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**CHARACTERISTICS OF THE NONLINEAR HYSTERESIS LOOP FOR
ROTOR-BEARING INSTABILITY**

by

JENQ-SHAN GUO

**Submitted in partial fulfillment of the requirements
for the degree of Doctor of Philosophy**

Thesis Advisor: Prof. M.L. Adams

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CHARACTERISTICS OF THE NONLINEAR HYSTERESIS LOOP FOR ROTOR-BEARING INSTABILITY

Abstract

by

JENQ-SHAN GUO

The nonlinear hysteresis loop of a simple-cylindrical-journal-bearing supported rotordynamic system is characterized by the Hopf-bifurcation and Saddle-Node bifurcation speeds. The nonlinearity of this system occurs in the journal-bearing fluid-film forces, and requires the imbedding of a solution for the Reynolds lubrication equation within a numerical integration scheme of the coupled motion equations in order to perform proper simulations. These show that under a light bearing static load, the Saddle-Node and Hopf bifurcations coalesce to a single speed at essentially two times the self-excited vibration frequency (i.e., the lowest natural frequency). At higher bearing loads, the classical instability threshold speed (i.e., Hopf bifurcation) occurs at progressively higher rotor speeds. However, the disappearance speed (Saddle Node of the periodic orbit) of the nonlinear limit cycle occurs at progressively lower rotor speeds, asymptotically approaching approximately 1.725 times the lowest natural frequency. By adding a rotor unbalance force to the model, exploratory simulations have been made to determine the extent to which chaos signal processing in the normal speed-up or coast-down vibration of actual machines could be used to locate this lower speed bound of the instability hysteresis loop. Laboratory experiments were performed and these agree quite well with simulation results.

DEDICATION

To my parents

and

my wife, Mei-Ling

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Table of Contents

	Page
Abstract	ii
Dedication	iii
Acknowledgments	iv
Table of Contents	v
List of Figures	vii
List of Photographs	xxi
List of Tables	xxii
Chapter 1 Introduction	1
1.0 Background	1
1.1 Literature Review	3
1.2 Objectives and the Scope of Work	8
Chapter 2 Mathematical Model	10
2.0 Description of the Rotor-Bearing System	10
2.1 Equations of Motion	11
2.2 Non-Dimensionalization of the Equations of Motion	14
2.3 Hydrodynamic Fluid Film Force	17
2.4 System Parameters	24
Chapter 3 Linear Stability Analysis	25
3.0 Introduction	25
3.1 Static Equilibrium Point of the System	26
3.2 Linear Dynamical Coefficients of the Journal Bearing	28
3.3 Linearization of the System Equations	31
3.4 Computation of the Linear Instability Threshold	34
3.5 Numerical Results and Discussion	37
Chapter 4 Nonlinear Hysteresis Loop of Rotor-Bearing System	57
4.0 Description of the Hysteresis Behavior	57
4.1 Hysteresis Loop of Rotor-Bearing System	58

4.2 Computation of the Nonlinear Hysteresis Loop	60
4.3 Numerical Results and Discussion	61
Chapter 5 Rotor Unbalance Response within the Hysteresis Loop	96
5.0 Introduction	96
5.1 Numerical Results and Discussion	98
5.2 Chaos Measurement	115
Chapter 6 Experiments on the Nonlinear Hysteresis Loop	124
6.0 Introduction	124
6.1 Experimental Facility, System Assembly and Testing Procedure	125
6.1.1 Experimental Facility	125
6.1.2 System Assembly	129
6.1.3 Testing Procedure	131
6.2 System Dimension	133
6.3 Data Reduction	134
6.4 Experimental Results and Discussion	135
Chapter 7 Conclusions and Future Works	138
References	140
Nomenclatures	145
Appendix A QR Solver	148
Appendix B Summary of the Hopf Bifurcation Theory	150
Appendix C Runge-Kutta-Fehlberg Method	153
Appendix D Experimental Results of Nonlinear Hysteresis Loop	159
Appendix E Boundary Conditions for Reynolds Equation and Pressure Distribution within Fluid Film	172
Photographs	174

List of Figures

	Page
Figure 1.0.1 Conceptual Hysteresis Loop of Rotor-Bearing System	2
Figure 2.0.1 Geometry and coordinate system of the rotor-bearing system.	11
Figure 2.3.1 Geometry and coordinates of the journal bearing.	17
Figure 3.4.1 Computation procedure of finding the instability threshold speed.	36
Figure 3.5.1 Stability Map for $L/D=0.5$, $\Delta=0.1$.	40
Figure 3.5.2 Stability Map for $\Omega_n=1.0$, $L/D=0.5$ with different point mass ratio.	41
Figure 3.5.3 Stability Map for $\Omega_n=1.0$, $L/D=0.5$ with different viscosity but the same amount of static load.	42
Figure 3.5.4 Root- locus for $B_p=5.0$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$.	44
Figure 3.5.5 Root- locus for $B_p=1.0$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$.	45
Figure 3.5.6 Root- locus for $B_p=0.5$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$.	46
Figure 3.5.7 Root- locus for $B_p=0.2$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$.	47
Figure 3.5.8 Root- locus for $B_p=0.15$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$.	48
Figure 3.5.9 Root- locus for $B_p=0.1$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$.	49
Figure 3.5.10 Root- locus for $B_p=0.05$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$.	50
Figure 3.5.11 Root- locus for $B_p=0.02$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$.	51

- Figure 3.5.12 a and b A diverging trajectory of the center of the journal integrating from $(\Delta x_j, \Delta y_j) = (0.05, 0.05)$ and a converging trajectory integrating from $(\Delta x_j, \Delta y_j) = (0.10, 0.10)$ for $B_p = 0.5$, $\Omega_n = 1.0$, $L/D = 0.5$, $\Delta = 0.1$, $\omega/\omega_c = 2.0$. 53
- Figure 3.5.12.c Journal orbit for $B_p = 0.5$, $\Omega_n = 1.0$, $L/D = 0.5$, $\Delta = 0.1$, $\omega/\omega_c = 2.0$. 53
- Figure 3.5.13 a and b Journal orbit and time history for $B_p = 5.0$, $\Omega_n = 1.0$, $L/D = 0.5$, $\Delta = 0.10$, at $\omega/\omega_c = 2.0$ were obtained by integration from $(X_j/C, Y_j/C) = (0.01, 0.01)$. A converging trajectory was shown. 54
- Figure 3.5.13 c and d Journal orbit and time history for $B_p = 5.0$, $\Omega_n = 1.0$, $L/D = 0.5$, $\Delta = 0.10$, at $\omega/\omega_c = 2.0$ were obtained by integration from $(X_j/C, Y_j/C) = (0.1, 0.1)$. A diverging trajectory was shown. 54
- Figure 3.5.14.a and b Journal orbit and time history for $B_p = 1.0$, $\Omega_n = 1.0$, $\Delta = 0.1$, $L/D = 0.5$, at $\omega/\omega_c = 2.0$. An outward spiraling orbit is obtained integration from $(\Delta x_j, \Delta y_j) = (0.1, 0.1)$ 55
- Figure 3.5.14 c and d Journal orbit and time history for $B_p = 1.0$, $\Omega_n = 1.0$, $\Delta = 0.1$, $L/D = 0.5$, at $\omega/\omega_c = 2.0$. An inward spiraling orbit is obtained integration from $(\Delta x_j, \Delta y_j) = (0.005, 0.005)$. 56
- Figure 4.0.1 Hysteresis in the presence of several attractors. 57
- Figure 4.1.1 Conceptual Rotor-Bearing Instability Hysteresis Loop. 59
- Figure 4.3.1.a A stable node obtained integrating from $(\Delta X_j, \Delta Y_j) = (0.01, 0.01)$, for $B_p = 0.5$, $\Omega_n = 1.0$, $L/D = 0.5$, $\Delta = 0.1$, at $\omega/\omega_c = 1.90$. 64

Figure 4.3.1.b A stable limit is forming, that a diverging orbit is obtained integrating from $(\Delta x_f, \Delta y_f) = (0.05, 0.05)$ and a converging orbit is obtained integrating from $(\Delta x_f, \Delta y_f) = (0.10, 0.10)$, for $B_p = 0.5$, $\Omega_n = 1.0$, $L/D = 0.5$, $\Delta = 0.1$, $\omega/\omega_c = 2.0$.

64

Figure 4.3.1.c A spiraling outward orbit is obtained for $B_p = 0.5$, $\Omega_n = 1.0$, $L/D = 0.5$, $\Delta = 0.1$, at $\omega/\omega_c = 2.055$ integrating from $(\Delta x_f, \Delta y_f) = (0.1, 0.1)$.

65

Figure 4.3.1.d A spiraling outward orbit is obtained for $B_p = 0.5$, $\Omega_n = 1.0$, $L/D = 0.5$, $\Delta = 0.1$, at $\omega/\omega_c = 2.50$ integrating from $(\Delta x_f, \Delta y_f) = (0.1, 0.1)$

65

Figure 4.3.1.e A spiraling outward orbit is obtained for $B_p = 0.5$, $\Omega_n = 1.0$, $L/D = 0.5$, $\Delta = 0.1$, at $\omega/\omega_c = 4.00$ integrating from $(\Delta x_f, \Delta y_f) = (0.1, 0.1)$.

66

Figure 4.3.1.f An oil whip orbit is obtained for $B_p = 0.5$, $\Omega_n = 1.0$, $L/D = 0.5$, $\Delta = 0.1$, at $\omega/\omega_c = 2.00$ integrating from oil whip condition.

66

Figure 4.3.1.g A stable is obtained for $B_p = 0.5$, $\Omega_n = 1.0$, $L/D = 0.5$, $\Delta = 0.1$, at $\omega/\omega_c = 1.977$ integrating from oil whip condition.

67

Figure 4.3.2.a A stable node is obtained integrating from $(\Delta X_f, \Delta Y_f) = (0.01, 0.01)$, (for $B_p = 0.2$, $\Omega_n = 1.0$, $L/D = 0.5$, $\Delta = 0.1$), at $\omega/\omega_c = 1.90$.

68

Figure 4.3.2.b A stable limit cycle is forming, that a spiraling outward trajectory obtained integrating from $(\Delta x_f, \Delta y_f) = (0.005, 0.005)$ and a spiraling inward trajectory obtained integrating from $(\Delta x_f, \Delta y_f) = (0.01, 0.01)$, (for $B_p = 0.2$, $\Omega_n = 1.0$, $L/D = 0.5$, $\Delta = 0.1$), at $\omega/\omega_c = 1.923$.

68

Figure 4.3.2.c A stable big limit cycle is obtained, for $B_p=0.2$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$, at $\omega/\omega_c=1.928$, integrating from $(\Delta x_f, \Delta y_f) = (0.1, 0.1)$.

69

Figure 4.3.2.d A stable big limit cycle is obtained, for $B_p=0.2$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$, at $\omega/\omega_c=1.95$, integrating from $(\Delta x_f, \Delta y_f)=(0.1, 0.1)$.

69

Figure 4.3.2.e A stable big limit cycle is obtained, for $B_p=0.2$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$, at $\omega/\omega_c=2.00$, integrating from $(\Delta x_f, \Delta y_f)=(0.1, 0.1)$.

70

Figure 4.3.2.f A stable big limit cycle is obtained, for $B_p=0.2$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$, at $\omega/\omega_c=2.20$, integrating from $(\Delta x_f, \Delta y_f)=(0.1, 0.1)$.

70

Figure 4.3.2.g A stable big limit cycle is obtained, for $B_p=0.2$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$, at $\omega/\omega_c=2.50$, integrating from $(\Delta x_f, \Delta y_f)=(0.1, 0.1)$.

71

Figure 4.3.2.h An oil whip is obtained, for $B_p=0.2$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$, at $\omega/\omega_c=2.75$, integrating from $(\Delta x_f, \Delta y_f)=(0.1, 0.1)$.

71

Figure 4.3.2.i An oil whip occurred, for $B_p=0.2$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$, at $\omega/\omega_c=3.00$, integrating from $(\Delta x_f, \Delta y_f)=(0.1, 0.1)$.

72

Figure 4.3.2.j An oil whip at $\omega/\omega_c=4.00$, ($B_p=0.2$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$), as integrating from $(\Delta x_f, \Delta y_f)=(0.1, 0.1)$.

72

Figure 4.3.2.k An stable limit cycle is recovered, for $B_p=0.2$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$, at $\omega/\omega_c=2.2$, integrating from oil whip condition.

73

Figure 4.3.2.l An stable limit cycle is recovered, for $B_p=0.2$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$, at $\omega/\omega_c=1.95$, integrating from oil whip condition

73

Figure 4.3.2.m An stable limit cycle is recovered, for $B_p=0.2$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$, at $\omega/\omega_c=1.923$, integrating from oil whip condition.

74

Figure 4.3.2.k An stable limit cycle is recovered, for $B_p=0.2$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$, at $\omega/\omega_c=1.88$, integrating from oil whip condition

74

Figure 4.3.2.o The limit cycle disappears at $\omega/\omega_c=1.85$, for $B_p=0.2$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$, integrating from oil whip condition.

75

Figure 4.3.3.a A stable node obtained integrating from $(\Delta X_J, \Delta Y_J) = (0.01, 0.01)$, (for $B_p=0.15$, $\Omega_n=1.0$, $\Delta=0.1$, $L/D=0.5$), at $\omega/\omega_c=1.964$.

76

Figure 4.3.3.b An stable limit cycle is obtained, that an outward spiral progressing integrating from $(\Delta X_J, \Delta Y_J) = (0.01, 0.01)$, and an inward spiral developing integrating from $(\Delta X_J, \Delta Y_J) = (0.1, 0.1)$, (for $B_p=0.15$, $\Omega_n=1.0$, $\Delta=0.1$, $L/D=0.5$), at $\omega/\omega_c=2.039$.

76

Figure 4.3.3.c An oil whip is developed, integrating from $(\Delta X_J, \Delta Y_J) = (0.1, 0.1)$, (for $B_p=0.15$, $\Omega_n=1.0$, $\Delta=0.1$, $L/D=0.5$), at $\omega/\omega_c=2.048$.

77

Figure 4.3.3.d An oil whip is developed, integrating from $(\Delta X_J, \Delta Y_J) = (0.1, 0.1)$, (for $B_p=0.15$, $\Omega_n=1.0$, $\Delta=0.1$, $L/D=0.5$), at $\omega/\omega_c=4.0$.

77

- Figure 4.3.3 e A persisted oil whip is developed, integrating from oil whip condition, (for $B_p=0.15$, $\Omega_n=1.0$, $\Delta=0.1, L/D=0.5$), at $\omega/\omega_c=2.039$. 78
- Figure 4.3.3 f A big limit cycle is obtained, integrating from oil whip condition, (for $B_p=0.15$, $\Omega_n=1.0$, $\Delta=0.1, L/D=0.5$), at $\omega/\omega_c=1.823$. 78
- Figure 4.3.3 g A big limit cycle is obtained, integrating from oil whip condition, (for $B_p=0.15$, $\Omega_n=1.0$, $\Delta=0.1, L/D=0.5$), at $\omega/\omega_c=1.795$. 79
- Figure 4.3.3 h A big limit cycle is obtained, integrating from oil whip condition, (for $B_p=0.15$, $\Omega_n=1.0$, $\Delta=0.1, L/D=0.5$), at $\omega/\omega_c=1.767$ 79
- Figure 4.3.3 i Limit cycle disappears at $\omega/\omega_c=1.823$ as integrating from oil whip condition, (for $B_p=0.15$, $\Omega_n=1.0$, $\Delta=0.1, L/D=0.5$). 80
- Figure 4.3.4 a A stable node obtained integrating from $(\Delta X_J, \Delta Y_J) = (0.01, 0.01)$, (for $B_p=0.10$, $\Omega_n=1.0$, $\Delta=0.1, L/D=0.5$), at $\omega/\omega_c=2.318$ 81
- Figure 4.3.4 b An stable limit cycle is obtained, where an outward spiral progressing integrating from $(\Delta X_J, \Delta Y_J) = (0.01, 0.01)$, and an inward spiral developing integrating from $(\Delta X_J, \Delta Y_J) = (0.1, 0.1)$, (for $B_p=0.10$, $\Omega_n=1.0$, $\Delta=0.1, L/D=0.5$), at $\omega/\omega_c=2.358$. 81
- Figure 4.3.4 c Oil whip occurs, where an outward spiral progressively as integrating from $(\Delta X_J, \Delta Y_J) = (0.1, 0.1)$, (for $B_p=0.10$, $\Omega_n=1.0$, $\Delta=0.1, L/D=0.5$), at $\omega/\omega_c=2.372$. 82

- Figure 4.3.4.d Oil whip is developed as integrating from $(\Delta X_J, \Delta Y_J) = (0.1, 0.1)$, (for $B_p=0.10$, $\Omega_n=1.0$, $\Delta=0.1$, $L/D=0.5$), at $\omega/\omega_c=3.00$. 82
- Figure 4.3.4.e Oil whip is developed as integrating from $(\Delta X_J, \Delta Y_J) = (0.1, 0.1)$, (for $B_p=0.10$, $\Omega_n=1.0$, $\Delta=0.1$, $L/D=0.5$), at $\omega/\omega_c=4.00$. 83
- Figure 4.3.4.f An persisted oil whip is obtained as integrating from oil whip condition, (for $B_p=0.10$, $\Omega_n=1.0$, $\Delta=0.1$, $L/D=0.5$), at $\omega/\omega_c=2.318$. 83
- Figure 4.3.4.g A big limit cycle is obtained as integrating from oil whip condition, (for $B_p=0.10$, $\Omega_n=1.0$, $\Delta=0.1$, $L/D=0.5$), at $\omega/\omega_c=2.057$. 84
- Figure 4.3.4.h A big limit cycle is obtained as integrating from oil whip condition, (for $B_p=0.10$, $\Omega_n=1.0$, $\Delta=0.1$, $L/D=0.5$), at $\omega/\omega_c=1.947$. 84
- Figure 4.3.4.i A big limit cycle is obtained as integrating from oil whip condition, (for $B_p=0.10$, $\Omega_n=1.0$, $\Delta=0.1$, $L/D=0.5$), at $\omega/\omega_c=1.854$. 85
- Figure 4.3.4.j A big limit cycle is obtained as integrating from oil whip condition, (for $B_p=0.10$, $\Omega_n=1.0$, $\Delta=0.1$, $L/D=0.5$), at $\omega/\omega_c=1.801$. 85
- Figure 4.3.4.k A big limit cycle is obtained as integrating from oil whip condition, (for $B_p=0.10$, $\Omega_n=1.0$, $\Delta=0.1$, $L/D=0.5$), at $\omega/\omega_c=1.775$. 86

Figure 4.3.4.l A big limit cycle is obtained as integrating from oil whip condition, (for $B_p=0.10$, $\Omega_n=1.0$, $\Delta=0.1$, $L/D=0.5$), at $\omega/\omega_c=1.748$.

86

Figure 4.3.4.m Limit cycle disappears at $\omega/\omega_c=1.722$ as integrating from oil whip condition, (for $B_p=0.10$, $\Omega_n=1.0$, $\Delta=0.1$, $L/D=0.5$).

87

Figure 4.3.5.a A stable node obtained integrating from $(\Delta X_J, \Delta Y_J) = (0.01, 0.01)$, for $B_p=0.05$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$, at $\omega/\omega_c=3.364$.

88

Figure 4.3.5.b A stable limit is developed. A spiraling outward orbit obtained as integrating from $(\Delta X_J, \Delta Y_J) = (0.01, 0.01)$ and an inward spiraling orbit obtained as integrating from $(\Delta X_J, \Delta Y_J) = (0.1, 0.1)$, for $B_p=0.05$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$, at $\omega/\omega_c=3.479$.

88

Figure 4.3.5.c Oil whip occurs as integrating from $(\Delta X_J, \Delta Y_J) = (0.1, 0.1)$, for $B_p=0.05$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$, at $\omega/\omega_c=3.531$.

89

Figure 4.3.5.d Oil whip occurs as integrating from $(\Delta X_J, \Delta Y_J) = (0.1, 0.1)$, for $B_p=0.05$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$, at $\omega/\omega_c=4.00$.

89

Figure 4.3.5.e A persisted oil whip is obtained as integrating from oil whip condition and the rotating speed is reduced to $\omega/\omega_c=3.364$, for $B_p=0.05$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$.

90

Figure 4.3.5.f Oil whip is remaining as integrating from oil whip condition and the rotating speed is reduced to $\omega/\omega_c=2.058$, for $B_p=0.05$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$.

90

Figure 4.3.5.g A stable limit cycle is obtained as integrating from oil whip condition at $\omega/\omega_c=1.961$, for $B_p=0.05$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$.

