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Abstract

Deployment of an intelligent transportation systems (ITS) program such as a real-time travel guidance system requires the good understanding of people's travel choice process. The whole travel choice process includes a series of choices including trip choice, destination choice, mode choice, departure time choice and route choice. Traditionally, static travel choice models or transportation network models have been developed to model the travel choice process. However, the static models cannot provide the real time traffic volume and travel time and cannot reflect the time-dependent variation of traffic in a road network. Thus static travel choice models cannot model the dynamic process in travel choice. The dynamic models can provide the real time link and path traffic volume and link and path travel time and can model the dynamic process in travel choice. Dynamic models are also applicable to long-term transportation planning. Unfortunately, the current studies on dynamic travel choice/dynamic transportation network have limitations on either modeling method or solution algorithm, which impede their application in practice.

In this dissertation, I have conducted a comprehensive study on dynamic travel choice problems and have presented a series of variational inequality models and solution algorithms to these problems. The problems that the dissertation addresses include deterministic dynamic user optimal route choice problem (DUO), stochastic dynamic user optimal route choice problem (SDUO), dynamic user optimal simultaneous departure time and route choice problem (DUOSDTRC), combined mode split and dynamic user optimal simultaneous departure time and route choice problem (MS DUOSDTRC), combined trip distribution and dynamic user optimal simultaneous departure time and route choice problem (TD DUOSDTRC), and combined trip distribution mode split and dynamic user optimal simultaneous departure time and route choice problem (TD MS DUOSDTRC). The innovative work is reflective of the successful modeling and development of corresponding algorithms without time-space network expansion. As a result, simplified and potentially efficient solution algorithms to the dynamic travel choice problems over a large-scaled transportation network are developed. All the models and algorithms are validated by numerical examples.

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List of abbreviations

CTM: cell transmission model

DTA: dynamic traffic assignment

DUO: deterministic dynamic user optimal route choice model

DUOIM: dynamic user optimal route choice with incident management

DUOST : dynamic user optimal route choice model integrated with signal timing system

DUOSDTRC: dynamic user optimal simultaneous departure time and route choice model

F-W: Frank-Wolfe

GP: gradient projection

MS DUOSDTRC: combined mode split and dynamic user optimal simultaneous departure time and route choice model

O-D: origin-destination

PQ: point-queue

SDUO: stochastic dynamic user optimal route choice model

SUE: stochastic user equilibrium

TD DUOSDTRC: combined trip distribution and dynamic user optimal simultaneous departure time and route choice model

TD MS DUOSDTRC: combined trip distribution mode split and dynamic user optimal simultaneous departure time and route choice model

UE: user equilibrium

UO : user optimal

VI: variational inequality

List of symbols

$f^{rs}(k)$: departure flow from origin r toward destination s during interval k

$u_a(k)$: inflow into link a during interval k

$u_a^{rs}(k)$**:** inflow into link a on a route between O-D pair rs at time k

$v_a(k)$: exit flow from link a during interval k

$v_a^{rs}(k)$**:** outflow from link a on a route between O-D pair rs at time k

$x_a(k)$: number of vehicles on link a at beginning of interval k

$\tau_a(k)$: actual travel time over link a for flows entering link a at time k

$\eta_p^{rs}(k)$: actual travel time for route p between O-D pair rs for flows departing origin r at time k

$\eta_p^{ri}(k)$: actual travel time for route p between origin r and node i for flows departing origin r at time k

$\pi^{rs}(k)$: minimal actual route travel time between O-D pair rs for flows departing origin r at time k

Chapter 1: Introduction

1.1 Urban Transportation Network Analysis

Transportation network models or travel choice models can be classified into two categories: static models and dynamic models. The dynamic models are the dynamic generalization of their static counterparts.

The fundamental static model is User Equilibrium (UE) or User Optimal (UO) traffic assignment model proposed by Beckmann (1956). The UE model adopts Wardrop's first principle (Wardrop, 1952), which states that at UE, all the used paths of an Origin-Destination (O-D) pair have minimum travel cost and the travel times on all the unused paths of the same O-D pair are equal to or more than the minimum travel cost of the O-D pair. To consider the heterogeneity in drivers' perception of travel time, Daganzo and Sheffi (1977) proposed Stochastic User Equilibrium (SUE) traffic assignment model. At SUE, no driver can improve his or her perceived travel time by unilaterally changing routes. Another kind of model is System Optimal (SO) traffic assignment model. The SO model adopts Wardrop's second principle which states that at SO, the total travel time of all drivers on a transportation network is minimum. UE and SUE are stable status of a transportation network because they are consistent with drivers' behavior in route choice. SO is an ideal status of a transportation network from the systematic point of view. It is not a stable status because such an ideal status does not comply with realistic drivers' behaviors in route choice.

The UE model (Beckmann, 1956) was studied extensively by Dafermos and Sparrow

(1969). But it was not solved until 1975 when LeBlanc et al. (1975) provides an efficient solution by applying Frank-Wolfe (F-W) algorithm with the minimum cost route algorithm. To consider the asymmetric link interactions, variational inequality (VI) was used to formulate transportation network problems and projection algorithm and diagonalization algorithm can be used to solve the VI problem (Nagurney, 1993).

A whole travel choice process includes the following series of choices: whether to make a trip (Trip Choice), where to go (Destination Choice), what mode to use (Mode Choice), when to begin the trip (Departure Time Choice), which route to use (Route Choice). Figure 1.1 shows the hierarchy of a travel choice process. To ensure the consistence of travel choices at different stages, combined travel choice models are proposed in this dissertation. A combined travel choice model incorporates route choice/traffic assignment with at least one of the other stages including trip generation, trip distribution, mode split, and departure time. Evans (1973, 1976) firstly formulated a model to combine trip distribution with traffic assignment. Abdulaal and LeBlanc (1979), Florian (1977), LeBlanc and Farhangian (1981), LeBlanc and Abdulaal (1982) studied models combing modal split and equilibrium assignment models. Lam et al. (1992) studied the combined distribution-assignment of traffic. Florian and Nguyen (1978), Friesz (1981), LeBlanc and Abdulaal (1982) studied combined trip distribution modal split and trip assignment model. Safwat and Magnanti (1988) developed combined trip generation, trip distribution, modal split, and trip assignment model. Boyce et al. (1984) studied combined location, mode, and route-choice problem. Departure time choice has been studied by Hendrickson (1981, 1984), Small (1982), etc. A comprehensive account of static transportation network models is given by Sheffi (1985).

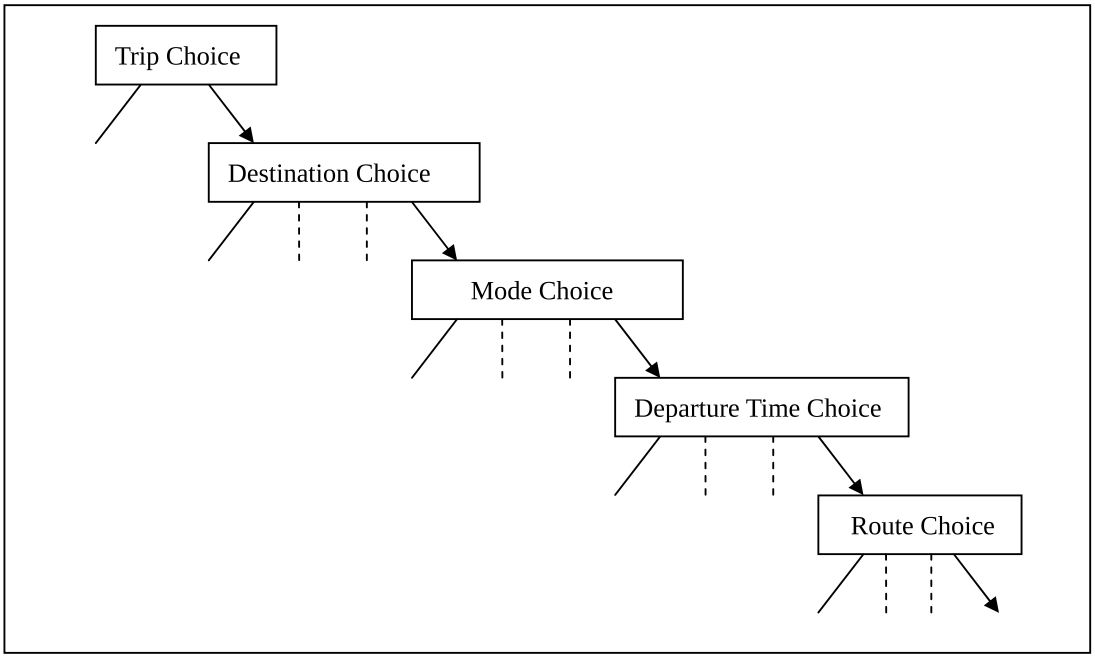

Figure 1-1. A Hierarchy of Travel Choice Models

Static models assume vehicles move concurrently and find the equilibrium flow volumes on each link or path. They are applicable to long-term transportation planning. However, the resultant link volume of a static model may be several times more than the capacity of a link, which is not consistent with actual situation. In addition, the static models can not provide the real time traffic volume and travel time and cannot reflect the time-dependent variation of traffic in a road network. Thus, the application of static models on the operation of a transportation network is limited. On the contrary, the dynamic models can provide the real time link and path traffic volumes and link and path travel times, which are necessary inputs for any travel guidance systems. Thus, dynamic transportation network models are useful in managing the real time operation or assessing the performance of a transportation system, whereas static transportation network models are the foundation of their dynamic counterparts. Many dynamic models assume similar structures to their static counterparts in both model forms and solution algorithms on the time-space network (The time-space network will be explained in Chapter 4). Dynamic models are also applicable to long-term transportation planning, though the study of their application on transportation planning is still limited (one of the reasons for this is the lack of an efficient solution algorithm for dynamic transportation network models).

All the static transportation network models have their dynamic counterparts. Dynamic transportation network models incorporate dynamic travel choice problems such as traveler's trip choice, destination choice, mode choice, departure time/arrival time choice and route choice. A detailed review of dynamic transportation network models is given in Chapter 2.

1.2 Travel Time Choice Research Problems

Different models on dynamic transportation networks can be formulated based on the following fundamental travel choice problems:

1) The actual/instantaneous travel time of each driver of the same O-D pair departing at the same time is equal and minimum;
2) The perceived actual travel time of each driver of the same O-D pair departing at the same time is equal and minimum;

3) Drivers of the same O-D pair choose departure time and route such that the actual/generalized travel time of each driver of the same O-D pair departing at any time is equal and minimum;
4) Same as 3), but consider the change of mode split of an O-D pair with the change of travel cost of the O-D pair;
5) Same as 3), but consider the change of trip distribution among O-D pairs with the change of travel cost of O-D pairs;
6) Same as 3), but consider the change of trip distribution among O-D pairs with the change of travel cost of O-D pairs and the change of mode split of an O-D pair with the change of travel cost of the O-D pair.

Problem 1) is ideal/ instantaneous Dynamic User Optimal (DUO) route choice problem or ideal/instantaneous dynamic user optimal traffic assignment problem. It can be further described in more detail as follows: given time-dependent O-D demand of each O-D pair, determine the flow pattern on the network such that for each O-D pair at each instant of time, the actual travel times experienced by travelers departing at the same time are equal and minimal (this state is called ideal or predictive user optimal state), or for any departure flow from a decision node to a destination node at each instant of time, the instantaneous travel times of all possible routes with the same O-D are equal to the minimal instantaneous route travel time (this state is called reactive or instantaneous user optimal state) (Ran, 1996b).

Problem 2) is Stochastic Dynamic User Optimal (SDUO) route choice problem. It differs from DUO in that the perceived actual travel times experienced by travelers departing at the same time are considered.

Problem 3 is Dynamic User Optimal Simultaneous Departure Time and Route Choice (DUOSDTRC) problem. It extends the DUO route choice problem in one respect: both departure time and route over a road network must be chosen. Each departure time choice is based on the actual minimum O-D travel times at each departure time. In a DUOSDTRC problem, the total O-D demand is given while the time-dependent O-D demand is a variable that needs to be solved for. At equilibrium, the actual travel cost of vehicles departing at any time through any used path is equal and minimum and no traveler can reduce his travel cost by unilaterally changing his departure time and route choice combination (Lim, et al., 2005). Any departure flow pattern different from the equilibrium pattern will incur more travel cost for some travelers.

Problem 4), 5) and 6) assume the transportation network consists of a transit network and an auto/road network. Problem 4) is Combined Mode Split and Dynamic User Optimal Simultaneous Departure Time and Route Choice (MS DUOSDTRC) Problem. It extends the DUOSDTRC problem in one respect: transportation mode, departure time and route over a road network must be chosen. In MS DUOSDTRC problem, the total O-D demand includes demands of transit and passenger car and is given, while the share of each mode needs to be solved. At equilibrium of MS DUOSDTRC, the same travel cost should be incurred for all passenger car drivers of the same O-D pair departing at all time, and should be equal to the transformed O-D travel cost of the transit of the same O-D pair.

Problem 5) is Combined Trip Distribution and Dynamic User Optimal Simultaneous Departure Time and Route Choice (TD DUOSDTRC) Problem. It extends the DUOSDTRC problem with additional consideration where the destination, departure time and route over a road network must be chosen. In TD DUOSDTRC problem, the trip generation of each origin and trip attraction of each destination is given and fixed, while the total demand of each O-D pair needs to be estimated. At equilibrium, not only the conditions for DUOSDTRC are satisfied, the consistency of trip distribution and dynamic travel impedance among zones are also guaranteed.

Problem 6) is Combined Trip Distribution Mode Split and Dynamic User Optimal Simultaneous Departure Time and Route Choice (TD MS DUOSDTRC) Problem. It extends the DUOSDTRC route choice model by assuming that the destination, mode, departure time and route over a road network must be chosen. In TD MS DUOSDTRC problem, the trip generation of each origin and trip attraction of each destination is given and fixed, while the demand of each mode of each O-D pair needs to be solved for. At equilibrium of TD MS DUOSDTRC, the same cost should be incurred for all passenger car drivers of the same O-D pair departing at all time, and should be equal to the transformed O-D cost of the transit of the same O-D pair, and the consistency of trip distribution and dynamic travel impedance among zones are guaranteed.

Different methods have been used in modeling DUO, SDUO, and DUOSDTRC problems. They include simulation-based method, mathematical programming, optimal control, and variational inequality. The literature review of this research covers more details about these models. The variational inequality (VI) method overcomes the drawbacks of the other methods and has been identified a useful tool to model dynamic transportation network problem. The application of VI in modeling dynamic transportation networks has been studied extensively since the early ninetieth (e.g., Friesz et al., 1993; Wie et al., 1995; Ran and Boyce, 1996b; Ran et al., 1996a; Chen and Hsueh, 1998; Bellei et al., 2006; Boyce et al., 2001; Bliemer et al, 2000; Ran et al, 2002a,b; Akamatsu, 2001; Han, 2004; etc.). A comprehensive account of VI formulation and diagonalization algorithm for dynamic transportation network problems was reviewed by Ran and Boyce (1996b).

The VI formulation (model) of dynamic transportation network problems can be link-based or route-based, with variables defined based on links in link-based formulation and on routes in route-based formulation. Correspondingly, solution algorithm for a VI model can be link-based or route-based. If an algorithm requires enumeration of all the paths for all O-D pairs, it is obvious a lousy, time-consuming work, and of course not efficiently applicable to a large-scaled network because the number of paths is huge on the large network and the huge amount of computation times will be required to find the optimal solution using modern computers. Path enumeration can be avoided for an algorithm by adopting the technique of column generation, which will be explained in chapter 4. The link-based and route-based formulations/algorithms are actually consistent. By recording the dynamic shortest paths and the corresponding path flows in any iteration of the calculation, a link-based algorithm is enabled to identify the path flows. Because of the link-path incidence relationship in the time-space network, link flows can be obtained based on the path flows and a route-based algorithm can also obtain link flows. In the dynamic case, the link flows refer to link inflow and link outflow. The link flows can be O-D based inflow and outflow or total inflow and outflow (the sum of all O-D based inflow and outflow). Only path flows and O-D based link inflow and outflow are useful in a general multi-origin-multi-destination transportation network.

The analysis of a dynamic transportation network model on a time-space network makes the traffic dynamic process on a transportation network easier to understand. A time-space network is the network combining original network and time dimension. When solving a VI model, the time period has to be discretized into short time intervals. Consequently, the size of the time-space network will be thousands of times bigger than the original physical network. The solution process will be very complicated and time-consuming if a solution algorithm is performed over time-space network. Thus the solution algorithm that needs time-space network expansion is not efficient for a large size transportation network. A solution algorithm that avoids time-space network expansion is appealing because it will be much more efficient and benefit the implementation of a real-time traveler information system.

Define departure horizon as the time period when there is a departure flow from any origin entering the network. Define assigning horizon as the time period from starting time to the time point when the last vehicle in the network reach its destination. In reality, only departure horizon is known. Assigning horizon is to be found by solving a VI model. Thus, a solution algorithm should be able to treat departure horizon freely without constraint on assigning horizon.

It can be concluded that an algorithm for solving a dynamic transportation network VI model should 1) be able to find the time-dependent path flows or O-D based link inflow/outflow without path enumeration 2) not be performed on a time-space network 3) be able to treat departure horizon freely without constrain on assigning horizon. Unfortunately, an algorithm satisfying the above conditions is still lack though different models and algorithms on dynamic transportation networks have been studied for decades.

Plenty of research on dynamic simultaneous departure time and route choice (DUOSDTRC) models has been reported (e.g., Hendrickson and Plank, 1984; Palma et al., 1983; Ben-Akiva et al., 1986; Mahmassani and Herman, 1984; Mahmassani and Chang, 1987; Zijpp et al, 2002; Szeto and Lo, 2004; Yang and Meng, 1998). Unfortunately, such models are either limited to solving departure time choice problems on simple networks, or not efficient for larger networks. Research on DUOSDTRC also includes Janson (1992). However their research is more applicable for long-term transportation planning, rather than dynamic traffic analysis (Huang and Lam, 2002). Among other researches, some do not provide a solution algorithm (Friesz et al., 1993); some provide a heuristic algorithm only (Huang and Lam, 2002; Bernstein et al., 1993); some require path enumeration (Lim, 2005; Ran et al. , 1996b); others need to be performed on time-space network (Ran et al., 1996b). Chapter 3 also covers the detailed review on DUOSDTRC problem. My literature review indicates that there is still no promising analytical solution algorithm for DUOSDTRC model.

Research on combined dynamic travel choice models includes Stathopoulos (2003) and

Bellei et al (2006). In Stathopoulos (2003)'s study, dynamic user optimal assignment is used in the estimation of travel demand and departure time choice is not considered. Bellei et al. (2006) presented a mixed discrete/continuous nested Logit dynamic demand model with five choice levels including generation, destination, mode, departure time and path choices. Unfortunately, these methods require path enumeration and no solution really that satisfies the combined dynamic travel choice conditions have been shown.

1.3 Objective/Contribution

The objective of this study is to develop new solution algorithms that have potential to solve VI-based dynamic transportation network models in a simplified and even efficient way. The associated dynamic transportation network models for such an improvement include DUO, SDUO, DUOSDTRC, MS DUOSDTRC, TD DUOSDTRC, and TD MS DUOSDTRC. One of the major advantages of the new algorithms are with the following capabilities to 1) find out the time-dependent path flows and O-D based link inflow/outflow without path enumeration; 2) need no time-space expansion of the network; and 3) treat departure horizon freely. Thus, they theoretically sound to be efficient and applicable for implementation in a general multiple origin-destination transportation network.

The main contribution of this study includes the following:

- New diagonalization/relaxation algorithms based on Frank-Wolf (F-W) and Gradient Projection (GP) algorithm for solving DUO model are proposed.
- A link-based VI SDUO model is proposed. New relaxation with MSA algorithm for

solving it is proposed.

- An efficient analytical relaxation with GP algorithm for DUOSDTRC model is proposed.
- A MS DUOSDTRC model and its solution algorithm are proposed.
- A TD DUOSDTRC model and its solution algorithm are proposed.
- A TD MS DUOSDTRC model and its solution algorithm are proposed.
- A VI DUO model integrated with signal timing system (DUOST) and its solution algorithm are proposed.

1.4 Dissertation Outline

The remaining part of the dissertation is organized as follows.

Chapter 2 provides a detailed review of the current models and algorithms for dynamic transportation network problems including DUO, SDUO, DUOSDTRC, and other combined dynamic travel choice problems.

Chapter 3 briefly summaries network flow constraints, First-In-First-Out constraints, definition of travel time, etc. It also introduces some traffic flow models which are useful in determining link travel times.

Chapters 4 through 8 are reflective of innovative components addressed in my research. In Chapter 4, both link-based and route-based relaxation with F-W algorithms are proposed for the models. Numerical examples showing the application of the new algorithms are exhibited.

In Chapter 5, a new link-based VI formulation of stochastic dynamic user optimal route choice problem and a link-base relaxation with MSA algorithm is proposed for it. A route-base relaxation with MSA algorithm is also proposed. Numerical examples showing the application of the new model and the algorithms are exhibited.

In Chapter 6, a route-based relaxation with GP algorithms is proposed for the DUO model. Then, DUOSDTRC and its VI formulation are introduced. Finally, an analytical relaxation with GP algorithm is proposed for the DUOSDTRC model. Numerical examples showing the application of the new algorithms are also exhibited.

In Chapter 7, three combined dynamic travel choice models and their solution algorithms are proposed. The models considered include combined MS DUOSDTRC, TD DUOSDTRC, and TD MS DUOSDTRC. Analytical solution algorithms for them are presented in detail. Numerical examples showing the application of the algorithms are given.

Chapter 8 covers two problems: DUOIM and DUOST. VI formulations of the two problems are given. A relaxation with GP algorithm for each model is presented. A numerical example showing the application of the algorithm is exhibited.

Chapter 9 concludes the dissertation and presents the potential future research direction.

Chapter 2: Literature Review on Transportation Network Modeling

This chapter summarizes the literature reviews on dynamic transportation network models/dynamic travel choice models and their solution algorithms. The focus is on the problems as stated in Chapter 1. Section 1 presents the review results on deterministic Dynamic Traffic Assignment (DTA) problems. Section 2 covers stochastic DTA problems. Section 3 covers Dynamic User Optimal Simultaneous Departure Time and Route Choice problems (DSDTRC). And Section 4 discuss other combined dynamic travel choice problems such as combined Trip Distribution/Mode/Departure Time/Route Choice problems.

2.1 Deterministic Dynamic Traffic Assignment (DTA)

Models and algorithms for Dynamic Traffic Assignment (DTA) problems (which include dynamic user optimal traffic assignment/route choice and dynamic system optimal traffic assignment) are the basis for developing models and algorithms for other combined dynamic travel choice problems. The approaches used to model DTA in the past literature can be classified into two types: simulation-based approach and analytical approach. The analytical approach includes mathematical programming, optimal control, and variational inequality.

2.1.1 Simulation-based DTA Models

In simulation-based DTA models, a traffic simulator is used to replicate the complex traffic flow dynamics and vehicle/driver's movement/characteristics are simulated. Given the substantial computational burden associated with the use of a simulator, the choice of granularity (macroscopic, microscopic or mesoscopic) has significant implications for the real-time computational tractability of simulation-based models. An example of the simulation-based approach is the model of Jayakrishnan and Mahmassani et al. (1994), which embraces an assignment module and a mesoscopic traffic simulator called DYNASMART. Ben-Akiva *et al.* (1997) also proposed DynaMIT as a dynamic traffic assignment system to estimate and predict in real-time current and future traffic conditions. Li et al (2000) introduce an internet-based GIS system that integrates data and models into one framework using traffic simulator RouteSim, which is a mesoscopic model based on cell transmission (Daganzo, 1994) for traffic propagation.

Based on DYNASMART-X, Chiu and Mahmassani (2002, 2003) studied the hybrid dynamic traffic assignment (HDTA) which considers the interplay between a centralized DTA (CDTA) model and a decentralized DTA (DDTA) capability. Lu et al. (2006) studied the bicriterion dynamic user equilibrium (BDUE) problem that allows for heterogeneous users with different value-of-time (VOT) preferences. Sbayti et al. (2007) conducted a vehicle-based simulation study to improve upon the performance of the MSA heuristic for UE and SO DTA problems on large congested networks models. The simulation method can also be used to evaluate the impacts of traffic incidents and to model incident management strategies and relevant intelligent transportation system (ITS) technologies (Sisiopiku et al., 2007), and to evaluate network performance under various schemes for the design and operation of high-occupancy toll (HOT) lanes (Abdelghany et al., 2000). Other research on simulation-based DTA include Peeta (2003, 2006a, 2006b), Ghali and Smith (1995), Smith et al. (1995), Tong and Wong (2000), Wang et al. (2001), Varia and Dhingra (2004a,b), Mahut et al. (2004, 2008), Sisiopiku et al. (2007), Hu et al. (2008), Wen et al. (2008), etc.

The simulation-based model usually lacks a sound mathematical background, the resulting solution property is heuristic, and the convergence of the solution procedure is not guaranteed (Ran, 2002b). The constraints of the problem are not strictly followed for a simulation-based algorithm. Considerable computational times and complexities inevitably occur in the simulation process. The solvable network scale of the simulation-based DTA model is limited (Ran, 2002b). For these reasons, analytical approach is adopted in this study.

2.1.2 Mathematical Programming

HO (1980) presented a linear optimization approach to the dynamic traffic assignment model problem. JANSON (1991a, b) presented a bi-level nonlinear optimization formulation of the dynamic user equilibrium assignment problem (DUE) for urban road networks with multiple trip origins and destinations. GHALI and SMITH (1995) developed a model for the dynamic system optimum traffic assignment. The model is approximate and is applicable to networks with many origin-destination pairs and many bottlenecks. Jayakrishnan et al. (1994) extended Janson's method (1991) and presented a dynamic traffic assignment model with traffic-flow relationships based on a bi-level optimization framework. They also presented a heuristic solution algorithm which resembles a Stackelberg leader-follower problem. Drissikaitouni (1992) expressed dynamic traffic assignment problem as a static traffic assignment problem over a temporal expansion of the base network and presented a solution algorithm for it over the Static Temporal Expanded Network (STEN).

Akamatsu (2000) studied a type of capacity paradox on one-to-many network and many-to-one to network for dynamic equilibrium assignment. Li et al. (2000) proposed a solution algorithm for the linear programming model for DTA by applying the Dantzig-Wolfe decomposition scheme. The algorithm solves a minimum-cost-flow problem as the sub-problem and a restricted optimization as the master. Akamatsu et al. (2003) studied the Braess paradox in the dynamic case on a more general network and gave a graph-theoretic interpretation of the condition for the paradox to occur. Golani et al. (2004) proposed an algorithm for solving the user optimal dynamic traffic assignment problem with multiple destinations. The algorithm selects a destination for equilibration, fixes the paths of the vehicles assigned to the other destinations, and finds an optimal dynamic traffic assignment for the destination of interest. The spatial path set obtained for this destination is then fixed, and another destination is relaxed. The process is repeated iteratively among the destinations. The approach is a heuristic for finding the multiple-destination user optimal path set. Waller and Ziliaskopoulos (2006) introduced a polynomial combinatorial optimization algorithm for the dynamic user optimal problem. The approach is applicable to single destination networks. Laval (2007) studied the user optimum dynamic traffic assignment in the same network and presented a simplified graphical solution method. Yang et al. (2007) proposed a linear programming model for a novel steepest-descent dynamic toll scheme that minimizes the total system cost at each day. Durlin et al. (2008) presents a dynamic network loading (DNL) model that can be used both for Dynamic Traffic Assignment (DTA) and for an accurate description of traffic. Ramadurai et al. (2006) explored the existence of equilibrium solutions in single bottleneck models with homogenous travelers having same preferred arrival times from both theoretical and experimental frameworks.

The limitation of mathematical programming DTA (either DUO or DSO) formulation is lack of clear understanding of solution properties for realistic problem scenarios (Ziliaskopoulos et al., 2002). Since an equivalent mathematical model exists only when the Jacobian of a mapping is symmetric, which does not hold in a general case for a DUO problem, an equivalent mathematical model for DUO problem does not always exist. None of above studies showed the validation of their model and algorithm. For this reason, I did not formulate DUO and other problems using mathematical programming.

2.1.3 Optimal Control

SMITH (1984) studied the stability of a dynamic model of traffic assignment using Lyapunov method. Friesz et al. (1989) proposed a link-based optimal control formulations for both the SO and UE with single destination. Ran and Shimazaki use the optimal control approach to develop a link-based SO (1989a) and UE (1989b) for an urban transportation network with multiple origins and destinations. Wie (1990) extends the UE model to include elastic time-varying travel demand. Ran et al. (1993) use the optimal control approach to obtain a convex model for the instantaneous UE DTA. Ran et al. (1993) proposed two instantaneous DUO traffic assignment models for a congested transportation network using the optimal control theory approach. Wie and Friesz (1994) developed an augumented Lagrangian method for solving dynamic traffic assignment models formulated as optimal control problems. The algorithm obviates the need for path enumeration and exploits the natural decomposition of the traffic assignment problem by time period. Boyce et al. (1995) presented a methodology to solve the problem using the Frank-Wolfe algorithm over an expanded time-space network representation. Gartner et al. (1998) presented a framework to integrate dynamic traffic assignment with real-time traffic adaptive control system. Peeta et al. (2003) explored stability issues for operational route guidance control strategies for vehicular traffic networks equipped with advanced information systems, and develops a general procedure for the stability analysis of the associated dynamic traffic assignment (DTA) problems.

The optimal control model has some drawbacks (Boyce et al., 2001), including: a) if the exit flow function is concave, it is not possible to establish an optimal control model of the dynamic User Optimal traffic assignment problem with multiple origin-destination (O-D) pairs; b) if the initial flow is zero, it causes unrealistic flow propagation. For this reason, optimal control is not considered in this study.

2.1.4 Variational Inequality

To remedy the limitations of mathematical programming and optimal control theory in the DTA (DUO or DSO) context, variational inequality (VI) formulations is introduced to model dynamic traffic assignment problems. VI problem is a generalization of constrained optimization problems, complementarity problems, and fixed point problems. It can tackle the problem when the Jacobian of a mapping is asymmetric, as is the case for DUO problem. An equivalent VI model for DUO problem always exists.

Smith (1995) introduced a smooth day-to-day dynamic user-equilibrium assignment VI model in which the day-to-day stability of the route-swapping process is considered in a continuous setting. Wei et al. (1995) formulated the dynamic network user equilibrium problem as a variational inequality problem in discrete time in terms of unit path cost functions and presented a heuristic algorithm to solve the model. To avoid path enumeration, the Frank-Wolfe algorithm was used to generate the set of paths that has competitive travel times in a congested network. Ran and Boyce (1996a) proposed a link-based discretized VI formulation for the ideal DUO problem with fixed departure times. In the paper, the traffic network constraints and link-based DUO route choice conditions are presented. The necessity and sufficiency of the VI is proved. Ran and Boyce (1996b) proved that the F-W algorithm is appropriate to solve the dynamic traffic assignment problem if a time-space network is considered. Chen and Hsueh (1998) proposed a link-based VI formulation for the UE DTA problem and presented a nested diagonalization algorithm for the model. In Chen et al. (1998), the dynamic traffic control problem and the dynamic traffic assignment problem are integrated as a noncooperative game between a traffic authority and highway users to find a mutually consistent dynamic system optimal signal setting and dynamic User Optimal traffic flow. The combined control-assignment problem is formulated as a one-level Cournot game, a bi-level Stackelberg game, and a Monopoly game.

Bliemer et al. (2000) proposed a quasi-variational inequality multiple-user-class macroscopic dynamic traffic assignment model to deal with various asymmetries such as intra-user-class interaction and interspatial and intertemporal asymmetries. A nested modified projection method requiring path enumeration is proposed to solve the assignment problem. Ran et al. (2002a) proposed an analytical dynamic traffic assignment model with the extended capability of performing rolling horizon implementation. The model is formulated as a link-based variational inequality and can be solved efficiently to convergence by a relaxation /diagonalization algorithm. Akamatsu (2001) presents an efficient algorithm for solving nonlinear complementarity formulation of the dynamic user equilibrium (DUE) traffic assignment. The algorithm is capable of dealing with very large-scale networks with a one-to-many origin-destination (O-D) pattern. Ran et al. (2002b) presented an algorithm for solving the dynamic traffic assignment/route choice problem without time-space network expansion but it cannot find the time-dependent path flows. Lo et al. (2002) developed a cell-based nonlinear complementarity formulation of ideal dynamic DUO traffic assignment (DTA). The formulation was transformed as an equivalent optimizationl program by defining an appropriate gap function.

Liu et al. (2003) considered the uncertain factors in the subjective recognition of travel times by travelers and proposed a fuzzy dynamic traffic assignment model. A fuzzy shortest path algorithm is used to find the group of fuzzy shortest paths and to assign traffic to each of them by using C-logit method. Hamdouch et al. (2004) proposed a VI model of dynamic traffic assignment where strategic choices are an integral part of user behaviour. Han et al. (2004) developed a descent direction of the merit function for co-coercive variational inequality (VI) problems and implemented the solution method for traffic assignment problems with nonadditive route costs. Jang et al. (2005) proposed a discrete ideal dynamic user optimal (DUO) route choice model using a route-based variational inequality approach and presented a projection-based approach to solve the model which aviated path enumeration by using column generation. One of the drawbacks with the model is the link propagation is too complicated and difficult to implement. Bellei et al. (2005) formulated within-day dynamic traffic assignment as a fixed-point problem and solved the problem through the Bather's method. In the solution process, an implicit path enumeration network loading procedure is used as an extension of Dial's algorithm. Kim and Jayakrishnan (2006) studied dynamic traffic assignment based on arrival time-based O-D demand. Mahut et al. (2008) formulated dynamic traffic assignment model as a time discrete variational inequality problem used MSA and a gradient-like method to solve the model. Ramadurai et al. (2008) developed a linear complementarity formulation for the single bottleneck model.

It was concluded in Chapter 1 that an efficient algorithm for solving a DUO model should 1) be able to find the time-dependent path flows or O-D based link inflow/outflow without path enumeration 2) not be performed on a time-space network 3) be able to treat departure horizon freely without constrain on assigning horizon. Unfortunately, none of the above algorithms satisfy the three conditions. The study presents new algorithms, termed as Relaxation with Frank-Wolfe (F-W) and Relaxation with Gradient Projection (GP), for both link-based and route-based VI DUO model. Our new algorithms satisfy the three conditions and are efficient for DUO problems in a large size transportation network.

2.2 Stochastic Dynamic Traffic Assignment (SDTA)

Ben-Akiva et al. (1986) extended the stochastic model of Palma et al. (1983) to a within and between days dynamic version. Cascetta and Cantarella (1991) also developed within day and between-days dynamic assignment with a stochastic process. Ran and Boyce (1996b) proposed route-based VI formulation dynamic stochastic models. Two popular route choice functions including logit route choice function and probit route choice function are analyzed in the study. He et al. (2000) proposed a new approach to calibrate and validate a dynamic traffic assignment (DTA) model. The paper derives the likelihood functions for estimating dynamic route choice and actual flow propagation by presenting approximate joint probability distribution functions of the temporal link traffic flows on a network. Sawaya et al. (2000) proposed a multistage stochastic mathematical model with recourse to compute and disseminate real-time traffic control actions, which account for system uncertainties such as demand variation and incident severity. Ran (2002b) presented an algorithm for stochastic dynamic user optimal route choice problem without 3-D time-space expansion of the network.

Liu et al. (2002) presented a variational inequality DTA model over a stochastic network. The model captures the travelers' decision making among discrete choices in a probabilistic and uncertain environment, in which both probabilistic travel times and random perception errors that are specific to individual travelers are considered. A solution algorithm was proposed by combining a relaxation approach, stochastic network loading, and the MSA. Barbara et al. (2006) formulated a stochastic equilibrium to address two types of uncertainty in travelers daily commutes: uncertainty in the actual travel time due to random link capacity degradations and perception variations in their travel time budget due to imperfect traffic information. Peeta et al. (2006) proposed a stochastic quasi-gradient (SQG) based algorithm to solve the off-line stochastic dynamic traffic assignment (DTA) problem that explicitly incorporates randomness in O–D demand, as part of a hybrid DTA deployment framework for real-time operations. The problem is formulated as a stochastic programming DTA model with multiple user classes. A simulation-based SQG method is proposed to solve the problem. In Barceló et al. (2006), a stochastic heuristic dynamic assignment algorithm is proposed in the case of a microscopic simulation using AIMSUN, a route-based microscopic simulator. The k-shortest paths of each OD pair is calculated at each iteration and the C-logit route choice model is used to determined the path-dependent flow rates on the paths in the network. Balijepalli et al. (2007) presents a doubly dynamic simulation assignment model which involves specifying a day-to-day route choice model as a discrete time stochastic process, combining a between-day driver learing and adjusting model with a continuous time, within-day dynamic network loading. Li et al. (2007) presented a dynamic user equilibrium model considering how cognitive map of transportation network's configuration shapes the state of equilibrium traffic flow. The equilibrium is formulated as an equivalent nonlinear complementarily problem and a heuristic route/time-swapping approach is adapted to solve the problem.

Since none of the existing algorithms for stochastic dynamic user optimal traffic assignment (SDUO) satisfy the three conditions aforementioned, I present in this study new algorithms, termed as Relaxation with MSA (method of successive average), for both link-based and route-based VI DUO model. In addition, I also present a new link-based VI SDUO model. Our algorithms satisfy the three conditions and are efficient for large size transportation network.

2.3 Dynamic User Optimal Simultaneous Departure Time and Route Choice problem (DUOSDTRC)

Hendrickson and Plank (1984) developed work trip scheduling models. In their study, mode and departure time choices are treated as a simultaneous interactive decision based upon maximization of individual traveler's utility or satisfaction with each alternative mode and departure time combination. The probability of an individual selecting each mode/departure time alternative is assumed to be of the logit form. Palma et al. (1983) developed a model to predict the pattern of traffic volumes and travel times during a peak period at a single bottleneck. In the model, a trip maker can shift his/her trip forward or backward in time to avoid a long delay. Ben-Akiva et al. (1986) developed a dynamic model of peak period traffic congestion that considers a limited number of bottlenecks. The model predicts the temporal distribution of traffic volumes with an elastic demand model. In response to changes in the traffic conditions travelers can switch to a different mode, divert to an alternate route, or shift the trip forward or backward in time to avoid a long delay. Mahmassani et al. (1984) analyzed the time-dependent departure pattern arising in an idealized situation of a pool of commuters going from a single origin to a single destination along one or two routes under user equilibrium conditions. Mahmassani et al. (1987) introduced a boundedly rational user equilibrium (BRUE) at a single bottleneck, with particular reference to the departure time decision problem. Unfortunately, the above models are limited to solving departure time choice problems for simple networks.

Friesz et al. (1993) first formulated a continuous time, infinite-dimensional VI model for the departure time/route choice problem but did not provide solution to the model. Wie et al. (1995) presented a discretized VI formulation for the simultaneous route/departure equilibrium problem and presented a heuristic algorithm whose convergence was not established. Yang et al. (1998) presented a model for departure time, route choice and congestion toll in a queuing network with elastic demand. The departure time and route choice of commuters and the optimal variable tolls of bottlenecks were determined jointly by solving a system optimization problem over the space-time expanded network (STEN).

Janson (1992) formulated a user-equilibrium traffic assignment model with variable departure times and scheduled arrival times. Bernstein et al. (1993) formulated the simultaneous route and departure time choice (SRD) equilibrium problem as a variational control problem. A path-swapping process and a heuristic procedure for solving the SRD problem were presented in the paper. However, such models adopt long time intervals and are more applicable for long-term transportation planning, rather than dynamic traffic analysis.

Ran et al. (1996a, 1996b) presented a link-based variational inequality formulation of simultaneous departure time and route choice problem. The equivalence of the formulation to the link-based DUO departure time/route choice conditions was proved. A diagonalization algorithm is presented to solve the model over time-space expansion network. Chen and Hsueh (1998) and Chen et al. (2001) also presented link-based formulations of simultaneous departure time and route choice problem. Huang and Lam (2002) presented a simultaneous path-based route and departure (SRD) time choice equilibrium assignment problem in network with queues. The problem is modeled on discrete-time basis variational inequality and formulated as an equivalent ‘zero-extreme value’ minimization problem. They also presented a heuristic algorithm which based on a route/time-swapping process for the problem. The solution needs path enumeration. Szeto and Lo (2004) developed a cell-based formulation for the simultaneous ideal dynamic user optimal route and departure time choice problem with elastic demands through a variational inequality problem. The cell transmission model (CTM) was used to model link propogation and link travel time. A descent method was adopted to solve the variational inequality problem. However, the method is not ideal for large network. Lim and Heydecker (2005) investigated a logit-based combined departure time and dynamic stochastic user equilibrium assignment problem and presented a solution algorithm to solve the problem which required path enumeration within a reasonable path set.

MUN (2006) presented a route-based combined model of dynamic route and departure time choice using variational inequality approach. He showed that solving the model was equivalent to solving systems of simultaneous non-linear equations and also proposed a Newton-type algorithm for solving the model. Zhou et al. (2007) described the development of a dynamic trip micro-assignment and (meso) simulation system that incorporates individual trip maker choices of travel mode, departure time and route in multimodal urban transportation networks. A variational inequality model and a heuristic procedure are developed to describe and solve the stochastic time dependent traffic user equilibrium problem. Zhang et al. (2007) investigated some new dynamic phenomena of Braess's paradox considering simultaneous departure time and route choices in transportation networks.

Our literature review indicates that there is still no analytical solution algorithm for dynamic user optimal simultaneous departure time and route choice (DUOSDTRC) model and all the existing algorithms need some heuristic process. In addition, none of the existing algorithms for DUOSDTRC satisfy the three conditions aforementioned. In our study, I present an efficient analytical algorithm, called Relaxation with Gradient Projection algorithm, for the route-based VI DUOSDTRC model. Our algorithm satisfies the three conditions and are efficient for large size transportation network. It is also the first analytical solution algorithm for VI DUOSDTRC model.

2.4 Other Combined Travel Choice Models

Stathopoulos (2003) described a methodology for analyzing the evolution of travel demand pattern within different timescales in the long run in general dynamic transportation networks. The estimation process uses as input a maximum-entropy (adjusted) time-dependent O-D trip matrix, whose estimation is based on a set of link traffic counts, and the corresponding (adjusted) dynamic user optimal path travel costs, as obtained from a suitable dynamic network assignment procedure. Bellei et al. (2006) presented a mixed discrete/continuous nested Logit dynamic demand model with five choice levels including generation, destination, mode, departure time and path choices. The logit type models are adopted to model each of the five choice levels. A continuous version of the logit model is adopted for departure time choice, thus not requiring to enumerate explicitly the desired departure time intervals. The dynamic traffic assignment model is formulated through a fixed point problem and solved through an efficient implicit path MSA algorithm. The study provides a modeling framework for the simulation of elastic demand in the context of within-day dynamic traffic assignment. However, the model requires path enumeration from each node to all destinations and is not applicable to large network. And it did not provide an example showing the solution really satisfy the dynamic travel choice conditions. Research on combined dynamic travel choice models includes Stathopoulos (2003) and Bellei et al. (2006). In Stathopoulos (2003), dynamic user optimal assignment is used in the estimation of travel demand and departure time choice is not considered. Bellei et al. (2006) presented a mixed discrete/continuous nested Logit dynamic demand model with five choice levels including generation, destination, mode, departure time and path choices. Unfortunately, their method requires path enumeration and they did not show the solution really satisfy the combined dynamic travel choice conditions.

In this study, I present new models on problem 4), 5), and 6) stated in Chapter 1, or combined mode split and dynamic user optimal simultaneous departure time and route choice (MS DUOSDTRC) problem, Combined trip distribution and dynamic user optimal simultaneous departure time and route choice (TD DUOSDTRC) problem, and combined trip distribution mode split and dynamic user optimal simultaneous departure time and route choice (TD MS DUOSDTRC) problem. I present an efficient algorithm for each model. The algorithms satisfy the three conditions aforementioned and are efficient for problems on a large size transportation network.

Chapter 3: Current Theories for Network Flow Constraints and Determining Link/Path Travel Times

This chapter provides other fundamentals about current theories and assumptions about transportation network flow constraints for dynamic transportation models, as well as models for determining link and path travel times. The content of this chapter is tended to provide more critical prerequisites or basics for understanding of innovative work in my research which will be described in Chapters 4 through 8.

3.1 Network Flow Constraints

The network flow constraints for dynamic transportation network models are briefly introduced in this section.

Inflow conservation equation:

$$\sum_{rsp} u_{ap}^{rs}(t) = u_a(t) \quad \forall\, a \tag{3.1}$$

(3.1) states that the number of vehicles entering link a at time t is the sum of vehicles entering link a over route p passing link a with origin r and destination s at time t.

Similarly, the following hold for $v_a(t)$ and $x_a(t)$:

$$\sum_{rsp} v_{ap}^{rs}(t) = v_a(t) \quad \forall\, a \tag{3.2}$$

$$\sum_{rsp} x_{ap}^{rs}(t) = x_a(t) \quad \forall\, a \tag{3.3}$$

The link state equation is

$$\frac{dx_{ap}^{rs}(t)}{dt} = u_{ap}^{rs}(t) - v_{ap}^{rs}(t) \quad \forall\ a\ p\ r\ s \tag{3.4}$$

The number of vehicles on link a can be stated as

$$x_{ap}^{rs}(t) = x_{ap}^{rs}(0) + \int_0^t \left[u_{ap}^{rs}(\omega) - v_{ap}^{rs}(\omega)\right] d\omega \quad \forall\ a\ p\ r\ s \tag{3.5}$$

Node flow conservation constraint is

$$\sum_{a \in B(j)} v_{ap}^{rs}(t) = \sum_{a \in A(j)} u_{ap}^{rs}(t) \quad \forall\ j \neq r,s;\ p,r,s \tag{3.6}$$

where $A(j)$ is the set of links after j and $B(j)$ is the set of links before j. Similarly, the following node constraints hold for origin r and destination s

$$\sum_{a \in A(r)} \sum_{p} u_{ap}^{rs}(t) = f^{rs}(t) \quad \forall\ r \neq s; s. \tag{3.7}$$

$$\sum_{a \in B(s)} \sum_{p} v_{ap}^{rs}(t) = e^{rs}(t) \quad \forall\ r; s \neq r. \tag{3.8}$$

where $f^{rs}(t)$ is the flow departing from origin r toward destination s at time t and $e^{rs}(t)$ is the flows arriving destination s from origin r at time t.

Flow propagation constraint is

$$u_{ap}^{rs}(t) = v_{ap}^{rs}(t + \tau_a(t)) \quad \forall\ a\ p\ r\ s \tag{3.9}$$

(3.9) states that the inflow rate $u_{ap}^{rs}(t)$ at t equals the exit flow rate $v_{ap}^{rs}(t + \tau_a(t))$ after the link travel time $\tau_a(t)$.

Assume the flow rate in each time interval is constant, then,

$$U_a(k) = U_a(k-1) + u_a(k) \qquad \forall a, k \tag{3.10}$$

where $u_a(k)$ is inflow into link a during interval k, $U_a(k)$ and $U_a(k-1)$ are the cumulative number of vehicles entering link a at the end of interval k and interval $k-1$

Similarly,

$$V_a(k) = V_a(k-1) + v_a(k) \qquad \forall a, k \tag{3.11}$$

where $v_a(k)$ is exit flow from link a during interval k, $V_a(k)$ and $V_a(k-1)$ are the cumulative number of vehicles exiting link a at the end of interval k and interval $k-1$. Let $\tau_a(k)$ be actual travel time over link a for flows entering link a at time k, then flow entering link a at time k will exit link a at time interval $k+\tau_a(k)$, so the following flow propagation holds:

$$u_a(k) = v_a(k+\tau_a(k)) \qquad \forall a, k \tag{3.12}$$

The above flow propagation is different from Huang and Lam's study (2002). In their study, $u_a(k)$ and $v_a(k)$ is taken as inflow rate and exit flow rate on link a at interval k, which has length δ. The flows entering link a at interval $k-1$ leave the link before the end of interval $k-1+\tau_a(k-1)$ by the departure rate $v_a(k-1+\tau_a(k-1))$. The flow propagation in their study says flows entering at interval k will leave the link during $[k-1+\tau_a(k-1), k+\tau_a(k)]$ by the departure rate $v_a(k+\tau_a(k))$. It is argued that this flow propagation confuses departure rate $v_a(\cdot)$ at interval $k+\tau_a(k)$ ($[k+\tau_a(k)]th$ interval) with departure rate $v_a(\cdot)$ during interval $[k-1+\tau_a(k-1), k+\tau_a(k)]$. These two intervals are generally not the same. The length of $[k-1+\tau_a(k-1), k+\tau_a(k)]$ is $\tau_a(k)-\tau_a(k-1)+1$. If $\tau_a(k)-\tau_a(k-1)+1=\delta$, then interval $[k-1+\tau_a(k-1), k+\tau_a(k)]$ is exactly the $[k+\tau_a(k)]th$ interval. Otherwise, they are different. This means $u_a(k) = v_a(k+\tau_a(k))[\tau_a(k)-\tau_a(k-1)+1]$ does not always hold as in Huang and Lam (2002).

With flow propagation constraint (3.12), exit flow v and link volume x can be expressed by inflow u as follows:

$$v_a(t) = \sum_k u_a(k)\delta_a^k(k) \tag{3.13}$$

where

$$\delta_a^k(t)=\begin{cases}1, & k+\tau_a(k)=t\\ 0, & otherwise\end{cases} \tag{3.14}$$

and

$$x_a(t)=\sum_k u_a(k)\delta_a^k(k) \tag{3.15}$$

where

$$\delta_a^k(t)=\begin{cases}1, & k<t, k+\tau_a(k)\ge t\\ 0, & otherwise\end{cases} \tag{3.16}$$

Equation (3.13) states that the exit flows of link a at time interval t are equal to the sum of flows entering link a at time interval k ($k \le t$) and exiting link a at time interval t [$k+\tau_a(k)=t$]. Equation (3.15) states that the flows on link a at time interval t are equal to those flows entering link a before time interval t and exiting link a after time interval t.

Another constraint is termed as causality, which states that the travel behaviour of vehicles is affected by some of the vehicles already on the link at the time of entry, but not by any future entering vehicles.

3.2 First-In-First-Out Constraints (FIFO)

Link FIFO states that vehicles entering link a at time t exit link a earlier than vehicles entering link a at time $t+\Delta t$. It reads (Ran and Boyce, 1996):

$$t+\tau_a(t)<t+\Delta t+\tau_a(t+\Delta t) \tag{3.17}$$

or, if $\tau_a(t)$ is differentiable,

$$\dot{\tau}_a(t) > -1 \tag{3.18}$$

Path FIFO states vehicles entering path p at time t exit path p earlier than vehicles entering path p at time $t+\Delta t$. It reads (Lo and Szeto, 2002):

$$t + \eta_p^{rs}(t) < t + \Delta t + \eta_p^{rs}(t+\Delta t), \quad \forall\, r, s; p \in P^{rs} \tag{3.19}$$

OD FIFO states that vehicles entering the actual minimum path between origin r and destination s at time t reach destination s earlier than vehicles entering the actual minimum path at time $t+\Delta t$. It reads

$$t + \pi^{rs}(t) < t + \Delta t + \pi^{rs}(t+\Delta t), \quad \forall\, r, s \tag{3.20}$$

It can be shown that if the link FIFO hold then the path FIFO also hold.

When time period $[0, T]$ is discretized into small time increments, each increments being an unit of time, then the following flow propagation constraints hold.

In general case, it follows that

$$\sum_{i+\tau_a(i)=k} u_{ap}^{rs}(i) = v_{ap}^{rs}(k) \quad \forall\, k,\ p,a,r,s \tag{3.21}$$

When FIFO condition holds, it follows that

$$\sum_{i^0 \le i \le i^1} u_{ap}^{rs}(i) = v_{ap}^{rs}(k) \quad \forall\, k,\ p,a,r,s \tag{3.22}$$

where i^0 and i^1 are the minimum and maximum of increments such that the following hold

$$i + \tau_a(i) = k \tag{3.23}$$

When strongly FIFO (SFIFO) holds, it follows that

$$u_{ap}^{rs}(i) = v_{ap}^{rs}(i + \tau_a(i)) \quad \forall\, a\ p\ r\ s \tag{3.24}$$

3.3 Link Capacity and Outflow Capacity

Maximal number of vehicles on a link is

$$x_a(t) \le l_a e_{am} \qquad \forall a \tag{3.25}$$

where l_a is the length of link a and e_{am} is the maximal traffic density.

Maximal exit flow from a link is

$$v_a(t) \le v_{am} \qquad \forall a \tag{3.26}$$

where v_{am} is the exit flow capacity of link a.

3.4 Link Travel Time Models

3.4.1 Speed-density Function Models

The speed-density function model to be introduced includes Greenshield's model, Triangle model, Trapezoid model, Greenberg model, and Underwood model. The following relation of volume q (veh/hr), speed s (m/hr) and density k (veh/m) holds for speed-density models:

$$q = ks \tag{3.27}$$

Greenshileds's model has been long used since it was published in 1935. The model reads

$$s = s_f \left(1 - \frac{k}{k_j}\right) \tag{3.28}$$

where s is speed (m/hr), k is density (veh/m), s_f is free flow speed (m/hr), k_j is jam density (veh/m). Figure 3.1 shows the speed-density curve and flow density curve for

Greenshield model.

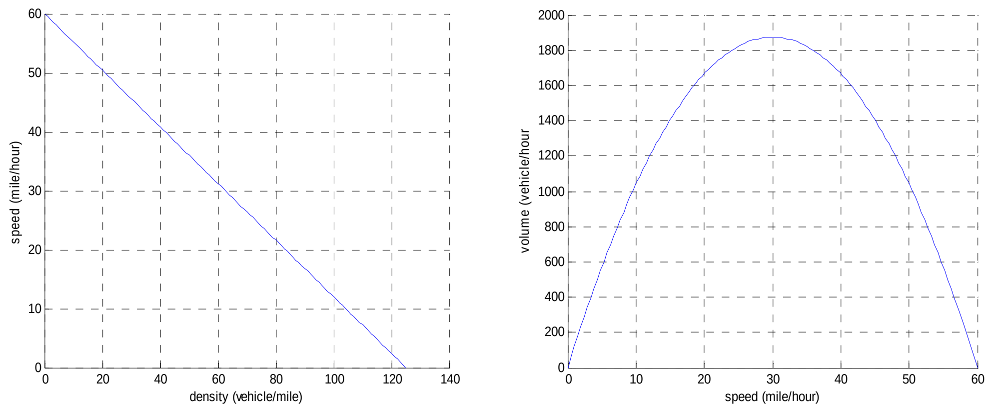

Figure 3-1 Speed-density curve and flow density curve for Greenshield model

Greenshields' model has been retained in the Highway Capacity Manual till 1994. However, it has been observed (Highway Capacity Manual 1994) that speed keeps constant until density reaches certain threshold and then drops quickly. The following model reflects this observation

$$s = \begin{cases} s_f & k \le k_c \\ \alpha s_f \left(\frac{1}{k} - \frac{1}{k_j} \right) & k > k_c \end{cases} \tag{3.29}$$

where $\alpha = \left(\frac{1}{k_c} - \frac{1}{k_j} \right)^{-1}$ Model (3.29) is named Triangle model because its volume-density curve is triangle-shaped. Figure 3.2 shows the speed-density curve and flow density curve for Triangle model.

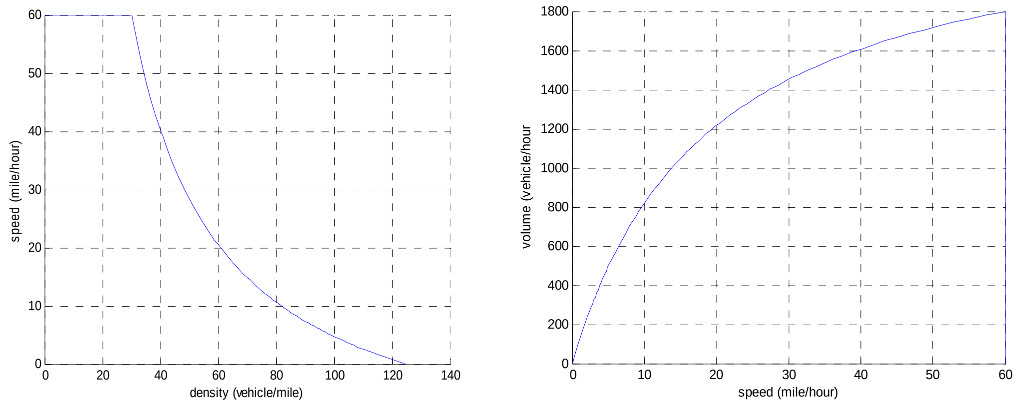

Figure 3-2 Speed-density curve and flow density curve for Triangle model

The Triangle model is a special case of Trapezoid model defined as follows:

$$s = \begin{cases} s_f & k \le k_{c1} \\ \dfrac{s_f k_{c1}}{k} & k_{c1} < k \le k_{c2} \\ \left(\dfrac{1}{k} - \dfrac{1}{k_j}\right)\left(\dfrac{1}{k_{c2}} - \dfrac{1}{k_j}\right)^{-1} \dfrac{s_f k_{c1}}{k_{c2}} & k > k_{c2} \end{cases} \tag{3.30}$$

Figure 3.3 shows the speed-density curve and flow density curve for Trapezoid model.

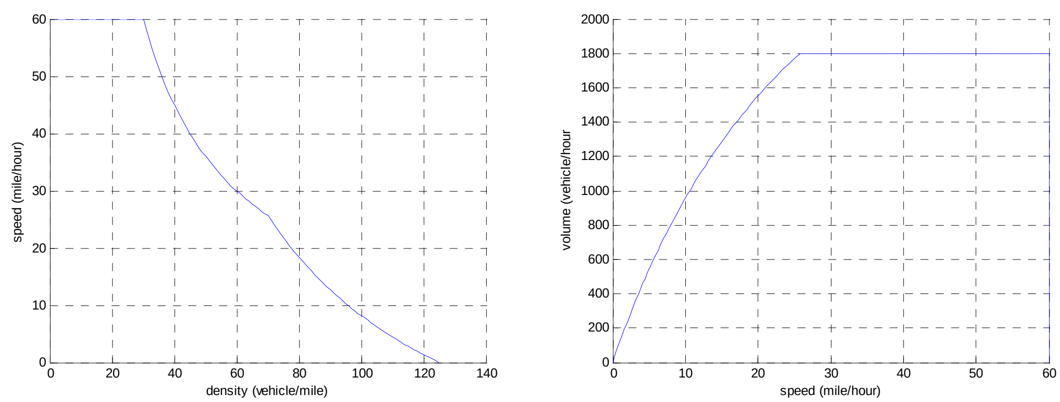

Figure 3-3 Speed-density curve and flow density curve for Trapezoid model

When traffic is congested, the following Greenberg model can be used:

$$s = s_m \ln\left(\frac{k_j}{k}\right) \tag{3.31}$$

where s_m is optimal speed (m/hr) corresponding to the maximum volume, k_j is jam density (veh/m). Figure 3.4 Speed-density curve and flow density curve for Greenberg model.

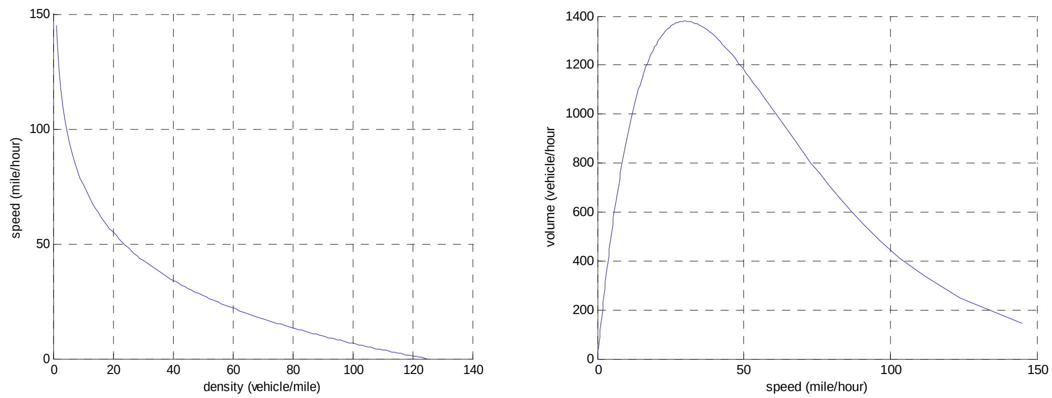

Figure 3-4 Speed-density curve and flow density curve for Greenberg model

When traffic is light, the following Underwood model can be used:

$$s = s_f e^{-k/k_m} \tag{3.32}$$

where k_m is optimal density corresponding to the maximum volume, s_f is free flow speed (m/hr). Figure 3.5 shows the speed-density curve and flow-density curve for Underwood model.

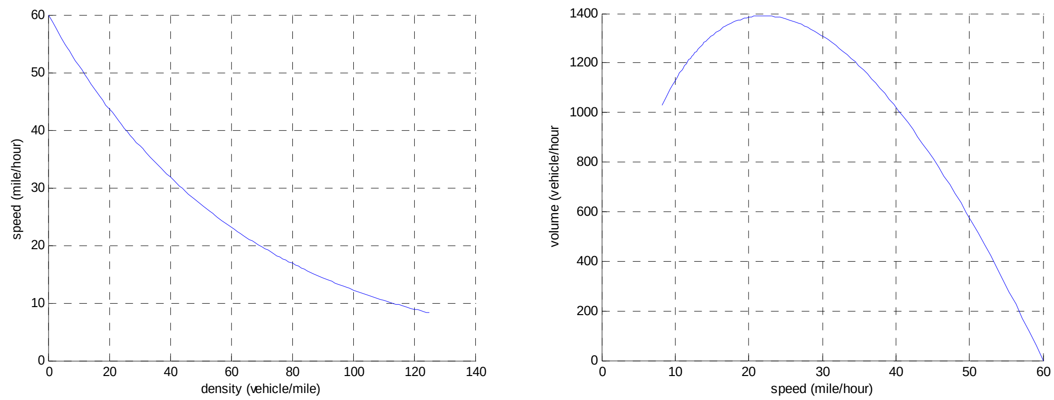

Figure 3-5 Speed-density curve and flow-density curve for Underwood model

3.4.2 Bottleneck Model

In bottleneck-type models, vehicles move at the free flow speed before arriving at the exit node where they join the queue if vehicles ahead are queued and exit the link otherwise. The original bottleneck model (Vickrey, W., 1969) assumes that vehicles do not take physical space and is also known as point-queue (PQ) model. It is stated as follows:

$$\frac{dx_q(t)}{dt} = \begin{cases} 0 & \text{if } x_q(t) = 0 \text{ and } u(t-\tau_0) < C \\ u(t-\tau_0) - C & \text{otherwise} \end{cases} \tag{3.33}$$

$$v(t) = \begin{cases} u(t-\tau_0) & \text{if } x_q(t) = 0 \text{ and } u(t-\tau_0) < C \\ C & \text{otherwise} \end{cases} \tag{3.34}$$

$$\tau(t) = \tau_0 + x_q(t+\tau_0)/C \tag{3.35}$$

where $u(t)$ is the entry rate at time t, $v(t)$ is the exit rate at time t, $x_q(t)$ is the total number of vehicles queued at the exit node, τ_0 is the free flow travel time, C is the bottleneck capacity.

