by choosing events that produced photo-electrons less than 2 ($\text{hcer_npe} < 2$), the green area in the middle plot of Fig. 54 represents the pion sample, whereas

the purple area represents all the events that passed the Čerenkov cut. The pion peak (green) was, then, scaled to match the pion peak (purple) that survived the Čerenkov cut. The bottom plot of Fig. 54 shows the tail of the pion distribution that extends beyond the calorimeter cut leading to a source of contamination. This background can not be removed by any cut, therefore it has to be estimated and corrected for.

If the π^+ and π^- yields are equal, then the number of pions remaining in the negative polarity run (after the cuts) will be the same as the pions in the positive polarity run. When subtracting the CSB (see Sec. 4.4.3.1), much of the pion background will be subtracted. The uncertainty of extracting the positron cross section was already included as a CSB uncertainty, and therefore no uncertainty was assigned for the pion contamination.

Figure 54: HMS calorimeter track energy spectrum. (Top) The black line represents the events before applying any PID cut, whereas the violet shaded area represents the events that pass the Čerenkov cut ($hcer_npe > 2$). (Middle) Events that produced $hcer_npe < 2$ formed the pion sample (green area). (Bottom) The pion sample (solid green area) was scaled down to estimate the remaining pions after Čerenkov cut. The extended tail (red area) that passes the calorimeter cut (blue line) is considered the pion contamination.

4.4.4 Corrections

4.4.4.1 $\delta p/p$ Correction

During the cross section analysis, the reconstructed target variables histograms from the data were compared to those from a Monte Carlo (MC) simulation (see Fig. 35). However, the ratio of the data reconstructed $\delta p/p$ to MC histograms was found to have systematic variations from unity, which suggested that some events were mis-reconstructed in $\delta p/p$ (see Fig. 55). This is a common effect that appears when using a standard set of HMS optics reconstruction matrix elements for many experiments (see [50], [53] and [61]). Studying the yield ratio Y_{Data}/Y_{MC} *vs.* δ , it was found that the effect was the same across many targets and kinematics, confirming that this δ effect was not due to inaccurate modeling of the acceptance or physics, but rather due to the δ reconstruction from the data. This has been attributed to inaccuracies in the δ reconstruction (on the order of $\times 10^{-4}$) due to overfitting of the data. In order to reduce the uncertainties in E' when extracting the cross section, the δ correction had to be implemented.

Figure 55: Data/MC yield ratio for each $\delta p/p$ bin before applying the $\delta p/p$ correction. Each ratio is an averaged sum of many (over 40) Carbon and Iron runs. The unit of $\delta p/p$ is %.

To apply the δ correction, a new map had to be built to reconstruct the events back to the true δ bin. We start with the assumption that the events reconstructed at $\delta = 0$ are accurately reconstructed. In other words, $\delta_{rec} = 0 = \delta_{real}$, where δ_{rec} is the reconstructed δ value from the nominal optics that needs to be corrected, and δ_{real} is the true value that would result in the data/MC ratio = unity. Next, we move to the neighboring bin edge, $\delta = 1$, and check the data/MC ratio at the center of that bin ($\delta = 0.5$) and define the shift on δ edge based on it. For example, if the data/MC ratio at the center of the bin was 0.98, that suggests that 2% of data events were moved to the adjacent bin because the current bin is reconstructed 2% smaller than it should. The δ bin width was, then, increased by 2% by adjusting the position of the outer bin edge. In general:

$$\begin{aligned}\Delta_i^{(+)} &= \left(\frac{Y_{data}}{Y_{MC}} - 1 \right) + \Delta_{i-1}^{(+)} \\ \Delta_i^{(-)} &= \left(1 - \frac{Y_{data}}{Y_{MC}} \right) + \Delta_{i-1}^{(-)}\end{aligned}\tag{4-25}$$

where the superscripts (+) and (-) represent the shift in the positive i bin, $\delta > 0$, and negative i bin, $\delta < 0$, respectively. The term $\Delta_{i-1}^{(\pm)}$ takes into account the shift performed on the previous bin. The same procedure was implemented on a bin-by-bin basis to build a mapping from δ_{rec} to δ_{real} at the bin edges (Fig. 56). The resulting average correction was found to be less than 5×10^{-4} .

Applying the δ correction leads to a significantly improved shape in the data/MC yield ratios vs. δ (Fig. 57). Figure 58 shows $\delta_{real} - \delta_{rec}$ after applying the δ correction, where the δ reconstruction was performed at the bin edges over the acceptance range (-8,+9)% used in this analysis. The data/MC ratio before and after applying δ correction is shown in Fig. 59 for two typical runs. The systematic uncertainty on the cross section due to the δ correction was estimated to be 0.4%, which was estimated from the spread of data to the fit (see Fig. 55).

Figure 56: $\delta_{real} - \delta_{rec}$ as determined from the procedure discussed in the text. The unit of $\delta p/p$ is %.

Figure 57: Data/MC yield ratio for each δ bin after applying the $\delta p/p$ correction. The unit of $\delta p/p$ is %.

Figure 58: $\delta p_{\text{real}} - \delta p_{\text{rec}}$ after applying the $\delta p/p$ correction at the bin edges over the acceptance range $(-8, +9)\%$ used in this analysis. The unit of $\delta p/p$ is %.

Figure 59: Plots of data/MC ratio for *vs.* $\delta p/p$ bin before and after applying $\delta p/p$ correction for Iron (top) and Carbon (bottom).

During the data taking, the horizontal beam position relative to the spectrometer pivot, x , and the z position of the target center determine the interaction point relative to the spectrometer central ray and can affect the acceptance. Therefore, any offset of the beam or target position needs to be determined and corrected for. As seen from Fig. 60, the total in-plane position offset of the target relative to the spectrometer central ray,

Δy_{tar} , is given by:

$$\Delta y_{tar} = \Delta x_{beam} \cdot \cos\theta + \Delta z_{tar} \cdot \sin\theta \quad , \quad (4-26)$$

which can be rewritten as:

$$\frac{\Delta y_{tar}}{\cos\theta} = \Delta x_{beam} + \Delta z_{tar} \cdot \tan\theta \quad , \quad (4-27)$$

where Δx_{beam} is the beam position offset, Δz_{tar} is the offset of the target z position relative to the pivot, and θ is the spectrometer central angle. A linear fit of $\frac{\Delta y_{tar}}{\cos\theta}$ vs. $\tan\theta$ gives Δx_{beam} and Δz_{tar} as the slope. In order to determine Δy_{tar} , a comparison of y_{tar} between the simulation and data was performed for several angles (see Fig. 63), where the expected peak position in y_{tar} was given by the simulation in order to account for any physics biasing. The difference between the two mean values was calculated ($\Delta y_{tar} = y_{tar}^{Data} - y_{tar}^{MC}$), which was then fit to determine the target and beam offsets to be utilized in the simulation. This procedure was done for both targets with the results shown in Table 6. The fit residuals for Carbon are shown in Fig. 62. The fits for both targets give a consistent beam offset as expected.

Target	Δx_{beam} (mm)	Δz_{tar} (mm)
Carbon	0.1279 ± 0.0237	-0.2443 ± 0.0406
Iron	0.136 ± 0.035	0.1347 ± 0.0575

Table 6: Target z and beam x position offsets for Carbon and Iron.

Figure 61: Comparison between data and simulated y_{tar} to determine Δy_{tar} .

Figure 62: Determination of the target z and beam x position offsets for Carbon.

After determining the position offsets for all targets, these were input to the simulation. Fig. 63 shows a comparison between the data and the simulation before and after applying the target and beam offsets for a sample kinematic point.

Figure 63: The difference between data and simulated y_{tar} before (left) and after (right) applying the target and beam offsets for the Carbon target at an angle of 10° .

4.4.4.3 Kinematic Offsets (E , E' , and θ)

The kinematic uncertainties are dominant contributors to the systematic uncertainties for LT separations. Therefore, it is crucial to determine any kinematic offsets and estimate the resulting uncertainties. In order to check the kinematic offsets, e - p elastic data was taken at 16 different kinematic settings. For scattering on hydrogen, the reconstructed W value for the elastic peak should be the proton mass, $W = M_p$, in the absence of Bremsstrahlung. One can use this fact and measure the shift in W from the proton mass, which would be due to the offsets in the quantities (E , E' , and θ).

The beam energy, E , was measured (see Sec. 3.1.3) using the Hall C Arc energy measurements. For each HMS central momentum, E' , a program called "field" was utilized to determine the set current and field for the HMS dipole, as well as the quadrupoles. The HMS angle θ , on the other hand, was read from a scale on the floor observed by a remote camera.

The W^2 value is given by Eq. 2-7:

$$W^2 = M^2 + 2M(E - E') - 4EE'\sin^2(\theta/2) . \quad (4-28)$$

The shift in W^2 for the e - p elastic peak center due to offsets in the energies and angles is given by:

$$\Delta W^2 = \frac{\partial W^2}{\partial E} \delta E + \frac{\partial W^2}{\partial E'} \delta E' + \frac{\partial W^2}{\partial \theta} \delta \theta . \quad (4-29)$$

The beam energy measurements have shown to be accurate to the 4×10^{-4} level, and any beam energy offset larger than this is not expected. An elastic peak study was performed on hydrogen for the E06-109 experiment [50], and concluded that any observed ΔW^2 offset was due to a $\delta \theta$ offset. The estimated value of the angle offset was 0.3 mrad,

and it was used as an input to correct the data set of this experiment. The same study was performed again for this data set to examine the consistency of the result between two data sets.

The values of ΔW^2 were measured for this experiment following the same steps used for the E06-109 experiment. First, the radiative effects were estimated using the Hall C Monte Carlo simulation program SIMC (see Fig. 64), where the simulation was ran with and without the radiative corrections. In Fig. 64, the blue histogram is the simulated W without the radiative corrections (W_{SIMC}), whereas the red histogram includes the radiative corrections (W_{SIMC}^{rad}). The vertical lines give the mean values of W , and the difference between those two values (Δrad) gives the radiative contribution. Subtracting this value from the data elastic peak value removes the radiative effects from data, and the difference between the data elastic peak and the proton mass gives the elastic peak offset ($\Delta_{peak} = W_{Data} - \Delta rad - M_p$). Figure 65 shows the elastic peak shift for one run, while Table 7 shows the elastic peak shifts for all elastic $e-p$ runs.

E (GeV)	E' (GeV)	θ	Δ_{peak} (GeV)
4.612	4.200	10.65°	1.0014
4.612	3.252	25.00°	-1.708
3.475	3.2	14.00°	-0.842
3.475	2.752	20.00°	0.574
3.475	2.367	28.00°	0.222
3.475	2.035	36.00°	-2.030
2.338	2.28	13.00°	-2.492
2.338	2.24	15.00°	-2.685
2.338	2.15	18.00°	-2.697
2.338	1.72	30.00°	-0.961
1.204	1.15	19.00°	-0.765
1.204	1.15	16.00°	-0.206
1.204	1.15	13.00°	-0.225
1.204	0.811	45.00°	0.684
1.204	1.070	20.00°	0.025
1.2 04	1.070	25.00°	-1.014

Table 7: Elastic peak offset at different kinematics for the E04-001 experiment.

Figure 64: Determination of radiative contributions to the proton elastic peak using Monte Carlo simulation (SIMC) with and without simulation of Bremsstrahlung emission.

Figure 65: Determination of the elastic peak offset by taking the difference of the elastic peak position (before and after subtracting the radiative effects) and the proton mass.

The above δW^2 values have been minimized with respect to the values of E' and θ (see Fig. 66). It was found that the results are consistent with what was found in Ref. [50].

Figure 66: Plot showing the calculated total χ^2 for the elastic peak analysis as a function of $\Delta E'/E'$ and θ offsets.

4.4.4.4 Energy Loss Correction

When electrons pass through matter, they lose energy due to collisions with atomic electrons. In this experiment, the electrons passed through several layers of materials: 1) the target material before the scattering occurs, 2) the target material after the scattering, 3) the target chamber exit window, 4) an air gap between the target chamber and the spectrometer vacuum entrance, and 5) the spectrometer entrance window. Therefore, the measured incident (scattering) energy is more (less) than the energy at the vertex. For a precise measurement of the cross section, this energy loss needed to be accounted for.

The mean energy loss for electrons can be calculated using Bethe-Bloch formula [62]:

$$-\frac{dE}{dx} = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A} \left[\ln \frac{\tau^2(\tau + 2)}{2(I/m_e c^2)^2} + F(\tau) - \delta \right] , \quad (4-30)$$

where,

r_e is the classical electron radius,

m_e is the electron mass,

N_A is Avogadro's number,

A is the atomic weight of the material and Z is the atomic number of absorbing material,

δ is the density correction,

$$\gamma = 1/\sqrt{1 - \beta^2},$$

$\beta = v/c$ of the incident electron,

$$F(\tau) = 1 - \beta^2 + \frac{\frac{\tau^2}{8} - (2\tau + 1)\ln 2}{(\tau + 1)^2}, \text{ for electrons,}$$

ρ is the density of absorbing material,

I is the mean excitation energy, which is given by:

$$I = 12Z + 7 \text{ eV} \quad \text{for } Z < 13 \quad (4-31)$$

$$I = 9.76Z + 58.8Z^{-0.19} \text{ eV} \quad \text{for } Z \geq 13 . \quad (4-32)$$

The values of the density correction, δ , are given by the Sternheimer parametrization [63], [64]:

$$\delta = \begin{cases} 0 & X < X_0 \\ 4.6052X + C + a(X_1 - X)^m & X_0 \leq X < X_1 \\ 4.6052X + C & X \geq X_1 , \end{cases} \quad (4-33)$$

where $X = \log_{10}(\beta\gamma)$, and the quantities X_0 , X_1 and m depend on the absorbing material.