91

Figure 4.3.5.h A stable limit cycle is obtained as integrating from oil whip condition at $\omega/\omega_c=1.811$, for $B_p=0.05$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$.	91
Figure 4.3.5.i A stable limit cycle is obtained as integrating from oil whip condition at $\omega/\omega_c=1.759$, for $B_p=0.05$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$.	92
Figure 4.3.5.j Big limit cycle disappears at $\omega/\omega_c=1.725$ integrating from oil whip condition , for $B_p=0.05$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$.	92
Figure 4.3.6.a Rotor free vibration response for $B_p=0.5$	93
Figure 4.3.6.b Rotor free vibration response for $B_p=0.2$	93
Figure 4.3.6.c Rotor free vibration response for $B_p=0.15$.	94
Figure 4.3.6.d Rotor free vibration response for $B_p=0.1$.	94
Figure 4.3.6.e Rotor free vibration response for $B_p=0.05$.	95
Figure 5.1.1.a Journal Orbit and Poincare' points.(for $B_p=0.2$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$), at $\omega/\omega_c=2.5$, $u/C=0.9$.	100
Figure 5.1.1.b Journal orbit and Poincare' points, (for $B_p=0.1$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$), at $\omega/\omega_c=1.95$, $u/C=0.5$.	100
Figure 5.1.1.c Journal orbit and Poincare' points, (for $B_p=0.05$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$), at $\omega/\omega_c=1.75$, $u/C=0.9$.	101
Figure 5.1.1.d Power Spectrum, (for $B_p=0.05$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$), at $\omega/\omega_c=1.75$, $u/C=0.9$	101
Figure 5.1.2.a Journal orbit, Poincare' map and power spectrum, (for $B_p=0.2$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$), at $\omega/\omega_c=1.95$, $u/C=0.9$.	103

Figure 5.1.2 b Journal orbit, Poincare' map and power spectrum, (for $B_p=0.1$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$), at $\omega/\omega_c=1.75$, $u/C=0.9$.	104
Figure 5.1.3 An Oil Whip orbit, (for $B_p=0.1$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$), at $\omega/\omega_c=2.20$, $u/C=0.3$	105
Figure 5.1.4 An Oil Whip orbit, (for $B_p=0.05$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$), at $\omega/\omega_c=3.0$, $u/C=0.3$.	105
Figure 5.1.5 Bifurcation diagram, $B_p=0.005$, $\Omega_n=5.0$, $L/D=0.5$, $\Delta=0.1$, $\omega/\omega_c=2.15$	110
Figure 5.1.6.a and b Journal orbit and Poincare' map of journal orbit, $B_p=0.005$, $\Omega_n=5.0$, $L/D=0.5$, $\Delta=0.1$, $\omega/\omega_c=2.15$, $u/C=0.134$.	111
Figure 5.1.6.c Power spectrum for $B_p=0.005$, $\Omega_n=5.0$, $L/D=0.5$, $\Delta=0.1$, $\omega/\omega_c=2.15$, $u/C=0.134$.	111
Figure 5.1.6.d and e Journal orbit and Poincare' map, $B_p=0.005$, $\Omega_n=5.0$, $L/D=0.5$, $\Delta=0.1$, $\omega/\omega_c=2.15$, $u/C=0.14$.	112
Figure 5.1.7.a Poincare' points and trajectory of the center of journal, $B_p=0.005$, $\Omega_n=5.0$, $L/D=0.5$, $\Delta=0.1$, $\omega/\omega_c=2.15$, $u/C=0.144$.	113
Figure 5.1.7.b Power spectrum, for $B_p=0.005$, $\Omega_n=5.0$, $L/D=0.5$, $\Delta=0.1$, at $\omega/\omega_c=2.15$, $u/C=0.144$.	113
Figure 5.1.8.a and b Journal orbit and Poincare' map, for $B_p=0.005$, $\Omega_n=5.0$, $L/D=0.5$, $\Delta=0.1$, at $\omega/\omega_c=2.15$, $u/C=0.15$, $Tol = 10^{-5}$.	114
Figure 5.1.8.c Linear averaging power spectrum.	114

Figure 5.1.8 d Poincare' map of the center of the journal , for Tol = 10^{-6} .	114
Figure 5.2.1 a The logarithm of the correlation integral $\text{Log } C(\gamma, \tau, m)$ as $N=2000$, $\gamma = 0.5$ and $dt = \pi/30$, Tol = 10^{-5} .	120
Figure 5.2.1 b The logarithm of the correlation integral $\text{Log } C(\gamma, \tau, m)$ as a function of the number N of used data points. Also $\gamma = 0.5$, $m = 3$ and $dt = \pi/30$, Tol = 10^{-5}	120
Figure 5.2.2 Estimated correlation dimension D2, for $B_p=0.005$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$, at $\omega/\omega_c=2.15$, $u/C=0.15$, and Tol = 10^{-5} . diamond: $N=8192$, triangular : $N=16384$, dot: $N=32768$.	121
Figure 5.2.3 Max. Lyapunov exponent, for $B_p=0.005$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$, at $\omega/\omega_c=2.15$, $u/C=0.15$, and Tol = 10^{-5} .	121
Figure 5.2.4 The logarithm of the correlation integral $\text{Log } C(\gamma, \tau, m)$ as $N=2000$, $\gamma = 0.5$ and $dt = \pi/20$, for $B_p=0.005$, $\Omega_n=5.0$, $L/D=0.5$, $\Delta=0.1$ at $\omega/\omega_c=2.15$ and $u/C=0.15$, Tol = 10^{-6} .	122
Figure 5.2.5 Estimated correlation dimension D2, for $B_p=0.005$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$, at $\omega/\omega_c=2.15$, $u/C=0.15$, and Tol = 10^{-6} . diamond: $N=8192$, triangular : $N=16384$, dot: $N=32768$.	122
Figure 5.2.6 Max. Lyapunov exponent, for $B_p=0.005$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$, at $\omega/\omega_c=2.15$, $u/C=0.15$, and Tol = 10^{-6} .	123
Figure 6.1.1 Basic Rotor Kit.	126
Figure 6.1.2 Oil Whirl Option Setup.	126
Figure 6.1.3 a General Arrangement of the Hysteresis Loop Test Rig.	127
Figure 6.1.3 b Cross-Section of Preload Unit.	128

Figure 6.1.4 Preload Bearing.	128
Figure 6.1.2.1 Proximity Calibration Curves.	130
Figure 6.1.3.1 Spring Calibration Curve.	132
Figure 6.2.1 Schematic configuration of the rotor-bearing system.	134
Figure B.1 Root locus with Hopf Bifurcation.	151
Figure B.2 2-Dimensional Hopf bifurcation phase space.	152
Figure C.1 Predator-Prey Results	156
Figure C.2 a Exact solution of system	157
Figure C.2 b The Relationship between Controlled Tolerance and Adaptive Time Marching Step-Size	158
Figure D.1 Journal vibration response for 3.459 lb preload at speed up about 4,110 rpm.	159
Figure D.2 Journal orbit and vibration response for 3.459 lb preload at instability threshold speed about 4,170 rpm.	159
Figure D.3 Journal orbit and vibration response for 3.459 lb preload at speed up about 4,500 rpm.	160
Figure D.4 Journal orbit and vibration response for 3.459 lb preload at coast down about 4,085 rpm.	160
Figure D.5 Journal orbit and vibration response for 3.459 lb preload at speed up about 3,800 rpm.	161

Figure D.6 Journal orbit and vibration response for 3.459 lb preload at Saddle-Node speed about 3,710 rpm.	161
Figure D.7 Journal orbit and vibration response for 3.904lb preload at speed up about 4,330 rpm.	162
Figure D.8 Journal orbit and vibration response for 3.904lb preload at instability threshold speed about 4,360 rpm.	162
Figure D.9 Journal orbit and vibration response for 3.904lb preload at speed up about 5,060 rpm.	163
Figure D.10 Journal orbit and vibration response for 3.904lb preload at coast down about 4,210 rpm.	163
Figure D.11 Journal orbit and vibration response for 3.904lb preload at speed up about 3,900 rpm.	164
Figure D.12 Journal orbit and vibration response for 3.904lb preload at Saddle-Node speed about 3,810 rpm.	164
Figure D.13 Journal orbit and vibration response for 4.349 lb preload at speed up about 4,770 rpm.	165
Figure D.14 Journal orbit and vibration response for 4.349 lb preload at instability threshold speed at about 4,800 rpm.	165
Figure D.15 Journal orbit and vibration response for 4.349 lb preload at speed up about 5,500 rpm	166
Figure D.16 Journal orbit and vibration response for 4.349 lb preload at coast down about 4,700 rpm.	166

Figure D.17 Journal orbit and vibration response for 4.349 lb preload at coast down about 4,400 rpm.	167
Figure D.18 Journal orbit and vibration response for 4.349 lb preload at coast down about 4,100 rpm.	167
Figure D.19 Journal orbit and vibration response for 4.349 lb preload at coast down about 4,000 rpm.	168
Figure D.20 Journal orbit and response for 4.794 lb preload at speeding up about 4,930 rpm.	168
Figure D.21 Journal orbit and response for 4.794 lb preload at instability threshold speed about 5,020 rpm.	169
Figure D.22 Journal orbit and response for 4.794 lb preload at speeding up about 5,570 rpm.	169
Figure D.23 Journal orbit and response for 4.794 lb preload at coast down about 4,820 rpm.	170
Figure D.24 Journal orbit and response for 4.794 lb preload at coast down about 4,385 rpm.	170
Figure D.25 Journal orbit and response for 4.794 lb preload at coast down about 4,100rpm.	171
Figure D.26 Oil whirl disappeared at Saddle-Node Speed bout 4,080rpm.	171

List of Photographs

	Page
Photo. 1 Hysteresis Loop Test Rig.	174
Photo. 2 Preload Bearing and Spring	175
Photo. 3 1 x 4 Frame.	176
Photo. 4 Two-in-One oil collector, oil reservoir and bulb pump.	177
Photo. 5 Real-Time monitoring system	178
Photo. 6 Data acquisition system.	179

List of Tables

	Page
Table 3.5.1 Linear instability threshold for $\Omega_n = 1.0$, $L/D=0.5$, $\Delta = 0.1$.	38
Table 3.5.2 Summary of Linear Analysis for $\Omega_n = 1.0$, $L/D=0.5$, $\Delta = 0.1$.	39
Table 4.3.1 Summary of results of the hysteresis loop computation for system parameters, $\Omega_n = 1.0$, $L/D=0.5$, $\Delta=0.1$	63
Table 5.1.1 a Forcing response for $B_p=0.2$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$ with linear instability threshold $\omega_{th}/\omega_c=1.92$ and Saddle-Node speed $\omega_{sn}/\omega_c=1.85$.	108
Table 5.1.1 b Forcing response for $B_p=0.1$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$ with linear instability threshold $\omega_{th}/\omega_c=2.36$ and Saddle-Node speed $\omega_{sn}/\omega_c=1.72$.	108
Table 5.1.1 c Forcing response for $B_p=0.05$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$ with linear instability threshold $\omega_{th}/\omega_c=3.48$ and Saddle-Node speed $\omega_{sn}/\omega_c=1.73$.	108
Table 5.1.2 Forcing response for $B_p=0.005$, $\Omega_n=5.0$, $L/D=0.5$, $\Delta=0.1$ with linear instability threshold $\omega_{th}/\omega_c=19.12$.	109
Table 5.1.3 Birth of chaos, $B_p=0.005$, $\Omega_n=5.0$, $L/D=0.5$, $\Delta=0.1$, $\omega = 2.15 \omega_c$.	109
Table 6.4.1 System parameters for different loading conditions.	137
Table 6.4.2 Summary of experimental results	137
Table A.1 Eigenvalues comparison between QR method and Mathematica computation.	149

Chapter 1

Introduction

1.0 Background

Rotors supported in fluid film journal bearings can develop an instability known as oil-whip which may occur when the rotating speed is above the associated instability threshold speed. This instability manifests itself by the growth of a co-rotational rotor orbital vibration about the static equilibrium position, at a sub-rotative frequency, i.e., a self-excited vibration. Energy is transferred by the fluid-film forces to the rotor to sustain this orbital vibration in a stable nonlinear limit cycle. Such vibration is detrimental to rotating machinery and requires elimination, hopefully at the design stage, but sometimes in the field.

In a 1988 I. Mech. E. international conference Keynote Address paper, Hori (1988) dealt with several aspects of large seismic-induced vibrations of large rotating machinery, including a triggering of the stable nonlinear limit-cycle that exists within the upper and lower speed boundaries of the hysteresis loop associated with the classical rotor-bearing dynamic instability problem (see Fig. 1.0.1). For example, imagine a large steam turbine-generator running at its 3,000 or 3,600 rpm synchronous 2-pole operating speed, and which has an instability threshold speed (Hopf bifurcation) at say 4,200 rpm, faster than the machine ever operates, even at over speed condition. A high intensity earthquake occurs, which by itself does not produce high enough amplitude oscillations to cause machine

damage. However, a Saddle-Node speed, unknown to the designers, is at say 2,450 rpm and the initial seismic disturbance is sufficient to "kick" the system up to the stable nonlinear limit cycle. The resulting high amplitude rotor vibration could then very well produce a condition where the machine is essentially destroyed during the couple of minutes it takes to coast down below the 2,450 rpm Saddle-Node speed. In fact, most modern two-pole steam turbogenerators do operate below a small-perturbation instability threshold speed and above the Saddle-Node speed. When these machines were designed and built, such a scenario was not considered. In Japan, a highly active seismic region, this scenario has now received serious attention.

Figure 101 Conceptual Hysteresis Loop of Rotor-Bearing System

1.1 Literature Review

"Shaft Whipping", called by shop persons and engineering in General Electric Company, phenomenon was first reported by Newkirk (1924). The shaft was observed to vibrate at a sub-rotative (subsynchronous) frequency which in some instances could be followed by the eye. After performing many tests, they found that the whipping was independent of the unbalance and in some cases would build up of its own accord, and that in other cases a shock was necessary to start it. The whipping would not happen when the rotor was running below its critical speed (lowest natural frequency). It was found that damping resistance to the vibration applied at any point along the shaft or at the bearings would suppress the whip, indicating that the basic phenomena was a dynamic instability, not a forced vibration.

A violent whip of shafts due to "the action of oil film in the bearing" was first reported by Newkirk and Taylor in 1925. It was actually discovered accidentally that the whipping was stopped immediately by shutting off the oil supply to the bearing and could be brought back to full amplitude immediately by turning on the oil again. They called it the "Oil Whip" to distinguish it from whipping due to cramping, as shown in Newkirk (1924). They found that the whipping that took the same direction as the rotation started at speeds higher than twice the critical speed and persisted at even higher speeds, but did not occur at heavy bearing loading conditions. They also reported that the shaft could be made to whip by subjecting it to a shock. They gave a qualitative theory for this phenomenon, and moreover, provided several means to avoid this trouble, such as confining

operation to a speed range below twice critical speed, or by the use of a friction - damped bearing, or by avoiding very light bearing loading.

Hagg (1946) developed the stability criteria by analyzing the characteristic equation for certain common rotor-bearing systems. He found that even though the natural frequency of a vertical rotor with unloaded journal bearings is zero, the driving mechanism is sufficiently strong to produce whirling near the upper limit of frequency, i.e., one-half running speed. He also observed that an eccentric running of the unloaded vertical rotor system may in some cases eliminate whirling. For the horizontal rotor, the upper limit of whirl frequency is also about one-half running speed with lightly loaded bearings and this factor could be less than 1/2 for more heavily loaded bearings. Poritsky (1953) was able to show, by introducing an additional radial force to the oil-film, that the instability occurs only at speeds exceeding twice the first critical speed which, however, might decrease even to zero depending upon the oil-film condition.

Pinkus (1953 and 1956) performed very extensive experiments and reported that in a number of cases a phenomenon that might be called an inertia effect in which it was observed and that the stable and unstable states were separated by a region of transient whip. Also, he observed even more complicated cases in which the whippings disappeared and resume again. But he did not find the whip, such as reported by Newkirk and Taylor (1925), initiated by a sudden shock. The effect of oil-inlet temperature on oil whip also was investigated and he observed that some of the test bearings whipped at higher inlet-oil temperatures had a narrow whipping range at intermediate temperatures, and did not whip at all on cold oil.

Newkirk and Lewis (1956) found that the rotor could safely operate at speeds above twice the critical speed in various bearing configurations. In their experiments, an instability threshold speed as high as 6 times the critical speed was produced, although in most cases operation was not stable at speeds much above twice critical speed. They also found that for any given rotor-bearing combination, the static eccentricity increases and the unstable speeds increase as the oil viscosities decrease, which is consistent with present time understanding of this field

Hori (1959) investigated the oil whip phenomenon theoretically and experimentally. The vibrations of the rotor are divided into two parts, i.e., "small" vibrations and "large" vibrations. For "small" vibrations, he showed that the shaft is always stable if the displacement of the journal center from the bearing center is more than approximately 80 per cent of the radial clearance of the bearing, or if the vertical displacement of the journal center from its lowest position is less than 50 per cent of the clearance. Therefore, large static eccentricity will be favorable for stable operation. For "large" vibrations, he concluded that a large vibration of the rotor can continue to exist only when the speed of rotation of the shaft is higher than twice its critical speed. He also found that the oil whip, for light and medium loaded cases, occurs only when the rotational speeds are above twice the critical. While for heavily loaded bearings, the oil-whip threshold speed could occur at the rotating speeds considerably above twice the critical speed. Moreover, the oil whip will remain even when decreasing the rotating speed below the threshold speed, until it transverses twice the critical speed, i.e., hysteresis loop exists in heavily loaded bearing systems. He also pointed out that lower viscosity gives greater

stability for small vibration and higher viscosity is effective in depressing the oil whip after it once develops.

Jennings and Ocvirk (1962) used an electronic-analog computer to solve the nonlinear dynamic equations of a rigid rotor-bearing system in which the Ocvirk's short-bearing approximation was used for the fluid-film pressure distribution and they were able to simulate the bearing whirl behavior by tracing the journal center. They reported that stability is independent of the initial conditions, the orbit size increases as the speed of the journal is increased, the size of the orbit is not determined by the static equilibrium position, the orbit size becomes significantly larger as the mass of the journal is increased, and the whirl frequency is approximately half journal speed. This infers the existence of the stable limit cycle of the rotor-bearing system.

Lund and Saibel (1967) used the "method of averaging" to compute the size, orientation and stability of the limit cycle of a journal bearing supported rotor-bearing system, and the π -film Ocvirk short bearing approximation was applied for the fluid film force. They found that small stable self-excited whirl orbits only exist in a narrow speed range above the linear instability threshold speed under certain operating conditions. Hence, it is not easy to investigate a stable small whirl orbit in experiments.

Adams (1980) developed a general analysis to computer simulate steady state and transient vibration phenomena of complex rotor-bearing supported systems.

He showed a nonlinear jump phenomenon leading to a large amplitude $1/3$ subharmonic, initiated by large unbalance.

Hopf bifurcation analysis was used by Myers (1984) and Hollis and Taylor (1986) to investigate the self-excited periodic solutions of the nonlinear equations of motion for rotor speeds close to the instability threshold speed. Hollis and Taylor were able to find the supercritical and subcritical limit cycle using the BIFOR2 software (referred to Hassard et al 1981)

Crooijmans et al. (1990) presented an analysis of the self-excited vibrations of a journal carried in a cylindrical fluid film bearing. Solutions of the nonlinear system equations were obtained by time discretization and by an arc-continuation method for solving the obtained nonlinear algebra equations. A limit cycle was obtained, at near the linear stability threshold, as a function of system parameters. This stable limit cycle jumped to a big orbit when the linear stability threshold was exceeded

An earthquake-induced instability of an oil film bearing supported rotor system was studied by Hori (1988) and Hori and Kato (1990), using the Runge-Kutta-Gill method to integrate the nonlinear equations of motion. They showed that a linearly stable rotor system could become unstable under a strong shock or seismic wave, if the system is running at speeds above twice the first critical speed. They pointed out that the linear analysis is insufficient to examine the stability of a rotor-bearing system if the rotor is going to be operated above twice the critical

speed and a strong shock such as due to an earthquake or turbine blade loss occurs.

Recently, Khonsari and Chang (1993) showed through their computations that there exists a closed initial position boundary within which the system is stable (i.e., decays to the static equilibrium state) and outside which a stable large-amplitude nonlinear limit cycle occurs, even at rotating speeds below the linear instability threshold speed. The results showed that, for a given Sommerfeld number, the area of the region is reduced as the operating speed is increased. Above the threshold speed, the stability region reduces to a very small circle (nearly to a point) which corresponds to the equilibrium point. Additional relevant papers are also listed in the references.

1.2 Objectives and Scope of the Work

There are three main objectives in this work. The first is to computationally characterize the rotor-bearing instability hysteresis loop more comprehensively than prior investigations. The second is to computationally explore a safe means of determining the lower speed bound of the hysteresis loop on actual running machines, such as large steam turbogenerators. To fully appreciate this second objective, it is important to realize that computational approaches would be unreliable, due to machine complexities and the machine - to - machine variations between so called "identical machines". Furthermore, it is obviously not advisable, to say the least, that such machines be intentionally pushed into the stable nonlinear

limit cycle (e.g. via an imposed large perturbation), simply to determine on subsequent coast-down the speed at which the limit cycle "kicks out". The approach here is to explore the degree to which chaos signal processing on speed-up and coast-down might be used to expose the hysteresis loop lower speed bound (Saddle-Node speed). The third objective is to experimentally investigate the oil-whip hysteresis loop on a laboratory test rig and compare the experimental and simulation results

To accomplish these three objectives, the following approach was taken. First, the equations of motion were developed for a two-bearing symmetrical flexible rotor (Chapter 2). Second, linear stability analysis (Chapter 3) was then performed numerically to the linearized system equations in which the linear dynamical coefficients of the journal bearing were calculated and used. Then the nonlinear hysteresis loop of the system is obtained (Chapter 4) by integrating the full nonlinear equations of motion which incorporate the Reynolds lubrication equation. Superimposed rotor unbalance response within the hysteresis loop was then studied (Chapter 5). An experiment was designed and implemented to obtain test results of a journal-bearing supported rotor test rig (Chapter 6).