3.4.3 Exit-flow Model or Outflow Model

The exit-flow model assumes that the link exit flow rate at any time is a function of current link volume. It is first used by Merchant and Nemhauser (1978) and later on by Carey (1986, 1987), Frieze et al. (1989) and Wei et al. (1995). It is stated as follows:

$$\frac{dx(t)}{dt} = u(t) - v(t) \tag{3.36}$$

$$v(t) = g_e(x(t)) \tag{3.37}$$

$$g_e(x(t)) \le x(t) \tag{3.38}$$

where $g_e(x(t))$ is a nondecreasing and concave function of current link volume.

In Merchant and Nemhauser (1978), the follow exit function is used:

$$g_e(x) = \begin{cases} x, & 0 \le x \le 50 \\ 50, & 50 \le x \end{cases} \tag{3.39}$$

In Wie et al. (1995), the following exit function is used:

$$g_e(x(t)) = Q(t)(1 - \exp(-x(t)/\bar{x})) \tag{3.40}$$

where $Q(t)$ is capacity at time t; and $\bar{x}$ is the product of $Q(t)$ and time increment Δt. Because outflow models do not explicitly define the travel times on the link, travel time is usually calculated by using the flow propagation function.

Study on exit-flow model can also be found in Careya (2004).

3.4.4 Delay-function Model

Delay-function model (also known as whole link model) assumes that the traverse time experienced by vehicles entering a link at time t is a function of the number of vehicles on the link at time t. The travel time for any path can be explicitly expressed in recursive form with the delay-function-based link models, which brings considerable tractability in the formulation, analysis and solution of DUE problems. Delay-function model reads

$$\frac{dx(t)}{dt} = u(t) - v(t) \tag{3.41}$$

$$\tau(t) = g_d\left(x(t)\right) \tag{3.42}$$

$$v(t + \tau(t)) = \frac{u(t)}{1 + \dot{\tau}(t)} \tag{3.43}$$

Equation is derived from the following equation under FIFO constraint:

$$\int_{-\infty}^{t} u(\omega) d\omega = \int_{-\infty}^{t+\tau(t)} v(\omega) d\omega \tag{3.44}$$

Nie and Zhang (2005) studied the delay-function-based link models and found that (1) the linear delay function, the only proven FIFO consistent delay function, substantially overestimates link travel time due to the so-called double-counting effect (2) the piece-wise linear delay function, an improvement over the linear delay function in reducing double-counting, does not always respect FIFO.

Some whole-link travel time models assume that the travel time from the beginning to the end of a link of the network can be expressed as an increasing function of the whole-link variables such as link inflows, outflows or link volume (the number of vehicles on the link) at each time point. For example, Ran and Boyce (1996, 1997) have used more general nonlinear whole-link travel time function. It is the sum of two components: (i) a flow-dependent cruise time which depends on inflow and on the number of vehicles in the link; and (ii) a queuing delay which depends on the outflows and the number of vehicles in the link. Study on whole link model can also be found in Careya (2002, 2003).

3.4.5 Hydrodynamic Models

The hydrodynamic model is also known as LWR model because it was first presented in Lighthill & Whitham (1955) and Richards (1956). Hydrodynamic models take traffic as continuous fluid represented by volume (q), speed (s), and density (k). The fundamental hypothesis of the theory is that at any point of the road the traffic volume q is a function of the density k, or

$$q(x,t) = k(x,t) * s(x,t) = q(k(x,t)) \qquad (3.45)$$

The movement of traffic on uniform road segment is decided by the following flow conservation equation:

$$\frac{\partial q}{\partial x} + \frac{\partial k}{\partial t} = g(x,t) \qquad (3.46)$$

where x is space, t is time, $g(x,t)$ is exit function of the segment. The equation can also be written as

$$\dot{q}\frac{\partial k}{\partial x} + \frac{\partial k}{\partial t} = g(x,t), \quad \dot{q} = \frac{\partial q}{\partial k} \qquad (3.47)$$

When $g(x,t)$=0, the flow conservation equation becomes:

$$\frac{\partial q}{\partial x} + \frac{\partial k}{\partial t} = 0 \qquad (3.48)$$

Or

$$\dot{q}\frac{\partial k}{\partial x} + \frac{\partial k}{\partial t} = 0, \quad \dot{q} = \frac{\partial q}{\partial k} \qquad (3.49)$$

(3.48) and (3.49) states that the quantity in a small element of length changes at a rate equal to the difference between the inflow and outflow. Model (3.49) was presented by Lighthill and Whitham (1955) and Richards (1956) separately so it was called LWR model.

The LWR model assumes the existence of an equilibrium speed-density relationship

$$s = s_e(k), \tag{3.50}$$

where s_e is equilibrium speed. The equilibrium condition is defined as (Zhang, 1998)

$$\frac{dq}{dx} = 0 \tag{3.51}$$

Given initial value of $k(x,0) = f(x)$ and an equilibrium speed-density relationship (3.48), the solution of flow conservation equation (3.47) is

$$k(x,t) = f\left(x - q'\left(f(x_0)\right)t\right) \tag{3.52}$$

where $q' = dq/dk$, thus one knows traffic status anywhere at any time.

Figure 3.6 and Figure 3.7 show an example of the characteristics and the surface of Hydrodynamic model, respectively. In the example, Greenshield model (3.28) is used, where free flow speed s_f =45 m/hr, jam density k_j =200 veh/m. 3-mile long road and 15 minutes is considered. The initial density $k(x,0)$=100+10*x*, where *x* is distance (mile).

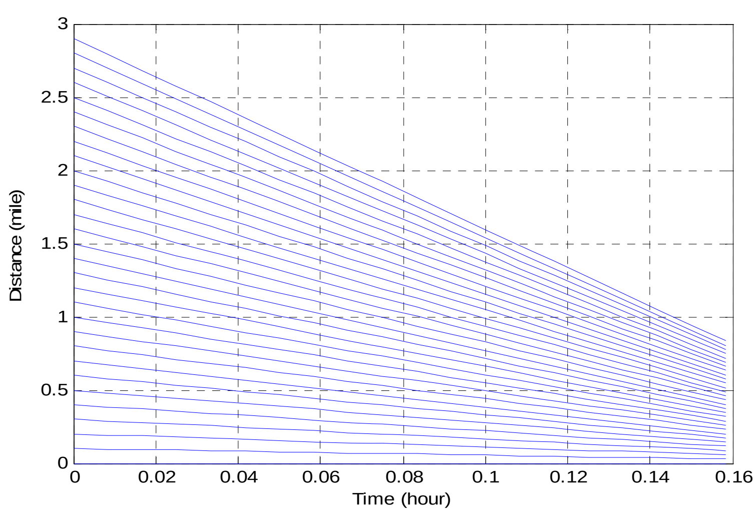

Figure 3-6 Characteristics of Hydrodynamic model in the example

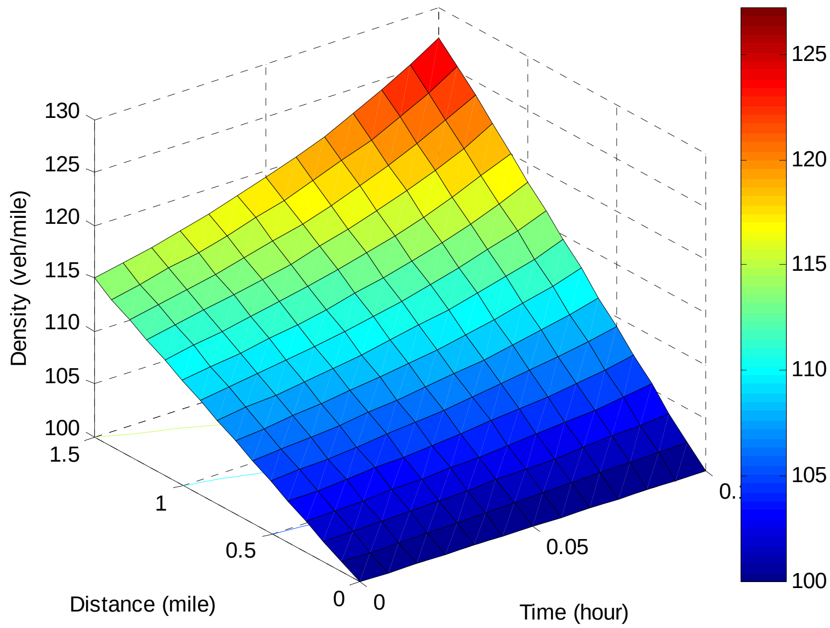

Figure 3-7 Density surface of Hydrodynamic model in the example

3.4.6 Cell Transmission Model

Daganzo (1994, 1995) presented Cell Transmission model (CTM) based on LWR model by assuming a trapezoid-shaped volume and density diagram. The CTM adopting the following relationship between traffic volume q and density k :

$$q = \min\{s_f k, Q, W(k_j - k)\} \tag{3.53}$$

where s_f, Q, W, k, k_j denote free-flow speed, inflow capacity (or maximum allowable inflow), backward shock wave, density, and jam density, respectively. By dividing road into uniform seg s ments and time into intervals, the CTM uses the following recursive equations to approximate the solution of LWR:

$$n_j(\omega+1) = n_j(\omega) + q_j(\omega) - q_{j+1}(\omega) \tag{3.54}$$

$$q_j(\omega) = \min\left\{n_{j-1}(\omega), Q_j(\omega), \frac{W}{s_f}\left[N_j(\omega) - n_j(\omega)\right]\right\} \tag{3.55}$$

where the subscript j refers to cell j, and $j+1$ and $j-1$ represents the cell downstream (upstream) of j. The variables $n_j(\omega)$, $q_j(\omega)$, $N_j(\omega)$ denote the number of vehicles, the actual inflow, and the maximum number of vehicles that can be held in cell j at time ω, respectively.

Table 3.1 shows the solution of an example of Cell Transmission model. Figure 3.8 shows the surface of the density of the example. In the example, an isosceles trapezoid flow-density model is used with k_{c1}=55 vehicle unit per distance unit, s_f=1 distance unit per time tick, $Q = 55$, $N = k_j = 160$.

Table 3-1 Numerical results of the example

Time (Tick)

42	43	44	45	46	47	48	62	63	64	65	66	67	68	69	70
43	44	45	46	47	48	55	62	63	64	65	66	67	68	69	70
44	45	46	47	48	49	61	62	63	64	65	66	67	68	69	70
45	46	47	48	49	55	61	62	63	64	65	66	67	68	69	70
46	47	48	49	50	60	61	62	63	64	65	66	67	68	69	70
47	48	49	50	55	60	61	62	63	64	65	66	67	68	69	70
48	49	50	51	59	60	61	62	63	64	65	66	67	68	69	70
49	50	51	55	59	60	61	62	63	64	65	66	67	68	69	70
50	51	52	58	59	60	61	62	63	64	65	66	67	68	69	70
51	52	55	58	59	60	61	62	63	64	65	66	67	68	69	70
52	53	57	58	59	60	61	62	63	64	65	66	67	68	69	70
53	55	57	58	59	60	61	62	63	64	65	66	67	68	69	70
54	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70
55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70

Distance (unit distance)

Hydrodynamic model and cell transmission model have the advantage of offering plausible descriptions of flow, including the propagation of congestion, whereas it has the disadvantage of being both analytically and computationally demanding. NIE and ZHANG (2005) studied four link models—the linear delay-function (DF) model, the MN model, the PQ model and the CTM model. Using the CTM model as a benchmark, they found that the PQ model behaves identically as the CTM model; the DF model, however, was found to systematically overestimate link traversal times, and the EF model was found to overestimate link traversal times when inflow rate decreases suddenly but underestimates link traversal times when inflow rate increases suddenly.

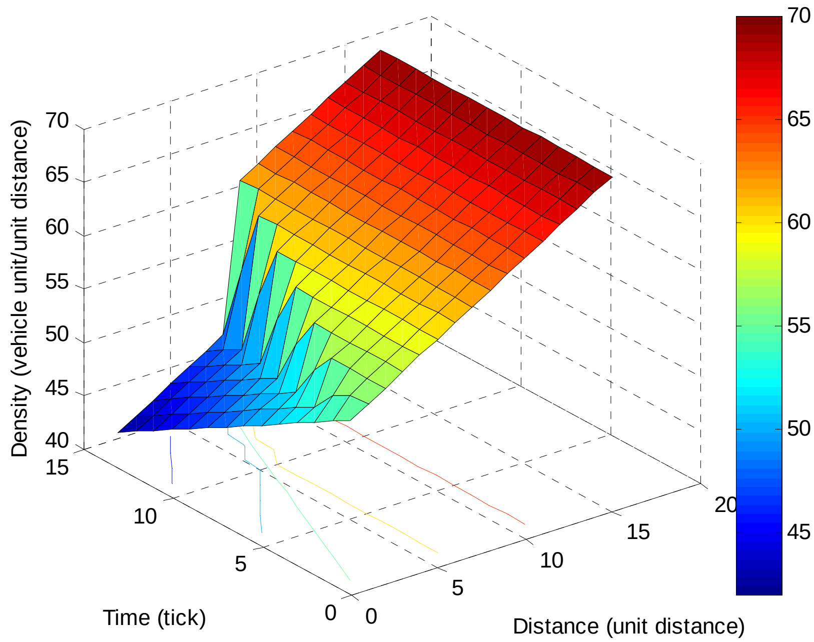

Figure 3-8 Density surface of Cell Transmission model in the example

3.5 Path Travel Times

There are two types of time-dependent path travel time: naïve path travel time and recursive path travel time.

3.5.1 Naïve Path Travel Time

In Naïve path travel time, the travel time required to traverse path $p = \{a_1, a_2, \cdots, a_m\}$ for vehicles entering the network at time t, is calculated using the following formula:

$$\eta_p^{rs}(t) = \tau_{a_1}(t) + \tau_{a_2}(t) + \cdots + \tau_{a_m}(t) \qquad (3.56)$$

where $\tau_a(t)$ is travel time on link a at time t.

3.5.2 Recursive Path Travel Time

In recursive path travel time, the travel time required to traverse path $p = \{a_1, a_2, \cdots, a_m\}$ for vehicles entering the network at time t, is calculated using the following recursive formula:

$$\eta_p^{rs}(t) = \tau_{a_1}(t) + \tau_{a_2}(t + \tau_{a_1}(t)) + \cdots + \tau_{a_m}(t + \tau_{a_1}(t) + \cdots + \tau_{a_{m-1}}(t + \tau_{a_1}(t) + \cdots + \tau_{a_{m-2}}(t))) \qquad (3.57a)$$

For simplicity, let $\tau_{a_1} = \tau_{a_1}(t)$, $\tau_{a_2} = \tau_{a_2}(t + \tau_{a_1}(t))$, etc, (3.57a) can be rewritten as

$$\eta_p^{rs}(t) = \sum_{a \in p} \sum_{l > k} \tau_a(l) \delta_{apk}^{rs}(l) \qquad (3.57b)$$

where $\delta_{apk}^{rs}(l)$ is equal to 1, if the flow on path p of pair (r, s) entering the network at interval k arrives link a at interval l; otherwise, 0. The following equations hold (Huang and Lam, 2002).

$$\delta_{a_i pk}^{rs}(l) = \begin{cases} 1 & \text{if} \quad k + \tau_{a_1} + \tau_{a_2} + \cdots + \tau_{a_{i-1}} = l \\ 0 & \text{otherwise} \end{cases} \qquad (3.58a)$$

and

$$\sum_{l > k} \delta_{apk}^{rs}(l) = 1 \qquad \forall p \in P_{rs}, r \in R, s \in S, k \in K \qquad (3.58b)$$

Chapter 4: Concepts and New Algorithm for Ideal Dynamic User Optimal Route Choice (DUO) Problem

In this chapter, the ideal dynamic user optimal (DUO) route choice problem is studied. At DUO state, the actual travel times experienced by travelers of the same O-D pair departing at the same time are equal and minimal. Some basic definitions are given in Section 4.1. Section 4.2 presents the link-based variation inequality DUO model and development of its algorithm. Section 4.3 presents the route-based variation inequality DUO model and development of its algorithm.

4. 1 Some Definitions

Some definitions are given as follows:

Departure Horizon: The time period in which there are vehicles departing from an origin and entering the network. Denote it as $[0, T_0]$. All departing flow rate from any origins is zero after T_0.

Assigning Horizon: The time point at which the last vehicle entering the network reaches its destination. Denote it as T . $[0, T]$ is the whole analysis time period.

Time Increment: The length of the time interval used to partition $[0, T_0]$ and $[0, T]$. Denote it as Δt . Each time increment is a unit of time. The kth time interval is k .

Let $K = [T/\Delta t]_+ \equiv \operatorname{argmin}\{i > T/\Delta t, i \in \overline{Z}\}$, where $\overline{Z}$ is the set of natural number. Similarly, let $K_0 = [T_0/\Delta t]_+$.

Time-Space Network: The network with time dimension, showing the network state at each time interval k .

Figure 4.2 shows an example of time-space network with 4 time interval for the 3-link network in Figure 4.1. $x(a,k)$ is the number of vehicles on link a at interval k .

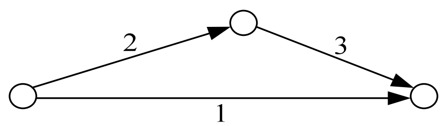

Figure 4-1 A 3-link network.

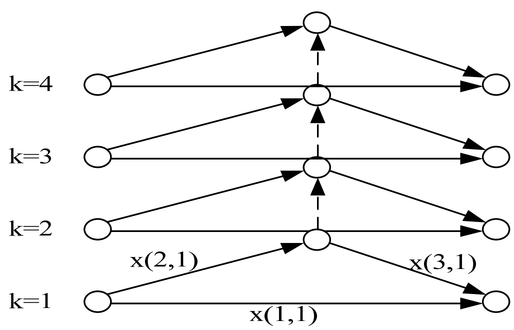

Figure 4-2 Time-space network with 4 time intervals for the 3-link network

4.2 Link-based Variational Inequality (VI) DUO Model

4.2.1 Link-based VI Formulation of DUO

Assume the network is empty at $t = 0$, and only travel demands departing within the departure horizon are considered. The link-based DUO continuous VI model can be expressed as

$$\int_0^T \left\langle \overline{\mathbf{\Omega}}(t), \overline{\mathbf{u}}(t) - \overline{\mathbf{u}}^*(t) \right\rangle dt \geq 0 \tag{4.1a}$$

where $\overline{\mathbf{\Omega}} \in \Re_+^{|R\times S|\times|A|}$, $\overline{\mathbf{u}} \in \Re_+^{|R\times S|\times|A|}$, $|N|$, $|A|$, and $|R\times S|$ are the cardinalities of the set nodes, links and O-D pairs, etc. $\langle \mathbf{a}, \mathbf{b} \rangle = \mathbf{a}^T \mathbf{b}$,

or in expanded form as

$$\int_0^T \left\langle \sum_{rs} \sum_a \Omega_a^{rs*}(t) \left\{ u_a^{rs}\left[t + \pi^{ri*}(t)\right] - u_a^{rs*}\left[t + \pi^{ri*}(t)\right] \right\} \right\rangle dt \geq 0 \tag{4.1b}$$

where

$$\Omega_a^{rs*}(t) = \pi^{ri*}(t) + \tau_a\left[t + \pi^{ri*}(t)\right] + \pi^{js*}\left[t + \pi^{rj*}(t)\right] - \pi^{rs*}(t), \quad a = (i, j) \tag{4.1c}$$

This formulation is equivalent to the following link-based DUO route choice conditions:

$$\Omega_a^{rs*}(t) \geq 0 \quad \forall\, a = (i, j), r, s; \tag{4.2a}$$

$$u_a^{rs*}\left[t + \pi^{ri*}(t)\right]\Omega_a^{rs*}(t) = 0 \quad \forall\, a = (i, j), r, s; \tag{4.2b}$$

$$u_a^{rs}\left[t + \pi^{ri*}(t)\right] \geq 0 \;\; \forall\, a = (i, j), r, s; \tag{4.2c}$$

The above formulation and conditions comes from Ran and Boyce (1996b) with some modification. In Ran and Boyce (1996b), the link cost term is defined as

$$\Omega_a^{rj*}(t) = \pi^{ri*}(t) + \tau_a\left[t + \pi^{ri*}(t)\right] - \pi^{rj*}(t) \tag{4.3}$$

which is different from (4.1c).

(4.2a) states that if time-space link $a\left[t + \pi^{ri*}(t)\right]$ is on the minimal actual route (dynamic shortest path) from origin r to destination s at time t, $\Omega_a^{rs*}(t) = 0$; otherwise, $\Omega_a^{rs*}(t) > 0$. (4.2b) states that if time-space link $a\left[t + \pi^{ri*}(t)\right]$ is on the minimal actual route from origin r to destination s at time t, or if $\Omega_a^{rs*}(t) = 0$, $u_a^{rs*}\left[t + \pi^{ri*}(t)\right] \geq 0$; otherwise, or if $\Omega_a^{rs*}(t) > 0$, $u_a^{rs*}\left[t + \pi^{ri*}(t)\right] = 0$. (4.2c) is nonnegative condition for inflow.

Below proving traffic status satisfying (4.1) is in a DUO status or equivalent to (4.2a), (4.2b), (4.2c).

Proof:

(i) Necessity. By (4.2a) and (4.2c), $\overline{\mathbf{\Omega}} \geq 0$, $\overline{\mathbf{u}} \geq 0$, this implies $\langle \overline{\mathbf{\Omega}}, \overline{\mathbf{u}} \rangle \geq 0$. By (4.2b), $\langle \overline{\mathbf{\Omega}}, \overline{\mathbf{u}}^* \rangle = 0$. Thus, $\langle \overline{\mathbf{\Omega}}(t), \overline{\mathbf{u}}(t) - \overline{\mathbf{u}}^*(t) \rangle \geq 0$ holds. Integrating it over $[0, T]$, we have (4.1).

(ii) Sufficiency. (4.2a) and (4.2c) hold by definition. Let the optimal solution of (4.1) be $\overline{\mathbf{u}}^*$. To prove (4.2b) holds for $\overline{\mathbf{u}}^*$, we first find a feasible solution $\overline{\mathbf{u}}^\oplus$ such that (4.2b) holds, or $\langle \overline{\mathbf{\Omega}}, \overline{\mathbf{u}}^\oplus \rangle = 0$. Suppose (4.2b) does not hold for $\overline{\mathbf{u}}^*$, we have $\langle \overline{\mathbf{\Omega}}, \overline{\mathbf{u}}^* \rangle > 0$. We further has $\langle \overline{\mathbf{\Omega}}, \overline{\mathbf{u}}^\oplus - \overline{\mathbf{u}}^* \rangle < 0$, or $\int_0^T \langle \overline{\mathbf{\Omega}}(t), \overline{\mathbf{u}}^\oplus(t) - \overline{\mathbf{u}}^*(t) \rangle dt < 0$. This contradicts (4.1). Thus (4.2b) holds for $\overline{\mathbf{u}}^*$.

4.2.2 Solution Algorithms for Link-based VI DUO Model

Discrete Link-based VI DUO Model

To solve the DUO problem, the continuous VI formulation is discretized with each time interval being time increment. The estimated actual travel time on each time-space link a is a multiple of the time increment and is fixed at each time increment, i.e.,

$$\overline{\tau}_a(k) = i \qquad \text{if} \quad (i - 0.5)\Delta t \leq \tau_a(k) \leq (i + 0.5)\Delta t \tag{4.4}$$

where i is an integer and $0 \leq i \leq K$, Δt is time increment.. This round-off method is used only in the flow propagation constraints. The round-off error can be made as small as desired by making the time increment smaller (Ran and Boyce, 1996b).

The link-based DUO discrete-time VI formulation is

$$\left\langle \overline{\Omega}(k), \mathbf{u}(k) - \mathbf{u}^*(k) \right\rangle \geq 0 \tag{4.5a}$$

or in expanded form as

$$\sum_{k=1}^{K_0} \sum_{rs} \sum_{a} \Omega_a^{rs*}(k) \left\{ u_a^{rs}\left[k + \pi^{ri*}(k)\right] - u_a^{rs*}\left[k + \pi^{ri*}(k)\right] \right\} \geq 0 \tag{4.5b}$$

where $\Omega \in \Re_+^{|R \times S| \times |A| \times K_0}$, $\mathbf{u} \in \Theta$, and

$$\Omega_a^{rs*}(k) = \pi^{ri*}(k) + \tau_a\left[k + \pi^{ri*}(k)\right] + \pi^{js*}\left[k + \pi^{rj*}(k)\right] - \pi^{rs*}(k), a = (i, j) \tag{4.6}$$

Θ is the feasible region defined by the following constraints:

Path flow conservation constraint:

$$\sum_{p} f_p^{rs}(k) = f^{rs}(k) \ \forall\, k, r, s \tag{4.7}$$

Link inflow conservation constraint:

$$\sum_{rs} u_a^{rs}(k) = u_a(k) \quad \forall\, a, k$$

(4.8)

Link outflow conservation constraint:

$$\sum_{rs} v_a^{rs}(k) = v_a(k) \quad \forall\, a, k$$

(4.9)

Node flow conservation constraint:

$$\sum_{a \in B(j)} v_a^{rs}(k) = \sum_{a \in A(j)} u_a^{rs}(k) \quad \forall\, j \neq r,s; r,s; k \tag{4.10}$$

where $A(j)$ is the set of links after j and $B(j)$ is the set of links before j.

Link flow propagation constraint:

$$u_a^{rs}(k) = v_a^{rs}\left(k + \tau_a(k)\right) \quad \forall\, a,\ r, s, k \tag{4.11}$$

The link state equation:

$$x_a(k+1) = x_a(k) + u_a(k) - v_a(k) \quad \forall\, a, k \tag{4.12a}$$

or

$$x_a(k+1) = x_a(k) + u_a(k+1) - v_a(k+1) \quad \forall\, a, k \tag{4.12b}$$

(4.12a) is forward formula, (4.12b) is backward formula.

Path-link flow incidence constraint:

$$u_a^{rs}(n) = \sum_{rs}\sum_{p}\sum_{k=1}^{K_0} f_p^{rs}(k)\delta_{rsa}^{pkn} \quad \forall\, a, n \tag{4.13}$$

where $\delta_{rsa}^{pkn} \in \{0,1\}$ is defined as:

$$\delta_{rsa}^{pkn} = \begin{cases} 1 & \text{if traffic departing origin } r \text{ at any time interval } k \\ & \text{heading for destination } s \text{ on path } p \text{ arrives at link } a \\ & \text{during the } n\text{th time interval.} \\ 0 & \text{otherwise} \end{cases} \tag{4.14}$$

Nonnegative constraint:

$$f_p^{rs}(k) \geq 0,\, u_a^{rs}(k) \geq 0,\ \forall\, k, r, s, a, p \tag{4.15}$$

With flow propagation constrain (4.11), exit flow $v_a^{rs}(t)$ and link volume $x_a(t)$ can be expressed by inflow u_a^{rs} as follows (Ran, 2002b; Chen, 1998):

$$v_a^{rs}(t) = \sum_k u_a^{rs}(k)\delta_{a'}^{k}(k) \tag{4.16}$$

where

$$\delta_{a'}^{k}(t) = \begin{cases} 1, & k + \tau_a(k) = t \\ 0, & \text{otherwise} \end{cases} \tag{4.17}$$

and

$$x_a(t) = \sum_k \sum_{rs} u_a^{rs}(k)\delta_{a''}^{k}(k) \tag{4.18}$$

where

$$\delta_{a''}^{k}(t) = \begin{cases} 1, & k < t, k + \tau_a(k) \geq t \\ 0, & \text{otherwise} \end{cases} \tag{4.19}$$

Relaxation

At each relaxation, it is to temporarily fix (Ran and Boyce, 1996b; Ran, 2002b): 1) Actual travel time $\tau_a(k)$ in the link flow propagation constraints as $\bar{\tau}_a(k)$ and corresponding actual route travel time $\eta_p^{rs}(k)$ as $\bar{\eta}_p^{rs}(k)$; 2) Actual travel time $\tau_a(k)$ in the VI cost term $\Omega_a^{ri}(k)$ as $\tau_a\left[k+\bar{\pi}^{ri}(k)\right]$ and 3) Minimal travel times $\pi^{ri}(k)$ as $\bar{\pi}^{ri}(k)$, $\pi^{js}\left[k+\bar{\pi}^{rj}(k)\right]$ as $\bar{\pi}^{js}\left[k+\bar{\pi}^{rj}(k)\right]$ and $\pi^{rs}(k)$ as $\bar{\pi}^{rs}(k)$ for each link and each origin and destination. At each relaxation, a time-space network is implicitly formed with fixed link flow propagation constraints and fixed actual route travel time.

Via relaxation, the VI cost term becomes

$$\overline{\Omega}_a^{rs}(k)=\bar{\pi}^{ri}(k)+\tau_a\left[x_a\left[k+\bar{\pi}^{ri}(k)\right]\right]+\bar{\pi}^{js}\left[k+\bar{\pi}^{rj}(k)\right]-\bar{\pi}^{rs}(k) \tag{4.20}$$

Optimization Problem

An optimization problem which is equivalent to the discrete VI under relaxation can thus be formulated, as follows:

$$\min_{u} Z=\sum_{k=1}^{K_0}\sum_{rs}\sum_{a}\left\{\int_0^{u_a^{rs}\left(k+\bar{\pi}^{ri}(k)\right)}\tau_a\left(x_a\left[k+\bar{\pi}^{ri}(k)\right]\right)d\omega+u_a^{rs}\left(k+\bar{\pi}^{ri}(k)\right)\left[\bar{\pi}^{ri}(k)+\bar{\pi}^{js}\left(k+\bar{\pi}^{rj}(k)\right)-\bar{\pi}^{rs}(k)\right]\right\} \tag{4.21}$$

The gradient of (4.21) is shown to be

$$\frac{\partial Z}{\partial u_a^{rs}\left(k+\bar{\pi}^{ri}(k)\right)}=\bar{\pi}^{ri}(k)+\tau_a\left(k+\bar{\pi}^{ri}(k)\right)+\bar{\pi}^{js}\left(k+\bar{\pi}^{rj}(k)\right)-\bar{\pi}^{rs}(k) \tag{4.22}$$

(4.21) is equivalent to the cost term of discrete VI (4.5b) under relaxation. This indicates the above optimization program is equivalent to the discrete VI (4.5).

By using (4.18), we have

$$x_a\left[k+\bar{\pi}^{ri}(k)\right]=\sum_{\bar{k}=1}^{K}\sum_{rs}u_a^{rs}(\bar{k})\delta_{a''}^{\bar{k}}\left[k+\bar{\pi}^{ri}(k)\right] \quad (4.23a)$$

where $\delta_{a''}^{\bar{k}}(t)=\begin{cases}1, & \bar{k}\le t, \bar{k}+\tau_a(\bar{k})\ge t\\ 0, & \text{otherwise}\end{cases}$

Letting $\bar{k}_{\min}=\min_{\delta_a^{\bar{k}}(t)=1}\{\bar{k}\}$ and $\bar{k}_{\max}=\max_{\delta_a^{\bar{k}}(t)=1}\{\bar{k}\}$, (4.23a) can be expressed as

$$x_a\left[k+\bar{\pi}^{ri}(k)\right]=\sum_{\bar{k}=\bar{k}_{\min}}^{\bar{k}=\bar{k}_{\max}}\sum_{rs}u_a^{rs}(\bar{k}) \quad (4.23b)$$

(4.23b) can be rewritten as

$$x_a\left[k+\bar{\pi}^{ri}(k)\right]=u_a^{\dot{r}\dot{s}}\left(k+\bar{\pi}^{ri}(k)\right)+\sum_{rs\neq\dot{r}\dot{s}}u_a^{rs}\left(k+\bar{\pi}^{ri}(k)\right)+\sum_{\bar{k}=\bar{k}_{\min}}^{\bar{k}=\bar{k}_{\max}-1}\sum_{rs}u_a^{rs}(\bar{k}) \quad (4.23c)$$

Letting $X_a^{\dot{r}\dot{s}}\left(k+\bar{\pi}^{ri}(k)\right)=\sum_{rs\neq\dot{r}\dot{s}}u_a^{rs}\left(k+\bar{\pi}^{ri}(k)\right)+\sum_{\bar{k}=\bar{k}_{\min}}^{\bar{k}=\bar{k}_{\max}-1}\sum_{rs}u_a^{rs}(\bar{k})$, (4.23c) can be rewritten as

$$x_a\left[k+\bar{\pi}^{ri}(k)\right]=u_a^{\dot{r}\dot{s}}\left(k+\bar{\pi}^{ri}(k)\right)+X_a^{\dot{r}\dot{s}}\left(k+\bar{\pi}^{ri}(k)\right) \quad (4.23d)$$

Substitute (4.23d) into (4.21), we have

$$\min_{u} Z=\sum_{k=1}^{K_0}\sum_{rs}\sum_{a}\left\{\int_0^{u_a^{rs}\left(k+\bar{\pi}^{ri}(k)\right)}\left[\tau_a\left(u_a^{rs}\left(k+\bar{\pi}^{ri}(k)\right)+X_a^{rs}\left(k+\bar{\pi}^{ri}(k)\right)\right)+\bar{\pi}^{ri}(k)+\bar{\pi}^{js}\left(k+\bar{\pi}^{rj}(k)\right)-\bar{\pi}^{rs}(k)\right]d\omega\right\} \quad (4.24)$$

Since all cross effects (cross link, cross time interval, cross O-D) are fixed in each relaxation, $u_a^{rs}\left(k+\bar{\pi}^{ri}(k)\right)$ is the only variable for each summation term of (4.21) and (4.24).

At each relaxation, the VI formulation of DUO problem was transformed into a series of static user equilibrium traffic assignment problems over the time-space network of the relaxation, which can be solved by Frank-Wolfe algorithm. Call the relaxation as outer iteration and solving static user equilibrium traffic assignment problems over the time-space network of the relaxation as inner iteration.

At the mth iteration of the inner iteration (Frank-Wolfe algorithm), the descending direction of nonlinear programming (4.21) can be found by solving the following linear program:

$$\min_{h} \ \hat{Z} = h^T \nabla_u Z^{(m)} \tag{4.25}$$

in Θ, where h is subproblem variable, $\nabla_u Z^{(m)}$ is gradient of Z with respect to $\mathbf{u}$ evaluated at $\mathbf{u}^{(m)}$.

(4.25) is equivalent to:

$$\min_{h} \ \hat{Z} = \sum_{k=1}^{K_0} \sum_{rs} \sum_{a} \left[t_a^{rs(m)}(k) h_a^{rs}\left(k + \bar{\pi}^{ri}(k)\right) \right] \tag{4.26}$$

in Θ, where

$$t_a^{rs(m)}(k) = \frac{\partial Z\left(\mathbf{u}^{(m)}\right)}{\partial u_a^{rs}\left(k + \bar{\pi}^{ri}(k)\right)} = \bar{\pi}^{ri}(k) + \tau_a\left[x_a^{(m)}\left(k + \bar{\pi}^{ri}(k)\right)\right] + \bar{\pi}^{js}\left(k + \bar{\pi}^{rj}(k)\right) - \bar{\pi}^{rs}(k) \tag{4.27}$$

(4.27) can be decomposed by origin-destination pair. The resulting subproblem for O-D pair rs is:

$$\min_{h} \ \hat{Z} = \sum_{k=1}^{K_0} \sum_{a} \left[t_a^{rs(m)}(k) h_a^{rs}\left(k + \bar{\pi}^{ri}(k)\right) \right] \tag{4.28}$$

in Θ.

(4.28) can be further decomposed by each O-D flow $f^{rs}(k), k = 1, \cdots, K_0$. The resulting subproblem for O-D flow $f^{rs}(k)$ is:

$$\min_{h} \ \hat{Z} = \sum_{a} \left[t_a^{rs(m)}(k) h_a^{rs}\left(k + \bar{\pi}^{ri}(k)\right) \right] \tag{4.29}$$

in Θ.

(4.29) can be viewed as a shortest path problem over the time-space network of the relaxation. The minimum of (4.29) is found by assigning $f^{rs}(k)$ to the actual minimum cost route (dynamic shortest path) of O-D pair rs at time interval k. The cost of each time-space link is defined as (4.27). The shortest path for (4.27) can be found on the original network, with the time interval for each link recorded on the original network to track the shortest path on time-space network. As an example, Figure 4.3 shows how to record time interval on the original network for demand $f^{13}(1)$.

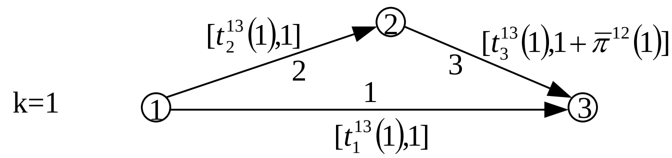

Figure 4-3 An example of recording time intervals on original network.

Cost term (4.27) contains the fixed actual travel time $\bar{\pi}^{ri}(k)$ and $\bar{\pi}^{js}(k+\bar{\pi}^{rj}(k))$ at each relaxation for every link $a=(i,j), \forall rs \in R \times S, k=1,\cdots,K_0$. They are dynamic shortest path on time-space network. Section (4.2.3) describes an efficient algorithm to find dynamic shortest paths on the original network based on time-space link travel times.

Notice the difference between cost term (4.27) and $\tau_a[k+\bar{\pi}^{ri}(k)]$. If $u_a^{rs}(k+\bar{\pi}^{ri}(k))$ does not contribute to $\tau_a(k+\bar{\pi}^{ri}(k))$, the shortest paths based on (4.27) and $\tau_a[k+\bar{\pi}^{ri}(k)]$ are the same. To see this, let $p_{rsk}=\{p_{rsk}^1,\cdots,p_{rsk}^{N_{rsk}}\}$ be the set of the actual minimum cost route of O-D pair rs at time interval k at the mth iteration of the inner iteration, where N_{rsk} is the number of the actual minimum cost route of p_{rsk}. Consider the path cost of any $p_{rsk}^I \in p_{rsk}$, $I=1,\cdots,N_{rsk}$, with $p_{rsk}^I=(a_1,\cdots,a_{\hat{I}})$, where $a_1=(i_1,j_1),\cdots,a_{\hat{I}}=(i_{\hat{I}},j_{\hat{I}})$ are sequential links on route p_{rsk}^I, $\hat{I}$ is the number of links on route p_{rsk}^I. The path cost of p_{rsk}^I (denote it as c_{rsk}^I) is the sum of all the cost of time-space links on the path, or

$$\begin{aligned} c_{rsk}^{I} = \quad 0 \quad &+ \tau_{a_1}\left[k + \overline{\pi}^{ri_1}(k)\right] + \overline{\pi}^{j_1 s}\left(k + \overline{\pi}^{rj_1 *}(k)\right) - \overline{\pi}^{rs}(k) + \\ \overline{\pi}^{ri_2}(k) &+ \tau_{a_2}\left[k + \overline{\pi}^{ri_2}(k)\right] + \overline{\pi}^{j_2 s}\left(k + \overline{\pi}^{rj_2}(k)\right) - \overline{\pi}^{rs}(k) + \\ &\cdots\cdots \qquad\qquad + \\ \overline{\pi}^{ri_{\hat{I}}}(k) &+ \tau_{a_{\hat{I}}}\left[k + \overline{\pi}^{ri_{\hat{I}}}(k)\right] + \quad 0 \quad - \overline{\pi}^{rs}(k) \end{aligned} \tag{4.30a}$$

If $p_{rsk}^{I} = \left(a_1, \cdots, a_{\hat{I}}\right)$ is the same path as the minimum route (with path cost $\pi^{rs}(k)$) under the relaxation, then we have

$$\begin{aligned} \overline{\pi}^{ri_2}(k) + \overline{\pi}^{j_1 s}\left(k + \overline{\pi}^{rj_1}(k)\right) &= \overline{\pi}^{rs}(k) \\ \cdots \quad \cdots& \\ \overline{\pi}^{ri_{\hat{I}}}(k) + \overline{\pi}^{j_{\hat{I}-1} s}\left(k + \overline{\pi}^{rj_{\hat{I}-1}}(k)\right) &= \overline{\pi}^{rs}(k) \end{aligned} \tag{4.30b}$$

(4.16a) reduces to

$$c_{rsk}^{I} = \tau_{a_1}\left[k + \overline{\pi}^{ri_1}(k)\right] + \tau_{a_2}\left[k + \overline{\pi}^{ri_2}(k)\right] + \cdots + \tau_{a_{\hat{I}}}\left[k + \overline{\pi}^{ri_{\hat{I}}}(k)\right] - \overline{\pi}^{rs}(k) = 0 \tag{4.31}$$

Since $c_{rsk}^{I} \geq 0$ and $\overline{\pi}^{rs}(k)$ is fixed at each relaxation, equation (4.31) implies p_{rsk}^{I} is also the minimum cost route if cost term $\tau_a\left[k + \overline{\pi}^{ri}(k)\right]$ is used.

However, if $u_a^{rs}\left(k + \overline{\pi}^{ri}(k)\right)$ contributes to $\tau_a\left(k + \overline{\pi}^{ri}(k)\right)$, the shortest paths based on (4.27) and $\tau_a\left[k + \overline{\pi}^{ri}(k)\right]$ are not necessarily the same. To see this, now let $p_{rsk} = \left\{p_{rsk}^{1}, \cdots, p_{rsk}^{N_{rsk}}\right\}$ be the actual minimum cost route of O-D pair rs at time interval k at the mth iteration of the inner iteration based on cost term $\tau_a\left[k + \overline{\pi}^{ri}(k)\right]$. For any $p_{rsk}^{I} \in p_{rsk}$ with $p_{rsk}^{I} = \left(a_1, \cdots, a_{\hat{I}}\right)$, $a_1 = \left(i_1, j_1\right), \cdots, a_{\hat{I}} = \left(i_{\hat{I}}, j_{\hat{I}}\right)$, its path cost based on cost term $\tau_a\left[k + \overline{\pi}^{ri}(k)\right]$ is

$$\hat{c}_{rsk}^{I} = \tau_{a_1}\left[x_{a_1}^{(m)}\left(k + \overline{\pi}^{ri_1}(k)\right)\right] + \tau_{a_2}\left[x_{a_2}^{(m)}\left(k + \overline{\pi}^{ri_2}(k)\right)\right] + \cdots + \tau_{a_{\hat{I}}}\left[x_{a_{\hat{I}}}^{(m)}\left(k + \overline{\pi}^{ri_{\hat{I}}}(k)\right)\right]$$

Its path cost based on cost term (4.27) is

$$c^I_{rsk} = 0 + \tau_{a_1}\left[x^{(m)}_{a_1}\left(k+\bar{\pi}^{ri_1}(k)\right)\right] + \bar{\pi}^{j_1 s}\left(k+\bar{\pi}^{rj_1}(k)\right) - \bar{\pi}^{rs}(k) +$$
$$\bar{\pi}^{ri_2}(k) + \tau_{a_2}\left[x^{(m)}_{a_2}\left(k+\bar{\pi}^{ri_2}(k)\right)\right] + \bar{\pi}^{j_2 s}\left(k+\bar{\pi}^{rj_2}(k)\right) - \bar{\pi}^{rs}(k) +$$
$$\cdots \cdots +$$
$$\bar{\pi}^{ri_{\hat{I}}}(k) + \tau_{a_{\hat{I}}}\left[x^{(m)}_{a_{\hat{I}}}\left(k+\bar{\pi}^{ri_{\hat{I}}}(k)\right)\right] + 0 - \bar{\pi}^{rs}(k)$$

Because $p^I_{rsk} = \left(a_1, \cdots, a_{\hat{I}}\right)$ may not be the same path as the minimum route (with path cost $\pi^{rs}(k)$) under the relaxation, (4.30b) do not necessarily hold, and the shortest paths based on (4.27) and $\tau_a\left[k+\bar{\pi}^{ri}(k)\right]$ are not necessarily the same.

The step size along the descending direction can be decided by solving the following one-dimensional search problem:

$$\min_{0\le\alpha\le1} Z = \sum_{k=1}^{K_0}\sum_{rs}\sum_{a}\left\{\int_0^{u_a^{rs(l+1)}\left(k+\bar{\pi}^{ri}(k)\right)}\left[\tau_a\left(\omega+X^{rs}_a\left(k+\bar{\pi}^{ri}(k)\right)\right)+\bar{\pi}^{ri}(k)+\bar{\pi}^{js}\left(k+\bar{\pi}^{rj}(k)\right)-\bar{\pi}^{rs}(k)\right]d\omega\right\} \tag{4.32}$$

After the optimal step size α^m is found, the solution at the inner iteration can be updated as

$$u_a^{rs(m+1)}(n) = u_a^{rs(m+1)}(n) + \alpha^m\left[h_a^{rs(m+1)}(n) - u_a^{rs(m+1)}(n)\right] \quad \forall a, r, s, n = 1, \cdots, K \tag{4.33}$$

Algorithm

According to above rational analysis, a new algorithm for solving the ideal DUO route choice model is developed and summarized as follows.

Step 0: Outer Initialization.

Compute $k_{\max} = \max_{\forall rs}\{\pi^{rs}\}$, where π^{rs} is the static minimum travel time of O-D rs.

Set $K' = K_0 + C\cdot[k_{\max}]_+$. Set $\hat{\tau}_a^{(0)}(k) = \tau_a[0]$, $\forall a \in A$, $k = 1, \cdots, K'$. Find an initial feasible solution $\left[u_a^{rs(0)}(k)\right]$. Set outer iteration counter $l = 0$. Set an outer iteration convergence criterion ε_{out} .

Step 1: Relaxation.

Step 1.0: Find a new estimation of actual link travel times: $\hat{\tau}_a^{(l)}(k) = \tau_a\left[x_a^*(k)\right]$, find $\bar{\tau}_a^{(l)}(k)\ \forall\ a \in A,\quad k = 1,\cdots,K'$, where * denotes the solution obtained from the most recent inner iteration or from outer initialization. Find $\delta_{a'}^{k(l)}(t)$ and $\delta_{a''}^{k(l)}(t)$.

Step 1.1: Find $\bar{\pi}^{rs}(k), \bar{\pi}^{ri}(k)$, and $\bar{\pi}^{js}\left(k + \bar{\pi}^{rj}(k)\right)$ by using dynamic shortest path algorithm, $\forall\ rs \in R \times S, a \in A, k = 1,\cdots,K_0$.

Step 2: Inner Iteration

Step 2.0: Inner Initialization. Compute and reset the inner initial feasible solution to be consistent with the flow propagation constrain at the current relaxation. Set an inner iteration counter $m = 1$ (or a convergence criterion ε_{in}).

Step 2.1: Update. Compute $\tau_a^{(m)}(k)$. Update $\overline{\Omega}_a^{rs(m)}(k)$ by equation (4.27).

Step 2.2: Direction Finding. Based on $\overline{\Omega}_a^{rs(m)}(k)$, search for shortest routes for all OD pairs over the physical network without time-space expansions. Perform an all-or-nothing assignment following the link flow propagation constrain, yielding subproblem solution

$h_a^{rs(m)}\left(k + \bar{\pi}^{ri}(k)\right)$.

Step 2.3: Line Search. Solve the one-dimensional search problem (4.32) using a line search procedure such as the bisection method and find the optimal step size $\alpha^{(m)}$.

Step 2. 4: Move. Find a new solution $u_a^{rs(m+1)}(k)$ by (4.33).

Step 2. 5: Convergence Test for Inner Iteration.

If $\sqrt{\sum_a \sum_k^{K^{(l)}} \left(u_a^{(m+1)}(k) - u_a^{(m)}(k)\right)} \Big/ \sum_a \sum_k^{K^{(l)}} u_a^{(m)}(k)\ \tau_a^{(m)}(k) > \varepsilon$, set $m = m+1$, go to Step 2.1;

otherwise, set $\hat{u}_a^{rs(l)}(k) = u_a^{rs(m+1)}(k)$, $\hat{x}_a^{(l)}(k) = x_a^{(m+1)}(k)$, go to Step 3.

Step 3: Convergence Test for Outer Iteration. If $\hat{\tau}_a^{(l)}(k) \cong \hat{\tau}_a^{(l-1)}(k)$, stop. The current

solution $u_a^{rs}(k)$, $v_a^{rs}(k)$, $x_a^{rs}(k)$ is in a near optimal state; otherwise, set

$l = l+1$ and go to

Step 1.

In the above algorithm, $\hat{u}_a^{rs(l)}(k)\ \hat{x}_a^{(l)}(k)$ are solutions at outer iteration l. $\hat{\tau}_a^{(l)}(k)$ is the estimation of link travel time at outer iteration l. $\bar{\tau}_a^{(l)}(k)$ is the floored link travel time. $u_a^{rs(m)}(k)$ and $x_a^{(m)}(k)$ are solutions at inner iteration m. $\tau_a^{(m)}(k)$ is the estimation of link travel time based on them.

The length of the initial assignment horizon does not affect the solution as long as it is sufficiently long. But if it is too long, some time-space link may never be used and storage of them is wasted. Depending on the congestion of the network, C may be set as 2 or 3, etc. All inflow of the time-space link is zero and the corresponding link travel time is free flow travel time unless the link is assigned flow. The initial feasible solution in outer initialization can be found by performing all-or-nothing assignment on the dynamic shortest path based on free flow link cost for all OD pairs.

At each relaxation, a time-space network is implicitly formed. The algorithm then performs F-W iteration on the time-space network. The $\delta_{a'}^{k(l)}(t)$, $\delta_{a''}^{k(l)}(t)$ and $K^{(l)}$ at the lth relaxation are calculated using the solution $\hat{u}_a^{rs(l-1)}(k)$ at the $(l-1)$th relaxation. Notice the solution $\hat{u}_a^{rs(l-1)}(k)$ at the $(l-1)$th outer iteration cannot be used as the initial solution in the inner iteration of the lth relaxation unless $\delta_{a}^{k}(t)$ and $\delta_{a}^{k}(t)$ at the two relaxations are exactly the same (which indicates the implicit time-space networks of the two relaxations are the same). If $\delta_{a}^{k}(t)$ and $\delta_{a}^{k}(t)$ at the two relaxations are different, the solution $\hat{u}_{a}^{rs(l-1)}(k)$ at the $(l-1)$th outer iteration is not a feasible solution in the inner iteration of the lth relaxation. A procedure to reset the initial feasible solution for the inner iteration at each relaxation is needed to make the initial feasible solution consistent with the current flow propagation. In inner iterations, the shortest paths with link cost term $\overline{\Omega}_{a}^{rs}(k)$ can be found by dynamic shortest path algorithm. Or they can be found by static shortest path algorithm with arrival time interval for each link recorded on the original network as shown in Figure 4.3.

When performing all-or-nothing assignment for $f^{rs}(k), k=1,\cdots,K_0$, the assigned value should be $h_{a}^{rs}(k+\overline{\pi}^{ri}(k))$ instead of $h_{a}^{rs}(k)$. As an example, Figure 4.4 shows how $f^{rs}(i)$ should be assigned on the time-space network. The corresponding links are highlighted as thick black. The assigned volumes resulting from $f^{rs}(i)$ are $h(1,i)$, $h(2,i)$, and $h(3,i+\tau(2,i))$.

Since any route on the time-space network corresponds to a unique route on the original physical network, the assignment of any time-dependent demand $f^{rs}(i)$ can also be performed on the original network if arrival time interval for the link is recorded. Figure 4.5 shows how $f^{rs}(i)$ should be assigned on the original network for the 3-link network. The same method is used to assign all time dependent demand $f^{rs}(i), \ \forall r,s,i$ on the original network.

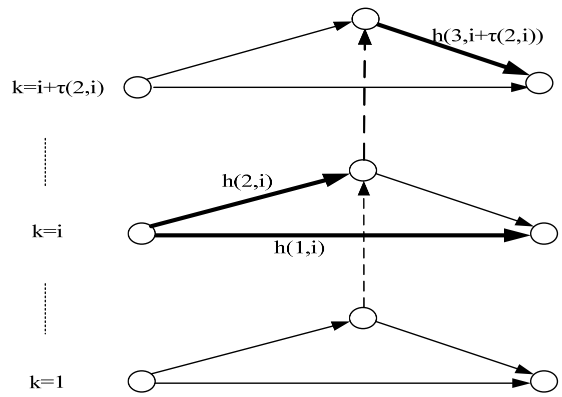

Figure 4-4 Assigned volumes on the simplified time-space expansion network.

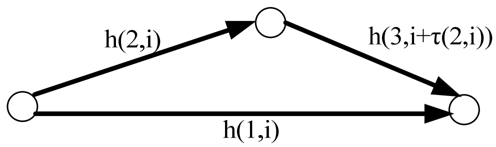

Figure 4-5 Assigned volumes on the original network.

The number of inner iterations at each relaxation can also be pre-specified. The departure horizon is the same for all relaxations. The assignment horizon and the time-space network are fixed at each relaxation but may change from relaxation to relaxation. The assignment horizon and time-space network will finally tend to be fixed. A necessary condition of the convergence of the algorithm is that the time-space network remains the same at successive relaxations. As explained above, the solution of DUO does not need to expand the physical network. The introduction of time-space network is for better explaining and understanding the solution process.

The actual assignment horizon at the end of the solution is $K = K_0 + k_{\max}$, where $k_{\max} = \max_{\forall rs, k=1,\cdots,K_0} \left\{ \overline{\pi}^{rs}(k) \right\}$. When FIFO condition holds, departure horizon $[0, K_0]$ and assignment horizon $[0,K]$ have the following relationship under DUO status: $K = K_0 + \hat{\pi}$, where $\hat{\pi} = \sup\{\pi^{rs}(K_0), \forall r,s\}$, $\pi^{rs}(K_0)$ is the minimal actual route travel time from origin r to destination s at time K_0.

4.2.3 Dynamic Shortest-Path Algorithm

Let $G = (V, A)$ be a directed network with node set V and arc set A. Any link $a \in A$ is indexed by (v_i, v_j), or $a = v_i v_j$, where v_i and v_j are the 'from node' and 'to node'. Denote link $a = v_i v_j$ at time interval k as $a(k)$ or $v_i v_j(k)$, node v at time interval k as $v(k)$, the travel time on link $v_i v_j$ at time interval k as $t(v_i v_j, k)$, $k = 1, \cdots, K$. $\bar{t}(v_i v_j, k)$ is its floored value. Denote by $\pi_{v_i}(t)$ the minimum travel time to destination s departing node v_i at time t. The optimality condition of minimum travel times are defined by the following functional form:

$$\pi_{v_i}(t) = \begin{cases} \min_{v_j \in A(v_i)} t(v_i v_j, t) + \pi_{v_j}(t + t(v_i v_j, t)); & v_i \neq s \\ 0 & ; v_i = s \end{cases}$$

When the FIFO condition is valid, the label-correcting algorithm can be generalized to solve the time-dependent minimum paths (dynamic shortest paths) problem with the same time complexity as the static shortest paths problem. Below we introduce an algorithm to find the dynamic shortest path without time-space network. In order to describe our algorithm, the following denotations are introduced.

Denote $O(N) = [N, V - N]$, where $N(\neq \phi) \subseteq V$, and

$$[N, V - N] = \{a = v_i v_j \mid a \in A, v_i \in N, v_j \in V - N\}$$

Further denote $O(n) = [n, V - N], \forall\ n \in N$, where

$$[n, V - N] = \{a = nv_j \mid a \in A, v_j \in V - N\}$$

Denote $\overline{N} = \{n \in N \mid O(n) = \phi\}$. Let $LT = \{n \in N \mid O(n) \neq \phi\}$, or $LT = N - \overline{N}$

Accordingly, an algorithm to find the dynamic shortest path between any node *r* and *s* at time k_0 has been developed and is described as follows:

Step 0: Initialization.

Set $l_r^1 = 0$, $l_r^2 = k_0$, $l_v^1 = \infty$, $l_v^2 = \infty$, $p_v = 0, \forall\ k, v \neq r$.

Set $LT^{(0)} = \{r\}$ and $N^{(0)} = \{r\}$.

Step 1: Set $N^{(0)} = \{r\}$, choose $rv_1 \in O(N^{(0)})$ such that

$t(rv_1, k_0) = \min\{t(rv', k_0) \mid rv' \in O(N^{(0)})\}$.

Label $l_1^1 = t(rv_1, k_0)$, $l_1^2 = k_0 + \bar{t}(rv_1, k_0)$, $p_1 = r$.

Set $N^{(1)} = \{r, v_1\}$ and $LT^{(1)} = N^{(1)} - \overline{N}^{(1)}$. If $N^{(1)} = V$ or $O(N^{(1)}) = \phi$

or $v_1 = s$, stop; otherwise, go to Step 2.

Step 2:

Step 2.1: Search among $LT^{(k)}$ and choose $v_{i0}v_{k+1} \in O(N^{(k)})$ such that

$l_{i0}^1 + t(v_{i0}v_{k+1}, l_{i0}^2) = \min\{l_i^1 + t(v_i v', l_i^2) \mid v_i v' \in O(N^{(k)})\}$

Label $l_{k+1}^1 = l_{i0}^1 + t(v_{i0}v_{k+1}, l_{i0}^2)$, $l_{k+1}^2 = l_{i0}^2 + \bar{t}(v_{i0}v_{k+1}, l_{i0}^2)$, $p_{k+1} = v_{i0}$.

Step 2.2: Set $N^{(k+1)} = \{r, v_1, \cdots, v_k, v_{k+1}\}$. Set $LT^{(k+1)} = N^{(k+1)} - \overline{N}^{(k+1)}$. If $N^{(k+1)} = V$ or

$O(N^{(K+1)}) = \phi$ or $v_{k+1} = s$, stop; otherwise, go to Step 2.1

The above shortest path algorithm is the forward label-correcting method. It finds the dynamic shortest path from a given origin r at time k_0 to any other nodes in the network. A travel cost $t(v_i v_j, k)$ is associated with each link $a = v_i v_j$ at k. Each node v has three labels: l_v^1, l_v^2 and p_v. l_v^1 is the minimum cost from the origin node r to node v along the shortest path at k_0. l_v^2 is the time interval when one departing node r at k_0 and traveling along the shortest path reaches node v. p_v is the node just preceding node v along the shortest path. A sequence list is used to help keep track of the nodes. The list includes all the nodes that have yet to be examined as well as the nodes requiring further examination. In initialization, the algorithm sets all l_v^1 and l_v^2 to infinity and all p_v to zero. And place the origin node r on the sequence list with label l_r^1=0, $l_r^2 = k_0$. Each iteration starts with the selection of a node v_i from the sequence list for examination. All nodes, v_j, that can be reach from v_i by traversing only a single link are tested in the examination process. If the minimum path to v_j through v_i at $l_{v_i}^2$ is shorter than the previous path to v_j, then $l_{v_j}^1$ and $l_{v_j}^2$ are updated. In other words, if $l_{v_i}^1 + t(v_i v_j, l_{v_i}^2) < l_{v_j}^1$, then the current shortest path form the origin node to v_j can be improved by going through node v_i. To reflect this change, the label list is updated by setting $l_{v_j}^1 := l_{v_i}^1 + t(v_i v_j, l_{v_i}^2)$, $l_{v_j}^2 := l_{v_i}^2 + \bar{t}(v_i v_j, l_{v_i}^2)$, the predecessor list is updated by setting $p_{v_j} := v_i$, and the sequence list is updated by adding v_j to it. Once all the nodes v_j (that can be reached from v_i) are tested, the examination of node v_i is complete and it is deleted from the sequence list. The algorithm terminates when the sequence list is empty. The dynamic shortest path from the origin at k_0 to any other node can be found by tracing the predecessor list back to the origin node. The corresponding time interval for each node v on the shortest path is given by l_v^2.

4.2.4 A Numerical Example

Example 4.1

An example is presented below to validate the above model and algorithm. The configuration of the network is shown in Figure 4.6. In the network, each link is assumed as an one-lane street with a length of 0.5 mi. The free flow speed is assumed to be 25 mile/hour. The following linear travel time function is used to enforce FIFO condition: $\tau_a(k) = L_a / s_f + 0.3 \cdot x_a(k)$, where L_a is the length of link a, s_f is free flow speed, $\tau_a(k)$ is link travel time on link a at time k, $x_a(k)$ is number of vehicles on link a at time k. Four O-D pairs are considered. Five 20 s departure time intervals are specified. The OD flows are 10 vehicle units per time interval. The O-D pairs and the time-dependent O-D demand are shown in Table 4.1. In this example, the departure horizon is 5 time increments, and the time increment is 20 seconds.

Table 4-1 O-D pairs and time-dependent O-D demand for example 4.1

O-D	Departure time interval *k*				
	1	2	3	4	5
1-9	10	10	10	10	10
9-1	10	10	10	10	10
3-7	10	10	10	10	10
7-3	10	10	10	10	10

The program of the algorithm was run on a computer with 1.5 GHz frequency processor. The inner iteration (F-W algorithm) convergence test method was set as a prespecified number n. The outer iteration (Relaxation) convergence test method was set as

$$\max\left\{ |\tau_a^{(l)}(k) - \tau_a^{(l-1)}(k)| \,\middle|\, a \in A, k = 1, \cdots, K \right\}$$

where $|\tau_a^{(l)}(k) - \tau_a^{(l-1)}(k)|$ is the actual travel time difference of link a at time k between successive relaxations. The operation of the program is shown in Table 4.2.

