**PROPAGATION OF GAUSSIAN BEAMS THROUGH
A MODIFIED VON KARMAN PHASE SCREEN**

Thesis

Submitted to

The School of Engineering of the
UNIVERSITY OF DAYTON

In Partial Fulfillment of the Requirement for

The Degree of
Master of Science in Electro-Optics

By

Erica Marie Whitfield

Dayton, Ohio

December 2012

**PROPAGATION OF GAUSSIAN BEAMS THROUGH
A MODIFIED VON KARMAN PHASE SCREEN**

Name: Whitfield, Erica Marie

APPROVED BY:

Partha P. Banerjee, Ph.D.
Advisory Committee Chairman
Professor
Electro-Optics Program

Joseph W. Haus, Ph.D.
Committee Member
Professor
Electro-Optics Program

Dean R. Evans, Ph.D.
Committee Member
Air Force Research Laboratories

John G. Weber, Ph.D.
Associate Dean
School of Engineering

Tony E. Saliba, Ph.D.
Dean, School of Engineering
&Wilke Distinguished Professor

ABSTRACT

PROPAGATION OF GAUSSIAN BEAMS THROUGH A MODIFIED VON KARMAN PHASE SCREEN

Name: Whitfield, Erica M.
University of Dayton

Advisor: Dr. Partha P Banerjee

Gaussian beam propagation through discrete phase screens and an extended random media has been studied extensively over the last four decades. In this thesis, we numerically examine the effects of a discrete phase screen during Gaussian beam propagation on the scintillation index for a single beam and on the fringe visibility for the two transversely separated beams. We use the modified von Karman spectrum model to describe the phase screen statistics, utilizing the Fried parameter, inner scale and the outer scale. The scintillation index is analyzed as a function of the structure constant, phase screen location, the initial width of the Gaussian beam, etc. Similarly, the numerical simulations are extended using a pair of transversely separated Gaussian beams. We examine the interference of the beams and measure the fringe visibility at the target. We then correlate the scintillation index and the fringe visibility results and suggest the possible applications.

I dedicate this work to my peers Maureen Crotty and Micah Gatz
for their friendship, encouragement, and continuous wisdom.

ACKNOWLEDGEMENTS

I would like to express my appreciation to my advisory committee: Dr. Partha Banerjee, Dr. Joseph Haus, and Dr. Dean Evans. I would especially like to thank my advisor Dr. Partha Banerjee for his patience, thoughtfulness and meticulous editing and guidance. The insight, advice, and continued interest in my thesis of Dr. Joseph Haus have been academically valuable. Nick Miller also provided a great deal of resource suggestions and help for which I am most indebted.

I am most grateful for the help of my friends Max Lepper and Daniel Christiansen for offering their assistance in collecting data by running my simulated model on their personal computers. Their help shaved a multitude of hours off my data collection and without them, my data collection would have taken three to four times longer. Thank you.

I would also like to thank my graduate student peers Maureen Crotty, Ryan Feaver, Micah Gatz, Ighor Idehenne, Jeffrey Kraczek, and Gui Min Wu for always being willing to listen and discuss my questions and concerns and in promoting a stimulating and welcoming academic and social environment.

Without teachers and professors to both encourage me and inspire me, I would not be here today. I would specifically like to mention Richard Fairchild, my high school biology and physics teacher, introduced me to optics and encouraged me to pursue my interests and dreams. I am deeply indebted to my Alma matter faculty, specifically Dr. Steve Bowling, Dr. James Cain, Dr. Tim DeClue, Dr. Kevin Hopkins, Mark Kellogg, Dr. Bob Kitchin, Dr. John Patton, and Dr. Hillary Glauser-Patton who graciously worked with me and shared me between three departments. Specifically I would like to thank Dr. Bob Kitchin who inspired and demanded me to reach for my full potential.

Lastly, I would like to thank my eldest siblings Robert and Christina who have always been my biggest fans, been with me, and encouraged me through a great deal of hardships, victories, and life.

TABLE OF CONTENTS

ABSTRACT	iii
DEDICATION	iv
ACKNOWLEDGEMENTS	v
TABLE OF CONTENTS	vi
LIST OF FIGURES	ix
LIST OF TABLES	xiii
LIST OF ABBREVIATIONS	xiv
LIST OF SYMBOLS	xv
CHAPTER 1 INTRODUCTION	1
1.1 Primary Goals	1
1.2 Thesis Outline	2
CHAPTER 2 BACKGROUND AND THEORY	4
2.1 A Mathematical Description of Gaussian Beams	4
2.2 The Split-Step Beam Propagation Method	5
2.3 Power Spectral Density Models and the Modified von Karman Spectrum	7
2.4 Modeling Atmospheric Turbulence with a Thin Phase Screen	8
2.5 Numerical Determination of the Scintillation Index of Weak Turbulence Utilizing the Rytov Variance	9

2.6 A Discussion Concerning the Interchangeability of the Structure Constant and Fried Parameter	12
2.7 Mathematical Description of Two-Beam Interference.....	14
2.8 Measuring Fringe Visibility	16
CHAPTER 3 DEVELOPING A SIMULATION MODEL.....	18
3.1 Introduction.....	18
3.2 Modeling Single and Double Beam Forward Propagation through a Single Phase Screen.....	19
CHAPTER 4 SINGLE GAUSSIAN BEAM PROPAGATION THROUGH A PHASE SCREEN.....	25
4.1 Introduction.....	25
4.2 Simulated Results on the Effects of Propagation Distance on SI and Single Pass of a Gaussian Beam through a Phase Screen	25
4.3 Simulated Results on the Effects of Normalized Phase Screen Location on SI of a Single Pass of a Gaussian Beam Through a Phase Screen	27
4.4 Simulated Results on the Effects of Gaussian Beam Width on SI of a Single Pass of a Gaussian Beam Through a Phase Screen.....	29
4.5 Simulated Results on the Effects of the Fried Parameter on SI of a Single Pass of a Gaussian Beam Through a Phase Screen.....	31
4.6 Simulated Results on the Effects of the Inner Scale on SI of a Single Pass of a Gaussian Beam Through a Phase Screen	32
4.7 Concluding Remarks	33
CHAPTER 5 TWIN GAUSSIAN BEAM PROPAGATION THROUGH A PHASE SCREEN	35
5.1 Introduction.....	35
5.2 Simulated Results on the Effects of Propagation Distance and Transverse Beam Separation on FV of a Single Pass of Two Transversely Separated Gaussian Beams through a Phase Screen	35

5.3 Simulated Results on the Effects Normalized Phase Screen Location on FV of a Single Pass of Two Transversely Separated Gaussian Beams Through a Phase Screen	37
5.4 Concluding Remarks	38
CHAPTER 6 CONCLUSION AND DISCUSSION	40
6.1 Summary of Single Beam and Double Beam Results with Possible Applications	40
6.2 Future Work.....	41
BIBLIOGRAPHY	43
APPENDIX	45
A. Matlab© Simulated Model for Single Gaussian Beam.....	45
B. Matlab© Simulated Model for Double Gaussian Beams.....	48

LIST OF FIGURES

Figure 2.1: Two slits separated by a distance d interfere on a screen located a distance L from slits. The slits produce an interference pattern due to phase differences δ caused by path length differences. Point P results from constructive interference.....	14
Figure 3.1: Schematic showing the transit plane, phase screen plane and the target plane. A Gaussian beam transmits at the transmit plane a distance d_1 to a phase screen. The phase screen introduces phase variations that result in scintillation at the target plane a distance d_2 from the phase screen. The ratio of d/L is used to describe the normalized phase screen location.....	19
Figure 3.2: Spatial frequency profile for Gaussian beam of initial waist of 30 mm.....	20
Figure 3.3: A typical single Gaussian beam is transmitted through a phase screen placed at half the propagation distance of 1 km. The beam waist is 30 mm. Both images are shown on a logarithmic scale. The left-hand image shows a single exposure of fifty unique frame realizations. The right-hand image is a single frame realization contributing to the exposure on the left-hand side.....	22
Figure 3.4: Double beam interference through a phase screen placed at half the propagation distance of 4 km. Each beam has a beam width of 30 mm and are transversely separated by 70 mm. The interference pattern is observed at the target plane. Both images are shown on a logarithmic scale. The left-hand image shows a single exposure of fifty unique frame realizations and is digitally enhanced to clearly show the fringe visibility that is otherwise washed out. The right-hand image is a single frame realization contributing to the exposure on the left-hand side. ...	22
Figure 3.5: A phase screen results in a scintillation index between zero and unity. Typical orders of magnitude are 10^{-1} . Without the phase screen, the effect of diffraction on the scintillation index is observed to be several orders of magnitude smaller and thus miniscule.....	23

Figure 3.6: The center 10x10 pixels obtained from Figure 3.3 are used to determine the on-axis scintillation index. Both images are shown on a logarithmic scale and are digitally enhanced to show contrast between pixel intensity. Looking at beam center of the left-hand image of Figure 3.3 the reasoning for this enhancement becomes clear. The left-hand image shows the intensity of the center 10x10 pixels of an exposure, whereas the right-hand figure shows the variance of the intensity of these pixels..... 23

Figure 3.7: Cross-sectional plot of intensity variations during interference of twin Gaussian beams propagating through a random medium. A 1x1024 slice is taken from the average of 50 frames. The background, shown to be close to zero in the left figure, must be removed to get an accurate value for the FV. The case shown above for two 10mm beams separated by 30mm propagated through a phase screen located at the midpoint of the total propagation distance of 1km. The Fried parameter is held constant at $r_0 = 246\text{mm}$. The maximum intensity occurs where the two beams constructively interfere in the center with an intensity of approximately 2.5. The two outer fringes and the background are removed and displayed in the right plot. The array obtained from the right hand figure is used to calculate the fringe visibility as outlined in Equation (2.44)..... 24

Figure 4.1: The SI is plotted as a function of total propagation distance, where the phase screen is characterized by a Fried parameter of 1.42 m and an inner and outer scale size of 1 mm and 3 m respectively. The phase screen is located at a normalized propagation distance of 0.5 of the total propagation distance for each point. The beam width is held constant at 30 mm, where the Rayleigh range is approximately 5.5 km and the beam size in the absence of turbulence at 1, 3, and 5 km is 30.12, 31.04 and 32.80 mm respectively. The points along the line are the mean of 30 exposures of 50 frames as outlined in Chapter 3. The error bars represent the standard deviation from the mean of the 30 exposures..... 26

Figure 4.2: The SI is plotted as a function of phase screen location along the propagation path, where the transmit plane is located at the normalized location $z/L = 0$ and the target plane is at $z/L = 1$. The location of the phase screen is plotted for a range of distances given in the legend in kilometers. The beam width is held constant at 10 mm, the Fried parameter is a constant 1.421

m and the inner and outer scale is held at 1 mm and 3 m respectively. A maximum in the SI occurs for most propagation distances between 0.50 and 0.75 of the total propagation distance.