Chapter 2

Mathematical Model

2.0 Description of the Rotor-Bearing system

The rotor-bearing system considered here is shown as Figure 2.0.1, and consists of a central disk of mass M_D , located at the mid-span of a flexible massless shaft with stiffness K_S , supported at each end by two identical journal bearings, and with a point mass $M_J/2$ at each end of the shaft. The elastic force in one half of the flexible shaft and the hydrodynamic fluid film force between the journal and bearing are applied to each point mass, and the reaction force of the shaft on the disk produces the acceleration of its center of mass according to Newton's law. The geometry and coordinates are defined as following,

- O_B is the origin of the fixed Cartesian coordinates (X, Y, Z) with unit vectors $(\underline{e}_x, \underline{e}_y, \underline{e}_z)$. O_B is also the center of the bearing, (i.e., we assume that bearing is fixed).
- O_J is the center of journal with coordinates (X_J, Y_J) .
- O_D is the geometric center of disk with coordinates (X_D, Y_D) .
- O_G is the gravitational center of disk with coordinates (X_G, Y_G) .
- ϵ is the journal eccentricity in the bearing.
- δ is the shaft static deflection.
- u is the static imbalance of disk.
- ω is the rotating speed of shaft.
- ϕ is the attitude angle of the journal bearing.

Figure 2.0.1 Geometry and coordinate system of the rotor-bearing system

2.1 Equations of Motion

Applying mode shape symmetry, motion is described by 4 degree of freedom and can be derived by Newton's Law as follows.

The position vector of the gravitational center of the disk is $\vec{r}_G$ and

$$\vec{r}_G = X_G \underline{e}_x + Y_G \underline{e}_y = (X_D + u \sin(\omega t)) \underline{e}_x + (Y_D - u \cos(\omega t)) \underline{e}_y$$

Then, the velocity vector $\dot{\vec{r}}_G$ is,

$$\dot{\vec{r}}_G = (\dot{X}_D + u \omega \cos(\omega t)) \underline{e}_x + (\dot{Y}_D + u \omega \sin(\omega t)) \underline{e}_y$$

and the acceleration vector $\ddot{\vec{r}}_G$ will be

$$\ddot{\vec{r}}_G = (\ddot{X}_D - u \omega^2 \sin(\omega t))\underline{e}_x + (\ddot{Y}_D + u \omega^2 \cos(\omega t))\underline{e}_y$$

Similarly, the position, velocity and acceleration vectors of the journal are

$$\vec{r}_J = X_J \underline{e}_x + Y_J \underline{e}_y$$

$$\dot{\vec{r}}_J = \dot{X}_J \underline{e}_x + \dot{Y}_J \underline{e}_y$$

$$\ddot{\vec{r}}_J = \ddot{X}_J \underline{e}_x + \ddot{Y}_J \underline{e}_y$$

Also the position vector of the center of the disk is $\vec{r}_D$ and

$$\vec{r}_D = X_D \underline{e}_x + Y_D \underline{e}_y$$

Then, according to Newton's Law we have the following equations of motion, for the disk,

$$\frac{M_D}{2} \ddot{\vec{r}}_G = \sum F_D = -\frac{K_s}{2} (\vec{r}_D - \vec{r}_J) - \frac{M_D}{2} g \underline{e}_y$$

and for the journal,

$$\frac{M_J}{2} \ddot{\vec{r}}_J = \sum F_J = -\frac{K_s}{2} (\vec{r}_J - \vec{r}_D) + F_x \underline{e}_x + F_y \underline{e}_y - \frac{M_J}{2} g \underline{e}_y$$

After substituting those positions and acceleration vectors into the above equations, one obtains the following equations of motion.

For the disk we have,

$$\frac{M_D}{2} \ddot{X}_D + \frac{K_s}{2} (X_D - X_J) = \frac{M_D}{2} u \omega^2 \sin(\omega t) \quad (2.1.1)$$

$$\frac{M_D}{2} \ddot{Y}_D + \frac{K_s}{2} (Y_D - Y_J) = -\frac{M_D}{2} u \omega^2 \cos(\omega t) - \frac{M_D}{2} g \quad (2.1.2)$$

and

for the journal we have,

$$\frac{M_J}{2} \ddot{X}_J - F_x - \frac{K_s}{2}(X_D - X_J) = 0 \quad (2.1.3)$$

$$\frac{M_J}{2} \ddot{Y}_J - F_y - \frac{K_s}{2}(Y_D - Y_J) = - \frac{M_J}{2} g \quad (2.1.4)$$

where, F_x and F_y are the X and Y components of the hydrodynamic force of the journal bearing obtained by solving the Reynolds lubrication equation.

Equations (2.1.1) to (2.1.4) are the governing equations of the rotor-bearing system. Also the center of the journal can be represented by the eccentricity of the journal as,

$$X_J = \varepsilon \cos(\phi) \quad (2.1.5)$$

$$Y_J = \varepsilon \sin(\phi) \quad (2.1.6)$$

where, ϕ is the attitude angle of the journal.

2.2 Non-Dimensionalization of the Equations of Motion

The equations of motion can be non-dimensionalized as follows. Choose the mean radial clearance of journal and bearing, C , as the characteristic length and τ as the non-dimensional time scale. Then the non-dimensional quantities as follows.

$$X_D = \frac{X_D}{C}, Y_D = \frac{Y_D}{C}, X_J = \frac{X_J}{C}, Y_J = \frac{Y_J}{C}, u = \frac{u}{C}, \tau = \omega t$$

$$\frac{d(\)}{dt} = \frac{d(\)}{d\tau} \frac{d\tau}{dt} = \omega \frac{d(\)}{d\tau} \quad \text{and}$$

$$\frac{d}{dt} \left(\frac{d(\)}{dt} \right) = \frac{d}{d\tau} \left(\frac{d(\)}{d\tau} \frac{d\tau}{dt} \right) = \frac{d}{d\tau} \left(\omega \frac{d(\)}{d\tau} \right) = \omega^2 \frac{d^2(\)}{d\tau^2}$$

$$\text{also let } (\dot{\ }) = \frac{d}{dt} \quad \text{and} \quad (\prime) = \frac{d}{d\tau}$$

Then equation (2.1.1) becomes

$$M_D C \omega^2 X_D'' + K_s C (X_D - X_J) = M_D C \omega^2 u \sin(\tau) \quad (2.2.1)$$

Set the bearing load W as,

$$W = \frac{(M_D + M_J)}{2} g.$$

the nondimensional mass of disk M_D as

$$M_D = \frac{M_D C \omega^2}{W}$$

and the dimensionless stiffness of rotor K_s , as

$$K_s = \frac{K_s C}{W}$$

Eqn. (2.2.1) divides W and also substitutes M_D and K_s , then we have

$$M_D \ddot{X}_D + K_s (X_D - X_J) = M_D u \sin(\tau) \quad (2.2.2)$$

The natural frequency of rotor is ω_n and

$$\omega_n = \sqrt{\frac{K_s}{M_D}}$$

Also define the speed ratio Ω as

$$\Omega = \frac{\omega}{\omega_n}$$

$$\text{Then } \frac{K_s}{M_D} = \frac{K_s}{M_D \omega^2} = \frac{\omega_n^2}{\omega^2} = \frac{1}{\Omega^2}$$

Finally, the X component of the equations of motion for the disk, Eqn. (2.2.2), can be simplified as

$$\ddot{X}_D + \frac{1}{\Omega^2} (X_D - X_J) = u \sin(\tau) \quad (2.2.3)$$

Similarly, equations (2.1.2) to (2.1.6) non-dimensionalize as follows,

$$\ddot{Y}_D + \frac{1}{\Omega^2} (Y_D - Y_J) = -u \cos(\tau) - \frac{1}{\Omega^2 \Omega_n^2} \quad (2.2.4)$$

$$\ddot{X}_J - \frac{2f_x}{M_J} - \frac{1}{\Delta \Omega^2} (X_D - X_J) = 0 \quad (2.2.5)$$

$$Y_j'' - \frac{2f_r}{M_j} - \frac{1}{\Delta\Omega^2} (Y_D - Y_j) = -\frac{1}{\Omega^2 \Omega_n^2} \quad (2.2.6)$$

$$X_j = \kappa \cos(\phi) \quad (2.2.7)$$

$$Y_j = \kappa \sin(\phi) \quad (2.2.8)$$

where,

$$M_j = \frac{M_i C \omega^2}{W} \quad \text{is the non-dimensional mass of journal.}$$

$$\Omega_n = \omega_n \sqrt{\frac{C}{g}} \quad \text{is the stiffness parameter of rotor.}$$

$$f_x = \frac{F_x}{W}, \quad f_r = \frac{F_y}{W} \quad \text{is the dimensionless film force.}$$

$$\Delta = \frac{M_i}{M_D} \quad \text{is the point mass ratio of journal and mass.}$$

$$\kappa = \frac{\varepsilon}{C} \quad \text{is the the eccentricity ratio of journal.}$$

Equations (2.2.3) to (2.2.6) are then the non-dimensional governing equations of the system.

2.3 Hydrodynamic Fluid Film Force

The bearing geometry and coordinates refer to Figure 2.3.1.

Figure 2.3.1 Geometry and coordinates of the journal bearing

The classical Reynolds lubrication equation for incompressible laminar isoviscous films is given as follows.

$$\frac{\partial}{\partial \eta} \left[\frac{h^3}{\mu} \frac{\partial P}{\partial \eta} \right] + \frac{\partial}{\partial Z} \left[\frac{h^3}{\mu} \frac{\partial P}{\partial Z} \right] = 6 \frac{\partial}{\partial \eta} [hU] + 12 \frac{dh}{dt} \quad (2.3.1)$$

where,

Z is the axial coordinate.

η is the circumferential coordinate $= R\theta$.

h is the local film thickness.

dh/dt is the instantaneous local rate of change in h .

U is the sliding velocity $= R\omega$.

L is the length of bearing.

R is the mean radius of the bearing.

and

$$h = C - X_j \cos(\theta) - Y_j \sin(\theta) \quad (2.3.2)$$

$$\frac{\partial h}{\partial \eta} = \frac{1}{R} \frac{\partial h}{\partial \theta} = \frac{1}{R} [X_j \sin(\theta) - Y_j \cos(\theta)] \quad (2.3.3)$$

$$\frac{dh}{dt} = -\dot{X}_j \cos(\theta) - \dot{Y}_j \sin(\theta) \quad (2.3.4)$$

where (X_j, Y_j) are the inertial coordinates of the journal referenced to the bearing center. The Reynolds lubrication equation is non-dimensionalized as follows.

Let

$$\zeta = \frac{\eta}{C}, \quad z = \frac{Z}{L}, \quad h = \frac{h}{C} \quad \text{and} \quad \tau = \omega t$$

Then the non-dimensional Reynolds lubrication equation becomes

$$\frac{1}{C} \frac{\partial}{\partial \zeta} \left[\frac{h^3 C^3}{\mu} \frac{1}{C} \frac{\partial P}{\partial \zeta} \right] + \frac{1}{L} \frac{\partial}{\partial z} \left[\frac{h^3 C^3}{\mu} \frac{1}{L} \frac{\partial P}{\partial z} \right] = 6 \frac{1}{C} \frac{\partial}{\partial \zeta} [hCU] + 12\omega \frac{d(hC)}{d\tau} \quad (2.3.5)$$

and simplified as follows.

$$C \frac{\partial}{\partial \zeta} \left[\frac{h^3}{\mu} \frac{\partial P}{\partial \zeta} \right] + \frac{C^3}{L^2} \frac{\partial}{\partial z} \left[\frac{h^3}{\mu} \frac{\partial P}{\partial z} \right] = 6 \frac{\partial}{\partial \zeta} [hU] + 12\omega C \frac{d(h)}{d\tau} \quad (2.3.6)$$

Define the dimensionless fluid film pressure, sliding velocity and radius of bearing as follows.

$$p = \frac{P}{\mu N} \left(\frac{C}{L}\right)^2, \quad = \frac{U}{R^* (2\pi N)}, \quad r = \frac{R}{C}$$

where, N is the rotating speed of shaft (rps).

Then the non-dimensional Reynolds lubrication equation becomes.

$$\frac{\partial}{\partial \zeta} \left[h^3 \frac{\partial p}{\partial \zeta} \right] + \left(\frac{C}{L}\right)^2 \frac{\partial}{\partial z} \left[h^3 \frac{\partial p}{\partial z} \right] = \left(\frac{C}{L}\right)^2 \left[12 \pi U r \frac{\partial h}{\partial \zeta} + 24 \pi \frac{d(h)}{d\tau} \right] \quad (2.3.7)$$

with

$$h = 1 - X_J \cos(\theta) - Y_J \sin(\theta) \quad (2.3.8)$$

$$\frac{\partial h}{\partial \zeta} = \frac{1}{r} \frac{\partial h}{\partial \theta} = \frac{1}{r} [X_J \sin(\theta) - Y_J \cos(\theta)] \quad (2.3.9)$$

$$\frac{dh}{d\tau} = -X_J' \cos(\theta) - Y_J' \sin(\theta) \quad (2.3.10)$$

An efficient and accurate approximate solution method for the Reynolds equation is described by Adams et al., (1982,1983), and is applied in this work to solve this 2-D transient partial differential equation.

Assume an axially symmetric axial pressure distribution at every circumferential location as,

$$P(\theta, Z) = P(\theta, 0) \left(1 - \frac{4Z^2}{L^2} \right)$$

or in non-dimensional form as,

$$p(\theta, z) = p(\theta, 0) (1 - 4z^2)$$

which satisfies the boundary condition $p(\theta, -\frac{1}{2}) = p(\theta, \frac{1}{2}) = 0$.

Then, the partial differential equation can be reduced to an ordinary differential equation which is,

$$\frac{d}{d\zeta} \left[h^3 \frac{dp}{d\zeta} \right] = \left(\frac{C}{L} \right)^2 \left[12\pi l r \frac{d(h)}{d\zeta} + 24\pi \frac{d(h)}{d\tau} + 8ph^3 \right] \quad (2.3.11)$$

This is actually a first-order Fourier series approximation using the parabola as the single approximative function.

Then, using a 3-point central finite difference approach to discretize (2.3.11). The first and second order derivatives are expressed as following.

$$\left(\frac{dp}{d\zeta} \right)_i \cong \frac{p_{i+1} - p_{i-1}}{2\Delta\zeta}$$

$$\left(\frac{d^2p}{d\zeta^2} \right)_i \cong \frac{p_{i+1} - 2p_i + p_{i-1}}{\Delta\zeta^2}$$

So, the difference equation becomes,

$$h_i^3 \left(\frac{p_{i+1} - 2p_i + p_{i-1}}{(\Delta\zeta)^2} \right) + 3h_i^2 \frac{dh_i}{d\zeta} \left(\frac{p_{i+1} - p_{i-1}}{2\Delta\zeta} \right) = \left(\frac{C}{L} \right)^2 \left[12\pi U r \frac{dh_i}{d\zeta} + 24\pi \frac{dh_i}{d\tau} + 8p_i h_i^3 \right]$$

and after gathering, we have,

$$p_{i+1} \left[\frac{h_i^3}{(\Delta\zeta)^2} + \frac{3h_i^2}{2\Delta\zeta} \frac{dh_i}{d\zeta} \right] + p_i \left[-\frac{2h_i^3}{(\Delta\zeta)^2} - \left(\frac{C}{L} \right)^2 8h_i^3 \right] + p_{i-1} \left[\frac{h_i^3}{(\Delta\zeta)^2} - \frac{3h_i^2}{2\Delta\zeta} \frac{dh_i}{d\zeta} \right] = \left(\frac{C}{L} \right)^2 \left[12\pi U r \frac{dh_i}{d\zeta} + 24\pi \frac{dh_i}{d\tau} \right]$$

This is condensed to the following.

$$C_i p_i + E_i p_{i+1} + D_i p_{i-1} = R_i \quad (2.3.12)$$

where,

$$C_i = -\frac{2h_i^3}{(\Delta\zeta)^2} - \left(\frac{C}{L} \right)^2 8h_i^3$$

$$D_i = \frac{h_i^3}{(\Delta\zeta)^2} + \frac{3h_i^2}{2\Delta\zeta} \frac{dh_i}{d\zeta}$$

$$E_i = \frac{h_i^3}{(\Delta\zeta)^2} - \frac{3h_i^2}{2\Delta\zeta} \frac{dh_i}{d\zeta}$$

$$R_i = \left(\frac{C}{L} \right)^2 \left[12\pi U r \frac{dh_i}{d\zeta} + 24\pi \frac{dh_i}{d\tau} \right] \quad (2.3.13)$$

Employing the recursion relationship

$$p_{i+1} = A_i p_i + B_i \quad (2.3.14)$$

Then, one obtains a pressure expression in terms of only two adjacent grid points,

$$C_i p_i + E_i (A_i p_i + B_i) + D_i p_{i-1} = R_i$$

or

$$p_j(C_j + E_j A_j) + E_j B_j + D_j p_{j+1} = R_j$$

Therefore

$$p_j = \left(\frac{-D_j}{C_j + E_j A_j} \right) p_{j+1} + \frac{R_j - E_j B_j}{(C_j + E_j A_j)} \quad (2.3.15)$$

So,

$$\begin{aligned} A_{j+1} &= \frac{-D_j}{C_j + E_j A_j} \\ B_{j+1} &= \frac{R_j - E_j B_j}{(C_j + E_j A_j)} \end{aligned} \quad (2.3.16)$$

Where $\{A\}$ & $\{B\}$ vectors are determined by starting with $A_2 = 0, B_2 = P_1$ (called the forward sweep). The downstream boundary condition is inserted at the beginning of the backward sweep as follows,

$$p_{M+1} = A_M p_M + B_M$$

$$p_{M+2} = A_{M+1} p_{M+1} + B_{M+1}$$

$$p_2 = A_1 p_1 + B_1 \quad (2.3.17)$$

Also, the cavitation condition is considered as : if $p_j < p_{\text{vapor}}$,set $p_j = p_{\text{vapor}}$ before computing p_{j-1} (i.e., $\frac{\partial p}{\partial n} = 0$ at the film rupture boundary, as is standard).

The boundary conditions and two examples of pressure distribution within the fluid film are summarized in Appendix E.

The hydrodynamic bearing force components on the journal (Fig. 2.3.1) are then obtained by integrating the vector decomposed fluid-film pressure distribution over the bearing surface as follows, where $\hat{n}_\theta$, ($\hat{n}_x, \hat{n}_y$) are the local and global unit radial vectors, respectively.

$$F_x = \iint_A P(\eta, Z) \hat{n}_\theta \cdot \hat{n}_x dA$$

$$F_y = \iint_A P(\eta, Z) \hat{n}_\theta \cdot \hat{n}_y dA$$

(2.3.18)

and becomes,

$$F_x = -LR \int_{\theta_1}^{\theta_2} p(\theta) \left(\mu N \left(\frac{L}{C} \right)^2 \right) \cos(\theta) d\theta$$

(2.3.19)

$$F_y = -LR \int_{\theta_1}^{\theta_2} p(\theta) \left(\mu N \left(\frac{L}{C} \right)^2 \right) \sin(\theta) d\theta$$

and are calculated by numerical integration (using Simpson's Rule) of the instantaneous fluid film pressure distribution. Where, and are the angular bounds of the non-zero pressure distribution.

2.4 System Parameters

The rotor-bearing system can be specified by five parameters defined as follows.

The bearing parameter B_p ,

$$B_p = \left(\frac{\mu L}{\pi} \right) \left(\frac{R}{C} \right)^3 \left(\frac{g}{W} \right) \sqrt{\frac{C}{g}} = S * \frac{\omega_n}{\Omega_n \omega}$$

where, S is the bearing Sommerfeld number. By using this definition, for a given bearing configuration B_p is only dependent on the bearing load. The higher bearing load, the smaller B_p .

The non-dimensional shaft stiffness Ω_n and $\Omega_n = \omega_n \sqrt{\frac{C}{g}}$.

The length/diameter ratio of journal bearing L / D

The point mass ratio Δ and $\Delta = \frac{M_f}{M_D}$.

The frequency ratio Ω and $\Omega = \frac{\omega}{\omega_n}$.

Also the rotor-bearing system dimensional parameters are chosen the same as Hori and Kato (1990) as follows, $L = 0.331$ m, $D = 0.662$ m, $C = 0.993 * 10^{-3}$ m, $\mu = 5.6 * 10^{-3}$ Pa. S. These values are typical for large steam turbogenerators and the Sommerfeld number $S = B_p * 3.77$ for rotation at 3,600 rpm.

Chapter 3

Linear Stability Analysis

3.0 Introduction

The basic idea of representing the dynamic response characteristics of a journal bearing by means of equivalent stiffness and damping coefficients to improve the calculation of the critical speed of a rotor originated quite early (Stodola 1925) as was the discovery of self-excited vibration induced by oil film journal bearings (Newkirk in 1925). However, it was not until 1956 (Hagg) that the basic was implemented. Hagg modeled bearing as radial linearized springs and dampers. Hagg's idea was extended by Sternlicht (1959) and others (ala Taylor series expansion) into a general linearized mathematical model for oil film journal bearings. For aligned journal bearings, this linearized model gave birth to the now classical eight spring and damping coefficients which are commonly used to model the dynamic radial interaction force components between journal and bearing. Of course, the underlying assumption is that orbital motion about the equilibrium state is sufficiently small that higher order terms in the Taylor series expansion of bearing dynamical forces are vanishingly small.

The characteristic equation of the system is then used to compute the linear instability threshold speed of the system by eigenvalue extraction as a function of rotor spin speed. Static equilibrium state of the system is first evaluated by balancing the fluid force and against the static load of the bearings. The linear

dynamical coefficients of the journal bearing are then derived through small displacement and velocity perturbations about the corresponding equilibrium point. Linearized equations of motion are then derived about the static equilibrium point. Instability threshold speed was then computed in terms of the system dimensionless parameters. The Hopf bifurcation is confirmed at the instability threshold speed by tracing the train of the eigenvalues (i.e., the root locus) of the system.

