**SIGNATURE STABILITY IN LASER DOPPLER
VIBROMETRY**

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SIGNATURE STABILITY IN LASER DOPPLER VIBROMETRY

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ABSTRACT

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Speckle can complicate signal acquisition in coherent laser systems such as Laser Doppler Vibrometry (LDV). Variations in the speckle pattern at the receiver due to fluctuations in the system such as beam pointing can lead to impulsive events in the signature. The beam size at the object has a direct influence on the size of the speckle at the receiving aperture. Increasing the beam spot size reduces the average speckle size, but also decreases the strength of the signal coupled with the local oscillator in the LDV. In this thesis, we derive the relationship between scattering spot size at the object and average speckle size at the receiver. Theory is presented on how increasing the beam diameter at the object can reduce the fluctuations of the heterodyned signal coupled with the Local Oscillator (LO). The Antenna theorem is presented to show the tradeoff between angular field of view and capture area. An equation is established for the Doppler shift that is used to calculate the frequency shift contained in a heterodyned signature. The heterodyne signature is derived and the intensity imaged on the photodetector inside an LDV device is discussed. A signature stability simulation is conducted and the effects of pointing jitter on a generated electric field are investigated by numerically changing beam spot diameter. Two methods in which speckle spot size is calculated are composed. We

show experimental results on the effects of speckle size and decreasing signal strength have on the stability of an LDV signature. A kurtosis metric previously reported in the literature is used to assess the stability and quality of the return signature.

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CHAPTER 1

INTRODUCTION

Laser-Doppler Vibrometry (LDV) is an established technique for evaluating the Doppler shift of coherent laser light to measure a target object's motion without contact ^[1]. Illuminating a driven target with the laser vibrometer produces scattered light that contains a phase shift proportional to the velocity of the target in the direction of the laser. The vibrometer collects a region of the scattered light and uses the principle of optical heterodyning, or “beating”, to mix it with a coherent reference beam. The mixed beam of light is then imaged on to a photodetector and the collected intensity produces a time dependent velocity measurement. LDV has become a standard tool in a wide range of industrial and military applications. Specialized designs of the measurement head allow for a variety of purposes ranging from vibrational analysis of mechanical structures to 3D mapping of local environments. Advancing LDV systems are capable of covering distances ranging from several centimeters to distances kilometers away ^[2].

Optically rough surfaces will produce scattered light that varies in phase when illuminated by a coherent laser source. The scattered light is of the same frequency as the coherent laser beam and interferes constructively and destructively as it propagates through space. A camera can be used to capture a region of the scattered light to produce an interference pattern known as a speckle pattern. Speckle can complicate signal acquisition

in coherent laser systems such as LDV. The random intensity pattern of speckle is a manifestation of the random phase at the receiving aperture. This random phase directly affects the phase measurements made by an LDV. The LDV integrates the electric field over the receiving aperture. Variations in the integrated electric field are introduced by beam pointing or atmospheric disturbances. The intensity variations can lead to impulsive events in the signature at the receiver. These impulsive events lead to a strongly fluctuating signal power. Theories on the effects speckle noise have on LDV measurements have been developed^[3], and experimentally verified^[4, 5].

1.1 Problem Statement

The laser beam size at the object has a direct influence on the variations of the electric field at the receiver plane. One measure of the variation is the size of the speckle at the receiving aperture. Chapters 2 and 3 prove that increasing the beam spot size at the target by defocusing the beam reduces the average speckle size as well as decrease the strength of the signal coupled with the local oscillator in the LDV. The size of speckle with respect to the integrating aperture will affect the stability of the signature. Fluctuations in the phase and amplitude measurements of the electric field across the integrating aperture can lead to impulsive behavior in the signature. Such fluctuations can be produced by external motions of the instrument itself. Speckle roughly the size of the receiving aperture may experience significant changes in the integrated magnitude and phase of the electric field, potentially causing signature instability.

The specific problem in this thesis is to quantify the improvement in stability of the heterodyned signal measured with an LDV device by increasing the stability in the integrated phase measurements. The goal of this thesis is to investigate the effects speckle

size and decreasing signal strength have on the stability of an LDV signature. These effects will be investigated by manipulating the LDV beam spot size at the target object through both Matlab simulations and laboratory experiments. The compiled Matlab code will simulate the effects of pointing jitter to demonstrate theoretically the impact speckle has on signature stability. Stability will be determined by calculating the first and second moments of the amplitude and phase components of the integrated electric field generated in the plane of the receiving aperture.

Stability has been experimentally tested in the laboratory by collecting vibration signatures from an LDV device of a driven target object. In examining the vibration signatures, the main focus is to determine how defocusing the beam on target influences the stability of the collected measurements. Several metrics, including a kurtosis metric previously reported in the literature^[7], will be used to assess the stability and quality of the return signature. A comparison between the simulation and the laboratory results will be discussed.

1.2 Literature

Drain^[1] introduces the techniques used in Laser Doppler Vibrometry. The Doppler Shift technique is first discussed and derivations are presented for various LDV setups. Drain^[1] and Boyd^[13] then present the principle of optical heterodyning to derive the irradiance distribution leading to the current across the optical detector in the LDV. Drabenstedt et al^[6] explore the principals of LDV and investigate a Laser Vibrometer operating at the 1550nm wavelength.

Siegman ^[12] reviews the properties of an optical heterodyne receiver and states that they are in principle both a receiver and antenna. Within the article, he begins with a review of the heterodyning of two electric fields leading to the intensity distribution across an LDV receiver. From the intensity distribution he derives coherence area and integrates over the solid angle of the receiver leading to the well-known antenna theorem. This theorem states the limiting constraint for wavefront alignment between signal and Local Oscillator (LO).

The origins of speckle patterns have been an area of interest in many research projects and have been well developed and understood. The effects speckle have on Laser Vibrometer systems have been numerous studied and methods have been proposed to quantify and reduce the added noise speckle have on such systems. Various textbooks and published works produce excellent resources to better understand speckle theory and help to confirm acquired experimental results.

Statistical properties necessary for understanding the theory behind speckle phenomena are discussed in the Goodman Statistical Optics textbook ^[11]. This textbook derives important theories, for example the van Cittert-Zernike theorem and the Wiener-Khinchin theorem. In Speckle Phenomena in Optics, Goodman ^[8] introduces speckle theory through the investigation of random phasor sums and probability density functions along with other statistical properties. Characteristics of the speckle field such as Power Spectral Density and average speckle size are derived from the intensity correlation function of two points in a speckle pattern. Martin and Rothberg ^[4] experimentally verify the characteristics of average speckle size introduced in Goodman ^[8].

Cristalli et al ^[7] demonstrates a new technique for measuring stability and quality in vibration signatures measured by an LDV. The purpose of the study was to discover mechanical faults in manufacturing appliances that were moving across an assembly line. These faults were observed through vibration signals which are comprised of periodic components with a characteristic frequency corresponding to a particular defect. It was important to suppress the speckle-noise effects present from the electro mechanical components and the movement from the assembly line. This was accomplished by implementing a statistical approach based on kurtosis to select an undistorted region within the signal. The proposed method outlined in the paper is used to determine if a vibration signature contains impulses which in turn demonstrate stability between the return signature and LO.

1.3 Thesis Overview

This paper is organized into six chapters. The first chapter introduces the Laser Doppler Vibrometry (LDV) technique and describes the manifestation of speckle and how it affects the stability of vibration measurements. Chapter one contains the problem statement and indicates the points of interest in the research. A literature review is presented of past research/work that is referenced through the paper.

Chapter two derives important statistical properties of speckle and discusses the applications to the stability of vibration measurements. The free space propagation geometry is first outlined to aid in the following derivations. An intensity correlation of the field in the observation plane between two different points is then derived in terms of complex field correlation. The power spectral density of a speckle pattern is established and relationship between the scattering spot diameter on the target object and spatial

fluctuations of the return signal is discussed. The final section of this chapter discusses speckle size and how it can be used to investigate signature stability in vibration measurements.

Chapter three contains a discussion on Laser Doppler Vibrometry theory. It begins with a short discussion on the Doppler shift equation. Optical heterodyne detection theory is then presented which contains a discussion on the current illuminating the photodetector in an LDV system. The last section of chapter three derives the optical antenna theorem and how it relates to signature stability.

Chapter four begin the discussion of the research by means of simulations compiled in Matlab. It begins with a discussion on how speckle is simulated by generating electric fields using Matlab code. The generated electric fields are then used to simulate pointing jitter and the effects speckle has on the stability of vibration signatures is investigated. Further Matlab code is presented to investigate the average speckle size using two different approaches: Averaging over Single and Multiple realizations of the generated electric fields. The method of each approach is discussed in detail and the results are presented and compared.

Chapter five discusses the experimental methods and results from the experiments performed in the laboratory. Images of the speckle captured in the laboratory are presented and average speckle size relative to the receiving aperture is presented. The signal strength of the heterodyned signal is presented and discussed in detail. The last section presents a quantitative metric to show an increase in stability and confirms the results of the signal strength measurements.

Chapter six contains a short summary followed by a discussion on future work to be performed. The conclusion drawn from this research are then summarized and presented in the final section of chapter six.

CHAPTER 2

STATISTICAL PROPERTIES OF SPECKLE

2.1 Free Space Speckle Geometry

When a coherent laser beam illuminates an optically rough surface, the rays of light will scatter along different optical paths. The reflected laser light will then interfere constructively and destructively to produce a granular pattern known as speckle. If laser light has a coherence time greater than the observation time, the speckle pattern will persist. An example of speckle can be seen in Fig. 2-1.

Figure 2-1: Example of a speckle image

For fully developed speckle, the received field is composed of multitude complex contributions. Each contribution has a random amplitude and random phase that are independent of one another. The independent random phase is uniformly distributed on

the interval $(-\pi, \pi)$. The free space propagation geometry for the following theory is presented in Fig. 2-2. We will assume our observation plane is located in the Fresnel region and that the roughness of our object is on the order of several wavelengths.

Figure 2-2: Free space propagation geometry for the target object and observation plane when object is illuminated by coherent light.

