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THE EFFECTS OF DIFFERENTIALLY COMPUTED
FACTOR SCORES ON STATISTICAL DECISIONS

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ABSTRACT

Five factor score algorithms were investigated in order to determine whether they yielded composites which produced different statistical results. Factor scores generated via the ideal variable, regression, Z-sum, raw-sum, and fractional weight computational procedures were used, independently, as dependent variables in 117 univariate analyses of variance, 18 multivariate analyses of variance, 18 multiple discriminant analyses, and 14 canonical correlation analyses. Significance probabilities and their associated F-ratios and chi-square values were examined across scoring methods to determine which of the differentially computed scores produced the smallest probability in each statistical test. Obtained frequencies (of smallest p-values) were submitted to an appropriate chi-square test to contrast the effectiveness of the scoring procedures in the various statistical analyses.

The results indicated no significant difference (at $\alpha = .05$) in the observed versus expected frequencies across the five factor score algorithms in the univariate or multivariate tests. However, a recurrent trend evidenced in the data suggests that the regression factor score technique may be the most effective way to compute subjects' factor scores when they are to be subsequently used as criterion variables in statistical analyses.

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Chapter I

THEORETICAL BACKGROUND

The purpose of this study was to investigate the relative effects of different factor score procedures on the results of statistical analyses. If it could be demonstrated that the scoring method did not significantly influence the statistical decision, then researchers would be legitimate in selecting the most convenient factor score algorithm to calculate their composites. However, if different decisions were reached because of the method used to derive the criterion variable(s), then investigators would be compelled to defend their choice in light of the available evidence.

INTRODUCTION AND STATEMENT OF THE PROBLEM

Factor analysis has been used by communication researchers for two purposes: they have factor analyzed a battery of variables to discover the underlying meaning(s) of the set; and they have employed the technique to parsimoniously reduce the amount of data collected on the cases. These dual intents are commonly used in conjunction with one another in empirical investigations in the field of speech

communication. A domain of variables is factor analyzed to determine its structure. Then the obtained structure is manipulated to compute factor scores for inputting into various statistical tests. As Kahn (1973) points out:

... the purpose of factor analysis is not only to reduce the dimensionality of an observed R matrix and interpret the emerged dimensions, but to relate scores on these dimensions to scores on a new set of variables to test hypotheses of scientific interest (p. 103).

In essence, factor scores represent a subject's measurements on the underlying dimensions. Through a series of transformations, individual responses on the n variables are changed into p responses on the factors. There are a variety of ways to make the data transformation, but each method involves weighting the raw scores (which are often standardized) and then linearly combining them to obtain a single composite score. Algebraically, the basic factor score equation is:

$$F = ZW$$

where:

F = an N by p matrix of factor scores

Z = the obtained scores on the n variables

W = an n by p matrix of weights.

Much has been written about the advantages of using functional linear composites in statistical tests in place of single variable scores. Two of the most salient criticisms leveled against individual items is that they contain considerable specificity, and are imbued with measurement error. That is, individual variables not only correlate with the attribute being measured, but likewise relate to other attributes. Moreover, single variables (i.e. variables that are not linear combinations) contain a degree of random error which makes them unreliable measures.

Both of these problems can be reduced by computing composite scores. Specificity of items is averaged out when several variables are combined; and measurement error diminishes as the number of variables in a combination increases. Hence, the most sensible way to measure attributes of people is to sum scores on variables (Nunnally, 1967, p. 74).

The researcher, however, is still faced with a problem. Which competing factor score algorithm should be used to compute the composite variables?

THEORETICAL DEVELOPMENT AND REVIEW OF RELEVANT LITERATURE

There are a number of recommended techniques for calculating such scores, many of which rest upon different theoretical bases and produce composites with varying characteristics. Factor score procedures are generally dichotomized as being exact or inexact. The categories differ in two major respects: The exact methods use the factor loading matrix as it is given by the usual calculations in factor analysis, and also utilize a least squares procedure (involving the computation of an inverse) to minimize the error of estimation. By contrast, the inexact factor score techniques are not based upon a least squares procedure, and do not necessarily make use of the maximum amount of information provided in the factor loading matrix (Horn, 1965).

Three of the exact factor score algorithms have gained considerable attention among empirical researchers. They are all basically variations on the same theme. That is, each is derived mathematically from the basic equations of factor analysis. Assuming a principal components model, the

primary equation is:

$$Z = FP'$$

where:

Z = Standard score data matrix (N individuals by n variables)

F = Scores for the N individuals on the p factors

P = Factor structure matrix containing correlations between the variables and the factors.

The above formula implies that the raw score data matrix can be perfectly reproduced by finding the product of two basic matrices: (1) the transpose of the factor loading matrix, and (2) the factor score matrix. It further indicates that the variables were perfectly measured and contain no unique factor variance. Therefore, with minor manipulations, $Z = FP'$ will yield component scores that are directly determinable from the scores on the observed variables. This is true for all models of component analysis. The three exact techniques that are commonly used to calculate component

scores follow:

$$(1) F = ZP^{-1}$$

This solution for component scores is only applicable when the model is of full rank, i.e. all of the principal components have been extracted. When this is the case, p will be a square, non-singular matrix and an inverse can be calculated.

When it is desirable to extract only a few of the larger components (truncated model) then the idealized variable equation may be solved for the component scores:

$$(2) F = ZP(P'P)^{-1}$$

And, finally, perhaps the best known of the exact procedures which can operate on a full or reduced rank model, states the problem as one of linear multiple regression analysis (Horn, 1965):

$$(3) F = ZR^{-1}P$$

where: R = correlations among the variables.

Again, it should be noted that for each of the preceding formulas, the component scores are described mathematically with no question of statistical estimation. Another way of stating it is that given equations (1) through (3) above and the principal components model, the resultant set of scores are unique and invariant. This means for the observed matrix of raw data and obtained factor coefficient matrix, the produced component scores are the only set that is possible. Moreover, except for rounding error, the composites that are calculated by the exact equations (assuming principal components) will be identical (Gorsuch, 1974, p.228).

Factor scores that are determined via an exact principal components model have the additional advantage of embodying all of the desired characteristics of constructed factor scores. McDonald and Burr (1967) summarize these properties:

First, constructed factor scores should have high correlations with the factors that they are measuring; second, in the case of an orthogonal factor solution, the factor scores should be uncorrelated (orthogonality); third, the constructed factor scores need to be conditionally unbiased estimators of the corresponding true factor scores; and fourth, if the factors are orthogonal, the constructed

factor scores should have zero correlations with noncorresponding factors (univocality) (p.382).

The ideal variable and regression formulas can be used to compute factor scores given the common factor analysis model as well. With this model, the primary equation of factor analysis is expanded to include both common and unique parts, as reflected in the following equation:

$$Z = F_c P' + U f_u$$

where:

Z = an n by N matrix of standardized variable scores

F_c = an m by N matrix of common factor scores

P = an n by m matrix of common factor loadings

U = an n by n diagonal matrix of uniqueness

f_u = an m by N matrix of unique factor scores.

The unique portion of the preceeding equation is not determinable, and consequently, the matrix of standardized variable scores cannot be exactly determined. Instead, the common part of Z is determined by finding the product of P and P'.

$$Z_c = F_c P'$$

However, the common parts of the data are likewise unavailable, only the obtained Z matrix can be employed in the factor score calculations. To the extent that Z_c does not equal Z, the estimated composite variables will only approximate the 'theoretical true factor scores.' Moreover, it has been further demonstrated that there are a number of legitimate factor score matrices that exist which can be input as the prefactor in the equation to solve for Z_c . For these reasons, common factor scores are indeterminate. They are considered only estimates of the undefined theoretical scores.

Only the underlying assumptions of the ideal variable and regression factor score algorithms are altered given the common factor analysis model. The two equations remain the same:

$$\text{(Ideal Variable)} \quad -- \quad F = ZP(P'P)^{-1}$$

$$\text{(Regression)} \quad -- \quad F = ZR^{-1}P$$

However, unlike their principal component counterparts, these estimates do not analogously satisfy all of the conditions specified by McDonald and Burr (1967) for good factor scores. Gorsuch (1974) criticizes the factor scores estimated by the ideal variable equation on the four criteria:

The ideal variable equation will generally give factor scores which correlate well with the desired factors. Idealized variable estimates of common factor scores are univocal and are considered unbiased estimators. However, the common factor scores will often correlate among themselves even though the factors are uncorrelated (p. 232).

Similarly, regression estimators differ from their principal component counterparts on the desired characteristics. Whereas regression component scores fulfill each of the four properties, regression estimators suffer from two defects: (1) the factor scores are intercorrelated, and (2) they are contaminated by non-zero correlations with factors other than the one they were designed to estimate. These dual problems are more than simply a nuisance for researchers. They seriously confound the simplicity and clarity of

interpretation afforded by an orthogonal factor solution. Guilford (1951) offers an explanation for the difficulties encountered by regression estimates:

There are several reasons for the correlation of factor scores. One is the fact of intercorrelations of the factors themselves. In addition...there is the overlap of specific and error variances. ..contributing to the correlations among the scores of the factors. There is also the fact that the items themselves are factorially complex. The items designed to measure factor A, on the whole, may be frequently involved with factor B and items designed to measure factor B may be involved with factor A.

Guilford concludes:

When the intercorrelations of factor scores are distorted by all of the sources of error that have just been described...it can be seen that an analysis will yield results that are difficult to interpret, at best (p. 30).

Nevertheless, the choice of any factor score algorithm entails certain weaknesses which the researcher should compare with the solution's strengths. In the present case, although regression estimators sacrifice univocality and orthogonality, they have the advantage of possessing the highest correlation with the actual factor, i.e., the investigator achieves the maximum validity by utilizing regression estimates as his composite scores.

If factor scores are to be computed for the same set of subjects on which the initial factor analysis was conducted, then an exact procedure is the investigator's 'best' choice for obtaining composite scores. The justification for this decision is that wherever scores are computed on the same set of individuals that were used to validate the instrument (i.e. determine the structure of a battery of variables) there is bound to be error variation included in the measurement, regardless of the factor score algorithm employed. Therefore, if the composite variables are to contain error no matter if an exact or inexact technique is implemented, it's advisable to use an exact procedure which makes use of the maximum amount of information provided in the data. The exact procedures result in factor scores that give the best representation of the data matrix.

On the other hand, if factor scores are to be used on a different sample of individuals with a subset of the variables, an exact factor score technique is not the most appropriate procedure. There are several reasons why the complete methods are not legitimate in this case. First, the exact factor score algorithms use all of the variables to compute the composite scores. This practice not only violates the principle of parsimony, but also contradicts the purpose of the initial factor analysis which was to reduce the rank of the data while maintaining a maximum

amount of information. A second explanation of why exact factor score algorithms are not suited to a reduced variable set and another sample of individuals is that the exact factor score procedures capitalize on chance variation in the initial analysis. Alwin (1973) explains the importance of this error of estimation:

Factor pattern matrices are subject to sampling error as are any parameter estimates based on sample data. To the extent that the sample correlations are due to chance variation, the coefficients in the factor pattern will also be subject to sampling error (p. 206).

Hence, because exact factor score methods capitalize on chance variation and require the complete variable set for computation, an inexact procedure is recommended when factor scores are to be used with another sample of subjects and a reduced set of variables. Inexact techniques utilize a simplistic weighting schema which reduces error variation, and they demand only a subset of the original n variables in order to approximate the factor scores.

One of the most commonly used of the inexact techniques, and the object of much criticism in the literature, uses the complete factor loading matrix as the weight matrix for estimating the factor scores. Harris (1967) calls this procedure "a kind of 'quick and dirty' method that seems right even though it actually is wrong most of the time"

(cited in: Alwin, 1973, p. 199). Using the complete factor loading matrix is an incorrect way of approximating factor scores, however, not for the reasons posited by Glass and Maquire (1966). The authors claim that estimates computed in this fashion are 'bogus factor scores' because "they can have large non-zero intercorrelations" (p. 298) and "erratic correlations with the correctly computed vari-max factor scores" (p. 301). These criticisms should be viewed as faults associated with the technique, but are not sufficient to call the method illegitimate. Indeed, not every factor score algorithm maintains orthogonality and univocality.

Instead, the method lacks justification on the basis of the factor analytic model ($Z = FP$). The equation illustrates that the intact factor loading matrix supplies the weights to reproduce the variables. However, it is not proportional to the weight matrix used to estimate the factors from the variables (Harris, 1971, p. 361). For this reason, then, $F = ZP'$ should not be a preferred method for approximating common factor scores.

Two inexact factor score procedures which are important in many research situations and are often as accurate as the exact techniques are presented below. For both methods, salient variables (cutoff value determined by the research-

er) are distinguished from the nonsalient ones by inspection of the factor loading matrix or the weight matrix of one of the more exact procedures. For the first inexact method, fractional weights, which are equal or proportional to the corresponding factor loading coefficients, are used to weight the salient variables before they are linearly combined into the composite score. Nonsalient items are either weighted zero or else they are discarded from the analysis. The second inexact procedure simply involves summing the standardized raw scores of the variables that are salient on a given factor. In this instance, the weight matrix would consist only of the elements one and zero. Salient variables are weighted one; nonsalient variables zero.

Considerable controversy has developed over the issue of fractional versus unit weights. Fractional weighting implies that there are significant differences between salient variables. Therefore, the factor score should reflect these differences by being more dependent upon those salient variables with high correlations with the factors than it is with the salients which have loadings of a lesser magnitude (Gorsuch, 1974, p. 238).

Although it has been demonstrated that fractional weighting improves the reliability and validity of the

composite score, many researchers still suggest that there is little to be gained from differential weighting. Criticism of the practice is found as early as 1941 when Sewell notes that:

The problem of assigning weights to items in a scale is one which is rather annoying but not of great practical significance in light of the roughness of most sociometric devices at the present time. Several studies have shown that essentially the same final results are obtained with arbitrary common sense weighting as with more complicated, but still arbitrary statistical techniques (cited in: Alwin, 1973, p. 209).

There are two types of information which can aid the researcher when deciding whether differential or unit weights should be used to compute his factor scores. A more technical source of help is supplied by Gulliksen (1950):

...from a practical point of view, 50 to 100 variables are probably sufficient to make differential weighting unprofitable, and the same conclusion holds if the variables are very highly correlated. Weighting may be worthwhile... when there are few (3 to 10) variables and when their average intercorrelation is low (.50 or less) (quoted in: Wang and Stanley, 1965, p. 679).

A more commonsense-type recommendation is provided by Alwin (1973):

If a researcher intends to tap some new domain of content and has by virtue of this fact, little prior experience with the particular set of items sampled, he will probably not want to consider weighting his items. In contrast, variables for which there is some consensus among knowledgeable researchers regarding their representation of a conceptual domain may be candidates for differential weighting (pp. 205-6).

Factor scores computed by either of the preceeding inexact techniques have several defects. Although these estimators correlate substantially with the factor they were designed to estimate, they are not orthogonal or univocal. However, as noted earlier, inexact factor score techniques operate on a reduced variable set, and involve a less elaborate weighting system (diminishing the effects of chance variation) which makes for easy computation.

The foregoing exact and inexact factor score procedures are the ones that have been most fully developed in the literature. From a review of the research in which factor scores were computed it appears that investigators are inclined to favor the inexact, unit weight algorithm (Alwin, 1973). Nonetheless, oftentimes a factor score technique is chosen without an accompanying justification for selection, or investigators report that factor scores were calculated but fail to specify how they were derived. These behaviors seem to imply that factor scores computed via the various procedures are equivalent, and therefore, the researcher's

selection is an arbitrary one.

Before adopting this position, however, two questions demand investigation: (1) How well do the factor scores computed by the various procedures correlate with each other? and (2) When input into a statistical model, do the differentially computed factor scores produce results which lead to the same conclusions?

RELEVANT EMPIRICAL RESEARCH

During the last ten years some attention has been directed to the first of these issues. Horn (1965) inter-correlated composites derived by three exact factor score techniques and three inexact techniques. His findings indicated that the factor scores computed by the three exact procedures correlated approximately .90. The variates produced by the inexact procedures were similarly highly correlated. Horn's results further showed that the correlations between the two broad categories of factor scores averaged between .79 and .88. These general findings have been replicated in subsequent studies.

Moseley and Klett (1964) compared factor scores computed by unit weighting, fractional weighting by factor loadings, and weighting by multiple regression analysis. On

the basis of their results the authors deduced that: "the methods of scoring factors are roughly equivalent insofar as the intercorrelations among factor scores or reliabilities are concerned" (p. 179). Furthermore, the correlation coefficients were correspondingly high when Alwin (1973) interrelated factor scores derived by four least square methods and a procedure of equal weighting. Alwin concluded:

... the factor score estimation procedures yield highly intercorrelated scores...the factor scores are highly intercorrelated with the score using equal weights, and...all composite scores yield nearly equivalent correlations with other theoretically relevant variables (p. 208).

On the basis of the preceeding research findings alone, investigators would be justified in selecting any of the factor score algorithms. However, equal attention has not been given to the second of the previously specified research questions, that of the relative effects of the various methods on experimental results. Provided that significant differences do in fact exist between treatment groups, an experimenter is enjoined to employ that (those) statistical technique(s) capable of detecting these differences. It is entirely possible that the various factor score algorithms do not yield composite variables that are equally responsive to mean differences. If this is true,

one way that a researcher can increase his chances in detecting variations is by selecting a factor score procedure which is maximally sensitive to variations between the experimental groups.

If it can be demonstrated that the techniques used to calculate factor scores do not introduce error variation which influences the statistical decision, then researchers are legitimate in selecting the most convenient factor score procedure to compute their composites. However, if different decisions are reached because of the method used to derive the criterion variable(s), then investigators are compelled to defend their choice in light of the available evidence. Kerlinger (1973) addresses the topic as follows:

One goal of science...is to discover the relations among variables. Since unreliable measurement is measurement overloaded with error, the discovery of these relations becomes a difficult and tenuous business...Is an analysis of variance F-ratio not significant because the hypothesized relationship does not exist or because the measure of the dependent variables is too crude, too unreliable? (p. 455).