..... 27

Figure 4.3: The SI is plotted as a function of phase screen location along the propagation path, where the transmit plane is located at the normalized location $z/L = 0$ and the target plane is at $z/L = 1$. The location of the phase screen is plotted for a range of distances given in the legend in kilometers. The beam width is held constant at 30 mm, the Fried parameter is a constant 1.421 m and the inner and outer scale is held at 1 mm and 3 m respectively. A local maximum in the SI occurs for most propagation distances around 0.5 of the total propagation distance. 28

Figure 4.4: The SI is plotted as a function of beam width at varying propagation distances. The normalized location of the phase screen is 0.5, the Fried parameter is 1.42 m and the inner and outer scale is 1 mm and 3 m respectively. The propagation distance is indicated in the legend below the graph. The dips in the graph match expectations using the three criteria mentioned earlier and the assistance of Table 4.1. 31

Figure 4.5: The SI index is shown as a function of Fried parameter. The inner scale size is 1 mm, the beam width of 30 mm, and a normalized phase screen location of 0.5 of the total propagation distance of 1 km. Note the decrease in the SI as the phase screen is moved closer to the target. The normalized phase screen location is indicated by the legend and corresponds to what is shown in Figure 4.3 for a 30 mm beam. 32

Figure 4.6: The SI index is shown as a function of inner scale size, l_0 , as viewed in the legend located below the graphic. This relationship is shown for a variety of normalized phase screen locations as shown on the vertical axis where the total propagation distance is 1 km and the outer scale is 3 m. The beam width is 30 mm. The Fried parameter is also varied as seen in the legend. The graph clearly shows a decrease in the SI index with an increase in the inner scale. 33

Figure 5.1: Variation of FV as a function of distance of propagation. The two beams have a width of 30 mm and are transversely separated by a variety of distances as shown in the plot's legend. The

normalized phase screen location is 0.5 with a Fried parameter of 1.42 m and an inner scale size of 1 mm. 36

Figure 5.2: Variation of FV as a function of transverse beam separation for a variety of propagation distances as indicated in the legend in meters. The two beams have a 30 mm beam waist and travel through the phase screen located at half the total propagation distance. The Fried parameter is held constant at 1.42 m and the inner scale values is 1 mm. 36

Figure 5.3: The FV is plotted as a function of normalized phase screen location for a variety of propagation distances as shown in the legend (in meters). The two beams have a width of 30 mm and are transversely separated by 60 mm. The phase screen has a Fried parameter of 1.42 m and an inner and outer scale size of 1 mm and 3 m respectively..... 38

LIST OF TABLES

Table 4.1: The table includes the Rayleigh range z_R , the beam width $w_{L/2}$ (in the absence of turbulence) at $z = L/2$, and the Fresnel ratio β at $z = L/2$ for various propagation distances L and initial width w_0	30
--	----

LIST OF ABBREVIATIONS

FV	Fringe visibility
IWP	Initial wave profile
MVKS	Modified von Karman spectrum
PDE	Partial differential equation
PSD	Power spectral density
RMS	Root mean squared
SI	Scintillation index
SSBPM	Split-step propagation beam method
PPS	Phase power spectrum
FFT	Discrete Fast Fourier Transform
IFFT	Discrete Inverse Fast Fourier Transform

LIST OF SYMBOLS

(x, y)	Spatial domain variables
L	Total propagation distance
z	Propagation distance along path
Δz	Discrete portion of z
n_{int}	Positive integer equal to the number of sections
$\Psi_e(x, y; z)$	Initial wave profile
(k_x, k_y)	Spatial frequencies corresponding with x and y
k_0	Electromagnetic wavenumber
$\mathfrak{F}\{ \}, \mathfrak{F}^{-1}\{ \}$	Continuous Fourier transform, continuous inverse Fourier transform
$\hat{S}$	Inhomogeneous operator
Δn	Random index fluctuations
k_v	Kinematic viscosity
$\vec{l}$	Characteristic volume
v_{avg}	Average wind velocity
R_e	Reynolds number
$\vec{\kappa}$	Spatial wavenumber vector/ turbulence wavenumber vector
l_0	Inner scale length
κ_m	Modified inner-scale wavenumber cutoff $\kappa_m = \frac{5.92}{l_0}$
L_0	Outer scale length
κ_0	Outer scale wavenumber cutoff: $\kappa_0 = \frac{2\pi}{L_0}$
C_n^2	Structure constant
r_0	Fried parameter

$\Phi_p(\kappa)$	Phase power spectrum
$\mathfrak{a} + i\mathfrak{b}$	Complex random variable matrix
$\phi(x, y)$	Phase function
$\langle \rangle$	Ensemble average
$I(\mathbf{r}, L)$	Local irradiance of electric field
$\sigma^2(\mathbf{r}, L)$	Phase variance / Scintillation index
$\langle \chi^2 \rangle$	Logarithmic amplitude variations
$\delta\epsilon(\mathbf{r}, t)$	Dielectric variation
$\psi_1(\mathbf{R})$	First term of Rytov variance
$A(\mathbf{R}, \mathbf{r})$	Real part of scattering kernel
$E_1(\mathbf{R})$	Born-series term representing one scattering
$E_0(\mathbf{R})$	Unperturbed scalar field strength
$\mathbf{r}'$	Position vector defining a point in a random medium
t	Time in seconds or minutes
B_1	First term of Born approximation
$G(\mathbf{R}, \mathbf{r})$	Green's function
$B(\mathbf{R}, \mathbf{r})$	Imaginary part of the scattering kernel
$\langle \varphi^2 \rangle$	Phase variance
σ_R^2	Rytov variance
I	Intensity
I_{max}	Maximum intensity
I_{min}	Minimum intensity
$g(x, y)$	Simulated Gaussian beam
w_0	Beam waist
y_0	Separation between twin Gaussian beam centers
ξ	Normalized distance variable
χ	Real part of Born approximation/ logarithmic amplitude variance

φ	Imaginary part of Born approximation/ signal phase variance
ξ	Azimuthal angle
δ	Path length difference
d	Transverse beam separation distance
ω	Angular frequency
t	Time in seconds
ϕ	Phase difference
$\delta(x)$	Dirac delta function
γ	Coherence function
A	Amplitude of normalized electric field
β	Fresnel Ratio
w_L	Beam width at distance L
Δn	Variance of the refractive index
z_R	Rayleigh range
α	Complex beam parameter; multiplicative constant
R_0	Radius of curvature
$E(x, y; z)$	Electric field vector
$\hat{S}$	Inhomogeneous operator
Φ_n	Refractive index power spectrum
$\phi_{i,j}$	Random phase matrix
Φ_ε	Dielectric power spectrum
x_0	Half the physical distance between two slits/Gaussian beams
r_{sim}	Artificial Fried parameter value as stated in simulated model
Δh	Grid spacing
$\Delta \kappa$	Spatial frequency separation
$\Delta \kappa_x, \Delta \kappa_y$	Increments in spatial frequencies in x and y

CHAPTER 1

INTRODUCTION

1.1 Primary Goals

As the frontier of optics continues to expand there is an increased interest in horizontal propagation through atmospheric conditions using coherent laser sources. To this point, a great deal of work characterizes propagation of light through the atmosphere pioneered by such figures as Kolmogorov, Tartarski and Fried [1]. A better understanding of atmospheric effects on horizontal propagating optical beams will enable applications of free space optical communications, remote imaging, surveillance, etc. [1]. Power spectral density models have developed over the past half century to account for the effects of turbulence on electromagnetic propagation. Discussed later in this work are several models of atmospheric turbulence, along with the strength and weaknesses of each model.

A vast amount of work has been done regarding propagation through phase screens, however few are devoted to the effects that the modified von Karman spectrum (MVKS) has on laser beam propagation. This may be due to more simplified atmospheric models of the power spectral density such as the Kolmogorov spectrum and the von Karman spectrum, which are mathematically easier to use. The MVKS spectrum is chosen for its simplicity while still including both the inner and outer scales defining the inertial range for the turbulent eddies [1,2]. The outer scale influences the amplitude variance even in weak turbulence. It can affect the off-axis scintillation index (SI) for larger beams [3], but in our work the outer scale provides a filter at long spatial wavelengths [4].

In this thesis, we report simulation results on the effect of the MVKS using a single phase screen placed at different positions over the entire propagation distance. The motivation for our study is the use of phase screens to describe turbulence in a compact turbulence testbed [5]. Our theoretical calculations follow a widely used beam propagation algorithm [2,6]. In the simulations, we include the initial beam width, the propagation distance, and the location of the phase screen in the propagation path. Other

parameters include the inner scale size, Fried parameter, and the transverse separation distance for the two beam case. The phase fluctuations of the phase screen are parameterized by the Fried parameter, which describes the transverse coherence length, and the inner and outer scales determine the amount of aberration seen by the propagating beam. A simple equation connects the structure constant to the Fried parameter [1,2]. The scintillation index (SI) is computed for a range of values for each of these parameters.

While single beam and multi-beam propagation has been compared in the presence of a phase screen, this comparison was done only utilizing SI [7]. The SI of the single beam case is compared with the fringe visibility (FV) due to interference between a twin set of Gaussian beams. In both cases, a single phase screen is located along the propagation path causing fluctuations in the irradiance.

We gain exciting insight into the effects of the SI and how we can counteract the effects of turbulence over a given propagation distance by using the beam width, spacing of two in-phase Gaussian beams, and how we can utilize the SI and FV to determine where the turbulence is located along the propagation path. Should the results extend to extended turbulence, their usefulness would be profound in various free-space applications.

1.2 Thesis Outline

The second Chapter in this thesis explores key areas utilized in developing a simulated model. This discussion includes descriptions of Gaussian beams, a common beam propagation algorithm and different atmospheric models. The models of generating phase screens with a fixed atmospheric model using the Fried parameter and the effects of this choice are also detailed. Mathematical descriptions of the SI and FV are discussed along with their physical meanings.

A comprehensive description of the simulated model is included in Chapter 3. The model is applied to two cases: single beam and double beam. Each case undergoes a similar process of propagation to the target plane and differs in the processing of the wavefront at this plane. The results of the simulated model are reported in Chapters 4 and 5.

Chapter 4 details the results of the single beam simulation. Here, we examine the effects of various parameters on the SI at the target plane and discuss the reasoning behind what is observed. It is expected the total propagation distance, the normalized phase screen location, initial beam width, Fried parameter and inner scale will have visible effects on the SI.

Chapter 5 details the results of the double beam simulation. In this case, we explore the effects of various parameters on the FV at the target plane and discuss the reasoning behind what is observed in the simulated data. It is hypothesized the FV results will be inversely proportional to the single beam SI results. This stems from the differing definitions where a high SI indicates a high level of atmospheric fluctuations and a high FV indicates strong coherence between the two beams (less phase fluctuations). The parameters examined in this Chapter are total propagation distance, normalized phase screen location, and the transverse separation distance between the two beams.

Finally, in Chapter 6, comparisons between the single beam SI index results and the double beam FV results are discussed. This includes a discussion of the implications of the results if extended to extended turbulence models. Lastly, ideas are suggested to further expand the work.

CHAPTER 2

BACKGROUND AND THEORY

This Chapter summarizes the necessary background of a variety of key concepts and considerations used in the development of simulation models. The eight key areas discussed include: beams, the split-step propagation method, power spectral density models, modeling turbulence with phase screens, the relationship to the scintillation index and Rytov variance, the structure constant and Fried parameter, two beam interference, and fringe visibility. Each concept accumulates to a greater understanding of basic principles governing this work.

2.1 A Mathematical Description of Gaussian Beams

It is common to use a Gaussian beam to represent laser beams. The most basic beam is the TEM_{00} with a parabolic phase distribution at the transmitter located at $z = 0$. The beam profile can be represented as

$$E_0(r, z) = \mathcal{E}_0 e^{\left\{ -r^2 \left(\frac{1}{w_0^2} + i \frac{k_0}{2R_0} \right) \right\}}, \quad (2.1)$$

with an initial radius of w_0 , and an initial radius of curvature of R_0 . The on-axis initial amplitude is $\mathcal{E}_0$, r is the radial distance from on-axis center, $k_0 = 2\pi/\lambda$ is the vacuum electromagnetic wavenumber where λ is the wavelength. As the beam leaves the transmitter, the field is subject to the effect of diffraction during propagation [4]. The field strength after a distance z of propagation is

$$E_0(r, z) = \frac{\mathcal{E}_0}{1 + i\alpha z} e^{\left\{ ik_0 z - \frac{1}{2} k_0 \alpha \frac{r^2}{1 + i\alpha z} \right\}} \quad (2.2)$$

where α is defined as

$$\alpha = \alpha_1 + i\alpha_2 = \frac{\lambda}{\pi w_0^2} + i \frac{1}{R_0}. \quad (2.3)$$

If $R_0 = \infty$, this expression is valid for distances within a vacuum of

$$\frac{1}{\alpha} = z_R. \quad (2.4)$$

where z_R is the Rayleigh range of the Gaussian beam. At the Rayleigh range, the beam width is $\sqrt{2}$ times the initial width w_0 , which is commonly referred to as the beam waist. For a 30 mm beam waist and a wavelength of 0.50 micron, this implies that the Rayleigh range is about 5.5 km. In our simulations, we have propagated the Gaussian beam through distances ranging from zero to $1.2z_R$.

