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**Numerical simulation of a shock wave/turbulent boundary layer
interaction in a duct**

Yang, Wei-Li, Ph.D.

Case Western Reserve University, 1992

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NUMERICAL SIMULATION OF A SHOCK WAVE
/TURBULENT BOUNDARY LAYER INTERACTION IN A DUCT

by

WEI-LI YANG

Submitted in partial fulfillment of the requirements
for the Degree of Doctor of Philosophy

Thesis Advisor: Dr. Isaac Greber

Department of Mechanical and Aerospace Engineering
CASE WESTERN RESERVE UNIVERSITY

May, 1992

CASE WESTERN RESERVE UNIVERSITY

GRADUATE STUDIES

We hereby approve the thesis of

Wei-Li Yang
candidate for the Ph. D.

degree.*

Signed:

Isaac Rubin
(Chairman)

Sci. Keshotko

P. K. ...

...

Peter M. Locks

Date

10 April 1992

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W. L. Yang

NUMERICAL SIMULATION OF A SHOCK WAVE/
TURBULENT BOUNDARY LAYER INTERACTION IN A DUCT

ABSTRACT

by

WEI-LI YANG

A numerical investigation of the interaction of an incident oblique shock wave with a turbulent duct flow is presented. The investigation consisted of solving the three dimensional, unsteady, compressible, mass averaged Navier-Stokes equations. An implicit finite volume, lower-upper time marching code (RPLUS) has been employed and modified. A three dimensional Baldwin-Lomax turbulence model has been programmed in conjunction with the code.

Computed results are obtained at Mach number 2.9 for a turning angle of 12.99 degrees and Reynolds number based on duct width of 1.36×10^7 . Under various inlet conditions, the results clearly depict the flow characteristics including the shock structure, the separated flow region, the wall pressure distribution and the skin friction distribution.

The findings provide a physical understanding

of the three dimensional vortex structure of the flow in a duct in which a shock wave interacts with a turbulent boundary layer. The results show that the ratio of the boundary layer thickness to the duct width is the critical parameter in determining the separation structure. The computational results also show that the corner vortices and center separation structures are closely related. However, the non-uniformities and secondary flows in the approach boundary layer do not change the separation structure qualitatively.

ACKNOWLEDGEMENTS

I would like to express my sincere appreciation and gratitude to my advisor, Dr. Isaac Greber for his advise, guidance and supervision throughout my graduate studies at Case Western Reserve University. Special thanks are given to Dr. Peter Sockol of NASA Lewis Research Center for his support and supervision for the project.

I also want to thank my thesis committee Dr. Eli Reshotko, Dr. Y. Kamotani and Dr. Robert Mullen for taking the time to review the document and giving their remarks and suggestions.

The author wishes to express his gratitude to Dr. Fang-Pei Liang and Mr. Soo-Yong Cho of Department of Mechanical and Aerospace Engineering, Dr. Peter Tsi and Dr. Sheng-Tao Yu of Sverdrup Technology, Inc for the their suggestions in many respects in this work.

I am most grateful to my wife Qing-Ping Song for her love, support and encouragement to make this possible. Special appreciation and gratitude is given to my parents He-Xuan Yang and Guang Jun Cheng and my

sister Virginia Yang for their love and encouragement. All of this will not be possible without my uncle Yun-Hsien Yang's initial support for my education in the United States. I also like to mention my son Theodore Yang, who gives the special meaning of this effort.

Appreciation is also given to the colleagues in Joy Technologies Inc., Smith and Harris Division for their encouragement and help.

This project is supported by NASA Lewis Research Center.

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NOMENCLATURE

a	Sonic speed
A	Jacobian of x-direction flux vector
B	Jacobian of y-direction flux vector
C	Jacobian of z-direction flux vector
d	Adaptive artificial dissipation
D	Spacial difference
D^+	Forward difference
D^-	Forward difference
e	Total energy per unit mass
E	X-direction flux
F	Y-direction flux
G	Z-direction flux
J	Jacobian of coordinate transformation
k	Conductivity
k_∞	Free stream conductivity
l	Weighted length from the distances from the walls
L	Characteristic length
M	Mach number
M_∞	Mach number at free stream
p	Thermodynamic pressure

Pr	Prandtl number
Pr_t	Turbulent Prandtl number
q_x	Heat flux in x-direction
q_y	Heat flux in y-direction
q_z	Heat flux in z-direction
Re	Reynolds number
Re_w	Reynolds number based on duct width
t	Time
T	Temperature
T_w	Wall Temperature
T_∞, T_e	Free steam temperature
u	Velocity in x-direction
u_τ	Friction velocity = $(\tau_w/\rho_w)^{1/2}$
U_∞, U_e	Free stream velocity in x-direction
v	Velocity in y-direction
w	Velocity in z-direction
x, y, z	Cartesian coordinates
γ	Ratio of specific heats
$\gamma_A, \gamma_B, \gamma_C$	Upper bounds of the eigenvalues of Jacobians
δ	Boundary thickness
$\delta()$	Temporal difference
δ^*	Displacement thickness

Δ	Difference
ϵ	Eddy viscosity
$\epsilon^{(2)}$	Coefficient of second order artificial dissipation
$\epsilon^{(4)}$	Coefficient of fourth order artificial dissipation
κ	Von Karman constant = 0.4
λ, Λ	Eigenvalue of Jacobian
μ	Dynamic viscosity
μ_∞	Free stream dynamic viscosity
ν	Kinematic viscosity
ξ, η, ζ	General coordinates
ρ	Density
ρ_∞	Free stream density
$\tau_{xx}, \tau_{yy}, \tau_{zz}$	Normal stresses
$\tau_{xy}, \tau_{yz}, \tau_{zx}$	Shear stresses
ω	Magnitude of vorticity

Subscripts

e	Free stream condition
i, j, k	Tensor index
v	Viscous terms
w	Duct width
∞	Free stream Condition

CHAPTER 1

INTRODUCTION

The interaction between a shock wave and a boundary layer has been investigated for about half a century. It is still an important problem in aerodynamics today. In the design of supersonic inlets of airbreathing propulsion systems, these interactions are known to induce changes in the thickness of the boundary layer and the shape of the velocity profiles. With a sufficient pressure rise associated with the incident and reflected shocks, separation of the boundary layer may occur. These effects will result in lower inlet total pressure recovery and increased flow distortion. The physical understanding of the three dimensional flow structure is crucial in providing direction in of controlling the interactions.

The investigation of shock/boundary layer interactions began in 1940s and 50s. Liepmann, Roshko

and Dhawan[1] measured the rate of wall pressure increase near the point on a plate where the oblique shock produced by a wedge interacts with the boundary layer. The results showed the wall pressure distribution was smoothed out near the interaction region in comparison with the inviscid solution; and the pressure gradient is considerably steeper for the turbulent boundary layer than for the laminar boundary layer. Bogdonoff and Kepler [2] investigated the reflection of oblique shock waves from a flat wall. They pointed out the occurrence of separation is independent of deflection angle but dependent on the ratio of the wall pressure behind reflected shock wave and the pressure upstream of the incident shock wave . On shock wave and two dimensional laminar boundary layer interactions, Hakkinen, Greber, Trilling and Abarbanel [3] theoretically predicted the pressure levels of the separation region and the length of the separated region with good agreement with the experimental results.

Due to the development of new experimental techniques, more phenomena of the interactions

between shock waves and flat plate boundary layers have been presented and studied recently. Jurkovich [4] showed that in a wind tunnel, a strong separation vortex pair appeared at high deflection angles. The pair is symmetric about the tunnel center line and was attributed to the non-uniformity in the approach boundary layer. Systematic measurements on two-dimensional oblique shock waves interacting with turbulent flat plate boundary layers were reported in detail by Skebe, Greber and Hingst [5] and Skebe [6]. Continuing advances in computational fluid dynamics and computer capabilities have provided the possibility to solve two-dimensional, and even three-dimensional, Navier-Stokes equations. For two-dimensional flow, the MacCormack's [7] predictor-corrector method successfully solved the shock wave and boundary layer interaction problems for both laminar flows and turbulent flows with proper turbulence model. Benhachmi [8] applied MacCormack's method to the shock wave and turbulent boundary layer interaction with suction; the results showed good agreement with his experimental results.

Beam and Warming [9] developed an implicit factored scheme for solving the compressible Navier-Stokes equations called the alternating-direction implicit (ADI) algorithms. This method became one of the most popular method for its efficiency as well as its accuracy. Beam and Warming also solve the two-dimensional shock/boundary layer problem.

Despite the success of these methods in solving two-dimensional flow problems, they either required impracticably long CPU time in current computer technology or diverged in iteration when they were extended to solve three-dimensional compressible flow equations. An important reason for this difficulty is that the equation set is a mixed type of elliptic and hyperbolic equations.

In order to solve three-dimensional compressible flow, Jameson and Turkel [10] first applied the implicit LU decomposition scheme. This scheme was extended to solve a three-dimensional wave equations by Walters, Dwoyer and Hassan [11]. The scheme was shown stable for solving the three-dimensional equations. Jameson and Yoon [12]

modified the LU scheme with multiple grid and added artificial viscosity [13] into the scheme to compute unsteady Euler equations. Shuen and Yoon [14] applied the LU decomposition scheme to solve the two-dimensional chemically reacting flows. Yu and Shuen [15] developed the three-dimensional version of LU code called RPLUS for three-dimensional flows with chemical reaction.

The LU decomposition method made it possible to use current computation capability to solve rather complicated three-dimensional flows. On the other hand, scientists were working hard to understand turbulent flows and to search for the proper model for turbulence. Rubesin [16] has reviewed most of the recent developments in turbulence modeling with emphasis on compressible flows. Even though one equation and two equation models are sounder physically, the algebraic models are still being widely used because of their simplicity and their easy applicability. Cebeci, Smith and Mosinskis [17] presented an eddy-viscosity model for compressible turbulent boundary layer. The model has been applied

to variety types of flows, such as shock/turbulent boundary layer interaction with suction [8]. Baldwin and Lomax [18] modified Cebeci's model to make the scheme more efficient for numerical simulation. Visbal and Knight [19] pointed out that some constants in the Baldwin-Lomax model need to be adjusted with Mach number. Knight also carefully extended the Baldwin-Lomax turbulent model to a three-dimensional model with wall distance weighted mixing length [20, 21]. The model has been coded in conjunction with the Knight's hybrid explicit-implicit scheme and successfully applied to a three-dimensional wedge generated shock wave interacting with boundary layer and corner flows [36, 37].

The objective of this thesis is to simulate an incident shock wave interacting with the turbulent boundary layer and corner flow of a wind tunnel. In particular, the study is focused on understanding of the vortex structure in the interaction region, such as was found in Jurkovich's experiment. Efforts are made to determine the critical factors which effect

the flow structure in the separated region. The research concentrates on the physical understanding of the phenomena instead of matching the experimental data quantitatively.

An implicit finite volume, lower-upper time marching code (RPLUS) [15] which was developed in NASA Lewis Research Center, is employed. The program was modified in order to apply it to the specific problem. The three-dimensional Baldwin-Lomax turbulence model is coded in conjunction with RPLUS. In order to test the applicability of the code, a number of related two-dimensional and three-dimensional laminar and turbulent flows are tested.

CHAPTER 2

GOVERNING EQUATIONS AND BOUNDARY CONDITIONS

2.1 GOVERNING EQUATIONS

The governing equations of are a set of conservation equations for a compressible, viscous, continuous, Newtonian fluid flow. The equations are cast into the vector form in Cartesian coordinates, as follows:

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x} (E - E_v) + \frac{\partial}{\partial y} (F - F_v) + \frac{\partial}{\partial z} (G - G_v) = 0 \quad (2-1)$$

where

$$Q = \begin{pmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \\ \rho e \end{pmatrix} \quad (2-2)$$

$$E = \begin{pmatrix} \rho u \\ \rho u^2 + p \\ \rho uv \\ \rho uw \\ u(\rho e + p) \end{pmatrix} \quad (2-2)$$

$$F = \begin{pmatrix} \rho v \\ \rho uv \\ \rho v^2 + p \\ \rho vw \\ v(\rho e + p) \end{pmatrix} \quad (2-3)$$

$$G = \begin{pmatrix} \rho w \\ \rho uw \\ \rho vw \\ \rho w^2 + p \\ w(\rho e + p) \end{pmatrix} \quad (2-4)$$

$$E_v = \begin{pmatrix} 0 \\ \tau_{xx} \\ \tau_{xy} \\ \tau_{xz} \\ u\tau_{xx} + v\tau_{xy} + w\tau_{zx} - q_x \end{pmatrix} \quad (2-5)$$

$$F_v = \begin{pmatrix} 0 \\ \tau_{xy} \\ \tau_{yy} \\ \tau_{yz} \\ u\tau_{xy} + v\tau_{yy} + w\tau_{yz} - q_y \end{pmatrix} \quad (2-6)$$

$$G_v = \begin{pmatrix} 0 \\ \tau_{zx} \\ \tau_{yz} \\ \tau_{zz} \\ u\tau_{zx} + v\tau_{zy} + w\tau_{zz} - q_z \end{pmatrix} \quad (2-7)$$

In (2-1) to (2-7), e is the summation of internal energy and kinetic energy per unit mass.

τ_{xy} , τ_{yz} , τ_{zx} , τ_{xx} , τ_{yy} and τ_{zz} are shear and normal stresses, and can be expressed as

$$\tau_{xy} = \mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)$$

$$\tau_{yz} = \mu \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right)$$

$$\tau_{zx} = \mu \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right)$$

$$\tau_{xx} = 2\mu \frac{\partial u}{\partial x} + \lambda \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right)$$

$$\tau_{yy} = 2\mu \frac{\partial v}{\partial y} + \lambda \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right)$$

$$\tau_{zz} = 2\mu \frac{\partial w}{\partial z} + \lambda \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right)$$

(2-8)

Due to the lack of data for the second viscous factor λ , Stokes' hypothesis $\lambda = -2/3 \mu$ is used in the computation.

In the energy equation, heat flux q_x , q_y and q_z is expressed as

$$q_x = -k \frac{\partial T}{\partial x}$$

$$q_y = -k \frac{\partial T}{\partial y} \quad (2-9)$$

$$q_z = -k \frac{\partial T}{\partial z}$$

To close the system, the state equation obtained from the perfect gas assumption

$$p = \rho RT \quad (2-10)$$

is employed.