Table 4-2 Convergence criterion and computation time for Example 4.1

Inner iteration convergence criterion	Outer iteration convergence criterion	Total relaxations	Total computation time (minute)
n=4	0.002	8	25.8

The assignment horizon K is found to be 21 time increments. Table 4.3a shows the output of $u_a^{rs}(k)$. Table 4.3b shows the output of $v_a^{rs}(k)$. Table 4.3c shows the output of $u_a(k)$. Table 4.3d shows the output of $v_a(k)$. Table 4.3e shows the output of $x_a(k)$. Table 4.3f shows the output of $\tau_a(k)$. Table 4.3g shows the output of $f_p^{rs}(k)$, $c_p^{rs}(k)$ on each path and the arrival time interval for each link on a path. For conciseness, only Table 4.3g is attached to this dissertation.

Table 4-3 The resultant path flow and path travel time for example 4.1

Path number	O	D	k	Path flow	Path time	Links on the path	Arrival time for each link on the path
This table is appendix 1 of this thesis.							

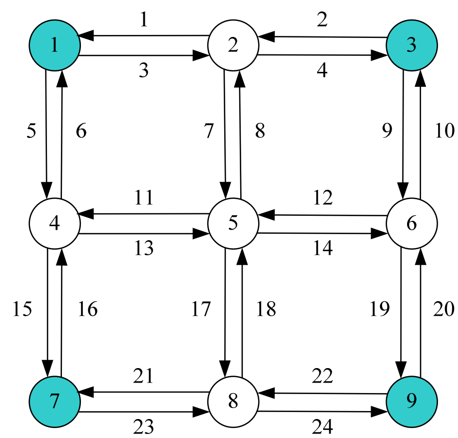

Figure 4-6 Simulation network for example 4.1

The following examples are taken to verify that the solution satisfy the constraints and the dynamic User Optimal conditions.

Path flow conservation constraint (4.7):

$$f^{19}(1) = f_1^{19}(1) + f_2^{19}(1) + f_3^{19}(1) + f_4^{19}(1) + f_5^{19}(1) + f_6^{19}(1)$$

=3.4424+1.865+3.1786+1.1275+0.2891+0.0974

=10

Link inflow conservation constraint (4.8):

$$u_8^{91}(10) + u_8^{73}(10) = 2.1353 + 2.1353 = 4.2706 = u_8(10)$$

Link outflow conservation constraint (4.9):

$$v_8^{91}(14) + v_8^{73}(14) = 2.1353 + 2.1353 = 4.2706 = v_8(14)$$

Node flow conservation constraint (4.10):

$$\sum_{a \in B(6)} v_a^{rs}(k) = \sum_{a \in B(6)} v_a(k) \quad = v_9(8) + v_{14}(8) + v_{20}(8) = 4.7216 + 0 + 5.2784 = 10$$

$$\sum_{a \in A(6)} u_a^{rs}(k) = \sum_{a \in A(6)} u_a(k) = u_{10}(8) + u_{12}(8) + u_{19}(8) = 3.3945 + 3.1047 + 3.5008 = 10$$

Link flow propagation constraint (4.11):

$$u_8^{91}(10) = v_8^{91}(10 + \bar{\tau}_8(10)) = v_8^{91}(14) = 2.1353$$

$$u_8^{73}(10) = v_8^{73}(10 + \bar{\tau}_8(10)) = v_8^{73}(14) = 2.1353$$

Where $\tau_8(10)$=1.2428 minutes. For a time increment of 20 seconds, $\bar{\tau}_8(10)$=4.

The link state equation (4.12b):

$$x_8(10) = x_8(9) + u_8(10) - v_8(10) = 4.3082 + 4.2706 - 0 = 8.5788$$

The actual travel times on the used paths from origin 1 toward destination 9 departing at time increment 1 are as follows:

$$c_1^{19}(1) =$$

$$\tau_5(1) + \tau_{15}(1 + \bar{\tau}_5(1)) + \tau_{23}(1 + \bar{\tau}_5(1) + \bar{\tau}_{15}(1 + \bar{\tau}_5(1))) + \tau_{24}(1 + \bar{\tau}_5(1) + \bar{\tau}_{15}(1 + \bar{\tau}_5(1)) + \bar{\tau}_{23}(1 + \bar{\tau}_5(1) + \bar{\tau}_{15}(1 + \bar{\tau}_5(1))))$$

$$= \tau_5(1) + \tau_{15}(5) + \tau_{23}(9) + \tau_{24}(13)$$

= 1.2232+1.2171+1.2171+1.2242

= 4.8816 minutes

Similarly,

$c_1^{19}(1) = 4.8816$ minutes, $c_2^{19}(1) = 4.8888$ minutes, $c_3^{19}(1) = 4.8841$ minutes,

$c_4^{19}(1) = 4.878$ minutes, $c_5^{19}(1) = 4.8871$ minutes, $c_6^{19}(1) = 4.8798$ minutes

They are nearly equal.

As can be checked in the same way, all the solution output satisfies the constraints and the dynamic user optimal conditions. This verifies the rationale of the above model and solution algorithm.

4.3 Route-based Variational Inequality (VI) DUO Model

In this section, a route-based VI formulation of DUO is introduced. A route-based algorithm is proposed to solve the model. A numerical example showing the application of the algorithm is presented.

4.3.1 Route-based VI Formulation of DUO Model

The route-time-based DUO route choice conditions can be expressed as (Ran and Boyce, 1996b):

$$\eta_p^{rs*}(t) - \pi^{rs*}(t) \geq 0 \quad \forall\ p = p, r, s; \tag{4.34a}$$

$$f_p^{rs*}(t)\left[\eta_p^{rs*}(t) - \pi^{rs*}(t)\right] = 0 \quad \forall\ p = p, r, s; \tag{4.34b}$$

$$f_p^{rs}(t) \geq 0 \quad \forall\ p = p, r, s; \tag{4.34c}$$

The asterisk in the above equations denotes that the flow variables are the optimal solutions under the travel-time-based ideal DUO state.

(4.34a) states that any actual route travel time $\eta_p^{rs*}(t)$ is no less than the minimal actual route (dynamic shortest path) $\pi^{rs*}(t)$ from origin r to destination s at time t. (4.34b) states that if an actual route travel time $\eta_p^{rs*}(t)$ equals $\pi^{rs*}(t)$, $f_p^{rs*}(t) \geq 0$; otherwise, if an actual route travel time $\eta_p^{rs*}(t)$ is larger than $\pi^{rs*}(t)$, $f_p^{rs*}(t) = 0$.

(4.34c) is nonnegative condition for path flow.

Assume the network is empty at $t = 0$, and only travel demands departing within the departure horizon are considered. Our route-based DUO continuous VI model is given as

$$\int_0^T \left\langle \left[\mathbf{\eta}^*(t) - \mathbf{\pi}^*(t)\right], \mathbf{f}(t) - \mathbf{f}^*(t) \right\rangle dt \geq 0 \tag{4.35a}$$

Or in expanded form, as

$$\int_0^T \left\langle \sum_{rs}\sum_{p}\left[\eta_p^{rs^*}(t)-\pi^{rs^*}(t)\right]\cdot\left[f_p^{rs}(t)-f_p^{rs^*}(t)\right]\right\rangle dt \geq 0 \tag{4.35b}$$

Below proving traffic status satisfying (4.35) is in a DUO status or equivalent to (4.34a), (4.34b), (4.34c).

Proof:

(i) **Necessity.** By (4.34a) and (4.34c), $\left[\boldsymbol{\eta}^*(t)-\boldsymbol{\pi}^*(t)\right]\geq 0$, $\mathbf{f}\geq 0$, this implies $\left\langle\left[\boldsymbol{\eta}^*(t)-\boldsymbol{\pi}^*(t)\right],\mathbf{f}(t)\right\rangle\geq 0$. By (4.34b), $\left\langle\left[\boldsymbol{\eta}^*(t)-\boldsymbol{\pi}^*(t)\right],\mathbf{f}^*(t)\right\rangle = 0$. Thus, $\left\langle\left[\boldsymbol{\eta}^*(t)-\boldsymbol{\pi}^*(t)\right],\mathbf{f}(t)-\mathbf{f}^*(t)\right\rangle\geq 0$ holds. Integrating it over $[0,T]$, we have (4.35).

(ii) **Sufficiency.** (4.34a) and (4.34c) hold by definition. Let the optimal solution of (4.35) be $\mathbf{f}^*$. To prove (4.34b) holds for $\mathbf{f}^*$, we first find a feasible solution $\mathbf{f}^{\oplus}$ such that (4.24b) holds, or $\left\langle\left[\boldsymbol{\eta}^*(t)-\boldsymbol{\pi}^*(t)\right],\mathbf{f}^{\oplus}(t)\right\rangle = 0$. Suppose (4.34b) does not hold for $\mathbf{f}^*$, we have $\left\langle\left[\boldsymbol{\eta}^*(t)-\boldsymbol{\pi}^*(t)\right],\mathbf{f}^*(t)\right\rangle > 0$. We further have $\left\langle\left[\boldsymbol{\eta}^*(t)-\boldsymbol{\pi}^*(t)\right],\left[\mathbf{f}^{\oplus}(t)-\mathbf{f}^*(t)\right]\right\rangle < 0$, or $\int_0^T\left\langle\left[\boldsymbol{\eta}^*(t)-\boldsymbol{\pi}^*(t)\right],\left[\mathbf{f}^{\oplus}(t)-\mathbf{f}^*(t)\right]\right\rangle dt < 0$. This contradicts (4.35). Thus (4.44b) holds for $\mathbf{f}^*$.

4.3.2 Solution Algorithms for Route-based VI DUO Model

To solve the DUO problem, the continuous VI formulation is discretized with each time interval being the assignment increment. The estimated actual travel time on each link a is a multiple of the time increment and is fixed at each time increment, i.e.

$$\bar{\tau}_a(k)=i \qquad \text{if} \quad (i-0.5)\Delta t\leq\tau_a(k)\leq(i+0.5)\Delta t \tag{4.36}$$

where i is an integer and $0\leq i\leq K$, Δt is time increment. This round-off method is used only in the flow propagation constraints. The round-off error can be made as small as desired by making the assignment increment smaller.

The route-based DUO discrete-time VI formulation is

$$\left\langle \left[\boldsymbol{\eta}^*(k) - \boldsymbol{\pi}^*(k)\right], \mathbf{f}(k) - \mathbf{f}^*(k) \right\rangle \geq 0 \tag{4.37a}$$

Or in expanded form, as

$$\sum_{rs} \sum_{p} \sum_{k=1}^{K_0} \left[\eta_p^{rs^*}(k) - \pi^{rs^*}(k)\right]\left[f_p^{rs}(k) - f_p^{rs^*}(k)\right] \geq 0 \tag{4.37b}$$

where $\boldsymbol{\eta}, \mathbf{f} \in \Re_+^{|P| \times K_0}$,.

$$\eta_p^{ri}(k) = \eta_p^{r(i-1)}(k) + \tau_a\left[k + \eta_p^{r(i-1)}(k)\right] \quad \forall\ p = p, r, i;\ i = 1, 2, \cdots, s;$$

$$p = (r, 1, 2, \cdots, i, \cdots s), \quad x \in \Theta \tag{4.37c}$$

Θ is the feasible region defined by the following constraints:

Path flow conservation constraint:

$$\sum_p f_p^{rs}(k) = f^{rs}(k)\ \forall\ k, r, s \tag{4.38}$$

Link inflow conservation constraint:

$$\sum_{rs} u_a^{rs}(k) = u_a(k) \quad \forall\ a, k \tag{4.39}$$

Link outflow conservation constraint:

$$\sum_{rs} v_a^{rs}(k) = v_a(k) \quad \forall\ a, k \tag{4.40}$$

Node flow conservation constraint:

$$\sum_{a \in B(j)} v_a^{rs}(k) = \sum_{a \in A(j)} u_a^{rs}(k) \quad \forall\ j \neq r, s; r, s; k \tag{4.41}$$

where $A(j)$ is the set of links after j and $B(j)$ is the set of links before j .

Link flow propagation constraint:

$$u_a^{rs}(k) = v_a^{rs}(k + \tau_a(k)) \quad \forall\ a,\ r, s, k \tag{4.42}$$

The link state equation:

$$x_a(k+1) = x_a(k) + u_a(k) - v_a(k) \quad \forall\, a, k \tag{4.43a}$$

or

$$x_a(k+1) = x_a(k) + u_a(k+1) - v_a(k+1) \quad \forall\, a, k \tag{4.43b}$$

(4.12a) is forward formula, (4.12b) is backward formula.

Path-link flow incidence constraint:

$$u_a^{rs}(n) = \sum_{rs}\sum_{p}\sum_{k=1}^{K_0} f_p^{rs}(k)\delta_{rsa}^{pkn} \quad \forall\, a, n \tag{4.44}$$

where $\delta_{rsa}^{pkn} \in \{0,1\}$ is defined as:

$$\delta_{rsa}^{pkn} = \begin{cases} 1 & \text{if traffic departing origin } r \text{ at any time interval } k \\ & \text{heading for destination } s \text{ on path } p \text{ arrives at link } a \\ & \text{during the } n\text{th time interval.} \\ 0 & \text{otherwise} \end{cases} \tag{4.45}$$

Nonnegative constraint:

$$f_p^{rs}(k) \geq 0, u_a^{rs}(k) \geq 0, \; \forall\, k, r, s, a, p \tag{4.46}$$

Relaxation

At each relaxation, we temporarily fix 1) Actual travel time $\tau_a(k)$ in the link flow propagation constraints as $\bar{\tau}_a(k)$; 2) Actual travel time $\tau_a(k)$ as $\tau_a\left[k + \bar{\pi}^{ri}(k)\right]$ and 3) Minimal travel times $\pi^{rs}(k)$ as $\bar{\pi}^{rs}(k)$ for each origin and destination. At each relaxation, the time-space network is fixed with fixed link flow propagation constraints.

Via relaxation, the VI cost term becomes

$$\eta_p^{rs}(k) - \bar{\pi}^{rs}(k) \tag{4.47a}$$

where

$$\eta_p^{rs}(k) = \sum_{k=1}^{K_0} \sum_a \tau_a(n) \delta_{rsa}^{pkn} \tag{4.47b}$$

$$= \tau_{a_1}(k) + \tau_{a_2}\left(k + \overline{\tau}_{a_1}(k)\right) + \cdots + \tau_{a_{\overline{p}}}\left(k + \overline{\eta}_p^{ra_{(\overline{p}-1)}}(k)\right) \tag{4.47c}$$

where $p = (a_1, a_2, \cdots\cdots a_{\overline{p}})$, a_i is the link number of path p of O-D pair rs at time k . and ,

$$\delta_{rsa}^{pkn} = \begin{cases} 1 & \text{if traffic departing origin } r \text{ at any time interval } k \text{ heading for destination } s \text{ on path } p \text{ arrives at link } a \text{ during the } n\text{th time interval.} \\ 0 & \text{otherwise} \end{cases} \tag{4.48}$$

Optimization Problem

An optimization problem which is equivalent to the discrete VI under relaxation can thus be formulated, as follows:

$$\min_{\mathbf{f}} Z = \sum_{k=1}^{K_0} \sum_{rs} \sum_p \left\{ \int_0^{f_p^{rs}(k)} \left[\eta_p^{rs}(\omega, \mathbf{f}_p^{rs}) - \overline{\pi}^{rs}(k) \right] d\omega \right\} \tag{4.49}$$

in Θ .

where $\mathbf{f}_p^{rs}$ denotes the path flow vector $\mathbf{f}$ without component f_p^{rs} .

The gradient of (4.49) is

$$\frac{\partial Z}{\partial f_p^{rs}(k)} = \eta_p^{rs}(k) - \overline{\pi}^{rs}(k) \tag{4.50}$$

(4.50) is equivalent to the cost term of discrete VI (4.37b) under relaxation. This indicates the above optimization program is equivalent to the discrete VI (4.37).

By (4.47b), we have

$$\eta_p^{rs}\left(f_p^{rs}(k); \mathbf{f}_p^{rs}\right)$$

$$= \tau_{a_1}\left(f_p^{rs}(k);\mathbf{f}_p^{rs}\right) + \tau_{a_2}\left(f_p^{rs}(k);\mathbf{f}_p^{rs}\right) + \cdots + \tau_{a_{\bar{p}}}\left(f_p^{rs}(k);\mathbf{f}_p^{rs}\right)$$

$$= \tau_{a_1}\left(u_{a_1p}^{rs}(k);\mathbf{f}_p^{rs}\right) + \tau_{a_2}\left(u_{a_2p}^{rs}\left(k+\bar{\tau}_{a_1}(k)\right);\mathbf{f}_p^{rs}\right) + \cdots + \tau_{a_{\bar{p}}}\left(u_{a_{\bar{p}}p}^{rs}\left(k+\bar{\eta}_p^{ra_{(\bar{p}-1)}}(k)\right);\mathbf{f}_p^{rs}\right) \quad (4.51)$$

$$= \tau_{a_1}\left(k;u_{a_1p}^{rs};\mathbf{f}_p^{rs}\right) + \tau_{a_2}\left(\left(k+\bar{\tau}_{a_1}(k)\right);u_{a_2p}^{rs};\mathbf{f}_p^{rs}\right) + \cdots + \tau_{a_{\bar{p}}}\left(\left(k+\bar{\eta}_p^{ra_{(\bar{p}-1)}}(k)\right);u_{a_{\bar{p}}p}^{rs};\mathbf{f}_p^{rs}\right)$$

where $u_{a_ip}^{rs}\left(k+\bar{\eta}_p^{ri}(k)\right) = f_p^{rs}(k)$ is the inflow on link a at time interval $\left(k+\bar{\eta}_p^{ri}(k)\right)$ resultant from $f_p^{rs}(k)$, $a_i = (i,j)$.

Since all cross effects (cross path, cross time interval, cross O-D) are fixed in each relaxation, $f_p^{rs}(k)$ is the only variable for each summation term of (4.49). At each relaxation, the VI formulation of DUO problem was transformed into a series of static user equilibrium traffic assignment problems over the time-space network of the relaxation, which can be solved by Frank-Wolfe algorithm. Call the relaxation as outer iteration and solving static user equilibrium traffic assignment problems over the time-space network of the relaxation as inner iteration.

At the mth iteration of the inner iteration (Frank-Wolfe algorithm), the descending direction of nonlinear programming (4.49) can be found by solving the following linear program:

$$\min_g \ \hat{Z} = g^T \nabla_g Z^{(m)} \quad (4.51)$$

in Θ. where g is subproblem variable, $\nabla_g Z^{(m)}$ is gradient of Z with respect to $\mathbf{f}$ evaluated at $\mathbf{f}^{(m)}$.

Program (4.31)is equivalent to:

$$\min_g \ \hat{Z} = \sum_{k=1}^{K_0} \sum_{rs} \sum_{p} \left[t_p^{rs(m)}(k) g_p^{rs}(k)\right] \quad (4.52)$$

in Θ, where

$$t_p^{rs(m)}(k) = \frac{\partial Z\left(\mathbf{f}^{(m)}\right)}{\partial f_p^{rs}(k)} = \eta_p^{rs}(k) - \overline{\pi}^{rs}(k) \qquad (4.53)$$

Program (4.52) can be decomposed by origin-destination pair. The resulting subproblem for O-D pair rs is:

$$\min_{g} \ \hat{Z} = \sum_{k=1}^{K_0} \sum_{p} \left[t_p^{rs(m)}(k) g_p^{rs}(k) \right] \qquad (4.54)$$

in Θ.

Program (4.54) can be further decomposed by each O-D flow $f^{rs}(k), k = 1, \cdots, K_0$. The resulting subproblem for O-D flow $f^{rs}(k)$ is:

$$\min_{g} \ \hat{Z} = \sum_{p} \left[t_p^{rs(m)}(k) g_p^{rs}(k) \right] \qquad (4.55)$$

in Θ.

Program (4.55) can be viewed as a shortest path problem over the time-space network of the relaxation. The minimum of (4.55) is found by assigning $f^{rs}(k)$ to the actual minimum cost route (dynamic shortest path) of O-D pair rs at time interval k.

The step size along the descending direction can be found by solving the following one-dimensional search problem:

$$\min_{0 \le \alpha \le 1} Z = \sum_{k=1}^{K_0} \sum_{rs} \sum_{p} \left\{ \int_0^{f_p^{rs(m)}(k) + \alpha\left(g_p^{rs(m)}(k) - f_p^{rs(m)}(k)\right)} \left[\eta_p^{rs}\left(\omega, \mathbf{f}_p^{rs}\right) - \overline{\pi}^{rs}(k) \right] d\omega \right\} \qquad (4.56)$$

Since $\overline{\pi}^{rs}(k)$ is fixed for each O-D pair at each relaxation, it can be dropped from (4.56), the resultant problem is

$$\min_{0 \le \alpha \le 1} Z = \sum_{k=1}^{K_0} \sum_{rs} \sum_{p} \left\{ \int_0^{f_p^{rs(m)}(k) + \alpha\left(g_p^{rs(m)}(k) - f_p^{rs(m)}(k)\right)} \eta_p^{rs}\left(\omega, \mathbf{f}_p^{rs}\right) d\omega \right\} \qquad (4.57)$$

By using (4.51), (4.57) can be rewritten as

$$\min_{0\le\alpha\le 1} Z=\sum_{k=1}^{K_0}\sum_{rs}\sum_{p}\left\{\int_0^{f_p^{rs(m)}(k)+\alpha\left(g_p^{rs(m)}(k)-f_p^{rs(m)}(k)\right)}\left[\sum_{a=a_1}^{a_{\bar{p}}}\tau_a\left(n;\omega;\mathbf{f}_p^{rs}\right)\delta_{rsa}^{pkn}\right]d\omega\right\} \tag{4.58}$$

where $p=\left(a_1,a_2,\cdots\cdots a_{\bar{p}}\right)$, and

$$\sum_{a=a_1}^{a_{\bar{p}}}\tau_a\left(n;\omega;\mathbf{f}_p^{rs}\right)\delta_{rsa}^{pkn}$$

$$=\tau_{a_1}\left(k;\omega;\mathbf{f}_p^{rs}\right)+\tau_{a_2}\left(\left(k+\bar{\tau}_{a_1}(k)\right);\omega;\mathbf{f}_p^{rs}\right)+\cdots+\tau_{a_{\bar{p}}}\left(\left(k+\bar{\eta}_p^{ra_{(\bar{p}-1)}}(k)\right);\omega;\mathbf{f}_p^{rs}\right) \tag{4.59}$$

In solving (4.56), it is not necessary to enumerate all the paths of each O-D pair on the network. With the technique of column generation, (4.56) can be solved on the path set $P^{(m)}$ defined by

$$P^{(m)}=P^{(m-1)}\cup\bar{P}^{(m)} \tag{4.60}$$

where $P^{(m-1)}$ is the path set at the $(m-1)$th iteration, $\bar{P}^{(m)}$ is the path set composed of dynamic shortest paths of each O-D pair at the mth iteration. $P^{(1)}$ is defined as $\bar{P}^{(1)}$.

After the optimal step size α^m is found, the solution at the inner iteration can be updated as

$$f_p^{rs(m+1)}(k)=f_p^{rs(m)}(k)+\alpha^m\left[g_p^{rs(m)}(k)-f_p^{rs(m)}(k)\right]\quad\forall p,r,s,k=1,\cdots,K_0 \tag{4.61}$$

Algorithm

The algorithm for solving the ideal route-based DUO route choice model is summarized as follows.

Step 0: Outer Initialization.

Compute $k_{\max}=\max_{\forall rs}\left\{\pi^{rs}\right\}$, where π^{rs} is the static minimum travel time of O-D rs. Set $K^{'}=K_0+C\cdot\left[k_{\max}\right]_+$. Set $\hat{\tau}_a^{(0)}(k)=\tau_a[0]$, $\forall\, a\in A,\quad k=1,\cdots,K'$. Find an initial feasible solution $\left[f_p^{rs(0)}(k)\right]$. Set outer iteration counter $l=0$. Set an outer iteration convergence criterion ε_{out}.

Step 1: Relaxation.

Find a new estimation of actual link travel times: $\hat{\tau}_a^{(l)}(k)=\tau_a\left[x_a^*(k)\right]$, find $\overline{\tau}_a^{(l)}(k)\ \forall\ a\in A$, $k=1,\cdots,K'$, where * denotes the solution obtained from the most recent inner iteration or from outer initialization. Find $\delta_{a'}^{k(l)}(t)$ and $\delta_{a''}^{k(l)}(t)$.

Step 2: Inner Iteration

Step 2.0: Inner Initialization. Compute and reset the inner initial feasible solution to be consistent with the flow propagation constrain at the current relaxation. Set an inner iteration counter $m=1$ (or a convergence criterion ε_{in}).

Step 2.1: Direction Finding. Based on $\tau_a^{(m)}(k)$, search for dynamic shortest routes for all OD pairs without time-space expansions. Perform an all-or-nothing assignment following the link flow propagation constrain, yielding sub-problem solution $g_p^{rs(m)}(k)$.

Step 2.2: Line Search. Solve the one-dimensional search problem (4.56) using a line search procedure such as the bisection method and find the optimal step size $\alpha^{(m)}$.

Step 2. 3: Move. Find a new solution $f_p^{rs(m+1)}(k)$ by (4.61).

Step 2. 4: Convergence Test for Inner Iteration.

If $\sqrt{\sum_{rs}\sum_{k}^{K_0}\left(f_p^{rs(m+1)}(k)-f_p^{rs(m)}(k)\right)}\Big/\sum_{rs}\sum_{k}^{K_0}f_p^{rs(m)}(k)\ >\varepsilon$, set $m=m+1$, go to Step 2.1.; otherwise, set $\hat{f}_a^{rs(l)}(k)=f_p^{rs(m+1)}(k)$, $\hat{x}_a^{(l)}(k)=x_a^{(m+1)}(k)$, go to Step 3.

Step 3: Convergence Test for Outer Iteration. If $\hat{\tau}_a^{(l)}(k)\cong\hat{\tau}_a^{(l-1)}(k)$, stop. The current Solution $u_a^{rs}(k)$, $v_a^{rs}(k)$, $x_a^{rs}(k)$ is in a near optimal state; otherwise, set $l=l+1$ and go to

Step 1.

4.3.3 A Numerical Example

Example 4.2

An example is presented below to validate the above model and algorithm. The problem is the same as in Example 4.1. The program of the algorithm was run on a computer with 1.5GHz frequency processor. The inner iteration (F-W algorithm) convergence test method was set as a prespecified number n. The outer iteration (Relaxation) convergence test method was set as

$$\max\{\| \tau_a^{(l)}(k) - \tau_a^{(l-1)}(k) | \, | a \in A, k = 1, \cdots, K\}$$

where $| \tau_a^{(l)}(k) - \tau_a^{(l-1)}(k) |$ is the actual travel time difference of link a at time k between successive relaxations. The operation of the program is shown in Table 4.4.

Table 4-4 Convergence criterion and computation time for example 4.2

Inner iteration convergence criterion	Outer iteration convergence criterion	Total relaxations	Total computation time (minute)
n=4	0.002	8	26.4

Table 4-5 The resultant path flow and path travel time for example 4.2

Path number	O	D	k	Path flow	Path time	Links on the path	Arrival time for each link on the path
This table is appendix 2 of this thesis.							

The assignment horizon K is found to be 21 time increments. Table 4.5a shows the output of $u_a^{rs}(k)$. Table 4.5b shows the output of $v_a^{rs}(k)$. Table 4.5c shows the output of $u_a(k)$. Table 4.5d shows the output of $v_a(k)$. Table 4.5e shows the output of $x_a(k)$. Table 4.5f shows the output of $\tau_a(k)$. Table 4.5g shows the output of $f_p^{rs}(k)$, $c_p^{rs}(k)$, links on each path and the arrival time interval for each link on a path. For conciseness, only Table 4.5g is attached to this dissertation.

The following examples are taken to verify that the solution satisfy the constraints and the dynamic User Optimal conditions.

Path flow conservation constraint (4.38):

$$f^{19}(1) = f_1^{19}(1) + f_2^{19}(1) + f_3^{19}(1) + f_4^{19}(1)$$

$$=3.3372+1.7504+3.2694+1.643$$

$$=10$$

Link inflow conservation constraint (4.39):

$$u_8^{91}(10) + u_8^{73}(10) = 1.7504+1.7504=3.5008= u_8(10)$$

Link outflow conservation constraint (4.40):

$$v_8^{91}(14) + v_8^{73}(14) = 1.7504+1.7504=3.5008= v_8(14)$$

Node flow conservation constraint (4.41):

$$\sum_{a\in B(6)} v_a^{rs}(k) = \sum_{a\in B(6)} v_a(k) = v_9(8) + v_{14}(8) + v_{20}(8) = 4.9802+0+5.0198=10$$

$$\sum_{a\in A(6)} u_a^{rs}(k) = \sum_{a\in A(6)} u_a(k) = u_{10}(8) + u_{12}(8) + u_{19}(8) = 3.2694+3.3934+3.3372=10$$

Link flow propagation constraint (4.42):

$$u_8^{91}(10) = v_8^{91}(10 + \bar{\tau}_8(10)) = v_8^{91}(14) = 1.7504$$

$$u_8^{73}(10) = v_8^{73}(10 + \bar{\tau}_8(10)) = v_8^{73}(14) = 1.7504$$

Where $\tau_8(10) = 1.2349$ minutes. For a time increment of 20 seconds, $\bar{\tau}_8(10) = 4$.

The link state equation (4.43b):

$$x_8(10) = x_8(9) + u_8(10) - v_8(10) = 3.5008 + 3.5008 - 0 = 7.0016$$

The actual travel times on the used paths from origin 1 toward destination 9 departing at time increment 1 are as follows:

$$c_1^{19}(1) =$$

$$\tau_5(1) + \tau_{15}(1 + \bar{\tau}_5(1)) + \tau_{23}(1 + \bar{\tau}_5(1) + \bar{\tau}_{15}(1 + \bar{\tau}_5(1))) + \tau_{24}(1 + \bar{\tau}_5(1) + \bar{\tau}_{15}(1 + \bar{\tau}_5(1)) + \bar{\tau}_{23}(1 + \bar{\tau}_5(1) + \bar{\tau}_{15}(1 + \bar{\tau}_5(1))))$$

$$= \tau_5(1) + \tau_{15}(5) + \tau_{23}(9) + \tau_{24}(13)$$

$$= 1.2248 + 1.2166 + 1.2166 + 1.2248$$

$$= 4.8828 \text{ minutes}$$

Similarly, $c_2^{19}(1) = 4.8852$ minutes, $c_3^{19}(1) = 4.8831$ minutes, $c_4^{19}(1) = 4.8821$ minutes. They are nearly equal.

As can be checked in the same way, all the solution output satisfies the constraints and the dynamic user optimal conditions. This verifies the rationale of the above model and solution algorithm.

Chapter 5: Stochastic Dynamic User Optimal Route Choice (SDUO) Problem

In this chapter, the stochastic dynamic user optimal (SDUO) route choice problem is studied. At SDUO state, the perceived travel times experienced by travelers of the same O-D pair departing at the same time are equal and minimal. The randomness on dynamic transportation networks are given in Section 5.1. Section 5.2 presents the link-based variation inequality SDUO model and development of the algorithm. Section 5.3 presents the route-based variation inequality SDUO model and development of the algorithm.

5.1 Randomness on Dynamic Transportation Networks

The deterministic dynamic user optimal route choice model assumes that dynamic route guidance systems are deployed and all drivers have perfect information of the network traffic conditions and choose their routes based on dynamic route guidance information. However, some drivers may not rely on the information provided by the route guidance system to choose their route. Furthermore, drivers without navigation systems do not have perfect information on the road network and must use their own experience and perception of the current traffic conditions to make travel decisions (Ran and Boyce, 1996b). To model dynamic route choice under imperfect information, stochastic dynamic route choice models is proposed. A stochastic dynamic user optimal (SDUO) route choice model is a stochastic generalization of ideal deterministic DUO route choice model.

The randomness in dynamic transportation networks may include:

1) randomness in traveler's perceptions of travel times

Because of the perception error of each driver and the difference in the perception among drivers, there is variation in drivers' perceptions of travel time of a route. The perceived travel time is a random variable with a certain probability distribution among the population of drivers. Drivers make decision on route choice based on the perceived travel times and choose their perceived actual minimal routes, which are not necessarily the real actual minimal routes.

2) randomness of time-dependent origin-destination demand

The variation of O-D demand may arise due to the errors in the estimation and forecasting of O-D demand. The day-to-day variation of O-D demands may be another stochastic factor.

3) randomness of link traffic flow and link travel times

In reality, link capacity, link traffic flow and link travel time may be affected by random factors such as the degradation of a link's capacity due to double parking or vehicle's breakdown or road accidents. The road networks thus have random link traffic flow and random link travel time. Such networks are also called stochastic network.

In this study, only the randomness in traveler's perceptions of travel times is included. When the drivers' perception error on route travel time follows Gumbel distribution, a logit-type model can be formulated. When the drivers' perception error on route travel time follows Multinomial normal distribution, a probit-type model can be formulated. Since the logit-type route choice model has the limitation of IID, only probit-type route choice model is considered in this study.

Define the satisfaction function as follows (Sheffi, 1985; Ran, 2002b):

$$S^{rs}(t) = E\left[\min_{p} C_p^{rs}(t)\right] \; \forall r, s, p \tag{5.1}$$

where $C_p^{rs}(t)$ is the perceived actual travel time of route p of O-D pair *rs* at time t that a traveler derives from a set of route travel times. The satisfaction function $S^{rs}(t)$ captures the expected minimum travel time of route p of O-D pair *rs* at time t. Its partial derivative with respect to the mean actual travel time for route p of O-D pair *rs* for flows departing from origin r at time t, $\eta_p^{rs}(t)$, equals the proportion of flows of O-D pair *rs* that follow route p at time t:

$$\frac{\partial S^{rs}(t)}{\partial \eta_p^{rs}(t)} = P_p^{rs}(t); \;\; \forall r, s, p \tag{5.2}$$

The SDUO route choice condition is defined as:

$$f_p^{rs}(t) = f^{rs}(t) P_p^{rs}(t); \;\; \forall r, s, p \tag{5.3}$$

It states that the departure flows from r to s on route p at time t equals the total departure flows from r to s at time t times the proportion of flows of O-D pair *rs* using route p at time t.

5.2 Link-based Variational Inequality (VI) SDUO Model

In this section, a link-based VI formulation of SDUO is proposed. The relaxation method is used to solve it. At each relaxation, a link-based nonlinear program is constructed. A solution algorithm which avoids time-space network expansion is proposed.

5.2.1 Link-based VI Formulation of SDUO

Assume the mean actual link travel time $\tau_a(t)$ increases with link inflow $u_a^{rs}(t)$, or

$$\frac{d\tau_a(t)}{du_a^{rs}(t)} > 0\,; \quad \forall a \in A, r, s \tag{5.4}$$

Define a cost term $F_a^{rs}(t)$ for each link a corresponding O-D rs as follows:

$$F_a^{rs}(t) = \left[u_a^{rs}(t) - \int_0^t f^{rs}(\omega) \sum_p P_p^{rs}(\omega) \delta_{rsa}^{p\omega t}\, d\omega \right] \frac{d\tau_a(t)}{du_a^{rs}(t)} \tag{5.5}$$

where

$$\delta_{rsa}^{p\omega t} = \begin{cases} 1 & \text{if traffic departing origin } r \text{ at time } \omega \\ & \text{heading for destination } s \text{ on path } p \\ & \text{arrives at link } a \text{ at time } t \\ 0 & \text{otherwise} \end{cases} \tag{5.6}$$

Below showing the SDUO route choice condition (5.3) holds if and only if $F_a^{rs}(t)=0$, $\forall a \in A, r, s$.

Proof: Given $F_a^{rs}(t)=0$, $\forall a \in A, r, s$. Since $d\tau_a(t)/du_a^{rs}(t) > 0$, it follows that

$$u_a^{rs}(t) = \int_0^t f^{rs}(\omega) \sum_p P_p^{rs}(\omega) \delta_{rsa}^{p\omega t}\, d\omega \tag{5.7a}$$

Or

$$u_a^{rs}(t) = \sum_p \int_0^t f^{rs}(\omega) P_p^{rs}(\omega) \delta_{rsa}^{p\omega t}\, d\omega \tag{5.7b}$$

Since

$$u_a^{rs}(t) = \sum_p \int_0^t f_p^{rs}(\omega)\delta_{rsa}^{p\omega t} d\omega \qquad (5.8)$$

Thus we have:

$$\sum_p \int_0^t f_p^{rs}(\omega)\delta_{rsa}^{p\omega t} d\omega = \sum_p \int_0^t f^{rs}(\omega)P_p^{rs}(\omega)\delta_{rsa}^{p\omega t} d\omega \qquad (5.9a)$$

or

$$f_p^{rs}(t) = f^{rs}(t)P_p^{rs}(t) \qquad (5.9b)$$

This is SDUO condition. So $F_a^{rs}(t)$ =0, $\forall a \in A$, r, s contains the SDUO route choice conditions (5.3). Similarly, we can show SDUO route choice conditions (5.3) implies $F_a^{rs}(t)$=0, $\forall a \in A, r, s$.

Assume the network is empty at $t = 0$, and only travel demands departing within the departure horizon are considered. The link-based SDUO continuous VI model can be expressed as

$$\int_0^T \left\langle \mathbf{F}^*(t), \overline{\mathbf{u}}(t) - \overline{\mathbf{u}}^*(t) \right\rangle dt \geq 0 \qquad (5.10a)$$

where $\mathbf{F}^* \in \Re_+^{|R\times S|\times|A|}$, $\overline{\mathbf{u}} \in \Re_+^{|R\times S|\times|A|}$, $|N|$, $|A|$, and $|R \times S|$ are the cardinalities of the set nodes, links and O-D pairs, etc. Asterisk means the optimal inflow at SDUO state.

(5.10a) can be written in the expanded form as:

$$\int_0^T \sum_{rs} \sum_p F_a^{rs*}(t)\left[u_a^{rs}(t) - u_a^{rs*}(t)\right] dt \geq 0 \qquad (5.10b)$$

Theorem: The dynamic traffic flow pattern satisfying network constraint set (chapter 4) is an ideal SDUO route choice state if and only if it satisfies the variational inequality problem (5.10).

Proof: The following proof is similar to the procedure as in Nagurney (1993). If $F(\mathbf{u}^*(t)) = 0$,

$F\left(\mathbf{u}^*(t)\right)=\left[\cdots, F_a\left(\mathbf{u}^*(t)\right), \cdots\right]^T$ then inequality (5.10) holds with equality. Conversely, if $\mathbf{u}^*(t)$ satisfies (5.10), let $\mathbf{u}(t)=\mathbf{u}^*(t)-F_a\left(\mathbf{u}^*(t)\right)$, which implies that

$$\int_0^T F\left(\mathbf{u}^*(t)\right)^T \cdot \left(-F\left(\mathbf{u}^*(t)\right)\right) dt \geq 0,$$

or $-\int_0^T \left\|F\left(\mathbf{u}^*(t)\right)\right\|^2 dt \geq 0$, therefore, $F\left(\mathbf{u}^*(t)\right)=0$.

5.2.2 Solution Algorithm for Link-based VI SDUO Model

Discrete Link-based VI SDUO Model

To solve the SDUO problem, the continuous VI formulation is discretized with each time interval being time increment. The estimated actual travel time on each time-space link a is a multiple of the time increment and is fixed at each time increment, i.e.,

$$\bar{\tau}_a(k)=i \qquad \text{if} \quad (i-0.5)\Delta t \leq \tau_a(k) \leq (i+0.5)\Delta t \tag{5.11}$$

where i is an integer and $0 \leq i \leq K$, Δt is time increment.. This round-off method is used only in the flow propagation constraints. The round-off error can be made as small as desired by making the time increment smaller (Ran and Boyce, 1996b).

The link-based SDUO discrete-time VI formulation is

$$\left\langle \mathbf{F}^*(k), \mathbf{u}(k)-\mathbf{u}^*(k) \right\rangle \geq 0 \tag{5.12a}$$

or in expanded form as

$$\sum_{k=1}^{K_0} \sum_{rs} \sum_{a} F_a^{rs*}\left[k+\pi^{ri*}(k)\right] \left\{ u_a^{rs}\left[k+\pi^{ri*}(k)\right] - u_a^{rs*}\left[k+\pi^{ri*}(k)\right] \right\} \geq 0 \tag{5.12b}$$

where $F \in \Re_+^{|R \times S| \times |A| \times K_0}$, $\mathbf{u} \in \Theta$, and

$$F_a^{rs*}\left[k+\pi^{ri*}(k)\right]=\left[u_a^{rs*}\left[k+\pi^{ri*}(k)\right]-f^{rs}(k)\sum_p P_p^{rs}(k)\delta_{rsa}^{pk\left[k+\pi^{ri*}(k)\right]}\right]\frac{d\tau_a}{du_a^{rs}}\bigg|_{\left[k+\pi^{ri*}(k)\right]} \quad (5.12c)$$

in Θ. $a=(i,j)$. Θ is the feasible region defined by the following constraints:

Path flow conservation constraint:

$$\sum_p f_p^{rs}(k)=f^{rs}(k)\ \forall\, k,r,s \quad (5.13)$$

Link inflow conservation constraint:

$$\sum_{rs} u_a^{rs}(k)=u_a(k)\ \ \forall\, a,k \quad (5.14)$$

Link outflow conservation constraint:

$$\sum_{rs} v_a^{rs}(k)=v_a(k)\ \ \forall\, a,k \quad (5.15)$$

Node flow conservation constraint:

$$\sum_{a\in B(j)} v_a^{rs}(k)=\sum_{a\in A(j)} u_a^{rs}(k) \quad \forall\, j\neq r,s;r,s;k \quad (5.16)$$

where $A(j)$ is the set of links after j and $B(j)$ is the set of links before j.

Link flow propagation constraint:

$$u_a^{rs}(k)=v_a^{rs}\left(k+\tau_a(k)\right) \quad \forall\, a,\ r,s,k \quad (5.17)$$

The link state equation:

$$x_a(k+1)=x_a(k)+u_a(k)-v_a(k) \quad \forall\, a,k \quad (5.18a)$$

or

$$x_a(k+1)=x_a(k)+u_a(k+1)-v_a(k+1) \quad \forall\, a,k \quad (5.18b)$$

(5.18a) is forward formula, (5.18b) is backward formula.

Path-link flow incidence constraint:

$$u_a^{rs}(n)=\sum_{rs}\sum_p\sum_{k=1}^{K_0} f_p^{rs}(k)\delta_{rsa}^{pkn} \quad \forall\, a,n \quad (5.19)$$

where $\delta_{rsa}^{pkn} \in \{0,1\}$ is defined as:

$$\delta_{rsa}^{pkn} = \begin{cases} 1 & \text{if traffic departing origin } r \text{ at any time interval } k \\ & \text{heading for destination } s \text{ on path } p \text{ arrives at link } a \\ & \text{during the } n\text{th time interval.} \\ 0 & \text{otherwise} \end{cases} \tag{5.20}$$

Nonnegative constraint:

$$f_p^{rs}(k) \geq 0, u_a^{rs}(k) \geq 0, \;\forall\, k, r, s, a, p \tag{5.21}$$

Relaxation

At each relaxation, I temporarily fix (Ran and Boyce, 1996b): 1) Actual travel time $\tau_a(k)$ in the link flow propagation constraints as $\bar{\tau}_a(k)$ and corresponding actual route travel time $\eta_p^{rs}(k)$ as $\bar{\eta}_p^{rs}(k)$; 2)Actual travel time in the VI cost term $F_a^{rs*}(k)$ as $\tau_a\left[k + \bar{\pi}^{ri*}(k)\right]$ and 3) Minimal travel times $\pi^{ri*}(k)$ as $\bar{\pi}^{ri*}(k)$ for each link and each origin. Via relaxation, the auxiliary VI cost term for each link a at each time interval n becomes

$$F_a^{rs*}(n) = \left[u_a^{rs*}(n) - \sum_k f^{rs}(k) \sum_p P_p^{rs}(k) \delta_{rsa}^{pkn}\right] \frac{d\tau_a(n)}{du_a^{rs}(n)} \tag{5.22}$$

Optimization Problem

A nonlinear program which is equivalent to the discrete VI under relaxation can thus be formulated, as follows:

$$\min_u Z(\mathbf{u}) = \sum_{k=1}^{K_0} \sum_{rs} \left[- f^{rs}(k) S^{rs}(k)\right] + Z_1 - Z_2 \tag{5.23a}$$

where

$$Z_1 = \sum_{k=1}^{K_0} \sum_{rs} \sum_{a} u_a^{rs}\left(k+\bar{\pi}^{ri}(k)\right)\tau_a\left(k+\bar{\pi}^{ri}(k)\right) \tag{5.23b}$$

$$Z_2 = \sum_{k=1}^{K_0} \sum_{rs} \sum_{a} \left\{ \int_0^{u_a^{rs}\left(k+\bar{\pi}^{ri}(k)\right)} \tau_a\left(\omega + X_a^{rs}\left(k+\bar{\pi}^{ri}(k)\right)\right) d\omega \right\} \tag{5.23c}$$

$S^{rs}(k)$ is satisfaction function defined as (5.1).

Next, I demonstrate that the gradient of the objective function is equivalent to auxiliary cost term of the VI, i.e.,

$$\nabla_u Z(\mathbf{u}) = \mathbf{F} \tag{5.24}$$

where $\mathbf{F} = [\cdots, F_a(k), \cdots]$.

The derivative of the first term with respect to $u_a^{rs}(n)$ can be calculated as follows:

$$\frac{\partial}{\partial u_a^{rs}(n)} \left\{ -\sum_{rs} \sum_{k} f^{rs}(k) S^{rs}\left[\mathbf{c}_k^{\mathbf{rs}}(\mathbf{u})\right] \right\} = -\sum_{rs} \sum_{k} f^{rs}(k) \sum_{p} \frac{\partial S^{rs}\left(\mathbf{c}_k^{\mathbf{rs}}\right)}{\partial c_p^{rs}(k)} \frac{\partial c_p^{rs}(k)}{\partial u_a^{rs}(n)} \tag{5.25a}$$

where $n = k + \bar{\pi}^{ri*}(k)$ and

$$\frac{\partial S^{rs}\left(\mathbf{c}_k^{\mathbf{rs}}\right)}{\partial c_p^{rs}(k)} = P_p^{rs}(k) \tag{5.25b}$$

$$\frac{\partial c_p^{rs}(k)}{\partial u_a^{rs}(n)} = \frac{\partial}{\partial u_a^{rs}(n)} \left[\sum_{b} \sum_{k} \tau_b(k) \delta_{rsa}^{pkn} \right] = \frac{d\tau_a(n)}{du_a^{rs}(n)} \delta_{rsa}^{pkn} \tag{5.25c}$$

$$\frac{\partial}{\partial u_a^{rs}(n)} \left[-\sum_{rs} \sum_{k} f^{rs}(k) S^{rs}(\cdot) \right] = -\sum_{k} f^{rs}(k) \sum_{p} P_p^{rs}(k) \frac{d\tau_a(n)}{du_a^{rs}(n)} \delta_{rsa}^{pkn} \tag{5.25d}$$

The derivative of the last term in the objective function is

$$\frac{\partial Z_1}{\partial u_a^{rs}(n)} = \tau_a\left(k+\bar{\pi}^{ri}(k)\right) + u_a^{rs}(n) \frac{d\tau_a(n)}{du_a^{rs}(n)} \tag{5.26}$$

$$\frac{\partial Z_2}{\partial u_a^{rs}(n)} = \tau_a\left(k+\bar{\pi}^{ri}(k)\right) \tag{5.27}$$

The derivative of the objective function of the nonlinear program with respect to

$u_a^{rs}(n)$ can be given by combining equation (5.25d), (5.26), (5.27), as follows,

$$\frac{\partial Z(\mathbf{u})}{\partial u_a^{rs}(n)} = \left[u_a^{rs*}(n) - \sum_k f^{rs}(k) \sum_p P_p^{rs}(k) \delta_{rsa}^{pkn} \right] \frac{d\tau_a(n)}{du_a^{rs}(n)} \tag{5.28}$$

(5.28) is equivalent to the cost term of discrete VI (5.12b) under relaxation. This indicates the optimization program (5.23) is equivalent to the discrete VI (5.12).

At each relaxation, the VI formulation of SDUO problem was transformed into a series of static stochastic user equilibrium traffic assignment problems over the time-space network of the relaxation, which can be solved by Method of Successive Average (MSA). Call the relaxation as outer iteration and solving static stochastic user equilibrium traffic assignment problems over the time-space network of the relaxation as inner iteration.

The negative gradient of the objective function (5.23) can be written as

$$\mathbf{d} = -\nabla Z(\mathbf{u}) = \left[\sum_k f^{rs}(k) \mathbf{P}^{rs}(k) (\boldsymbol{\Delta}^{rs})^T - \mathbf{u} \right] \nabla_u \boldsymbol{\tau} \tag{5.29a}$$

where $\mathbf{P}^{rs}(k) = (\cdots, P_p^{rs}(k), \cdots)$ is the route choice probability for O-D rs at time k, $\boldsymbol{\Delta}^{rs}$ is the time-space link path incidence matrix with element δ_{rsa}^{pkn}.

Dropping $\nabla_u \boldsymbol{\tau}$ from (5.29a), we obtain a simpler direction as follows

$$\mathbf{d} = \left[\sum_{rs} \sum_k f^{rs}(k) \mathbf{P}^{rs}(k) (\boldsymbol{\Delta}^{rs})^T - \mathbf{u} \right] \tag{5.29b}$$

It is easy to show that $\nabla Z(\mathbf{u}) \cdot (\mathbf{d})^T < 0$, so (5.29b) is a descent direction of the objective function.

The component of $\mathbf{d}$ (5.29b) is

$$d_a^{rs}(k) = \sum_k f^{rs}(k) \sum_p P_p^{rs}(k) \delta_{rsa}^{pkn} - u_a^{rs}(n) \tag{5.29c}$$

Let $$y_a^{rs}(k) = \sum_k f^{rs}(k) \sum_p P_p^{rs}(k) \delta_{rsa}^{pkn} \tag{5.30}$$

$\mathbf{y}$ can be obtained by performing probit-based stochastic network loading on the original network without time-space network expansion.

The probit-based stochastic network loading procedure is developed as follows:

Step 1: Initialization. Set $l := 1$

Step 2: Sampling. Sample $\tau_a^{(l)}(k)$ from $\tau_a(k) \sim N(\tau_a(k), \rho\tau_a(k))$ for each link a at time interval k.

Step 3: All-or-nothing assignment. Find dynamic shortest path of OD pair $r-s$ at time interval k based on $\{\tau_a^{(l)}(k)\}$, assign $\{f^{rs}(k)\}$ to the dynamic shortest path. This step yields the set of link flows $Y_a^{(l)}(k)$.

Step 4: Flow averaging. Let $y_a^{(l)}(k) = \left[(l-1)y_a^{(l-1)}(k) + Y_a^{(l)}(k)\right]/l$, $\forall a, k$.

Step 5: Stopping test

(a) Let $$\sigma_a^{(l)}(k) = \sqrt{\frac{1}{l(l-1)} \sum_{m=1}^{l} \left[Y_a^{(m)}(k) - y_a^{(l)}(k)\right]^2} \qquad \forall a, k$$

or

$$\sigma_a^{(l)}(k) = \sqrt{\frac{1}{l(l-1)} \left[Y_a^{(l)}(k) - y_a^{(l)}(k) + (l-1)(l-2)\sigma_a^{(l)}(k)^2\right]}$$

(b) If max $\{\sigma_a^{(l)}(k) / y_a^{(l)}(k)\} \le \varepsilon$, stop. The solution is $\{y_a^{(l)}(k)\}$. Otherwise, set $l := l+1$ and go to Step 2.

At the mth iteration of the inner iteration (MSA), the descending direction of nonlinear programming (5.23) can be found by performing the above probit-based stochastic network loading.

The step size along the descending direction is simply $1/m$. The solution at the inner iteration can be updated as

$$u_a^{rs(m+1)}(n) = u_a^{rs(m)}(n) + (1/m)\left[y_a^{rs(m)}(n) - u_a^{rs(m)}(n)\right] \quad \forall a, r, s, n \tag{5.31}$$

Algorithm

Based on the rational analysis as described above, the algorithm for solving the ideal SDUO route choice model (5.12) is developed and summarized as follows.

Step 0: Outer Initialization.

Compute $k_{\max} = \max_{\forall rs}\{\pi^{rs}\}$, where π^{rs} is the static minimum travel time of O-D rs. Set $K^{'} = K_0 + C \cdot [k_{\max}]_+$. Set $\hat{\tau}_a^{(0)}(k) = \tau_a[0]$, $\forall\, a \in A,\quad k = 1, \cdots, K'$. Find an initial feasible solution $\left[u_a^{rs(0)}(k)\right]$. Set outer iteration counter $l = 0$. Set an outer iteration convergence criterion ε_{out}.

Step 1: Relaxation.

Find a new estimation of actual link travel times: $\hat{\tau}_a^{(l)}(k) = \tau_a\left[x_a^*(k)\right]$, find $\bar{\tau}_a^{(l)}(k)$ $\forall\, a \in A,\, k = 1, \cdots, K'$, where * denotes the solution obtained from the most recent inner iteration or from outer initialization. Find $\delta_{a'}^{k(l)}(t)$ and $\delta_{a''}^{k(l)}(t)$.

Step 2: Inner Iteration

Step 2.0: Inner Initialization. Compute and reset the inner initial feasible solution to be consistent with the flow propagation constrain at the current relaxation. Set an inner iteration counter $m = 1$ (or a convergence criterion ε_{in}).

Step 2.1: Direction Finding. Perform probit-based stochastic network loading without time-space network, yielding subproblem solution $y_a^{rs(m)}(k)$.

Step 2. 2: Move. Find a new solution $u_a^{rs(m+1)}(k)$ by (5.31).

Step 2. 3: Convergence Test for Inner Iteration.

If $\sqrt{\sum_{rs}\sum_{k}\left(u_a^{rs(m+1)}(k) - u_a^{rs(m)}(k)\right)} \Big/ \sum_{rs}\sum_{k} u_a^{rs(m)}(k) > \varepsilon$, set $m = m+1$, go to Step 2.1.;

otherwise, set $\hat{u}_a^{rs(l)}(k) = u_a^{rs(m+1)}(k)$, $\hat{x}_a^{(l)}(k) = x_a^{(m+1)}(k)$, go to Step 3.

Step 3: Convergence Test for Outer Iteration. If $\hat{\tau}_a^{(l)}(k) \cong \hat{\tau}_a^{(l-1)}(k)$, stop. The current Solution $u_a^{rs}(k)$, $v_a^{rs}(k)$, $x_a^{rs}(k)$ is in a near optimal state; otherwise, set $l = l+1$ and go to

Step 1.

5.2.3 A Numerical Example

Example 5.1

An example is presented below to validate the above model and algorithm. The configuration of the network is shown in Figure 5.1. In the network, each link is assumed as an one-lane street with a length of 0.5 mi. The free flow speed is assumed to be 25 mile/hour. The following linear travel time function is used to enforce FIFO condition: $\tau_a(k) = L_a / s_f + 0.3 \cdot x_a(k)$, where L_a is the length of link a, s_f is free flow speed, $\tau_a(k)$ is link travel time on link a at time k, $x_a(k)$ is number of vehicles on link a at time k. Four O-D pairs are considered. Five 20 s departure time intervals are specified. The OD flows are 10 vehicle units per time interval. The O-D pairs and the time-dependent O-D demand are shown in Table 5.1. In this example, the departure horizon is 5 time increments, and the time increment is 20 seconds.

Table 5-1 O-D pairs and time-dependent O-D demand for example 5.1

O-D	Departure time interval *k*				
	1	2	3	4	5
1-9	10	10	10	10	10
9-1	10	10	10	10	10
3-7	10	10	10	10	10
7-3	10	10	10	10	10

The program of the algorithm was run on a computer with 1.5GHz frequency processor. The inner iteration (MSA algorithm) convergence test method was set as a pre-specified number *m*. The outer iteration (Relaxation) convergence test method was set as

$$\max\left\{ |\tau_a^{(l)}(k) - \tau_a^{(l-1)}(k)| \,\middle|\, a \in A, k = 1, \cdots, K \right\}$$

where $|\tau_a^{(l)}(k) - \tau_a^{(l-1)}(k)|$ is the actual travel time difference of link a at time k between successive relaxations. The operation of the program is shown in Table 5.2.

Table 5-2 Convergence criterion and computation time for Example 5.1

Inner iteration convergence criterion	Outer iteration convergence criterion	Total relaxations	Total computation time (minute)
m=12	0.02	16	43.8

The assignment horizon K is found to be 21 time increments. Table 5.3a shows the output of $u_a^{rs}(k)$. Table 5.3b shows the output of $v_a^{rs}(k)$. Table 5.3c shows the output of $u_a(k)$. Table 5.3d shows the output of $v_a(k)$. Table 5.3e shows the output of $x_a(k)$. Table 5.3f shows the output of $\tau_a(k)$. Table 5.3g shows the output of $f_p^{rs}(k)$, $c_p^{rs}(k)$, links on each path and the arrival time interval for each link on a path. For conciseness, only Table 5.3g is attached to this dissertation.

Table 5-3 The resultant path flow and path travel time for example 5.1

Path number	O	D	k	Path flow	Path time	Links on the path	Arrival time for each link on the path
This table is appendix 2 of this thesis.							

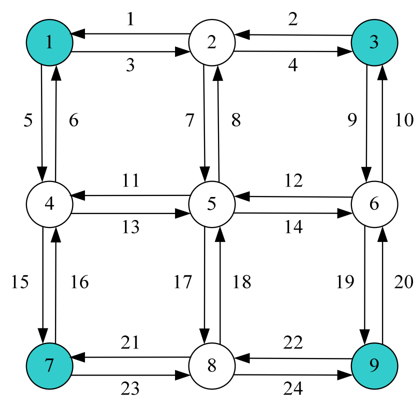

Figure 5-1 Simulation network for Example 5.1

We take the following examples to verify that the solution satisfy the constraints and the dynamic User Optimal conditions.

Path flow conservation constraint (5.13):

$$f^{19}(2)= f_1^{19}(2)+ f_2^{19}(2)+ f_3^{19}(2)+ f_4^{19}(2)+ f_5^{19}(2)+ f_6^{19}(2)$$

=1.8182+2.7273+1.8182+0.9091+1.8182+0.9091

=10

Link inflow conservation constraint (5.14):

$$u_8^{91}(10)+u_8^{73}(10)=0.9091+1.8182=2.7273=u_8(10)$$

Link outflow conservation constraint (5.15):

$$v_8^{91}(14)+v_8^{73}(14)=0.9091+1.8182=2.7273=v_8(14)$$

Node flow conservation constraint (5.16):

$$\sum_{a\in B(6)} v_a^{rs}(9)=\sum_{a\in B(6)} v_a(9) \quad =v_9(9)+v_{14}(9)+v_{20}(9)=5.4545+0+4.5455=10$$

$$\sum_{a\in A(6)} u_a^{rs}(9)=\sum_{a\in A(6)} u_a(9)= u_{10}(9)+u_{12}(9)+u_{19}(9)=2.7273+3.6364+3.6364=10$$

Link flow propagation constraint (5.17):

$$u_8^{91}(10) = v_8^{91}(10 + \bar{\tau}_8(10)) = v_8^{91}(14) = 0.9091$$

$$u_8^{73}(10) = v_8^{73}(10 + \bar{\tau}_8(10)) = v_8^{73}(14) = 1.8182$$

Where $\tau_8(10) = 1.2499$ minutes. For a time increment of 20 seconds, $\bar{\tau}_8(10) = 4$.

The link state equation (5.18b):

$$x_8(10) = x_8(9) + u_8(10) - v_8(10) = 7.2727 + 2.7273 - 0 = 10.0000$$

The actual travel times on the fifth used path from origin 1 toward destination 9 departing at time increment 1 are as follows:

$c_5^{19}(1) =$

$\tau_5(1) + \tau_{15}(1 + \bar{\tau}_5(1)) + \tau_{23}(1 + \bar{\tau}_5(1) + \bar{\tau}_{15}(1 + \bar{\tau}_5(1))) + \tau_{24}(1 + \bar{\tau}_5(1) + \bar{\tau}_{15}(1 + \bar{\tau}_5(1)) + \bar{\tau}_{23}(1 + \bar{\tau}_5(1) + \bar{\tau}_{15}(1 + \bar{\tau}_5(1))))$

$= \tau_5(1) + \tau_{15}(5) + \tau_{23}(9) + \tau_{24}(13)$

=1.2317+1.2090+1.2090+1.2226

= 4.8723 minutes

Similarly, we have $c_1^{19}(1) = 4.9041$ minutes, $c_2^{19}(1) = 4.9041$ minutes, $c_3^{19}(1) = 4.8814$ minutes, $c_4^{19}(1) = 4.8814$ minutes, $c_6^{19}(1) = 4.8859$ minutes. They are quite close but not equal. They are roughly normally distributed, which is consistent with SDUO condition.

As can be checked in the same way, all the solution output satisfies the constraints and the dynamic stochastic user optimal conditions. This verifies the rationale of the above model and solution algorithm.

5.3 Route-based Variational Inequality (VI) SDUO Model

In this section, a route-based VI formulation of SDUO is proposed. The relaxation method is used to solve it. At each relaxation, a route-based nonlinear program is constructed. A solution algorithm which avoids time-space network expansion is proposed.

5.3.1 Route -based VI Formulation of SDUO

Assume the mean actual link travel time $\tau_a(t)$ increases with link inflow $u_a^{rs}(t)$, or the mean actual route travel time $\eta_p^{rs}(t)$ increases with route departure flows as shown in the following:

$$\frac{d\tau_a(t)}{du_a^{rs}(t)} > 0\,;\ \ \forall a \in A, r, s \tag{5.32a}$$

Or

$$\frac{\partial \eta_p^{rs}(t)}{\partial f_p^{rs}(t)} > 0\,;\ \ \forall r, s, p \tag{5.32b}$$

Define a cost term $F_p^{rs}(t)$ for each route p and OD pair O-D rs as follows (Ran and Boyce, 1996b):

$$F_p^{rs}(t) = \left(f_p^{rs}(t) - f^{rs}(t)P_p^{rs}(t)\right)\frac{\partial \eta_p^{rs}(t)}{\partial f_p^{rs}(t)} = 0\,;\ \ \forall r, s, p \tag{5.33}$$

Since $\partial \eta_p^{rs}(t) / \partial f_p^{rs}(t) > 0$, the above equality states the SDUO route choice condition (5.3).

