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An implantable transducer for two-degree-of-freedom joint angle sensing

Johnson, Mark William, Ph.D.

Case Western Reserve University, 1994

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Ann Arbor, MI 48106**

AN IMPLANTABLE TRANSDUCER FOR
TWO-DEGREE-OF-FREEDOM JOINT ANGLE SENSING.

by

MARK WILLIAM JOHNSON

Submitted in partial fulfillment of the requirements
for the degree of Doctor of Philosophy

Thesis Advisor: Dr. P. Hunter Peckham

Department of Biomedical Engineering
CASE WESTERN RESERVE UNIVERSITY

May, 1994

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GRADUATE STUDIES

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degree.*

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AN IMPLANTABLE TRANSDUCER FOR TWO-DEGREE-OF-FREEDOM JOINT ANGLE SENSING.

Abstract

by

MARK WILLIAM JOHNSON

An implantable joint angle transducer (IJAT) was developed for chronic use with upper extremity functional neuromuscular stimulation (FNS) neuroprostheses. The IJAT uses a permanent magnet surgically implanted in one bone, and an array of Hall effect sensors surgically implanted in the opposing bone. As the joint is moved, the sensors detect the relative position and orientation changes of the magnet, producing voltages related to the movement. A mathematical transformation is used to convert the sensor voltages into measures of the joint angle. Titanium encapsulation protects the magnet, sensors, and support electronics, and secures the packages in the bone. The IJAT provides consistent, reliable, high quality signals that reflect joint movement from mid-sized two-degree-of-freedom joints. Mathematical modeling techniques were developed for predicting the transfer characteristics of an IJAT when the magnet and sensor array are surgically placed into joints having different geometric configurations, and for calibrating and characterizing operation of the IJAT after implantation. IJATs were implanted using a chronic *in-vivo* dog model to demonstrate the feasibility of implantation and periodic measurement techniques, and to validate use of the modeling techniques prediction of function and calibration. The flexion resolution ranged from 0.4° to 3.0° , the maximum deviation from a linear response was 9° , and over joints ranges up to 108° . The resolution and linearity depend on several transducer and joint geometry parameters, and can be predicted pre-implantation and calibrated post-implantation using the modeling techniques. The IJAT is suitable for use as either a command control or feedback sensor.

I dedicate this dissertation to my loving wife, Jeanne ♥
and
to Bud, Snooter, and the Nipster. 🐾

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Chapter I

Introduction

Functional Neuromuscular Stimulation: FNS systems provide quadriplegic individuals with restored grasp and release function, and the ability to independently accomplish tasks that can not be accomplished with alternative assistive devices [31,32,38,39,40]. Shoulder or wrist movement sensed by an externally mounted transducer controls a percutaneous or implanted electrical stimulator that provides hand function. Externally mounted transducers have been viable for first generation systems, but have inherent limitations. They provide inconsistent signal quality because of mounting issues, encumber other activities, require donning, maintenance, and removal, and may become dislodged during use. Such limitations suggest replacement of external transducers by implantable alternatives. An implanted transducer in the wrist for persons with C6 level injuries would enhance the clinical acceptance of this command control source and potentially enable bilateral function to be restored. In individuals with no retained voluntary wrist control (C5 level injuries), an implanted transducer in the sterno-clavicular joint would replace an externally mounted transducer for sensing shoulder command control movements. An implanted transducer in the wrist would provide for closed loop feedback regulation of wrist angle and free these individual of orthotic supports on the forearm. Thus, an implanted transducer could supply command or feedback information to an FNS system for several important and immediate clinical applications.

Command Control: The lack of adequate command control sources presents major limitations to assistive devices for the severely disabled. The lack of such sources becomes increasingly apparent with the high level spinal cord injury population, since increasingly complex signals are required to interface with assistive devices for individuals with the

highest level injuries. Although many sources such as speech, infrared detection, switches, etc. have been used, these sources generally lack adequate information transfer rates for control of newly emerging systems, such as those employing functional neuromuscular stimulation (FNS). For these applications a one or two-degree-of-freedom proportional signal with fast response time and high signal to noise ratio is required.

In studies comparing the information content of command control signals obtained from a myoelectric signal, pressure, and position, Vodovnik and Rebersek [50] found a significant difference in the quality of position over pressure, and similarly of pressure over myoelectric signals. Although enhanced signal processing schemes for myoelectric control have been developed [20,21], it is insufficient to overcome this difference. Our studies have thus focused on joint movement as a command control source. In studies of quadriplegic subjects evaluating our FNS hand assist systems, shoulder movement has been shown to be a viable command source for quadriplegic users [22,23,26], capable of providing at least one proportional command signal and logical command signals. Users find that using proportional movement of their shoulder to open and close their hand is both easy to learn, and fairly natural to use. Externally mounted shoulder movement transducers have been developed in our center for control of the hand in C5 and C6 quadriplegics [40]. The most substantial deficiency of the external transducer can be traced to the mounting of the device. With an external mounting scheme, a transducer can be moved or dislodged by clothing and perspiration. A particularly difficult problems to overcome has been a shifting of the transducer baseline with postural changes, due primarily to the compliance of the mounting interface (i.e. the skin). The goal in developing an implantable joint angle transducer (IJAT) is to provide consistent, reliable, high quality signals that reflect the movement of either paralyzed or non-paralyzed joints. An implanted transducer would not require user mounting or maintenance, would not be dislodged by clothing or perspiration, and would be entirely

cosmetic. Also, since the IJAT would be rigidly fixed to the bone (a non-compliant interface), the inconsistency of the signals should be eliminated.

While use of the IJAT to control of the hand in spinal cord injury is our intended application, there are other persons with handicaps who potentially could benefit from such developments. In particular, the use of shoulder control for control of powered wheelchairs is a viable technique which has been implemented in our center [8]. Utilization of this command control source relieves the user (C4 quadriplegic) of interfacing with the wheelchair through any external control source. IJAT technology also could be applied to the lower extremity to provide joint angle information for feedback purposes in paraplegic walking, or in the upper extremity for joint angle feedback information of the elbow [5]. In addition, such a transduction system might be applied in multifunctional upper extremity prostheses, such as those providing elbow and hand function [13]. These applications dictate a transduction technique which is implantable, repeatable, and reliable over many decades.

Implantable FNS Systems: A great deal of experience has been obtained with implanted FNS systems. These systems have demonstrated the advantages of the implanted approach. Multichannel implantable stimulators have been surgically implanted in fifteen users. The stimulator is placed in the axillary region with electrodes led subcutaneously to the forearm and hand [45]. The current implanted stimulator systems still derive command control signals from externally mounted transducers. Although the function achieved with an implanted stimulator is similar to that obtained with the earlier percutaneous electrode systems, users greatly prefer the implanted systems simplicity of use and the elimination of electrode maintenance. These are the primary issues addressed by the development of an IJAT. We expect that implantation of the command control transducer will enhance FNS systems by further simplifying their use and reducing the required maintenance.

Implantable Sensing Technologies: Implantable sensors for measuring movement and forces in limbs and joints generally have been developed for acute or intra-operative measurements in humans and animals, or short term chronic animal studies. Sensing technologies used to measure muscle or tendon distension have included: elastic strain gages - which utilize the impedance through silicone rubber tubes filled with a conducting fluid [18,43]; sonomicrometry - which uses the transit time of ultrasound pulses to measure distance [18,51]; and Hall effect transducer based devices that measure the relative displacement between a sensor and a magnet [4,6,14,42,46]. A number of authors have measured tendon tension using a so called "buckle" or "E shaped" transducer, where the tendon is threaded between bars of a metal transducer instrumented with strain gages [5, 18, 30]. Troyk [48], described a method for using a Hall effect sensor and two magnets to measure flexion in a one degree-of-freedom joint such as the knee. These devices were successful only for short term implantation, due to packaging limitations. Encapsulants were all non-hermetic, and included Paralene, silicone rubber, and medical grade epoxy. Several authors have reviewed the use of these types of non-hermetic encapsulation as well as hermetic encapsulation techniques for biomedical implants [7,12,34].

A second class of sensing technologies utilizes bio-electric potentials from muscles or nerves as a command or feedback signal. Myoelectric activity has been used widely in prosthetics control for many years. Myoelectric systems utilize the electrical activity associated with muscle contraction as a measure of muscle activity. Myoelectric applications range from use as a control source for prosthetic and orthotic devices to use in clinical diagnostics for assessing neuro-motor disorders. More recently, there has been increased interest in the use of the electrical activity recorded from sensory nerves from the tactile sensors of the hand and foot ("natural sensors") as an alternative to artificial sensors [19].

Research is still in an early phase, but may be able to provide feedback information for hand grasp.

Objectives

The objectives of this study were to:

- (1) Develop an implantable transducer for measuring joint angles that is suitable for chronic use in humans;
- (2) Characterize the functional characteristics of the transducer;
- (3) Develop and evaluate mathematical modeling techniques to describe the transfer characteristics of a transducer; and
- (4) Demonstrate the feasibility of implantation and periodic measurement techniques in a chronic *in-vivo* model.

Chapter II, entitled "An Implantable Transducer for Two-Degree-of-Freedom Joint Angle Sensing," describes the development of the implantable joint angle transducer (IJAT), bench test results from the IJAT, and *in-vivo* test results from the IJAT. Chapter II also introduces some of the modeling techniques that were developed. Chapter III, entitled "A Model for Describing the Characteristics of a Two-Degree-of-Freedom Angle Transducer based on Hall Effect Sensors and a Permanent Magnet," describes the details of the modeling techniques that were developed, shows transducer characteristics based on simulations, and describes the factors that affect transducer linearity and resolution. Chapter IV outlines some future directions of this work and concludes the dissertation.

CHAPTER II:
An Implantable Transducer for
Two-Degree-of-Freedom Joint Angle Sensing.

Abstract

An implantable joint angle transducer (IJAT) was developed for chronic use with upper extremity functional neuromuscular stimulation (FNS) neuroprostheses. The IJAT uses a permanent magnet surgically implanted in one bone, and an array of Hall effect sensors surgically implanted in the opposing bone. Titanium encapsulation protects the magnet, sensors, and support electronics, and secures the packages in the bone. The IJAT provides consistent, reliable, high quality signals that reflect joint movement from mid-sized two-degree-of-freedom joints. IJATs were implanted using a chronic *in-vivo* dog model to demonstrate the feasibility of implantation and periodic measurement techniques, and to validate modeling techniques used for prediction of function and calibration. The flexion resolution ranged from 0.4° to 3.0°. The maximum deviation from a linear response was 9°. The resolution and linearity depend on several transducer and joint geometry parameters, and can be predicted pre-implantation and calibrated post-implantation. The IJAT is suitable for use as either a command control or feedback sensor.

Introduction

Functional Neuromuscular Stimulation: FNS systems provide quadriplegic individuals with restored grasp and release function, and the ability to independently accomplish tasks that can not be accomplished with alternative assistive devices [15,16,21,22,23]. Shoulder or wrist movement sensed by an externally mounted transducer controls a percutaneous or implanted electrical stimulator that provides hand function. Externally mounted transducers have been viable for first generation systems, but have inherent limitations [12]. They provide inconsistent signal quality because of mounting issues, encumber other activities,

require donning, maintenance, and removal, and may become dislodged during use. Such limitations suggest replacement of external transducers by implantable alternatives. An implanted transducer in the wrist for persons with C6 level injuries would enhance the clinical acceptance of this command control source and potentially enable bilateral function to be restored. In individuals with no retained voluntary wrist control (C5 level injuries), an IJAT implanted in the sterno-clavicular joint would replace an externally mounted transducer for sensing shoulder command control movements. An implanted transducer in the wrist would provide for closed loop feedback regulation of wrist angle and free these individual of orthotic supports on the forearm. Thus, an IJAT could supply command or feedback information to an FNS system for several important and immediate clinical applications.

While use of the IJAT to control of the hand in spinal cord injury is our intended application, there are other persons with handicaps who potentially could benefit from such developments. In particular, the use of shoulder control for control of powered wheelchairs is a viable technique which has been implemented in our center [4]. Utilization of this command control source relieves the user (C4 quadriplegic) of interfacing with the wheelchair through any external control source. IJAT technology also could be applied to the lower extremity to provide joint angle information for feedback purposes in paraplegic walking, or in the upper extremity for joint angle feedback information of the elbow [5]. These applications dictate a transduction technique which is implantable, repeatable, and reliable over many decades.

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sonomicroscopy [11, 29], and Hall effect transducer based devices [1, 2, 7, 25, 27]. Troyk [28], described a method for using a Hall effect sensor and two magnets to measure flexion in a single degree-of-freedom joint, such as the knee. These devices were successful only for short term implantation, due to packaging limitations. Encapsulants were all non-hermetic, and included Paralene, silicone rubber, and medical grade epoxy [3, 6, 17]. These limitations were overcome by the present development.

Objectives

The objectives of this study were to:

- (1) Develop an implantable transducer for measuring joint angles that is suitable for chronic use in humans;
- (2) Characterize the functional characteristics of the transducer;
- (3) Develop and evaluate mathematical modeling techniques to describe the transfer characteristics of a transducer; and
- (4) Demonstrate the feasibility of implantation and periodic measurement techniques in a chronic *in-vivo* model.

Implanted Joint Angle Transducer Requirements

The IJAT must produce command control or feedback information, and be safely implantable surgically. The specifications that are derived thus must address both IJAT performance specifications and implantation.

Performance Requirements: The IJAT should provide signals that can be monotonically related to joint angle over the full two-degree-of-freedom range of the joint. The range needs to be the normal range of the joint, which is approximately 50° at the sternoclavicular joint, and 135° at the wrist. The IJAT should provide a resolution of better than 2° for feedback and command control tasks (or as otherwise dictated by the force and position resolution required from the hand grasp) [5]. The frequency response should be at least 30 Hz to allow sufficient bandwidth under a variety of movement conditions.

The power consumption of the device must be within the limitations imposed by the implantable telemeter providing power. Nominally, the telemeter will be able to provide 50mW average power, but lower transducer power consumption is preferable.

Stability should be sufficient such that regular recalibration is not required and measurements do not drift appreciably during a usage session. The accuracy should be limited only by the resolution, stability, and calibration to account for non-linearities.

Implantation Requirements: The transduction technique should not restrict the range of motion of the joint. It is desirable to have no physical connection across the joint that might interfere with joint motion or result in mechanical failure of the IJAT. The IJAT must be small enough to allow implantation at the desired joint without damaging the joint capsule or interfering with the joint, and be implantable using standard surgical techniques.

The IJAT should be packaged in materials that are biocompatible and provide hermetic protection for the electronics, but that do not interfere with the operation of the device. Ideally, the IJAT's lifetime should be the user's lifetime, up to 50 years. Once implanted, the IJAT should require no regular maintenance, but should be replaceable if necessary. The IJAT should be readily manufacturable, following the quality assurance standards necessary for any device to be implanted in humans. Additionally, manufacturing cost should be reasonable in comparison with the telemeter to which the IJAT is attached and the surgical costs to implant the device.

Methods

Principles of Implantable Joint Angle Transducer Operation

A magnetic sensing technique was utilized for the IJAT, in which a permanent magnet is surgically implanted in one bone and an array of Hall effect sensors is surgically implanted in the opposing bone. The IJAT developed operates by sensing the relative movement

between two opposing bones of a joint, as shown in Figure 1. The sensor array and magnet are each encapsulated in a non-ferromagnetic titanium capsule, which protects the device and provides a biocompatible interface without interference to the magnetic field. The magnet produces a local magnetic field with identifiable shape and intensity characteristics, and the Hall effect sensors detect the magnetic field produced by this magnet. As the joint moves, the relative position and orientation of the sensors in the magnetic field changes, producing sensor voltages that are related to the joint movement. Sensor leadwires provide power for the sensors and allow external measurement of the voltages produced by the sensors. The Hall effect sensor array is designed to be connected to a telemeter device, which

provides power to the sensors, amplifies the sensor signals, and encodes and transmits the signals to an external receiver.

Theoretical Model of IJAT Characteristics

Mathematical modeling techniques were developed for predicting the transfer characteristics of an IJAT when the magnet and sensor array are surgically placed in different joints having different geometric configurations. The modeling techniques were also used for calibrating and characterizing the IJAT after implantation. The modeling

Figure 1. Schematic diagram of implantable joint angle transducer showing magnet implanted in one bone, and sensor array implanted in opposing bone, with leadwire running proximally. Magnet is at center of rotation. Sensor plane is perpendicular to magnetic axis. Magnetic axis is approximately perpendicular to two axes of joint rotation.

techniques include: 1) a technique for modeling the movement of a joint and the placement of the magnet and sensor components in the joint; 2) a mathematical equation for the voltages of sensors placed near a permanent magnet; 3) an inversion technique for calculating estimates of the joint angle from sensor voltages; and 4) techniques to characterize the IJAT output, such as angular resolution, linearity, and variations of these parameters over the range of motion and with different placements of the IJAT components in the joint. These modeling techniques are used to predict the transfer characteristics of an IJAT and to calibrate and characterize the operation of an IJAT that has been implanted. A complete description of the modeling techniques is given in Johnson [13]. A summary is given below.

Technique for Describing the Joint, Sensor, and Magnet Movements: A technique was developed to describe the relative position and orientation between two bones in a joint, or between a magnet and sensor array. A **transducer configuration** is a particular arrangement of Hall effect sensors and a magnet placed in a particular mechanical or biological joint. A **geometric model** is a detailed mathematical description of the transducer configuration. The position and orientation of a sensor relative to a magnet are represented by a coordinate or reference frame transformation technique similar to those utilized in the robotics field [20]. Using this technique, a complex joint movement is separated into a sequence of simple translations and rotations, which provide a mechanism for converting the coordinates of an object in its own reference frame to equivalent coordinates in another object's reference frame.

Geometric models were used to describe the movement of a biological joint, and the relative positions and orientations of Hall effect sensors and a magnet within the joint. Variable parameters represented relative movement between two sides of a joint, and fixed parameters represented the geometric relationships between objects within a rigid body. The

modeling techniques can describe one-degree-of-freedom rotational (hinge joint) movement, two-degree-of-freedom (universal joint) movement, or screw displacement axis movement commonly used to describe movement of human joints, where rotation around an axis is coupled to translation along the axis. If these simple models are insufficient to describe the movement of a particular joint, a more complex model can be utilized.

An idealized configuration of a magnet and sensor array implanted in a joint is represented schematically in Figure 1. In this configuration, a magnet is positioned at the center of rotational movement. When the joint is at the center of its range: 1) the magnetic axis (arrow) is directed toward a planar array of Hall effect sensors; 2) the plane of the sensor array is perpendicular to the vector between the center of the magnet and the center of the sensor array (the **z**-axis); and 3) joint rotation occurs about the **x** and **y** axes, each perpendicular to the **z**-axis and each other. A more complex model is required to describe movement in a more realistic joint. The parameters defined below were used to describe deviations from this idealized configuration.

Definition of Primary Normalized Descriptive Geometry Parameters: The IJAT characteristics are inherently independent of absolute sizes and distances, depending only on the relative sizes of and distances between the IJAT components. This normalization property extends throughout all characteristics, including sensitivity, resolution, and linearity. Thus the models of specific joints were presented using normalized parameters with no loss of generality.

The primary geometric parameters used to model the position and orientation of a magnet and sensor array in a joint are illustrated in Figure 2. All distances were normalized by D , the root mean square (RMS) average distance between each sensor and the center of rotation. The following parameters were used to characterize the geometry:

θ_x and θ_y represent joint movement around the **x** and **y** axes;

Figure 2. Illustration of the primary geometric parameters used to model the position and orientation of a magnet and sensor array in a joint.

r is the normalized magnet radius and half-length (for a cylindrical magnet with equal length and diameter);

s is the normalized distance of each sensor from the center of the array;

$\mathbf{m}$ is the normalized vector distance from the center of the magnet to the center of rotation;

$\mathbf{d}$ is the normalized vector distance from the center of rotation to the center of the sensor array. d_x and d_y represent the lateral shift of the sensor array in the plane of the array. For symmetrical sensor arrangements, $\sqrt{d^2 + s^2} = 1$;

$\mathbf{b}$ is the orientation of the magnetic axis;

$\mathbf{n}$ is the orientation of the Hall effect sensors; and

$\mathbf{d}_i$ (not shown) equals $\mathbf{d} + \mathbf{s}_i$, is the vector from the center of rotation to the i th sensor;

$\mathbf{p}_i$ and $\mathbf{o}$ (not shown) are the position and orientation of the i th sensor relative to the magnet, which vary as the joint is rotated.

Screw axis displacement was modeled by allowing $\mathbf{m}$ to vary linearly with joint angle. Variants of this model were used in analyzing the IJAT characteristics presented below.

In the *in-vivo* data analysis, the zero angles were defined relative to the sensor array and magnet rather than the bones in which they were implanted, since X-rays measurements from the precise geometric shapes of the sensor array and magnet were more accurate than from the curved, less defined, irregular shape of the bones. Additional model parameters can be added to allow transformation to a bone based coordinate system, but if each IJAT component is rigidly fixed to those bones, the transformations do not change the fundamental movement of the joint or the relative movement between the sensors and magnet. In the model used to describe the dog elbow, flexion was defined such that at 90° the elbow was flexed to a right angle, and angles greater than 90° were in the extension direction. In the model used to describe dog wrist movement, the zero angles ($\theta_x = \theta_y = 0$) were defined by the orientation where the magnetic axis was perpendicular to the plane of the sensors. $+\theta_x$ was used to describe wrist extension and $+\theta_y$ to describe ulnar deviation. There was also a significant screw displacement along the x -axis with flexion/extension. The wrist flexion/extension was defined such that $+90^\circ$ was approximately a straight wrist (near full extension in the dog), and 0° was a flexed posture.

Magnet and Sensor Voltage Model: Far-field magnetic equations were used to model the transducer characteristics. Several properties of the magnetic field are important in determining how a Hall effect based transducer will operate. First, the magnetic field drops off in proportion to the inverse of the distance cubed. Second, the magnetic field is proportional to the volume and magnetization of the magnet. Third, the magnitude and direction of the magnetic field are related to the cosine and sine of the angle from the magnetic axis. And finally, the magnetic field is symmetric around the magnetization axis.

Hall effect sensors [8, 9, 10] were used in this transducer design to sense the field produced by the magnet. When a material that exhibits the Hall effect is placed in a magnetic field and a current is applied through the material, an electric field perpendicular to the current flow is generated, resulting in a voltage appearing across the material. In various commercially available Hall effect sensors, a Hall effect sensing element is integrated together with a differential amplifier to provide a single-ended sensor, whose output voltage is proportional to the magnetic field component perpendicular to the surface of the sensor.

Magnet and sensor voltage equations were combined to give an expression for the voltage produced by the i th Hall effect sensor placed at a given position, $\mathbf{p}_i$, and orientation, $\mathbf{o}$, relative to the magnet:

$$v_i = v_{oi} + \frac{k_{si} \mu_0 M_0 r^3}{p_i^3} \left[\frac{3}{2} \frac{\mathbf{p}_i \cdot \mathbf{o}}{p_i^2} p_{iz} - \frac{1}{2} o_z \right] \quad (1)$$

where: v_{oi} and k_{si} are the Hall effect sensor's offset and sensitivity;

r is the radius and half-length of a cylindrical magnet;

$\mu_0 M_0$ is the magnetization of the magnet;

and p_{iz} and o_z are the $\mathbf{z}$ -axis components of $\mathbf{p}_i$ and $\mathbf{o}$.

This equation shows the sensor voltage's dependence on the sensor's sensitivity, the magnet's strength and radius cubed, and the inverse sensor to magnet distance cubed, plus the directional component shown in brackets. This is a general expression for the voltage produced by a sensor at distances greater than about three magnet radii from the magnet [13].

Inverse Model of Overall IJAT Characteristics:

Combining Equation 1 with the techniques for describing the movement of a joint provides a powerful modeling tool for calculating or predicting sensor output voltages.

However, in actual transducer applications we are interested in the inverse problem of deriving measures of the movement angles from the sensor voltages. The inversion techniques utilized to calculate the transducer output from the sensor voltages are given in Johnson [13]. In *in-vivo* applications of the IJAT, the inversion coefficients were calculated using model simulations derived from X-ray measurements. Using simulations rather than actual data to invert the sensor voltage equations results in a more robust relationship that doesn't depend on sensor noise, and can be used to extrapolate IJAT function between and beyond the set of data points collected. The *in-vivo* analysis given below shows actual IJAT function and simulated IJAT function for comparison.

Definition of Transducer Output: The ideal transducer output, which arises from Equation 1 and the definition of the joint movement [13], can be defined as points on the surface of a unit sphere. Points on the sphere can be described in spherical coordinates by the two movement angles, θ_x and θ_y , (latitude and longitude by analogy), or in rectangular coordinates by the vector $\mathbf{u}$:

$$\mathbf{u} = \begin{bmatrix} -C_x S_y & S_x & C_x C_y \end{bmatrix}^t \quad (2)$$

where: $C_x = \cos(\theta_x)$; $C_y = \cos(\theta_y)$; $S_x = \sin(\theta_x)$; and $S_y = \sin(\theta_y)$.

The spherical coordinate system, designated $\mathbf{a}$, and the rectangular coordinate system are equivalent, but one or the other may be more suitable under differing circumstances. Data presented in rectangular coordinates looks like a segment of a sphere, whereas data presented in spherical coordinates flattens out the sphere. Near the poles of the sphere, where θ_x is near $\pm 90^\circ$, the axis of the magnet is aligned with θ_y 's axis of rotation, and θ_y becomes indeterminant because of the axial symmetry of the magnetic field. Under operating conditions near the poles, data were generally presented in rectangular coordinates. Otherwise, spherical coordinates were used. Actual transducer outputs, designated $\hat{\mathbf{a}}$ or $\hat{\mathbf{u}}$, are estimates of the true joint angles given by $\mathbf{a}$ or $\mathbf{u}$. Under non-ideal geometric conditions

of a transducer implanted in a joint, the transducer output sphere is distorted, both in terms of radius and longitude and latitude lines. Characterization of the non-linearities is shown below.