In case of air, it is possible to use so called Sutherland's model of viscosity to calculate viscosity. This can be written

$$\frac{\mu}{\mu_0} = \left(\frac{T}{T_0} \right)^{\frac{3}{2}} \frac{T_0 + S_1}{T + S_1} \quad (2-11)$$

where S_1 is a constant which is assumed 110 K for air; μ_0 denotes the viscosity at the reference temperature T_0 .

At temperature range of 150 K to 2500 K, equation (2-11) can be approximated by

$$\mu = 1.468 \times 10^{-6} \frac{T^{1.5}}{T + 110.6} \quad (\text{kg}/(\text{m sec})) \quad (2-12)$$

To non-dimensionalize the governing equations,

we chose the values at free stream upstream of the incident shock as reference quantities. We then obtain a set of non-dimensional equations which is similar to equation (2-1). In the dimensionless equations, μ in the momentum equations is replaced by $1/Re \mu'$ and k and μ in the energy equation are replaced by $1/(Pr Re) k'$ and $(\gamma - 1) M^2/Re \mu'$ respectively. Here k' and μ' are the dimensional conductivity and viscosity; Re is the Reynolds number

$$Re = \frac{\rho_{\infty} U_{\infty} L}{\mu_{\infty}} \quad (2-13)$$

, Pr is the Prandtl number

$$Pr = \frac{\mu_{\infty} C_p}{k_{\infty}} \quad (2-14)$$

and the Mach number is defined as

$$M = \frac{U_{\infty}}{a_{\infty}} \quad (2-15)$$

The non-dimensional equation set leads to the conclusion that the above system of equations for the density, velocity and temperature fields only depends on the three dimensionless parameters defined above

and the boundary conditions. In the temperature range of interest in this work, the variation of the Prandtl number is negligible. Therefore the solution of the dimensionless governing equation with defined boundary conditions depends solely on the Reynolds number and the Mach number.

2.2 JACOBIAN MATRICES AND THEIR EIGENVALUES

The Jacobians of the flux vectors, with the state equation of a perfect gas, are given by the following matrices

$$A = \frac{\partial E}{\partial Q} =$$

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ -u^2 + (\gamma-1)q^2/2 & -(\gamma-3)u & -(\gamma-1)v & -(\gamma-1)w & \gamma-1 \\ -uv & v & u & 0 & 0 \\ -uw & w & 0 & u & 0 \\ [-\gamma e + (\gamma-1)q^2]u & \gamma e - (\gamma-1)q^2/2 - u^2(\gamma-1) & -(\gamma-1)uv & -(\gamma-1)uw & \gamma u \end{pmatrix}$$

$$B = \frac{\partial F}{\partial Q} =$$

$$\begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ -uv & v & u & 0 & 0 \\ -v^2 + (\gamma-1)q^2/2 & -(\gamma-1)u & -(\gamma-3)v & -(\gamma-1)w & \gamma-1 \\ -vw & 0 & w & v & 0 \\ [-\gamma e + (\gamma-1)q^2]v & -(\gamma-1)uv & \gamma e - (\gamma-1)q^2/2 - v^2(\gamma-1) & -(\gamma-1)vw & \gamma v \end{pmatrix}$$

$$C = \frac{\partial G}{\partial Q} =$$

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ -uw & w & 0 & u & 0 \\ -vw & 0 & w & v & 0 \\ -w^2 + \frac{(\gamma-1)q^2}{2} & -(\gamma-1)u & -(\gamma-1)v & -(\gamma-3)w & \gamma-1 \\ [-\gamma e + \frac{(\gamma-1)q^2}{2}]w & -(\gamma-1)uw & -(\gamma-1)vw & \gamma e - \frac{(\gamma-1)q^2}{2} - w^2(\gamma-1) & \gamma w \end{pmatrix}$$

(2-16)

where $q = u^2 + v^2 + w^2$.

To analyze the stability of a numerical scheme, we often need to know the eigenvalues of the Jacobians shown above. In order to obtain the eigenvalues of the Jacobians matrices, we may build a simpler form called the inviscid gas-dynamic equations in nonconservative form [22] as a set of transformed linearized Euler equations. The nonconservative equations can be written as:

$$\frac{\partial \bar{Q}}{\partial t} + \bar{A} \frac{\partial}{\partial x} \bar{Q} + \bar{B} \frac{\partial}{\partial y} \bar{Q} + \bar{C} \frac{\partial}{\partial z} \bar{Q} = 0$$

(2-17)

where

$$\bar{Q} = \begin{pmatrix} \rho \\ u \\ v \\ w \\ p \end{pmatrix} \quad (2-18)$$

$$\bar{A} = \begin{pmatrix} u & \rho & 0 & 0 & 0 \\ 0 & u & 0 & 0 & 1/\rho \\ 0 & 0 & u & 0 & 0 \\ 0 & 0 & 0 & u & 0 \\ 0 & \rho a^2 & 0 & 0 & u \end{pmatrix} \quad (2-19)$$

$$\bar{B} = \begin{pmatrix} v & \rho & 0 & 0 & 0 \\ 0 & v & 0 & 0 & 0 \\ 0 & 0 & v & 0 & 1/\rho \\ 0 & 0 & 0 & v & 0 \\ 0 & 0 & \rho a^2 & 0 & v \end{pmatrix} \quad (2-20)$$

$$\bar{C} = \begin{pmatrix} w & \rho & 0 & 0 & 0 \\ 0 & w & 0 & 0 & 0 \\ 0 & 0 & w & 0 & 0 \\ 0 & 0 & 0 & w & 1/\rho \\ 0 & 0 & 0 & \rho a^2 & w \end{pmatrix} \quad (2-21)$$

Now we call

$$\frac{\partial Q}{\partial \bar{Q}} = M \quad (2-22)$$

$$M = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ u & \rho & 0 & 0 & 0 \\ v & 0 & \rho & 0 & 0 \\ w & 0 & 0 & \rho & 0 \\ q^2/2 & \rho u & \rho v & \rho w & 1/(\gamma-1) \end{pmatrix} \quad (2-23)$$

$$M^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ -u/\rho & 1/\rho & 0 & 0 & 0 \\ -v/\rho & 0 & 1/\rho & 0 & 0 \\ -w/\rho & 0 & 0 & 1/\rho & 0 \\ (\gamma-1)q^2/2 & -(\gamma-1)u & -(\gamma-1)v & -(\gamma-1)w & (\gamma-1)^2 \end{pmatrix} \quad (2-24)$$

By matrix multiplication, we obtain

$$\bar{A}, \bar{B} \text{ and } \bar{C} = M^{-1}(A, B \text{ and } C) M \quad (2-25)$$

This means the matrices A , B and C are similar to $\bar{A}$, $\bar{B}$ and $\bar{C}$, so they have the same eigenvalues respectively.

By finding the eigenvalues of $\bar{A}$, we obtain

$$\lambda_{1,2,3} = u$$

$$\lambda_4 = u + a \quad (2-26)$$

$$\lambda_5 = u - a$$

which are also the eigenvalues of A. The eigenvalues of B and C are similar to that of A, we only need to replace u in the eigenvalues of A to v and w respectively.

2.3 BOUNDARY CONDITIONS

The problems of interest in this thesis all consist of the wind tunnel type of boundary conditions which include solid wall conditions, upstream conditions, outer boundary conditions and downstream conditions.

2.3.1 SOLID SURFACE

On solid surfaces, the no slip condition is applied. The velocity on the solid wall is taken as zero. The wall static pressure can be derived from the Navier-Stokes equations at the wall as follows:

$$\left. \frac{\partial p}{\partial y} \right|_{y=y_w} = \left\{ \frac{\partial}{\partial x} \left(\mu \frac{\partial u}{\partial y} \right) + \frac{\partial}{\partial y} \left(2 \mu \frac{\partial v}{\partial y} - \frac{2}{3} \mu \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) \right) + \frac{\partial}{\partial z} \left(\mu \frac{\partial w}{\partial y} \right) \right\} \Big|_{y=y_w}$$

$$\left. \frac{\partial p}{\partial z} \right|_{z=z_w} = \left\{ \frac{\partial}{\partial x} \left(\mu \frac{\partial u}{\partial z} \right) + \frac{\partial}{\partial y} \left(\mu \frac{\partial v}{\partial z} \right) + \frac{\partial}{\partial z} \left(2 \mu \frac{\partial w}{\partial z} - \frac{2}{3} \mu \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) \right) \right\} \Big|_{z=z_w}$$

(2-27)

The wall temperature is obtained by using the assumption of the adiabatic condition:

$$\left. \frac{\partial T}{\partial y} \right|_{y=y_w} = 0$$

$$\left. \frac{\partial T}{\partial z} \right|_{z=z_w} = 0 \quad (2-28)$$

Then the wall air density can be found from the state equation.

2.3.2 UPSTREAM CONDITIONS

Flow quantities are the free stream values, at a location upstream of an imagined leading edge of a plate. This choice was used in the two-dimensional test computations.

In the three-dimensional computations, approach flow was taken to be uniform in a central region, with specified boundary layer profiles on the side walls and bottom wall.

2.3.3 OUTER BOUNDARY CONDITIONS

In two dimensional computations, the outer boundary conditions for present analysis consists of

two segments. In the region upstream of the incident shock, the free stream condition ρ_e , u_e , T_e , p_e and zero normal velocity were prescribed. Behind the incident shock wave, the post-shock condition were specified using inviscid oblique shock wave theory. Due to the limitation on the computer memory and CPU time, the viscous effects of the shock generator is neglected.

In the computations on three dimensional flows, the boundary layer profiles are specified at the top mesh boundary along the side wall to avoid singularity. The outer edge flow values of the boundary layer profiles upstream of the incident shock are the free stream values; and the outer edge values of the boundary layer downstream of the shock are the values obtained from the inviscid solution of the flow.

In order to avoid specifying downstream condition of the reflected shock at outer boundary, the anticipated reflected shock is permitted to pass through the downstream boundary in the computational domain. (see Figure 1)

2.3.4 DOWNSTREAM CONDITION

At the downstream or exit boundary of the computational domain, we set the first derivatives of all quantities to equal zero. This condition forces a streamwise zero gradient for all values for the flow. For shock and boundary layer interactions, the condition is consistent with the inviscid solution at the exit except the shock area. Inside of the boundary layer, the condition is an approximation which may induce some error in the boundary layer profile. Fortunately, the flow is supersonic in most of the computational domain, the effect of the downstream disturbance has very little feed back to upstream solution. According to the similarity solution of the laminar boundary layer, the error introduced by the exit condition is proportional to the inverse of the distance from leading. In general, the error decreases while the distance between leading edge and exit increases.

2.4 TURBULENCE EQUATIONS

A strong conservation form of the mass-averaged Navier-Stokes equations [23, 24] is adopted in calculating turbulent flow. The form takes all turbulence effect into account by adding the eddy viscosity ϵ to the equations. The equations can be obtained by replacing dynamic viscosity μ by $(\epsilon + \mu)$ and conductivity k by $c_p \left(\frac{\mu}{Pr} + \frac{\epsilon}{Pr_t} \right)$ in (2-1). The expression of the turbulent normal stresses in the equation is incorrect. To resolve the problem requires a turbulent model which reflects compressible turbulent flow characteristics. In the computations, the molecular Prandtl number is $Pr = 0.72$ for air and the turbulent Prandtl number is assigned $Pr_t = 0.9$, and the eddy viscosity will be discussed in Chapter 3.

CHAPTER 3

NUMERICAL SCHEME

This chapter presents the scheme that is used to numerically solve the governing equations described in the previous chapter. The scheme employed in this work is called lower-upper symmetric successive overrelaxation (LU-SSOR) scheme. This implicit time marching technique is a finite volume form of linear integration on each face of an element. The scheme is based on lower-upper (LU) factorization as well as a form of a symmetric successive overrelaxation (SSOR) method. Although the system of equations is formulated in a fully implicit and fully coupled manner, the LU-SSOR scheme requires only scalar diagonal inversion for the solution. Unlike explicit schemes, this scheme is unconditionally stable. Since most cases we are interested in require only the steady state solutions, we can use any large number of Courant-Friedrichs-Lewy (CFL) number to speed up the

code to converge.

In turbulence modeling, a three-dimensional Baldwin-Lomax model will be presented in this chapter. The Baldwin-Lomax model is patterned after that of Cebeci [25] with modifications that avoid the necessity for finding the edge of the boundary layer.

3.1 COORDINATE TRANSFORMATION

In numerical computation, it is easier to solve the governing equations on a uniformly spaced computation grid; but in most practical computations, the grid need to be distributed to fit the physical geometry and flow characters. In order to use a non-uniform grid, consider a general transformation of the form

$$\begin{aligned}\xi &= \xi(x,y,z) \\ \eta &= \eta(x,y,z) \\ \zeta &= \zeta(x,y,z)\end{aligned}\tag{3-1}$$

which can be used to transform the governing equations from the physical domain (x,y,z) to the computational domain (ξ,η,ζ) . Using the chain rule of partial differentiation, the partial derivatives becomes

$$\begin{aligned}\frac{\partial}{\partial x} &= \xi_x \frac{\partial}{\partial \xi} + \eta_x \frac{\partial}{\partial \eta} + \zeta_x \frac{\partial}{\partial \zeta} \\ \frac{\partial}{\partial y} &= \xi_y \frac{\partial}{\partial \xi} + \eta_y \frac{\partial}{\partial \eta} + \zeta_y \frac{\partial}{\partial \zeta} \\ \frac{\partial}{\partial z} &= \xi_z \frac{\partial}{\partial \xi} + \eta_z \frac{\partial}{\partial \eta} + \zeta_z \frac{\partial}{\partial \zeta}\end{aligned}\tag{3-2}$$

We can also write the differential expressions in matrix form

$$\begin{pmatrix} d\xi \\ d\eta \\ d\zeta \end{pmatrix} = \begin{pmatrix} \xi_x & \xi_y & \xi_z \\ \eta_x & \eta_y & \eta_z \\ \zeta_x & \zeta_y & \zeta_z \end{pmatrix} \begin{pmatrix} dx \\ dy \\ dz \end{pmatrix} \quad (3-3)$$

In the same manner we can write

$$\begin{pmatrix} dx \\ dy \\ dz \end{pmatrix} = \begin{pmatrix} x_\xi & x_\eta & x_\zeta \\ y_\xi & y_\eta & y_\zeta \\ z_\xi & z_\eta & z_\zeta \end{pmatrix} \begin{pmatrix} d\xi \\ d\eta \\ d\zeta \end{pmatrix} \quad (3-4)$$