Below we show (5.3) holds if and only if $F_p^{rs}(t)$=0, $\forall r, s, p$.

Proof: Given $F_p^{rs}(t)$=0, $\forall a \in A, r, s$. Since $\partial \eta_p^{rs}(t) / \partial f_p^{rs}(t) > 0$, it follows that

$$f_p^{rs}(t) - f^{rs}(t)P_p^{rs}(t) = 0 \tag{5.34}$$

This is SDUO condition. So $F_p^{rs}(t)$=0, $\forall r, s, p$ contains the SDUO route choice conditions (5.3). Similarly, we can show SDUO route choice conditions (5.3) implies $F_p^{rs}(t)$=0,

$\forall r, s, p$.

Assume the network is empty at $t = 0$, and only travel demands departing within the departure horizon are considered. The route-based SDUO continuous VI model can be expressed as

$$\int_0^T \left\langle \mathbf{F}^*(t), \bar{\mathbf{f}}(t) - \bar{\mathbf{f}}^*(t) \right\rangle dt \geq 0 \tag{5.35a}$$

where $\mathbf{F}^* \in \Re_+^{|P|}$, $\bar{\mathbf{f}} \in \Re_+^{|P|}$, $|P|$ is the cardinality of the set path, etc. Asterisk means the optimal inflow at SDUO state, or in the expanded form as:

$$\int_0^T \sum_{rs} \sum_p F_p^{rs*}(t) \left[f_p^{rs}(t) - f_p^{rs*}(t) \right] dt \geq 0 \tag{5.35b}$$

Theorem: The dynamic traffic flow pattern satisfying network constraint set (chapter4) is an ideal SDUO route choice state if and only if it satisfies the variational inequality problem (5.35).

Proof: The following proof is similar to the procedure as in Nagurney (1993). If $F(\mathbf{u}^*(t)) = 0$, $F(\mathbf{u}^*(t)) = [\cdots, F_a(\mathbf{u}^*(t)), \cdots]^T$ then inequality (5.35) holds with equality. Conversely, if $\mathbf{u}^*(t)$ satisfies (5.35), let $\mathbf{u}(t) = \mathbf{u}^*(t) - F_a(\mathbf{u}^*(t))$, which implies that

$$\int_0^T F(\mathbf{u}^*(t))^T \cdot \left(-F(\mathbf{u}^*(t)) \right) dt \geq 0,$$

or $-\int_0^T \left\| F(\mathbf{u}^*(t)) \right\|^2 dt \geq 0$, therefore, $F(\mathbf{u}^*(t)) = 0$.

5.3.2 Solution Algorithm for Route-based VI SDUO Model

Discrete Route-based VI SDUO Model

To solve the SDUO problem, the continuous VI formulation is discretized with each time interval being time increment. The estimated actual travel time on each time-space link a is a multiple of the time increment and is fixed at each time increment, i.e.,

$$\bar{\tau}_a(k) = i \qquad \text{if} \quad (i - 0.5)\Delta t \le \tau_a(k) \le (i + 0.5)\Delta t \tag{5.36}$$

where i is an integer and $0 \le i \le K$, Δt is time increment.. This round-off method is used only in the flow propagation constraints. The round-off error can be made as small as desired by making the time increment smaller (Ran and Boyce, 1996b).

The link-based SDUO discrete-time VI formulation is

$$\left\langle \mathbf{F}^*(k), \mathbf{f}(k) - \mathbf{f}^*(k) \right\rangle \ge 0 \tag{5.37a}$$

or in expanded form as

$$\sum_{k=1}^{K_0} \sum_{rs} \sum_{p} F_p^{rs*}[k] \left\{ f_p^{rs}(k) - f_p^{rs*}(k) \right\} \ge 0 \tag{5.37b}$$

where $F \in \Re_+^{|P| \times K_0}$, $\mathbf{f} \in \Theta$, and

$$F_p^{rs*}(k) = \left[f_p^{rs*}(k) - f^{rs}(k) P_p^{rs}(k) \right] \frac{\partial \eta_p^{rs}(k)}{\partial f_p^{rs}(k)} \tag{5.37c}$$

in Θ. $a = (i, j)$. Θ is the feasible region defined by the following constraints:

Path flow conservation constraint:

$$\sum_p f_p^{rs}(k) = f^{rs}(k) \ \forall\, k, r, s \tag{5.38}$$

Link inflow conservation constraint:

$$\sum_{rs} u_a^{rs}(k) = u_a(k) \quad \forall\, a, k \tag{5.39}$$

Link outflow conservation constraint:

$$\sum_{rs} v_a^{rs}(k) = v_a(k) \quad \forall\, a, k \tag{5.40}$$

Node flow conservation constraint:

$$\sum_{a \in B(j)} v_a^{rs}(k) = \sum_{a \in A(j)} u_a^{rs}(k) \quad \forall\ j \neq r,s; r,s; k \tag{5.41}$$

where $A(j)$ is the set of links after j and $B(j)$ is the set of links before j.

Link flow propagation constraint:

$$u_a^{rs}(k) = v_a^{rs}(k + \tau_a(k)) \quad \forall\ a,\ r, s, k \tag{5.42}$$

The link state equation:

$$x_a(k+1) = x_a(k) + u_a(k) \text{-} v_a(k) \quad \forall\ a, k \tag{5.43a}$$

or

$$x_a(k+1) = x_a(k) + u_a(k+1) \text{-} v_a(k+1) \quad \forall\ a, k \tag{5.43b}$$

(5.43a) is forward formula, (5.43b) is backward formula.

Path-link flow incidence constraint:

$$u_a^{rs}(n) = \sum_{rs} \sum_{p} \sum_{k=1}^{K_0} f_p^{rs}(k) \delta_{rsa}^{pkn} \quad \forall\ a, n \tag{5.44}$$

where $\delta_{rsa}^{pkn} \in \{0,1\}$ is defined as:

$$\delta_{rsa}^{pkn} = \begin{cases} 1 & \begin{array}{l} \text{if traffic departing origin } r \text{ at any time interval } k \\ \text{heading for destination } s \text{ on path } p \text{ arrives at link } a \\ \text{during the } n\text{th time interval.} \end{array} \\ 0 & \text{otherwise} \end{cases} \tag{5.45}$$

Nonnegative constraint:

$$f_p^{rs}(k) \geq 0, u_a^{rs}(k) \geq 0,\ \forall\ k, r, s, a, p \tag{5.46}$$

Relaxation

At each relaxation, we temporarily fix (Ran and Boyce, 1996b): 1)Actual travel time $\tau_a(k)$ in the link flow propagation constraints as $\bar{\tau}_a(k)$ and corresponding actual route travel time $\eta_p^{rs}(k)$ as $\overline{\eta}_p^{rs}(k)$;2)Actual travel time in the VI cost term $F_a^{rs*}(k)$ as $\tau_a\left[k+\overline{\pi}^{ri*}(k)\right]$ and 3)Minimal travel times $\pi^{ri*}(k)$ as $\overline{\pi}^{ri*}(k)$ for each link and each origin. Via relaxation, the auxiliary VI cost term for each link *p* at each time interval *k* becomes

$$F_p^{rs*}(k)=\left[f_p^{rs*}(k)-f^{rs}(k)P_p^{rs}(k)\right]\frac{\partial\eta_p^{rs}(k)}{\partial f_p^{rs}(k)} \tag{5.47a}$$

At each relaxation, the time-space network is fixed with fixed link flow propagation constraints.

At each relaxation, we have

$$\eta_p^{rs}(k)=\sum_{k=1}^{K_0}\sum_a \tau_a(n)\delta_{rsa}^{pkn} \tag{5.47b}$$

$$=\tau_{a_1}(k)+\tau_{a_2}\left(k+\overline{\tau}_{a_1}(k)\right)+\cdots+\tau_{a_{\bar{p}}}\left(k+\overline{\eta}_p^{ra_{(\bar{p}-1)}}(k)\right) \tag{5.47c}$$

where $p=\left(a_1,a_2,\cdots\cdots a_{\bar{p}}\right)$, a_i is the link number of path p of O-D pair rs at time k . and ,

$$\delta_{rsa}^{pkn}=\begin{cases}1 & \begin{array}{l}\text{if traffic departing origin } r \text{ at any time interval } k\\ \text{heading for destination } s \text{ on path } p \text{ arrives at}\\ \text{link } a \text{ during the } n\text{th time interval.}\end{array}\\ 0 & \text{otherwise}\end{cases} \tag{5.48}$$

Optimization Problem

An optimization problem which is equivalent to the discrete VI under relaxation can thus be formulated, as follows:

$$\min_{\mathbf{f}}\ Z\ =\sum_{k=1}^{K_0}\sum_{rs}\left\{-f^{rs}(k)S^{rs}(k)+\sum_p f_p^{rs}(k)\eta_p^{rs}\left[f_p^{rs}(k);\mathbf{f}_p^{rs}\right]-\sum_p\int_0^{f_p^{rs}(k)}\eta_p^{rs}\left(\omega;\mathbf{f}_p^{rs}\right)d\omega\right\} \tag{5.49}$$

in Θ , where $\mathbf{f}_p^{rs}$ denotes the path flow vector $\mathbf{f}$ without component f_p^{rs} .

The gradient of (5.49) is

$$\frac{\partial Z}{\partial f_p^{rs}(k)} = -f^{rs}(k)\frac{\partial S^{rs}(k)}{\partial \eta_p^{rs}(k)}\frac{\partial \eta_p^{rs}(k)}{\partial f_p^{rs}(k)} + \eta_p^{rs}(k) + f_p^{rs}(k)\frac{\partial \eta_p^{rs}(k)}{\partial f_p^{rs}(k)} - \eta_p^{rs}(k)$$

$$= \left[f_p^{rs}(k) - f^{rs}(k)P_p^{rs}(k)\right]\frac{\partial \eta_p^{rs}(k)}{\partial f_p^{rs}(k)} \qquad (5.50)$$

(5.50) is equivalent to the cost term of discrete VI (5.37b) under relaxation. This indicates the above optimization program is equivalent to the discrete VI (5.37). Since all cross effects are fixed in each relaxation, $f_p^{rs}(k)$ is the only variable for each summation term of (5.49).

At each relaxation, the VI formulation of DUO problem was transformed into a series of static stochastic user equilibrium traffic assignment problems over the time-space network of the relaxation, which can be solved by Method of Successive Average (MSA). Call the relaxation as outer iteration and solving static stochastic user equilibrium traffic assignment problems over the time-space network of the relaxation as inner iteration.

The negative gradient of the objective function (5.49) can be written as

$$\mathbf{d} = -\nabla Z(\mathbf{f}) = [\boldsymbol{\Delta}\mathbf{f} \cdot \mathbf{P} - \mathbf{f}]\nabla_f \boldsymbol{\eta} \qquad (5.51a)$$

Where

$\boldsymbol{\Delta}\mathbf{f} = \mathrm{diag}\left(\cdots, \boldsymbol{\Delta}\mathbf{f}^{rs}(k), \cdots\right)$, $\boldsymbol{\Delta}\mathbf{f}^{rs}(k) = \mathrm{diag}\left(f^{rs}(k), \cdots, f^{rs}(k)\right)$, $\mathbf{P} = \left(\cdots, \mathbf{P}^{rs}(k), \cdots\right)^T$,
$\mathbf{P}^{rs}(k) = \left(\cdots, P_p^{rs}(k), \cdots\right)$

Dropping $\nabla_f \boldsymbol{\eta}$ from (5.51a), we obtain a simpler direction as follows

$$\mathbf{d} = \boldsymbol{\Delta}\mathbf{f} \cdot \mathbf{P} - \mathbf{f} \qquad (5.51b)$$

It is easy to show that $\nabla Z(\mathbf{u}) \cdot (\mathbf{d})^T < 0$, so (5.51b) is a descent direction of the objective function.

The component of **d** (5.51b) is

$$d_p^{rs}(k) = f^{rs}(k)P_p^{rs}(k) - f_p^{rs}(k) \qquad (5.51c)$$

Let

$$g_p^{rs}(k) = f^{rs}(k)P_p^{rs}(k) \qquad (5.52)$$

g can be obtained by performing route-based probit-based stochastic network loading on the original network without time-space network expansion.

The route-based probit-based stochastic network loading procedure is similar to link-based probit-based stochastic network loading procedure except in Step 3 and Step 4. In Step 3, path flows $G_p^{rs(l)}(k)$ is yielded. In Step 4, the path flow is averaged according to the following equation

$$g_p^{rs(l)}(k) = \left[(l-1)g_p^{rs(l-1)}(k) + G_p^{rs(l)}(k)\right]/l \qquad (5.53)$$

where $g_p^{rs(l)}(k)$ is path flow.

At the mth iteration of the inner iteration (MSA), the descending direction of nonlinear programming (5.49) can be found by performing the route-based probit-based stochastic network loading.

In solving (5.49), it is not necessary to enumerate all the paths of each O-D pair on the network. With the technique of column generation, (5.49) can be solved on the path set $P^{(m)}$ defined by

$$P^{(m)} = P^{(m-1)} \cup \overline{P}^{(m)} \qquad (5.54)$$

where $P^{(m-1)}$ is the path set at the $(m-1)$th iteration, $\overline{P}^{(m)}$ is the path set composed of dynamic shortest paths of each O-D pair at the mth iteration. $P^{(1)}$ is defined as $\overline{P}^{(1)}$.

The step size along the descending direction is simply $1/m$. The solution at the inner iteration can be updated as

$$f_p^{rs(m+1)}(k) = f_p^{rs(m)}(k) + (1/m)\left[g_p^{rs(m)}(k) - f_p^{rs(m)}(k)\right] \quad \forall a, r, s, n \qquad (5.55)$$

Solution Algorithm

Based on above rational analysis, an algorithm for solving the ideal SDUO route choice model (5.37) has been developed and is summarized as follows.

Step 0: Outer Initialization.

Compute $k_{\max} = \max_{\forall rs}\{\pi^{rs}\}$, where π^{rs} is the static minimum travel time of O-D rs. Set $K' = K_0 + C \cdot [k_{\max}]_+$. Set $\hat{\tau}_a^{(0)}(k) = \tau_a[0]$, $\forall a \in A, \quad k = 1, \cdots, K'$. Find an initial feasible solution $\left[f_p^{rs(0)}(k)\right]$. Set outer iteration counter $l = 0$. Set an outer iteration convergence criterion ε_{out}.

Step 1: Relaxation.

Find a new estimation of actual link travel times: $\hat{\tau}_a^{(l)}(k) = \tau_a\left[x_a^*(k)\right]$, find $\bar{\tau}_a^{(l)}(k)$ $\forall a \in A$, $k = 1, \cdots, K'$, where * denotes the solution obtained from the most recent inner iteration or from outer initialization. Find $\delta_{a'}^{k(l)}(t)$ and $\delta_{a''}^{k(l)}(t)$.

Step 2: Inner Iteration

Step 2.0: Inner Initialization. Compute and reset the inner initial feasible solution to be consistent with the flow propagation constrain at the current relaxation. Set an inner iteration counter $m = 1$ (or a convergence criterion ε_{in}).

Step 2.1: Direction Finding. Perform probit-based stochastic network loading without time-space network, yielding subproblem solution $g_p^{rs(m)}(k)$.

Step 2. 2: Move. Find a new solution $f_p^{rs(m+1)}(k)$ by (5.55).

Step 2. 3: Convergence Test for Inner Iteration.

If $\sqrt{\sum_{rs}\sum_{k}\left(u_a^{rs(m+1)}(k)-u_a^{rs(m)}(k)\right)} \Big/ \sum_{rs}\sum_{k} u_a^{rs(m)}(k) > \varepsilon$, set $m=m+1$, go to Step 2.1.;

otherwise, set $\hat{u}_a^{rs(l)}(k)=u_a^{rs(m+1)}(k)$, $\hat{x}_a^{(l)}(k)=x_a^{(m+1)}(k)$, go to Step 3.

Step 3: Convergence Test for Outer Iteration. If $\hat{\tau}_a^{(l)}(k)\cong\hat{\tau}_a^{(l-1)}(k)$, stop. The current solution $f_p^{rs}(k)$, $u_a^{rs}(k)$, $v_a^{rs}(k)$, $x_a^{rs}(k)$ is in a near optimal state; otherwise, set $l=l+1$ and go to Step 1.

5.3.3 A Numerical Example

Example 5.2

An example is presented below to validate the above model and algorithm. The configuration of the network is shown in Figure 5.1. In the network, each link is assumed as a one-lane street with a length of 0.5 mi. The free flow speed is assumed to be 25 mile/hour. The following linear travel time function is used to enforce FIFO condition: $\tau_a(k)=L_a/s_f+0.3\cdot x_a(k)$, where L_a is the length of link a, s_f is free flow speed, $\tau_a(k)$ is link travel time on link a at time k, $x_a(k)$ is number of vehicles on link a at time k. Four O-D pairs are considered. Five 20 s departure time intervals are specified. The OD flows are 10 vehicle units per time interval. The O-D pairs and the time-dependent O-D demand are shown in Table 5.1. In this example, the departure horizon is 5 time increments, and the time increment is 20 seconds.

The program of the algorithm was run on a computer with 1.5GHz frequency processor. The inner iteration (MSA algorithm) convergence test method was set as a pre-specified number *m*. The outer iteration (Relaxation) convergence test method was set as

$$\max\left\{ \left| \tau_a^{(l)}(k) - \tau_a^{(l-1)}(k) \right| \middle| a \in A, k = 1, \cdots, K \right\}$$

where $|\tau_a^{(l)}(k) - \tau_a^{(l-1)}(k)|$ is the actual travel time difference of link *a* at time *k* between successive relaxations. The operation of the program is shown in Table 5.4.

Table 5-4 Convergence criterion and computation time for Example 5.2

Inner iteration convergence criterion	Outer iteration convergence criterion	Total relaxations	Total computation time (minute)
m=12	0.02	19	45.6

The assignment horizon *K* is found to be 21 time increments. Table 5.5a shows the output of $u_a^{rs}(k)$. Table 5.5b shows the output of $v_a^{rs}(k)$. Table 5.5c shows the output of $u_a(k)$. Table 5.5d shows the output of $v_a(k)$. Table 5.5e shows the output of $x_a(k)$. Table 5.5f shows the output of $\tau_a(k)$. Table 5.5g shows the output of $f_p^{rs}(k)$, $c_p^{rs}(k)$, links on each path and the arrival time interval for each link on a path. For conciseness, only Table 5.5g is attached to this dissertation.

Table 5-5 The resultant path flow and path travel time for example 5.2

Path number	*O*	*D*	*k*	Path flow	Path time	Links on the path	Arrival time for each link on the path
This table is appendix 4 of this thesis.							

We take the following examples to verify that the solution satisfy the constraints and the dynamic User Optimal conditions.

Path flow conservation constraint (5.38):

$f^{19}(2) = f_1^{19}(2) + f_2^{19}(2) + f_3^{19}(2) + f_4^{19}(2) + f_5^{19}(2) + f_6^{19}(2)$

=3.6364+0.9091+1.8182+1.8182+0+1.8182

=10

Link inflow conservation constraint (5.39):

$$u_8^{91}(10) + u_8^{73}(10) = 1.8182 + 2.7273 = 4.5455 = u_8(10)$$

Link outflow conservation constraint (5.40):

$$v_8^{91}(14) + v_8^{73}(14) = 1.8182 + 2.7273 = 4.5455 = v_8(14)$$

Node flow conservation constraint (5.41):

$$\sum_{a \in B(6)} v_a^{rs}(9) = \sum_{a \in B(6)} v_a(9) \quad = v_9(9) + v_{14}(9) + v_{20}(9) = 4.5455+0+3.6364=8.1819$$

$$\sum_{a \in A(6)} u_a^{rs}(9) = \sum_{a \in A(6)} u_a(9) = u_{10}(9) + u_{12}(9) + u_{19}(9) = 0.9091+5.4545+1.8182=8.1819$$

Link flow propagation constraint (5.42):

$$u_8^{91}(10) = v_8^{91}(10 + \bar{\tau}_8(10)) = v_8^{91}(14) = 1.8182$$

$$u_8^{73}(10) = v_8^{73}(10 + \bar{\tau}_8(10)) = v_8^{73}(14) = 2.7273$$

Where $\tau_8(10)$=1.2499 minutes. For a time increment of 20 seconds, $\bar{\tau}_8(10)$=4.

The link state equation (5.43b):

$$x_8(10) = x_8(9) + u_8(10) - v_8(10) = 5.4545 + 4.5455 - 0 = 10.0000$$

The actual travel times on the fifth used path from origin 1 toward destination 9 departing at time increment 1 are as follows:

$c_1^{19}(1) =$

$$\tau_5(1) + \tau_{15}(1 + \bar{\tau}_5(1)) + \tau_{23}(1 + \bar{\tau}_5(1) + \bar{\tau}_{15}(1 + \bar{\tau}_5(1))) + \tau_{24}(1 + \bar{\tau}_5(1) + \bar{\tau}_{15}(1 + \bar{\tau}_5(1)) + \bar{\tau}_{23}(1 + \bar{\tau}_5(1) + \bar{\tau}_{15}(1 + \bar{\tau}_5(1))))$$

$= \tau_5(1) + \tau_{15}(5) + \tau_{23}(9) + \tau_{24}(13)$

=1.2317+1.2135+1.2135+1.2226

= 4.895 minutes

Similarly, we have $c_2^{19}(1) = 4.8905$ minutes, $c_3^{19}(1) = 4.9359$ minutes, $c_4^{19}(1) = 4.8405$ minutes, $c_5^{19}(1) = 4.9132$ minutes, $c_6^{19}(1) = 4.9132$ minutes. They are quite close but not equal. They are roughly normally distributed, which is consistent with SDUO condition.

As can be checked in the same way, all the solution output satisfies the constraints and the dynamic stochastic user optimal conditions. This verifies the rationale of the above model and solution algorithm.

Chapter 6: Dynamic User Optimal Simultaneous Departure Time and Route Choice (DUOSDTRC) Problem

Generally an arrival time interval is required for work trips. Road users may choose alternative routes or shift their departure time to avoid congestion and arrive at work on time. The dynamic user optimal simultaneous departure time and route choice problem (DUOSDTRC) extends the DUO route choice model in one respect: both departure time and route over a road network must be chosen. Each departure time choice is based on actual minimum origin-destination travel times at each departure time. Any change in departure times alters the time-dependent O-D pattern in the network, so route and departure time decisions of other travelers will be affected.

There are two main differences between DUO problem and DUOSDTRC problem. First, the time-dependent O-D demand is given for DUO problem, while it is a variable needs to be solved for in DUOSDTRC problem. Second, different costs can be incurred for drivers of the same O-D pair departing at different times in DUO problem, while the same cost should be incurred for all drivers of the same O-D pair departing at all time in DUOSDTRC problem.

This chapter presents a relaxation with gradient projection algorithm for solving a route-based dynamic User Optimal simultaneous departure time and route choice model (DUOSDTRC) for a general network with multiple origin-destination pairs. Section 6.1

introduces a relaxation with gradient projection algorithm for the DUO model, which is an important stage in constructing relaxation with gradient projection algorithm for DUOSDTRC model. Section 6.2 introduces disutility function or schedule delay function, generalized time-dependent path travel cost, and additional network constraints for DUOSDTRC problem. Section 6.3 presents the route-based DUOSDTRC model and development of the relaxation with gradient projection algorithm for solving the above model. A numerical example showing the application of the algorithm is exhibited.

6.1 Relaxation with Gradient Projection Algorithm for DUO

6.1.1 Relaxation-Gradient Projection Algorithm for DUO

In this section, we present a relaxation algorithm to solve the route-based VI formulation of dynamic user optimal (DUO) problem. The discrete VI formulation DUO is given as

$$\left\langle \left[\boldsymbol{\eta}^*(k) - \boldsymbol{\pi}^*(k)\right], \mathbf{f}(k) - \mathbf{f}^*(k) \right\rangle \geq 0 \tag{6.1a}$$

in Θ. Or in expanded form, as

$$\sum_{rs}\sum_{p}\sum_{k=1}^{K_0}\left[\eta_p^{rs^*}(k) - \pi^{rs^*}(k)\right]\left[f_p^{rs}(k) - f_p^{rs^*}(k)\right] \geq 0 \tag{6.1b}$$

in Θ, where $\boldsymbol{\eta}, \mathbf{f} \in \Re_+^{|P| \times K_0}$,.

$$\eta_p^{ri}(k) = \eta_p^{r(i-1)}(k) + \tau_a\left[k + \eta_p^{r(i-1)}(k)\right] \quad \forall\ p = p, r, i;\ i = 1, 2, \cdots, s;$$

$p = (r, 1, 2, \cdots, i, \cdots s)$,

Θ is the feasible region defined by constraints by (3.38) --- (3.46).

Relaxation

At each relaxation, we temporarily fix: 1) Actual travel time $\tau_a(k)$ in the link flow propgation constraints as $\bar{\tau}_a(k)$; 2) Actual travel time $\tau_a[k+\pi^{ri}(k)]$ as $\tau_a[k+\bar{\pi}^{ri}(k)]$ and 3)Minimal travel times $\pi^{rs}(k)$ as $\bar{\pi}^{rs}(k)$ for each origin and destination. At each relaxation, the time-space network is fixed with fixed link flow propagation constraints.

Via relaxation, the VI cost term becomes

$$\eta_p^{rs}(k) - \bar{\pi}^{rs}(k) \quad (6.2a)$$

where

$$\eta_p^{rs}(k) = \sum_{k=1}^{K_0} \sum_a \tau_a(n) \delta_{rsa}^{pkn} \quad (6.2b)$$

$$= \tau_{a_1}(k) + \tau_{a_2}\left(k + \bar{\tau}_{a_1}(k)\right) + \cdots + \tau_{a_{\bar{p}}}\left(k + \bar{\eta}_p^{ra_{(\bar{p}-1)}}(k)\right) \quad (6.2c)$$

where $p = (a_1, a_2, \cdots\cdots a_{\bar{p}})$, a_i is the link number of path p of O-D pair rs at time k . and ,

$$\delta_{rsa}^{pkn} = \begin{cases} 1 & \text{if traffic departing origin } r \text{ at any time interval } k \\ & \text{heading for destination } s \text{ on path } p \text{ arrives at} \\ & \text{link } a \text{ during the } n\text{th time interval.} \\ 0 & \text{otherwise} \end{cases} \quad (6.3)$$

Optimization Problem

An optimization problem which is equivalent to the discrete VI under relaxation can thus be formulated, as follows:

$$\min_{\mathbf{f}} Z = \sum_{k=1}^{K_0} \sum_{rs} \sum_p \left\{ \int_0^{f_p^{rs}(k)} \left[\eta_p^{rs}(\omega, \mathbf{f}_p^{rs}) - \bar{\pi}^{rs}(k) \right] d\omega \right\} \quad (6.4)$$

in Θ, where $\mathbf{f}_{p}^{rs}$ denotes the path flow vector $\mathbf{f}$ without component f_{p}^{rs}. Θ is the feasible set defined by (4.38) --- (4.46).

Since $\bar{\pi}^{rs}(k)$ is fixed for each O-D pair at each relaxation, it can be dropped from (6.4), the resultant problem is

$$\min_{\mathrm{f}} Z = \sum_{k=1}^{K_0} \sum_{rs} \sum_{p} \left\{ \int_0^{f_p^{rs}(k)} \eta_p^{rs}\left(\omega, \mathbf{f}_p^{rs}\right) d\omega \right\} \tag{6.5}$$

in Θ.

The gradient of (6.5) is

$$\frac{\partial Z}{\partial f_p^{rs}(k)} = \eta_p^{rs}(k) \tag{6.6}$$

At each relaxation, the VI formulation of DUO problem was transformed into a series of static user equilibrium traffic assignment problems over the time-space network of the relaxation, which can be solved by Gradient Projection algorithm. Call the relaxation as outer iteration and solving static user equilibrium traffic assignment problems over the time-space network of the relaxation as inner iteration.

The gradient projection (GP) algorithm for UE problem is given by Jayakrishnan et al. (1994), which is generalized to solve the series of UE problem on the implicit time-space network as described below.

The formulation of the algorithm focuses on the time-dependent traffic demand constraints:

$$\sum_{p \in P_{rs}^k} f_p^{rs}(k) = f^{rs}(k) \tag{6.7}$$

where P_{rs}^k is the set of paths (with positive flow) between origin *r* and destination *s* at time intervals *k*.

If we express the shortest-path flows $f_{\bar{p}_{rs}(k)}$ in terms of other path flows

$$f_{\bar{p}_{rs}(k)} = f^{rs}(k) - \sum_{\substack{p \in P_{rs}^{k} \\ p \neq \bar{p}_{rs}(k)}} f_p^{rs}(k) \tag{6.8}$$

The optimization problem (6.5) at Θ can be restated as

$$\min \tilde{Z}\left(\tilde{f}\right) \tag{6.9}$$

at $\tilde{\Theta}$. $\tilde{\Theta}$ is the feasible set defined by (4.39) --- (4.46). $\tilde{Z}$ is the new objective function, and $\tilde{f}$ is the set of non-shortest-path flows between all of the O-D pairs at any departure time intervals *k*.

The gradient of the objective function written in terms of the non-shortest-path variables can be found using

$$\frac{\partial \tilde{Z}}{\partial f_p^{rs}(k)} = \frac{\partial Z}{\partial f_p^{rs}(k)} - \frac{\partial Z}{\partial f_{\bar{p}_{rs}(k)}} \quad \text{where} \quad p \in P_{rs} \text{ and } \quad p \neq \bar{p}_{rs}(k) \tag{6.10}$$

which results from the definition of $\tilde{Z}$. Each component of the gradient vector is the difference between the first derivative lengths of a path and the corresponding shortest path and the first derivative lengths are simply the dynamic path cost at that flow solution.

The second derivative is simply the sum of the second derivative lengths of the links on either path *p* or path $\bar{p}_{rs}(k)$, but not on both. Once the second derivatives of $\tilde{Z}$ with respect to each path flow are calculated, the inverse of Hessian matrix each second derivative gives an approximate quasi-Newton step size for updating each path flow.

It is better to keep α^n a constant (i.e., $\alpha^n = \alpha$, $\forall\, n$). It can be shown that given any starting set of path flows there exists an $\overline{\alpha}$ such that if $\alpha \in \left(0, \overline{\alpha}\right)$ the sequence generated by this algorithm converges to the optimum (), provided that the link-cost functions are convex. Our experience shows that α equal to 0.5 achieves a very good convergence rate.

Algorithm

According to the above rational analysis, the algorithm for solving the ideal route-based DUO route choice model is developed and summarized as follows.

Step 0: Outer Initialization.

Compute $k_{\max} = \max\limits_{\forall rs}\{\pi^{rs}\}$, where π^{rs} is the static minimum travel time of O-D rs. Set $K^{'} = K_0 + C \cdot [k_{\max}]_+$. Set $\hat{\tau}_a^{(0)}(k) = \tau_a[0]$, $\forall\, a \in A, \quad k = 1, \cdots, K'$. Find an initial feasible solution $[f_p^{rs(0)}(k)]$. Set outer iteration counter $l = 0$. Set an outer iteration convergence criterion ε_{out}.

Step 1: Relaxation.

Find a new estimation of actual link travel times: $\hat{\tau}_a^{(l)}(k) = \tau_a[x_a^*(k)]$, find $\bar{\tau}_a^{(l)}(k)$ $\forall\, a \in A$, $k = 1, \cdots, K'$, where * denotes the solution obtained from the most recent inner iteration or from outer initialization. Find $\delta_{a'}^{k(l)}(t)$ and $\delta_{a''}^{k(l)}(t)$.

Step 2: Inner Iteration

Step 2.0: Inner Initialization. Compute and reset the inner initial feasible solution to be consistent with the flow propagation constrain at the current relaxation. Set an inner iteration counter $m = 1$.

In the first relaxation, set $\tau_a^{(1)}(k)$ equal to free flow travel time $\tau_a(0), \forall\, a$, k=1,…, K. and perform all-or-nothing assignments. This yields initial path flows $f_{(1)}^{rs}(k), \forall\; r, s$, k=1,…,K_0.

In other relaxations, reset the most inner iteration solution to be consistent with the flow propagation constrain at the current relaxation, and set them as initial path flows $f_{(1)}^{rs}(k), \forall\ r,s$, k=1,…, K_0, at current relaxation (Initialize the path set P_{rs}^k with the shortest path for each O-D pair *rs* at time *k*).

Step 2.1: Update. Set $\tau_a^{(m)}(k)$ equal to $\tau_a^{(m)}\left[x_a^{(m)}(k)\right]$. Update the first derivative lengths $d_p^{rs(m)}(k)$(i.e., path cost at current flow) of all of the paths in P_{rs}^k, $\forall\ r,s$.

Step 2.2: Direction finding. Find the shortest-path $\bar{p}_{rs}^{(m)}(k)$ from each origin *r* to each destination *s* at *k* on the basis of $\tau_a^{(m)}(k)$. If different from all the paths in the existing path set in P_{rs}^k, (no need for path comparison here; just compare $d_p^{rs(m)}(k)$, add it to in P_{rs}^k and record $d_{\bar{p}_{rs}^{(m)}(k)}$. If not tag the shortest among the paths in P_{rs}^k in $d_{\bar{p}_{rs}^{(m)}(k)}$.

Step 2.3: Move. Set the new path flows.

$$f_p^{rs(m+1)}(k) = \max\left[0, f_p^{rs(m)}(k) - \frac{\alpha^n}{s_p^{rs(m)}(k)}\left(d_p^{rs(m)}(k) - d_{\bar{p}_{rs}^{(m)}(k)}\right)\right] \forall\ r,s,p \in P_{rs}^k, p \neq \bar{p}_{rs}^{(m)}(k) \qquad (6.11)$$

where

$$s_p^{rs(m)}(k) = \sum_a \sum_k \frac{\partial \tau_a^{(m)}(k)}{\partial x_a^{(m)}(k)}, \forall\ p \in P_{rs}^k \qquad (6.12)$$

a and k denotes time-space links that are on either p or $\bar{p}_{rs}^{(m)}(k)$, but not on both, and α^n is a scalar step-size modifier.

Also,

$$f_{\bar{p}_{rs}(k)}^{(m+1)} = f^{rs}(k) - \sum_{\substack{p \in P_{rs}^k \\ p \neq \bar{p}_{rs}(k)}} f_p^{rs(m+1)}(k) \quad \forall\ p \in P_{rs}^k, p \neq \bar{p}_{rs}^{(m)}(k) \qquad (6.13)$$

Assign the flows on the trees and find the link flows $u_a^{(m+1)}(k)$.

Step 2.4: Convergence Test for Inner Iteration.

If $\sqrt{\sum_{rs}\sum_{k}^{K_0}\left(f_p^{rs(m+1)}(k)-f_p^{rs(m)}(k)\right)}\Big/\sum_{rs}\sum_{k}^{K_0} f_p^{rs(m)}(k) > \varepsilon$, set $m=m+1$, go to Step 2.1; otherwise, set $\hat{f}_a^{rs(l)}(k)=f_p^{rs(m+1)}(k), \hat{x}_a^{(l)}(k)=x_a^{(m+1)}(k)$, go to Step 3.

Step 3: Convergence Test for Outer Iteration. If $\hat{\tau}_a^{(l)}(k)\cong\hat{\tau}_a^{(l-1)}(k)$, stop. The current solution $u_a^{rs}(k)$, $v_a^{rs}(k)$, $x_a^{rs}(k)$ is in a near optimal state; otherwise, set $l=l+1$ and go to Step 1.

6.1.2 A Numerical Example

Example 6.1

An example is presented below to validate the algorithm. The problem is the same as in Example 4.1. The program of the algorithm was run on a computer with 1.5GHz frequency processor. The inner iteration (GP algorithm) convergence test method was set as a prespecified number n. The outer iteration (Relaxation) convergence test method was set as

$$\max\left\{|\tau_a^{(l)}(k)-\tau_a^{(l-1)}(k)|\,\middle|\,a\in A, k=1,\cdots,K\right\}$$

where $|\tau_a^{(l)}(k)-\tau_a^{(l-1)}(k)|$ is the actual travel time difference of link a at time k between successive relaxations. The operation of the program is shown in Table 6.1.

Table 6-1 Convergence criterion and computation time for Example 6.1

Inner iteration convergence criterion	Outer iteration convergence criterion	Total relaxations	Total computation time (minute)
n=4	0.0002	8	3.1

The assignment horizon K is found to be 21 time increments. Table 6.2a shows the output of $u_a^{rs}(k)$. Table 6.2b shows the output of $v_a^{rs}(k)$. Table 6.2c shows the output of $u_a(k)$. Table 6.2d shows the output of $v_a(k)$. Table 6.2e shows the output of $x_a(k)$. Table 6.2f shows the output of $\tau_a(k)$. Table 6.2g shows the output of $f_p^{rs}(k)$, $c_p^{rs}(k)$, links on each path and the arrival time interval for each link on a path. For conciseness, only Table 6.2g is attached to this dissertation.

Table 6-2 The resultant path flow and path travel time for example 6.1

Path number	O	D	k	Path flow	Path time	Links on the path	Arrival time for each link on the path
This table is appendix 6 of this thesis.							

We take the following examples to verify that the solution satisfy the constraints and the dynamic User Optimal conditions.

Path flow conservation constraint is automatically satisfied:

$$f^{19}(1) = f_1^{19}(1) + f_2^{19}(1) + f_3^{19}(1) + f_4^{19}(1)$$

=3.3333+1.6667+3.3333+1.6667

=10

Link inflow conservation constraint (4.39):

$$u_8^{91}(10) + u_8^{73}(10) = 1.6666+1.6666=3.3332= u_8(10)$$

Link outflow conservation constraint (4.40):

$$v_8^{91}(14) + v_8^{73}(14) = 1.6666+1.6666=3.3332= v_8(14)$$

Node flow conservation constraint (4.41):

$$\sum_{a\in B(6)} v_a^{rs}(k) = \sum_{a\in B(6)} v_a(k) \quad = v_9(8) + v_{14}(8) + v_{20}(8) = 5.0002+0+4.9998=10$$

$$\sum_{a\in A(6)} u_a^{rs}(k) = \sum_{a\in A(6)} u_a(k) = u_{10}(8) + u_{12}(8) + u_{19}(8) =3.3334+3.3337+3.3329=10$$

Link flow propagation constraint (4.42):

$$u_8^{91}(10) = v_8^{91}(10 + \bar{\tau}_8(10)) = v_8^{91}(14) =1.6666$$

$$u_8^{73}(10) = v_8^{73}(10 + \bar{\tau}_8(10)) = v_8^{73}(14) =1.6666$$

where $\tau_8(10)$=1.2332 minutes. For a time increment of 20 seconds, $\bar{\tau}_8(10)$=4.

The link state equation (4.43b):

$$x_8(10) = x_8(9) + u_8(10) - v_8(10) = 3.3333+3.3332 - 0 = 6.6665$$

The actual travel times on the used paths from origin 1 toward destination 9 departing at time increment 1 are as follows:

$c_1^{19}(1) =$

$\tau_5(1) + \tau_{15}(1+\bar{\tau}_5(1)) + \tau_{23}(1+\bar{\tau}_5(1) + \bar{\tau}_{15}(1+\bar{\tau}_5(1))) + \tau_{24}(1+\bar{\tau}_5(1) + \bar{\tau}_{15}(1+\bar{\tau}_5(1)) + \bar{\tau}_{23}(1+\bar{\tau}_5(1) + \bar{\tau}_{15}(1+\bar{\tau}_5(1))))$

$= \tau_5(1) + \tau_{15}(5) + \tau_{23}(9) + \tau_{24}(13)$

= 1.2249+1.2166+1.2166+1.2249

= 4.8829 minutes

Similarly, we have $c_2^{19}(1) = 4.8829$ minutes, $c_3^{19}(1) = 4.8829$ minutes, $c_4^{19}(1) = 4.8829$ minutes.

They are nearly equal.

As can be checked in the same way, all the solution output satisfies the constraints and the dynamic user optimal conditions. This verifies the validity of the solution algorithm.

6.2 Generalized Path Travel Cost and Additional Network Constraints for DUOSDTRC problem

6.2.1 Disutility Function or Schedule Delay Function

A disutility function or schedule delay function based on departure times is defined for travelers departing from origin *r* to destination *s* on route *p* at time *t*. The disutility function is weight sum of waiting time at the origin, driving time during the journey, and utility and disutility for early arrival or late arrivals (Ran and Boyce, 1996b).

We assume that travelers going to the same destination s have the same desired arrival times, expressed as the desired arrival time interval $[\tilde{t}_s - \Delta_s, \tilde{t}_s + \Delta_s]$, where $\tilde{t}_s$ and Δ_s are desired arrival time and indifference interval of arrival time for travelers going to destination s. If early arrival is not encouraged, Travelers arrive at destination s earlier than $[\tilde{t}_s - \Delta_s]$ incurs early arrival disutility/penalty. If early arrival is encouraged, Travelers arrive at destination s earlier than $[\tilde{t}_s - \Delta_s]$ incurs early arrival utility/bonus. Travelers arrive at destination s later than $[\tilde{t}_s + \Delta_s]$ incurs disutility/late penalty since later arrival should not be encouraged.

Different disutility function/schedule delay function were defined in previous studies (Bernstein et al., 1993; Wie et al. , 1995; Ran and Boyce, 1996b). A popularly used piecewise linear schedule delay function (Hendrickson and Kocur, 1981; Bernstein et al., 1993; Yang and Meng, 1998) is adopted in this study, which is as follows:

$$c_s(t)=\begin{cases}\rho_s\left[\left(\tilde{t}_s-\Delta_s\right)-\left(t+\pi^{rs}(t)\right)\right] & \text{if} \quad \tilde{t}_s-\Delta_s>t+\pi^{rs}(t)\\ 0 & \text{if} \quad \left|t+\pi^{rs}(t)-\tilde{t}_s\right|\le\Delta_s\\ \beta_s\left[\left(t+\pi^{rs}(t)\right)-\left(\tilde{t}_s+\Delta_s\right)\right] & \text{if} \quad \tilde{t}_s+\Delta_s<t+\pi^{rs}(t)\end{cases} \tag{6.14}$$

The schedule delay function is shown in Figure 6.1.

We assume the unit cost of late arrival is higher than the cost of unit travel time and the cost of unit travel time is again higher than the unit cost of early arrival, then the following relationship hold:

$$\beta_s>\alpha_s>\rho_s>0 \tag{6.15}$$

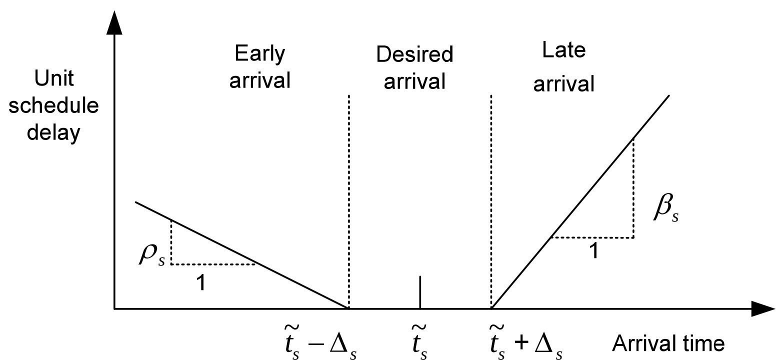

Figure 6-1 An example of disutility function

6.2.2 Generalized Time-dependent Path Travel Cost

The generalized time-dependent path travel cost for route p between O-D pair rs for flows departing origin r at time t is defined as

$$\phi_p^{rs}(t)=\eta_p^{rs}(t)+c_s\left(t+\eta_p^{rs}(t)\right),\quad\forall r,s,p,t \tag{6.16}$$

where $\eta_p^{rs}(t)$ is actual travel time for route p between O-D pair rs for flows departing origin r at time t, $c_s(t)$ is the schedule delay cost for travelers arriving destination s at t.

Let $(\mathrm{Y}, \Sigma, \vartheta)$ be a measure space, and let $F : \mathrm{Y} \to R$ be a function defined on Y and with real values, then the essential infimum of F on Y is defined by

$$\operatorname{ess\,inf} F = \sup\{b \in \Re : \vartheta(\{y : F(y) < b\}) = 0\}$$

and

$$\hat{\phi}_p^{rs}(t) = \operatorname{ess\,inf} \{\phi_p^{rs}(t) : t \in [0, T]\} \qquad \forall r, s$$

The generalized cost between OD pair rs at time t is defined as

$$\phi^{rs}(t) = \min \{\phi_{\mathrm{p}}^{\mathrm{rs}}(t) : p \in P_{rs}\}$$

The minimum generalized cost between OD pair rs during period $[0, T]$ is defined as

$$\pi_{\min}^{rs} = \min \{\hat{\phi}_{\mathrm{p}}^{\mathrm{rs}}(t) : p \in P_{rs}\}$$

6.2.3 Additional Network Constraints

In addition to the network constraints for the DUO problem, the DUOSDTRC problem requires the following additional network constraints

$$q^{rs} = \int_0^T f^{rs}(w) dw \qquad \forall r, s \tag{6.17}$$

(6.17) states that the integral, or the sum in the discrete case, of the time-dependent O-D demand $f^{rs}(t)$ of O-D pair rs over time period $[0, T]$ equals demand q^{rs}, where q^{rs} is the number of vehicles to depart from origin r to destination s during time period $[0, T]$. If the vector $\mathbf{q} = (\cdots, q^{rs}, \cdots)$ is set as the fixed travel demand vector, the problem is dynamic User Optimal simultaneous departure time and route choice with fixed demand. Otherwise, if q^{rs} is

$$q^{rs} = D_{rs}(\pi_{\min}^{rs}) \text{ or } \pi_{\min}^{rs} = D_{rs}^{-1}(q^{rs}) \qquad \forall r, s \tag{6.18}$$

where $D_{rs}(\cdot)$ is the demand function of OD pair rs and $D_{rs}^{-1}(\cdot)$ is the inverse of demand function of OD pair, the problem is dynamic User Optimal simultaneous departure time and route choice with elastic demand (Yang and Meng, 1998; Szeto and Lo, 2004). In this study, the dynamic User Optimal simultaneous departure time and route choice with fixed demand is studied.

6.3 Route-based Variational Inequality (VI) DUOSDTRC Model

6.3.1 DUOSDTRC Conditions

The dynamic User Optimal simultaneous departure time and route choice condition can be written as

$$\eta_p^{rs*}(t) - \pi^{rs*}(t) \geq 0 \quad \forall\ p, r, s; \tag{6.19a}$$

$$f_p^{rs*}(t)\left[\eta_p^{rs*}(t) - \pi^{rs*}(t)\right] = 0 \quad \forall\ p, r, s; \tag{6.19b}$$

$$\pi^{rs*}(t) - \pi_{\min}^{rs*} \geq 0 \quad \forall\ r, s; \tag{6.19c}$$

$$f^{rs*}(t)\left[\pi^{rs*}(t) - \pi_{\min}^{rs*}\right] = 0 \quad \forall\ r, s; \tag{6.19d}$$

$$f_p^{rs}(t) \geq 0 \quad \forall\ p, r, s;$$

(6.19e)

$$f^{rs}(t) \geq 0 \ \forall\ r, s; \tag{6.19f}$$

where the asterisk denotes that the travel disutility is computed using DUOSDTRC time-dependent demand and route flows.

The dynamic User Optimal departure time and route choice conditions require that for each O-D pair *rs* at any time *t*, if there is a positive departure flow $f_p^{rs*}(t) > 0$ over route *p*, the disutility $\phi_p^{rs*}(t)$ for route p must equal the minimal *rs* disutility $\pi_{\min}^{rs^\bullet}$ over time *t*. Furthermore, if the departure flow $f_p^{rs*}(t)$ over route *p* equal 0 at time *t*, the disutility $\phi_p^{rs*}(t)$ over route *p* at time *t* must be greater than or equal to the minimal *rs* disutility $\pi_{\min}^{rs^\bullet}$. In other words, at equilibrium, the actual travel cost of vehicles departing at any time through any used path is equal and minimum and no traveler can reduce his travel cost by unilaterally changing his departure time and route choice combination (Lim et al., 2005). Any departure flow pattern different from the equilibrium pattern will incur more travel cost for some travelers.

6.3.2 Route-based VI Formulation of DUOSDTRC Model

Assume the network is empty at $t = 0$, and only travel demands departing within the departure horizon are considered. The route-based DUOSDTRC continuous VI model can be expressed as

$$\int_0^T \left\{ \left\langle \boldsymbol{\eta}^*(t), \left[\mathbf{f}(t) - \mathbf{f}^*(t)\right] \right\rangle + \left\langle \boldsymbol{\pi}^*(t), \left[\breve{\mathbf{f}}(t) - \breve{\mathbf{f}}^*(t)\right] \right\rangle \right\} dt \geq 0 \qquad (6.20a)$$

or in expanded form as

$$\int_0^T \left\{ \left\langle \sum_{rs} \sum_p \eta_p^{rs^*}(t) \cdot \left[f_p^{rs}(t) - f_p^{rs^*}(t) \right] \right\rangle + \left\langle \sum_{rs} \pi^{rs^\bullet}(t) \left[f^{rs}(t) - f^{rs^*}(t) \right] \cdot \right\rangle \right\} dt \geq 0 \qquad (6.20b)$$

Below we prove traffic status satisfying (6.20) is in a DUO status or equivalent to (6.19).

Proof:

Since at equilibrium $\pi^{rs^\bullet}(t)$ is the same for all flows departing origin *r* toward destination *s* at time *t* and $\pi_{\min}^{rs^\bullet}$ is the same for all flows departing origin *r* toward destination *s*, we have

$$\int_0^T \left\langle \boldsymbol{\pi}^*(t), \left[\mathbf{f}(t) - \mathbf{f}^*(t)\right] \right\rangle dt = 0 \qquad (6.21a)$$

And

$$\int_0^T \left\langle \boldsymbol{\pi}_{\min}^*, \left[\breve{\mathbf{f}}(t) - \breve{\mathbf{f}}^*(t)\right] \right\rangle dt = 0 \qquad (6.21b)$$

Thus, variational inequality (6.20a) and (6.20b) are equivalent to the following variational inequality

$$\int_0^T \left\{ \left\langle \left[\boldsymbol{\eta}^*(t) - \boldsymbol{\pi}^*(t)\right], \left[\mathbf{f}(t) - \mathbf{f}^*(t)\right] \right\rangle + \left\langle \left[\boldsymbol{\pi}^*(t) - \boldsymbol{\pi}_{\min}^*\right], \left[\breve{\mathbf{f}}(t) - \breve{\mathbf{f}}^*(t)\right] \right\rangle \right\} dt \geq 0 \qquad (6.22a)$$

or in expanded form as

$$\int_0^T \left\{ \left\langle \sum_{rs} \sum_p \left[\eta_p^{rs^*}(t) - \pi^{rs^\bullet}(t)\right] \cdot \left[f_p^{rs}(t) - f_p^{rs^*}(t)\right] \right\rangle + \left\langle \sum_{rs} \left[\pi^{rs^\bullet}(t) - \pi_{\min}^{rs^\bullet}\right] \left[f^{rs}(t) - f^{rs^*}(t)\right] \cdot \right\rangle \right\} dt \geq 0 \qquad (6.22b)$$

It is only needed to prove traffic status satisfying (6.22) is in a DUOSDTRC status or equivalent to (6.19).

(iii) **Necessity.** By (6.19a) and (6.19e), $\left[\boldsymbol{\eta}^*(t) - \boldsymbol{\pi}^*(t)\right] \geq 0$, $\mathbf{f} \geq 0$, this implies $\left\langle \left[\boldsymbol{\eta}^*(t) - \boldsymbol{\pi}^*(t)\right], \mathbf{f}(t) \right\rangle \geq 0$. By (6.19b), $\left\langle \left[\boldsymbol{\eta}^*(t) - \boldsymbol{\pi}^*(t)\right], \mathbf{f}^*(t) \right\rangle = 0$.

Thus, we have

$$\left\langle \left[\boldsymbol{\eta}^*(t) - \boldsymbol{\pi}^*(t)\right], \left[\mathbf{f}(t) - \mathbf{f}^*(t)\right] \right\rangle \geq 0 \qquad (6.23a)$$

Similarly, by (6.19c) and (6.19f), $\left[\boldsymbol{\pi}^*(t) - \boldsymbol{\pi}_{\min}^*\right] \geq 0$, $\breve{\mathbf{f}} \geq 0$, this implies $\left\langle \left[\boldsymbol{\pi}^*(t) - \boldsymbol{\pi}_{\min}^*\right], \breve{\mathbf{f}}(t) \right\rangle \geq 0$. By (6.19d), $\left\langle \left[\boldsymbol{\pi}^*(t) - \boldsymbol{\pi}_{\min}^*\right], \breve{\mathbf{f}}^*(t) \right\rangle = 0$

Thus, we have

$$\left\langle \left[\boldsymbol{\pi}^*(t) - \boldsymbol{\pi}_{\min}^*\right], \left[\breve{\mathbf{f}}(t) - \breve{\mathbf{f}}^*(t)\right] \right\rangle \geq 0 \qquad (6.23b)$$

Sum up (6.23a) and (6.23b) and integrating it over $[0, T]$, we have (6.22).

(iv) Sufficiency. (6.19a), (6.19c), (6.19e), and (6.19f) hold by definition. Let the optimal solution of (6.22) be $\mathbf{f}^*$ and $\breve{\mathbf{f}}^*$. To prove (6.19b) holds for $\mathbf{f}^*$ and (6.19d) holds for $\breve{\mathbf{f}}^*$, we first find a feasible solution $\mathbf{f}^{\oplus}$ and $\breve{\mathbf{f}}^{\oplus}$ such that (6.19b) and (6.19d) hold, or $\left\langle \left[\boldsymbol{\eta}^*(t) - \boldsymbol{\pi}^*(t)\right], \mathbf{f}^{\oplus}(t) \right\rangle = 0$ and $\left\langle \left[\boldsymbol{\pi}^*(t) - \boldsymbol{\pi}^*_{\min}\right], \breve{\mathbf{f}}^{\oplus}(t) \right\rangle = 0$.

Suppose (6.19b) does not hold for $\mathbf{f}^*$, we have $\left\langle \left[\boldsymbol{\eta}^*(t) - \boldsymbol{\pi}^*(t)\right], \mathbf{f}^*(t) \right\rangle > 0$. It follows that

$$\left\langle \left[\boldsymbol{\eta}^*(t) - \boldsymbol{\pi}^*(t)\right], \left[\mathbf{f}^{\oplus}(t) - \mathbf{f}^*(t)\right] \right\rangle < 0 \tag{6.24a}$$

Furthermore, we need to consider two cases: 1) (6.19d) holds for $\breve{\mathbf{f}}^*$; 2) (6.19d) does not hold for $\breve{\mathbf{f}}^*$.

In case 1), we have $\left\langle \left[\boldsymbol{\pi}^*(t) - \boldsymbol{\pi}^*_{\min}\right], \breve{\mathbf{f}}^*(t) \right\rangle = 0$. It follows that

$$\left\langle \left[\boldsymbol{\pi}^*(t) - \boldsymbol{\pi}^*_{\min}\right], \left[\breve{\mathbf{f}}^{\oplus}(t) - \breve{\mathbf{f}}^*(t)\right] \right\rangle = 0 \tag{6.24b}$$

Sum up (6.24a) and (6.24b) and integrate the result over $[0, T]$, it follows that

$$\int_0^T \left\{ \left\langle \left[\boldsymbol{\eta}^*(t) - \boldsymbol{\pi}^*(t)\right], \left[\mathbf{f}^{\oplus}(t) - \mathbf{f}^*(t)\right] \right\rangle + \left\langle \left[\boldsymbol{\pi}^*(t) - \boldsymbol{\pi}^*_{\min}\right], \left[\breve{\mathbf{f}}^{\oplus}(t) - \breve{\mathbf{f}}^*(t)\right] \right\rangle \right\} dt < 0 \tag{6.24c}$$

which contradicts (6.22). Thus (6.19b) holds for $\mathbf{f}^*$.

In case 2), we have $\left\langle \left[\boldsymbol{\pi}^*(t) - \boldsymbol{\pi}^*_{\min}\right], \breve{\mathbf{f}}^*(t) \right\rangle > 0$. It follows that

$$\left\langle \left[\boldsymbol{\pi}^*(t) - \boldsymbol{\pi}^*_{\min}\right], \left[\breve{\mathbf{f}}^{\oplus}(t) - \breve{\mathbf{f}}^*(t)\right] \right\rangle < 0 \tag{6.24d}$$

Sum up (6.24a) and (6.24d) and integrate the result over $[0, T]$, we again have (6.24c). Thus (6.19b) holds for $\mathbf{f}^*$.

Similarly, suppose (6.19d) does not hold for $\breve{\mathbf{f}}^*$, we have $\left\langle \left[\boldsymbol{\pi}^*(t) - \boldsymbol{\pi}^*_{\min}\right], \breve{\mathbf{f}}^*(t) \right\rangle > 0$. It follows that

$$\left\langle \left[\boldsymbol{\pi}^*(t) - \boldsymbol{\pi}^*_{\min}\right], \left[\breve{\mathbf{f}}^{\oplus}(t) - \breve{\mathbf{f}}^*(t)\right] \right\rangle < 0 \tag{6.25a}$$

Furthermore, we need to consider two cases: 1) (6.19b) holds for $\mathbf{f}^*$; 2) (6.19b) does not hold for $\mathbf{f}^*$.

In case 1), we have $\left\langle \left[\boldsymbol{\eta}^*(t) - \boldsymbol{\pi}^*(t)\right], \mathbf{f}^*(t) \right\rangle = 0$. It follows that

$$\left\langle \left[\boldsymbol{\eta}^*(t) - \boldsymbol{\pi}^*(t)\right], \left[\mathbf{f}^{\oplus}(t) - \mathbf{f}^*(t)\right] \right\rangle = 0 \qquad (6.25b)$$

Sum up (6.25a) and (6.25b) and integrate the result over $[0, T]$, we have (6.24c), which contradicts (6.22). Thus (6.19d) holds for $\breve{\mathbf{f}}^*$.

In case 2), we have $\left\langle \left[\boldsymbol{\eta}^*(t) - \boldsymbol{\pi}^*(t)\right], \mathbf{f}^*(t) \right\rangle > 0$. It follows that

$$\left\langle \left[\boldsymbol{\eta}^*(t) - \boldsymbol{\pi}^*(t)\right], \left[\mathbf{f}^{\oplus}(t) - \mathbf{f}^*(t)\right] \right\rangle < 0 \qquad (6.25c)$$

Sum up (6.25a) and (6.25c) and integrate the result over $[0, T]$, we have (6.24c), which contradicts (6.22). Thus (6.19d) holds for $\breve{\mathbf{f}}^*$.

6.3.3 Solution Algorithms for Route-based VI DUOSDTRC Model

To solve the DUOSDTRC problem, the continuous VI formulation is discretized with each time interval being the assignment increment. The estimated actual travel time on each link a is a multiple of the time increment and is fixed at each time increment, i.e.

$$\bar{\tau}_a(k) = i \qquad \text{if} \quad (i - 0.5)\Delta t \le \tau_a(k) \le (i + 0.5)\Delta t \qquad (6.26)$$

where i is an integer and $0 \le i \le K$, Δt is time increment. This round-off method is used only in the flow propagation constraints. The round-off error can be made as small as desired by making the assignment increment smaller.

The route-based DUOSDTRC discrete-time VI formulation is

$$\left\langle \boldsymbol{\eta}^*, \left[\mathbf{f} - \mathbf{f}^*\right] \right\rangle + \left\langle \boldsymbol{\pi}^*, \left[\breve{\mathbf{f}} - \breve{\mathbf{f}}^*\right] \right\rangle \ge 0 \qquad (6.27a)$$

where $\boldsymbol{\eta} \in \Re_+^{|P| \times K_0}$, $\mathbf{f} \in \Re_+^{|P| \times K_0}$, $\breve{\mathbf{f}} \in \Re_+^{|R \times S| \times K_0}$, $|P|$ and $|R \times S|$ are the cardinalities of the path set and O-D pairs, etc., or in expanded form as

$$\sum_{rs}\sum_{k=1}^{K_0}\left\{\sum_{p}\eta_p^{rs^*}(k)\cdot\left[f_p^{rs}(k)-f_p^{rs^*}(k)\right]+\pi^{rs^\bullet}(k)\cdot\left[f^{rs}(k)-f^{rs^*}(k)\right]\right\}\geq 0 \qquad (6.27b)$$

in Θ . Θ is the feasible region defined by the following constraints:

O-D demand conservation constraints:

$$\sum_{k=1}^{K_0} f^{rs}(k)=q^{rs} \quad \forall\, r,s \qquad (6.28)$$

Path flow conservation constraints:

$$\sum_{p} f_p^{rs}(k)= f^{rs}(k) \;\; \forall\, k,r,s \qquad (6.29)$$

Link inflow conservation constraints:

$$\sum_{rs} u_a^{rs}(k)=u_a(k) \quad \forall\, a,k \qquad (6.30)$$

Link outflow conservation constraints:

$$\sum_{rs} v_a^{rs}(k)=v_a(k) \quad \forall\, a,k \qquad (6.31)$$

Node flow conservation constraints:

$$\sum_{a\in B(j)} v_a^{rs}(k)= \sum_{a\in A(j)} u_a^{rs}(k) \quad \forall\; j\neq r,s; r,s\,;k \qquad (6.32)$$

where $A(j)$ is the set of links after j and $B(j)$ is the set of links before j .

Link flow propagation constraints:

$$u_a^{rs}(k)=v_a^{rs}\left(k+\tau_a(k)\right) \quad \forall\, a,\; r,s,k \qquad (6.33)$$

The link state equations:

$$x_a(k+1)=x_a(k)+u_a(k)-v_a(k) \quad \forall\, a\,,k \qquad (6.34a)$$

or

$$x_a(k+1)=x_a(k)+u_a(k+1)-v_a(k+1) \quad \forall\, a\,,k \qquad (6.34b)$$

(6.34a) is forward formula, (6.34b) is backward formula.