In general, the irradiance of a beam can be expressed as

$$I_0(r, z) \propto |E_0(r, z)|^2 = I_0 \left(\frac{w_0}{w(z)} \right)^2 e^{\left\{ -\frac{2r^2}{w^2(z)} \right\}} \quad (2.5)$$

where the width of the beam as a function of distance, $w(z)$, is related to w_0 in the following way [4]

$$w^2(z) = w_0^2 \left[\left(\frac{z\lambda}{\pi w_0^2} \right)^2 + \left(1 - \frac{z}{R_0} \right)^2 \right]. \quad (2.6)$$

2.2 The Split-Step Beam Propagation Method

The key ideas behind the split-step beam propagation method (SSBPM) are two-fold. First, the total expected propagation distance is sectioned into smaller portions. Secondly, the linear proportional processes are performed in the spatial frequency domain or k -space, whereas the nonlinearities and random processes are carried out in the space domain (x, y) [5]. This method enables diffraction, and the effects of an inhomogeneous medium caused by turbulence, to be separately treated in the same algorithm. The most basic form of the split-step propagation method is accurate to the first order. The simulations described later utilize a method that is accurate to the second order. Outlined below are the first and second order methods.

The SSBPM is quite conducive to the use of multiple phase screens. To realize this, the total propagation distance, L , is sectioned off into smaller distances $\Delta z = \frac{L}{n_{int}}$ where n_{int} is a positive integer equal to the number of desired phase screens. Multiple phase screens can be placed along the propagation path because they accurately mimic extended turbulence [2].

In the first order SSBPM, the profile $E(x, y; z)$ at an arbitrary plane z is Fourier transformed, and the angular plane wave spectrum is multiplied by the transfer function of propagation in the spatial frequency domain (k_x, k_y) , to incorporate the effects of diffraction:

$$\tilde{E}(k_x, k_y; z + \Delta z) = \mathfrak{F}\{E(x, y; z)\} e^{\frac{i(k_x^2 + k_y^2)\Delta z}{2k_0}}. \quad (2.7)$$

The inverse Fourier transform returns the spectrum to the space domain, where the inhomogeneous operator multiplies it:

$$E(x, y; z + \Delta z) = \mathfrak{F}^{-1}\{\tilde{E}(k_x, k_y; z + \Delta z)\} e^{-i\hat{S}\Delta z}, \quad (2.8)$$

where

$$\hat{S} = \Delta n k_0. \quad (2.9)$$

This operator $e^{-i\Delta z \hat{S}}$ accounts for the random nature of the phase screen where Δn is the change in the index of refraction from a mean value, n . The profile propagates through each slice of z until the total length of propagation is reached.

The second order SSBPM is similar to the first order SSBPM in that in both cases, the homogenous (diffraction) effects are incorporated in the spatial frequency domain, while inhomogeneous effects are incorporated in the spatial domain. While the first order accuracy takes account for the entire Δz and then the effects of the phase screen, the second order method splits the diffraction effects into two sections of length $\frac{\Delta z}{2}$ that sandwich the phase screen step. This has been shown to increase the accuracy of the SSBPM [5].

As stated above, the second order SSBPM again begins with the profile, $E(x, y; z)$. This undergoes a Fourier transform and is multiplied by the linear effects of diffraction through a distance $\frac{\Delta z}{2}$ in the spatial frequency domain:

$$\tilde{E}\left(k_x, k_y; z + \frac{\Delta z}{2}\right) = \mathfrak{F}\{E(x, y; z)\} e^{\frac{i(k_x^2 + k_y^2)(\frac{\Delta z}{2})}{2k_0}}. \quad (2.10)$$

The inverse Fourier transform returns the spectrum to the space domain where the inhomogeneous operator as multiplies it:

$$E'(x, y; z + \frac{\Delta z}{2}) = \mathfrak{F}^{-1} \left\{ \tilde{E} \left(k_x, k_y; z + \frac{\Delta z}{2} \right) \right\} e^{-i\delta \Delta z}. \quad (2.11)$$

Another Fourier transform returns the beam profile to the spatial frequency domain to account for the remaining effects of diffraction and the inverse Fourier transform returns it back to the space domain:

$$E(x, y; z + \Delta z) = \mathfrak{F}^{-1} \left\{ \mathfrak{F} \left\{ E' \left(x, y; z + \frac{\Delta z}{2} \right) \right\} e^{\frac{i(k_x^2 + k_y^2)(\frac{\Delta z}{2})}{2k_0}} \right\}. \quad (2.12)$$

The process repeats for each slice of the total distance of propagation until the initial profile successfully propagates through the total length.

2.3 Power Spectral Density Models and the Modified von Karman Spectrum

A phenomenon commonly known as (optical) turbulence occurs due to local temperature and humidity fluctuations which result in fluctuations of the atmospheric refractive index [6]. The local random fluctuations result in large inhomogeneous pockets of air that are further broken into smaller homogeneous and isotropic pockets, known as eddies, by fluctuating wind velocities [1]. A good indicator of fluid motion, stable or unstable, is the Reynolds number.

If air is treated as a fluid with a kinematic viscosity of $k_v = 1.5 \times 10^{-5} \frac{m^2}{s}$, and average wind velocity of $v_{avg} = 1 \frac{m}{s}$ (~2.2 mph) and a scale size of the characteristic length of $l = 10m$, the Reynolds number mathematically described as [1]:

$$R_e = \frac{v_{avg} l}{k_v}, \quad (2.13)$$

yields a value $R_e \sim 6.7 \times 10^5$. For this high value of R_e , random processes govern atmospheric turbulence. Therefore, in most cases, the atmosphere is considered (optically) turbulent. Typically, Reynolds numbers greater than 2500 to 5000 imply turbulent flow [7]. The power spectral density is used to describe the size, number, and distribution of these eddies [1]. The atmosphere is typically modeled in terms of a power spectral density (PSD); the most common models are Kolmogorov, Tatarskii, and the von Karman [1].

In this simulation, the modified von Karman spectrum (MVKS) has been used and is described by

$$\Phi_n(\kappa) = \frac{0.033 C_n^2}{(\kappa^2 + \kappa_0^2)^{11/6}} e^{-\left\{ \frac{\kappa^2}{\kappa_m^2} \right\}}, \quad (2.14)$$

where $\kappa = \sqrt{\kappa_x^2 + \kappa_y^2}$ is the spatial wavenumber. The inner scale, l_0 , is inversely proportional to κ_m . Similarly the outer scale, L_0 , is inversely proportional to κ_0 . This model is chosen because it includes the inner and outer scale factors used to describe the size of turbulent eddies [2,8]. The strength of turbulence is contained within a structure constant, C_n^2 , and typical values are approximately $10^{-17} \text{ m}^{-2/3}$ for weak turbulence to $10^{-13} \text{ m}^{-2/3}$ for cases of strong turbulence [1,7].

The rationale for using the MVKS in this thesis is outlined below. The simplest of the models for turbulence, known as the Kolmogorov spectrum [1,2,4,6], is

$$\Phi_n(\kappa) = 0.033 C_n^2 \kappa^{-\frac{11}{3}}. \quad (2.15)$$

However, this model has numerous drawbacks. First, the Kolmogorov spectrum does not take into account the inner or outer scale resulting in inaccuracies when the wavenumber, κ , is less than $2\pi/l_0$. It also is inconsistent with physical observations for large wavenumbers [1]. Furthermore, mathematically, Equation (2.15) suffers from a non-integrable pole at $\kappa = 0$.

The Tartarskii spectrum accounts for the inner scale parameter and has the form [2]:

$$\Phi_n(\kappa) = 0.033 C_n^2 \kappa^{-\frac{11}{3}} e^{-\frac{\kappa^2}{\kappa_m^2}}. \quad (2.16)$$

This spectrum suffers from some of the same complications of the Kolmogorov spectrum. It cannot be integrated at $\kappa = 0$ and the covariance cannot be calculated utilizing this spectrum [2]. The von Karman spectrum accounts for the outer scale with the following form [4]:

$$\Phi_n(\kappa) = \frac{0.033 C_n^2}{(\kappa^2 + \kappa_0^2)^{11/6}}. \quad (2.17)$$

Combining the von Karman and the Tartarskii spectrums, the MVKS [1,2] is obtained and performs closer to that physically observed experimentally. Additional models such as the Modified Atmospheric spectrum also exist in the literature [2]. The Modified Atmospheric model has not been considered since it is more complicated than the MVKS.

2.4 Modeling Atmospheric Turbulence with a Thin Phase Screen

The purpose of a phase screen is to simulate the random phase perturbations resulting from by random index fluctuations, Δn , in extended atmospheric turbulence [1,3]. A phase screen is considered “thin” if its thickness is considerably smaller than the Rayleigh range of the Gaussian beam. Our

simulations can incorporate one or more phase screens, but the general method for generating distinct phase screens remains the same. Each transverse “point” along the wavefront sees a random phase perturbation, which is different from an adjacent point. Following the Equation (2.19), the phase screen is generated by starting from using a complex random set of spatial frequencies with a Gaussian distribution, and a phase power spectrum (PPS) [1]. The random numbers generated to have zero mean and unit variance, a process that is easily accomplished with common pseudo-random number generators in any standard programming language. Now, the PPS is related to the index power spectrum in Equation (2.14) through the Fried parameter, r_0 , as [1]

$$\Phi_p(\kappa) = \frac{0.23r_0^{-\frac{5}{3}}}{(\kappa^2 + \kappa_0^2)^{11/6}} e^{-\left\{\frac{\kappa^2}{\kappa_m^2}\right\}}. \quad (2.18)$$

The square-root of the PPS is multiplied by a complex random variable matrix, $\mathfrak{a} + i\mathfrak{b}$, and inverse Fourier transformed. The real part of this result is taken to be the random phase function, $\phi(x, y)$, due to atmospheric fluctuations. The phase screen is then a randomly distributed matrix generated by

$$\phi_{ij} = \text{Re} \left\{ IFFT_{ij,kl} \left((\mathfrak{a}_{kl} + i\mathfrak{b}_{kl}) \sqrt{\Phi_{pkl} \Delta\kappa_x \Delta\kappa_y} \right) \right\}, \quad (2.19)$$

where $IFFT$ denotes the inverse fast Fourier transform and $\Delta\kappa_x, \Delta\kappa_y$ denote increments in spatial frequencies in x and y . If multiple phase screens are desired this process is repeated with multiple matrices of complex random variables, each unique to every phase screen.

For application of the SSBPM, it is instructive to note, upon comparison with Equation (2.9) that

$$\hat{S}\Delta z = n_0 k_0 \Delta z = \phi(x, y). \quad (2.20)$$

Thus the inhomogenous operator easily relates to random phase fluctuations as in Equation (2.19) induced by the phase screen.

2.5 Numerical Determination of the Scintillation Index of Weak Turbulence Utilizing the Rytov

Variance

For free space optical communications it is important to understand the effects the atmosphere has on propagating beams, because the atmosphere creates beam wander and scintillation. Beam fluctuations lead to signal fading and bit error rates that may severely impair the system performance. One common

metric used to measure intensity fluctuations caused by atmospheric turbulence is the scintillation index. The scintillation index is the “normalized variance of irradiance” [2]. From experimental data or numerical calculations, this value can be obtained. Theoretical calculations require knowledge of the fourth-order cross-coherence function [2,4,8]. Mathematically defined, the scintillation index, SI, is

$$\sigma^2(r, L) = \frac{\langle I^2(\mathbf{r}, L) \rangle}{\langle I(\mathbf{r}, L) \rangle^2} - 1 \quad (2.21)$$

where $\langle \rangle$ denotes the ensemble average, $I(\mathbf{r}, L)$ is the local irradiance, L is the propagation distance of the beam, and $\mathbf{r}$ is the radial distance from the symmetry axis [2]. The turbulence is weak if the SI is less than unity.

The variation of the irradiance may also be described by the Born approximation which accounts for the effects of scattering; however this approximation is insufficient and inaccurate for millimeter, infrared and optical wavelengths [8]. The Rytov approximation is a modification of the Born approximation with weaker constraints on the phase term. The Rytov approximation describes the effects of scattering as well, and is used to calculate the theoretical expectations of the variance of the irradiance [2,4]. The Rytov variance may sometimes be used in place of the SI in cases of weak turbulence, thus simplifying theoretical calculations if the variance of the logarithm of the amplitude is less than unity [2,4].