Only two studies have been undertaken that investigate the effects of the scoring procedure on the statistical outcome. The first study by Dagenais and Marascuilo (1973), explored the effects of factor scores, simple sum scores and Guttman scores on the magnitude of the F-ratio in a one-way analysis

of variance design. In this study, simple sum scores are analogous to the composites derived via the unit weight algorithm; factor scores were computed by the formula, $F = Z P^{-1}$; and Guttman scores were generated by means of the Biomedical computer program, BMD 05S. The authors hypothesized that the F-ratio would not be significantly influenced by scales based on simple sums, factor scores, or Guttman scores.

Three different scales composed of 3, 5, and 7 dichotomous items were scored by each of the three procedures across N subjects. An individual response in the favored direction was scored '1'; in the unfavored direction, '0'.

The obtained data were submitted to a locally available ANOVA program to generate F-ratios and significance probabilities. "Inspection of the probabilities of the resulting F-ratios across the three methods for each group of subjects reveals very little difference in numerical size and no apparent pattern of rank differences" (Dagenais and Marascuilo, 1973, p. 497). 123 F-ratios were produced; 42 of them sufficiently large to reject the null hypothesis at $\alpha = .05$. Of the 123 tests, there was only a discrepancy about the decision in seven cases; in four tests Guttman scoring led to a rejection of the null hypothesis where alternative procedures did not; and in three instances sum scores

yielded an F value high enough to reject H_0 , whereas factor scores and Guttman scores did not. Based on these findings, Dagenais and Marascuilo concluded that:

...variation in scoring does not materially affect the amount of variance explained by the independent variables.. and...within the limits of scaling technique and data used, no one method of deriving scores offers any statistical advantage over the other methods when resultant scores are used in a simple one-way analysis of variance experiment... (1973, pp. 498-500).

Contradictory results were obtained in a second study which investigated the effects of differential factor scoring methods on statistical decisions. In the article, "An Empirical Comparison of Factor Scoring Methods," Moseley and Klett (1964) explored treatment group change as a function of the scoring method employed.

Four-hundred, forty-one schizophrenic patients completed the "Inpatient Multidimensional Psychiatric Scale" before and after eight weeks of treatment in order to determine the effects of six tranquilizing drugs. These data were then input into a common factor analysis followed by a varimax rotation of the ten syndromes previously identified by factor analysis. Three factors were extracted that were interpretable and also accounted for a sizeable portion of the common factor variance. For each subject,

composite scores were computed on all three of the common factors. Six different scoring methods were used to estimate factor scores:

(1) unit weights for all variables having factor loadings $\geq .30$;

(2) unit weights for all variables having factor loadings $\geq .45$;

(3) fractional weights (equal to the factor loadings) for all variables having loadings $\geq .30$;

(4) fractional weights (equal to the factor loadings) for all variables having loadings $\geq .45$;

(5) all variables weighted by their factor loadings;

(6) weights derived for all variables via multiple regression analysis.

Factor scores derived by the above methods were subsequently used as the criterion variables in a one-way analysis of

variance design in which F-ratios were generated for the six drug groups.

The results of the study showed that only the factor scores produced by the regression equation yielded an F value great enough to reject the null hypothesis. Although the composites of the other five methods produced F-ratios that approached the tabled cutoff value, only the regression scores detected the differences operating between the six drug groups. Moreover, even though the numerical value of the difference between the significant and nonsignificant F values is slight, Moseley and Klett claim that the difference "should not be interpreted as being trivial as all methods were derived from the same data and any difference is important when the factor score is being considered as a dependent variable" (1964, p. 183).

Furthermore, when comparisons were made of the two most discrepant drugs, even though all of the scoring methods led to the same decision (reject H_0), the regression procedure yielded the greatest t value. The results of the comparison, likewise, indicated that the higher factor loading cutoff value produced a larger F for both unit and fractional weighting procedures, and that the fractional methods yielded F values which were greater than the ones produced by unit weights. Identical conclusions were reached when

the authors submitted the data to an oblique rotation before factor scores were computed by the six methods..

Based on the results of their study, Moseley and Klett (1964) concluded that:

The evidence does favor the regression approach...in detecting differences among groups when factor scores are to be used as dependent variables. Computational ease, particularly in this era of high speed data processing, should not be the deciding factor in those situations where the regression method has real advantage (p. 184).

LIMITATIONS OF PREVIOUS RESEARCH

The research by Dagenais and Marascuilo (1973), and Moseley and Klett (1964) furnishes the only evidence concerning the impact of the various factor score computational algorithms on statistical significance testing. The generalizability of the foregoing studies is diminished, however, because of a number of limiting characteristics.

The first difficulty associated with the prior research on this topic involves the idea of empirical replication. It is commonly known that the replication of empirical results is crucial to the advancement of scientific knowledge. Only when scientific findings are repeated several

times can one accept the "realness" of the observed differences. Geoffrey Keppel (1973) expresses this point as follows:

...when a particular pattern of results is duplicated in spite of many changes occurring from replication to replication, it must represent a relatively "robust" phenomenon. A significant finding in a single experiment may always be the result of a set of fortuitous circumstances, favoring some conditions over others, producing the particular pattern observed (p. 546).

Both of the studies which investigated the effects of the different scoring methods on statistical decisions were single experiments. Neither the authors of the articles, nor any other researcher, has attempted to replicate the findings generated by the two studies. Hence, it is unknown whether 'fortuitous circumstances' or chance variation was a significant factor in either investigation. Moreover, the value of the available research on this issue is further reduced because of the contradictory conclusions derived by the two independent studies. Dagenais and Marascuilo (1973) found that varying factor score algorithms generated composites which led to the same statistical decision when used as dependent variables in an analysis of variance design. On the other hand, Moseley and Klett (1964) discovered that different decisions were reached depending upon the technique used to calculate the composite variables. Indeed,

additional research is required before any judgment is passed on whether or not the factor score procedure significantly influences statistical results.

Another limitation connected with the past studies is that both employed a univariate analysis of variance. There is no ready information on the effects of the composite scoring technique on the experimental results in a multivariate analysis. More specifically, researchers do not know if or how the different factor score algorithms influence the multivariate F-ratio when used in a MANOVA. Likewise, the impact of the various factor score computational algorithms on results associated with other multivariate techniques, i.e. multiple discriminant analysis and canonical correlation, has yet to be explored.

Finally, no research has been undertaken within the field of Speech/Communication which investigates this methodological issue. Refining experimental instruments is vitally important since the value of any scientific investigation depends upon the adequacy of its tools. Underwood (1957) expresses the idea this way:

Once we have identified the dependent variable, the response, which we wish to investigate, and once we have determined that some level of quantification is feasible, the next step is ascertaining the reliability of the scale. Unfortun-

ately, this is a matter which too many of us overlook and yet as far as the actual research is concerned, it is of the greatest importance. If our response measure is not reliable, no further investigative procedures should be undertaken. Science attempts to discover and understand reproducible phenomena; lack of reliability in our attempts at measurement precludes this reproducibility (p. 22).

Research which is aimed at refining measuring instruments (psychometric research) is just as important as that which advances theory construction. Moreover, methodological research is the responsibility of every discipline. It is not merely the task of the psychometricians and mathematicians; it is an obligation which is likewise imposed upon communication scholars. Researchers in the field of speech oftentimes compute factor scores to be input into a subsequent statistical analysis. Consequently, they should be equally concerned with the effects that the scoring procedures have on their findings. However, to date, no communication scholar has addressed this issue.

STATEMENT OF RESEARCH QUESTIONS

In order to investigate whether or not differentially computed factor scores produce results which lead to the same statistical conclusions, as well as to alleviate the limitations of previous studies in this area, the following

research program was undertaken:

Two exact and three inexact factor score algorithms were compared to determine if they yielded significantly different results when input into a statistical model. The two exact computational procedures employed were : (1) the ideal variable algorithm (given a principal components model -- while the composites generated by this procedure are actually 'component scores,' to avoid confusion they will henceforth be referred to as 'factor scores') and, (2) the regression factor score technique (assuming a common factor analytic model); the three inexact methods were: (1) fractional weights (equal to the rotated factor loadings) for all standardized salient variables, (2) unit weights for all standardized salient variables, and (3) unit weights for all unstandardized salient variables.

The constructed factor scores were input into a univariate analysis of variance, multivariate analysis of variance, multiple discriminant analysis, and a canonical correlation analysis to ascertain what impact (if any) the various methods have on statistical results. More specifically, the investigation was guided by three main research questions:

(1) Does the method used to calculate factor scores have a significant effect on the magnitude of the p-value in a multivariate or univariate one-way analysis of variance design; if so, which composite most frequently yields the smallest probability?

(2) Does the method used to calculate factor scores have a significant effect on the magnitude of the p-value associated with the first discriminant function in a multiple discriminant analysis; if so, which composite most frequently yields the smallest probability?

(3) Does the method used to calculate factor scores have a significant effect on the magnitude of the p-value associated with the first canonical root in a canonical correlation analysis; if so, which composite most frequently yields the smallest probability?

The manner in which this research program was executed will be explained in the following chapter.

Chapter II

METHOD

The purpose of this chapter is to detail the preparatory procedures that were engaged in prior to the factor score computations, outline the scoring techniques used in the calculation of the composite variables, and describe the statistical analyses performed in order to compare the various factor scores. Included in the account will also be an explanation of the type of data (and its source) employed in the computational algorithms. It is this latter issue to which this author shall initially address herself.

DATA SOURCE

Experimental data that was collected at an earlier time was reanalyzed to investigate the questions proposed by this study. This data was gathered over a period of eight years by another researcher, Dr. Raymond K. Tucker (Professor of Speech-Communication at Bowling Green State University), who was investigating experimental hypotheses unrelated to the questions presently under consideration.

Data reanalysis is a practice that has achieved widespread national acceptance. It not only ensures that a maximum amount of information will be extracted from a single body of data, but also leads to the discovery of many valuable insights which were overlooked in the initial investigations. Data reanalysis is recommended by Underwood:

I usually think it is a good idea to analyze the data of completed experiments in many different ways. One may make analyses of subgroups within the general groups or fractionate the data in many other ways to obtain all the information possible from a single experiment (cited in: Wilke, 1971, p. 7).

Tukey (1962) advocates 'praying-over' experimental findings and suggests that progress in the physical sciences would have advanced at a much slower rate without data reanalysis.

Twenty empirical studies within the area of Speech/Communication were reanalyzed in order to determine the effects of the factor score methods on various statistical results. Different numbers of distinct sets of criterion variables, each measuring a different construct, comprised the twenty communication experiments. Seven or more items formed the variable sets, and these items were measured via equal interval scales, the number of steps fluctuating between studies. The number of subjects who completed the

scales likewise varied from study to study. Total N's together with other pertinent information concerning the data is provided in Table 2.1 below:

Table 2.1
Data Summary Table

<u>Study</u>	<u>Total N</u>	<u>Number of DV Sets</u>	<u>Total Number of Variables</u>	<u>Number of Variables in Each DV Set</u>
1	248	3	56	12, 22, 22
2	99	2	22	12, 10
3	56	4	81	10, 12, 24, 35
4	47	2	24	12, 12
5	61	3	36	12, 12, 12
6	19	8	96	12,12,12, 12, 12,12,12,12
7	60	5	66	12,10,15,19,10
8	73	5	66	12,10,15,19,10
9	62	5	66	12,10,15,19,10
10	63	5	66	12,10,15,19,10
11	68	5	66	12,10,15,19,10
12	43	3	36	12,12,12
13	52	3	36	12,12,12
14	24	7	84	12,12,12,12, 12,12,12
15	51	4	48	12,12,12,12
16	66	5	60	12,12,12,12,12
17	49	6	72	12,12,12,12, 12,12
18	60	6	76	10,12,27,7, 10,10
19	200	4	56	12,10,10,24
20	450	1	23	23

PRELIMINARY PROCEDURES

Before the factor scores could be computed via the different computational procedures, and compared in subsequent statistical analyses, two preliminary procedures were required. The prerequisites entailed: (1) discovering a reduced number of highly correlated variables to approximate each factor in the inexact factor score calculations; and (2) identifying those factors whose underlying structures remained basically the same regardless of the type of factor analysis performed on the data.

Salient Variables

Because the inexact factor score procedures operate on a reduced number of highly correlated variables instead of the complete Z matrix, prior to the factor score computations, a small subset of the original n variables was identified to approximate each factor. This entailed two related operations which sought to (1) differentiate salient variables from trivial or nonsignificant ones; and (2) divide salient items into two groups according to their correlations with a factor.

Both of the preceeding data-reducing operations utilize the information provided in the rotated factor loading matrix. Therefore, subjects' responses to each set of criterion variables were input into a principal components factor analysis (orthogonal varimax rotation). Subprogram "Factor" of the Statistical Package For the Social Sciences was used to perform the analysis on each separate set of data. To be considered for rotation, factors were required to have an eigenvalue equal to or greater than 1.00 and also contain at least three variables with correlations of .30 or higher.

Two examinations of the obtained rotated factor loadings were conducted in order to discover the salient variables used in the inexact factor score calculations (one examination by factor, the other by dependent variable). The purpose of the first examination was to distinguish those variables with correlations of .30 or greater from the nonsigniificant elements on each factor. Implementing these results, the next examination compared the salient loadings of each dependent variable across all factors to find that factor with which the variable had its strongest relationship. It was then used only in that factor's measurement, and was regarded as a nonsalient variable, (despite its loading) on all other factors.

One primary advantage of proceeding in this fashion is that an investigator is assured that his obtained set of factor scores is experimentally independent (Gorsuch, 1974, p. 238). To the extent that variables are allowed to relate with a number of factors, spurious correlations will result among the composite scores. Moreover, interpretation of the underlying dimensions likewise becomes more difficult when the same dependent variable is used to define several different factors.

The result of the two preceding inspections of the factor structure matrix was the discovery of a reduced number of highly correlated variables which, theoretically, could be used in the inexact factor score procedures. However, Nunnally (1967), among others, has claimed that it is seldom feasible to use more than three or four variables to calculate composite scores. Moreover, the results of Norman's investigation of factor scores consisting of various numbers of test items indicates that:

...factor scores are more reliable than single ratings and ...most of the gain in test - retest stability is accomplished by averaging just three or four scales (cited in: Heise, 1969, p. 410).

Therefore, instead of inputting all of a factor's salient items into the inexact computational procedures, only five variables were used to approximate the factor. The five included in the factor score estimation were differentiated by rank ordering each factor's salient items according to their rotated factor loadings and then extracting the five most highly correlated variables. If a factor did not contain a total of five salient variables, three or four salients were accepted to measure it. However, it was required that each of the factors included in the factor score calculations contained a minimum of three salient elements. If this criterion was not met, then another principal components analysis was performed on the data set. One less factor was extracted than had been in the original analysis, and the obtained rotated factor loading matrix was again examined by the procedure previously detailed. If the "three-salient-variable" criterion was achieved, the process was complete; if not, the set was reanalyzed until an acceptable factor solution was produced.

Component Versus Common Factor Model: The Determination Of Comparable Factor Structures

Each of the various factor score algorithms uses, completely or in part, information contained in the rotated factor loading matrix to derive its composite scores. However, while some scoring methods utilize the principal

components factor loading matrix to calculate composite scores, others require the common factor structure. For this reason, prior to the actual factor score computations, a principal components analysis and a common factor analysis were performed on the data sets to determine whether they yielded similar factor structures. This comparison was crucial to this research, since obviously, the use of discrepant factor loading matrices in the computational procedures would result in incomparable factor scores. Only when identical salient variables were produced for the respective factors by both analyses was it possible to contrast 'principal components scores' with their corresponding 'common factor analysis scores' in any subsequent statistical test.

Subprogram "Factor" of SPSS generated both the principal components and common factor analysis rotated factor loading matrices which were compared in this final preparatory procedure. Squared multiple correlations served as the initial communality estimates in the common factor analysis model, and a maximum of twenty-five iterations were requested to obtain the final estimates (unity, of course, was inserted in the main diagonal of the correlation matrix in the principal components model). Factors which had eigenvalues of 1.00 or greater and also were composed of a minimum of three salient variables (determined by the

process detailed previously) were extracted and rotated to an orthogonal varimax solution.

For each factor, salient variables generated via a principal components analysis were compared with the corresponding group of salients produced in the common factor analytic model. If the salient variables did not coincide between analyses, the factor in question was totally deleted from this research. Only those factors whose highly correlated variables remained the same, regardless of the type of factor analysis performed on the data, were included in the factor score calculations. Presented in Tables 2.2 -- 2.21 below is the list of factors which were measured via the different scoring procedures and their composites compared in subsequent statistical tests. The reduced set of salient items used to estimate each factor in the inexact procedures are designated with an asterisk. Included in the tables are the rotated factor loadings, the percent of total variance accounted for, and the eigenvalues for each of the respective factors.