2.2 Correlation of Intensity

The intensity correlation of the field in the observation plane between two different points can be written as:

$$\Gamma_I = \langle u_n u_n^* \cdot u_m u_m^* \rangle \quad (2-1)$$

where $u \cdot u^*$ represents the intensity at two arbitrary points (n, m) in the observation plane and the brackets indicate an ensemble average. Applying the complex Gaussian moment theorem, we can rewrite Eq. 2-1 as the following ^[11]:

$$\begin{aligned} \langle u_n u_n^* \cdot u_m u_m^* \rangle &= \langle u_n^* u_n \rangle \langle u_m^* u_m \rangle + \langle u_n^* u_m \rangle \langle u_m^* u_n \rangle \\ &= \langle I_n \rangle \langle I_m \rangle + |\Gamma_A|^2 \end{aligned} \quad (2-2)$$

where Γ_A is the correlation function of the complex fields and can be written in terms of the complex correlation coefficient. We will assume $\langle I_n \rangle = \langle I_m \rangle = \langle I \rangle$, where the

average intensity is the same regardless of the position in the field. The intensity correlation can then be written as follows [8]:

$$\begin{aligned}\Gamma_I = \langle I_n I_m \rangle &= \langle I \rangle^2 + \langle I \rangle^2 |\mu_{n,m}|^2 \\ &= \langle I \rangle^2 [1 + |\mu_{n,m}|^2]\end{aligned}\tag{2-3}$$

The relationship describes the intensity correlation in terms of mean irradiance and the correlation coefficient of the fields in the observation plane. Thus because the intensity correlation can be directly measured in the laboratory, the magnitude squared of the underlying complex field correlation can be calculated without difficulty. This is important because as we will see the magnitude squared of the field correlation is related to the average speckle size

2.3 Field Correlation and Power Spectrum of Speckle

In the previous section, the correlation of the intensities, Γ_I , was derived in terms of the correlation coefficient μ_A , as seen in Eq. 2-3. Using the free-space propagation geometry introduced in Fig. 2-2, an expression for the complex correlation coefficient will be derived in order to find an exact expression for the intensity correlation. By applying the Wiener-Khinchin theorem, a power spectral density of the speckle pattern will be found.

We start by considering a coherent source illuminating a rough target object surface, and observe the scattered light some distance z away. We will make the assumption that the target object is stationary and will assume paraxial propagation. The complex amplitude $A(x,y)$ of the light at position (x,y) in the observation plane can then

be related to the complex amplitude $a(\alpha, \beta)$ in the field just right of the target object at coordinates (α, β) though the Fresnel diffraction integral^[8]:

$$A(x, y) = \frac{e^{jkz}}{j\lambda z} e^{j\frac{k}{2z}(x^2+y^2)} \iint_{-\infty}^{\infty} a(\alpha, \beta) e^{j\frac{k}{2z}(\alpha^2+\beta^2)} e^{-j\frac{2\pi}{\lambda z}(x\alpha+y\beta)} d\alpha d\beta \quad (2-4)$$

where λ represents the wavelength of the coherent light source. Eq. 2-4 describes the speckle field at one point in the observation plane, thus we will use the autocorrelation function to find the correlation of $A(x, y)$ at two points (x_1, y_1) and (x_2, y_2) in the observation plane:

$$\Gamma_A(x_1, y_1; x_2, y_2) = \langle A(x_1, y_1) A^*(x_2, y_2) \rangle \quad (2-5)$$

Substituting Eq. 2-4 into Eq. 2-5, the field correlation between to point become:

$$\begin{aligned} \Gamma_A(x_1, y_1; x_2, y_2) & \quad (2-6) \\ &= \frac{1}{\lambda^2 z^2} e^{j\frac{k}{2z}(x_1^2+y_1^2-x_2^2-y_2^2)} \iint_{-\infty}^{\infty} \iint_{-\infty}^{\infty} \langle a(\alpha_1, \beta_1) a^*(\alpha_2, \beta_2) \rangle \\ & \times e^{j\frac{k}{2z}(\alpha_1^2+\beta_1^2-\alpha_2^2-\beta_2^2)} e^{-j\frac{2\pi}{\lambda z}(x_1\alpha_1+y_1\beta_1-x_2\alpha_2-y_2\beta_2)} d\alpha_1 d\beta_1 d\alpha_2 d\beta_2 \end{aligned}$$

This allows for the autocorrelation of two points in the target object plane to be introduced, where:

$$\Gamma_a(\alpha_1, \beta_1; \alpha_2, \beta_2) = \langle a(\alpha_1, \beta_1) a^*(\alpha_2, \beta_2) \rangle \quad (2-7)$$

We now write the autocorrelation Γ_a in more practical terms by assuming the correlation of the roughness of the target object is so small it can be approximated with a delta function.

The correlation of the wavefront upon reflection is then given by ^[8]:

$$\Gamma_a(\alpha_1, \beta_1; \alpha_2, \beta_2) = \kappa I(\alpha_1, \beta_1) \delta(\alpha_1 - \alpha_2, \beta_1 - \beta_2) \quad (2-8)$$

where κ is a constant and $I(\alpha_1, \beta_1)$ is the intensity of the light just right of the target object, together multiplied by a two dimensional delta function. Substituting Eq. 2-8 into Eq. 2-6 we obtain:

$$\Gamma_A(x_1, y_1; x_2, y_2) = \frac{\kappa}{\lambda^2 z^2} e^{j\frac{k}{2z}\psi} \iint_{-\infty}^{\infty} I(\alpha, \beta) e^{-j\frac{2\pi}{\lambda z}(\alpha\Delta x + \beta\Delta y)} d\alpha d\beta \quad (2-9)$$

where $\psi = \frac{\pi}{\lambda z} [(x_1^2 + y_1^2) - (x_2^2 + y_2^2)]$ and $\Delta x = x_1 - x_2, \Delta y = y_1 - y_2$. We are able to drop the quadratic-phase term proceeding the integral because the modulus of the correlation function is what is of interest here, as will be seen in the following equations. The result is equivalent to the well know van Cittert-Zernike theorem^[11]:

$$\Gamma_A(\Delta x, \Delta y) = \frac{\kappa}{\lambda^2 z^2} \iint_{-\infty}^{\infty} I(\alpha, \beta) e^{-j\frac{2\pi}{\lambda z}(\alpha\Delta x + \beta\Delta y)} d\alpha d\beta \quad (2-10)$$

The van Cittert-Zernike theorem states Fourier relationship between the irradiance distribution across the object plane and the speckle field correlation. The complex correlation coefficient is defined by normalizing the correlation function^[8]:

$$\mu_A(\Delta x, \Delta y) = \Gamma_A(\Delta x, \Delta y) / \Gamma_A(0, 0) \quad (2-11)$$

which takes the form:

$$\mu_A(\Delta x, \Delta y) = \frac{\iint_{-\infty}^{\infty} I(\alpha, \beta) e^{-j\frac{2\pi}{\lambda z}(\alpha\Delta x + \beta\Delta y)} d\alpha d\beta}{\iint_{-\infty}^{\infty} I(\alpha, \beta) d\alpha d\beta} \quad (2-12)$$

The result is the correlation coefficient of the fields shown in Eq. 2-3, which describes the quantitative measure of how the fields at two points are related. A value of one represents two perfectly correlated points while a value of zero identifies no correlation.

Now that an expression for μ_A is obtained, the autocorrelation function for the intensity distribution of the speckle pattern in the observation plane can be developed. Substituting Eq. 2-12 into Eq. 2-3, we get an expression for the correlation function of the intensity distribution across the observation plane^[8]:

$$\Gamma_I(\Delta x, \Delta y) = \langle I \rangle^2 \left[1 + \left| \frac{\iint_{-\infty}^{\infty} I(\alpha, \beta) e^{-j\frac{2\pi}{\lambda z}(\alpha\Delta x + \beta\Delta y)} d\alpha d\beta}{\iint_{-\infty}^{\infty} I(\alpha, \beta) d\alpha d\beta} \right|^2 \right] \quad (2-13)$$

The power spectral density $\chi_I(v_x, v_y)$ of the speckle pattern represents the distribution of intensity fluctuating power over spatial frequency. The Wiener-Khinchin theorem states the Fourier relationship between the autocorrelation function and the power spectral density function^[11], written as follows:

$$\chi_I(v_x, v_y) = \iint_{-\infty}^{\infty} \Gamma_I(\Delta x, \Delta y) e^{-j2\pi(v_x\Delta x + v_y\Delta y)} d\Delta x d\Delta y \quad (2-14)$$

Applying the Wiener-Khinchin theorem to Eq. 2-13 we find the power spectral density of the speckle pattern becomes^[8]:

$$\chi_I(v_x, v_y) = \langle I \rangle^2 \left[\delta(v_x, v_y) + (\lambda z)^2 \frac{\iint I(\alpha, \beta) I(\alpha + \lambda z v_x, \beta + \lambda z v_y) d\alpha d\beta}{[\iint I(\alpha, \beta) d\alpha d\beta]^2} \right] \quad (2-15)$$

The delta function in this expression correspond to the constant zero frequency power contributed by the average intensity. The second term is the normalized and scaled autocorrelation function of the intensity distribution $I(\alpha, \beta)$ at the scattering spot with value at $v_x = v_y = 0$ of:

$$\tilde{\chi}(0,0) = \langle I \rangle^2 (\lambda z)^2 \frac{\iint_{-\infty}^{\infty} I^2(\alpha, \beta) d\alpha d\beta}{[\iint_{-\infty}^{\infty} I(\alpha, \beta) d\alpha d\beta]^2} \quad (2-16)$$

If the intensity at the target object $I(\alpha, \beta)$ is uniform over the geometrical area then the intensity ratio reduces to $1/A$, where A is the area of the scattering spot on the target object.

Thus the power spectral density can be written as follows:

$$\tilde{\chi}_I(0,0) = \langle I \rangle^2 \frac{(\lambda z)^2}{A} = \langle I \rangle^2 \frac{\lambda^2}{\Omega_s} \quad (2-17)$$

where Ω_s is the solid angle subtended from the scattering spot viewed in the observation plane. We are presented with an inverse relationship between the scattering spot area and power spectral density of the speckle pattern intensity. Thus, a larger scattering spot illuminated the target object will reduced spatial fluctuations of the signal at the observation plane.

2.4 Average Speckle Spot Size

Due to the random intensity pattern of speckle, a statistical approach is used to calculate the average size and shape of the speckle field. As mentioned in the previous

section, the correlation of the intensity field is important and can be used as a measure of average speckle size. Size of speckle with respect to the integrating aperture will be calculated to show how speckle size affects the stability of the heterodyned signature. Fluctuations in phase and amplitude measurement across the integrating aperture can lead to impulsive events in the heterodyned signature. These impulsive event directly affects the phase measurements made by an LDV resulting in a strongly fluctuating signal power. The average size of a fully developed speckle pattern will be shown to depend on the area of the scattering spot or “beam” spot size on the target object similar to the previous derivation.