Therefore,

$$\begin{pmatrix} \xi_x & \xi_y & \xi_z \\ \eta_x & \eta_y & \eta_z \\ \zeta_x & \zeta_y & \zeta_z \end{pmatrix} = \begin{pmatrix} x_\xi & x_\eta & x_\zeta \\ y_\xi & y_\eta & y_\zeta \\ z_\xi & z_\eta & z_\zeta \end{pmatrix}^{-1}$$

$$= J \begin{pmatrix} y_\eta z_\zeta - y_\zeta z_\eta & x_\zeta z_\eta - x_\eta z_\zeta & x_\eta y_\zeta - x_\zeta y_\eta \\ y_\zeta z_\xi - y_\xi z_\zeta & x_\xi z_\zeta - x_\zeta z_\xi & x_\zeta y_\xi - x_\xi y_\zeta \\ y_\xi z_\eta - y_\eta z_\xi & x_\eta z_\xi - x_\xi z_\eta & x_\xi y_\eta - x_\eta y_\xi \end{pmatrix}$$

(3-5)

where J is the Jacobian of the transformation

$$J = \det \begin{pmatrix} \xi_x & \xi_y & \xi_z \\ \eta_x & \eta_y & \eta_z \\ \zeta_x & \zeta_y & \zeta_z \end{pmatrix} = 1 / \det \begin{pmatrix} x_\xi & x_\eta & x_\zeta \\ y_\xi & y_\eta & y_\zeta \\ z_\xi & z_\eta & z_\zeta \end{pmatrix}$$

(3-6)

If we apply the above standard transformation to the governing equation (2-1), the transformed equation is obtained

$$\begin{aligned} Q_t + \xi_x E_\xi + \eta_x E_\eta + \zeta_x E_\zeta + \xi_y F_\xi + \eta_y F_\eta + \zeta_y F_\zeta \\ + \xi_z G_\xi + \eta_z G_\eta + \zeta_z G_\zeta = \xi_x E_{v\xi} + \eta_x E_{v\eta} + \zeta_x E_{v\zeta} \\ + \xi_y F_{v\xi} + \eta_y F_{v\eta} + \zeta_y F_{v\zeta} + \xi_z G_{v\xi} + \eta_z G_{v\eta} + \zeta_z G_{v\zeta} \end{aligned}$$

(3-7)

The transformed equation (3-7) can be rewritten in conservation-law form

$$\frac{\partial Q_1}{\partial t} + \frac{\partial E_1}{\partial \xi} + \frac{\partial F_1}{\partial \eta} + \frac{\partial G_1}{\partial \zeta} = \frac{\partial E_{v1}}{\partial \xi} + \frac{\partial F_{v1}}{\partial \eta} + \frac{\partial G_{v1}}{\partial \zeta} \quad (3-8)$$

where

$$\begin{aligned} Q_1 &= \frac{Q}{J} \\ E_1 &= \frac{1}{J}(E\xi_x + F\xi_y + G\xi_z) \\ F_1 &= \frac{1}{J}(E\eta_x + F\eta_y + G\eta_z) \\ G_1 &= \frac{1}{J}(E\zeta_x + F\zeta_y + G\zeta_z) \\ E_{v1} &= \frac{1}{J}(E_v\xi_x + F_v\xi_y + G_v\xi_z) \\ F_{v1} &= \frac{1}{J}(E_v\eta_x + F_v\eta_y + G_v\eta_z) \\ G_{v1} &= \frac{1}{J}(E_v\zeta_x + F_v\zeta_y + G_v\zeta_z) \end{aligned} \quad (3-9)$$

It should be remembered all the terms in E_v , F_v and G_v that contain partial derivatives should be also transformed. For example, the shear stress term τ_{xy} , would be transformed to

$$\tau_{xy} = \mu \left(\xi_y \frac{\partial u}{\partial \xi} + \eta_y \frac{\partial u}{\partial \eta} + \zeta_y \frac{\partial u}{\partial \zeta} + \xi_x \frac{\partial v}{\partial \xi} + \eta_x \frac{\partial v}{\partial \eta} + \zeta_x \frac{\partial v}{\partial \zeta} \right) \quad (3-10)$$

From (3-9) and (2-16), we can obtain the

Jacobians of the flux vectors

$$\begin{aligned}
A_1 &= \frac{\partial E_1}{\partial Q_1} = (A \xi_x + B \xi_y + C \xi_z) \\
B_1 &= \frac{\partial F_1}{\partial Q_1} = (A \eta_x + B \eta_y + C \eta_z) \\
C_1 &= \frac{\partial G_1}{\partial Q_1} = (A \zeta_x + B \zeta_y + C \zeta_z)
\end{aligned} \tag{3-11}$$

Also from (2-25) we can derive the eigenvalues of A_1 , B_1 and C_1 by

$$\begin{aligned}
A_1 &= M (\bar{A} \xi_x + \bar{B} \xi_y + \bar{C} \xi_z) M^{-1} \\
B_1 &= M (\bar{A} \eta_x + \bar{B} \eta_y + \bar{C} \eta_z) M^{-1} \\
C_1 &= M (\bar{A} \zeta_x + \bar{B} \zeta_y + \bar{C} \zeta_z) M^{-1}
\end{aligned} \tag{3-12}$$

It is not hard to find the eigenvalue for A_1

$$\begin{aligned}
\Lambda_1 &= \Lambda_2 = \Lambda_3 = u \xi_x + v \xi_y + w \xi_z \\
\Lambda_4 &= u \xi_x + v \xi_y + w \xi_z + a (\xi_x^2 + \xi_y^2 + \xi_z^2)^{\frac{1}{2}} \\
\Lambda_5 &= u \xi_x + v \xi_y + w \xi_z - a (\xi_x^2 + \xi_y^2 + \xi_z^2)^{\frac{1}{2}}
\end{aligned} \tag{3-13}$$

The eigenvalues of B_1 and C_1 are similar to that of A_1 ; we only need to replace ξ_x , ξ_y and ξ_z to η_x , η_y and η_z for B_1 and to replace ξ_x , ξ_y and ξ_z to

ζ_x , ζ_y and ζ_z for C_1 respectively.

In order to examine the numerical stability of a scheme, we often need to know the bound of the above eigenvalues. Let us now define γ_A , γ_B and γ_C are the bound of the eigenvalues of A_1 , B_1 and C_1 . Since

$$\begin{aligned} & \text{Max}(\Lambda_1)_{1=1,5} \\ & \leq |u| |\xi_x| + |v| |\xi_y| + |w| |\xi_z| + a (\xi_x^2 + \xi_y^2 + \xi_z^2)^{\frac{1}{2}} \\ & \leq \text{AAA} (|u| + |v| + |w| + \sqrt{3a}) \\ & \leq \text{AAA} (2 \sqrt{u^2 + v^2 + w^2} + 2a) \end{aligned} \quad (3-14)$$

where $\text{AAA} = \text{Max}(\xi_x, \xi_y, \xi_z)$, so we obtain the bounds of the eigenvalues

$$\begin{aligned} \gamma_A &= \text{AAA} (2 \sqrt{u^2 + v^2 + w^2} + 2a) \\ \gamma_B &= \text{BBB} (2 \sqrt{u^2 + v^2 + w^2} + 2a) \\ \gamma_C &= \text{CCC} (2 \sqrt{u^2 + v^2 + w^2} + 2a) \end{aligned} \quad (3-15)$$

where $\text{BBB} = \text{Max}(\eta_x, \eta_y, \eta_z)$ and

$\text{CCC} = \text{Max}(\zeta_x, \zeta_y, \zeta_z)$.

3.2 LU-SSOR METHOD

A prototype implicit scheme for a system such as the Navier-Stokes equation set (3-8) can be formulated as

$$\begin{aligned} Q_1^{n+1} = & Q_1^n - \beta \Delta t \{ D_\xi E_1(Q_1^{n+1}) + D_\eta F_1(Q_1^{n+1}) + D_\zeta G_1(Q_1^{n+1}) \} \\ & - (1-\beta) \Delta t \{ D_\xi E_1(Q_1^n) + D_\eta F_1(Q_1^n) + D_\zeta G_1(Q_1^n) \} + \\ & \Delta t \{ D_\xi E_{v1}(Q_1^n) + D_\eta F_{v1}(Q_1^n) + D_\zeta G_{v1}(Q_1^n) \} \end{aligned} \quad (3-16)$$

where D_ξ , D_η and D_ζ are difference operators that approximate $\partial/\partial\xi$, $\partial/\partial\eta$ and $\partial/\partial\zeta$.

The scheme (3-16) can linearized by setting

$$\begin{aligned} E_1(Q_1^{n+1}) &= E_1(Q_1^n) + A_1 \delta Q_1 + O(\|\delta Q_1\|^2) \\ F_1(Q_1^{n+1}) &= F_1(Q_1^n) + B_1 \delta Q_1 + O(\|\delta Q_1\|^2) \\ G_1(Q_1^{n+1}) &= G_1(Q_1^n) + C_1 \delta Q_1 + O(\|\delta Q_1\|^2) \end{aligned} \quad (3-17)$$

where $\delta Q_1 = Q_1^{n+1} - Q_1^n$. If we drop the terms of the second and higher order in equations (3-17), the equation (3-16) yields

$$\{I + \beta \Delta t (D_\xi A_1 + D_\eta B_1 + D_\zeta C_1)\} \delta Q_1 = \Delta t \text{ RHS} \quad (3-18)$$

where the right hand side RHS is the residual,

$$\begin{aligned} \text{RHS} = & - D_{\xi} E_1(Q_1^n) - D_{\eta} F_1(Q_1^n) - D_{\zeta} G_1(Q_1^n) \} + D_{\xi} E_{v1}(Q_1^n) \\ & + D_{\eta} F_{v1}(Q_1^n) + D_{\zeta} G_{v1}(Q_1^n) \end{aligned} \quad (3-19)$$

Physically, the RHS term also presents the difference form of the steady part of the Navier-Stokes equations. So if we are only interested in steady solutions, as long as $\delta Q_1/\Delta t$ tends to zero, the numerical solutions approach the steady state solutions plus higher order truncation errors and run off errors.

If we neglect the viscous terms and set $\beta = 1/2$, then the scheme is second-order time accurate. This implies if we set $\beta = 1/2$, even though the solution inside of the boundary layer is only first-order time accurate in iteration, the unsteady inviscid solutions of the Euler equation can still remain second-order time accurate.

If $\beta = 0$, the scheme becomes an Euler explicit algorithm. Due to the CFL stability requirement, the iteration step has to be made sufficiently small to ensure the convergence. This limitation made it

impractical to calculate the three-dimensional compressible flows.

The RPLUS code used the $\beta = 1$, which allows a large iteration step and still maintain the numerical stability in time iteration.

In the limit of $\Delta t \rightarrow \infty$ of this scheme, the scheme reduces to a Newton iteration

$$(D_{\xi}A_1 + D_{\eta}B_1 + D_{\zeta}C_1) \delta Q_1 = \text{RHS} \quad (3-20)$$

But with Newton iteration, the required number of iterations grows rapidly when the number of grid points increases. Even though the iteration remained stable, it becomes impractical because of slow convergence.

An alternative way to solve the system (3-18) is to use a relaxation algorithm. The scheme then becomes an alternative to approximate factorization. One way of factorization is well known as ADI scheme or alternating direction implicit scheme. The ADI scheme replaces the operator of equation (3-18) a product of one-dimensional operators, for example; for two-dimensional flows, the ADI scheme becomes

$$(I + \beta \Delta t D_{\xi} A_1) (I + \beta \Delta t D_{\eta} B_1) \delta Q_1 = \Delta t \text{ RHS} \quad (3-21)$$

In two-dimensional cases, the scheme converges and has been widely used. But in three-dimensional cases, the third operator $(I + \beta \Delta t D_{\eta} B_1)$ introduces additional errors into the solver, which makes the steady state solution depend on Δt . Even though the scheme avoids the huge matrix inversion needed to solve (3-18) directly, it still requires many inversion operations of small block matrices. Additionally, the convergence speed is severely limited by CFL restriction due to the error from the third operator.

An alternative way of factorization is based on flux vector splitting. An LU implicit scheme can be derived from the symmetric successive overrelaxation (SSOR) method.

The linearized implicit scheme (3-18) can be altered as

$$\{I + \beta \Delta t (D_{\xi}^{-} A_1^{+} + D_{\xi}^{+} A_1^{-} + D_{\eta}^{-} B_1^{+} + D_{\eta}^{+} B_1^{-} + D_{\zeta}^{-} C_1^{+} + D_{\zeta}^{+} C_1^{-})\} \delta Q_1 = \Delta t \text{ RHS} \quad (3-22)$$

where

$$\begin{aligned}
 A_1^+ &= \frac{1}{2} (A_1 + \gamma_A I) \\
 A_1^- &= \frac{1}{2} (A_1 - \gamma_A I) \\
 B_1^+ &= \frac{1}{2} (B_1 + \gamma_B I) \\
 B_1^- &= \frac{1}{2} (B_1 - \gamma_B I) \\
 C_1^+ &= \frac{1}{2} (C_1 + \gamma_C I) \\
 C_1^- &= \frac{1}{2} (C_1 - \gamma_C I)
 \end{aligned} \tag{3-23}$$

γ_A , γ_B and γ_C are the upper bound of the eigenvalues of A_1 , B_1 and C_1 given in (3-15). So the eigenvalues of A^+ , B^+ and C^+ are non-negative and the eigenvalues of A^- , B^- and C^- are non-positive. D_ξ^- , D_η^- and D_ζ^- are backward-difference operators and D_ξ^+ , D_η^+ and D_ζ^+ are forward-difference operators. The derivatives in RHS are replaced by central differences. We chose the grid size 1 x 1 x 1 in computational domain. So equation (3-22) can be rewritten as

$$\begin{aligned}
 &\delta Q_1(i,j,k) + \beta \Delta t (A_1^+(i,j,k) \delta Q_1(i,j,k) \\
 &- A_1^+(i-1,j,k) \delta Q_1(i-1,j,k) +
 \end{aligned}$$

$$\begin{aligned}
& A_1^-(i+1,j,k) \delta Q_1(i+1,j,k) - A_1^-(i,j,k) \delta Q_1(i,j,k) \\
& + B_1^+(i,j,k) \delta Q_1(i,j,k) - B_1^+(i,j-1,k) \delta Q_1(i,j-1,k) \\
& + B_1^-(i,j+1,k) \delta Q_1(i,j+1,k) - B_1^-(i,j,k) \delta Q_1(i,j,k) \\
& + C_1^+(i,j,k) \delta Q_1(i,j,k) - C_1^+(i,j,k-1) \delta Q_1(i,j,k-1) \\
& + C_1^-(i,j,k+1) \delta Q_1(i,j,k+1) - C_1^-(i,j,k) \delta Q_1(i,j,k) \\
& = \Delta t \text{ RHS} \tag{3-24}
\end{aligned}$$