3.1 Static Equilibrium Point of the System

Initially, a condition of zero unbalance is assumed (i.e., $u = 0$). The equilibrium point, $(X_J^*, Y_J^*, X_D^*, Y_D^*)$, of the system can be obtained by substituting it into the equations of motion, (2.2.3) to (2.2.6), and also set all the derivative terms $\dot{X}_J^* = \dot{Y}_J^* = \dot{X}_D^* = \dot{Y}_D^* = 0$. Then we have,

$$X_D^* - X_J^* = 0 \quad (3.1.1)$$

$$Y_D^* - Y_J^* = -\frac{1}{\Omega_n^2} = -\frac{1}{\omega_n^2 \frac{C}{g}} = -\frac{M_u g}{K_s} \frac{1}{C} = -\frac{\delta}{C} = -\delta^* \quad (3.1.2)$$

$$f_N^* = 0 \quad (3.1.3)$$

$$f_1^* = \left(\frac{M_J}{2}\right) \left[\frac{1}{\Omega^2 - \Omega_n^2} - \frac{1}{\Delta \Omega^2} (Y_D^* - Y_J^*) \right] = \left(\frac{M_J}{2}\right) \left[\frac{1}{\Omega^2 - \Omega_n^2} + \frac{1}{\Delta \Omega^2} \delta^* \right] \quad (3.1.4)$$

$$X_J^* = \kappa^* \cos(\phi^*) \quad (3.1.5)$$

$$Y_J^* = \kappa^* \sin(\phi^*) \quad (3.1.6)$$

where,

$$\delta = \frac{M_D g}{K_s} \quad \text{is the static deflection of the shaft.}$$

$$\delta^* = \frac{\delta}{C} \quad \text{is the non-dimensional static deflection of the shaft.}$$

Equations (3.1.3) and (3.1.4) represent that the fluid force on the journal is equal to the static load on the bearing. The pressure distribution within the fluid thickness of the journal bearing is unique for a given static load and shaft rotating speed. For a simple cylindrical journal bearing, there is only one equilibrium point for a given operating condition. The Newton-Raphson iteration has been employed to solve these equations coupled with the Reynolds equations to obtain the journal equilibrium point, (X_J^*, Y_J^*) , κ^* , and ϕ^* . Also, the equilibrium point of the disk, (X_D^*, Y_D^*) can be obtained by substituting (X_J^*, Y_J^*) into equations (3.1.1) and (3.1.2). The eccentricity ratio and attitude angle of the center of the journal are as following,

$$\kappa^* = \sqrt{(X_J^{*2} + Y_J^{*2})} \quad (3.1.7)$$

$$\phi^* = \tan^{-1}\left(\frac{Y_J^*}{X_J^*}\right) \quad (3.1.8)$$

3.2 Linear Dynamical Coefficients of the Journal Bearing

The hydrodynamic fluid film force can be expanded by the Taylor series at given state $(X_j^*, Y_j^*, X_j', Y_j')$ as follows,

$$f_x = f_x^* + (X_j - X_j^*) \frac{\partial f_x}{\partial X_j} + (Y_j - Y_j^*) \frac{\partial f_x}{\partial Y_j} + (X_j') \frac{\partial f_x}{\partial X_j'} + (Y_j') \frac{\partial f_x}{\partial Y_j'} + H.O.T. \quad (3.2.1)$$

$$f_y = f_y^* + (X_j - X_j^*) \frac{\partial f_y}{\partial X_j} + (Y_j - Y_j^*) \frac{\partial f_y}{\partial Y_j} + (X_j') \frac{\partial f_y}{\partial X_j'} + (Y_j') \frac{\partial f_y}{\partial Y_j'} + H.O.T. \quad (3.2.2)$$

where,

$$X_j' = Y_j' = 0, \quad \text{at equilibrium point.}$$

$$f_x^* \text{ and } f_y^*, \quad \text{are the static loads of the bearing.}$$

$$H.O.T., \quad \text{means the higher order terms.}$$

The linear dynamic coefficients, (i.e., stiffness and damping coefficients), of the journal bearing are defined as follows,

$$K_{xx} = -\frac{\partial f_x}{\partial X_j}, \quad K_{xy} = -\frac{\partial f_x}{\partial Y_j}, \quad K_{yx} = -\frac{\partial f_y}{\partial X_j}, \quad K_{yy} = -\frac{\partial f_y}{\partial Y_j} \quad (3.2.3)$$

$$C'_{xx} = -\frac{\partial f_x}{\partial X_j'}, \quad C'_{xy} = -\frac{\partial f_x}{\partial Y_j'}, \quad C'_{yx} = -\frac{\partial f_y}{\partial X_j'}, \quad C'_{yy} = -\frac{\partial f_y}{\partial Y_j'} \quad (3.2.4)$$

Then equations (3.2.1) and (3.2.2) can be replaced as

$$f_x = f_x^* - (X_j - X_j^*)K_{xx} - (Y_j - Y_j^*)K_{xy} - (X_j')C_{xx} - (Y_j')C_{xy} + H.O.T. \quad (3.2.5)$$

and

$$f_y = f_y^* - (X_j - X_j^*)K_{yx} - (Y_j - Y_j^*)K_{yy} - (X_j')C_{yx} - (Y_j')C_{yy} + H.O.T. \quad (3.2.6)$$

Obviously, the evaluations of the first derivative of the fluid force is required to obtain the stiffness and damping coefficients of the bearing. In this work, we use the five-point formula to evaluate those derivatives. In general form, we have,

$$f'(x_0) = [f(x_0 - 2l) - 8f(x_0 - l) + 8f(x_0 + l) - f(x_0 + 2l)] / (12l) + l^4/30 f^{(5)}(\xi) \\ , x_0 - 2l < \xi < x_0 + 2l$$

where,

f is the function to be evaluated. In our case, it represents the fluid forces f_x or f_y .

x_0 is the state position where we want to evaluate the function. In here, it is (X_j^*, Y_j^*) for obtaining stiffness matrix and (X_j', Y_j') for damping matrix

l is the perturbation quantity in displacement or in velocity.

The error of this derivative is

$$|e(x_0)| = |f'(x_0) - \tilde{f}'(x_0)| \leq l^4/30 f^{(5)}(\xi) + r/l$$

where,

$\tilde{f}'(x_0)$ is the approximated value

r is the round-off error $\approx 5 * 10^{-16}$ for double precision real operation.

Assume M is the maximum value of $f'(\xi)$, then the error function of the derivative becomes

$$|e(x_n)| \leq l^{1/30} M + r/l \quad (3.2.7)$$

The derivative of the error function with respect to l is

$$\frac{\partial |e(x_n)|}{\partial l} \leq \frac{1}{15} l^{-2/30} M - \frac{r}{l^2} \quad (3.2.8)$$

To obtain a minimum error, we need $\frac{\partial |e(x_n)|}{\partial l} = 0$. With this equation, we get the optimal perturbation quantity needed to preserve the minimum error of the derivative. The relationship between the optimal perturbation quantity and the error is

$$l = \sqrt[3]{\frac{15r}{2M}} = 1.3026 * 10^{-3} M^{-0.2} \quad (3.2.9)$$

So, for example,

$$\text{if } M = 0.01, \quad l = 3.2720 * 10^{-3}$$

$$\text{if } M = 1000, \quad l = 0.3272 * 10^{-3}$$

That is the perturbation quantity is somewhere around 0.001 to 0.0001 , although we do not know what the exact value of M is. However, an "experienced" approach is applied to determine the value of l , which is as following.

Given an equilibrium state variables $(X_j^*, Y_j^*, X_D^*, Y_D^*)$, we are looking for a small perturbation displacement, (to compute [K] matrix) or a small perturbation velocity, (to compute [C] matrix), which makes the elements of the [K] and [C] matrices satisfy the following conditions,

$$0.001\% < \left| \frac{\Delta K_y}{K_y} \right| < 0.5\%, \quad 0.001\% < \left| \frac{\Delta C_y}{C_y} \right| < 0.5\%$$

And these small perturbation quantities are also located within the above reasonable regions. Other perturbation techniques have also been shown to work well, such as controlling the number of significant figures the perturbed and unperturbed forces have in common, (e.g., first four out of eight total in single precision).

3.3 Linearization of System Equations

As a small motion, we can assume

$$X_D = X_D^* + x_D$$

$$Y_D = Y_D^* + y_D$$

$$X_J = X_J^* + x_J$$

$$Y_J = Y_J^* + y_J$$

After substituting (X_J, Y_J, X_D, Y_D) into equations (2.2.3) to (2.2.6), replacing the fluid film force by equation (3.2.5) and (3.2.6) and assuming there is no

unbalance (i.e., $u = 0$), the linearized equations of motion are obtained as following,

$$x_D'' = -\frac{1}{\Omega^2}(x_D - x_J) \quad (3.3.1)$$

$$y_D'' = -\frac{1}{\Omega^2}(y_D - y_J) \quad (3.3.2)$$

$$x_J'' = \frac{1}{\Omega^2 \Delta}(x_D - x_J) + \frac{2}{M_J}(-x_J K_{xx} - y_J K_{xy} - x_J' C_{xx} - y_J' C_{xy}) + H.O.T. \quad (3.3.3)$$

$$y_J'' = \frac{1}{\Omega^2 \Delta}(y_D - y_J) + \frac{2}{M_J}(-x_J K_{xy} - y_J K_{yy} - x_J' C_{xy} - y_J' C_{yy}) + H.O.T. \quad (3.3.4)$$

These four second order ordinary differential system equations are then transferred into first order systems by the following substitutions,

$$x_1 = x_D, \quad x_2 = x_D' = x_1', \quad x_3 = y_D, \quad x_4 = y_D' = x_3', \quad x_5 = x_J, \quad x_6 = x_J' = x_5', \quad x_7 = y_J, \quad x_8 = y_J' = x_7',$$

The new first order system equations become

$$\underline{x}' = \begin{Bmatrix} x_1' \\ x_2' \\ x_3' \\ x_4' \\ x_5' \\ x_6' \\ x_7' \\ x_8' \end{Bmatrix} = [J] \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{Bmatrix} + \{H.O.T.\} \quad (3.3.5)$$

where, $[J]$ is the Jacobian matrix of the system at the corresponding equilibrium point.

$$[J] = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{\Omega^2} & 0 & 0 & 0 & \frac{1}{\Omega^2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{\Omega^2} & 0 & 0 & 0 & \frac{1}{\Omega^2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ \frac{1}{\Omega^2 \Delta} & 0 & 0 & 0 & \left(-\frac{1}{\Omega^2 \Delta} - \frac{2K_{\psi\psi}}{M_j} \right) & -\frac{2C'_{\psi\psi}}{M_j} & -\frac{2K_{\psi\gamma}}{M_j} & -\frac{2C'_{\psi\gamma}}{M_j} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{1}{\Omega^2 \Delta} & 0 & -\frac{2K_{\gamma\psi}}{M_j} & -\frac{2C'_{\gamma\psi}}{M_j} & \left(-\frac{1}{\Omega^2 \Delta} - \frac{2K_{\gamma\gamma}}{M_j} \right) & -\frac{2C'_{\gamma\gamma}}{M_j} \end{bmatrix}$$

The linearized four DOF - second order system becomes an 8-dimensional phase space system and is rewritten in vector form as

$$\underline{x}' = \underline{v}(\underline{x}, [K], [C], \Omega, \Delta, M_j) \quad , \underline{x} \in \mathcal{R}^8 \quad (3.3.6)$$

where, $\underline{v}$ is the vector field of the system.

3.4 Compute the Linear Instability Threshold

The linearized system equation (3.3.6), can be rewritten in the matrix form as following, (neglecting the higher order terms since they do not effect the linear stability)

$$[I]\{\underline{x}'\} = [J]\{\underline{x}\} \quad (3.4.1)$$

where

$[I]$ is an 8*8 identity matrix.

$[J] = \left[\frac{\partial^2 v_i}{\partial^2 x_j} \right]_{i,j=1,8}$ is the Jacobian matrix of vector field v at any given state

After substituting the general solution $e^{\lambda t}$ into equation (3.4.1), we can end up with the following eigenvalue equation.

$$([J] - \lambda [I])\{\underline{x}\} = 0 \quad (3.4.2)$$

So, for non-trivial solution of equation (3.4.2) we need

$$\| [J] - \lambda [I] \| = 0 \quad (3.4.3)$$

This equation is called the characteristic equation of the rotor-bearing system. Also, because that the Jacobian matrix $[J]$ is an unsymmetric matrix, complex eigenvalues λ are expected. They are

$$\lambda_i = \text{Re}(\lambda_i) + \text{Im}(\lambda_i), \quad i = 1, 8 \quad (3.4.4)$$

and can be found numerically.

The linear instability threshold ω_{th} of this rotor-bearing system is the lowest rotating speed where one of the real parts of an eigenvalue changes from negative to positive. And the imaginary part of this eigenvalue is the natural frequency associated zero-damped vibration mode. In nondimensional form of characteristic equation, the frequency ratio $\Omega_c = \omega_c / \omega_{th} = |\text{Im}(\lambda)|$ is obtained at the instability threshold speed, ω_{th} . Thus, for a given rotor-bearing configuration, we can obtain the linear instability threshold by evaluating the eigenvalues of the system and searching for the lowest rotating speed where one of the real parts of an eigenvalue changes from negative to positive.

The corresponding complex eigenvalues in equation (3.4.3) are obtained numerically by applying QR method, referred to by Richard et al. (1978). A brief introduction of the QR solver and two samples test are summarized in appendix A. In those sample tests, two arbitrary chosen Jacobian matrices are solved by QR and Mathematica separately. Excellent agreement is obtained between both methods. Fig 3.4.1 summarizes the procedure of finding the linear instability threshold speed of a given rotor-bearing system.

Figure 3.4.1 Computation procedure of finding the instability threshold speed.

3.5 Numerical Results and Discussions

Table 3.5 1 is the sensitivity analysis result of the linear instability threshold speed to the finite-difference grid points at different bearing load conditions. The grid spaces are used to discretize the spatial domain in the circumferential direction of the journal bearing in solving the Reynolds lubrication equation as described earlier. As can be observed, that the instability threshold speed is less sensitive to grid number for light loading conditions and few grid points, (for example, 71 grid points for $B_p=5.0$ and $B_p=1.0$), already obtain very consistent result. Conversely, for high bearing load cases, more grids are needed, such as 501 grid points for $B_p=0.02$

Table 3.5 2 is the summary of the result of the linear analysis which shows the relations between instability threshold frequency ratio, ω_{th}/ω_c , eccentricity ratio (ϵ/C) and attitude angle (ϕ) of the journal with respect to the loading conditions (B_p) and number of grid points. In light bearing load cases (for example, $B_p=0.5$), the center of journal is stabilized at low eccentric position and the system becomes unstable as the shaft rotating speed is above two times of the critical speed. The instability threshold speed progressively increases above $2\omega_c$ when system is at high bearing load condition. These results are consistent with Hori (1959) and other previous experimental results Newkirk and Taylor (1925) and Newkirk and Lewis (1956)

Bp	GRID No.	Ω_{ih}	Ω_c	ω_h / ω_c
5.0	71	1.068	0.500	2.000
5.0	101	1.068	0.500	2.000
5.0	201	1.068	0.500	2.000
Average		1.068	0.500	2.000
1.0	71	1.055	0.500	2.000
1.0	101	1.054	0.500	2.000
1.0	201	1.054	0.500	2.000
Average		1.054	0.500	2.000
0.5	71	1.026	0.499	2.004
0.5	101	1.026	0.499	2.004
0.5	201	1.026	0.499	2.004
Average		1.026	0.499	2.004
0.4	71	1.011	0.496	2.018
0.4	101	1.010	0.496	2.016
0.4	201	1.011	0.498	2.018
Average		1.011	0.498	2.018
0.2	101	1.197	0.522	1.916
0.2	201	1.213	0.513	1.951
0.2	301	1.209	0.520	1.923
0.2	401	1.205	0.518	1.931
0.2	501	1.207	0.518	1.930
Average		1.206	0.517	1.933
0.15	201	1.450	0.491	2.038
0.15	301	1.459	0.489	2.044
0.15	401	1.453	0.490	2.039
0.15	501	1.448	0.492	2.032
0.15	601	1.451	0.491	2.036
0.15	701	1.455	0.490	2.040
Average		1.453	0.491	2.038

Table 3.5 | Linear instability threshold for $\Omega_n = 1.0$, $L/D=0.5$, $\Delta = 0.1$.

0.1	201	1.742	0.433	2.307
0.1	301	1.765	0.428	2.338
0.1	401	1.779	0.424	2.358
0.1	501	1.783	0.423	2.365
0.1	601	1.787	0.422	2.372
0.1	701	1.787	0.422	2.370
Average		1.780	0.425	2.352
0.05	301	2.638	0.293	3.413
0.05	401	2.688	0.287	3.479
0.05	501	2.730	0.283	3.537
0.05	601	2.675	0.289	3.461
0.05	701	2.685	0.287	3.480
Average		2.683	0.288	3.474
0.02	501	5.565	0.140	7.165
0.02	601	5.502	0.141	7.079
0.02	701	5.643	0.138	7.269
0.02	801	5.582	0.139	7.188
0.02	901	5.540	0.140	7.131
0.02	1001	5.632	0.138	7.256
Average		5.577	0.139	7.181

Table 3 5 1 continued Linear instability threshold for $\Omega_n = 1.0$, $L/D=0.5$, $\Delta = 0.1$.

Bp	Grid Number	instability threshold ($m_{th} m_c$)	Eccentricity Ratio (e/C)	Attitude Angle (Degree)
5.00	71	2.000	0.053	- 0.153
1.00	71	2.000	0.248	- 0.677
0.50	71	2.004	0.424	- 1.051
0.40	71	2.018	0.488	- 1.147
0.20	301	1.923	0.622	- 2.106
0.15	201	2.038	0.654	- 9.889
0.10	401	2.358	0.704	- 22.615
0.05	401	3.479	0.756	- 38.274
0.02	501	7.181	0.784	- 48.770

Table 3 5 2 Linear instability threshold for $\Omega_n = 1.0$, $L/D=0.5$, $\Delta = 0.1$.

Fig. 3.5.1 shows the linear stability map of the rotor-bearing system with different dimensionless shaft rigidity values. A rigid shaft system has the smaller value of the instability threshold frequency ratio, ω_{th}/ω_c , than a soft shaft system does, when both of them are subjected to the same amount of static load. This is because the rigid shaft is operating at lower eccentric position than the soft shaft system. An asymptotic stability bound at eccentricity ratio about 0.8 is also illustrated in Fig. 3.5.1. This is also consistent with Hori's (1959) result and infers that the rotor-bearing system is always stable, for simple cylindrical bearings, if the equilibrium eccentricity ratio is higher than 0.8.

Figure 3.5.1 Stability Map for $L/D=0.5$, $\Delta=0.1$

Fig. 3.5.2 is the linear stability map for different point mass ratio conditions. Clearly, the stability is not affected within these small mass ratio variations. Also, Fig. 3.5.4 shows the effect of the viscosity to the linear stability of the system. For the same static load, (i.e., B_p is varied proportional to the variation of viscosity), a higher instability frequency ratio is obtained for lower viscosity system because this system will be stabilized at higher eccentric position just like heavier bearing load conditions. This suggests that lower viscosity is beneficial from the stability point of view, despite that lower viscosity will provide less damping at resonance speed.

Figure 3.5.2 Stability Map for $\Omega_n=1.0$, $L/D=0.5$ with different point mass ratio.

Figure 3.5.3 Stability Map for $\Omega_n=1.0$, $L/D=0.5$ with different viscosity but the same amount of static load.

Figs. 3.5.4 to 3.5.11 show the root-locus of the rotor-bearing system when the system is subjected to different bearing load conditions and the rotation speed is from $\omega/\omega_n=0.4$ to $\omega/\omega_n=10.0$, (here, we use ω_n as the reference speed). The real part of the eigenvalues is moving from left-half plane to right-half plane as we increase the rotation speed. A pure imaginary pair of eigenvalue is obtained at the instability threshold speed, ω_{th} , and the magnitude of the imaginary part of this eigenvalue is just the orbital frequency or called it the damped natural frequency of the system (in here, it is the frequency ratio of orbital frequency to rotational speed for the nondimensional system). This orbital frequency ratio decreased slowly (changing about from 0.5 to 0.45) when the rotation speed was increased in light bearing load condition (e.g., $Bp=5.0$ to $Bp=0.2$). These two, with positive real part, complex eigenvalues are somehow turning back and approaching to another

complex eigenvalue set, with the negative real part but with the same magnitude as the positive's) at the vicinity of zero, as shown. There also exists another pair of eigenvalues across the imaginary axis from negative plane to positive plane at very high rotation speed, (for example, rotation speed at $\omega/\omega_n=7.8$ for $Bp=5.0$, and rotation speed at $\omega/\omega_n=9.6$ for $Bp=0.05$) and the corresponding orbital frequency ratio is about 0.5. At this high speed, the first pair with positive real part eigenvalues are already approaching the origin and with a very low orbital frequency (for example, about 0.128 for $Bp=5.0$ to $Bp=15$ and about 0.10 for $Bp=0.1$ and $Bp=0.05$). Two limit cycles with different orbital frequency could occur at that high speed. There also exist pure negative but large in magnitude real eigenvalues around the point of instability threshold speed in which the pure imaginary eigenvalues in magnitude are less than 0.5. This implies that we could have a large magnitude ratio of maximum eigenvalue to minimum eigenvalue, for example $|\lambda_{\max} / \lambda_{\min}| = 900$ at $Bp=5.0$ and $|\lambda_{\max} / \lambda_{\min}| = 11$ at $Bp=0.02$. That is, we have a "stiff" system at light bearing load system and a moderate "stiff" system at high bearing load condition. A tiny integration time marching step would be needed for very light bearing load case, although large mesh size can be used in solving Reynolds equation at this time. On the contrary, large integration time step is allowed to be used for high bearing load, but more grid points are needed in this case. So, intensive CPU time consumption is expected in performing the time marching integration.