Table 2.2
Rotated Principal Components Extracted From Study 1
Data and Used in the Factor Score Calculations

Set 1			Set 2		Set 3	
<u>Variables</u>	<u>Factor 1</u>	<u>Factor 2</u>	<u>Variables</u>	<u>Factor 1</u>	<u>Variables</u>	<u>Factor 1</u>
1	*0.84732	0.10659	13	0.29822	35	0.17486
2	0.70998	0.28370	14	0.30668	36	0.14225
3	*0.84218	0.22088	15	0.65359	37	*0.82448
4	*0.78630	0.30950	16	-0.08461	38	-0.03113
5	0.21367	*0.43732	17	0.15116	39	0.16547
6	0.16304	*0.80871	18	0.14684	40	-0.03403
7	*0.76063	0.19699	19	0.68754	41	0.71823
8	*0.82921	0.27920	20	0.29761	42	0.31805
9	0.27880	*0.77895	21	0.64228	43	0.57813
10	0.74469	0.32293	22	*0.72079	44	*0.77066
11	0.12037	*0.83593	23	*0.79634	45	0.48633
12	0.46995	*0.57269	24	0.30131	46	0.29594
% Total Variation 51.5%			25	*0.77625	47	*0.84522
Eigenvalue 6.19			26	0.29380	48	0.28562
			27	*0.75068	49	*0.84803
			28	0.33277	50	0.37974
			29	*0.75138	51	*0.80810
			30	0.26043	52	0.33737
			31	-0.02482	53	0.16546
			32	-0.01706	54	0.09394
			33	0.14435	55	0.29299
			34	0.59691	56	0.36499
			% Total Variation		% Total Variation	
			36.5%		44.9%	
			Eigenvalue 8.03		Eigenvalue 9.88	

Table 2.3
Rotated Principal Components Extracted from Study 2
Data and Used in the Factor Score Calculations

Set 1	
<u>Variables</u>	<u>Factor 1</u>
13	*0.80541
14	*0.77290
15	*0.82394
16	*0.81587
17	*0.79276
18	0.73795
19	0.71643
20	0.70723
21	0.74796
22	0.74872
% Total Variation	59.0%
Eigenvalue	5.897

Table 2.4
Rotated Principal Components Extracted from Study 3
Data and Used in the Factor Score Calculations

Set 1		Set 2		
<u>Variables</u>	<u>Factor 1</u>	<u>Variables</u>	<u>Factor 1</u>	<u>Factor 2</u>
1	0.88948	11	0.06436	*-0.62472
2	0.88653	12	*0.72370	-0.26401
3	*0.92573	13	0.10954	*-0.56312
4	0.89586	14	0.47003	*-0.56273
5	0.89742	15	*0.69605	0.15371
6	*-0.92034	16	*0.80603	-0.28465
7	-0.79517	17	0.05337	*0.57683
8	*-0.91521	18	-0.19789	*0.74134
9	*-0.91990	19	*-0.89007	0.18701
10	*-0.91120	20	-0.17820	0.34223
		21	*-0.85525	0.12745
		22	-0.62783	0.50455
% Total Variation		% Total Variation		
	80.4%			
Eigenvalue	8.04		40.0%	13.2%
		Eigenvalue	4.80	1.59

Table 2.5
Rotated Principal Components Extracted from Study 4
Data and Used in the Factor Score Calculations

Set 1			
<u>Variables</u>	<u>Factor 1</u>	<u>Factor 2</u>	<u>Factor 3</u>
1	0.17061	0.05840	*0.83312
2	0.27896	0.38130	*0.57110
3	0.14625	*0.61055	0.08976
4	0.08744	*0.87654	-0.00554
5	*0.38094	0.14589	0.28367
6	*0.89765	0.12390	0.16245
7	-0.02366	*0.84403	0.22861
8	0.16332	*0.58497	0.33870
9	*0.89062	0.10712	0.10453
10	0.11628	0.17448	*0.88320
11	*0.81875	-0.07197	0.13412
12	*0.54187	0.34166	0.11550
% Total Variation	36.2%	16.2%	10.5%
Eigenvalue	4.35	1.95	1.26

Table 2.6
Rotated Principal Components Extracted from Study 5
Data and Used in the Factor Score Calculations

Set 1		Set 2		
<u>Variables</u>	<u>Factor 1</u>	<u>Variables</u>	<u>Factor 1</u>	<u>Factor 2</u>
1	*0.88955	13	*0.81954	0.06552
2	*0.91855	14	0.67703	0.49886
3	0.78148	15	0.70058	0.47045
4	0.85231	16	0.81499	0.39975
5	*0.89848	17	*0.85497	0.35608
6	0.87595	18	*0.84538	0.37157
7	0.81740	19	*0.87636	0.347099
8	0.80939	20	*0.84622	0.38862
9	0.74408	21	0.27138	*0.87347
10	*0.89058	22	0.22614	*0.89357
11	0.78319	23	0.53467	*0.64797
12	*0.91381	24	0.70696	0.47757
% Total Variation		% Total Variation		
	72.2%		70.3%	8.9%
Eigenvalue	8.67	Eigenvalue	8.44	1.07

Table 2.7
Rotated Principal Components Extracted from Study 6
Data and Used in the Factor Score Calculations

Set 1		Set 2		
<u>Variables</u>	<u>Factor 1</u>	<u>Variables</u>	<u>Factor 1</u>	<u>Factor 3</u>
1	0.63603	37	-0.01092	0.09678
2	0.83937	38	0.28284	*0.85836
3	0.77675	39	0.20346	0.12226
4	*0.93469	40	*0.78010	-0.37276
5	0.76631	41	*0.70023	0.13456
6	0.77824	42	*0.63128	0.11230
7	*0.87052	43	0.23386	*0.79521
8	0.85709	44	*0.77350	0.25022
9	0.86621	45	0.08618	0.09897
10	*0.92355	46	*0.54797	0.28276
11	*0.91991	47	0.45666	0.28167
12	*0.89230	48	-0.36250	*0.51957
% Total Variation		% Total Variation		
	71.0%		27.1%	13.7%
Eigenvalue	8.52	Eigenvalue	3.25	1.64

Table 2.8
Rotated Principal Components Extracted from Study 7
Data and Used in the Factor Score Calculations

Set 1			Set 2			Set 3		
<u>Variables</u>	<u>Factor 1</u>	<u>Factor 2</u>	<u>Variables</u>	<u>Factor 1</u>	<u>Factor 2</u>	<u>Variables</u>	<u>Factor 1</u>	<u>Factor 2</u>
1	0.14675	*0.85595	13	0.02686	*0.65420	23	0.66871	0.39867
2	0.11701	*0.87590	14	0.18603	*0.75817	24	0.64817	0.39792
3	0.17193	*0.87457	15	0.06047	*0.84207	25	0.67099	0.36392
4	-0.01848	*0.86856	16	0.10946	*0.84301	26	0.14340	*0.68972
5	0.02818	*0.80139	17	0.04946	*0.79551	27	*0.69613	0.13802
6	0.11734	0.74987	18	*0.82988	0.04250	28	*0.76713	0.33481
7	*0.90281	0.11532	19	*0.88342	-0.01622	29	*0.88905	0.26063
8	*0.91926	0.11276	20	*0.87750	0.15406	30	*0.82716	0.14435
9	*0.90328	0.15636	21	*0.85447	0.13582	31	0.67587	0.45965
10	*0.95369	0.11902	22	*0.80822	0.14268	32	*0.75328	0.40974
11	*0.90872	0.07761				33	0.10624	*0.82795
12	0.89058	0.02122				34	0.36134	*0.71099
% Total Variation			% Total Variation			35	0.38502	*0.65567
	48.2%	29.9%		41.8%	26.2%	36	0.26768	*0.63544
Eigenvalue			Eigenvalue			37	0.68409	0.10939
	5.79	3.59		4.18	2.62	% Total Variation		
							52.5%	9.5%
						Eigenvalue		
							7.87	1.43

Table 2.9
Rotated Principal Components Extracted from Study 8
Data and Used in the Factor Score Calculations

Set 1 Variables	Factor 1	Factor 2	Set 2 Variables	Factor 1	Factor 2	Set 3 Variables	Factor 1	Factor 2
1	0.07889	*0.81900	13	0.27277	*0.84418	23	*0.79095	0.24858
2	0.26587	*0.78719	14	0.17885	*0.79874	24	0.72178	0.41732
3	0.17649	*0.82737	15	0.30286	*0.83538	25	0.55943	0.28358
4	0.32364	*0.77277	16	0.24993	*0.83035	26	0.04328	*0.84228
5	0.29264	0.75405	17	0.10500	*0.75225	27	*0.78478	-0.06497
6	0.26423	*0.80967	18	*0.82812	0.27860	28	0.66607	0.42498
7	*0.83067	0.21321	19	*0.92171	0.18569	29	*0.87040	0.20639
8	*0.85682	0.13969	20	*0.86783	0.26646	30	*0.82434	0.04775
9	*0.87916	0.25370	21	*0.89355	0.20561	31	0.61716	0.50413
10	*0.81554	0.26451	22	*0.85085	0.20639	32	*0.80031	0.24605
11	0.69997	0.34959				33	0.16989	*0.82830
12	*0.86287	0.20763				34	0.43892	*0.58475
% Total Variation			% Total Variation			35	0.63591	0.38474
	54.9%	17.0%		57.5%	19.1%	36	0.23036	*0.81735
Eigenvalue	6.58	2.04	Eigenvalue	5.75	1.91	37	0.58770	0.26110
						% Total Variation		
							51.1%	12.1%
						Eigenvalue	7.66	1.82

Table 2.9 (continued)

Set 4					Set 5		
Variables	Factor 1	Factor 2	Factor 3	Factor 4	Variables	Factor 1	Factor 2
38	0.39507	0.20883	*0.69759	-0.03294	57	*0.82956	0.00469
39	0.33306	0.22019	*0.73774	-0.07426	58	*0.80220	0.31161
40	-0.04831	0.13866	-0.21444	*0.72360	59	*0.78875	0.27046
41	0.18974	0.23206	0.38885	*0.65415	60	*0.81198	0.38959
42	0.42916	-0.11379	0.39215	*0.63620	61	*0.83700	0.32262
43	*0.56470	0.21884	0.00898	0.37649	62	0.74625	0.42685
44	0.18472	0.21662	0.15422	*0.62215	63	0.17918	*0.83975
45	*0.39465	0.04622	0.38630	0.29678	64	0.30802	*0.72865
46	*0.86974	0.09442	0.14709	0.20033	65	0.19189	*0.78117
47	*0.74385	0.30304	0.25060	-0.00998	66	0.28348	*0.82669
48	*0.85123	0.19139	0.15916	0.06744			
49	0.38149	*0.70720	0.32200	0.01413			
50	0.49935	*0.73379	0.19535	-0.00249			
51	-0.03016	*0.63851	0.33301	0.17976			
52	0.39699	*0.62794	0.23261	0.26103			
53	0.17587	*0.73662	0.08644	0.18867			
54	0.00657	0.50278	0.42195	0.34667			
55	0.04895	0.42914	*0.67213	0.31165			
56	0.05717	0.34420	*0.76917	0.16854			
% Total Variation					% Total Variation		
	41.8%	9.3%	8.7%	6.4%		58.1%	14.4%
Eigenvalue	7.94	1.78	1.65	1.22	Eigenvalue	5.81	1.44

Table 2.10
Rotated Principal Components Extracted from Study 9
Data and Used in the Factor Score Calculations

Set 1			Set 2			Set 3		
<u>Variables</u>	<u>Factor 1</u>	<u>Factor 2</u>	<u>Variables</u>	<u>Factor 1</u>	<u>Factor 2</u>	<u>Variables</u>	<u>Factor 1</u>	<u>Factor 2</u>
1	*0.85363	0.26152	13	0.10349	*0.84540	23	*0.83403	0.14457
2	*0.86146	0.22574	14	0.05584	*0.68490	24	*0.84696	0.23483
3	*0.80871	0.25936	15	-0.11680	*0.75706	25	*0.82444	0.25337
4	*0.82881	0.14060	16	0.06420	*0.87089	26	-0.01472	*0.73180
5	0.78092	0.06304	17	-0.01747	*0.89771	27	0.63446	0.16001
6	*0.83011	0.16229	18	*0.91033	-0.01924	28	0.68036	0.20435
7	0.38640	*0.76806	19	*0.94730	0.00494	29	*0.86232	0.18847
8	0.13333	*0.92942	20	*0.90160	0.01066	30	0.64508	0.22844
9	0.08514	*0.88316	21	*0.90689	0.03217	31	0.68579	0.34141
10	0.16765	*0.92940	22	*0.87255	0.06836	32	*0.83055	0.11740
11	0.16614	*0.82553				33	0.26743	*0.77238
12	0.52060	0.65651				34	0.22963	*0.64251
						35	0.35105	*0.54264
						36	-0.01322	*0.74919
						37	0.21499	0.21034
% Total Variation			% Total Variation			% Total Variation		
	54.9%	20.6%		41.9%	33.0%		43.0%	13.3%
Eigenvalue	6.589	2.47	Eigenvalue	4.19	3.30	Eigenvalue	6.45	1.99

Table 2.10 (continued)

Set 4				Set 5		
<u>Variables</u>	<u>Factor 1</u>	<u>Factor 2</u>	<u>Factor 3</u>	<u>Variables</u>	<u>Factor 1</u>	<u>Factor 2</u>
38	0.27838	*0.78842	-0.05104	57	*0.70614	0.15188
39	0.17548	*0.71412	-0.24876	58	*0.75939	0.36330
40	-0.12284	-0.12550	*0.78277	59	*0.66761	0.36085
41	0.36483	0.25446	*0.56476	60	*0.80976	0.24516
42	0.49972	*0.50672	0.28341	61	*0.84394	0.15546
43	0.05283	0.37223	*0.57665	62	0.35378	*0.58382
44	0.30989	-0.19910	*0.49960	63	0.23084	*0.81358
45	0.50185	*0.67569	0.21735	64	0.16809	*0.74443
46	0.31883	*0.78592	0.26151	65	0.12016	*0.57669
47	0.56797	0.45517	0.33355	66	0.27201	*0.66254
48	0.61386	0.46252	0.33460			
49	0.70959	0.31506	0.19312			
50	*0.80850	0.31444	0.11343			
51	*0.80560	0.03855	0.27919			
52	*0.84187	0.32633	0.18740			
53	*0.82406	0.09572	0.06150			
54	*0.80586	0.33676	-0.13372			
55	0.77074	0.42629	-0.08984			
56	0.75075	0.44351	0.05104			
% Total Variation				% Total Variation		
	50.2%	9.7%	7.5%		46.8%	12.0%
Eigenvalue	9.54	1.84	1.42	Eigenvalue	4.68	1.20

Table 2.11
Rotated Principal Components Extracted from Study 10
Data and Used in the Factor Score Calculations

Set 1			Set 2			Set 3	
Variables	Factor 1	Factor 2	Variables	Factor 1	Factor 2	Variables	Factor 1
1	0.08808	*0.83761	13	0.22097	*0.82002	23	*0.78887
2	0.13505	*0.84567	14	0.11812	*0.84004	24	*0.73615
3	0.12129	*0.82793	15	0.22618	*0.83833	25	0.39138
4	0.14740	*0.85725	16	0.24731	*0.82433	26	0.31258
5	0.15222	0.77070	17	0.12755	*0.80484	27	0.26929
6	0.20928	*0.78365	18	*0.78412	0.18710	28	*0.70061
7	*0.88282	0.15561	19	*0.93741	0.14556	29	*0.81220
8	*0.90479	0.15912	20	*0.89802	0.14248	30	0.37912
9	*0.91137	0.13248	21	*0.87237	0.29551	31	0.25035
10	*0.89227	0.20676	22	*0.90806	0.22850	32	0.42959
11	*0.84751	0.10626	% Total Variation			33	0.46545
12	0.83181	0.15183	55.2% 21.8%			34	-0.02613
% Total Variation			Eigenvalue			35	0.03601
50.0% 24.6%			5.52 2.18			36	0.11786
Eigenvalue						37	*0.62140
6.00 2.95						% Total Variation	
						46%	
						Eigenvalue	
						6.9	

Table 2.12
Rotated Principal Components Extracted from Study 11
Data and Used in the Factor Score Calculations

Set 1			Set 2		
<u>Variables</u>	<u>Factor 1</u>	<u>Factor 2</u>	<u>Variables</u>	<u>Factor 1</u>	<u>Factor 2</u>
1	-0.04606	*0.86120	13	0.18424	*0.81052
2	-0.00352	*0.88476	14	0.06333	*0.83275
3	-0.01548	*0.91723	15	0.09352	*0.81361
4	0.13214	*0.83696	16	0.13489	*0.88488
5	0.13337	0.67354	17	0.19503	*0.76685
6	-0.00841	*0.70101	18	*0.93729	0.10482
7	*0.86314	0.01695	19	*0.92439	0.12375
8	*0.91473	-0.01845	20	*0.94598	0.08061
9	*0.88238	0.14821	21	*0.89353	0.23058
10	*0.88238	0.09142	22	*0.90025	0.21533
11	*0.85045	0.01989			
12	0.82856	-0.03215			
% Total Variation			% Total Variation		
	39.6%	32.4%		52.3%	26.3%
Eigenvalue	4.75	3.88	Eigenvalue	5.23	2.63

Table 2.12 (continued)

Set 3				Set 4		Set 5		
<u>Variables</u>	<u>Factor 1</u>	<u>Factor 2</u>	<u>Factor 3</u>	<u>Variables</u>	<u>Factor 1</u>	<u>Variables</u>	<u>Factor 1</u>	<u>Factor 2</u>
23	0.29519	*0.84389	-0.00275	38	*0.59375	57	*0.84294	0.19101
24	0.33658	*0.79782	0.09491	39	*0.69170	58	*0.81192	0.17447
25	0.24265	*0.81952	0.13375	40	-0.16160	59	*0.69981	0.45642
26	-0.02245	-0.21195	*0.70366	41	0.19477	60	*0.84149	0.24245
27	*0.87647	0.14464	-0.09309	42	0.10811	61	*0.76370	0.26191
28	*0.71680	0.35896	0.18431	43	0.00240	62	0.51434	*0.63180
29	*0.68245	0.39269	0.01058	44	0.17018	63	0.25412	*0.74208
30	*0.86745	0.24743	0.02150	45	0.21829	64	0.14158	*0.73642
31	0.40436	*0.53023	0.33325	46	0.10254	65	0.45533	*0.60595
32	*0.74140	0.28902	0.03000	47	0.24478	66	0.13779	*0.75759
33	0.13929	0.17478	*0.81575	48	0.14086			
34	0.08479	0.37435	*0.65778	49	0.35992			
35	0.19069	*0.68296	0.25267	50	0.47264			
36	-0.09816	0.20436	*0.69104	51	0.10045			
37	0.24783	0.44699	0.04304	52	0.27516			
				53	0.56728			
				54	*0.69967			
				55	*0.87455			
				56	*0.84198			
% Total Variation				% Total Variation		% Total Variation		
	41.4%	14.7%	7.9%		3.78%		53.5%	12.1%
Eigenvalue	6.21	2.21	1.18	Eigenvalue	7.18	Eigenvalue	5.35	1.21

Table 2.13
Rotated Principal Components Extracted from Study 12
Data and Used in the Factor Score Calculations

Set 1				Set 2			Set 3	
Variables	Factor 1	Factor 2	Factor 3	Variables	Factor 1	Factor 2	Variables	Factor 1
1	0.06503	*0.90872	0.05700	13	0.20182	*0.88610	25	0.70821
2	0.21841	0.18621	*0.84416	14	0.64936	0.33250	26	*0.89763
3	0.35441	*0.53001	0.33352	15	0.34407	*0.77198	27	0.86450
4	0.26047	0.19052	*0.71297	16	0.50985	0.59540	28	0.87576
5	*0.72848	0.16462	0.11396	17	*0.72269	0.15363	29	*0.92699
6	*0.81345	0.23846	0.17084	18	*0.76283	0.36674	30	0.88579
7	0.13448	0.07329	*0.85553	19	0.39098	*0.71099	31	*0.93528
8	0.18300	*0.89269	0.17008	20	0.38176	*0.77881	32	*0.90336
9	*0.77506	0.02033	0.33000	21	*0.77557	0.33493	33	0.83561
10	0.17335	*0.85063	0.13908	22	0.22876	*0.70064	34	0.80465
11	*0.84798	0.07079	0.14187	23	*0.83248	0.33091	35	*0.89553
12	*0.70260	0.22753	0.15461	24	*0.71641	0.33158	36	0.88809
% Total Variation				% Total Variation			% Total Variation	
	44.0%	15.7%	11.2%		58.4%	9.4%		75.8%
Eigenvalue	5.28	1.88	1.35	Eigenvalue	7.00	1.23	Eigenvalue	9.09