Path-link flow incidence constraints:

$$u_a^{rs}(n) = \sum_{rs}\sum_{p}\sum_{k=1}^{K_0} f_p^{rs}(k)\delta_{rsa}^{pkn} \quad \forall\, a, n \tag{6.35}$$

where $\delta_{rsa}^{pkn} \in \{0,1\}$ is defined as:

$$\delta_{rsa}^{pkn} = \begin{cases} 1 & \text{if traffic departing origin } r \text{ at any time interval } k \\ & \text{heading for destination } s \text{ on path } p \text{ arrives at link } a \\ & \text{during the } n\text{th time interval.} \\ 0 & \text{otherwise} \end{cases} \tag{6.36}$$

Nonnegative constraints:

$$f^{rs}(k) \geq 0,\; f_p^{rs}(k) \geq 0, u_a^{rs}(k) \geq 0,\; \forall\, k, r, s, a, p \tag{6.37}$$

Relaxation

At each relaxation, we temporarily fix: 1) Actual travel time $\tau_a(k)$ in the link flow propagation constraints as $\bar{\tau}_a(k)$; 2) Actual travel time $\tau_a[k + \pi^{ri}(k)]$ as $\tau_a[k + \bar{\pi}^{ri}(k)]$. At each relaxation, the time-space network is fixed with fixed link flow propagation constraints and fixed time dependent O-D demand.

Via relaxation, the VI cost term becomes $\eta_p^{rs}(k)$ and $\pi^{rs}(k)$, where

$$\eta_p^{rs}(k) = \sum_{k=1}^{K_0}\sum_{a} \tau_a(n)\delta_{rsa}^{pkn} \tag{6.38a}$$

$$= \tau_{a_1}(k) + \tau_{a_2}\left(k + \bar{\tau}_{a_1}(k)\right) + \cdots + \tau_{a_{\bar{p}}}\left(k + \bar{\eta}_p^{ra_{(\bar{p}-1)}}(k)\right) \tag{6.38b}$$

where $p = (a_1, a_2, \cdots\cdots a_{\bar{p}})$, a_i is the link number of path p of O-D pair rs at time k, and

$$\pi^{rs}(k) = \min_{p}\left\{\phi_p^{rs}(k) := \eta_p^{rs}(k) + c_s\left(k + \eta_p^{rs}(k)\right)\right\} \quad \forall r, s, k \tag{6.38c}$$

Optimization Problem

An optimization problem which is equivalent to the discrete VI under relaxation can thus be formulated, as follows:

$$\min_{\mathbf{f},\breve{\mathbf{f}}} Z = \sum_{k=1}^{K_0} \sum_{rs} \left\{ \sum_{p} \int_0^{f_p^{rs}(k)} \eta_p^{rs}\left(\omega, \mathbf{f}_p^{rs}\right) d\omega + \int_0^{f^{rs}(k)} \pi^{rs}\left(\omega, \breve{\mathbf{f}}^{rs}\right) d\omega \right\} \quad (6.39)$$

in Θ, where $\mathbf{f}_p^{rs}$ denotes the path flow vector $\mathbf{f}$ without component $f_p^{rs}(k)$, $\breve{\mathbf{f}}^{rs}$ denotes the path flow vector $\breve{\mathbf{f}}$ without component $f^{rs}(k)$.

The gradient of (6.39) is

$$\frac{\partial Z}{\partial f_p^{rs}(k)} = \eta_p^{rs}(k) \quad (6.40a)$$

$$\frac{\partial Z}{\partial f^{rs}(k)} = \pi^{rs}(k) \quad (6.40b)$$

(6.40) is equivalent to the cost term of discrete VI (6.27b) under relaxation. This indicates the above optimization program is equivalent to the discrete VI (6.27). Since all cross effects are fixed in each relaxation, $f_p^{rs}(k)$ and $f^{rs}(k)$ are the only variables for each summation term of (6.39).

Program (6.39) can be solved by gradient projection (GP) algorithm (Jayakrishnan et al, 1994). The formulation of the GP algorithm focuses on the time-dependent traffic demand constraints

$$\sum_{p \in P_{rs}^k} f_p^{rs}(k) = f^{rs}(k) \quad \forall\, k, r, s \quad (6.41)$$

where P_{rs}^k is the set of paths (with positive flow) between origin *r* and destination *s* at time intervals *k*, and O-D demand constraints:

$$\sum_{k=1}^{K_0} f^{rs}(k) = q^{rs} \quad \forall\, r,s \tag{6.42}$$

If we express the shortest-path flows $f_{\bar{p}_{rs}(k)}$ in terms of other path flows

$$f_{\bar{p}_{rs}(k)} = f^{rs}(k) - \sum_{\substack{p\in P_{rs}^k \\ p\neq \bar{p}_{rs}(k)}} f_p^{rs}(k) \qquad \forall\, k,r,s \tag{6.43}$$

and the shortest generalized time-dependent O-D demand $f_{\min}^{rs}$ (the time-dependent O-D demand corresponding to $\pi_{\min}^{rs}$) in terms of other time-dependent O-D demands

$$f_{\min}^{rs} = q^{rs} - \sum_{\substack{k=1 \\ f^{rs}(k)\neq f_{\min}^{rs}}}^{K_0} f^{rs}(k) \qquad \forall\, r,s \tag{6.44}$$

The optimization problem (6.5) at Θ can be restated as

$$\min \tilde{Z}\left(\tilde{\mathbf{f}}, \tilde{\tilde{\mathbf{f}}}\right) \tag{6.45}$$

at $\tilde{\Theta}$. $\tilde{\Theta}$ is the feasible set defined by (6.29) --- (6.37). where $\tilde{Z}$ is the new objective function, $\tilde{\mathbf{f}}$ is the set of non-shortest-path flows between all of the O-D pairs at any departure time intervals *k*, and $\tilde{\tilde{\mathbf{f}}}$ is the set of non-shortest generalized time-dependent O-D demand of all the O-D pairs.

The gradient of the objective function written in terms of the non-shortest-path variables can be found using

$$\frac{\partial \tilde{Z}}{\partial f_p^{rs}(k)} = \frac{\partial Z}{\partial f_p^{rs}(k)} - \frac{\partial Z}{\partial f_{\bar{p}_{rs}(k)}} \quad \text{where} \quad p \in P_{rs} \text{ and } \quad p \neq \bar{p}_{rs}(k) \tag{6.46}$$

which results from the definition of $\tilde{Z}$. Each component of the gradient vector is the difference between the first derivative lengths of a path and the corresponding shortest path and the first derivative lengths are simply the dynamic path cost at that flow solution.

The gradient of the objective function written in terms of the non-shortest-demand variables can be found using

$$\frac{\partial \tilde{Z}}{\partial f^{rs}(k)} = \frac{\partial Z}{\partial f^{rs}(k)} - \frac{\partial Z}{\partial f^{rs}_{\min}} \tag{6.47}$$

which results from the definition of $\tilde{Z}$. Each component of the gradient vector is the difference between the average generalized path cost of the shortest paths of a non-shortest generalized time-dependent O-D demand of a O-D pair and the average generalized path cost of the shortest paths of the shortest generalized time-dependent O-D demand of the O-D pair.

The second derivative with respect to $f_p^{rs}(k)$ is simply the sum of the second derivative lengths of the links on either path p or path $\overline{p}_{rs}(k)$, but not on both. The second derivative with respect to $f^{rs}(k)$ is the sum of the second derivative lengths of the links on either shortest paths of a non-shortest generalized time-dependent O-D demand of a O-D pair $f^{rs}(k)$ or shortest paths of a shortest generalized time-dependent O-D demand $f^{rs}_{\min}$ of the O-D pair, but not on both.

Once the second derivatives of $\tilde{Z}$ with respect to each path flow $f_p^{rs}(k)$ and time-dependent O-D demand $f^{rs}(k)$ are calculated, the inverse of each second derivative gives an approximate quasi-Newton step size for updating each path flows and time-dependent O-D demand.

It is better to keep α^n a constant (i.e., $\alpha^n = \alpha$, $\forall n$). It can be shown that given any starting set of path flows there exists an $\overline{\alpha}$ such that if $\alpha \in (0, \overline{\alpha})$ the sequence generated by this algorithm converges to the optimum point, provided that the link-cost functions are convex. Our experience shows that α equal to 0.5 achieves a very good convergence rate.

Algorithm

Based on the above rational analysis, the algorithm for solving the ideal route-based DUOSDTRC model is developed and summarized as follows.

Step 0: Outer Initialization.

Set an initial feasible solution of $\left[f^{rs(0)}(k)\right]$. Compute $k_{\max} = \max_{\forall rs}\{\pi^{rs}\}$, where π^{rs} is the static minimum travel time of O-D rs. Set $K' = K_0 + C \cdot [k_{\max}]_+$. Set $\hat{\tau}_a^{(0)}(k) = \tau_a[0]$, $\forall a \in A$, $k = 1, \cdots, K'$. Set outer iteration counter $L = 0$. Set an outer iteration convergence criterion ε_{out}.

Step 1: Middle Initialization.

Find an initial feasible solution $\left[f_p^{rs(0)}(k)\right]$. Set middle iteration counter $l = 0$. Set an middle iteration convergence criterion ε_{mid}.

Step 2: Relaxation.

Find a new estimation of actual link travel times: $\hat{\tau}_a^{(l)}(k) = \tau_a\left[x_a^*(k)\right]$, find $\bar{\tau}_a^{(l)}(k)$ $\forall a \in A$, $k = 1, \cdots, K'$, where * denotes the solution obtained from the most recent inner iteration or from middle initialization. Find $\delta_{a'}^{k(l)}(t)$ and $\delta_{a''}^{k(l)}(t)$.

Step 3: Inner Iteration

Step 3.0: Inner Initialization. Compute and reset the inner initial feasible solution to be consistent with the flow propagation constrain at the current relaxation. Set an inner iteration counter $m = 1$.

In the first relaxation, set $\tau_a^{(1)}(k)$ equal to free flow travel time $\tau_a(0), \forall a$, k=1,…, K. and perform all-or-nothing assignments. This yields initial path flows $f_{(1)}^{rs}(k), \forall\ r, s$, k=1,…,K_0.

In other relaxations, reset the most inner iteration solution to be consistent with the flow propagation constrain at the current relaxation, and set them as initial path flows $f_{(1)}^{rs}(k), \forall\ r,s$, k=1,…,K_0, at current relaxation (Initialize the path set P_{rs}^k with the shortest path for each O-D pair *rs* at time *k*).

Step 3.1: Update. Set $\tau_a^{(m)}(k)$ equal to $\tau_a^{(m)}\left[x_a^{(m)}(k)\right]$. Update the first derivative lengths $d_p^{rs(m)}(k)$(i.e., path cost at current flow) of all of the paths in P_{rs}^k, $\forall\ r,s$.

Step 3.2: Direction finding for $\left[f_p^{rs}(k)\right]$. Find the shortest-path $\bar{p}_{rs}^{(m)}(k)$ from each origin *r* to each destination *s* at *k* on the basis of $\tau_a^{(m)}(k)$. If different from all the paths in the existing path set in P_{rs}^k, (no need for path comparison here; just compare $d_p^{rs(m)}(k)$, add it to in P_{rs}^k and record $d_{\bar{p}_{rs}^{(m)}(k)}$. If not tag the shortest among the paths in P_{rs}^k in $d_{\bar{p}_{rs}^{(m)}(k)}$.

Step 3.3: Move for $\left[f_p^{rs}(k)\right]$. Set the new path flows.

$$f_p^{rs(m+1)}(k)=\max\left[0, f_p^{rs(m)}(k)-\frac{\alpha^n}{s_p^{rs(m)}(k)}\left(d_p^{rs(m)}(k)-d_{\bar{p}_{rs}^{(m)}(k)}\right)\right]\forall\ r,s,p\in P_{rs}^k, p\neq \bar{p}_{rs}^{(m)}(k) \tag{6.48}$$

Where

$$s_p^{rs(m)}(k)=\sum_a\sum_k\frac{\partial\tau_a^{(m)}(k)}{\partial x_a^{(m)}(k)}, \forall\ p\in P_{rs}^k \tag{6.49}$$

a and *k* denotes time-space links that are on either *p* or $\bar{p}_{rs}^{(m)}(k)$, but not on both, and α^n is a scalar step-size modifier.

Also,

$$f_{\bar{p}_{rs}(k)}^{(m+1)} = f^{rs}(k)-\sum_{\substack{p\in P_{rs}^k\\ p\neq\bar{p}_{rs}(k)}} f_p^{rs(m+1)}(k) \quad \forall\ r,\mathrm{s},\mathrm{k} \tag{6.50}$$

Assign the flows on the trees and find the link flows $u_a^{(m+1)}(k)$.

Step 3.4: Convergence Test for Inner Iteration.

If $f_p^{rs(m+1)}(k) \cong f_p^{rs(m)}(k)$, set $\hat{f}_a^{rs(l)}(k) = f_p^{rs(m+1)}(k)$, $\hat{x}_a^{(l)}(k) = x_a^{(m+1)}(k)$, go to Step 4; otherwise, set $m = m+1$, go to Step 3.1.

Step 4: Convergence Test for Middle Iteration. If $\hat{\tau}_a^{(l)}(k) \cong \hat{\tau}_a^{(l-1)}(k)$, go to step 5; otherwise, set $l = l+1$ and go to Step 2.

Step 5: Outer Iteration

Step 5.1: Direction finding for $\left[f^{rs}(k)\right]$. Find the average generalized path cost $d^{rs(L)}(k)$ of the shortest paths for all the time-dependent O-D demand. Tag the average generalized path cost $d_{\min}^{rs(L)}$ of the shortest paths of the shortest generalized time-dependent O-D demand for all the O-D pairs.

Step 5.2: Move for $\left[f^{rs}(k)\right]$. Set the new time-dependent O-D demand.

$$f^{rs(L+1)}(k) = \max\left[0, f^{rs(L)}(k) - \frac{\alpha^L}{s^{rs(L)}(k)}\left(d^{rs(L)}(k) - d_{\min}^{rs(L)}\right)\right] \forall\ r, s, f^{rs}(k) \neq f_{\min}^{rs} \qquad (6.51)$$

Where

$$s^{rs(L)}(k) = \sum_a \sum_k \frac{\partial \tau_a(k)}{\partial x_a(k)} \quad \forall\ r, \mathrm{s}, \mathrm{k} \qquad (6.52)$$

a and k denotes time-space links on either shortest paths of the non-shortest generalized time-dependent O-D demand $f^{rs}(k)$ or shortest paths of the shortest generalized time-dependent O-D demand $f_{\min}^{rs}$ of the O-D pair, but not on both. α^L is a scalar step-size modifier.

Also,

$$f_{\min}^{rs(L+1)} = q^{rs} - \sum_{f^{rs}(k) \neq f_{\min}^{rs}} f^{rs(L+1)}(k) \quad \forall\ r, \mathrm{s} \qquad (6.53)$$

Step 5.3: Convergence Test for Outer Iteration. If $f^{rs(L)}(k) \cong f^{rs(L-1)}(k)$, stop; otherwise, set $L = L + 1$ and go to Step 1.

The flowchart of the solution algorithm is shown in Figure 6.2.

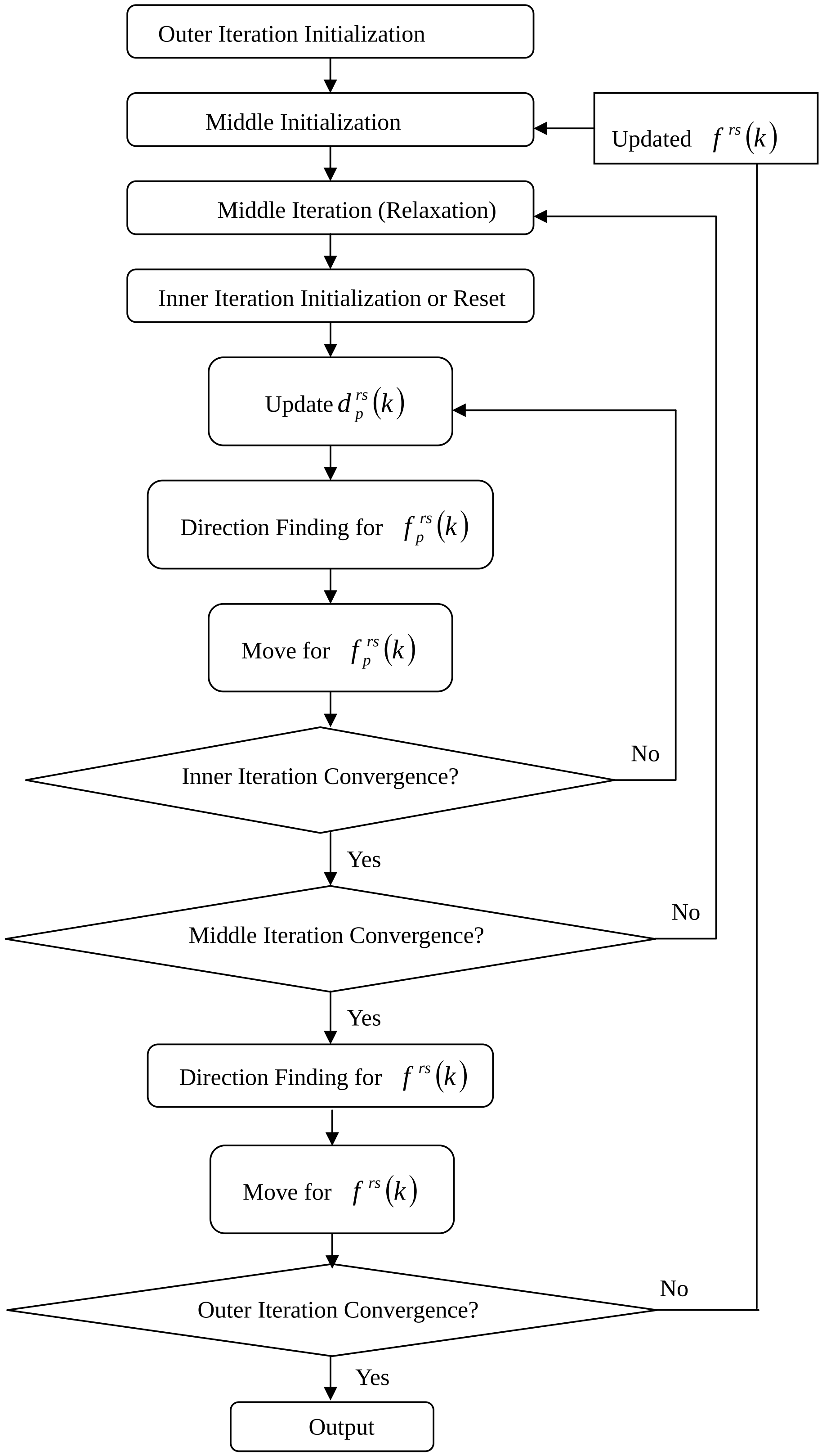

Figure 6-2 Flowchart of the Solution Algorithm

6.3.4 A Numerical Example

Example 6.2

An example is presented below to validate the above model and algorithm. The configuration of the network is the same as Figure 4.6. In the network, each link is assumed as a one-lane street with a length of 0.5 mi. The free flow speed is assumed to be 25 mile/hour. The following linear travel time function is used to enforce FIFO condition: $\tau_a(k) = L_a / s_f + 0.3 \cdot x_a(k)$, where L_a is the length of link a, s_f is free flow speed, $\tau_a(k)$ is link travel time on link a at time k, $x_a(k)$ is number of vehicles on link a at time k. Four O-D pairs are considered. The total O-D demand for each O-D pair is 50 vehicle units. Five 20 s departure time intervals are specified. The initial time-dependent O-D demand are 10 vehicle units per time interval. The O-D pairs and initial time-dependent O-D demand are shown in Table 6.3.

Table 6-3 O-D pairs and initial time-dependent O-D demand for example 4.1

O-D	Departure time interval *k*				
	1	2	3	4	5
1-9	10	10	10	10	10
9-1	10	10	10	10	10
3-7	10	10	10	10	10
7-3	10	10	10	10	10

We considered two cases: 1) the disutility function is not considered; 2) the disutility function is considered. At optimal point, the actual path travel time for all the path flows are equal when the disutility function is not considered, while the generalized path travel time for all the path flows are equal when the disutility function is considered.

The program of the algorithm was run on a computer with 1.5 GHz frequency processor. The inner iteration convergence test method was set as a prespecified number n. The middle iteration convergence test method was set as $\max\{|\tau_a^{(l)}(k)-\tau_a^{(l-1)}(k)|\,|a\in A, k=1,\cdots,K\}$, where $|\tau_a^{(l)}(k)-\tau_a^{(l-1)}(k)|$ is the actual travel time difference of link a at time k between successive middle iterations. The outer iteration convergence test method was set as $\max\{|f^{rs(L)}(k)-f^{rs(L-1)}(k)|\,|rs\in R\times S, k=1,\cdots,K_0\}$, where $|f^{rs(L)}(k)-f^{rs(L-1)}(k)|$ is the difference of time-dependent O-D demand of O-D pair rs at time k between successive outer iterations.

The operation of the program when disutility function is not considered is shown in Table 6.4. The corresponding optimal time-dependent O-D demand is shown in Table 6.5. The assignment horizon K is found to be 21 time increments. Table 6.6a shows the output of $u_a^{rs}(k)$. Table 6.6b shows the output of $v_a^{rs}(k)$. Table 6.6c shows the output of $u_a(k)$. Table 6.6d shows the output of $v_a(k)$. Table 6.6e shows the output of $x_a(k)$. Table 6.6f shows the output of $\tau_a(k)$. Table 6.6g shows the output of $f_p^{rs}(k)$, $c_p^{rs}(k)$, links on each path and the arrival time interval for each link on a path. For conciseness, only Table 6.6g is attached to this dissertation.

Table 6-4 Convergence criterion and computation time for Example (no disutility function)

ε_{in}	ε_{mid}	ε_{out}	Number of Outer iterations	Total computation time (minute)
n=4	0.002	0.095	32	101

Table 6-5 Resultant time-dependent O-D demand (without disutility function)

O-D	Departure time interval k				
	1	2	3	4	5
1-9	25.1083	1.2949	0	0	23.5967
9-1	25.1083	1.2949	0	0	23.5967
3-7	25.1083	1.2949	0	0	23.5967
7-3	25.1083	1.2949	0	0	23.5967

Table 6-6 The resultant path flow and path travel time for example 6.2

Path number	O	D	k	Path flow	Path time	Links on the path	Arrival time for each link on the path
This table is appendix 7 of this thesis.							

We take the following examples to verify that the solution satisfy the constraints and conditions of DUOSDTRC.

O-D demand conservation constraints (6.28):

$$q^{19} = f^{19}(1) + f^{19}(2) + f^{19}(3) + f^{19}(4) + f^{19}(5)$$

$= 25.1083+1.2949+0+0+23.5967$

$= 50$

Path flow conservation constraint (6.29):

$f^{19}(1) = f_1^{19}(1) + f_2^{19}(1) + f_3^{19}(1) + f_4^{19}(1)$

$= 8.3774+4.1892+8.3627+4.179$

$= 25.1083$

$f^{19}(2) = f_1^{19}(2) + f_2^{19}(2) + f_3^{19}(2) + f_4^{19}(2)$

$= 0.3638+0.2152+0.4591+0.2568$

$= 1.2949$

Link inflow conservation constraint (6.30):

$$u_8^{91}(10) + u_8^{73}(10) = 0.2152+0.2152=0.4305= u_8(10)$$

Link outflow conservation constraint (6.31):

$$v_8^{91}(14) + v_8^{73}(14) = 0.2152+0.2152=0.4305= v_8(14)$$

Node flow conservation constraint (6.32):

$$\sum_{a \in B(6)} v_a^{rs}(k) = \sum_{a \in B(6)} v_a(k) = v_9(9) + v_{14}(9) + v_{20}(9) = 11.7981+0+11.7986=23.5967$$

$$\sum_{a \in A(6)} u_a^{rs}(k) = \sum_{a \in A(6)} u_a(k) = u_{10}(9) + u_{12}(9) + u_{19}(9) = 7.7337+7.8498+8.0132=23.5967$$

Link flow propagation constraint (6.33):

$$u_8^{91}(10) = v_8^{91}(10 + \bar{\tau}_8(10)) = v_8^{91}(14) = 0.2152$$

$$u_8^{73}(10) = v_8^{73}(10 + \bar{\tau}_8(10)) = v_8^{73}(14) = 0.2152$$

where $\tau_8(10)=1.2439$ minutes. For a time increment of 20 seconds, $\bar{\tau}_8(10)=4$.

The link state equation (6.34b):

$$x_8(10) = x_8(9) + u_8(10) - v_8(10) = 8.3785 + 0.4305 - 0 = 8.8089$$

The actual travel times on the used paths from origin 1 toward destination 9 departing at time increment 1 are as follows:

$c_1^{19}(1) =$

$\tau_5(1) + \tau_{15}(1+\bar{\tau}_5(1)) + \tau_{23}(1+\bar{\tau}_5(1)+\bar{\tau}_{15}(1+\bar{\tau}_5(1))) + \tau_{24}(1+\bar{\tau}_5(1)+\bar{\tau}_{15}(1+\bar{\tau}_5(1))+\bar{\tau}_{23}(1+\bar{\tau}_5(1)+\bar{\tau}_{15}(1+\bar{\tau}_5(1))))$

$= \tau_5(1) + \tau_{15}(5) + \tau_{23}(9) + \tau_{24}(13)$

= 1.2627+1.2418+1.2418+1.2627

= 5.009 minutes $\cong$ 5.0 minutes

Similarly, we have $c_p^{rs}(k) \cong 5.0$ minutes, $\forall\, r,s,k$. They are nearly equal, which is consistent with the DUOSDTRC condition.

As can be checked in the same way, all the solution output satisfies the constraints and conditions for DUOSDTRC problem. This verifies the rationale of the above model and solution algorithm.

In the second case, we assume that the beginning of the departure horizon is time 0 and define the parameters of the disutility function as $\tilde{t}_s = 370$, $\Delta_s = 30$, $\rho_s = 0.5$, $\beta_s = 1, \forall\, s$. The interval that incurs zero disutility is [340, 400] in seconds, or [5.6667, 6.6667] in minutes. Vehicles arriving before 340 second or 5.6667 minute incur early arrival disutility. Vehicles arriving after 400 second or 6.6667 minute incur late arrival disutility. The operation of the program is shown in Table 6.7. The corresponding resultant time-dependent O-D demand is shown in Table 6.8. The assignment horizon K is found to be 21 time increments. Table 6.9a shows the output of $u_a^{rs}(k)$. Table 6.9b shows the output of $v_a^{rs}(k)$. Table 6.9c shows the output of $u_a(k)$. Table 6.9d shows the output of $v_a(k)$. Table 6.9e shows the output of $x_a(k)$. Table 6.9f shows the output of $\tau_a(k)$. Table 6.9g shows the output of $f_p^{rs}(k), c_p^{rs}(k), \phi_p^{rs}(k)$, links on each path and the arrival time interval for each link on a path. For conciseness, only Table 6.9g is attached to this dissertation.

Table 6-7 Convergence criterion and computation time for Example (with disutility function)

ε_{in}	ε_{mid}	ε_{out}	Number of Outer iterations	Total computation time (minute)
n=4	0.002	0.05	20	62

Table 6-8 Resultant time-dependent O-D demand (with disutility function)

O-D	Departure time interval k				
	1	2	3	4	5
1-9	16.6399	28.2918	4.1235	0.9448	0
9-1	17.5245	27.6569	4.0812	0.7375	0
3-7	16.6399	28.2918	4.1235	0.9448	0
7-3	17.5245	27.6569	4.0812	0.7375	0

Table 6-9 The resultant path flow and path travel time for example 6.2

Path number	O	D	k	Path flow	Path time	Links on the path	Arrival time for each link on the path
This table is appendix 8 of this thesis.							

We take the following examples to verify that the solution satisfy the constraints and conditions of DUOSDTRC.

O-D demand conservation constraints (6.28):

$$q^{19} = f^{19}(1) + f^{19}(2) + f^{19}(3) + f^{19}(4) + f^{19}(5)$$

= 16.6399+28.2918+4.1235+0.9448+0

= 50

Path flow conservation constraint (6.29):

$$f^{19}(1) = f_1^{19}(1) + f_2^{19}(1) + f_3^{19}(1) + f_4^{19}(1) + f_5^{19}(1) + f_6^{19}(1)$$

= 5.6186+2.5586+5.6217+2.561+0.2126+0.0675

= 16.64

$$f^{19}(2) = f_1^{19}(2) + f_2^{19}(2) + f_3^{19}(2) + f_4^{19}(2) + f_5^{19}(2) + f_6^{19}(2)$$

= 9.3766+4.6948+9.382+4.7102+0.1215+0.0068

= 28.2919

Link inflow conservation constraint (6.30):

$$u_8^{91}(10) + u_8^{73}(10) = 4.6182+4.6182=9.2364= u_8(10)$$

Link outflow conservation constraint (6.31):

$$v_8^{91}(14) + v_8^{73}(14) = 4.6182+4.6182=9.2364= v_8(14)$$

Node flow conservation constraint (6.32):

$$\sum_{a \in B(6)} v_a^{rs}(k) = \sum_{a \in B(6)} v_a(k) = v_9(8) + v_{14}(8) + v_{20}(8) = 0.4876+0+0.4072=0.8948$$

$$\sum_{a \in A(6)} u_a^{rs}(k) = \sum_{a \in A(6)} u_a(k) = u_{10}(8) + u_{12}(8) + u_{19}(8) = 0.2417+0.3392+0.3139=0.8948$$

Link flow propagation constraint (6.33):

$$u_8^{91}(10) = v_8^{91}(10 + \bar{\tau}_8(10)) = v_8^{91}(14) = 4.6182$$

$$u_8^{73}(10) = v_8^{73}(10 + \bar{\tau}_8(10)) = v_8^{73}(14) = 4.6182$$

where $\tau_8(10)$=1.2753 minutes. For a time increment of 20 seconds, $\bar{\tau}_8(10)$=4.

The link state equation (6.34b):

$$x_8(10) = x_8(9) + u_8(10) - v_8(10) = 5.8449 + 9.2364 - 0 = 15.0813$$

The actual travel times on the used paths from origin 1 toward destination 9 departing at time increment 1 are as follows:

$c_1^{19}(1) =$

$\tau_5(1) + \tau_{15}(1+\bar{\tau}_5(1)) + \tau_{23}(1+\bar{\tau}_5(1)+\bar{\tau}_{15}(1+\bar{\tau}_5(1))) + \tau_{24}(1+\bar{\tau}_5(1)+\bar{\tau}_{15}(1+\bar{\tau}_5(1))+\bar{\tau}_{23}(1+\bar{\tau}_5(1)+\bar{\tau}_{15}(1+\bar{\tau}_5(1))))$

$= \tau_5(1) + \tau_{15}(5) + \tau_{23}(9) + \tau_{24}(13)$

= 1.2627+1.2418+1.2418+1.2627

=1.2411+1.2280+1.2280+1.2419

= 4.939 minutes

The arrival time for $f_1^{19}(1)$ is $k \cdot \Delta t / 60 + c_1^{19}(1) = 1 * \frac{20}{60} + 4.939 < 5.6667$

The early arrival disutility is

$$\rho_s \cdot \left[5.6667 - k \cdot \Delta t / 60 - c_1^{19}(1)\right] = 0.5\left[5.6667 - 1 * \frac{20}{60} - 4.939\right] = 0.19717$$

The generalized path cost $\phi_1^{19}(1) = 4.939 + 0.19717 = 5.1361$ minutes.

Similarly, the path cost for $f_1^{19}(2)$ is $c_1^{19}(2) = 5.174$, the arrival time is

$k \cdot \Delta t / 60 + c_1^{19}(2) = 2 * \frac{20}{60} + 5.174 \in [5.6667, 6.6667]$,

Since no disutility incurred, $\phi_1^{19}(2) = c_1^{19}(2)$.

The arrival time for $f_1^{19}(5)$ is

$$k \cdot \Delta t / 60 + c_1^{19}(5) = 5 * \frac{20}{60} + 5.0772 = 6.7437 > 6.6667$$

The late arrival disutility is

$$\beta_s \cdot \left[k \cdot \Delta t / 60 + c_1^{19}(5) - 6.6667\right] = 1 \cdot (6.7437 - 6.6667) = 0.0772$$

The generalized path cost $\phi_1^{19}(5)$ =5.0772+0.0772=5.1544

The generalized path costs for all the departure flows from the same O-D pair are approximately equal, which is consistent with the DUOSDTRC condition.

As can be checked in the same way, all the solution output satisfies the constraints and conditions for DUOSDTRC problem. This verifies the rationale of the above model and solution algorithm.

Chapter 7: NEW Combined Dynamic Travel Choice Models

When both transit and passenger cars are available, a shift of travelers from cars to transit or from low-occupancy cars to high-occupancy cars may significantly decrease road congestions and increase the efficiency of the overall transportation system. In order to balance allocations to various departure times and different modes, an integrated model including all elements (mode, departure time, and route choice) needs to be constructed.

Apart from choosing departure time to begin their trip and choosing alternative routes toward their destinations, people may choose different transportation modes to travel. The combined mode split and dynamic user optimal simultaneous departure time and route choice problem (MS DUOSDTRC) extends the DUOSDTRC route choice model in one respect: transportation mode, departure time and route over a road network must be chosen. For simplicity it is assumed with two modes: transit and passenger car. In DUOSDTRC problem, the total O-D demand of each O-D pair is in the mode of passenger car and is given. In MS DUOSDTRC problem, the total O-D demand includes demands of transit and passenger car and is given, while the share of each mode needs to be solved. Any change in the demand share alters the time-dependent O-D pattern in the network, so route and departure time decisions of other travelers will be affected. At equilibrium of MS DUOSDTRC, the same travel cost should be incurred for all passenger car drivers of the same O-D pair departing at all times, and should be equal to the transformed O-D travel cost of the transit of the same

O-D pair.

When routes from an origin to a destination are congested, people may choose another destination to fulfill their need. For example, if the routes to a shopping center are congested, people may choose another shopping center as their destinations. In such case, the distributions of trips change. The combined trip distribution and dynamic user optimal simultaneous departure time and route choice problem (TD DUOSDTRC) extends the DUOSDTRC route choice model in another respect: destination, departure time and route over a road network must be chosen. In DUOSDTRC problem, the total O-D demand of each O-D pair is given and fixed. In TD DUOSDTRC problem, the trip generation of each origin and trip attraction of each destination is given and fixed, while the demand of each O-D pair needs to be solved for. At equilibrium, not only the conditions for DUOSDTRC are satisfied, the consistency of trip distribution and dynamic travel impedance among zones are also guaranteed.

Based on the travel information provided, the available travel mode, and the congestion level of the roads, people may choose different destinations, travel modes, departure times and routes to fulfill their travel needs. The combined trip distribution mode split and dynamic user optimal simultaneous departure time and route choice problem (TD MS DUOSDTRC) extends the DUOSDTRC route choice model in two respects: destination, mode, departure time and route over a road network must be chosen. In TD MS DUOSDTRC problem, the trip generation of each origin and trip attraction of each destination is given and fixed, while the demand of each mode of each O-D pair needs to be solved for. At equilibrium of TD MS

DUOSDTRC, the same cost should be incurred for all passenger car drivers of the same O-D pair departing at all time, and should equal the transformed O-D cost of the transit of the same O-D pair, and the consistency of trip distribution and dynamic travel impedance among zones are guaranteed.

This chapter presents a series of combined dynamic travel choice models and solution algorithms for them for a general network with multiple origin-destination pairs. Section 7.1 presents a combined mode split and dynamic user optimal simultaneous departure time and route choice model (MS DUOSDTRC) and its solution algorithm. Section 7.2 presents a combined trip distribution and dynamic user optimal simultaneous departure time and route choice model (TD DUOSDTRC) and its solution algorithm. Section 7.3 presents a combined trip distribution mode split and dynamic user optimal simultaneous departure time and route choice model (TD MS DUOSDTRC) and its solution algorithm. In each section, a numerical example showing the application of the algorithm is exhibited.

7.1 Combined Mode Split and Dynamic User Optimal Simultaneous Departure Time and Route Choice (MS DUOSDTRC) Problem

In this section, we present a combined mode split and dynamic user optimal simultaneous departure time and route choice model (MS DUOSDTRC) for a general transportation network. We assume that a given number of travelers are ready for departure between each O-D pair at the beginning of each departure horizon. Travelers may choose either transit or passenger car to travel. For simplicity we only consider the case in which the transportation network consists of a transit network (which can be some exclusive routes/lanes for bus) and an auto network. We further assume the actual travel time on the transit network is fixed. We do not consider the departure time and route choice problem on transit network. Instead, we only consider the departure time and route choice problem on auto network. At equilibrium, the same travel cost is incurred for all passenger car drivers of the same O-D pair departing at all time, and equals the transformed O-D travel cost of the transit of the same O-D pair.

7.1.1 MS DUOSDTRC Model

Chapter 7 has presented the detailed explanation of dynamic user optimal simultaneous departure time and route choice problem (DUOSDTRC). To explain the combined mode split and dynamic user optimal simultaneous departure time and route choice problem (MS DUOSDTRC), we further introduce some denotations. For each O-D pair rs, let $\overline{q}^{rs}$ be the total demand, q^{rs} be the automobile demand and $\hat{q}^{rs}$ be the transit demand. Both q^{rs} and $\hat{q}^{rs}$ are variables. Then the following O-D demand conservation equations hold:

$$\overline{q}^{rs} = q^{rs} + \hat{q}^{rs} \quad \forall\ r, s \tag{7.1}$$

Assume the share of transit demand is given by the following logit modal split function:

$$\hat{q}^{rs} = \overline{q}^{rs} \frac{1}{1 + \exp\theta\left(\pi_{\min}^{rs} - \hat{\pi}^{rs}\right)} \quad \forall\ r, s \tag{7.2}$$

where $\pi_{\min}^{rs}$ is the (general) travel cost incurred for all passenger car drivers of O-D pair

rs departing at all time interval, $\hat{\pi}^{rs}$ is the fixed travel impedance for transit, and θ is nonnegative parameters which has effect on the demand share of each mode.

Define the transformed O-D cost for transit as the inverse demand function of (12) as follows.

$$W^{rs}(\omega) = \frac{1}{\theta} \ln \frac{\omega}{\bar{q}^{rs} - \omega} + \hat{\pi}^{rs} \tag{7.3}$$

$W^{rs}(\cdot)$ is a function of $\hat{q}^{rs}$. At equilibrium, the (general) travel cost $\pi^{rs}_{\min}$ incurred for all passenger car of each O-D pair rs departing at all time interval over auto network should equal the transformed O-D cost W^{rs} over transit network. It follows that

$$W^{rs*} - \pi^{rs*}_{\min} = 0 \quad \forall \ r, s; \tag{7.4}$$

The dynamic User Optimal simultaneous departure time and route choice condition over auto network can be written as

$$\eta^{rs*}_{p}(t) - \pi^{rs*}(t) \geq 0 \quad \forall \ p, r, s; \tag{7.5a}$$

$$f^{rs*}_{p}(t)\left[\eta^{rs*}_{p}(t) - \pi^{rs*}(t)\right] = 0 \quad \forall \ p, r, s; \tag{7.5b}$$

$$\pi^{rs*}(t) - \pi^{rs*}_{\min} \geq 0 \quad \forall \ r, s; \tag{7.5c}$$

$$f^{rs*}(t)\left[\pi^{rs*}(t) - \pi^{rs*}_{\min}\right] = 0 \quad \forall \ r, s; \tag{7.5d}$$

$$f^{rs}_{p}(t) \geq 0 \quad \forall \ p, r, s; \tag{7.5e}$$

$$f^{rs}(t) \geq 0 \ \forall \ r, s; \tag{7.5f}$$

where the asterisk denotes that the travel disutility is computed using DUOSDTRC time-dependent demand and route flows.

The combined mode split and dynamic user optimal simultaneous departure time and route choice problem (MS DUOSDTRC) can be expressed as follows.

Find $\mathbf{q} \in \Re_{+}^{|R\times S|}$, $\mathbf{f} \in \Re_{+}^{|R\times S|\times P}$, and $\breve{\mathbf{f}} \in \Re_{+}^{|R\times S|}$, such that condition (7.1), (7.4), and (7.5) holds simultaneously.

Conditions (7.5) is equivalent to the following variational inequality problem

$$\int_0^T \left\{ \left\langle \boldsymbol{\eta}^*(t), \left[\mathbf{f}(t) - \mathbf{f}^*(t)\right] \right\rangle + \left\langle \boldsymbol{\pi}^*(t), \left[\breve{\mathbf{f}}(t) - \breve{\mathbf{f}}^*(t)\right] \right\rangle \right\} dt \geq 0 \tag{7.6a}$$

or in expanded form as

$$\int_0^T \left\{ \left\langle \sum_{rs} \sum_{p} \eta_p^{rs^*}(t) \cdot \left[f_p^{rs}(t) - f_p^{rs^*}(t) \right] \right\rangle + \left\langle \sum_{rs} \pi^{rs^*}(t) \left[f^{rs}(t) - f^{rs^*}(t) \right] \cdot \right\rangle \right\} dt \geq 0 \tag{7.6b}$$

Thus, the MS DUOSDTRC problem can be expressed as follows.

Find $\mathbf{q} \in \Re_{+}^{|R\times S|}$, $\mathbf{f} \in \Re_{+}^{|R\times S|\times P}$, and $\breve{\mathbf{f}} \in \Re_{+}^{|R\times S|}$, such that equation (7.1), (7.4), and variational inequality (7.6) hold simultaneously.

7.1.2 Solution Algorithm for MS DUOSDTRC Model

To solve the MS DUOSDTRC problem, the continuous VI formulation is discretized with each time interval being the assignment increment. The estimated actual travel time on each link a is a multiple of the time increment and is fixed at each time increment, i.e.

$$\bar{\tau}_a(k) = i \qquad \text{if} \quad (i - 0.5)\Delta t \leq \tau_a(k) \leq (i + 0.5)\Delta t \tag{7.7}$$

where i is an integer and $0 \leq i \leq K$, Δt is time increment. This round-off method is used only in the flow propagation constraints. The round-off error can be made as small as desired by making the assignment increment smaller.

The MS DUOSDTRC problem is to find

$$\mathbf{q} \in \Re_{+}^{|R\times S|}, \quad \mathbf{f} \in \Re_{+}^{|P|\times K_0}, \quad \breve{\mathbf{f}} \in \Re_{+}^{|R\times S|\times K_0}$$

such that

$$\bar{q}^{rs} = q^{rs} + \hat{q}^{rs} \quad \forall\ r, s \tag{7.8}$$

$$W^{rs*} - \pi_{\min}^{rs*} = 0 \quad \forall\ r, s; \tag{7.9}$$

and

$$\left\langle \boldsymbol{\eta}^*, \left[\mathbf{f} - \mathbf{f}^*\right]\right\rangle + \left\langle \boldsymbol{\pi}^*, \left[\breve{\mathbf{f}} - \breve{\mathbf{f}}^*\right]\right\rangle \geq 0 \tag{7.10a}$$

or in expanded form as

$$\sum_{rs}\sum_{k=1}^{K_0}\left\{\sum_{p}\eta_p^{rs^*}(k)\cdot\left[f_p^{rs}(k) - f_p^{rs^*}(k)\right] + \pi^{rs^*}(k)\cdot\left[f^{rs}(k) - f^{rs^*}(k)\right]\right\} \geq 0 \tag{7.10b}$$

in Θ. Θ is the feasible region defined by the following constraints:

O-D demand conservation constraints:

$$\sum_{k=1}^{K_0} f^{rs}(k) = q^{rs} \quad \forall\ r, s \tag{7.11}$$

Path flow conservation constraints:

$$\sum_{p} f_p^{rs}(k) = f^{rs}(k) \quad \forall\ k, r, s \tag{7.12}$$

Link inflow conservation constraints:

$$\sum_{rs} u_a^{rs}(k) = u_a(k) \quad \forall\ a, k \tag{7.13}$$

Link outflow conservation constraints:

$$\sum_{rs} v_a^{rs}(k) = v_a(k) \quad \forall\ a, k \tag{7.14}$$

Node flow conservation constraints:

$$\sum_{a\in B(j)} v_a^{rs}(k) = \sum_{a\in A(j)} u_a^{rs}(k) \quad \forall\ j \neq r, s; r, s; k \tag{7.15}$$

where $A(j)$ is the set of links after j and $B(j)$ is the set of links before j.

Link flow propagation constraints:

$$u_a^{rs}(k) = v_a^{rs}(k + \tau_a(k)) \quad \forall a,\ r, s, k \tag{7.16}$$

The link state equations:

$$x_a(k+1) = x_a(k) + u_a(k) - v_a(k) \quad \forall a, k \tag{7.17a}$$

or

$$x_a(k+1) = x_a(k) + u_a(k+1) - v_a(k+1) \quad \forall a, k \tag{7.17b}$$

(6.34a) is forward formula, (6.34b) is backward formula.

Path-link flow incidence constraints:

$$u_a^{rs}(n) = \sum_{rs}\sum_{p}\sum_{k=1}^{K_0} f_p^{rs}(k)\delta_{rsa}^{pkn} \quad \forall a, n \tag{7.18}$$

where $\delta_{rsa}^{pkn} \in \{0,1\}$ is defined as:

$$\delta_{rsa}^{pkn} = \begin{cases} 1 & \begin{array}{l}\text{if traffic departing origin } r \text{ at any time interval } k \\ \text{heading for destination } s \text{ on path } p \text{ arrives at link } a \\ \text{during the } n\text{th time interval.}\end{array} \\ 0 & \text{otherwise} \end{cases} \tag{7.19}$$

Nonnegative constraints:

$$f^{rs}(k) \geq 0,\ f_p^{rs}(k) \geq 0, u_a^{rs}(k) \geq 0,\ \forall k, r, s, a, p \tag{7.20}$$

Relaxation for VI Problem

At each relaxation, we temporarily fix: 1) Actual travel time $\tau_a(k)$ in the link flow propagation constraints as $\bar{\tau}_a(k)$; 2) Actual travel time $\tau_a[k + \pi^{ri}(k)]$ as $\tau_a[k + \bar{\pi}^{ri}(k)]$. At each relaxation, the time-space network is fixed with fixed link flow propagation constraints and fixed time dependent O-D demand.

Via relaxation, the VI cost term becomes $\eta_p^{rs}(k)$ and $\pi^{rs}(k)$, where

$$\eta_p^{rs}(k) = \sum_{k=1}^{K_0} \sum_a \tau_a(n) \delta_{rsa}^{pkn} \tag{7.21a}$$

$$= \tau_{a_1}(k) + \tau_{a_2}\left(k + \overline{\tau}_{a_1}(k)\right) + \cdots + \tau_{a_{\overline{p}}}\left(k + \overline{\eta}_p^{ra_{(\overline{p}-1)}}(k)\right) \tag{7.21b}$$

where $p = \left(a_1, a_2, \cdots\cdots a_{\overline{p}}\right)$, a_i is the link number of path p of O-D pair rs at time k, and

$$\pi^{rs}(k) = \min_p \left\{ \phi_p^{rs}(k) := \eta_p^{rs}(k) + c_s\left(k + \eta_p^{rs}(k)\right) \right\} \quad \forall r, s, k \tag{7.21c}$$

Optimization Problem for VI problem

An optimization problem which is equivalent to the discrete VI under relaxation can thus be formulated, as follows:

$$\min_{\mathrm{f},\breve{\mathrm{f}}} Z = \sum_{k=1}^{K_0} \sum_{rs} \left\{ \sum_p \int_0^{f_p^{rs}(k)} \eta_p^{rs}\left(\omega; \mathbf{f}_p^{rs}\right) d\omega + \int_0^{f^{rs}(k)} \pi^{rs}\left(\omega; \breve{\mathbf{f}}^{rs}\right) d\omega \right\} \tag{7.22}$$

in Θ, where $\mathbf{f}_p^{rs}$ denotes the path flow vector $\mathbf{f}$ without component $f_p^{rs}(k)$, $\breve{\mathbf{f}}^{rs}$ denotes the path flow vector $\breve{\mathbf{f}}$ without component $f^{rs}(k)$.

Algorithm

The algorithm for solving the MS DUOSDTRC model is summarized as follows.

Step 0: Mode Split Initialization. Finding the static shortest paths over the auto network and calculate the initial demand share $q_{rs}^{(0)}$ and $\hat{q}_{rs}^{(0)}$ based on (7.2). Set mode split iteration counter $M := 0$. Set a mode split iteration convergence criterion $\varepsilon_{\mathrm{mode}}$.

Step 1: Departure Time Initialization.

Set an initial feasible solution of $\left[f^{rs(0)}(k)\right]$. Compute $k_{\max} = \max_{\forall rs}\left\{\pi^{rs}\right\}$, where π^{rs} is the static minimum travel time of O-D rs. Set $K' = K_0 + C \cdot \left[k_{\max}\right]_+$. Set $\hat{\tau}_a^{(0)}(k) = \tau_a[0]$, $\forall a \in A$,

$k=1,\cdots,K'$. Set departure time iteration counter $L:=0$. Set an departure time iteration convergence criterion ε_{dep}.

Step 2: DUO Initialization.

Find an initial feasible solution $\left[f_p^{rs(0)}(k)\right]$. Set DUO iteration counter $l:=0$. Set DUO iteration convergence criterion ε_{DUO}.

Step 3: Relaxation.

Find a new estimation of actual link travel times: $\hat{\tau}_a^{(l)}(k)=\tau_a\left[x_a^*(k)\right]$, find $\bar{\tau}_a^{(l)}(k)\ \forall\ a\in A,\ k=1,\cdots,K'$, where * denotes the solution obtained from the most recent UE iteration or from DUO initialization. Find $\delta_{a'}^{k(l)}(t)$ and $\delta_{a''}^{k(l)}(t)$.

Step 4: UE Iteration

Step 4.0: UE Initialization. Compute and reset the inner initial feasible solution to be consistent with the flow propagation constrain at the current relaxation. Set an UE iteration counter $m:=1$.

In the first relaxation, set $\tau_a^{(1)}(k)$ equal to free flow travel time $\tau_a(0), \forall\ a$, k=1,…, K. and perform all-or-nothing assignments. This yields initial path flows $f_{(1)}^{rs}(k), \forall\ r,s$, k=1,…,K_0. In other relaxations, reset the most recent UE iteration solution to be consistent with the flow propagation constrain at the current relaxation, and set them as initial path flows $f_{(1)}^{rs}(k), \forall\ r,s$, k=1,…,K_0, at current relaxation (Initialize the path set P_{rs}^k with the shortest path for each O-D pair *rs* at time *k*).

Step 4.1: Update. Set $\tau_a^{(m)}(k)$ equal to $\tau_a^{(m)}\left[x_a^{(m)}(k)\right]$. Update the first derivative lengths $d_p^{rs(m)}(k)$(i.e., path cost at current flow) of all of the paths in P_{rs}^k, $\forall\ r,s$.

Step 4.2: Direction finding for $\left[f_p^{rs}(k)\right]$. Find the shortest-path $\bar{p}_{rs}^{(m)}(k)$ from each origin *r* to each destination *s* at *k* on the basis of $\tau_a^{(m)}(k)$. If different from all the paths in the existing path set in P_{rs}^k, (no need for path comparison here; just compare $d_p^{rs(m)}(k)$, add it to in P_{rs}^k and record $d_{\bar{p}_{rs}^{(m)}(k)}$. If not tag the shortest among the paths in P_{rs}^k in $d_{\bar{p}_{rs}^{(m)}(k)}$.

Step 4.3: Move for $\left[f_p^{rs}(k)\right]$. Set the new path flows.

$$f_p^{rs(m+1)}(k)=\max\left[0, f_p^{rs(m)}(k)-\frac{\alpha^n}{s_p^{rs(m)}(k)}\left(d_p^{rs(m)}(k)-d_{\bar{p}_{rs}^{(m)}(k)}\right)\right] \forall\ r,s,p\in P_{rs}^k, p\neq \bar{p}_{rs}^{(m)}(k) \qquad (7.23)$$

where

$$s_p^{rs(m)}(k)=\sum_a\sum_k \frac{\partial \tau_a^{(m)}(k)}{\partial x_a^{(m)}(k)}, \forall\ p\in P_{rs}^k \qquad (7.24)$$

a and *k* denotes time-space links that are on either *p* or $\bar{p}_{rs}^{(m)}(k)$, but not on both, and α^n is a scalar step-size modifier.

Also,

$$f_{\bar{p}_{rs}(k)}^{(m+1)} = f^{rs}(k)-\sum_{\substack{p\in P_{rs}^k\\ p\neq \bar{p}_{rs}(k)}} f_p^{rs(m+1)}(k) \quad \forall\, r, \mathrm{s}, \mathrm{k} \qquad (7.25)$$

Assign the flows on the trees and find the link flows $u_a^{(m+1)}(k)$.

Step 4.4: Convergence Test for UE Iteration.

If $f_p^{rs(m+1)}(k)\cong f_p^{rs(m)}(k)$, set $\hat{f}_a^{rs(l)}(k)= f_p^{rs(m+1)}(k), \hat{x}_a^{(l)}(k)= x_a^{(m+1)}(k)$, go to Step 5; otherwise, set $m=m+1$, go to Step 4.1.

Step 5: Convergence Test for DUO Iteration. If $\hat{\tau}_a^{(l)}(k)\cong\hat{\tau}_a^{(l-1)}(k)$, go to step 6; otherwise, set $l=l+1$ and go to Step 3.

Step 6: Departure Time Iteration

Step 6.1: Direction finding for $\left[f^{rs}(k)\right]$. Find the average generalized path cost $d^{rs(L)}(k)$ of the shortest paths for all the time-dependent O-D demand. Tag the average generalized path cost $d_{\min}^{rs(L)}$ of the shortest paths of the shortest generalized time-dependent O-D demand for all the O-D pairs.

Step 6.2: Move for $\left[f^{rs}(k)\right]$. Set the new time-dependent O-D demand.

$$f^{rs(L+1)}(k)=\max\left[0, f^{rs(L)}(k)-\frac{\alpha^L}{s^{rs(L)}(k)}\left(d^{rs(L)}(k)-d_{\min}^{rs(L)}\right)\right] \forall\ r,s,\ f^{rs}(k)\neq f_{\min}^{rs} \tag{7.26}$$

Where

$$s^{rs(L)}(k)=\sum_a\sum_k\frac{\partial\tau_a(k)}{\partial x_a(k)} \quad \forall\ r,\text{s},\text{k} \tag{7.27}$$

a and *k* denotes time-space links on either shortest paths of the non-shortest generalized time-dependent O-D demand $f^{rs}(k)$ or shortest paths of the shortest generalized time-dependent O-D demand $f_{\min}^{rs}$ of the O-D pair, but not on both. α^L is a scalar step-size modifier. Also,

$$f_{\min}^{rs(L+1)} = q^{rs} - \sum_{f^{rs}(k)\neq f_{\min}^{rs}} f^{rs(L+1)}(k) \quad \forall\ r,\text{s} \tag{7.28}$$

Step 6.3: Convergence Test for Departure Time Iteration. If $f^{rs(L)}(k)\cong f^{rs(L+1)}(k)$, go to Step 7; otherwise, set $L := L+1$ and go to Step 2.

Step 7: Convergence Test for Mode Split Iteration. Calculate $q_{rs}^{(M)}$ and $\hat{q}_{rs}^{(M)}$ based on $d_{\min}^{rs}$ of most recent departure time iteration. If $q_{rs}^{(M)}\cong q_{rs}^{(M+1)}$, stop; or set $M := M+1$ and go to Step 1.

The flowchart of the solution algorithm is shown in Figure 7.1.

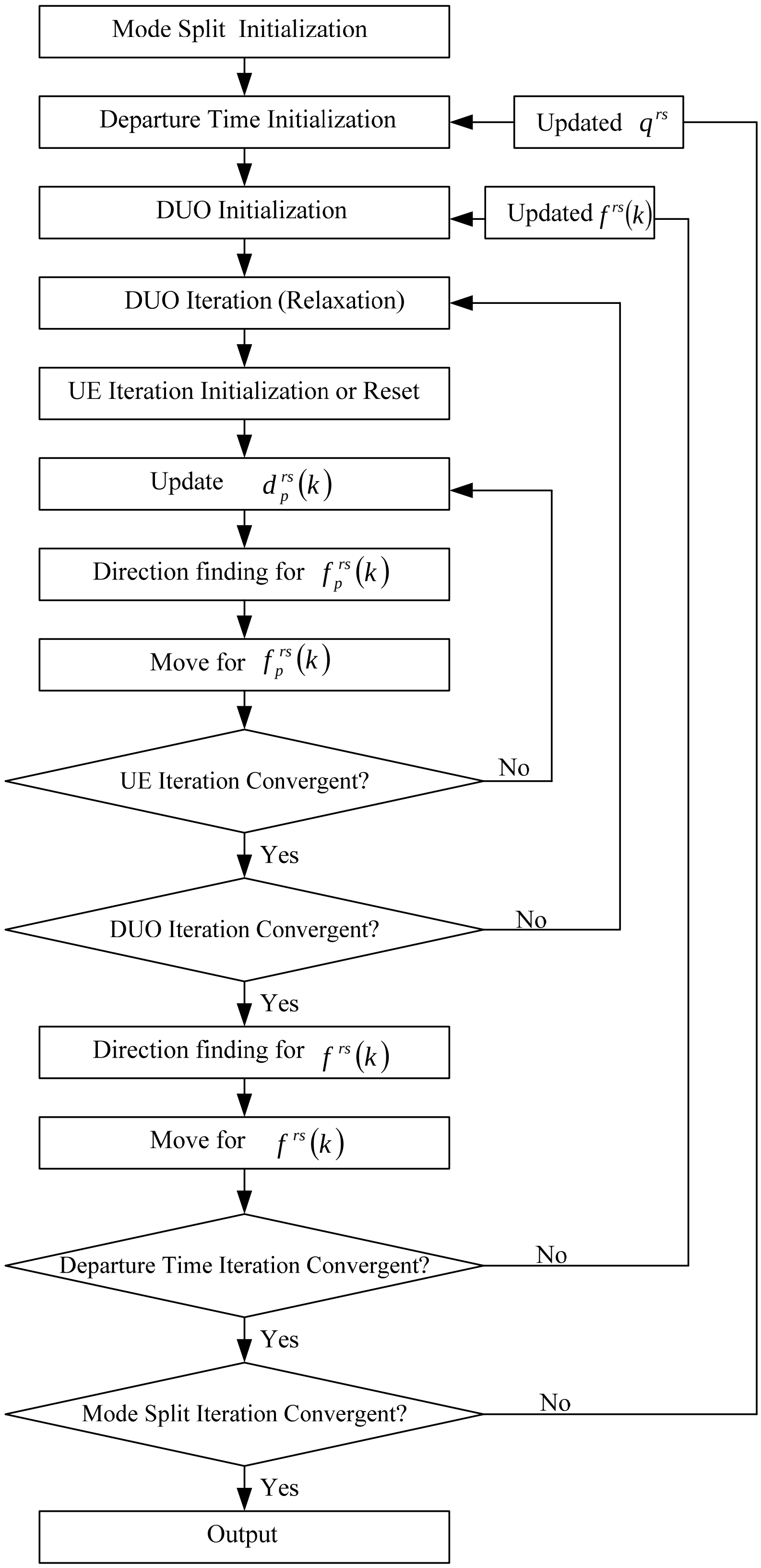

Figure 7-1 Flowchart of the Solution Algorithm

7.1.3 A Numerical Example

Example 7.1

Below we present an example to validate the above model and algorithm. The network is show in Figure 7.2. Link 1, 2, 5, 6 are 0.75 mile one lane street. Link 3, 4, 7, 8, 9, 10 are 0.35 mile one lane street. The free flow speed is assumed to be 25 mile/hour. The following linear travel time function is used to enforce FIFO condition: $\tau_a(k) = L_a / s_f + 0.3 \cdot x_a(k)$, where L_a is the length of link a, s_f is free flow speed, $\tau_a(k)$ is link travel time on link a at time k, $x_a(k)$ is number of vehicles on link a at time k. Six O-D pairs are considered. The O-D pairs, total O-D demand for each O-D pair, and fixed O-D travel cost for transit are shown in Table 7.1. Set $\theta = 0.1$. Five 20 s departure time intervals are specified. For simplicity we do not use disutility function, so all link and path costs are actual link and path costs.

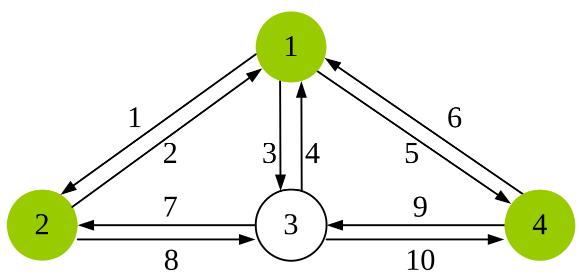

Figure 7.2 Simulation network for Example 7.1

Table 7-1 O-D pairs, O-D demand, and fixed O-D travel cost for transit

O-D	1-2	1-4	2-1	2-4	4-1	4-2
$\bar{q}^{rs}$	50	50	50	50	50	50
$\hat{u}^{rs}$	5	5	5	5	5	5

The program of the algorithm was run on a computer with 1.5 GHz frequency processor. The UE iteration convergence test method was set as a prespecified number *m*. The DUO iteration convergence test method was set as

$$\varepsilon_{\mathrm{DUO}} = \max\left\{ \left| \tau_a^{(l)}(k) - \tau_a^{(l-1)}(k) \right| \middle| a \in A, k = 1, \cdots, K \right\},$$

where $|\tau_a^{(l)}(k) - \tau_a^{(l-1)}(k)|$ is the actual travel time difference of link a at time k between successive DUO iterations. The departure time iteration convergence test method was set as

$$\varepsilon_{\mathrm{dep}} = \max\left\{ \left| f^{rs(L)}(k) - f^{rs(L-1)}(k) \right| \middle| rs \in R \times S, k = 1, \cdots, K_0 \right\},$$

where $|f^{rs(L)}(k) - f^{rs(L-1)}(k)|$ is the difference of time-dependent O-D demand of O-D pair rs at time k between successive departure time iterations. The mode split iteration convergence test method was set as

$$\varepsilon_{\mathrm{mode}} = \max\left\{ \left| q^{rs(M)} - q^{rs(M+1)} \right| \middle| rs \in R \times S \right\},$$

where $|q^{rs(M)} - q^{rs(M+1)}|$ is the difference of passenger car O-D demand of O-D pair rs between successive mode split iterations.