First 6 Eigenvalues

The 7th and 8th Eigenvalues.

Figure 3.5.4 Root-locus for $Bp=5.0$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$.

First 6 Eigenvalues.

The 7th and 8th eigenvalues.

Figure 3.5.5 Root-locus for $B_p=1.0$, $B_p=5.0$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$.

First 6 Eigenvalues

The 7th and 8th Eigenvalues

Figure 3.5.6 Root-locus for $B_p=0.5$, $B_p=5.0$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$.

First 6 Eigenvalues.

The 7th and 8th Eigenvalues

Figure 3.5.7 Root-locus for $B_p=0.2$, $B_p=5.0$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$.

First 6 Eigenvalues

The 7th and 8th Eigenvalues

Figure 3.5.8 Root-locus for $Bp=0.15$, $Bp=5.0$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$.

First 6 Eigenvalues

The 7th and 8th Eigenvalues

Figure 3.5.9 Root-locus for $B_p=0.10$, $B_p=5.0$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$.

Eigenvalues around (0.,0.)

Figure 3.5.10 Root-locus for $B_p=0.05$, $B_p=5.0$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$.

Eigenvalues around (0.,0.)

Figure 3.5.11 Root-locus for $B_p=0.02$, $B_p=5.0$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$.

From the foregoing eigenvalues analyses, it was found that one pair of the eigenvalues changes from negative real part to positive real part and all other eigenvalues are negative real part at the instability threshold speed. This suggests that a Hopf bifurcation could happen at the threshold point, a limit cycle is expected and the orbital frequency is just the magnitude of that pure imaginary pair of eigenvalues (in fact it is a frequency ratio in nondimensional system). A Hopf bifurcation theory is summarized in appendix B.

Lund (1967) used the " method of averaging " and Hallis and Taylor (1986) used BIFOR2 software to compute the size and the stability of the limit cycle. In this work, because there is no explicit form for the hydrodynamic fluid film force, we directly integrate the full nonlinear equations of motion to investigate the stability and size of the limit cycle. A limit cycle is stable when an outward spiraling trajectory is obtained integrating from inside the limit cycle and an inward orbit integrating from outside the limit cycle, (see Fig. 3.5.12). The converse of these would constitute an unstable limit cycle, (see Fig. 3.5.13 and Fig. 3.5.14). More detail computation about limit cycle at $\omega > \omega_{th}$ is going to investigate in next Chapter about nonlinear hysteresis loop.

The Runge-Kutta-Fehlberg scheme is used here to perform integration of the motion equations, with the time step size for time marching changed adaptively according to the required tolerance between the fourth and fifth order solutions. The RKF scheme and sample test are summarized in appendix C. An efficient and sufficiently accurate solution is obtained when the tolerance is set to 10^{-5} . Also, the hydrodynamic fluid force is updated at every time marching step.

Figure 3.5.12 a and b A diverging trajectory of the center of the journal integrating from $(\Delta x_j, \Delta y_j) = (0.05, 0.05)$ and a converging trajectory integrating from $(\Delta x_j, \Delta y_j) = (0.10, 0.10)$ for $B_p = 0.5$, $\Omega_n = 1.0$, $L/D = 0.5$, $\Delta = 0.1$, $\omega/\omega_c = 2.0$.

Figure 3.5.12.c Journal orbit for $B_p = 0.5$, $\Omega_n = 1.0$, $L/D = 0.5$, $\Delta = 0.1$, $\omega/\omega_c = 2.0$.

According to a and b, we conclude that the journal orbit is a stable limit cycle.

Figure 3.5.13 a and b Journal orbit and time history for $B_p=5.0$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.10$, at $\omega/\omega_c=2.0$ were obtained by integration from $(X_j/C, Y_j/C) = (0.01, 0.01)$. A converging trajectory was shown.

Figure 3.5.13 c and d Journal orbit and time history for $B_p=5.0$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.10$, at $\omega/\omega_c=2.0$ were obtained by integration from $(X_j/C, Y_j/C) = (0.1, 0.1)$. A diverging trajectory was shown.

According to the above two integration conditions, we concluded that the limit cycle at the equilibrium point is unstable.

Figure 3.5.14.a

Figure 3.5.14.b

Figure 3.5.14.a and b Journal orbit and time history for $B_p=1.0$, $\Omega_n=1.0$, $\Delta=0.1$, $L/D=0.5$, at $\omega/\omega_c=2.0$. An outward spiraling orbit is obtained integration from $(\Delta x_j, \Delta y_j) = (0.1, 0.1)$.

Figure 3.5.14.c

Figure 3.5.14.d

Figure 3.5.14.c and d Journal orbit and time history for $B_p=1.0$, $\Omega_n=1.0$, $\Delta=0.1$, $L/D=0.5$, at $\omega/\omega_c=2.0$. An inward spiraling orbit is obtained integration from $(\Delta x_j, \Delta y_j)=(0.005, 0.005)$.

Combining the above two integration conditions, we conclude that the limit cycle at the equilibrium point is unstable.

Chapter 4

Nonlinear Hysteresis Loop of Rotor-Bearing System

4.0 Description of the Hysteresis Behavior

Suppose that the system (f'_μ) has an attractor A_μ (a node or periodic solution) that changes continuously with the real parameter μ . If $x_0 \in A_{\mu_0}$, and if μ is increased slowly with t , we may expect that $f'_\mu(x_0)$ will remain very close to A_μ . If the attractor A_μ disappears for some value μ' of μ , $f'_\mu(x_0)$ may jump to another attractor B_μ . If μ is now decreased, $f'_\mu(x_0)$ will not jump back to A_μ , but will stay near B_μ (see Fig. 4.1.1). This nonreversible behavior is called hysteresis and the path b-c-d-e-b forms the hysteresis loop of the system. It is one expression of the fact that the choice of attractor made by a system depends on the past history of the system. That is, multivalues exist within the hysteresis loop. In Fig. 4.1.1, A_μ and B_μ represent two stable solutions while within them the path e-c is an unstable solution.

Figure 4.0.1 Hysteresis in the presence of several attractors

4.1 Hysteresis Loop of Rotor-Bearing System

The conceptual Hysteresis Loop for a cylindrical-journal-bearing supported rotor-bearing system is shown in Fig. 4.1.1, in which the system loses the linear stability (i.e., stable "small" solution) at the point of instability threshold speed, ω_{th} , by a Hopf bifurcation. Beyond the Hopf bifurcation point, the system becomes unstable, a stable limit cycle could exist when the rotational speed is not far away from the Hopf bifurcation speed and eventually the journal orbit tends to grow larger and larger until it reaches the full clearance circle. Oil whip is the generic label for this kind of self-excited vibration. In a spin coast-down, of the the rotational speed from the oil whip limit cycle condition, the big limit cycle will remain there until the rotation speed is reaches the Saddle Node bifurcation speed. There, a stable limit cycle and an unstable limit cycle co-exist and the journal orbit shrinks suddenly when the rotational speed is below the Saddle-Node speed, as shown. The Saddle-Node speed and the Hopf bifurcation speed characterize the hysteresis loop of the rotor-bearing system. An unstable solution exists within the hysteresis loop. An initial disturbance above the unstable solution will grow to become the stable limit cycle, whereas, an initial disturbance below the unstable solution will decay to the stable-in-the-small (zero vibration) solution.

Khonsari and Chang (1993) used numerical integration of the nonlinear time-transient motion to show the existence of a closed position boundary, (within the bearing clearance - circle boundary and encircling the static equilibrium point) which delineates between initial positions which die out and initial positions which grow to a stable large amplitude orbit. In fact, Khonsari and Chang have

encountered the unstable solution which exists within the hysteresis loop and which provides a boundary between initial-position regimes for the two stable solutions. However they do not set their results within the framework of the hysteresis loop. Furthermore, the complete initial condition boundary provided by the unstable solution would have to be described in the appropriate dimensional phase space, (for example, we have 8 Dimension in this case), of which the orbital position space is a subset. Finding the elusive unstable intermediate solution is not an objective of this research.

Figure 4.1.1 Conceptual Rotor-Bearing Instability Hysteresis Loop

4.2 Computation of the Nonlinear Hysteresis Loop

The nonlinear hysteresis loop of the journal bearing supported rotor-bearing system is obtained by the following solution scenario. First, a small perturbation of the journal is imposed as an initial condition and integration of the motion equations is carried computationally forward in time until the motion resolves itself, (it could take a hundred to thousands of shaft revolutions, depending upon the operating condition). A stable node is obtained when $\omega < \omega_{th}$, and a limit cycle is obtained when $\omega > \omega_{th}$. Oil whip occurs, as the rotation speed is increased, and the journal orbit finally approaches the full clearance circle of the bearing. This process is compared to prior the linear stability analysis map for confirmation of the instability threshold speed, ω_{th} (i.e., Hopf bifurcation speed). Provided the same numerical solution scheme for the Reynolds lubrication equation is used in both the (K_{ij}, C_{ij}) determination in the small perturbation stability analysis and in the time - transition motion simulation, then this is a valid confirmation.

Second, the stable limit cycle is used as an initial condition and the spin speed is reduced in small increments. This process is repeated to progressively lower speeds, and the Saddle-Node bifurcation speed is located by a sharp disappearance of the limit cycle. The Saddle-Node and the Hopf bifurcation speeds form the hysteresis loop of the journal bearing supported rotor-bearing system. A jump phenomenon is possible when the shaft rotating speed is beyond the Saddle-Node speed. Within the hysteresis loop, the system could be stable in-the-small or oscillating with a typically large amplitude nonlinear limit cycle, depending on whether the magnitude of the initial disturbance places it above or below the unstable intermediate solution.

4.3 Numerical Results and Discussions

The Hysteresis loop is developed as a function of bearing number, B_p , as follows. Fig. 4.3.1 to Fig. 4.3.5 are the orbits of the center of the journal that show the birth of the nonlinear hysteresis loop of the journal bearing supported rotor-bearing system, when it is for different loading conditions, (i.e., B_p is varied from 0.5 to 0.05). A stable node is obtained when $\omega < \omega_{th}$, as shown in Fig. 4.3.1.a, Fig. 4.3.2.a, Fig. 4.3.3.a, Fig. 4.3.4.a and Fig. 4.3.5.a. Hopf bifurcation accompanied by a limit cycle is obtained at the instability threshold speed, ω_{th} , which is consistent with the result of the linear stability analysis. Fig. 4.3.1.b, Fig. 4.3.2.b, Fig. 4.3.3.b, Fig. 4.3.4.b and Fig. 4.3.5.b show the forming of the stable limit cycle which an outward spiraling orbit is obtained integrating from inside the limit cycle and an inward trajectory integrating from outside the limit cycle, as shown. It is not our intention to indicate the size of the small limit cycle at the Hopf bifurcation speed. In stead, we are more interesting in the stability of the limit cycle and the developing of the limit cycle when the spin speed is above this Hopf bifurcation speed. The system becomes unstable at $\omega > \omega_{th}$, and the limit cycle becomes bigger and bigger very fast when we simulate the speed up process. Then, finally the journal orbit will approach the full clearance circle of the bearing, (i.e., Oil Whip occurs). Fig. 4.3.1.c, Fig. 4.3.3.c, Fig. 4.3.4.c and Fig. 4.3.5.c show that oil whip usually occurs instantaneously as soon as the rotating speed is beyond the instability threshold speed. This kind of behavior is consistent with previous authors, such as Lund (1967) and Hallis and Taylor (1986) who noticed that the shaft spin speed zone, (above ω_{th}), with stable small limit cycle was very small and was hard to be indicated in experiment. In this work, we found an one exception, (at the moderate bearing load case at $B_p=0.2$), that the evolving stable limit cycle is obtained with the increasing of the rotating speed, as shown in Fig. 4.3.2.c to Fig. 4.3.2.h.

The instability threshold for $B_p=0.2$ is $\omega_{th}/\omega_c=1.923$. Stable big limit cycle is continuously existing until the rotating speed is beyond about $2.5 \omega_c$, as shown. Also, a more convincing indication comes from comparing the stable limit cycle at Fig. 4.3.2.d, in which an outward spiraling orbit and reaching a stable limit cycle is obtained integrating from small perturbation at $\omega/\omega_c=1.95$ to Fig. 4.3.2.l, in which a stable limit cycle is obtained integrating from oil whip condition also at $\omega/\omega_c=1.95$. The same size of the limit cycle is obtained. The similar result, in Fig. 4.3.2.f and Fig. 4.3.2.k at $\omega/\omega_c=2.2$, is also obtained and it proves the existence of the stable limit cycle at the supercritical speed, (i.e., the rotation speed is above the instability threshold speed).

A coast down process then was carried out by starting the integration from the oil whip conditions. The journal orbit is either remaining at oil whip orbit or converging to a big size of limit cycle, as shown. The big limit cycle is persisting until the rotation speed reaches the so called Saddle-Node speed, in which a stable limit cycle and an unstable limit cycle co-exist theoretically. A disappearance of the big limit cycle occurs as soon as the shaft rotating speed is past the Saddle-Node speed, ω_{sn} , as shown in Fig. 4.3.1.g for $B_p=0.5$, Fig. 4.3.2.o for $B_p=0.2$, Fig. 4.3.3.i for $B_p=0.15$, Fig. 4.3.4.m for $B_p=0.1$ and Fig. 4.3.5.j for $B_p=0.05$.

One should notice that the journal orbit, which is used to illustrate the nonlinear hysteresis loop, is only a subset of our 8-dimensional phase space system. Therefore, crossing line occurred in the developing of the limit cycle, is just one of the projecting effect that in our case is actually projected from 8 dimensional trajectory to a 2 dimensional section. Ideally, there should not have any line crossing each other in a limit cycle trajectory.

Table 4.3.1 summarizes the results of Hopf bifurcation and Saddle-Node speeds at different bearing load cases. Fig. 4.3.6.a to Fig. 4.3.6.e show the corresponding nonlinear hysteresis loops. The results show that for light loads, for example $B_p=0.5$, there is no hysteresis loop (Fig. 4.3.6.a). The system becomes unstable when the rotating speed is $\omega > 2\omega_c$ and stable when $\omega < 2\omega_c$. As bearing load is progressively increased (Figs. 4.3.6.b, c, d and e), a hysteresis loop is “born”. The trend with bearing load is obvious and informative. The small perturbation linear instability threshold speed (i.e., Hopf bifurcation) increases with bearing load, as is well known by rotating machinery designers. Conversely, the Saddle Node (which is unknown to most designers) decreases with increasing bearing load, as shown. The Saddle Node appears to asymptotically approach 1.725 as bearing load is increased. This is interesting also from a practical point of view. A value of 2 is thought to be the worst-case (lowest) of the stability limit possible (when bearing loads become small). These results show that while increasing bearing load does increase the classical instability threshold speed, as is well known, the lowest rotor speed at which a large nonlinear orbit can occur is actually lowered somewhat.

B_p	ω_{th}/ω_c	ω_{sn}/ω_c
0.50	2.00	2.00
0.20	1.92	1.85
0.15	2.04	1.74
0.10	2.36	1.72
0.05	3.48	1.73

Table 4.3.1 Summary of results of the hysteresis loop computation for system parameters, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$.

Figure 4.3.1.a A stable node obtained integrating from $(\Delta X_J, \Delta Y_J) = (0.01, 0.01)$, for $B_p=0.5$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$, at $\omega/\omega_c=1.90$.

Figure 4.3.1.b A stable limit is forming, that a diverging orbit is obtained integrating from $(\Delta x_J, \Delta y_J) = (0.05, 0.05)$ and a converging orbit is obtained integrating from $(\Delta x_J, \Delta y_J) = (0.10, 0.10)$, for $B_p=0.5$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$, $\omega/\omega_c=2.0$.

Figure 4.3.1.c A spiraling outward orbit is obtained for $B_p=0.5$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$, at $\omega/\omega_c=2.055$ integrating from $(\Delta x_f, \Delta y_f)=(0.1, 0.1)$.

Figure 4.3.1.d A spiraling outward orbit is obtained for $B_p=0.5$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$, at $\omega/\omega_c=2.50$ integrating from $(\Delta x_f, \Delta y_f)=(0.1, 0.1)$.

Figure 4.3.1.e A spiraling outward orbit is obtained for $B_p=0.5$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$, at $\omega/\omega_c=4.00$ integrating from $(\Delta x_p \Delta y_p)=(0.1, 0.1)$.

Figure 4.3.1.f An oil whip orbit is obtained for $B_p=0.5$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$, at $\omega/\omega_c=2.00$ integrating from oil whip condition.

Figure 4.3.1.g A stable node is obtained for $B_p=0.5$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$, at $\omega/\omega_c=1.977$ integrating from oil whip condition.

Figure 4.3.2.a A stable node is obtained integrating from $(\Delta X_J, \Delta Y_J) = (0.01, 0.01)$, (for $B_p=0.2, \Omega_n=1.0, L/D=0.5, \Delta=0.1$), at $\omega/\omega_c=1.90$.

Figure 4.3.2.b A stable limit cycle is forming, that a spiraling outward trajectory obtained integrating from $(\Delta x_j, \Delta y_j) = (0.005, 0.005)$ and a spiraling inward trajectory obtained integrating from $(\Delta x_j, \Delta y_j) = (0.01, 0.01)$, (for $B_p=0.2, \Omega_n=1.0, L/D=0.5, \Delta=0.1$), at $\omega/\omega_c=1.923$.

Figure 4.3.2.c A stable big limit cycle is obtained, for $Bp=0.2$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$, at $\omega/\omega_c=1.928$, integrating from $(\Delta x_f, \Delta y_f) = (0.1, 0.1)$.

Figure 4.3.2.d A stable big limit cycle is obtained, for $Bp=0.2$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$, at $\omega/\omega_c=1.95$, integrating from $(\Delta x_f, \Delta y_f) = (0.1, 0.1)$.

Figure 4.3.2.e A stable big limit cycle is obtained, for $Bp=0.2$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$, at $\omega/\omega_c=2.00$, integrating from $(\Delta x_j, \Delta y_j)=(0.1, 0.1)$.

Figure 4.3.2.f A stable big limit cycle is obtained, for $Bp=0.2$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$, at $\omega/\omega_c=2.20$, integrating from $(\Delta x_j, \Delta y_j)=(0.1, 0.1)$.

Figure 4.3.2.g A stable big limit cycle is obtained, for $Bp=0.2$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$, at $\omega/\omega_c=2.50$, integrating from $(\Delta x_j, \Delta y_j)=(0.1, 0.1)$.

Figure 4.3.2.h A stable big limit cycle is obtained, for $Bp=0.2$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$, at $\omega/\omega_c=2.75$, integrating from $(\Delta x_j, \Delta y_j)=(0.1, 0.1)$.

Figure 4.3.2.i An oil whip occurred, for $Bp=0.2$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$, at $\omega/\omega_c=3.00$, integrating from $(\Delta x, \Delta y)=(0.1, 0.1)$.

Figure 4.3.2.j An oil whip at $\omega/\omega_c=4.00$, ($Bp=0.2$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$), as integrating from $(\Delta x, \Delta y)=(0.1, 0.1)$.

Figure 4.3.2.k An stable limit cycle is recovered, for $Bp=0.2$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$, at $\omega/\omega_c=2.2$, integrating from oil whip condition.

Figure 4.3.2.l An stable limit cycle is recovered, for $Bp=0.2$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$, at $\omega/\omega_c=1.95$, integrating from oil whip condition.

Figure 4.3.2.m An stable limit cycle is recovered, for $Bp=0.2$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$, at $\omega/\omega_c=1.923$, integrating from oil whip condition.

Figure 4.3.2.n An stable limit cycle is recovered, for $Bp=0.2$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$, at $\omega/\omega_c=1.88$, integrating from oil whip condition.

Figure 4.3.2.o The limit cycle disappears at $\omega/\omega_c=1.85$., for $Bp=0.2$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$, integrating from oil whip condition.

Figure 4.3.3.a A stable node obtained integrating from $(\Delta X_J, \Delta Y_J) = (0.01, 0.01)$, (for $B_p=0.15$, $\Omega_n=1.0$, $\Delta=0.1$, $L/D=0.5$), at $\omega/\omega_c=1.964$.

Figure 4.3.3.b An stable limit cycle is obtained, that an outward spiral progressing integrating from $(\Delta X_J, \Delta Y_J) = (0.01, 0.01)$, and an inward spiral developing integrating from $(\Delta X_J, \Delta Y_J) = (0.1, 0.1)$, (for $B_p=0.15$, $\Omega_n=1.0$, $\Delta=0.1$, $L/D=0.5$), at $\omega/\omega_c=2.039$.

Figure 4.3.3.c An oil whip is developed, integrating from $(\Delta X_J, \Delta Y_J) = (0.1, 0.1)$, (for $B_p=0.15$, $\Omega_n=1.0$, $\Delta=0.1$, $L/D=0.5$), at $\omega/\omega_c=2.048$.

Figure 4.3.3.d An oil whip is developed, integrating from $(\Delta X_J, \Delta Y_J) = (0.1, 0.1)$, (for $B_p=0.15$, $\Omega_n=1.0$, $\Delta=0.1$, $L/D=0.5$), at $\omega/\omega_c=4.0$.

Figure 4.3.3.e A persisted oil whip is developed, integrating from oil whip condition, (for $Bp=0.15$, $\Omega_n=1.0$, $\Delta=0.1$, $L/D=0.5$), at $\omega/\omega_c=2.039$.

Figure 4.3.3.f A big limit cycle is obtained, integrating from oil whip condition, (for $Bp=0.15$, $\Omega_n=1.0$, $\Delta=0.1$, $L/D=0.5$), at $\omega/\omega_c=1.823$.