By operating the backward and forward relaxation sweeps on equation (3-24), we obtain the symmetric successive overrelaxation method, which can be written in two steps as

$$\begin{aligned}
& \delta Q_1^*(i,j,k) + \beta \Delta t (A_1^+(i,j,k) - A_1^-(i,j,k)) \\
& \delta Q_1^*(i,j,k) - \beta \Delta t A_1^+(i-1,j,k) \delta Q_1^*(i-1,j,k) \\
& + \beta \Delta t (B_1^+(i,j,k) - B_1^-(i,j,k)) \delta Q_1^*(i,j,k) \\
& - \beta \Delta t B_1^+(i,j-1,k) \delta Q_1^*(i,j-1,k) \\
& + \beta \Delta t (C_1^+(i,j,k) - C_1^-(i,j,k)) \delta Q_1^*(i,j,k) \\
& - \beta \Delta t C_1^+(i,j,k-1) \delta Q_1^*(i,j,k-1) = \Delta t \text{ RHS} \tag{3-25}
\end{aligned}$$

followed by

$$\begin{aligned}
& \delta Q_1(i,j,k) + \beta \Delta t (A_1^+(i,j,k) - A_1^-(i,j,k)) \\
& \delta Q_1(i,j,k) + \beta \Delta t A_1^-(i+1,j,k) \delta Q_1(i+1,j,k) \\
& - \beta \Delta t A_1^+(i-1,j,k) \delta Q_1^*(i-1,j,k) \\
& + \beta \Delta t (B_1^+(i,j,k) - B_1^-(i,j,k)) \delta Q_1(i,j,k)
\end{aligned}$$

$$\begin{aligned}
& + \beta \Delta t B_1^-(i, j+1, k) \delta Q_1(i, j+1, k) \\
& - \beta \Delta t B_1^+(i, j-1, k) \delta Q_1^*(i, j-1, k) \\
& + \beta \Delta t (C_1^+(i, j, k) - C_1^-(i, j, k)) \delta Q_1(i, j, k) \\
& + \beta \Delta t C_1^-(i, j, k+1) \delta Q_1(i, j, k+1) \\
& - \beta \Delta t C_1^+(i, j, k-1) \delta Q_1^*(i, j, k-1) \\
& = \Delta t \text{ RHS} \tag{3-26}
\end{aligned}$$

Subtract equation (3-25) from (3-26) to get

$$\begin{aligned}
& \delta Q_1(i, j, k) + \beta \Delta t (A_1^+(i, j, k) - A_1^-(i, j, k)) \\
& \delta Q_1(i, j, k) + \beta \Delta t A_1^-(i+1, j, k) \delta Q_1(i+1, j, k) \\
& + \beta \Delta t (B_1^+(i, j, k) - B_1^-(i, j, k)) \delta Q_1(i, j, k) \\
& + \beta \Delta t B_1^-(i, j+1, k) \delta Q_1(i, j+1, k) \\
& + \beta \Delta t (C_1^+(i, j, k) - C_1^-(i, j, k)) \delta Q_1(i, j, k) \\
& + \beta \Delta t C_1^-(i, j, k+1) \delta Q_1(i, j, k+1) \\
& = \delta Q_1^*(i, j, k) + \beta \Delta t (A_1^+(i, j, k) - A_1^-(i, j, k)) \\
& \delta Q_1^*(i, j, k) + \beta \Delta t (B_1^+(i, j, k) - B_1^-(i, j, k)) \\
& \delta Q_1^*(i, j, k) + \beta \Delta t (C_1^+(i, j, k) - C_1^-(i, j, k)) \\
& \delta Q_1^*(i, j, k) \tag{3-27}
\end{aligned}$$

This may be rewritten in matrix form as

$$\begin{aligned}
& \{I + \beta \Delta t (D_\xi^+ A_1^- + D_\eta^+ B_1^- + D_\zeta^+ C_1^-) \\
& + A_1^+ + B_1^+ + C_1^+\} \delta Q_1 \\
& = \{I + \beta \Delta t (A_1^+ + B_1^+ + C_1^+ \\
& - A_1^- - B_1^- - C_1^-)\} \delta Q_1^* \tag{3-28}
\end{aligned}$$

where

$$\delta Q_1^* = \{I + \beta \Delta t (D_{\xi}^{-} A_1^* + D_{\eta}^{-} B_1^* + D_{\zeta}^{-} C_1^* - A_1^- - B_1^- - C_1^-)\}^{-1} \Delta t \text{ RHS} \quad (3-29)$$

From equation (3-23), we can obtain

$$\begin{aligned} A_1^+ - A_1^- &= \gamma_A I \\ B_1^+ - B_1^- &= \gamma_B I \\ C_1^+ - C_1^- &= \gamma_C I \end{aligned} \quad (3-30)$$

Thus we obtain the LU-SSOR implicit scheme

$$\begin{aligned} &\{I + \beta \Delta t (D_{\xi}^+ A_1^- + D_{\eta}^+ B_1^- + D_{\zeta}^+ C_1^- + A_1^+ + B_1^+ + C_1^+)\} \\ &\{I + \beta \Delta t (D_{\xi}^- A_1^+ + D_{\eta}^- B_1^+ + D_{\zeta}^- C_1^+ - A_1^- - B_1^- - C_1^-)\} \\ \delta Q_1 &= \{1 + \beta \Delta t (\gamma_A + \gamma_B + \gamma_C)\} \Delta t \text{ RHS} \end{aligned} \quad (3-31)$$

It is important to note that both lower and upper operators in (3-31) only require scalar inversions. Therefore, the computation time in each iteration step is actually of the same order of magnitude as explicit methods.

The RHS term is second-order accurate in space since we used central difference. So the steady state solutions are in second-order accuracy in space. The time marching only has first-order accurate. The LU scheme takes advantage of the diagonal dominance in

the formulation of the implicit operator; the convergence trend of the calculation is monotonic and stable in laminar calculation.

3.3 ARTIFICIAL DAMPING

Far away from the wall, the supersonic flows in our study behave approximately as inviscid flows. The spurious oscillations which are triggered by discontinuities cannot be smoothed out totally by the viscous and dissipation terms. One practical and effective scheme to prevent the numerical oscillation is adding artificial damping into the LU scheme.

This work adopted A. Jameson's [26] adaptive artificial dissipation scheme. The dissipative term is constructed so that it is of fourth order in smooth regions of the flow and second order near the shock waves. The term is explicitly added to the RHS term as an additional residual. For the density equation, for example, the dissipation has the form

$$\begin{aligned} d(i+1/2, j, k) - d(i-1/2, j, k) + d(i, j+1/2, k) - \\ d(i, j-1/2, k) + d(i, j, k+1/2) - d(i, j, k-1/2) \end{aligned} \quad (3-32)$$

where

$$\begin{aligned} d(i+1/2, j, k) = \epsilon^{(2)}(i+1/2, j, k) [\rho(i+1, j, k) - \\ \rho(i, j, k)] - \epsilon^{(4)}(i+1/2, j, k) [\rho(i+2, j, k) - \\ 3 \rho(i+1, j, k) + 3 \rho(i, j, k) - \rho(i-1, j, k)] \end{aligned} \quad (3-33)$$

The adaptive coefficients $\epsilon^{(2)}(i,j,k)$ and $\epsilon^{(4)}(i,j,k)$ are proportional to the spectral radius of the Jacobian matrix

$$\begin{aligned} \text{rad}(i,j,k) = J * \\ (|u| |\xi_x| + |v| |\xi_y| + |w| |\xi_z| + a (\xi_x^2 + \xi_y^2 + \xi_z^2))^{\frac{1}{2}} \\ + |u| |\eta_x| + |v| |\eta_y| + |w| |\eta_z| + a (\eta_x^2 + \eta_y^2 + \eta_z^2))^{\frac{1}{2}} \\ + |u| |\zeta_x| + |v| |\zeta_y| + |w| |\zeta_z| + a (\zeta_x^2 + \zeta_y^2 + \zeta_z^2))^{\frac{1}{2}} \end{aligned} \quad (3-34)$$

$\epsilon^{(2)}(i+1/2,j,k)$ and $\epsilon^{(4)}(i+1/2,j,k)$ are proportional to $\min(\text{rad}(i+1,j,k), \text{rad}(i,j,k))$.

An effective sensor to detect a shock wave is also constructed by taking the second difference of the pressure. Define

$$\text{dpx}(i,j,k) = \left| \frac{p(i+1,j,k) - 2 p(i,j,k) + p(i-1,j,k)}{p(i+1,j,k) + 2 p(i,j,k) + p(i-1,j,k)} \right| \quad (3-35)$$

$\epsilon^{(2)}(i+1/2,j,k)$ is also proportional to $\max(\text{dpx}(i+2,j,k), \text{dpx}(i+1,j,k), \text{dpx}(i,j,k), \text{dpx}(i-1,j,k))$. A switch is set to turn $\epsilon^{(4)}$ off near the shock by the value of dpx as a measure.

In the same manner, the artificial damping is used on the momentum and energy equations by replacing ρ in (3-33) with ρu , ρv , ρw and ρe respectively.

3.4. EDDY VISCOSITY MODEL

In section 2.4, we introduced eddy viscosity in the mass-averaged Navier-Stokes equations. The present work adopted Baldwin-Lomax algebraic eddy viscosity model and generalized it to a three dimensional model in accordance with Knights [20]. The model is based on a two-layer eddy viscosity model in which

$$\epsilon = \begin{cases} \epsilon_1 & \ell \leq \ell_{\text{crossover}} \\ \epsilon_0 & \ell_{\text{crossover}} < \ell \end{cases} \quad (3-36)$$

where ℓ is a weighted length of the distances from the walls. In the wind tunnel flow, the computation domain only includes two perpendicular walls due to the symmetry assumption. The length is defined as

$$\ell = \frac{2 y z}{y + z + (y^2 + z^2)^{1/2}} \quad (3-37)$$

where y and z are the normal distances of the two walls of the wind tunnel.

In the inner layer, the eddy viscosity

$$\epsilon_1 = \rho (\kappa \ell D)^2 \omega \quad (3-38)$$

where D is the Van Driest damping factor which is defined as

$$D = 1 - \exp\left(-\frac{\ell}{26} \frac{u_\tau}{\nu_w}\right) \quad (3-39)$$

and ω is the magnitude of the vorticity

$$\omega = \sqrt{\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial z} - \frac{\partial w}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)^2} \quad (3-40)$$

and u_τ is defined as

$$u_\tau = \left(\frac{|\tau_w|}{\rho_w}\right)^{1/2} \quad (3-41)$$

The wall shear stress τ_w and the wall density ρ_w are also weighted by the normal distances from the walls.

In place of the Clauser formulation for the outer region

$$\varepsilon_o = K C_{cp} \rho F_{wake} F_{Kleb}(\ell) \quad (3-42)$$

where $K = 0.0168$ is the Clauser constant, C_{cp} is another constant suggested by Knight [20] that should vary based on Mach number. For Mach 3 flow, $C_{cp} = 2.08$ and for an equilibrium incompressible turbulent boundary layer $C_{cp} = 1.2$.

$$F_{wake} = \ell_{max} F_{max} \quad (3-43)$$

where $F_{max} = \max(\ell \omega D)$ and ℓ_{max} is the value of ℓ where $(\ell \omega D)$ is a maximum.

$$F_{\text{Kleb}} = \left\{ 1 + 5.5 \left(\frac{C_{\text{Kleb}} \ell}{\ell_{\text{max}}} \right)^6 \right\}^{-1} \quad (3-44)$$

where $C_{\text{Kleb}} = 0.3$.

The turbulent eddy viscosity is implemented by dividing the y-z plane into two regions by a plane of $y = z$. In the region that y is less than z, the inner and outer eddy viscosity profiles are obtained along $z = \text{constant}$ lines, and the eddy viscosity is switched from the inner to the outer form at the location where $\varepsilon_1 > \varepsilon_0$. In the region that y is greater than z, the same approach is employed on lines of $y = \text{constant}$.

3.5 GRID DISTRIBUTION

A computational domain in either two dimensional or three dimensional flow calculation is divided into two regions. A stretched mesh is used near the walls to resolve the part of flow where the gradients are high; the rest of the domain is described by using a uniform coarse mesh. The grid points in the region with stretched mesh were distributed exponentially as follows [8]:

$$\Delta y = \Delta y_{\max} \left(\frac{\Delta y_{\min}}{\Delta y_{\max}} \right)^{(J_{in}-J)/(J_{in}-1)} \quad \text{for } J \leq J_{in} \quad (3-45)$$

$$\Delta z = \Delta z_{\max} \left(\frac{\Delta z_{\min}}{\Delta z_{\max}} \right)^{(K_{in}-K)/(K_{in}-1)} \quad \text{for } K \leq K_{in} \quad (3-46)$$

where $\Delta y_{\max}$ and $\Delta z_{\max}$ are also the mesh width in uniform grids in the non-stretched region. In streamwise direction, uniform mesh is used throughout the computational domain.

CHAPTER 4

NUMERICAL RESULTS AND DISCUSSION

4.1 INTERACTION STRUCTURE

The flow under investigation is a supersonic wind tunnel flow with an incident oblique shock wave interacting with a turbulent boundary layer along the floor and side walls of the wind tunnel. The oblique shock wave was generated by an angle plate inside the wind tunnel. A simplified sketch of the flow configuration is shown in Figure 2. When the shock strength reaches a certain value, the flow separates. At high enough shock strength the structure of the separation was found to be three-dimensional. Reda and Murphy described in their paper [31] "a pair of large standing vortices within the reverse flow region". Jurkovich presented in his M.S. thesis [4] a set of clear pictures showing a strong separation vortex pair symmetric about the tunnel center line (Figure 3). On the side of the vortex pair, there exists a pair of corner separation bubbles. The

vortices near the corners are counter to the direction of the vortices in the center. The structure was captured by oil traces on the surface of the wind walls.

The three-dimensional structure may be formed due to the existence of the side walls. It may also be due to the non-uniform approaching boundary layer and secondary flows, since the flow has travelled through a complex geometric duct section, including a nozzle before it reaches the test section. Jurkovich concluded that the non-uniformity of the approaching boundary layer contributed to the formation of the three-dimensional vortex pair. This did not agree with Reda and Murphy, because they observed a similar structure even in a flow with fairly uniform inlet boundary layer.

The results of the oil flow studies raised questions about the full structure of the interaction as well as how the structure is formed. To answer these questions is the main objective of this project. Before we are able to simulate the flow, we have to verify the numerical scheme on known

solutions.