The parameter C is given by:

$$C = -2 \left(\ln \frac{I}{h\nu_p} + 1 \right) , \quad (4-34)$$

where $h\nu_p$ is the plasma frequency of the material and is defined as:

$$h\nu_p = h \sqrt{\frac{N_e e^2}{\pi m_e}} . \quad (4-35)$$

The values of the above parameters are given in Ref. [62]. A general parametrization for all material was used in the replay code for E04-001 data, where: $X_0 = 0$, $X_1 = 3$, $m = 3$, and $a = -C/27$. The Hall C Analyzer corrected for the mean energy loss by subtracting (adding) the average energy loss for the incoming (scattering) energy.

In order to ensure that the simulation was consistently treated with the data, the energy loss was computed as follows:

1. Assuming that the scattering occurs at the target center, the energy that the incident electrons loses due to passing through half of the target thickness was sampled for each event, and the average energy loss, E_{loss} , was subtracted from the measured beam energy, i.e. $E_{true} = E - E_{loss}$.
2. The energy loss of the scattered electrons due to passing through the remaining half of the material, the target chamber exit window, and the spectrometer entrance window was calculated and sampled via the Vavilov distribution (see Ref. [62]). The average value of the distribution was, then, saved in a separate Ntuple as a correction factor, E'_{loss} .
3. During the analysis, the average energy loss was added to the reconstructed particle's energy to derive the energy of the particle at the vertex, i.e. $E'_{true} = E' + E'_{loss}$.
4. E'_{true} was used to correct the reconstructed physics quantities and obtain the vertex values.

4.4.4.5 Radiative Correction

The differential cross section for electron-nucleon scattering (Eq. 2-11) is written in the one photon exchange (Born) approximation. The Born approximation (see Fig. 67 (1)) is the lowest order contributing process in the fine structure function α . However, in the nucleon resonance region, higher order electromagnetic processes in α may have a large contribution to the cross section. These processes include vacuum polarization (creation of particle-antiparticle pairs), vertex processes (emission and reabsorption of a virtual photon), and bremsstrahlung (emission of real photons in the nucleon field during interaction).

Figure 67: 1) Lowest order Feynman diagram for inclusive electron-nucleon scattering (Born approximation). 2-5) Feynman diagrams for higher orders electromagnetic processes in α that contribute to the measured cross section.

One can divide the radiative processes into two categories: internal and external processes. The internal processes occur at the scattering vertex and include vacuum polarization, vertex processes, bremsstrahlung and multiple photon exchange. The external processes, on the other hand, occur either before or after the primary scattering takes

place and within the target material.

In order to extract the Born cross section, the other radiative processes need to be calculated and corrected for. Such correction is called "radiative correction". The internal corrections were calculated using Bardin formalism [67], and the external corrections were calculated using the Mo-Tsai formalism [68].

A program was used to calculate the radiative corrections and the Born model cross sections, which were obtained from a fit [60]. Then, each event in the Monte Carlo was weighted with a radiative correction factor RC that depends on the true vertex kinematics and is given by:

$$RC = \frac{\sigma_{Born}^{Mod}}{\sigma_{Rad}^{Mod}} , \quad (4-36)$$

where σ_{Born}^{Mod} is the Born model cross section, and σ_{Rad}^{Mod} is the total radiated model cross section that includes all internal and external radiative corrections. The radiated model cross section includes the radiated elastic, quasielastic and inelastic cross sections:

$$\sigma_{Rad}^{Mod} = \sigma_{Rad,el}^{Mod} + \sigma_{Rad,qel}^{Mod} + \sigma_{Rad,inel}^{Mod} . \quad (4-37)$$

The uncertainty on the cross sections due to the radiative correction is estimated at 1% for the ϵ dependence (see [53]).

4.5 Cross Section Extraction

The double differential cross section is proportional to the probability that the scattered electron will be detected with a scattering energy E' at a solid angle $d\Omega$ around θ . Therefore, we can write:

$$N^{e^-} = L \cdot \frac{d^2\sigma}{dE'd\Omega}(\Delta E' \cdot \Delta\Omega) \cdot \epsilon_{tot} \cdot A(E', \theta) + BG \ , \quad (4-38)$$

where,

- $\frac{d^2\sigma}{dE'd\Omega}$ is the total differential cross section,
- L is the integrated Luminosity and it is obtained by multiplying the total number of incident electrons on the target, and the total number of target particles in the path of the beam as:

$$L = N_{\text{electrons}} \times N_{\text{scattering centers}}/\text{Area} \ . \quad (4-39)$$

The total number of incident electrons is determined by the formula:

$$N_{\text{electrons}} = \frac{C_{tot}}{1.6022 \times 10^{-19} \text{C}/e} \ , \quad (4-40)$$

where C_{tot} is the total charge passed by the beam in Coulombs, and,

$$N_{\text{scattering centers}}/\text{Area} = A \cdot \frac{\tau}{M_A} \ , \quad (4-41)$$

where τ is the target thickness (gm/cm²), M_A is the nucleus mass (gm), and A is the target mass number.

- $A(E', \theta)$ is the detector acceptance.
- ϵ_{tot} is the total efficiency factor, which takes care of the missing electrons due to detector, trigger, electronics, computer inefficiencies.
- BG represents the background events that did not come from inclusive electron scattering, yet were detected and recorded during the data taking (see Sec. 4.4.1).

The data yield, Y_{Data} , (see Sec. 4.4) is determined from efficiency correcting N^{e^-} in Eq. 4–38, and subtracting the background events:

$$Y_{Data}(E', \theta) = \frac{(N^{e^-} - BG)}{\epsilon_{tot}} = L_{Data} \cdot \sigma_{Data} \cdot (\Delta E' \cdot \Delta \Omega) \cdot A(E', \theta) \quad , \quad (4-42)$$

where $\sigma_{Data} \equiv d^2\sigma/dE'd\Omega$.

At each kinematic, the single-arm Monte Carlo is used to generate electron events based on the spectrometer phase space (y_{tar} , x'_{tar} , y'_{tar} and δ). The MC includes multiple scattering at the target and in the spectrometer. However, it does not include the radiative effects or physics weight. In order to generate a realistic MC yield, Y_{MC} , a separate model is used to weight the MC events based on the generated vertex kinematics. Then, the generated yield can be written as:

$$Y_{MC}(E', \theta) = L_{MC} \cdot \sigma_{model} \cdot (\Delta E' \cdot \Delta \Omega) \cdot A_{MC}(E', \theta) \quad , \quad (4-43)$$

where $\sigma_{model} \equiv d^2\sigma_{mod}/dE'd\Omega$ is a model cross section obtained from a global fit [60], $A_{MC}(E', \theta)$ and L_{MC} are the generated acceptance and luminosity, respectively.

Taking the ratio of data yield (Eq. 4–42) to the simulated yield (Eq. 4–43) gives:

$$\frac{Y_{Data}(E', \theta)}{Y_{MC}(E', \theta)} = \frac{L_{Data} \cdot A(E', \theta)}{L_{MC} \cdot A_{MC}(E', \theta)} \times \frac{\sigma_{Data}}{\sigma_{model}} \quad . \quad (4-44)$$

Good understanding of the spectrometer acceptance allows us to assume $A(E', \theta) = A_{MC}(E', \theta)$. In addition, one can scale the generated luminosity so that it equals the data luminosity. Therefore, using Eq. 4-44, the cross section can be determined from:

$$\sigma_{Data} = \frac{Y_{Data}}{Y_{MC}} \cdot \sigma_{model} \quad . \quad (4-45)$$

This method of extracting the cross section is called *Monte Carlo Ratio Method*, and the results of the extracted cross section using this method will be presented in Chapter 5.

4.6 Coulomb Correction

When an electron approaches the target nucleon in the nucleus, the electron's energy will be affected by the Coulomb field generated by the target nucleus. The incoming electron will accelerate as it gets closer to the nucleus, whereas the scattered electron will decelerate. This acceleration (deceleration) will cause an increase (decrease) in the electron's momentum at the vertex from the measured values. In the case of an incoming beam electron with initial energy E and a scattered electron with measured energy E' , the energies at the vertex will be $E + \Delta E$ and $E' + \Delta E$, respectively. Since the true cross section value is being extracted at the vertex, one needs to apply a correction taking into account the electron's momentum shift at the vertex due to the nucleus Coulomb field. Otherwise, the measured cross section would be at different kinematics than the one at the vertex. This correction is called *Coulomb Correction*. For high beam energies this correction can be negligible, however, at the beam energies of this experiment the Coulomb correction need to be taken into account.

One efficient method to approximate the Coulomb correction is the Effective Momentum Approximation (EMA) [65]. In the EMA, the Born approximation loses its validity and requires a correction arising from the nuclear Coulomb field, which is estimated as follows: Assuming that the nuclear charge is uniform and spherically distributed, the electrostatic potential inside the charge sphere is given by:

$$V(r) = -\frac{3\alpha Z}{2R} + \frac{\alpha Z}{2R} \left(\frac{r}{R} \right)^2, \quad (4-46)$$

where Z is the atomic number of the nucleus, α is the fine-structure constant, and R is the effective radius of the spherical charge distribution, which is given by:

$$R = 1.1A^{1/3} + 0.68A^{-1/3}. \quad (4-47)$$

The energy shift ΔE to the electron's beam energy arises from this change in the potential energy the electron experiences when coming from far away and then falling into the target's Coulomb field. Assuming that the electron scatters from a nucleon at the center of that sphere, i.e. $z \rightarrow 0$, then the energy shift is given by:

$$\Delta E = -V_0 = \frac{3\alpha(Z-1)}{2R} . \quad (4-48)$$

In Eq. 4-46, we replace Z with $Z-1$ for the charge of the residual nucleus that causes acceleration. Since most of the nucleons of heavy nuclei are located at the periphery of the nucleus, taking the potential energy at the center to calculate the electron's energy shift would be an overestimate of the Coulomb correction. In the EMA, the effective potential used is $V_{eff} \approx (0.75 - 0.8)V_0$, which agrees with the extracted effective potential from studies of positron and electron inclusive scattering experiments [66]. When taking V_{eff} into account, Q^2 should be replaced by Q_{eff}^2 where,

$$\begin{aligned} Q_{eff}^2 &= 4(E + \Delta E)(E' + \Delta E)\sin^2\left(\frac{\theta}{2}\right) \\ &= 4(E + V_{eff})(E' + V_{eff})\sin^2\left(\frac{\theta}{2}\right) . \end{aligned} \quad (4-49)$$

In order to apply the Coulomb correction when extracting the cross section, we change the Mott cross section by replacing Q^2 with Q_{eff}^2 and E' with $E' + V_{eff}$, and change the nuclear spectral function. In addition, a focusing factor for the incoming electron should be applied, which is given by:

$$F_{foc} = \frac{E + V_{eff}}{E} . \quad (4-50)$$

The extracted cross section after applying the Coulomb correction then becomes:

$$\sigma_{data}^{CC}(E, E') = \sigma_{data}(E, E') \times \frac{\sigma_{model}(E, E')}{\sigma_{model}^{CC}(E + \Delta E, E' + \Delta E)} \times \frac{1}{F_{foc}^2} . \quad (4-51)$$

The Coulomb correction *vs.* W^2 is shown in Fig. 68 for ^{12}C and ^{56}Fe for a beam energy 4.628 GeV and for the four angles of 10.6° , 16° , 20° , and 25° . The Coulomb correction becomes larger with increasing Z and at low W^2 , below the quasi-elastic peak. This is due to the large variation in the cross section in this region.

Figure 68: The Coulomb correction *vs.* W^2 for ^{12}C and ^{56}Fe for a beam energy 4.628 GeV and for the four angles of 10.6° , 16° , 20° , and 25° .

4.7 Extracting F_L , F_1 , F_2 and R

The Rosenbluth separation technique was used in order to extract R and the structure functions F_L , F_1 , and F_2 from the measured cross section. After obtaining the cross section data, a program searches through all kinematics to find points that have the same W^2 and fall within a small range in Q^2 . Each group of points that satisfy this condition define an LT set. For E04-001, around 500 Rosenbluth separations (LT sets) were obtained for both Carbon and Iron. Figure. 69 shows the kinematic coverage for the cross section for the E04-001 experiment and the selection of LT sets, with each rectangle representing an LT set of points that have the same W^2 and a maximum Q^2 range of $\pm 20\%$ about a central value Q_c^2 . For this analysis, the LT separation was implemented only for $Q_c^2 \leq 2.5 \text{ GeV}^2$ because the aim this work is to study the medium modifications on the structure functions in the resonance region and at low Q^2 .

Figure 69: Selection of data points to obtain Rosenbluth separations (LT sets).