Figure 4.3.3.g A big limit cycle is obtained, integrating from oil whip condition, (for $Bp=0.15$, $\Omega_n=1.0$, $\Delta=0.1$, $L/D=0.5$), at $\omega/\omega_c=1.795$.

Figure 4.3.3.h A big limit cycle is obtained, integrating from oil whip condition, (for $Bp=0.15$, $\Omega_n=1.0$, $\Delta=0.1$, $L/D=0.5$), at $\omega/\omega_c=1.767$.

Figure 4.3.3.i Limit cycle disappears at $\omega/\omega_c=1.739$ as integrating from oil whip condition, (for $Bp=0.15$, $\Omega_n=1.0$, $\Delta=0.1$, $L/D=0.5$).

Figure 4.3.4.a A stable node obtained integrating from $(\Delta X_J, \Delta Y_J) = (0.01, 0.01)$,
(for $B_p=0.10$, $\Omega_n=1.0$, $\Delta=0.1$, $L/D=0.5$), at $\omega/\omega_c=2.318$.

Figure 4.3.4.b An stable limit cycle is obtained, where an outward spiral
progressing integrating from $(\Delta X_J, \Delta Y_J) = (0.01, 0.01)$, and an inward spiral
developing integrating from $(\Delta X_J, \Delta Y_J) = (0.1, 0.1)$, (for $B_p=0.10$, $\Omega_n=1.0$,
 $\Delta=0.1$, $L/D=0.5$), at $\omega/\omega_c=2.358$.

Figure 4.3.4.c Oil whip occurs, where an outward spiral progressively as integrating from $(\Delta X_J, \Delta Y_J) = (0.1, 0.1)$, (for $B_p=0.10$, $\Omega_n=1.0$, $\Delta=0.1$, $L/D=0.5$), at $\omega/\omega_c=2.372$.

Figure 4.3.4.d Oil whip is developed as integrating from $(\Delta X_J, \Delta Y_J) = (0.1, 0.1)$, (for $B_p=0.10$, $\Omega_n=1.0$, $\Delta=0.1$, $L/D=0.5$), at $\omega/\omega_c=3.00$.

Figure 4.3.4.e Oil whip is developed as integrating from $(\Delta X_J, \Delta Y_J) = (0.1, 0.1)$, (for $B_p=0.10$, $\Omega_n=1.0$, $\Delta=0.1$, $L/D=0.5$), at $\omega/\omega_c=4.00$.

Figure 4.3.4.f An persisted oil whip is obtained as integrating from oil whip condition, (for $B_p=0.10$, $\Omega_n=1.0$, $\Delta=0.1$, $L/D=0.5$), at $\omega/\omega_c=2.318$.

Figure 4.3.4.g A big limit cycle is obtained as integrating from oil whip condition, (for $Bp=0.10$, $\Omega_n=1.0$, $\Delta=0.1$, $L/D=0.5$), at $\omega/\omega_c=2.057$.

Figure 4.3.4.h A big limit cycle is obtained as integrating from oil whip condition, (for $Bp=0.10$, $\Omega_n=1.0$, $\Delta=0.1$, $L/D=0.5$), at $\omega/\omega_c=1.947$.

Figure 4.3.4.i A big limit cycle is obtained as integrating from oil whip condition, (for $Bp=0.10$, $\Omega_n=1.0$, $\Delta=0.1$, $L/D=0.5$), at $\omega/\omega_c=1.854$.

Figure 4.3.4.j A big limit cycle is obtained as integrating from oil whip condition, (for $Bp=0.10$, $\Omega_n=1.0$, $\Delta=0.1$, $L/D=0.5$), at $\omega/\omega_c=1.801$.

Figure 4.3.4.k A big limit cycle is obtained as integrating from oil whip condition, (for $Bp=0.10$, $\Omega_n=1.0$, $\Delta=0.1$, $L/D=0.5$), at $\omega/\omega_c=1.775$.

Figure 4.3.4.l A big limit cycle is obtained as integrating from oil whip condition, (for $Bp=0.10$, $\Omega_n=1.0$, $\Delta=0.1$, $L/D=0.5$), at $\omega/\omega_c=1.748$.

Figure 4.3.4.m Limit cycle disappears at $\omega/\omega_c=1.722$ as integrating from oil whip condition, (for $B_p=0.10$, $\Omega_n=1.0$, $\Delta=0.1$, $L/D=0.5$).

Figure 4.3.5.a A stable node obtained integrating from $(\Delta X_J, \Delta Y_J) = (0.01, 0.01)$, for $B_p=0.05$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$, at $\omega/\omega_c=3.364$.

Figure 4.3.5.b A stable limit is developed. A spiraling outward orbit obtained as integrating from $(\Delta X_J, \Delta Y_J) = (0.01, 0.01)$ and an inward spiraling orbit obtained as integrating from $(\Delta X_J, \Delta Y_J) = (0.1, 0.1)$, for $B_p=0.05$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$, at $\omega/\omega_c=3.479$.

Figure 4.3.5.c Oil whip occurs as integrating from $(\Delta X_J, \Delta Y_J) = (0.1, 0.1)$, for $Bp=0.05$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$, at $\omega/\omega_c=3.531$.

Figure 4.3.5.d Oil whip occurs as integrating from $(\Delta X_J, \Delta Y_J) = (0.1, 0.1)$, for $Bp=0.05$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$, at $\omega/\omega_c=4.00$.

Figure 4.3.5.e A persisted oil whip is obtained as integrating from oil whip condition and the rotating speed is reduced to $\omega/\omega_c=3.364$, for $Bp=0.05$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$.

Figure 4.3.5.f Oil whip is remaining as integrating from oil whip condition and the rotating speed is reduced to $\omega/\omega_c=2.058$, for $Bp=0.05$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$.

Figure 4.3.5.g A stable limit cycle is obtained as integrating from oil whip condition at $\omega/\omega_c=1.961$, for $Bp=0.05$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$.

Figure 4.3.5.h A big stable limit cycle is obtained as integrating from oil whip condition at $\omega/\omega_c=1.811$, for $Bp=0.05$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$.

Figure 4.3.5.i A big stable limit cycle is obtained as integrating from oil whip condition at $\omega/\omega_c=1.759$, for $Bp=0.05$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$.

Figure 4.3.5.j Big limit cycle disappears at $\omega/\omega_c=1.725$ integrating from oil whip condition, for $Bp=0.05$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$.

Figure 4.3.6.a Rotor free vibration response for $B_p=0.5$.

Figure 4.3.6.b Rotor free vibration response for $B_p=0.2$.

Figure 4.3.6.c Rotor free vibration response for $B_p=0.15$.

Figure 4.3.6.d Rotor free vibration response for $B_p=0.1$.

Figure 4.3.6.e Rotor free vibration response for $B_p=0.05$.

Chapter 5

Rotor Unbalance Response within the Hysteresis Loop

5.0 Introduction

Nonlinear hysteresis loop, characterized by the Saddle-Node and Hopf bifurcation speeds, was found by using free-vibration and applying appropriate initial conditions, as described in the previous Chapter. The procedure that is used to find the Saddle-Node speed in an actual machine is important but is presently an unsolved engineering problem. In this work, exploring for safe means of locating the Saddle-Node speed on actual running machines was researched. Clearly, such means would provide valuable knowledge concerning actual risks for operating machines. To that end, signal processing for identifying chaos is explored here as a safe way to obtain the Saddle-Node speed on a running machine, using the simulation model as a "test rig". Simulation cases were focused at rotor speeds within the hysteresis loop speed range. Both static bearing load and unbalance magnitude were varied in this investigation. Three categories of long-terms nonlinear response were obtained, (i) periodic (synchronous and subharmonic), (ii) quasi-periodic (two or more incommensurate frequencies) and (iii) chaotic, by integrating the equations of motion, (i.e., Eqns (2.2.3) to (2.2.6)).

Li and Taylor (1987) obtained the Saddle-Node type bifurcation by analyzing an autonomous squeeze film damper system with employing the short bearing

approximation and including the unbalance force in the dynamics equations. Nonsynchronous and higher order subharmonic motions (second, third and sixth subharmonic orbits) were found for certain system parameters by numerical integration with different initial conditions (i.e., eccentricity ratio and attitude angle). The nonsynchronous motion was interpreted as a mathematical limit cycle but not a physical self-excited phenomenon because its existence directly relates to external excitation. However, the analysis was not constructed on the instability domain, which is much more interesting for rotating machinery engineering.

Chaotic oscillation in a rotating shaft was first studied theoretically by Shaw (1988). He studied a slender beam rotating about its longitudinal axis with a geometrical stiffening nonlinearity, and shows theoretically the existence of chaotic motion. This has been a well known phenomena for several decades, although not within the context of chaos. Later, Ehrich (1991 and 1992) observed chaotic motions in aircraft gas-turbines and studied it using numerical simulation. Kim and Noah (1990 and 1991) investigated bifurcation phenomena in a modified Jeffcott rotor theoretically and showed the possibility of chaotic response.

For a journal-bearing supported rotor-bearing system, in 1972 Holmes already found a very interesting behavior for a squeeze-film bearing damper for certain system parameters. He stated that " a steady-state orbit is never achieved; the journal merely rattles about in the clearance space in a never-ending transient ". This could actually be the first article that mentions something akin to chaotic motion in fluid-film supported rotor, although he did not use the term chaos.

Zhao, J. Y. and Hahn, E. J. (1993) studied the unbalance response of a rigid rotor supported by an eccentric squeeze film damper numerically using a fourth order Runge-Kutta integration. The complexity of the system behavior were elucidated and illustrated by using the techniques of Poincare' maps, rotor trajectories, bifurcation diagrams and power spectra. The jump phenomenon and subharmonic, quasi-period and even chaotic motions were found and were used to explain the experimentally observed subharmonic motions in Botman, M. (1976), Nikolajsen, J. L. and Holems, R. (1979) and Sykes, J. E. H. and Holems, R. (1990)

Recently, an exploratory research on chaos concepts as diagnostic tools for assessing rotating machinery vibration signatures had been made by Adams and Abu-Mahfouz (1994). They found that the chaotic motion could occur within a wide range of system parameters in three fundamental rotordynamical systems, (which are rub-impact, cylindrical, and tilting-pad journal bearing) and suggested a possibility of using chaos content - Poincare' map as a diagnostic toll for rotating machine.

5.1 Numerical Results and Discussions

The system was started at the equilibrium point, with the synchronous rotating unbalance force added, and neither displacement nor velocity perturbations. Thus, the question of multiple steady - state solutions was neither explored nor encountered. The steady - state solution was analyzed after allowing transient

behavior to die out. The time history of journal orbit and Poincare' points in-phase with the unbalance force were tracked and used to identify that a stable response was achieved during the time transient integration. To achieve this purpose, the algorithm program is coded to allow restart after any time-period. This is also necessary because of the time-limit restriction for interactive access the Ohio Supercomputer (Cray Y-MP8/864).

Three examples of periodic motion are shown in Fig. 5.1.1, which show the journal orbit and Poincare' points. The first of these cases involves a moderate static bearing load (Fig. 5.1.1.a) and shows a large amplitude journal orbit with a period-2 (P2) $1/2$ subharmonic resonance dominating the motion. In the second case (Fig. 5.1.1.b) at twice the previous static bearing load, but smaller unbalance, a period-2 motion is also obtained, albeit not dominated by the subharmonic, as in the previous case. In the third of these three periodic cases, (Fig. 5.1.1.c), the static load is made four times that for the first case, and the resulting steady state is a fairly large amplitude journal vibration with a large synchronous component that reflects the "contact-problem" nature of the high static load condition. Also, it is noteworthy in this third case that the motion is period-5, i.e., a significant $1/5$ subharmonic component is present and evident in the power spectrum Fig. 5.1.1.d as well.

Figure 5.1.1.a Journal Orbit and Poincare' points,(for $B_p=0.2$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$), at $\omega / \omega_c = 2.5$, $u/C=0.9$.

Figure 5.1.1.b Journal orbit and Poincare' points, (for $B_p=0.1$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$), at $\omega / \omega_c=1.95$, $u/C=0.5$.

Figure 5.1.1.c Journal orbit and Poincaré points, (for $B_p=0.05$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$), at $\omega/\omega_c=1.75$, $u/C=0.9$.

Figure 5.1.1.d Power Spectrum, (for $B_p=0.05$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$), at $\omega/\omega_c=1.75$, $u/C=0.9$.

Two examples of quasi-period response are shown in Fig. 5.1.2 (the journal orbit, Poincare' map and power spectrum). Both results from nonlinear limit cycle subsynchronous and the unbalance synchronous frequencies being incommensurate (as shown in the power spectrum). The only parameter difference between these two cases is a two-fold change in static bearing load. The formation of a single closed loop in the obtained Poincare' map is the well known characteristic of two incommensurate frequencies (called it quasi-2H). The Poincare' mapping used here is strictly from the obtained orbital position, stroboscopically deposited at the spin frequency of the rotor and in-phase with the rotating unbalance force vector.

Fig. 5.1.3 and Fig. 5.1.4 are two interesting results. In Fig. 5.1.3, oil whip orbit was created when journal is operating at $\omega/\omega_c=2.2$ with 0.3 C unbalance, although the linear instability threshold speed is $\omega_{th}/\omega_c=2.36$. In Fig. 5.1.4, oil whip occurred at $\omega/\omega_c=3.0$ less than the linear threshold speed $\omega_{th} = 3.48 \omega_c$. These results show again that a nonlinear big limit cycle could be excited by a small amount of unbalance even though the rotating speed is still less than linear instability threshold speed.

Figure 5.1.2.a Journal orbit, Poincare' map and power spectrum, (for $B_p=0.2$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$), at $\omega / \omega_c = 1.95$, $u/C=0.9$.

Journal orbit

Poincare' map

Figure 5.1.2.b Journal orbit, Poincare' map and power spectrum, (for $B_p=0.1$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$), at $\omega/\omega_c=1.75$, $w/C=0.9$.

Figure 5.1.3 An Oil Whip orbit, (for $B_p=0.1$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$), at $\omega / \omega_c=2.20$, $u/C=0.3$.

Figure 5.1.4 An Oil Whip orbit, (for $B_p=0.05$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$), at $\omega/\omega_c=3.0$, $u/C=0.3$.

Figs. 5.1.5 through Fig. 5.1.8 show results for a high static bearing load, ($B_p=0.005$, $\Omega_n=5.0$, $L/D=0.5$, $\Delta=0.1$) case in which only the unbalance is varied. Fig. 5.1.5 shows the bifurcation diagram for both X and Y orbital displacement signals. By mapping the synchronous Poincare' points of orbital motion, the nature of a specific case can be properly identified, as indicated in Figs. 5.1.6 - 5.1.8.

Of particular interest here are the results for the dimensionless unbalance range 0.134 to 0.15. The three cases shown in Figs. 5.1.6 - 5.1.8 cover this narrow unbalance range, in which the transition is from quasi-periodic motion, Fig. 5.1.6 (two Poincare' map loops, indicating three incommensurate frequencies (i.e. quasi-3H)), to period-10 motion, Fig. 5.1.7, to chaotic motion, Fig. 5.1.8.. In this third case, the chaos content is evidenced by the fractal nature of the Poincare' map, wide range frequencies of power spectrum and more rigorously by the positive Lyapunov exponent ($\lambda>0$) and fractal dimension, D_2 , as shown. The computation of the maximum Lyapunov exponent and the correlation dimension will be introduced later. This case was recomputed with a order of ten reduction in integration tolerance, achieving nearly the same Poincare' mapping and chaos measures, verifying the validity of the numerical integration for the motion equations. Tables 5.1.1, 5.1.2, and 5.1.3 provide a summary of cases with unbalance.

The bifurcation diagrams in Fig. 5.1.5 correspond to the information summarized in Tables 5.1.2 and 5.1.3. At small unbalance magnitudes, the nonlinear response is period-1 and bifurcates into period-2 as the unbalance is progressively increased. In the narrow range of unbalance, 0.132 - 0.134, the

period-2 motion changes to a three-frequency quasi-periodic motion. Note from Fig. 5.1.6.b that the two closed loops are nearly the two coalesced points of period-2 motion at slightly lower unbalance magnitude. That is, there is an intrusion of a third harmonic (at frequency about 0.1ω , see the power spectrum at Fig. 5.1.6.c) which is incommensurate with the two harmonics of the period-2 motion. This intrusion is increased as unbalance is increased slightly more, as evidenced from the Poincare' map in Fig. 5.1.6.d

A further small increase in unbalance magnitude (0.14 to 0.144) causes the steady-state motion to emerge from quasi-periodic to period-10, as shown by ten repeating Poincare' points superimposed on the orbital trajectory of one period of motion, Fig. 5.1.7.a and power spectrum at Fig. 5.1.7.b, in which 0.1ω subharmonic becomes more clearly. And a further small increase in unbalance to 0.15 causes this period-10 motion to transition into chaotic motion, as shown in Fig. 5.1.8. It is interesting to note the apparent genesis of the fractal Poincare' map of Fig. 5.1.8 is the two closed loops of the quasi-periodic Poincare' map of Fig. 5.1.6.e, even though they are separated by a narrow band of unbalance magnitude in which the motion is period-10 (Fig. 5.1.7).

ω/ω_c u/C	1.85	1.90	1.95	2.2	2.5	2.75
0.1	P1	P1	QP	QP	P2	oil whip
0.3	P1	P1	P1	P1	P2	
0.5	P1	P1	P1	P1	P2	
0.7	QP	P1	P1	P1	P2	
0.9	P1	QP	QP	QP	P2	

Table 5.1.1.a Forcing response for $B_p=0.2$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$ with linear instability threshold $\omega_{th}/\omega_c=1.92$ and Saddle-Node speed $\omega_{sn}/\omega_c=1.85$.

ω/ω_c u/C	1.75	1.85	1.95	2.2
0.1	P1	P1	P1	P1
0.3	P1	P1	P2	oil whip
0.5	P1	P1	P2	oil whip
0.7	QP	QP	QP	
0.9	QP	QP	QP	

Table 5.1.1.b Forcing response for $B_p=0.1$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$ with linear instability threshold $\omega_{th}/\omega_c=2.36$ and Saddle-Node speed $\omega_{sn}/\omega_c=1.72$.

ω/ω_c u/C	1.75	1.85	1.95	2.2	3.0
0.1	P1	P1	P1	P1	P1
0.3	P1	P1	P2	P1	oil whip
0.5	P1	P2	P2	QP	oil whip
0.7	QP	QP	QP	QP	
0.9	P5	QP	QP	QP	

Table 1.c Forcing response for $B_p=0.05$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$ with linear instability threshold $\omega_{th}/\omega_c=3.48$ and Saddle-Node speed $\omega_{sn}/\omega_c=1.73$.

where, P_n represents the period n subharmonic motion.
QP represents the quasi-periodic motion.

ω/ω_c u/C	2.15	4.30
0.05	P1	P5
0.10	P2	oil whip
0.15	chaos	
0.20	chaos	

Table 5.1.2 Forcing response for $B_p=0.005$, $\Omega_n=5.0$, $L/D=0.5$, $\Delta=0.1$ with linear instability threshold $\omega_{th}/\omega_c=19.12$.

u/C	Poincare Section
0.05	P1
0.06	P1
0.07	P1
0.08	P1
0.09	P1
0.10	P2
0.11	P2
0.12	P2
0.13	P2
0.132	P2
0.134	QP
0.14	QP
0.144	P10
0.15	chaos

Table 5.1.3 Birth of chaos, $B_p=0.005$, $\Omega_n=5.0$, $L/D=0.5$, $\Delta=0.1$, $\omega = 2.15 \omega_c$.

(a)

(b)

Figure 5.1.5 Bifurcation diagram, $B_p=0.005$, $\Omega_n=5.0$, $L/D=0.5$, $\Delta=0.1$, $\omega/\omega_c=2.15$

Figure 5.1.6.a

Figure 5.1.6.b

Figure 5.1.6.a and b Journal orbit and Poincaré map of journal orbit, $B_p=0.005$, $\Omega_n=5.0$, $L/D=0.5$, $\Delta=0.1$, $\omega/\omega_c=2.15$, $u/C=0.134$.

Figure 5.1.6.c Power spectrum for $B_p=0.005$, $\Omega_n=5.0$, $L/D=0.5$, $\Delta=0.1$, $\omega/\omega_c=2.15$, $u/C=0.134$.

Figure 5.1.6.d

Figure 5.1.6.e

Figure 5.1.6.d and e Journal orbit and Poincare' map , $B_p=0.005$, $\Omega_n=5.0$, $L/D=0.5$, $\Delta=0.1$, $\omega/\omega_c=2.15$, $w/C=0.14$.

Figure 5.1.7.a Poincare' points and trajectory of the center of journal , $B_p=0.005$, $\Omega_n=5.0$, $L/D=0.5$, $\Delta=0.1$, $\omega/\omega_c=2.15$, $u/C=0.144$.

Figure 5.1.7.b Power spectrum, for $B_p=0.005$, $\Omega_n=5.0$, $L/D=0.5$, $\Delta=0.1$, at $\omega/\omega_c=2.15$, $u/C=0.144$.

Figure 5.1.8.a

Figure 5.1.8.b

Journal orbit and Poincaré map, for $B_p=0.005$, $\Omega_n=5.0$, $L/D=0.5$, $\Delta=0.1$, at $\omega/\omega_c=2.15$, $w/C=0.15$, $Tol = 10^{-5}$.

Figure 5.1.8.c Linear averaging
power spectrum

Figure 5.1.8.d Poincaré map of the
center of the journal, for $Tol = 10^{-6}$.