FLOW CONFIGURATION

Fig. 2

OIL FLOW PATTERN ($\alpha=9^\circ$)

Floor and corner oil flow pattern for 9 degrees angle of attack, after Jurkovich [4]

Fig. 3

4.2 TEST CASES

4.2.1 TWO DIMENSIONAL LAMINAR BOUNDARY LAYER

The solution of a boundary layer within a supersonic flow passing a flat plate at zero incidence were given by Crocco [33] half a century ago. The flow field was calculated using power law for viscosity ($\mu/\mu_0=T/T_0$) and Prandtl number equal to unity; the results agree well with experiment [5] in air; the deviation is within 5%. There have been solutions for the flow with different numerical methods, but for the purpose of validation of the RPLUS code, we are only comparing the numerical results with Crocco's solution and Skebe's experiment.

The test case of the flow is calculated using the laminar two-dimensional version of the LU-SSOR scheme. The initial condition for the interior grid points is uniform flow. The lower surface was either a plane of symmetry or a wall no-slip surface. The symmetry condition ($u_{1,1} = u_{1,2}$, $v_{1,1} = -v_{1,2}$, $p_{1,1} = p_{1,2}$ and $e_{1,1} = e_{1,2}$) was enforced along the lower surface from first grid point to the grid point ahead

of the leading edge. Along the surface of the flat plate, the wall condition was implemented by using $u_{1,1} = -u_{1,2}$, $v_{1,1} = -v_{1,2}$, $p_{1,1} = p_{1,2}$ and $e_{1,1} = e_{1,2}$. The flow values inlet boundary are held constant. At the top boundary and the exit boundary, zero-order extrapolations in the y direction ($Q_{1,j} = Q_{1,j-1}$) and in the x direction ($Q_{1,j} = Q_{1-1,j}$) were used respectively, where Q is the vector defined in equation (2-2).

The mesh contained 80 x 80 points and distributed evenly in the x direction and stretched in the y direction. The ratio of the length to the width of the computational domain $x_{\max}/y_{\max}$ is 4. $\Delta x/y_{\max}$ is .05, $\Delta y_{\max}/y_{\max}$ is .0208 and $\Delta y_{\min}/\Delta y_{\max} = 9.208 \times 10^{-4}$. From the first grid point to the fortieth grid point from the wall in the y direction, Δy was stretched following equation (3-45). The grid points above the fortieth grid point in the y direction are evenly distributed with $\Delta y = \Delta y_{\max}$.

The calculated results for laminar flow at Mach 2 are shown on Figure 4 and Figure 5. The dash-dotted line are the result calculated by Crocco. The dashed

line represent Skebe's experiment result. The solid lines are the current computational results. The velocity and temperature profiles from the computation plotted on the figures are picked at the 15th grid point ($x/y_{\max} = 1.1108$) from the leading edge. The computed boundary layer thickness at that location is $\delta/y_{\max} = 1.283 \times 10^{-3}$. There are only twenty five grid points distributed within the boundary layer. This coarse grid distribution resulted a fuller velocity profile and a smoother temperature profile in comparison with the experiment and Crocco's solution. The maximum deviation is about five percent. As shown in Figure 4, the temperature profile of the computation is closer to experiment than Crocco's solution. The deviation of the Crocco solution for the temperature probably is due to the choice of $Pr = 1$.

The two dimensional test computation was purposely carried out using a coarse grid as a test of the adequacy of the coarse grid, because a finer grid was not possible in the later three dimensional computations.

VELOCITY DISTRIBUTION OF LAMINAR BOUNDARY LAYER

(M=2)

Compressible laminar boundary layer, adiabatic wall
 $M=2$, $\eta=y(U_e/\nu_e x)^{0.5}$

— Current Computation
 - - - Solution of boundary layer equation [33]
 - . - Experiment [5]

Fig. 4

TEMPERATURE DISTRIBUTION OF LAMINAR BOUNDARY LAYER
($M=2$)

Compressible laminar boundary layer, adiabatic wall
 $M=2$, $\eta=y(U_e/\nu_e x)^{0.5}$

- Current Computation
- - - Solution of boundary layer equation [33]
- . - Experiment [5]

Fig. 5

4.2.2 TWO DIMENSIONAL SHOCK/LAMINAR BOUNDARY LAYER INTERACTION

With regard to laminar flat plate boundary layers interacting with oblique shocks, the most detailed set of experiments is Greber's work [3]. Skebe [6] extended the experiments to the flows at different Mach numbers. The results of their experiments are not only significant for the fundamental understanding of the interacting flow, but became an important reference for testing the numerical schemes. Numerous authors such as MacCormack [7], Beam and Warming [9] and many others selected some of the results to validate their numerical codes.

Our test cases are calculated under two sets of inlet and boundary conditions. The results of one case is shown on Figure 6 and Figure 7. The initial condition of this case for the interior grid points is uniform flow. The flow variables at the top mesh boundary were set to either free stream values or values for a given shock strength so that the shock wave would impinge on the plate surface at the lower

boundary at a specified point. As we assumed in the computation of flat plate boundary layer, the lower surface was either a plane of symmetry or a wall surface; at the exit boundary, zero-order extrapolation was used. The length of the computational domain was carefully arranged so that the reflected shock could go through the exit boundary instead of hitting the top mesh boundary.

In this case, the mesh contained 94×45 points. In x direction, the grid points are uniformly distributed with $\Delta x/L = .0208$, where L is the distance between the leading edge and the shock impinging point. In y direction, from the first grid point to the thirty second grid point from the wall, Δy was stretched following equation (3-45). The grids above thirty second grid point in the y direction are evenly distributed with $\Delta y = \Delta y_{\max}$. Here $\Delta y_{\max}/L$ is .0506 and $\Delta y_{\min}/\Delta y_{\max}$ is .0157. The length of the computation domain $x_{\max}/L$ is 2.0094 and the height $y_{\max}/L$ is .9637.

The Reynolds number based on the distance between the leading edge and the shock impinging

point is 2.96×10^5 , the Mach number is 2, and the deflection angle is 3.1° .

The solid lines of Figure 6 and Figure 7 are the calculated results. The structures of the profiles of the three variables are very close to that of the experiment and MacCormack's computation. The size of the separated region calculated is almost the same as that by MacCormack's predictor-corrector method, but the experimental results show a thirty percent smaller separated region. The results also show that the separation bubble represented by skin friction and wall pressure profiles is somewhat shifted downstream in comparison with the experimental results. A larger deviation at the exit reflects the numerical error induced by zero extrapolation boundary condition at the exit. These problems led the second test case show on Figure 8.

After careful study of the experimental results, we found the experiment results are closer to a Mach 2.03 flow instead of Mach 2. With the same pressure ratio between the pressure behind the reflected shock and the pressure upstream of the

incident shock in inviscid flow, which is 1.4, the deflection angle can be found to be 3.06° . In this case, we doubled the grid density in both x and y direction by using 100 x 100 grid points in a smaller computation domain, $x_{\max}/L$ is 1.784, and $y_{\max}/L$ is .574. We arranged the shock impingement point in the middle of the computational domain. The inlet condition was changed, we specified the supersonic boundary layer profile calculated from the similarity solution at the inlet flow near the wall. The results of this calculation show closer agreement with the experiment in both skin friction and wall pressure profiles. The separation bubble size become very close to the experimental results. The exit deviation is resolved. There is still a small deviation near the separation point in the wall pressure profile.

WALL PRESSURE AND SKIN FRICTION DISTRIBUTION OF SHOCK
/LAMINAR BOUNDARY LAYER INTERACTION ($M=2$, $\alpha=3.1^\circ$)

Interaction of shock/laminar boundary layer
 $M=2$, $\alpha=3.1^\circ$, $Re_{sk}=2.96 \times 10^5$

— Current Computation
 ◻ Experiment [3]
 - - MacCormack [7]

Fig. 6

VELOCITY DISTRIBUTION OF SHOCK/LAMINAR
BOUNDARY LAYER INTERACTION ($M=2$, $\alpha=3.1^\circ$)

Interaction of shock/laminar boundary layer
 $M=2$, $\alpha=3.1^\circ$, $Re_{sk}=2.96 \times 10^5$, at $Re_x=2.87 \times 10^5$

— Current Computation
◻ Experiment [3]

Fig. 7

WALL PRESSURE AND SKIN FRICTION DISTRIBUTION OF SHOCK
/LAMINAR BOUNDARY LAYER INTERACTION ($M=2.03$, $\alpha=3.06^\circ$)

WALL PRESSURE

SKIN FRICTION

Interaction of shock/laminar boundary layer
 $M=2.03$, $\alpha=3.06^\circ$, $Re_{sk}=2.96 \times 10^5$

— Current Computation
◦ Experiment [3]

Fig. 8

4.2.3 TWO DIMENSIONAL TURBULENT BOUNDARY LAYER

Due to its simplicity and easy applicability, the Baldwin-Lomax algebraic model is adopted for calculation of turbulent eddy viscosity. In section 3.4, we described the generalized equation set of the model for three dimensional flow. If one changes the weighted length ' l ' in the equations to ' y ', the equation then reduces to the original two dimensional Baldwin-Lomax model.

Baldwin and Lomax [18] validated their model through comparison between the numerical solutions and experimental results of boundary layers of supersonic flows on a flat plate. One of these case which is validated in this work is a Mach 2.85 flow. The turbulent eddy viscosity model was turned on at a Reynolds number based on distance from the leading edge of about 300,000 to match the results from the Hopkins-Inouye correlation [34].

Based on the Baldwin-Lomax scheme, a subroutine was made to calculate eddy viscosity and modify the viscous terms and heat flux term in the RPLUS code. In accordance to Knight [19], the constants C_{cp} and

C_{kleb} in the Baldwin-Lomax model are dependent on the Mach number of the free stream flow. The subroutine we programmed uses Knight's modification. For Mach 2.85 flow, we used $C_{cp} = 1.96$ instead of 1.6. We still kept $C_{kleb} = 0.3$. The initial flow field is uniform and boundary conditions of our calculation for the turbulent boundary layer are the same as that used in the laminar calculation.

Within the computational domain, 94 x 45 grid points were used. The ratio of the length of the computational domain to the height x_{max}/y_{max} is 2.86. The grid points are uniformly distributed in the x direction with $\Delta x/y_{max} = .0308$. From the first grid point to the thirty second grid point from the wall in the y direction, Δy is stretched according to equation (3-45). The grid points above the thirty second grid point in the y direction are evenly distributed with $\Delta y = \Delta y_{max}$. Here $\Delta y_{max}/y_{max}$ is .059 and the $\Delta y_{min}/\Delta y_{max} = .00313$. The leading edge is ten grid points upstream of the inlet of the computational domain.

Figure 9 presents the skin friction profiles

of the flow. The dashed lines are the results calculated by Baldwin and Lomax, the squares represent Hopkins and Inouye experimental results, and the solid lines are the current computational results. The figure shows the current results on skin friction are extremely close to Baldwin and Lomax result in turbulent region; the maximum deviation is about 1.5%. Both computational results show skin friction values ten percent higher than the experimental results. The deviation may be caused by the inaccurate turbulent model, the damping of the coarse grid or the artificial viscosity. The deviation is also reflected in the other computed results.

Figure 10 and Figure 11 show plots of the velocity profiles at Reynolds number of 1.6×10^6 against the law of the wall and law of the wake, where

$$u^* = (u / m_e)^{1/2} \sin^{1/2} (m_e u / u_e)$$

$$m = \frac{(\gamma-1)^2}{2} M_e^2 / [1 + \frac{(\gamma-1)^2}{2} M_e^2]$$

$$u_p^+ = (T_e/T_w)^{1/2} \int C_f/2 [1 + \phi^2 (T_e - T_w)/T_w]^{-1/2} d\phi$$

$$\phi = u/u_e$$

The current results show the logarithm region in both figures with the correct slope. The deviation remains the same level as shown in skin friction. Since Hopkins and Inouye did not present their velocity profiles on Mach 2.85, we plotted the numerical results against the experimental results of Mach 2.57 flow from Maise and McDonald experimental result [28] to have an approximate comparison with the law of wake (Fig. 10); and we used the results of Mach 3.55 flow of Squire [35] to compare with the law of the wall. As expected, the profiles show a logarithm region with similar amount of deviation to current results as in skin friction. In the viscous sublayer and in the outer layer, flows with different Mach number behave differently.

SKIN FRICTION DISTRIBUTION OF
TURBULENT BOUNDARY LAYER ($M=2.85$)

Turbulent boundary layer, $M=2.85$
 — Current computation
 - - - Baldwin and Lomax [18]
 □ Hopkins and Inouye [34]

Fig. 9

LAW OF THE WAKE OF
TURBULENT BOUNDARY LAYER ($M=2.85$)

Turbulent boundary layer, $M=2.85$
 — Current computation
 - - - Law of the wake
 o Maise and McDonald [28] $M=2.57$

Fig. 10

LAW OF THE WALL OF
TURBULENT BOUNDARY LAYER ($M=2.85$)

Turbulent boundary layer, $M=2.85$

— Current computation

- - - Law of the wall

o Squire [35] $M=3.55$

Fig. 11

4.2.4 TWO DIMENSIONAL SHOCK/TURBULENT BOUNDARY LAYER INTERACTION

In Figure 12, calculated surface pressure and skin friction are compared with Reda and Murphy's measurement [31]. The experiment was performed in a rectangular channel, so that the experimental results contain the side wall and corner flow effects. Nevertheless, we make comparison with the experiment because of the lack of a pure two dimensional experiment.

A turbulent velocity profile based on empirical law (A-8) and (A-9) is specified at the inlet boundary. The connection between the viscous sublayer and outer layer is at $y^+ = 11.5$. The temperature profile is then given by the Crocco-Busemann equation (A-10) with a turbulent recovery factor of 0.896. The flow variables at the top mesh boundary were set to either the free stream values or values for a given shock strength so that the shock wave would impinge on the plate surface on lower boundary at a given point.

In this case, the mesh contained 100 x 100

points. The ratio of the length of the domain to the height $x_{\max}/y_{\max}$ is 2.711. In the x direction, the grid points are uniformly distributed with $\Delta x/y_{\max} = .02711$. In the y direction, from the first grid point to the seventieth grid point from the wall, Δy was stretched following equation (3-45). The grid points above the seventieth grid point in the y direction are evenly distributed with $\Delta y = \Delta y_{\max}$. Here $\Delta y_{\max}/y_{\max}$ is 0.0249 and $\Delta y_{\min}/\Delta y_{\max}$ is .00789.