Table 2.14
Rotated Principal Components Extracted from Study 13
Data and Used in the Factor Score Calculations

Set 1			Set 2		
<u>Variables</u>	<u>Factor 1</u>	<u>Factor 2</u>	<u>Variables</u>	<u>Factor 1</u>	<u>Factor 2</u>
1	*0.86554	0.23073	13	*0.81610	0.33211
2	0.81674	0.31730	14	*0.86439	0.17960
3	*0.90761	0.10550	15	*0.78353	0.45633
4	*0.86484	0.18657	16	*0.85464	0.30064
5	0.25952	*0.79326	17	0.52899	0.51738
6	0.39769	*0.68377	18	0.42770	*0.76294
7	*0.92069	0.15486	19	0.58924	*0.63070
8	*0.91417	0.15260	20	*0.77800	0.46850
9	0.38049	*0.46584	21	0.21176	*0.83583
10	0.58899	0.38993	22	0.66040	0.56315
11	-0.19300	*0.81938	23	0.27631	*0.85588
12	0.49503	0.50253	24	0.35743	*0.77746
% Total Variation			% Total Variation		
	55.4%	13.9%		66.8%	8.50%
Eigenvalue	6.64	1.66	Eigenvalue	8.01	1.02

Table 2.15
Rotated Principal Components Extracted from Study 14
Data and Used in the Factor Score Calculations

Set 1			Set 2				Set 3		
Variables	Factor 1	Factor 2	Variables	Factor 1	Factor 2	Factor 3	Variables	Factor 1	Factor 2
1	0.31066	*0.69788	13	*0.81886	-0.22664	0.11262	25	*0.89750	0.28867
2	0.28820	*0.86476	14	*0.77010	-0.17565	0.04987	26	*0.78794	0.43147
3	0.26832	*0.87444	15	0.68581	-0.03457	0.26014	27	0.38954	*0.65203
4	*0.80960	0.37403	16	0.16833	*0.53234	0.13303	28	0.66943	0.43134
5	*0.84803	0.40298	17	0.04289	0.43186	*0.81005	29	*0.89673	0.27793
6	*0.89590	0.13059	18	0.43051	-0.01795	*0.80787	30	*0.82995	0.37907
7	0.78280	0.42101	19	0.04287	*0.86982	-0.02622	31	0.42040	*0.77104
8	*0.81622	0.35349	20	-0.09477	*0.85087	-0.04691	32	0.73309	0.29755
9	0.73053	0.21997	21	*0.76004	0.27314	0.03524	33	0.34091	*0.76245
10	0.69705	0.24980	22	-0.04697	-0.14704	*0.90857	34	*0.82355	0.38849
11	*0.82807	0.38666	23	*0.76532	0.38686	0.04024	35	0.22557	*0.89114
12	0.80190	0.44683	24	*0.75290	0.31773	-0.02676	36	0.33415	*0.81331
% Total Variation			% Total Variation				% Total Variation		
66.7%			34.4%				66.1%		
9.5%			18.7%				10.2%		
16.3%									
Eigenvalue	8.00	1.34	Eigenvalue	4.13	2.24	1.95	Eigenvalue	7.94	1.23

Table 2.15 (continued)

Set 4			Set 5			Set 6	
<u>Variables</u>	<u>Factor 1</u>	<u>Factor 2</u>	<u>Variables</u>	<u>Factor 1</u>	<u>Factor 2</u>	<u>Variables</u>	<u>Factor 1</u>
37	0.12782	*0.48822	61	*0.90114	0.06231	73	*0.91181
38	*0.88888	0.11794	62	0.77680	0.30306	74	0.88049
39	*0.48293	0.00046	63	*0.93244	0.09153	75	0.86223
40	-0.22292	*0.68381	64	*0.89624	0.02355	76	0.88079
41	0.04863	*0.67212	65	0.12564	*0.30541	77	*0.89311
42	*0.42065	-0.21389	66	0.37409	*0.76406	78	0.87350
43	*0.88079	0.22860	67	0.82130	0.20488	79	0.82301
44	-0.06148	*0.60088	68	*0.85486	0.20337	80	*0.89350
45	*0.66703	0.24167	69	-0.20230	*0.91977	81	0.76974
46	-0.03355	0.38246	70	*0.90137	0.15931	82	*0.88754
47	0.26787	*0.82013	71	0.12600	*0.90611	83	0.71495
48	0.41598	-0.17528	72	0.18306	*0.87724	84	*0.88663
% Total Variation			% Total Variation			% Total Variation	
	25.6%	18.4%		51.2%	22.8%		73.7%
Eigenvalue	3.06	2.21	Eigenvalue	6.15	2.73	Eigenvalue	8.84

Table 2.16
Rotated Principal Components Extracted from Study 15
Data and Used in the Factor Score Calculations

Set 1		Set 2			Set 3		Set 4	
<u>Variables</u>	<u>Factor 1</u>	<u>Variables</u>	<u>Factor 1</u>	<u>Factor 2</u>	<u>Variables</u>	<u>Factor 1</u>	<u>Variables</u>	<u>Factor 3</u>
1	0.72911	13	0.24648	0.52569	25	0.79884	37	0.12512
2	0.87899	14	*0.58478	0.05343	26	0.85530	38	0.14692
3	0.86473	15	-0.12785	*0.68620	27	0.81500	39	*-0.73256
4	*0.89892	16	0.44297	*0.53474	28	*0.90346	40	0.39762
5	*0.91952	17	0.54503	0.05213	29	*0.92275	41	-0.08222
6	0.87244	18	*0.81275	0.28707	30	0.89560	42	*0.75234
7	*0.89326	19	0.28435	*0.83824	31	0.85465	43	-0.08935
8	*0.94651	20	0.05180	*0.62290	32	*0.92752	44	0.24559
9	0.79784	21	*0.82516	0.14099	33	0.78979	45	*-0.60697
10	0.83148	22	0.19107	*0.80931	34	*0.89998	46	-0.00739
11	0.89032	23	*0.77635	0.04794	35	*0.90258	47	-0.13397
12	*0.90111	24	*0.57235	0.23678	36	0.83983	48	*0.61935
% Total Variation		% Total Variation			% Total Variation		% Total Variation	
75.8%		37.5% 15.6%			75.4%		14.4%	
Eigenvalue	9.09	Eigenvalue	4.50	1.87	Eigenvalue	9.05	Eigenvalue	1.72

Table 2.17
Rotated Principal Components Extracted from Study 16
Data and Used in the Factor Score Calculations

Set 1			Set 2	
<u>Variables</u>	<u>Factor 1</u>	<u>Factor 2</u>	<u>Variables</u>	<u>Factor 1</u>
13	0.66940	0.29157	25	0.80181
14	*0.80957	0.32419	26	0.86183
15	0.67781	0.06209	27	0.81119
16	*0.81713	0.26242	28	*0.90395
17	0.19314	*0.66217	29	*0.87298
18	0.04180	*0.77480	30	0.83425
19	*0.81949	0.18732	31	*0.86340
20	*0.85361	0.10860	32	0.85881
21	0.19616	*0.85583	33	0.82597
22	*0.69582	0.17614	34	*0.89689
23	0.23373	*0.87300	35	0.80997
24	0.38867	*0.60695	36	*0.86828
% Total Variation			% Total Variation	
	48.0%	15.7%		72.5%
Eigenvalue	5.76	1.88	Eigenvalue	8.70

Table 2.17 (continued)

Set 3			Set 4		
<u>Variables</u>	<u>Factor 1</u>	<u>Factor 2</u>	<u>Variables</u>	<u>Factor 1</u>	<u>Factor 2</u>
37	0.32668	0.31293	49	0.72624	0.40297
38	*0.69472	0.07084	50	*0.81932	0.26762
39	0.21546	*0.69633	51	*0.89123	0.21369
40	*0.64877	0.06930	52	*0.85452	0.20707
41	*0.73545	-0.03159	53	0.09708	*0.86344
42	0.08262	0.18778	54	0.26047	*0.84246
43	*0.72425	0.18162	55	*0.85339	0.36687
44	0.13441	*0.73588	56	*0.89755	0.24346
45	-0.05154	*0.76583	57	0.48508	*0.6949
46	0.08536	*0.72071	58	0.79525	0.33142
47	*0.53981	0.36264	59	0.39182	*0.69604
48	0.21579	0.11802	60	0.55295	0.47450
% Total Variation			% Total Variation		
	28.1%	13.9%		62.9%	11.5%
Eigenvalue	3.37	1.67	Eigenvalue	7.55	1.38

Table 2.18
Rotated Principal Components Extracted from Study 17
Data and Used in the Factor Score Calculations

Set 1			Set 2		Set 3	
<u>Variables</u>	<u>Factor 1</u>	<u>Factor 2</u>	<u>Variables</u>	<u>Factor 1</u>	<u>Variables</u>	<u>Factor 2</u>
1	0.11794	*0.82507	25	*0.92606	37	-0.25505
2	0.37111	*0.82648	26	0.88499	38	0.29815
3	0.36974	*0.83318	27	0.81832	39	*0.79673
4	*0.88683	0.20735	28	*0.91479	40	-0.38492
5	*0.82751	0.35811	29	*0.90592	41	-0.02279
6	0.80215	0.46952	30	0.87204	42	0.17038
7	*0.82592	0.41030	31	0.87183	43	0.19133
8	*0.80837	0.44567	32	0.89726	44	0.23413
9	*0.83063	0.17050	33	0.77536	45	*0.71328
10	0.57695	0.56936	34	*0.91608	46	*0.55649
11	0.46661	*0.78940	35	*0.89985	47	-0.17380
12	0.67028	0.56567	36	0.81457	48	0.08531
% Total Variation			% Total Variation		% Total Variation	
	69.3%	10.2%		76.7%		15.9%
Eigenvalue	8.32	1.22	Eigenvalue	9.21	Eigenvalue	1.91

Table 2.18 (continued)

Set 4			Set 5	
<u>Variables</u>	<u>Factor 1</u>	<u>Factor 2</u>	<u>Variables</u>	<u>Factor 1</u>
49	*0.86401	0.35241	61	*0.89063
50	0.58281	0.49944	62	*0.92127
51	*0.89151	0.29598	63	0.76104
52	*0.76250	0.16039	64	*0.94541
53	0.22505	*0.63675	65	0.85067
54	0.21287	*0.86986	66	0.86140
55	*0.85854	0.26616	67	0.86147
56	*0.85154	0.21000	68	*0.89373
57	0.20197	*0.81887	69	0.82933
58	0.67137	0.34486	70	*0.91419
59	0.28149	*0.78562	71	0.86493
60	0.43758	*0.50999	72	0.77578
% Total Variation			% Total Variation	
	56.2%	12.2%		75.0%
Eigenvalue	6.75	1.46	Eigenvalue	8.99

Table 2.19
Rotated Principal Components Extracted from Study 18
Data and Used in the Factor Score Calculations

Set 1		Set 2			Set 3		
<u>Variables</u>	<u>Factor 1</u>	<u>Variables</u>	<u>Factor 1</u>	<u>Factor 2</u>	<u>Variables</u>	<u>Factor 1</u>	<u>Factor 2</u>
1	0.86956	11	*0.89724	0.19853	50	*0.66530	0.34551
2	*0.87453	12	*0.86639	0.32342	51	*0.89923	0.04785
3	*0.92504	13	*0.83852	0.29912	52	*0.81113	0.45301
4	*0.91273	14	*0.79711	0.42621	53	*0.84558	0.17775
5	*0.88451	15	0.25497	*0.83130	54	0.07674	*0.66872
6	*0.89850	16	0.23985	*0.86143	55	0.21840	*0.80164
7	0.82592	17	*0.69793	0.51512	56	0.30041	*0.74439
8	0.85425	18	0.69602	0.58333			
9	0.83115	19	0.54310	*0.62524			
10	0.83249	20	0.69748	0.52496			
		21	0.58236	*0.63949			
		22	0.37012	*0.76513			
% Total Variation		% Total Variation			% Total Variation		
	76.0%		69.2%	8.6%		52.6%	15.5%
Eigenvalue	7.60	Eigenvalue	8.31	1.04	Eigenvalue	3.68	1.09

Table 2.19 (continued)

Set 4			Set 5		
<u>Variables</u>	<u>Factor 1</u>	<u>Factor 2</u>	<u>Variables</u>	<u>Factor 1</u>	<u>Factor 2</u>
57	0.19853	*0.75038	67	*0.71613	0.33809
58	*0.67575	0.48025	68	*0.59395	0.49471
59	0.59790	0.39093	69	*0.70581	0.19634
60	*0.75594	0.44642	70	*0.85273	0.25719
61	0.65581	0.32583	71	*0.86168	0.06709
62	0.14627	*0.88819	72	0.58126	0.55501
63	0.38436	*0.73029	73	0.18772	*0.72548
64	*0.75152	0.40766	74	0.18492	*0.79078
65	*0.88450	0.11100	75	0.25235	*0.71205
66	*0.83265	0.06991	76	0.20006	*0.84526
% Total Variation			% Total Variation		
	56.4%	11.8%		51.5%	13.4%
Eigenvalue	5.64	1.18	Eigenvalue	5.15	1.34

Table 2.20
Rotated Principal Components Extracted from Study 19
Data and Used in the Factor Score Calculations

Set 1			Set 2			Set 3		
<u>Variables</u>	<u>Factor 1</u>	<u>Factor 2</u>	<u>Variables</u>	<u>Factor 1</u>	<u>Factor 2</u>	<u>Variables</u>	<u>Factor 1</u>	<u>Factor 2</u>
1	0.70152	0.51248	13	0.68209	0.28531	23	*0.71135	0.14821
2	*0.72751	0.42944	14	*0.75432	0.33671	24	0.68895	0.25862
3	*0.77160	0.48359	15	*0.79916	0.10743	25	*0.72865	0.20492
4	*0.82130	0.30763	16	*0.85140	0.31195	26	*0.85041	0.19851
5	0.56381	-0.06884	17	*0.80224	0.30447	27	*0.72927	0.29637
6	0.31953	*0.75310	18	0.06904	*0.89851	28	*0.70880	0.39802
7	*0.71182	0.52833	19	0.42355	*0.73681	29	0.53095	0.45968
8	*0.72141	0.50116	20	*0.71980	0.47197	30	0.41733	*0.60151
9	0.37127	*0.71815	21	0.54438	*0.67122	31	0.16856	*0.86250
10	0.63020	0.60939	22	0.53463	*0.67347	32	0.21649	*0.83079
11	0.14458	*0.85213						
12	0.23240	*0.84493						
% Total Variation			% Total Variation			% Total Variation		
	62.0%	9.1%		61.9%	10.1%		51.5%	10.7%
Eigenvalue	7.45	1.10	Eigenvalue	6.19	1.01	Eigenvalue	5.15	1.07

Table 2.21
Rotated Principal Components Extracted from Study 20
Data and Used in the Factor Score Calculations

Set 1 Variables	Factor 1
1	*0.71870
2	0.05123
3	0.07684
4	0.37863
5	-0.12181
6	0.03067
7	-0.06825
8	0.36515
9	-0.10452
10	0.47294
11	-0.16192
12	*0.62088
13	-0.05786
14	*0.55441
15	-0.33698
16	*0.65750
17	0.28336
18	0.04948
19	*0.64303
20	0.17731
21	0.28245
22	-0.30085
23	-0.03684
% Total Variation	
	20.5%
Eigenvalue	4.71

FACTOR SCORE ALGORITHMS TO BE CONSIDERED

After the factors to be measured had been determined, scores for each of them were generated by the following methods:

Ideal Variable Algorithm. Factor scores were calculated using the Statistical Package For the Social Sciences, Subprogram "Factor," principal components analysis, and the Bowling Green State University computer facilities. Factors, which were designated in the preparatory procedures for the factor score calculations, were extracted and rotated to an orthogonal varimax solution, and then measured via the idealized variable scoring method. Subjects' raw data was transformed into Z-scores prior to being used in the factor score computational procedures.

(2) Regression Factor Score Technique. Common factor scores were generated with the SPSS factor analysis computer Subprogram as described by Kim (1975). Squared multiple correlations served as the initial communality estimates in the main diagonal of the correlation matrix, and were replaced after twenty-five iterations with the final estimates. Factors, identified in the preliminary operations, were extracted and rotated to an orthogonal varimax solu-

tion, and then measured by the regression estimate algorithm. Raw data was, again, converted to standard scores before being used in the factor score estimations.

(3) Z-Sum Scores. Sum of items (or Z-sum scores) were formed by adding those variables (in their standardized form) which had been designated as a factor's defining group of salients in the preliminary procedures (See Tables 2.2 -- 2.21). A standard Fortran computer program, written by the researcher, performed the addition of variable scores (and generated the composites for each factor). An example of the program is included in Appendix A.

(4) Raw-Sum Scores. Sum of items (or raw-sum scores) were formed by adding those variables (in their unstandardized form) which had been designated as a factor's defining group of salients in the preliminary procedures (See Tables 2.2 -- 2.21). A standard Fortran computer program, written by the researcher, performed the addition of the variable scores and generated the composites for each factor (as was the case with the Z-Sum scores -- the program is presented in Appendix A).

(5) Fractional Weight Scores. Scores were determined by assigning each salient variable a weight equal to its correlation with the factor, and then summing the variables.

Principal components rotated factor loadings, produced by SPSS, Subprogram "Factor," formed the weighting matrix, and a Fortran computer program (written by this researcher) performed the multiplication and addition of responses. Subjects' raw data was transformed into standard scores before being used in the factor score arithmetic operations. An example of this program is, again, included in Appendix A.