To see this in more detail, readers are reminded that the objective is to solve the Helmholtz equation

$$\nabla^2 E + k_0^2 [1 + \delta\epsilon(\mathbf{r}, t)] E = 0 \quad (2.22)$$

where $\delta\epsilon(\vec{r}, t)$ is the random dielectric variation and E is the electric field [4]. This is achieved by defining a surrogate function that must meet certain criteria. First, it must satisfy a partial differential equation. Secondly, the dielectric variations must “appear” as an additional source term [4]. Finally, the function must account for a complex phase and amplitude. The resulting surrogate function that satisfies these conditions is

$$\psi_1(\mathbf{R}) = B_1 = -k_0^2 \int d^3r \mathbb{G}(\mathbf{R}, \mathbf{r}) \delta\epsilon(\mathbf{r}, t) \frac{E_0(\mathbf{r})}{E_0(\mathbf{R})} \quad (2.23)$$

where $\mathbb{G}(\mathbf{R}, \mathbf{r}) \frac{E_0(\mathbf{r})}{E_0(\mathbf{R})} = A(\mathbf{R}, \mathbf{r}) + iB(\mathbf{R}, \mathbf{r})$ is the first term of the separated complex Born approximation and $\mathbb{G}$ is Green's Function [4] and

$$B_1 = \chi + i\varphi \quad (2.24)$$

is the rewritten first term of the Born approximation [4]. The unperturbed scalar field strength is given by $E_0(\mathbf{r})$. The position vector defining a point is represented as $\mathbf{r}$, and $\mathbf{R}$ is the vector position of the receiver. From this, the basic Rytov approximation can be written as:

$$E_1(\mathbf{R}) = E_0(\mathbf{R}) \exp \left\{ -k_0^2 \int d^3r \mathbb{G}(\mathbf{R}, \mathbf{r}) \delta\varepsilon(\mathbf{r}, t) \frac{E_0(\mathbf{r})}{E_0(\mathbf{R})} \right\} = E_0(\mathbf{R}) \exp(B_1). \quad (2.25)$$

This approximation satisfies the Helmholtz Equation with a random source term [4]. From the scattering integral, the phase expression results from the imaginary portion whereas the real part yields the logarithmic signal amplitude as shown in Equation (2.25). The signal phase is

$$\varphi(\vec{R}) = -k_0^2 \int d^3r \delta\varepsilon(\mathbf{r}) \Im \left\{ \mathbb{G}(\mathbf{R}, \mathbf{r}) \frac{E_0(\mathbf{r})}{E_0(\mathbf{R})} \right\}, \quad (2.26)$$

and the logarithmic amplitude is measured as:

$$\chi = \log \left(\frac{A}{E_0} \right) = -k_0^2 \int d^3r \delta\varepsilon(\vec{r}) \left(\text{Re} \left\{ \mathbb{G}(\vec{R}, \vec{r}) \frac{E_0(\vec{r})}{E_0(\vec{R})} \right\} \right), \quad (2.27)$$

where A is the real portion of the scattering kernel. From basic Rytov approximation the phase variance $\langle \varphi^2 \rangle$ is given by

$$\langle \varphi^2 \rangle = k_0^4 \int d^3r B(\vec{R}, \vec{r}) \int d^3r' B(\vec{R}, \vec{r}') \langle \delta\varepsilon(\vec{r}, t) \delta\varepsilon(\vec{r}', t) \rangle, \quad (2.28)$$

where the imaginary part of the scattering kernel is $B(\vec{R}, \vec{r}')$, and the real part is given by $A(\vec{R}, \vec{r}')$. The logarithmic amplitude variance $\langle \chi^2 \rangle$ is

$$\langle \chi^2 \rangle = k_0^4 \int d^3r A(\vec{R}, \vec{r}) \int d^3r' A(\vec{R}, \vec{r}') \langle \delta\varepsilon(\vec{r}, t) \delta\varepsilon(\vec{r}', t) \rangle. \quad (2.29)$$

As atmospheric turbulence is not homogenous, and the spatial covariance of $\delta\varepsilon$ is only valid for homogenous random media, we must revisit our descriptions of the phase variance and the logarithmic amplitude variance. The turbulent spectrum now depends on the average position of two points $(\vec{r}_1, \vec{r}_2)$, and the spatial covariance rewritten is

$$\langle \delta \varepsilon(\vec{r}_1) \delta \varepsilon(\vec{r}_2) \rangle = \int d^3 \kappa \Phi_\varepsilon \left(\kappa, \frac{\vec{r}_1 + \vec{r}_2}{2} \right) e^{i \kappa \cdot (\vec{r}_1 - \vec{r}_2)}. \quad (2.30)$$

The power spectrum is noted as $\Phi_\varepsilon(\kappa)$, and $\Phi_n(\kappa)$ specifically refers to the power spectrum within the troposphere. The logarithmic amplitude variance is also dependent on the average position of two points and the spatial covariance must be written as the wave number integral [4]. Henceforth, the logarithmic amplitude variance rewritten is a nine-fold integral that includes the power spectral density $\Phi_n(\kappa)$:

$$\langle \chi^2 \rangle = k_0^4 \int d^3 \kappa \int d^3 r A(\vec{R}, \vec{r}) \exp\{i \kappa \cdot \vec{r}\} \int d^3 r' A(\vec{R}, \vec{r}') \exp\{-i \kappa \cdot \vec{r}'\} \Phi_n(\kappa). \quad (2.31)$$

The logarithmic amplitude variance becomes a nine-fold integral that cannot be decoupled. As a result, other options are explored to circumvent this problem. Equation (2.26) expands from its basic form to a more usable form. The more basic form of the Rytov variance for a plane wave is [3,8]

$$\sigma_R^2 = \int_0^\infty d\kappa \int_0^1 d\xi \kappa \left(1 - \cos \left[\frac{L \kappa^2 \xi}{k_0} \right] \right) \Phi_n(\kappa). \quad (2.32)$$

2.6 A Discussion Concerning the Interchangeability of the Structure Constant and Fried Parameter

The model discussed in subsequent Sections relies heavily upon the Fried parameter, r_0 . The Fried parameter and structure constant are intricately related; however, the structure constant is more frequently used. Thus, it is important not only to discuss the relation of the parameters, but the manner in which they affect results obtained in simulated models. The structure constant, C_n^2 , describes the index of refraction fluctuations and is used to characterize the strength of these fluctuations in the atmosphere [1]. The Fried parameter describes the transverse distance over which the rms wavefront aberration is less than 1 radian. The relationship connecting the structure constant to the Fried Parameter for extended turbulence is [1]:

$$r_0 = 0.185 \left(\frac{32\pi^2}{3 k_0^2 L C_n^2} \right)^{\frac{3}{5}} \quad (2.33)$$

for a spherical wave. While the structure constant is well suited for describing turbulence in an extended medium, this concept does not translate well into the physical manifestation of atmospheric turbulence of fixed phase screen plates. Investigations into the design and construction of physical phase screens reveal

common use of the Fried parameter as a way to characterize turbulence by these means [9]. The pre-dominant form of the Fried parameter is thus used in our simulation model.

It is vital to acknowledge the implications of using the Fried parameter, and understand how the results will differ from extended turbulence results. A phase screen, with a set Fried parameter, placed along the propagation path with a set distance encapsulates the structure constant and the extended turbulence model. Andrews and Philips go into detail regarding this model of phase screens [2,3,8]. This however, is not the purpose of this work. It is assumed in this thesis that a variety of phase screens, with different constant Fried parameters, placed in a wide variety of locations along the total propagation path for a wide variety of propagation distances can adequately model extended turbulence and is suitable for laboratory implementations. As a result, the level aberration within a radian will remain constant but the turbulence will be subject to fluctuations and a set C_n^2 value will be impossible to maintain. An extended distance results in a greater SI, but the value for the structure constant will decrease implying weaker index of refraction fluctuations as a whole.

Another consideration is the effects of the Fried parameter and the structure constant on a varying Gaussian beam width. Equation (2.33) has no obvious connection to the beam width. The ratio of the beam width and r_0 is important contributor to beam jitter [2]. This connection is explored by Andrews and Phillips in Ref. [2].

2.7 Mathematical Description of Two-Beam Interference

Figure 2.1: Two slits separated by a distance d interfere on a screen located a distance L from slits. The slits produce an interference pattern due to phase differences δ caused by path length differences. Point P results from constructive interference.

When a random phase screen interrupts the spatial coherency of beams, it is easy to model the effects up to the plane of the screen in a variety of ways. For simplicity, if each beam propagates without a phase screen into the Fraunhofer regime, it will produce a result similar to that obtained by Young's double slit experiment. Indeed, the same is true for propagation of the beams up to the phase screen. Using Figure 2.1 as a guide, the separation of the slits causes a path difference of

$$\delta = r_1 - r_2 = d \sin \theta \quad (2.34)$$

and determines if the two waves will arrive in phase at a point P. Expressing this difference in the form of the electric field of each wave [10]:

$$\begin{aligned} E_1 &= E_0 \sin(\omega t) \\ E_2 &= E_0 \sin(\omega t - \phi) \end{aligned} \quad (2.35)$$

where the phase difference, ϕ , is dependent upon the path difference, Equation (2.35) the amplitude is E_0 , and ω is the angular frequency. The phase difference and path difference relate in the following expression:

$$\phi = \frac{2\pi}{\lambda} \delta. \quad (2.36)$$

From here, the superposition principle is applied to Equation (2.36) yielding the resultant electric field at point P as

$$E_p = 2E_0 \cos\left(\frac{\phi}{2}\right) \sin\left(\omega t - \frac{\phi}{2}\right). \quad (2.37)$$

The intensity is easily expressed at point P as

$$I \propto E_p^2 = 4E_0^2 \cos^2\left(\frac{\phi}{2}\right) \sin^2\left(\omega t - \frac{\phi}{2}\right). \quad (2.38)$$

We replace Equation (2.38) with a time-averaged value for the intensity and substituting for ϕ to arrive at a final approximation of the intensity as [10]

$$I = I_{max} \cos^2\left(\frac{\pi d \sin(\theta)}{\lambda}\right), \quad (2.39)$$

or for the small angle approximation

$$I = I_{max} \cos^2\left(\frac{\pi d}{L \lambda} y\right). \quad (2.40)$$

The simple plane wave optics based approach may be readily extended to accommodate two Gaussian beams; however utilizing Fourier optics enables a more simplified procedure. For two identical Gaussian beams separated by a given distance the composite profile can be written as

$$E_{intial}(x) = e^{-ax^2} [\delta(x + x_0) + \delta(x - x_0)] \quad (2.41)$$

where $\delta(x \pm x_0)$ denotes a pair of transversely displaced Dirac delta functions and where $x_0 = d/2$ is half the distance of separation of the two beams. Equation (2.41) undergoes a Fourier transformation, and in the far field the optical field becomes

$$E_{far}(x) = \sqrt{\frac{2}{\pi}} e^{-ax_0^2} \cos(x_0 \kappa), \quad (2.42)$$

which implies that a fringe pattern is easily observed. The intensity, which is the square of the magnitude of the electric field is given as

$$I = \frac{2}{\pi} \cos^2(x_0 \kappa). \quad (2.43)$$

The above models do well in approximating the intensity pattern in the absence of turbulence. Thus, this approximation is valid up to the first phase screen. Unfortunately, the phase screen is expected

to cause phase distortions to the optical fields and interrupt the spatial coherence of the beams. After passing through the phase screen the beams are expected to become partially coherent, which should deteriorate the fringe visibility (FV), to be defined below.

2.8 Measuring Fringe Visibility

Interferometry has a rich history in astronomy and provides a good understanding of beam interference. Conceptually, the idea of (FV) comes from the superposition of two plane waves, one from each arm of a Michelson interferometer [11]. As explained in the previous Section, the path length differences result in phase differences that manifest themselves as light and dark areas where constructive and destructive interference has occurred.