The average size of a speckle can be defined as the equivalent area of the normalized covariance function of speckle intensity ^[8]. By normalizing Eq. 2-3, the covariance function of speckle intensity, c_I , can be related to the autocorrelation function by^[8]:

$$c_I(\Delta x, \Delta y) = \frac{\Gamma_I(\Delta x, \Delta y) - \langle I \rangle^2}{\langle I \rangle^2} \quad (2-18)$$

where it is assumed $\langle I_n \rangle = \langle I_m \rangle = \langle I \rangle$. The equivalent area, also called “correlation area” or “coherence area”, is written as:

$$\mathcal{A}_c = \iint_{-\infty}^{\infty} c_I(\Delta x, \Delta y) d\Delta x d\Delta y = \iint_{-\infty}^{\infty} |\mu_A(\Delta x, \Delta y)|^2 d\Delta x d\Delta y \quad (2-19)$$

This expression is similar to temporal coherence and coherence time except coherence area is associated with spatial coherence. For the case of a uniform circular source we find coherence area to take the following form ^[8]:

$$\mathcal{A}_c = \frac{(\lambda z)^2}{A} = \frac{\lambda^2}{\Omega_s} \quad (2-20)$$

where A is the area of the scattering spot and Ω_s is the solid angle subtended by that spot. It is observed that due to the inverse relation between scattering spot size and coherence area, a larger illuminated region will produce smaller speckle and a smaller illuminated region will produce larger speckle. As Goodman points out, the free space geometry is fundamentally two-dimensional, thus a reasonable approximation of the one-dimensional width of a speckle is the square root of the coherence area presented above.

The inverse relationship between scattering spot size at the target object and size of speckle in the integrating aperture is an important result, however exactly how the scattering spot size is increased will be critical. Simply increasing scattering spot on target by means of moving the target further away will not be sufficient. This is due to the optical transfer function, which can be used to show that speckle size has a dependence on λz , where z is the distance between the target object and integrating aperture. Moving the target away will increase the scattering spot size, but also increase the speckle size at the receiving aperture as well. The solution will be to increase the scattering spot size by means of defocusing the beam and keeping the target object stationary. This will result in a larger beam spot size, and because λz does not change the inverse relationship between beam spot size and speckle spot size will remain true.

CHAPTER 3

LASER DOPPLER VIBROMETRY THEORY

The change or apparent change in the frequency of wave motion (electromagnetic or any other type) owing to the relative motion of the source and receiver is known as the Doppler shift ^[1]. Laser Doppler measurements of velocity are made by calculating these relative frequency shifts produced from the movement of an object through which wave motion is transferred from source to receiver. The fundamental Doppler shift equation is^[1]:

$$\frac{\Delta v}{v} = 2 \frac{v/c}{1 - v/c} \quad (3-1)$$

where Δv is the change in frequency of light and c is the speed of light.

3.1 Optical Heterodyne Detection

In heterodyne detection systems, illustrated schematically in Fig. 3-1, a Doppler shift is used to create a difference in frequencies between the local oscillator (LO) and the reference beam. The LO and reference beam interfere with one another on the photodetector and the frequency differences show up as periodic intensity fluctuations at a heterodyned frequency. The photo detector produces an output current signal from the fluctuations in intensity which then is converted into a voltage signal. Thus, the phase of the optical field is preserved in the phase of the electric signal.

Figure 3-1: Heterodyne Detection

Consider a coherent monochromatic laser field of frequency ω_{LO} serving as a local oscillator. The LO is combined with a signal field of frequency ω_S with a beam splitter, the combined electric field then illuminates a detector. The expression for the complex field at the detector can be written as a combination of the LO field and the return signal field^[10]:

$$E(t) = E_{LO}e^{-i\omega_{LO}t} + E_S e^{-i\omega_S t} \quad (3-2)$$

where E_{LO} and E_S are the amplitudes of the LO and return signal fields, respectively. For now, we are considering the two optical beams are well collimated and are aligned so the wave fronts are parallel. The interference intensity from the returning signal and LO electric fields is then measured on the detector by:

$$\begin{aligned} I(t) &= |E_{LO} + E_S|^2 \\ &= E_{LO}^2 + E_S^2 + 2E_{LO} E_S \cos[(\omega_S + \omega_{LO})t + \phi] \end{aligned} \quad (3-3)$$

where ϕ represents the relative phase between E_{LO} and E_S and is given by:

$$\phi = \arctan\left(\frac{\text{Im } E_{LO}}{\text{Re } E_{LO}}\right) - \arctan\left(\frac{\text{Im } E_S}{\text{Re } E_S}\right) \quad (3-4)$$

The first two terms in Eq. 3-3 represent the constant unmodulated light that creates the dc current in the detector which leads to unavoidable shot noise. The third term in Eq. 3-3 represents the intensity modulation, the heterodyne signal is contained entirely in this frequency component. This component of the heterodyne signal oscillates at the beat frequency $\omega_{i.f.} = \omega_S - \omega_{LO}$, referred to as the intermediate frequency (i.f.). The detector converts the light intensity into a proportional current by pairing a charge carrier to every absorbed photon. There is some loss associated with this process, the ratio of created charge carrier out of every absorbed photon is known as quantum efficiency η . The conversion from light intensity to current is given by ^[10]:

$$i(t) = \frac{\eta e \lambda P(t)}{hc} \quad (3-6)$$

where e is the charge of an electron, h is the Plank constant and c is the speed of light. By using the impedance of free space, z_0 , the power integrated by the detector can be written as^[10]:

$$P(t) = \frac{E^2(t)A}{z_0} \quad (3-7)$$

where A is the area of the detector. Rewriting Eq. 3-3 in terms of detector current, we obtain:

$$i(t) = i_{LO} + i_s + 2\sqrt{i_{LO}i_s} \cos[(\omega_S + \omega_{LO})t + \phi] \quad (3-8)$$

where the individual local oscillator and signal power dc currents equal:

$$i_{LO} = \frac{\eta e \lambda A}{h c z_0} E_{LO}^2 \quad (3-9)$$

$$i_S = \frac{\eta e \lambda A}{h c z_0} E_S^2$$

Thus, the strength of the heterodyned signal can be characterized by taking the mean-square i.f. current at frequency $\omega_{i.f.}$ ^[10]:

$$\langle i_{i.f.} \rangle^2 = \frac{(2\sqrt{i_{LO}i_S})^2}{2} = 2i_{LO}i_S \quad (3-6)$$

Note that we assume optimum photomixing resulting in maximum sensitivity.

3.2 The Antenna Theorem

Optical heterodyne detectors, for example an LDV system, are in principle both a receiver and an antenna. As an antenna, it can be characterized by the effective aperture or capture cross section that is present to a signal plane wave arriving from any specified direction^[12]. Figure 3(a) illustrates this concept by presenting a signal wave front inclined at an angle θ to the local oscillator at the receiving aperture. When $\theta=0$, the returned signal at all points on the surface of the receiver contribute with the same phase to the LO signal. However for $\theta \neq 0$, different points on the surface of the receiver will contribute intermediate frequency (i.f.) signals with different phases, and the integrated signal will decrease. We can observe the heterodyne signal drop to zero when^[13]:

$$\theta L = \lambda \quad (3-7)$$

where λ is the wavelength of the signal.

Figure 3-2: (a) Illustration of Local Oscillator wave fronts and Signal wave fronts at the receiving aperture. (b) Geometry of the solid angle defined by four adjacent transverse field modes.

To understand the limitation for the integrated cross section of the receiving aperture, we define a projected solid angle that may be associated with the signal transverse field mode.

The solid angle geometry is presented in Fig. 3(b), and is given by^[13]:

$$\Omega_{proj} = \frac{\Delta k_x \Delta k_y}{k^2} = \frac{\lambda^2}{L_x L_y} \quad (3-8)$$

A detector of aperture $A = L_x L_y$ can be inserted, leading to the relation:

$$\Omega_{proj} = \frac{\lambda^2}{A} \quad (3-9)$$

The results can be interpreted physically as a statement that no source of projected solid angle greater than λ^2/A can provide spatially coherent illumination over a region of area A

[13].

CHAPTER 4

SIGNATURE STABILITY SIMULATION

The primary goal of the stability simulation was to simulate pointing jitter in the integrated electric field at the receiver plane of an LDV device. The electric field was then used to investigate the effects speckle has on the stability of vibration signatures. Stability was measured by calculating the integrated phase and amplitude of the electric field across the receiving aperture. Stability is discussed in section 4.2.

To investigate the effects speckle has on signature stability, Matlab code was compiled to generate speckle. This was accomplished by simulating a laser beam reflecting off a rough target object. Speckle was attained by propagating an electric field from a transmitter to a target object, with variable beam spot sizes at the object. The electric field was then scattered off a rough surface and propagated back to a receiver with a set diameter. The speckle field illuminating the receiver is found to be dependent on beam spot size on the target object. The generated speckle is discussed in section 4.1.

The last part of the simulation computed the average speckle size associated with the different beam widths. This was achieved by calculating the correlation of intensities as shown in Eq. 2-3 and measuring the equivalent area of the correlation coefficient as described in section 2.4. Two different methods were performed in calculating the average speckle size: averaging over multiple realizations and averaging over a single realization.

These two methods will be discussed along with the average speckle size results in section 4.3.

4.1 Speckle Generation

The simulation was compiled in Matlab and the parameters were set to simulate the conditions in the laboratory. The first step was to initialize the transmitted wavefront by assuming the beam waist was located at the transmitter pupil. This was important to ensure the radius of curvature of the aperture distribution could be considered infinity. The widths of the transmit beam were arranged such that the target object was located in the far field. This was accomplished by fulfilling the condition of:

$$z > \frac{2D^2}{\lambda} \quad (4-1)$$

where the aperture of linear dimension D is less than the product of the wavelength λ multiplied by z , the distance between transmitter and receiver. The transmitted beam is then written in the following form:

$$U(x, y) = e^{-\pi(\frac{x^2+y^2}{\sigma^2})} \quad (4-2)$$

where x and y represent the coordinates of the transmitter. The size of the transmitted Gaussian beam was manipulated with σ , smaller sigma values correspond to larger beam spot sizes at the target object and vice versa. Images of the simulated transmitted Gaussian beams can be seen in Figure 4-1 along with their respective sigma values in Table 1.