ANALYSES OF THE DATA

In order to determine whether differentially computed factor scores yield significantly different statistical results, composite scores generated via the five algorithms were input into the following analyses:

(1) univariate analysis of variance;

(2) multivariate analysis of variance (in those cases where two or more factor scores existed for each case);

(3) multiple discriminant analysis (in the multivariate case); and

(4) canonical correlation analysis (in those cases where four or more factor scores existed for each case).

These four analyses were performed by Finn's (1972) Multivariate Program, Version V, utilizing the Bowling Green State University computer facilities. For each test, exact probability levels, as well as F-ratios, chi-square values and other pertinent summary information, were provided. Table 2.22 categorizes the types of statistical analyses performed on the differentially computed factor scores for each study.

Table 2.22
Type of Statistical Analysis Performed on
the Factor Scores of Each Study

Study	Number of Factor Scores (DV'S)	Type of Statistical Test(s) Performed			
		ANOVA	MANOVA	DISCRIM	CANCORR
1	4	x	x	x	x
2	1	x			
3	3	x	x	x	
4	3	x	x	x	
5	3	x	x	x	
6	3	x	x	x	
7	6	x	x	x	x
8	12	x	x	x	x
9	11	x	x	x	x
10	5	x	x	x	x
11	10	x	x	x	x
12	6	x	x	x	x
13	4	x	x	x	x
14	12	x	x	x	x
15	5	x	x	x	x
16	7	x	x	x	x
17	7	x	x	x	x
18	9	x	x	x	x
19	6	x	x	x	x
20	1	x			
Total Number of Tests		117	18	18	14

Significance probabilities resulting from the foregoing statistical analyses were compared across the scoring methods. The factor scoring method which yielded the smallest p-value was awarded one point, representing the statistical test in which its obtained p was smallest. In the event that two or more composites produced identical probabilities, associated F-ratios (in the univariate and multivariate analyses of variance) and chi-square values (in the multiple discriminant and canonical correlation analyses) were compared and the scoring method yielding the largest F (or chi-square) was awarded the point. However, if these values were also the same, the statistical test was excluded from the study.

Scores were summed for the five scoring methods associated with the multivariate analyses of variance, univariate analyses of variance, multiple discriminant analyses, and canonical correlation analyses. A table of observed frequencies (of smallest p-values) was prepared, across scoring methods, for each statistical test. These classification matrices provided the basis for determining whether the method used to generate the factor scores had a significant effect on the results of these analyses.

Chi-square tests were used to contrast the effectiveness of the scoring methods in each of the formerly described statistical tests. H_0 was rejected if the associated p-value of the obtained chi-square was equal to or less than $\alpha = .05$. The results of these analyses are presented in Chapter III.

Chapter III

RESULTS

The purpose of this chapter is to present the results of the statistical analyses associated with each of the research questions stated in Chapter I. So as not to confuse the results with their interpretation, discussion of the findings will be reserved for the fourth chapter.

The first question which this study sought to investigate was the effect of the differentially computed factor scores on the magnitude of the probability-value in a univariate and multivariate analysis of variance design. Specifically stated, the question being examined was:

- (1) Does the method used to calculate factor scores have a significant effect on the magnitude of the p-value in a multivariate or univariate one-way analysis of variance design: if so, which composite most frequently yields the smallest probability?

In order to investigate this research question, factor scores generated via five different computational algorithms were separately used as dependent variables in 117 univariate and eighteen multivariate one-way analysis of variance designs. In each statistical test, resultant F-ratios and their associated significance probabilities were compared across scoring methods to find that procedure which yielded the smallest p-value. Tables 3.1 to 3.20 below, categorize the results of the analyses of variance. The factor score algorithm which produced the smallest probability in each statistical test is designated with an asterisk next to its corresponding p-value.

Table 3.1
Univariate and Multivariate Analyses of Variance
Significance Probabilities and F-Ratios Generated
by Five Differentially Computed Factor Scores
Study 1

Ideal		Z	Raw	Frac	Ideal		Z	Raw	Frac
<u>Variable</u>	<u>Regression</u>	<u>Sum.</u>	<u>Sum.</u>	<u>Weights</u>	<u>Variable</u>	<u>Regression</u>	<u>Sum.</u>	<u>Sum.</u>	<u>Weights</u>
	<u>Significance</u>	<u>Probability</u>				<u>Multivariate</u>	<u>F-Ratio</u>		
.001*	.002	.002	.001	.002	2.594	2.362	2.404	2.449	2.381
<u>Variable</u>	<u>Significance</u>	<u>Probability</u>				<u>Univariate</u>	<u>F-Ratio</u>		
1 .068*	.068	.073	.073	.072	2.213	2.212	2.167	2.174	2.177
2 .083*	.104	.145	.146	.133	2.090	1.943	1.724	1.721	1.781
3 .002*	.007	.006	.006	.007	4.332	3.636	3.667	3.730	3.643
4 .597	.546*	.656	.620	.660	.694	.770	.610	.661	.605
df = 16/733.85									

Table 3.2
Study 2
Univariate Analysis of Variance Significance
Probabilities and F-Ratios Generated by
Five Differentially Computed Factor Scores

<u>Method</u>	<u>Significance Probability</u>	<u>Univariate F-Ratio</u>
Ideal Variable	.205	1.614
Regression	.201	1.630
Z Sum	.251	1.403
Raw Sum	.245	1.428
Weights	.120*	2.173
df = 2/93		

Table 3.3
Study 3
Univariate and Multivariate Analyses of Variance
Significance Probabilities and F-Ratios Generated
by Five Differentially Computed Factor Scores

Ideal		Z	Raw	Frac	Ideal		Z	Raw	Frac
<u>Variable</u>	<u>Regression</u>	<u>Sum.</u>	<u>Sum.</u>	<u>Weights</u>	<u>Variable</u>	<u>Regression</u>	<u>Sum.</u>	<u>Sum.</u>	<u>Weights</u>
	<u>Significance Probability</u>					<u>Multivariate F-Ratio</u>			
	.372	.364	.209*	.226					
				.218	1.065	1.084	1.564	1.497	1.529
<u>Variable</u>		<u>Significance Probability</u>					<u>Univariate F-Ratio</u>		
1	.082	.081*	.209	.225	3.151	3.173	1.618	1.510	1.608
2	.880	.977	.061	.066	.023	.001	3.667	3.530	3.760
3	.789	.985	.616	.671	.073	.0004	.255	.182	.284
df = 3/52									

Table 3.4
Study 4
Univariate and Multivariate Analyses of Variance
Significance Probabilities and F-Ratios Generated
by Five Differentially Computed Factor Scores

Ideal		Z	Raw	Frac	Ideal		Z	Raw	Frac
<u>Variable</u>	<u>Regression</u>	<u>Sum.</u>	<u>Sum.</u>	<u>Weights</u>	<u>Variable</u>	<u>Regression</u>	<u>Sum.</u>	<u>Sum.</u>	<u>Weights</u>
	<u>Significance Probability</u>					<u>Multivariate F-Ratio</u>			
	.023*	.050	.056	.048					
				.052	3.340	2.817	2.720	2.852	2.795
<u>Variable</u>		<u>Significance Probability</u>					<u>Univariate F-Ratio</u>		
1	.924	.889	.274*	.290	.009	.020	1.229	1.146	.613
2	.006	.007	.007	.005*	8.200	7.992	8.172	8.572	8.421
3	.210	.436	.110	.094*	1.616	.618	2.660	2.922	2.468
df = 3/43									

Table 3.5
Study 5
Univariate and Multivariate Analyses of Variance
Significance Probabilities and F-Ratios Generated
by Five Differentially Computed Factor Scores

Ideal		Z	Raw	Frac	Ideal		Z	Raw	Frac	
<u>Variable</u>	<u>Regression</u>	<u>Sum.</u>	<u>Sum.</u>	<u>Weights</u>	<u>Variable</u>	<u>Regression</u>	<u>Sum.</u>	<u>Sum.</u>	<u>Weights</u>	
		<u>Significance</u>	<u>Probability</u>				<u>Multivariate</u>	<u>F-Ratio</u>		
	.134*	.166	.309	.265	.224	1.937	1.756	1.226	1.358	1.500
<u>Variable</u>		<u>Significance</u>	<u>Probability</u>				<u>Univariate</u>	<u>F-Ratio</u>		
1	.056*	.085	.104	.107	.105	3.795	3.063	2.727	2.645	2.717
2	.444	.334	.259	.244*	.258	.595	.951	1.301	1.388	1.307
3	.157	.163	1.000	.161	.132*	2.052	2.000	.000	2.018	2.336
df = 3/57										

Table 3.6
Study 6
Univariate and Multivariate Analyses of Variance
Significance Probabilities and F-Ratios Generated
by Five Differentially Computed Factor Scores

Ideal		Z	Raw	Frac	Ideal		Z	Raw	Frac
<u>Variable</u>	<u>Regression</u>	<u>Sum.</u>	<u>Sum.</u>	<u>Weights</u>	<u>Variable</u>	<u>Regression</u>	<u>Sum.</u>	<u>Sum.</u>	<u>Weights</u>
	<u>Significance</u>	<u>Probability</u>				<u>Multivariate</u>	<u>F-Ratio</u>		
.994	.965*	.994	.989	.990	.027	.090	.027	.040	.036
<u>Variable</u>	<u>Significance</u>	<u>Probability</u>				<u>Univariate</u>	<u>F-Ratio</u>		
1 .769	.854	.774	.741*	.774	.089	.035	.085	.113	.085
2 .855	.807*	.890	.960	.998	.035	.062	.020	.003	.000
3 .903	.693*	.835	.882	.806	.015	.161	.045	.023	.062
df = 3/15									

Table 3.7
Study 7
Univariate and Multivariate Analyses of Variance
Significance Probabilities and F-Ratios Generated
by Five Differentially Computed Factor Scores

Ideal		Z	Raw	Frac	Ideal		Z	Raw	Frac
<u>Variable</u>	<u>Regression</u>	<u>Sum.</u>	<u>Sum.</u>	<u>Weights</u>	<u>Variable</u>	<u>Regression</u>	<u>Sum.</u>	<u>Sum.</u>	<u>Weights</u>
	<u>Significance</u>	<u>Probability</u>					<u>Multivariate</u>	<u>F-Ratio</u>	
	.001	.0004	.001	.001		5.000	4.526	4.609	4.734
<u>Variable</u>	<u>Significance</u>	<u>Probability</u>					<u>Univariate</u>	<u>F-Ratio</u>	
1#	.382	.371	.293	.293		.777	1.124	1.125	1.125
2	.114	.151	.117	.130		2.582	2.540	2.366	2.481
3	.319	.291	.364	.366		1.012	.838	.830	.840
4	.245	.251	.285	.270		1.382	1.163	1.240	1.195
5	.475	.770	.446	.446		.517	.589	.588	.564
6	.0001	.0002	.001	.001		18.184	13.803	14.090	14.266
df = 6/53					#p-values & F's identical-variable deleted				

Table 3.8
Study 8
Univariate and Multivariate Analyses of Variance
Significance Probabilities and F-Ratios Generated
by Five Differentially Computed Factor Scores

Ideal		Z	Raw	Frac	Ideal		Z	Raw	Frac
<u>Variable</u>	<u>Regression</u>	<u>Sum.</u>	<u>Sum.</u>	<u>Weights</u>	<u>Variable</u>	<u>Regression</u>	<u>Sum.</u>	<u>Sum.</u>	<u>Weights</u>
	<u>Significance</u>	<u>Probability</u>				<u>Multivariate</u>	<u>F-Ratio</u>		
.132	.135	.050	.046*	.063	1.551	1.542	1.918	1.948	1.830
<u>Variable</u>	<u>Significance</u>	<u>Probability</u>				<u>Univariate</u>	<u>F-Ratio</u>		
1 .136	.125*	.180	.176	.181	2.274	2.406	1.835	1.872	1.823
2 .414*	.428	.637	.641	.638	.676	.635	.225	.220	.223
3 .471	.444*	.766	.761	.759	.525	.594	.089	.093	.095
4 .112	.111*	.186	.182	.185	2.587	2.602	1.788	1.815	1.794
5 .134	.110	.094	.095	.093*	2.296	2.621	2.880	2.871	2.906
6 .038	.035	.025*	.030	.027	4.472	4.639	5.251	4.896	5.124
7 .521	.521	.304	.315	.231*	.416	.416	1.070	1.023	1.462
8 .046	.020	.001	.001*	.002	4.145	5.712	11.082	11.485	10.714
9 .045	.041	.008*	.009	.009	4.163	4.326	7.456	7.257	7.348
10 .482	.324	.045*	.045	.046	.501	.985	4.179	4.160	4.119
11 .004	.006	.001*	.001	.001	8.937	8.088	12.295	12.193	12.243
12 .082	.036	.009	.008*	.009	3.123	4.571	7.139	7.377	7.163
df = 12/60									

Table 3.9
Study 9
Univariate and Multivariate Analyses of Variance
Significance Probabilities and F-Ratios Generated
by Five Differentially Computed Factor Scores

Ideal		Z	Raw	Frac	Ideal		Z	Raw	Frac
<u>Variable</u>	<u>Regression</u>	<u>Sum.</u>	<u>Sum.</u>	<u>Weights</u>	<u>Variable</u>	<u>Regression</u>	<u>Sum.</u>	<u>Sum.</u>	<u>Weights</u>
	<u>Significance</u>	<u>Probability</u>					<u>Multivariate F-Ratio</u>		
.264	.230	.227	.208	*.191	1.280	1.342	1.349	1.388	1.427
<u>Variable</u>	<u>Significance</u>	<u>Probability</u>					<u>Univariate F-Ratio</u>		
1 .170*	.187	.277	.276	.280	1.926	1.785	1.201	1.207	1.190
2 .352	.342*	.522	.521	.514	.880	.918	.415	.417	.432
3 .687	.673*	.725	.733	.727	.164	.181	.125	.118	.123
4 .927*	.960	.933	.966	.955	.008	.003	.007	.002	.003
5 .977	.882	.622	.610*	.627	.001	.022	.245	.263	.239
6 .503	.665	.413	.415	.402*	.454	.189	.681	.673	.712
7 .009*	.011	.024	.024	.024	7.237	6.986	5.393	5.364	5.400
8 .555*	.676	.734	.749	.814	.353	.176	.117	.104	.056
9 .687*	.884	.917	.822	.997	.164	.022	.011	.051	.000
10 .482	.491	.482	.467	.466*	.502	.481	.500	.536	.540
11 .515	.535	.299	.275*	.314	.428	.389	1.099	1.215	1.030
df = 11/50									

Table 3.10
Study 10
Univariate and Multivariate Analyses of Variance
Significance Probabilities and F-Ratios Generated
by Five Differentially Computed Factor Scores

Ideal		Z	Raw	Frac	Ideal		Z	Raw	Frac
<u>Variable</u>	<u>Regression</u>	<u>Sum.</u>	<u>Sum.</u>	<u>Weights</u>	<u>Variable</u>	<u>Regression</u>	<u>Sum.</u>	<u>Sum.</u>	<u>Weights</u>
	<u>Significance Probability</u>					<u>Multivariate F-Ratio</u>			
.173	.144	.106	.091*	.095	1.609	1.724	1.916	2.008	1.984
<u>Variable</u>	<u>Significance Probability</u>					<u>Univariate F-Ratio</u>			
1 .606	.549*	.680	.658	.674	.269	.363	.172	.198	.179
2 .433	.383	.431	.336*	.431	.624	.772	.628	.939	.629
3 .369	.427	.212	.212*	.225	.819	.641	1.590	1.591	1.503
4 .099	.095	.065*	.068	.066	2.807	2.879	3.537	3.455	3.500
5 .010	.006	.004	.004	.003*	7.159	8.036	8.933	8.941	9.381
df = 5/57									

Table 3.11
Study 11
Univariate and Multivariate Analyses of Variance
Significance Probabilities and F-Ratios Generated
by Five Differentially Computed Factor Scores

<u>Ideal</u>		<u>Z</u>	<u>Raw</u>	<u>Frac</u>	<u>Ideal</u>		<u>Z</u>	<u>Raw</u>	<u>Frac</u>
<u>Variable</u>	<u>Regression</u>	<u>Sum.</u>	<u>Sum.</u>	<u>Weights</u>	<u>Variable</u>	<u>Regression</u>	<u>Sum.</u>	<u>Sum.</u>	<u>Weights</u>
	<u>Significance Probability</u>					<u>Multivariate F-Ratio</u>			
	.102	.037*	.048	.047					
				.048	1.703	2.123	2.015	2.028	2.038
<u>Variable</u>		<u>Significance Probability</u>					<u>Univariate F-Ratio</u>		
1 .581	.552*	.652	.651	.650	.308	.358	.206	.207	.208
2 .258	.184*	.260	.259	.251	1.300	1.804	1.291	1.298	1.341
3 .534	.522*	.537	.541	.535	.390	.414	.385	.378	.390
4 .985	.857*	.906	.901	.897	.0003	.033	.014	.016	.017
5 .621	.649	.411	.431	.409*	.247	.206	.686	.630	.692
6 .334	.131*	.305	.306	.249	.946	2.340	1.069	1.069	1.353
7 .245	.363	.088	.087*	.091	1.376	.838	3.000	3.029	2.937
8 .015	.002*	.003	.003	.003	6.222	10.559	9.393	9.550	9.625
9 .090	.044	.006*	.006	.006	2.964	4.212	8.162	8.131	8.119
10 .003	.005	.001	.001	.001*	9.444	8.408	11.841	12.089	12.501
df = 10/57									

Table 3.12
Study 12
Univariate and Multivariate Analyses of Variance
Significance Probabilities and F-Ratios Generated
by Five Differentially Computed Factor Scores

Ideal		Z	Raw	Frac	Ideal		Z	Raw	Frac
<u>Variable</u>	<u>Regression</u>	<u>Sum.</u>	<u>Sum.</u>	<u>Weights</u>	<u>Variable</u>	<u>Regression</u>	<u>Sum.</u>	<u>Sum.</u>	<u>Weights</u>
	<u>Significance</u>	<u>Probability</u>					<u>Multivariate F-Ratio</u>		
	.022	.019*	.023	.025					
				.021	2.870	2.966	2.850	2.780	2.906
<u>Variable</u>		<u>Significance</u>	<u>Probability</u>				<u>Univariate F-Ratio</u>		
1	.206	.233	.103	.094*	1.652	1.463	2.789	2.941	2.701
2	.762	.773	.263	.240*	.093	.085	1.291	1.423	.880
3	.068	.043*	.050	.053	3.522	4.357	4.072	3.989	4.140
4	.030	.033	.003*	.004	5.090	4.881	9.704	9.533	9.555
5	.007	.005	.0005	.0005*	8.185	8.801	14.298	14.514	14.038
6	.942	.904	.664	.647*	.005	.015	.191	.213	.195
df = 6/36									