The Reynolds number based inlet boundary layer thickness is 9.7×10^5 , the Mach number is 2.9, and the deflection angle is 12.99° . The inlet boundary layer thickness $\delta/y_{\max}$ is .0832. With Mach number 2.9, we used $C_{cp} = 2.08$. We still kept $C_{kleb} = 0.3$.

The solid line of Figure 12 is the calculated result. The result shows the size of the separation bubble agrees with the experiment. The wall pressure profile has the same jump interval as the experimental result, but the shape of the wall pressure profile is somewhat different. The deviation may be caused by the three dimensionality of the experiment. The artificial viscosity effects will

also be higher than that of the laminar flows since the Reynolds number is much higher in turbulent cases. The accuracy of the Baldwin-Lomax model in capturing the separation also needs further investigation.

WALL PRESSURE AND SKIN FRICTION DISTRIBUTION OF SHOCK/
TURBULENT BOUNDARY LAYER INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)

Interaction of shock/turbulent boundary layer
 $M=2.9$, $\alpha=12.99^\circ$, $Re_{\delta_0}=9.7 \times 10^5$

— Current Computation

° Experiment [31] at center plane of a duct

Fig. 12

4.3 THREE DIMENSIONAL SHOCK/LAMINAR BOUNDARY LAYER INTERACTION

Figure 2 sketches the geometry of this calculation. A boundary layer profile based on similarity solution of Mach 2.03 flat plate boundary layer is specified at the inlet section of the computational domain, both on the side wall and on the tunnel floor. At the corner, the boundary layer thickness is weighted on the distances from the two walls by the following equation:

$$\delta = 2 \delta_y \delta_z / \{ \delta_y + \delta_z + [\delta_y^2 + \delta_z^2]^{1/2} \} \quad (4-1)$$

The flow variables at the upper boundary are set either as the inlet flow values or values for the given shock strength. Boundary layer profiles are specified along the side wall at top mesh boundary to avoid a singularity.

On the tunnel floor and side walls the zero slip condition is imposed. A symmetry condition is applied at the center plane of the duct. A zero-order extrapolation is used for exit. The reflected shock is allowed to exit through the exit plane without interaction with the top boundary.

A 40 x 40 x 40 grid is used in this calculation. The grid points are uniformly distributed in the x direction with $\Delta x/w = .0072$, where w is the duct width. In the y and the z directions, from the first grid point to the twenty seventh grid point from the ground and side wall, Δy and Δz are stretched following equation (3-45) and (3-46). The grid points farther from the walls are equally distributed with $\Delta y_{\max}/w = .057$ and $\Delta z_{\max}/w = .0285$. $\Delta y_{\min}/\Delta y_{\max} = .00156$ and $\Delta z_{\min}/\Delta z_{\max} = .00156$. A computational domain of $x_{\max}/w : y_{\max}/w : z_{\max}/w = 2.93:1:.5$ is used in this calculation

The Reynolds number based on the distance between leading edge and shock impinging point is 2.96×10^5 , where the distance L/w is 1.74, the Mach number is 2.03 and the deflection angle is 3.06° .

The numerical results show the flow contains two major separation structures. One is the center separation shown on Figure 13. If one looks at a cross section near the center plane of the duct (Figure 15), the center separation behaves very close to the two dimensional separation. In fact in this

case, a wide spanwise range of the center separation behaves two dimensionally. If one compares Figure 15 to Figure 16, the cross section shape of the separation bubble changes very little. The separation behaves two dimensionally in the central thirty percent of the width of the duct.

Nevertheless, the separation is three dimensional; the separation height is lower at cross sections closer to the side walls. At seven percent of duct width from the side wall, the cross section of the streamwise velocity shows no reverse flow (Figure 18). Then there is a pair of separation bubbles at the corners of between side walls and the tunnel ground. The corner bubbles indicate the interaction of the shock wave of side walls and the corners. From Figure 14, one can see clearly a three dimensional separation bubble sitting on the side wall. The peak of the bubble is very close to the tunnel ground at approximately .6% of the duct width from the side wall as shown on Figure 20. The cross section size of the corner separation bubble increases towards the wall, the largest streamwise

cross section of the reverse flow region is .8 of the duct width wide and .17 of the duct width high (Figure 20 to 23). The flow direction changes dramatically from the inviscid flow on the side wall (compare Figure 15 with Figure 23). The flow near the incident and reflected shocks changes its direction by the pressure gradient; and the direction of the flow near the wall changes more towards the counter direction of the pressure gradient because of the low initial velocity.

The top view of the flow field (from Figure 24 to Figure 28) gives a clear picture of the relationship between the center separation structure and the corner separation structure. Here we clearly captured the vortex pair observed by Jurkovich, Reda and Murphy. The center of the trace of the vortex pair is within the central separation region. The vortex pair at the higher cross section (Figure 25) is much closer to the center line of the duct than the vortex pair near the tunnel ground (Figure 28). The numerical results show the horseshoe structure of the center separation. The top view also tells us

even though the center separation bubble looks quite two dimensional in side view, it in fact is a totally three dimensional bubble. The shear stress and wall pressure distribution is smoother near the side wall than at the center line because of the corner interaction (Figure 29 to figure 33). The stream lines near the tunnel ground (Figure 34) show the flow near the center line of the tunnel is lifted by the separation. The flow near the corner flows into the central separation region helically and then exits near the center line of the duct.

The corner separation structure is shown in cross sections perpendicular to the main stream (Figure 35 to 42). In the corner separation bubble, there are two counter rotating vortex cells aligned approximately in the streamwise direction (see Figure 39). The cell size increases with downstream distance (compare Figure 39 with Figure 41). The upper cell was initiated by the interaction of impinging shock and side wall boundary layer. In figure 36, the pressure in the upper portion of the flow field is the high pressure behind the shock, and the flow

direction is towards the tunnel ground. On the other hand, the flow near ground boundary layer tends to flow upward. At some point, the flow near the side wall has less momentum moves along the counter direction of the pressure gradient while the flow outside of the side wall boundary layer has larger momentum to move up. This is why the velocity profile have an over shooting peak. The flow travels down from the side is redirected towards the center line by the corner. Then the flow encounters reflected shock, the pressure near the ground now behind the reflected shock which is higher than the pressure above. The pressure difference is upward and the flow momentum outside of the boundary layer was now towards the ground. The flow near the side wall which has less momentum is accelerated upward while the flow outside the boundary layer carried larger momentum is still moving downwards. Thus the lower vortex cell was formed. The flow towards the center encountered the center separation. The three dimensional vortex pair is then formed. Downstream of the interaction, the flow tends to recover from the

separation. The flow near the center reattaches the tunnel floor. The reattachment pushes flow towards the corner. This tendency lifts the lower vortex cell and formed upper vortex cell.

If one thinks the duct wall is a vortex sheet which contains a family of vortex rings, the shock impingement line on the tunnel floor lifts the vortex sheet away from the floor; and the incident and reflected shock lift the sheet off the side walls. From the top view, the lifted vortex ring near the center separation has a component in the z-direction. This component is the vortex observed by Jurkovich. Due to the corner interaction, the vortex ring in fact is three dimensional. The outer vortex cell shown in the spanwise view (Figure 40) indicates that there is an x-direction component of the vortex near the center separation. The corner vortex cell in that view shows the vortex line has a counter direction component near the corner.

From the structure we analyzed already, one found the side walls play a very important role in the three dimensional separation structure.

SHOCK/LAMINAR DUCT FLOW INTERACTION ($M=2.03$, $\alpha=3.06^\circ$)

STREAMWISE VELOCITY FIELD

(Separation Structure near the Center Plane)

Laminar shock/duct flow interaction
 $M=2.03$, $Re_w=1.666 \times 10^5$, $\alpha=3.06^\circ$, $\delta/w=.00484$

Fig. 13

SHOCK/LAMINAR DUCT FLOW INTERACTION ($M=2.03$, $\alpha=3.06^\circ$)

STREAMWISE VELOCITY FIELD

(Separation Structure near the Side Wall)

(Z-direction not Scale)

Laminar shock/duct flow interaction
 $M=2.03$, $Re_w = 1.666 \times 10^5$, $\alpha = 3.06^\circ$, $\delta/w = .00484$

Fig. 14

SHOCK/LAMINAR DUCT FLOW INTERACTION ($M=2.03$, $\alpha=3.06^\circ$)VELOCITY IN XY-PLANE ($Z/W=.486$)

EXPANDED VIEW

Laminar shock/duct flow interaction
 $M=2.03$, $Re_w=1.666 \times 10^5$, $\alpha=3.06^\circ$, $\delta/w=.00484$

Fig. 15

SHOCK/LAMINAR DUCT FLOW INTERACTION ($M=2.03$, $\alpha=3.06^\circ$)
 VELOCITY IN XY-PLANE ($Z/W=.343$)

EXPANDED VIEW

Laminar shock/duct flow interaction
 $M=2.03$, $Re_w=1.666 \times 10^5$, $\alpha=3.06^\circ$, $\delta/w=.00484$

Fig. 16

SHOCK/LAMINAR DUCT FLOW INTERACTION ($M=2.03$, $\alpha=3.06^\circ$)VELOCITY IN XY-PLANE ($Z/W=.2$)

EXPANDED VIEW

Laminar shock/duct flow interaction
 $M=2.03$, $Re_w=1.666 \times 10^5$, $\alpha=3.06^\circ$, $\delta/w=.00484$

Fig. 17

SHOCK/LAMINAR DUCT FLOW INTERACTION ($M=2.03$, $\alpha=3.06^\circ$)VELOCITY IN XY-PLANE ($Z/W=.07$)

EXPANDED VIEW

Laminar shock/duct flow interaction
 $M=2.03$, $Re_w=1.666 \times 10^5$, $\alpha=3.06^\circ$, $\delta/w=.00484$

Fig. 18

SHOCK/LAMINAR DUCT FLOW INTERACTION ($M=2.03$, $\alpha=3.06^\circ$)
 VELOCITY IN XY-PLANE ($Z/W=.0201$)

EXPANDED VIEW

Laminar shock/duct flow interaction
 $M=2.03$, $Re_w=1.666 \times 10^5$, $\alpha=3.06^\circ$, $\delta/w=.00484$

Fig. 19

SHOCK/LAMINAR DUCT FLOW INTERACTION ($M=2.03$, $\alpha=3.06^\circ$)
 VELOCITY IN XY-PLANE ($Z/W=.00568$)

EXPANDED VIEW

Laminar shock/duct flow interaction
 $M=2.03$, $Re_w=1.666 \times 10^5$, $\alpha=3.06^\circ$, $\delta/w=.00484$

Fig. 20

SHOCK/LAMINAR DUCT FLOW INTERACTION ($M=2.03$, $\alpha=3.06^\circ$)VELOCITY IN XY-PLANE ($Z/W=.00153$)

EXPANDED VIEW

Laminar shock/duct flow interaction
 $M=2.03$, $Re_w=1.666 \times 10^5$, $\alpha=3.06^\circ$, $\delta/w=.00484$

Fig. 21

SHOCK/LAMINAR DUCT FLOW INTERACTION ($M=2.03$, $\alpha=3.06^\circ$)VELOCITY IN XY-PLANE ($Z/W=.00032$)

EXPANDED VIEW

Laminar shock/duct flow interaction
 $M=2.03$, $Re_w=1.666 \times 10^5$, $\alpha=3.06^\circ$, $\delta/w=.00484$

Fig. 22

SHOCK/LAMINAR DUCT FLOW INTERACTION ($M=2.03$, $\alpha=3.06^\circ$)
 VELOCITY IN XY-PLANE ($Z/W=.000022$)

EXPANDED VIEW

Laminar shock/duct flow interaction
 $M=2.03$, $Re_w=1.666 \times 10^5$, $\alpha=3.06^\circ$, $\delta/w=.00484$

Fig. 23

SHOCK/LAMINAR DUCT FLOW INTERACTION ($M=2.03$, $\alpha=3.06^\circ$)VELOCITY IN XZ-PLANE ($Y/W=.01136$)

EXPANDED VIEW

Laminar shock/duct flow interaction

 $M=2.03$, $Re_w=1.666 \times 10^5$, $\alpha=3.06^\circ$, $\delta/w=.00484$

— Incident shock line
 - - Shock impinging line
 . . . Reflected shock line

Fig. 24

SHOCK/LAMINAR DUCT FLOW INTERACTION ($M=2.03$, $\alpha=3.06^\circ$)
 VELOCITY IN XZ-PLANE ($y/w=.00523$)

EXPANDED VIEW

Laminar shock/duct flow interaction
 $M=2.03$, $Re_w=1.666 \times 10^5$, $\alpha=3.06^\circ$, $\delta/w=.00484$

— Incident shock line
 — — Shock impinging line
 - - - Reflected shock line

Fig. 25

SHOCK/LAMINAR DUCT FLOW INTERACTION ($M=2.03$, $\alpha=3.06^\circ$)
 VELOCITY IN XZ-PLANE ($\gamma/W=.00306$)

EXPANDED VIEW

Laminar shock/duct flow interaction
 $M=2.03$, $Re_w=1.666 \times 10^5$, $\alpha=3.06^\circ$, $\delta/w=.00484$

— Incident shock line
 — Shock impinging line
 - - - Reflected shock line

Fig. 26

SHOCK/LAMINAR DUCT FLOW INTERACTION ($M=2.03$, $\alpha=3.06^\circ$)VELOCITY IN XZ-PLANE ($\gamma/w=.000066$)

EXPANDED VIEW

Laminar shock/duct flow interaction

 $M=2.03$, $Re_w=1.666 \times 10^5$, $\alpha=3.06^\circ$, $\delta/w=.00484$

— Incident shock line
 — — Shock impinging line
 - - - Reflected shock line

Fig. 27

SHOCK/LAMINAR DUCT FLOW INTERACTION ($M=2.03$, $\alpha=3.06^\circ$)
 VELOCITY IN XZ-PLANE ($y/w=.000044$)

EXPANDED VIEW

Laminar shock/duct flow interaction
 $M=2.03$, $Re_w=1.666 \times 10^5$, $\alpha=3.06^\circ$, $\delta/w=.00484$

— Incident shock line
 — — Shock impinging line
 - - - Reflected shock line

Fig. 28

SHOCK/LAMINAR DUCT FLOW INTERACTION ($M=2.03$, $\alpha=3.06^\circ$)
 WALL PRESSURE ON TUNNEL FLOOR

Laminar shock/duct flow interaction
 $M=2.03$, $Re_w=1.666 \times 10^5$, $\alpha=3.06^\circ$, $\delta/w=.00484$