Table 1: Sigma Values for transmitted Gaussian beams.

a) $\sigma = 0.0025$	b) $\sigma = 0.001$
c) $\sigma = 0.00025$	d) $\sigma = 0.000125$

Figure 4-1: Generated transmitted Gaussian beams with decreasing σ values over images a)-d)

In order to compute the field at the target object a 2-dimensional Fourier Transform was performed in Matlab on Eq.4-1. This can be written numerically, ignoring the propagation constants preceding the integral, as simply the Fourier transform of the aperture distribution:

$$U(\xi, \eta) = \iint_{-\infty}^{\infty} U(x, y) \exp \left[-i \frac{2\pi}{\lambda z} (\xi x + \eta y) \right] dx dy \quad (4-3)$$

where (ξ, η) represent the coordinates in the object plane, wavelength λ is 1550nm and the distance between object and receiver, z , is 10m.

Optically rough surfaces will produce scattered light that is random in phase when illuminated by a coherent laser source. The scattered light is simulated in Matlab by first generating a large matrix of random phases on the interval between $[0, 2\pi]$ as seen in Eq. 4-4.

$$\exp[i * 2\pi * rnumph] \quad (4-4)$$

where $rnumph$ is a matrix of random numbers uniformly distributed on an interval between $[0, 1]$ with dimensions equal the electric field. The transmitted electric field in the object plane is then multiplied by this random phase matrix to produce the reflected scattered light. The roughness of the object's surface was coded to be delta correlated, identical to the assumptions made in Eq. 2-8, since each cell in the electric field is multiplied by a random phase component. Figure 4-2 shows images of the reflected electric field simulating scattered light off a rough object surface.

Figure 4-2: Generated reflected electric field upon reflection off a rough object surface with decreasing σ values over images a)-d).

Due to the inverse relationship of Fourier Transforms, larger Gaussian beams in the transmitter coordinates produce smaller beam spot sizes on the target object. This relationship is highlighted in Figure 4-2. To simulate propagation of the electric field back to the receiver plane, a 2-dimensional Fourier transform of the reflected field, which includes the random phase, is computed in Matlab. The field at the receiver was contained in large $[1500 \times 1500]$ matrixes that are used to investigate the stability of the detected heterodyne signal. An intensity matrix is then produced by multiplying the field at the receiver by the complex conjugate to obtain a speckle image. The images in Fig. 4-3 are cropped regions of the speckle generated for the electric field matrix in the receiver plane with dimensions $[250 \times 300]$, which is similar to the dimension of the pixel array used to capture speckle in the laboratory.

Figure 4-3: Images of generated speckle simulated in Matlab

The generated speckle images clearly show the inverse relationship of beam spot size and speckle spot size presented in section 2.4. A decrease in the average speckle spot size is

observed as the beam spot size on target is increased. Quantitative average speckle spot sizes of the images in Fig. 4-3 will be calculated in section 4.3 using two different methods.

4.2 Signature Stability

To investigate the effects speckle have on the stability of vibration signatures, pointing jitter was simulated for each beam spot size. We expect that one of the primary causes of instability in the observed Vibrometry signatures is that the integrated electric field in the plane of the receiving aperture can vary significantly with pointing jitter. Pointing jitter refers to the unwanted line-of-sight motion sustained by a pointing system over time resulting from external vibrations. The effects of pointing jitter produce variations to the integrated electric field which in turn cause the speckle to translate across the receiving aperture. When speckle that is comparatively large to the integrating aperture drifts in and out of focus, fluctuations are observed in the heterodyned signal power. However, as the size of speckle relative to the receiving aperture is decreased the power fluctuations are considerably reduced.

Pointing jitter effects are typically observed when coherent laser light wanders across a stationary target. As the beam drifts on the target object, the imaged speckle is translated across the receiving aperture. This effect was simulated in Matlab by capturing several different locations of the electric field in the plane of the receiver. This was accomplished by first initializing a [250 x 300] matrix at the center of the large electric field matrix inside a “for loop” command. For each iteration of the “for loop”, the smaller matrix was randomly shifted away from the central part of the larger matrix, and the intensity and amplitude of the integrated electric field across the receiver were recorded. A statistical analysis was performed on the pointing jitter results of each speckle spot size to

find the first and second moments of the intensity and amplitude measurements. Example magnitude plots of the speckle field from varying beam spot sizes can be seen in Figure 4-4.

Figure 4-4: Magnitude plots from cropped electric field with increasing spot sizes over images a)-d)

A reduction the fluctuations in magnitude and intensity over multiple realizations of electric field signatures reduce as the size of speckle across the receiving aperture decreases. It is observed that in the presence of pointing jitter, fluctuations in the speckle field become less extreme as the speckle size decrease. The pointing jitter effect was programed to collect the electric field measurements at 750 different locations of the large [1500 x 1500] array. Amplitude and intensity measurements of the integrating the electric field were collected at each beam drift location. The codes output consists of two [750 x 4] arrays that contain the amplitude and intensity measurements of 750 different beam drift locations for each of the four speckle spot sizes. As a metric to quantify the increase in

stability as the speckle spot size is reduced, the first and second moments of the integrated electric field measurements were calculated over the 750 drift locations. The magnitude and intensity results can be found in Fig 4-5 and 4-6, respectively.

Figure 4-5: First and second statistics of the magnitude of the integrated electric field in the plane of the receiver over.

Figure 4-6: First and second statistics of the intensity in the plane of the receiver.

The analytical results of the simulation in Fig. 4-5 & 4-6, suggest that the electric field is driven toward wide-sense stationarity as the beam spot size on target is reduced. This is indicative of the first and second moments becoming more stationary as the speckle sizes are reduced. The increase in stationarity confirms that stability of the speckle field in

the plane of the receiving aperture is improved. The effects of pointing jitter are reduced leading to less impulsive events when the return signature couples to the LO.

The reduction to the electric field magnitude and corresponding intensity fluctuations is analogous of the results derived in Eq. 2-15 and 2-17. These results stated that larger scattering spots illuminating the target object will reduce fluctuations to the collected signal at the observation plane. This inverse relationship is observed in the second order statistics of the integrated electric field calculations. As the speckle drifts across the receiving aperture due to the effects of pointing jitter, the reduction in the electric field magnitude at each speckle spot lower the impulses in the return signature.

4.3 Average Speckle size

The final portion of the simulation computes the average size of the generated speckle outlined in section 4.1. Size of speckle with respect to the integrating aperture will be important in determining stability of the heterodyned signature. The fluctuations in phase and amplitude measurements across the integrating aperture can lead to impulsive events in the heterodyned signature as will be shown in the following chapter. Reducing the size of speckle in the integrating receiver will be shown to reduce the impulsive events introduced by pointing jitter. Thus an effective method for calculating the average size of speckle is necessary. The average speckle size will be determined by calculating one-dimensional widths along the lateral and longitudinal dimensions of the speckle matrix. Two different methods are used to compute speckle spot sizes: averaging over a single electric field realization and averaging over multiple electric field realizations. Theoretical errors contained in the single realization method will be presented leading to the multiple realization method to be established as more accurate. Both methods will employ the

correlation of irradiances derived in section 2.2. By increasing the separation between two points of a single speckle realization, an intensity correlation coefficient array will be constructed. The intensity correlations will then be integrated to find the average speckle size as outlined in section 2.4.

4.3.1 Single Realization Method

This method calculates average speckle size by performing an average from a single realization of the generated electric field. This approach assumes that the speckle field is ergodic. The electric field is multiplied by its complex conjugate to obtain a [1500 x 1500] intensity matrix in the plane of the receiver. A [250 x 300] matrix is cropped from the central portion of the [1500 x 1500] intensity matrix to obtain a speckle image similar to those seen in Fig. 4-3. The smaller matrix is then used to compute intensity correlations along the lateral and longitudinal dimensions to determine a speckle size. Since the method for calculating is identical along both lateral and longitudinal dimensions, the following will demonstrate calculations along the longitudinal directions. Rewriting Eq. 2-3, the correlation coefficient can be calculated by:

$$|\mu_{n,m}|^2 = \frac{\langle I_n I_m \rangle}{\langle I_n \rangle \langle I_m \rangle} - 1 \quad (4-5)$$

where indices (n, m) refer to two different points in a single speckle realization separated by varying lateral distances, Δx , from each other. A value of $\Delta x = 0$ indicates there is zero spacing between points (n, m) and therefore all values of I_n are equal to I_m . The normalized correlation in Eq. 4-5 will then take the form:

$$|\mu_{n,m}|^2 = \frac{\langle I^2 \rangle}{\langle I \rangle^2} - 1 \quad (4-6)$$

where the second moment of intensity can then be written as ^[8]:

$$\langle I^2 \rangle = 2\langle I \rangle^2 \quad (4-7)$$

Inserting Eq. 4-7 into Eq. 4-6, we find the correlation coefficient equals one. Therefore, theory states that the correlation coefficient should approach one as the separation between the two points (n, m) is decreased.

The first step in calculating the intensity correlations is to obtain the initial intensity values, I_n . These are collected from the left half of the [250 x 300] matrix as depicted in Figure 4-7.

Figure 4-7: First half of intensity matrix is stored to compute initial I_n values.

Intensity values are stored to a [250 x 150] matrix and the average intensity, $\langle I_n \rangle$, is calculated. The next step is to collect the shifted intensity values, I_m , and apply Eq. 4-5 to compute the normalized correlation between points I_n and I_m . Figure 4-8 illustrates the process in which the intensity values I_m are shifted and stored.

Figure 4-8: Process illustrating how intensity values I_m are shifted and stored

To perform the shift and calculate the normalized correlation, a “for loop” command is implemented. With each iteration of the loop, the size of the shift is increased and the shifted intensity values, I_m , are stored to a new $[250 \times 150]$ matrix. The normalized correlation value is calculated by forming an element by element product of the I_n and I_m array. The product of the arrays is averaged and the resulting value is appended to an array that contains the intensity correlation values. For example, the first iteration of the loop stores a zero shift I_m value and average intensity $\langle I_m \rangle$ is calculated. While still inside the loop command, the normalized correlation value $\frac{\langle I_n I_m \rangle}{\langle I_n \rangle \langle I_m \rangle}$ is calculated and appended to an array. The next iteration of the loop, every (m) point is shifted over one cell and all calculations are repeated. This process is reiterated until the maximum shifted distance between points (n, m) is 150. This number is chosen based on the dimensional constraints of the $[250 \times 300]$ matrix (larger shifted distances would cause the dimensions of the array to be exceeded).