After defining an LT set, the measured cross section of each point, $\sigma^{mod}(W^2, Q_c^2)$, was

bin centered to a Q_c^2 value using:

$$\sigma(W^2, Q_c^2) = \frac{\sigma^{mod}(W^2, Q_c^2)}{\sigma^{mod}(W^2, Q^2)} \times \sigma(W^2, Q^2) \quad , \quad (4-52)$$

where σ^{mod} is the model cross section. After the bin centering in Q^2 , the reduced cross section was calculated for each LT set (Eq. 2-28):

$$\sigma_R = \frac{1}{\Gamma} \frac{d^2\sigma}{dE'd\Omega} = \frac{8\pi^2\alpha}{(W^2 - M^2)} [F_1 + \epsilon F_L/2x] \quad . \quad (4-53)$$

where Γ is the flux as given by Eq. 2-18. The reduced cross section was then plotted verses ϵ (Fig. 70) and fitted linearly to extract F_1 (the intercept) and F_L (the slope $\times 2x$). After extracting F_1 and F_L , one can calculate F_2 and R using Eqs. 2-27 and 2-29:

$$F_2(x, Q^2) = \left(\frac{\nu^2}{\nu^2 + Q^2} \right) (F_L + 2xF_1) \quad , \quad (4-54)$$

$$R(x, Q^2) = \frac{F_L}{2xF_1} \quad . \quad (4-55)$$

When applying the Rosenbluth separation, each data point was used only once, to avoid correlated uncertainties between different LT sets. In addition, in order to have enough ϵ range, each Rosenbluth separation was done only if $\Delta\epsilon \geq 0.25$.

Figure 70: The Rosenbluth LT separation performed at a fixed W^2 and central Q_c^2 for Carbon.

4.8 Extracting $R_{A1} - R_{A2}$

In order to study the medium modification of R , one should look at the difference between the R value for different nuclei, say $A1$ and $A2$. One way to measure such a difference, is to use the measured R values obtained from the extracted reduced cross section as described in section 4.7. However, systematic uncertainties may have big impact when using this method. Therefore, $R_{A1} - R_{A2}$ was extracted by applying the Rosenbluth separation on the *ratio* of the cross sections.

The cross section ratio of two targets $A1$ and $A2$ is given by :

$$\begin{aligned} \frac{\sigma_{A1}}{\sigma_{A2}} &= \frac{\sigma_T^{A1}(1 + \epsilon R_{A1})}{\sigma_T^{A2}(1 + \epsilon R_{A2})} \\ &= \frac{\sigma_T^{A1}}{\sigma_T^{A2}} \times \left[1 + \epsilon'(R_{A1} - R_{A2}) \right] , \end{aligned} \quad (4-56)$$

where $\epsilon' = \epsilon/(1 + \epsilon R_{A2})$. The Rosenbluth separation technique can be implemented on the cross section ratio in a similar way as in section 4.7. The linear fit of the cross section ratio *vs.* ϵ' gives $\sigma_T^{A1}/\sigma_T^{A2}$ as intercept, and a slope proportional to $R_{A1} - R_{A2}$. Most systematic uncertainties cancel out using this method, such as acceptance, beam charge, beam energy offset, HMS angle and momentum and model-dependent uncertainties. The Rosenbluth separation was implemented only on the points that have an ϵ' range equal or bigger than 0.25.

4.9 Systematic Uncertainties

Systematic uncertainties on the cross section can be divided into two categories: uncorrelated (point-to-point) uncertainties and correlated (normalization) uncertainties. The former come from changing the experimental conditions and their effect is uncorrelated between data points. The latter, on the other hand, are correlated between measurements at different ϵ , such as uncertainties in target thickness and the systematic shift in the offset of HMS central angle.

The total systematic uncertainty in the extracted cross section was taken as the quadratic sum of all point-to-point uncertainties that contribute to the cross section measurement. A break down of these uncertainties is given in Table 8 and were discussed in the related sections.

A 5% correlated uncertainty was assigned to the CSB, this value was estimated based on the parametrization of the positron cross section. For positive polarity runs, the extraction of the positron cross section is not done with very high precision. The value of 5% was chosen to cover the uncertainty on the e^+ cross section.

Section	Quantity	Uncertainty
4.4.4.3	Beam Energy E	4×10^{-4} GeV
4.4.4.3	Scattering Energy E'	4×10^{-4} GeV
4.4.4.3	Scattering Angle θ	0.35 mrad
3.1.4	Beam Charge	0.23 μ A
4.4.1.1	Trigger Efficiency	0.46 %
4.4.1.2	Calorimeter Efficiency	0.1%
4.4.1.2	Cerenkov Efficiency	0.04 %
4.4.2	Computer Deadtime	0.2 %
4.4.2	Electronic Deadtime	20 %
4.4.3.1	Charge Symmetric Background	0.05 - 2 %
4.2	Optics	0 - 1%
4.4.4.1	Acceptance	0.6 %
4.4.4.5	Radiative Corrections	1 %

Table 8: The point-to-point systematic uncertainties in the differential cross section.

Chapter5: Results

In this chapter, the following results from experiment E04-001 are presented:

- The extracted differential cross section for the Carbon and Iron targets at 4 beam energies and 13 total spectrometer angle settings;
- The structure functions F_L , F_1 and F_2 obtained after applying the Rosenbluth separation technique on each target;
- The ratios F_L^C/F_L^{Fe} , F_1^C/F_1^{Fe} , and F_2^C/F_2^{Fe} from the Rosenbluth separation on each target;
- The nuclear dependence of $R_C - R_{Fe}$, and the ratio σ_T^C/σ_T^{Fe} ($=F_1^C/F_1^{Fe}$) obtained from the Rosenbluth separation on the Carbon to Iron cross section ratio,

5.1 Results for the Differential Cross Sections

The differential cross sections were extracted for two targets: Carbon and Iron, using the Monte Carlo Ratio Method that was discussed in section 4.5. Figures 71 and 72 show the ratio of data to simulated yield ratio and the extracted cross section, respectively, for the Fe target at $E = 3.5$ GeV and $\theta = 20^\circ$. This example shows the typical cross section uncertainties of better than 2%.

For the phase I E04-001 experiment (see Fig. 13), Carbon and Iron data were taken for four beam energies ($E = 1.204, 2.346, 3.488, \text{ and } 4.612$ GeV). The scattering angles and energies were set to cover the range $1 < W^2 < 4.5$ GeV², and $0.5 < Q^2 < 2.5$ (GeV/ c)². The results of the extracted cross sections at all kinematic settings are shown in Figures 73 through 80. All the values were measured at the center of each W^2 bin

with a fixed width of 0.04 GeV^2 . The colored markers are the measured cross section values and the different colors represent different runs with different HMS central momentum settings. The dashed black lines represent the model [60], which was utilized for the radiative corrections and other parts of the analysis. The model fit includes the preliminary results of this data set. In general, for $W^2 > 1 \text{ GeV}^2$, the cross section points agree within 4% with the corresponding model values, but with large deviations for $W^2 < 1 \text{ GeV}^2$. This is likely due to the contribution from quasielastic scattering peak and two-body contributions in the transverse cross section which can be modeled only approximately. The inner error bars represent the combined statistical and uncorrelated systematic uncertainties (see Table 8).

Figure 71: Data to Monte Carlo electron yield for Iron target at $E = 3.4889 \text{ GeV}$ and $\theta = 20^\circ$. The pink dashed lines indicate a $\pm 2\%$ range. The different colors represent different runs with different HMS central momentum settings.

Figure 72: The measured cross section for Iron target at $E = 3.4889 \text{ GeV}$ and $\theta = 20^\circ$. The colored markers represent the data, while the black dashed line represents the model. The measurements were taken at the center of each W^2 bin with a fixed width of 0.04 GeV^2 .

Figure 73: The cross section results for Carbon target at $E = 4.6286$ GeV. The colored markers represent the measure cross sections whereas the black dashed lines represent the model. The uncertainties represent both systematic and uncorrelated uncertainties.

Figure 74: The cross section results for Carbon target at $E = 3.4886$ GeV. The colored markers represent the measure cross sections whereas the black dashed lines represent the model.

Figure 75: The cross section results for Carbon target at $E = 2.3466$ GeV. The colored markers represent the measure cross sections whereas the black dashed lines represent the model.

Figure 76: The cross section results for Carbon target at $E = 1.2044$ GeV. The colored markers represent the measure cross sections whereas the black dashed lines represent the model.

Figure 77: The cross section results for Iron target at $E = 4.6289$ GeV. The colored markers represent the measure cross sections whereas the black dashed lines represent the model.

Figure 78: The cross section results for Iron target at $E = 3.4889$ GeV. The colored markers represent the measure cross sections whereas the black dashed lines represent the model.

Figure 79: The cross section results for Iron target at $E = 2.3469$ GeV. The colored markers represent the measure cross sections whereas the black dashed lines represent the model.

Figure 80: The cross section results for Iron target at $E = 1.2047$ GeV. The colored markers represent the measure cross sections whereas the black dashed lines represent the model.

5.2 Results for F_L , F_1 , F_2 and R for Carbon and Iron

The extraction of the purely longitudinal and transverse structure functions was accomplished via the Rosenbluth separation technique. The separation of the cross section into the structure functions F_L, F_1 is given by (see Sec. 4.7):

$$\frac{1}{\Gamma} \frac{d^2\sigma}{dE'd\Omega} \times \frac{(W^2 - M^2)}{8\pi^2\alpha} = [F_1 + \epsilon F_L/2x] \quad . \quad (5-1)$$

In order to apply the Rosenbluth separation, the cross section had to be measured for the same W^2 and Q^2 but different values of ϵ (different beam energies). F_2 can then be calculated for the same W^2 and Q^2 using:

$$F_2 = \left(\frac{\nu^2}{\nu^2 + Q^2} \right) (F_L + 2xF_1) \quad . \quad (5-2)$$

Figures 81 through 86 show examples of typical Rosenbluth plots for Carbon and Iron. The extracted structure functions were then bin centered to five separate Q^2 values of 0.5, 0.8, 1.25, 1.75 and 2.2 (GeV/c)². This was accomplished using the model, via

$$F_i(Q_c^2) = F_i(Q^2) \frac{F_i^{model}(Q_c^2)}{F_i^{model}(Q^2)} \quad , \quad (5-3)$$

where i represents 1, 2 or L, Q^2 is the true value at which the separation was performed and Q_c^2 is the central value. For the bin centering, the maximum ΔQ^2 was taken to be ± 0.2 (GeV/c)² for $Q^2 < 1$, and ± 0.25 (GeV/c)² for $Q^2 > 1$. The top panels of Figures 87 through 104 show the extracted structure functions F_L , F_1 , and F_2 per nucleon *vs.* W^2 for Carbon and Iron and are compared to the model values obtained from the fit [60]. The errors include both statistical and uncorrelated systematic uncertainties in ϵ . The correlated uncertainties that arise from both charge symmetric background

(CSB) and the absolute uncertainty on the HMS central angle are shown in the bottom panels of Figures 87 through 104. To estimate the contribution of the CSB uncertainty on the structure functions, the Rosenbluth separation was performed twice. First, the separation was performed on the nominal values of the extracted cross sections. Then, the cross section values were shifted by the CSB (see Sec. 4.9), leading to shifted values of the structure functions. The difference between the nominal and the shifted values of structure functions give the correlated uncertainty due to CSB (red bars). The correlated uncertainty due to the HMS angle offset (0.35 mrad) was estimated in the same way as the CSB correlated uncertainty and is typically larger at the more forward angles. The green bars shown in the bottom panels of Fig. 87 - 104 show the total correlated uncertainty for each W^2 bin, and it was obtained by adding both correlated uncertainties in quadratures.

Figure 81: Sample Rosenbluth plots for Carbon at fixed W^2 and central Q_c^2 . The data set of phase I (*Jan05*) of E04-001 was combined with the data set of phase II (*Rosen07*).

Figure 82: Same as Fig. 81.

Figure 83: Same as Fig. 81.

(a)

Figure 84: Sample Rosenbluth plots for Iron at fixed W^2 and central Q_c^2 . The data set of phase I (*Jan05*) of E04-001 was combined with the data set of phase II (*Rosen07*).

Figure 85: Same as Fig. 84.

Figure 86: Same as Fig. 84.

Figure 87: (Top) The colored circles represent the extracted longitudinal structure function F_L for Carbon versus W^2 for fixed Q^2 ranges. The black dashed line is the fit obtained from [60]. (Bottom) The red bars represent the CSB uncertainties and the green bars represent the total correlated uncertainties that come from CSB and the HMS angle offset.

Figure 88: Same as Fig. 87.

Figure 89: Same as Fig. 87.

Figure 90: (Top) The colored circles represent the extracted longitudinal structure function F_L for Iron versus W^2 for fixed Q^2 ranges. The black dashed line is the fit obtained from [60]. (Bottom) The red bars represent the CSB uncertainties and the green bars represent the total correlated uncertainties that come from CSB and the HMS angle offset.

Figure 91: Same as Fig. 90.

Figure 92: Same as Fig. 90.

Figure 93: (Top) The colored circles represent the extracted transverse structure function F_1 for Carbon versus W^2 for fixed Q^2 ranges. The black dashed line is the fit obtained from [60]. (Bottom) The red bars represent the CSB uncertainties and the green bars represent the total correlated uncertainties that come from CSB and the HMS angle offset.

Figure 94: Same as Fig. 93.

Figure 95: Same as Fig. 93.

Figure 96: (Top) The colored circles represent the extracted transverse structure function F_1 for Iron versus W^2 for fixed Q^2 ranges. The black dashed line is the fit obtained from [60]. (Bottom) The red bars represent the CSB uncertainties and the green bars represent the total correlated uncertainties that come from CSB and the HMS angle offset.

Figure 97: Same as Fig. 96.

Figure 98: Same as Fig. 96.

Figure 99: (Top) The colored circles represent the extracted structure function F_2 for Carbon versus W^2 for fixed Q^2 ranges. The black dashed line is the fit obtained from [60]. (Bottom) The red bars represent the CSB uncertainties and the green bars represent the total correlated uncertainties that come from CSB and the HMS angle offset.

Figure 100: Same as Fig. 99.

Figure 101: Same as Fig. 99.