— — — Inviscid pressure profile

Fig. 29

SHOCK/LAMINAR DUCT FLOW INTERACTION ($M=2.03$, $\alpha=3.06^\circ$)
 SKIN FRICTION ON TUNNEL FLOOR

Laminar shock/duct flow interaction
 $M=2.03$, $Re_v=1.666 \times 10^5$, $\alpha=3.06^\circ$, $\delta/w=.00484$

— — — Inviscid pressure profile

Fig. 30

SHOCK/LAMINAR DUCT FLOW INTERACTION ($M=2.03$, $\alpha=3.06^\circ$)
 WALL PRESSURE AND SKIN FRICTION DISTRIBUTION ($Z/W=.486$)

Laminar shock/duct flow interaction
 $M=2.03$, $Re_w=1.666 \times 10^5$, $\alpha=3.06^\circ$, $\delta/w=.00484$
 — — — Inviscid pressure profile
 □ Experiment [3] (2-d flat plate)

Fig. 31

SHOCK/LAMINAR DUCT FLOW INTERACTION ($M=2.03$, $\alpha=3.06^\circ$)
 WALL PRESSURE AND SKIN FRICTION DISTRIBUTION ($Z/W=.00568$)

Laminar shock/duct flow interaction
 $M=2.03$, $Re_w=1.666 \times 10^5$, $\alpha=3.06^\circ$, $\delta/w=.00484$
 — — — Inviscid pressure profile

Fig. 32

SHOCK/LAMINAR DUCT FLOW INTERACTION ($M=2.03$, $\alpha=3.06^\circ$)
 WALL PRESSURE AND SKIN FRICTION DISTRIBUTION ($Z/W=.000022$)

Laminar shock/duct flow interaction
 $M=2.03$, $Re_w=1.666 \times 10^5$, $\alpha=3.06^\circ$, $\delta/w=.00484$
 — — — Inviscid pressure profile

Fig. 33

SHOCK/LAMINAR DUCT FLOW INTERACTION ($M=2.03$, $\alpha=3.06^\circ$)
STREAMLINES

Laminar shock/duct flow interaction
 $M=2.03$, $Re_w=1.666 \times 10^5$, $\alpha=3.06^\circ$, $\delta/w=.00484$

Fig. 34

SHOCK/LAMINAR DUCT FLOW INTERACTION ($M=2.03$, $\alpha=3.06^\circ$)
 VELOCITY IN YZ-PLANE ($x/w=.0366$)

Laminar shock/duct flow interaction
 $M=2.03$, $Re_w=1.666 \times 10^5$, $\alpha=3.06^\circ$, $\delta/w=.00484$

— — Incident shock line

Fig. 35

SHOCK/LAMINAR DUCT FLOW INTERACTION ($M=2.03$, $\alpha=3.06^\circ$)
 VELOCITY IN YZ-PLANE ($X/W=1.063$)

Laminar shock/duct flow interaction
 $M=2.03$, $Re_w=1.666 \times 10^5$, $\alpha=3.06^\circ$, $\delta/w=.00484$

— — Incident shock line

Fig. 36

SHOCK/LAMINAR DUCT FLOW INTERACTION ($M=2.03$, $\alpha=3.06^\circ$)VELOCITY IN YZ-PLANE ($x/w=1.43$)

Laminar shock/duct flow interaction
 $M=2.03$, $Re_w=1.666 \times 10^5$, $\alpha=3.06^\circ$, $\delta/w=.00484$

— — Incident shock line

Fig. 37

SHOCK/LAMINAR DUCT FLOW INTERACTION ($M=2.03$, $\alpha=3.06^\circ$)VELOCITY IN YZ-PLANE ($X/W=1.796$)

Laminar shock/duct flow interaction
 $M=2.03$, $Re_w=1.666 \times 10^5$, $\alpha=3.06^\circ$, $\delta/w=.00484$

— — Incident shock line

Fig. 38

SHOCK/LAMINAR DUCT FLOW INTERACTION ($M=2.03$, $\alpha=3.06^\circ$)
 VELOCITY IN YZ-PLANE ($X/W=2.1624$)

Laminar shock/duct flow interaction
 $M=2.03$, $Re_w=1.666 \times 10^5$, $\alpha=3.06^\circ$, $\delta/w=.00484$
 — — Reflected shock line

Fig. 39

SHOCK/LAMINAR DUCT FLOW INTERACTION ($M=2.03$, $\alpha=3.06^\circ$)
 VELOCITY IN YZ-PLANE ($X/W=2.529$)

Laminar shock/duct flow interaction
 $M=2.03$, $Re_w=1.666 \times 10^5$, $\alpha=3.06^\circ$, $\delta/w=.00484$

— — Reflected shock line

Fig. 40

SHOCK/LAMINAR DUCT FLOW INTERACTION ($M=2.03$, $\alpha=3.06^\circ$)
 VELOCITY IN YZ-PLANE ($x/w=2.89$)

Laminar shock/duct flow interaction
 $M=2.03$, $Re_w=1.666 \times 10^5$, $\alpha=3.06^\circ$, $\delta/w=.00484$
 — — Reflected shock line

Fig. 41

SHOCK/LAMINAR DUCT FLOW INTERACTION ($M=2.03$, $\alpha=3.06^\circ$)

SPANWISE VELOCITY FIELD

(Vortex Structure near the Corner)

Laminar shock/duct flow interaction
 $M=2.03$, $Re_w=1.666 \times 10^5$, $\alpha=3.06^\circ$, $\delta/w=.00484$

Fig. 42

4.4 SHOCK WAVE/TURBULENT BOUNDARY LAYER INTERACTION IN A DUCT

Similarly to the geometry of the computation of interaction of shock wave and laminar boundary layer in section 4.3, a boundary layer profile based equations (A-8), (A-9) and (A-10) are specified at the inlet cross section. All the lengths from the wall such as ' y ' and ' δ ' used in the computation are replaced by wall distances weighted as defined in (4-1).

The variables at the top boundary are set to either inlet flow values or values for the given shock strength with a boundary layer profile specified near the wall based on the flow values after the given shock. A no-slip condition was enforced on the tunnel floor and the side wall were considered as wall surfaces. A symmetry condition was applied at the center plane of the duct. A zeroth-order extrapolation was used at the exit boundary. The reflected shock was arranged to exit through the exit plane.

40 x 40 x 40 grid points are used in this

calculation. The grids are uniformly distributed in x direction with $\Delta x/w = .029$. In the y direction, from the first grid point to the twenty seventh grid point from the ground and side walls, Δy and Δz were stretched following equation (3-45) and (3-46). The grid prints farther from the walls are equally distributed with $\Delta y_{\max}/w = .0321$ $\Delta z_{\max}/w = .0321$. Near the wall, the grid was stretched with $\Delta y_{\min}/\Delta y_{\max} = .0057$ and $\Delta z_{\min}/\Delta z_{\max} = .0057$. We specified a computational domain of $x_{\max}:y_{\max}:z_{\max} = 1.15:.5:.5$.

The Reynolds number based on duct width is 1.36×10^7 , the Mach number is 2.9, and the deflection angle is 12.99° . The inlet boundary layer thickness δ/w is .0712. In accordance with Knight [21], we used $C_{cp} = 2.08$, $C_{kleb} = 0.3$.

The numerical results show that the fundamental interaction structure of the turbulent flow is similar to that of the laminar flow. Figure 43 shows a clear three dimensional separation bubble around the center of the wind tunnel. From the detailed cross section view (Figure 45 to 48), one observes

that at the cross section at around eight percent of the duct width from the side wall, the reverse flow no longer exists. The side wall separation is much weaker in the turbulent case than in the laminar flow. Using the same ideas as we discussed in laminar flow, the flow near the side wall redirects itself along the incident and reflected shocks (Figure 53). Near the corner and before the shock impinging point, reverse flow exists (Figure 50 to Figure 53). From the top views (Figure 54 to Figure 58), one observes two vortex structures. If one compares the turbulent vortex pairs (Figure 54) to laminar vortex pairs (Figure 27), the center vortex of turbulent flow has a more curved shape than the laminar flow. The horseshoe vortex pair qualitatively similar to both Reda, Murphy's and Jurkovich's results. The higher cross section has a central vortex pair closer to the center line than the lower cross sections (Figure 54 and Figure 55), as expected. The cross sections near the wall (Figure 57 to Figure 58) show the flow from the corner flows into the central separation region. The streamlines near the tunnel floor (Figure 64)

show the center separation structure three dimensionally. The upstream flow near the corner travels under the central separation and spins close to the center line helically then exits near the center line. The upstream flow farther away from the corners is lifted by the separation bubble and travels through the exit cross section. The wall pressure gradient was smoothed out due to corner interactions. In comparison with the experiment, the computational result shows a wider interaction region (Figure 59 to Figure 63). The disagreement may due to the inaccurate turbulent model for separation, the high artificial damping relative to the physical viscosity and coarse grid distribution.

The corner interaction can also be seen in spanwise view. Figure 72 shows the vortex cell growing along the corner. The structure of the corner interaction of the turbulent flow also resembles that of the laminar flow. While the flow interacts with impinging shock, the upper portion of the cross section has higher pressure, the flow direction is downward (Figure 66). The boundary layer flow flows

upward. When they encounter each other, the flow near the wall with less momentum enforces by pressure difference flowing down and the flow outside of boundary layer containing larger momentum is only decelerated by the pressure difference and keeps moving up. This counter direction movement induced a vortex cell which directs flow toward the center line of the tunnel near the wall. Downstream of the shock impinging line, the flow in the lower portion of the cross section contains higher pressure. The flow near the wall which has less momentum is accelerated upward while the outside of boundary layer is still moving down. Thus the corner vortex cell is formed (Figure 67). The cell pushes the flow near the wall further toward the center line. Then the flow encounters center separation which lead the flow circulate three dimensionally as shown on Figure 64. The corner vortex diffused at downstream (Figure 67 to Figure 71).

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)

STREAMWISE VELOCITY FIELD

(Separation Structure near the Center Plane)

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^5$, $\alpha=12.99^\circ$, $\delta/w=.0712$

Fig. 43

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)

STREAMWISE VELOCITY FIELD

(Separation Structure near the Side Wall)

(Z-direction not Scale)

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0712$

Fig. 44

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)VELOCITY IN XY-PLANE ($Z/W=.485$)

EXPANDED VIEW

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0712$

Fig. 45

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)VELOCITY IN XY-PLANE ($Z/W=.338$)

EXPANDED VIEW

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0712$

Fig. 46

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)VELOCITY IN XY-PLANE ($Z/W=.191$)

EXPANDED VIEW

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0712$

Fig. 47

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)VELOCITY IN XY-PLANE ($Z/W=.0775$)

EXPANDED VIEW

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0712$

Fig. 48

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)VELOCITY IN XY-PLANE ($Z/W=.01375$)

EXPANDED VIEW

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0712$

Fig. 49

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)VELOCITY IN XY-PLANE ($Z/W=.00323$)

EXPANDED VIEW

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0712$

Fig. 50

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)VELOCITY IN XY-PLANE ($Z/W=.00073$)

EXPANDED VIEW

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0712$

Fig. 51

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)VELOCITY IN XY-PLANE ($Z/W=.0001353$)

EXPANDED VIEW

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0712$

Fig. 52

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)VELOCITY IN XY-PLANE ($Z/W=.0000168$)

EXPANDED VIEW

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0712$

Fig. 53

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)VELOCITY IN XZ-PLANE ($Y/W=.01835$)

EXPANDED VIEW

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0712$

— Incident shock line
 - - Shock impinging line
 - - - Reflected shock line

Fig. 54

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)VELOCITY IN XZ-PLANE ($y/w=.01376$)

EXPANDED VIEW

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0712$

— Incident shock line
 - - Shock impinging line
 - - - Reflected shock line

Fig. 55

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)VELOCITY IN XZ-PLANE ($y/w=.00432$)

EXPANDED VIEW

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0712$

- Incident shock line
- Shock impinging line
- - - Reflected shock line

Fig. 56

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)VELOCITY IN XZ-PLANE ($y/w=.00099$)

EXPANDED VIEW

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0712$

— Incident shock line
 — — Shock impinging line
 - - - Reflected shock line

Fig. 57

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)VELOCITY IN XZ-PLANE ($Y/W=.0000168$)

EXPANDED VIEW

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0712$

— Incident shock line
 — — Shock impinging line
 - - - Reflected shock line

Fig. 58

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)
 WALL PRESSURE ON TUNNEL FLOOR

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0712$

— — — Inviscid pressure profile

Fig. 59

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)
 SKIN FRICTION ON TUNNEL FLOOR

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0712$

— — — Inviscid pressure profile

Fig. 60

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)
 WALL PRESSURE AND SKIN FRICTION DISTRIBUTION ($Z/W=.485$)

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0712$
 — — — Inviscid pressure profile
 □ Experiment [3] (2-d flat plate)

Fig. 61

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)
 WALL PRESSURE AND SKIN FRICTION DISTRIBUTION ($Z/W=.0032$)

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0712$
 — — — Inviscid pressure profile

Fig. 62

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)
 WALL PRESSURE AND SKIN FRICTION DISTRIBUTION ($Z/W=.0000168$)

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0712$
 — — — Inviscid pressure profile

Fig. 63

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)

STREAMLINES

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0712$

Fig. 64

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)VELOCITY IN YZ-PLANE ($x/w=.0145$)

EXPANDED VIEW

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0712$

— Incident shock line

Fig. 65

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)VELOCITY IN YZ-PLANE ($x/w=.275$)

EXPANDED VIEW

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0712$

— Incident shock line

Fig. 66

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)VELOCITY IN YZ-PLANE ($x/w=.565$)

EXPANDED VIEW

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0712$

— Incident shock line

Fig. 67

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)VELOCITY IN YZ-PLANE ($x/w=.710$)

EXPANDED VIEW

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0712$

— Incident shock line

Fig. 58

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)VELOCITY IN YZ-PLANE ($x/w=.842$)

EXPANDED VIEW

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0712$

— — Reflected shock line

Fig. 69

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)VELOCITY IN YZ-PLANE ($x/w=1.$)

EXPANDED VIEW

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0712$

— — Reflected shock line

Fig. 70

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)VELOCITY IN YZ-PLANE ($X/W=1.145$)

EXPANDED VIEW

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0712$

— — Reflected shock line

Fig. 71

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)

SPANWISE VELOCITY FIELD

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0712$

Fig. 72

4.5 EFFECTS OF BOUNDARY LAYER THICKNESS

From the results of three dimensional shock/duct flow interaction in both laminar and turbulent conditions, one can conclude that the three dimensional vortex pair at the center of the separation structure is strongly related to the side wall and the corner flows. In fact, the formation of the standing vortex pair was mostly due to the corner interaction. Nevertheless, researchers still claim there are "two dimensional" shock and boundary layer interactions in wind tunnels. Jurkovich [4] compared the results of a shock wave interacted with a wind tunnel floor and with a 90 degree rotated wind tunnel floor. One turned out be a three dimensional separation and the other showed a large two dimensional separation area (Figure 73 and Figure 74). What are the critical parameters that control the shape of the center separation? As we described in the first chapter, the flow structure depends on the Reynolds number, the Mach number and certainly the deflection angle. These conclusions follow in both two dimensional and three dimensional flow. This

both two dimensional and three dimensional flow. This work focuses on a few factors that mainly appear in three dimensional flows. First, we are presenting the result of a thinner approach boundary layer with the same shock strength as described in section 4.4.