The square modulus correlation coefficient, $|\mu_{nm}|^2$, is computed by subtracting one from the normalized correlation values. The correlation coefficient varies on an interval

between (0, 1), however only the positive correlation values are of interest because they express similarity between the two points (n, m). Figure 4-9 displays plots of the correlation coefficient in terms of Δx , representing the spacing between points (n, m). The different plots are associated with different beam spot sizes on target.

Figure 4-9: Single realization Correlation Coefficient versus longitudinal separation.

The plots in Fig. 4-9 accurately represent the theory defined in Eqs. 4-6 and 4-7 with the exception of plot a), which is produced from large speckle in the plane of the receiver. The limited quantity of large speckle spots contained within a receiver aperture width in a single realization of the electric field does not contain enough information to effectively calculate the correlation coefficient. In this case our implicit assumption of stationarity is not valid. When speckle is small however, larger numbers of speckle are contained in a single

realization leading to more accurate correlation measurements. Our assumption of stationarity appears valid for these cases.

The average size of speckle as described in Eq. 2-19, is defined by calculating the equivalent area of the squared modulus correlation coefficient function. The shift was performed one cell at a time, therefore the average speckle size calculations were obtained from a summation of all values contained in the correlation coefficient array. As a result, the code output an average speckle size along the longitudinal dimensions for four simulated beam diameters on a target object. To compute speckle size and demonstrate the accuracy of the measurements, the code was run through ten iterations. Each iteration of the code calculated the average speckle size for one realization of the electric field. The results in Table 2 were obtained by calculating the mean and standard deviation from the ten iterations of the code.

Table 2: Average speckle size and standard deviation in the longitudinal direction using single realization method.

Average Speckle Size	Standard Deviation
23.031	10.111
12.682	1.599
4.318	0.358
2.194	0.137

The standard deviation results in Table 2 establishes the method's ability to reproduce speckle size calculations. The code performs well for smaller speckle illustrated by the relatively low standard deviation results. Larger speckle have a greater standard deviation, indicating the calculated speckle spots fluctuate in size over each iteration. Thus, performing an ergodic averaging over a single realization of the electric field will correctly

determine the size of smaller speckle spots, however the method does not accurately reproduce calculations of the average size of larger speckle spots with repeated iterations. Therefore to account for the fluctuations in size over each iteration, an ensemble average over multiple electric field realizations will be implemented in the following section.

4.3.2 Multiple Realization Method

This method calculates average speckle size by performing an ensemble average over multiple realization of the generated electric field. First, the electric field is multiplied by its complex conjugate to obtain a $[1500 \times 1500]$ intensity matrix in the plane of the receiver. This process is repeated 400 times to compile a $[1500 \times 1500 \times 400]$ 3 Dimensional intensity matrix. A $[250 \times 300 \times 400]$ matrix is cropped from the central portion of the $[1500 \times 1500 \times 400]$ intensity matrix. Intensity correlations will again be calculated along the lateral and longitudinal dimensions in order to compute the correlation coefficient, stated in Eq. 4-5. Since the method for calculating is identical along both lateral and longitudinal dimensions, the following will demonstrate calculations along the lateral directions.

The first step in calculating the intensity correlation is to obtain the initial intensity values, I_n where n will range from zero to 150. These values will be obtained from one row located in center of the intensity matrix. In each of the 400 iterations of the intensity matrix, the first 150 cells of the central rows will be collected to compile a $[1 \times 150 \times 400]$ matrix. Average intensities are calculated of each I_n value by performing an ensemble average on each cell across all 400 iteration of the field intensity. The result will be a $[1 \times 150]$ array which will then be averaged to obtain $\langle I_n \rangle$, the average initial intensity value.

Figure 4-10 is presented to help illustrate the 3-Dimensional matrix to aid in the explanation of the calculations.

Figure 4-10: First half of intensity matrix for each realization is stored to compile an initial I_n matrix with dimensions $[1 \times 150 \times 400]$.

The next step is to compile a secondary matrix of intensities, I_m , located in the same row of the intensity matrix used to obtain the initial intensity values, I_n . The separation between points (n, m) will vary between zero and 150 for each iteration of the 400 original intensity fields. The intensity matrix, I_m , will have dimensions $[150 \times 150 \times 400]$ when compiled; successive rows indicate the level of separation between points (n, m). For example, the first row in the I_m matrix specifies a separation of zero, indicating no shift has occurred. Therefore all values of I_m are equal to I_n , satisfying the conditions outlined in Eq. 4-6 and 4-7. The second row of the I_m matrix specifies a separation of one, indicating values I_m are compiled from cells 2 through 151. This process is iterated until the maximum separation between points (n, m) is 150, thus values contained in cell 151 through 300 are

stored in the 150th row of the I_m matrix. Figure 4-11 is presented to help illustrate the 3-Dimensional matrix to aid in the explanation of the calculations.

Figure 4-11: Shifted I_m values are stored to a matrix with dimensions [150 x 150 x 400]. Each row of the matrix represents an increase in separation between points (n, m)

The normalized intensity correlation between points (n, m) can now be calculated from the compiled I_n and I_m matrixes. This is performed by computing the intensity correlations, $\langle I_n I_m \rangle$, along with the respective ensemble average, $\langle I_m \rangle$, at each level of separation. The intensity correlations are calculated by multiplying the [1 x 150 x 400] initial intensity matrix by each individual row of the [150 x 150 x 400] secondary intensity matrix. Calculating an ensemble average across all realizations results in an [150 x 150] matrix, which is then averaged across all columns to obtain a [1 x 150] matrix containing $\langle I_n I_m \rangle$ for each level of separation between points (n, m). Average intensities are calculated of each I_m value by performing an ensemble average across all 400 iteration of the field

intensity. The result will be a $[150 \times 150]$ array which will then be averaged across all columns to obtain a single $\langle I_m \rangle$ value for each level of separation. The normalized correlation value $\frac{\langle I_n I_m \rangle}{\langle I_n \rangle \langle I_m \rangle}$ is then calculated for each level of separation and appended to an array. As presented in Eq. 4-5, the square modulus correlation coefficient, $|\mu_{nm}|^2$ is computed by subtracting one from each of the normalized correlation values. The correlation coefficient varies on an interval between $(-1, 1)$, however only the positive correlation values are of interest because they express similarity between the two points (n, m) . Figure 4-12 displays plots of the correlation coefficient in terms of Δx , representing the separation between points (n, m) .

Figure 4-12: Multiple realization Correlation Coefficient versus longitudinal separation.

The plots in Fig. 4-12 accurately represent the theory defined in Eqs. 4-6 and 4-7, which states the square modulus correlation coefficient should achieve a value of one when I_n is equal to I_m . The limited quantity of large speckle spots contained in a single realization of the electric field does not impact the calculation of the correlation coefficient as observed in plot a) of Fig. 4-12. The previous method confirmed that large speckle had a tendency to fluctuate in size over multiple realizations. Thus, performing an average over many realizations of the electric field provides a more accurate representation of speckle size since the fluctuations in size are accounted for.

The average size of speckle as described in Eq. 2-19, is defined by calculating the equivalent area of the squared modulus correlation coefficient function. The shift was performed one cell at a time, therefore the average speckle size calculations were obtained from a summation of all values contained in the correlation coefficient array. As a result, the code output an average speckle size along the longitudinal dimensions for four simulated beam diameters on a target object. To compute speckle size and demonstrate the accuracy of the measurements, the code was run through ten iterations. Each iteration of the code calculated the average speckle size for 400 realizations of the electric field. The results in Table 3 were obtained by calculating the mean and standard deviation from the ten iterations of the code

Table 3: Average speckle size and standard deviation in the longitudinal direction using multiple realization method.

Average Speckle Size	Standard Deviation
26.085	2.265
11.844	0.672
3.468	0.097
2.059	0.035

The standard deviation results in Table 3 establishes the method's ability to reproduce speckle size calculations with each execution of the code. As illustrated by the relatively low standard deviation results, the code performs relatively well for both smaller and larger speckle spots. The benefit of the multiple realization method lies in the ability to account for the fluctuation in large speckle size. The single realization method was able to compute the size of speckle contained in the intensity field, however because of the fluctuations the method failed to reproduce these calculations with each execution of the code. Since a large number of realization used in the multiple realization method account for the fluctuations, a more accurate speckle size is anticipated with each execution of the code.

CHAPTER 5

EXPERIMENT

There are three important measurements obtained from the experimental set up in the laboratory and will be discussed in the first two sections of chapter 5. First, the general setup for the LDV hardware is discussed in section 5.1. This section will include a discussion on how return vibration signatures are collected and analyzed as well as a discussion on how the strength of the heterodyned signal is measured and analyzed. Second, the imaging set up is discussed in section 5.2 and will include a discussion on how images are captured using the Matlab IMAQ tool as well as a brief discussion on how they are analyzed using Matlab code. Lastly, the results of the experiments are discussed in section 5.3.

5.1 Experimental Setup

To study the effects of speckle size and decreasing signal strength on the stability of an LDV signature, a Polytec RSV-150 laser vibrometer with wavelength 1550 nm was set up in the laboratory^[6]. The Polytec's beam was aligned to illuminate a target roughly 6 meters away on an opposing optical table. The target was a thick piece of cardboard firmly attached to a computer fan that was driven by a Hewlett Packard Triple Output DC Power Supply (Model: E3630A). The cardboard target provided a rough surface that produced a vibration signature along with a speckle field when illuminated by the Polytec LO. A focusing ring on the front lens of the sensor head allowed for manual adjustments to the focused distance of the laser beam.

A Goodrich SWIR Camera was aligned with an optical system consisting of one 2” diameter lens with 150mm focal length to capture images of the beam spot at each focused distance. The images were analyzed using Matlab Image Region Analyzer application to determine how many pixels the beam covered in both the lateral and longitudinal directions. Using the dimensions of the pixel and the magnification equation, the beam spot diameter was measured for each focused distance. Table 4 shows the four measured spot diameters at the target object.

Table 4: Measured spot diameter

Spot Diameter (measured) mm
0.65
1.40
2.59
3.70

At each focused distance, images of the speckle field were captured simultaneously with measurements of the return signal strength and vibration signature. From these measurements, it is demonstrated that defocusing the Polytec LO on target increases the stability of the return signature coupled with the LO.