Linearity: For the case where the magnet is positioned at the center of rotation of the joint, the two IJAT output angles are linear with the movement angles of the joint. However, as the magnet is shifted away from the ideal location or if the joint does not have a constant center of rotation, the linearity of the relationship is degraded, roughly in proportion to how far off center the magnet is shifted. The **linearity error**, η , was defined as the difference between the IJAT output angles and the actual joint angles. For specific IJAT configurations, the RMS, and worst case linearity error were calculated from the actual IJAT data and from numerical simulations based on joint models derived from X-ray measurements.

Resolution: Each nominal IJAT data point defines a point on the surface of a distorted sphere. Sensor voltage noise will produce sample-to-sample deviations around that point. The sensor voltage noise can be statistically represented by a multi-dimensional spherical or ellipsoidal Gaussian distribution [14]. The inversion process maps the voltage noise distribution into a different ellipsoidally shaped distribution of errors in $\hat{\mathbf{u}}$. The shape, size, and orientation of the ellipsoid is determined directly by the RMS magnitude of the voltage noise and the inversion process [13]. The **resolution** of the IJAT was defined as the solid angular region statistically spanning 95% of the noise around a nominal operating point. IJAT resolution ellipses were calculated using actual or simulated sensor output voltages, measured sensor noise, and the inversion coefficients calculated from simulations. Description of the resolution was generally given as the span of the ellipse, in the rectangular or spherical coordinate system.

IJAT Design and Packaging

Design and Packaging of Sensor Array: The IJAT design used an array of three Sprague UGN-3503C linear Hall effect sensors, with nominal sensitivities of 1.3mV/Gauss, for sensing the field produced by the magnet. The three sensors and an LM336-5.0V voltage regulator were assembled into a hybrid circuit on an alumina substrate, which was attached to the lid of the titanium sensor capsule (Figure 3). The sensors were arranged as an equilateral triangle. The sensors were connected to feedthrough pins, consisting of platinum wires sealed into holes in the lid using a ceramic insulator, to provide power and signal connections. The capsule lid was laser-welded to a mating titanium shell, sealing the sensor electronics inside. Stainless steel

leadwires were welded to the outer portion of the feedthrough pins (Figure 4). Three dual-conductor leadwires were used to provide two power, one signal ground, and three signal connections to the sensors. The leadwires each consisted of two helically wound, teflon insulated, 316LVM stainless steel wires enclosed in silastic tubing. A titanium shroud was welded over the lead weld area, and partially filled with epoxy to provide rigid protection around the weld junction.

Medical Adhesive-A (silicone rubber) was used to fill the remainder of the

Figure 3. Hall effect sensor array hybrid circuit on titanium package lid, showing internal connections to feedthroughs.

Figure 4. Packaged IJAT components, showing threaded titanium magnet capsule with slot, titanium sensor array package and shroud over leadwire connections, silicone rubber strain relief, three dual-conductor leadwires exiting package, and in-line connector pins of one leadwire.

shroud and was tapered where the leadwires exited the shroud to minimize the concentration of bending stresses in the leadwires. The capsule lid/feedthrough assemblies were custom manufactured by Kyle Technology Corp. The overall dimensions of the main sensor capsule were 7.6 x 7.6 x 3.8mm, with three 1.4mm diameter dual-conductor leadwires exiting along one edge, plus the titanium shroud over the lead connection site.

Magnets: Cylindrical Neodymium-Iron-Boron (NdFeB) permanent magnets, packaged in titanium capsules were used in the IJAT design. Three different packaged magnet configurations have been fabricated to date. The magnets had maximum energy-product specifications in the 32-40 MGOe (250-320 KJ/m³) range, with magnetizations from 11.5-13KG (1.15-1.3T) and oriented either parallel or perpendicular to the cylindrical axis [24]. For all magnets, the length was equal to the diameter, with diameters from 3.2mm to

4.75mm. Magnet capsules of different sizes and surface textures were used. Smooth sided capsules were fabricated initially, followed by threaded cylindrical capsules, with a slot in the end of the capsule for easier insertion and alignment at implantation. The capsule sizes ranged from 6.1mm long by 4.75mm diameter to 8.1mm long by 6.4mm diameter. For each *in-vivo* application, the largest, strongest magnet available and suitable for the joint was utilized.

External Instrumentation: The Hall effect sensor array was designed to be connected to an implanted telemeter that will provide pulsed power for the sensors, amplify the sensor signals, and encode and transmit the signals to an external receiver. In this study, a hard wired, percutaneous interface was used to connect the IJAT to external amplification and powering circuitry. Due to the fairly high, length dependent, impedance of the stainless steel leadwires (60-250 Ω), the sensor array circuitry included a 5.0 volt shunt regulator, and the powering circuitry was designed to provide a regulated current to the sensor array. This configuration delivers correct power to the sensors independent of the length of the lead wires. Since the voltage drop along the current carrying power and ground leadwires depends on their length, a second signal ground wire was externally attached to the sensor package to sense the ground potential at the sensors. In the external amplification circuitry, the signal ground was level shifted by 2.5 volts to the mid-point of the Hall effect sensor output range to provide a reference voltage. The three Hall effect voltages were differentially amplified relative to the reference voltage. Pulsed powering was used. The regulated current to the sensor was turned on, and the sensors and amplifiers were allowed to settle. The amplified voltages were sampled using a sample and hold circuit, the supply current was turned off, then each of the voltages was digitized using a 12-bit A/D converter. The sensor outputs were sampled repetitively at 30 Hz. Data was collected and stored from the instrumentation using either an oscilloscope or a laboratory computer interface.

Bench Testing

Bench testing of the Hall effect sensors was carried out using combinations of the pulse powering instrumentation described above, standard bench test equipment, specially constructed test circuitry, a mechanical test fixture, and a laboratory computer based data acquisition system.

The minimum current required to power packaged sensor arrays and the offset sensitivity to current level above the minimum current were measured using the pulse powering instrumentation and computer interface, with an oscilloscope to measure the current levels. The current levels were manually adjusted from 40mA to 15mA in 1mA intervals, with 0.5mA increments near the minimum current required for regulation. At each current level, the sensor offset voltages were sampled 100 times at 30Hz. Below the minimum required current the sensor offset voltages drop abruptly. Above the minimum required current, the offset sensitivity to current level was measured using a linear regression of offset voltage versus current.

The same test setup was used to measure the relative sensitivity between sensors within a packaged array. The sensor array was placed in a nearly uniform alternating magnetic field generated by a copper coil driven by a 15Hz square-wave from a voltage waveform generator. The 15Hz frequency was slow enough to allow inductive transients to damp out, leaving a static magnetic field that alternated polarity on successive samples of the Hall effect sensors. The relative magnetic sensitivity of each sensor was taken as the average difference between samples over 1000 samples normalized by the average sensitivity of all three sensors.

A similar test setup was used to measure the temperature sensitivity of packaged sensor arrays, but the sensor array was placed in a temperature controlled bath together with a temperature probe, rather than in an alternating magnetic field. The temperature probe's

output was also sampled by the A/D converter. The bath temperature was increased at a rate of approximately $2^{\circ}\text{C}/\text{min.}$ over a range of at least 24°C to 37°C . The temperature sensitivity was calculated using a linear regression between sensor offset voltage and temperature.

The power-up settling time and background noise measured from any sensor generally depends on the bandwidth of the amplifier used, with a lower high frequency cutoff decreasing the background noise, but increasing the settling time. The power-up settling time and RMS noise were measured using an Analog Devices AD620 instrumentation amplifier followed by a single order 8KHz low-pass filter, and a Tektronix TDS420 digital storage oscilloscope with waveform measurement capabilities. Two phases of settling were measured from the oscilloscope: a primary phase due to the low-pass filter, and a small ($<0.5\text{mV}$) but slower phase due to the sensors. After a sensor had settled to within its background noise, the RMS sensor noise was measured using the oscilloscope. The sensors were also replaced by resistors to verify that the background noise of the instrumentation was at least an order of magnitude lower than the sensor noise, and that the slow component of settling was not due to the instrumentation.

Long term sensor drift was measured using a test setup with a non-pulsed power supply, differential amplifiers, a temperature probe, and a laboratory data acquisition system. Data was sampled at 1Hz, and the mean and variance of every 100 points was calculated and stored over a 51 day period. The sensor drift was measured from the collected data after removing the sensor offset component correlated with daily temperature cycling. The sensor with the largest drift showed an exponential decay. The asymptotic offset shift was calculated from a least squares best fit to an exponential curve.

The input/output characteristics of a sensor array and magnet were characterized in a mechanical test jig, which consisted of a ball and socket joint mounted above an X-Y

plotter bed, and a rod assembly connecting the ball to the plotter arm to translate movement of the plotter arm into rotation of the ball. A magnet was mounted inside the ball, and a sensor array was mounted above the ball. The test setup also included a non-pulsed power supply, differential amplifiers, and a laboratory computer system which controlled the X-Y plotter and sampled the sensor signals. In a test sequence, the plotter arm was moved through a two-dimensional grid pattern of points, and the sensors were sampled at each position where the plotter arm stopped. The angular movement of the ball was calculated based on the height of the ball above the plotter arm and the coordinates of the grid pattern.

In-vivo Studies

In-vivo testing was performed in three adult Labrador Retriever dogs. Table I summarizes the three implantation, including implantation sites, device characteristics, and implantation chronology.

Implantation Procedure: All implantation procedures were performed by an orthopaedic surgeon (MWK) using standard sterile orthopaedic techniques. A C-arm fluoroscopic X-ray imaging system was used throughout the procedures for visualization of the bones and device placement. In each procedure, the implant sites were surgically exposed. A hole was drilled in one bone, and a magnet capsule was pressed or threaded into the hole. A rectangular cavity was created in the other bone, and the packaged sensor array was pressed into place. Leadwires were tunneled subcutaneously to a percutaneous exit site at the shoulder, and incisions were closed. The limb and torso were casted to immobilize the joint for a period of healing.

Evaluation: Evaluation of the IJAT was carried out to assess its characteristics. For the first dog, implanted at the elbow, an experiment was performed at sacrifice when a lead fracture occurred at the package. Output of the IJAT was compared to that of an externally affixed goniometer similar to Long et al.[18]. A double four bar linkage configuration with

Table I. Summary of *in-vivo* implantation sites, device characteristics, and implantation chronology.

Implantation number	1	2	3
Joint implanted	Elbow	Wrist	Wrist
Sensor array site	Olecranon process of ulna	Distal Radius	Distal Radius
Magnet site	Trochlea of humerus	Radial carpal bone	Radial carpal bone
Sensor capsule shroud	No – prototype	Yes	Yes
Magnet size (mm) field orientation strength (MGOe)	4.75 Perpendicular 35	3.2 Parallel 32	3.2 Parallel 32
Magnet capsule texture	threaded	smooth	smooth
Days casted	18	23	23
Evaluation method	Goniometer	Bi-planar imaging	Bi-planar imaging
Days until failure	37	28*	45
Days implanted	37	44	—
Reason for failure	lead failure at package	percutaneous interface	percutaneous interface
*initial failure of one percutaneous leadwire			

a potentiometer at the joint between the two linkages was used to produce a signal proportional to the rotation angle, but independent of displacement of the two anchoring pieces and independent of the location of the center of rotation of the joint.

X-ray Imaging Evaluation: For the second and third dogs, implanted at the wrist, bi-planar radiographic imaging was used to measure joint movement. The dog was sedated using a mixture of Xylazine and Ketamine, then transferred to a limb stabilizing apparatus for fluoroscopic imaging. The orientation of the forelimb was adjusted for well aligned lateral and anterior-posterior (AP) views of the wrist. Images were taken at a number flexion

angles in approximately neutral, radial, and ulnar deviation. At each wrist position, a lateral view X-ray image was recorded first, followed approximately ten seconds later by an anterior-posterior (AP) view. During each trial, data was collected from the IJAT using the instrumentation described above. Each sensor was sampled four times per X-ray image (eight per trial), stored by an oscilloscope, then photographically recorded. All data from the IJAT and the X-ray images were transcribed and analyzed off-line. Experiments were performed six times over the three weeks following cast removal. Results from the second dog were incomplete because of failures of the percutaneous lead wires and are not included.

X-ray Analysis of Joint Movement: Bi-planar X-rays were used to analyze the joint movement and to measure the position of the magnet and sensor array in the joint. The joint and IJAT were characterized for each session by fitting a geometric model to the sequence of X-rays. The model parameters included: flexion/extension, radial/ulnar deviation, and screw-axis displacement of the carpal bones; an estimate of the joint center of rotation; the position of the magnet and sensor array relative to the center of rotation; and X-ray view parameters that corresponded to image magnification, forearm supination, humeral internal rotation, and elbow flexion. Initial estimates of the model parameters were measured from the X-ray images. The model and its parameters were used to make computer generated images to superimpose over each X-ray image. The parameters were iteratively adjusted until a good match was obtained between the X-ray and the computer generated images. The resulting model parameters were used as reference measurements for calibrating the IJAT, for simulating IJAT performance to compare with actual IJAT performance, and for assessing the stability of the IJAT components in the bone.

Techniques for Analyzing IJAT characteristics: For each joint, geometric model parameters of the joint and IJAT placement were determined from X-rays. Reference measurements of the joint angles were derived from a goniometer or X-rays. Sensor offset voltages and

scaling coefficients were derived by fitting measured sensor voltages to simulated sensor voltages. Inversion coefficients were calculated from model simulations. IJAT outputs in rectangular or spherical coordinates were calculated using the inversion techniques. Finally, the IJAT linearity and resolution were calculated and compared with simulation results.

Results

Hall Effect Sensor Characteristics

Table II summarizes most of the bench measurements of the Hall effect sensor array characteristics. The information in the table can be used to estimate the contributions of various error sources under a given set of conditions.

Figure 5 shows transducer output characteristics measured using the X-Y plotter test jig. The graph shows the y-axis versus the x-axis angular movement of the ball. The dashed lines show the angular movement calculated from the grid pattern traced out by the plotter. The solid lines show the estimates of the two angles derived from the sensor voltages and inversion coefficients. In this example, the magnet was mounted at the center of rotation of the ball, and the ball was ranged over 74° along both axes.

Figure 5. Transducer output characteristics measured from a sensor array and magnet mounted in an X-Y plotter test jig. Dashed lines show angles calculated from the plotter movement. Solid lines show the angles derived from the sensor voltages and inversion coefficients.

Table II. Summary of Hall effect sensor bench test results.

Parameter Measured	minimum	average	maximum	units	# of sensors (arrays)
Minimum current required	31.5	—	33	mA	6 (2)
Average power @ 200 μ s pulse, 50Hz, 33mA, 36V	—	12	—	mW	
Offset voltage versus supply current sensitivity	.03	—	.45	mV/mA	6 (2)
Relative sensitivity	0.912	—	1.050		6 (2)
RMS deviation of relative sensitivities	—	4.6	—	%	6 (2)
Temperature sensitivity (all R values ≥ 0.9987)	+0.20	+0.48	+0.69	mV/ $^{\circ}$ C	6 (2)
Settling time @ 8kHz cutoff -to within 0.5mA -to within 0.1mA	150 —	— —	175 250	μ s μ s	6 (2)
Steady state RMS voltage @ 8kHz cutoff	0.05	—	0.1	mV	6 (2)
Drift over 51 day period -asymptotic offset -time constant of drift	-0.3 — —	— — 48	+1.3 +2.0 —	mV mV days	3 (1) 1 1

Table III. Summary of primary geometry parameters used to characterize joint movement and IJAT placement. All parameters except D are normalized by D .

Description	symbol	Dog #1	Dog #3
RMS average distance between sensors and center of rotation	D	18.8mm	10.1mm
Sensor array position	$\mathbf{d}$	{0.13, 0, -0.986}	{0.39 -0.05, -0.90}
Sensor separation	s	0.10	0.18
Magnet radius/half-length	r	0.127	0.15
Magnet offset position	$\mathbf{m}$	{0.093, 0.046, 0.020}	{-0.25, m_y^* , 0.24}
Magnet offset magnitude	m	0.106	0.34 to 0.44
Sensor to magnet distance range	p	0.89 to 1.05	1.10 to 1.41

* m_y varied with θ_x and θ_y . A least-squares best fit of the data measured from X-rays gave the relationship: $m_y(\theta_x, \theta_y) = -0.41 - 0.28\theta_x - 0.55\theta_y$

In-vivo Results

Geometric Model Parameters: Table III summarizes the primary geometry parameters that were used to model the joint and IJAT movement in the two dogs presented here. Figure 6 shows an example of the type of computer generated line image used to determine the model parameters, superimposed over a lateral X-ray image of the wrist and implanted IJAT. The wrist angle was 47° in this X-ray.

Figure 6. Lateral X-ray image of the wrist and implanted IJAT superimposed by computer generated line image representing model parameters.

In-vivo Results of Implantation at Elbow: Figure 7a shows a graph of elbow flexion derived from the IJAT versus elbow flexion measured from a goniometer rigidly fixed to the dog's humerus and ulna. The graph shows the simulated IJAT transfer characteristics (dashed line) superimposed by IJAT data from twelve trials (dots) where the elbow was ranged from full flexion to full extension or the reverse at approximately $30^\circ/\text{sec}$. The figure also shows an estimate of the relative elbow deviation (valgus/varus) angle derived from the IJAT. Although this angle was not measured by the goniometer, the second IJAT output angle is related to it. The raw sensor voltages were pre-processed to remove occasional relatively large amplitude spikes due the instrumentation power supply. Excluding the spikes, the sensor voltage errors were normally distributed (R value > 0.997 on a Q-Q plot), with an average standard deviation of 0.05mV .

Figure 7. Input/Output characteristics of IJAT implanted at the elbow. (a) shows simulated and actual elbow angles versus elbow flexion measured from a goniometer. (b) shows elbow flexion error on an expanded scale.

Table IV. Summary of elbow flexion range, linearity error, and resolution specified in degrees.			
Description	minimum	RMS	maximum
Joint range	39	—	147
Linearity error from simulations	-8.3	3.6	4.8
Linearity discrepancy between simulations and actual	—	0.8	—
Resolution based on simulation and sensor voltage noise	0.4	—	1.7

Figure 7b expands the detail of Figure 7a, showing one trial of elbow flexion angle error from the IJAT (dots) versus elbow flexion angle from the goniometer. The graph also shows the linearity error predicted using simulations (solid line), and the simulated resolution (dashed lines) based on the RMS noise of the actual sensor voltages. Table IV numerically summarizes the elbow range and the IJAT linearity error and resolution for this dog.

In-vivo results of implantation at wrist: Table V summarizes the wrist range and the IJAT linearity error and resolution for the third dog implanted. 10° carpal flexion corresponds to the wrist and elbow fully flexed, and 125° corresponds to the dog standing, with the wrist hyper-extended. Data showing the linearity and resolution are presented in Figure 8.

Figure 8 shows a summary of data collected during one session. The graphs show IJAT output in the rectangular coordinate representation because the pole of the coordinate system is included within the range of the joint movement. The pole was included in the range of wrist motion in both wrist IJATs that have been implanted to date because at the time of implant, magnets that were both sufficiently small to implant at the wrist and magnetized across their diameter were unavailable, and because of the limited directional access available when surgically implanting the magnet. The coordinate system pole the operating point where one of the movement axes is aligned with the magnetic axis. In this

Table V. Summary of wrist range, linearity error, and resolution specified in degrees.			
Joint range:	minimum	RMS	maximum
overall flexion/extension:	10	—	125
flexion/extension during X-ray sessions	39	—	97
deviation change during X-ray sessions	—	—	5
magnitude of maximum change in screw axis displacement during X-ray sessions	—	—	3.2mm
Flexion/extension linearity error:			
from simulations over 39° to 97° flexion/extension, 0° to 10° ulnar deviation	-4.7	2.4	3.3
from simulations over 20° to 120° flexion/extension, 20° ulnar to 10° radial deviation	-8.7	3.6	8.1
from IJAT over X-ray sessions 3, 4, and 6	-4.5	3.9	9.0
discrepancy between simulations and actual	-3.7	1.7	3.7
Resolution:			
flexion/extension (rectangular or spherical coordinates)	1.0	—	3.0
radial/ulnar deviation (rectangular coordinates)	1.6	—	2.6
radial/ulnar deviation (spherical coordinates)	1.6 cos(extension)	— cos(extension)	2.6 cos(extension)

case, radial/ulnar deviation degenerates when the wrist is near full extension (vertical axis equals zero).

Figure 8a shows an idealized representation of the transducer output, plotting the z component versus the x component of the rectangular coordinate transducer output. The grid pattern (dashed lines) shows a small segment of the sphere representing the transducer output, viewed from the polar direction. The grid represents a simulated array of points that span the range of motion of the joint, with 10° between grid lines, ranging from 120° (extension) to 20° (flexion), and from 20° ulnar deviation (counterclockwise) to 10° radial

Figure 8. Output characteristics of IJAT implanted at the wrist. (a) shows idealized IJAT characteristics in rectangular coordinates. The dashed grid lines represent a segment of a sphere viewed from the pole. The symbols represent the angular position of the wrist measured from X-rays in ten trials. (b) shows simulated and actual IJAT output for the points and grid shown in (a). The grid and circles are simulated output calculated from model parameters for this dog. The symbol $\times$ designates the average IJAT output for the ten trials. (c) illustrates the IJAT noise and resolution for the same trials. The individual IJAT samples are indicated by the dots, with the trial averages again indicated by the symbol $\times$. The superimposed ellipses represent the IJAT resolution.

deviation (clockwise). The square symbols represent the flexion and deviation measured from X-rays, mapped into rectangular coordinates. The symbols show seven trials over the range of wrist flexion with the wrist at approximately neutral deviation, and three trials with the paw was in a more radially deviated position.

Figure 8b shows three types of IJAT output. The grid lines are a simulation of the IJAT output for the grid points shown in Figure 8a, calculated using the geometric model parameters for this dog. The circles show the simulated IJAT output for each of the X-ray points shown in Figure 8a. The symbol $\times$ designates the actual IJAT output based on the average of the eight data samples from each trial. The length of the lines connecting the symbols to the circles represents the discrepancy between the simulations and the actual IJAT output. The linearity error of either the simulations or the actual data is represented by the difference between comparable points in Figure 8a and 8b. A numerical summary of the linearity errors are given in Table V.

Figure 8c is similar to Figure 8b, but illustrates the IJAT noise and resolution. The individual IJAT samples are indicated by the dots, with the trial averages again indicated by the symbol $\times$. The superimposed ellipses are a measure of the IJAT resolution, indicating the region that should statistically encompass 95% of the IJAT sample data points for each wrist position. The ellipses were calculated using the inverse weighting matrix coefficients and measurements of the sensor noise voltages. The size and shape of the ellipses vary over the range of motion, which is the two-degree-of-freedom analog to the thickness of the error band varying with elbow flexion in Figure 7. In the lower-most group of samples, all eight samples fall outside the error ellipse. The four samples below the ellipse which were taken during the lateral view X-ray, and the four samples above the ellipse which were taken during the AP view X-ray, indicating a slight change in the wrist angle between the two X-rays. Table V includes a numerical summary of the resolution.

Figure 9. Output from wrist IJAT in awake and freely moving dog.

Some data was recorded from the IJAT implanted in this dog while it was awake and freely moving. Figure 9 shows two segments of wrist flexion data derived from the IJAT during this session. The first segment shows two steps followed by a period of standing. During the second segment, the dog's limb was manually ranged from full elbow and wrist flexion to full elbow and wrist extension, without the dog bearing weight on the limb. No alternate measures of the movement were made during these recordings. The inverse model coefficients used to calculate the IJAT output were derived using the model parameters from the X-ray sessions.

Repeatability of characteristics across sessions: Repeatability of the IJAT characteristics across sessions was recorded from the third dog. Figure 10 shows wrist flexion/extension angle over five sessions. The horizontal axis shows wrist angles measured from X-rays. The vertical axis shows the X-ray angle measurements for reference (dashed lines), simulated IJAT output derived from the X-ray measurements (solid lines), and the actual IJAT output measured during the X-rays (symbols), with the middle bar giving the average angle, and the vertical extensions showing the calculated resolution at that point. Inversion

Figure 10. Repeatability of the wrist flexion/extension from the IJAT across five sessions. The horizontal axis shows wrist angles measured from X-rays. The vertical axis shows the X-ray angle measurements for reference (dashed lines), simulated IJAT output derived from the X-ray measurements (solid lines), and the actual IJAT output measured during the X-rays (symbols), with the middle bar giving the average angle, and the vertical extensions showing the calculated resolution at that point.

coefficients calculated for the final session were used to calculate the IJAT output for all five sessions. Sensor offset voltages were calculated separately for each session. The number of days after cast removal are shown in brackets for each session. The most prominent change across the sessions is the increase in range of motion. A sixth X-ray session, 17 days after cast removal, was excluded because of an intermittent connection to one of the percutaneous leadwires.

Discussion

Implantation Requirements: Most of the implantation requirements given in the Introduction were met by this IJAT design. The IJAT components were packaged in well established biocompatible materials that do not interfere with the operation of the device. The IJAT allows full range of joint motion, and has no physical connection across the joint that might eventually lead to mechanical failure. The IJAT components were small enough to be implanted in the wrist and elbow of large dogs using orthopaedic techniques comparable to those used for implantation of artificial joint replacements. Since the bulk of the components is implanted deep in the bone, cosmetically the IJAT is nearly unnoticeable.

The IJAT packaging was designed for configurations where the sensor plane is perpendicular to the subcutaneous orientation of the leadwires. The size and shape of the package should be appropriate to the initial joints targeted for human use, the wrist and sterno-clavicular joints. The human joints are roughly comparable in size to the dog bones implanted with the IJAT.

The hermetic seal of the feedthrough pins manufactured for the sensor package was delicate. The hermetic seal could not be consistently maintained during attachment of wires to the inside and outside of the pins, and leak testing was not feasible after attachment of the external leadwires. However, the general design was viable, and we are pursuing re-manufacture of the feedthroughs along with some minor package design changes to improve

the robustness of the feedthroughs and simplify the IJAT assembly and implantation procedures.