For two identical Gaussian beams, separated by a given transverse spatial distance, the beams constructively and destructively interfere resulting in fringes as the beams propagate to the far field. The contrast and quality of the fringes is generally a measure of optical coherence and defined with a metric commonly referred to as FV. FV, mathematically defined in terms of the intensity I on the observation plane, is

$$FV = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}. \quad (2.44)$$

The FV is closely related to the coherence function (γ), which “relates the wave fields received at two neighboring points in space,” [11]. For two complex fields $E(\mathbf{r}, t)$ the coherence function is

$$\gamma(\mathbf{r}_1, \mathbf{r}_2) = \frac{\langle E(\mathbf{r}_1, t)E^*(\mathbf{r}_2, t) \rangle}{[\langle |E(\mathbf{r}_1, t)|^2 \rangle \langle |E(\mathbf{r}_2, t)|^2 \rangle]^{\frac{1}{2}}}. \quad (2.45)$$

For mean intensities of A_1^2 and A_2^2 and normalized complex fields the coherence function becomes

$$\gamma = \frac{\langle A_1 E(\mathbf{r}_1) \cdot A_2 E^*(\mathbf{r}_2) \rangle}{A_1 A_2} = \langle E(\mathbf{r}_1) E^*(\mathbf{r}_2) \rangle. \quad (2.46)$$

The phase difference, ϕ , varies within the fringe pattern as

$$g(\phi) = A_1 E(\mathbf{r}_1) + A_2 E(\mathbf{r}_2) e^{i\phi} \quad (2.47)$$

and the intensity is

$$\begin{aligned}
I(\phi) &= |g(\phi)|^2 = A_1^2 |E(\mathbf{r}_1)|^2 + A_2^2 |E(\mathbf{r}_2)|^2 \\
&+ A_1 A_2 [|E(\mathbf{r}_1)E^*(\mathbf{r}_2)|e^{i\phi} + |E^*(\mathbf{r}_1)E(\mathbf{r}_2)|e^{-i\phi}].
\end{aligned} \tag{2.48}$$

With time averaging, and substituting the maximum and minimum of the intensity into Equation (2.44) the FV is

$$FV = \frac{2A_1 A_2}{A_1^2 A_2^2} \cdot |\gamma(\mathbf{r}_1, \mathbf{r}_2)|, \tag{2.49}$$

showing a clear connection between coherence and FV [11].

This Chapter provides fundamental basics used in gathering and explaining phenomenon to be included in later chapters. Further concepts are discussed as necessary in succeeding chapters. In the following Chapter, all key concepts developed here are applied to the development of a model for single beam and multi-beam transmission into the far-field through a single phase screen. Two independent, but similar, models are developed for each case.

CHAPTER 3

DEVELOPING A SIMULATION MODEL

3.1 Introduction

In order to model a round trip pass of a single Gaussian beam through a single phase screen and free space, it is important to understand the effects of three key components. These components are forward propagation from the transmitter to the target, the target surface, and backward propagation from the target to the receiver. Lastly, the received signal must be processed to gather data of interest. Each component contributes to the overall understanding of the consequences of a turbulent atmosphere modeled in this thesis by the MVKS. Our simulated model explores the first component of this process in detail.

The understanding of the effects of the atmosphere on forward propagating beams facilitates the understanding of target illumination. Forward propagation theory also lends to beneficial insight in a large number of applications involving transmit/receive where back propagation is not used or unnecessary. Furthermore, understanding the effects of forward beam propagation, i.e. transmitting, is easily transformable to backward beam propagation, i.e. reflection. Studying forward propagation can therefore greatly aid in the understanding of a transmit-reflect-receive system.

As stated above, the goal of this work is to investigate in detail the effects of forward propagation. The SI and FV simulation data is collected for a range of values of various parameters, such as beam width, strength of the phase screen, characteristics lengths defining the phase screen, propagation distance, phase screen location, etc.. The atmospheric turbulence is modeled as a single phase screen exhibiting the MVKS throughout or computer simulations. The theoretical calculation for forward propagation follow a model outlined by Andrews and Phillips for the Kolmogorov spectrum [2,3]. The phase screen is characterized by the Fried parameter, which is related to the structure constant via Equation (2.32) [1,2]. The SI values

obtained from the simulations are compared with expected values of the numerically integrated Rytov variance for the given set of parameters.

Simulation results on the effect of the MVKS using a single phase screen placed at different positions over the entire propagation distance are presented and discussed in Chapters 4 and 5. The motivation for studying forward propagation using discrete phase screens is to compare simulations with experimental data for laser propagation in a compact turbulence testbed [5], as shown schematically in Figure 3.1.

Figure 3.1: Schematic showing the transmit plane, phase screen plane and the target plane. A Gaussian beam transmits at the transmit plane a distance d_1 to a phase screen. The phase screen introduces phase variations that result in scintillation at the target plane a distance d_2 from the phase screen. The ratio of d/L is used to describe the normalized phase screen location.

The theoretical calculations follow SSBPM algorithm [2,3]. The simulations include the initial beam width and curvature, the propagation distance, and the location of the phase screen in the propagation path. The phase fluctuations of the phase screen are parameterized by the Fried parameter, which describes the transverse coherence length, and the inner and outer scales.

3.2 Modeling Single and Double Beam Forward Propagation through a Single Phase Screen

In the previous chapter, we have discussed important fundamentals in simulating forward propagation through phase screens. These concepts are used within MATLAB programs, listed in the Appendix, which models this phenomenon. The processes within the program are broken down into four

categories: declaration of all relevant parameters; definition and use of the power spectral density to generate turbulence; Gaussian beam propagation, before, through, and after the phase screen; and post processing for specified results, i.e. SI and FV. Additional features have been incorporated as needed to create figures and movies.

Figure 3.2: Spatial frequency profile for Gaussian beam of initial waist of 30 mm

First, a single Gaussian beam, using a MATLAB script included in Appendix A, has been propagated in a transverse domain of physical size of 750 mm x 750 mm sampled 1024 points along each axis. The wavelength, λ , is constant in all simulations. The spatial frequency map is shown in Figure 3.2. The spatial frequency corresponding to a 30 mm dimension is of the order of 0.2/mm. The phase screen is governed by four primary principles: inner scale parameter, outer scale parameter, Fried parameter, and the PPS constant. We have determined that the outer scale has little effect on the on-axis SI and thus held constant at 3 meters in our simulations, while the inner scale is varied about a nominal value of 1 mm. The Fried parameter as stated in the program is not the “true” value. MATLAB’s two-dimensional FFT algorithm is done discretely and, therefore, dimensionless. As a result, the true Fried parameter must be derived. If the simulated value is r_{sim} , the grid spacing is Δh , and the spatial frequency separation $\Delta\kappa = 2\pi/\Delta h$, the true value of the Fried parameter can be found from

$$r_0 = \frac{r_{sim}}{\lambda^5 (\Delta\kappa)^5} \quad (3.1)$$

A simulation value of $r_{sim} = 0.0002$ thus results in an approximate value of $r_0 = 1.42$ m for the Fried parameter for a grid size of 750 mm and a wavelength of 0.5 microns. The spectrum used in simulation has a PSD constant of 0.033 when using the structure constant C_n^2 . This value differs when the PSD is translated into the PPS and Fried parameter terms as a result of this relationship outlined by Equation (2.32). This value is varied to account for different phase screen widths and a nominal value of 0.23 is used in simulations.

The Gaussian beam is constructed initially have a plane wavefront and is represented by

$$g(x, y) = e^{\frac{-(x^2+y^2)}{w_0^2}} \quad (3.2)$$

where w_0 denotes the beam waist. Beam separation is an important parameter for the two beam case used to determine the FV, and has a minimum value twice that of the beam width of the largest beam. The phase curvature and beam widths are taken to be identical for the two beam case. The two beam case is represented as the sum of two single Gaussian beams:

$$g(x, y) = g_1(x, y) + g_2(x, y); \quad g_{1,2}(x, y) = e^{\frac{(x^2+(y \pm y_0)^2)}{w_0^2}}. \quad (3.3)$$

Beam propagation for both the single and double beam cases follows the basic algorithm outlined by the second order SSBPM. As only one phase screen is used, the total propagation distance is separated into two sections, d_1 and d_2 as seen in Figure 3.1. The phase screen's location is determined by the length of section d_1 . After propagating a total distance of L data i.e. the FV and SI are calculated.

We define a “frame” as a single iteration of the SSBPM for a unique phase screen and an “exposure” is defined as a time average of a set number of frames. Examples of a frame and exposure for the single and double beam cases are shown in Figures 3.3 and 3.4, respectively. Each frame involves a different phase screen resulting from the same PPS and a constant Fried parameter. The differing phase screens arise due to a unique draw of complex random variables with a zero mean and unit variance for each frame [1]. The intensity profile corresponding to an exposure is derived by adding the single frame intensities pixel by pixel at the target plane.

Figure 3.3: A typical single Gaussian beam is transmitted through a phase screen placed at half the propagation distance of 1 km. The beam waist is 30 mm. Both images are shown on a logarithmic scale. The left-hand image shows a single exposure of fifty unique frame realizations. The right-hand image is a single frame realization contributing to the exposure on the left-hand side.

Figure 3.4: Double beam interference through a phase screen placed at half the propagation distance of 4 km. Each beam has a beam width of 30 mm and are transversely separated by 70 mm. The interference pattern is observed at the target plane. Both images are shown on a logarithmic scale. The left-hand image shows a single exposure of fifty unique frame realizations and is digitally enhanced to clearly show the fringe visibility that is otherwise washed out. The right-hand image is a single frame realization contributing to the exposure on the left-hand side.

The reported scintillation index (SI) is determined from the mean of thirty exposures with set values for all changeable variables. The SI is first calculated in the absence of the phase screen and the effect of the diffraction alone on the SI is expected and seen to be several orders of magnitude smaller than the atmospheric phase fluctuations and thus is considered negligible. The effects of diffraction on SI are shown in Figure 3.5. As is to be expected, the SI values are less than 10^{-4} . We consider the central, near on-axis points of the beam in calculating the SI. For each frame the mean of the central 10x10 points

(shown in Figure 3.6), 512 ± 5 in each dimension, are used in calculating the on-axis SI using Equation (2.21).

Figure 3.5: A phase screen results in a scintillation index between zero and unity. Typical orders of magnitude are 10^{-1} . Without the phase screen, the effect of diffraction on the scintillation index is observed to be several orders of magnitude smaller and thus miniscule.

Figure 3.6: The center 10x10 pixels obtained from Figure 3.3 are used to determine the on-axis scintillation index. Both images are shown on a logarithmic scale and are digitally enhanced to show contrast between pixel intensity. Looking at beam center of the left-hand image of Figure 3.3 the reasoning for this enhancement becomes clear. The left-hand image shows the intensity of the center 10x10 pixels of an exposure, whereas the right-hand figure shows the variance of the intensity of these pixels.

The mean of thirty “exposures” are now used to determine the final FV for the given beam waist, separation between the beams, phase screen location, and propagation distance. A 1x1024 slice along the

center of each frame is first taken to get a (y) cross-section of the interference profile, and fringes near the highest peaks are examined for visibility calculations (see Figure 3.7). This eliminates erroneous values for the fringe visibility, which would likely result from near-zero intensities at the wings of the Gaussian(s). The three central peaks are found (see Figure 3.7(b)) and the FV is calculated from the minimum and maximum of the truncated slice.

Figure 3.7: Cross-sectional plot of intensity variations during interference of twin Gaussian beams propagating through a random medium. A 1x1024 slice is taken from the average of 50 frames. The background, shown to be close to zero in the left figure, must be removed to get an accurate value for the FV. The case shown above for two 10mm beams separated by 30mm propagated through a phase screen located at the midpoint of the total propagation distance of 1km. The Fried parameter is held constant at $r_0 = 246\text{mm}$. The maximum intensity occurs where the two beams constructively interfere in the center with an intensity of approximately 2.5. The two outer fringes and the background are removed and displayed in the right plot. The array obtained from the right hand figure is used to calculate the fringe visibility as outlined in Equation (2.44)

CHAPTER 4

SINGLE GAUSSIAN BEAM PROPAGATION THROUGH A PHASE SCREEN

4.1 Introduction

The simulation model has been outlined in Chapter 3. This Chapter details the results obtained with the single Gaussian beam propagation program. The simulations include the initial beam width and curvature, the propagation distance, and the location of the phase screen in the propagation path. The phase fluctuations of the phase screen are parameterized by the Fried parameter, which describes the transverse coherence length, and the inner and outer scales. We observe the effects of various parameters on the SI at the target plane. The Chapter contains 5 additional Sections, each discuss a particular parameter's effect on the SI and a possible reasoning. The six individual parameters discussed are propagation distance, normalized phase screen location, beam width, Fried parameter, and the inner scale.