The operation of the program is shown in Table 7.2. The resultant O-D demand for each mode and O-D travel impedance is shown in Table 7.3. The corresponding optimal time-dependent O-D demand is shown in Table 7.4. The assignment horizon K is found to be 14 time increments. Table 7.5a shows the output of $u_a^{rs}(k)$. Table 7.5b shows the output of $v_a^{rs}(k)$. Table 7.5c shows the output of $u_a(k)$. Table 7.5d shows the output of $v_a(k)$. Table 7.5e shows the output of $x_a(k)$. Table 7.5f shows the output of $\tau_a(k)$. Table 7.5g shows the output of $f_p^{rs}(k), c_p^{rs}(k)$, links on each path and the arrival time interval for each link on a path. For conciseness, only Table 7.5g is attached to this dissertation.

Table 7-2 Convergence criterion and computation time for Example 7.1

ε_{UE} or m	ε_{DUO}	ε_{dep}	$\varepsilon_{\text{mode}}$	Mode Split iterations	Computation time (minute)
$m = 4$	0.01	0.1	0.01	3	38

Table 7-3 The resultant O-D demand for each mode and O-D travel impedance

O-D	q^{rs}	$\pi^{rs}_{\min}$	$\hat{q}^{rs}$	W^{rs}
1-2	28.8043	1.9328	21.1957	1.9328
1-4	30.0372	0.9143	19.9628	0.9143
2-1	30.0248	0.9247	19.9752	0.9247
2-4	28.9601	1.8050	21.0399	1.8050
4-1	30.0179	0.9304	19.9821	0.9304
4-2	27.7318	2.8058	22.2682	2.8058

Table 7-4 Resultant time-dependent O-D demand

O-D	Departure time interval k				
	1	2	3	4	5
1-2	21.9154	3.7810	2.1850	0.9218	0
1-4	12.7305	3.9368	2.2727	6.4509	4.6460

2-1	14.5340	3.9080	1.7738	6.0916	3.7170
2-4	10.3225	3.8005	4.0613	5.0701	5.7050
4-1	15.6909	3.9392	2.2759	4.6597	3.4516
4-2	8.6144	4.5720	3.8735	5.7226	4.9480

Table 7-5 The resultant path flow and path travel time for example 7.1

Path number	*O*	*D*	*k*	Path flow	Path time	Links on the path	Arrival time for each link on the path
This table is appendix 9 of this thesis.							

We take the following examples to verify that the solution satisfy the constraints and conditions of MS DUOSDTRC.

Total O-D demand conservation constraints (7.8):

$$\bar{q}^{12} = q^{12} + \hat{q}^{12} = 28.8043+21.1957=50$$

Mode O-D travel cost constraints (7.9):

$$\pi_{\min}^{12} = 1.9328 = W^{12}$$

Mode O-D demand conservation constraints (7.11):

$$q^{12} = f^{12}(1) + f^{12}(2) + f^{12}(3) + f^{12}(4) + f^{12}(5)$$

$$= 21.8315+3.8000+2.2151+0.9567+0$$

$$= 28.8043$$

Path flow conservation constraint (7.12):

$$f^{24}(4) = f_1^{24}(4) + f_2^{24}(4) = 1.1021+3.9687 = 5.0708$$

Link inflow conservation constraint (7.13):

$$u_8^{24}(2) = 3.8197 = u_8(2)$$

Link outflow conservation constraint (7.14):

$$v_8^{24}(5) = 3.8197 = v_8(5)$$

Node flow conservation constraint (7.15):

$$\sum_{a \in B(3)} v_a^{rs}(5) = \sum_{a \in B(3)} v_a(5) \ = \ v_8^{24}(5) \ = \ v_8(5) \ = 3.8197$$

$$\sum_{a \in A(3)} u_a^{rs}(5) = \sum_{a \in A(3)} u_a(5) \ = \ u_{10}^{24}(5) \ = \ u_{10}(5) \ = 3.8197$$

Link flow propagation constraint (7.16):

$$u_8^{24}(2) = v_8^{24}(2 + \bar{\tau}_8(2)) = v_8^{24}(5) = 3.8197$$

where $\tau_8(2)$ = 0.9106 minutes. For a time increment of 20 seconds, $\bar{\tau}_8(2) = 3$.

The link state equation (7.17b):

$$x_8(5) = x_8(4) + u_8(5) - v_8(5) = 10.5865+3.5340-3.8197=10.3008$$

The actual travel times on the used paths from origin 2 toward destination 4 departing at time increment 4 are as follows:

$$c_1^{24}(4) = \tau_2(4) + \tau_5(4 + \bar{\tau}_2(4)) = \tau_2(4) + \tau_5(7) = 0.9106+0.8751=1.7857 \cong 1.8 \text{ minutes}$$

Similarly, we have $c_p^{24}(k) \cong 1.8$ minutes, $\forall\, p,k$. They are nearly equal, which is consistent with the DUOSDTRC condition.

In order to decrease the computation time of each departure time iteration, the convergence criterion for departure time iteration is set as a relatively large value (ε_{dep}=0.1). The resultant actual path travel times of the same O-D rs are approximately equal but not exactly equal, and $\pi_{\min}^{rs}$ is set as the average of them. If ε_{dep} is sufficiently small, the actual path travel times of the same O-D rs will be exactly equal. In real implementation of the algorithm, ε_{dep} can be decreased when the mode split iteration is nearly convergent.

As can be checked in the same way, all the solution output satisfies the constraints and conditions for MS DUOSDTRC problem. This verifies the rationale of the above model and solution algorithm.

7.2 Combined Trip Distribution and Dynamic User Optimal Simultaneous Departure Time and Route Choice (TD DUOSDTRC) Problem

In this section, we present a combined trip distribution and dynamic user optimal simultaneous departure time and route choice model (TD DUOSDTRC) for a general transportation network. We assume that the number of travelers to be departed from each origin is given and fixed and the number of travelers to be attracted to each destination is also given and fixed. The number of travelers from each origin to each destination (or the demand of each O-D pair) is a function of the actual travel cost of the O-D pair and needs to be solved for.

7.2.1 TD DUOSDTRC Model

Let O_r be the trip generation in origin r and D_s be the trip attraction in destination s. Let $\bar{q}^{rs}$ be the total demand of O-D pair rs. It follows that:

$$\sum_{s} \bar{q}_{rs} = O_r \quad \forall\ r \tag{7.29}$$

$$\sum_r \bar{q}_{rs} = D_s \quad \forall\ s \tag{7.30}$$

Assume the total demand of O-D pair rs is given by the doubly constrained gravity model defined by

$$\bar{q}_{rs} = A_r B_s \exp\left(-\gamma \pi_{\min}^{rs}\right) \quad \forall\ r, s; \tag{7.31}$$

where $\pi_{\min}^{rs}$ is the (general) travel cost incurred for all passenger car drivers of O-D pair rs departing at all time interval.

The dynamic User Optimal simultaneous departure time and route choice condition over auto network can be written as

$$\eta_p^{rs*}(t) - \pi^{rs*}(t) \geq 0 \quad \forall\ p, r, s; \tag{7.32a}$$

$$f_p^{rs*}(t)\left[\eta_p^{rs*}(t) - \pi^{rs*}(t)\right] = 0 \quad \forall\ p, r, s; \tag{7.32b}$$

$$\pi^{rs*}(t) - \pi_{\min}^{rs*} \geq 0 \quad \forall\ r, s; \tag{7.32c}$$

$$f^{rs*}(t)\left[\pi^{rs*}(t) - \pi_{\min}^{rs*}\right] = 0 \quad \forall\ r, s; \tag{7.32d}$$

$$f_p^{rs}(t) \geq 0 \quad \forall\ p, r, s; \tag{7.32e}$$

$$f^{rs}(t) \geq 0 \ \forall\ r, s; \tag{7.32f}$$

where the asterisk denotes that the travel disutility is computed using DUOSDTRC time-dependent demand and route flows.

Conditions (7.32) is equivalent to the following variational inequality problem

$$\int_0^T \left\{ \left\langle \boldsymbol{\eta}^*(t), \left[\mathbf{f}(t) - \mathbf{f}^*(t)\right] \right\rangle + \left\langle \boldsymbol{\pi}^*(t), \left[\breve{\mathbf{f}}(t) - \breve{\mathbf{f}}^*(t)\right] \right\rangle \right\} dt \geq 0 \tag{7.33a}$$

or in expanded form as

$$\int_0^T \left\{ \left\langle \sum_{rs} \sum_p \eta_p^{rs^*}(t) \cdot \left[f_p^{rs}(t) - f_p^{rs^*}(t) \right] \right\rangle + \left\langle \sum_{rs} \pi^{rs^*}(t) \left[f^{rs}(t) - f^{rs^*}(t) \right] \cdot \right\rangle \right\} dt \geq 0 \tag{7.33b}$$

The combined trip distribution and dynamic user optimal simultaneous departure time and route choice problem (TD DUOSDTRC) can be expressed as follows. Find $\overline{\mathbf{q}} \in \Re_+^{|R\times S|}$, $\mathbf{f} \in \Re_+^{|R\times S|\times P}$, and $\breve{\mathbf{f}} \in \Re_+^{|R\times S|}$, such that equation (7.29), (7.30), (7.31), and variational inequality (7.33) hold simultaneously.

7.2.2 Solution Algorithm for TD DUOSDTRC Model

To solve the TD DUOSDTRC problem, the continuous VI formulation is discretized with each time interval being the assignment increment. The estimated actual travel time on each link a is a multiple of the time increment and is fixed at each time increment, i.e.

$$\overline{\tau}_a(k) = i \qquad \text{if} \quad (i-0.5)\Delta t \le \tau_a(k) \le (i+0.5)\Delta t \qquad (7.34)$$

where i is an integer and $0 \le i \le K$, Δt is time increment. This round-off method is used only in the flow propagation constraints. The round-off error can be made as small as desired by making the assignment increment smaller.

The TD DUOSDTRC problem is to find

$$\overline{\mathbf{q}} \in \Re_+^{|R\times S|},\ \ \mathbf{f} \in \Re_+^{|P|\times K_0},\ \ \breve{\mathbf{f}} \in \Re_+^{|R\times S|\times K_0}$$

such that

$$\sum_s \overline{q}^{rs} = O_r \qquad \forall\ r \qquad (7.35)$$

$$\sum_r \overline{q}^{rs} = D_s \qquad \forall\ s \qquad (7.36)$$

$$\overline{q}^{rs} = A_r B_s \exp\left(-\gamma \pi_{\min}^{rs}\right) \qquad \forall\ r, s \qquad (7.37)$$

and

$$\left\langle \boldsymbol{\eta}^*, \left[\mathbf{f} - \mathbf{f}^*\right]\right\rangle + \left\langle \boldsymbol{\pi}^*, \left[\breve{\mathbf{f}} - \breve{\mathbf{f}}^*\right]\right\rangle \ge 0 \qquad (7.38a)$$

or in expanded form as

$$\sum_{rs}\sum_{k=1}^{K_0}\left\{\sum_{p}\eta_p^{rs^*}(k)\cdot\left[f_p^{rs}(k)-f_p^{rs^*}(k)\right]+\pi^{rs^*}(k)\cdot\left[f^{rs}(k)-f^{rs^*}(k)\right]\right\}\geq 0 \tag{7.38b}$$

in Θ. Θ is the feasible region defined by the following constraints:

O-D demand conservation constraints:

$$\sum_{k=1}^{K_0} f^{rs}(k)=\bar{q}^{rs} \quad \forall\, r,s \tag{7.39}$$

Path flow conservation constraints:

$$\sum_{p} f_p^{rs}(k)= f^{rs}(k) \ \forall\, k,r,s \tag{7.40}$$

Link inflow conservation constraints:

$$\sum_{rs} u_a^{rs}(k)=u_a(k) \quad \forall\, a,k \tag{7.41}$$

Link outflow conservation constraints:

$$\sum_{rs} v_a^{rs}(k)=v_a(k) \quad \forall\, a,k \tag{7.42}$$

Node flow conservation constraints:

$$\sum_{a\in B(j)} v_a^{rs}(k)=\sum_{a\in A(j)} u_a^{rs}(k) \quad \forall\, j\neq r,s;r,s\,;k \tag{7.43}$$

where $A(j)$ is the set of links after j and $B(j)$ is the set of links before j.

Link flow propagation constraints:

$$u_a^{rs}(k)=v_a^{rs}\left(k+\tau_a(k)\right) \quad \forall\, a,\ r,s,k \tag{7.44}$$

The link state equations:

$$x_a(k+1)=x_a(k)+u_a(k)\text{-}v_a(k) \quad \forall\, a\,,k \tag{7.45a}$$

or

$$x_a(k+1)=x_a(k)+u_a(k+1)\text{-}v_a(k+1) \quad \forall\, a\,,k \tag{7.45b}$$

(7.45a) is forward formula, (7.45b) is backward formula.

Path-link flow incidence constraints:

$$u_a^{rs}(n)=\sum_{rs}\sum_{p}\sum_{k=1}^{K_0} f_p^{rs}(k)\delta_{rsa}^{pkn} \qquad \forall\, a, n \tag{7.46}$$

where $\delta_{rsa}^{pkn} \in \{0,1\}$ is defined as:

$$\delta_{rsa}^{pkn} = \begin{cases} 1 & \text{if traffic departing origin } r \text{ at any time interval } k \\ & \text{heading for destination } s \text{ on path } p \text{ arrives at link } a \\ & \text{during the } n\text{th time interval.} \\ 0 & \text{otherwise} \end{cases} \tag{7.47}$$

Nonnegative constraints:

$$\bar{q}^{rs}(k)\geq 0,\ f^{rs}(k)\geq 0,\ f_p^{rs}(k)\geq 0, u_a^{rs}(k)\geq 0,\ \forall\, k, r, s, a, p \tag{7.48}$$

Relaxation for VI Problem

At each relaxation, we temporarily fix: 1) Actual travel time $\tau_a(k)$ in the link flow propagation constraints as $\bar{\tau}_a(k)$; 2) Actual travel time $\tau_a\left[k+\pi^{ri}(k)\right]$ as $\tau_a\left[k+\bar{\pi}^{ri}(k)\right]$. At each relaxation, the time-space network is fixed with fixed link flow propagation constraints and fixed time dependent O-D demand.

Via relaxation, the VI cost term becomes $\eta_p^{rs}(k)$ and $\pi^{rs}(k)$, where

$$\eta_p^{rs}(k)=\sum_{k=1}^{K_0}\sum_{a}\tau_a(n)\delta_{rsa}^{pkn} \tag{7.49a}$$

$$=\tau_{a_1}(k)+\tau_{a_2}\left(k+\bar{\tau}_{a_1}(k)\right)+\cdots+\tau_{a_{\bar{p}}}\left(k+\bar{\eta}_p^{ra_{(\bar{p}-1)}}(k)\right) \tag{7.49b}$$

where $p=\left(a_1, a_2, \cdots\cdots a_{\bar{p}}\right)$, a_i is the link number of path p of O-D pair rs at time k, and

$$\pi^{rs}(k)=\min_p\left\{\phi_p^{rs}(k):=\eta_p^{rs}(k)+c_s\left(k+\eta_p^{rs}(k)\right)\right\} \quad \forall r, s, k \tag{7.49c}$$

Optimization Problem for VI problem

An optimization problem which is equivalent to the discrete VI under relaxation can thus be formulated, as follows:

$$\min_{\mathbf{f},\breve{\mathbf{f}}} Z = \sum_{k=1}^{K_0} \sum_{rs} \left\{ \sum_p \int_0^{f_p^{rs}(k)} \eta_p^{rs}\left(\omega, \mathbf{f}_p^{rs}\right) d\omega + \int_0^{f^{rs}(k)} \pi^{rs}\left(\omega, \breve{\mathbf{f}}^{rs}\right) d\omega \right\} \tag{7.50}$$

in Θ, where $\mathbf{f}_p^{rs}$ denotes the path flow vector $\mathbf{f}$ without component $f_p^{rs}(k)$, $\breve{\mathbf{f}}^{rs}$ denotes the path flow vector $\breve{\mathbf{f}}$ without component $f^{rs}(k)$.

Algorithm

The algorithm for solving the TD DUOSDTRC model is summarized as follows.

Step 0: Trip Distribution Initialization. Find shortest paths for each OD pair based on free flow travel time. Solve doubly constrained gravity model (7.37) constrained by (7.35) and (8.36) to get $\overline{q}_{rs}^{(0)}$**.** Set trip distribution iteration counter $D := 0$. Set a trip distribution iteration convergence criterion $\varepsilon_{\text{trip}}$.

Step 1: Departure Time Initialization.

Set an initial feasible solution of $\left[f^{rs(0)}(k)\right]$. Compute $k_{\max} = \max_{\forall rs}\left\{\pi^{rs}\right\}$, where π^{rs} is the static minimum travel time of O-D rs . Set $K^{'} = K_0 + C \cdot \left[k_{\max}\right]_+$. Set $\hat{\tau}_a^{(0)}(k) = \tau_a[0]$, $\forall\, a \in A$, $k = 1, \cdots, K'$. Set departure time iteration counter $L := 0$. Set an departure time iteration convergence criterion ε_{dep} .

Step 2: DUO Initialization.

Find an initial feasible solution $\left[f_p^{rs(0)}(k)\right]$. Set DUO iteration counter $l := 0$. Set DUO iteration convergence criterion ε_{DUO} .

Step 3: Relaxation.

Find a new estimation of actual link travel times: $\hat{\tau}_a^{(l)}(k) = \tau_a\left[x_a^*(k)\right]$, find $\overline{\tau}_a^{(l)}(k)$ $\forall\, a \in A$,

$k = 1, \cdots, K'$, where * denotes the solution obtained from the most recent UE iteration or from DUO initialization. Find $\delta_{a'}^{k(l)}(t)$ and $\delta_{a''}^{k(l)}(t)$.

Step 4: UE Iteration

Step 4.0: UE Initialization. Compute and reset the inner initial feasible solution to be consistent with the flow propagation constrain at the current relaxation. Set an UE iteration counter $m := 1$.

In the first relaxation, set $\tau_a^{(1)}(k)$ equal to free flow travel time $\tau_a(0), \forall a$, k=1,…, K. and perform all-or-nothing assignments. This yields initial path flows $f_{(1)}^{rs}(k)$, $\forall\ r, s$, k=1,…, K_0. In other relaxations, reset the most recent UE iteration solution to be consistent with the flow propagation constrain at the current relaxation, and set them as initial path flows $f_{(1)}^{rs}(k), \forall\ r, s$, k=1,…, K_0, at current relaxation (Initialize the path set P_{rs}^k with the shortest path for each O-D pair *rs* at time *k*).

Step 4.1: Update. Set $\tau_a^{(m)}(k)$ equal to $\tau_a^{(m)}\left[x_a^{(m)}(k)\right]$. Update the first derivative lengths $d_p^{rs(m)}(k)$(i.e., path cost at current flow) of all of the paths in P_{rs}^k, $\forall\ r, s$.

Step 4.2: Direction finding for $\left[f_p^{rs}(k)\right]$. Find the shortest-path $\bar{p}_{rs}^{(m)}(k)$ from each origin *r* to each destination *s* at *k* on the basis of $\tau_a^{(m)}(k)$. If different from all the paths in the existing path set in P_{rs}^k, (no need for path comparison here; just compare $d_p^{rs(m)}(k)$, add it to in P_{rs}^k and record $d_{\bar{p}_{rs}^{(m)}(k)}$. If not tag the shortest among the paths in P_{rs}^k in $d_{\bar{p}_{rs}^{(m)}(k)}$.

Step 4.3: Move for $\left[f_p^{rs}(k)\right]$. Set the new path flows.

$$f_p^{rs(m+1)}(k) = \max\left[0, f_p^{rs(m)}(k) - \frac{\alpha^n}{s_p^{rs(m)}(k)}\left(d_p^{rs(m)}(k) - d_{\bar{p}_{rs}^{(m)}(k)}\right)\right] \forall\ r, s, p \in P_{rs}^k, p \neq \bar{p}_{rs}^{(m)}(k) \qquad (7.51)$$

Where

$$s_p^{rs(m)}(k) = \sum_a \sum_k \frac{\partial \tau_a^{(m)}(k)}{\partial x_a^{(m)}(k)}, \forall\ p \in P_{rs}^k \quad (7.52)$$

a and k denotes time-space links that are on either p or $\bar{p}_{rs}^{(m)}(k)$, but not on both, and α^n is a scalar step-size modifier.

Also,

$$f_{\bar{p}_{rs}(k)}^{(m+1)} = f^{rs}(k) - \sum_{\substack{p \in P_{rs}^k \\ p \neq \bar{p}_{rs}(k)}} f_p^{rs(m+1)}(k) \quad \forall\ r, \mathrm{s}, \mathrm{k} \quad (7.53)$$

Assign the flows on the trees and find the link flows $u_a^{(m+1)}(k)$.

Step 4.4: Convergence Test for UE Iteration.

If $f_p^{rs(m+1)}(k) \cong f_p^{rs(m)}(k)$, set $\hat{f}_a^{rs(l)}(k) = f_p^{rs(m+1)}(k), \hat{x}_a^{(l)}(k) = x_a^{(m+1)}(k)$, go to Step 5; otherwise, set $m = m+1$, go to Step 4.1.

Step 5: Convergence Test for DUO Iteration. If $\hat{\tau}_a^{(l)}(k) \cong \hat{\tau}_a^{(l-1)}(k)$, go to step 6; otherwise, set $l = l+1$ and go to Step 3.

Step 6: Departure Time Iteration

Step 6.1: Direction finding for $[f^{rs}(k)]$. Find the average generalized path cost $d^{rs(L)}(k)$ of The shortest paths for all the time-dependent O-D demand. Tag the average generalized path cost $d_{\min}^{rs(L)}$ of the shortest paths of the shortest generalized time-dependent O-D demand for all the O-D pairs.

Step 6.2: Move for $[f^{rs}(k)]$. Set the new time-dependent O-D demand.

$$f^{rs(L+1)}(k) = \max\left[0, f^{rs(L)}(k) - \frac{\alpha^L}{s^{rs(L)}(k)}\left(d^{rs(L)}(k) - d_{\min}^{rs(L)}\right)\right] \forall\ r, s, f^{rs}(k) \neq f_{\min}^{rs} \quad (7.54)$$

where

$$s^{rs(L)}(k) = \sum_a \sum_k \frac{\partial \tau_a(k)}{\partial x_a(k)} \quad \forall\, r, \mathrm{s}, \mathrm{k} \qquad (7.55)$$

a and k denotes time-space links on either shortest paths of the non-shortest generalized time-dependent O-D demand $f^{rs}(k)$ or shortest paths of the shortest generalized time-dependent O-D demand $f^{rs}_{\min}$ of the O-D pair, but not on both. α^L is a scalar step-size modifier.

Also,

$$f^{rs(L+1)}_{\min} = q^{rs} - \sum_{f^{rs}(k) \neq f^{rs}_{\min}} f^{rs(L+1)}(k) \quad \forall\, r, \mathrm{s} \qquad (7.56)$$

Step 6.3: Convergence Test for Departure Time Iteration. If $f^{rs(L)}(k) \cong f^{rs(L+1)}(k)$, go to Step 7; otherwise, set $L := L + 1$ and go to Step 2.

Step 7: Solve doubly constrained gravity model (7.37) constrained by (7.35) and (8.36) based on $d^{rs}_{\min}$ of most recent departure time iteration to get $\overline{q}^{(D+1)}_{rs}$.

Step 8: Convergence Test for Trip Distribution Iteration. If $q^{(D)}_{rs} \cong q^{(D+1)}_{rs}$, stop; or set $D := D + 1$ and go to Step 1.

The flowchart of the solution algorithm is shown in Figure 7.3.

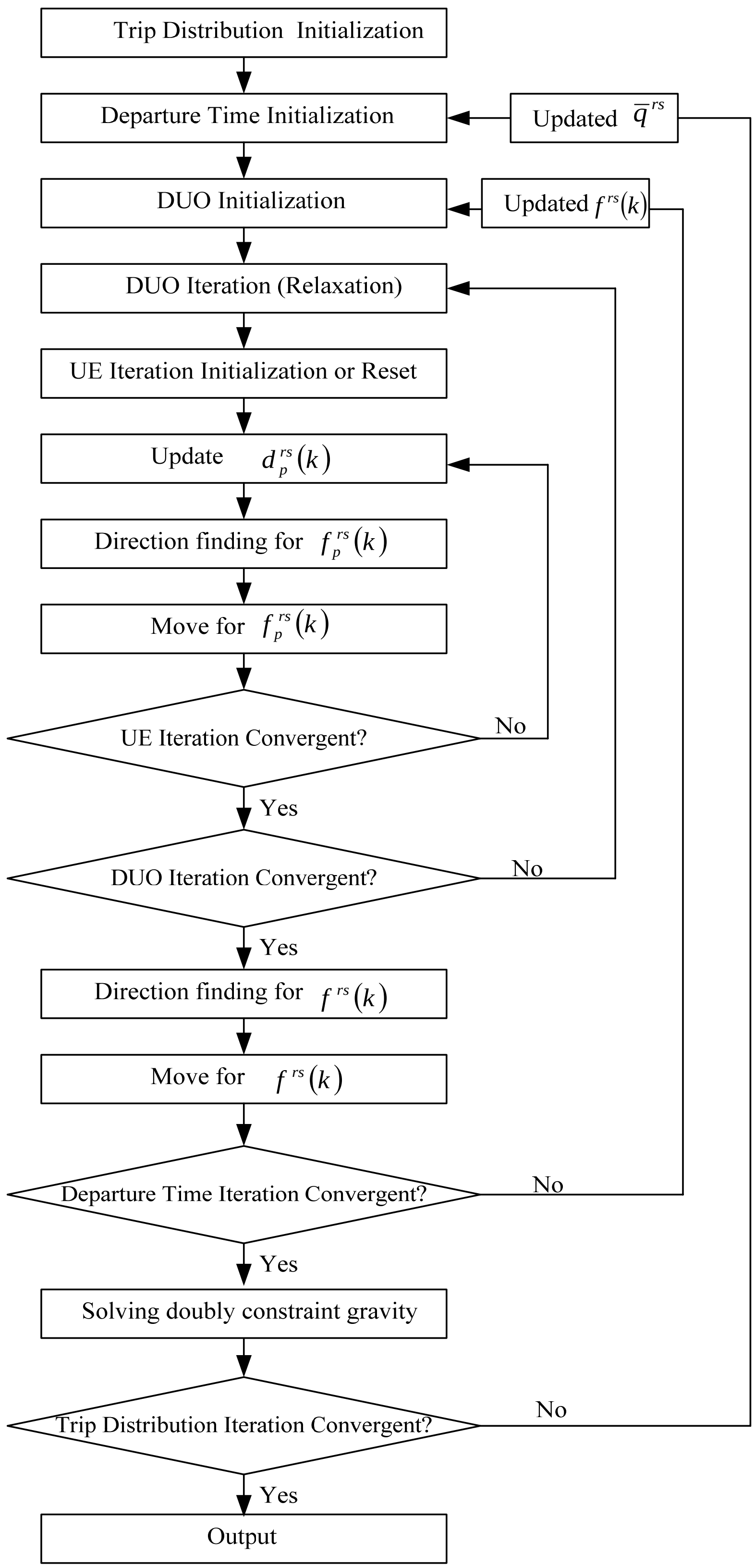

Figure 7.3 Flowchart of the Solution Algorithm

7.2.3 A Numerical Example

Example 7.2

Below we present an example to validate the above model and algorithm. The network is show in Figure 7.4. Link 1, 2, 5, 6 are 0.75 mile one lane street. Link 3, 4, 7, 8, 9, 10 are 0.35 mile one lane street. The free flow speed is assumed to be 25 mile/hour. The following linear travel time function is used to enforce FIFO condition: $\tau_a(k) = L_a/s_f + 0.3 \cdot x_a(k)$, where L_a is the length of link a, s_f is free flow speed, $\tau_a(k)$ is link travel time on link a at time k, $x_a(k)$ is number of vehicles on link a at time k. Six O-D pairs are considered. The trip generations of each origin and trip attraction of each destination are shown in Table 7.6. Set γ =0.01 in equation (). Five 20 s departure time intervals are specified. For simplicity we do not use disutility function, so all path costs are actual path costs.

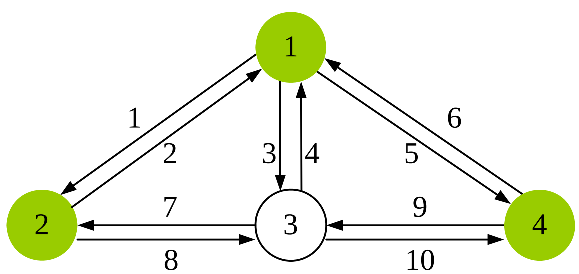

Figure 7.4 Simulation network for Example 7.1

The program of the algorithm was run on a computer with 1.5 GHz frequency processor. The UE iteration convergence test method was set as a prespecified number *m*. The DUO iteration convergence test method was set as

$$\varepsilon_{\text{DUO}} = \max\left\{ |\tau_a^{(l)}(k) - \tau_a^{(l-1)}(k)| \,\middle|\, a \in A, k = 1, \cdots, K \right\},$$

where $|\tau_a^{(l)}(k) - \tau_a^{(l-1)}(k)|$ is the actual travel time difference of link a at time k between successive DUO iterations. The departure time iteration convergence test method was set as

$$\varepsilon_{\text{dep}} = \max\left\{ \left| f^{rs(L)}(k) - f^{rs(L-1)}(k) \right| \middle| rs \in R \times S, k = 1, \cdots, K_0 \right\},$$

where $| f^{rs(L)}(k) - f^{rs(L-1)}(k) |$ is the difference of time-dependent O-D demand of O-D pair rs at time k between successive departure time iterations. The trip distribution iteration convergence test method was set as

$$\varepsilon_{\text{trip}} = \max\left\{ \left| \bar{q}^{rs(D)} - \bar{q}^{rs(D+1)} \right| \middle| rs \in R \times S \right\},$$

where $| \bar{q}^{rs(D)} - \bar{q}^{rs(D+1)} |$ is the difference of O-D demand of O-D pair rs between successive trip distribution iterations.

The operation of the program is shown in Table 7.7. The Resultant $\bar{q}^{rs}$, A_r, B_s, and $\pi^{rs}_{\min}$ are shown in Table 7.8. The corresponding optimal time-dependent O-D demand is shown in Table 7.9. The assignment horizon K is found to be 14 time increments. Table 7.10a shows the output of $u^{rs}_a(k)$. Table 7.10b shows the output of $v^{rs}_a(k)$. Table 7.10c shows the output of $u_a(k)$. Table 7.10d shows the output of $v_a(k)$. Table 7.10e shows the output of $x_a(k)$. Table 7.10f shows the output of $\tau_a(k)$. Table 7.10g shows the output of $f^{rs}_p(k), c^{rs}_p(k)$, links on each path and the arrival time interval for each link on a path. For conciseness, only Table 7.10g is attached to this dissertation.

Table 7-6 Trip generation of each origin and trip attraction of each destination

Origin/Destination	1	2	4
O_r	110	100	90
D_s	105	100	95

Table 7-7 Convergence criterion and computation time for Example 7.2

ε_{UE} or m	ε_{DUO}	ε_{dep}	$\varepsilon_{\text{trip}}$	Trip Distribution iterations	Computation time (minute)
m= 4	0.01	0.1	0.001	3	120

Table 7-8 Resultant $\bar{q}^{rs}$, A_r, B_s, and $\pi^{rs}_{\min}$ for Example 7.2

Origin	Destination			A_r
	1	2	4	
1	0	58.3179 2.0764	51.6821 0.9644	59.5329
2	56.6819 0.9970	0	43.3181 1.8765	50.3416
4	48.3174 0.9697	41.6826 2.8442	0	42.8904
B_s	1.1374	1.0000	0.8767	

Note: for each O-D pair, the upper value is O-D demand $\bar{q}^{rs}$, the lower value in the frame is $\pi^{rs}_{\min}$.

Table 7-9 Resultant time-dependent O-D demand

O-D	Departure time interval k				
	1	2	3	4	5

1-2	51.0196	5.6955	1.6028	0	0
1-4	22.1226	5.0224	1.4006	15.6977	7.4388
2-1	27.5240	5.5006	0.2833	18.7365	4.6374
2-4	17.3334	4.2211	4.6820	9.7373	7.3443
4-1	22.4335	4.1487	0.9010	17.3096	3.5246
4-2	11.7099	5.8088	5.0334	6.5872	12.5434

Table 7-10 The resultant path flow and path travel time for example 7.2

Path number	*O*	*D*	*k*	Path flow	Path time	Links on the path	Arrival time for each link on the path
This table is appendix 10 of this thesis.							

We take the following examples to verify that the solution satisfy the constraints and conditions of TD DUOSDTRC.

Trip generation constraint (7.35):

$$O_1 = \bar{q}^{12} + \bar{q}^{14} = 58.3179+51.6821==110$$

Trip attraction constraint (7.36):

$$D_1 = \bar{q}^{21} + \bar{q}^{41} = 56.6819+48.3174=105$$

O-D demand equation (7.37):

$$\bar{q}^{12} = A_1 B_2 \exp\left(-\gamma\pi_{\min}^{12}\right)=59.5329*1.0000*\exp\left(-0.01*2.0764\right)=58.3179$$

O-D demand conservation constraints (7.39):

$$\bar{q}^{12} = f^{12}(1) + f^{12}(2) + f^{12}(3) = 50.9542+5.7241+1.6396= 58.3179$$

Path flow conservation constraint (7.40):

$$f^{24}(5) = f_1^{24}(5) + f_2^{24}(5) = 6.6940+0.6525 = 7.3465$$

Link inflow conservation constraint (7.41):

$$u_8^{24}(2) = 4.2424 = u_8(2)$$

Link outflow conservation constraint (7.42):

$$v_8^{24}(5) = 4.2424 = v_8(5)$$

Node flow conservation constraint (7.43):

$$\sum_{a \in B(3)} v_a^{rs}(5) = \sum_{a \in B(3)} v_a(5) = v_8^{24}(5) = v_8(5) = 4.2424$$

$$\sum_{a \in A(3)} u_a^{rs}(5) = \sum_{a \in A(3)} u_a(5) = u_{10}^{24}(5) = u_{10}(5) = 4.2424$$

Link flow propagation constraint (7.44):

$$u_8^{24}(2) = v_8^{24}(2 + \bar{\tau}_8(2)) = v_8^{24}(5) = 4.2424$$

where $\tau_8(2)$ = 0.9478 minutes. For a time increment of 20 seconds, $\bar{\tau}_8(2)$=3.

The link state equation (7.45b):

$$x_8(5) = x_8(4) + u_8(5) - v_8(5) = 15.7071+6.6940-4.2424=18.1587$$

The actual travel times on the used paths from origin 2 toward destination 4 departing at time increment 3 are as follows:

$$c_1^{24}(3) = \tau_2(3) + \tau_5(3 + \bar{\tau}_2(3)) = \tau_2(3) + \tau_5(7) = 1.0215+0.8918=1.9133 \cong 1.9 \text{ minutes}$$

Similarly, we have $c_p^{24}(k) \cong 1.9$ minutes, $\forall\, p,k$. They are nearly equal, which is consistent with the DUOSDTRC condition.

In order to decrease the computation time of each departure time iteration, the convergence criterion for departure time iteration is set as a relatively large value (ε_{dep}=0.1).

The resultant actual path travel times of the same O-D rs are approximately equal but not exactly equal, and $\pi_{\min}^{rs}$ is set as the average of them. If ε_{dep} is sufficiently small, the actual path travel times of the same O-D rs will be exactly equal.

As can be checked in the same way, all the solution output satisfies the constraints and conditions for TD DUOSDTRC problem. This verifies the rationale of the above model and solution algorithm.

7.3 Combined Trip Distribution Mode Split and Dynamic User Optimal Simultaneous Departure Time and Route Choice (TD MS DUOSDTRC) Problem

In this section, we present a combined trip distribution mode split and dynamic user optimal simultaneous departure time and route choice model (TD MS DUOSDTRC) and its solution algorithm. The TD MS DUOSDTRC extends the DUOSDTRC route choice model in two respects: destination, mode, departure time and route over a road network must be chosen. In TD MS DUOSDTRC problem, the total demand and the demand of each mode of each O-D pair need to be solved for. At equilibrium of TD MS DUOSDTRC, the same cost should be incurred for all passenger car drivers of the same O-D pair departing at all time, and should equal the transformed O-D cost of the transit of the same O-D pair.

7.3.1 TD MS DUOSDTRC Model

Let O_r be the trip generation in origin r and D_s be the trip attraction in destination s.

Let $\bar{q}^{rs}$ be the total demand of O-D pair rs. It follows that:

$$\sum_s \bar{q}_{rs} = O_r \quad \forall\ r \tag{7.57}$$

$$\sum_r \bar{q}_{rs} = D_s \quad \forall\ s \tag{7.58}$$

Assume the total demand of O-D pair rs is given by the doubly constrained gravity model defined by

$$\bar{q}_{rs} = A_r B_s \exp\left(-\gamma \pi_{\min}^{rs*}\right) \qquad \forall\ r, s; \tag{7.59}$$

where $\pi_{\min}^{rs*}$ is the (general) travel cost incurred for all passenger car drivers of O-D pair rs departing at all time interval.

For each O-D pair rs, let q^{rs} be the automobile demand and $\hat{q}^{rs}$ be the transit demand. Both q^{rs} and $\hat{q}^{rs}$ are variables. Then the following O-D demand conservation equations hold:

$$\bar{q}^{rs} = q^{rs} + \hat{q}^{rs} \quad \forall\ r, s \tag{7.60}$$

Assume the share of transit demand is given by the logit modal split function defined by (7.2). And define the transformed O-D cost for transit $W^{rs}(\cdot)$ as shown in (7.3). At equilibrium, it holds that

$$W^{rs*} - \pi_{\min}^{rs*} = 0 \quad \forall\ r, s; \tag{7.61}$$

The dynamic user optimal simultaneous departure time and route choice condition over auto network can be written as

$$\eta_p^{rs*}(t) - \pi^{rs*}(t) \geq 0 \qquad \forall\ p, r, s; \tag{7.62a}$$

$$f_p^{rs*}(t)\left[\eta_p^{rs*}(t) - \pi^{rs*}(t)\right] = 0 \quad \forall\ p, r, s; \tag{7.62b}$$

$$\pi^{rs*}(t) - \pi_{\min}^{rs*} \geq 0 \quad \forall\ r, s; \tag{7.62c}$$

$$f^{rs*}(t)\left[\pi^{rs*}(t)-\pi_{\min}^{rs*}\right]=0 \quad \forall \ r,s; \tag{7.62d}$$

$$f_p^{rs}(t)\geq 0 \quad \forall \ p,r,s; \tag{7.62e}$$

$$f^{rs}(t)\geq 0 \ \forall \ r,s; \tag{7.62f}$$

where the asterisk denotes that the travel disutility is computed using DUOSDTRC time-dependent demand and route flows.

Conditions (7.62) is equivalent to the following variational inequality problem

$$\int_0^T \left\{ \left\langle \boldsymbol{\eta}^*(t), \left[\mathbf{f}(t)-\mathbf{f}^*(t)\right]\right\rangle + \left\langle \boldsymbol{\pi}^*(t), \left[\breve{\mathbf{f}}(t)-\breve{\mathbf{f}}^*(t)\right]\right\rangle \right\} dt \geq 0 \tag{7.63a}$$

or in expanded form as

$$\int_0^T \left\{ \left\langle \sum_{rs}\sum_{p} \eta_p^{rs^*}(t)\cdot\left[f_p^{rs}(t)-f_p^{rs^*}(t)\right]\right\rangle + \left\langle \sum_{rs} \pi^{rs^*}(t)\left[f^{rs}(t)-f^{rs^*}(t)\right]\cdot\right\rangle \right\} dt \geq 0 \tag{7.63b}$$

The combined mode split and dynamic user optimal simultaneous departure time and route choice problem (TD MS DUOSDTRC) can be expressed as follows.

Find $\mathbf{q} \in \Re_+^{|R\times S|}$, $\mathbf{f} \in \Re_+^{|R\times S|\times P}$, and $\breve{\mathbf{f}} \in \Re_+^{|R\times S|}$, such that condition (7.57)-(7.61) and variational inequality (7.63) hold simultaneously.

7.3.2 Solution Algorithm for TD MS DUOSDTRC Model

To solve the TD MS DUOSDTRC problem, the continuous VI formulation is discretized with each time interval being the assignment increment. The estimated actual travel time on each link a is a multiple of the time increment and is fixed at each time increment, i.e.

$$\bar{\tau}_a(k)=i \qquad \text{if} \quad (i-0.5)\Delta t \leq \tau_a(k) \leq (i+0.5)\Delta t \tag{7.64}$$

where i is an integer and $0 \le i \le K$, Δt is time increment. This round-off method is used only in the flow propagation constraints. The round-off error can be made as small as desired by making the assignment increment smaller.

The TD MS DUOSDTRC problem is to find

$\mathbf{q} \in \Re_+^{|R \times S|}$, $\mathbf{f} \in \Re_+^{|P| \times K_0}$, $\breve{\mathbf{f}} \in \Re_+^{|R \times S| \times K_0}$, such that

$$\sum_s \bar{q}_{rs} = O_r \quad \forall\ r \tag{7.65}$$

$$\sum_r \bar{q}_{rs} = D_s \quad \forall\ s \tag{7.66}$$

$$\bar{q}_{rs} = A_r B_s \exp\left(-\gamma \pi_{\min}^{rs*}\right) \qquad \forall\ r, s; \tag{7.67}$$

$$\bar{q}^{rs} = q^{rs} + \hat{q}^{rs} \quad \forall\ r, s \tag{7.68}$$

$$W^{rs*} - \pi_{\min}^{rs*} = 0 \quad \forall\ r, s; \tag{7.69}$$

and

$$\left\langle \boldsymbol{\eta}^*, \left[\mathbf{f} - \mathbf{f}^*\right] \right\rangle + \left\langle \boldsymbol{\pi}^*, \left[\breve{\mathbf{f}} - \breve{\mathbf{f}}^*\right] \right\rangle \ge 0 \tag{7.70a}$$

or in expanded form as

$$\sum_{rs} \sum_{k=1}^{K_0} \left\{ \sum_p \eta_p^{rs^*}(k) \cdot \left[f_p^{rs}(k) - f_p^{rs^*}(k) \right] + \pi^{rs^*}(k) \cdot \left[f^{rs}(k) - f^{rs^*}(k) \right] \right\} \ge 0 \tag{7.70b}$$

in Θ. Θ is the feasible region defined by the following constraints:

O-D demand conservation constraints:

$$\sum_{k=1}^{K_0} f^{rs}(k) = q^{rs} \quad \forall\ r, s \tag{7.71}$$

Path flow conservation constraints:

$$\sum_p f_p^{rs}(k) = f^{rs}(k) \ \ \forall\ k, r, s \tag{7.72}$$

Link inflow conservation constraints:

$$\sum_{rs} u_a^{rs}(k) = u_a(k) \quad \forall\, a, k \tag{7.73}$$

Link outflow conservation constraints:

$$\sum_{rs} v_a^{rs}(k) = v_a(k) \quad \forall\, a, k \tag{7.74}$$

Node flow conservation constraints:

$$\sum_{a \in B(j)} v_a^{rs}(k) = \sum_{a \in A(j)} u_a^{rs}(k) \qquad \forall\; j \neq r,s; r,s\,; k \tag{7.75}$$

where $A(j)$ is the set of links after j and $B(j)$ is the set of links before j.

Link flow propagation constraints:

$$u_a^{rs}(k) = v_a^{rs}(k + \tau_a(k)) \qquad \forall\, a,\; r, s, k \tag{7.76}$$

The link state equations:

$$x_a(k+1) = x_a(k) + u_a(k) - v_a(k) \qquad \forall\, a\,, k \tag{7.77a}$$

or

$$x_a(k+1) = x_a(k) + u_a(k+1) - v_a(k+1) \qquad \forall\, a\,, k \tag{7.77b}$$

(7.77a) is forward formula, (7.77b) is backward formula.

Path-link flow incidence constraints:

$$u_a^{rs}(n) = \sum_{rs} \sum_{p} \sum_{k=1}^{K_0} f_p^{rs}(k) \delta_{rsa}^{pkn} \qquad \forall\, a, n \tag{7.78}$$

where $\delta_{rsa}^{pkn} \in \{0,1\}$ is defined as:

$$\delta_{rsa}^{pkn} = \begin{cases} 1 & \text{if traffic departing origin } r \text{ at any time interval } k \\ & \text{heading for destination } s \text{ on path } p \text{ arrives at link } a \\ & \text{during the } n\text{th time interval.} \\ 0 & \text{otherwise} \end{cases} \tag{7.79}$$

Nonnegative constraints:

$$f^{rs}(k) \geq 0,\; f_p^{rs}(k) \geq 0, u_a^{rs}(k) \geq 0,\; \forall\, k, r, s, a, p \tag{7.80}$$

Relaxation for VI Problem

At each relaxation, we temporarily fix: 1) Actual travel time $\tau_a(k)$ in the link flow propagation constraints as $\bar{\tau}_a(k)$; 2) Actual travel time $\tau_a\left[k+\pi^{ri}(k)\right]$ as $\tau_a\left[k+\bar{\pi}^{ri}(k)\right]$. At each relaxation, the time-space network is fixed with fixed link flow propagation constraints and fixed time dependent O-D demand.

Via relaxation, the VI cost term becomes $\eta_p^{rs}(k)$ and $\pi^{rs}(k)$, where

$$\eta_p^{rs}(k)=\sum_{k=1}^{K_0}\sum_a \tau_a(n)\delta_{rsa}^{pkn} \tag{7.81a}$$

$$=\tau_{a_1}(k)+\tau_{a_2}\left(k+\bar{\tau}_{a_1}(k)\right)+\cdots+\tau_{a_{\bar{p}}}\left(k+\bar{\eta}_p^{ra_{(\bar{p}-1)}}(k)\right) \tag{7.81b}$$

where $p=\left(a_1,a_2,\cdots\cdots a_{\bar{p}}\right)$, a_i is the link number of path p of O-D pair rs at time k, and

$$\pi^{rs}(k)=\min_p\left\{\phi_p^{rs}(k):=\eta_p^{rs}(k)+c_s\left(k+\eta_p^{rs}(k)\right)\right\} \quad \forall r,s,k \tag{7.81c}$$

Optimization Problem for VI problem

An optimization problem which is equivalent to the discrete VI under relaxation can thus be formulated, as follows:

$$\min_{\mathrm{f},\breve{\mathrm{f}}} Z=\sum_{k=1}^{K_0}\sum_{rs}\left\{\sum_p\int_0^{f_p^{rs}(k)}\eta_p^{rs}\left(\omega,\mathbf{f}_p^{rs}\right)d\omega+\int_0^{f^{rs}(k)}\pi^{rs}\left(\omega,\breve{\mathbf{f}}^{rs}\right)d\omega\right\} \tag{7.82}$$

in Θ, where $\mathbf{f}_p^{rs}$ denotes the path flow vector $\mathbf{f}$ without component $f_p^{rs}(k)$, $\breve{\mathbf{f}}^{rs}$ denotes the path flow vector $\breve{\mathbf{f}}$ without component $f^{rs}(k)$.

Algorithm

The algorithm for solving the TD MS DUOSDTRC model is summarized as follows.

Step 0: Trip Distribution Initialization. Find shortest paths for each OD pair based on free flow travel time. Solve doubly constrained gravity model (7.67) constrained by (7.65) and (8.66) to get $\bar{q}_{rs}^{(0)}$**.** Set trip distribution iteration counter $D := 0$. Set a trip distribution iteration convergence criterion $\varepsilon_{\text{trip}}$.

Step 1: Mode Split Initialization. Finding the static shortest paths over the auto network and calculate the initial demand share $q_{rs}^{(0)}$ and $\hat{q}_{rs}^{(0)}$ based on (7.2). Set mode split iteration counter $M := 0$. Set a mode split iteration convergence criterion $\varepsilon_{\text{mode}}$.

Step 2: Departure Time Initialization.

Set an initial feasible solution of $\left[f^{rs(0)}(k)\right]$. Compute $k_{\max} = \max_{\forall rs}\{\pi^{rs}\}$, where π^{rs} is the static minimum travel time of O-D rs. Set $K' = K_0 + C \cdot [k_{\max}]_+$. Set $\hat{\tau}_a^{(0)}(k) = \tau_a[0]$, $\forall a \in A$, $k = 1, \cdots, K'$. Set departure time iteration counter $L := 0$. Set an departure time iteration convergence criterion ε_{dep}.

Step 3: DUO Initialization.

Find an initial feasible solution $\left[f_p^{rs(0)}(k)\right]$. Set DUO iteration counter $l := 0$. Set DUO iteration convergence criterion ε_{DUO}.

Step 4: Relaxation.

Find a new estimation of actual link travel times: $\hat{\tau}_a^{(l)}(k) = \tau_a\left[x_a^*(k)\right]$, find $\bar{\tau}_a^{(l)}(k)$ $\forall a \in A$, $k = 1, \cdots, K'$, where * denotes the solution obtained from the most recent UE iteration or from DUO initialization. Find $\delta_{a'}^{k(l)}(t)$ and $\delta_{a''}^{k(l)}(t)$.

Step 5: UE Iteration

Step 5.0: UE Initialization. Compute and reset the inner initial feasible solution to be consistent with the flow propagation constrain at the current relaxation. Set an UE iteration counter $m := 1$.

In the first relaxation, set $\tau_a^{(1)}(k)$ equal to free flow travel time $\tau_a(0), \forall\, a$, k=1,…, K. and perform all-or-nothing assignments. This yields initial path flows $f_{(1)}^{rs}(k), \forall\ r, s$, k=1,…,K_0.

In other relaxations, reset the most recent UE iteration solution to be consistent with the flow propagation constrain at the current relaxation, and set them as initial path flows $f_{(1)}^{rs}(k), \forall\ r, s$, k=1,…,K_0, at current relaxation (Initialize the path set P_{rs}^k with the shortest path for each O-D pair *rs* at time *k*).

Step 5.1: Update. Set $\tau_a^{(m)}(k)$ equal to $\tau_a^{(m)}\left[x_a^{(m)}(k)\right]$. Update the first derivative lengths $d_p^{rs(m)}(k)$(i.e., path cost at current flow) of all of the paths in P_{rs}^k, $\forall\ r, s$.

Step 5.2: Direction finding for $\left[f_p^{rs}(k)\right]$. Find the shortest-path $\bar{p}_{rs}^{(m)}(k)$ from each origin *r* to each destination *s* at *k* on the basis of $\tau_a^{(m)}(k)$. If different from all the paths in the existing path set in P_{rs}^k, (no need for path comparison here; just compare $d_p^{rs(m)}(k)$, add it to in P_{rs}^k and record $d_{\bar{p}_{rs}^{(m)}(k)}$. If not tag the shortest among the paths in P_{rs}^k in $d_{\bar{p}_{rs}^{(m)}(k)}$.

Step 5.3: Move for $\left[f_p^{rs}(k)\right]$. Set the new path flows.

$$f_p^{rs(m+1)}(k) = \max\left[0, f_p^{rs(m)}(k) - \frac{\alpha^n}{s_p^{rs(m)}(k)}\left(d_p^{rs(m)}(k) - d_{\bar{p}_{rs}^{(m)}(k)}\right)\right] \forall\ r, s, p \in P_{rs}^k, p \neq \bar{p}_{rs}^{(m)}(k) \tag{7.83}$$

Where

$$s_p^{rs(m)}(k) = \sum_a \sum_k \frac{\partial \tau_a^{(m)}(k)}{\partial x_a^{(m)}(k)}, \forall\ p \in P_{rs}^k \tag{7.84}$$

a and *k* denotes time-space links that are on either *p* or $\bar{p}_{rs}^{(m)}(k)$, but not on both, and α^n is a scalar step-size modifier.

Also,

$$f_{\bar{p}_{rs}(k)}^{(m+1)} = f^{rs}(k) - \sum_{\substack{p \in P_{rs}^{k} \\ p \neq \bar{p}_{rs}(k)}} f_p^{rs(m+1)}(k) \quad \forall\, r, \mathrm{s}, \mathrm{k} \tag{7.85}$$

Assign the flows on the trees and find the link flows $u_a^{(m+1)}(k)$.

Step 5.4: Convergence Test for UE Iteration.

If $f_p^{rs(m+1)}(k) \cong f_p^{rs(m)}(k)$, set $\hat{f}_a^{rs(l)}(k) = f_p^{rs(m+1)}(k)$, $\hat{x}_a^{(l)}(k) = x_a^{(m+1)}(k)$, go to Step 6; otherwise, set $m = m+1$, go to Step 5.1.

Step 6: Convergence Test for DUO Iteration. If $\hat{\tau}_a^{(l)}(k) \cong \hat{\tau}_a^{(l-1)}(k)$, go to step 7; otherwise, set $l = l+1$ and go to Step 4.

Step 7: Departure Time Iteration

Step 7.1: Direction finding for $\left[f^{rs}(k)\right]$. Find the average generalized path cost $d^{rs(L)}(k)$ of the shortest paths for all the time-dependent O-D demand. Tag the average generalized path cost $d_{\min}^{rs(L)}$ of the shortest paths of the shortest generalized time-dependent O-D demand for all the O-D pairs.

Step 7.2: Move for $\left[f^{rs}(k)\right]$. Set the new time-dependent O-D demand.

$$f^{rs(L+1)}(k) = \max\left[0, f^{rs(L)}(k) - \frac{\alpha^L}{s^{rs(L)}(k)}\left(d^{rs(L)}(k) - d_{\min}^{rs(L)}\right)\right] \forall\, r, s, f^{rs}(k) \neq f_{\min}^{rs} \tag{7.86}$$

where

$$s^{rs(L)}(k) = \sum_a \sum_k \frac{\partial \tau_a(k)}{\partial x_a(k)} \quad \forall\, r, \mathrm{s}, \mathrm{k} \tag{7.87}$$

a and k denotes time-space links on either shortest paths of the non-shortest generalized time-dependent O-D demand $f^{rs}(k)$ or shortest paths of the shortest generalized time-dependent O-D demand $f_{\min}^{rs}$ of the O-D pair, but not on both. α^L is a scalar step-size modifier.

Also,

$$f_{\min}^{rs(L+1)} = q^{rs} - \sum_{f^{rs}(k) \neq f_{\min}^{rs}} f^{rs(L+1)}(k) \quad \forall r, s \tag{7.88}$$

Step 7.3: Convergence Test for Departure Time Iteration. If $f^{rs(L)}(k) \cong f^{rs(L+1)}(k)$, go to Step 8; otherwise, set $L := L+1$ and go to Step 3.

Step 8: Convergence Test for Mode Split Iteration. Calculate $q_{rs}^{(M)}$ and $\hat{q}_{rs}^{(M)}$ based on $d_{\min}^{rs}$ of most recent departure time iteration. If $q_{rs}^{(M)} \cong q_{rs}^{(M+1)}$, go to Step 9; or set $M := M+1$ and go to Step 2.

Step 9: Solve doubly constrained gravity model (7.67) constrained by (7.65) and (7.66) based on $d_{\min}^{rs}$ of most recent departure time iteration in most recent mode split iteration to get $\overline{q}_{rs}^{(D+1)}$.

Step 10: Convergence Test for Trip Distribution Iteration. If $q_{rs}^{(D)} \cong q_{rs}^{(D+1)}$, stop; or set $D := D+1$ and go to Step 1.

The flowchart of the solution algorithm is shown in Figure 7.5.

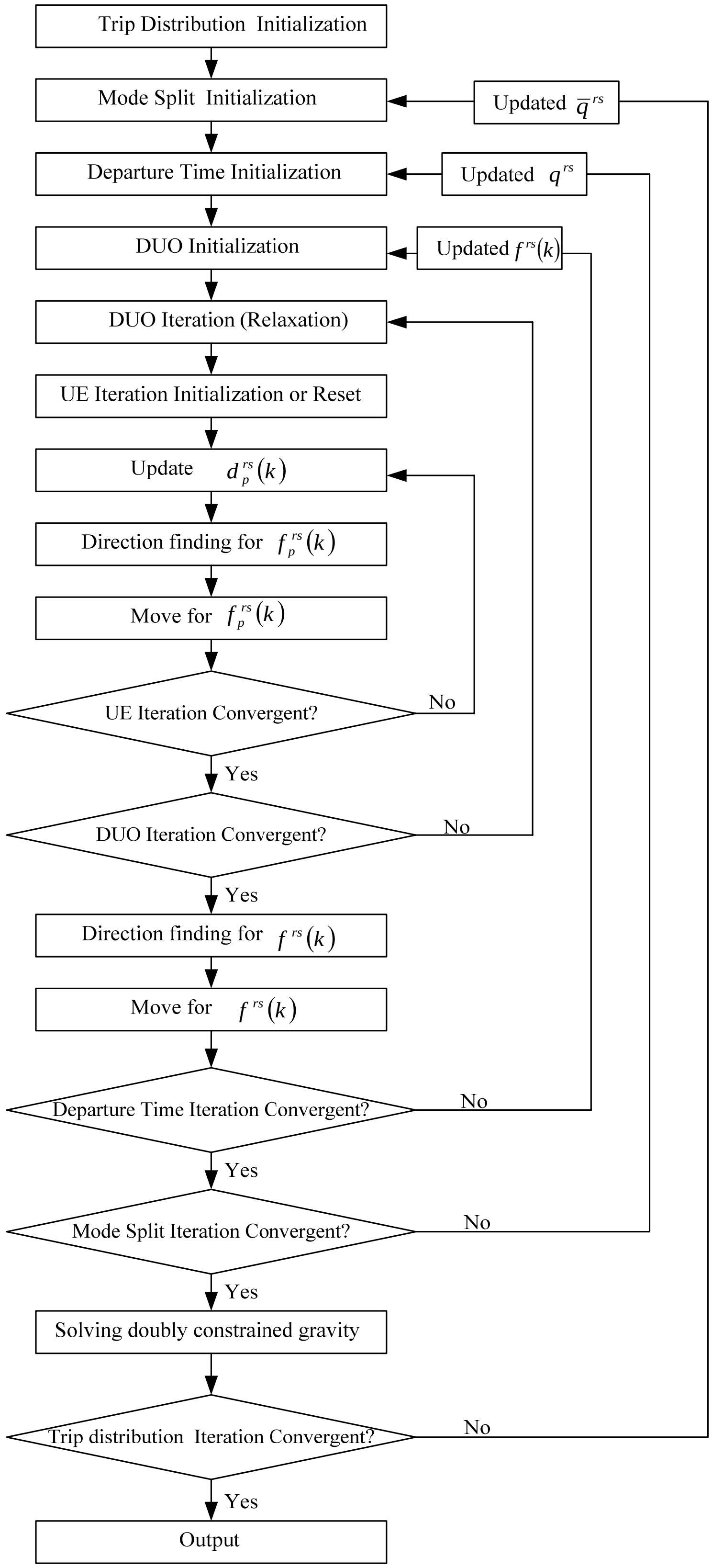

Figure 7.5 Flowchart of the Solution Algorithm

7.3.3 A Numerical Example

Example 7.3

Below we present an example to validate the above model and algorithm. The network is show in Figure 7.6. Link 1, 2, 5, 6 are 0.75 mile one lane street. Link 3, 4, 7, 8, 9, 10 are 0.35 mile one lane street. The free flow speed is assumed to be 25 mile/hour. The following linear travel time function is used to enforce FIFO condition: $\tau_a(k) = L_a / s_f + 0.3 \cdot x_a(k)$, where L_a is the length of link a, s_f is free flow speed, $\tau_a(k)$ is link travel time on link a at time k, $x_a(k)$ is number of vehicles on link a at time k. Six O-D pairs are considered. The O-D pairs and fixed O-D travel cost for transit are shown in Table 7.11. The trip generation of each origin and trip attraction of each destination are shown in Table 7.12. Set $\theta = 0.1$ in mode split function (7.2).Set γ =0.01 in equation (7.2). Five 20 s departure time intervals are specified. For simplicity we do not use disutility function, so all link and path costs are actual link and path costs.

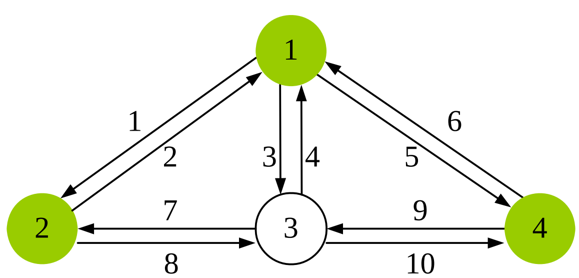

Figure 7.6 Simulation network for Example 7.3

Table 7-11 O-D pairs and fixed O-D travel cost for transit

O-D	1-2	1-4	2-1	2-4	4-1	4-2
$\hat{u}^{rs}$	5	5	5	5	5	5

Table 7-12 Trip generation of each origin and trip attraction of each destination

Origin/Destination	1	2	4
O_r	110	100	90
D_s	105	100	95

The program of the algorithm was run on a computer with 1.5 GHz frequency processor. The UE iteration convergence test method was set as a prespecified number *m*. The DUO iteration convergence test method was set as

$$\varepsilon_{\text{DUO}} = \max\left\{ |\tau_a^{(l)}(k) - \tau_a^{(l-1)}(k)| \,\middle|\, a \in A, k = 1, \cdots, K \right\},$$

where $|\tau_a^{(l)}(k) - \tau_a^{(l-1)}(k)|$ is the actual travel time difference of link *a* at time *k* between successive DUO iterations. The departure time iteration convergence test method was set as

$$\varepsilon_{\text{dep}} = \max\left\{ |f^{rs(L)}(k) - f^{rs(L-1)}(k)| \,\middle|\, rs \in R \times S, k = 1, \cdots, K_0 \right\},$$

where $|f^{rs(L)}(k) - f^{rs(L-1)}(k)|$ is the difference of time-dependent O-D demand of O-D pair rs at time k between successive departure time iterations. The mode split iteration convergence test method was set as

$$\varepsilon_{\text{mode}} = \max\left\{ |q^{rs(M)} - q^{rs(M+1)}| \,\middle|\, rs \in R \times S \right\},$$

where $|q^{rs(M)} - q^{rs(M+1)}|$ is the difference of passenger car O-D demand of O-D pair rs between successive mode split iterations. The trip distribution iteration convergence test method was set as

$$\varepsilon_{\text{trip}} = \max\left\{ |\overline{q}^{rs(D)} - \overline{q}^{rs(D+1)}| \,\middle|\, rs \in R \times S \right\},$$

where $|\overline{q}^{rs(D)} - \overline{q}^{rs(D+1)}|$ is the difference of O-D demand of O-D pair rs between successive trip distribution iterations.