Table 3.13
Study 13
Univariate and Multivariate Analyses of Variance
Significance Probabilities and F-Ratios Generated
by Five Differentially Computed Factor Scores

Ideal		Z	Raw	Frac	Ideal		Z	Raw	Frac	
<u>Variable</u>	<u>Regression</u>	<u>Sum.</u>	<u>Sum.</u>	<u>Weights</u>	<u>Variable</u>	<u>Regression</u>	<u>Sum.</u>	<u>Sum.</u>	<u>Weights</u>	
	<u>Significance Probability</u>					<u>Multivariate F-Ratio</u>				
	.321	.301*	.407	.433	.518	1.206	1.255	1.019	.970	.822
<u>Variable</u>		<u>Significance Probability</u>					<u>Univariate F-Ratio</u>			
1	.188	.084	.073*	.074	.086	1.783	3.107	3.349	3.325	3.062
2	.263*	.294	.326	.336	.443	1.282	1.125	.984	.944	.598
3	.455*	.564	.737	.755	.738	.568	.338	.114	.099	.113
4	.537*	.633	.781	.805	.993	.387	.231	.078	.0062	.0001
df = 4/47										

Table 3.14
Univariate and Multivariate Analyses of Variance
Significance Probabilities and F-Ratios Generated
by Five Differentially Computed Factor Scores

Ideal		Z	Raw	Frac	Ideal		Z	Raw	Frac
<u>Variable</u>	<u>Regression</u>	<u>Sum.</u>	<u>Sum.</u>	<u>Weights</u>	<u>Variable</u>	<u>Regression</u>	<u>Sum.</u>	<u>Sum.</u>	<u>Weights</u>
<u>Multivariate F-Ratios</u>					<u>Significance Probability</u>				
.526	1.043	.763	.447	.695	.852	.469*	.670	.904	.723
<u>Variable</u>	<u>Univariate F-Ratios</u>					<u>Significance Probability</u>			
1 .100	.100	.573	.003	.593	.755	.756	.457	.958	.450*
2 .357	.690	.044	.116	.038	.556	.415*	.835	.737	.847
3 .151	.120	3.104	.350	3.100	.701	.733	.092*	.560	.092
4 3.518	3.956	2.652	2.532	3.098	.074	.059*	.118	.126	.092
5 .005	.010	3.668	.027	4.621	.942	.920	.069	.872	.043*
6 .253	.474	.067	.873	.061	.620	.498	.798	.360*	.807
7 .214	.174	.176	.389	.191	.648	.681	.679	.539*	.666
8 .124	.804	1.469	.495	1.624	.728	.380	.238	.489	.216*
9 .295	4.021	1.698	.646	2.264	.592	.058*	.206	.430	.147
10 1.635	.023	3.500	1.240	3.494	.214	.882	.075*	.278	.075
11 1.337	.004	5.652	1.083	5.041	.260	.948	.027*	.309	.035
df = 11/12									

Table 3.15
Study 15

Univariate and Multivariate Analyses of Variance
Significance Probabilities and F-Ratios Generated
by Five Differentially Computed Factor Scores

Ideal		Z	Raw	Frac	Ideal		Z	Raw	Frac	
<u>Variable</u>	<u>Regression</u>	<u>Sum.</u>	<u>Sum.</u>	<u>Weights</u>	<u>Variable</u>	<u>Regression</u>	<u>Sum.</u>	<u>Sum.</u>	<u>Weights</u>	
	<u>Significance</u>	<u>Probability</u>				<u>Multivariate</u>	<u>F-Ratio</u>			
	.422	.473	.301*	.318	.310	1.012	.926	1.252	1.215	1.233
<u>Variable</u>		<u>Significance</u>	<u>Probability</u>			<u>Univariate</u>	<u>F-Ratio</u>			
1	.159	.144*	.159	.153	.159	2.044	2.209	2.051	2.110	2.050
2	.143*	.200	.167	.189	.166	2.220	1.687	1.973	1.779	1.974
3	.801	.783	.700	.666*	.725	.064	.077	.150	.188	.125
4	.735	.697	.640*	.661	.644	.116	.153	.221	.194	.216
5	.329	.315	.267	.246*	.280	.971	1.031	1.264	1.380	1.193
df = 5/45										

Table 3.16
Study 16
Univariate and Multivariate Analyses of Variance
Significance Probabilities and F-Ratios Generated
by Five Differentially Computed Factor Scores

Ideal		Z	Raw	Frac	Ideal		Z	Raw	Frac
<u>Variable</u>	<u>Regression</u>	<u>Sum.</u>	<u>Sum.</u>	<u>Weights</u>	<u>Variable</u>	<u>Regression</u>	<u>Sum.</u>	<u>Sum.</u>	<u>Weights</u>
	<u>Significance</u>	<u>Probability</u>				<u>Multivariate</u>	<u>F-Ratios</u>		
.036	.049	.033	.032*	.036	1.880	1.787	1.900	1.915	1.879
<u>Variable</u>	<u>Significance</u>	<u>Probability</u>				<u>Univariate</u>	<u>F-Ratios</u>		
1 .840	.897	.829	.834	.827*	.175	.109	.189	.182	.190
2 .118*	.291	.146	.124	.180	2.215	1.258	1.986	2.158	1.761
3 .424	.445	.357	.354*	.359	.817	.821	1.047	1.055	1.041
4 .853	.872	.776*	.823	.808	.160	.137	.255	.196	.215
5 .784	.778	.768	.792	.757*	.244	.253	.266	.235	.280
6 .002	.002	.0002	.0002*	.0002	6.721	7.002	9.971	9.995	9.928
7 .030	.016	.002	.002	.002	3.732	4.454	6.682	6.653	6.886
df = 14/114									

Table 3.17
Study 17
Univariate and Multivariate Analyses of Variance
Significance Probabilities and F-Ratios Generated
by Five Differentially Computed Factor Scores

Ideal		Z	Raw	Frac	Ideal		Z	Raw	Frac	
<u>Variable</u>	<u>Regression</u>	<u>Sum.</u>	<u>Sum.</u>	<u>Weights</u>	<u>Variable</u>	<u>Regression</u>	<u>Sum.</u>	<u>Sum.</u>	<u>Weights</u>	
	<u>Significance</u>	<u>Probability</u>				<u>Multivariate</u>	<u>F-Ratios</u>			
	.062	.039*	.105	.099	.098	1.751	1.895	1.573	1.592	1.597
<u>Variable</u>	<u>Significance</u>	<u>Probability</u>				<u>Univariate</u>	<u>F-Ratios</u>			
1	.889	.834	.721	.703*	.719	.118	.182	.329	.355	.333
2	.851	.814	.713	.693*	.709	.162	.207	.341	.370	.346
3	.261	.242	.181	.177*	.180	1.382	1.462	1.773	1.798	1.780
4	.890	.642*	.735	.710	.652	.117	.447	.310	.345	.432
5	.119	.068	.041	.036*	.039	2.231	2.858	3.419	3.565	3.475
6	.172	.180	.047*	.047	.051	1.829	1.779	3.279	3.277	3.176
7	.036*	.055	.073	.076	.072	3.590	3.092	2.780	2.732	2.795
df = 14/80										

Table 3.18
Study 18
Univariate and Multivariate Analyses of Variance
Significance Probabilities and F-Ratios Generated
by Five Differentially Computed Factor Scores

Ideal		Z	Raw	Frac	Ideal		Z	Raw	Frac
<u>Variable</u>	<u>Regression</u>	<u>Sum.</u>	<u>Sum.</u>	<u>Weights</u>	<u>Variable</u>	<u>Regression</u>	<u>Sum.</u>	<u>Sum.</u>	<u>Weights</u>
<u>Multivariate F-Ratios</u>					<u>Significance Probabilities</u>				
	.9579	.8889	1.0957	1.163	1.016		.5134	.5928	.3682 .3486 .4490
<u>Variable</u>	<u>Univariate F-Ratios</u>				<u>Significance Probabilities</u>				
1	.327	.251	.162	.164	.159	.723*	.779	.851	.849 .853
2	.124	.127	.087	.088	.070	.883	.881*	.917	.916 .853
3	1.325	1.348	.638	.677	.669	.244*	.268	.532	.512 .516
4#	1.351	.928	1.331	1.351	1.234	.267	.402	.272	.267 .299
5	.045	.009	.103	.116	.076	.956	.991	.903*	.891 .927
6	.948	1.433	1.918	1.867	1.736	.394	.247	.156*	.164 .185
7	.940	.475	1.018	.837	.957	.397	.624	.368*	.438 .390
8	1.182	1.015	1.447	1.492	1.352	.314	.369	.244	.234* .267
9	.319	.799	.385	.380	.396	.728	.455*	.682	.686 .675
df = 18/98					#F's & p-value identical-variable deleted				

Table 3.19
Study 19
Univariate and Multivariate Analyses of Variance
Significance Probabilities and F-Ratios Generated
by Five Differentially Computed Factor Scores

Ideal		Z	Raw	Frac	Ideal		Z	Raw	Frac
<u>Variable</u>	<u>Regression</u>	<u>Sum.</u>	<u>Sum.</u>	<u>Weights</u>	<u>Variable</u>	<u>Regression</u>	<u>Sum.</u>	<u>Sum.</u>	<u>Weights</u>
	<u>Significance</u>	<u>Probability</u>				<u>Multivariate</u>	<u>F-Ratios</u>		
.001	.0001	.0001*	.0001	.0001	2.206	2.071	2.468	2.413	2.2931
<u>Variable</u>	<u>Significance</u>	<u>Probability</u>				<u>Univariate</u>	<u>F-Ratios</u>		
1 .076	.044	.014	.012	.014	1.772	1.978	2.388	2.429	2.394
2 .010*	.059	.019	.021	.023	2.506	1.865	2.275	2.240	2.210
3 .007*	.016	.019	.022	.030	2.647	2.348	2.279	2.227	2.116
4 .301	.107	.083	.080*	.161	1.194	1.638	1.739	1.750	1.473
5 .0003*	.001	.009	.007	.007	3.766	3.220	2.546	2.649	2.616
6 .021*	.042	.029	.038	.057	2.247	1.998	2.134	2.036	1.879
df = 54/947.91									

Table 3.20
Study 20
Univariate Analysis of Variance Significance
Probabilities and F-Ratios Generated by
Five Differentially Computed Factor Scores

<u>Method</u>	<u>Significance Probability</u>	<u>F-Ratios</u>
Ideal Variable	.0002	8.901
Regression	.0001	10.570
Z Sum	.0001	13.860
Raw Sum	.0001	13.902
Frac Weights	.0001*	13.946
df = 2/447		

Frequencies were computed for each factor score procedure in both the univariate and multivariate analyses of variance, and then compared via two chi-square tests; one analyzing the observed univariate test frequencies, and the other comparing the multivariate test scores. Tables 3.21 and 3.22, and Figures 3.1 and 3.2 depict the results of these analyses:

Table 3.21
Number of Times Factor Score Methods Yielded
the Smallest Significance Probability;
Univariate Analyses of Variance

Factor Score Algorithm	Number of Times Produced Smallest p	Number of Times Did Not Produced Smallest p
Ideal Variable	Observed = 25 Expected = 23 Percentage = 22%	Observed = 90 Expected = 92 Percentage = 78%
Regression	Observed = 25 Expected = 23 Percentage = 22%	Observed = 90 Expected = 92 Percentage = 78%
Z Sum	Observed = 19 Expected = 23 Percentage = 17%	Observed = 96 Expected = 92 Percentage = 83%
Raw Sum	Observed = 28 Expected = 23 Percentage = 24%	Observed = 87 Expected = 92 Percentage = 76%
Fractional Weights	Observed = 18 Expected = 23 Percentage = 16%	Observed = 97 Expected = 92 Percentage = 84%
$\chi^2 = 3.22$; $df = 4$; $.70 > p > .50$		

Figure 3.1
Histogram for Data of Table 3.21. Number of Times
Each Factor Score Method Yielded the Smallest
ANOVA P - value

Table 3.22
Number of Times Factor Score Methods Yielded
the Smallest Significance Probability;
Multivariate Analyses of Variance

Factor Score Algorithm	Number of Times Produced Smallest p	Number of Times Did Not Produce Smallest p
Ideal Variable	Observed = 3 Expected = 36 Percentage = 17%	Observed = 15 Expected = 14.4 Percentage = 83%
Regression	Observed = 6 Expected = 3.6 Percentage = 33%	Observed = 12 Expected = 14.4 Percentage = 67%
Z Sum	Observed = 3 Expected = 3.6 Percentage = 17%	Observed = 15 Expected = 14.4 Percentage = 83%
Raw Sum	Observed = 5 Expected = 3.6 Percentage = 28%	Observed = 13 Expected = 14.4 Percentage = 72%
Fractional Weights	Observed = 1 Expected = 3.6 Percentage = 6%	Observed = 17 Expected = 14.4 Percentage = 94%
$\chi^2 = 4.22$; $df = 4$; $.50 > p > .30$		

Figure 3.2
Histogram for Data of Table 3.22 Number of Times
Each Factor Score Method Yielded the Smallest
MANOVA P - value

The results of the first chi-square indicate no significant difference in the observed versus expected frequencies across the five factor score procedures in a one-way univariate analysis of variance design (chi-square = 3.22; .70 > p > .50). This finding supports the results of Dagenais and Marascuilo (1973): that the method used to calculate subjects' composite scores does not significantly influence the F-distribution in a univariate analysis of variance test. The results of this analysis do not, however, support those reported by Moseley and Klett (1964), who found a significant difference between scoring methods and statistical outcomes. Although the results of the present analysis appear to confirm those of Dagenais and Marascuilo while contradicting those findings obtained by Moseley and Klett, trends evident in the data which serve to integrate the findings of all three analyses will be discussed in Chapter IV.

Inspection of the observed frequencies across the five scoring procedures reveals that raw sum scores most frequently produced the smallest significance probability in the univariate analyses of variance; the ideal variable and regression factor score algorithms (both exact methods) were next most effective; and Z-sum scores and fractional weight

scores accounted for the fewest percentage of smallest p-values in the univariate tests.

Inspection of the results of the second test (Table 3.22) suggests that variation in scoring, likewise, does not materially affect the number of times each method yields the smallest probability in a multivariate analysis of variance design ($\chi^2 = 4.22$; $.50 > p > .30$). Significance probabilities are distributed across the scoring methods in much the same way as would be expected by chance alone. In this case the regression method produced the highest percentage of smallest p-values (33%). The other four methods, in order of percentage of lowest p-values, were as follows: Raw-Sum (28%), Ideal Variable and Z-Sum (17% each), and Fractional Weights (6%).

In addition to the determination of the most sensitive scoring procedure in univariate and multivariate analyses of variance, this study also sought to investigate the effects of differentially computed factor scores on the statistical results in a multiple discriminant analysis. Explicitly stated, the second research question being examined was as follows:

(2) Does the method used to calculate factor scores have a significant effect on the magnitude of the p-value associated with the first discriminant function in a multiple discriminant analysis; if so, which composite most frequently yields the smallest probability?

In response to this research question, factor scores generated by the five different procedures were used as dependent variables in eighteen multiple discriminant analyses. Chi-square values and their associated significance probabilities were calculated and compared across scoring methods in each test. Tables 3.23 -- 3.40 summarize the results of these analyses (variates producing the lowest p-value for a given test are indicated with an asterisk).