4.2 Simulated Results on the Effects of Propagation Distance on SI and Single Pass of a Gaussian Beam through a Phase Screen

Within an extended turbulence model the SI is expected to increase as the distance of propagation increases [2]. A similar result is expected from the use of a single phase screen placed at a fixed normalized phase screen location, in this case 0.5 of the total propagation distance. The beam width, Fried parameter, and inner and outer scale size are all set to be constant as the effects of propagation distance are measured. The results are graphically shown in Figure 4.1.

Figure 4.1: The SI is plotted as a function of total propagation distance, where the phase screen is characterized by a Fried parameter of 1.42 m and an inner and outer scale size of 1 mm and 3 m respectively. The phase screen is located at a normalized propagation distance of 0.5 of the total propagation distance for each point. The beam width is held constant at 30 mm, where the Rayleigh range is approximately 5.5 km and the beam size in the absence of turbulence at 1, 3, and 5 km is 30.12, 31.04 and 32.80 mm respectively. The points along the line are the mean of 30 exposures of 50 frames as outlined in Chapter 3. The error bars represent the standard deviation from the mean of the 30 exposures.

As expected, the SI does indeed increase as the total propagation distance increases. First, the beam must travel further to encounter the phase screen resulting in a larger beam width, especially as the beam surpasses the Rayleigh range. The Rayleigh range for a 30 mm beam width is approximately 5.5 km. Increases of the standard deviation of the collected data set is also observed as the total propagation distance increases. The standard deviation increases more rapidly after the Rayleigh range is surpassed. This increase may be attributed to the increase in the area of the phase screen an expanding Gaussian encounters resulting in greater fluctuations measured at the target plane.

4.3 Simulated Results on the Effects of Normalized Phase Screen Location on SI of a Single Pass of a Gaussian Beam Through a Phase Screen

It is strongly suspected the location of a phase screen along the propagation path affect the SI. A phase screen placed close to the target plane or transmit plane is expected to have a minimal effect for the following reasons. If we consider the beam to be partially incoherent after passing through a phase screen, it is expected at a far enough distance these fluctuations begin to appear coherent as suggested by the Van Cittert–Zernike theorem. When the phase screen is located closer to the target plane, the random effects have less propagation distance for phase changes to manifest themselves in the intensity of the optical field. We therefore expected to see the SI to be highest when the phase screen is placed near a central normalized phase screen location. The beam width, Fried parameter, and the inner and outer scale size, are all set to be constant as the effects of the normalized phase screen location are measured for a variety of propagation distances.

Figure 4.2: The SI is plotted as a function of phase screen location along the propagation path, where the transmit plane is located at the normalized location $z/L = 0$ and the target plane is at $z/L = 1$. The location of the phase screen is plotted for a range of distances given in the legend in kilometers. The beam width is held constant at 10 mm, the Fried parameter is a constant 1.421 m and the inner and outer scale is held at 1 mm and 3 m respectively. A maximum in the SI occurs for most propagation distances between 0.50 and 0.75 of the total propagation distance.

The results, as expected, in Figure 4.2 for a beam width of 10 mm a normalized phase screen located near the transmit or target plane results in a low SI. For the 10 mm beam width a maximum is observed for all propagation distances to be between 50 to 75 percent of the total propagation distance. A similar result is observed in Vorontsov and Carhart [12] for an image quality metric where a scene is imaging through turbulence located at various points along the propagation path. The maximum effect of the turbulence occurred at a normalized propagation distance of 0.73 in Vorontsov and Carhart [12].

Figure 4.3: The SI is plotted as a function of phase screen location along the propagation path, where the transmit plane is located at the normalized location $z/L = 0$ and the target plane is at $z/L = 1$. The location of the phase screen is plotted for a range of distances given in the legend in kilometers. The beam width is held constant at 30 mm, the Fried parameter is a constant 1.421 m and the inner and outer scale is held at 1 mm and 3 m respectively. A local maximum in the SI occurs for most propagation distances around 0.5 of the total propagation distance.

The corresponding result for a 30 mm beam width is shown in Figure 4.3. The 30 mm beam shows a local maximum near 0.5 of the total propagation distance with the maximum occurring near the transmit plane $z = 0$. The 30 mm beam experiences more of the phase screen than the 10 mm beam due to its initial size resulting in a lower degree of coherence. This may account for the high scintillation index of the 30 mm beam near the transmit plane. However, the global behavior shows a monotonic decrease in the SI. The lack of a well-defined maximum as in the case of a 10 mm beam implies that even when the phase

screen is close to the transmit plane, the beam does not regain partial coherence after propagation behind the phase screen, unlike the 10 mm width case. This is to be expected since lower coherence resulting from a wider beam needs more propagation distance to regain coherence, again as a consequence of the van-Cittert Zernike theorem [13].

4.4 Simulated Results on the Effects of Gaussian Beam Width on SI of a Single Pass of a Gaussian Beam Through a Phase Screen

As with other variables, it is expected that the beam width should have an effect on the SI. There are a few reasoning points for this assumption. First, all Gaussians are subject to beam expansion based on diffraction alone. Secondly, the Rayleigh range, z_R , differs with different beam widths. Thirdly, the theoretical formula for the von Karman spectrum given in ref. [2], hints the Fresnel ratio is directly related to the SI. This ratio is given as:

$$\beta = \frac{2L}{k_0 w_L^2}, \quad (4.1)$$

where w_L is the beam width at a distance L and is given by the formula

$$w_L = w_0 \sqrt{1 + \left(\frac{L\lambda}{\pi w_0^2} \right)^2}. \quad (4.2)$$

It appears that knowledge of the Rayleigh range and Fresnel ratio, enables us to make a rough guess to what beam size is most appropriate for the lowest SI, given a propagation distance, $z = L$. For a phase screen placed at $z = L/2$, the smallest SI is found if the following conditions are simultaneously satisfied: (a) $z_R \geq L/2$, (b) $w_{L/2}$ minimum, and (c) $\beta \sim 0.5$. Table 4.1 shows L , w_0 , z_R , $w_{L/2}$, and β for various L and w_0 . For instance, for $L = 3$ km, $w_0 = 20$ mm gives a $z_R \sim 2.5$ km, $w_{L/2} \sim 23$ mm and $\beta \sim 0.44$. If the Rayleigh range of a given beam width does not exceed the total propagation distance the beam will not likely have a minimized SI when compared to beam widths with Rayleigh ranges that exceed the total propagation distance. Furthermore, we expect the minimum to occur when the Fresnel ratio is near 0.5. Using these principles we can predict which beam widths will have the minimal SI for a given propagation distance. These predictions are easily determined from Table 4.1 below.

Table 4.1: The table includes the Rayleigh range z_R , the beam width $w_{L/2}$ (in the absence of turbulence) at $z = L/2$, and the Fresnel ratio β at $z = L/2$ for various propagation distances L and initial width w_0 .

L (km)	w_0 (mm)	z_R (km)	$w_{z/2}$ (mm)	β	L (km)	w_0 (mm)	z_R (km)	$w_{z/2}$ (mm)	β
1	10	.628319	12.78	0.49	4	10	.628319	33.36	0.29
1	15	1.413717	15.91	0.31	4	15	1.413717	25.99	0.47
1	20	2.513274	20.39	0.19	4	20	2.513274	25.56	0.49
1	25	3.926991	25.20	0.13	4	25	3.926991	28.06	0.40
1	30	5.654867	30.12	0.09	4	30	5.654867	31.82	0.31
1	35	7.696902	35.07	0.06	4	35	7.696902	36.16	0.24
1	40	10.053096	40.05	0.05	4	40	10.053096	40.78	0.19
1	45	12.723450	45.03	0.04	4	45	12.723450	45.55	0.15
1	50	15.707963	50.03	0.03	4	50	15.707963	50.40	0.13
2	10	.628319	18.80	0.45	5	10	.628319	41.03	0.24
2	15	1.413717	18.37	0.47	5	15	1.413717	30.47	0.43
2	20	2.513274	21.53	0.34	5	20	2.513274	28.21	0.50
2	25	3.926991	25.80	0.24	5	25	3.926991	29.64	0.45
2	30	5.654867	30.47	0.17	5	30	5.654867	32.80	0.37
2	35	7.696902	35.29	0.13	5	35	7.696902	36.80	0.29
2	40	10.053096	40.20	0.10	5	40	10.053096	41.22	0.23
2	45	12.723450	45.14	0.08	5	45	12.723450	45.86	0.19
2	50	15.707963	50.10	0.06	5	50	15.707963	50.63	0.16
3	10	.628319	25.88	0.36	6	10	.628319	48.78	0.20
3	15	1.413717	21.87	0.50	6	15	1.413717	35.19	0.39
3	20	2.513274	23.29	0.44	6	20	2.513274	31.14	0.49
3	25	3.926991	26.76	0.33	6	25	3.926991	31.46	0.48
3	30	5.654867	31.04	0.25	6	30	5.654867	33.96	0.41
3	35	7.696902	35.66	0.19	6	35	7.696902	37.56	0.34
3	40	10.053096	40.44	0.15	6	40	10.053096	41.74	0.27
3	45	12.723450	45.31	0.12	6	45	12.723450	46.23	0.22
3	50	15.707963	50.23	0.09	6	50	15.707963	50.90	0.18

It should be noted that these predictions do not take into account the effects of the inner scale or the Fried parameter, thus we expect some inaccuracy when relating predictions to actual simulated data where a phase screen is in place. However we expect to see the SI to be at a minimum when the beam

width is close to that predicted by Table 4.1. The variation of the SI with the initial beam width w_0 is shown in Figure 4.4 for various distances of propagation when the phase screen is located at $z = L/2$. Upon comparing Figure 4.4 with Table 4.1, we observe that the simulated data matches quite well with our predictions for propagation distances between 1 km to 5 km. If this prediction technique can be broadened to extended turbulence models the implications would be quite meaningful. Essentially knowing the total propagation distance, the SI could be minimized with the appropriate initial beam width.

Figure 4.4: The SI is plotted as a function of beam width at varying propagation distances. The normalized location of the phase screen is 0.5, the Fried parameter is 1.42 m and the inner and outer scale is 1 mm 3 m respectively. The propagation distance is indicated in the legend below the graph. The dips in the graph match expectations using the three criteria mentioned earlier and the assistance of Table 4.1.

We also notice the SI plateaus with increasing beam width. Comparing this trend to Table 4.1 we notice this correlates with small changes in the Fresnel Ratio between the plateaued points.

4.5 Simulated Results on the Effects of the Fried Parameter on SI of a Single Pass of a Gaussian

Beam Through a Phase Screen

The Fried parameter is defined as the transverse distance over which the rms wavefront aberration is less than 1 radian, implying consequently, smaller turbulence for larger Fried parameters. Therefore, it is expected that an increase in the Fried parameter will result in a lower SI measurement due to a larger area

in which the aberrations add up to 1 radian. This is also verified by discussions in Chapter 2 in which the relationship between the Fried parameter and the structure constant was shown to be inversely proportional (see Equation (2.33)). A larger Fried parameter corresponds to a weaker structure constant and thus weaker turbulence.

Figure 4.5: The SI index is shown as a function of Fried parameter. The inner scale size is 1 mm, the beam width of 30 mm, and a normalized phase screen location of 0.5 of the total propagation distance of 1 km. Note the decrease in the SI as the phase screen is moved closer to the target. The normalized phase screen location is indicated by the legend and corresponds to what is shown in Figure 4.3 for a 30 mm beam.

The results, graphically displayed in Figure 4.5, agree with theoretical arguments regardless of phase screen location. Note that a Fried parameter smaller than 1.136 m would eventually approach the strong turbulence case yielding $SI \rightarrow 1$. Hence, smaller values for the Fried parameter could have been used in our simulations.