Figure 102: (Top) The colored circles represent the extracted structure function F_2 for Iron versus W^2 for fixed Q^2 ranges. The black dashed line is the fit obtained from [60]. (Bottom) The red bars represent the CSB uncertainties and the green bars represent the total correlated uncertainties that come from CSB and the HMS angle offset.

Figure 103: Same as Fig. 102.

Figure 104: Same as Fig. 102.

The ratios of the extracted structure functions to the model values are shown in Figures 105 through 110. As can be seen from the F_L structure function plots (Figs. 105 and 106), the extracted values are higher than the model by about 40-50% for the lowest Q^2 . With increasing Q^2 , the data to model ratio gets closer to unity, and the data becomes less than the model by about 40-50% for the highest Q^2 . The same trend on F_L is observed on both Carbon and Iron.

In contrast to F_L , the extracted F_1 values are less than the model by $\sim 10\%$ for the lowest Q^2 value. The ratio reaches unity with increasing Q^2 and drops by 10% for the highest Q^2 value (see Figs. 107 and 108). Again, the same trend for F_1 was observed on both targets. Looking at Figs. 109 and 110, the extracted F_2 values seem to agree pretty well with the model (within $\pm 2.5\%$). This suggests that the model has the right inputs for $\epsilon = 1$, but has difficulties describing the nuclear dependence of the separated F_L and F_1 structure functions. This could be due to inadequacies in the functional form for the modification factors for F_1 and F_2 , which was chosen to provide a good description of existing measurements of F_2^A/F_2^D in the region of DIS.

Figure 105: The ratio of the extracted of F_L values to the model versus W^2 for Carbon. The red dashed line represents unity.

Figure 106: Same as Fig. 105 but for Iron.

Figure 107: The ratio of the extracted of F_1 values to the model versus W^2 for Carbon. The red dashed line represents unity.

Figure 108: Same as Fig. 107 but for Iron.

Figure 109: The ratio of the extracted of F_2 values to the model versus W^2 for Carbon. The red dashed line represents unity.

Figure 110: Same as Fig. 109 but for Iron.

As mentioned above, the Rosenbluth separation technique gives the values of F_1 and F_L versus W^2 and Q^2 . Each bin in Figures 87 through 104 gives a value of the structure functions at fixed W^2 and Q^2 . The photonuclear ratio R , can be obtained using the formula:

$$R(W^2, Q^2) = \frac{F_L(W^2, Q^2)}{2xF_1(W^2, Q^2)} \quad , \quad (5-4)$$

where x is the Bjorken scaling variable introduced in Eq. 2-21. Figures 111 and 112 show the measured values of R *vs.* W^2 for Carbon and Iron respectively, and are compared to the nucleon R parametrization (R^N) introduced by Ref. [5]. The inner error bars represent the total uncertainty in R , which was estimated using the propagation of errors of two correlated variables F_1 and F_L . One can see that, away from the QE peak ($W^2 > 2.5 \text{ GeV}^2$), the extracted R_C and R_{Fe} are higher than the parametrized R^N for the lowest Q^2 by $\sim 60\%$, and lower at the highest Q^2 . In another words, R increases with decreasing Q^2 and perhaps displays a nuclear momentum dependence.

Figure 111: Values of R as a function of W^2 for Carbon for the Q^2 ranges of $Q^2 = 0.5, 0.8, 1.25, 1.75,$ and 2.2 GeV^2 . The colored markers are the measured values, whereas the black dashed lines represent the nucleon R parametrization (R^N) as introduced by Whitlow *et al.* [5].

Figure 112: Same as Fig. 111 but for Iron.

5.3 Medium Modification of the Separated F_L , F_1 and F_2

The extracted structure functions for each nuclei (Sec. 5.2) can be utilized to search for any nuclear dependence in F_L , F_1 and F_2 . Figures 113 through 115 show the Carbon to Iron structure function ratios *versus* x for fixed Q^2 values as well as the fit utilized in the analysis [60]. It can be seen from Fig. 113 that, for $Q^2 > 1 \text{ (GeV/c)}^2$ and $0.4 \leq x \leq 0.6$, the ratio F_L^C/F_L^{Fe} is above unity on average by $\sim 30\%$. This suggests that F_L^C is larger than F_L^{Fe} in the EMC region ($0.3 \leq x \leq 0.7$) in the nucleon resonance region.

Looking at the behavior of F_1^C/F_1^{Fe} (Fig. 114) in the same region, it is clear that the ratio is below unity. This indicates that F_1^C is larger than F_1^{Fe} in this region by $\sim 5\%$. The F_2^C/F_2^{Fe} ratio, on the other hand, (Fig. 115) is found to be below unity in this region, which is as expected. It is worth mentioning that the measured values include all contributions at large x , e.g. from the QE region.

There is a clear indication that the structure functions F_L and F_1 have different nuclear dependence in the EMC region (in the nucleon resonance region), which is not well produced by the fit. Even though the fit has the right nuclear dependence of F_2 , it is likely not robust enough to describe the medium modifications of F_L and F_1 in the resonance region. It is not clear yet whether the observed difference in the medium modification of F_L and F_1 is due to an isoscalar effect or due to an A -dependence effect. Final analysis of the remaining targets on Deuterium and Aluminum of this data set will help address this question.

The relative uncertainties of the structure functions were added in quadrature to get the total relative uncertainty on the ratio. Therefore, the systematics assigned for the ratios in Figures 113 through 115 have been over estimated, as much of the correlated uncertainties that should cancel with the ratio are still included. In the following section, the nuclear dependence of F_1 and R will be re-examined by applying the Rosenbluth

separation on the cross section ratios, which allows for the correlated uncertainties to be more easily handled.

Figure 113: The F_L ratio for Carbon to Iron *vs.* x at fixed Q^2 values (blue marker). The pink dashed lines are the fit obtained from [60].

Figure 114: The F_1 ratio for Carbon to Iron *vs.* x at fixed Q^2 values. The pink dashed line is the same as Fig. 113.

Figure 115: The F_2 ratio for Carbon to Iron *vs.* x at fixed Q^2 values. The pink dashed line is the same as Fig. 113.

5.4 Extracting $R_C - R_{Fe}$ and σ_T^C/σ_T^{Fe}

The extracted Carbon to Iron cross section ratio per nucleon, σ_C/σ_{Fe} , is shown in Fig. 116, where it is plotted versus W^2 for five Q^2 ranges. The different colors in each panel corresponds to different ϵ ranges of $\epsilon < 0.4$, $0.4 < \epsilon < 0.7$, and $\epsilon > 0.7$. A clear ϵ dependence can be observed in the ratio, which gets larger with increasing ϵ within the same range of Q^2 . It should be noted here that the cross sections for both targets are Coulomb corrected, however, no isoscaler correction was applied to Iron to account for the neutron excess. The extracted cross section ratio to the model ratio, $(\sigma_C/\sigma_{Fe})/(\sigma_C^{mod}/\sigma_{Fe}^{mod})$, is shown in Fig. 117. The double ratio is about unity, however, it is clear that the model does not have the right ϵ dependence for the cross section as mentioned before.

The extraction of $R_C - R_{Fe}$ and σ_T^C/σ_T^{Fe} was done by performing the Rosenbluth separation technique on the cross section ratio as discussed in Sec. 4.8. The resulting $R_C - R_{Fe}$ values versus x for fixed Q^2 values are shown in Fig. 118. The dashed pink line and the dashed purple line correspond to $\Delta R = 0.04$ and $\Delta R = 0.3 \times R_{1998}$, respectively. R_{1998} is the parametrization of R for the nucleon in the DIS region that was obtained as the result of a global analysis [69], whereas $\Delta R = 0.04$ is the value obtained by the New Muon Collaboration (NMC) [7]. In general, the extracted values of $R_C - R_{Fe}$ are positive. The relative medium modification of R to fitted R^N [5] is shown in Fig. 119. As one can see from the figure, a hint of nuclear dependence of R can be seen. On average, $(R_C - R_{Fe})/R^N \sim 30\%$, which suggests a different medium modification of F_L and F_1 , consistent with what was observed in the previous section.

The extracted values of σ_T^C/σ_T^{Fe} ($= F_1^C/F_1^{Fe}$) versus x are shown in Fig. 120. The extracted values were bin centered to five values of Q^2 and compared to the model [60]. For $Q^2 > 1$, σ_T^C seems to be less than σ_T^{Fe} by $\sim 5\%$. This nuclear dependence observed

on $\sigma_T (F_1)$ confirms the result obtained in the previous section. When comparing the nuclear dependence on $\sigma_T (F_1)$ obtained from applying the Rosenbluth separation on the cross section of each target (Fig. 114) to the one obtained from applying the Rosenbluth separation on the cross section ratio (Fig. 120), it can be seen that the second has less points. This is because the condition $\Delta\epsilon' \geq 0.25$ was utilized in selecting the W^2 , Q^2 kinematics to perform the Rosenbluth separation on the cross section ratio, which is more strict than the condition $\Delta\epsilon \geq 0.25$ that was applied for the Rosenbluth separation on the cross sections for the individual nuclei.

In order to compare to previous measurements at larger Q^2 in the DIS region, one should look into the inelastic contribution to the cross section and remove the quasi elastic contribution. However, this can not be cleanly separated from the DIS contribution in a model independent way. Therefore, we chose to show the experimental observables including all contributions.

Miller [11] predicted a significant pion enhancement in the F_L^A/F_L^D ratio, which could explain the medium modification of R (see Sec. 2.5). According to Miller's calculations, the ratio F_L^C/F_L^{Fe} is approximately 67% for $Q^2 = 0.5 \text{ GeV}^2$ at the region $0.3 < x < 0.5$, which suggests that F_L^C is less than F_L^{Fe} at that region. A way to test Miller's theory is to look at the extracted F_L^C/F_L^{Fe} ratio versus x for $Q^2 = 0.5 \text{ GeV}^2$ (first panel in Fig. 113). The weighted averaged F_L^C/F_L^{Fe} in the region $0.3 < x < 0.5$ is approximately 1.026 ± 0.14 , which suggests that, even within 2σ , the existence of nuclear pions in this regions cannot be confirmed. However, in this region there is a significant contribution from the QE cross section. Therefore, the search of nuclear pions can be tested after subtracting the QE contribution and after finalizing the analysis of the Deuterium target.

Figure 116: The extracted cross section ratio of Carbon to Iron versus W^2 and ϵ for five ranges of Q^2 .

Figure 117: The ratio of the extracted σ_C/σ_{Fe} to the model $\sigma_C^{model}/\sigma_{Fe}^{model}$, versus W^2 and ϵ for five ranges of Q^2 .

Figure 118: The extracted $R_C - R_{Fe}$ values versus x for fixed Q^2 values. The dashed pink line and the dashed purple line correspond to $\Delta R = 0.04$ and $\Delta R = 0.3 R_{1998}$, respectively. R_{1998} is the parametrization of R for the nucleon that was obtained as the result of a global analysis [69], whereas $\Delta R = 0.04$ is the value obtained by NMC [7].

Figure 119: The relative $R_C - R_{Fe} = \Delta R / R^N$ versus W^2 and for five fixed Q_c^2 values. R^N is the global fit obtained from [5].

Figure 120: The extracted σ_T^C/σ_T^{Fe} versus x for fixed Q^2 values. The black dashed line is the fit obtained from [60].

Chapter6: Summary and Conclusions

The inclusive electron-nucleon cross sections were extracted in the nucleon resonance region at $1 < W^2 < 4.5 \text{ GeV}^2$ for four momentum transfer values between $0.5 < Q^2 < 2.5 \text{ (GeV}/c)^2$ for both Carbon and Iron. The data were collected at four beam energies 1.204, 2.346, 3.488 and 4.628 GeV at scattering angles between 10.65° and 70° . The high precision extracted cross sections have a total statistical and uncorrelated systematic uncertainties typically better than 2%.

The Rosenbluth separation was performed on both Carbon and Iron cross sections to extract the structure functions F_L, F_1, F_2 and the ratio of the longitudinal to transverse structure functions R . The results indicate a difference on the nuclear dependence of F_L and F_1 in the EMC region.

The results of this dissertation represent the most precise and complete survey of the inclusive cross sections and separations of the longitudinal and transverse structure functions on ^{12}C and ^{56}Fe across the nucleon resonance region ever performed in the range $0.5 < Q^2 < 2.5 \text{ (GeV}/c)^2$.

The extracted Carbon and Iron cross sections were used to study the nuclear modifications of R and σ_T . The Rosenbluth separation technique was performed on the cross section ratios to extract $\Delta R = R_C - R_{Fe}$ and σ_T^C/σ_T^{Fe} . The results suggest possible medium modifications of R and σ_T , and are consistent with what was observed from performing the Rosenbluth separation on individual targets.

It is necessary to complete the analysis of Aluminum and Deuterium targets of the experiment 04-001 and combine the results obtained from analyzing all targets. However, the results of this work should be a fertile ground for modeling the nuclear dependence

of R , and improving the neutrino cross section modeling.