5.1.1 Signal Measurements

Velocity measurements were collected using the VibSoft-20 data acquisition system specific to the Laser Vibrometer. The acquisition setting in the VibSoft 5.2 software were manipulated to satisfy the Nyquist sampling criteria. Adjustable settings for the VibSoft 5.2 software include selecting the number of samples (N_{sample}) and a Sample

frequency (f_{sample}). The sample time and resolution of the signal are dependent on these parameters based on the following equations:

$$t_{sample} = \frac{N_{sample}}{f_{sample}} \quad (5-1)$$

And

$$\Delta t = \frac{1}{f_{sample}} \quad (5-2)$$

Setting the number of samples to $N_{sample} = 16384$ with a sampling frequency of $f_{sample} = 1.2 \text{ kHz}$ produced a sampling time of $t_{sample} = 13.65 \text{ s}$ and a resolution of $\Delta t = 1.042 \text{ ms}$.

The Polytec Vibrometer is capable of recording and digitally processing an analog time signal with the VibSoft software. In order to digitally process analog time signals, a correct sampling frequency must be chosen to sample over a time period. The number of samples in a given period is limited, thus limiting the information digitally obtained. For correct digital processing, the sampling theorem (also known as the Nyquist theorem) must be achieved. This is fulfilled when:

$$f_{sample} > 2f_{signal}$$

where f_{signal} is the maximum signal frequency obtained from the target object. When impinge on this theorem, the spectrum of digitally sampled signal is falsified by the alias effect. The alias effect creates artificial spectral lines at frequencies $f_{Alias} < \frac{f_{sample}}{2}$. To avoid infringing the Nyquist theorem, increase the sampling frequency or suppress

frequency higher than half the sampling frequencies. All VibSoft system provide built in alias suppression, this is achieved by suppressing the frequencies above the chosen bandwidth.

The RSV-150 vibrometer controller measures the amount of light scattered back from an object's surface that effectively mixes with the local oscillator internal to the instrument. A voltage proportional to the signal strength is output through a BNC connector on the back of the control unit. A Keysight InfiniiVision Oscilloscope (Model: DSOX3021T) was connected to the controller unit to collect 10 seconds of the optical signal strength at each focused distance. The collected measurements were plotted in MATLAB[®] and spectrogram plots were created for each return signature. The signatures were divided into segments and a statistical analysis was performed to show the increase in stability at each focus. The VibSoft 5.2 software contains MATLAB[®] code that allows the user to upload the data for analysis. A kurtosis method ^[7] was implemented to estimate the less impulsive behavior present in the signal as a function of the beam spot size at the object.

5.2 Imaging Setup

A Goodrich (Model: SU320KTS-1.7RT) SWIR Camera was placed in line with an optical system consisting of a 2" and 1" diameter lens with focal lengths 150mm and 38.5mm, respectively. The 2" lens was placed in the same plane as the Polytec receiver aperture, but offset from it. This lens defines the location of the speckle field that is sampled by the camera and also acts as a field lens to increase image brightness. The 1" lens images the speckle in the aperture plane on the camera detector array.

It was necessary to confirm the speckle captured by the SWIR camera was representative of the speckle in the plane of the Polytec receiving aperture. To measure this relationship, images of the speckle field, in the same plane of the receiving aperture, were captured at various positions to the left and right of the Polytec. At each position the SWIR camera captured 100 frames of the specular field; the Polytec was focused on the target and focused at infinity for each position. A Matlab code was compiled to output average speckle size, average intensity, and standard deviation for both the x and y directions of the speckle images at the various locations. The calculated statistics remained consistent for both focused distances, thus it was concluded that the speckle field captured by the camera was indicative of what the Polytec receiving aperture was capturing.

After determining the speckle in the Polytec plane was representative of what the camera was capturing, the optical system was setup on a tripod off the optical table. The SWIR camera was positioned to capture images of the speckle field in the same plane as the Polytec measurement head. Using the MATLAB[®] Image Acquisition Tool, 850 frames of the speckle field were captured at different focused distances. A MATLAB[®] code was compiled to calculate the average speckle size for each of the laser beam spot sizes on target.

5.3 Experimental Results

This section presents three important measurements captured in the laboratory from the experimental procedures outlined in the previous section. The first set of results consists of images that were captured of the speckle field. The images and calculated speckle size in terms of pixels are presented along with a ratio between speckle size and receiving aperture size are presented in section 5.3.1.

The second set of results consists of measurements of the strength of the heterodyned signature. Voltage measurements representing the heterodyned signal strength are presented. An increase in signature stability of the heterodyned signal as the beam diameter is increased is observed and discussed. A tradeoff between signature stability and signal strength is presented and spectrogram plots demonstrating and optimal focus are displayed.

The last set of results presents the kurtosis ratio as a metric to quantify an increase in stability. This metric measures the impulsive characteristics contained in the voltage measurements presented in section 5.3.2.

5.3.1 Speckle Images

Images of the specular field captured in the laboratory are presented in Figure 5-1. These images show the strong dependence speckle size has on laser beam diameter. Laser spot size is at a minimum when focused on the cardboard target, producing larger speckle as seen in image a). As we turn the focusing ring of the Polytec, we are increasing the focusing distance of the laser beam resulting in a larger beam diameter on target. The larger illumination area reflects greater numbers of randomly distributed phase leading to smaller sized speckles as seen in images b)-d).

Figure 5-1: Speckle images captured in the laboratory. a) Polytec focused on target b) One turn of Polytec focusing ring. c) Two turns of Polytec focusing ring. d) Three turns of Polytec focus ring

These images were captured using an optical set up that images speckle in the plane of a 2" lens. Each image is comprised of a [300 x 250] matrix where 300 pixels is related to the diameter of the 2" lens. Table 5 consists of the calculated size of speckle along the longitudinal and lateral dimensions of the images presented above. A ratio is presented comparing the size of the speckle with respect to the receiving aperture of the Polytec device.

Table 5: Longitudinal and Lateral speckle sizes in terms of Pixels. The ratio between speckle size and aperture width is presented to the right of the respective dimensions.

Spot Diameter (measured) mm	<u>Longitudinal</u> Speckle size (Pixels)	Speckle to Aperture ratio	<u>Lateral</u> Speckle size (Pixels)	Speckle to Aperture ratio
0.65	36.12	12.25%	36.75	14.70%
1.40	6.28	2.12%	6.42	2.62%
2.59	3.62	1.25%	3.68	1.51%
3.70	3.41	1.16%	3.21	1.36%

5.3.2 Heterodyne Strength Measurements

Figure 5-2 (left) shows the return signal strength captured by the receiving aperture of the sensor head. The signal strength is in terms of voltage which is proportional to the heterodyne signal strength. The different colors represent the successive turns of the focusing control on the Polytec, starting with black indicating the beam is focused on target. As can be seen, the black signature contains fluctuations in the return voltage measurements as a function of time. This is attributed to fluctuations in the integrated phase and amplitude captured by the Polytec receiving aperture. These fluctuations give rise to impulsive behavior. The impulsive behaviors are reduced as the beam is defocused on the target. There are less fluctuations, however there is also less power in the returned signals coupled with the LO. The green signatures shows an increase in the stability of the return measurements, however the magnitude of the heterodyned signal is greatly reduced. To quantify the increase in stability, the standard deviation was calculated of each return signature and plotted as seen in Fig. 5-2 (right). This plot shows that as we defocus the beam on target, the fluctuations in the return measurements are reduced.

Figure 5-2: (left) Voltage measurements representing heterodyned signal strength. Black signal is target in focus. Blue is one turn of focus lens. Red is two turns of focus lens. Green is three turns of focus lens. (right) Standard Deviation of the strength measurements. Position 1 is target in focus. Position 2 is one turn of focus lens. Position 3 is two turns of focus lens. Position 4 is three turns of focus lens.

Spectrogram plots of the heterodyne signal measurements are presented in Figure 5-3 to show the frequency characteristics of the signatures presented in Fig. 5-2. Figure 5-3(a) was obtained by focusing the beam on target without the target being driven by an external vibration source. What is seen is the signature resulting from natural vibration differences between the vibrometer and the target. We see that these natural vibrations (noise) are mostly lower frequency. Figure 5-3(b) is the spectrum obtained when the Polytec beam is focused on target and the target is driven by an external fan. We see frequencies present that are characteristic of the driven target. Figures 5-4 show the spectrum of the signature as the Polytec beam is progressively defocused. The plot in Fig. 5-4(a) & 5-4(b) still exhibit the strong signals that are representative of the driven target, but the low frequency noise components are reduced in relative strength. In Fig. 5-4(c) the strength of the driven target signals are greatly reduced making their detection difficult. We see, then, that there is an optimum defocused beam diameter in which the signals of interest are measurable but the noise components are reduced.

Figure 5-3: Spectrogram with the Polytec beam focused on target. a) Target is undriven b) Target is driven to produce a vibration signature.

Figure 5-4: Spectrogram of the driven target object with a) One turn of Polytec focusing ring. b) Two turns of Polytec focusing ring. c) Three turns of Polytec focus ring.

5.3.3 Vibration Signature

A kurtosis metric was employed to measure the stability of the signatures plotted in Fig. 5-2. The Kurtosis Ratio (KR) of each signature is presented equation 5-3 and the results are presented in Table 6.

$$KR(x) = \frac{\beta_2(x)}{\beta_2(x_t)} \quad (5-3)$$

where $\beta_2(x)$ is the kurtosis of the a raw voltage signature and $\beta_2(x_t)$ is the kurtosis of the trimmed voltage signature. If the condition $KR < 2$ is fulfilled, the analyzed signature is regarded as undistorted^[7]. The several impulsive events present in the black signature in Fig.5-2 produce a high KR of 5.61 and l is labeled distorted. As the beam is defocused the signature becomes more stable and the KR decreases. This indicates that defocusing the beam will produce quality return signatures that contain less impulsive characteristics.

Table 6: Kurtosis Ratio measurements presented with Beam diameter at object

Beam Diameter (mm)	Kurtosis Ratio
0.65	5.61
1.40	1.80
2.59	1.65
3.70	1.63

CHAPTER 6

CONCLUSION

6.1 Future Work

The results obtained in this thesis outline a method that describe an increase in stability of vibration signatures by mean of defocusing the target object. There is however a tradeoff between signature stability and the strength of the signal due to the antenna theorem. Future research can be conducted to better understand the limits of this trade off, or reduce it altogether.

In order to better understand the limitations of the tradeoff between signal strength and signature stability, further testing could be conducted outside the laboratory environment. It would be advantageous to set up the Polytec device outside the building where the effects of the atmosphere and external vibrations would have a greater impact on the device. This would give rise to more appropriate testing equipment such as a vehicle or building structure that contain interesting vibration signatures to analyze. In this environment, it would be possible to find an optimal defocus that results in the best signature stability verses signal strength.