5.2 Chaos Measurement

Two invariant quantities, Lyapunov exponent and fractal dimension, were estimated from the time series data in this work to identify the chaotic motion, which was presented in previous Chapter. Lyapunov exponents are related to the average rates of convergence or divergence of nearby trajectories in the phase space. They measure how predictable or unpredictable the system is. It is believed that when at least one of Lyapunov exponents is positive, then the system is chaotic. The fractal dimension refers to static properties and reflects the geometric character of a system. In general, the dimensionality of a map gives us an estimate of the number of active degrees of freedom for the system. For example, the dimensionality of a fixed point is equal to 0 because a point is a 0-dimensional object in geometry. A line or a simple closed curve has a dimensionality of 1. Similarly, a surface has a dimensionality of 2, etc. . A chaotic system will then have a dimensionality of " Fractal " number, (i.e., not an integer).

Takens' (1981) method of time delay is applied and allows us to reconstruct the new phase space coordinates from one variable data, that the x-coordinate of the center of the journal, x_j , was chosen in this work. This new m-dimensional state vector $X(t_i)$ are as follows,

$$X(t_i) = (x_j(t_i), x_j(t_i+\tau), \dots, x_j(t_i+(m-1)\tau)), \quad (5.2.1)$$

where $t_i = i \text{ (dt)}$, $i = 1, 2, \dots, N$, m is the embedding dimension and τ is the time delay. The time delay can be chosen at which we have the first minimum

mutual information or the first minimum of the generalized correlation integral $C(\gamma, \tau, m)$ according to Liebert and Schuster (1989) or Fraser and Swinney (1986). The mutual information measures the general dependence of two variables. Thus, with the choice of time delay at the minimum mutual information allows us to make the new coordinates as independent as possible. Fig. 5.2.1.a and Fig. 5.2.1.b show the logarithm correlation integral versus the time delay as a function of the embedding dimension and number of used data points for the chaotic motion, $B_p=0.005$, $\Omega_n=5.0$, $L/D=0.5$, $\Delta=0.1$, $\omega/\omega_c=2.15$ and $u/C=0.15$. The first minimum is occurred at 28 (dt), i.e., time delay $\tau = 28$ (dt), and preserved with respect to the number of used data points. Then the new phase space coordinates can be generated by Eq. 5.2.1 with time delay $\tau = 28$ (dt) and used to compute the fractal dimension and Lyapunov exponent.

The fractal dimension or more precisely correlation dimension D_2 is computed by applying Grassberger and Procaccia's (1983) approach and obtained by using the more efficient computation technique of Grassberger (1990). The correlation integral is defined as,

$$C(\gamma, \tau, m) \equiv \lim_{N \rightarrow \infty} \frac{1}{N^2} \times \{ \text{number of pairs } (i,k) \text{ with } \|\tilde{X}(t_i) - \tilde{X}(t_k)\| < \gamma \} \quad (5.2.2)$$

and the correlation integral $C(\gamma, \tau, m)$ behaves as a power of γ for small γ :

$$C(\gamma, \tau, m) \propto \gamma^{d_2} \quad (5.2.3)$$

Moreover, the scaling exponent d_2 is close to the fractal dimension. We compute the correlation dimension by varying γ and evaluate the scaling exponent d_2 at different embedding dimension m and also at different number of used data points N . Fig. 5.2.2 shows the convergence of the estimated correlation dimension D_2 . The obtained correlation dimension D_2 at embedding dimension $m = 6$ and number of used data points $N = 32768$ is equal to 2.43 for the chaotic motion, $B_p=0.005$, $\Omega_n=5.0$, $L/D=0.5$, $\Delta=0.1$, at $\omega/\omega_c=2.15$ and $u/C=0.15$. The relation between embedding dimensions m and numbers of used data points N is consistent with Nerenberg and Essex's (1990) conclusion about the number of data needed at different embedding dimension.

The Lyapunov exponents, which measure the average exponential rates of divergence or convergence of nearby trajectories of motion with slightly different initial conditions, is estimated according to Wolf et al. (1985) accompanied with the previous time delay and embedding dimension information. The average largest Lyapunov exponent, $\lambda_{\max}$, is defined by

$$\lambda_{\max} = \frac{1}{N \Delta t} \sum_{i=1}^N \log_2 \frac{d(t_i)}{d(t_i - \Delta t)} \quad (5.2.4)$$

where, Δt is the step size, N is the total number of point used along the trajectory, $d(t, -\Delta t)$ is the distance between two trajectories of motion at a starting time $t, -\Delta t$ and at a time step Δt later the distance is $d(t,)$. The noise effect considered in computing the Lyapunov exponent is only the suddenly " spike " data or too " small " data which actual did not exist in our analysis because that we are using the

numerical integration data rather than experimental data. For experimental signals, noise detection and reduction could usually be a problem, a nonlinear averaging scheme introduced by Sidorowich (1990), could be useful about noise reduction from a possibly chaotic time series data. Fig. 5.2.3 shows that a temporal converging of the maximum Lyapunov exponent (only the maximum one needed to identify a chaotic motion) $\lambda_{\max} = 2.00 \pm 0.06$ (Bits/sec) is obtained for the chaotic motion, $B_p=0.005$, $\Omega_n=5.0$, $L/D=0.5$, $\Delta=0.1$, at $\omega/\omega_c=2.15$ and $u/C=0.15$. Which means that we will lose the initial information about 2.00 bits at every second. Then, for a twenty bits accuracy system, (12 bits accuracy is common in the current laboratory), the initial information will totally be lost after 10 seconds. This tells us the sensitivity of the initial condition of the system as operating at that condition and long term prediction of the orbit of any initial point is impossible.

A similar procedure was also applied to the simulation results, but the numerical tolerance has a ten-time reduction. Fig. 5.2.4 shows the correlation integral as a function of embedding dimension and the first minimum correlation integral occurred at 20 (dt), i.e time delay $\tau = 20$ (dt). It is different with the previous result at $Tol = 10^{-5}$. This is because of the different sampling rate, $dt = \pi/20$ for $Tol = 10^{-6}$ and $dt = \pi/30$ for $Tol = 10^{-5}$. So, actually the time delays in these two cases are almost at about $\tau = \pi$. That is the intention of choosing different sampling rate in two cases, in order to study the sensitivity of the sampling rate. It looks like that the sampling rate did not have much effect. Fig. 5.2.5 shows the converging of the correlation dimension D_2 and $D_2=2.38$. This is quite the same as the high tolerance result, which implies that the Poincare' map Fig. 5.1.8.b and Fig. 5.1.8.d have the same geometric structure. Fig. 5.2.6 is the result of the maximum

Lyapunov exponent $\lambda_{\max}$ and $\lambda_{\max} = 1.61 \pm 0.06$. The $\lambda_{\max}$ is 24% lower than the high tolerance result ($\lambda_{\max} = 2.00$). It could be because that there still exists some difference in the local structure, which is dominated by the negative Lyapunov exponents (difficult to be estimated), although it is a close fractal dimension.

The boundedness and density of the Poincare' section (Fig. 5.1.8.b), topological transition of journal orbit (Fig. 5.1.8.a), and sensitive dependence on initial data (i.e., the positive Lyapunov exponent) prove the existence of the chaotic motion of this rotor-bearing system when operating at very high load for high rigidity of shaft.

Figure 5.2.1.a The logarithm of the correlation integral $\text{Log } C(\gamma, \tau, m)$ as $N=2000$, $\gamma = 0.5$ and $dt = \pi/30$, $\text{Tol} = 10^{-5}$.

Figure 5.2.1.b The logarithm of the correlation integral $\text{Log } C(\gamma, \tau, m)$ as a function of the number N of used data points. Also $\gamma = 0.5$, $m = 3$ and $dt = \pi/30$, $\text{Tol} = 10^{-5}$.

The logarithm correlation integral for $B_p=0.005$, $\Omega_n=5.0$, $L/D=0.5$, $\Delta=0.1$ at $\omega/\omega_c=2.15$ and $u./C=0.15$

Figure 5.2.2 Estimated correlation dimension D_2 , for $B_p=0.005$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$, at $\omega/\omega_c=2.15$, $u/C=0.15$, and $\text{Tol} = 10^{-5}$. diamond: $N=8192$, triangular : $N=16384$, dot: $N=32768$.

Figure 5.2.3 Max. Lyapunov exponent, for $B_p=0.005$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$, at $\omega/\omega_c=2.15$, $u/C=0.15$, and $\text{Tol} = 10^{-5}$.

Figure 5.2.4 The logarithm of the correlation integral $\text{Log } C_l^m(\epsilon, \tau)$ as $N=2000$, $\gamma = 0.5$ and $dt = \pi/20$, for $Bp=0.005$, $\Omega_n=5.0$, $L/D=0.5$, $\Delta=0.1$ at $\omega/\omega_c=2.15$ and $u/C=0.15$, $\text{Tol} = 10^{-6}$.

Figure 5.2.5 Estimated correlation dimension $D2$, for $Bp=0.005$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$, at $\omega/\omega_c=2.15$, $u/C=0.15$, and $\text{Tol} = 10^{-6}$. diamond: $N=8192$, triangular : $N=16384$, dot: $N=32768$.

Figure 5.2.6 Max. Lyapunov exponent, for $B_p=0.005$, $\Omega_n=1.0$, $L/D=0.5$, $\Delta=0.1$, at $\omega/\omega_c=2.15$, $u/C=0.15$, and $\text{Tol} = 10^{-6}$.

Chapter 6

Experiments on the Nonlinear Hysteresis Loop

6.0 Introduction

A Bently rotor-bearing test rig (set up for oil whip option), was modified to investigate the nonlinear hysteresis loop phenomenon for a journal bearing supported rotor-bearing system. The two purposes in this experiment are (i) to confirm the existence of the hysteresis loop, (i.e., to identify the instability threshold speed, ω_{th} , and the Saddle-Node speed, ω_{sn}), and (ii) to correlate the Saddle-Node speed to the lowest damped natural frequency, ω_c , that was shown to asymptotically approach to $\omega_{sn}/\omega_c=1.725$ in simulations. The instability threshold speed is the rotating speed at which the subsynchronous vibration becomes dominant and the orbital subsynchronous is the lowest natural frequency, ω_c , of the system. These two frequencies and Saddle-Node speed were carefully investigated during each test. The oscillation of the journal was measured by two proximitors probes, (X and Y directions) and was real-time monitored both in orbit, (by an oscilloscope), and in spectrum, (by a HP 3582 spectrum analyzer which is connected to a portable PC to capture the real time spectrum). These two displacement signals and a synchronous trigger signal were also recorded by a 486 PC data acquisition system with 12-bits precision. Journal orbit and the

corresponding spectrum are then analyzed accompanied with the nondimensional system parameters.

6.1 Experimental Facility, System Assembly and Testing Procedure

6.1.1 Experimental Facility

A modified Bently Rotor-Kit made by Bently Nevada Co. was used to perform the experiments on the nonlinear hysteresis loop of a rotor-bearing system. The Bently Rotor-Kit was designed to demonstrate various basic rotor vibration phenomenon, such as natural frequency, oil whip, unbalance response, balancing technique, and rub impact etc.. The original " oil whip option " was modified and enhanced to scientifically investigate (not just demonstrate) the oil whip behavior of a journal bearing system and to be able to match the simulation of the nonlinear hysteresis loop. The basic rotor kit, as shown in Fig. 6.1.1, consists of the following parts; power driving and speed control system, (a motor with maximum 10,000 rpm rotating speed), a flexible coupling, and the key phaser wheel. The shaft has a 3/4" in diameter and 22" length and is supported by a bush bearing (as shown in the inboard bearing housing spot) and a fluid-film bearing at the other end, precisely balanced rotor masses, and measurement struts.

The lubrication system (Fig. 6.1.2) includes a lubricant supply bottle, a throttle valve, a transparent bearing with proximity probes at 90° apart, an oil collector and

a pump to return the lubricant from the oil collector, and is installed at the outer board bearing spot of the basic rotor kit.

Figure 6.1.1 Basic Rotor Kit

Figure 6.1.2 Oil Whirl Option Setup

A preload application unit was designed and built (by Michael Adams, M. S. student) and used in conjunction the original oil whip setup system and the whole experimental system for the hysteresis loop investigation, as shown in Fig. 6.1.3.a, b and Photo. 1. The preload supply unit consists of a 180 degree precision drilled journal bearing (called the preload bearing, shown in Fig. 6.1.4 and Photo. 2, with 0.001" radius clearance to the shaft), an extension spring (see Photo. 2) to supply the required preload, a 1 x 4 frame (with a bolt through the center (Photo. 3) and mounted on the plate at the bottom) to hang up the spring, two hooks (one for connecting with journal bearing and the other for hanging the spring (as show in Photos. 1, 3) and a manual control lubrication system, including one two-box-in-one oil collector (Photo. 4) in which the preload bearing is immersing into the upper oil collector, a reservoir (Photo. 4) with one feeding tube, one draining tube and one tube connected to a bulb pump, (see Photo. 4).

Figure 6.1.3.a General Arrangement of the Hysteresis Loop Test Rig

Figure 6.1.3.b Cross-Section of Preload Unit

Figure 6.1.4 Preload Bearing

6.1.2 System Assembly

The whole system was assembled in the following procedure.

1. Remove the shaft from the rotor kit.
2. Remove end cap from oil whirl bearing and slide it over oil whirl shaft. Install the balanced rotor mass to the approximately desired position on the oil whirl shaft. Install the two-in-one box and preload bearing (with bearing in the middle of the box) on the oil whirl shaft. Install the shaft into the motor coupling.
3. Position the oil whirl bearing housing on the base, insert the journal, and slide the bearing housing inboard until the journal contact the axial face of the bearing. Slide the bearing housing outboard to provide approximately 1/16 inch axial clearance. Install end cap into oil whirl bearing.
4. Check that no rub, preload, unbalance, or bow exists along the shaft. Also, the journal must move freely within the whirl bearing clearance.
5. Position the two-in-one box and the frame to the desired position and secure the frame on the plate. Hook up the preload bearing and the spring. Ensure the bearing and the spring is aligned by giving a little preload and adjusting the hook-up position on the bearing.
6. Position the rotor mass to the desired position. Ensure the 0 degree mark aligned with the phase marker on the coupling.
7. Fill the oil whip bearing reservoir and the upper oil collector of the two-in-one box with the lubricant, similar to SAE5. Ensure the preload bearing is fully immersed into the oil.

8. Adjust the gap between the proximator and the surface of the journal to 0.020", in which the linear proportional range is applicable as shown in Fig. 6.1.2.1.

9. Connect the X and Y proximators to the proximator amplifier that connected to the DVF3 (Digital Vector Filter 3). Connect the X and Y outputs from the DVF3 to an oscilloscope operating in X-Y mode, also bypass those signal to the spectrum analyzer to monitor the oscillating spectrum. Ensure the DVF3 is set to the direct display mode to bypass the filter.

Proximator - X calibration curve

Proximator - Y calibration curve

Figure 6.1.2.1 Proximators Calibration Curves

6.1.3 Testing Procedure

The testing procedure is summarized as follows.

1. Turn on the measurement system and the monitoring equipment (see Photo. 5), and recheck the system especially the oil level of the preload bearing.
2. Stretch the spring to the desired preload position according to the spring constant, as shown in Fig. 6.1.3.1.
3. Open the oil flow valve fully (for the oil whirl bearing). Ensure the oil drips from the bearing into the oil collector and operate the oil return pump as necessary to maintain a supply in the reservoir.
4. Start the motor at idle speed (about 300 rpm) and recheck all of the signals.
5. Set the maximum speed, set the ramp rate to 5000 rpm / minute and turn the ramping switch to the “ramp up “ position. As the rotor accelerates, monitor the journal orbit and the spectrum. Record the approximate instability threshold speed (where the journal orbit expands suddenly and the subsynchronous frequency becomes dominant.) Switch to “ramp down “ when the journal orbit nearly fills the bearing clearance. Record the approximate speed where the journal orbit suddenly collapses to a stable linear mode. This procedure provides a rough scope of the hysteresis loop.
6. Increase the speed slowly (step by step) to accurately find the instability threshold speed. Journal orbit signals and the spectrum data were taken at a rotating speed lower than the instability threshold speed, at the instability threshold speed, and at higher than instability threshold speed during the ramp up process. Data points within hysteresis loop and in the vicinity of Saddle-Node speed were also taken at the ramp down process. The X and Y proximitors signals

and a trigger signal were recorded during a 2 sec period at a sampling rate of 1,000 Hz sampling rate (each channel) for each steady state data point.

7. Change the preload and repeat the steps 3 to 6.

The oil level of the preload bearing should be constantly monitoring, and it may be necessary to feed the oil from the reservoir and to drain the oil from the bottom oil collector of the two-box-in-one during long periods of running. The temperature of the oil in the oil whirl bearing and in the preload bearing should not have large change during the testing period to ensure the constant viscosity operation.

Figure 6.1.3.1 Spring Calibration Curve

6.2 System Dimension

The schematic system configuration and the position (relative to bush bearing) of the components are shown in Fig. 6.2.1 with the following component dimensions.

Oil whirl bearing :

Length : 0.5 inch.

Diameter : 1.0 inch.

Radius clearance : 0.0056 inch.

Oil viscosity : 37.78 mPa S (calibrated by the Cannon-Ubbelohde viscometer made by Cannon Instrument Co.).

Mass of oil whirl bearing : 0.110 lbm.

Mass of rotor disk : 1.340 lbm.

Mass of shaft : 0.656 lbm.

Mass of preload bearing : 0.057 lbm

Mass of spring : 0.136 lbm

Mass of hook and extension bar : 0.059 lbm

Spring force = $0.890 * (\text{stretched length}) + 3.014 \text{ lbf}$.

(the spring force was calibrated by force calibration bar)

Room temperature : 70 degree F.

Figure 6.2.1 Schematic Configuration of the Rotor-Bearing system

6.3 Data Reduction

Two kinds of data sets were obtained during the experiment. One was the analog signals, including the trigger and the relative displacement of the journal bearing for the journal orbit from the X and Y proximitors. These signals were taken by a 486 PC data acquisition system (see Photo. 6) and were converted to digital data by a 12 bits precision A/D converter with 1,000 Hz sampling rate at each channel. The other set was the oscillation spectrum of the journal and was directly down-loaded to a portable PC during the test.

The spectrum data was used to directly analyze the journal vibration in the frequency domain. The X and Y proximitors measured the relative displacement of the probe tip to the surface of the journal. The actual journal displacement was obtained by subtract the total displacement by the starting DC value (which

indicated the initial position of the proximator). The trigger signal was taken by an optical device and the signal was triggered from high voltage to low voltage as the optical device picked up reflected light which was generated by a bright strip installed on the surface of the coupling. This high to low trigger was used as the starting flag of an orbit cycle, and the data points within a forcing cycle were then counted and recorded until the next high to low signal occurred, (for example, 10 points were counted within a cycle for the rotating speed of 6,000 rpm by 1,000 Hz sampling rate). Averaging process was then performed if the preceding cycle had the same number of data points. The number of data points within a cycle varied often because the sampling rate divided by the rotating speed was usually not an integer. To overcome this problem, we can use the trigger signal as an external clock to trigger the data acquisition system. The orbit averaging was performed if the preceding cycle had the same number of data points. As many averaged cycles as possible were taken in the averaging process in order to obtain a more precise journal orbit. The cubic spline interpolation method was applied to create the final orbit by interpolating more points within the averaged data points. The interpolation process allowed us to create a much more smooth orbit without bias.

6.4 Experimental Results and Discussions

Four preloads testing conditions (3.46lb, 3.90lb, 4.35lb and 4.79 lb) had been investigated. Table 6.4.1 summarized the corresponding system parameters, in which the stiffness and natural frequency of the shaft were estimated by the

assumption that the shaft was supported at each end by a simple support, (which were a bushing bearing and an oil whirl bearing), and was subjected to the concentrated load at the positions of the rotor mass and the preload bearing. The net load of the oil whirl bearing was computed by considering the contribution from the rotor mass, preload, mass of the shaft and the mass of the oil whirl bearing itself. The bearing parameter, B_p , was varied from 0.94 to 0.46, and the shaft rigidity was varied from 0.98 to 0.81 with the preload was increasing from 3.46 lb to 4.79 lb. The instability threshold speed, ω_{th} , and the lowest natural frequency of the system, ω_c , and the Saddle-Node speed, ω_{sn} , were obtained at different testing conditions by analyzing the journal orbit and vibration spectrum, which were shown in Appendix D. Table 6.4.2 summarized the correlated speed ratio, ω_{sn}/ω_c , to the bearing load. The speed ratio is decreased from 1.78 at low bearing load to 1.66 at high bearing load. This is very similar to the previous simulation results, in which the speed ratio is decreased from 1.85 at a low bearing load to 1.72 at a high bearing load. Qualitative trend and quantitative value prove again the existence of the hysteresis loop and its correlation to the bearing load.

A sudden jump behavior was also observed during testing, in that when we provided a small disturbance (by slightly touching the shaft) to the system, we noticed that the journal orbit suddenly expanded and remained so when the shaft was operating within the hysteresis loop. The jump behavior did not occur when the system parameters lay outside the hysteresis loop.

Testing for much higher preload (with higher instability threshold speed) was attempted, but a stable orbit could not be obtained at the instability threshold

speed. The journal orbit would switch between oil whip and high order subharmonic oscillation. This may occur because the upper portion (at the edge) of the preload bearing was squeezed (it was not designed to do so) and then a downward force was created to push the journal to the center at the same time that the oil whirl bearing was tending to be unstable. These two forces were coincidentally imposed on the system, so a stable orbit could not be obtained. This behavior was also observed at even higher rotating speeds.