The computation is based on the same geometry, same grid configuration and same incident shock strength as the interaction of shock wave and three dimensional turbulent boundary layer described in section 4.4. The difference is that the inlet boundary layer thickness at the tunnel floor and side wall is only one third that at the previous case. δ/w in this case is .0237.

Figure 75 shows the structure of the center separation bubble. In comparison with Figure 43, the bubble height is much lower than the flow with thicker approach boundary layer. Observing Figure 77 to Figure 79, the height of the separation bubble is only half of the separation height shown on Figure 44 to Figure 47. Near the side wall, it shows a weaker corner separation on Figure 85 than that on Figure 53.

An obviously structural difference between the two cases is shown on the top views of the separation bubbles. Comparing Figures 86 to 90 with Figures 54 to 58, one finds that by reducing the approach boundary layer thickness, the horseshoe center separation shape faded. A large area of nearly two dimensional center separation built up. The top view shows that due to the weaker corner separation produced by the thinner boundary layer, the corner interaction structure was not strong enough to push the flow from the side wall to the tunnel center line. The vortex pair stood near the side wall; and it has no effect on a large portion of the center separation.

Similar conclusions are also obtained from the streamlines on Figure 96. The streamlines away from the side walls are almost parallel to each other. The streamlines from the inlet corner only reaches a third of the duct width from the side wall. In contrast, Figure 66 shows a dramatic three dimensional separation bubble. The streamlines from the inlet corner form the standing vortex pair which

is under the center separation. The three dimensional shape of the center separation structure was mostly contributed by this vortex pair.

A thinner approach boundary layer also results in a smaller interaction zone. The pressure and skin friction profiles (Figure 91 to Figure 95) show that the pressure jump at the separation area took only half the distance of the flow with the thicker boundary layer. The distance from the separation point to the reattachment point at the center separation area is also half that in the previous case.

With the same mechanism discussed in section 4.3 and 4.4, the corner vortex structure remains the same (Figure 99 to Figure 104). But the corner vortex pair stayed closer to the side wall in this case. When one examines the flow near the tunnel floor, one finds that in the flow with thicker approach boundary layer, the streamlines from the inlet corner (Figure 69 to 73) reaches much farther toward the center line than the flow with thinner approach boundary layer (Figure 99 to Figure 103).

The calculations demonstrate that the shape of the separation bubble strongly depends on the thickness of the approach boundary layer. Shock wave interaction with a thin boundary layer may lead nearly a two dimensional center separation; while a thick boundary layer results in a three dimensional center separation. The qualitative vortex structures in all the cases remain the same.

OIL FLOW PATTERN ($\alpha=8^\circ$, $\delta_{\max}^* = .9 \text{ cm}$, $\delta_{\min}^* = .54 \text{ cm}$)

Floor and corner oil flow pattern for
 8 degrees angle of attack
 Non-uniform approach boundary later
 $\delta_{\max}^* = .9 \text{ cm}$, $\delta_{\min}^* = .54 \text{ cm}$, after Jurkovich [4]

Fig. 73

OIL FLOW PATTERN ($\alpha=8^\circ$, $\delta^*=.54$ cm)

Floor and corner oil flow pattern for
8 degrees angle of attack
Non-uniform approach boundary later
 $\delta^*=.54$ cm, after Jurkovich [4]

Fig. 74

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)

STREAMWISE VELOCITY FIELD

(Separation Structure near the Center Plane)

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^5$, $\alpha=12.99^\circ$, $\delta/w=.0237$

Fig. 75

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)

STREAMWISE VELOCITY FIELD

(Separation Structure near the Side Wall)

(Z-direction not Scale)

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0237$

Fig. 76

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)VELOCITY IN XY-PLANE ($Z/W=.485$)

EXPANDED VIEW

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0237$

Fig. 77

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)VELOCITY IN XY-PLANE ($Z/W=.338$)

EXPANDED VIEW

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0237$

Fig. 78

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)VELOCITY IN XY-PLANE ($Z/W=.191$)

EXPANDED VIEW

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0237$

Fig. 79

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)VELOCITY IN XY-PLANE ($Z/W=.0775$)

EXPANDED VIEW

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0237$

Fig. 80

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)VELOCITY IN XY-PLANE ($Z/W=.01375$)

EXPANDED VIEW

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0237$

Fig. 81

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)VELOCITY IN XY-PLANE ($Z/W=.00323$)

EXPANDED VIEW

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0237$

Fig. 82

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)VELOCITY IN XY-PLANE ($Z/W=.00073$)

EXPANDED VIEW

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0237$

Fig. 83

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)VELOCITY IN XY-PLANE ($Z/W=.0001353$)

EXPANDED VIEW

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0237$

Fig. 84

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)VELOCITY IN XY-PLANE ($Z/W=.0000168$)

EXPANDED VIEW

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0237$

Fig. 85

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)VELOCITY IN XZ-PLANE ($Y/W=.01835$)

EXPANDED VIEW

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0237$

— Insident shock line
 - - Shock impinging line
 - - - Reflected shock line

Fig. 86

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)VELOCITY IN XZ-PLANE ($y/w=.01376$)

EXPANDED VIEW

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0237$

— Insident shock line
 — Shock impinging line
 - - - Reflected shock line

Fig. 87

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)VELOCITY IN XZ-PLANE ($y/w=.00432$)

EXPANDED VIEW

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0237$

—— Insident shock line
 - - - Shock impinging line
 - - - Reflected shock line

Fig. 88

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)VELOCITY IN XZ-PLANE ($y/w=.00099$)

EXPANDED VIEW

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0237$

—— Insident shock line
 - - Shock impinging line
 - - - Reflected shock line

Fig. 89

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)VELOCITY IN XZ-PLANE ($Y/W=.0000168$)

EXPANDED VIEW

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0237$

— Insident shock line
 — Shock impinging line
 - - - Reflected shock line

Fig. 90

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)
 WALL PRESSURE ON TUNNEL FLOOR

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0237$
 — — — Inviscid pressure profile

Fig. 91

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)
SKIN FRICTION ON TUNNEL FLOOR

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0237$

— — — Inviscid pressure profile

Fig. 92

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)WALL PRESSURE AND SKIN FRICTION DISTRIBUTION ($Z/W=.485$)

Turbulent shock/duct flow interaction

 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0237$

— — — Inviscid pressure profile

□ Experiment [3] (2-d flat plate)

Fig. 93

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)
 WALL PRESSURE AND SKIN FRICTION DISTRIBUTION ($Z/W=.0032$)

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0237$
 — — — Inviscid pressure profile

Fig. 94

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)
 WALL PRESSURE AND SKIN FRICTION DISTRIBUTION ($Z/W=.0000168$)

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0237$
 — — — Inviscid pressure profile

Fig. 95

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)
STREAMLINES

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0237$

Fig. 96

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)VELOCITY IN YZ-PLANE ($x/w=.0145$)

EXPANDED VIEW

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0237$

— — Incident shock line

Fig. 97

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)VELOCITY IN YZ-PLANE ($x/w=.275$)

EXPANDED VIEW

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0237$

— Incident shock line

Fig. 98

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)VELOCITY IN YZ-PLANE ($x/w=.565$)

EXPANDED VIEW

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0237$

— — Incident shock line

Fig. 99

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0237$

— — Incident shock line

Fig. 100

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)VELOCITY IN YZ-PLANE ($X/W=.842$)

EXPANDED VIEW

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0237$

— — Reflected shock line

Fig. 101

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0237$

— — Reflected shock line

Fig. 102

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)VELOCITY IN YZ-PLANE ($x/w=1.145$)

EXPANDED VIEW

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0237$

— — Reflected shock line

Fig. 103

SHOCK/TURBULENT DUCT FLOW INTERACTION ($M=2.9$, $\alpha=12.99^\circ$)

SPANWISE VELOCITY FIELD

(Vortex Structure near the Corner)

Turbulent shock/duct flow interaction
 $M=2.9$, $Re_w=1.36 \times 10^7$, $\alpha=12.99^\circ$, $\delta/w=.0237$

Fig. 104

4.6 EFFECTS OF NON-UNIFORMITY OF APPROACH BOUNDARY LAYER

Jurkovich [4] measured the profile of the approach boundary layer thickness across the wind tunnel floor. He observed that the three dimensional horseshoe separation occurred with a non-uniform approach boundary layer. By rotating the shock generator 90 degrees to allow the shock to impinge on the side wall of the wind tunnel, the approach boundary layer was found uniform and also thinner, the separation on that side wall turned out to be mostly two dimensional.

A case of a shock/turbulent boundary layer interaction with a non-uniform approach boundary layer is tested. The profile of the inlet boundary layer thickness of this case is plotted on Figure 105. It has the same maximum inlet boundary layer thickness as the thickness in the case described in section 4.4, $\delta_{\max}/w = .0712$. The ratio of the boundary thickness along the inlet tunnel floor to the maximum inlet boundary layer thickness was specified as the same as the profile reported on

Jurkovich's thesis [4]. The computation was run under the same conditions as the case in section 4.4 except the spanwise distribution of the inlet boundary layer thickness. The results show no essential differences from that of the flow with a uniform approach boundary layer. Whether these results are physical and how much of the numerical damping effects the results requires further investigation. From the numerical results, the non-uniform approach boundary layer did not change any of the vortex structure shown in the previous case. Thus it appears that the non-uniformity the approach boundary layer is not the fundamental cause of the three dimensional shape of the separation bubble.

As we displayed in section 4.5, a thinner approach boundary layer thickness forms an almost two dimensional separation. A similar reason could be the explanation for Jurkovich's experimental results. The flow shows two dimensional separation bubble in fact has a thinner approach boundary layer. The thickness ratio of his two experiments is .602.

LATERAL DISTRIBUTION OF INLET
APPROACH BOUNDARY LAYER THICKNESS

Fig. 105

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

The numerical results of this work show the phenomena of the separated flow that is consistent with experimental observation. The standing vortex pairs, the center separation bubble and corner separation bubbles, the shock angle and flow direction change near the side walls, all these structures have been witnessed in experimental studies. The test cases shows the computation code employed is applicable for simulating the qualitative behavior of the three dimensional flow under investigation.

The numerical results also demonstrate that the basic interaction structure remains same for both laminar and turbulent flow. The large standing vortex pair exist in both laminar and turbulent shock/duct flow interactions. The three dimensional flow structure exhibits standing vortices and a horseshoe shaped separation bubble strongly related to the

interaction between shock wave and corner flow. A critical factor in determination of the shape of the center separation structure is the thickness of the approach boundary layer. With a thick approach boundary layer, the center separation forms a three dimensional horseshoe shape; while with a thin approach boundary layer, the center separation appears to nearly two dimensional. From the numerical results, the non-uniformity of the approach boundary layer dose not appreciably change the structure of the interaction; more study is needed to verify this results.

Further efforts on turbulent modeling are necessary for quantitative investigation. More sophisticated experiments on the corner flow structure will be necessary to verify the corner vortex structure presented in this work.

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APPENDIX A

INLET TURBULENT BOUNDARY LAYER PROFILE

The inlet conditions for the computation of shock waves and turbulent boundary layer are based on empirical laws for two-dimensional turbulent boundary layer profiles for zero pressure gradient flows.

The streamwise velocity profiles are usually considered to be represented in terms of two similarity laws, the law of the wall and the law of the wake. The law of the wall is formulated as

$$u^+ = \frac{1}{\kappa} \ln y^+ + c \quad (A-1)$$

where

$$u^+ = \frac{u}{u_\tau} \quad (A-2)$$

and

$$y^+ = \frac{y u_\tau}{\nu_w} \quad (A-3)$$

The friction velocity, u_τ , is defined by

$$u_\tau = \sqrt{\frac{\tau_w}{\rho_w}} \quad (A-4)$$

Nominal values of the constants are $\kappa = 0.4$,

and $c = 5.1$. Here the Van Driest's velocity with compressibility effect is used

$$u^* = \frac{u_e}{\sqrt{\sigma}} \arcsin\left(\sqrt{\sigma} \frac{u}{u_e}\right) \quad (A-5)$$

where

$$\sigma = \frac{[(\gamma-1)/2] M_e^2}{1 + [(\gamma-1)/2] M_e^2} \quad (A-6)$$

The law of wake can be expressed as

$$\frac{u_e^* - u^*}{u_\tau} = - \frac{1}{\kappa} \ln\left(\frac{y}{\delta}\right) + C (2 - w) \quad (A-7)$$

where $C = 1.25$, δ is boundary layer thickness and w is Coles's tabulated wake function [27].

In the laminar sublayer, for $y^* < 6.0$, the laminar equation is used

$$u^* = y^* \quad (A-8)$$

In the overlap and outer layers, Maise and McDonald [28] followed by Mathews and Childs [29] suggested a general form of the wall-wake profile for compressible flow

$$\frac{u}{u_e} = \frac{1}{\sqrt{\sigma}} \sin \left\{ \arcsin \sqrt{\sigma} \left(1 + \frac{1}{\kappa} \frac{u_\tau}{u_e} \ln \left(\frac{y}{\delta} \right) - 1.25 \frac{u_\tau}{u_e} \left(1 + \cos \frac{\pi y}{\delta} \right) \right) \right\} \quad (\text{A-9})$$

The static temperature in two-dimensional boundary layer is assumed given by the Crocco-Busemann equation

$$T = T_w + (T_{aw} - T_w) \frac{u}{u_e} - \frac{r u^2}{2 C_p} \quad (\text{A-10})$$

where the adiabatic wall temperature is computed, assuming a recovery factor of 0.896, from

$$T_{aw} = T_e + \frac{0.896 u_e^2}{2 C_p} \quad (\text{A-11})$$