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Appendix A:

A.1 Tables of Rosenbluth Separated F_L, F_1, F_2 and R

W^2	Q^2	x	F_L	ΔF_L	F_1	ΔF_1	F_2	ΔF_2	R	ΔR
0.35	1.75	1.43	0.0085	0.0039	0.0058	0.0011	0.0049	0.0002	0.5055	0.3239
0.39	1.75	1.39	0.0063	0.0044	0.0082	0.0012	0.0059	0.0002	0.2799	0.2350
0.51	2.20	1.20	0.0092	0.0016	0.0097	0.0004	0.0098	0.0002	0.3957	0.0834
0.59	1.25	1.30	0.0324	0.0058	0.0381	0.0016	0.0228	0.0004	0.3267	0.0722
0.63	0.80	1.46	0.0374	0.0230	0.0896	0.0062	0.0289	0.0007	0.1434	0.0979
0.67	0.80	1.36	0.0370	0.0251	0.1227	0.0074	0.0407	0.0009	0.1112	0.0821
0.67	1.25	1.20	0.0324	0.0074	0.0567	0.0022	0.0333	0.0006	0.2372	0.0630
0.67	2.20	1.11	0.0085	0.0023	0.0168	0.0006	0.0155	0.0003	0.2272	0.0687
0.71	0.80	1.27	0.0586	0.0252	0.1458	0.0080	0.0529	0.0011	0.1581	0.0766
0.71	1.25	1.16	0.0248	0.0071	0.0679	0.0023	0.0381	0.0006	0.1576	0.0504
0.71	2.20	1.08	0.0111	0.0042	0.0177	0.0016	0.0172	0.0003	0.2888	0.1352
0.75	0.80	1.19	0.0729	0.0346	0.1708	0.0129	0.0660	0.0012	0.1786	0.0982
0.75	1.25	1.12	0.0286	0.0073	0.0757	0.0024	0.0438	0.0007	0.1689	0.0484
0.75	1.75	1.08	0.0157	0.0080	0.0358	0.0030	0.0278	0.0004	0.2034	0.1204
0.75	2.20	1.06	0.0025	0.0038	0.0229	0.0015	0.0182	0.0003	0.0518	0.0823
0.79	0.80	1.13	0.0569	0.0312	0.2052	0.0126	0.0788	0.0012	0.1229	0.0749
0.79	1.25	1.08	0.0417	0.0069	0.0788	0.0024	0.0495	0.0007	0.2455	0.0477
0.79	2.20	1.04	0.0113	0.0028	0.0208	0.0011	0.0199	0.0003	0.2614	0.0771
0.83	1.25	1.04	0.0299	0.0081	0.0891	0.0028	0.0531	0.0012	0.1612	0.0487

W^2	Q^2	x	F_L	ΔF_L	F_1	ΔF_1	F_2	ΔF_2	R	ΔR
0.87	0.80	1.01	0.1079	0.0190	0.2228	0.0083	0.1014	0.0011	0.2390	0.0508
0.87	1.25	1.01	0.0281	0.0067	0.0948	0.0025	0.0567	0.0007	0.1471	0.0390
0.87	1.75	1.01	0.0220	0.0045	0.0411	0.0018	0.0345	0.0003	0.2664	0.0662
0.91	0.80	0.96	0.1242	0.0174	0.2225	0.0083	0.1087	0.0012	0.2894	0.0511
0.91	1.25	0.98	0.0284	0.0064	0.0997	0.0026	0.0605	0.0008	0.1457	0.0364
0.91	1.75	0.98	0.0144	0.0089	0.0469	0.0037	0.0362	0.0006	0.1562	0.1091
0.91	2.20	0.99	0.0127	0.0018	0.0245	0.0006	0.0239	0.0002	0.2618	0.0426
0.99	0.80	0.88	0.1250	0.0214	0.2272	0.0125	0.1191	0.0014	0.3129	0.0705
0.99	1.25	0.92	0.0375	0.0067	0.1018	0.0030	0.0665	0.0009	0.2005	0.0415
0.99	1.75	0.94	0.0193	0.0045	0.0493	0.0019	0.0403	0.0004	0.2078	0.0563
1.03	0.80	0.84	0.1111	0.0146	0.2310	0.0081	0.1213	0.0013	0.2853	0.0473
1.03	1.25	0.89	0.0270	0.0067	0.1102	0.0029	0.0689	0.0009	0.1370	0.0378
1.03	1.75	0.92	0.0173	0.0042	0.0517	0.0018	0.0416	0.0004	0.1818	0.0507
1.07	0.80	0.81	0.1055	0.0132	0.2298	0.0082	0.1231	0.0014	0.2840	0.0453
1.07	1.25	0.87	0.0276	0.0065	0.1104	0.0029	0.0702	0.0009	0.1439	0.0376
1.07	1.75	0.90	0.0217	0.0042	0.0517	0.0019	0.0436	0.0004	0.2330	0.0533
1.11	0.80	0.78	0.1048	0.0127	0.2228	0.0079	0.1233	0.0014	0.3027	0.0473
1.11	1.25	0.84	0.0326	0.0064	0.1104	0.0030	0.0728	0.0009	0.1748	0.0389
1.11	1.75	0.88	0.0255	0.0042	0.0531	0.0019	0.0464	0.0004	0.2714	0.0542
1.11	2.20	0.91	0.0159	0.0019	0.0312	0.0008	0.0313	0.0003	0.2807	0.0409
1.15	0.80	0.75	0.0976	0.0123	0.2228	0.0079	0.1244	0.0014	0.2930	0.0470
1.15	1.25	0.82	0.0352	0.0064	0.1108	0.0030	0.0749	0.0010	0.1930	0.0400
1.15	1.75	0.87	0.0194	0.0042	0.0568	0.0019	0.0469	0.0004	0.1969	0.0490
1.15	2.20	0.89	0.0144	0.0019	0.0322	0.0008	0.0316	0.0003	0.2516	0.0401

W^2	Q^2	x	F_L	ΔF_L	F_1	ΔF_1	F_2	ΔF_2	R	ΔR
1.19	0.80	0.72	0.0786	0.0131	0.2318	0.0078	0.1256	0.0014	0.2351	0.0468
1.19	1.75	0.85	0.0278	0.0043	0.0548	0.0020	0.0493	0.0005	0.2985	0.0568
1.23	0.80	0.70	0.0677	0.0117	0.2326	0.0074	0.1250	0.0012	0.2092	0.0425
1.23	0.80	0.70	0.1022	0.0125	0.2186	0.0061	0.1298	0.0012	0.3358	0.0502
1.23	1.75	0.83	0.0261	0.0045	0.0581	0.0020	0.0512	0.0006	0.2691	0.0555
1.23	2.20	0.86	0.0192	0.0021	0.0339	0.0009	0.0354	0.0004	0.3284	0.0435
1.27	0.80	0.67	0.0903	0.0123	0.2200	0.0079	0.1291	0.0017	0.3051	0.0521
1.27	1.75	0.82	0.0234	0.0044	0.0611	0.0020	0.0526	0.0006	0.2339	0.0519
1.31	0.80	0.65	0.0628	0.0171	0.2392	0.0114	0.1306	0.0019	0.2017	0.0643
1.31	1.75	0.80	0.0250	0.0045	0.0628	0.0020	0.0548	0.0007	0.2474	0.0522
1.31	2.20	0.84	0.0175	0.0021	0.0378	0.0009	0.0380	0.0004	0.2766	0.0401
1.35	2.20	0.82	0.0185	0.0021	0.0390	0.0009	0.0396	0.0004	0.2871	0.0389
1.39	0.80	0.61	0.0540	0.0141	0.2674	0.0077	0.1441	0.0022	0.1654	0.0478
1.39	0.80	0.61	0.0538	0.0108	0.2675	0.0069	0.1440	0.0014	0.1645	0.0371
1.39	1.75	0.77	0.0115	0.0048	0.0740	0.0024	0.0572	0.0006	0.1007	0.0455
1.39	2.20	0.81	0.0181	0.0022	0.0406	0.0010	0.0409	0.0004	0.2748	0.0390
1.43	0.80	0.59	0.0535	0.0104	0.2721	0.0071	0.1477	0.0014	0.1660	0.0363
1.47	0.80	0.58	0.0359	0.0100	0.2820	0.0064	0.1467	0.0015	0.1107	0.0332
1.47	1.75	0.75	0.0287	0.0031	0.0706	0.0016	0.0632	0.0005	0.2713	0.0352
1.51	0.50	0.44	0.0705	0.0152	0.4975	0.0149	0.2147	0.0018	0.1601	0.0393
1.51	1.75	0.74	0.0237	0.0084	0.0748	0.0044	0.0640	0.0012	0.2151	0.0886
1.51	2.20	0.78	0.0210	0.0023	0.0464	0.0009	0.0474	0.0006	0.2908	0.0366
1.55	0.50	0.43	0.0754	0.0149	0.4891	0.0152	0.2158	0.0021	0.1803	0.0410
1.55	1.75	0.72	0.0132	0.0085	0.0846	0.0046	0.0661	0.0010	0.1081	0.0752

W^2	Q^2	x	F_L	ΔF_L	F_1	ΔF_1	F_2	ΔF_2	R	ΔR
1.55	2.20	0.77	0.0203	0.0023	0.0480	0.0010	0.0484	0.0006	0.2760	0.0364
1.59	0.80	0.53	0.0391	0.0119	0.2973	0.0069	0.1584	0.0020	0.1242	0.0403
1.59	2.20	0.76	0.0262	0.0033	0.0477	0.0012	0.0514	0.0008	0.3633	0.0552
1.63	0.50	0.40	0.0588	0.0140	0.5121	0.0154	0.2203	0.0022	0.1435	0.0384
1.67	0.50	0.39	0.0364	0.0134	0.5322	0.0151	0.2181	0.0022	0.0882	0.0348
1.71	0.50	0.38	0.0481	0.0127	0.5093	0.0146	0.2160	0.0022	0.1256	0.0367
1.71	1.75	0.68	0.0314	0.0036	0.0881	0.0019	0.0783	0.0007	0.2625	0.0359
1.75	0.50	0.37	0.0618	0.0124	0.4978	0.0144	0.2194	0.0022	0.1701	0.0390
1.75	1.75	0.67	0.0331	0.0037	0.0894	0.0019	0.0804	0.0008	0.2771	0.0366
1.75	2.20	0.72	0.0142	0.0060	0.0627	0.0029	0.0571	0.0009	0.1583	0.0738
1.79	0.50	0.35	0.0613	0.0121	0.4926	0.0142	0.2178	0.0022	0.1754	0.0394
1.79	1.75	0.66	0.0353	0.0037	0.0920	0.0020	0.0836	0.0008	0.2918	0.0367
1.83	1.75	0.65	0.0380	0.0038	0.0949	0.0020	0.0873	0.0009	0.3092	0.0373
1.83	1.75	0.65	0.0382	0.0037	0.0949	0.0020	0.0873	0.0007	0.3104	0.0366
1.87	1.25	0.56	0.0580	0.0065	0.1736	0.0044	0.1341	0.0013	0.2991	0.0406
1.87	1.75	0.64	0.0386	0.0038	0.0990	0.0021	0.0906	0.0008	0.3050	0.0364
1.91	1.75	0.63	0.0351	0.0038	0.1038	0.0021	0.0922	0.0009	0.2686	0.0339
1.91	2.20	0.68	0.0232	0.0062	0.0705	0.0030	0.0684	0.0011	0.2417	0.0746
1.95	1.25	0.54	0.0509	0.0082	0.1885	0.0049	0.1397	0.0019	0.2504	0.0465
1.95	1.75	0.62	0.0400	0.0036	0.1054	0.0020	0.0963	0.0009	0.3057	0.0332
1.99	1.25	0.53	0.0396	0.0081	0.1946	0.0049	0.1372	0.0020	0.1922	0.0441
1.99	1.75	0.61	0.0445	0.0038	0.1083	0.0021	0.1010	0.0009	0.3355	0.0345
2.03	1.75	0.60	0.0415	0.0039	0.1131	0.0022	0.1027	0.0009	0.3040	0.0345
2.07	1.25	0.51	0.0599	0.0084	0.1972	0.0051	0.1506	0.0023	0.2963	0.0487

W^2	Q^2	x	F_L	ΔF_L	F_1	ΔF_1	F_2	ΔF_2	R	ΔR
2.11	1.25	0.50	0.0381	0.0080	0.2119	0.0050	0.1467	0.0021	0.1782	0.0415
2.15	1.25	0.50	0.0458	0.0081	0.2136	0.0051	0.1522	0.0022	0.2161	0.0429
2.19	2.20	0.63	0.0312	0.0067	0.0896	0.0036	0.0881	0.0013	0.2775	0.0708
2.23	1.25	0.48	0.0596	0.0086	0.2224	0.0056	0.1656	0.0024	0.2788	0.0469
2.27	1.25	0.47	0.0529	0.0085	0.2306	0.0056	0.1663	0.0025	0.2420	0.0442
2.27	1.75	0.56	0.0479	0.0047	0.1343	0.0025	0.1216	0.0015	0.3199	0.0370
2.27	2.20	0.61	0.0233	0.0070	0.0994	0.0038	0.0906	0.0016	0.1913	0.0649
2.31	2.20	0.61	0.0163	0.0071	0.1071	0.0039	0.0920	0.0017	0.1256	0.0588
2.35	1.75	0.54	0.0515	0.0044	0.1433	0.0025	0.1300	0.0014	0.3305	0.0336
2.39	1.25	0.45	0.0695	0.0085	0.2382	0.0057	0.1808	0.0027	0.3221	0.0464
2.39	1.75	0.54	0.0527	0.0046	0.1471	0.0028	0.1333	0.0014	0.3338	0.0354
2.39	2.20	0.59	0.0233	0.0074	0.1115	0.0041	0.0995	0.0016	0.1763	0.0621
2.43	1.25	0.45	0.0593	0.0117	0.2576	0.0105	0.1853	0.0019	0.2579	0.0611
2.43	1.25	0.45	0.0661	0.0063	0.2497	0.0046	0.1851	0.0016	0.2964	0.0333
2.47	0.50	0.24	0.1399	0.0188	0.5005	0.0339	0.2704	0.0027	0.5841	0.1177
2.47	0.80	0.33	0.0962	0.0104	0.3999	0.0117	0.2437	0.0022	0.3594	0.0490
2.47	1.75	0.52	0.0563	0.0046	0.1541	0.0029	0.1403	0.0015	0.3485	0.0347
2.51	0.80	0.33	0.0911	0.0104	0.4146	0.0116	0.2465	0.0023	0.3335	0.0472
2.51	1.25	0.43	0.0754	0.0086	0.2555	0.0058	0.1942	0.0030	0.3399	0.0459
2.51	1.75	0.52	0.0624	0.0046	0.1552	0.0029	0.1449	0.0016	0.3884	0.0354
2.55	0.50	0.23	0.1487	0.0202	0.5048	0.0364	0.2775	0.0035	0.6389	0.1326
2.55	1.75	0.51	0.0620	0.0046	0.1583	0.0029	0.1467	0.0016	0.3825	0.0350
2.59	0.50	0.23	0.1150	0.0200	0.5826	0.0367	0.2783	0.0035	0.4362	0.1031
2.59	1.25	0.42	0.0727	0.0067	0.2685	0.0056	0.1994	0.0018	0.3206	0.0361