Performance Characteristics: The IJAT output can be represented by points on the surface of a sphere. The poles of this coordinate system occur at any position where one of the axes of joint movement is parallel to the axis of the magnet. The IJAT output is capable of spanning the range of virtually any one or two degree-of-freedom joint, except near a pole, where only one degree-of-freedom can be resolved. An example of a poor configuration would be implantation of a magnet in the distal humerus for measuring elbow flexion where the magnetic axis were parallel to the axis of rotation of the elbow. Under these conditions, the IJAT would produce little to no output for any movement of the elbow. The key to good implantation is to orient the magnetic axis perpendicular to both axes of rotation of the joint when the joint is at the center of its range. In the elbow example, this can be accomplished with a cylindrical magnet magnetized across its diameter. Figure 1 illustrates a second good configuration. The data shown in Figure 5, collected using a mechanical test jig, show the type of two-degree-of-freedom output that can be obtained under these conditions and when the magnet is at the center of rotation. Examples of two-degree-of-freedom function for a variety of simulated configurations are given in Johnson [13].

Using a pulsed powering approach, the IJAT power consumption is within the power limitations of the telemeter to which it will be connected. At a 50Hz sampling rate, the anticipated powering duty cycle will be 1%, or 100 fold decrease in power consumption over a D.C. powering approach. At 50Hz sampling rate, under the anticipated operating conditions shown in Table II, the IJAT will consume 12mW, which will increase or decrease in direct proportion to the sampling rate. The maximum sampling rate capabilities using this IJAT will be limited by the average power that can be supplied across the inductive powering link. The manufacturer specified bandwidth of the sensors is 23Khz, which is well

above the mechanical bandwidth requirements for biological joint movement, and will not be a limiting factor.

The remaining IJAT performance characteristics relate to various error components. The error can be separated into four components: 1) the linearity error due to the geometric configuration; 2) the noise error due to random sensor voltage fluctuations that establish the resolution; 3) the discrepancy between the actual and simulated IJAT outputs; and 4) the day-to-day or repeatability error.

To assist in interpreting errors, the bench and *in-vivo* IJAT characteristics were presented together with simulated IJAT characteristics. Because of the wide variety of possible biological joint movements and IJAT placements, the characteristics of a particular configuration are not indicative of other configurations. Model simulations allow prediction of IJAT characteristics for any configuration, being particularly useful when the magnet is not at the center of rotation, or when the joint has a moving axis of rotation or screw axis displacement.

Modeling and simulations were utilized in many different phases of development and analysis. Simulations and analytical optimization were used to develop the methods for transforming sensor voltages into joint angle estimates. Simulations were used to determine joint geometry model parameters, sensor offset and scaling coefficients, and inversion and re-linearization coefficients. Modeling tools were used to calculate the linearity and resolution for specific configurations and to provide assistance in interpreting the results. Finally, equations for estimating the linearity and resolution from the primary geometry parameters were analytically derived from the IJAT models.

Equations for estimating IJAT linearity and resolution: The IJAT linearity error and resolution ellipses can be calculated numerically for a given transducer configuration if all joint geometry and IJAT parameters are known. However, to assist in estimating the

characteristics under general conditions, explicit expressions for the RMS linearity error and resolution were derived for two specific configurations (see Johnson [13]).

Linearity error: Expressions for the RMS linearity error were derived under the limiting conditions that the distance of each sensor from the center of the array, and the distance of the magnet from the center of rotation approach zero:

$$\eta_{x(\text{RMS})} = \sqrt{(c_{xx} m_x)^2 + (c_{xy} m_y)^2 + (c_{xz} m_z)^2 + c_{xx}^2 m_x m_z} \quad (3)$$

and
$$\eta_{y(\text{RMS})} = \sqrt{(c_{yx} m_x)^2 + (c_{yy} m_y)^2 + (c_{yz} m_z)^2 + c_{yz}^2 m_x m_z} \quad (4)$$

where $\mathbf{m}$ is offset of the magnet from the center of rotation, and the c coefficients depend on the lateral shift of the sensor array, d_x and d_y , and the selected ranges of θ_x and θ_y . These expressions provide a reasonable estimate of the linearity error for most practical IJAT configurations with co-planar sensors. Table VI gives numerical values for the c coefficients for the case where θ_y ranges over a full 360° and θ_x ranges over $\pm 60^\circ$, and for three ranges of d_x and d_y . The table entries are the worst case values over the given ranges of d_x and d_y . They can be used with Equations 3 and 4 to estimate the RMS linearity error if $\mathbf{m}$ and $\mathbf{d}$ are known for a particular configuration.

Table VI. Coefficients for estimating the RMS linearity error using Equations 3 and 4. Units of $\mathbf{c}_x$ and $\mathbf{c}_y$ are in degrees.		
range of d_x and d_y	worst case values of $\mathbf{c}_x$ { c_{xx} , c_{xy} , c_{xz} , c_{xxz} }	worst case values of $\mathbf{c}_y$ { c_{yx} , c_{yy} , c_{yz} , c_{yxz} }
0	{14.0, 9.09, 15.3, 0.0}	{8.64, 27.1, 31.9, 0.0}
± 0.25	{22.3, 61.4, 25.0, ± 2.64 }	{49.3, 28.7, 32.5, ± 24.3 }
± 0.5	{25.0, 99.1, 26.7, ± 5.05 }	{97.5, 33.3, 38.0, ± 39.4 }

When m , the magnitude of $\mathbf{m}$, equals zero, the linearity error equals zero. If only one component of $\mathbf{m}$ is non-zero, the expressions simplify. For example, if $m_x=0.1$ and $d_x=d_y=0$, then $\eta_x = c_{xx} m_x = 1.4^\circ$, and $\eta_y = c_{yx} m_x = 0.86^\circ$. The linearity error estimates derived from these expressions are proportional to m , but different terms dominate, depending on the configuration.

Resolution: Expressions for the resolution in rectangular coordinates were derived for the case where there is a symmetric arrangement of sensors and no lateral offset or tilt of the sensor array. Under these conditions, the primary axes of the ellipsoidal error distribution are aligned with the $\mathbf{x}$, $\mathbf{y}$, and $\mathbf{z}$ axes, with axis lengths given in degrees by:

$$\hat{e}_{u_x} = \hat{e}_{u_y} = \frac{152.8 \epsilon}{k_{s0} \mu_0 M_0 r^3 \hat{u} s \sqrt{1-s^2}} \quad (5)$$

and

$$\hat{e}_{u_z} = \frac{162.1 \sqrt{\epsilon^2 + 3\epsilon_{cm}^2}}{k_{s0} \mu_0 M_0 r^3 \hat{u} \left(1 - \frac{3}{2}s^2\right)} \quad (6)$$

where: ϵ_{cm} is the common mode RMS sensor voltage noise;

ϵ is the average uncorrelated RMS sensor voltage noise;

and $\hat{u}$ is the magnitude of $\hat{\mathbf{u}}$, the transducer output in rectangular coordinates.

Changes in resolution due to distance changes between the sensors and magnet are implicit in $\hat{u}$, which equals one when the magnet is at the center of rotation. When the magnet is not at the center of rotation, $\hat{u}$ typically ranges between $1/(1+m)^4$ and $1/(1-m)^4$. For small errors, the resolution can be mapped to spherical coordinates using:

$$\hat{e}_{\theta_x} \approx S_x S_y \hat{e}_{u_x} + C_x \hat{e}_{u_y} - S_x C_y \hat{e}_{u_z} \quad (7)$$

and

$$\hat{e}_{\theta_y} \approx -\frac{C_y}{C_x} \hat{e}_{u_x} - \frac{S_y}{C_x} \hat{e}_{u_z} \quad (8)$$

Equation 7 is a pure rotation of the elements of $\hat{\mathbf{e}}_u$. Equation 8 includes a pure rotation of $\hat{\mathbf{e}}_u$ around the $\mathbf{y}$ -axis, but also divides by the cosine of θ_x , which has only a minor effect

when θ_x is small, but makes the resolution very large when θ_x is near $\pm 90^\circ$. This corresponds to operating at the poles of the coordinate system, where θ_y degenerates.

Relationship between repeatability and sensor offset voltages changes: Sensor noise error sources can be separated into errors that are common to all the sensors (common mode noise) and errors that are unique to each sensor (uncorrelated noise), and can be separated into slowly varying and quickly varying errors. Slowly varying errors were treated as contributing to repeatability, whereas quickly varying errors were treated as contributing to the resolution. Slowly varying was loosely defined as much slower than the sampling rate of the IJAT signals. The resolution equations can be adapted for estimating the IJAT offset repeatability characteristics by dividing the equations by 4.9 and substituting other error sources for the RMS sensor noise. The factor 4.9 arises in the conversion from RMS error to 95% confidence interval.

The bench tests results summarized in Table II can be used to estimate the contributions of temperature changes, power supply current fluctuations, noise, and long term drift to sensor offset errors. Environmental magnetic and electrical fields can also contribute to the offset errors. The Earth's magnetic field is the most consistently present environmental interference. The strength of the Earth's average magnetic field in the continental United States ranges from 0.48 to 0.60 Gauss, with inclinations of 58° to 78° from horizontal [19]. With a typical sensor sensitivity of 1.3mV/Gauss, 0.6 Gauss translates into a ± 0.78 mV sensor offset range that depends on the orientation of the sensors. If the sensor sensitivities are approximately equal, the Earth's magnetic field will produce primarily a common mode error. Under normal operating conditions, the IJAT would not be exposed to other significant environmental magnetic fields. However, a strong magnetic field from a large magnet in the proximity of the IJAT could produce offsets sufficient to impede operation of the IJAT.

Table VII shows example calculations of the resolution and repeatability errors using some typical parameters. The table shows the resolution given a 0.05mV RMS noise, and repeatability or offset errors due to 1mV common mode and differential offset voltages. Under the conditions shown, differential errors have a greater effect on the x and y directions compared with the z direction. When the lateral shift of the sensor array is zero, common mode errors have no effect on the x and y directions, but have more effect than differential errors on the z direction. The common mode rejection along x and y is substantially reduced when the sensor array is shifted laterally. For a given magnet and sensor array, r and s decrease as the RMS average distance between the sensors and the magnet increases. The table shows the strong relationship between resolution or repeatability errors and these parameters.

Table VII. Hypothetical resolution and repeatability errors (in degrees) from Equations 5 and 6 for a case where $k_s=1.3\text{mV/G}$, $\mu_0 M_0=12\text{kG}$, the magnet is at the center of rotation, and there is no lateral shift or tilt of the sensor array.				
Error magnitude and description	$r = s = .15$		$r = s = .106$	
	x and y directions	z direction	x and y directions	z direction
0.05 mV RMS common mode and uncorrelated noise	1.0	0.32	3.9	0.89
1 mV net differential offset	4.0	0.65	16.	1.8
1 mV common mode offset	0.0	0.88	0.0	2.4

Discrepancy between simulations and in-vivo data: The discrepancy between the actual and simulated IJAT outputs may arise from several factors. In the elbow data shown in Figure 7, the discrepancy may be due to the Earth's magnetic field, since the absolute sensor orientation changed with elbow position. On the other hand, in the wrist data shown in Figures 8 and 10, the absolute sensor orientation did not change significantly with wrist

movement or from day-to-day, so the Earth's magnetic field probably did not result in significant error. The second factor affecting the discrepancy was inaccuracies in the mathematical modeling of the joint taken from X-rays, particularly in the wrist studies. The joints models used constant axes of rotation, which only approximated the true axes of rotation. Estimating moving axes of rotation was not feasible in this study. In addition, slight forearm pronation, flexion, and internal rotation variations between X-rays contributed to the uncertainty in measuring the carpal radial/ulnar deviation and screw axis displacement.

In-vivo repeatability: Several factors may have affected the repeatability data shown in Figure 10. The first was the process of recovery from surgery. There were dramatic changes in the joint range of motion characteristics over the three week testing period, as would be anticipated following any surgery that disrupted the joint capsule. During the period after surgery when the limb was casted, wound healing was taking place, and adhesions formed around the ligaments and tendons of the joint. When the cast was removed, normal use of the limb gradually freed the adhesions, returning the joint to a relatively normal condition. Three weeks after the cast removal, the dog's use of its limb appeared to be back to normal, with the dog naturally assuming typical sternal postures with one or the other of its front paws curled under (wrist fully flexed). The changes in the range of motion were accompanied by changes in the model parameters describing the movement of the bones.

The second factor potentially affecting repeatability of the IJAT characteristics is movement of the IJAT components in the bone. Careful measurements of the X-ray were made in the process of generating model parameters. Based on these measurements and the direct superposition of similar X-rays, there were no measurable changes in the position of the sensor array or magnet relative to the bones in which they were implanted. This observation is true across sessions and across different wrist flexion angles. On the other

hand, the X-rays showed measurable changes across sessions in the relative movement between the radius and carpal bones as the post-surgical adhesions loosened. Movement of the subcutaneous lead wires was also observed as the joint was flexed and extended.

The third factor that affected repeatability across sessions was changes in the sensor offset voltages. In the wrist IJAT repeatability data presented in Figure 10, day-to-day offset changes were compensated for in the data analysis. There was a large change in the recorded offset after the second session because of changes in the external circuitry that were made to reduce power supply induced noise. The maximum day-to-day offset variations over the remaining three sessions were 2.5mV, 0.9mV, and 0.6mV for the three individual sensors. The majority of the offset shift was probably due to body temperature changes and impedance changes in the external crimped connections to the percutaneous leadwires. Connection impedance changes introduced offset changes since the external amplifier was not designed to reject leadwire impedance changes. The actual impedance of each crimped connection could not be measured directly, but indirect measures indicated day-to-day shifts of several ohms. An impedance change of 2.2Ω would account for a 2.5mV offset shift. The range of body temperatures measured during the five sessions was 38.6 to 39.4°C. Based on the maximum temperature coefficients measured for the UGN-3503 Hall effect sensors, this temperature range could account for up to 0.6mV of offset shift.

Prediction and optimization of IJAT performance: IJAT performance must be analyzed on a case by case basis. Performance can be predicted using either numerical simulations or Equations 3-8. Numerical simulation is the more accurate method, but may require more detailed measurement of the movement of the joint. Equations 3-8 were derived for specific subset configurations, but are representative of the major factors affecting linearity and resolution for typical configurations with co-planar sensors.

The linearity error depends only on the normalized geometry parameters. The linearity is primarily dependent on the offset of the magnet from the center of rotation, and secondarily on the lateral offset of the sensor array. Analysis of the linearity is further complicated when m varies, as with screw displacement axis movement. In the elbow IJAT data presented above, m was approximately 0.1, resulting in a fairly mild non-linearity over the 108° joint range. When m exceeds approximately 0.35, regions of non-monotonic transducer characteristics occur. m can exceed 0.35, as in the wrist IJAT presented above, but the regions of non-monotonicity must be avoided. The relative resolution tends to be worst near non-monotonic regions. In general, m should be somewhat less than 0.35 or numerical simulations of the configuration should be evaluated to guarantee monotonic IJAT characteristics.

The resolution provided by the IJAT is probably the most critical factor in determining the IJAT's suitability for a given application. The resolution depends on many factors. In the design and fabrication of an IJAT the S/N ratio of the sensor, the separation between the sensors, and the strength and size of the magnet must all be maximized to optimize the resolution. To optimize resolution when implanting an IJAT, the distance between the magnet and the sensors must be minimized without compromising the integrity of the joint capsule or its blood supply. In addition, the variation of the IJAT resolution of the can be minimized by centering the magnet near the center of rotation. In the *in-vivo* testing that was done, the resolution ranged from 0.4° to 3.0° . Through refinement of the IJAT fabrication and implantation techniques, we anticipate being able to decrease the upper end of the resolution range considerably.

The repeatability, stability, and accuracy of the IJAT characteristics are closely tied to the IJAT resolution, but result from relatively slowly varying error sources such as long-term offset drift, day-to-day offset variations, temperature dependencies, and the Earth's or

other environmental magnetic fields. The effects of each of these error sources can be minimized by increasing the magnet strength and size, increasing the sensor separation within the array, and optimizing the placement of the IJAT components in the joint. Increasing the sensor sensitivity or S/N ratio will not necessarily have any effect, although increasing the signal to temperature sensitivity ratio would improve performance. The accuracy of the IJAT characteristics are limited primarily by the IJAT's repeatability and resolution, since the static non-linearities of the response can be corrected using look-up tables.

Conclusions

- A two-degree-of-freedom implantable joint angle transducer (IJAT) and support electronics were developed and fabricated for bench and *in-vivo* testing;
- Functional characteristics of IJAT components were measured, including: temperature sensitivity, background noise, power consumption, and dynamic characteristics.
- Mathematical modeling techniques were developed that to describe the transfer characteristics of an IJAT when the magnet and sensor array are surgically placed into different joints having different geometric configurations;
- Functional characteristics of the IJAT were described, including primarily the resolution, linearity, and repeatability characteristics, and how these characteristics vary with the characteristics of the IJAT components and placement of the components in the joint;
- The feasibility of implantation and periodic measurement techniques were demonstrated in a chronic *in-vivo* model. Longer term testing will require implantation along with a telemetry system, so that the percutaneous interface can be eliminated;
- Use of the modeling techniques for prediction and calibration was validated from the *in-vivo* testing. The quality of the signals obtained is highly dependent on the placement of the IJAT components in the joint, but the techniques developed can be used to optimize placement and predict IJAT performance. Individual decisions must be made regarding the suitability of the IJAT for a particular joint and a particular application;
- The IJAT packaging was suitable for housing the IJAT components and for implantation in certain joints. However, some technical refinements will be necessary to fabricate package feedthroughs that will maintain their hermetic seal.
- When coupled with an implantable telemeter, improved feedthroughs, and a careful pre-operative assessment of predicted IJAT performance, the implantable joint angle transducer should be well suited as a two-degree-of-freedom command control or feedback source for chronic use with neuroprosthetic systems in humans.

Chapter II References

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CHAPTER III:
A Model for Describing the Characteristics of a
Two-Degree-of-Freedom Implantable Joint Angle Transducer
based on Hall Effect Sensors and a Permanent Magnet.

Abstract

An implantable joint angle transducer (IJAT) was developed for chronic use with upper extremity functional neuromuscular stimulation (FNS) neuroprostheses. The IJAT uses a permanent magnet surgically implanted in one bone and an array of Hall effect sensors surgically implanted in the opposing bone. Mathematical modeling techniques were developed for predicting the transfer characteristics of an IJAT when the magnet and sensor array are surgically placed into different joints having different geometric configurations, and for calibrating the IJAT after implantation. The IJAT linearity, resolution, and sensitivity to errors depend on the joint geometry, the magnet strength and size, the configuration and signal-to-noise ratio of the sensors, and the positioning of the magnet and sensor array in the joint. The results of the modeling assist in IJAT design optimization and intraoperative placement of the IJAT components.

Introduction

Functional Neuromuscular Stimulation

FNS systems provide quadriplegic individuals with of restored grasp and release function, and the ability to independently accomplish tasks that can not be accomplished with alternative assistive devices [31,32,38,39,40]. Shoulder or wrist movement sensed by an externally mounted transducer controls a percutaneous or implanted electrical stimulator that provides hand function. Externally mounted transducers have been viable for first generation systems, but have inherent limitations. They provide inconsistent signal quality

because of mounting issues, encumber other activities, require donning, maintenance, and removal, and may become dislodged during use. Such limitations suggest replacement of external transducers by implantable alternatives.

Objectives

Johnson et al. [Chapter II] describe the design of an implantable joint angle transducer (IJAT) developed for chronic use with upper extremity functional neuromuscular stimulation (FNS) neuroprostheses. The objectives of this study were to:

- (1) Develop the mathematical modeling techniques to describe the transfer characteristics of the IJAT when the magnet and sensor array are surgically placed into joints having different geometric configurations;
- (2) Describe the performance properties of the IJAT, including: linearity, resolution, and sensitivity to errors, and their relationship to joint geometry, IJAT design parameters, and surgical placement of the IJAT components;
- (3) Provide tools for estimating the functional performance that can be anticipated from an IJAT, prior to surgical implantation.

Methods/Derivations

Principles of Implantable Joint Angle Transducer Operation

The IJAT developed operates by sensing the relative movement between two opposing bones of a joint, as shown in Figure 1. The IJAT uses a permanent magnet surgically implanted in one bone and an array of Hall effect sensors surgically implanted in the opposing bone. The sensor array and magnet are each packaged in a non-ferromagnetic titanium capsule that protects the device and provides a biocompatible interface without altering the magnetic field. The magnet produces a local magnetic field with certain shape and intensity characteristics, and the Hall effect sensors detect the magnetic field produced by the magnet. As the joint is moved, the relative position and orientation of the sensors in the magnetic field change, producing sensor voltages that are related to the joint movement.

Forward Model of Transducer Voltage versus Movement

Mathematical modeling techniques were developed for predicting the transfer characteristics of an IJAT when the magnet and sensor array are surgically placed in different joints having different geometric configurations, and for calibrating and characterizing the IJAT after implantation. The modeling techniques include: 1) a mathematical equation of the magnetic field produced by the permanent magnet; 2) a mathematical equation of the sensor voltages produced by the magnetic field; 3) a matrix transformation technique for modeling and describing the movement of a joint and the placement of the magnet and sensor components in the joint; 4) a technique for inverting the angle to sensor characteristics, so that estimates of the joint angles can be calculated from sensor voltages; and 5) techniques for describing characteristics of the angle estimates such as angular resolution, linearity, and how these parameters vary over the range of motion and with different placements of the IJAT components in the joint. These modeling techniques are used to predict the transfer characteristics of an IJAT as and to calibrate and characterize the operation of an IJAT that has been implanted.

Nomenclature: Scalar variables or constants and their subscripts are designated using italics, except for numerals or greek symbols (x, p_x, l, α, v_l). Vectors and matrices are designated using an alternate bold font, with vectors generally designated in lower case and matrices designated in upper case, except where standard convention dictates otherwise ($\mathbf{p}$, $\mathbf{A}$, $\mathbf{B}$ is magnetic field vector); The elements of an array or vector are designated using a subscripted scalar version of the same symbol. A vector's magnitude is designated using an unsubscripted scalar version of the same symbol. The transpose of $\mathbf{a}$ is designated as $\mathbf{a}^t$, and the matrix inverse of $\mathbf{A}$ is designated $\mathbf{A}^{-1}$. Functions are designated in standard typeface.

Magnetic Field and Hall Effect Sensor Equations

Definitions of Magnetic Field Related Vectors: The three primary magnetic field related vectors, **B**, **H**, and **M**, are used by different names in textbooks, literature and industry. The definitions used in this work are: The **magnetic field**, **B**, which is generally considered to be the primary magnetic field related vector, is defined by the directed force, **F**, generated on a particle of charge q , moving at velocity **v** through the field. The magnetic field can be generated by electric currents or permanent magnets, and can be measured using Hall effect sensors, among other techniques. The **magnetizing field**, **H**, is the driving force that produces a **B** field in the absence of magnetic materials, and polarizes or depolarizes magnetic materials when present. **H** fields are either induced using an electromagnet, or remain as an residual demagnetizing field in a magnetic material that has been removed from an applied magnetizing field. The **magnetization**, $\mu_0 \mathbf{M}$, is the polarization induced in a magnetic material due to an applied magnetizing field. μ_0 is the permeability constant, relating the units of the **H** and **M** fields to the units of the **B** field. A listing of some magnetic field related names, symbols, and conversions between the CGS and SI unit systems are given in Appendix A.

Magnetization of a Permanent Magnet: The magnetization of a permanent magnet depends on many factors, including the magnetization-demagnetization characteristics of the magnetic material, the shape of the magnet, the magnetic circuit, and the magnet's history. For a spherical magnet, the magnetization is uniform within the magnet, and estimation of the actual magnetization is straight forward to derive from a magnet manufacturers demagnetization curves. When no other ferromagnetic materials are in the immediate vicinity, the load line or circuit permeance for a spherical magnet is represented on a normal demagnetization curve (**B** versus **H**) by a straight line passing through the origin with a slope

of $-2\mu_0$ [44]. $\mathbf{B}$ and $\mathbf{H}$ at the point where the demagnetization curve intersects the load line can be used to calculate the magnetization:

$$\mu_0 \mathbf{M} = \mathbf{B} - \mu_0 \mathbf{H} \quad (\mathbf{H} \text{ is negative, so the magnitudes add}) \quad (1)$$

The magnetization of a sphere provides a reasonable estimate of the magnetization of other shaped magnets under the following restricted conditions: the magnetic material has a strong tendency to resist demagnetization (i.e. a recoil permeability near μ_0 over a wide $\mathbf{H}$ field range); the length and diameter or width are equal; and no other ferromagnetic materials are in the immediate vicinity. Under these conditions, the magnetization will be typically 96% to 98% of the residual induction specification for high strength magnetic materials such as Neodymium-Iron-Boron and Cobalt-Samarium. A more detailed treatment of the subject is given in Appendices A and B.

Magnetic Field Equations: The magnitude and direction of the $\mathbf{B}$ field surrounding the magnet are important in determining how a Hall effect based transducer will operate. A general expression (adapted from Montgomery [36]) for the magnetic field produced by a cylindrical magnet magnetized along its cylindrical axis is given in Appendix C. The expression has no general closed form solution, but can be used to numerically calculate the magnetic field, although computationally intensive. The equations converge rapidly to the far-field relationship at distances on the order of three times the magnet's radius and half-length (see Appendix B). In spherical coordinates, the far-field equation for the magnetic field at a point $\mathbf{p}(\theta, \phi, \rho)$ near a cylindrical magnet is:

$$\mathbf{B} = \frac{\mu_0 M_0 r_m^2 h_m}{2 \rho^3} \left(2 \cos(\theta) \mathbf{a}_r + \sin(\theta) \mathbf{a}_\theta + 0 \mathbf{a}_\phi \right) \quad (2)$$

where: $\mu_0 M_0$ is the magnitude of the magnetization of the magnetic material;

r_m and h_m are the radius and half-length of the magnet;

θ , ϕ , and ρ are the polar angle, azimuth angle, and radial distance of $\mathbf{p}$ relative to the center of the magnet and the magnetic axis;

and $\mathbf{a}_r$, $\mathbf{a}_\theta$, and $\mathbf{a}_\phi$ are spherical coordinate system unit vectors (the $\mathbf{a}_\phi$ component has zero magnitude).