Preload	Bp	$K_s^{(1)}$	$\omega_n^{(2)}$	Ω_n	W
3.46 lb	0.94	231.81	41.13	0.98	0.97 lb
3.90 lb	0.70	199.17	38.13	0.91	1.30 lb
4.35 lb	0.56	174.53	35.69	0.85	1.64 lb
4.79 lb	0.46	155.32	33.67	0.81	1.97 lb

(1) unit : lbf / in, (2) unit : Hz

Table 6.4.1 System parameters for different loading conditions

Preload	Bp	ω_{th}	ω_c	ω_{sn}	ω_{sn}/ω_c
3.46 lb	0.94	70.31 Hz	35.16 Hz	62.50 Hz	1.78
3.90 lb	0.70	72.27 Hz	36.14 Hz	64.45 Hz	1.78
4.35 lb	0.56	80.08 Hz	39.06 Hz	66.14 Hz	1.69
4.79 lb	0.46	83.98 Hz	41.02 Hz	68.00 Hz	1.66

Table 6.4.2 Experimental results

Chapter 7

Conclusions and Future Works

The dynamical character of a cylindrical journal bearing supported rotor-bearing system has been investigated by performing the linear analyses, nonlinear numerical simulations and experimentation. The conclusions drawn from this research extend significantly the pre-existing “ picture “ of the oil-whip instability phenomena, especially the associated hysteresis loop.

A clearer picture of the nonlinear hysteresis loop, characterized by the Hopf bifurcation (or the linear instability threshold speed) and Saddle-Node speeds has emerged, showing that the associated lower - speed bound (i.e., Saddle-Node speed) actually decreases to an asymptote (approaching about $\omega_{SN}/\omega_C = 1.725$ in this work) with increased bearing load, in contrast to the small perturbation instability threshold speed.

A journal bearing supported rotor-bearing test rig has been built and tested. The nonlinear hysteresis loop had been clearly confirmed, and the trend of the ratio of Saddle-Node speed to the instability threshold speed is consistent with the simulation results. The lower bound speed of the hysteresis loop is somewhat lower with an asymptote about 1.66 times of the lowest natural frequency with increasing bearing load in this experiment work (1.725 obtained from simulations).

By superimposing rotor unbalance on operating speeds within the hysteresis loop, it has been shown that periodic (synchronous and subharmonic), quasi-periodic (quasi-2H and quasi-3H) and chaotic motions are all possible. Furthermore, the occurrence of any of these and their transition to each other are highly sensitive to the operating parameters (e.g., bearing load, unbalance magnitude and proximity of the bifurcation points). This strongly suggests the utility of chaos measures and patterns as safe diagnostic tools in assessing risks associates with the stable nonlinear limit cycle within the hysteresis loop.

More works, (such as chaotic symptom detection from similar rotordynamic system and more parametric simulation studies (clearance, viscosity, L/D ratio)), still needs to be done to move this concept of using chaos signal processing methods as a diagnostic tools for practical machinery applications. While this work hopefully expands the picture, the basic oil-whip phenomenon still retains many of its secrets.

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Nomenclature in Rotor-Bearing Motion

B_p = dimensionless bearing parameter (= $(\mu L/\pi) (R/C)^3 (g/W) \sqrt{C/g}$)

C = mean radial clearance of journal bearing, m

C_{xx}, C_{xy} , = dimensionless damping coefficients of journal bearing

C_{yx}, C_{yy} ,($C_{xy} = C\omega(\partial F_x / \partial \dot{Y}) / W$ etc.)

D = inner diameter of journal bearing, m

f_x, f_y = dimensionless fluid film force, N

F_x, F_y = components of fluid film force, N

g = gravity acceleration, m/s^2

h = film thickness of journal bearing, m

K_s = shaft stiffness, N/m

K_{xx}, K_{xy} , = dimensionless stiffness coefficients of journal bearing

K_{yx}, K_{yy} ,($K_{xy} = C(\partial F_x / \partial Y) / W$ etc.)

L = bearing length, m

M_D = disk mass, Kg

M_j = total journal mass, Kg

M_j = dimensionless journal mass (= $M_j C\omega^2/W$)

N = shaft rotating speed, RPM

O_B = bearing center, also the origin of the fixed coordinate (X,Y,Z)

O_j = journal center

P = fluid film pressure, N/m^2

p = dimensionless film pressure (= $(P/\mu N) (C/L)^2$)

R = radius of fluid film bearing (= $D/2$), m

r = dimensionless radius of fluid film bearing (= R/C)

S = the bearing Sommerfeld number

t = time, sec

U = slip velocity of bearing, m/sec

U = dimensionless slip velocity ($= U/(R^*(2\pi N))$)

W = load capacity of fluid film bearing, N

(X_D, Y_D) = coordinates of disk center

(x_D, y_D) = dimensionless coordinates of disk center ($= (X_D/C, Y_D/C)$)

(x_D^*, y_D^*) = dimensionless coordinates of disk center in equilibrium point

(X_G, Y_G) = coordinates of the gravitational center of disk

(X_J, Y_J) = coordinates of journal center

(x_J, y_J) = dimensionless coordinates of journal center

(x_J^*, y_J^*) = dimensionless coordinates of journal center in equilibrium point

Z = axial coordinate of the bearing, m

z = dimensionless axial coordinate of the bearing, (Z/L)

ε = eccentricity of journal, m

λ_l = complex eigenvalue of the rotor bearing system

$\bar{\lambda}_l$ = complex conjugate of the eigenvalue λ_l

κ = eccentricity ratio of journal ($= \varepsilon/C$)

η = coordinate axis in bearing circumferential direction, m

μ = viscosity of oil film, Pa. S

ζ = dimensionless coordinate axis ($= \eta/C$)

θ = angular location of point in bearing film, rad

ϕ = attitude angle of journal, rad

τ = dimensionless time ($= \omega t$)

ω = rotational speed of rotor, rad/sec

ω_c = critical speed of rotor-bearing system, rad/sec

ω_n = natural frequency of rotor. rad/sec

ω_{pf} = Pitch-Fork bifurccation speed of rotor-bearing system, rad/sec

ω_{th} = linear instability threshold of rotor-bearing system, rad/sec

Ω_n = stiffness parameter of rotor ($= \omega_n * \sqrt{C/g}$) (imensionless)

Δ = mass ratio ($= M_J / M_D$)

Nomenclature in Chaos Measurement

$C(\gamma, \tau, m)$ = correlation integral

d_2 = scaling exponent

D_2 = correlation dimension

dt = sampling period

m = embedding dimension

N = number of used data points

X = reconstructed phase space coordinates

γ = radius of hypercubes

τ = time delay

$\lambda_{\max}$ = maximum of Lyapunov exponent

Appendix A

QR solver

The QR solver used in here is extracted from " Numerical Recipes" () and including **balanc**, **elmhes**, and **hqr** three routines. Where **balanc** routine is used to balance a non-symmetric matrix to make corresponding row and column of the matrix have comparable norms to reduce the rounding errors of the matrix while leaving the eigenvalues unchanged. **Elmhes** routine is used to reduce the nonsymmetric matrix to an upper Hessenberg matrix by a sequence of Householder transformation. **Hqr** routine is used to solve the complex eigenvalues of an upper Hessenberg matrix by QR algorithm.

To make sure the QR solver working properly, we use Mathematica software to check its accuracy. Table A.1 shows the computing results of using QR and Mathematica separately to find the complex eigenvalues of two arbitrarily chosen Jacobian matrices. An excellent agreement is found.

JACOBIAN MATRIX:

0	1	0	0	0	0	0	0
-0.54869684	0	0	0	0.54869684	0	0	0
0	0	0	1	0	0	0	0
0	0	-0.54869684	0	0	0	0.54869684	0
0	0	0	0	0	1	0	0
5.4869684	0	0	0	-15.586064	-57.650499	-12.089675	-0.50251809
0	0	0	0	0	0	0	1
0	0		0	21.396435	1.1245989	-7.9190342	-18.856539
5.48696840							

The corresponding eigenvalues solving by using QR and Mathematica.

Q R method's result		Mathematica's result	
REAL	IMAGE	REAL	IMAGE
-57.37318	0	-57.3732	0
-18.427423	0	-18.4274	0
-0.30519264	-0.73515293	-0.305193	0.735153
-0.30519264	0.73515293	-0.305193	-0.735153
-0.039618213	-0.73274034	-0.0396182	0.73274
-0.039618213	0.73274034	-0.0396182	-0.73274
-0.008406514	-0.48614663	-0.00840652	0.486147
-0.008406514	0.48614663	-0.00840652	-0.486147

JACOBIAN MATRIX:

0	1	0	0	0	0	0	0
-0.51020408	0	0	0	0.51020408	0	0	0
0	0	0	1	0	0	0	0
0	0	-0.51020408	0	0	0	0.51020408	0
0	0	0	0	0	1	0	0
5.1020408	0	0	0	-10.410477	-52.984409	-11.438495	-0.28641136
0	0	0	0	0	0	0	1
0	0	5.1020408	0	20.960668	1.0216089	-7.3910655	-17.978737

The corresponding eigenvalues solving by using QR and Mathematica.

Q R method's result		Mathematica's result	
REAL	IMAGE	REAL	IMAGE
-52.785961	0	-52.786	0
-17.560119	0	-17.5601	0
-0.29459237	-0.73917047	-0.294592	0.73917
-0.29459237	0.73917047	-0.294592	-0.73917
-0.03985596	-0.70905466	-0.039856	0.709055
-0.03985596	0.70905466	-0.039856	-0.709055
0.025914979	-0.46996277	0.025915	0.469963
0.025914979	0.46996277	0.025915	-0.469963

Table A.1 Eigenvalues comparison between QR method and Mathematica computation.

Summary of the Hopf Bifurcation Theory

A Hopf bifurcation occurs in a system of differential equations

$$\dot{\underline{x}} = \nu(\underline{x}, \mu) \quad \underline{x} \in \mathbb{R}^n \quad (\text{B.1})$$

if, when the parameter μ is varied through some critical value $\mu = \mu_c$, an equilibrium point $\underline{x} = \underline{x}^*(\mu)$ loses linear stability by having a complex conjugate pair of eigenvalues cross into the right half plane, such as Fig. B.1. That is, the real part of this complex conjugate pair changes the sign from negative to positive. The system is stable if all eigenvalues of the Jacobian at $\underline{x} = \underline{x}^*(\mu)$ with negative real parts.

In a two-dimensional phase space, a Hopf bifurcation would appear as in Fig. B.2. A stable nodes is obtained as $\mu \leq \mu_c$, and limit cycle appears as $\mu > \mu_c$. In higher dimension, the central manifold theory can be applied to reduce the order of the system. Where the original system can be linearized around the bifurcation point and transformed into in the coordinate of the eigenvector directions and an invariant surface (close to the equilibrium point) can be obtained. At Hopf bifurcation point, we will obtain a zero and two pure imaginary pair of eigenvalues in the transformed system. The invariant surface corresponding to the zero eigenvalue is called the central manifold and the dynamic of the bifurcation point can be determined locally.

For n th order systems, a Hopf bifurcation would appear as in two-dimensional case, at least locally around the bifurcation point. The technical conditions which must be met for a Hopf bifurcation to occur are:

$$(i) \lambda_1 = \bar{\lambda}_2 = \alpha(\eta) + i\beta(\eta), \quad \eta = \mu - \mu_c$$

$$(ii) \alpha(0) = 0, \beta_0 = \beta(0) > 0$$

$$(iii) \alpha'(0) \neq 0$$

$$(iv) \operatorname{Re} \lambda_j(\mu_c) < 0, j = 3, 4, \dots, n$$

where $\lambda_i(\mu_c)$, $i = 1, 2, \dots, n$, are the eigenvalues in order of decreasing real parts.

For more information on central manifold theory and Hopf bifurcation theory, please refer to the book by Hassard, Kazarinoff, and Wan (1981).

Figure B.1 Root locus with Hopf Bifurcation

Figure B.2 2-Dimensional Hopf bifurcation phase space

Runge-Kutta-Fehlberg Method

The Runge-Kutta-Fehlberg (or RKF) method is an explicit, one-step and multi-stages with error-controlled schemes. It allows us to change the needed integrating time-step adaptively by controlling the global error between the fourth and fifth order solutions by only 6 evaluation. The whole idea of the RKF scheme is as following.

Goal : To go from $t=a$ to $t=b$ and achieve a "global" tolerance TOL.

Input: $a, b, \text{TOL}, u(a)$; $h_{\max}, h_{\min}, h$

(1) $t = a$; $\text{Flag} = 1$; If $(a+h > b)$ $\bar{h} = b-a$
 else set $\bar{h} = h$
 (Note: $\bar{h}$ is the correct substep size)

(2) While (Flag = 1) do

(2.a) get $\bar{u}$, u^* by using time step $\bar{h}$

(2.b) set $T = (u^* - \bar{u}) / \bar{h}$

(2.c) set $q = (\frac{TOL}{safety * T})^{\frac{1}{n}}$, where n is the order of accuracy of u^* ,
and set $safety = 2$.

(2.d) If ($T < TOL$) (happy with current step)

```

t → t + h̄
u → u'
end if

```

(2.e) If ($q < q_{\min}$) $q = q_{\min}$; set min. magnification factor = .1
 If ($q > q_{\max}$) $q = q_{\max}$; set max. magnification factor = 4

(2.f) $h = q h$
 If ($h > h_{\max}$) $h = h_{\max}$
 If ($h < h_{\min}$) Print warning message, " system maybe stiff or
 TOL is too small ".

$$\bar{h} = h$$

(2.g) If ($t \geq b$) Flag = 0 (DONE !)

$$\text{else (} t + \bar{h} > b \text{) } \quad \bar{h} = b - t$$

(2.h) End While.

where,

$\bar{u}$ is the 4th order solution.

u^* is the 5th order solution.

In RKF method,

$$\bar{u}(t+h) = u(t) + \frac{25}{216} k_1 + \frac{1408}{2565} k_3 + \frac{2197}{4104} k_4 - \frac{1}{5} k_5$$

$$u^*(t+h) = u(t) + \frac{16}{135} k_1 + \frac{6656}{12825} k_3 + \frac{28561}{56430} k_4 - \frac{9}{50} k_5 + \frac{2}{55} k_6$$

and

$$k_1 = h f(t, u)$$

$$k_2 = h f(t + 1/4 h, u + 1/4 k_1)$$

$$k_3 = h f(t + 3/8 h, u + 3/32 k_1 + 9/32 k_2)$$

$$k_4 = h f(t + 12/13 h, u + 1932/2197 k_1 - 7200/2197 k_2 \\ + 7296/2197 k_3)$$

$$k_5 = h f(t + h, u + 439/216 k_1 - 8k_2 + 3680/513 k_3 - 845/4104 k_4)$$

$$k_6 = h f(t + 1/2 h, u - 8/27 k_1 + 2k_2 - 3544/2565 k_3 \\ + 1859/4104 k_4 - 11/40 k_5)$$

Two samples have been chosen to test the RKF scheme, as follows.

A. Predator - Prey problem

The system equations are given as follows

$$\begin{aligned}\frac{dx}{dt} &= \alpha x + \beta xy \\ \frac{dy}{dt} &= \gamma y + \delta xy\end{aligned}\tag{A.1}$$

Also, the system parameters and initial conditions are given as $(\alpha, \beta, \gamma, \delta) = (0.25, -0.01, -1.00, 0.01)$ and $x(0) = 80, y(0) = 30$.

According to the primary linear analysis, we know that there are two equilibrium points, $(0.0, 0.0)$ and $(100.0, 25.0)$ in this system. The equilibrium point $(0.0, 0.0)$ is actually a saddle point with one positive eigenvalue, 0.25, and one negative eigenvalue, -1.0. That is, we can not obtain a stable solution at this equilibrium point. While pure imaginary pair of eigenvalues, $0.5i$ and $-0.5i$, are obtained for the other equilibrium point $(100.0, 25.0)$. That is, $(100.0, 25.0)$ is a center and a limit cycle is expected. Furthermore, the size of the limit cycle is determined by the initial condition. Fig. C.1 shows two exactly the same limit cycles that tolerances are set to be 10^{-5} and 10^{-6} separately.

Figure C.1 Predator-Prey Results

B. An ill-behavior system with the following ordinary differential equation, (chosen from Burden, Fairs and Reynolds (1978)) is solved.

$$y'' - 2y' + 2y = e^{-2t} \sin(t) \quad (\text{B.1})$$

with initial conditions $y(0) = -0.4$, $y'(0) = -0.6$.

The exact solution of this system is $y(t) = 2e^{-2t}(\sin(t) - 2\cos(t))$. Fig. C.1 shows an excellent agreement between the exact solution and RKF result integrated from $t = 0$ to $t = 5$ where tolerance is controlled to be 10^{-5} .

While system becomes ill-behavior, (i.e., system becomes unbounded), when time is increased, for example $t > 6.017$ (where system has a relatively minimum value) in this case as shown in Fig. C.2.a. Fig. C.2.b shows the relationship

between controlled tolerance and the adaptive step-size, the time marching step-size becomes too small ($< 10^{-7}$) for tolerance = 10^{-5} at $t > 9.1$ and RKF will be forced to stop (a "stiff" condition occurred) unless large tolerance is allowed, for example tolerance is set to 10^{-4} or 10^{-3} , or uses a very tiny step-size (this is usually not allowed for CPU time consideration). This example proves that RKF scheme is still working well even for a mild ill-behavior (or mild stiff) problem.

Figure C.2.a Exact solution of the system

Figure C.2.b The Relationship between Controlled Tolerance and Adaptive Time Marching Step-Size

Experimental Results of Nonlinear Hysteresis Loop

Figure D.1 Journal vibration response for 3.459 lb preload at speed up about 4,110 rpm.

Figure D.2 Journal orbit and vibration response for 3.459 lb preload at instability threshold speed about 4,170 rpm.

Figure D.3 Journal orbit and vibration response for 3.459 lb preload at speed up about 4,500 rpm.

Figure D.4 Journal orbit and vibration response for 3.459 lb preload at coast down about 4,085 rpm.

Figure D.5 Journal orbit and vibration response for 3.459 lb preload at speed up about 3,800 rpm.

Figure D.6 Journal orbit and vibration response for 3.459 lb preload at Saddle-Node speed about 3,710 rpm.

Figure D.7 Journal orbit and vibration response for 3.904lb preload at speed up about 4,330 rpm.

Figure D.8 Journal orbit and vibration response for 3.904lb preload at instability threshold speed about 4,360 rpm.

Figure D.9 Journal orbit and vibration response for 3.904lb preload at speed up about 5,060 rpm.

Figure D.10 Journal orbit and vibration response for 3.904lb preload at coast down about 4,210 rpm.

Figure D.11 Journal orbit and vibration response for 3.904lb preload at speed up about 3,900 rpm.

Figure D.12 Journal orbit and vibration response for 3.904lb preload at Saddle-Node speed about 3,810 rpm.

Figure D.13 Journal orbit and vibration response for 4.349 lb preload at speed up about 4,770 rpm.

Figure D.14 Journal orbit and vibration response for 4.349 lb preload at instability threshold speed at about 4,800 rpm.

Figure D.15 Journal orbit and vibration response for 4.349 lb preload at speed up about 5,500 rpm

Figure D.16 Journal orbit and vibration response for 4.349 lb preload at coast down about 4,700 rpm.

Figure D.17 Journal orbit and vibration response for 4.349 lb preload at coast down about 4,400 rpm.

Figure D.18 Journal orbit and vibration response for 4.349 lb preload at coast down about 4,100 rpm.

Figure D.19 Journal orbit and vibration response for 4.349 lb preload at coast down about 4,000 rpm.

Figure D.20 Journal orbit and response for 4.794 lb preload at speeding up about 4,930 rpm.

Figure D.21 Journal orbit and response for 4.794 lb preload at instability threshold speed about 5,020 rpm.

Figure D.22 Journal orbit and response for 4.794 lb preload at speeding up about 5,570 rpm.

Figure D.23 Journal orbit and response for 4.794 lb preload at coast down about 4,820 rpm.

Figure D.24 Journal orbit and response for 4.794 lb preload at coast down about 4,385 rpm.

Figure D.25 Journal orbit and response for 4.794 lb preload at coast down about 4,100rpm.

Figure D.26 Oil whirl disappeared at Saddle-Node Speed about 4,080rpm.

Boundary conditions for Reynolds equation and pressure distribution within fluid film

A. Boundary conditions for Reynolds equation

The boundary conditions for the classic Reynolds equation are given as follows.

(a) $p(-l/2, \theta) = p(l/2, \theta) = 0$ at the boundary of bearing exposed to ambient pressure.

(b) $p(z, 0) = p(z, 2\pi) = 0$ for no cavitation case.

(c) $p(z, \theta) = p_{\text{vapor}}$ for rupture fluid film, (i.e., at $\theta_1 < \theta < \theta_2$) and also

applies $\frac{\partial p}{\partial \zeta} = 0$ to locate the position of

cavitation, (i.e., θ_1 and θ_2).

B. Pressure distribution

Two examples of pressure distributions are given as follows.

(a) For light bearing load case ; $B_p = 1.0$, $L/D = 0.5$, $\Delta = 0.1$, $\Omega_n = 1.0$, $\omega/\omega_n = 0.5$.

(b) For heavy bearing load case ; $B_p = 0.05$, $L/D = 0.5$, $\Delta = 0.1$, $\Omega_n = 1.0$, $\omega/\omega_n = 5.0$

Front View

Side View

Photo. 1 Hysteresis Loop Test Rig.

Photo. 2 Preload Bearing and Spring

Photo. 3 1 x 4 Frame.

Photo. 4 Two-in-One oil collector, oil reservoir and bulb pump.

Photo. 5 Real-Time monitoring system.

Photo. 6 Data acquisition system.