W^2	Q^2	x	F_L	ΔF_L	F_1	ΔF_1	F_2	ΔF_2	R	ΔR
2.59	1.75	0.51	0.0666	0.0046	0.1616	0.0029	0.1519	0.0016	0.4073	0.0354
2.59	2.20	0.56	0.0318	0.0102	0.1237	0.0056	0.1135	0.0031	0.2283	0.0834
2.63	0.50	0.22	0.1231	0.0195	0.5723	0.0359	0.2800	0.0035	0.4836	0.1066
2.63	0.80	0.31	0.0918	0.0082	0.4333	0.0099	0.2537	0.0019	0.3375	0.0377
2.63	1.75	0.50	0.0615	0.0047	0.1692	0.0030	0.1535	0.0017	0.3635	0.0339
2.63	2.20	0.56	0.0147	0.0098	0.1359	0.0053	0.1110	0.0032	0.0969	0.0685
2.67	0.50	0.22	0.1149	0.0191	0.5942	0.0356	0.2803	0.0036	0.4427	0.0999
2.67	1.25	0.41	0.0740	0.0063	0.2786	0.0055	0.2053	0.0017	0.3230	0.0335
2.67	1.25	0.41	0.0973	0.0067	0.2619	0.0052	0.2118	0.0019	0.4519	0.0394
2.67	1.75	0.49	0.0474	0.0084	0.1839	0.0054	0.1537	0.0024	0.2604	0.0538
2.71	0.50	0.21	0.0852	0.0189	0.6603	0.0360	0.2784	0.0036	0.3008	0.0830
2.71	0.80	0.30	0.1065	0.0096	0.4341	0.0117	0.2633	0.0023	0.4031	0.0469
2.71	1.75	0.49	0.0551	0.0096	0.1833	0.0067	0.1582	0.0026	0.3073	0.0645
2.71	1.75	0.49	0.0484	0.0095	0.1883	0.0062	0.1570	0.0023	0.2631	0.0603
2.71	2.20	0.55	0.0356	0.0061	0.1324	0.0035	0.1219	0.0017	0.2462	0.0484
2.75	0.50	0.21	0.1073	0.0190	0.6277	0.0366	0.2833	0.0036	0.4051	0.0950
2.75	1.25	0.40	0.0949	0.0067	0.2796	0.0056	0.2197	0.0021	0.4235	0.0379
2.79	0.50	0.21	0.1088	0.0183	0.6345	0.0355	0.2856	0.0037	0.4133	0.0925
2.79	0.80	0.30	0.0980	0.0109	0.4650	0.0139	0.2692	0.0025	0.3569	0.0502
2.79	1.25	0.40	0.0692	0.0123	0.3027	0.0107	0.2142	0.0028	0.2888	0.0612
2.79	2.20	0.54	0.0429	0.0050	0.1349	0.0027	0.1285	0.0015	0.2973	0.0401
2.83	0.50	0.20	0.1335	0.0166	0.5837	0.0333	0.2875	0.0031	0.5603	0.1014
2.83	0.80	0.29	0.0944	0.0107	0.4760	0.0138	0.2706	0.0025	0.3410	0.0484
2.83	1.25	0.39	0.0955	0.0076	0.2886	0.0058	0.2245	0.0028	0.4237	0.0417

W^2	Q^2	x	F_L	ΔF_L	F_1	ΔF_1	F_2	ΔF_2	R	ΔR
2.83	1.25	0.39	0.0610	0.0124	0.3178	0.0109	0.2163	0.0028	0.2455	0.0581
2.83	1.75	0.47	0.0565	0.0095	0.1968	0.0062	0.1674	0.0026	0.3038	0.0603
2.83	2.20	0.53	0.0431	0.0062	0.1377	0.0036	0.1305	0.0020	0.2953	0.0498
2.87	0.50	0.20	0.1105	0.0177	0.6376	0.0347	0.2855	0.0038	0.4316	0.0925
2.87	0.80	0.29	0.1041	0.0108	0.4691	0.0136	0.2740	0.0028	0.3869	0.0511
2.87	1.25	0.39	0.0674	0.0123	0.3172	0.0108	0.2200	0.0029	0.2755	0.0593
2.87	2.20	0.53	0.0372	0.0057	0.1462	0.0032	0.1323	0.0018	0.2420	0.0422
2.91	0.50	0.20	0.0960	0.0171	0.6620	0.0338	0.2805	0.0037	0.3669	0.0839
2.91	0.80	0.28	0.0951	0.0104	0.4815	0.0135	0.2717	0.0026	0.3493	0.0478
2.91	1.25	0.38	0.0706	0.0123	0.3227	0.0110	0.2246	0.0030	0.2869	0.0595
2.95	0.50	0.19	0.1213	0.0171	0.6410	0.0340	0.2927	0.0039	0.4862	0.0940
2.95	0.80	0.28	0.1134	0.0079	0.4659	0.0101	0.2781	0.0023	0.4366	0.0395
2.95	1.25	0.38	0.0773	0.0108	0.3220	0.0101	0.2285	0.0025	0.3187	0.0544
2.95	2.20	0.52	0.0440	0.0057	0.1503	0.0033	0.1395	0.0020	0.2838	0.0424
2.99	0.80	0.27	0.1045	0.0101	0.4811	0.0132	0.2769	0.0026	0.3950	0.0488
2.99	1.25	0.37	0.0690	0.0107	0.3370	0.0101	0.2301	0.0025	0.2752	0.0509
3.03	0.50	0.19	0.1227	0.0162	0.6328	0.0327	0.2890	0.0038	0.5137	0.0942
3.03	1.25	0.37	0.0859	0.0107	0.3262	0.0101	0.2360	0.0026	0.3581	0.0553
3.03	2.20	0.51	0.0503	0.0063	0.1532	0.0038	0.1456	0.0023	0.3247	0.0486
3.07	0.50	0.19	0.0943	0.0108	0.7060	0.0181	0.2869	0.0043	0.3591	0.0501
3.07	1.25	0.36	0.0813	0.0118	0.3321	0.0109	0.2352	0.0034	0.3367	0.0597
3.11	0.50	0.18	0.0966	0.0158	0.7005	0.0325	0.2857	0.0038	0.3764	0.0787
3.11	1.25	0.36	0.0795	0.0114	0.3404	0.0110	0.2377	0.0029	0.3251	0.0567
3.11	2.20	0.50	0.0513	0.0050	0.1595	0.0028	0.1504	0.0019	0.3239	0.0371

W^2	Q^2	x	F_L	ΔF_L	F_1	ΔF_1	F_2	ΔF_2	R	ΔR
3.15	0.50	0.18	0.1179	0.0153	0.6640	0.0312	0.2909	0.0039	0.4918	0.0865
3.15	1.25	0.36	0.0836	0.0113	0.3452	0.0110	0.2426	0.0030	0.3412	0.0567
3.15	1.75	0.44	0.0535	0.0085	0.2375	0.0062	0.1884	0.0026	0.2586	0.0479
3.15	2.20	0.49	0.0477	0.0058	0.1641	0.0029	0.1508	0.0025	0.2951	0.0404
3.19	0.50	0.18	0.1043	0.0105	0.6939	0.0175	0.2872	0.0044	0.4222	0.0528
3.19	1.25	0.35	0.0886	0.0112	0.3432	0.0109	0.2446	0.0030	0.3676	0.0580
3.23	0.50	0.18	0.1234	0.0146	0.6568	0.0302	0.2908	0.0038	0.5352	0.0873
3.23	0.80	0.25	0.1114	0.0090	0.5069	0.0120	0.2873	0.0027	0.4326	0.0450
3.23	1.25	0.35	0.0822	0.0119	0.3521	0.0112	0.2439	0.0037	0.3361	0.0592
3.23	2.20	0.48	0.0414	0.0058	0.1757	0.0029	0.1537	0.0025	0.2436	0.0376
3.27	0.50	0.17	0.1179	0.0125	0.6817	0.0275	0.2922	0.0030	0.4998	0.0727
3.27	1.75	0.42	0.0582	0.0086	0.2437	0.0064	0.1944	0.0027	0.2825	0.0488
3.31	0.50	0.17	0.1189	0.0122	0.6818	0.0272	0.2918	0.0030	0.5109	0.0726
3.31	1.25	0.34	0.0974	0.0121	0.3512	0.0116	0.2536	0.0038	0.4083	0.0640
3.35	0.50	0.17	0.1072	0.0082	0.7120	0.0146	0.2892	0.0033	0.4469	0.0428
3.35	1.25	0.34	0.0854	0.0113	0.3667	0.0114	0.2518	0.0032	0.3465	0.0564
3.35	1.75	0.41	0.0735	0.0084	0.2411	0.0063	0.2031	0.0030	0.3674	0.0511
3.35	2.20	0.47	0.0357	0.0066	0.1883	0.0038	0.1572	0.0029	0.2011	0.0408
3.39	0.50	0.17	0.0998	0.0080	0.7282	0.0144	0.2862	0.0033	0.4126	0.0405
3.39	2.20	0.47	0.0519	0.0062	0.1810	0.0036	0.1638	0.0030	0.3069	0.0422
3.43	1.75	0.41	0.0625	0.0085	0.2578	0.0064	0.2042	0.0029	0.2979	0.0474
3.43	2.20	0.46	0.0434	0.0060	0.1901	0.0033	0.1634	0.0027	0.2465	0.0378
3.47	2.20	0.46	0.0513	0.0062	0.1885	0.0035	0.1678	0.0030	0.2962	0.0411
3.51	1.25	0.32	0.0830	0.0101	0.3871	0.0107	0.2573	0.0031	0.3328	0.0494

W^2	Q^2	x	F_L	ΔF_L	F_1	ΔF_1	F_2	ΔF_2	R	ΔR
3.51	1.75	0.40	0.0666	0.0084	0.2640	0.0065	0.2100	0.0031	0.3155	0.0474
3.51	2.20	0.46	0.0562	0.0063	0.1902	0.0036	0.1723	0.0031	0.3246	0.0421
3.55	1.25	0.32	0.0896	0.0106	0.3892	0.0097	0.2626	0.0034	0.3607	0.0512
3.55	2.20	0.45	0.0466	0.0065	0.2017	0.0036	0.1725	0.0029	0.2558	0.0396
3.59	1.75	0.39	0.0654	0.0086	0.2742	0.0066	0.2142	0.0034	0.3040	0.0471
3.59	2.20	0.45	0.0446	0.0064	0.2061	0.0036	0.1735	0.0029	0.2413	0.0385
3.63	1.25	0.31	0.0923	0.0164	0.3987	0.0175	0.2678	0.0051	0.3705	0.0818
3.63	1.75	0.39	0.0666	0.0084	0.2755	0.0065	0.2153	0.0035	0.3108	0.0463
3.63	1.75	0.39	0.0674	0.0080	0.2774	0.0050	0.2171	0.0034	0.3123	0.0422
3.63	2.20	0.44	0.0409	0.0064	0.2107	0.0035	0.1734	0.0028	0.2184	0.0373
3.67	0.80	0.22	0.1018	0.0112	0.5782	0.0168	0.2950	0.0036	0.3951	0.0547
3.67	1.75	0.39	0.0629	0.0084	0.2836	0.0058	0.2167	0.0038	0.2875	0.0438
3.71	0.80	0.22	0.1071	0.0113	0.5811	0.0172	0.2992	0.0037	0.4180	0.0562
3.71	1.75	0.38	0.0787	0.0084	0.2800	0.0058	0.2262	0.0039	0.3681	0.0464
3.71	2.20	0.44	0.0369	0.0068	0.2268	0.0041	0.1801	0.0035	0.1861	0.0370
3.75	0.50	0.15	0.1118	0.0142	0.7495	0.0288	0.2893	0.0040	0.5026	0.0829
3.75	0.80	0.22	0.0943	0.0112	0.6051	0.0171	0.2962	0.0037	0.3576	0.0524
3.75	1.75	0.38	0.0769	0.0073	0.2843	0.0055	0.2268	0.0033	0.3568	0.0404
3.75	2.20	0.43	0.0357	0.0069	0.2272	0.0040	0.1790	0.0037	0.1809	0.0376
3.79	0.50	0.15	0.1150	0.0142	0.7602	0.0288	0.2935	0.0040	0.5159	0.0827
3.79	1.25	0.30	0.1168	0.0166	0.3899	0.0182	0.2799	0.0052	0.4985	0.0940
3.79	1.75	0.38	0.0813	0.0073	0.2865	0.0055	0.2309	0.0033	0.3778	0.0409
3.79	2.20	0.43	0.0184	0.0092	0.2330	0.0044	0.1689	0.0045	0.0919	0.0472
3.83	0.80	0.21	0.0981	0.0111	0.6065	0.0167	0.2974	0.0037	0.3791	0.0529