This equation illustrates several important relationships. First, the magnetic field drops off in proportion to the inverse of the distance cubed. Second, the magnetic field is proportional to the volume and magnetization of the magnet. Third, the magnitude and direction of the magnetic field are related to the cosine and sine of the angle from the magnetic axis (polar angle). Finally, because of symmetry, there is no dependence on the direction of the polar angle from the magnetic axis (azimuth angle). These four relationships are important characteristics in designing a transducer that detects movement based on the magnetic field.

The spherical coordinate form of the far-field magnetic equation is useful for illustration, but the same equation expressed in rectangular coordinates is more suited to the calculation of actual transducer characteristics. In rectangular coordinates, the far-field equation for the magnetic field at a point $\mathbf{p}$ near a cylindrical magnet is:

$$\mathbf{B} = \frac{\mu_0 M_0 r_m^2 h_m}{\rho^3} \left(\frac{3p_x p_z \mathbf{x} + 3p_y p_z \mathbf{y} + (3p_z^2 - \rho^2) \mathbf{z}}{2\rho^2} \right) \quad (3)$$

where: p_x , p_y , and p_z are the rectangular coordinates of $\mathbf{p}$;

$\rho = |\mathbf{p}|$ is the distance from the center of the magnet;

$\mathbf{x}$, $\mathbf{y}$, and $\mathbf{z}$ are rectangular coordinate system unit vectors;

and the magnetic axis of the magnet is oriented along the $\mathbf{z}$ axis.

This form of the magnetic field equation will be utilized throughout the remainder of this paper. For non-cylindrically shaped magnets the $r_m^2 h_m$ can be replaced by $v_m/2\pi$, where v_m is the volume of the magnet.

Hall Effect Sensor Voltage Equation: Hall effect sensors [15] are used in this transducer design to sense the field produced by the magnet. When a material that exhibits the Hall effect is placed in a magnetic field and a current is applied through the material, an electric field, $\mathbf{E}$, is generated:

$$\mathbf{E} = -\mathbf{j} \times \mathbf{B} / ne \quad (4)$$

where: $\mathbf{j}$ is the current density in the material;

n is the number of charge carriers per unit volume;

and e is the charge per carrier.

The electric field results in a voltage appearing across the material. In various commercially available Hall effect sensors, a Hall effect sensing element is integrated together with a differential amplifier to provide a single-ended sensor, whose output voltage, v , is proportional to the magnetic field component perpendicular to the surface of the sensor [16, 17], with v given by:

$$v = v_{oi} + k_{si} \mathbf{B} \cdot \mathbf{o} \quad (5)$$

where: v_{oi} and k_{si} are the Hall effect sensors offset and sensitivity coefficients;

$\mathbf{o}$ is the orientation of the vector normal to the sensor's surface;

and $v - v_o$ is proportional to the dot product of $\mathbf{B}$ and $\mathbf{o}$.

Combination of Sensor and Magnet Equations: By combining the magnet and sensor equations, a general expression for the voltage produced by a sensor placed at a given location and orientation relative to the magnet can be written:

$$v_i = v_{oi} + \frac{k_{si} \mu_0 M_0 r_m^2 h_m}{p_i^3} \left[\frac{3}{2} \frac{\mathbf{p}_i \cdot \mathbf{o}}{p_i^2} p_{iz} - \frac{1}{2} o_z \right] \quad (6)$$

where: $\mathbf{p}_i$ is the position of the sensor relative to the center of the magnet;

$\mathbf{o}$ is the orientation of the sensor normal vector relative to the magnetic axis;

p_i is the magnitude of $\mathbf{p}_i$;

and p_{iz} and o_z are the z axis components of $\mathbf{p}_i$ and $\mathbf{o}$.

This is the general expression for the voltage produced by a sensor at an arbitrary position and orientation relative to a permanent magnet at distances greater than about three magnet radii and half-lengths. Although this equation does not provide any additional intuitive insight into the transducer characteristics, it is the basis for all subsequent derivations and calculation of transducer characteristics. The following section describes a technique for calculating $\mathbf{p}$ and $\mathbf{o}$ within the constraints of movement of a joint.

Technique for Describing Joint Movements:

Technique for Describing the Geometric Relationships Between a Sensor and Magnet: A technique was developed to assist in mathematically describing the relative position and orientation between two bones in a joint, or between a magnet and sensor array.

Transducer configuration refers to a particular arrangement of Hall effect sensors and a magnet placed in a particular mechanical or biological joint, and the associated movement of the joint. A **geometric model** and its associated parameters provide a detailed description of the transducer configuration. Homogeneous transformations are utilized to provide the mathematical formalism for expressing the geometric model so that the position and orientation can be algebraically or numerically represented.

The position and orientation of a sensor relative to a magnet are represented by a coordinate or reference frame transformation. The formalized homogeneous transformation matrix techniques described here are similar to those utilized in the robotics field [37]. Using this technique, the relative displacement and orientation between two objects is separated into a sequence of component translations and rotations represented by standardized 4x4 transformation matrices. There are four forms of matrices used within the model presented here. They are: [Trans($\mathbf{a}$)], a translation in space by the vector quantity $\mathbf{a}$; and [Rot($\mathbf{x}, \alpha$)], [Rot($\mathbf{y}, \alpha$)], or [Rot($\mathbf{z}, \alpha$)], rotations by α around the $\mathbf{x}$, $\mathbf{y}$ or $\mathbf{z}$ axes. A

mathematical description of the overall relationship between the two objects is obtained by multiplying the component matrices together into a single 4x4 spatial transformation matrix. This matrix provides a mechanism for converting the coordinates of an object in its own reference frame to the equivalent coordinates in the other object's reference frame. Written in symbolic matrix notation:

$$\begin{bmatrix} \text{Sensor Coordinates} \\ \text{in Magnet} \\ \text{Reference Frame} \end{bmatrix} = \begin{bmatrix} \text{Spatial} \\ \text{Transformation} \\ \text{Matrix} \end{bmatrix} \begin{bmatrix} \text{Sensor Coordinates} \\ \text{in Sensor} \\ \text{Reference Frame} \end{bmatrix} \quad (7)$$

Closed form algebraic expressions of the matrix for a particular transformation can be derived, but do not provide any particular computational advantage over numerical matrix computation and require complete re-derivation if the structure of the model is changed even slightly.

Application of Matrix Transformations to Description of Joint Movement: Geometric models are used to describe the movement of a biological joint, and to describe the relative positions and orientations of Hall effect sensors and a magnet within the joint. Variable parameters represent relative movement between two sides of a joint, and fixed parameters represent the geometric relationships between objects within a rigid body. Transformation matrices can be used to describe one-degree-of-freedom rotational (hinge joint) movement, $[\text{Rot}(\mathbf{y}, \theta_y)]$, two-degree-of-freedom (universal joint) movement, $[\text{Rot}(\mathbf{x}, \theta_x)][\text{Rot}(\mathbf{y}, \theta_y)]$, or screw displacement axis (SDA) movement, $[\text{Trans}(0, k\theta_y, 0)][\text{Rot}(\mathbf{y}, \theta_y)]$. In SDA movement, commonly used to describe movement of human joints, rotation around an axis is coupled to translation along that axis. If these simple models are insufficient to describe the movement of a particular joint, a more complex model can be used. The model can include additional coupled varying parameters, or may utilize different sets of fixed parameters over different segments of the movement range, provided that continuity is maintained at the boundaries between segments. This type of model can represent what is referred to as

instantaneous (moving) screw displacement axes. A model of a joint can be made arbitrarily complex, but at the expense of an increased number of parameters, and an inability to use analytical techniques with the model.

Transducer Configurations: An idealized configuration of a magnet and sensor array implanted in a joint is represented schematically in Figure 1. In this configuration, the magnet is positioned at the center of rotational movement. When the joint is at the center of its range: 1) the magnetic axis (arrow) is directed toward a planar array of Hall effect sensors; 2) the plane of the sensor array is perpendicular to the vector between the center of the magnet and the center of the sensor array (the z axis); and 3) joint rotation occurs about the x and y axes, each perpendicular to the z axis and each other. A planar configuration of the Hall effect sensors was selected because of practical fabrication issues, although alternate configurations are possible.

Movement around the x and y axes are modeled using the variable parameters θ_x and θ_y . Fixed parameters describe the distance of the sensor array from the magnet, and the distance and direction of each sensor from the array center. Several additional parameters are required to model movement in a more realistic joint. These include fixed parameters to describe: the lateral shift of the sensor array (or

Figure 1. Schematic diagram of implantable joint angle transducer showing magnet implanted in one bone, and sensor array implanted in opposing bone, with leadwire running proximally. Magnet is at center of rotation. Sensor plane is perpendicular to magnetic axis. Magnetic axis is approximately perpendicular to two axes of joint rotation.

equivalently, tilt of the sensor array), the offset of the magnet from the center of rotation, and the **z** axis alignment of the magnet and sensor array relative to the movement axes. At least one additional varying parameter to may be necessary describe screw displacement axis movement. Variants of this model structure were used to model joint angle transducers implanted in the elbow or wrist of dogs as described in Chapter II.

Standard Configuration: A standard transducer configuration is presented here and used in examples through the remainder of the paper. The model describing this configuration has the minimum number of parameters required to describe the important characteristics of a two-degree-of-freedom joint angle transducer over a broad range of geometric conditions. The transducer characteristics, including: sensitivity, resolution, and linearity, are inherently independent of absolute sizes and distances, depending only on the relative sizes of and distances between the transducer components. Thus the standard configuration is presented using normalized distance parameters with no loss of generality.

Figure 2 illustrates geometric parameters used in the standard model. Two purely rotational movements, θ_x and θ_y , occur around two orthogonal axes designated **x** and **y**, with the **z** axis orthogonal to these two axes. The position of the magnet and each sensor are defined relative to the center of rotation. The configuration uses three co-planar sensors configured in an equilateral triangle, with one of the sensors oriented along the +**y** axis. The orientation of the sensor array and magnet are defined such that when $\theta_x = \theta_y = 0$, the magnetic axis vector, **b**, and the vectors normal to the plane of the sensors, **n**, are oriented along the **z** axis. An ordering of the two rotations must be specified to provide a coherent description, thus the orientation of the **y** axis is fixed relative to the sensor side of the joint, and the orientation of the **x** axis is fixed relative to the magnet side of the joint.

All distances are normalized by D , the root mean square (RMS) average distance between each sensor and the center of rotation. r is the normalized magnet radius and half-

length (for a cylindrical magnet with equal length and diameter). s is the normalized distance of each sensor from the center of the array. $\mathbf{m}$ is the normalized vector distance from the center of the magnet to the center of rotation. $\mathbf{d}$ is the normalized vector distance from the center of rotation to the center of the sensor array. d_x and d_y represent the lateral shift of the sensor array in the plane of the array. For symmetrical sensor arrangements, $\sqrt{d^2 + s^2} = 1$. Several additional parameters are shown in Figure 3. $\mathbf{d}_i$ equals $\mathbf{d} + \mathbf{s}_i$, which is the vector from the center of rotation to the i th sensor. $\mathbf{p}_i$ and $\mathbf{o}$ are the position and orientation of the i th sensor relative to the magnet, which vary as the joint is rotated. References to dimension will be in terms of the normalized distance, but can be re-scaled into any unit system with no change in result as long as all dimensions are re-scaled the same.

For convenience in analyzing the results, the transformation matrix representation of the standard configuration was separated into two halves, $\mathbf{A}_s$, the matrix describing the relationship between a sensor and the center of rotation of the movement, and $\mathbf{A}_m$, the matrix describing the movement between the center of rotation and the magnet. All of the parameters describing $\mathbf{A}_s$ are constant, while $\mathbf{A}_m$ includes both fixed and variable parameters. The transformation matrix for the i th sensor is:

$$[\mathbf{A}_{s_i}] = [\text{Trans}(\mathbf{d})][\text{Rot}(\mathbf{z}, t_i)][\text{Trans}(0, s, 0)] \quad (8)$$

where t_i is the direction angle of the i th sensor in radians. t_i is the only parameter that is different for each sensor, equaling $-2\pi/3$, 0 , and $+2\pi/3$ for the three sensors, to form an equilateral triangle. The third and fourth columns of $\mathbf{A}_{s_i}$ define $\mathbf{n}$ and $\mathbf{d}_i$, the orientation and position of the sensor relative to the center of rotation when θ_x and θ_y equal zero (see Figure 3). The orientation vectors are equal for all co-planar sensors.

The transformation matrix for the magnet is:

$$[\mathbf{A}_m] = [\text{Trans}(\mathbf{m})][\text{Rot}(\mathbf{x}, \theta_x)][\text{Rot}(\mathbf{y}, \theta_y)] \quad (9)$$

By definition, the transpose of the third row of $\mathbf{A}_m$, designated $\mathbf{u}$, determines the z components of $\mathbf{p}$ and $\mathbf{o}$, which are used in Equation 6. As shown below, the transducer output characteristics are directly related to $\mathbf{u}$. For the standard model, $\mathbf{u}$ is given by:

$$\mathbf{u} = \begin{bmatrix} -C_x S_y & S_x & C_x C_y & m_z \end{bmatrix}^t \quad (10)$$

where: $C_x = \cos(\theta_x)$; $C_y = \cos(\theta_y)$; $S_x = \sin(\theta_x)$; and $S_y = \sin(\theta_y)$;

Pre-multiplying $\mathbf{A}_{s_i}$ by $\mathbf{A}_m$ gives the overall transformation matrix, $\mathbf{A}_i$, from the i th sensor's coordinates into magnet's coordinates:

$$\begin{bmatrix} \mathbf{A}_i \end{bmatrix} = \begin{bmatrix} \mathbf{A}_m \end{bmatrix} \begin{bmatrix} \mathbf{A}_{s_i} \end{bmatrix} \quad (11)$$

The third and fourth columns of $\mathbf{A}_i$ define $\mathbf{o}$ and $\mathbf{p}_i$, the orientation and position of the i th sensor's normal vector relative to the magnet as a function of θ_x and θ_y (see Figure 3). The relationships between the transformation matrices, and the position and orientation vectors are summarized by the equation:

$$\begin{bmatrix} \mathbf{o} \mid \mathbf{p}_i \end{bmatrix} = \mathbf{A}_m \begin{bmatrix} \mathbf{n} \mid \mathbf{d}_i \end{bmatrix} = \mathbf{A}_m \mathbf{A}_{s_i} \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (12)$$

At the right-hand side: The vector $[0 \ 0 \ 0 \ 1]^t$ is the position of the sensor in its own coordinate system (it is located at its own origin); and the vector $[0 \ 0 \ 1 \ 0]^t$ is the orientation of the sensor's normal vector in its own coordinate system (namely the z axis). These vectors are pre-multiplied by $\mathbf{A}_{s_i}$, to give $\mathbf{d}_i$ and $\mathbf{n}$, which are in turn pre-multiplied by $\mathbf{A}_m$ to give $\mathbf{p}_i$ and $\mathbf{o}$.

Definition of Transducer Output:

The vector $\mathbf{u}$ defined above defines the ideal transducer output in rectangular coordinates. The fourth element of $\mathbf{u}$, m_z , can be thought of as an offset, and is constant in the treatment presented here. However, the first three elements of $\mathbf{u}$ are variable, depending on the sines and cosines of the two movement angles. If the two movement angles, θ_x and θ_y , are used to define a spherical coordinate system, representing, by analogy,

latitude and longitude on a unit radius sphere, then the first three elements of $\mathbf{u}$ are the rectangular coordinates of points on the surface of the sphere. A complete spherical coordinate system, $\mathbf{a} = [\theta_x, \theta_y, l]^t$, can be defined by adding the radius parameter, l . The transducer output in rectangular coordinates is designated $\hat{\mathbf{u}}$, an estimate of $\mathbf{u}$, and the transducer output in spherical coordinates is designated $\hat{\mathbf{a}}$, an estimate of $\mathbf{a}$. The following equations defining the transformation from $\mathbf{u}$ to $\mathbf{a}$ are valid over the entire range of possible angles except when θ_x equals $\pm 90^\circ$:

$$\theta_x = \text{ArcTan}_{(2)}\left(\frac{u_y}{\sqrt{u_x^2 + u_z^2}}\right) = \text{ArcTan}_{(2)}\left(\frac{S_x}{C_x \sqrt{S_y^2 + C_y^2}}\right) = \text{ArcTan}_{(2)}\left(\frac{S_x}{C_x}\right) = \theta_x \quad (13)$$

$$\theta_y = \text{ArcTan}_{(2)}\left(\frac{-u_x}{u_z}\right) = \text{ArcTan}_{(2)}\left(\frac{-(-C_x S_y)}{C_x C_y}\right) = \text{ArcTan}_{(2)}\left(\frac{S_y}{C_y}\right) = \theta_y \quad (14)$$

$$\text{and} \quad l = \sqrt{u_x^2 + u_y^2 + u_z^2} = \sqrt{S_x^2 + C_x^2 (S_y^2 + C_y^2)} = \sqrt{S_x^2 + C_x^2} = 1 \quad (15)$$

$\text{ArcTan}_{(2)}$ is the two parameter arc tangent function available in most computer programming environments which takes the numerator and the denominator as separate parameters so angles can be resolved in all quadrants. When θ_x equals $\pm 90^\circ$, the equation for θ_y becomes indeterminant. These points correspond to the north and south poles of the coordinate system, and result directly from the axial symmetry of the magnetic field.

Summarizing, $\mathbf{u}$ defines a rectangular coordinate system equivalent to the spherical coordinate system defined by $\mathbf{a}$. The transducer output derived below is an estimate of $\mathbf{u}$ or $\mathbf{a}$, and will be designated $\hat{\mathbf{u}}$ or $\hat{\mathbf{a}}$. And, the angles defined by $\mathbf{a}$ provide a two-degree-of-freedom representation of joint movement that is valid at all angles except near the poles of the coordinate system.

Techniques for Inversion of Transducer Model:

Combining Equation 6 for the voltage from a Hall effect sensor near a magnet with the transformation matrix technique for describing the movement of a joint provides a powerful modeling tool for calculating or predicting sensor output voltages. However, in actual transducer applications we are interested in the inverse problem of deriving measures of the movement angles from the sensor voltages. The following section describes techniques for inverting the model. The techniques are separated into two sections: 1) linear analysis that provides exact inversion under certain idealized conditions; and 2) non-linear analysis, that must be utilized under more general conditions.

Linear Matrix Inversion Technique When Magnet is at Center of Rotation: In an idealization of the standard configuration described above, if the magnet is centered at the center of rotation ($\mathbf{m} = 0$), then the inversion of the sensor voltages into joint angles is quite straight forward, with exactly one optimal set of inversion coefficients. Given the fixed vectors, $\mathbf{d}_i$ and $\mathbf{n}$, that describe the position and orientation of the i th sensor relative to the center of rotation, Equation 6 for the i th sensor voltage can be rewritten:

$$v_i - v_{o_i} = k_0 \mathbf{k}_i^t \mathbf{u} = \left(k_{s_0} \mu_0 M_0 r^3 \right) \left(\frac{k_{s_i}}{d_i^3} \left[\frac{3}{2} \mathbf{C}_{\alpha_i} \mathbf{C}_{\beta_i} - \frac{1}{2} \mathbf{n} \right]^t \right) \mathbf{u} \quad (16)$$

where: k_0 (left-hand parameter group) is the overall scaling coefficient common to all sensors;

$\mathbf{k}_i$ (middle parameter group) is the vector of parameters specific to the i th sensor;

k_{s_0} is the nominal or average sensor sensitivity;

k_{s_i} is the i th sensor's sensitivity relative to k_{s_0} ;

d_i is the i th sensor's distance from the center of rotation;

$\mathbf{C}_{\beta_i} = \cos(\beta_i) = \mathbf{d}_i / |\mathbf{d}_i|$ is the vector of direction cosines of $\mathbf{d}_i$;

and $\mathbf{C}_{\alpha_i} = \cos(\alpha_i) = \mathbf{d}_i \cdot \mathbf{n} / |\mathbf{d}_i|$ is the cosine of the angle between $\mathbf{d}_i$ and $\mathbf{n}$.

Referring to Figure 3, β_i is the vector of angles between $\mathbf{d}_i$ and the $\mathbf{x}$, $\mathbf{y}$, and $\mathbf{z}$ axes, and α_i is the angle between $\mathbf{d}_i$ and $\mathbf{n}$. $\mathbf{u}$, $\mathbf{d}_i$, and $\mathbf{n}$ are the same as defined in Equations 10 and 12, except that $\mathbf{m} = 0$ when the magnet is at the center of rotation.

Examining the elements of $\mathbf{k}_i$, by definition C_{α_i} and each element of $\mathbf{C}_{\beta_i}$ have values between ± 1 , and the magnitudes of $\mathbf{n}$ and $\mathbf{C}_{\beta_i}$ equal 1. Overall, $\left[\frac{3}{2} C_{\alpha_i} \mathbf{C}_{\beta_i} - \frac{1}{2} \mathbf{n}\right]$ is a direction vector with magnitude between 1/2 and 1. Under nominal circumstances k_{s_i} and d_i have values near 1. As a direction vector, $\mathbf{k}_i$ primarily provides relative weightings to the three components of $\mathbf{u}$, having a minor contribution to overall scaling. By comparison, k_0 is the primary scaling coefficient, which is affected by sensor sensitivity, magnet material, magnet size, and sensor to magnet distance. For a given configuration, $\mathbf{k}_i$ and k_0 are constant, and all of the variables related to movement have been separated into the vector $\mathbf{u}$, making it equivalent to the form:

$$v_i - v_{o_i} = k_0 \begin{bmatrix} k_{i1} & k_{i2} & k_{i3} & 0 \end{bmatrix} \mathbf{u} \quad (17)$$

which is a linear equation with constant coefficients k_0 , k_{i1} , k_{i2} , and k_{i3} . Equation 17 can be extended to j sensors at different locations around a magnet in (matrix notation) as:

$$\mathbf{v} - \mathbf{v}_o = \begin{bmatrix} v_1 - v_{o1} \\ \vdots \\ v_j - v_{oj} \end{bmatrix} = k_0 \begin{bmatrix} \mathbf{K}_{j \times 3} \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix} = k_0 \mathbf{K} \mathbf{u} \quad (18)$$

where each sensor voltage is a linear combination of the elements of $\mathbf{u}$, with the constant matrix, $\mathbf{K}$, giving the relative scaling coefficients, and k_0 giving the overall scale factor. The trailing zero was left off of $\mathbf{u}$ for convenience. If the inverse (or pseudo-inverse) matrix, $\mathbf{G} = \mathbf{K}^{-1}$, (or $(\mathbf{K}'\mathbf{K})^{-1} \mathbf{K}'$), exists, then the inverse equation for $\hat{\mathbf{u}}$, the estimate of $\mathbf{u}$, as a linear function of the sensor voltages can be written:

$$\hat{\mathbf{u}} = \frac{\mathbf{G}}{k_0} (\mathbf{v} - \mathbf{v}_o) \quad (19)$$

where $\hat{\mathbf{u}}$ represents the transducer output. Since three rows in $\mathbf{K}$ is the minimum number required to obtain an inverse matrix, three sensors are the minimum number required produce a complete inverse relationship. The relationships given in Equations 13-15 can be used to transform $\hat{\mathbf{u}}$ into its spherical equivalent, $\hat{\mathbf{a}}$.

In summary, for the case where the magnet is centered at an ideal center of rotation, each sensor voltage is a fixed linear combination of the elements of $\mathbf{u}$. Given a model of the joint movement and sensor and magnet placement, the weighting coefficients can be directly calculated from Equation 16. If the matrix formed by the relative weighting coefficients has a non-zero determinant, then the matrix can be inverted to form an inverse weighting matrix that gives $\hat{\mathbf{u}}$ as a function of the sensor voltages. Finally, arc tangent functions can be used to convert $\hat{\mathbf{u}}$ into two angles, θ_x and θ_y that are directly and linearly related to movement of the joint.

Non-linearities Introduced by Shifting Magnet Away From Center of Rotation: In actual applications, joint movement is more complex than can be described by a fixed CR. Additionally there are deviations from the ideal placement of transducer components in the joint. The resulting deviations from the ideal transducer characteristics can be thought of as distortion of the sphere represented by $\mathbf{u}$. The following sections describe the technique that was utilized for calculating the transducer output under non-ideal conditions, as well as techniques for characterization of the transducer resolution and linearity.

Non-linear Inversion Technique: In analyzing the transducer characteristics under actual conditions, Equation 6 and a modification of Equation 16 are still applicable. When the magnet is at the center of rotation, parameters such as C_{α_i} and C_{β_i} are constant, which dramatically simplifies the transducer equations. However, under general conditions, the distance between the magnet each sensor varies as the joint is rotated, and the distance is slightly different for each sensor. The distance cubed terms in the denominator of each

sensor equation, among other factors, introduce complex non-linearities into the transducer equations.