Another area of future research would be to implement a telephoto lens systems in front of the receiving aperture. This would allow for an increase of beam diameter at the target object without having to defocus the object from the LDV angular field of view.

Implementing the telephoto lens would eliminate the inverse tradeoff between signature stability and signal strength because the LDV could remain focused on the target object.

6.2 Conclusion

Impulsive return signatures due to fluctuations of the integrated speckle pattern at the receiver contribute to signature instability. In this paper, we have shown that fluctuations in the integrated signal coupled to the LO can be reduced by increasing the beam size at the object. The Antenna theorem was presented to show the tradeoff between increasing the angular field of view of an optical heterodyne receiver and its effective aperture area. The resulting loss of signal power must then be traded against the increase in stability.

A simulation was compiled to investigate the effects pointing jitter had on the stability of vibration measurements. The results of the simulation have shown that as the beam spot diameter on the target object is reduced, the electric field is driven towards wide sense stationarity. This conclusion was determined based on the first and second moments of the magnitude and intensity of the speckle field in the plane of the receiving aperture. The results show a decrease to the magnitude and intensity measurements of the integrated electric fields across the receiving aperture.

Two different Matlab code are presented for calculating the average speckle size of a speckle image: averaging over single and multiple realizations of the electric field in the plane of the receiving aperture. Both methods employ calculating the intensity correlation to compute a correlation coefficient that is integrated to find average speckle size. The single realization method contained results that didn't agree with the presented correlation

coefficient theory. The computed correlation coefficient for large speckle did not reach a value of one when no shift occurred, thus it was determined that one realization of the electric field did not contain enough information to effectively calculate the average size of speckle.

Images of speckle patterns produced from four beam spot diameters illustrate the decrease in speckle size as the beam size at object is increased. Measurements show a decrease in standard deviation as the beam spot size at the object is increased, indicating less fluctuation is present in the heterodyne signal. Spectrogram plots of the signal measurements show a decrease in the strength of the return signature coupled to the LO as the beam spot reaches a maximum diameter. A kurtosis metric previously reported in literature ^[7] was implemented to further show the stability of the return signatures. The results show that larger beam diameters at the object fulfill the KR metric resulting in the successful collection of stable measurements. From the experimental results we can conclude that an optimal beam spot diameter on the object exists that produces stable heterodyned signatures with sufficient signal strength.

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APPENDIX A

MATLAB CODE

Single Realization Code

```
%Program to investigate speckle effects in stability of vibration
%signature.

clear all
clc
fignum=1;      %Number for plots
for z=1:5
    c=3*10^8;   %Speed of light
    lambda=1550*10^-9; %Wavelength
    d=10;       %Distance between Transmitter/Receiver
    lambda*d=.1; %Wavelength times distance between object &
    transmitter/receiver

    %Initialize transmitted wavefront
    %Assume waist at exit pupil so that radius of curvature is infinite
    %Arrange width of transmit beam so that object in far field

    D=.075;     %Receiver aperture diameter
    T=.075;     %Transmitter diameter
    step=1500;  %Define step size in transmitter coordinate space
    xt=-T/2:T/step:T/2; %Transmitter x coordinates
    yt=-T/2:T/step:T/2; %Transmitter y coordinates
    [X,Y]=meshgrid(xt,yt);
    [XX,YY]=meshgrid(xt,yt);
    sig0 = 0.0025;
    sig1 = 0.001;
    sig2 = 0.00025;
    sig3 = 0.000125;
    sigmavalue=[sig0,sig1,sig2,sig3];
    eTran=exp(-pi.*(X.^2+Y.^2)./(sigmavalue(z)*sigmavalue(z)));
    %Transmitted electric field - Gaussian beam

    %    figure(fignum)
    %    mesh(X,Y,eTran)
    %    xlim([-0.01 0.01])
    %    ylim([-0.01 0.01])
    %    title('Transmit Beam Profile')
```

```

%      set(gcf,'color','w');
%      fignum=fignum+1;

%Use Fourier transform to compute field at target
eTar=fftshift(fft2(eTran)); %fftshift to center beam in field
matrix

%Generate array of random phases to simulate reflection off of a
rough surface
n=size(xt);
m=size(yt);
rnumph=rand(n(2),m(2));
ranphase=exp(1i*2*pi*rnumph); %Currently have the roughness
delta-correlated
eRef=eTar.*ranphase; %Reflected electric field; Uniform
reflectivity = 1 assumed

%Calculate units in object plane; spatial freq separation between
elements given by
% 1/T = dxo/lambdaz
dxo=(lambdaz/(T))./(step);
dyo=(lambdaz/(T))./(step);
sampnum=floor(n(2)/2);
sfx=-sampnum*dxo:(2*sampnum*dxo)/step:sampnum*dxo; sfxx=sfx; %X-
Coordinates in object plane;
sfy=-sampnum*dyo:(2*sampnum*dyo)/step:sampnum*dyo; sfyy=sfy; %Y-
Coordinates in object plane
[sfX, sfY]=meshgrid(sfx,sfy); [sfXX, sfYY]=meshgrid(sfxx,sfyy);

figure(fignum)
mesh(sfX,sfY,abs(eRef)./(step.^2)) %Reflected electric field -
Gaussian beam
xlim([-0.6 0.6])
ylim([-0.6 0.6])
title('Reflected Beam Profile')
set(gcf,'color','w');
fignum=fignum+1;

phasref=atan(imag(eRef)./real(eRef)); %Phase at object
figure(fignum)
mesh(sfX,sfY,phasref)
title('Phase at target')
fignum=fignum+1;

%Fourier transform reflected field to get field at receiver plane
eRec=fft2(eRef); %fft2 will perform a 2d fourier transform
eReciever = eRec;
output = eRec.*conj(eRec);
SpeckleIMG = eRec.*conj(eRec);

figure(fignum) %Magnitude at receiver
mesh(X,Y,abs(eRec))
zlim([0 200000])
title('Beam Profile')

```

```

fignum=fignum+1;

phaserec=atan(imag(eRec)./real(eRec)); %Phase at reciever
std(std(phaserec))
figure(fignum)
mesh(X,Y,phaserec)
title('Phase at object')
fignum=fignum+1;

figure(fignum) %Image of Speckle
imagesc(output)
title('output')
colormap gray
fignum=fignum+1;

% Start Pointing Jitter simulation
Right=901; Left=600; Bottem=876; Top=625;
for k=1:750
    eReciever=eRec;
    XX=X; YY=Y;
    p(k)=floor(30*randn(1,1));
    Right=Right+p(k); Left=Left+p(k); Bottem=Bottem+p(k);
    Top=Top+p(k);

    %Crop out electric field @ reciever
    eReciever(:,(Right):end)=[];
    eReciever(:,1:(Left))=[];
    eReciever((Bottem):end,:)=[];
    eReciever(1:(Top),:)=[];
    yh=size(eReciever);

    %Ensure not to step out of array
    if yh(1) ~= 250 || yh(2) ~= 300
        eReciever=eRec;
        Right=901; Left=600; Bottem=876; Top=625;
        p(k)=floor(30*randn(1,1));
        Right=Right+p(k); Left=Left+p(k); Bottem=Bottem+p(k);
        Top=Top+p(k);
        eReciever(:,(Right):end)=[];
        eReciever(:,1:(Left))=[];
        eReciever((Bottem):end,:)=[];
        eReciever(1:(Top),:)=[];
    end

    %
    pause

    %Crop step size array
    XX(:,(Right):end)=[]; YY(:,(Right):end)=[];
    XX(:,1:(Left))=[]; YY(:,1:(Left))=[];
    XX((Bottem):end,:)=[]; YY((Bottem):end,:)=[];
    XX(1:(Top),:)=[]; YY(1:(Top),:)=[];

phasrec(z,k)=atan(imag(sum(sum(eReciever)))/real(sum(sum(eReciever))))
; %Phase at Reciever
% figure(fignum)

```

```

%         fignum=fignum+1;
%         mesh(XX,YY,phasrec(z,k))
%         title('Phase at reciever')
%         set(gcf,'color','w');

amplitude(z,k)=sqrt(imag(sum(sum(eReciever))).^2+real(sum(sum(eReciever
))).^2); %Field magnitude at receiver
%         figure(fignum)
%         fignum=fignum+1;
%         mesh(XX,YY,abs(eReciever))
%         title('Field magnitude at receiver')
%         zlim([0 150000])
%         set(gcf,'color','w');
%         speckle_intensity(z,k)=amplitude(z,k).*conj(amplitude(z,k));
%Cropped Speckle image intensity
%         figure(fignum)
%         fignum=fignum+1;
%         imagesc(output)
%         colormap gray
%         set(gcf,'color','w');

        k
    end

    sizintensity=size(speckle_intensity);
    sizphase=size(phasrec);
    sizamplitude=size(amplitude);
    %calculate mean intensity, phase, and amplitude
    meanintensity(z)=sum(speckle_intensity(z,:))./(sizintensity(2));
    meanphase(z)=sum(phasrec(z,:))./(sizphase(2));
    meanamplitude(z)=sum(amplitude(z,:))./(sizamplitude(2));
    %calculate STD intensity, phase, and amplitude
    STDint(z)=std(speckle_intensity(z,:));
    STDphase(z)=std(phasrec(z,:));
    STDamplitude(z)=std(amplitude(z,:));

    % plot mean intensity, phase, amplitude
    figure(fignum)
    fignum=fignum+1;
    plot(meanintensity,'LineWidth',2)
    title('First moment Intensity')
    grid on
    set(gcf,'color','w');

    figure(fignum)
    fignum=fignum+1;
    plot(meanphase,'LineWidth',2)
    title('First moment Phase')
    ylim([-0.5 0.5])
    grid on
    set(gcf,'color','w');

    figure(fignum)
    fignum=fignum+1;
    plot(meanamplitude,'LineWidth',2)
    title('First moment Magnitude')
    grid on
    set(gcf,'color','w');