W^2	Q^2	x	F_L	ΔF_L	F_1	ΔF_1	F_2	ΔF_2	R	ΔR
3.83	1.25	0.30	0.1140	0.0124	0.3971	0.0132	0.2804	0.0042	0.4824	0.0682
3.83	1.75	0.37	0.0686	0.0096	0.2976	0.0058	0.2269	0.0047	0.3093	0.0487
3.83	2.20	0.43	0.0462	0.0077	0.2283	0.0041	0.1868	0.0037	0.2370	0.0432
3.87	0.80	0.21	0.0983	0.0109	0.6078	0.0166	0.2967	0.0037	0.3830	0.0524
3.87	1.75	0.37	0.0669	0.0095	0.2997	0.0059	0.2261	0.0045	0.3020	0.0484
3.87	2.20	0.42	0.0430	0.0074	0.2303	0.0042	0.1850	0.0038	0.2202	0.0415
3.91	0.50	0.14	0.0787	0.0097	0.8459	0.0228	0.2790	0.0037	0.3285	0.0489
3.91	1.25	0.29	0.0950	0.0139	0.4224	0.0140	0.2756	0.0052	0.3851	0.0687
3.95	0.50	0.14	0.0964	0.0109	0.8294	0.0241	0.2888	0.0034	0.4148	0.0585
3.95	1.25	0.29	0.0841	0.0104	0.4373	0.0085	0.2729	0.0045	0.3324	0.0473
3.95	1.75	0.36	0.0559	0.0091	0.3202	0.0055	0.2280	0.0042	0.2406	0.0428
3.99	0.50	0.14	0.1117	0.0108	0.8029	0.0237	0.2943	0.0034	0.5021	0.0628
3.99	1.25	0.29	0.1082	0.0120	0.4270	0.0111	0.2867	0.0051	0.4421	0.0603
3.99	1.75	0.36	0.0529	0.0089	0.3222	0.0056	0.2260	0.0043	0.2280	0.0419
4.03	0.50	0.14	0.1087	0.0102	0.8135	0.0234	0.2929	0.0030	0.4876	0.0593
4.11	0.80	0.20	0.0906	0.0117	0.6480	0.0198	0.2964	0.0039	0.3520	0.0560
4.11	1.25	0.28	0.0985	0.0115	0.4438	0.0105	0.2839	0.0052	0.3978	0.0555
4.11	1.75	0.35	0.0661	0.0102	0.3207	0.0064	0.2335	0.0051	0.2932	0.0506
4.15	1.75	0.35	0.0684	0.0101	0.3218	0.0065	0.2353	0.0053	0.3048	0.0506
4.19	1.25	0.27	0.0951	0.0128	0.4521	0.0128	0.2831	0.0059	0.3839	0.0621
4.23	0.80	0.19	0.0944	0.0182	0.6509	0.0317	0.2968	0.0057	0.3761	0.0906
4.23	1.25	0.27	0.1018	0.0120	0.4578	0.0102	0.2902	0.0057	0.4090	0.0567
4.23	1.75	0.34	0.0675	0.0102	0.3330	0.0070	0.2394	0.0053	0.2955	0.0503
4.27	0.80	0.19	0.0936	0.0180	0.6575	0.0312	0.2970	0.0058	0.3728	0.0889

W^2	Q^2	x	F_L	ΔF_L	F_1	ΔF_1	F_2	ΔF_2	R	ΔR
4.27	1.25	0.27	0.1119	0.0120	0.4494	0.0106	0.2940	0.0058	0.4622	0.0599
4.27	1.75	0.34	0.0966	0.0101	0.3147	0.0071	0.2521	0.0055	0.4509	0.0564
4.31	0.80	0.19	0.1126	0.0179	0.6318	0.0310	0.3038	0.0059	0.4713	0.0975
4.31	1.25	0.27	0.1166	0.0120	0.4480	0.0108	0.2964	0.0058	0.4874	0.0610
4.31	1.75	0.34	0.0940	0.0102	0.3199	0.0071	0.2522	0.0054	0.4348	0.0559
4.35	1.25	0.26	0.0918	0.0125	0.4706	0.0128	0.2848	0.0060	0.3681	0.0597
4.35	1.75	0.34	0.0947	0.0102	0.3219	0.0072	0.2532	0.0055	0.4385	0.0563
4.39	1.25	0.26	0.0935	0.0129	0.4786	0.0134	0.2888	0.0062	0.3720	0.0611

Table 9: Structure functions for Carbon.

W^2	Q^2	x	F_L	ΔF_L	F_1	ΔF_1	F_2	ΔF_2	R	ΔR
0.75	2.20	1.06	0.0045	0.0025	0.0201	0.0009	0.0168	0.0004	0.1046	0.0630
0.91	2.20	0.99	0.0025	0.0026	0.0265	0.0009	0.0214	0.0004	0.0472	0.0508
1.03	1.75	0.92	-0.0069	0.0115	0.0587	0.0052	0.0374	0.0008	-0.0638	0.1003
1.07	1.75	0.90	0.0104	0.0114	0.0538	0.0053	0.0408	0.0008	0.1072	0.1282
1.11	1.75	0.88	0.0101	0.0113	0.0566	0.0054	0.0428	0.0008	0.1004	0.1223
1.11	2.20	0.91	0.0131	0.0033	0.0305	0.0014	0.0295	0.0004	0.2365	0.0710
1.15	1.75	0.87	0.0238	0.0105	0.0511	0.0051	0.0447	0.0008	0.2686	0.1445
1.15	2.20	0.89	0.0042	0.0033	0.0357	0.0014	0.0299	0.0004	0.0659	0.0540
1.19	1.75	0.85	0.0194	0.0102	0.0566	0.0051	0.0471	0.0008	0.2020	0.1243
1.23	1.75	0.83	0.0152	0.0105	0.0613	0.0053	0.0489	0.0009	0.1483	0.1151
1.23	2.20	0.86	0.0112	0.0033	0.0362	0.0015	0.0337	0.0004	0.1797	0.0591
1.27	1.75	0.82	0.0149	0.0102	0.0634	0.0052	0.0506	0.0009	0.1436	0.1097
1.31	1.75	0.80	-0.0024	0.0105	0.0743	0.0053	0.0508	0.0010	-0.0205	0.0862
1.31	2.20	0.84	0.0125	0.0032	0.0384	0.0015	0.0362	0.0005	0.1947	0.0569
1.35	2.20	0.82	0.0128	0.0032	0.0406	0.0015	0.0382	0.0005	0.1910	0.0550
1.39	1.75	0.77	0.0076	0.0100	0.0742	0.0052	0.0555	0.0010	0.0662	0.0914
1.39	2.20	0.81	0.0134	0.0033	0.0417	0.0015	0.0395	0.0005	0.1984	0.0551
1.47	1.75	0.75	0.0159	0.0046	0.0767	0.0026	0.0615	0.0006	0.1382	0.0448
1.51	0.50	0.44	0.0296	0.0153	0.5405	0.0152	0.2135	0.0018	0.0619	0.0337
1.51	1.75	0.74	0.0166	0.0090	0.0790	0.0047	0.0636	0.0012	0.1428	0.0858
1.51	2.20	0.78	0.0147	0.0029	0.0480	0.0013	0.0454	0.0006	0.1969	0.0439
1.55	0.50	0.43	0.0576	0.0147	0.5116	0.0153	0.2165	0.0020	0.1318	0.0374
1.55	1.75	0.72	0.0057	0.0093	0.0879	0.0051	0.0648	0.0011	0.0451	0.0757
1.55	2.20	0.77	0.0132	0.0030	0.0508	0.0014	0.0469	0.0006	0.1689	0.0431

W^2	Q^2	x	F_L	ΔF_L	F_1	ΔF_1	F_2	ΔF_2	R	ΔR
1.59	2.20	0.76	0.0149	0.0041	0.0523	0.0017	0.0491	0.0009	0.1883	0.0574
1.63	0.50	0.40	0.0311	0.0138	0.5450	0.0156	0.2196	0.0021	0.0712	0.0337
1.67	0.50	0.39	0.0452	0.0130	0.5195	0.0148	0.2176	0.0021	0.1121	0.0354
1.71	0.50	0.38	0.0431	0.0125	0.5138	0.0146	0.2152	0.0021	0.1116	0.0354
1.71	1.75	0.68	0.0244	0.0046	0.0922	0.0026	0.0776	0.0008	0.1949	0.0421
1.75	0.50	0.37	0.0609	0.0122	0.4974	0.0144	0.2187	0.0022	0.1676	0.0383
1.75	1.75	0.67	0.0221	0.0047	0.0961	0.0026	0.0793	0.0009	0.1721	0.0411
1.75	2.20	0.72	0.0189	0.0082	0.0604	0.0040	0.0579	0.0013	0.2186	0.1086
1.79	0.50	0.35	0.0676	0.0120	0.4944	0.0143	0.2218	0.0022	0.1928	0.0395
1.79	1.75	0.66	0.0216	0.0047	0.1014	0.0027	0.0828	0.0009	0.1617	0.0391
1.83	1.75	0.65	0.0189	0.0049	0.1051	0.0027	0.0841	0.0010	0.1387	0.0391
1.83	1.75	0.65	0.0197	0.0046	0.1053	0.0027	0.0846	0.0008	0.1442	0.0374
1.87	1.25	0.56	0.0497	0.0077	0.1806	0.0057	0.1338	0.0012	0.2464	0.0458
1.87	1.75	0.64	0.0288	0.0047	0.1040	0.0027	0.0888	0.0009	0.2172	0.0406
1.91	1.75	0.63	0.0236	0.0049	0.1127	0.0029	0.0921	0.0010	0.1665	0.0385
1.91	2.20	0.68	0.0163	0.0083	0.0730	0.0042	0.0664	0.0014	0.1637	0.0923
1.95	1.25	0.54	0.0465	0.0096	0.1974	0.0064	0.1426	0.0020	0.2186	0.0521
1.95	1.75	0.62	0.0249	0.0045	0.1146	0.0026	0.0942	0.0010	0.1752	0.0356
1.99	1.25	0.53	0.0354	0.0095	0.2033	0.0064	0.1400	0.0020	0.1645	0.0489
1.99	1.75	0.61	0.0275	0.0047	0.1179	0.0028	0.0980	0.0010	0.1904	0.0371
2.03	1.75	0.60	0.0303	0.0053	0.1223	0.0033	0.1027	0.0011	0.2054	0.0410
2.07	1.25	0.51	0.0564	0.0098	0.2053	0.0064	0.1534	0.0024	0.2681	0.0545
2.11	1.25	0.50	0.0551	0.0094	0.2073	0.0064	0.1539	0.0022	0.2634	0.0525
2.15	1.25	0.50	0.0503	0.0096	0.2216	0.0065	0.1595	0.0024	0.2287	0.0500

W^2	Q^2	x	F_L	ΔF_L	F_1	ΔF_1	F_2	ΔF_2	R	ΔR
2.19	2.20	0.63	0.0266	0.0083	0.0907	0.0046	0.0862	0.0015	0.2342	0.0845
2.23	1.25	0.48	0.0551	0.0097	0.2288	0.0068	0.1666	0.0025	0.2503	0.0513
2.27	1.25	0.47	0.0341	0.0096	0.2449	0.0068	0.1630	0.0025	0.1471	0.0451
2.27	1.75	0.56	0.0383	0.0070	0.1414	0.0039	0.1206	0.0021	0.2432	0.0502
2.27	2.20	0.61	0.0279	0.0093	0.0975	0.0051	0.0921	0.0020	0.2339	0.0899
2.31	2.20	0.61	0.0234	0.0092	0.1032	0.0051	0.0935	0.0022	0.1868	0.0829
2.35	1.75	0.54	0.0424	0.0062	0.1486	0.0037	0.1279	0.0019	0.2625	0.0444
2.39	1.25	0.45	0.0719	0.0096	0.2414	0.0069	0.1841	0.0028	0.3287	0.0531
2.39	1.75	0.54	0.0387	0.0070	0.1575	0.0043	0.1315	0.0020	0.2286	0.0474
2.39	2.20	0.59	0.0040	0.0095	0.1202	0.0053	0.0938	0.0020	0.0284	0.0679
2.43	1.25	0.45	0.0215	0.0120	0.2919	0.0105	0.1807	0.0021	0.0824	0.0488
2.43	1.25	0.45	0.0610	0.0089	0.2607	0.0066	0.1881	0.0023	0.2619	0.0445
2.47	0.50	0.24	0.1120	0.0205	0.5649	0.0364	0.2725	0.0033	0.4143	0.1023
2.47	0.80	0.33	0.0984	0.0160	0.3994	0.0197	0.2450	0.0027	0.3680	0.0776
2.47	1.75	0.52	0.0479	0.0070	0.1616	0.0044	0.1399	0.0021	0.2829	0.0485
2.51	0.80	0.33	0.0962	0.0146	0.4143	0.0169	0.2498	0.0029	0.3524	0.0676
2.51	1.25	0.43	0.0559	0.0095	0.2683	0.0070	0.1887	0.0030	0.2401	0.0466
2.51	1.75	0.52	0.0439	0.0072	0.1677	0.0045	0.1414	0.0023	0.2527	0.0478
2.55	0.50	0.23	0.0985	0.0244	0.6031	0.0419	0.2740	0.0048	0.3542	0.1120
2.55	1.75	0.51	0.0400	0.0065	0.1756	0.0041	0.1439	0.0021	0.2228	0.0413
2.59	0.50	0.23	0.0850	0.0240	0.6433	0.0418	0.2764	0.0048	0.2920	0.1012
2.59	1.25	0.42	0.0545	0.0090	0.2835	0.0077	0.1957	0.0022	0.2277	0.0437
2.59	1.75	0.51	0.0541	0.0064	0.1672	0.0040	0.1474	0.0022	0.3195	0.0450
2.59	2.20	0.56	0.0178	0.0113	0.1304	0.0065	0.1092	0.0032	0.1211	0.0827