The approach adopted for inverting the transducer characteristics was to use the linear inversion technique, with some correction factors added to optimize the transducer characteristics. The technique requires a reasonably accurate model of the joint movement and reasonably accurate measurements of the placement of the magnet and sensor array in the joint. The technique calculates a forward sensor coefficient matrix by supposing that the center of rotation is located at the center of the magnet when the joint is at the center of its range, then calculating the matrix coefficients from Equation 16. The resulting 3×3 coefficient matrix, **K**, is inverted to give the inverse weighting coefficient matrix, **G**. The remaining steps can be carried out using either data collected from the transducer or simulated data generated from the geometric model of the joint. The data set must span the range of motion of the joint, and if empirically collected data is used, each data point must have an associated reference measurement of the joint angles taken from a medical imaging system or other sensors. The steps are:

- (1) Calculate $\hat{\mathbf{u}}$ for each data point using **G** and Equation 19;
- (2) Normalize $\hat{\mathbf{u}}$ by its magnitude, $\hat{u}$, to obtain $\hat{\mathbf{w}}$, the normalized transducer output, constraining the points to lie on the surface of a unit sphere, but not changing the angular components;
- (3) Use analytical or parameter estimation techniques to find the k parameters that minimize the RMS error between θ_x and $\hat{\theta}_x$, and between θ_y and $\hat{\theta}_y$, where:

$$\hat{\theta}_x = \text{ArcTan}_{(2)} \left(\frac{(\hat{w}_y + k_{xy} + k_{xyz} \hat{w}_z)}{\sqrt{(\hat{w}_x + k_{xx})^2 + (\hat{w}_z + k_{xz})^2}} \right) \quad (20)$$

$$\text{and } \hat{\theta}_y = \text{ArcTan}_{(2)} \left(\frac{-(\hat{w}_x + k_{yx} + k_{yz} \hat{w}_z)}{(\hat{w}_z + k_{yz})} \right) + (\hat{\theta}_x - k_{sk1}) k_{sk2} \quad (21)$$

Equations 20 and 21 are extensions of Equations 13 and 14, with the k parameters added to compensate for non-linearities. The rationale for selecting the specific k parameters in

Equations 20 and 21 is described below. Once the k parameters have been determined, the procedure for calculating the angle estimates, $\hat{\theta}_y$ and $\hat{\theta}_x$, for any set of voltages follows steps (1) through (3) excluding the parameter estimation.

The vector $\hat{\mathbf{W}}$, introduced above, represents a third form of transducer output. For cases where the magnet is at the center of rotation, $\hat{\mathbf{U}}$ and $\hat{\mathbf{W}}$ are the same. For more general cases, $\hat{\mathbf{W}}$ is often more useful than $\hat{\mathbf{U}}$ because it reflects only the rotational aspects of the movement, essentially ignoring the distance changes between the sensors and the magnet.

Technique for Calculating Transducer Resolution:

Each nominal transducer data point, $\hat{\mathbf{U}}$, defines a point on the surface of a distorted sphere. Sensor voltage noise produces sample-to-sample deviations around that point. The sensor voltage noise can be statistically represented by a multi-dimensional ellipsoidal normal distribution. The inversion process maps the voltage noise distribution into a differently shaped ellipsoidal error distribution in the $\mathbf{u}$ domain. The shape, size, and orientation of the ellipsoid are directly determined by the magnitude of the voltage noise and the inverse weighting coefficients. The following sections define use of the term resolution in this context, and describe the methods used to calculate resolution.

Resolution Definition: The transducer **resolution** is defined here as the solid angular region statistically spanning 95% of the noise around a nominal operating point, $\hat{\mathbf{U}}$. For single normally distributed variables, the region that includes 95% of all sampled data points corresponds to ± 1.96 standard deviations. Similarly, when the error ellipsoid is mapped to the two degrees of freedom at the surface of a sphere, the elliptical error region that includes 95% of the sample data points is defined statistically by a Chi-square distribution with $\nu = 2$ degrees of freedom and significance level $\alpha = 0.05$. The radii of the error ellipse must be scaled by $\chi = \sqrt{\chi_{\nu, \alpha}^2}$ or ± 2.45 standard deviations to include 95% of the data points. The resolution (95% confidence region) is given by 2χ or 4.90 times the radii of the error

ellipsoid. A summary of the relationships between different confidence intervals and the corresponding range (in standard deviations) are given in Table 1.

Table 1. Relationship between the confidence interval and the data range, 2χ , in number of standard deviations.			
Confidence Interval $100\times(1-\alpha)$	Univariate Range ($v=1$)	Bivariate Range ($v=2$)	Trivariate Range ($v=3$)
68.3%	$2x$	$3.03x$	$3.76x$
95%	$3.92x$	$4.90x$	$5.59x$
99%	$5.15x$	$6.07x$	$6.74x$
99.9%	$6.58x$	$7.43x$	$8.07x$

The signal-to-noise ratio (S/N) can be defined as magnitude of $\hat{\mathbf{u}}$ at a point divided by the components of noise tangent to the surface of the sphere defined by $\mathbf{u}$. For small errors, the resolution is essentially equal to the inverse of the S/N ratio.

Sensor Noise: The sensor voltage noise was modeled using two components: ϵ_{cm} , noise statistically correlated between all the sensors (common mode error), and ϵ , the vector of residual uncorrelated noise from the individual sensors after removing the common mode error. The correlated noise component is due to factors such as power supply induced noise, or common mode environmental magnetic fields. The uncorrelated noise component is primarily due to the random fluctuations inherent in the individual sensors. Equation 19 can be modified to include the errors as follows:

$$\hat{\mathbf{u}} + \hat{\mathbf{e}} = \frac{\mathbf{G}}{k_0} (\mathbf{v} - \mathbf{v}_o + \epsilon + \epsilon_{cm}) \quad (22)$$

Each error component was assumed have a normal distribution, with zero mean, and a measured RMS error. For modeling purposes, all the error components were taken to be statistically uncorrelated (the correlation is accounted for by ϵ_{cm}). If the magnitudes of the RMS error of all four components are equal, then the shape of the joint probability density function will be a four dimensional spheroid. In the process of transforming sensor voltages

into useable signals (Equation 22), the voltages are weighted and summed by the inverse matrix coefficients. The process also operates on the errors, resulting in stretching and rotation of the voltage error distribution. The size and shape of the resulting distribution determine the resolution characteristics of the transducer. The characteristics of $\hat{\mathbf{e}}$ do not depend on the particular value of $\hat{\mathbf{u}}$, since the first stage of the inversion process is a linear operation.

Technique for Calculating Resolution: To derive the ellipsoidal region encompassing a particular statistical confidence region, the inverse weighting matrix $\mathbf{G}/k_0$ is rearranged then scaled by χ and the RMS magnitudes of the voltage errors to give $\mathbf{E}$:

$$\mathbf{E} = \frac{\chi}{k_0} \mathbf{G}^* \text{Diag} \begin{bmatrix} \epsilon \\ \epsilon_{cm} \end{bmatrix} = \frac{\chi}{k_0} \begin{bmatrix} g_{11} & g_{12} & g_{13} & (g_{11} + g_{12} + g_{13}) \\ g_{21} & g_{22} & g_{23} & (g_{21} + g_{22} + g_{23}) \\ g_{31} & g_{32} & g_{33} & (g_{31} + g_{32} + g_{33}) \end{bmatrix} \begin{bmatrix} |\epsilon_1| & 0 & 0 & 0 \\ 0 & |\epsilon_2| & 0 & 0 \\ 0 & 0 & |\epsilon_3| & 0 \\ 0 & 0 & 0 & |\epsilon_{cm}| \end{bmatrix} \quad (23)$$

where g_{ij} is the element from the i th row and j th column of $\mathbf{G}$.

$\mathbf{E}$ describes the scaling and rotation of the voltage errors that occur when the sensor voltages are mapped to $\hat{\mathbf{u}}$. The characteristics of the ellipsoidal confidence region of $\mathbf{E}$ are given by the eigenvalues and eigenvectors of $\mathbf{E}\mathbf{E}^t$. The orientations of the axes of the ellipsoid are given by the eigenvectors, and the corresponding lengths of the axes are given by the square root of the eigenvalues. Analytical expressions for the eigenvalues and eigenvectors are only possible for the simplest configurations, but numerical evaluation is always feasible for a particular configuration.

The following technique was used to map the surface of the 3-D error ellipsoid onto the 2-D ellipse that defines resolution. If the eigenvalues of $\mathbf{E}\mathbf{E}^t$ are used to construct the diagonal matrix, $\boldsymbol{\lambda}$, with the square root of the eigenvalues along the diagonal and zeros

elsewhere, and the eigenvectors of $\mathbf{E}\mathbf{E}^t$ are used to form the columns of the matrix $\mathbf{Q}$, then by definition:

$$\mathbf{E}\mathbf{E}^t = (\mathbf{Q}\boldsymbol{\lambda})(\boldsymbol{\lambda}\mathbf{Q}^t) \quad (24)$$

In addition, $(\mathbf{Q}\boldsymbol{\lambda})^{-1}$ defines a transformation from the error ellipsoid in the $\mathbf{u}$ -domain into a domain where the error region is a unit sphere. The procedure used to calculate the ellipse that represents the angular resolution of the transducer is:

- (1) At a point $\hat{\mathbf{u}}$, transform the coordinates into a domain where the error ellipsoid is a unit sphere using:

$$\hat{\mathbf{u}}_1 = (\mathbf{Q}\boldsymbol{\lambda})^{-1} \hat{\mathbf{u}} \quad (25)$$

- (2) Define a sequence of n points, $\hat{\mathbf{u}}_1 + \hat{\mathbf{e}}_1$, that represents a cone tangent to the surface of the unit error sphere with the vertex passing through the origin, using the transformation:

$$\hat{\mathbf{u}}_1 + \hat{\mathbf{e}}_{1i} = \text{Cos}(\gamma) [\text{Rot}(\mathbf{y}, -\alpha)] [\text{Rot}(\mathbf{x}, -\beta)] [\text{Rot}(\mathbf{z}, \delta_i)] [\text{Rot}(\mathbf{x}, \gamma + \beta)] [\text{Rot}(\mathbf{y}, \alpha)] \hat{\mathbf{u}}_1 \quad (26)$$

where: i ranges from 1 to n ,

δ_i ranges from 0 to 2π , tracing out the circle,

$\gamma = \text{ArcSin}(1/\hat{u}_1)$ is the angle subtended by the cone where $\hat{u}_1$ is the magnitude of $\hat{\mathbf{u}}_1$,

$\text{Cos}(\gamma) = \sqrt{\hat{u}_1^2 - 1} / \hat{u}_1$ re-scales length of the vectors,

$\alpha = \text{ArcTan}(-u_{1x}/u_{1z})$ and $\beta = \text{ArcTan}(u_{1y}/\sqrt{u_{1x}^2 + u_{1z}^2})$ rotate the coordinate system to align it with the $\mathbf{z}$ axis,

and $-\alpha$ and $-\beta$ reverse the rotation of the coordinate system;

- (3) Re-map the points $\hat{\mathbf{u}}_1 + \hat{\mathbf{e}}_{1i}$ back to the $\mathbf{u}$ domain using:

$$\hat{\mathbf{u}} + \hat{\mathbf{e}}_u = (\mathbf{Q}\boldsymbol{\lambda})(\hat{\mathbf{u}}_1 + \hat{\mathbf{e}}_1) \quad (27)$$

- (4) Use the arc tangent relationships in Equations 20 and 21 to map the points into the $\mathbf{a}$ domain:

$$\hat{\mathbf{a}} + \hat{\mathbf{e}}_a \Leftarrow \hat{\mathbf{u}} + \hat{\mathbf{e}}_u \quad (28)$$

Properties of the Error Ellipse/Resolution: There are several generalizable properties of $\mathbf{E}$ that are reflected in the size of the error ellipsoid, and thus the transducer resolution. First,

the size of the ellipsoid scales proportionately with the RMS magnitude of the voltage errors, as would be anticipated from the linear weighted summation process. Second, the size of the error ellipsoid scales inversely proportionately with the scaling constant k_0 , so that if factors such as the strength of magnet increase k_0 , the resolution will be improved proportionately. Third, the resolution depends on the confidence interval chosen. For example if a 68% confidence region is used to define resolution, the transducer resolution will appear to be better than if a 95% confidence region is used, when in reality there will be no actual difference in the transducer output. Examples of the properties of the resolution are given below in the Results section.

Resolution with no lateral shift of sensor array: Using the standard configuration, a closed-form expression for $\hat{\mathbf{e}}_{\mathbf{u}}$, the resolution of $\hat{\mathbf{u}}$ was derived for the case when the lateral shift of the sensor array is zero, and the magnet is located at the center of rotation. Under these conditions, and with a symmetric arrangement of the sensors, the primary axes of the ellipsoidal error distribution are aligned with the $\mathbf{x}$, $\mathbf{y}$, and $\mathbf{z}$ axes, with the magnitude along the $\mathbf{x}$ and $\mathbf{y}$ axes being equal. The two primary geometry parameters that affect resolution are r , the normalized magnet radius, and s , the normalized sensor separation. The expressions for $\hat{\mathbf{e}}$ under these conditions are:

$$\hat{e}_{u_x} = \hat{e}_{u_y} = \frac{\chi}{k_0} g_{xy} \frac{180}{\pi} \frac{\epsilon}{\hat{u}} = \frac{4.9}{k_{s_0} \mu_0 M_0 r^3} \frac{2\sqrt{6}}{9s\sqrt{1-s^2}} \frac{180}{\pi} \frac{\epsilon}{\hat{u}} \quad (29)$$

$$\text{and } \hat{e}_{u_z} = \frac{\chi}{k_0} g_z \frac{180}{\pi} \frac{\sqrt{\epsilon^2 + 3\epsilon_{cm}^2}}{\hat{u}} = \frac{4.9}{k_{s_0} \mu_0 M_0 r^3} \frac{2}{\sqrt{3}(2-3s^2)} \frac{180}{\pi} \frac{\sqrt{\epsilon^2 + 3\epsilon_{cm}^2}}{\hat{u}} \quad (30)$$

where: $\chi = 4.9$ is derived from the Chi-square error distribution;

g_{xy} and g_z are derived from the eigenvalues of $\mathbf{GG}^t$;

$180/\pi$ converts radians to degrees;

ϵ_{cm} is the common mode RMS sensor voltage error;

ε is the average uncorrelated RMS sensor voltage error;

and the remaining parameters were defined earlier.

With the magnet at the center of rotation, $\hat{u} = 1$. If the magnet is not at the center of rotation, these equations approximate the resolution, accounting for the distance changes between the sensors and magnet are implicitly through $\hat{u}$. For small errors, the relationship between $\hat{\mathbf{e}}_a$ and $\hat{\mathbf{e}}_u$ can be approximated by taking the partial derivatives of the arc tangent relationships with respect to the elements of $\hat{\mathbf{e}}_u$:

$$\hat{e}_{\theta_x} \approx -\frac{\hat{u}_y \hat{u}_x}{\hat{u} \hat{u}_{xz}} \hat{e}_{u_x} + \frac{\hat{u}_{xz}}{\hat{u}} \hat{e}_{u_y} - \frac{\hat{u}_y \hat{u}_z}{\hat{u} \hat{u}_{xz}} \hat{e}_{u_z} \approx S_x S_y \hat{e}_{u_x} + C_x \hat{e}_{u_y} - S_x C_y \hat{e}_{u_z} \quad (31)$$

and

$$\hat{e}_{\theta_y} \approx -\frac{\hat{u} \hat{u}_z}{\hat{u}_{xz}^2} \hat{e}_{u_x} - \frac{\hat{u} \hat{u}_x}{\hat{y}_{xz}^2} \hat{e}_{u_z} \approx -\frac{C_y}{C_x} \hat{e}_{u_x} - \frac{S_y}{C_x} \hat{e}_{u_z} \quad (32)$$

where $\hat{u}_{xz} = \sqrt{\hat{u}_x^2 + \hat{u}_z^2}$. Equation 31 is a pure rotation of the elements of $\hat{\mathbf{e}}_u$. Equation 32 includes a pure rotation of $\hat{\mathbf{e}}_u$ around the $\mathbf{y}$ axis, but also divides by the cosine of θ_x , which has a minor effect when θ_x is small, but makes the resolution very large when θ_x is near $\pm 90^\circ$. This corresponds to operating at the poles of the coordinate system, where θ_y degenerates.

The separation between the sensors is the primary determinant of the resolution along the $\mathbf{x}$ and $\mathbf{y}$ -axes relative to the resolution along the $\mathbf{z}$ -axis. In Equations 29 and 30, if s is replaced by $\text{Sin}(\alpha)$, its angular equivalent, then g_{xy} and g_z can be rewritten:

$$\text{a) } g_{xy} = \frac{4\sqrt{6}}{9 \text{Sin}(2\alpha)}; \quad \text{b) } g_z = \frac{4}{\sqrt{3} (1 + 3 \text{Cos}(2\alpha))} \quad (33)$$

showing that the inverse of the resolution varies with the sine and cosine of twice α . For the fairly small angles typical of implanted joint angle transducer configurations, $\text{Sin}(2\alpha) \approx 2\alpha$ and $\text{Cos}(2\alpha) \approx 1$. Thus, the x and y resolution will be approximately inversely proportional

to α , improving as the separation between the sensors increases, with only a minor degradation of the z resolution.

Transducer Sensitivity to Other Offset Errors:

Sensor offset voltage noise was used to define resolution. Similarly, static or very low frequency sensor voltage offset shifts will result in transducer output errors. These offset shifts may result from sources such as temperature variations, power supply variations, sensor baseline drift, or environmental magnetic fields. The effects of these other types of errors can be calculated from the resolution calculation techniques by setting χ equal to one. χ was used to convert from the RMS noise magnitude into the 95% confidence region. For static errors, this scale factor is unnecessary. Equation 23 can be used to calculate the effect of a specific known offset shift on a known configuration. For errors of a given magnitude, but where the distribution of the error among the three sensor is not known or varies, the techniques for calculating the resolution ellipses can be used to describe the region of transducer output offsets resulting from the errors.

Derivation of Transducer Linearity Expressions:

For transducer configurations where the magnet is positioned at the center of rotation, the two transducer output angles, $\hat{\theta}_x$ and $\hat{\theta}_y$, are exactly linearly related to the joint movement angles, θ_x and θ_y . However, as the magnet is shifted away from the ideal location or if the joint does not have a constant center of rotation, the linearity of the relationship is degraded, roughly in proportion to how far off-center the magnet is located. The **linearity error**, η , is defined as the difference between the transducer output angles and the actual angles.

Numerical Versus Analytical Techniques for Describing Transducer Linearity: For cases where all of the mechanical parameters of a configuration are known, it is straight forward to use numerical simulations to predict or display the linearity of a transducer. In general,

numerical simulations will provide the most accurate estimate of transducer characteristics, but only if an accurate geometric model and parameters are available. However, numerical simulations are not well suited for looking at general interactions between model parameters, or for providing simple techniques for estimating transducer characteristics and how they change as different model parameters change.

In order to overcome the limitations of numerical techniques, analytical expressions for the linearity error were derived for a restricted set of conditions. First, analytical expressions for $\hat{\mathbf{u}}$ were derived for the limiting condition where the distance of each sensor from the center of the array approaches zero. Then, these expressions for $\hat{\mathbf{u}}$ were used to derive estimates of the RMS linearity error and optimal offset coefficients for the inversion process that minimize the RMS linearity error for the limiting condition where the distance of the magnet from the center of rotation approaches zero. The analytical techniques provide expressions for the transducer output and linearity that are strictly accurate only under restricted conditions, but provide reasonable estimates under more general conditions. The analytical expressions also show the interactions between the geometry parameters, and provide a mechanism for optimizing the transducer output.

Analytical Expression for Transducer Output Under Restricted Conditions: General expressions for $\hat{\mathbf{u}}$ were derived for a co-planar arrangement of three sensors under the limiting condition where s , the distance of each sensor from the center of the array, approaches zero. Under this condition, the denominators of the terms from the three sensors can be combined, yielding a closed form expression for the transducer output as a function of $\mathbf{d}$, $\mathbf{m}$, θ_x and θ_y .

The limiting case where s approaches zero is an impractical configuration because the sensitivities approach zero, but the derivation is possible in the "noise free" environment of algebra. Most importantly, the shape and magnitude of the transducer linearity

characteristics are preserved under these conditions. Qualitatively, the results provide a reasonable match to numerically obtained transducer characteristics from typical practical sensor configurations.

In summary, the derivation: 1) Calculated the inverse coefficient matrix, $\mathbf{G}/k_0$, as a function of s , for the case with the magnet at the center of rotation; 2) Calculated the actual sensor voltages as a function of s with the magnet shifted by $\mathbf{m}$ away from the center of rotation; 3) Multiplied the sensor voltages by $\mathbf{G}/k_0$; 4) Took the limit as s approached zero; and 5) Canceled, factored, and manipulated to simplify the expressions. The resulting closed-form equations show the non-linear effects of displacement of the magnet away from the center of rotation, and the compounding effects of lateral shifts of the sensor array. The resulting equations are:

$$\hat{u}_x = -\frac{f_{C_x S_y}}{p^5} - \frac{2d_x f_{C_x C_y}}{p^5} + \frac{5d_x f_1 f_3}{p^7} - \frac{(\mathbf{m} \cdot \mathbf{o}_x) f_3}{p^7} \quad (34)$$

$$\hat{u}_y = \frac{f_{S_x}}{p^5} - \frac{2d_y f_{C_x C_y}}{p^5} + \frac{5d_y f_1 f_3}{p^7} - \frac{(\mathbf{m} \cdot \mathbf{o}_y) f_3}{p^7} \quad (35)$$

and

$$\hat{u}_z = \frac{C_x C_y}{p^5} - \frac{3d_z f_{C_x C_y}}{p^5} + \frac{5d_z (f_1 + f_2) f_3}{p^7} \quad (36)$$

Full expressions for the f terms are given in Appendix 1, at the end of chapter.

In a qualitative comparison, the first term of each equation contains the primary dependence on $-C_x S_y$, S_x , and $C_x C_y$ respectively. These are the only terms that do not disappear when m equals zero. All of the remaining parts of the equations represent the shape of the distortion characteristics when m is non-zero. In the limit as m also tends toward zero, $f_1 \rightarrow 0$, $f_2 \rightarrow 0$, $f_{C_x C_y} \rightarrow 0$, $f_{S_x} \rightarrow S_x$, $f_{C_x S_y} \rightarrow C_x S_y$, and $p \rightarrow 1$, giving the result $\hat{\mathbf{u}} \rightarrow \mathbf{u}$. When m is non-zero, the second and third terms of Equations 34 and 35 are

proportional d_x and d_y respectively. These terms arise because of an imbalanced weighting of the sensor voltages when the sensor array is laterally shifted or tilted. They represent a cross-coupling of $\hat{u}_z$ into $\hat{u}_x$ and $\hat{u}_y$ that disappears when d_x and d_y are zero. The fourth term of Equations 34 and 35 are also cross-couplings of $\hat{u}_z$ into $\hat{u}_x$ and $\hat{u}_y$, which disappear when m is zero, but not when d_x and d_y are zero. The dominant linearity errors introduced by these terms are proportional to m_x and m_y respectively. The coefficients k_{yxx} and k_{xxz} in Equations 20 and 21 act primarily to minimize the the effects of these cross-coupling terms on the linearity. The second and third terms of Equation 36 have many similarities to the last three terms in Equations 34 and 35. However, when d_x and d_y equal zero, d_z equals one, so none of the major terms in Equation 36 disappear when there no sensor array tilt. The most dominant effect of these terms is to add an offset to $\hat{u}_z$ proportional to m_z .

The p^5 and p^7 terms in the denominators of Equations 34-36 are of conspicuously higher order than the overall p^3 relationship that might be anticipated from the magnetic field equations. In Equation 6, which describes the sensor voltage, the p_z term inside the brackets effectively has a p^5 term in its denominator. The numerator of the p_z term includes two terms that are proportionally related to $\mathbf{p}$, but do not fully cancel with the denominator. The o_z term in Equation 6, has only a p^3 term in its denominator. In deriving Equations 34-36, when taking differences between sensor voltages in the limit as s tends to zero, the series expansion of an expression generates denominators of higher order than the original expression. This is illustrated in the following simplified analogous example:

$$\text{Limit}_{\Delta \rightarrow 0} \left[\frac{k_1/\Delta + k_2}{(p^2 - \Delta)^{(5/2)}} + \frac{-k_1/\Delta + k_2}{(p^2 + \Delta)^{(5/2)}} \right] = \frac{2k_2}{p^5} - \frac{5k_1}{p^7} \quad (37)$$

where the sum of the k_2 terms results in a p^5 denominator, while the difference of the k_1 terms results in a p^7 denominator. If the denominators in the left-hand expression are equal or if the k_1 terms are not divided by Δ the resultant dependence on k_1 disappears.

Although the higher order denominators arise when taking the limit as s tends to zero, the distortion characteristics are still present for non-zero values of s , and are reasonably well described by Equations 34-36 under typical conditions. In Equations 34-36, the terms that depend explicitly on p^3 have canceled out, leaving no simple dependence on $1/p^3$. However, the numerators of remaining expressions have implicit dependencies on p , such that it is a gross oversimplification to state that Equations 34-36 only depend on $1/p^5$ and $1/p^7$.

When converting $\hat{\mathbf{u}}$ into polar coordinates, the arc tangent function utilizes ratios of the elements of $\hat{\mathbf{u}}$. Since $1/p^5$ is a major dependence that can be factored out of $\mathbf{u}$, the $1/p^5$ term is inherently canceled by taking the arc tangent of ratios of components of $\hat{\mathbf{u}}$, even though p is an unknown variable. Similarly, when $\hat{\mathbf{u}}$ is normalized to give $\hat{\mathbf{w}}$, the denominators in Equations 34-36 are largely canceled out, leaving primarily additive distortion or errors terms. The non-linear inversion techniques described above are based on these findings.

Analytical Expressions for RMS Linearity Error of Transducer Output Under Restricted Conditions: The analytical expressions for $\hat{\mathbf{u}}$ (Equations 34-36) were used to derive an expression for the RMS linearity error, η , and optimal offset coefficients for the inversion process that minimize the RMS linearity error. The goals in deriving an expression for the RMS linearity error were two-fold. The first was to provide an expression that can be used to provide a quick and simple estimate of the error without doing involved simulations. This simple expression should indicate the relative error contributions due to different geometric model parameters. The second goal was to demonstrate the existence of a unique set of optimum offset parameters that minimize the RMS linearity error.