4.6 Simulated Results on the Effects of the Inner Scale on SI of a Single Pass of a Gaussian Beam Through a Phase Screen

The inner scale is a measure of the smallest eddy that is statistically homogeneous and isotropic [2]. This implies that a larger inner scale would result in less turbulence as larger areas effectively have the same characteristics. It should be noted the inner scale has a different meaning than the Fried parameter.

Two inner scale eddies may differ slightly in refractive index, the accumulation of these effects finally define the Fried parameter.

Figure 4.6: The SI index is shown as a function of inner scale size, l_0 , as viewed in the legend located below the graphic. This relationship is shown for a variety of normalized phase screen locations as shown on the vertical axis where the total propagation distance is 1 km and the outer scale is 3 m. The beam width is 30 mm. The Fried parameter is also varied as seen in the legend. The graph clearly shows a decrease in the SI index with an increase in the inner scale.

We thus expect the SI index to decrease with an increased inner scale size. The beam width, Fried parameter, and inner scale size are all set to be constant as the effects of inner scale are computed for a variety of normalized phase screen locations for a propagation distance of 1 km. The results are graphically shown in Figure 4.6. We note that the simulated data matches quite well with predictions using intuitive reasoning. This is true regardless of the location of the phase screen or the size of the Fried parameter.

4.6 Concluding Remarks

Propagation of a single Gaussian beam through a phase screen has been analyzed in this Chapter. The SI increases with an increased propagation distance due to increasing beam width at the phase screen

plane due to diffraction. The SI behaves as a function of the normalized phase screen locations, with the maximum occurring at 0.5 - 0.7 of the total propagation distance or when the phase screen is placed near the transmit plane, and suggests that beam width impacts these results. The beam width does have an impact on the SI, a result that could prove quite advantageous if it holds true for the extended turbulence case. Lastly, both the inner scale and Fried parameter results are as expected. A larger inner scale size or larger Fried parameter will result in a lower scintillation index.

The next Chapter details the simulation results for the double Gaussian beam case. While FV and SI are not directly correlated, we expect to see results to be inversely proportional as discussed in Chapter 2. The correlations of the SI and FV for the propagation distance and phase screen location will be discussed in Chapter 6.

CHAPTER 5

TWIN GAUSSIAN BEAM PROPAGATION THROUGH A PHASE SCREEN

5.1 Introduction

The simulation model has been outlined in Chapter 3. This Chapter details the results obtained for double Gaussian beam propagation. The simulations include the initial beam width and curvature, the propagation distance, transverse separation distance, and the location of the phase screen in the propagation path. The phase fluctuations of the phase screen are parameterized by the Fried parameter, which describes the transverse coherence length, and the inner and outer scales. We observe the effects of various parameters on the FV at the target plane. The Chapter contains 3 additional Sections, each discuss a particular parameter's effect on the FV and a possible reasoning. The three individual parameters discussed are propagation distance, normalized phase screen location, and transverse separation of the beams.

5.2 Simulated Results on the Effects of Propagation Distance and Transverse Beam Separation on FV of a Single Pass of Two Transversely Separated Gaussian Beams through a Phase Screen

In Chapter 4, we observed the SI for a 30 mm beam increased with total propagation distance. As the SI and FV are conceptually inversely proportional, we expect a decrease in the FV as the interfering beams, propagating through turbulence, further propagates into the far field. The beam width, Fried parameter, and inner scale size, are all set to be constant as the effects of propagation distance are measured. The results are graphically shown in Figure 5.1.

Figure 5.1: Variation of FV as a function of distance of propagation. The two beams have a width of 30 mm and are transversely separated by a variety of distances as shown in the plot's legend. The normalized phase screen location is 0.5 with a Fried parameter of 1.42 m and an inner scale size of 1 mm.

Figure 5.2: Variation of FV as a function of transverse beam separation for a variety of propagation distances as indicated in the legend in meters. The two beams have a 30 mm beam waist and travel through the phase screen located at half the total propagation distance. The Fried parameter is held constant at 1.42 m and the inner scale values is 1 mm.

As expected, Figure 5.1 shows that the FV tends to decrease as the propagation distance increases. This is especially true for smaller separation between the beams, since the effect of turbulence manifests itself more with increased propagation distance. The reason for this is currently under investigation; however, one may surmise that the optical fields from the two Gaussian beams may tend to become more mutually coherent with increasing propagation distance beyond the phase screen, once again as a consequence of the van-Cittert Zernike theorem. The FV as a function of beam separation for different propagation distances is shown in Figure 5.2. A smaller transverse separation translates to fringes that are more closely spaced than larger transverse separations and coherently overlap sooner. The phase screen imparts this phase perturbations to the incoming optical field, and since the 60 mm separated beams are likely to be more strongly interfering at the phase screen plane they do not recover as well as beams with a larger separation. This implies weak mutual coherence between twin beams may in fact improve the FV at longer distances. We think that a stronger interference pattern incident on the phase screen suffers more degradation due to the fact that a stronger interference pattern can be regarded as a collection of many small, random scatterers behind the phase screen, which in turn requires a longer distance to become mutually coherent.

5.3 Simulated Results on the Effects Normalized Phase Screen Location on FV of a Single Pass of Two Transversely Separated Gaussian Beams Through a Phase Screen

As with the SI, it is strongly suspected the location of a phase screen along the propagation path will have an impact on the FV. If we utilize the same approach as in Section 5.2 we would expect the FV to yield inversely proportional results to its single beam counterpart's SI. A phase screen placed close to the target plane or transmit plane is expected to have a minimal effect for differing reasons. If we consider the beam to be partially incoherent after passing through a phase screen, it is expected at a far enough distance these fluctuations begin to appear coherent as suggested by the Van Cittert-Zernike theorem. When the phase screen is located closer to the target plane, the effects have less propagation distance for phase changes to manifest themselves in intensity of the field.

Upon comparing with the variation of the SI with the normalized screen location, we expect to see the FV to be lowest when the SI is the highest. For a 30 mm beam this occurs near the transmit plane and

for our simulations, the beam width, Fried parameter, and inner scale size are all set to be constant as the effects of the normalized phase screen location are measured for a variety of propagation distances. The results are graphically shown in Figure 5.3.

Figure 5.3: The FV is plotted as a function of normalized phase screen location for a variety of propagation distances as shown in the legend (in meters). The two beams have a width of 30 mm and are transversely separated by 60 mm. The phase screen has a Fried parameter of 1.42 m and an inner and outer scale size of 1 mm and 3 m respectively.

As seen from Figure 5.3 the increase in FV implies a higher contrast (or less effect from the phase screen). The FV increases as the normalized phase screen location is moved towards the target plane, in agreement with the decrease in SI, as described earlier. FV is commonly used a measurement of coherence. Lower FV for lower values of the screen location is likely due to the fact that interference between more incoherent sources is expected to yield lower quality fringes. Similar to Figure 5.1 and 5.2, the FV decreases with propagation distance.

5.4 Concluding Remarks

Propagation of double Gaussian beams through a random phase screen has been analyzed in this Chapter. Assuming that a single phase screen is placed midway during propagation, we find that the FV varies with the propagation distance, attaining its lowest value when the separation between the beams equals twice the beam width. We also see that transversely separating the beams a large distance apart may

in fact increase the FV. The FV monotonically increases when the screen location is moved away from the source, in agreement with the variation of SI shown in Chapter 4 for a 30 mm beam width. In some cases, SI or FV can be used as an equivalent characterization tools for turbulence, since they seem to show an inverse relationship. Work on variations of FV with beam separation for different locations of the phase screen will be attempted in the future.

CHAPTER 6

CONCLUSION AND DISCUSSION

6.1 Summary of Single Beam and Double Beam Results with Possible Applications

In this work we have developed a simulation method relying on statistical properties of the MVKS atmospheric model and a standard free-space propagation model. The model, discussed in Chapter 3, includes a single beam case, where the SI is determined and a double beam case where the FV is calculated. Generally, we have considered the propagation distance to be in the order of kilometers, and beam sizes of the order of tens of millimeters. The phase screen has been characterized by the MVKS where the inner scale and Fried parameters were used as variables.

The SI has been determined for the single beam case under the following variable conditions: total propagation distance, normalized phase screen location, beam width, inner scale and Fried parameter. These results have been presented in Chapter 4. The most notable results for the SI pertain to the normalized phase screen location and the beam width. We have found that the maximum SI for the phase screen location is at least partially depended upon the beam width as evident when the 10 mm beam width cases is compared to the 30 mm beam case. For pockets of turbulence along the propagation path, our computations could be used to locate where a majority of the turbulence lies. We have also found that the SI could be minimized by carefully choosing the appropriate beam width that meets a list of criteria. Again, if the results are applicable to extended turbulence, the application could be significant.

The FV has been determined for the double beam case under the following variable conditions: total propagation distance, normalized phase screen location, and transverse beam separation. These results were discussed in Chapter 5. The most notable results for the FV pertain to the normalized phase screen location and the interdependency of propagation distance and beam separation on the FV. We have found that the FV reaches a maximum as the phase screen moves farther from the transmission plane. Again, for

pockets of turbulence along the propagation path, this could be used to locate where a majority of the turbulence lies. We have also found that the FV suffers when the transverse separation is small and when the phase screen is closely located to the transmit plane. Again, if the results are applicable to extended turbulence, the application could be significant.

From our simulations in Chapter 4 and 5, it is conceivable that although SI and FV depend on the same parameters they provide different metrics for the turbulence and in fact seem to display an inversely proportional relationship as expected. One could exploit this to characterize turbulence more accurately, especially in a test bed configuration, which is the motivation for this work. The FV could be used to optimize a multi-aperture system to counteract the effects a phase screen with certain characteristics, and should the results be relevant to extended turbulence this application would quite fruitful in a wide variety of applications. Furthermore, should this work prove to be applicable in extended turbulence cases, the location of turbulence along the path could be determined knowing the initial beam width. For example, if a beam with a 30 mm width has a low SI and a high FV are detected at the target plane this would suggest the turbulent layer is located somewhere near the target plane. However, if a high SI and a low FV are detected at the transmit plane this would suggest the turbulent layer is located very close to the target plane.

6.2 Future Work

In this Section we list some areas of this thesis that could be further explored in future work. The following is a list of work that would improve upon this thesis:

- Verify work presented in this thesis with experimental work on a compact test bed.
- Investigate the double beam case further to include differing beam widths, inner scale sizes and Fried parameters.
- Investigate the SI and FV for differing radius of curvatures.
- Determine if this work is valid for an extended turbulence case via the following:
 - Expand simulated model to include multiple phase screens to mimic extended turbulence case. Compare these results with the results of the single phase screen model presented within this work.
 - Expand experimental test bed model to an outdoor range.

- Model and explore the SI and FV of double Gaussian beams. Determine if the inversely proportional nature of the SI and FV remain factual.
- Expand FV model to include three or more beams and determine if results are similar in nature. Model could include transverse separation, angle separation, beam width, etc.

BIBLIOGRAPHY

- [1] M. C. Roggemann and B. M. Welsh, *Imaging Through Turbulence*. Boca Raton, FL: CRC Press, 1996.
- [2] L. C. Andrews and R. L. Phillips, *Laser Beam Propagation through Random Medium*, 2nd ed. Bellingham, WA: SPIE Press, 2005.
- [3] W. B. Miller, J. C. Ricklin, and L. C. Andrews, "Effects of the refractive index spectral model on the irradiance variance of a Gaussian beam," *J. Opt. Soc. Amer. A*, vol. 11, pp. 2719-2726, 1994.
- [4] A. D. Wheelon, *Electromagnetic Scintillation*. Cambridge, U.K.: Cambridge Univ. Press, 2003, vol. II. Weak Scattering.
- [5] N. J. Miller, P. F. Widiker, P. F. McManamon, and J. W. Haus, "Active multi-aperture imaging through turbulence," in *Proc. SPIE*, vol. 8395, pp. 839504, 2012.
- [6] L.C. Andrews, R.L. Phillips, and A.R. Weeks, "Propagation of a Gaussian-beam wave through a random phase screen," *Waves in Random Media*, vol. 7, pp. 229-244, 1997.
- [7] X. Liu and J. Pu, "Investigation on the scintillation reduction of elliptical vortex beams propagating in atmospheric turbulence," *Optics Express*, vol. 19, pp. 26444-26450, 2011.
- [8] T. -C. Poon and T. Kim, *Engineering Optics with Matlab®*. Toh Tuck Link, Singapore: World Scientific Publishing Co., 2006.
- [9] R. Rampy and D. Gavel, "New methods of fabricating phase screens for simulated atmospheric turbulence," in *Proc. SPIE*, vol. 7736, pp. 77362Y, 2010.
- [10] R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers with Modern Physics*, 6th ed., Michelle Julet, Ed. Belmont, CA: Thomson Brooks/Cole, 2004, vol. 2.