W^2	Q^2	x	F_L	ΔF_L	F_1	ΔF_1	F_2	ΔF_2	R	ΔR
2.63	0.50	0.22	0.1029	0.0233	0.6105	0.0409	0.2777	0.0048	0.3790	0.1111
2.63	0.80	0.31	0.0696	0.0102	0.4673	0.0129	0.2532	0.0021	0.2374	0.0413
2.63	1.75	0.50	0.0387	0.0066	0.1835	0.0042	0.1478	0.0022	0.2109	0.0406
2.63	2.20	0.56	0.0230	0.0109	0.1308	0.0062	0.1128	0.0032	0.1580	0.0820
2.67	0.50	0.22	0.0751	0.0226	0.6643	0.0401	0.2734	0.0048	0.2586	0.0932
2.67	1.25	0.41	0.0551	0.0083	0.2946	0.0075	0.2014	0.0019	0.2273	0.0398
2.67	1.25	0.41	0.0634	0.0093	0.2930	0.0075	0.2061	0.0025	0.2631	0.0451
2.67	1.75	0.49	0.0435	0.0115	0.1835	0.0072	0.1508	0.0033	0.2398	0.0724
2.71	0.50	0.21	0.0844	0.0225	0.6650	0.0405	0.2792	0.0049	0.2957	0.0967
2.71	0.80	0.30	0.0996	0.0140	0.4429	0.0180	0.2622	0.0028	0.3695	0.0668
2.71	1.25	0.41	0.0641	0.0089	0.2955	0.0083	0.2076	0.0021	0.2671	0.0446
2.71	1.75	0.49	0.0318	0.0112	0.1991	0.0077	0.1529	0.0026	0.1631	0.0638
2.71	2.20	0.55	0.0303	0.0077	0.1349	0.0046	0.1202	0.0021	0.2053	0.0591
2.75	0.50	0.21	0.0821	0.0215	0.6582	0.0391	0.2740	0.0048	0.2956	0.0947
2.75	1.25	0.40	0.0856	0.0089	0.2906	0.0083	0.2193	0.0022	0.3678	0.0484
2.79	0.50	0.21	0.1154	0.0214	0.6278	0.0389	0.2884	0.0050	0.4428	0.1093
2.79	0.80	0.30	0.0888	0.0138	0.4729	0.0180	0.2660	0.0029	0.3180	0.0614
2.79	1.25	0.40	0.0654	0.0163	0.3078	0.0144	0.2145	0.0036	0.2687	0.0792
2.79	2.20	0.54	0.0319	0.0080	0.1411	0.0047	0.1254	0.0021	0.2115	0.0601
2.83	0.50	0.20	0.1114	0.0175	0.6362	0.0344	0.2869	0.0036	0.4288	0.0905
2.83	0.80	0.29	0.1058	0.0135	0.4596	0.0178	0.2719	0.0031	0.3956	0.0656
2.83	1.25	0.39	0.0956	0.0089	0.2927	0.0075	0.2268	0.0029	0.4183	0.0490
2.83	1.25	0.39	0.0660	0.0163	0.3165	0.0145	0.2191	0.0037	0.2669	0.0778
2.83	1.75	0.47	0.0341	0.0112	0.2133	0.0078	0.1627	0.0029	0.1692	0.0616

W^2	Q^2	x	F_L	ΔF_L	F_1	ΔF_1	F_2	ΔF_2	R	ΔR
2.83	2.20	0.53	0.0287	0.0079	0.1467	0.0047	0.1270	0.0025	0.1844	0.0565
2.87	0.50	0.20	0.1165	0.0170	0.6316	0.0337	0.2883	0.0035	0.4592	0.0913
2.87	0.80	0.29	0.1048	0.0135	0.4705	0.0177	0.2751	0.0032	0.3883	0.0644
2.87	1.25	0.39	0.0821	0.0160	0.3067	0.0144	0.2245	0.0038	0.3469	0.0838
2.87	2.20	0.53	0.0236	0.0079	0.1501	0.0047	0.1258	0.0022	0.1498	0.0546
2.91	0.50	0.20	0.0939	0.0166	0.6838	0.0332	0.2856	0.0035	0.3472	0.0779
2.91	0.80	0.28	0.1132	0.0131	0.4607	0.0174	0.2764	0.0030	0.4346	0.0665
2.91	1.25	0.38	0.0711	0.0161	0.3203	0.0144	0.2237	0.0039	0.2914	0.0790
2.95	0.50	0.19	0.1062	0.0161	0.6601	0.0326	0.2867	0.0035	0.4135	0.0830
2.95	0.80	0.28	0.0976	0.0101	0.4801	0.0135	0.2722	0.0026	0.3645	0.0476
2.95	2.20	0.52	0.0368	0.0079	0.1525	0.0048	0.1362	0.0025	0.2342	0.0574
2.99	0.80	0.27	0.1223	0.0125	0.4565	0.0169	0.2801	0.0030	0.4872	0.0675
3.03	0.50	0.19	0.1226	0.0155	0.6416	0.0318	0.2916	0.0035	0.5061	0.0890
3.03	2.20	0.51	0.0262	0.0074	0.1677	0.0047	0.1390	0.0024	0.1545	0.0476
3.07	0.50	0.19	0.0953	0.0123	0.7091	0.0218	0.2887	0.0044	0.3613	0.0575
3.07	1.25	0.36	0.0595	0.0202	0.3533	0.0203	0.2305	0.0044	0.2317	0.0918
3.11	0.50	0.18	0.1135	0.0152	0.6689	0.0317	0.2900	0.0035	0.4632	0.0836
3.11	2.20	0.50	0.0317	0.0076	0.1713	0.0049	0.1447	0.0021	0.1865	0.0497
3.15	0.50	0.18	0.1169	0.0145	0.6577	0.0304	0.2882	0.0035	0.4924	0.0836
3.15	2.20	0.49	0.0307	0.0082	0.1759	0.0050	0.1469	0.0027	0.1772	0.0520
3.19	0.50	0.18	0.1082	0.0119	0.6926	0.0214	0.2901	0.0045	0.4390	0.0614
3.23	0.50	0.18	0.1130	0.0141	0.6843	0.0299	0.2902	0.0035	0.4705	0.0789
3.23	0.80	0.25	0.1170	0.0115	0.4992	0.0161	0.2886	0.0031	0.4613	0.0600
3.23	1.25	0.35	0.0544	0.0195	0.3815	0.0200	0.2384	0.0047	0.2055	0.0843

W^2	Q^2	x	F_L	ΔF_L	F_1	ΔF_1	F_2	ΔF_2	R	ΔR
3.23	2.20	0.48	0.0320	0.0093	0.1818	0.0060	0.1513	0.0028	0.1821	0.0585
3.27	0.50	0.17	0.1185	0.0122	0.6784	0.0272	0.2917	0.0028	0.5048	0.0720
3.31	0.50	0.17	0.1207	0.0122	0.6896	0.0272	0.2955	0.0029	0.5126	0.0717
3.31	1.25	0.34	0.0923	0.0133	0.3550	0.0134	0.2517	0.0038	0.3828	0.0694
3.35	0.50	0.17	0.1067	0.0091	0.7165	0.0181	0.2901	0.0031	0.4423	0.0484
3.35	1.25	0.34	0.0662	0.0133	0.3872	0.0137	0.2476	0.0035	0.2543	0.0597
3.35	1.75	0.41	0.0569	0.0112	0.2502	0.0094	0.1964	0.0033	0.2740	0.0641
3.35	2.20	0.47	0.0314	0.0089	0.1896	0.0059	0.1550	0.0032	0.1757	0.0553
3.39	0.50	0.17	0.1134	0.0089	0.7102	0.0179	0.2925	0.0031	0.4808	0.0491
3.39	2.20	0.47	0.0406	0.0088	0.1893	0.0059	0.1612	0.0032	0.2293	0.0567
3.43	2.20	0.46	0.0375	0.0090	0.1939	0.0060	0.1617	0.0030	0.2090	0.0562
3.47	2.20	0.46	0.0376	0.0089	0.1980	0.0060	0.1641	0.0033	0.2067	0.0547
3.51	1.25	0.32	0.0921	0.0113	0.3811	0.0123	0.2613	0.0032	0.3752	0.0577
3.51	2.20	0.46	0.0352	0.0089	0.2016	0.0060	0.1644	0.0033	0.1919	0.0541
3.55	1.25	0.32	0.1021	0.0143	0.3751	0.0149	0.2653	0.0040	0.4266	0.0764
3.63	1.25	0.31	0.0925	0.0200	0.4023	0.0211	0.2697	0.0062	0.3679	0.0987
3.63	1.75	0.39	0.0614	0.0110	0.2857	0.0093	0.2175	0.0040	0.2764	0.0583
3.63	1.75	0.39	0.0673	0.0114	0.2788	0.0088	0.2178	0.0036	0.3102	0.0618
3.67	0.80	0.22	0.1130	0.0117	0.5693	0.0170	0.3010	0.0040	0.4455	0.0589
3.67	1.75	0.39	0.0505	0.0104	0.2930	0.0083	0.2127	0.0041	0.2235	0.0519
3.71	0.80	0.22	0.0947	0.0118	0.6010	0.0175	0.2963	0.0040	0.3576	0.0546
3.71	1.75	0.38	0.0536	0.0103	0.2947	0.0084	0.2155	0.0041	0.2381	0.0521
3.71	2.20	0.44	0.0448	0.0090	0.2161	0.0064	0.1790	0.0034	0.2371	0.0543
3.75	0.50	0.15	0.1011	0.0132	0.7893	0.0270	0.2903	0.0038	0.4316	0.0707

W^2	Q^2	x	F_L	ΔF_L	F_1	ΔF_1	F_2	ΔF_2	R	ΔR
3.75	1.75	0.38	0.0617	0.0103	0.2898	0.0083	0.2183	0.0043	0.2811	0.0545
3.75	2.20	0.43	0.0602	0.0090	0.2096	0.0064	0.1860	0.0035	0.3306	0.0591
3.79	0.50	0.15	0.1167	0.0129	0.7538	0.0263	0.2933	0.0038	0.5279	0.0762
3.79	1.25	0.30	0.1198	0.0212	0.3874	0.0227	0.2811	0.0068	0.5145	0.1210
3.79	1.75	0.38	0.0644	0.0107	0.3002	0.0087	0.2258	0.0045	0.2855	0.0551
3.83	1.25	0.30	0.1002	0.0174	0.4229	0.0183	0.2816	0.0059	0.3980	0.0860
3.83	1.75	0.37	0.0649	0.0110	0.2972	0.0087	0.2238	0.0040	0.2934	0.0581
3.83	2.20	0.43	0.0556	0.0097	0.2162	0.0071	0.1860	0.0034	0.3012	0.0618
3.87	1.75	0.37	0.0567	0.0112	0.3072	0.0088	0.2225	0.0040	0.2498	0.0561
3.87	2.20	0.42	0.0415	0.0097	0.2317	0.0073	0.1848	0.0036	0.2113	0.0558
3.91	0.50	0.14	0.0676	0.0206	0.8891	0.0531	0.2799	0.0057	0.2682	0.0975
3.95	0.50	0.14	0.0890	0.0107	0.8478	0.0247	0.2869	0.0033	0.3747	0.0556
3.95	1.75	0.36	0.0584	0.0129	0.3111	0.0091	0.2248	0.0051	0.2587	0.0641
3.99	0.50	0.14	0.0917	0.0105	0.8337	0.0242	0.2843	0.0033	0.3972	0.0565
3.99	1.75	0.36	0.0643	0.0129	0.3154	0.0092	0.2312	0.0054	0.2832	0.0644
4.03	0.50	0.14	0.0967	0.0101	0.8431	0.0240	0.2894	0.0030	0.4185	0.0554
4.11	0.80	0.20	0.0861	0.0142	0.6748	0.0235	0.3017	0.0048	0.3215	0.0638
4.11	1.75	0.35	0.0198	0.0212	0.3723	0.0181	0.2254	0.0078	0.0755	0.0847
4.15	1.75	0.35	0.0208	0.0209	0.3739	0.0180	0.2262	0.0080	0.0799	0.0840
4.23	0.80	0.19	0.1376	0.0215	0.5784	0.0364	0.3099	0.0071	0.6171	0.1344
4.23	1.75	0.34	0.0065	0.0215	0.3939	0.0188	0.2238	0.0081	0.0239	0.0805
4.27	0.80	0.19	0.1205	0.0212	0.6181	0.0360	0.3073	0.0071	0.5108	0.1190
4.31	0.80	0.19	0.1090	0.0208	0.6369	0.0355	0.3023	0.0071	0.4526	0.1112

Table 10: Structure functions for Iron.