The linearity errors $\eta_x = \hat{\theta}_x - \theta_x$ and $\eta_y = \hat{\theta}_y - \theta_y$ tend toward zero as m tends toward zero. All of the error subterms are proportional to m and powers of m . When m is small, the first order powers of m dominate over the higher order powers of m . In order to derive manageable equations for estimating the linearity error, the derivation below utilized the limiting case as m approached zero, dropping the higher order dependencies on m out of the equations. The minimization of the RMS linearity error was calculated by substituting the expressions for $\hat{\mathbf{u}}$ in Equations 34-36 into Equations 20 and 21, then solving:

$$\frac{\partial}{\partial \mathbf{k}} \left(\iint_{\theta_x, \theta_y} \left(\text{Limit}_{m \rightarrow 0} [\eta/m] \right)^2 \right) = 0 \quad (38)$$

for $\mathbf{k}$. The process started by taking the limit of η/m as m approached zero, resulting in asymptotically non-zero relationships with the higher order dependencies on m removed. The resulting error expressions were squared and integrated over the range of θ_x and θ_y . Finally, the expressions were differentiated with respect to each of the offset coefficients, solving for the value of the vector $\mathbf{k}$ where the derivatives equal zero. This corresponds to the point where the RMS error is minimized. The RMS error is given by:

$$\eta_{\text{RMS}} = \left(\iint_{\theta_x, \theta_y} \left(\text{Limit}_{m \rightarrow 0} [\eta/m] \right)^2 / \iint_{\theta_x, \theta_y} d\theta \right)^{1/2} \quad (39)$$

Three generalizations about the optimal offsets are given here. First, there is a definite single vector, $\mathbf{k}_{opt}$ that minimizes the RMS errors for a given set of conditions. The implication is that it is possible to calculate the optimal offset coefficients using numerical parameter estimation techniques. Second, the optimal offsets depend on the geometry parameters, $\mathbf{d}$ and $\mathbf{m}$, and thus are different for every configuration. Finally, the optimal offsets depend on the selected ranges of θ_x and θ_y .

Under the restricted conditions of this derivation, the RMS linearity error can be expressed:

$$\eta_{x(\text{RMS})} = \sqrt{(c_{xx} m_x)^2 + (c_{xy} m_y)^2 + (c_{xz} m_z)^2 + c_{xx}^2 m_x m_z} \quad (40)$$

and
$$\eta_{y(\text{RMS})} = \sqrt{(c_{yx} m_x)^2 + (c_{yy} m_y)^2 + (c_{yz} m_z)^2 + c_{yz}^2 m_x m_z} \quad (41)$$

where $\mathbf{c}_x = \{c_{xx}, c_{xy}, c_{xz}, c_{xxz}\}$ and $\mathbf{c}_y = \{c_{yx}, c_{yy}, c_{yz}, c_{yxz}\}$. $\mathbf{c}_x$ and $\mathbf{c}_y$ are dependent on d_x and d_y , and the selected ranges of θ_x and θ_y . If d_x and d_y are constant for a given configuration, then $\mathbf{c}_x$ and $\mathbf{c}_y$ are also constant. The units of the c parameters are in degrees, and the components of $\mathbf{m}$ are unitless (normalized by D). Values for the c parameters are given below.

Results/Examples

One degree of freedom example

Figure 4 shows a simple two sensor, one-degree-of-freedom configuration to illustrate some of the relationships between $\hat{u}$, the error ellipses, the linearity error, and the sensor and magnet positioning. Figure 4(a) shows u_z versus u_x for a rotational movement, θ_y , about the y -axis, which is pointing into the page. The position of the magnet and sensors and their respective magnetic axis and normal vectors when $\theta_x=0$ are shown by the heavy lines and arrows. The diagram can be interpreted by thinking of the magnet as fixed in space, with the sensors rotating in a circle around center of rotation (the point where the dotted lines converge). The line with triangular symbols shows the path of $\hat{u}_z$ versus $\hat{u}_x$ as θ_y ranges over 360° with symbols at 6° increments. The line with square symbols shows the comparable normalized transducer output, $\hat{w}_z$ versus $\hat{w}_x$. The relative spacing between the squares represents the sensitivity of the transducer characteristics at that point. The ellipses represent the transducer resolution with an arbitrarily selected noise magnitude. As

Figure 4. A two sensor, one-degree-of-freedom example. a) z vs x transducer output in rectangular coordinates, showing sensor and magnet positions; $\hat{u}$, unnormalized output; $\hat{w}$, normalized output; resolution ellipses; and angular span of ellipses. b) shows the angular transducer output error and resolution as a function of actual angle.

indicated above in the derivations, the size and shape of the ellipses is constant in the $\mathbf{u}$ -domain. However, $\hat{\theta}_a$, the angle spanned by the error ellipse (shown by the radial dotted lines), is very dependent on $\hat{\mathbf{u}}$ and θ_y . Specifically, when the magnitude of $\hat{\mathbf{u}}$ is large (magnet is close to sensors) and when the change in angle is aligned with the short axis of the ellipse, then the resolution is small ($\theta_y = -60^\circ$). Conversely, the resolution is much larger when the change in angle is aligned with the long axis of the ellipse ($\theta_y = 120^\circ$) or when the magnitude of $\hat{\mathbf{u}}$ is small ($\theta_y = 120^\circ$). Figure 4(b) shows the deviation of $\hat{\theta}_y$ (solid line) from the ideal angle, and how the angular transducer resolution (separation between dashed lines) varies over the range of θ_y in this example.

Results when magnet is at center of rotation

Linearity: Figures 5 and 6 introduce graphs of transducer characteristics from the configuration where the magnet is at the center of rotation. All graphs are shown as θ_y ranges over $\pm 180^\circ$ and θ_x ranges over $\pm 60^\circ$, with 30° grid increments. Each graph emphasizes different aspects of the transducer output. Figure 5(a) shows $\hat{\mathbf{u}}$, the unnormalized rectangular coordinate system transducer output components as the transducer is ranged through an angular grid pattern. Each component of $\hat{\mathbf{u}}$ is plotted against the others, showing three views of the sphere defined by the transducer output. Figure 5(b) shows $\hat{\mathbf{w}}$, the normalized transducer output. When the magnet is at the center of rotation, (a) and (b) are identical. Figure 5(c) shows $\hat{\mathbf{a}}$, the equivalent angular transducer output, plotting $\hat{\theta}_x$ versus $\hat{\theta}_y$. When the magnet is at the center of rotation, $\hat{\mathbf{a}}$ equals $\mathbf{a}$, and shows the grid pattern of angles through which the transducer was ranged. Figure 6 shows several transducer input/output characteristics. Each output parameter is plotted versus θ_x and θ_y . From bottom to top, Figure 6 shows the transducer output ($\hat{\theta}_x$ and $\hat{\theta}_y$), the linearity error (η_x and η_y), the magnitude of $\hat{\mathbf{u}}$, and the derivatives of $\hat{\theta}_x$ and $\hat{\theta}_y$ with respect to θ_x and θ_y . Graphs 6(a) and 6(d) illustrate the input/output characteristics in a standard form.

Figure 5. Transducer output with magnet at center of rotation over the range $-180^\circ \leq \theta_y \leq 180^\circ$ and $-60^\circ \leq \theta_x \leq 60^\circ$. (a) shows the unnormalized transducer output in rectangular coordinates. (b) shows the normalized transducer output. (c) shows the angular transducer output (spherical coordinates).

Figure 6. Transducer characteristics versus joint angles with magnet at center of rotation over the range $-180^\circ \leq \theta_y \leq 180^\circ$ and $-60^\circ \leq \theta_x \leq 60^\circ$. (a-d) input/output and cross interactions. (e-h) linearity error. (i-j) magnitude of $\hat{u}$. (k-l) sensitivity and cross-axis sensitivity.

Graphs 6(b) and 6(c) show the cross-axis interactions between x and y angles. Graphs 6(e) - 6(h) explicitly show the linearity error as a function of θ_x and θ_y . Graphs 6(i) and 6(j) show the magnitude of $\hat{\mathbf{U}}$, which is closely related to the inverse distance raised to a power and is strongly related to the relative resolution. $\hat{u}$ equals one in this example. Graphs 6(k) and 6(l) show the transducer sensitivity (upper) and the magnitude of the cross-axis sensitivity (lower) as a function of θ_x and θ_y . The sensitivity equals one, and the cross-axis sensitivity equals zero in this example. Overall, Figure 5(c) is the most compact method for displaying and comparing the transducer linearity characteristics, although the explicit display of θ_x and θ_y is lost to an implicit display represented by distortion of the grid pattern.

Resolution: Figure 7 illustrates the transducer resolution characteristics when the magnet is at the center of rotation for several sensor configurations. Each graph is similar to Figure 5(c), with ellipses added at selected vertices of the grid to represent the transducer resolution. The absolute size of the ellipses was chosen for illustrative purposes. The uncorrelated noise of each sensor and the common mode noise were all chosen to be equal. The relative shape and size of the ellipses across the five graphs illustrate how the resolution varies with s , the sensor separation, and $\mathbf{d}$, the sensor array position. In Figure 7(a), $d_x = d_y = 0$ and $s = 0.15$, which is a typical practical value. For this simple configuration, the relative size of the ellipses can be derived directly from Equations 29-32. In the $\mathbf{u}$ -domain, the elements of $\hat{\mathbf{e}}_{\mathbf{u}}$ are constant at $\hat{e}_{u_x} = \hat{e}_{u_y} = 12.8^\circ$ and $\hat{e}_{u_z} = 4.2^\circ$. Translated into the $\mathbf{a}$ -domain, this gives ellipses 12.8° high along $\hat{\theta}_x = 0$, with widths ranging between 4.2° and 12.8° . At $\hat{\theta}_x = \pm 60^\circ$, the widths are doubled because of the $1/C_x$ relationship shown in Equation 32. Eight-region symmetry of the resolution ellipses is evident under these conditions. Figure 7(b) is similar to 7(a), but d_x and d_y have been increased to 0.2, maintaining the RMS average distance between the sensors and the magnet. No distortion of the grid lines is introduced, but the resolution ellipses are slightly larger overall, the

Figure 7. The effects of sensor separation and lateral offset on transducer resolution. (a) nominal example with no lateral sensor array shift. (b) sensor array shifted laterally to moderate degree. (c) sensor array shifted laterally to strong degree. (d) no lateral shift, decreased sensor separation. (e) no lateral shift, sensor separation increased sufficiently to result in equal x and y resolution at $\hat{\theta}_x = 0^\circ$.

ellipse axes along $\hat{\theta}_x = 0$ and $\hat{\theta}_y = 0$ have been rotated, and the regional symmetry of the ellipses is gone. Equations 29-32 are only approximately true under these conditions. In Figure 7(c), d_x and d_y have been increased to 0.5. The overall size of the ellipses is considerably larger, and the ellipse rotation and asymmetry have increased. Distortion of the individual ellipses due to the arctangent has increased to a considerable degree. At this degree of lateral sensor offset, Equations 29-32 are not good estimates of the resolution, and the general methods described earlier must be used for estimating the resolution. In Figure 7(d), d_x and d_y have been set back to zero, but s has been decreased to 0.075. $\hat{e}_{u_x}$ and $\hat{e}_{u_y}$ are approximately doubled, and $\hat{e}_{u_z}$ is slightly decreased relative to Figure 7(a), resulting in a larger difference between the largest and smallest resolution. This divergence continues to increase as the sensors are moved closer to one another. Figure 7(e) shows the case where $s \approx 0.39$. Here, when the common mode noise equals the uncorrelated noise, all three elements of $\hat{\mathbf{e}}_u$ are equal, the resolution of $\hat{\theta}_x$ is constant, and the resolution of $\hat{\theta}_y$ is proportional to $1/C_x$. In the five configurations shown in Figure 7, the magnet is at the center of rotation resulting in linear input/output characteristics, but the resolution varies with the configuration and over the range of motion. Similar variation of the resolution is present when the magnet is not at the center of rotation, but is compounded by the varying distance between the sensors and the magnet. Non-linear examples are given below.

Results when magnet is shifted away from center of rotation

Distortion of $\hat{\mathbf{w}}$: Figure 8 shows graphs of the normalized transducer output, $\hat{\mathbf{w}}$, for several configurations to illustrate the distortion introduced by shifting the magnet off center. Figure 5(b) shows the comparable graphs when the magnet is at the center of rotation. Figure 8(a) shows $\hat{\mathbf{w}}$ for the case where $m_x = 0.1$. The dominant effect on the transducer output is best illustrated in 8(a.i) as a rotation of the radial grid lines toward the top of the graph. Ideally, the two radial grid lines just above the $\hat{w}_x$ zero line would coincide with the

Figure 8. Distortion introduced by shifting magnet away from center of rotation. Each group shows the three possible pairs of normalized transducer outputs plotted against each other, showing three equivalent views of a distorted three-dimensional sphere. Each group shows the magnet shifted in a different direction: (a) $m_x = 0.1$, (b) $m_y = 0.1$, (c) $m_z = 0.1$, and (d) $m_x = m_y = m_z = 0.1$.

zero line. The two simplest methods to reduce this distortion are to add an offset, k_{yx} , or the term $k_{yxz}|\hat{w}_z|$ to $\hat{w}_x$. These are two of the correction terms shown in Equation 20. A more complex correction factor can be derived from Equations 34-36, but would require an initial estimate of $\hat{\theta}_y$ to calculate $\hat{\theta}_y$, making the correction factor recursive. k_{yx} and k_{yxz} are fixed coefficients for a given configuration, and do not require recursive calculation.

Figure 8(b) shows $\hat{\mathbf{W}}$ for the case where $m_y = 0.1$. The dominant effect on the transducer output is best shown by the diagonal grid lines across the center of 8(b.iii). Ideally, these lines would be horizontal. The term $k_{xyz}\hat{w}_z$ in Equation 21 substantially reduces this distortion using a fixed coefficient for a given configuration without requiring a recursive calculation.

Figure 8(c) shows $\hat{\mathbf{W}}$ for the case where $m_z = 0.1$. The dominant transducer output error is best illustrated in 8(c.i) as a shift of the radial grid lines toward the left. This component of error can be eliminated by adding the offsets, k_{xz} and k_{yz} to $\hat{w}_z$ as shown in Equations 20 and 21. m_z was established in Equation 10 as an integral offset in the derivation of the sensor voltage equations.

Figure 8(d) shows $\hat{\mathbf{W}}$ for the case where $m_x = m_y = m_z = 0.1$. The dominant distortion components in this configuration appear as approximately the superposition of the distortion components shown in Figures 8(a)-8(c). The distortion components for this more complex configuration are significantly reduced by the same coefficients described above. The examples shown below illustrate the reduction in linearity error gained by using these correction coefficients. The distortion introduced when both $\mathbf{m}$ and d_x or d_y are non-zero result in more complex errors and harmonics. No additional correction coefficients were added to compensate for them, but examples are illustrated below.

Linearity: Figures 9-11 summarize the linearity results from a transducer configuration with $m_x = m_y = m_z = 0.1$, $d_x = d_y = 0.2$. Figure 9 is comparable to Figure 5, for this

configuration. Figure 9(a) shows the angular distortion of the grid lines and the radius changes of the sphere due to the varying sensor array to magnet distance. Figure 9(b) shows how the normalization process flattens the grid lines onto the surface of a unit sphere. The re-linearization coefficients do not affect these graphs. Figure 9(c) shows the angular transducer output grid after including the re-linearization coefficients. Because of the numerous harmonics in the transducer equations, the shape characteristics defy simple description. Figure 10 shows transducer input/output characteristics in more detailed form, better separating which aspects of the transducer characteristics are due to sensitivity changes versus cross-axis interactions. Figures 10(a) - 10(d) shows the input/output characteristics. Figures 10(e) - 10(h) show the linearity error. The RMS linearity errors are derived from these curves. Figures 10(i) and 10(j) show the magnitude of $\hat{\mathbf{u}}$ over the range of motion. These curves are strongly inversely related to the resolution shown in Figure 14. Figures 10(k) and 10(l) show the input/output sensitivity and the magnitude of the cross-axis interaction. In more extreme cases, non-monotonic transducer characteristics would be indicated by the transducer sensitivity curve dropping below zero, corresponding to a rift or collapse in a region of the sphere created by $\hat{\mathbf{u}}$. Regions of low but positive sensitivity are danger zones, generally corresponding to regions of poor resolution. Rapid shifts from low to high sensitivity such as between θ_y equals -180° and -120° (θ_x equals -60°) are an indication that the configuration is nearing a rift zone. This particular region corresponds to the u_x positive, u_y negative, and u_z negative region in Figure 9(a).

Re-Linearization: Figure 11 shows how the input/output characteristics are changed by introduction of re-linearization offset coefficients. Figures 11(a) and 11(b) show $\hat{\theta}_y$ versus θ_y and $\hat{\theta}_x$ versus θ_x . The vertical scales only show relative angle, with the three groups of curves displaced by 90° relative to each other for clarity. The upper groups show the transducer output with no re-linearization offsets. The RMS linearity values in this example

Figure 9. Transducer output with magnet shifted away from center of rotation and sensor array shifted laterally, over the range $-180^\circ \leq \theta_y \leq 180^\circ$ and $-60^\circ \leq \theta_x \leq 60^\circ$. (a) shows the unnormalized transducer output in rectangular coordinates. (b) shows the normalized transducer output. (c) shows the angular transducer output (spherical coordinates).

Figure 10. Transducer output with magnet shifted away from center of rotation and sensor array shifted laterally, over the range $-180^\circ \leq \theta_y \leq 180^\circ$ and $-60^\circ \leq \theta_x \leq 60^\circ$. (a-d) input/output and cross interactions. (e-h) linearity error. (i-j) magnitude of $\hat{u}$. (k-l) sensitivity and cross-axis sensitivity.

Figure 11. The effects of re-linearization in transducer input/output characteristics.

are $\eta_x = 13.4^\circ$ and $\eta_y = 20.1^\circ$. The middle groups show how fixed offsets (excluding k_{xyz} and k_{yxz}) change the linearity, reducing η_x to 13.0° and η_y to 10.2° . The primary reduction is effected by removing the first order harmonic from $\hat{\theta}_y$. The lower groups show a further reduction in the linearity error by the addition of k_{xyz} and k_{yxz} , reducing η_x to 5.0° and η_y to 8.1° .

Estimation of RMS Linearity Error: Equations 40 and 41 give formulas for estimating the RMS linearity error over range of conditions. The derivation of these equations determined optimum offset coefficients for Equations 20 and 21 by minimizing the RMS error under the limiting conditions that s and m approach zero. Figure 12 shows a summary of the

Figure 12. Coefficients, as a function of sensor array lateral shift parameters and re-linearization method, for estimating RMS linearity error using Equations 40 and 41. Re-linearization offsets: (a) none; (b) constant; (c) co-dependent.

resulting RMS error coefficients for Equations 40 and 41, and how they vary with respect to d_x and d_y . Figure 12 shows the coefficients for the case where θ_y ranges over a full 360° , and θ_x ranges over $\pm 60^\circ$. Figure 12(a) shows the RMS error coefficients when no offset coefficients are used to re-linearize the transducer characteristics. Figures 12(b) and 12(c) show the RMS error coefficients when fixed and co-dependent offsets are used to re-linearize the transducer characteristics. Each graph shows eight surfaces representing the eight parameters in Equations 40 and 41, arranged in two rows (one for each equation) by four columns (one for each dependency on m_x , m_y , m_z , and one for the co-variation between m_x and m_z). Within each surface, d_x and d_y vary from 0 to 0.5, with the arrows at the base of each surface showing the direction of increasing d_x and d_y . Symmetry can be used to extrapolate to negative values of d_x and d_y . All of the surfaces have even symmetry around $d_x = 0$ and $d_y = 0$, except for the co-varying coefficients which have odd symmetry around $d_y = 0$. Figure 12 graphically illustrates the dominant RMS error components, how they change with re-linearization, and how the effectiveness of re-linearization process is degraded as the magnitude of d_x and d_y are increased. Without re-linearization offsets the dominant coefficients are proportional to m_y and m_z for η_x and m_x and m_z for η_y . The constant re-linearization offsets substantially reduce the error due to m_z , with little

Table II. Coefficients for estimating the RMS linearity error using Equations 40 and 41. Units of $\mathbf{c}_x$ and $\mathbf{c}_y$ are in degrees.			
re-linearization offsets	range of d_x and d_y	worst case values of $\mathbf{c}_x$ { c_{xx} , c_{xy} , c_{xz} , c_{xz} }	worst case values of $\mathbf{c}_y$ { c_{yx} , c_{yy} , c_{yz} , c_{yz} }
no	0	{19.5, 119., 64.4, 0.0}	{131., 27.1, 110., 0.0}
yes	0	{14.0, 9.09, 15.3, 0.0}	{8.64, 27.1, 31.9, 0.0}
no	± 0.5	{30.7, 121., 73.5, ± 10.2 }	{153., 33.3, 115., ± 39.6 }
yes	± 0.5	{25.0, 99.1, 26.7, ± 5.05 }	{97.5, 33.3, 38.0, ± 39.4 }

dependence on d_x and d_y . Co-dependent re-linearization offsets substantially reduce c_{xy} and c_{yx} , but the reduction is substantially diminished as d_x and d_y are increased. Table II gives numerical values of c_x and c_y for estimating the RMS linearity error under four conditions: with and without re-linearization offsets, and with two ranges of values of d_x and d_y . The table entries are the worst case values over the range of d_x and d_y .

Re-linearization of the transducer characteristics changes the absolute magnitude of the resolution, as will be shown in Figure 14. However, since re-linearization is a one-to-one remapping, it should not affect the ability to distinguish two adjacent points, thus the real resolution should not be affected. Figures 13(a) and 13(b) illustrate this relationship. These figures show a calculated signal-to-noise ratio (S/N) parameter for $\hat{\theta}_y$ and $\hat{\theta}_x$ for the three offset conditions shown in Figures 11(a) and 11(b). The signal was calculated as the sensitivity at each point, and the noise was taken from the span of the error ellipse. The inverse of the S/N parameter has a magnitude equal to the resolution adjusted by the sensitivity. The re-linearization does not significantly affect the signal-to-noise ratio in this example (i.e. the 90% confidence intervals of linear regression parameters include slope = 1 and intercept = 0). However, re-linearization does result in minor point-to-point changes in the signal-to-noise ratio due to inversion non-linearities.

Resolution: Figure 14 shows a summary of the resolution and non-linearities from the same configuration shown in Figures 9-11 under four re-linearization conditions, with $s = 0.15$. Figure 14(a) shows the transducer output and resolution ellipses with no re-linearization offsets. Figure 14(b) shows same output using fixed relinearization offsets. η_y is improved, but the lower left region of the graph shows characteristics that are nearing degeneracy. Figure 14(c) shows the same output adding the co-dependent offsets k_{xyz} and k_{yxz} . The linearity error is further reduced, and the unstable characteristics in the lower left region are improved. Figure 14(d) reduces the linearity error of Figure 14(c) to zero using interpolation

Figure 13. Comparison between the x and y signal-to-noise ratios with and without re-linearization offsets.

in an inverse look-up table. The error ellipses represent the resolution after correcting for the linearity errors, and can be directly compared to the inverse of the S/N parameter shown in Figures 14(c) and 14(d). The ellipses in Figure 14(d) are closely related to the multiplicative combination of the ellipses in Figure 7(b) and the inverse of $\hat{u}$, shown in Figure 10(i) and 10(j).

Numerical versus Analytical Calculation: Figure 15 compares the transducer linearity characteristics calculated using numerical simulations with the characteristics calculated using the analytical expressions derived for the limiting case where s approaches zero. The Figure compares the transducer output both with and without re-linearization offsets. The graphs illustrate that when s is fairly small, the dominant linearity errors are due to $\mathbf{m}$ and $\mathbf{d}$, not s . In this example, the RMS differences between the two methods is 1.6° along $\hat{\theta}_x$ and 2.6° along $\hat{\theta}_y$ when $s = 0.15$, and are approximately proportional to s . So, although the resolution depends strongly on s , the linearity error is relatively independent of s . All figures other than Figure 15 show linearity for the limiting case where s approaches zero, but are reasonable estimates for all configurations where s is fairly small.

Figure 14. The effects of re-linearization method on the angular transducer output and resolution ellipses.

Figure 15. Comparison between angular transducer outputs calculated using numerical simulation and approximate closed-form analytical expressions for a typical configuration with and without re-linearization offsets.

Comparative Examples of Transducer Output: Figure 16 shows examples of the transducer non-linearities and resolution for a number of different configurations. Figure 16(a) shows the case where the magnet is at the center of rotation. Figures 16(b) - 16(d) show the cases where m_x , m_y , and m_z are individually set to 0.1. Figures 16(e) - 16(g) show the cases where m_x , m_y , and m_z are all set to 0.1, and d_x and d_y are varied between ± 0.2 . Figure 16(h) shows the case where m_x , m_y , and m_z are all set to 0.2. Appendix D and E give examples of transducer characteristics for a variety of configurations, and includes a complete set of graphs for each of these examples.

Figure 16. Angular transducer output and resolution ellipses for various configurations. (a) magnet at center of rotation. (b-d) magnet offset equal to 0.1 normalized units in the x , y , or z directions.

Figure 16 (continuation). Angular transducer output and resolution ellipses for various configurations. (e-g) magnet offset equal to 0.173 normalized units with equal components in the x , y , and z directions for three different lateral sensor array offsets. (h) magnet offset equal to 0.346 normalized units with equal components in the x , y , and z directions and no lateral sensor array offset.

Discussion

The first objective of this study was to develop mathematical modeling techniques to describe the transfer characteristics of joint angle transducers surgically implanted in two-degree-of-freedom joints of different geometric configurations. The techniques modeled the voltages from a Hall effect sensor array and magnet implanted in a joint using a far-field expression for the field produced by a magnet, an equation for the voltage produced by a Hall effect sensor at an arbitrary point, and a matrix transformation technique for describing the coordinates of the point. When the magnet is located at the center of rotation of the joint, the resulting equations show that the sensor voltages are a linear combination of three elements, defined as $\mathbf{u}$, that are sin and cosine functions of the two joint angles. In this case, a linear matrix inversion of the weighting coefficients from three sensors provides inverse weighting coefficients that can be used to calculate the elements of $\mathbf{u}$ from the three sensor voltages, provided the matrix is non-singular. The elements of $\mathbf{u}$ can be transformed into angular measures of the two joint angles. For the case where the magnet is not at the center of rotation, non-linearities are introduced that are roughly proportional to the distance of the magnet from the center of rotation. Inversion techniques applicable to this situation were developed, using the linear techniques as a starting point, with some adaptations to reduce the linearity errors. A mathematically simple inversion technique was utilized so that the algorithms would be feasible to execute in real-time on a small micro-controller based system, after the initial set-up and calibration.