Table 3.23
Study 1
Multiple Discriminant Analysis Summary
Information Generated by Five
Differentially Computed Factor Scores

<u>Method</u>	<u>Variance of Canonical Variate</u>	<u>% of Canonical Variation</u>	<u>χ^2</u>	<u>Significance Probability</u>
Ideal Variable	.144	80.76	40.752	.001
Regression	.127	78.96	37.201	.002
Z-Sum	.127	77.22	37.847	.002
Raw-Sum	.129	77.24	38.533	.001
Weights	.126	77.76	37.495	.002
df = 16				

Table 3.24
Study 3
Multiple Discriminant Analysis Summary
Information Generated by Five
Differentially Computed Factor Scores

<u>Method</u>	<u>Variance of Canonical Variate</u>	<u>% of Canonical Variation</u>	<u>χ^2</u>	<u>Significance Probability</u>
Ideal Variable	.061	100.00	3.129	.372
Regression	.063	100.00	3.186	.364
Z-Sum	.090	100.00	4.536	.209
Raw Sum	.086	100.00	4.350	.226
Weights	.088	100.00	4.439	.218
df = 3				

Table 3.25
Study 4
Multiple Discriminant Analysis Summary
Information Generated by Five
Differentially Computed Factor Scores

<u>Method</u>	<u>Variance of Canonical Variate</u>	<u>% of Canonical Variation</u>	<u>χ^2</u>	<u>Significance Probability</u>
Ideal Variable	.233	100.00	9.113	.028
Regression	.197	100.00	7.804	.050
Z-Sum	.190	100.00	7.558	.056
Raw Sum	.199	100.00	7.894	.048
Weights	.195	100.00	7.750	.052
df = 3				

Table 3.26
Study 5
Multiple Discriminant Analysis Summary
Information Generated by Five
Differentially Computed Factor Scores

<u>Method</u>	<u>Variance of Canonical Variate</u>	<u>% of Canonical Variation</u>	<u>χ^2</u>	<u>Significance Probability</u>
Ideal Variable	.102	100.00	5.582	.134
Regression	.092	100.00	5.081	.166
Z-Sum	.065	100.00	3.595	.309
Raw Sum	.071	100.00	3.968	.265
Weights	.079	100.00	4.369	.224
df = 3				

Table 3.27
Study 6
Multiple Discriminant Analysis Summary
Information Generated by Five
Differentially Computed Factor Scores

<u>Method</u>	<u>Variance of Canonical Variate</u>	<u>% of Canonical Variation</u>	<u>χ^2</u>	<u>Significance Probability</u>
Ideal Variable	.005	100.00	.084	.994
Regression	.018	100.00	.276	.965
Z-Sum	.005	100.00	.082	.994
Raw Sum	.008	100.00	.124	.989
Weights	.007	100.00	.113	.990
df = 3				

Table 3.28
Study 7
Multiple Discriminant Analysis Summary
Information Generated by Five
Differentially Computed Factor Scores

<u>Method</u>	<u>Variance of Canonical Variate</u>	<u>% of Canonical Variation</u>	<u>χ^2</u>	<u>Significance Probability</u>
Ideal Variable	.566	100.00	24.669	.0004
Regression	.566	100.00	24.675	.0004
Z Sum	.512	100.00	22.751	.001
Raw Sum	.522	100.00	23.096	.001
Weights	.536	100.00	23.601	.001
df = 6				

Table 3.29
Study 8
Multiple Discriminant Analysis Summary
Information Generated by Five
Differentially Computed Factor Scores

<u>Method</u>	<u>Variance of Canonical Variate</u>	<u>% of Canonical Variation</u>	<u>χ^2</u>	<u>Significance Probability</u>
Ideal Variable	.310	100.00	17.574	.130
Regression	.308	100.00	17.478	.133
Z Sum	.384	100.00	21.110	.049
Raw Sum	.390	100.00	21.388	.045
Weights	.366	100.00	20.270	.062
df = 12				

Table 3.30
Study 9
Multiple Discriminant Analysis Summary
Information Generated by Five
Differentially Computed Factor Scores

<u>Method</u>	<u>Variance Canonical Variate</u>	<u>% of Canonical Variation</u>	<u>χ^2</u>	<u>Significance Probability</u>
Ideal Variable	.282	100.00	13.518	.261
Regression	.295	100.00	14.101	.228
Z -Sum	.297	100.00	14.164	.224
Raw Sum	.305	100.00	14.525	.205
Weights	.314	100.00	14.880	.188
df = 11				

Table 3.31
Study 10
Multiple Discriminant Analysis Summary
Information Generated by Five
Differentially Computed Factor Scores

<u>Method</u>	<u>Variance of Canonical Variate</u>	<u>% of Canonical Variation</u>	<u>χ^2</u>	<u>Significance Probability</u>
Ideal Variable	.141	100.00	7.722	.172
Regression	.151	100.00	8.239	.144
Z Sum	.168	100.00	9.090	.106
Raw Sum	.176	100.00	9.491	.091
Weights	.174	100.00	9.386	.095
df = 5				

Table 3.32
Study 11
Multiple Discriminant Analysis Summary
Information Generated by Five
Differentially Computed Factor Scores

<u>Method</u>	<u>Variance of Canonical Variate</u>	<u>% of Canonical Variation</u>	<u>χ^2</u>	<u>Significance Probability</u>
Ideal Variable	.299	100.00	15.946	.101
Regression	.372	100.00	19.309	.037
Z Sum	.354	100.00	18.466	.048
Raw Sum	.356	100.00	18.567	.046
Weights	.358	100.00	18.648	.045
df = 10				

Table 3.33
Study 12
Multiple Discriminant Analysis Summary
Information Generated by Five
Differentially Computed Factor Scores

<u>Method</u>	<u>Variance of Canonical Variate</u>	<u>% of Canonical Variation</u>	<u>χ^2</u>	<u>Significance Probability</u>
Ideal Variable	.478	100.00	14.855	.023
Regression	.494	100.00	15.266	.018
Z Sum	.475	100.00	14.789	.022
Raw Sum	.463	100.00	14.461	.025
Weights	.484	100.00	15.010	.020
df = 6				

Table 3.34
Study 13
Multiple Discriminant Analysis Summary
Information Generated by Five
Differentially Computed Factor Scores

<u>Method</u>	<u>Variance of Canonical Variate</u>	<u>% of Canonical Variation</u>	<u>χ^2</u>	<u>Significance Probability</u>
Ideal Variable	.103	100.00	4.689	.321
Regression	.107	100.00	4.871	.301
Z Sum	.087	100.00	3.991	.407
Raw Sum	.083	100.00	3.806	.433
Weights	.070	100.00	3.247	.518
df = 4				

Table 3.35
Multiple Discriminant Analysis Summary
Information Generated by Five
Differentially Computed Factor Scores

<u>Method</u>	<u>Variance of Canonical Variate</u>	<u>% of Canonical Variation</u>	<u>χ^2</u>	<u>Significance Probability</u>
Ideal Variable	.482	100.00	6.49	.839
Regression	.956	100.00	11.07	.437
Z Sum	.699	100.00	8.75	.645
Raw Sum	.410	100.00	5.67	.895
Weights	.637	100.00	8.13	.702
df = 11				

Table 3.36
Multiple Discriminant Analysis Summary
Information Generated by Five
Differentially Computed Factor Scores

<u>Method</u>	<u>Variance of Canonical Variate</u>	<u>% of Canonical Variation</u>	<u>χ^2</u>	<u>Significance Probability</u>
Ideal Variable	.113	100.00	4.957	.421
Regression	.103	100.00	4.556	.473
Z Sum	.139	100.00	6.065	.301
Raw Sum	.135	100.00	5.889	.317
Weights	.137	100.00	5.972	.309
df = 5				

Table 3.37
Multiple Discriminant Analysis Summary
Information Generated by Five
Differentially Computed Factor Scores

<u>Method</u>	<u>Variance of Canonical Variate</u>	<u>% of Canonical Variation</u>	<u>χ^2</u>	<u>Significance Probability</u>
Ideal Variable	.383	80.12	24.29	.035
Regression	.376	82.31	23.81	.048
Z Sum	.377	78.31	25.17	.033
Raw Sum	.380	78.23	25.34	.031
Weights	.373	78.40	24.91	.036

df = 14

Table 3.38
Multiple Discriminant Analysis Summary
Information Generated by Five
Differentially Computed Factor Scores

<u>Method</u>	<u>Variance of Canonical Variate</u>	<u>% of Canonical Variation</u>	<u>χ^2</u>	<u>Significance Probability</u>
Ideal Variable	.365	59.28	22.98	.061
Regression	.400	69.98	24.63	.038
Z Sum	.290	52.71	20.910	.104
Raw Sum	.291	52.16	21.13	.098
Weights	.295	52.68	21.19	.097

df = 14

Table 3.39
Multiple Discriminant Analysis Summary
Information Generated by Five
Differentially Computed Factor Scores

<u>Method</u>	<u>Variance of Canonical Variate</u>	<u>% of Canonical Variation</u>	<u>χ^2</u>	<u>Significance Probability</u>
Ideal Variable	.222	62.63	17.18	.5108
Regression	.204	62.64	16.03	.5904
Z Sum	.261	64.45	19.44	.3655
Raw Sum	.265	64.21	19.77	.3459
Weights	.241	64.19	18.14	.4463

df = 18

Table 3.40
Multiple Discriminant Analysis Summary
Information Generated by Five
Differentially Computed Factor Scores

<u>Method</u>	<u>Variance of Canonical Variate</u>	<u>% of Canonical Variation</u>	<u>χ^2</u>	<u>Significance Probability</u>
Ideal Variable	.313	47.49	115.31	.0001
Regression	.243	39.62	108.59	.0001
Z Sum	.352	47.44	128.11	.0001
Raw Sum	.329	45.54	125.44	.0001
Weights	.303	44.26	119.57	.0001

df = 54

As with the univariate and multivariate analyses of variance, frequencies (of lowest p-values) were computed across the five scoring methods and then compared utilizing a chi-square procedure. Table 3.41 and Figure 3.3 depict the results of this analysis.

Table 3.41
Number of Times Factor Score Methods Yielded
the Smallest Significance Probability;
Multiple Discriminant Analyses

Factor Score Algorithm	Number of Times Produced Smallest p	Number of Times Did Not Produce Smallest p
Ideal Variable	Observed = 3 Expected = 3.6 Percentage = 17%	Observed = 15 Expected = 14.4 Percentage = 83%
Regression	Observed = 6 Expected = 3.6 Percentage = 33%	Observed = 12 Expected = 14.4 Percentage = 67%
Z Sum	Observed = 3 Expected = 3.6 Percentage = 17%	Observed = 15 Expected = 14.4 Percentage = 83%
Raw Sum	Observed = 5 Expected = 3.6 Percentage = 28%	Observed = 13 Expected = 14.4 Percentage = 72%
Fractional Weights	Observed = 1 Expected = 3.6 Percentage = 6%	Observed = 17 Expected = 14.4 Percentage = 94%
$\chi^2 = 4.22; df = 4; .50 > p > .30$		

Figure 3.3
Histogram for Data of Table 3.41. Number of Times
Each Factor Score Method Yielded the Smallest
Discrim P - value

An examination of the results of the chi-square test indicates no significant difference between factor score algorithms in terms of the frequency of lowest p-values associated with each method. The order of the various factor score procedures (highest percentage of smallest probability levels to lowest) was exactly the same as was the case for the multivariate analysis of variance: Regression (33%), Raw Sum (28%), Z-Sum and Ideal Variable (17% each), and Fractional Weights (6%).

On the basis of these findings one must again conclude that the method of arriving at factor scores does not significantly influence the statistical results obtained when these variates are input as dependent variables in a multiple discriminant analysis.

A similar conclusion may be derived in regard to the third research question posited by this investigator:

(3) Does the method used to calculate factor scores have a significant effect on the magnitude of the p-value associated with the first canonical root in a canonical correlation analysis; if so, which composite most frequently yields the smallest probability?

Again, factor scores generated by the five different computational procedures were input as criterion variables in fourteen canonical correlation analyses. Inasmuch as canonical correlation requires two sets of dependent measures containing at least two variates each, only fourteen of the possible twenty studies could be used for this analysis. Resultant chi-square values and their associated probabilities were compared across scoring procedures for each of the fourteen tests. Tables 3.42 -- 3.56 depict the results of these analyses (the variate yielding the smallest p-value in each test is designated with an asterisk).

As with the foregoing statistical tests, frequencies (of lowest p-values) were computed across the five scoring procedures and compared using an appropriate chi-square test. Table 3.57 and Figure 3.4 display the results of this analysis.

Table 3.42
Study 1
Canonical Correlation Analysis Summary
Information Generated by Five
Differentially Computed Factor Scores

<u>Method</u>	<u>Canonical Correlation</u>	<u>χ^2</u>	<u>Significance Probability</u>
Ideal Variable	.196	10.111	.039
Regression	.190	9.266	.055
Z Sum	.142	5.454	.244
Raw Sum	.142	5.456	.244
Frac Weights	.142	5.743	.219
df = 4			

Table 3.43
Study 2
Canonical Correlation Analysis Summary
Information Generated by Five
Differentially Computed Factor Scores

<u>Method</u>	<u>Canonical Correlation</u>	<u>χ^2</u>	<u>Significance Probability</u>
Ideal Variable	.357	10.477	.313
Regression	.385	12.366	.194
Z Sum	.397	11.399	.249
Raw Sum	.404	11.932	.217
Frac Weights	.411	12.269	.199

Table 3.44
Study 8
Canonical Correlation Analysis Summary
Information Generated by Five
Differentially Computed Factor Scores

<u>Method</u>	<u>Canonical Correlation</u>	<u>χ^2</u>	<u>Significance Probability</u>
Ideal Variable	.877	137.497	.0001
Regression	.888	145.829	.0001
Z Sum	.857	127.452	.0001
Raw Sum	.852	125.001	.0001
Frac Weights	.865	130.301	.0001
df = 36			

Table 3.45
Study 9
Canonical Correlation Analysis Summary
Information Generated by Five
Differentially Computed Factor Scores

<u>Method</u>	<u>Canonical Correlation</u>	<u>χ^2</u>	<u>Significance Probability</u>
Ideal Variable	.723	62.021	.001
Regression	.727	63.201	.0004
Z Sum	.727	56.312	.003
Raw Sum	.729	56.216	.003
Frac Weights	.733	58.169	.002
df = 30			

Table 3.46
Study 10
Canonical Correlation Analysis Summary
Information Generated by Five
Differentially Computed Factor Scores

<u>Method</u>	<u>Canonical Correlation</u>	<u>X²</u>	<u>Significance Probability</u>
Ideal Variable	.204	3.443	.752
Regression	.261	4.259	.642
Z Sum	.222	3.333	.766
Raw Sum	.239	3.687	.719
Frac Weights	.221	3.250	.777
df = 6			

Table 3.47
Canonical Correlation Analysis Summary
Information Generated by Five
Differentially Computed Factor Scores

<u>Method</u>	<u>Canonical Correlation</u>	<u>X²</u>	<u>Significance Probability</u>
Ideal Variable	.813	86.024	.0001
Regression	.773	76.547	.0001
Z Sum	.852	109.997	.0001
Raw Sum	.854	111.691	.0001
Frac Weights	.852	108.542	.0001
df = 25			

Table 3.48
Study 12
Canonical Correlation Analysis Summary
Information Generated by Five
Differentially Computed Factor Scores

<u>Method</u>	<u>Canonical Correlation</u>	<u>χ^2</u>	<u>Significance Probability</u>
Ideal Variable	.757	35.500	.0001
Regression	.641	23.421	.005
Z Sum	.801	42.697	.0001
Raw Sum	.799	41.788	.0001
Frac Weights	.796	42.273	.0001
df = 9			

Table 3.49
Study 13
Canonical Correlation Analysis Summary
Information Generated by Five
Differentially Computed Factor Scores

<u>Method</u>	<u>Canonical Correlation</u>	<u>χ^2</u>	<u>Significance Probability</u>
Ideal Variable	.575	20.119	.0005
Regression	.513	15.372	.004
Z Sum	.516	15.055	.005
Raw Sum	.504	14.259	.007
Frac Weights	.518	14.819	.005
df = 4			

Table 3.50
Study 14
Canonical Correlation Analysis Summary
Information Generated by Five
Differentially Computed Factor Scores

<u>Method</u>	<u>Canonical Correlation</u>	<u>χ^2</u>	<u>Significance Probability</u>
Ideal Variable	.914	42.25	.068
Regression	.835	29.28	.503
Z Sum	.736	28.38	.550
Raw Sum	.853	39.14	.123
Frac Weights	.761	31.32	.400
df = 30			

Table 3.51
Study 15
Canonical Correlation Analysis Summary
Information Generated by Five
Differentially Computed Factor Scores

<u>Method</u>	<u>Canonical Correlation</u>	<u>χ^2</u>	<u>Significance Probability</u>
Ideal Variable	.287	4.402	.622
Regression	.320	5.271	.510
Z Sum	.475	12.244	.057
Raw Sum	.455	11.264	.081
Frac Weights	.467	11.787	.067
df = 6			

Table 3.52
Study 16
Canonical Correlation Analysis Summary
Information Generated by Five
Differentially Computed Factor Scores

<u>Method</u>	<u>Canonical Correlation</u>	<u>χ^2</u>	<u>Significance Probability</u>
Ideal Variable	.434	17.66	.127
Regression	.419	19.01	.088
Z Sum	.406	13.38	.342
Raw Sum	.410	13.43	.338
Frac Weights	.414	14.16	.291
df = 12			

Table 3.53
Study 17
Canonical Correlation Analysis Summary
Information Generated by Five
Differentially Computed Factor Scores

<u>Method</u>	<u>Canonical Correlation</u>	<u>χ^2</u>	<u>Significance Probability</u>
Ideal Variable	.627	26.45	.009
Regression	.636	27.25	.007
Z Sum	.612	24.43	.018
Raw Sum	.618	25.09	.015
Frac Weights	.615	24.68	.017
df = 12			

Table 3.54
Study 18
Canonical Correlation Analysis Summary
Information Generated by Five
Differentially Computed Factor Scores

<u>Method</u>	<u>Canonical Correlation</u>	<u>χ^2</u>	<u>Significance Probability</u>
Ideal Variable	.7873	69.71	.0001
Regression	.7843	65.30	.0001
Z Sum	.7938	71.12	.0001
Raw Sum	.7992	72.10	.0001
Frac Weights	.7891	69.48	.0001
df = 20			

Table 3.55
Study 19
Canonical Correlation Analysis Summary
Information Generated by Five
Differentially Computed Factor Scores

<u>Method</u>	<u>Canonical Correlation</u>	<u>χ^2</u>	<u>Significance Probability</u>
Ideal Variable	.429	52.93	.0001
Regression	.444	53.96	.0001
Z Sum	.722	149.67	.0001
Raw Sum	.744	162.70	.0001
Frac Weights	.729	155.67	.0001
df = 9			

Table 3.56
Number of Times Factor Score Methods Yielded
the Smallest Significance Probability;
Canonical Corr. An.

Factor Score Algorithm	Number of Times Produced Smallest p	Number of Times Did Not Produce Smallest p
Ideal Variable	Observed = 3 Expected = 2.8 Percentage = 21%	Observed = 11 Expected = 11.2 Percentage = 79%
Regression	Observed = 6 Expected = 2.8 Percentage = 43%	Observed = 8 Expected = 11.2 Percentage = 57%
Z Sum	Observed = 2 Expected = 2.8 Percentage = 14%	Observed = 12 Expected = 11.2 Percentage = 86%
Raw Sum	Observed = 3 Expected = 2.8 Percentage = 21%	Observed = 11 Expected = 11.2 Percentage = 79%
Fractional Weights	Observed = 0 Expected = 2.8 Percentage = 0%	Observed = 14 Expected = 11.2 Percentage = 100%
$\chi^2 = 6.71; df = 4 \quad .20 > p > .10$		

Figure 3.4
Histogram for Data of Table 3.55. Number of Times
Each Factor Score Method Yielded the Smallest
CANCORR P - value

The results of the test indicate no significant differences between factor score methods in terms of the frequency of lowest p-values associated with each procedure. Inspection of the observed frequencies across the five scoring methods reveals that the regression factor score algorithm most often yielded the smallest probability; the ideal variable and raw sum procedures were next most effective; and Z-sum scores and fractional weight scores accounted for the smallest percentage of lowest p-values in the canonical correlation tests.

These findings again suggest that no one method of deriving subjects' composite scores offers any clear statistical advantage over the other methods when variates are used in a canonical correlation analysis.

Chapter IV

DISCUSSION

This chapter is devoted to three major areas: (1) a discussion of the statistical results associated with each of the stated research questions; (2) an analysis of the findings in order to offer some viable explanations and additional insights; and (3) a brief statement of preferences for the direction of future research.