- [11] A. Labeyrie, S. G. Lipson, and P. Nisenson, *An Introduction to Optical Stellar Interferometry*. Cambridge, UK: Cambridge University Press, 2006.
- [12] M. A. Vorontsov and G. W. Carhart, "Anisoplanatic imaging through turbulent media: image recovery by local information fusion from a set of short-exposure images," *J. Opt. Soc. Amer. A*, vol. 18, pp. 1312-1324, 2001.
- [13] C. S. Roychoudhuri and K. R. Lefebvre. SPIE. [Online]. http://spie.org/etop/1995/148_1.pdf
- [14] V. I. Tatarski, *Wave Propagation in a Turbulent Medium*. New York: McGraw-Hill Book Co., 1961.
- [15] Z. Wang, *Characterization of Random Scattering Media and Related Information Retrieval*, PhD Dissertation, Purdue University, 2008.

APPENDIX

A. Matlab© Simulated Model for Single Gaussian Beam

The purpose of the single Gaussian beam propagation simulated model was to determine the SI at the target plane. This section supplements the detail provided in Chapter 3. A primary key feature not discussed in Chapter 3 is the process in which the data is “physically” collected. For a given set of parameters (1500 frames equating to 30 exposures) a filename is generated to contain the inner scale size, the beam width, the propagation distance, the normalized phase screen location and the Fried parameter. The 30 exposure values are collected in a matrix and then written to the file. This process is repeated for the combination of the variable parameters. Each file is manually copied into a Microsoft Excel file for further processing such as statistical data (mean, standard deviation and outliers within a set of exposures) and graphing.

The program was written in Matlab version R2011a and requires the Image Processing Toolbox. Appendix C contains the necessary file from this toolbox to run the file (mean2) to run this model in the absence of this toolbox. All parameters are properly documented to avoid confusion. Information about each function can be found by searching the Matlab documentation center at <http://www.mathworks.com/help/documentation-center.html>.

```
%% 2D Gaussian Beam Propagation
% Split-step method with phase screen
% Single Gaussian Beams SI.
% Erica Whitfield
% Last Edit July 25, 2012

clear all; close all; format short; clc
tic
%% Preliminary Grid Set-up and Constants
N = 2^10; %number of sample points
L = 750; %Physical size of grid in mm
h = L/N; % mm/points-- space step
n = [-N/2:1:N/2 - 1]'; %Indexing
x = n*h; %Grid points
y = x;
[X,Y] = meshgrid(x,y);
lambda = 0.5*10^(-3); % wavelength in mm
```

```

k0 = 2*pi/lambda; % k vector
a = pi/h; % grid points in "frequency" domain--> {2*pi*(points/mm)}
kx = [-a:2*a/N:a-2*a/N]'; % "frequency" domain indexing
ky = kx;
[Kx, Ky] = meshgrid(kx,ky); % Freq. Meshgrid

% The following three lines deal with the diffraction limit,
% P is used later in the program for this purpose.
Numerator = - (Kx.^2 + Ky.^2);
Denominator = 1i*2*k0;
P = (Numerator/Denominator);

%% Phase screen parameters
mr = sqrt(Kx.^2 + Ky.^2);
lo = [10]; % inner scale
Lo = 3000; % outer scale

ko = 2*pi/Lo;
r0 = [0.0001, .0002, 0.0003]; % Modified Fried Parameter
% Power Spectral Density

%% Initial Gaussian set-up
w0 = [10, 15, 20, 25, 35, 40, 45, 50]; % Beam radius
d = [1,2,3,4,5,6,7,8]*1000*1000; % Distance of expected propagation in mm

x0 = 0; % x-shift, space domain
y0 = 60; % y-shift, space domain

%% Loop Parameters
numframes = 50; % average of frames
trials = 30; % number of values desired

%% The Grunt work of Beam propagation
for PP = 1:length(lo)

    km = 5.92/lo(PP);
    for QQ = 1:length(w0)
        gx = (exp(-(X.^2)+(Y.^2))/(w0(QQ)^2)); % Single Gaussian beam
        Zr = ((w0(QQ))^2*k0)/2; % Rayleigh Range

        for RR = 1:length(r0)
            psd = (0.23*r0(RR)^(-5/3)*exp(-(mr.^2)/(km.^2))./(mr.^2 + ko.^2).^(11/6));
            for Q = 1: length(d)
                d1 = [0.1,0.25,0.35,0.45, 0.65, 0.75, 0.85]*d(Q);
                trialSet = (zeros(length(d1), trials));
                for R= 1:length(d1)
                    d2 = d(Q)-d1(R);
                    u1 = exp(P.*d1(R)); u2 = exp(P.*d2);
                    format bank
                    filename = sprintf('Inner%.0fBeamRad%.0fProDist%1.1fscreen%1.2fFreid%1.5f.txt', lo(PP),
w0(QQ), d(Q)/1000000,d1(R)/d(Q), r0(RR));
                    format short
                    for tt = 1:trials
                        si = zeros(size(gx));
                        si2 = zeros(size(gx));
                        for val = 1:numframes

```

```

ce = randn(size(psd)) + 1i*randn(size(psd));
phi = real(iff2(fftshift(sqrt(psd).*ce)));
phasescreen = exp(1i*phi);
Gx = fftshift(fft2(gx)).*u1;
gxp = (iff2(fftshift(Gx))).*phasescreen;
Gxp = (fftshift(fft2(gxp))).*u2;
gx1 = (iff2shift(fftshift(Gxp)));
si = si + (abs(gx1)).^2;
si2 = si2 + (abs(gx1)).^4;
end % numframes
col = uint16(length(gx1)/2);
row = uint16(col);
sif = single(si(col-5:col+5, row-5:row+5));
sif2 = single(si2(col-5:col+5, row-5:row+5));
SI = single((mean2(sif2/(numframes))-
(mean2(sif/(numframes)))^2)/((mean2(sif/numframes))^2));
trialSet(R, tt) = SI;
clear sif
clear sif2
end % trials
% fid = fopen(filename, 'w');
B = transpose(trialSet(R,:));
dlmwrite(filename, B, 'delimiter', ',', 'precision', 6, 'newline', 'pc')
% fclose(fid);
toc
clear trialSet;
end
end
end
end
end
end
end

```


2. Matlab© Simulated Model for Double Gaussian Beams

The purpose of the double Gaussian beams propagation simulated model was to determine the FV at the target plane. This section supplements the detail provided in Chapter 3. The transverse beam separation is changed manually as is the beam width. A primary key feature not discussed in Chapter 3 is the process in which the data is “physically” collected. The 30 exposure values printed to the Matlab© workspace in order of the normalized phase screen location and each file is manually copied into a Microsoft Excel file for further processing such as statistical data (mean, standard deviation and outliers within a set of exposures) and graphing.

The program was written in Matlab© version R2011a and requires the Signal Processing Toolbox which contains the necessary file from this toolbox to run the file (findpeaks) to run this model in the absence of this toolbox. All parameters are properly documented to avoid confusion. Information about each function can be found by searching the Matlab© documentation center at <http://www.mathworks.com/help/documentation-center.html>.

```
%% 2D Twin Gaussian Beam Propagation
% Split-step method with phase screen
% Two Gaussian Beams interfere in the far field to create fringes.
% Erica Whitfield
% Last edited: July 3, 2012
```

```
clear all; close all; format short; clc
tic
```

```
%% Constants
N = 2^10; %number of sample points
L = 750; %Physical size of grid in mm
h = L/N; % mm/points-- space step
n = [-N/2:1:N/2 - 1]'; %Indexing
x = n*h; %Grid points
y = x;
[X,Y] = meshgrid(x,y);
lambda = 0.5*10^(-3); % wavelength in mm
k0 = 2*pi/lambda; % k vector

a = pi/h; % grid points in "frequency" domain--> {2*pi*(points/mm)}
kx = [-a:2*a/N:a-2*a/N]'; % "frequency" domain indexing
ky = kx;
```

```

[Kx, Ky] = meshgrid(kx,ky); % Freq. Meshgrid

% The following three lines deal with the diffraction limit, P is used later in the program for this purpose.
Numerator = - (Kx.^2 + Ky.^2);
Denominator = 1i*2*k0;
P = (Numerator/Denominator);
%% Phase screen parameters
sz = N/2;

mr = sqrt(Kx.^2 + Ky.^2);
lo = 1; % inner scale
Lo = 3000; % outer scale
km = 5.92/lo;
ko = 2*pi/Lo;
r0 = .0002; %Fried Parameter
% Power Spectral Density
psd = (0.23*r0^(-5/3)*exp(-(mr.^2)/(km.^2))./(mr.^2 + ko.^2).^(11/6));
%% Initial Gaussian set-up
w0 = 30; %Beam width as a percentage of grid in mm. If the number is sufficiently small it can
impersonate a "point source"
Zr = ((w0)^2*k0)/2; % Rayleigh Range where the beam is considered unchanged
d = [2,3,4,5]*1000*1000;%*1000;% const*Zr; % Distance of expected propagation in mm and as a
function of the Rayleigh Range

x0 = 0; % x-shift, space domain
y0 = 70; % y-shift, space domain

gx = (exp(-((X.^2)+(Y.^2))/(w0^2))); % Single Gaussian beam
gxa = (exp(-((X - x0).^2 + (Y + y0).^2)/(w0^2))); % shifted Gaussian beam
gxb = (exp(-((X + x0).^2 + (Y - y0).^2)/(w0^2))); % Mirror shift of gxa
hx = (gxa + gxb); % Two spaced Gaussian beams

%% Loop Parameters
numframes = 50; % average of frames
trials = 30; % number of values desired
% trialSet = zeros(length(d1),trials);
%% The Meat
for Q = 1:length(d)
    d1 = [0.15, 0.25, 0.35, 0.45, 0.5, 0.65, 0.75, 0.85]*d(Q);
    trialSet = zeros(length(d1),trials);
    for R = 1:length(d1)
        % Distance propagated to the phase screen
        d2 = d(Q) - d1(R); % Distance from phase screen to desired image plane
        u1 = exp(P.*d1(R)); u2 = exp(P.*d2);
        for ttt = 1:trials
            fv = zeros(length(gx),1);
            fv2 = zeros(length(gx)-1,1);
            for val = 1:numframes
                % Creation of the random phase screen
                ce = randn(size(psd)) + 1i*randn(size(psd));
                phi = (real(ifft2(fftshift(sqrt(psd).*ce)))); % Screen of phases
                phasescreen = exp(1i*phi);

                Hx = (fftshift(fft2(hx))).*u1;
                hxp = (ifft2(fftshift(Hx))).*phasescreen;
                Hxp = (fftshift(fft2(hxp))).*u2;
            end
        end
    end
end

```

```

hx1 = (ifft2(fftshift(Hxp)));

fv = fv + (abs(hx1(:,N/2))).^2; % Intensity
%   fv2 = fv2 + diff(fv); % Intensity squared
end

%% Fringe Visibility
[mmax,loc] = max(fv);
[pks,locs]=findpeaks(fv,'minpeakdistance',3, 'MINPEAKHEIGHT',0.35*mmax);

Fv = fv(locs(1):locs(end));
FV = (max(Fv)-min(Fv))/(max(Fv)+min(Fv));
trialSet(R, ttt) = FV;
%
clear Fv
clear FV
end
end

d(Q)
for L =1:length(d1)

    transpose(trialSet(L,:))
end
% transpose(trialSet);
toc
clear trialSet;
end

```