The equations and techniques resulting from the mathematical modeling were used to develop software programs for analyzing different transducer configurations, and for calculating the fixed parameters required for the transducer inversion calculations. The transducer linearity and resolution, and how they are affected by the primary geometry parameters were described and illustrated for a number of examples using the programs

developed. The analysis and presentation of the functional bench test and *in-vivo* data in Chapter II also utilized these programs. In Chapter II, actual transducer data and simulations of the transducer characteristics were presented together for comparison.

The equations developed were also used to derive analytical expressions for estimating transducer characteristics. If a particular configuration falls outside the scope of these expressions, numerical simulation techniques can be used to predict the transducer characteristics for that configuration. Table III summarizes the relative affect that different transducer parameters have on the linearity error, resolution, and immunity to other errors. Geometry parameters were normalized by the RMS average distance of each sensor from the center of rotation, since the transducer characteristics depend only on relative sizes and distances. The effects of the RMS average distance, D , are accounted for in that m , d , s , and r are normalized by D .

Conclusions

- Mathematical modeling techniques were developed to describe the transfer characteristics of the an implantable joint angle transducer based on a permanent magnet and an array of Hall effect sensors.
 - The transducer linearity, resolution, and sensitivity to errors were described along with their relationships to joint geometry, transducer design parameters, and surgical placement of the transducer components
 - Tools were developed for estimating transducer performance prior to surgical implantation, and for calibration and evaluation of transducer performance after implantation.
-

Table III. Summary of the effects of various parameters on transducer linearity error, resolution, and immunity to other errors. Table values are the approximate power to which parameters are raised in relationships, except as noted.

Parameter	Symbol and direction to improve	The relative effect on:		
		linearity error	resolution	error immunity
magnet offset from center of rotation	$\mathbf{m} \downarrow$	1	4*	4*
lateral shift of sensor array	d_x and $d_y \downarrow$	1†	>0	>0
separation between sensors	$s \uparrow$	<1	0 to 1	0 to 1
magnet size	$r \uparrow$	0	3	3
magnetization	$\mu_0 M_0 \uparrow$	0	1	1
sensor signal-to-noise ratio	S/N $\uparrow$	0	1	0
other sensor error coefficients	$\varepsilon \downarrow$	0	0	1
* variation over $(1 \pm m)^4$ † multiplicative proportionality with m				

Appendix 1. Full expressions for the f terms given in Equations 34-36.

$$p = \sqrt{1 + m^2 - 2d_x(\mathbf{m} \cdot \mathbf{o}_x) - 2d_y(\mathbf{m} \cdot \mathbf{o}_y) - 2d_z(\mathbf{m} \cdot \mathbf{o}_z)}$$

$$f_{S_x} = S_x \left(1 - (2m_x S_y + (\mathbf{m} \cdot \mathbf{o}_z)) / 3d_z \right) + 2m_y C_y / 3d_z$$

$$f_{C_x S_y} = C_x S_y \left(1 - (\mathbf{m} \cdot \mathbf{o}_z) / 3d_z \right) - 2m_x C_x / 3d_z$$

$$f_{C_x C_y} = \left(C_x C_y \left(m^2 / 3d_z + (\mathbf{m} \cdot \mathbf{o}_z) / 3 \right) + m_z \left(1 - (\mathbf{m} \cdot \mathbf{o}_z) / d_z \right) \right) / \left(2 - d_x^2 - d_y^2 \right)$$

$$f_1 = \left(m^2 - 2d_z(\mathbf{m} \cdot \mathbf{o}_z) + 3d_z^2(1 - p^2) \right) / \left(2 - d_x^2 - d_y^2 \right)$$

$$f_2 = \left(2m^2 - 3d_x(\mathbf{m} \cdot \mathbf{o}_x) - 3d_y(\mathbf{m} \cdot \mathbf{o}_y) - 4d_z(\mathbf{m} \cdot \mathbf{o}_z) \right) / \left(2 - d_x^2 - d_y^2 \right)$$

$$f_3 = d_x f_{C_x S_y} - d_y f_{S_x} - C_x C_y \left(d_z - 1 / 3d_z \right) + f_{C_x C_y}$$

Chapter IV

Overview of Results and Future Directions

Overview

Three main objectives were accomplished by this project:

- (1) A design concept utilizing Hall effect sensors and a magnet was developed into a packaged implantable transducer for two-degree-of-freedom joint angle sensing;
- (2) Mathematical modeling tools and techniques were developed to describe the transducer characteristics pre and post-operatively.
- (3) The functional characteristics of the transducer were demonstrated in bench tests, an *in-vivo* model, and simulations.

Device Development

The overall goal of this project was to develop an implantable joint angle transducer (IJAT) for the wrist or sterno-clavicular joint to provide command control or feedback information for a neuroprosthesis. The IJAT was designed to be connected to an implantable telemetry system that will power the sensors, and amplify, encode, and transmit the sensor signals to an external computer system. The implanted transducer would replace externally mounted transducers, and reduce or eliminate three of the biggest issues related to external transducers: cabling, attachment, and signal inconsistency.

The initial IJAT design concept was derived from inductive joystick transducers developed by Buckett[8] and Neuman[11], but replaced the alternating magnetic field and inductive pickup coils by a permanent magnetic field and Hall-effect sensors, Johnson[24,25]. Conceptually, the biological joint takes the place of the ball and socket joint in the joystick.

Two key developments strongly influenced the initial IJAT design. First, I determined that by centering the magnet at the center of rotation, a simple linear relationship between angular movement and sensor output could be obtained. Second, I determined that

three sensors was the minimum number required to detect two-degree-of-freedom movement of the magnet without loss of information, and that more than three sensors provides redundant rotational information. The following list highlights some of the design features incorporated into the resulting IJAT design.

- (1) Biocompatible, titanium sensor and magnet capsules that do not interfere with the magnetic sensing.
- (2) A magnet capsule in a threaded configuration with a slot to allow easy installation and adjustment during implantation. Thin capsule wall thickness and fine thread pitch to minimize the size of the capsule.
- (3) A strong magnet to optimize resolution. Magnetization across the diameter of the magnet to maximize the volume that will fit in a surgically drilled hole, and to optimize the orientation of the magnet.
- (4) A planar sensor array configuration for ease of fabrication, with the sensors separated far apart within the capsule for optimum resolution.
- (5) A sensor capsule configuration that orients the normal vector of the sensor plane parallel to the surface of the bone and perpendicular to the axes of joint rotation, with sensors implanted deep in the bone close to the joint capsule.
- (6) Thin sensor capsule walls to minimize size.
- (7) A linear arrangement of feedthroughs along the superficial edge of the main package, exiting perpendicular to the sensor plane and parallel to the surface of the bone so that lead wires are oriented well for subcutaneous tunneling.
- (8) A titanium shroud over the site where the lead wires are attached to the feedthroughs to provide mechanical stability and a cavity for epoxy and silicone rubber potting and strain relief.
- (9) Coiled stainless steel leadwires enclosed in silicone rubber to provide flexible but durable connections to the telemeter.
- (10) Dual conductor leadwires, to minimize the leadwire bulk.

The external circuitry used in this study served as a prototype for the design of implantable telemeter circuitry. The external circuitry utilized a pulsed current source to reduce power consumption 100 fold under anticipated conditions, and to provide correct power to the IJAT independent of lead length. The circuitry utilized a discrete op-amp implementation of a differential amplifier that amplified the sensor voltages relative to a

reference ground. The sensitivity of this design to changes in lead impedance indicated the need for a true instrumentation amplifier in the implantable telemeter design. A 12-bit A/D converter was used to provide sufficient sampling resolution together with a wide dynamic range.

Development of IJAT modeling tools

A set of modeling tools and analysis techniques were developed to provide a framework for understanding IJAT characteristics. The modeling was used to develop techniques, determine parameters, and derive formulas for estimating characteristics. Taken together, the modeling tools and techniques can be used to predict IJAT performance for specific configurations, estimate how performance is affected by various major device and configuration parameters, and analyze configurations after implantation. The modeling tools were used throughout Chapter II to analyze the characteristics of the IJATs implanted in dog joints. Chapter III and several appendices describe the details of the techniques used, show derivations of some of the results, and show a number of simulated results. Table I outlines specific tasks the modeling was used for.

In practice, the modeling tools were implemented as a set of subroutines used to carry out the procedures outlined in Table I. Throughout the development of the modeling tools and analysis of the data, the fact that the IJAT processing algorithms would need to be used for real-time processes executed by small micro-controllers was considered. Although the calibration process requires a laboratory computer for analysis, after calibration, the algorithms developed for calculating joint angles or IJAT output in rectangular coordinates are fairly rudimentary, requiring minimal computational power.

Demonstration of IJAT characteristics

The functional characteristics of the IJAT design were demonstrated in bench tests, an *in-vivo* model, and simulations. Each of these methods focused on different aspects of

Table I. Uses for modeling in analysis of IJAT characteristics.	
Use for Modeling	Method Used
To develop techniques for:	
inversion	Analytical derivations and numerical simulations
re-linearization	Analytical optimization and numerical simulations
calculating resolution	Derivation from inversion techniques
To determine:	
geometric model parameters	Computer generated images matched to X-rays
sensor scales and offsets	Fitting actual sensor voltages to simulated sensor voltages
inversion coefficients	Direct matrix inversion from geometric model parameters and sensor/magnet equations
re-linearization coefficients	Parameter fitting to minimize RMS linearity error of simulated data
To derive equations for estimating:	
linearity and resolution	Analytical derivations under restricted conditions

IJAT characterization.

Bench testing: Bench tests accomplished two goals. First, the major sources of error were characterized. This included measurements of the background noise under conditions determined by the pulsed powering approach, power-up settling time, offset sensitivity to power supply current and temperature, relative magnetic sensitivity, and long-term drift. Leadwire impedance changes were also factors affecting sensor measurement offsets in the *in-vivo* studies, but can be eliminated as a factor in future amplifier designs. The magnitude of the Earth's magnetic field was determined from the literature to ascertain its effect on sensor offsets. These are the device related factors that limit IJAT repeatability and resolution. The information provides the basis for calculating the magnitude of their effects on repeatability and resolution.

The second major goal of the bench tests was to demonstrate the two-degree-of-freedom function of the IJAT over a broader range than was possible in *in-vivo* studies. Figure 5 in Chapter II shows example results, implicitly demonstrating the validity of the IJAT model, which postulates that:

- (1) when a magnet at the center of rotation is rotated over two-degrees-of-freedom, each sensor voltage is a weighted sum of the elements of $\mathbf{u}$ and an offset, where $\mathbf{u}$ is

$$\mathbf{u} = \begin{bmatrix} -\cos(\theta_x)\sin(\theta_y) & \sin(\theta_x) & \cos(\theta_x)\cos(\theta_y) \end{bmatrix}^t$$

and θ_x and θ_y are the two angles;

- (2) the weighting coefficient matrix from three sensors can be inverted for calculating $\hat{\mathbf{u}}$, the IJAT's estimate of the joint movement in rectangular coordinates; and
- (3) $\hat{\mathbf{u}}$ can be transformed into angular estimates of θ_x and θ_y (Figure 5 – Chapter II).

In-vivo testing: The feasibility of IJAT implantation and periodic measurement techniques were demonstrated in an *in-vivo* dog model. The primary difficulty with the model was maintaining the percutaneous interface, not implantation of the IJAT. Orthopaedic surgeon M.W. Keith, M.D., performed the surgical IJAT installation using standard orthopaedic techniques under sterile conditions. The only special equipment required for the installation was a C-arm fluoroscopic imaging system that allowed visualization of the bones, surgical tools, and IJAT. This equipment is a standard operating room tool for an orthopaedic surgeon, but must be "borrowed" for animal testing.

The periodic *in-vivo* testing of the IJAT was also carried out using fluoroscopic imaging equipment. The techniques were suitable for collecting the necessary information, but off-line analysis was tedious and time consuming. Three refinements to the techniques will streamline future data collection. First, in the devices presented here, a small marker pin was placed in each bone to assist in making radiological measurements. In future implantation procedures, two to three pins placed in each bone will increase day-to-day fluoroscopic realignment accuracy and greatly speed up off-line analysis of the X-rays.

Second, a more portable computer interface will allow continuous sampling of the IJAT during X-rays, and will allow real-time display of the raw sensor voltages and processed IJAT output. Third, new equipment is now available for digitizing X-rays. The digitized images will speed up the process of generating joint model parameters from the X-rays.

The data collected from the *in-vivo* experiments were used to demonstrate IJAT function under actual conditions, and to demonstrate and validate use of the modeling techniques outlined in Table I for analyzing the results. Use of the modeling under these conditions demonstrates the validity of using modeling for predicting IJAT performance under other circumstances.

Simulations: The simulated results presented in Chapter III show examples of IJAT performance for wide variety of configurations. The simulations show:

- (1) how IJAT resolution is affected by sensor and magnet configuration parameters;
- (2) how IJAT linearity depends on the distance of the magnet from the center of rotation, with secondary effects from lateral displacement of the sensor array;
- (3) how simple correction techniques with constant coefficients can be used to improve the linearity; and
- (4) how different methods of presenting IJAT output (rectangular coordinates, spherical coordinates, linearity error, resolution ellipses, etc.) are interrelated.

The simulations presented in Chapter III cover a wide range of relatively simple configurations. The simulation data presented in Chapter II based on the *in-vivo* wrist configuration illustrates a more complicated example that includes screw axis displacement. Numerical simulations are a powerful tool that can be used to predict IJAT performance for even the most complex joint movement, provided a good description of the joint movement is available. Because of the wide variety of possible configurations, only a very small subset of examples could be presented. Just for fun, Appendix E shows some examples of simulation results that were accidentally encountered along the road.

Future Directions

Limitations of Current Design

Sensitivity of performance to geometric factors: One limitation of this IJAT design is that the performance is highly dependent on the joint configuration and implantation position.

The following design and implantation factors can be used to optimize IJAT performance.

- (1) Increasing the magnet strength and size will improve the resolution and decrease the sensitivity to all sources of error.
- (2) Improvements in sensor technology will help to improve the resolution and decrease the sensitivity to certain error sources. Improvements can be gained from a decreased temperature coefficient, and from an increased sensitivity which will decrease the relative effect of environmental electrical noise. Also, the following equation can be used to compare the signal to noise ratio of two different sensors with corrections for the power consumption and circuit bandwidth:

$$\frac{(\text{sensitivity}) \sqrt{\text{bandwidth}}}{(\text{RMS noise @ bandwidth}) (\text{supply current})}$$

Increases in this parameter will improve resolution for a given power consumption. None of these factors will improve the rejection of environmental magnetic fields.

- (3) Decreasing the sensor to magnet distance will improve resolution. To a first approximation, the z resolution is related to the distance raised to the third power, and the x and y resolution are related to the distance raised to the fourth power. This corresponds to a 100% increase in x and y resolution with a 19% increase in distance.
- (4) Increasing the separation between the sensors will improve x and y resolution proportionately with little effect on z resolution.
- (5) Decreasing the distance of the magnet from the center of rotation will improve the linearity and reduce the resolution variation.
- (6) Placing the sensor array closer to perpendicular to the line between the magnet and sensor array will improve the linearity and reduce the effects of common mode errors on the x and y IJAT output.

A careful preoperative assessment of these factors is necessary to ensure that postoperative IJAT performance will be suitable for the intended application.

Environmental interference: Environmental magnetic fields can be avoided in some situations, but cannot be eliminated completely. The current IJAT configuration does not

completely reject interference from common mode environmental magnetic fields. The addition of a fourth sensor to the IJAT would allow complete rejection of common mode environmental magnetic field. Ideally, the fourth sensor would be physically parallel to the other three, but significantly further from the magnet, which is impractical. Alternatively, a fourth sensor placed at the center of the equilateral triangle formed by the other sensors and at a slightly different distance from the substrate is a workable, alternative, configuration. Use of this type of configuration would require analysis to guarantee that the matrix equations will not degenerate for varying geometric joint models and IJAT placement. A disadvantage of the design is that one additional feedthrough would be required. However, the output of the fourth sensor would probably replace the reference ground leadwire, and the number of leadwires would be unchanged.

Feedthroughs: The sensor package feedthroughs are one of the primary limitations of the current IJAT design. The feedthrough seals are initially hermetic, but are fragile, and tend to acquire small leaks during IJAT assembly. Even if the feedthrough seals are not damaged in the fabrication, leak testing the package is not feasible after the external lead wires are attached or if a protective epoxy layer is placed around the base of the feedthroughs due to leak test contamination by the polymers, or inhibited flow of the helium tracer gas through the leak without a corresponding moisture barrier. Traeger, described some of the factors relating to moisture ingress into a package non-hermetically sealed with polymeric sealants such as silicones and epoxies[47]. He also warned against using helium leak testing where polymeric sealing is involved. In the polymerically sealed packages he tested, the times for package interiors to reach 50% of the exterior humidity were given in hours. The small pin-hole types of leaks that occur with the IJAT feedthroughs would increase the leakage times substantially, but cannot be calculated accurately. Using methods similar to Traeger's, the time constant for moisture ingress into an IJAT through a slightly leaky feedthrough

surrounded by an epoxy with a typical permeability and immersed in water at room temperature was: $\tau = (33.5 \text{ days}) / (\text{leak radius in } \mu\text{m})$

The difficulty with the formula is estimating the effective leak dimension.

The appropriate solution to the feedthrough issue is to pursue fabrication of a more robust feedthrough. Various alternative package designs have been reconsidered. Some of the options are:

- (1) Reducing the number of feedthroughs. This would only be possible by reducing the number of sensors or including additional circuitry inside the device. It would be possible to use a single sensor for a one-degree-of-freedom joint with an accessible center of rotation. However, reducing the number of feedthroughs while retaining three sensors would require additional circuitry with fairly complex logic to interpret incoming serial commands, and analog switching or data encoding capabilities. Without getting into very expensive custom circuitry, the circuitry would not fit into the existing package size, and the purpose of reducing the number of feedthroughs would be meaningless.
- (2) Using gold-brazed alumina ceramic feedthrough technology similar to the feedthroughs using in the CWRU/BioControl implant stimulators [45]. This feedthrough technology provides a very robust feedthrough, but existing technology would require considerable IJAT size increases in multiple dimensions.
- (3) Package reconfiguration. These include a) a staggered rather than linear arrangement of feedthrough pins, b) pins along the package edge, rather than the edge of the face, and c) pins distributed across the package face. All of these reconfiguration options increase at least one of the package's dimensions. They also tend to introduce new technical issues related to attaching the circuit to the feedthrough pins. Options a) and c) also introduce the issues of re-routing the leadwires outside the main capsule so they exit the package perpendicular to the sensor plane near the edge of the package.
- (4) A ceramic package. This option has the advantage of potentially allowing integration of the circuitry directly onto the inside of the package, and allowing brazing of the feedthrough pins directly into the package. This option would require considerable development cost, and would utilize biomaterials that are not as well characterized as titanium.
- (5) A titanium package, with machined holes, and feedthrough pins sealed into the holes using a glass or ceramic seal. This approach was utilized in the current IJAT design.

The conclusion is that using the style of feedthrough in option five, and maintaining the overall shape and configuration of the current IJAT sensor package is the best suited to

keeping the package as small as possible. Several manufacturers have been identified as potential sources for biocompatible feedthroughs of this variety.

Considerations for Clinical Implementation

Implantation related issues: The long-term viability of the implant-tissue interface is important to the long-term clinical success of a device such as the one presented here. The IJAT application is somewhat different than most orthopaedic implants in several respects. First, most orthopaedic implants are stress bearing. The IJAT does transmit some loading forces through its bulk, but the primary load transmission is through the bone surrounding the device. Certain types of cosmetic facial implants are similar to the IJAT in this respect. Second, most orthopaedic implants either replace the joint capsule, or are located remotely from the joint capsule. Conversely, in IJAT applications, the magnet and sensor packages must be implanted very close to the joint capsule for optimum resolution, but trauma that will damage the integrity of the joint surfaces cannot be risked. Issues are unanswered regarding maximum magnet size that can be implanted in the lunate or how much of the lunate's blood supply can be disrupted without causing avascular necrosis. The third difference between the IJAT and other orthopaedic implants is that the IJAT has leadwires, unlike other orthopaedic implants. Leadwires require a transition from hard tissue biomaterials to soft tissue biomaterials. Fortunately, the magnet capsule does not require leadwires.

Albrektsson, Carlsson, Kasemo, and their co-workers have published a number of articles related to osseointegration of titanium dental implants [1,2,3,9,10,29]. The IJAT magnet capsule is very similar in many respects to these dental implants. The sensor capsule is somewhat different in configuration, but many of their recommendations apply. The authors have outlined certain pre-requisites for establishing osseointegration — the direct contact between living bone and implant at a light microscope level. They are:

- (1) Implant material – non-alloyed titanium is best known biomaterial according to the authors;
- (2) Implant design (macrostructure) – an exact surgical fit between bone and implant is most important – a threaded cylindrical design allows a close intra-operative fit, has an increased surface area relative to a smooth cylinder, and provides inherent mechanical inter-locking;
- (3) Implant surface (microstructure) – rough surfaces have generally been shown to be better than smooth surfaces – clean, "high surface energy", uncontaminated, metal is probably the most important factor;
- (4) Healthy bone tissue;
- (5) Delicate surgical technique, that does "minimal tissue violence;" and
- (6) No loading of the implant until osseointegration has been achieved (6 weeks in rabbits, 3-4 months in humans).

Package refinements: In the process of re-fabrication of the IJAT feedthroughs, there are some package refinements that will assist in better achieving some of the Albrektsson's prerequisites for osseointegration. The primary modification planned for the sensor package is to change the cross-sectional profile to an oval. The package would have a semi-circular, single radii, edge along all bone interface regions, with flat surfaces along the sensor array plane. This will allow drilling of single radii holes in the bone at the two edges of the IJAT using simple alignment jigs. This should simplify the surgical fitting of the sensor capsule over the rectangular cavity with rounded corners required for the present IJAT design. In addition, we should investigate surface preparation, cleaning, sterile packaging, and sterilization methods appropriate for this device.

Suitability of package for other joints: In applying the IJAT to joints other than the wrist or sterno-clavicular joint two factors must be considered. First is the size of the joint, and second is the appropriateness of the package shape. With current packaging technology, the IJAT is too large for finger joints. However, a small single sensor package might be feasible for one-degree-of-freedom sensing. In larger joints, such as the gleno-humeral or various lower extremity joints, the current IJAT components may be too small to provide

acceptable resolution. Increasing the magnet size might be sufficient for these joints, but would need to be assessed.

The shape of the sensor array package is the second factor to consider in determining the suitability of the IJAT for other joints. The current L-shaped design was intended to have the leadwires running parallel to the surface of the bone. The elbow is an example of a joint where the L-shaped package is not particularly suitable. Here, leadwires exiting parallel to the sensor plane would be more appropriate. On the other hand, the threaded magnet capsule with the magnetization across the diameter is nearly universally appropriate for different joints, since the direction of surgical access is usually perpendicular to the direction toward the sensors. The primary difference among magnet capsules for different joints would be their size.

Leadwire issues: Leadwires and connectors are areas where new technology would be useful, not only for an IJAT, but also for stimulators and other sensing technologies. As the number of leadwires increases, the need for more conductors within a lead, and a simple, small, robust, multi-conductor connector may become imperative. A related issue is the need for different lead lengths for different patients or applications. If an IJAT and a stimulator-telemetry system are both manufactured with fixed length leadwires, then extension leads would be required for joining the two devices. This makes multi-conductor leadwires and connectors even more attractive. Finally, alternative materials to stainless-steel, such as MP35N, could be investigated to reduce the leadwire impedance. The high impedance of stainless-steel substantially increases the energy required to power an IJAT.

Neuroprosthesis system: A final clinically related issue is incorporation of the IJAT into a portable user based neuroprosthesis. The biggest change relative to the current implant stimulator based neuroprosthetic hand system is the requirement of an implanted telemetry system. A second issue relates to incorporation of algorithms for processing the IJAT

signals in a micro-controller based system. To setup and calibrate an IJAT after implantation, a laboratory computer will be required to perform simulation and modeling functions. However, after the calibration is complete, the mathematics required to convert the sensor signals into joint angle information requires primarily addition, multiplication, and division, with one square-root operation, and two optional arc tangent operations per sample. The algorithm could be implemented on the MC6805 micro-controller of the existing portable neuroprosthetic system, but would be cumbersome. An MC68HC11 micro-controller should handle the inversion algorithm with ease. One possible portable system configuration would include an MC68HC11 micro-controller to control stimulation and telemetry functions and to pre-process the IJAT signal for the main neuroprosthesis micro-controller.

Additional studies

Longer term in-vivo studies: Prior to human implantation, longer term chronic *in-vivo* studies of the IJAT will be required. The two main goals of these studies would be to investigate the repeatability of the IJAT characteristics, and to do a complete histological investigation of the implant/tissue interface. The studies would be carried out in conjunction with the implantable stimulator-telemeter (IST) that has been developed in our facilities. A prototype IST is scheduled to be implanted with an IJAT of the current design in the near future. Streamlined methods for determining joint geometry model parameters from X-rays will be utilized in this and future studies. The tools and techniques developed in this study will be used to analyze the IJAT linearity, resolution, and repeatability over sessions. A portable computer interface will allow continuous data streams to be collected during X-ray sessions. The fully implanted IJAT and telemetry components of the system will eliminate the technical difficulties of maintaining a percutaneous interface, allowing collection of repeatability data over a longer period.