The operation of the program is shown in Table 7.13. The Resultant $\bar{q}^{rs}$, A_r, B_s, and $\pi_{\min}^{rs}$ are shown in Table 7.14 .The resultant O-D demand for each mode and O-D travel impedance is shown in Table 7.15. The corresponding optimal time-dependent O-D demand is shown in Table 7.16. The assignment horizon K is found to be 14 time increments. Table 7.17a shows the output of $u_a^{rs}(k)$. Table 7.17b shows the output of $v_a^{rs}(k)$. Table 7.17c shows the output of $u_a(k)$. Table 7.17d shows the output of $v_a(k)$. Table 7.17e shows the output of $x_a(k)$. Table 7.17f shows the output of $\tau_a(k)$. Table 7.17g shows the output of $f_p^{rs}(k), c_p^{rs}(k)$, links on each path and the arrival time interval for each link on a path. For conciseness, only Table 7.17g is attached to this dissertation.

Table 7-13 Convergence criterion and computation time for Example 7.3

ε_{UE} or m	ε_{DUO}	ε_{dep}	$\varepsilon_{\text{mode}}$	$\varepsilon_{\text{trip}}$	Trip Distribution iterations	Computation time (minute)
m= 4	0.01	0.1	0.01	0.001	3	160

Table 7-14 Resultant $\bar{q}^{rs}$, A_r, B_s, and $\pi_{\min}^{rs}$ for Example 7.3

Origin	Destination			A_r
	1	2	4	
1	0	58.3190 1.9547	51.6810 0.9168	59.4842
	56.6808	0	43.3192	50.2966

2	0.9278		1.7889	
4	48.3185 0.9223	41.6815 2.7968	0	42.8738
B_s	1.1374	0.9998	0.8768	

Note: for each O-D pair, the upper value is O-D demand $\bar{q}^{rs}$, the lower value in the frame is $\pi_{\min}^{rs}$.

Table 7-15 The resultant O-D demand for each mode and O-D travel impedance

O-D	q^{rs}	$\pi_{\min}^{rs}$	$\hat{q}^{rs}$	W^{rs}
1-2	33.5654	1.9547	24.7536	1.9547
1-4	31.0440	0.9168	20.6370	0.9168
2-1	34.0323	0.9278	22.6485	0.9278
2-4	25.1075	1.7889	18.2117	1.7889
4-1	29.0179	0.9223	19.3006	0.9223
4-2	23.1273	2.7968	18.5542	2.7968

Table 7-16 Resultant time-dependent O-D demand on auto network

O-D	Departure time interval *k*				
	1	2	3	4	5
1-2	25.5376	4.4060	2.5462	1.0742	0

1-4	13.1572	4.0688	2.3489	6.6671	4.8017
2-1	14.9969	4.4137	2.5311	7.5389	4.5511
2-4	9.7100	3.2921	1.9012	4.4737	5.7301
4-1	14.0596	3.8073	2.1996	5.2461	3.7049
4-2	6.7213	3.6384	3.0365	5.0974	4.6328

Table 7-17 The resultant path flow and path travel time for example 7.3

Path number	*O*	*D*	*k*	Path flow	Path time	Links on the path	Arrival time for each link on the path
This table is appendix 11 of this thesis.							

We take the following examples to verify that the solution satisfy the constraints and conditions of TD MS DUOSDTRC.

Trip generation constraint (7.65):

$$O_1 = \bar{q}^{12} + \bar{q}^{14} = 58.3190+51.6810==110$$

Trip attraction constraint (7.66):

$$D_1 = \bar{q}^{21} + \bar{q}^{41} = 56.6808+48.3185=105$$

O-D demand equation (7.67):

$$\bar{q}^{12} = A_1 B_2 \exp\left(-\gamma \pi_{\min}^{12}\right) = 59.4842*0.9998*\exp\left(-0.01*1.9547\right)=58.3190$$

Total O-D demand conservation constraints (7.68):

$$\bar{q}^{12} = q^{12} + \hat{q}^{12} = 33.5654+24.7536=58.3190$$

Mode O-D travel cost constraints (7.69):

$$\pi_{\min}^{12} = 1.9547 = W^{12}$$

Mode O-D demand conservation constraints (7.71):

$$q^{12} = f^{12}(1) + f^{12}(2) + f^{12}(3) + f^{12}(4) + f^{12}(5)$$

$$= 25.4398+4.4281+2.5812+1.1148+0$$

$$= 33.5654$$

Path flow conservation constraint (7.72):

$$f^{24}(4) = f_1^{24}(4) + f_2^{24}(4) = 4.4152+0.0548 \cong 4.4737$$

Link inflow conservation constraint (7.73):

$$u_8^{24}(2) = 3.3090 = u_8(2)$$

Link outflow conservation constraint (7.74):

$$v_8^{24}(5) = 3.3090 = v_8(5)$$

Node flow conservation constraint (7.75):

$$\sum_{a \in B(3)} v_a^{rs}(5) = \sum_{a \in B(3)} v_a(5) = v_8^{24}(5) = v_8(5) = 3.3090$$

$$\sum_{a \in A(3)} u_a^{rs}(5) = \sum_{a \in A(3)} u_a(5) = u_{10}^{24}(5) = u_{10}(5) = 3.3090$$

Link flow propagation constraint (7.76):

$$u_8^{24}(2) = v_8^{24}(2 + \bar{\tau}_8(2)) = v_8^{24}(5) = 3.3090$$

where $\tau_8(2)$ =0.9050 minutes. For a time increment of 20 seconds, $\bar{\tau}_8(2)$=3.

The link state equation (7.77b):

$$x_8(5) = x_8(4) + u_8(5) - v_8(5) = 9.6518+3.3496-3.3090 = 9.6924$$

The actual travel times on the used paths from origin 2 toward destination 4 departing at time increment 4 are as follows:

$c_1^{24}(4) = \tau_8(4) + \tau_{10}(4 + \bar{\tau}_8(4)) = \tau_8(4) + \tau_{10}(7)$=0.8882+0.8882=1.7764 $\cong$ 1. 8 minutes

Similarly, we have $c_p^{24}(k) \cong$ 1. 8 minutes, $\forall$ *p,k*. They are nearly equal, which is consistent with the DUOSDTRC condition.

In order to decrease the computation time of each departure time iteration, the convergence criterion for departure time iteration is set as a relatively large value (ε_{dep}=0.1). The resultant actual path travel times of the same O-D rs are approximately equal but not exactly equal, and $\pi_{\min}^{rs}$ is set as the average of them. If ε_{dep} is sufficiently small, the actual path travel times of the same O-D rs will be exactly equal. In real implementation of the algorithm, ε_{dep} can be set high first and be decreased when the mode split iteration is nearly convergent.

As can be checked in the same way, all the solution output satisfies the constraints and conditions for TD MS DUOSDTRC problem. This verifies the rationale of the above model and solution algorithm.

Chapter 8: Some Applications of Dynamic User Optimal Route Choice Model

In this chapter, the dynamic user optimal route choice problem with incident management (DUOIM) and dynamic user optimal route choice problem integrated with signal timing system (DUOST) are studied. Section 8.1 presents the DUOIM model and development of its algorithm. Section 8.2 presents the DUOST model and development of its algorithm.

8.1 Dynamic User Optimal Route Choice Problem with Incident Management (DUOIM)

An incident is any nonrecurring event that impedes the flow of traffic. Traffic incidents annually account for approximately sixty percent of the delay (in vehicle-hours) on our highways in the country, causing disruption and reduction in road capacities (Sherali and Subramanian, 1999). The objective of an incident traffic management methodology is to determine an ideal traffic flow pattern that would minimize total network delay and congestion (system optimal) under the effect of traffic incidents or would minimize each vehicle's delay (user optimal) under the effect of traffic incidents. Different traffic control and guidance devices such as variable message signs, traffic signals, ramp metering, or in-vehicle devices are then employed to force the actual network traffic to be closer to this ideal traffic pattern. DTA is particularly appropriate for assessing the impacts of designed incident scenarios, evaluating the effectiveness of candidate incident management plans, as well as the impacts of different traffic operation and control strategies, for the analysis period.

Sawaya and Doan et al. (2000) presented a multistage stochastic mathematical model with recourse to compute and disseminate time-dependent alternate routes around freeway incidents. Sisiopiku et al. (2007) conducted simulation tests to use the DTA capabilities to support decision making for incident management on the Birmingham regional network and the Greater Chicago network using VISTA. Their study includes the impact of the duration of incident presence, partial closure of the freeway (instead of full closure), the response of individual drivers to the incidents, and the impact of the dissemination of incident information. The study confirmed the availability of information on incident presence, along with availability of alternative routes with residual capacity for rerouting of vehicles around incident congested locations, could considerably assist in improving incident management practices. Emphasizing the urgency of traffic incidents, Zografos partitioned the service-time duration into four phases: detection time, dispatch time, response-vehicle travel-time, and incident-clearance time (Zografos and Michalopoulos, 1993).

In this section, we study the dynamic User Optimal route choice problem when there are incidents occurring on some links during the analysis period. If the location and the lasting time $[t_1, T_1]$ of an incident are known at the beginning of departure horizon, the incident is predictable; otherwise, it is not predictable. Under predictable incidents, recourse or reroute for vehicles is not needed, and the ideal User Optimal route choice flow pattern can be achieved on the network avoiding the links during the incident lasting time period. Under unpredictable incidents, recourse or reroute for vehicles is needed, and the ideal User Optimal route choice flow pattern can not necessarily be achieved on the network. An ideal of multi-period routing procedure for unpredictable incidents is briefly presented by Ran and Boyce (1996b). In this section, a discrete route-based ideal User Optimal route choice model under predictable incidents is presented. A relaxation with gradient projection algorithm is presented for the model. A numerical example is given.

8.1.1 Discrete Route-based Variational Inequality (VI) DUOIM Model

To present the discrete VI DUOIM Model, we discretize the time domain with each time interval being the assignment increment. The estimated actual travel time on each link a is a multiple of the time increment and is fixed at each time increment, i.e.

$$\bar{\tau}_a(k) = i \qquad \text{if} \quad (i-0.5)\Delta t \le \tau_a(k) \le (i+0.5)\Delta t \tag{8.1}$$

where i is an integer and $0 \le i \le K$, Δt is time increment. This round-off method is used only in the flow propagation constraints. The round-off error can be made as small as desired by making the assignment increment smaller.

Assume the network is empty at $k = 0$, and only travel demands departing within the departure horizon are considered. Assume an incident occurs on the entrance of link b at k_1 and will be cleared at k_2. The discrete route-based VI DUOIM Model is

$$\left\langle \left[\boldsymbol{\eta}^*(k) - \boldsymbol{\pi}^*(k)\right], \mathbf{f}(k) - \mathbf{f}^*(k) \right\rangle \ge 0 \tag{8.2a}$$

Or in expanded form, as

$$\sum_{rs}\sum_{p}\sum_{k=1}^{K_0}\left[\eta_p^{rs^*}(k)-\pi^{rs^\bullet}(k)\right]\left[f_p^{rs}(k)-f_p^{rs^*}(k)\right]\geq 0 \quad (8.2b)$$

where $\boldsymbol{\eta},\mathbf{f}\in\Re_+^{|P|\times K_0}$,.

$$\eta_p^{ri}(k)=\eta_p^{r(i-1)}(k)+\tau_a\left[k+\eta_p^{r(i-1)}(k)\right] \ \ \forall\ p=p,r,i;\ i=1,2,\cdots,s;$$

$$p=(r,1,2,\cdots,i,\cdots s),\ \ x\in\Theta \quad (8.2c)$$

Θ is the feasible region defined by the following constraints:

Path flow conservation constraint:

$$\sum_{p} f_p^{rs}(k)=f^{rs}(k)\ \forall\ k,r,s \quad (8.3)$$

Link inflow conservation constraint:

$$\sum_{rs} u_a^{rs}(k)=u_a(k)\ \ \forall\ a,k \quad (8.4)$$

Link outflow conservation constraint:

$$\sum_{rs} v_a^{rs}(k)=v_a(k)\ \ \forall\ a,k \quad (8.5)$$

Node flow conservation constraint:

$$\sum_{a\in B(j)} v_a^{rs}(k)=\sum_{a\in A(j)} u_a^{rs}(k)\quad \forall\ j\neq r,s;r,s;k \quad (8.6)$$

where $A(j)$ is the set of links after j and $B(j)$ is the set of links before j.

Link flow propagation constraint:

$$u_a^{rs}(k)=v_a^{rs}(k+\tau_a(k))\quad \forall\ a,\ r,s,k \quad (8.7)$$

The link state equation:

$$x_a(k+1)=x_a(k)+u_a(k)-v_a(k)\quad \forall\ a,k \quad (8.8a)$$

or

$$x_a(k+1)=x_a(k)+u_a(k+1)-v_a(k+1)\quad \forall\ a,k \quad (8.8b)$$

(8.8a) is forward formula, (8.8b) is backward formula.

Path-link flow incidence constraint:

$$u_a^{rs}(n) = \sum_{rs}\sum_{p}\sum_{k=1}^{K_0} f_p^{rs}(k)\delta_{rsa}^{pkn} \qquad \forall\, a, n \tag{8.9}$$

where $\delta_{rsa}^{pkn} \in \{0,1\}$ is defined as:

$$\delta_{rsa}^{pkn} = \begin{cases} 1 & \text{if traffic departing origin } r \text{ at any time interval } k \\ & \text{heading for destination } s \text{ on path } p \text{ arrives at link } a \\ & \text{during the } n\text{th time interval.} \\ 0 & \text{otherwise} \end{cases} \tag{8.10}$$

Nonnegative constraint:

$$f_p^{rs}(k) \ge 0, u_a^{rs}(k) \ge 0, \ \forall\, k, r, s, a \ne b, p \tag{8.11a}$$

$$u_b^{rs}(k) \ge 0, \ \forall\, r, s, k = 1, \cdots, k_1, \text{or } k = k_2, \cdots, K \tag{8.11b}$$

$$u_b^{rs}(k) = 0, \ \forall\, r, s, k = k_1, \cdots, k_2 \tag{8.11c}$$

The difference between the discrete route-based VI DUOIM model and VI DUO is that there is an extra constrain (8.11c) for VI DUOIM model.

8.1.2 Relaxation-Gradient Projection Algorithm for DUOIM

The relaxation with gradient projection algorithm for DUO can be modified to solve the DUOIM model. At each relaxation, we temporarily fix: 1) Actual travel time $\tau_a(k)$ in the link flow propagation constraints as $\bar{\tau}_a(k)$; 2) Actual travel time $\tau_a[k + \pi^{ri}(k)]$ as $\tau_a[k + \bar{\pi}^{ri}(k)]$ and 3) Minimal travel times $\pi^{rs}(k)$ as $\bar{\pi}^{rs}(k)$ for each origin and destination. At each relaxation, the time-space network excluding link b at from k_1 to k_2 is fixed with fixed link flow propagation constraints.

An optimization problem which is equivalent to the discrete VI under relaxation can be formulated, as follows:

$$\min_{\mathbf{f}} Z = \sum_{k=1}^{K_0}\sum_{rs}\sum_{p}\left\{\int_0^{f_p^{rs}(k)} \eta_p^{rs}\left(\omega, \mathbf{f}_p^{rs}\right) d\omega\right\} \tag{8.12}$$

in Θ . where $\mathbf{f}_p^{rs}$ denotes the path flow vector $\mathbf{f}$ without component f_p^{rs}. Θ is the feasible set defined by (8.3) --- (8.11).

At each relaxation, the VI formulation of DUOIM problem was transformed into a series of static user equilibrium traffic assignment problems over the time-space network excluding link *b* from k_1 to k_2, which can be solved by Gradient Projection algorithm.

The algorithm for solving the ideal route-based DUOIM route choice model is summarized as follows.

Step 0: Outer Initialization.

Compute $k_{\max} = \max_{\forall rs}\{\pi^{rs}\}$, where π^{rs} is the static minimum travel time of O-D rs . Set $K^{'} = K_0 + C\cdot[k_{\max}]_+$. Set $\hat{\tau}_a^{(0)}(k) = \tau_a[0], \quad \forall\, a \neq b, \quad k = 1,\cdots,K'$.

$\hat{\tau}_b^{(0)}(k) = \tau_b[0], \ \forall\, k = 1,\cdots,k_1,$ and $\hat{\tau}_b^{(0)}(k) = M, \ \forall\, k = k_1,\cdots,k_2$, or $k = k_2,\cdots,K$ M is a constant which is bigger enough. Find an initial feasible solution $\left[f_p^{rs(0)}(k)\right]$. Set outer iteration counter $l = 0$. Set an outer iteration convergence criterion ε_{out} .

Step 1: Relaxation.

Find a new estimation of actual link travel times: $\hat{\tau}_a^{(l)}(k) = \tau_a\left[x_a^*(k)\right], \ \forall\, a \neq b, \quad k = 1,\cdots,K'$, $\hat{\tau}_b^{(l)}(k) = \tau_b\left[x_b^*(k)\right], \ \forall\, k = 1,\cdots,k_1$, or $k = k_2,\cdots,K$, where * denotes the solution obtained from the most recent inner iteration or from outer initialization. Set $\hat{\tau}_b^{(l)}(k) = M, \ \forall\, k = k_1,\cdots,k_2$, Find $\delta_{a^{'}}^{k(l)}(t)$ and $\delta_{a^{''}}^{k(l)}(t)$.

Step 2: Inner Iteration

Step 2.0: Inner Initialization. Compute and reset the inner initial feasible solution to be consistent with the flow propagation constrain at the current relaxation. Set an inner iteration counter $m = 1$.

In the first relaxation, set $\tau_a^{(1)}(k)$ equal to free flow travel time $\tau_a(0), \forall\, a, \forall\, a \neq b$, $k = 1, \cdots, K'$. $\tau_b(k) = \tau_b[0]$, $\forall\, k = 1, \cdots, k_1$, or $k = k_2, \cdots, K'$, and $\tau_b(k) = M$, $\forall\, k = k_1, \cdots, k_2$, and perform all-or-nothing assignments. This yields initial path flows $f_{(1)}^{rs}(k)$, $\forall\, r, s$, k=1,…, K_0. In other relaxations, reset the most inner iteration solution to be consistent with the flow propagation constrain at the current relaxation, and set them as initial path flows $f_{(1)}^{rs}(k), \forall\, r, s$, k=1,…, K_0, at current relaxation (Initialize the path set P_{rs}^k with the shortest path for each O-D pair *rs* at time *k*).

Step 2.1: Update. Set $\tau_a^{(m)}(k)$ equal to $\tau_a^{(m)}\left[x_a^{(m)}(k)\right]$. Update the first derivative lengths $d_p^{rs(m)}(k)$ (i.e., path cost at current flow) of all of the paths in P_{rs}^k, $\forall\, r, s$.

Step 2.2: Direction finding. Find the shortest-path $\bar{p}_{rs}^{(m)}(k)$ from each origin *r* to each destination *s* at *k* on the basis of $\tau_a^{(m)}(k)$. If different from all the paths in the existing path set in P_{rs}^k, (no need for path comparison here; just compare $d_p^{rs(m)}(k)$, add it to in P_{rs}^k and record $d_{\bar{p}_{rs}^{(m)}(k)}$. If not tag the shortest among the paths in P_{rs}^k in $d_{\bar{p}_{rs}^{(m)}(k)}$.

Step 2.3: Move. Set the new path flows.

$$f_p^{rs(m+1)}(k) = \max\left[0, f_p^{rs(m)}(k) - \frac{\alpha^n}{s_p^{rs(m)}(k)}\left(d_p^{rs(m)}(k) - d_{\bar{p}_{rs}^{(m)}(k)}\right)\right] \forall\, r, s, p \in P_{rs}^k, p \neq \bar{p}_{rs}^{(m)}(k) \tag{8.13}$$

Where

$$s_p^{rs(m)}(k) = \sum_a \sum_k \frac{\partial \tau_a^{(m)}(k)}{\partial x_a^{(m)}(k)}, \forall\, p \in P_{rs}^k \tag{8.14}$$

a and *k* denotes time-space links that are on either *p* or $\bar{p}_{rs}^{(m)}(k)$, but not on both, and α^n is a scalar step-size modifier.

Also,

$$f_{\bar{p}_{rs}(k)}^{(m+1)} = f^{rs}(k) - \sum_{\substack{p \in P_{rs}^k \\ p \neq \bar{p}_{rs}(k)}} f_p^{rs(m+1)}(k) \quad \forall\ p \in P_{rs}^k, p \neq \bar{p}_{rs}^{(m)}(k) \tag{8.15}$$

Assign the flows on the trees and find the link flows $u_a^{(m+1)}(k)$.

Step 2. 4: Convergence Test for Inner Iteration.

If $\sqrt{\sum_{rs}\sum_{k}^{K_0}\left(f_p^{rs(m+1)}(k) - f_p^{rs(m)}(k)\right)} \Big/ \sum_{rs}\sum_{k}^{K_0} f_p^{rs(m)}(k) > \varepsilon$, set $m = m+1$, go to Step 2.1;

otherwise, set $\hat{f}_a^{rs(l)}(k) = f_p^{rs(m+1)}(k), \hat{x}_a^{(l)}(k) = x_a^{(m+1)}(k)$, go to Step 3.

Step 3: Convergence Test for Outer Iteration. If $\hat{\tau}_a^{(l)}(k) \cong \hat{\tau}_a^{(l-1)}(k)$, stop. The current solution $u_a^{rs}(k)$, $v_a^{rs}(k)$, $x_a^{rs}(k)$ is in a near optimal state; otherwise, set $l = l+1$ and go to Step 1.

8.1.3 A Numerical Example

Example 8.1

An example is presented below to validate the above model and algorithms. The configuration of the network is shown in Figure 8.1. In the network, each link is assumed as an one-lane street with a length of 0.5 mi. The free flow speed is assumed to be 25 mile/hour. The following linear travel time function is used to enforce FIFO condition: $\tau_a(k) = L_a / s_f + 0.3 \cdot x_a(k)$, where L_a is the length of link a, s_f is free flow speed, $\tau_a(k)$ is link travel time on link a at time k, $x_a(k)$ is number of vehicles on link a at time k. Four

O-D pairs are considered. Five intervals of 20 seconds are specified. The OD flows are 10 vehicle units per time interval. The O-D pairs and the time-dependent O-D demand are shown in Table 8.1. In this example, the departure horizon is 5 time increments, and the time increment is 20 seconds. Assume an incident occurs at the entrance of link 11 at interval 9 and is cleared at interval 12.

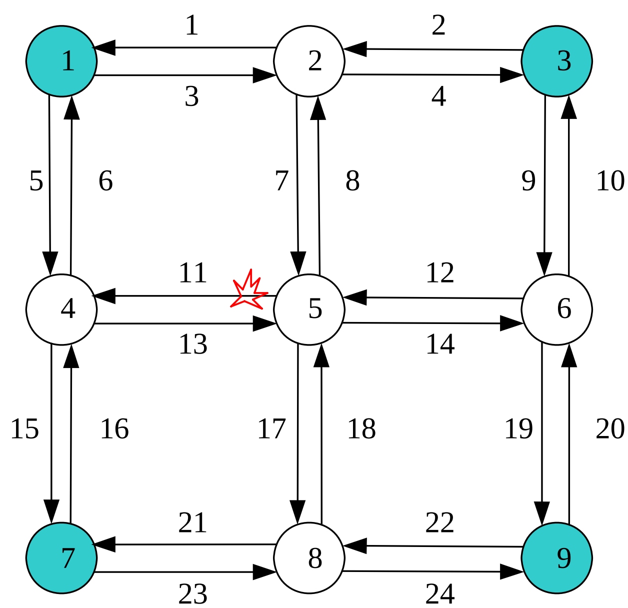

Figure 8.1 Simulation Network with Incident for Example 8.1

Table 8-1 O-D pairs and time-dependent O-D demand for Example 8.1

O-D	Departure time interval *k*				
	1	2	3	4	5
1-9	10	10	10	10	10
9-1	10	10	10	10	10
3-7	10	10	10	10	10
7-3	10	10	10	10	10

The program of the algorithm was run on a computer with 1.5GHz frequency processor. The inner iteration (GP algorithm) convergence test method was set as a pre-specified number n. The outer iteration (Relaxation) convergence test method was set as

$$\max\left\{|\tau_a^{(l)}(k)-\tau_a^{(l-1)}(k)|\,\middle|\,a\in A, k=1,\cdots,K\right\}$$

where $|\tau_a^{(l)}(k)-\tau_a^{(l-1)}(k)|$ is the actual travel time difference of link a at time k between successive relaxations. The operation of the program is shown in Table 8.2.

Table 8-2 Convergence criterion and computation time for Example 8.1

Inner iteration convergence criterion	Outer iteration convergence criterion	Total relaxations	Total computation time (minute)
n=4	0.001	16	7.1

The assignment horizon K is found to be 21 time increments. Table 8.3a shows the output of $u_a^{rs}(k)$. Table 8.3b shows the output of $v_a^{rs}(k)$. Table 8.3c shows the output of $u_a(k)$. Table 8.3d shows the output of $v_a(k)$. Table 8.3e shows the output of $x_a(k)$. Table 8.3f shows the output of $\tau_a(k)$. Table 8.3g shows the output of $f_p^{rs}(k)$, $c_p^{rs}(k)$, links on each path and the arrival time interval for each link on a path. For conciseness, only Table 8.3g is attached to this dissertation.

Table 8-3 The resultant path flow and path travel time for example 8.1

Path number	O	D	k	Path flow	Path time	Links on the path	Arrival time for each link on the path
This table is appendix 11 of this thesis.							

We take the following examples to verify that the solution satisfy the constraints and the dynamic User Optimal route choice with incident management conditions.

Path flow conservation constraint is automatically satisfied:

$f^{19}(1) = f_1^{19}(1) + f_2^{19}(1) + f_3^{19}(1) + f_4^{19}(1) + f_5^{19}(1) + f_6^{19}(1)$

=3.4259+1.8491+3.2407+1.4787+0.0028+0.0028

=10

Link inflow conservation constraint (8.4):

$u_8^{91}(10) + u_8^{73}(10)$=2.4076+1.4814=3.8890=$u_8(10)$

Link outflow conservation constraint (8.5):

$v_8^{91}(14) + v_8^{73}(14)$=2.4076+1.4814=3.8890=$v_8(14)$

Node flow conservation constraint (8.6):

$\sum_{a \in B(6)} v_a^{rs}(k) = \sum_{a \in B(6)} v_a(k) = v_9(8) + v_{14}(8) + v_{20}(8) =$ 4.9073+0+4.9076=9.8149

$\sum_{a \in A(6)} u_a^{rs}(k) = \sum_{a \in A(6)} u_a(k) = u_{10}(8) + u_{12}(8) + u_{19}(8)$=3.4255+2.9630+3.4263= 9.8148

Link flow propagation constraint (8.7):

$u_8^{91}(10) = v_8^{91}(10 + \bar{\tau}_8(10)) = v_8^{91}(14)$=2.4076

$u_8^{73}(10) = v_8^{73}(10 + \bar{\tau}_8(10)) = v_8^{73}(14)$=1.4814

where $\tau_8(10)$=1.2388 minutes. For a time increment of 20 seconds, $\bar{\tau}_8(10)$=4.

The link state equation (8.8b):

$x_8(10) = x_8(9) + u_8(10) - v_8(10)$ = 3.8889+3.8889 – 0 = 7.7778

Incident constrain (8.11c):

$u_{11}^{rs}(k) = 0, \ \forall\, r, s, k = 9, \cdots, 12$

The actual travel times on the used paths from origin 1 toward destination 9 departing at time increment 1 are as follows:

$c_1^{19}(1) =$

$\tau_5(1) + \tau_{15}(1+\bar{\tau}_5(1)) + \tau_{23}(1+\bar{\tau}_5(1)+\bar{\tau}_{15}(1+\bar{\tau}_5(1))) + \tau_{24}(1+\bar{\tau}_5(1)+\bar{\tau}_{15}(1+\bar{\tau}_5(1)) + \bar{\tau}_{23}(1+\bar{\tau}_5(1)+\bar{\tau}_{15}(1+\bar{\tau}_5(1))))$

$= \tau_5(1) + \tau_{15}(5) + \tau_{23}(9) + \tau_{24}(13)$

= 1.2244+1.2170+1.2170+1.2244

= 4.8829 minutes

Similarly, we have $c_2^{19}(1) = c_3^{19}(1) = c_4^{19}(1) = c_5^{19}(1) = c_6^{19}(1) = 4.8829$ minutes. They are nearly equal.

As can be checked in the same way, all the solution output satisfies the constraints and the dynamic user optimal route choice with incident management conditions. This verifies the validity of the solution algorithm.

8.2 Dynamic User Optimal Route Choice Problem Integrated with Signal Timing System (DUOST)

Since most intersections are signalized, it is important to develop DUO model integrated with signal timing system. Unfortunately, our literature view shows that study in this field is still scarce. Sun et al. (2006) developed a bi-level programming formulation and heuristic solution approach (HSA) for dynamic traffic signal optimization in etworks with time dependent demand and stochastic route choice. In the bi-level programming model, the upper level problem represents the decision-making behavior (signal control) of the system manager, while the user travel behavior is represented at the lower level. The HSA consists of a Genetic Algorithm (GA) and a Cell Transmission Simulation (CTS) based Incremental Logit Assignment (ILA) procedure. GA is used to seek the upper level signal control variables. ILA is developed to find user optimal flow pattern at the lower level, and CTS is implemented to propagate traffic and collect real-time traffic information. Varia and Dhingra (2004) proposed a dynamic system optimal traffic assignment model for a congested urban road network with a number of signalized intersections. A simulation-based approach is employed for the case of multiple-origin-multiple-destination traffic flows. Genetic algorithm is used to minimize the overall travel cost in the network with fixed signal timing and optimization of signal timing.

In this section, we present a discrete route-based ideal User Optimal route choice model integrated with signal timing system at under-saturated condition for a multiple origin multiple destination road network. A relaxation with gradient projection algorithm is presented for the model. A numerical example is given.

8.2.1 Discrete Route-based Variational Inequality (VI) DUOST Model

To present the discrete VI DUOST Model, we discretize the time domain with each time interval being the assignment increment. The estimated actual travel time on each link a is a multiple of the time increment and is fixed at each time increment, i.e.

$$\bar{\tau}_a(k) = i \quad \text{if} \quad (i-0.5)\Delta t \le \tau_a(k) \le (i+0.5)\Delta t \tag{8.16}$$

where i is an integer and $0 \le i \le K$, Δt is time increment. This round-off method is used only in the flow propagation constraints. The round-off error can be made as small as desired by making the assignment increment smaller.

We define the incoming legs for each signalized intersection as the streets entering the intersection. Figure 8.2 shows the incoming legs of the intersection and their network denotation.

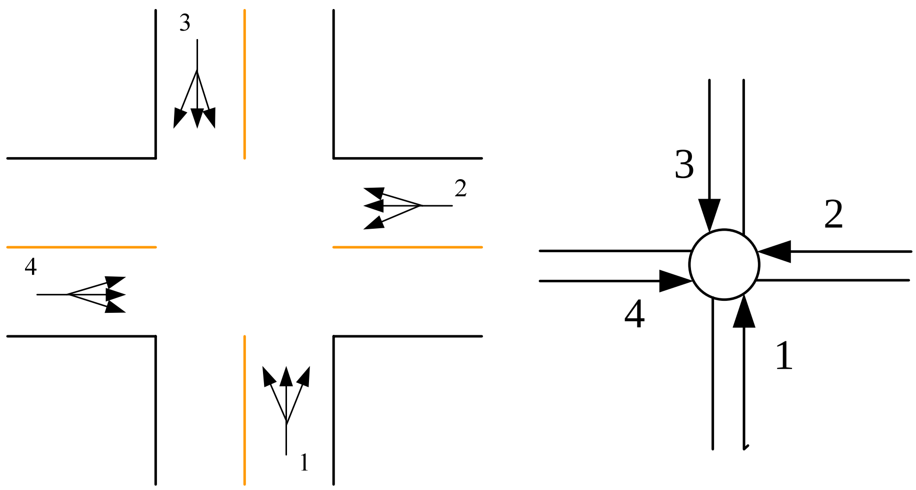

Figure 8.2 Incoming Legs of an Intersection and their Network Denotation

We assume the signal timing for all the signalized intersection are preset and fixed in the analysis period. We consider the under-saturated situation, in which all vehicles queued during red time and coming during green time of a cycle can pass the intersection in the same cycle and no vehicle queued at the end of the cycle. We assume the cycle time, green time and red time of all the signals are multiples of the time increment. For simplicity we assume a cycle has two time increments: one for red time and another for green time. Refer to the ℓth cycle of link a as the ℓth cycle of the signal of the intersection of which link a is an incoming leg. Let $q_a^r(k)$ be the number of vehicles queued at the end of the red time increment of link *a*. Let $q_a^g(k)$ be the number of vehicles joining the queue or entering the intersection during the green time increment of link *a*.

We use the point-queue (PQ) model to evaluate the link travel time for its accuracy in evaluating link travel time and its respect of First-In-First-Out (FIFO) condition. According to Nie and Zhang (2005), PQ model is as accurate as Cell Transmission Model (CTM) in evaluate link travel time. The latter is a finite approximation to hydrodynamic model and is a benchmark in evaluating link travel time. Since PQ model is a linear model it respects FIFO condition. Our point-queue model is stated as

$$\tau_a(t) = \begin{cases} \tau_a(0) + q_a(t+\tau_a(0))/S_a, & (t+\tau_a(0)) \in \text{green} \\ \tau_a(0) + \Delta t + q_a(t+\tau_a(0))/S_a, & (t+\tau_a(0)) \in \text{red} \end{cases} \tag{8.17}$$

$q_a(t+\tau_a(0))$ is the total number of vehicles queued at the exit of link a at $(t+\tau_a(0))$, $\tau_a(0)$ is the free flow travel time, S_a is the saturation flow rate for link *a*.

The point-queue model in discrete form is

$$\tau_a(k) = \begin{cases} \tau_a(0) + q_a(k+\bar{\tau}_a(0))/S_a, & (k+\bar{\tau}_a(0)) \in \text{green} \\ \tau_a(0) + \Delta t + q_a(k+\bar{\tau}_a(0))/S_a, & (k+\bar{\tau}_a(0)) \in \text{red} \end{cases} \tag{8.18}$$

The equation for the inflow and the queue of link *a* can be expressed as

$$u_a(k) = \begin{cases} q_a^g(k+\bar{\tau}_a(0)), & (t+\bar{\tau}_a(0)) \in \text{green} \\ q_a^r(k+\bar{\tau}_a(0)), & (t+\bar{\tau}_a(0)) \in \text{red} \end{cases} \tag{8.19}$$

The equation for the outflow and the queue of link *a* can be expressed as

$$v_a(k+\bar{\tau}_a(0)) = \begin{cases} q_a^r(k+\bar{\tau}_a(0)-1) + q_a^g(k+\bar{\tau}_a(0)), & (t+\bar{\tau}_a(0)) \in \text{green} \\ 0, & (t+\bar{\tau}_a(0)) \in \text{red} \end{cases} \tag{8.20}$$

In under-saturated situation, we have

$$q_a\left(k+\bar{\tau}_a(0)\right)=\begin{cases} q_a^r\left(k+\bar{\tau}_a(0)-1\right)+q_a^g\left(k+\bar{\tau}_a(0)\right), & \left(t+\bar{\tau}_a(0)\right)\in \text{green} \\ q_a^r\left(k+\bar{\tau}_a(0)\right), & \left(t+\bar{\tau}_a(0)\right)\in \text{red}\end{cases} \quad (8.21)$$

Assume the network is empty at $k=0$, and only travel demands departing within the departure horizon are considered. The discrete route-based VI DUOST Model is

$$\left\langle\left[\boldsymbol{\eta}^*(k)-\boldsymbol{\pi}^*(k)\right],\mathbf{f}(k)-\mathbf{f}^*(k)\right\rangle \geq 0 \quad (8.22a)$$

Or in expanded form, as

$$\sum_{rs}\sum_{p}\sum_{k=1}^{K_0}\left[\eta_p^{rs^*}(k)-\pi^{rs^\bullet}(k)\right]\left[f_p^{rs}(k)-f_p^{rs^*}(k)\right]\geq 0 \quad (8.22b)$$

where $\boldsymbol{\eta},\mathbf{f}\in\Re_+^{|P|\times K_0}$,.

$$\eta_p^{ri}(k)=\eta_p^{r(i-1)}(k)+\tau_a\left[k+\eta_p^{r(i-1)}(k)\right] \quad \forall\ p=p,r,i;\ i=1,2,\cdots,s;$$

$$p=(r,1,2,\cdots,i,\cdots s),\quad x\in\Theta \quad (8.21c)$$

Θ is the feasible region defined by the following constraints:

Path flow conservation constraint:

$$\sum_p f_p^{rs}(k)=f^{rs}(k)\ \forall\, k,r,s \quad (8.23)$$

Link inflow conservation constraint:

$$\sum_{rs}u_a^{rs}(k)=u_a(k)\ \ \forall\, a,k \quad (8.24)$$

Link outflow conservation constraint:

$$\sum_{rs}v_a^{rs}(k)=v_a(k)\ \ \forall\, a,k \quad (8.25)$$

Node flow conservation constraint:

$$\sum_{a\in B(j)}v_a^{rs}(k)=\sum_{a\in A(j)}u_a^{rs}(k)\quad \forall\ j\neq r,s;r,s;k \quad (8.26)$$

where $A(j)$ is the set of links after j and $B(j)$ is the set of links before j.

Link flow propagation constraint:

$$u_a^{rs}(k) = v_a^{rs}(k + \tau_a(k)) \quad \forall\, a,\ r, s, k \tag{8.27}$$

Inflow and queue equation

$$u_a(k) = \begin{cases} q_a^g(k + \bar{\tau}_a(0)), & (t + \bar{\tau}_a(0)) \in \text{green} \\ q_a^r(k + \bar{\tau}_a(0)), & (t + \bar{\tau}_a(0)) \in \text{red} \end{cases} \tag{8.28}$$

Outflow and queue equation

$$v_a(k + \bar{\tau}_a(0)) = \begin{cases} q_a^r(k + \bar{\tau}_a(0) - 1) + q_a^g(k + \bar{\tau}_a(0)), & (t + \bar{\tau}_a(0)) \in \text{green} \\ 0, & (t + \bar{\tau}_a(0)) \in \text{red} \end{cases} \tag{8.29}$$

Path-link flow incidence constraint:

$$u_a^{rs}(n) = \sum_{rs} \sum_{p} \sum_{k=1}^{K_0} f_p^{rs}(k) \delta_{rsa}^{pkn} \quad \forall\, a, n \tag{8.30}$$

where $\delta_{rsa}^{pkn} \in \{0,1\}$ is defined as:

$$\delta_{rsa}^{pkn} = \begin{cases} 1 & \text{if traffic departing origin } r \text{ at any time interval } k \\ & \text{heading for destination } s \text{ on path } p \text{ arrives at link } a \\ & \text{during the } n\text{th time interval.} \\ 0 & \text{otherwise} \end{cases} \tag{8.31}$$

Nonnegative constraint:

$$f_p^{rs}(k) \geq 0, u_a^{rs}(k) \geq 0, \ \forall\, k, r, s, a, p \tag{8.32}$$

8.2.2 Relaxation-Gradient Projection Algorithm for DUOST

The relaxation with gradient projection algorithm for DUO can be modified to solve the DUOST model. At each relaxation, we temporarily fix: 1) Actual travel time $\tau_a(k)$ in the link flow propagation constraints as $\bar{\tau}_a(k)$; 2) Actual travel time $\tau_a[k + \pi^{ri}(k)]$ as $\tau_a[k + \bar{\pi}^{ri}(k)]$; and 3) Minimal travel times $\pi^{rs}(k)$ as $\bar{\pi}^{rs}(k)$ for each origin and destination. At each relaxation, the time-space network is fixed with fixed signal timing and fixed link flow propagation constraints.

An optimization problem which is equivalent to the discrete VI under relaxation can be formulated, as follows:

$$\min_{\mathbf{f}} Z = \sum_{k=1}^{K_0} \sum_{rs} \sum_{p} \left\{ \int_0^{f_p^{rs}(k)} \eta_p^{rs}\left(\omega, \mathbf{f}_p^{rs}\right) d\omega \right\} \tag{8.33}$$

in Θ, where $\mathbf{f}_p^{rs}$ denotes the path flow vector $\mathbf{f}$ without component f_p^{rs}. Θ is the feasible set defined by (8.23) --- (8.32).

At each relaxation, the VI formulation of DUOST problem was transformed into a series of static user equilibrium traffic assignment problems over the time-space network with fixed signal timing, which can be solved by Gradient Projection algorithm.

The algorithm for solving the ideal route-based DUOST route choice model is summarized as follows.

Step 0: Outer Initialization.

Compute $k_{\max} = \max_{\forall rs}\{\pi^{rs}\}$, where π^{rs} is the static minimum travel time of O-D rs. Set $K^{'} = K_0 + C \cdot [k_{\max}]_+$. Set $\hat{\tau}_a^{(0)}(k) = \tau_a[0]$ based on (8.18), $\forall\, a,\quad k = 1, \cdots, K'$. Find an initial feasible solution $\left[f_p^{rs(0)}(k)\right]$. Set outer iteration counter $l = 0$. Set an outer iteration convergence criterion ε_{out}.

Step 1: Relaxation.

Find a new estimation of actual link travel times: $\hat{\tau}_a^{(l)}(k) = \tau_a\left[q_a^*(k)\right],\quad \forall\, a \neq b,\quad k = 1, \cdots, K'$, where * denotes the solution obtained from the most recent inner iteration or from outer initialization. Find $\delta_{a'}^{k(l)}(t)$ and $\delta_{a''}^{k(l)}(t)$.

Step 2: Inner Iteration

Step 2.0: Inner Initialization. Compute and reset the inner initial feasible solution to be consistent with the flow propagation constrain at the current relaxation. Set an inner iteration counter $m = 1$.

In the first relaxation, set $\tau_a^{(1)}(k)$ equal to free flow travel time $\tau_a(0)$, $\forall\, a$, $\forall\, a \neq b$, $k = 1, \cdots, K'$, and perform all-or-nothing assignments. This yields initial path flows $f_{(1)}^{rs}(k), \forall\ r, s$, k=1,…, K_0. In other relaxations, reset the most inner iteration solution to be consistent with the flow propagation constrain at the current relaxation, and set them as initial path flows $f_{(1)}^{rs}(k), \forall\ r, s$, k=1,…, K_0, at current relaxation (Initialize the path set P_{rs}^k with the shortest path for each O-D pair *rs* at time *k*).

Step 2.1: Update. Set $\tau_a^{(m)}(k)$ equal to $\tau_a^{(m)}\left[q_a^{(m)}(k)\right]$. Update the first derivative lengths $d_p^{rs(m)}(k)$(i.e., path cost at current flow) of all of the paths in P_{rs}^k, $\forall\ r, s$.

Step 2.2: Direction finding. Find the shortest-path $\bar{p}_{rs}^{(m)}(k)$ from each origin *r* to each destination *s* at *k* on the basis of $\tau_a^{(m)}(k)$. If different from all the paths in the existing path set in P_{rs}^k, (no need for path comparison here; just compare $d_p^{rs(m)}(k)$, add it to in P_{rs}^k and record $d_{\bar{p}_{rs}^{(m)}(k)}$. If not tag the shortest among the paths in P_{rs}^k in $d_{\bar{p}_{rs}^{(m)}(k)}$.

Step 2.3: Move. Set the new path flows.

$$f_p^{rs(m+1)}(k) = \max\left[0, f_p^{rs(m)}(k) - \frac{\alpha^n}{s_p^{rs(m)}(k)}\left(d_p^{rs(m)}(k) - d_{\bar{p}_{rs}^{(m)}(k)}\right)\right] \forall\ r, s, p \in P_{rs}^k, p \neq \bar{p}_{rs}^{(m)}(k) \qquad (8.34)$$

Where

$$s_p^{rs(m)}(k) = \sum_a \sum_k \frac{\partial \tau_a^{(m)}(k)}{\partial q_a^{(m)}(k)}, \forall\ p \in P_{rs}^k \qquad (8.35)$$

a and *k* denotes time-space links that are on either *p* or $\bar{p}_{rs}^{(m)}(k)$, but not on both, and α^n is a scalar step-size modifier.

Also,

$$f_{\bar{p}_{rs}(k)}^{(m+1)} = f^{rs}(k) - \sum_{\substack{p \in P_{rs}^{k} \\ p \neq \bar{p}_{rs}(k)}} f_{p}^{rs(m+1)}(k) \quad \forall\, p \in P_{rs}^{k},\, p \neq \bar{p}_{rs}^{(m)}(k) \tag{8.36}$$

Assign the flows on the trees and find the link flows $u_a^{(m+1)}(k)$.

Step 2. 4: Convergence Test for Inner Iteration.

If $\sqrt{\sum_{rs}\sum_{k}^{K_0}\left(f_p^{rs(m+1)}(k) - f_p^{rs(m)}(k)\right)} \Big/ \sum_{rs}\sum_{k}^{K_0} f_p^{rs(m)}(k) > \varepsilon$, set $m = m+1$, go to Step 2.1; otherwise, set $\hat{f}_a^{rs(l)}(k) = f_p^{rs(m+1)}(k), \hat{q}_a^{(l)}(k) = q_a^{(m+1)}(k)$, go to Step 3.

Step 3: Convergence Test for Outer Iteration. If $\hat{\tau}_a^{(l)}(k) \cong \hat{\tau}_a^{(l-1)}(k)$, stop. The current solution $u_a^{rs}(k)$, $v_a^{rs}(k)$, $q_a^{rs}(k)$ is in a near optimal state; otherwise, set $l = l+1$ and go to Step 1.

8.2.3 A Numerical Example

Example 8.2

An example is presented below to validate the above model and algorithms. The configuration of the network is shown in Figure 8.3. In the network, each link is assumed as an two-lane or one-lane street with a length of 0.5 mi. Links without signal timing include link 1, 4, 6, 10, 15, 19, 21, and 24. All the other links have signal timing. Links whose signal is red-green alternation from the 1st time increment include link 2, 3, 5, 9, 12, 13, 16, 20, 22, and 23. Links whose signal is green-red alternation from the 1st time increment include link 7, 8, 11, 14, 17, and 18. Links whose saturation flow rate is 2 vehicle units/second include link 7, 8, 11, 12, 13, 14, 17, and 18. All the other links have saturation flow rate of 4 vehicle units/second. The free flow speed of all the links is assumed to be 25 mile/hour or 36.67 feet/second. We assume links without signal timing have constant link travel time $\tau_a(k) = L_a / s_f + 0.35\Delta t$, where L_a is the length of link a, s_f is free flow speed, $\tau_a(k)$ is link travel time on link a at time k. The link travel times on other links are defined by equation (8.18), where $\tau_a(0) = L_a / s_f$.

Four O-D pairs are considered. Five 30 s departure time intervals are specified. The OD flows are 10 vehicle units per time interval. The O-D pairs and the time-dependent O-D demand are shown in Table 8.4. In this example, the departure horizon is 5 time increments, and the time increment is 30 seconds.

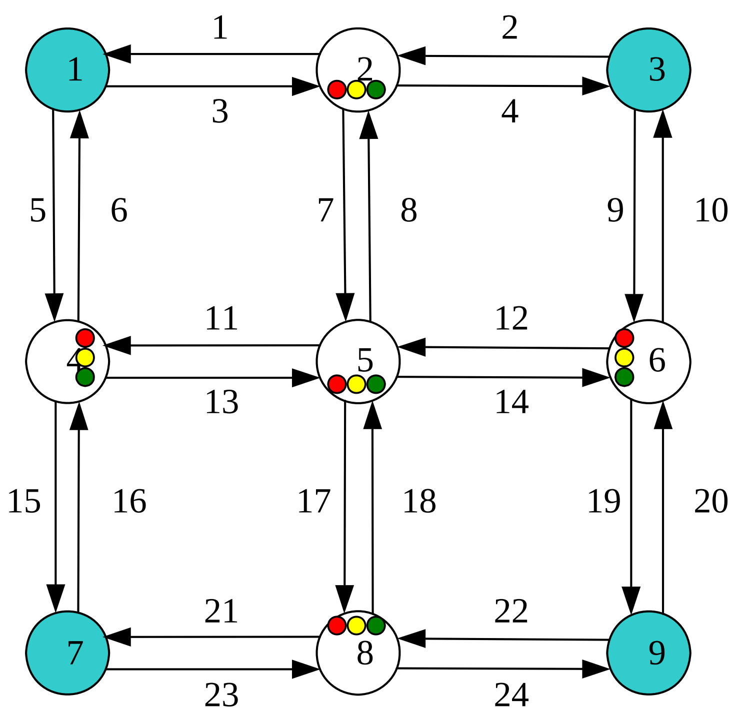

Figure 8.3 Simulation Network with Signal Timing

Table 8-4 O-D pairs and time-dependent O-D demand for Example 8.2

O-D	Departure time interval k				
	1	2	3	4	5
1-9	10	10	10	10	10
9-1	10	10	10	10	10
3-7	10	10	10	10	10
7-3	10	10	10	10	10

The program of the algorithm was run on a computer with 1.5 GHz frequency processor. The inner iteration (GP algorithm) convergence test method was set as a pre-specified number n . The outer iteration (Relaxation) convergence test method was set as

$$\max\left\{\left|\tau_a^{(l)}(k)-\tau_a^{(l-1)}(k)\right| \middle| a \in A, k=1,\cdots,K\right\}$$

where $|\tau_a^{(l)}(k)-\tau_a^{(l-1)}(k)|$ is the actual travel time difference of link a at time k between successive relaxations. The operation of the program is shown in Table 8.5.

Table 8-5 Convergence criterion and computation time for Example 8.2

Inner iteration convergence criterion	Outer iteration convergence criterion	Total relaxations	Total computation time (minute)
n=4	0.000001	13	3.3

The assignment horizon K is found to be 21 time increments. Table 8.6a shows the output of $u_a^{rs}(k)$. Table 8.6b shows the output of $v_a^{rs}(k)$. Table 8.6c shows the output of $u_a(k)$. Table 8.6d shows the output of $v_a(k)$. Table 8.6e shows the output of $q_a^r(k)$. Table 8.6f shows the output of $q_a^g(k)$. Table 8.6g shows the output of $q_a(k)$. Table 8.6hshows the output of $\tau_a(k)$. Table 8.6i shows the output of $f_p^{rs}(k)$, $c_p^{rs}(k)$, links on each path and the arrival time interval for each link on a path. For conciseness, only Table 8.6i is attached to this dissertation.

Table 8-6 The resultant path flow and path travel time for example 8.2

Path number	*O*	*D*	*k*	Path flow	Path time	Links on the path	Arrival time for each link on the path
This table is appendix 12 of this thesis.							

The following examples are exhibited to verify that the solution satisfy the constraints and the dynamic User Optimal route choice with signal timing conditions.

Path flow conservation constraint (8.23):

$f^{19}(2)= f_1^{19}(2)+ f_2^{19}(2)+ f_3^{19}(2)$

=5.7435+1.3338+2.9227=10

Link inflow conservation constraint (8.24):

$u_8^{91}(9)+u_8^{73}(9)$=6.8333+6.8333=13.6666=$u_8(9)$

Link outflow conservation constraint (8.25):

$v_8^{91}(13)+v_8^{73}(13)$=6.8333+6.8333=13.6666=$v_8(13)$

Node flow conservation constraint (8.26):

$$\sum_{a\in B(6)} v_a^{rs}(k) = \sum_{a\in B(6)} v_a(k) \quad = v_9(8)+v_{14}(8)+v_{20}(8) = 6.3333+0+6.3333=13.6666$$

$$\sum_{a\in A(6)} u_a^{rs}(k) = \sum_{a\in A(6)} u_a(k) = u_{10}(8)+u_{12}(8)+u_{19}(8) = 6.3333+0+6.3333=13.6666$$

Link flow propagation constraint (8.27):

$$u_8^{91}(9) = v_8^{91}(9+\bar{\tau}_8(9)) = v_8^{91}(13) = 6.8333$$

$$u_8^{73}(9) = v_8^{73}(9+\bar{\tau}_8(9)) = v_8^{73}(13) = 6.8333$$

where $\tau_8(9)$=1.7568 minutes. For a time increment of 30 seconds, $\bar{\tau}_8(10)$=4.

Inflow and queue equation (8.28):

$u_2(4) = q_2^r(7)$=8.6662, for link 2, time $7 \in$ red ,

$u_2(5) = q_2^g(8)$=5.0005, for link 2, time $8 \in$ green

Outflow and queue equation (8.29)

$v_2(7)$=0, for link 2, time $7 \in$ red

$v_2(8) = q_2^r(7) + q_2^g(8) = q_2(8)$=13.6667, for link 2, time $8 \in$ green

The actual travel times on the used paths from origin 1 toward destination 9 departing at time increment 1 are as follows:

$c_1^{19}(1) =$

$\tau_3(1)+\tau_7(1+\bar{\tau}_3(1))+\tau_{17}(1+\bar{\tau}_3(1)+\bar{\tau}_7(1+\bar{\tau}_3(1)))+\tau_{24}(1+\bar{\tau}_3(1)+\bar{\tau}_7(1+\bar{\tau}_3(1))+\bar{\tau}_{17}(1+\bar{\tau}_3(1)+\bar{\tau}_7(1+\bar{\tau}_3(1))))$

$= \tau_3(1)+\tau_7(4)+\tau_{17}(7)+\tau_{24}(11)$

= 1.2416+1.2832+1.7416+1.3749

= 5.6413 minutes

Similarly, we have $c_2^{19}(1) = 5.6413$ minutes. They are equal.

As can be checked in the same way, all the solution output satisfies the constraints and the dynamic user optimal route choice with signal timing conditions. This verifies the validity of the solution algorithm.

Chapter 9: Conclusions and Future Research

9.1 Conclusions

In this dissertation, I have made a comprehensive study on dynamic travel choice problems and have presented a series of variational inequality models and solution algorithms for them. Problems covered include deterministic dynamic user optimal route (DUO) choice problem, stochastic dynamic user optimal route (SDUOC) choice problem, dynamic user optimal simultaneous departure time and route choice (DUOSDTRC) problem, combined mode split and dynamic user optimal simultaneous departure time and route choice (MS DUOSDTRC) problem, combined trip distribution and dynamic user optimal simultaneous departure time and route choice (TD DUOSDTRC) problem, and combined trip distribution mode split and dynamic user optimal simultaneous departure time and route choice (TD MS DUOSDTRC) problem, dynamic user optimal route choice with incident management (DUOIM) problem, and dynamic user optimal route choice integrated with signal timing (DUOST) system.

The ideal DUO model describes the ideal user optimal route choice under the assumption of perfect routing information to roadway users. Combined dynamic travel choice modeling is built on the basis of the DUO model. In this study, newly developed Relaxation with F-W algorithms and Relaxation with GP algorithms are proposed for both the link-based and route-based VI DUO models.

Due to incapability of providing perfect traffic information to road users at any time, stochastic factors are introduced in estimating drivers' perceptions of their travel times, and as a result, the SDUO model is introduced. In this study, a new link-based VI formulation of stochastic dynamic user optimal route choice problem and a link-base relaxation with MSA algorithm are developed. A route-base relaxation with MSA algorithm is also proposed.

In a real-world transportation system, alternative routes are available to the commuters, and they may choose different departure times and alternative routes, depending on the times of a day to avoid recurring congestions so as to arrive at destinations within the anticipated time intervals. The DUOSDTRC is extended from the DUO route choice model under such an additional assumption that both the departure time and route over a road network must be chosen simultaneously. In this study, the DUOSDTRC problem and its VI formulation are integrated. An analytical Relaxation with multilevel GP algorithm is proposed for the DUOSDTRC model.

In addition to choosing departure time to begin their trip and choosing alternative routes toward their destinations, people may choose different transportation modes to travel when both transit and passenger car are available. The combined MS DUOSDTRC extends the DUOSDTRC route choice model in one respect: transportation mode, departure time and route must be chosen. When routes from an origin to a destination are congested, people may choose another destination to fulfill their need. This will alter the trip distributions pattern. The TD DUOSDTRC extends the DUOSDTRC route choice model in another respect: destination, departure time and route must be chosen. Based on the travel information provided, the available travel mode, and the congestion level of the road, people may choose different destination, travel mode, departure time and route to fulfill their travel need. The combined TD MS DUOSDTRC extends the DUOSDTRC route choice model in two respects: destination, mode, departure time and route must be chosen. In this study, these three combined dynamic travel choice models and their solution algorithms are developed and the validations of the model and algorithms are conducted through numerical examples.

The objective of an incident traffic management methodology is to determine an ideal traffic flow pattern that would minimize total network delay and congestion (system optimal) under the effect of traffic incidents or would minimize each vehicle's delay (user optimal) under the effect of traffic incidents. The dynamic user optimal route choice with incident management (DUOIM) accounts for the dynamic user optimal route choice problem under the influence of the incidents. Since most intersections are signalized, it is important to develop DUO model integrated with signal timing system (DUOST). In this study, the VI formulations of both DUOIM and DUOST under unsaturated conditions are developed. A relaxation with GP algorithm for each model is validated by numerical examples.

Through studies presented in the dissertation, all developed algorithms are proven to be capable of overcoming the drawbacks of other existing algorithms with the following functionalities to: (1) find the time-dependent path flows without path enumeration; 2) avoid time-space network expansion; and 3) treat departure horizon freely.

9.2 Future Research

The potential future research is to consider the development of the models and solution algorithms for combined location choice mode split and dynamic user optimal simultaneous departure time and route choice problem (LC MS DUOSDTRC). The change of actual travel impedance among zones may affect the residential or industrial location choice. The combined LC MS DUOSDTRC extends the DUOSDTRC route choice model with the assumption that location, mode, departure time and route must be chosen.

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Appendices

Appendix 1 Variational Inequality

Definition of Variational Inequality

Definition 1. Let $F : D \subset \Re^n \rightarrow \Re^n$ be a vector-valued and continuous mapping on a nonempty, closed and convex set. The variational inequality problem is to find a vector $\mathbf{x}^* \in D$ such that

$$\left\langle F\left(\mathbf{x}^*\right), \mathbf{x} - \mathbf{x}^* \right\rangle \geq 0 \quad \forall \, \mathbf{x} \in D \tag{1}$$

where $\langle a, b \rangle = a^T b$

Theorem 1. Let $F : D \subset \Re^n \rightarrow \Re^n$ be a vector-valued and continuous differentiable mapping on a nonempty , closed and convex set and the Jacobian matrix of $F(\mathbf{x})$ is symmetric and positive semidefinite. Then there exists a real-valued function $Z(\mathbf{x})$ with $\nabla Z(\mathbf{x}) = F(\mathbf{x})$ such that the solution to variational inequality problem (1) is also the solution to the optimization problem:

$$\min \quad g(\mathbf{x}) \tag{2}$$

s.t $\mathbf{x} \in D$

Existence and Uniqueness

Theorem 2. If D is compact convex set and $F(\mathbf{x})$ is continuous on D, the variational inequality problem has at least one solution $\mathbf{x}^*$.

Theorem 3. Suppose $F(\mathbf{x})$ is strictly monotone on D. Then, the solution is unique, if one exists.

Theorem 4. Suppose $F(\mathbf{x})$ is continuous differentiable on D and the Jacobian matrix of $F(\mathbf{x})$ is positive semidefinite (or positive definite), then $F(\mathbf{x})$ is monotone (or strictly monotone).

Theorem 5. Assume $F(\mathbf{x})$ is continuous differentiable at some $\bar{\mathbf{x}}$. Then $F(\mathbf{x})$ is locally strictly (or strongly) monotone at $\bar{\mathbf{x}}$ if the Jacobian matrix of $F(\mathbf{x})$, denoted as $\nabla F(\bar{\mathbf{x}})$, is positive definite, that is,

$$\mathbf{v}\nabla F(\bar{\mathbf{x}})\mathbf{v} > \mathbf{0}, \qquad \forall\ \mathbf{v} \in R^n, \mathbf{v} \neq \mathbf{0} \tag{3a}$$

or strongly positive definite, that is,

$$\mathbf{v}\nabla F(\bar{\mathbf{x}})\mathbf{v} > \alpha\|\mathbf{v}\|^2, \qquad \text{for some} \quad \alpha > 0, \forall\ \mathbf{v} \in R^n \tag{3b}$$

Theorem 6. Suppose $F(\mathbf{x})$ is continuous differentiable on D and the Jacobian matrix of $F(\mathbf{x})$ is strongly positive definite, then $F(\mathbf{x})$ is strongly monotone.

Theorem 7. Suppose $F(\mathbf{x})$ is strongly monotone on D. Then there exists precisely one solution $\mathbf{x}^*$ to the variational inequality (1)

Diagonalization/Relaxation Algorithm

Consider the variational inequality problem (1). If there exists a mapping f : $D \times D \rightarrow \Re^n$ such that the following properties hold:

1. $f(\mathbf{x}, \mathbf{y}) = F(\mathbf{x}) \quad \forall\ \mathbf{x} \in D$

2. $\nabla f(\mathbf{x},\mathbf{y})$ is positive definite, and $\nabla f(\mathbf{x},\mathbf{y}) = \nabla f(\mathbf{x},\mathbf{y})^T$.

where $D \times D$ is Cartesian product of D, ∇ is gradient operator, then $f(\mathbf{x},\mathbf{y})$ is a gradient mapping and there exists a mapping $Z: D \times D \to \Re^1$ such that the following holds:

$$f(\mathbf{x},\mathbf{y}) = \nabla_{\mathbf{x}} Z(\mathbf{x},\mathbf{y}) \tag{4}$$

Further, solving the variational inequality problem

$$\left\langle f\left(\mathbf{x}^{(n)}, \mathbf{x}^{(n-1)}\right), \mathbf{x} - \mathbf{x}^{(n)} \right\rangle \geq 0 \quad \forall\ \mathbf{x} \in D \tag{5}$$

is equivalent to solving the mathematical programming problem

$$\min \quad Z\left(\mathbf{x}, \mathbf{x}^{(n-1)}\right) \tag{6}$$

s.t $\mathbf{x} \in D$

where n is the iteration number.