```

```

    % plot STD intensity, phase, amplitude
    figure(fignum)
    fignum=fignum+1;
    plot(STDint,'LineWidth',2)
    title('Second moment Intensity')
    grid on
    set(gcf,'color','w');

    figure(fignum)
    fignum=fignum+1;
    plot(STDphase,'LineWidth',2)
    title('Second moment Phase')
    ylim([0 1])
    grid on
    set(gcf,'color','w');

    figure(fignum)
    fignum=fignum+1;
    plot(STDamplitude,'LineWidth',2)
    title('Second moment Magnitude')
    grid on
    set(gcf,'color','w');

    %This portion of the program will find the average Speckle size for
    %single realization of Electric field
    output=SpeckleIMG;
    output(:,901:end)=[]; %right side crop
    output(:,1:600)=[]; %left side crop
    output(876:end,:)=[]; %bottom crop
    output(1:625,:)=[]; %top crop

    figure(fignum)
    fignum=fignum+1;
    imagesc(output)
    colormap gray

    %Compute correlation of pixel to determine speckle size
    [y1,x1]=size(output); %Number of pixels in x and y directions

    % Correlation with x-direction shift
    corrInt=zeros(1,floor(x1/2)); % Array to hold the correlation
    values as function of shift
    Inx=output(:,1:floor(x1/2)); % Half of array to serve as first
    intensity pattern
    szIn=size(Inx);
    avgInx=sum(sum(Inx))/(szIn(1)*szIn(2)); % Average intensity of
    first pattern
    for dx=1:floor(x1/2)
        Imx=output(:,dx:floor(x1/2)+dx-1); % Shifted array for second
        intensity pattern
        szImx=size(Imx);
        avgImx=sum(sum(Imx))/(szImx(1)*szImx(2)); % Average intensity
        of second pattern

```



```

corrInt(dx)=sum(sum(Inx.*Imx))/(avgInx*avgImx*szImx(1)*szImx(2)); %
Normalized correlation value <InIm>
end
Unmx=corrInt-1; %Computes Correlation Coefficient
a=Unmx>0; %Creates a logical array of correlation data greater
than 0
speckle_x(z)=sum(Unmx.*a) %Speckle size in x direction

figure(fignum)
fignum=fignum+1;
plot(Unmx.*a,'LineWidth',2)
title('Unmx')
grid on
xlabel('\Delta x')
ylabel('|\mu_{n_m}|^2')
ylim([0 1.1])
set(gcf,'color','w');

%Correlation with y-direction shift
corrInty=zeros(1,floor(y1/2)); % Array to hold the correlation
values as function of shift
Iny=output(1:floor(y1/2),:); % Half of array to serve as first
intensity pattern
szIn=size(Iny);
avgIn=sum(sum(Iny))/(szIn(1)*szIn(2)); % Average intensity of
first pattern
for dy=1:floor(y1/2)
    Imy=output(dy:floor(y1/2)+dy-1,:); % Shifted array for second
intensity pattern
    szImy=size(Imy);
    avgImy=sum(sum(Imy))/(szImy(1)*szImy(2)); % Average intensity
of second pattern

corrInty(dy)=sum(sum(Iny.*Imy))/(avgIn*avgImy*szImy(1)*szImy(2)); %
Normalized correlation value <InIm>
end

Unmy=corrInty-1; % Computes Correlation Coefficient
a=Unmy>0; % Creates a logical array of correlation data greater
than 0
speckle_y(z)=sum(Unmy.*a) % Speckle size in y direction

figure(fignum)
fignum=fignum+1;
plot(Unmy.*a)
title('Unmy')
grid on
ylim([-0.1 1.1])
end

```

Multiple Realization Code

```
%Program to investigate speckle effects in stability of vibration
signature
%%
clear all
clc
fignum=1;    %Number for plots
for z=1:4
    clear output Inx Imx Iny Imy
    output=zeros(1501,1501,400);
    Inx=zeros(1,150,400); Imx=zeros(150,150,400);
    Iny=zeros(125,1,400); Imy=zeros(125,125,400);

    c=3*10^8;    %Speed of light
    lambda=1550*10^-9; %Wavelength
    d=10;        %Distance between Transmitter/Receiver
    lambdaz=.1; %Product of wavelength times distance between object &
    transmitter/receiver

    %Initialize transmitted wavefront
    %Assume waist at exit pupil so that radius of curvature is infinite
    %Arrange width of transmit beam so that object in far field

    D=.075;      %Receiver aperture diameter
    T=.075;      %Transmitter diameter
    step=1500;   %Define step size in transmitter coordinate space
    xt=-T/2:T/step:T/2;    %Transmitter x coordinates
    yt=-T/2:T/step:T/2;    %Transmitter y coordinates
    [X,Y]=meshgrid(xt,yt);
    sig0 = 0.0025;
    sig1 = 0.001;
    sig2 = 0.00025;
    sig3 = 0.000125;
    sigmavalue=[sig0,sig1,sig2,sig3];
    eTran=exp(-pi.*(X.^2+Y.^2)./(sigmavalue(z)*sigmavalue(z)));
    %Transmitted electric field - Gaussian beam

    figure(fignum)
    mesh(X,Y,eTran)
    xlim([-0.01 0.01])
    ylim([-0.01 0.01])
    title('Transmit Beam Profile')
    set(gcf,'color','w');
    fignum=fignum+1;

    for realizations=1:400;
        realizations
        %Use Fourier transform to compute field at target
        eTar=fftshift(fft2(eTran));    %fftshift to center beam in field
matrix

        %Generate array of random phases to simulate reflection off of
        a rough surface
```

```

n=size(xt);
m=size(yt);
rnumph=rand(n(2),m(2));
ranphase=exp(1i*2*pi*rnumph); %Currently have the roughness
delta-correlated
eRef=eTar.*ranphase; %Reflected electric field; Uniform
reflectivity = 1 assumed

%Calculate units in object plane; spatial freq separation
between elements given by
% 1/T = dxo/lambdaz
dxo=(lambdaz/(T))./(step);
dyo=(lambdaz/(T))./(step);
sampnum=floor(n(2)/2);
sfx=-sampnum*dxo:(2*sampnum*dxo)/step:sampnum*dxo; sfix=sfx;
%X-Coordinates in object plane;
sfy=-sampnum*dyo:(2*sampnum*dyo)/step:sampnum*dyo; sfyy=sfy;
%Y-Coordinates in object plane
[sfX, sfY]=meshgrid(sfx,sfy);

figure(fignum)
mesh(sfX,sfY,abs(eRef)./(step.^2)) %Reflected electric field -
Gaussian beam
xlim([-0.6 0.6])
ylim([-0.6 0.6])
title('Reflected Beam Profile')
set(gcf,'color','w');
fignum=fignum+1;

phaseref=atan(imag(eRef)./real(eRef)); %Phase at object
figure(fignum)
mesh(sfX,sfY,phaseref)
title('Phase at target')
fignum=fignum+1;

%Fourier transform reflected field to get field at receiver
plane
eRec=fft2(eRef); %fft2 will perform a 2d fourier transform
output(:, :, realizations)= eRec.*conj(eRec);

figure(fignum)
mesh(X,Y,abs(eRec))
zlim([0 200000])
title('Beam Profile')
fignum=fignum+1;

figure(fignum)
imagesc(output)
xlim([1 264])
ylim([1 256])
title('output')
colormap gray
fignum=fignum+1;
end

```

```

%Crop Speckle Image for Speckle Size calculation
output(:,901:end,:)=[]; %right side crop
output(:,1:600,:)=[]; %left side crop
output(876:end,:,:)=[]; %bottom crop
output(1:625,:,:)=[]; %top crop

%This program will find the average Speckle size over Multiple
Image
%Realizations
for realize=1:realizations
    figure(fignum)
    fignum=fignum+1;
    imagesc(output(:,:,realize))
    colormap gray

    %Compute correlation of pixel to determine speckle size
    [y1,x1]=size(output(:,:,realize)); %Number of pixels in x and y
directions
    % Correlation with x-direction shift
    Inx(1,:,realize)=output(floor(y1/2),1:floor(x1/2),realize); %
Half of array to serve as first intensity pattern
    for dx=1:floor(x1/2)
        Imx(dx,:,realize)=output(floor(y1/2),dx:floor(x1/2)+dx-
1,realize); % Shifted array for second intensity pattern
    end
    % Correlation with y-direction shift
    Iny(:,1,realize)=output(1:floor(y1/2),floor(x1/2),realize); %
Half of array to serve as first intensity pattern
    for dy=1:floor(y1/2)
        Imy(:,dy,realize)=output(dy:floor(y1/2)+dy-
1,floor(x1/2),realize); % Shifted array for second intensity pattern
    end
end

%X Direction Averages
corrIntx=zeros(1,floor(x1/2)); % Array to hold the correlation
values as function of shift
szInx=size(Inx);
szImx=size(Imx);
avgInx=sum(sum(Inx,3),2)/(szInx(2)*szInx(3)); % Average intensity
of first pattern
for d=1:szImx(1)
    avgImx=sum(sum(Imx(d,:,:),3),2)/(szImx(2)*szImx(3)); %
Average intensity of second pattern

corrIntx(d)=sum(sum(Inx.*Imx(d,:,:),3),2)/(avgInx*avgImx*szImx(2)*szImx
(3)); % Normalized correlation value <InIm>
end
Unmx=corrIntx-1; %Computes Correlation Coefficient
a=Unmx>0; %creates a logical array of correlation data
greater than 0
speckle_x(z)=sum(Unmx.*a) %speckle size in x direction

figure(fignum)
fignum=fignum+1;
plot(Unmx.*a,'LineWidth',2)

```

```

    title('Unmx')
    xlabel('\Delta x')
    ylabel('|\mu_{n_m}|^2')
    grid on
    ylim([0 1.1])
    set(gcf,'color','w');

    %Y Direction Averages
    corrInty=zeros(1,floor(y1/2)); % Array to hold the correlation
values as function of shift
    szIny=size(Iny);
    szImy=size(Imy);
    avgIny=sum(sum(Iny,3),1)/(szIny(1)*szIny(3)); % Average intensity
of first pattern
    for d=1:szImy(2)
        avgImy=sum(sum(Imy(:,d,:),3),1)/(szImy(1)*szImy(3)); %
Average intensity of second pattern

    corrInty(d)=sum(sum(Iny.*Imy(:,d,:),3),1)/(avgIny*avgImy*szImy(1)*szImy
(3)); % Normalized correlation value <InIm>
    end
    Unmy=corrInty-1; % Computes Correlation Coefficient
    a=Unmy>0; % Creates a logical array of correlation data
greater than 0
    speckle_y(z)=sum(Unmy.*a) %speckle size in y direction

    figure(fignum)
    fignum=fignum+1;
    plot(Unmy.*a,'LineWidth',2)
    title('Unmy')
    grid on
    ylim([0 1.1])
    set(gcf,'color','w');
end

```