The studies would also include histological investigation of the implant/tissue interface, and non-invasive assessment of the stability of the IJAT components in the bones over a longer period of time. In study presented here, the primary assessment of device stability was made from X-ray measurements. However, a preliminary histological evaluation was carried out on the second dog implanted. The bones, surrounding tissue, and implant components were harvested 44 days after implantation, fixed and dehydrated in ethanol, embedded in methylmethacrylate, cut into 100-400 μ m thick sections using a diamond wafering saw, and surface stained with toluidine blue for assessing bone formation. The tissue showed minimal cellular response, with areas of new bone formation adjacent to the metal and loose connective tissue filling areas void of bone. The areas void of bone indicate the need for improving the implant shape and implantation tools. The areas of new bone adjacent to the titanium and the minimal cellular response showed the good general biocompatibility of the device. The studies to test the tissue response should utilize a modified IJAT package with improved shape profile, and should be run for at least six months using a sufficient number of animals to demonstrate confidence in the results.

Human studies: Assessment of the kinematics of human wrist and sterno-clavicular joint movement would be required prior to human implantation. Some information about wrist kinematics has been published in the literature. Much of the literature focuses on the movement of wrist as a whole, rather than on the movement of individual carpal bones. Berger, et al. have published information about the three-dimensional rotational behaviors of the carpal bones in human cadavers. Their paper describes the complex rotational motions of the carpal bones, including screw displacement axes orientations that are not aligned with the primary axes of motions, and vary over the range of motion. Their publication does not contain sufficiently detailed information to model the characteristics of an IJAT, but the authors may have additional information available for more detailed study.

My preliminary observations of human cadaver wrists are consistent with Berger's findings. I have found no publications describing the detailed kinematics of the sterno-clavicular joint.

An alternative method for acquiring human wrist and sterno-clavicular joint movement information would involve MRI studies of normals. These studies would utilize similar techniques to those used in the *in-vivo* animal studies described in Chapter II, matching computer generated model data to measured MRI data. In the wrist, studies would focus on the radius, third metacarpal, and candidate magnet implant sites, initially the lunate.

Concluding Summary

This dissertation presents the design of an implantable joint angle transducer that was developed for chronic sensing of two-degree-of-freedom joint movement. *In-vivo* testing was used to demonstrate the feasibility of implantation and periodic measurement techniques. Modeling techniques were developed for analyzing and predicting IJAT functional characteristics. The modeling techniques were validated using data from functional bench tests and *in-vivo* experiments. Simulations were used to show how IJAT function depends on configuration parameters. Together, the device, modeling, and experimental verification provide important tools for proceeding toward the final goal, a two-degree-of-freedom implantable joint angle transducer suitable for acquiring command control or feedback information in disabled individuals.

Future directions of this work include refinement of the IJAT package, integration with telemetry and neuroprosthetic systems, longer term *in-vivo* studies to assess IJAT repeatability and the device-tissue interface, and assessment of the function that can be anticipated from implantation of the joint angle transducer in the human wrist and sterno-clavicular joints.

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Appendices

Appendix A: Magnetic Properties of Materials and Estimation of the Magnetization of a Particular Magnet

Definitions of Magnetic Field Related Vectors: The three primary magnetic field related vectors, **B**, **H**, and **M**, are used by widely varying names in textbooks, literature and industry. The following defines the conventions used here. The **magnetic field**, **B**, which is generally considered to be the primary magnetic field related vector, is defined by the force, **F**, generated on a particle of charge q , moving at velocity **v** through the field:

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B}$$

The magnetic field can be generated by electric currents or permanent magnets, and can be measured using Hall effect sensors or from the forces acting on current carrying loops of wire, among other techniques. The **magnetizing field**, **H**, is the driving force that produces a **B** field in the absence of magnetic materials, and polarizes or depolarizes magnetic materials when present. **H** fields are either induced using an electromagnet, or remain as an residual demagnetizing field in a magnetic material that has been removed from an applied magnetizing field. The **magnetization**, $\mu_0\mathbf{M}$, is the polarization induced in a magnetic material due to a presently or previously applied magnetizing field. μ_0 is the permeability constant, relating the units of the **H** and **M** fields to the units of the **B** field. Inside a magnetic material:

$$\mathbf{B} = \mu_0\mathbf{H} + \mu_0\mathbf{M}.$$

When dealing with permanent magnets, this form is preferable to combining **H** and **M** because it keeps the magnetizing field distinct from the magnetization. In free-space, outside any magnetic material, $\mu_0\mathbf{M} = 0$. Hall effect sensors sense the **B** fields induced by magnetized materials and magnetizing fields. A listing of some magnetic field related names, symbols, and conversions between the CGS and SI unit systems are given in Table I.

Table I. Magnetic Units			
Description	symbol	CGS Units	SI Units
magnetic flux	ϕ	10^8 maxwell	1 weber(Wb)
magnetomotive force	$\mathcal{F}$	0.4π gilbert(Gb)	1 amp(A)
permeability constant	μ_0	1 G/Oe	$4\pi 10^{-7}$ Wb/Am
magnetic field, flux density, or induction	B	10^4 gauss(G) or maxwell/cm ²	1 tesla(T) or Wb/m ²
magnetizing field or magnetic field strength or intensity	H	$0.4\pi/100$ oersted(Oe) or 1/100 (A/cm)	1 A/m
magnetization	$\mu_0 \mathbf{M}$	10^4 gauss(G)	1 tesla(T)

Magnetic Properties of Materials: The permeability of a material is defined as the change in **B** divided by the change in **H**. When a magnetizing field, **H**, is applied to a high permeability (ferromagnetic) material such as iron, the atoms of the material are readily aligned with the field, producing a magnetization, $\mu_0 \mathbf{M}$, orders-of-magnitude larger than the **H** field, with a resulting magnetic field, **B**, that is also orders-of-magnitude larger than would be produced by the **H** field alone. However, these materials are also readily demagnetized, making them unsuitable for permanent magnets.

The strongest modern permanent magnet materials actually have low recoil permeability (approximately $1.1\mu_0$) at operating temperatures, which is an indication of their strong tendency to resist changes in their magnetization. These ferromagnetic materials must be magnetized above their Curie temperature (typically 300-750°C) and cooled with the magnetizing field still applied, producing an atomic structure that has a large residual magnetization, but is stable and resistant to demagnetization at temperatures up to at least 100°C.

In paramagnetic materials, the magnitude of $\mu_0 \mathbf{M}$ is proportional to, but orders-of-magnitude smaller than $\mu_0 \mathbf{H}$, and returns to zero when the magnetizing field is removed.

In most applications, paramagnetic materials have a negligible effect on a magnetic circuit. Titanium, a paramagnetic material, was used to package the permanent magnet and Hall effect sensors of the implantable joint angle transducer described in Johnson, 1993c, in part, because it has a negligible effect on the magnetic field (in addition to its excellent biocompatibility).

Magnetic materials are generally characterized by several parameters, which are illustrated in the example demagnetization curves shown in Figure 1. When a large magnetizing field has been applied to the material, then the field is gradually reduced, the point where the internal $\mathbf{H}$ field equals zero is generally referred to as the residual induction, B_r . As the applied magnetizing field is further reduced, the internal $\mathbf{H}$ field becomes negative and the $\mathbf{B}$ field decreases. The slope of the relationship is referred to as the recoil permeability. When the applied magnetizing field is reduced to zero, the internal $\mathbf{B}$ and $\mathbf{H}$ fields reach an equilibrium point. The equilibrium point for a spherical magnet is the point on the demagnetization curve that intersects with a line passing through the origin with a slope of $-2\mu_0$ (see discussion below). If the polarity of the applied magnetizing field is reversed and the magnitude increased until the $\mathbf{B}$ field inside the magnet equals zero, a point is reached that is generally referred to as the coercive force, H_c , which is a measure of the tendency of the magnet to resist demagnetization. At some point along the $\mathbf{B}$ - $\mathbf{H}$ curve, the product $\mathbf{B} \cdot \mathbf{H}$ reaches a maximum. The magnitude of $\mathbf{B} \cdot \mathbf{H}$ at this point is generally referred to as the maximum energy product and is related to the maximum pulling force of the material. Frequently, two types of demagnetization curves are given for a magnetic material, normal and intrinsic. The intrinsic demagnetization curve is a graph of $\mu_0 \mathbf{M}$ versus $\mathbf{H}$, and the normal demagnetization curve is a graph of $\mathbf{B}$ versus $\mathbf{H}$. The two types of curves are equivalent, and one can be derived directly from the other.

Magnetization of a Permanent Magnet: The magnetization of a permanent magnet depends

on many factors, including the magnetization-demagnetization characteristics of the magnetic material, the shape of the magnet, the magnetic circuit, and the magnet's history.

There are very few magnet shapes where all three magnetic field vectors can be determined analytically both inside and outside the magnet. The ellipsoid is one such shape, of which the sphere is the simplest. The spherical magnet is useful for illustrating the relationships between the **B**, **H**, and **M** fields. A magnetic sphere has the property that the **B**, **H**, and **M** field inside the magnet are uniform when the sphere is placed in an externally applied uniform magnetizing field. If the magnetic material is not ferromagnetic then the fields will return to zero when the applied magnetizing field is removed. In ferromagnetic materials, when the applied magnetizing field returns to zero, there will be a residual magnetization, $\mu_0 \mathbf{M}$, which results in uniform non-zero **B**, **H**, and **M** fields within the sphere. The relationships between **B**, **H**, and **M** for this magnetized sphere are [43]:

$$\text{a) } \mu_0 \mathbf{M} \text{ is constant; b) } \mu_0 \mathbf{H} = -1/3 \mu_0 \mathbf{M}; \text{ and c) } \mathbf{B} = 2/3 \mu_0 \mathbf{M} \quad (1)$$

Outside the spherical magnet, $\mu_0 \mathbf{M}$ is zero; $\mathbf{B} = \mu_0 \mathbf{H}$; and **B** is described exactly by the far-field magnetic field equations. The value of $\mu_0 \mathbf{M}$, depends on the magnetizing field that was applied to the magnet and on the empirically measured demagnetization properties of the magnetic material. To determine the equilibrium position for the spherical magnet from demagnetization curves, a line passing through the origin can be placed on an intrinsic demagnetization curve with a slope of $-3\mu_0$, or placed on a normal demagnetization curve with a slope of $-2\mu_0$. The two methods are equivalent, and are established by Equations 1(a-c). The magnitude of the magnetization at the equilibrium point will be referred to as $\mu_0 M_0$, and will be used in many equations throughout this paper.

For comparison with spherical magnets, the two extremes in magnet shapes are an infinitely broad, flat, disk shaped magnet, where the load line on the $\mu_0 \mathbf{M}$ vs $\mu_0 \mathbf{H}$ curve has a slope of $-\mu_0$, and an infinitely long, thin needle shaped magnet where the slope is $-\infty$

($\mu_0 M_0 = B_r$) [43]. All other magnet shapes will fall between these extremes. For general magnet shapes, the magnetization will not have a single value, but will vary throughout the body of the material.

Figure 2 shows typical normal and intrinsic demagnetization curves for several magnet materials distributed by Permag [40]. Determination of the value of $\mu_0 M_0$ for a particular magnet material is the simplest from the intrinsic demagnetization curve, since the magnetization value read from the graph substitutes directly for $\mu_0 M_0$ in the magnetic field equations. For spherical magnets in free-space or a non-ferromagnetic environment, the magnetization can be directly determined from the demagnetization curve. For high strength magnetic materials such as Neodymium-Iron-Boron (NdFeB) or Cobalt-Samarium (CoSm) with a low recoil permeability, the demagnetization will be minimal. If a material such as Alnico 5, is used, the demagnetization will be substantial, with $\mu_0 \mathbf{M} = 0.16 * \mathbf{B}_r$.

For non-spherical magnet shapes, such as a cylinder, exact determination of the magnetization is not feasible because the magnetization is not uniform throughout the magnet. However, a reasonable estimate can be obtained using the spherical magnet methods under the conditions that: the magnetic material has a strong tendency to resist demagnetization (i.e. a recoil permeability near μ_0 over a wide $\mathbf{H}$ field range); and the length and diameter or width are equal (see Appendix B, "Approximate Numerical Calculation of Magnetization"). Using the more exact methods shown in Appendix B, the demagnetization was calculated for a high strength magnetic material, such as Neodymium-Iron-Boron (NdFeB) or Cobalt-Samarium (CoSm), with a recoil permeability of $1.1\mu_0$. Under these conditions, the demagnetization is minimal, and $\mu_0 \mathbf{M} \approx 0.96 * \mathbf{B}_r$. Alnico magnets are too far from ideal to rely on approximate analysis, but a cylinder of length and diameter equal to the diameter of a given sphere would have a total strength greater than the sphere, because the sphere is a subset of the volume of the cylinder.

Figure 1. Example demagnetization curve showing the definitions of various magnet parameters.

Figure 2. Normal and intrinsic demagnetization curves for several magnet materials available from Permag [40].

Appendix B: Some Derivations Related to Magnetization and Magnetic Fields in the Near-Field Region of a Magnet:

This appendix give a few Mathematica derivations for:

- the magnetization of a magnet with different recoil permeabilities;
- shows near fields for magnets of equal volumes, but different aspect ratios; and
- shows how the field just outside a cylindrical magnet converges to the $1/\text{distance}^3$ relationship;

A little bit of imagination must be used to separate the three lines that represent the **B**, **H**, and **M** fields in the first group of graphs. The magnet has extends from -1 to 1. The magnetization, **M**, is the highest curve bewteen ± 1 , but zero elsewhere. The **H** field is negative between ± 1 , and equal to the **B** field elsewhere. The **B** field varies with no discontinuities between the inside and the outside of the magnet.

(* axial B field along central axis of cylindrical magnet *)

$B[z, a, l] := ((z + l/2)/\text{Sqrt}[a^2 + (z + l/2)^2] - (z - l/2)/\text{Sqrt}[a^2 + (z - l/2)^2])/2$

(* magnetization is 1 inside, 0 outside magnet *)

$M[z, l] := \text{If}[\text{Abs}[z] \leq l/2, 1, 0]$

(* B, M, & H fields for ideal cylindrical magnet *)

$\text{Plot}\{B[z, 1, 2], M[z, 2], B[z, 1, 2] - M[z, 2]\}, \{z, -4, 4\}$

(* B, M, & H fields for ideal spherical magnet*)

$Bs[z, a, l] = \text{If}[\text{Abs}[z] \leq l/2, 2/3 * a^2 * l/2, 2/3 * a^2 * l/2 / \text{Abs}[z]^3]$

$\text{Plot}\{Bs[z, 1, 2], M[z, 2], Bs[z, 1, 2] - M[z, 2]\}, \{z, -4, 4\}$

(* average magnetization of cylindrical magnet for different recoil permeabilities (1+1, 1+1/2, 1+1/10, 1+1/20) based on B field along central axis *

```
Print [coeffs = {Expand[Integrate[1-1+B[z,1,2],{z,-1,1}]],
Expand[Integrate[1-1/2+B[z,1,2]/2,{z,-1,1}]],
Expand[Integrate[1-1/10+B[z,1,2]/10,{z,-1,1}]],
Expand[Integrate[1-1/20+B[z,1,2]/20,{z,-1,1}]]]}
Print [N[coeffs/2]]
```

```

      1      Sqrt[5]  17      Sqrt[5]  37      Sqrt[5]
{-1 + Sqrt[5], - + -----, -- + -----, -- + -----}
      2          2    10      10      20      20
{0.618034, 0.809017, 0.961803, 0.980902}
```

(* approximation to B, M, & H fields for cylindrical magnet with recoil permeability of

```
Plot[{B[z,1,2]*.961, M[z,2]*(.9*M[z,2]+.1*(B[z,1,2])),
B[z,1,2]*.961 - M[z,2]*(.9*M[z,2]+.1*(B[z,1,2]))}, {z, -4,4}]
```

(* approximation to B, M, & H fields for cylindrical magnet with recoil permeability of

```
Plot[{B[z,1,2]*.809, M[z,2]*(.5*M[z,2]+.5*(B[z,1,2])),
B[z,1,2]*.809 - M[z,2]*(.5*M[z,2]+.5*(B[z,1,2]))}, {z, -4,4}]
```

(* approximation to B, M, & H fields for cylindrical magnet with recoil permeability of
 Plot[{B[z,1,2]*.618, M[z,2]*B[z,1,2],
 B[z,1,2]*.618 - M[z,2]*B[z,1,2]}, {z, -4,4}]

(* Integrals of the B field over different regions inside and outside the magnet *)

iB = Integrate[B[z,a,2a],z]

(-(a^2 + (-a + z)^2)^(1/2) + (a^2 + (a + z)^2)^(1/2))/2

bmo = Collect[Together[(iB/.z->a) - Limit[iB/.z->z1, z1->Infinity]],a]

(a*(3 - 5^(1/2)))/2

bmi = Factor[(iB/.z->0) - (iB/.z->a)]

(a*(-1 + 5^(1/2)))/2

bpi = Factor[(iB/.z->a) - (iB/.z->0)]

(a*(-1 + 5^(1/2)))/2

bpo = Collect[Together[Limit[iB,z->Infinity] - (iB/.z->a)],a]

(a*(3 - 5^(1/2)))/2

Expand[bmo + bmi + bpi + bpo] (* Integral over all z *)

2*a

hpo=bpo (* H field outside magnet *)

(a*(3 - 5^(1/2)))/2

hpi=Collect[Distribute[Together[bpi - Integrate[1, {z, 0, a}]]],a] (* H field inside magne

a*(-3/2 + 5^(1/2)/2)

Expand[hpi+hpo] (*the sum of H inside and out should equal zero *)

0

(* Log-log plot of B field vs distance of 3 cyln. magnets of equal volume, but having 3 different length/diameter ratios of 1/4, 2, & 16. Long magnet at top, short at bottom.
 Plot[{{Log[10,B[10^z,4,1]],Log[10,B[10^z,2,4]],Log[10,B[10^z,1,16]]},{z,0,2},
 PlotRange->All]

(*Detail showing how axial field drops off relative to $1/z^3$ outside a cylindrical magn
 Plot[{{3/2*B[z,(2/3)^(1/3)],(2/3)^(1/3)*2]*z^3},{z,0,8},
 PlotRange->{{0,8},{.975,1.025}}]

(* Full definition of H field based on elliptic integrals *)

Clear[Hz,Hr,Phi]

Phi[z_,r_]:=4r/((1+r)^2+z^2)

Hz[z_,r_]:= 1/2/Pi*(EllipticK[Phi[z,r]] + (EllipticE[Phi[z,r]]*(1 - r^2 - z^2)/
 (z^2 + (1 - r)^2))/(z^2 + (1 + r)^2)^(1/2)

Hr[z_,r_]:= 1/2/Pi*z/r*(-EllipticK[Phi[z,r]] + (EllipticE[Phi[z,r]]*
 (1 + r^2 + z^2))/(z^2 + (1 - r)^2))/(z^2 + (1 + r)^2)^(1/2)

Appendix C: Near-Field Magnetic Equations

Magnetic Field Equations: The following is a general expression (adapted from Montgomery, 1969) for the magnetic field, $\mathbf{B}$, from a cylindrical magnet magnetized along its axis (the z axis), which is obtained by integrating the contributions to the magnetic field, $\partial\mathbf{B}$, from a stack of infinitesimally thin magnetic disks:

$$\mathbf{B} = \int_{p_z-h_m}^{p_z+h_m} \frac{\partial\mathbf{B}}{\partial z} dz \quad (1)$$

where

$$\partial\mathbf{B} = \begin{bmatrix} \partial B_x \\ \partial B_y \\ \partial B_z \end{bmatrix} = \frac{\mu_0 M_0}{\pi} \begin{bmatrix} \left(\frac{p_x z / p_r^2}{\sqrt{(r_m + p_r)^2 + z^2}} \right) \left(-K(\psi) + \frac{r_m^2 + p_r^2 + z^2}{(r_m - p_r)^2 + z^2} E(\psi) \right) \\ \left(\frac{p_y z / p_r^2}{\sqrt{(r_m + p_r)^2 + z^2}} \right) \left(-K(\psi) + \frac{r_m^2 + p_r^2 + z^2}{(r_m - p_r)^2 + z^2} E(\psi) \right) \\ \left(\frac{1}{\sqrt{(r_m + p_r)^2 + z^2}} \right) \left(K(\psi) + \frac{r_m^2 - p_r^2 - z^2}{(r_m - p_r)^2 + z^2} E(\psi) \right) \end{bmatrix} \partial z \quad (2)$$

and μ_0 is the permeability constant;

$\mu_0 M_0$ is the magnitude of the magnetization of the magnetic material;

r_m and h_m are the radius and one half the length of the magnet;

$\{p_x, p_y, p_z\} = \mathbf{p}$ is the position of a point in space relative to the center of the magnet;

$$p_r = \sqrt{p_x^2 + p_y^2};$$

z , the variable of integration, is used to integrate over the length of the magnet;

and $K(\psi)$ and $E(\psi)$ are the complete elliptic integrals of the first and second kind,

$$\text{where } \sin(\psi) = 2p_r / \sqrt{(r_m + p_r)^2 + p_z^2}.$$

The equation has no general closed form solution, but can be calculated numerically, although computationally intensive. However, the equations converge rapidly to the much simpler far-field relationship at distances on the order of two to three times the magnet's radius and half-length [Appendix B].

Appendix D: Examples of Transducer Characteristics for a Variety of Configurations

This appendix gives the set of graphs equivalent to Figures 9, 10, 11, 13, and 14 in Chapter III for the eight configurations given in Figure 16. Explanations of the graphs are the same as those given in Chapter III.

Table of page number for Appendix D.								
Mechanical parameters					Figure number in Chapter III			
m_x	m_y	m_z	d_x	d_y	9	10	11 & 13	14
0.0	0.0	0.0	0.0	0.0	138	146	154	162
0.1	0.0	0.0	0.0	0.0	139	147	155	163
0.0	0.1	0.0	0.0	0.0	140	148	156	164
0.0	0.0	0.1	0.0	0.0	141	149	157	165
0.1	0.1	0.1	-0.2	-0.2	142	150	158	166
0.1	0.1	0.1	0.0	0.0	143	151	159	167
0.1	0.1	0.1	0.2	0.2	144	152	160	168
0.2	0.2	0.2	0.0	0.0	145	153	161	169

Transducer output from
analytical equations for
case when $s \rightarrow 0$.

Configuration Parameters:

$$\mathbf{m} = \{0.1, 0, 0\}$$

$$\mathbf{d} = \{0, 0, 1\}$$

Offset Coefficients:

$$\mathbf{k}_x = \{-0.055 - 0.000\hat{w}_z, -0.000, 0.002\}$$

$$\mathbf{k}_y = \{0.033 - 0.426|\hat{w}_z|, -0.000\}$$

$$\mathbf{k}_x = \{-0.113 - 0.323\hat{w}_z, -0.047, 0.254\}$$

$$\mathbf{k}_y = \{0.049 - 0.489|\hat{w}_z|, 0.151\}$$

Transducer output from
analytical equations for
case when $s \rightarrow 0$.

Configuration Parameters:

$$\mathbf{m} = \{0.1, 0.1, 0.1\}$$

$$\mathbf{d} = \{0, 0, 1\}$$

Offset Coefficients:

$$\mathbf{k}_x = \{-0.068 - 0.379\hat{w}_z, -0.023, 0.272\}$$

$$\mathbf{k}_y = \{0.002 - 0.342|\hat{w}_z|, 0.203\}$$

Transducer output from
analytical equations for
case when $s \rightarrow 0$.

Configuration Parameters:

$$\mathbf{m} = \{0.1, 0.1, 0.1\}$$

$$\mathbf{d} = \{0.2, 0.2, 0.9592\}$$

Offset Coefficients:

$$\mathbf{k}_x = \{0.046 - 0.412\hat{w}_z, 0.026, 0.218\}$$

$$\mathbf{k}_y = \{-0.030 - 0.185|\hat{w}_z|, 0.241\}$$

Transducer output from analytical equations for case when $s \rightarrow 0$.

Configuration Parameters:

$$\mathbf{m} = \{0.2, 0.2, 0.2\}$$

$$\mathbf{d} = \{0, 0, 1\}$$

Offset Coefficients:

$$\mathbf{k}_x = \{-0.190 - 0.730\hat{w}_z, -0.093, 0.497\}$$

$$\mathbf{k}_y = \{0.023 - 0.565|\hat{w}_z|, 0.406\}$$

Comparison of the effects of offsets on linearity and relative noise

Comparison of the effects of offsets on linearity and relative noise

Comparison of the effects of offsets on linearity and relative noise

Comparison of the effects of offsets on linearity and relative noise

Comparison of the effects of offsets on linearity and relative noise

Comparison of the effects of offsets on linearity and relative noise

Comparison of the effects of offsets on linearity and relative noise

Comparison of the effects of offsets on linearity and relative noise

Comparison of the effects of offsets on linearity (grid) and relative resolution (ellipses)

$\mathbf{m} = \{0, 0, 0\}$
 $d_x = 0, d_y = 0$
 $s = 0.15, \text{ noise} = 0.01$

Comparison of the effects of offsets on linearity (grid) and relative resolution (ellipses)

Comparison of the effects of offsets on linearity (grid) and relative resolution (ellipses)

$m = \{0, 0.1, 0\}$
 $d_x = 0, d_y = 0$
 $s=0.15, \text{ noise}=0.01$

Comparison of the effects of offsets on linearity (grid) and relative resolution (ellipses)

$m = \{0, 0, 0.1\}$
 $d_x = 0, d_y = 0$
 $s = 0.15, \text{noise} = 0.01$

Comparison of the effects of offsets on linearity (grid) and relative resolution (ellipses)

Comparison of the effects of offsets on linearity (grid) and relative resolution (ellipses)

Comparison of the effects of offsets on linearity (grid) and relative resolution (ellipses)

Comparison of the effects of offsets on linearity (grid) and relative resolution (ellipses)

Appendix E: Examples of Transducer Characteristics Accidentally Encountered Along the Road

This appendix gives some examples of simulated transducer characteristics that were accidentally encountered, but had striking resemblances to various objects. I printed each of them at the time, and except by guessing, could not necessarily reproduce them. I suppose that makes them candidates for the Journal of Irreproducible Results. They are: a) fish, b) sea shell, c) slug, d) heart, and e) snowman. A few embellishments were added to some of the pictures.