FINDINGS

Analysis of Results by Research Question

This study explored the relative effects of different factor score procedures on the empirical results of various statistical analyses. The first relationship investigated, questioned the effects of five differentially computed factor scores on the magnitude of the F-value and its associated significance probability in a univariate and multivariate one-way analysis of variance design. Inspection of the results of the univariate tests indicates no statistical difference between factor score algorithms in

terms of the frequency of lowest p-values associated with each method. These findings support the conclusion posited by Dagenais and Marascuilo (1973) that the method used to calculate subjects' composite scores does not significantly influence the F-distribution in a one-way univariate analysis of variance design; they do not support the findings reported by Moseley and Klett (1964) who discovered a significant difference between scoring procedures and statistical outcomes.

While statistical significance was not attained in the present investigation, trends evident in the data still provide an indication of the 'best' factor score procedure to derive criterion variables for use in a univariate analysis of variance. Examination of the observed frequencies of smallest p-values across the factor score algorithms reveals little difference between raw sum, ideal variable, and regression procedures in terms of their obtained frequencies ($f_o = 28, 25, 25$ respectively). The results also indicate that composites generated via the Z-sum and fractional weight methods are likewise highly similar, although their observed frequencies of smallest p-values are less than would be expected on the basis of chance alone ($f_o = 19, 18$). These results seem to suggest that the exact factor score methods (and the raw sum procedure) offer a slight advantage over Z-sum and fractional weight scores when

utilized in a one-way analysis of variance design.

This grouping of factor score procedures according to their observed frequencies (of smallest probabilities) was not entirely unexpected. Correlational studies have indicated that while factors scores generated by the ideal variable and regression procedures were highly correlated ($r > .90$) as were those approximated by the Z-sum and fractional weight methods ($r > .90$), the exact and inexact factor score techniques yielded scores which correlated .80 on the average (Raw sum scores have generally not been included in these analyses). The arrangement of factor score procedures according to their observed frequencies in the univariate analyses of variance is directly analogous to the correlations of the various factor scores derived in previous research. Whereas the observed frequencies of both the exact and inexact factor score procedures are highly similar, a sizeable difference exists between the frequency (of smallest p-values) of the two broad categories of scoring methods.

Although Dagenais and Marascuilo (1973) and Moseley and Klett (1964) limited their investigations primarily to the obtained statistical results, inspection of the order of factor score procedures in terms of highest frequency of lowest p-values yielded in the previous studies reveals that

they parallel the order of scoring methods produced in the present investigation. Dagenais and Marascuilo (1973) explored the effects of the ideal variable, Z-sum, and Guttman scoring procedures on the statistical results in a univariate analysis of variance design (Guttman scores, however, are not relevant to the present research purposes). Their results indicate that ideal variable scores yield the smallest significance probability more frequently than Z-sum scores, a finding consistent with the present investigation. Similarly, the results of Moseley and Klett's (1964) study on the effects of the scoring method on the F-distribution in an analysis of variance design indicate that, "...fractional weights were somewhat better than unit weighting, and the regression method had the highest F-value of all methods" (p. 183). Although Moseley and Klett's findings regarding the Z-sum and fractional weight algorithms were exactly opposite to the results obtained in the present study, the difference between these two methods was slight in both instances. More importantly, as with Moseley and Klett's investigation, the present research effort also found that the regression method more frequently produced the smallest p-value than did either Z-sum or fractional weight scores in a one-way univariate analysis of variance design.

As with the univariate analyses, inspection of the

results for the multivariate tests reveals no statistical difference in the obtained versus expected frequencies across the five factor score methods ($\chi^2 = 4.22$; $.50 > p > .30$). Although the same statistical decision was attained by both analyses of variance, trends evident in this test deviate from those associated with the univariate findings. Whereas in the univariate case both of the exact methods (and the raw sum procedure) yielded the highest observed frequency of lowest p-values, in the multivariate case the regression factor score technique was clearly most effective in producing the smallest probability ($f_o = 6$) with Raw sum scores again resulting in a higher observed than expected frequency. Ideal variable scores and Z-sum scores yielded an observed number of frequencies slightly smaller than would be anticipated by chance alone ($f_o = 3$ each). The fractional weighting method again produced the lowest observed cell frequency (accounting for only 6% of the total number of multivariate tests).

Whereas the grouping of the factor score procedures in terms of their observed frequencies (of smallest p-values) was consistent with the results of the correlational studies in the univariate analysis of variance, the order of methods yielded by the multivariate tests does not coincide with these findings. Based upon prior factor score correlational research it would be predicted that composites calculated by

the ideal variable and regression procedures would have similar frequencies, as would Z-sum scores and fractional weight scores. However, inspection of the observed frequencies reveals no apparent order or pattern evidenced among the scoring procedures. Nevertheless, since the number of multivariate analyses included in the study is much smaller than the number of univariate tests (18 versus 115), it is possible that the addition of more tests would produce the predicted pattern of factor score frequencies.

The second research question which this study investigated was the effects of the various factor score algorithms on the statistical results associated with the first root in a multiple discriminant analysis. As in the analysis of variance tests, results of the chi-square analysis indicate no significant statistical difference between the five factor score algorithms in terms of their observed frequencies of smallest p-values (chi-square = 4.22; $.50 > p > .30$). Nonetheless, an examination of the findings reveals a recurrent trend evidenced among the scoring methods. As in the multivariate analysis of variance, composites calculated via the regression factor score technique once again yielded the highest frequency of smallest p-values ($f_o = 6$), and fractional weight scores produced the lowest observed frequency (accounting for only 6% of the total number of discriminant analysis tests). Moreover, the order of the

remaining factor score procedures (namely: raw sum, Z-sum, and ideal variable scores) was precisely the same as that generated in the multivariate tests. Raw sum score were, again, the second most effective procedure in terms of yielding the highest frequency (of smallest p-values -- $f_0 = 5$). Both the exact ideal variable scores and the inexact Z-sum scores accounted for 17% of the total discriminant analyses, their observed frequencies slightly smaller than would be anticipated by chance alone.

It is not surprising that the order of factor score procedures generated by this statistical test parallels the one found in the multivariate analysis of variance. The significance probability associated with the first discriminant root is often very similar to the p-value generated in the corresponding multivariate analysis of variance. This is due to the fact that the first root is the best linear combination of the dependent measures, a statistical relationship which is also reflected in the multivariate F-ratio. Discrepancies can occur, however, inasmuch as the chi-square value associated with the discriminant function is Rao's approximation of the multivariate F-distribution.

The final research question which this study sought to investigate explored the relative effects of differentially computed factor scores on the magnitude of the p-value in a

canonical correlation analysis. As in the foregoing statistical tests, the results of the chi-square analysis indicate no significant difference in the observed versus expected frequencies of smallest probabilities across the factor score procedures (chi-square = 6.71; $.20 > p > .10$). However, also resembling the analyses of variance and multiple discriminant analysis, is the trend evidenced in these data. Once again, factor scores generated via the regression algorithm yielded the highest frequency of smallest p-values ($f_o = 6$), and fractional weight scores the lowest ($f_o = 0$). The order of the remaining three methods varies slightly from the multivariate analysis of variance and discriminant analysis results. In this case, ideal variable scores and raw sum scores were the next most effective factor score procedures in terms of producing the smallest probabilities ($f_o = 3$ each), followed by Z-sum scores which accounted for 14% of the total number of canonical correlation analyses.

Conclusions

The data concerning the relative effects of the differentially computed factor scores on experimental results suggests the following general conclusions:

(1) The method used to compute subjects' factor scores does not significantly influence the resultant probability values in a univariate or multivariate analysis of variance, multiple discriminant analysis, or canonical correlation analysis.

(2) Trends evident in the data suggest that factor scores generated via the exact regression procedure may be more effective in detecting small group differences when used as criterion variables in a multivariate analysis of variance, discriminant analysis, or canonical correlation analysis. In the univariate analysis of variance, raw sum scores, ideal variable scores, and regression scores appear to be equally sensitive to mean differences.

(3) Factor scores produced by the inexact fractional weight procedure seem to be the least sensitive of the various composites investigated in the present study, regardless of the type of statistical test in which they are input as dependent variables.

Recommendations

Based upon the trends evident in these data and the results of former investigations in this area of research, a researcher is advised to derive his subjects' factor scores via the exact regression procedure if they are to be input as criterion variables in a univariate or multivariate analysis of variance, multiple discriminant analysis, or canonical correlation analysis. However, this recommendation is contingent upon whether the characteristics of the resultant set of factor score, and underlying assumptions of

the algorithm are consistent with an investigator's research purposes.

As noted earlier, although factor scores generated by the regression procedure possess a high correlation with their respective factors, they often correlate with one another, and with factors other than the one they were designed to measure. Therefore, if a study's theoretical foundation or statistical analysis assumes orthogonal, univocal composites, then regardless of how sensitive regression estimates are to small group differences, they should not be employed in the measurement of the factor. Similarly, since the regression factor score algorithm requires the complete data matrix be used in its estimation of factor scores, if future studies will not employ the full set of variables, then an inexact method would be an investigator's only choice.

The results of the present study indicate that raw sum scores yielded a higher frequency (of smallest p-values) than did either Z sum scores or fractional weight scores across all of the statistical tests. This finding would seem to suggest that whenever the use of factor scores necessitates a reduced sample of variables, composites should be approximated by the raw sum method. However, despite the method's high frequency of smallest probabili-

ties, it is not a recommended way to compute factor scores. While the raw sum procedure implies that all variables are assigned unit weights, in fact, they are disproportionately weighted by their variances. Such differences in variances may create a critical difference in terms of changing the basic definition of the construct involved, or in terms of external prediction. For this reason, a researcher is not advised to use raw sum scores in his statistical tests. Rather, if an inexact method is demanded, Z-sum scores (which have a slight advantage over fractional weights in terms of highest frequency of lowest p values across the statistical tests) would be a researcher's best choice.

INTERPRETATION AND ADDITIONAL INSIGHTS

The reader may recall that while no significant difference was found in terms of the various factor scores producing different statistical results in either the univariate or multivariate tests, a recurring trend was evidenced across the analyses. An inspection of the observed frequencies of lowest p-values for the different factor score procedures revealed that the regression algorithm yielded the highest frequency (of smallest probabilities) in every statistical test except the univariate analysis of variance, and that even in that case, ranked high in the order of factor score methods. One implication

drawn from this trend in the data was that regression estimates are more effective in detecting mean group differences when used as dependent variables in statistical analyses than are the composites generated via the other scoring procedures.

These results can perhaps be explained in terms of the error variance comprising each of the differentially computed factor scores. As was stated in Chapter I, composites estimated by an inexact procedure, because of their simplistic weighting schemas, contain less sample specific variation than the scores generated by more sophisticated factor score algorithms. However, these reduced weight matrices are responsible for another type of error variation afflicting the inexact factor scores, that is, error due to the sampling of items. Psychometric research has suggested that the precision of a linear combination is directly related to the number of variables which it combines. Whereas the combination of a few variables is a more precise measure than a single item, so is a composite based upon a large number of variables more reliable than one which is based upon only a few. Hence, since inexact factor score procedures utilize only a few salient items in their factor score calculations, this type of error is more of a distinct possibility for them than it is for the exact regression estimates whose calculations employ the complete

Z-matrix.

This increased amount of error contained in the inexact scores can diminish their effectiveness in detecting group differences in various statistical analyses. For example, inasmuch as the method of analysis of variance pits variance due to the experimental manipulations against variance due to error or randomness, it would not be unexpected if the inexact scores resulted in smaller F-ratios than the corresponding regression estimates. For although the mean differences produced by the differentially computed factor scores may be the same, the error variance in the inexact composites is greater (due to the reduced number of variables included in its calculations), thus resulting in a smaller F-value. This same general principle may also be applicable in multiple discriminant analysis and canonical correlation analysis.

In order to investigate this possibility the standard deviations and error mean squares of the differentially computed factor scores were contrasted for those analyses in which the regression procedure was found to yield the lowest significance probability. The results of this examination indicate that in those tests in which regression estimates produced the smallest p-value, their corresponding error variances were generally less than those of the inexact

factor scores. Thus, this finding favors the proposed explanation that error variance is responsible for the superiority of the regression scoring procedure over the inexact methods in terms of yielding a higher frequency (of smallest significance probabilities) in the various statistical tests.

Adopting this same line of reasoning, it would likewise be expected that the ideal variable algorithm would also be more effective in producing the smallest p-values than the inexact techniques. This is due to the fact that, like the regression computational procedure, it too utilizes the entire data matrix to calculate subjects' factor scores. However, inspection of the observed frequencies (of smallest p-values) across scoring procedures does not confirm this expectation. Whereas the results indicate that the ideal variable scores have a higher observed frequency than fractional weight scores in each statistical analysis, they also reveal that this method's observed frequencies are equal to or less than those obtained by the inexact Z-sum and raw sum scores.

These findings seem to discredit the proposed explanation which cites error variance as the factor differentiating the regression procedure from the inexact scoring methods. A closer examination of the significance probabi-

lities yielded by the regression estimates and the ideal variable scores, however, offers a plausible explanation for this discrepancy. While a high degree of similarity exists between the p-values associated with these two methods (the average difference equals approximately .04 across the statistical tests), the regression probabilities were generally smaller than the ones produced by the ideal variable scores. Therefore, because of a slight numerical advantage, regression estimates were often awarded the point (for having the lowest p-value), while ideal variable composites (despite their similarity) were not. Therefore, analogous to the idea of suppressor variables in canonical correlation analysis, regression estimates obtained a high observed frequency (of smallest p-values); ideal variable scores yielded a low frequency. For this reason it appeared as if only the regression method was more effective than the inexact scoring procedures. However, when the regression factor score method was deleted from the statistical analyses and the tests in which it produced the lowest significance probability were recategorized (according to the factor score method with the next smallest p-value), the ideal variable algorithm then yielded the highest frequency of smallest probabilities in all of the various analyses (i.e., univariate and multivariate analysis of variance, multiple discriminant analysis, and canonical correlation analysis). These findings further support the theory that

factor score methods utilizing reduced data matrices in the calculation of composite variables contain more error than the exact scoring procedures and, therefore, are less effective measures in detecting small group differences.

While the foregoing analysis accounts for the superiority of the regression factor score method over the less sophisticated inexact procedures, it does not explain why regression scores generally yielded slightly smaller p-values than the ideal variable scores, since both of these exact methods use the complete variable set in their factor score calculations. Rather, the difference may be explained in terms of the various factor analytic models which are used to derive the composite variables. Whereas the ideal variable scores are based upon a principal components solution, the corresponding regression estimates are founded upon a common factor analytic model. Marked differences can be expected between the scores generated from the two analyses, especially when the number of variables is relatively small ($n < 20$), and the communalities are low. This is due to the varying assumptions underlying the factor models. In a principal components analysis all of a variable's variance is regarded as relevant, and therefore not only common factor variance but also specific factor variance and random error are included in the factoring process. The inclusion of "unwanted" sources of variation can yield

inflated factor loadings which ultimately can effect the resultant ideal variable factor scores. Because the rotated loadings are part of the factor score computational procedures, the inclusion of error variance in these loadings can seriously impede the effectiveness of the derived composites in detecting small mean differences in statistical analyses.

Regression estimates, however, are not based upon a factor model which assumes error-free variables. Instead, they are derived via a common factor solution which attempts to partial out specific and random error variance, and factor only common (or shared) variance. This usually results in more conservative but also more accurate factor loadings. Again, these variable-factor correlations are used to derive the regression estimates. Since the loadings are generally more precise, so are the resultant factor scores. Hence, regression estimates would often be more sensitive to mean group differences than the corresponding exact ideal variable scores.

However, in the case of a large number of variables and high communalities, common factor procedures merge toward the principal components. Therefore, the factor scores generated by the analyses would be nearly identical. In this instance, one could expect the smallest probabilities associated with the various statistical tests to be distri-

buted across the two exact scoring methods in much the same way as would be anticipated by chance alone.

SUGGESTIONS FOR FUTURE RESEARCH

In most research problems it is usually desirable to derive more than the first discriminant function or canonical root (when more than a single significant root exists) in order to obtain the clearest understanding of the relationships operating between the dependent variables. While trends evident in the present investigation suggest that regression factor scores are most effective in yielding the smallest p-value in the first relationship, it is still unknown what effects the differential scoring procedures have on the statistical results in successive discriminant and canonical roots. In addition, the effects of the various factor score procedures on the results associated with a factorial univariate and/or multivariate analysis of variance has yet to be explored.

This study included five factor score procedures which are commonly used in empirical investigations. However, Bartlett (1937) and Anderson and Rubin (1956), among others, have proposed alternative ways to construct subjects' factor scores which only recently are becoming well-known. As these methods gain recognition in the psychometric litera-

ture and their computational procedures programmed onto computer facilities, these factor score algorithms should be investigated to determine whether they are more sensitive to mean group differences than the methods presently examined.

In addition to the foregoing suggestions, this author advocates a replication of the present study as well as Monte Carlo investigations in this area of research. Such endeavors are demanded in order to more clearly understand the relative effects of the factor score algorithm on the results of statistical tests.

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APPENDIX A

EXAMPLE OF THE FORTRAN PROGRAM USED
 TO CALCULATE INEXACT FACTOR SCORES:
 (Data taken from Study 19; Dependent Variable Set 1; N =
 200; n = 12; p = 2).

```

    DIMENSION VAP (12), WEIGHT (12,2)

    READ (5,100) ((WEIGHT(N,M),M=1,2) N=1,12)
100  FORMAT (3X,2F6.5)

    DO 300 I = 1,200

    READ (5,200) VAR
200  FORMAT (3X,6F10.5/6F10.5)

    FACSC1 = VAR(2) +VAR(3) +VAP(4) +VAR(7) +VAR(8)

    FACSC2 = VAR(6) +VAR(9) +VAR(11) +VAR(12)

    FACSC3 = VAR(2)*WEIGHT(2,1) +VAR(3)*WEIGHT(3,1) +
1VAR(4)*WEIGHT(4,1) +VAR(7)*WEIGHT(7,1) +VAR(8)*WEIGHT(8,1)

    FACSC4 = VAR(6)*WEIGHT(6,2) +VAR(9)*WEIGHT(9,2) +
1VAR(11)*WEIGHT(11,2) +VAR(12)*WEIGHT(12,2)

    WRITE (6,400) I, FACSC1, FACSC2, FACSC3, FACSC4

    WRITE (7,400) I, FACSC1, FACSC2, FACSC3, FACSC4
300  CONTINUE
400  FORMAT (I3,4F10.5)

    STOP

    END

```