The relaxation method for solving (1) is as follows:

Step 0: Initialization

Find a set of variables $\mathbf{x}^{(0)} \in D$. Set $n := 0$

Step 1: Relaxation

Solve the mathematical problem:

$$\min \quad Z\left(\mathbf{x}^{(n)}, \mathbf{x}^{(n-1)}\right) \tag{7}$$

s.t $\mathbf{x} \in D$

Step 2: Convergence test

If $\left|\mathbf{x}^{(n)} - \mathbf{x}^{(n-1)}\right| \leq \varepsilon$, ε is stopping criterion, stop; otherwise set $n := n+1$, and go to Step 1.

Appendix 2: Output for Example 4.1

Table 4.3g The resultant path flow and path travel time for example 4.1.

Path number	*r*	*s*	*k*	$f_p^{rs}(k)$	$c_p^{rs}(k)$	Links on the path				Arrival time for each link on the path			
1	1	9	1	3.4424	4.8816	5	15	23	24	1	5	9	13
2	1	9	1	1.865	4.8888	3	7	14	19	1	5	9	13
3	1	9	1	3.1786	4.8841	3	4	9	19	1	5	9	13
4	1	9	1	1.1275	4.878	5	13	17	24	1	5	9	13
5	1	9	1	0.2891	4.8871	3	7	17	24	1	5	9	13
6	1	9	1	0.0974	4.8798	5	13	14	19	1	5	9	13
7	1	9	2	3.4424	4.9728	5	15	23	24	2	6	10	14
8	1	9	2	1.865	4.9737	3	7	14	19	2	6	10	14
9	1	9	2	2.3457	4.9516	3	4	9	19	2	6	10	14
10	1	9	2	1.9793	4.9777	5	13	17	24	2	6	10	14
11	1	9	2	0.2703	4.9827	3	7	17	24	2	6	10	14
12	1	9	2	0.0974	4.9686	5	13	14	19	2	6	10	14
13	1	9	3	3.0312	5.0523	5	15	23	24	3	7	11	15
14	1	9	3	1.865	5.0622	3	7	14	19	3	7	11	15
15	1	9	3	3.1398	5.0339	3	4	9	19	3	7	11	15
16	1	9	3	1.9249	5.0657	5	13	17	24	3	7	11	15
17	1	9	3	0.039	5.0583	5	13	14	19	3	7	11	15
18	1	9	4	3.5008	5.1337	5	15	23	24	4	8	12	16

19	1	9	4	2.1353	5.1539	3	7	14	19	4	8	12	16
20	1	9	4	3.1431	5.119	3	4	9	19	4	8	12	16
21	1	9	4	1.163	5.1395	5	13	17	24	4	8	12	16
22	1	9	4	0.0578	5.1392	5	13	14	19	4	8	12	16
23	1	9	5	3.5398	5.1328	5	15	23	24	5	9	13	17
24	1	9	5	1.865	5.1522	3	7	14	19	5	9	13	17
25	1	9	5	3.3945	5.122	3	4	9	19	5	9	13	17
26	1	9	5	1.1818	5.1352	5	13	17	24	5	9	13	17
27	1	9	5	0.0188	5.14	5	13	14	19	5	9	13	17
28	9	1	1	3.5398	4.8805	22	21	16	6	1	5	9	13
29	9	1	1	2.1541	4.8917	20	12	8	1	1	5	9	13
30	9	1	1	3.1786	4.8855	20	10	2	1	1	5	9	13
31	9	1	1	1.1275	4.8722	22	18	11	6	1	5	9	13
32	9	1	2	3.4424	4.97	22	21	16	6	2	6	10	14
33	9	1	2	2.1353	4.9793	20	12	8	1	2	6	10	14
34	9	1	2	2.3457	4.9545	20	10	2	1	2	6	10	14
35	9	1	2	2.0766	4.9664	22	18	11	6	2	6	10	14
36	9	1	3	3.0703	5.0495	22	21	16	6	3	7	11	15
37	9	1	3	1.865	5.0678	20	12	8	1	3	7	11	15
38	9	1	3	3.1398	5.0367	20	10	2	1	3	7	11	15
39	9	1	3	1.9249	5.0544	22	18	11	6	3	7	11	15

40	9	1	4	3.4424	5.1308	22	21	16	6	4	8	12	16
41	9	1	4	1.8839	5.1559	20	12	8	1	4	8	12	16
42	9	1	4	3.3945	5.1244	20	10	2	1	4	8	12	16
43	9	1	4	1.2604	5.1297	22	18	11	6	4	8	12	16
44	9	1	4	0.0188	5.1438	22	18	8	1	4	8	12	16
45	9	1	5	3.4814	5.1315	22	21	16	6	5	9	13	17
46	9	1	5	1.865	5.1515	20	12	8	1	5	9	13	17
47	9	1	5	3.3945	5.1259	20	10	2	1	5	9	13	17
48	9	1	5	1.2402	5.1314	22	18	11	6	5	9	13	17
49	9	1	5	0.0188	5.1419	22	18	8	1	5	9	13	17
50	3	7	1	3.4424	4.8819	9	19	22	21	1	5	9	13
51	3	7	1	1.865	4.8888	2	7	11	15	1	5	9	13
52	3	7	1	3.1786	4.8841	2	1	5	15	1	5	9	13
53	3	7	1	1.1275	4.878	9	12	17	21	1	5	9	13
54	3	7	1	0.2891	4.8871	2	7	17	21	1	5	9	13
55	3	7	1	0.0974	4.8798	9	12	11	15	1	5	9	13
56	3	7	2	3.4424	4.9728	9	19	22	21	2	6	10	14
57	3	7	2	1.865	4.9737	2	7	11	15	2	6	10	14
58	3	7	2	2.3457	4.9516	2	1	5	15	2	6	10	14
59	3	7	2	1.9793	4.9777	9	12	17	21	2	6	10	14
60	3	7	2	0.2703	4.9827	2	7	17	21	2	6	10	14

61	3	7	2	0.0974	4.9686	9	12	11	15	2	6	10	14
62	3	7	3	3.0312	5.0523	9	19	22	21	3	7	11	15
63	3	7	3	1.865	5.0622	2	7	11	15	3	7	11	15
64	3	7	3	3.1398	5.0339	2	1	5	15	3	7	11	15
65	3	7	3	1.9249	5.0657	9	12	17	21	3	7	11	15
66	3	7	3	0.039	5.0583	9	12	11	15	3	7	11	15
67	3	7	4	3.5008	5.1337	9	19	22	21	4	8	12	16
68	3	7	4	2.1353	5.1539	2	7	11	15	4	8	12	16
69	3	7	4	3.1431	5.119	2	1	5	15	4	8	12	16
70	3	7	4	1.163	5.1395	9	12	17	21	4	8	12	16
71	3	7	4	0.0578	5.1392	9	12	11	15	4	8	12	16
72	3	7	5	3.5398	5.1328	9	19	22	21	5	9	13	17
73	3	7	5	1.865	5.1522	2	7	11	15	5	9	13	17
74	3	7	5	3.3945	5.122	2	1	5	15	5	9	13	17
75	3	7	5	1.1818	5.1352	9	12	17	21	5	9	13	17
76	3	7	5	0.0188	5.14	9	12	11	15	5	9	13	17
77	7	3	1	3.5398	4.8805	23	24	20	10	1	5	9	13
78	7	3	1	2.1541	4.8917	16	13	8	4	1	5	9	13
79	7	3	1	3.1786	4.8855	16	6	3	4	1	5	9	13
80	7	3	1	1.1275	4.8722	23	18	14	10	1	5	9	13
81	7	3	2	3.4424	4.97	23	24	20	10	2	6	10	14

82	7	3	2	2.1353	4.9793	16	13	8	4	2	6	10	14
83	7	3	2	2.3457	4.9545	16	6	3	4	2	6	10	14
84	7	3	2	2.0766	4.9664	23	18	14	10	2	6	10	14
85	7	3	3	3.0703	5.0495	23	24	20	10	3	7	11	15
86	7	3	3	1.865	5.0678	16	13	8	4	3	7	11	15
87	7	3	3	3.1398	5.0367	16	6	3	4	3	7	11	15
88	7	3	3	1.9249	5.0544	23	18	14	10	3	7	11	15
89	7	3	4	3.4424	5.1308	23	24	20	10	4	8	12	16
90	7	3	4	1.8839	5.1559	16	13	8	4	4	8	12	16
91	7	3	4	3.3945	5.1244	16	6	3	4	4	8	12	16
92	7	3	4	1.2604	5.1297	23	18	14	10	4	8	12	16
93	7	3	4	0.0188	5.1438	23	18	8	4	4	8	12	16
94	7	3	5	3.4814	5.1315	23	24	20	10	5	9	13	17
95	7	3	5	1.865	5.1515	16	13	8	4	5	9	13	17
96	7	3	5	3.3945	5.1259	16	6	3	4	5	9	13	17
97	7	3	5	1.2402	5.1314	23	18	14	10	5	9	13	17
98	7	3	5	0.0188	5.1419	23	18	8	4	5	9	13	17

Appendix 3: Output for Example 4.2

Table 4.5g The resultant path flow and path travel time for example 4.2.

Path number	*r*	*s*	*k*	$f_p^{rs}(k)$	$c_p^{rs}(k)$	Links on the path				Arrival time for each link on the path			
1	1	9	1	3.3372	4.882	5	15	23	24	1	5	9	13
2	1	9	1	1.7504	4.8852	3	7	14	19	1	5	9	13
3	1	9	1	3.2694	4.8831	3	4	9	19	1	5	9	13
4	1	9	1	1.643	4.8821	5	13	17	24	1	5	9	13
5	1	9	2	3.3372	4.9644	5	15	23	24	2	6	10	14
6	1	9	2	1.7504	4.9708	3	7	14	19	2	6	10	14
7	1	9	2	3.2694	4.9666	3	4	9	19	2	6	10	14
8	1	9	2	1.643	4.9646	5	13	17	24	2	6	10	14
9	1	9	3	3.3372	5.0468	5	15	23	24	3	7	11	15
10	1	9	3	1.7504	5.0564	3	7	14	19	3	7	11	15
11	1	9	3	3.2694	5.0501	3	4	9	19	3	7	11	15
12	1	9	3	1.643	5.0472	5	13	17	24	3	7	11	15
13	1	9	4	3.3372	5.1292	5	15	23	24	4	8	12	16
14	1	9	4	1.7504	5.142	3	7	14	19	4	8	12	16
15	1	9	4	3.2694	5.1336	3	4	9	19	4	8	12	16

16	1	9	4	1.643	5.1297	5	13	17	24	4	8	12	16
17	1	9	5	3.3372	5.1292	5	15	23	24	5	9	13	17
18	1	9	5	1.7504	5.142	3	7	14	19	5	9	13	17
19	1	9	5	3.2694	5.1336	3	4	9	19	5	9	13	17
20	1	9	5	1.643	5.1297	5	13	17	24	5	9	13	17
21	9	1	1	3.3372	4.882	22	21	16	6	1	5	9	13
22	9	1	1	1.7504	4.8852	20	12	8	1	1	5	9	13
23	9	1	1	3.2694	4.8831	20	10	2	1	1	5	9	13
24	9	1	1	1.643	4.8821	22	18	11	6	1	5	9	13
25	9	1	2	3.3372	4.9644	22	21	16	6	2	6	10	14
26	9	1	2	1.7504	4.9708	20	12	8	1	2	6	10	14
27	9	1	2	3.2694	4.9666	20	10	2	1	2	6	10	14
28	9	1	2	1.643	4.9646	22	18	11	6	2	6	10	14
29	9	1	3	3.3372	5.0468	22	21	16	6	3	7	11	15
30	9	1	3	1.7504	5.0564	20	12	8	1	3	7	11	15
31	9	1	3	3.2694	5.0501	20	10	2	1	3	7	11	15
32	9	1	3	1.643	5.0472	22	18	11	6	3	7	11	15
33	9	1	4	3.3372	5.1292	22	21	16	6	4	8	12	16
34	9	1	4	1.7504	5.142	20	12	8	1	4	8	12	16
35	9	1	4	3.2694	5.1336	20	10	2	1	4	8	12	16
36	9	1	4	1.643	5.1297	22	18	11	6	4	8	12	16

37	9	1	5	3.3372	5.1292	22	21	16	6	5	9	13	17
38	9	1	5	1.7504	5.142	20	12	8	1	5	9	13	17
39	9	1	5	3.2694	5.1336	20	10	2	1	5	9	13	17
40	9	1	5	1.643	5.1297	22	18	11	6	5	9	13	17
41	3	7	1	3.3372	4.882	9	19	22	21	1	5	9	13
42	3	7	1	1.7504	4.8852	2	7	11	15	1	5	9	13
43	3	7	1	3.2694	4.8831	2	1	5	15	1	5	9	13
44	3	7	1	1.643	4.8821	9	12	17	21	1	5	9	13
45	3	7	2	3.3372	4.9644	9	19	22	21	2	6	10	14
46	3	7	2	1.7504	4.9708	2	7	11	15	2	6	10	14
47	3	7	2	3.2694	4.9666	2	1	5	15	2	6	10	14
48	3	7	2	1.643	4.9646	9	12	17	21	2	6	10	14
49	3	7	3	3.3372	5.0468	9	19	22	21	3	7	11	15
50	3	7	3	1.7504	5.0564	2	7	11	15	3	7	11	15
51	3	7	3	3.2694	5.0501	2	1	5	15	3	7	11	15
52	3	7	3	1.643	5.0472	9	12	17	21	3	7	11	15
53	3	7	4	3.3372	5.1292	9	19	22	21	4	8	12	16
54	3	7	4	1.7504	5.142	2	7	11	15	4	8	12	16
55	3	7	4	3.2694	5.1336	2	1	5	15	4	8	12	16
56	3	7	4	1.643	5.1297	9	12	17	21	4	8	12	16
57	3	7	5	3.3372	5.1292	9	19	22	21	5	9	13	17

58	3	7	5	1.7504	5.142	2	7	11	15	5	9	13	17
59	3	7	5	3.2694	5.1336	2	1	5	15	5	9	13	17
60	3	7	5	1.643	5.1297	9	12	17	21	5	9	13	17
61	7	3	1	3.3372	4.882	23	24	20	10	1	5	9	13
62	7	3	1	1.7504	4.8852	16	13	8	4	1	5	9	13
63	7	3	1	3.2694	4.8831	16	6	3	4	1	5	9	13
64	7	3	1	1.643	4.8821	23	18	14	10	1	5	9	13
65	7	3	2	3.3372	4.9644	23	24	20	10	2	6	10	14
66	7	3	2	1.7504	4.9708	16	13	8	4	2	6	10	14
67	7	3	2	3.2694	4.9666	16	6	3	4	2	6	10	14
68	7	3	2	1.643	4.9646	23	18	14	10	2	6	10	14
69	7	3	3	3.3372	5.0468	23	24	20	10	3	7	11	15
70	7	3	3	1.7504	5.0564	16	13	8	4	3	7	11	15
71	7	3	3	3.2694	5.0501	16	6	3	4	3	7	11	15
72	7	3	3	1.643	5.0472	23	18	14	10	3	7	11	15
73	7	3	4	3.3372	5.1292	23	24	20	10	4	8	12	16
74	7	3	4	1.7504	5.142	16	13	8	4	4	8	12	16
75	7	3	4	3.2694	5.1336	16	6	3	4	4	8	12	16
76	7	3	4	1.643	5.1297	23	18	14	10	4	8	12	16
77	7	3	5	3.3372	5.1292	23	24	20	10	5	9	13	17
78	7	3	5	1.7504	5.142	16	13	8	4	5	9	13	17

79	7	3	5	3.2694	5.1336	16	6	3	4	5	9	13	17
80	7	3	5	1.643	5.1297	23	18	14	10	5	9	13	17

Appendix 4: Output for Example 5.1

Table 5.3g The resultant path flow and path travel time for example 5.1.

Path number	*r*	*s*	*k*	$f_p^{rs}(k)$	$c_p^{rs}(k)$	Links on the path				Arrival time for each link on the path			
1	1	9	1	2.7273	4.9041	5	13	17	24	1	5	9	13
2	1	9	1	0	4.9041	3	7	17	24	1	5	9	13
3	1	9	1	1.8182	4.8814	5	13	14	19	1	5	9	13
4	1	9	1	0.9091	4.8814	3	7	14	19	1	5	9	13
5	1	9	1	1.8182	4.8723	5	15	23	24	1	5	9	13
6	1	9	1	2.7273	4.8859	3	4	9	19	1	5	9	13
7	1	9	2	1.8182	4.9223	5	15	23	24	2	6	10	14
8	1	9	2	2.7273	5.0268	3	7	17	24	2	6	10	14
9	1	9	2	1.8182	4.995	3	4	9	19	2	6	10	14
10	1	9	2	0.9091	4.9905	5	13	17	24	2	6	10	14
11	1	9	2	1.8182	4.9905	3	7	14	19	2	6	10	14
12	1	9	2	0.9091	4.9541	5	13	14	19	2	6	10	14
13	1	9	3	3.6364	5.0677	3	4	9	19	3	7	11	15

14	1	9	3	2.7273	5.0041	5	15	23	24	3	7	11	15
15	1	9	3	0.9091	5.095	5	13	17	24	3	7	11	15
16	1	9	3	0.9091	5.0859	3	7	14	19	3	7	11	15
17	1	9	3	0.9091	5.1268	3	7	17	24	3	7	11	15
18	1	9	3	0.9091	5.0541	5	13	14	19	3	7	11	15
19	1	9	4	0	5.1677	3	7	14	19	4	8	12	16
20	1	9	4	0.9091	5.1859	5	13	17	24	4	8	12	16
21	1	9	4	0.9091	5.145	5	13	14	19	4	8	12	16
22	1	9	4	2.7273	5.095	5	15	23	24	4	8	12	16
23	1	9	4	3.6364	5.1405	3	4	9	19	4	8	12	16
24	1	9	4	1.8182	5.2087	3	7	17	24	4	8	12	16
25	1	9	5	0	5.1723	5	13	17	24	5	9	13	17
26	1	9	5	3.6364	5.1041	5	15	23	24	5	9	13	17
27	1	9	5	1.8182	5.1223	3	4	9	19	5	9	13	17
28	1	9	5	1.8182	5.1859	3	7	14	19	5	9	13	17
29	1	9	5	1.8182	5.1905	3	7	17	24	5	9	13	17
30	1	9	5	0.9091	5.1677	5	13	14	19	5	9	13	17
31	9	1	1	1.8182	4.8677	20	10	2	1	1	5	9	13
32	9	1	1	0.9091	4.895	20	12	11	6	1	5	9	13
33	9	1	1	1.8182	4.8677	22	21	16	6	1	5	9	13
34	9	1	1	0.9091	4.8996	22	18	11	6	1	5	9	13

35	9	1	1	2.7273	4.9087	22	18	8	1	1	5	9	13
36	9	1	1	1.8182	4.9041	20	12	8	1	1	5	9	13
37	9	1	2	0	4.9996	20	12	11	6	2	6	10	14
38	9	1	2	3.6364	4.9496	22	21	16	6	2	6	10	14
39	9	1	2	0.9091	5.0087	22	18	8	1	2	6	10	14
40	9	1	2	0	4.995	20	12	8	1	2	6	10	14
41	9	1	2	0.9091	5.0132	22	18	11	6	2	6	10	14
42	9	1	2	4.5455	4.9496	20	10	2	1	2	6	10	14
43	9	1	3	2.7273	5.0268	20	10	2	1	3	7	11	15
44	9	1	3	0.9091	5.0905	22	18	8	1	3	7	11	15
45	9	1	3	2.7273	5.0268	22	21	16	6	3	7	11	15
46	9	1	3	0	5.0905	20	12	11	6	3	7	11	15
47	9	1	3	0.9091	5.1041	22	18	11	6	3	7	11	15
48	9	1	3	2.7273	5.0768	20	12	8	1	3	7	11	15
49	9	1	4	0.9091	5.1587	22	18	8	1	4	8	12	16
50	9	1	4	1.8182	5.1359	22	21	16	6	4	8	12	16
51	9	1	4	0	5.1677	20	12	11	6	4	8	12	16
52	9	1	4	4.5455	5.1087	20	10	2	1	4	8	12	16
53	9	1	4	1.8182	5.1768	22	18	11	6	4	8	12	16
54	9	1	4	0.9091	5.1496	20	12	8	1	4	8	12	16
55	9	1	5	2.7273	5.1087	20	10	2	1	5	9	13	17

56	9	1	5	2.7273	5.1268	22	21	16	6	5	9	13	17
57	9	1	5	0.9091	5.1859	20	12	11	6	5	9	13	17
58	9	1	5	0.9091	5.1814	20	12	8	1	5	9	13	17
59	9	1	5	1.8182	5.1677	22	18	11	6	5	9	13	17
60	9	1	5	0.9091	5.1632	22	18	8	1	5	9	13	17
61	3	7	1	1.8182	4.8814	9	19	22	21	1	5	9	13
62	3	7	1	1.8182	4.8723	2	1	5	15	1	5	9	13
63	3	7	1	0.9091	4.8996	2	7	17	21	1	5	9	13
64	3	7	1	2.7273	4.9132	9	12	17	21	1	5	9	13
65	3	7	1	2.7273	4.8859	2	7	11	15	1	5	9	13
66	3	7	1	0	4.8996	9	12	11	15	1	5	9	13
67	3	7	2	1.8182	5.0041	9	12	11	15	2	6	10	14
68	3	7	2	1.8182	4.945	2	1	5	15	2	6	10	14
69	3	7	2	0.9091	4.9359	9	19	22	21	2	6	10	14
70	3	7	2	1.8182	5.0132	9	12	17	21	2	6	10	14
71	3	7	2	2.7273	5.0132	2	7	17	21	2	6	10	14
72	3	7	2	0.9091	5.0041	2	7	11	15	2	6	10	14
73	3	7	3	0.9091	5.0814	2	7	11	15	3	7	11	15
74	3	7	3	4.5455	4.9996	2	1	5	15	3	7	11	15
75	3	7	3	0.9091	5.1223	9	12	17	21	3	7	11	15
76	3	7	3	2.7273	5.045	9	19	22	21	3	7	11	15

77	3	7	3	0	5.0814	9	12	11	15	3	7	11	15
78	3	7	3	0.9091	5.1223	2	7	17	21	3	7	11	15
79	3	7	4	0.9091	5.1632	2	7	11	15	4	8	12	16
80	3	7	4	2.7273	5.1314	9	19	22	21	4	8	12	16
81	3	7	4	0.9091	5.1632	9	12	11	15	4	8	12	16
82	3	7	4	2.7273	5.0768	2	1	5	15	4	8	12	16
83	3	7	4	1.8182	5.1996	2	7	17	21	4	8	12	16
84	3	7	4	0.9091	5.1996	9	12	17	21	4	8	12	16
85	3	7	5	0	5.1814	2	7	17	21	5	9	13	17
86	3	7	5	3.6364	5.0768	9	19	22	21	5	9	13	17
87	3	7	5	0.9091	5.1723	9	12	17	21	5	9	13	17
88	3	7	5	0.9091	5.1905	9	12	11	15	5	9	13	17
89	3	7	5	1.8182	5.1996	2	7	11	15	5	9	13	17
90	3	7	5	2.7273	5.1223	2	1	5	15	5	9	13	17
91	7	3	1	3.6364	4.8723	16	6	3	4	1	5	9	13
92	7	3	1	2.7273	4.8905	23	24	20	10	1	5	9	13
93	7	3	1	1.8182	4.895	16	13	8	4	1	5	9	13
94	7	3	1	0	4.895	23	18	14	10	1	5	9	13
95	7	3	1	0.9091	4.9041	23	18	8	4	1	5	9	13
96	7	3	1	0.9091	4.8859	16	13	14	10	1	5	9	13
97	7	3	2	3.6364	4.9496	23	24	20	10	2	6	10	14

98	7	3	2	0.9091	4.9632	16	13	14	10	2	6	10	14
99	7	3	2	2.7273	4.9587	16	6	3	4	2	6	10	14
100	7	3	2	0.9091	4.9768	23	18	14	10	2	6	10	14
101	7	3	2	1.8182	5.0087	23	18	8	4	2	6	10	14
102	7	3	2	0	4.995	16	13	8	4	2	6	10	14
103	7	3	3	3.6364	5.0359	23	24	20	10	3	7	11	15
104	7	3	3	0	5.0905	23	18	8	4	3	7	11	15
105	7	3	3	2.7273	5.0268	16	6	3	4	3	7	11	15
106	7	3	3	0	5.0677	16	13	14	10	3	7	11	15
107	7	3	3	0.9091	5.0723	16	13	8	4	3	7	11	15
108	7	3	3	2.7273	5.0859	23	18	14	10	3	7	11	15
109	7	3	4	0.9091	5.1541	23	18	8	4	4	8	12	16
110	7	3	4	0.9091	5.1632	16	13	14	10	4	8	12	16
111	7	3	4	3.6364	5.1041	16	6	3	4	4	8	12	16
112	7	3	4	1.8182	5.1223	23	24	20	10	4	8	12	16
113	7	3	4	1.8182	5.1587	23	18	14	10	4	8	12	16
114	7	3	4	0.9091	5.1587	16	13	8	4	4	8	12	16
115	7	3	5	4.5455	5.1223	16	6	3	4	5	9	13	17
116	7	3	5	0.9091	5.1041	23	24	20	10	5	9	13	17
117	7	3	5	0.9091	5.1587	23	18	8	4	5	9	13	17
118	7	3	5	0	5.1587	23	18	14	10	5	9	13	17

119	7	3	5	1.8182	5.1768	16	13	8	4	5	9	13	17
120	7	3	5	1.8182	5.1768	16	13	14	10	5	9	13	17

Appendix 5: Output for Example 5.2

Table 5.5g The resultant path flow and path travel time for example 5.2.

Path number	*r*	*s*	*k*	$f_p^{rs}(k)$	$c_p^{rs}(k)$	Links on the path				Arrival time for each link on the path			
1	1	9	1	2.7273	4.895	5	15	23	24	1	5	9	13
2	1	9	1	1.8182	4.8905	3	7	14	19	1	5	9	13
3	1	9	1	1.8182	4.9359	5	13	17	24	1	5	9	13
4	1	9	1	1.8182	4.8405	3	4	9	19	1	5	9	13
5	1	9	1	1.8182	4.9132	5	13	14	19	1	5	9	13
6	1	9	1	0	4.9132	3	7	17	24	1	5	9	13
7	1	9	2	3.6364	4.9541	3	4	9	19	2	6	10	14
8	1	9	2	0.9091	5.0132	3	7	14	19	2	6	10	14
9	1	9	2	1.8182	4.945	5	15	23	24	2	6	10	14
10	1	9	2	1.8182	4.9905	5	13	14	19	2	6	10	14
11	1	9	2	0	4.995	5	13	17	24	2	6	10	14
12	1	9	2	1.8182	5.0177	3	7	17	24	2	6	10	14

13	1	9	3	0	5.0814	5	13	17	24	3	7	11	15
14	1	9	3	1.8182	5.0996	3	7	17	24	3	7	11	15
15	1	9	3	3.6364	4.995	5	15	23	24	3	7	11	15
16	1	9	3	0.9091	5.095	5	13	14	19	3	7	11	15
17	1	9	3	1.8182	5.0587	3	4	9	19	3	7	11	15
18	1	9	3	1.8182	5.1132	3	7	14	19	3	7	11	15
19	1	9	4	0.9091	5.1587	5	13	14	19	4	8	12	16
20	1	9	4	0.9091	5.1905	3	7	14	19	4	8	12	16
21	1	9	4	2.7273	5.0905	5	15	23	24	4	8	12	16
22	1	9	4	3.6364	5.145	3	4	9	19	4	8	12	16
23	1	9	4	0.9091	5.1496	5	13	17	24	4	8	12	16
24	1	9	4	0.9091	5.1814	3	7	17	24	4	8	12	16
25	1	9	5	0	5.1723	3	7	17	24	5	9	13	17
26	1	9	5	3.6364	5.0587	5	15	23	24	5	9	13	17
27	1	9	5	4.5455	5.1859	3	4	9	19	5	9	13	17
28	1	9	5	0.9091	5.1132	5	13	17	24	5	9	13	17
29	1	9	5	0.9091	5.1359	5	13	14	19	5	9	13	17
30	1	9	5	0	5.195	3	7	14	19	5	9	13	17
31	9	1	1	1.8182	4.8768	22	18	11	6	1	5	9	13
32	9	1	1	1.8182	4.8905	20	10	2	1	1	5	9	13
33	9	1	1	2.7273	4.8814	22	21	16	6	1	5	9	13

34	9	1	1	1.8182	4.8859	20	12	8	1	1	5	9	13
35	9	1	1	0.9091	4.8859	20	12	11	6	1	5	9	13
36	9	1	1	0.9091	4.8768	22	18	8	1	1	5	9	13
37	9	1	2	3.6364	4.9723	22	21	16	6	2	6	10	14
38	9	1	2	2.7273	4.9541	20	10	2	1	2	6	10	14
39	9	1	2	0.9091	4.9768	22	18	8	1	2	6	10	14
40	9	1	2	0.9091	4.9814	22	18	11	6	2	6	10	14
41	9	1	2	0.9091	4.9768	20	12	11	6	2	6	10	14
42	9	1	2	0.9091	4.9723	20	12	8	1	2	6	10	14
43	9	1	3	4.5455	5.045	20	10	2	1	3	7	11	15
44	9	1	3	0.9091	5.0632	22	18	8	1	3	7	11	15
45	9	1	3	3.6364	5.0359	22	21	16	6	3	7	11	15
46	9	1	3	0.9091	5.0859	20	12	11	6	3	7	11	15
47	9	1	3	0	5.0859	22	18	11	6	3	7	11	15
48	9	1	3	0	5.0632	20	12	8	1	3	7	11	15
49	9	1	4	0	5.1587	20	12	8	1	4	8	12	16
50	9	1	4	1.8182	5.1132	22	21	16	6	4	8	12	16
51	9	1	4	2.7273	5.1814	20	12	11	6	4	8	12	16
52	9	1	4	4.5455	5.1223	20	10	2	1	4	8	12	16
53	9	1	4	0.9091	5.1587	22	18	8	1	4	8	12	16
54	9	1	4	0	5.1814	22	18	11	6	4	8	12	16

55	9	1	5	0.9091	5.0996	20	10	2	1	5	9	13	17
56	9	1	5	1.8182	5.2041	22	18	11	6	5	9	13	17
57	9	1	5	0.9091	5.1632	22	18	8	1	5	9	13	17
58	9	1	5	1.8182	5.1587	20	12	8	1	5	9	13	17
59	9	1	5	3.6364	5.1268	22	21	16	6	5	9	13	17
60	9	1	5	0.9091	5.1996	20	12	11	6	5	9	13	17
61	3	7	1	2.7273	4.8632	2	1	5	15	1	5	9	13
62	3	7	1	1.8182	4.8859	9	12	11	15	1	5	9	13
63	3	7	1	1.8182	4.8905	9	19	22	21	1	5	9	13
64	3	7	1	1.8182	4.9087	2	7	17	21	1	5	9	13
65	3	7	1	0.9091	4.8905	2	7	11	15	1	5	9	13
66	3	7	1	0.9091	4.9041	9	12	17	21	1	5	9	13
67	3	7	2	1.8182	4.9496	2	1	5	15	2	6	10	14
68	3	7	2	1.8182	4.9814	9	12	17	21	2	6	10	14
69	3	7	2	0	5.0132	2	7	17	21	2	6	10	14
70	3	7	2	0	4.9723	9	12	11	15	2	6	10	14
71	3	7	2	4.5455	4.9587	9	19	22	21	2	6	10	14
72	3	7	2	1.8182	5.0041	2	7	11	15	2	6	10	14
73	3	7	3	0.9091	5.0814	9	12	17	21	3	7	11	15
74	3	7	3	2.7273	5.0496	9	19	22	21	3	7	11	15
75	3	7	3	2.7273	5.0223	2	1	5	15	3	7	11	15

76	3	7	3	1.8182	5.0905	2	7	11	15	3	7	11	15
77	3	7	3	0.9091	5.0723	9	12	11	15	3	7	11	15
78	3	7	3	0.9091	5.0996	2	7	17	21	3	7	11	15
79	3	7	4	0.9091	5.1632	9	12	11	15	4	8	12	16
80	3	7	4	2.7273	5.1314	9	19	22	21	4	8	12	16
81	3	7	4	4.5455	5.1132	2	1	5	15	4	8	12	16
82	3	7	4	1.8182	5.1768	2	7	17	21	4	8	12	16
83	3	7	4	0	5.1859	2	7	11	15	4	8	12	16
84	3	7	4	0	5.1541	9	12	17	21	4	8	12	16
85	3	7	5	1.8182	5.1723	2	7	17	21	5	9	13	17
86	3	7	5	3.6364	5.1041	2	1	5	15	5	9	13	17
87	3	7	5	1.8182	5.1768	9	12	11	15	5	9	13	17
88	3	7	5	1.8182	5.1314	9	19	22	21	5	9	13	17
89	3	7	5	0.9091	5.1723	9	12	17	21	5	9	13	17
90	3	7	5	0	5.1768	2	7	11	15	5	9	13	17
91	7	3	1	0.9091	4.8632	23	18	8	4	1	5	9	13
92	7	3	1	3.6364	4.8859	23	24	20	10	1	5	9	13
93	7	3	1	1.8182	4.8587	16	6	3	4	1	5	9	13
94	7	3	1	0.9091	4.9314	16	13	14	10	1	5	9	13
95	7	3	1	1.8182	4.8905	16	13	8	4	1	5	9	13
96	7	3	1	0.9091	4.9041	23	18	14	10	1	5	9	13

97	7	3	2	0	5.0132	23	18	14	10	2	6	10	14
98	7	3	2	3.6364	4.9268	16	6	3	4	2	6	10	14
99	7	3	2	0	4.9632	16	13	8	4	2	6	10	14
100	7	3	2	2.7273	4.9677	23	18	8	4	2	6	10	14
101	7	3	2	0.9091	5.0087	16	13	14	10	2	6	10	14
102	7	3	2	2.7273	4.9723	23	24	20	10	2	6	10	14
103	7	3	3	5.4545	5.0677	23	24	20	10	3	7	11	15
104	7	3	3	3.6364	4.9768	16	6	3	4	3	7	11	15
105	7	3	3	0	5.0314	16	13	8	4	3	7	11	15
106	7	3	3	0	5.1405	23	18	14	10	3	7	11	15
107	7	3	3	0	5.1041	16	13	14	10	3	7	11	15
108	7	3	3	0.9091	5.0677	23	18	8	4	3	7	11	15
109	7	3	4	0.9091	5.1223	16	13	8	4	4	8	12	16
110	7	3	4	0.9091	5.1405	23	24	20	10	4	8	12	16
111	7	3	4	1.8182	5.1768	23	18	8	4	4	8	12	16
112	7	3	4	0.9091	5.2177	23	18	14	10	4	8	12	16
113	7	3	4	1.8182	5.1632	16	13	14	10	4	8	12	16
114	7	3	4	3.6364	5.0587	16	6	3	4	4	8	12	16
115	7	3	5	1.8182	5.1996	23	18	14	10	5	9	13	17
116	7	3	5	3.6364	5.0768	16	6	3	4	5	9	13	17
117	7	3	5	1.8182	5.1405	23	24	20	10	5	9	13	17

118	7	3	5	1.8182	5.1177	16	13	14	10	5	9	13	17
119	7	3	5	0.9091	5.2041	23	18	8	4	5	9	13	17
120	7	3	5	0	5.1223	16	13	8	4	5	9	13	17

Appendix 6: Output for Example 6.1

Table 6.2g The resultant path flow and path travel time for example 6.1.

Path number	*r*	*s*	*k*	$f_p^{rs}(k)$	$c_p^{rs}(k)$	Links on the path				Arrival time for each link on the path			
1	1	9	1	3.3333	4.8829	5	15	23	24	1	5	9	13
2	1	9	1	1.6667	4.8829	3	7	14	19	1	5	9	13
3	1	9	1	3.3333	4.8829	3	4	9	19	1	5	9	13
4	1	9	1	1.6667	4.8829	5	13	17	24	1	5	9	13
5	1	9	2	3.3333	4.9662	5	15	23	24	2	6	10	14
6	1	9	2	1.6666	4.9662	3	7	14	19	2	6	10	14
7	1	9	2	3.3334	4.9662	3	4	9	19	2	6	10	14
8	1	9	2	1.6667	4.9662	5	13	17	24	2	6	10	14
9	1	9	3	3.3336	5.0496	5	15	23	24	3	7	11	15
10	1	9	3	1.6668	5.0496	3	7	14	19	3	7	11	15
11	1	9	3	3.3331	5.0496	3	4	9	19	3	7	11	15

12	1	9	3	1.6665	5.0496	5	13	17	24	3	7	11	15
13	1	9	4	3.3329	5.1329	5	15	23	24	4	8	12	16
14	1	9	4	1.6665	5.1329	3	7	14	19	4	8	12	16
15	1	9	4	3.3334	5.1329	3	4	9	19	4	8	12	16
16	1	9	4	1.6672	5.1329	5	13	17	24	4	8	12	16
17	1	9	5	3.3329	5.1329	5	15	23	24	5	9	13	17
18	1	9	5	1.6683	5.1329	3	7	14	19	5	9	13	17
19	1	9	5	1.6656	5.1329	5	13	17	24	5	9	13	17
20	1	9	5	3.3333	5.1329	3	4	9	19	5	9	13	17
21	9	1	1	3.3333	4.8829	22	21	16	6	1	5	9	13
22	9	1	1	1.6667	4.8829	20	12	8	1	1	5	9	13
23	9	1	1	3.3333	4.8829	20	10	2	1	1	5	9	13
24	9	1	1	1.6667	4.8829	22	18	11	6	1	5	9	13
25	9	1	2	3.3333	4.9662	22	21	16	6	2	6	10	14
26	9	1	2	1.6666	4.9662	20	12	8	1	2	6	10	14
27	9	1	2	3.3334	4.9662	20	10	2	1	2	6	10	14
28	9	1	2	1.6667	4.9662	22	18	11	6	2	6	10	14
29	9	1	3	3.3336	5.0496	22	21	16	6	3	7	11	15
30	9	1	3	1.6668	5.0496	20	12	8	1	3	7	11	15
31	9	1	3	3.3331	5.0496	20	10	2	1	3	7	11	15
32	9	1	3	1.6665	5.0496	22	18	11	6	3	7	11	15

33	9	1	4	3.3329	5.1329	22	21	16	6	4	8	12	16
34	9	1	4	1.6665	5.1329	20	12	8	1	4	8	12	16
35	9	1	4	3.3334	5.1329	20	10	2	1	4	8	12	16
36	9	1	4	1.6672	5.1329	22	18	11	6	4	8	12	16
37	9	1	5	3.3329	5.1329	22	21	16	6	5	9	13	17
38	9	1	5	1.6683	5.1329	20	12	8	1	5	9	13	17
39	9	1	5	1.6656	5.1329	22	18	11	6	5	9	13	17
40	9	1	5	3.3333	5.1329	20	10	2	1	5	9	13	17
41	3	7	1	3.3333	4.8829	9	19	22	21	1	5	9	13
42	3	7	1	1.6667	4.8829	2	7	11	15	1	5	9	13
43	3	7	1	3.3333	4.8829	2	1	5	15	1	5	9	13
44	3	7	1	1.6667	4.8829	9	12	17	21	1	5	9	13
45	3	7	2	3.3333	4.9662	9	19	22	21	2	6	10	14
46	3	7	2	1.6666	4.9662	2	7	11	15	2	6	10	14
47	3	7	2	3.3334	4.9662	2	1	5	15	2	6	10	14
48	3	7	2	1.6667	4.9662	9	12	17	21	2	6	10	14
49	3	7	3	3.3336	5.0496	9	19	22	21	3	7	11	15
50	3	7	3	1.6668	5.0496	2	7	11	15	3	7	11	15
51	3	7	3	3.3331	5.0496	2	1	5	15	3	7	11	15
52	3	7	3	1.6665	5.0496	9	12	17	21	3	7	11	15
53	3	7	4	3.3329	5.1329	9	19	22	21	4	8	12	16

54	3	7	4	1.6665	5.1329	2	7	11	15	4	8	12	16
55	3	7	4	3.3334	5.1329	2	1	5	15	4	8	12	16
56	3	7	4	1.6672	5.1329	9	12	17	21	4	8	12	16
57	3	7	5	3.3329	5.1329	9	19	22	21	5	9	13	17
58	3	7	5	1.6683	5.1329	2	7	11	15	5	9	13	17
59	3	7	5	1.6656	5.1329	9	12	17	21	5	9	13	17
60	3	7	5	3.3333	5.1329	2	1	5	15	5	9	13	17
61	7	3	1	3.3333	4.8829	23	24	20	10	1	5	9	13
62	7	3	1	1.6667	4.8829	16	13	8	4	1	5	9	13
63	7	3	1	3.3333	4.8829	16	6	3	4	1	5	9	13
64	7	3	1	1.6667	4.8829	23	18	14	10	1	5	9	13
65	7	3	2	3.3333	4.9662	23	24	20	10	2	6	10	14
66	7	3	2	1.6666	4.9662	16	13	8	4	2	6	10	14
67	7	3	2	3.3334	4.9662	16	6	3	4	2	6	10	14
68	7	3	2	1.6667	4.9662	23	18	14	10	2	6	10	14
69	7	3	3	3.3336	5.0496	23	24	20	10	3	7	11	15
70	7	3	3	1.6668	5.0496	16	13	8	4	3	7	11	15
71	7	3	3	3.3331	5.0496	16	6	3	4	3	7	11	15
72	7	3	3	1.6665	5.0496	23	18	14	10	3	7	11	15
73	7	3	4	3.3329	5.1329	23	24	20	10	4	8	12	16
74	7	3	4	1.6665	5.1329	16	13	8	4	4	8	12	16

75	7	3	4	3.3334	5.1329	16	6	3	4	4	8	12	16
76	7	3	4	1.6672	5.1329	23	18	14	10	4	8	12	16
77	7	3	5	3.3329	5.1329	23	24	20	10	5	9	13	17
78	7	3	5	1.6683	5.1329	16	13	8	4	5	9	13	17
79	7	3	5	1.6656	5.1329	23	18	14	10	5	9	13	17
80	7	3	5	3.3333	5.1329	16	6	3	4	5	9	13	17

Appendix 7: Output for Example 6.2

Table 6.6g The resultant path flow and path travel time for example 6.2 (no disutility function)

Path number	r	s	k	$f_p^{rs}(k)$	$c_p^{rs}(k)$	Links on the path				Arrival time for each link on the path			
1	1	9	1	8.3774	5.0091	5	15	23	24	1	5	9	13
2	1	9	1	4.1892	5.0086	3	7	14	19	1	5	9	13
3	1	9	1	8.3627	5.0086	3	4	9	19	1	5	9	13
4	1	9	1	4.179	5.0089	5	13	17	24	1	5	9	13
5	1	9	2	0.3638	5.0195	5	15	23	24	2	6	10	14
6	1	9	2	0.2152	5.0182	3	7	14	19	2	6	10	14

7	1	9	2	0.4591	5.02	3	4	9	19	2	6	10	14
8	1	9	2	0.2568	5.0204	5	13	17	24	2	6	10	14
9	1	9	3	0	5.0195	5	15	23	24	3	7	11	15
10	1	9	3	0	5.0182	3	7	14	19	3	7	11	15
11	1	9	3	0	5.02	3	4	9	19	3	7	11	15
12	1	9	3	0	5.0204	5	13	17	24	3	7	11	15
13	1	9	4	0	5.0195	5	15	23	24	4	8	12	16
14	1	9	4	0	5.0182	3	7	14	19	4	8	12	16
15	1	9	4	0	5.02	3	4	9	19	4	8	12	16
16	1	9	4	0	5.0204	5	13	17	24	4	8	12	16
17	1	9	5	8.0132	5.0096	5	15	23	24	5	9	13	17
18	1	9	5	4.0649	5.0048	3	7	14	19	5	9	13	17
19	1	9	5	7.7337	5.0055	3	4	9	19	5	9	13	17
20	1	9	5	3.7849	5.0069	5	13	17	24	5	9	13	17
21	9	1	1	8.3774	5.0091	22	21	16	6	1	5	9	13
22	9	1	1	4.1892	5.0086	20	12	8	1	1	5	9	13
23	9	1	1	8.3627	5.0086	20	10	2	1	1	5	9	13
24	9	1	1	4.179	5.0089	22	18	11	6	1	5	9	13
25	9	1	2	0.3638	5.0195	22	21	16	6	2	6	10	14
26	9	1	2	0.2152	5.0182	20	12	8	1	2	6	10	14
27	9	1	2	0.4591	5.02	20	10	2	1	2	6	10	14

28	9	1	2	0.2568	5.0204	22	18	11	6	2	6	10	14
29	9	1	3	0	5.0195	22	21	16	6	3	7	11	15
30	9	1	3	0	5.0182	20	12	8	1	3	7	11	15
31	9	1	3	0	5.02	20	10	2	1	3	7	11	15
32	9	1	3	0	5.0204	22	18	11	6	3	7	11	15
33	9	1	4	0	5.0195	22	21	16	6	4	8	12	16
34	9	1	4	0	5.0182	20	12	8	1	4	8	12	16
35	9	1	4	0	5.02	20	10	2	1	4	8	12	16
36	9	1	4	0	5.0204	22	18	11	6	4	8	12	16
37	9	1	5	8.0132	5.0096	22	21	16	6	5	9	13	17
38	9	1	5	4.0649	5.0048	20	12	8	1	5	9	13	17
39	9	1	5	7.7337	5.0055	20	10	2	1	5	9	13	17
40	9	1	5	3.7849	5.0069	22	18	11	6	5	9	13	17
41	3	7	1	8.3774	5.0091	9	19	22	21	1	5	9	13
42	3	7	1	4.1892	5.0086	2	7	11	15	1	5	9	13
43	3	7	1	8.3627	5.0086	2	1	5	15	1	5	9	13
44	3	7	1	4.179	5.0089	9	12	17	21	1	5	9	13
45	3	7	2	0.3638	5.0195	9	19	22	21	2	6	10	14
46	3	7	2	0.2152	5.0182	2	7	11	15	2	6	10	14
47	3	7	2	0.4591	5.02	2	1	5	15	2	6	10	14
48	3	7	2	0.2568	5.0204	9	12	17	21	2	6	10	14

49	3	7	3	0	5.0195	9	19	22	21	3	7	11	15
50	3	7	3	0	5.0182	2	7	11	15	3	7	11	15
51	3	7	3	0	5.02	2	1	5	15	3	7	11	15
52	3	7	3	0	5.0204	9	12	17	21	3	7	11	15
53	3	7	4	0	5.0195	9	19	22	21	4	8	12	16
54	3	7	4	0	5.0182	2	7	11	15	4	8	12	16
55	3	7	4	0	5.02	2	1	5	15	4	8	12	16
56	3	7	4	0	5.0204	9	12	17	21	4	8	12	16
57	3	7	5	8.0132	5.0096	9	19	22	21	5	9	13	17
58	3	7	5	4.0649	5.0048	2	7	11	15	5	9	13	17
59	3	7	5	7.7337	5.0055	2	1	5	15	5	9	13	17
60	3	7	5	3.7849	5.0069	9	12	17	21	5	9	13	17
61	7	3	1	8.3774	5.0091	23	24	20	10	1	5	9	13
62	7	3	1	4.1892	5.0086	16	13	8	4	1	5	9	13
63	7	3	1	8.3627	5.0086	16	6	3	4	1	5	9	13
64	7	3	1	4.179	5.0089	23	18	14	10	1	5	9	13
65	7	3	2	0.3638	5.0195	23	24	20	10	2	6	10	14
66	7	3	2	0.2152	5.0182	16	13	8	4	2	6	10	14
67	7	3	2	0.4591	5.02	16	6	3	4	2	6	10	14
68	7	3	2	0.2568	5.0204	23	18	14	10	2	6	10	14
69	7	3	3	0	5.0195	23	24	20	10	3	7	11	15

70	7	3	3	0	5.0182	16	13	8	4	3	7	11	15
71	7	3	3	0	5.02	16	6	3	4	3	7	11	15
72	7	3	3	0	5.0204	23	18	14	10	3	7	11	15
73	7	3	4	0	5.0195	23	24	20	10	4	8	12	16
74	7	3	4	0	5.0182	16	13	8	4	4	8	12	16
75	7	3	4	0	5.02	16	6	3	4	4	8	12	16
76	7	3	4	0	5.0204	23	18	14	10	4	8	12	16
77	7	3	5	8.0132	5.0096	23	24	20	10	5	9	13	17
78	7	3	5	4.0649	5.0048	16	13	8	4	5	9	13	17
79	7	3	5	7.7337	5.0055	16	6	3	4	5	9	13	17
80	7	3	5	3.7849	5.0069	23	18	14	10	5	9	13	17

Table 6.9g The resultant path flow, path travel time, and generalized path cost for example 6.2 (with disutility function).

Path number	r	s	k	$f_p^{rs}(k)$	$c_p^{rs}(k)$	$\phi_p^{rs}(k)$	Links on the path				Arrival time for each link on the path			
1	1	9	1	5.6186	4.9389	5.1361	5	15	23	24	1	5	9	13
2	1	9	1	2.5586	4.939	5.1361	3	7	14	19	1	5	9	13
3	1	9	1	5.6217	4.939	5.1362	3	4	9	19	1	5	9	13

4	1	9	1	2.561	4.9389	5.1361	5	13	17	24	1	5	9	13
5	1	9	1	0.2126	4.939	5.1362	3	7	17	24	1	5	9	13
6	1	9	1	0.0675	4.9389	5.1361	5	13	14	19	1	5	9	13
7	1	9	2	9.3766	5.174	5.174	5	15	23	24	2	6	10	14
8	1	9	2	4.6948	5.1741	5.1741	3	7	14	19	2	6	10	14
9	1	9	2	9.382	5.1743	5.1743	3	4	9	19	2	6	10	14
10	1	9	2	4.7102	5.1742	5.1742	5	13	17	24	2	6	10	14
11	**1**	9	2	0.1215	5.1742	5.1742	5	13	14	19	2	6	10	14
12	1	9	2	0.0068	5.1741	5.1741	3	7	17	24	2	6	10	14
13	1	9	3	1.3709	5.2085	5.2085	5	15	23	24	3	7	11	15
14	1	9	3	0.6626	5.2084	5.2084	3	7	14	19	3	7	11	15
15	1	9	3	1.3671	5.2086	5.2086	3	4	9	19	3	7	11	15
16	1	9	3	0.67	5.2087	5.2087	5	13	17	24	3	7	11	15
17	1	9	3	0.0203	5.2089	5.2089	3	7	17	24	3	7	11	15
18	1	9	3	0.0326	5.2083	5.2083	5	13	14	19	3	7	11	15
19	1	9	4	0.3139	5.2166	5.2166	5	15	23	24	4	8	12	16
20	1	9	4	0.1165	5.2159	5.2159	3	7	14	19	4	8	12	16
21	1	9	4	0.2976	5.2163	5.2163	3	4	9	19	4	8	12	16
22	1	9	4	0.1107	5.2159	5.2159	5	13	17	24	4	8	12	16
23	1	9	4	0.063	5.2161	5.2161	5	13	14	19	4	8	12	16
24	1	9	4	0.0431	5.2157	5.2157	3	7	17	24	4	8	12	16

25	1	9	5	0	5.0772	5.1544	5	15	23	24	5	9	13	17
26	1	9	5	0	5.0765	5.153	3	7	14	19	5	9	13	17
27	1	9	5	0	5.0769	5.1537	3	4	9	19	5	9	13	17
28	1	9	5	0	5.0766	5.1531	5	13	17	24	5	9	13	17
29	1	9	5	0	5.0768	5.1536	5	13	14	19	5	9	13	17
30	1	9	5	0	5.0763	5.1525	3	7	17	24	5	9	13	17
31	9	1	1	5.7671	4.9449	5.1391	22	21	16	6	1	5	9	13
32	9	1	1	2.7356	4.9448	5.1391	20	12	8	1	1	5	9	13
33	9	1	1	5.7682	4.9449	5.1391	20	10	2	1	1	5	9	13
34	9	1	1	2.733	4.9449	5.1391	22	18	11	6	1	5	9	13
35	9	1	1	0.3338	4.9448	5.1391	20	12	11	6	1	5	9	13
36	9	1	1	0.1868	4.9449	5.1391	22	18	8	1	1	5	9	13
37	9	1	2	9.2659	5.176	5.176	22	21	16	6	2	6	10	14
38	9	1	2	4.5125	5.1755	5.1755	20	12	8	1	2	6	10	14
39	9	1	2	9.2721	5.1759	5.1759	20	10	2	1	2	6	10	14
40	9	1	2	4.5006	5.1762	5.1762	22	18	11	6	2	6	10	14
41	9	1	2	0.1057	5.1759	5.1759	22	18	8	1	2	6	10	14
42	9	1	3	1.3378	5.21	5.21	22	21	16	6	3	7	11	15
43	9	1	3	0.7097	5.2092	5.2092	20	12	8	1	3	7	11	15
44	9	1	3	1.3462	5.2097	5.2097	20	10	2	1	3	7	11	15
45	9	1	3	0.6875	5.2106	5.2106	22	18	11	6	3	7	11	15

46	9	1	3	0	5.21	5.21	22	18	8	1	3	7	11	15
47	9	1	3	0	5.2098	5.2098	20	12	11	6	3	7	11	15
48	9	1	4	0.2438	5.2167	5.2167	22	21	16	6	4	8	12	16
49	9	1	4	0.1655	5.2144	5.2144	20	12	8	1	4	8	12	16
50	9	1	4	0.2417	5.2159	5.2159	20	10	2	1	4	8	12	16
51	9	1	4	0.0866	5.2174	5.2174	22	18	11	6	4	8	12	16
52	9	1	4	0	5.2156	5.2156	22	18	8	1	4	8	12	16
53	9	1	5	0	5.0714	5.1428	22	21	16	6	5	9	13	17
54	9	1	5	0	5.0691	5.1383	20	12	8	1	5	9	13	17
55	9	1	5	0	5.0706	5.1413	20	10	2	1	5	9	13	17
56	9	1	5	0	5.0721	5.1442	22	18	11	6	5	9	13	17
57	3	7	1	5.6186	4.9389	5.1361	9	19	22	21	1	5	9	13
58	3	7	1	2.5586	4.939	5.1361	2	7	11	15	1	5	9	13
59	3	7	1	5.6217	4.939	5.1362	2	1	5	15	1	5	9	13
60	3	7	1	2.561	4.9389	5.1361	9	12	17	21	1	5	9	13
61	3	7	1	0.2126	4.939	5.1362	2	7	17	21	1	5	9	13
62	3	7	1	0.0675	4.9389	5.1361	9	12	11	15	1	5	9	13
63	3	7	2	9.3766	5.174	5.174	9	19	22	21	2	6	10	14
64	3	7	2	4.6948	5.1741	5.1741	2	7	11	15	2	6	10	14
65	3	7	2	9.382	5.1743	5.1743	2	1	5	15	2	6	10	14
66	3	7	2	4.7102	5.1742	5.1742	9	12	17	21	2	6	10	14

67	3	7	2	0.1215	5.1742	5.1742	9	12	11	15	2	6	10	14
68	3	7	2	0.0068	5.1741	5.1741	2	7	17	21	2	6	10	14
69	3	7	3	1.3709	5.2085	5.2085	9	19	22	21	3	7	11	15
70	3	7	3	0.6626	5.2084	5.2084	2	7	11	15	3	7	11	15
71	3	7	3	1.3671	5.2086	5.2086	2	1	5	15	3	7	11	15
72	3	7	3	0.67	5.2087	5.2087	9	12	17	21	3	7	11	15
73	3	7	3	0.0203	5.2089	5.2089	2	7	17	21	3	7	11	15
74	3	7	3	0.0326	5.2083	5.2083	9	12	11	15	3	7	11	15
75	3	7	4	0.3139	5.2166	5.2166	9	19	22	21	4	8	12	16
76	3	7	4	0.1165	5.2159	5.2159	2	7	11	15	4	8	12	16
77	3	7	4	0.2976	5.2163	5.2163	2	1	5	15	4	8	12	16
78	3	7	4	0.1107	5.2159	5.2159	9	12	17	21	4	8	12	16
79	3	7	4	0.063	5.2161	5.2161	9	12	11	15	4	8	12	16
80	3	7	4	0.0431	5.2157	5.2157	2	7	17	21	4	8	12	16
81	3	7	5	0	5.0772	5.1544	9	19	22	21	5	9	13	17
82	3	7	5	0	5.0765	5.153	2	7	11	15	5	9	13	17
83	3	7	5	0	5.0769	5.1537	2	1	5	15	5	9	13	17
84	3	7	5	0	5.0766	5.1531	9	12	17	21	5	9	13	17
85	3	7	5	0	5.0768	5.1536	9	12	11	15	5	9	13	17
86	3	7	5	0	5.0763	5.1525	2	7	17	21	5	9	13	17
87	7	3	1	5.7671	4.9449	5.1391	23	24	20	10	1	5	9	13

88	7	3	1	2.7356	4.9448	5.1391	16	13	8	4	1	5	9	13
89	7	3	1	5.7682	4.9449	5.1391	16	6	3	4	1	5	9	13
90	7	3	1	2.733	4.9449	5.1391	23	18	14	10	1	5	9	13
91	7	3	1	0.3338	4.9448	5.1391	16	13	14	10	1	5	9	13
92	7	3	1	0.1868	4.9449	5.1391	23	18	8	4	1	5	9	13
93	7	3	2	9.2659	5.176	5.176	23	24	20	10	2	6	10	14
94	7	3	2	4.5125	5.1755	5.1755	16	13	8	4	2	6	10	14
95	7	3	2	9.2721	5.1759	5.1759	16	6	3	4	2	6	10	14
96	7	3	2	4.5006	5.1762	5.1762	23	18	14	10	2	6	10	14
97	7	3	2	0.1057	5.1759	5.1759	23	18	8	4	2	6	10	14
98	7	3	3	1.3378	5.21	5.21	23	24	20	10	3	7	11	15
99	7	3	3	0.7097	5.2092	5.2092	16	13	8	4	3	7	11	15
100	7	3	3	1.3462	5.2097	5.2097	16	6	3	4	3	7	11	15
101	7	3	3	0.6875	5.2106	5.2106	23	18	14	10	3	7	11	15
102	7	3	3	0	5.21	5.21	23	18	8	4	3	7	11	15
103	7	3	3	0	5.2098	5.2098	16	13	14	10	3	7	11	15
104	7	3	4	0.2438	5.2167	5.2167	23	24	20	10	4	8	12	16
105	7	3	4	0.1655	5.2144	5.2144	16	13	8	4	4	8	12	16
106	7	3	4	0.2417	5.2159	5.2159	16	6	3	4	4	8	12	16
107	7	3	4	0.0866	5.2174	5.2174	23	18	14	10	4	8	12	16
108	7	3	4	0	5.2156	5.2156	23	18	8	4	4	8	12	16

109	7	3	5	0	5.0714	5.1428	23	24	20	10	5	9	13	17
110	7	3	5	0	5.0691	5.1383	16	13	8	4	5	9	13	17
111	7	3	5	0	5.0706	5.1413	16	6	3	4	5	9	13	17
112	7	3	5	0	5.0721	5.1442	23	18	14	10	5	9	13	17

Appendix 8: Output for Example 7.1

Table 7.5g The resultant path flow and path travel time for example 7.1.

Path number	*r*	*s*	*k*	$f_p^{rs}(k)$	$c_p^{rs}(k)$	link on the path		Arrival time for each link on the path	
1	1	2	1	21.8315	1.9090	1	0	1	0
2	1	2	2	3.8000	1.9280	1	0	2	0
3	1	2	3	2.2151	1.9391	1	0	3	0
4	1	2	4	0.9567	1.9439	1	0	4	0
5	1	2	5	0	1.9439	1	0	5	0
6	1	4	1	12.7309	0.9036	5	0	1	0
7	1	4	2	3.9570	0.9234	5	0	2	0
8	1	4	3	2.3044	0.9349	5	0	3	0
9	1	4	4	6.3952	0.9032	5	0	4	0
10	1	4	5	4.6495	0.9067	5	0	5	0

11	2	1	1	14.5360	0.9126	2	0	1	0
12	2	1	2	3.9296	0.9323	2	0	2	0
13	2	1	3	1.8108	0.9477	2	0	3	0
14	2	1	4	6.0213	0.9106	2	0	4	0
15	2	1	5	3.7266	0.9203	2	0	5	0
16	2	4	1	10.3226	1.7831	8	10	1	4
17	2	4	2	3.8197	1.8213	8	10	2	5
18	2	4	3	2.7981	1.8493	8	10	3	6
19	2	4	3	1.2798	1.8493	2	5	3	6
20	2	4	4	1.1021	1.7857	2	5	4	7
21	2	4	4	3.9687	1.7857	8	10	4	7
22	2	4	5	3.5340	1.7829	8	10	5	8
23	2	4	5	2.1343	1.7828	2	5	5	8
24	4	1	1	15.6912	0.9184	6	0	1	0
25	4	1	2	3.9592	0.9382	6	0	2	0
26	4	1	3	2.3075	0.9497	6	0	3	0
27	4	1	4	4.5983	0.9181	6	0	4	0
28	4	1	5	3.4612	0.9277	6	0	5	0
29	4	2	1	8.6188	2.7259	9	7	1	7
30	4	2	2	4.5845	2.7718	9	7	2	8
31	4	2	3	3.8924	2.8107	9	7	3	9

32	4	2	4	0.9095	2.7767	9	7	4	10
33	4	2	4	4.7767	2.7767	6	1	4	7
34	4	2	5	2.5417	2.7792	9	7	5	11
35	4	2	5	2.4070	2.7793	6	1	5	8

Appendix 9: Output for Example 7.2

Table 7.5g The resultant path flow and path travel time for example 7.2.

Path number	r	s	k	$f_p^{rs}(k)$	$c_p^{rs}(k)$	link on the path		Arrival time for each link on the path	
1	1	2	1	50.9542	2.0546	1	0	1	0
2	1	2	2	5.7241	2.0832	1	0	2	0
3	1	2	3	1.6396	2.0914	1	0	3	0
4	1	2	4	0	2.0914	1	0	4	0
5	1	2	5	0	2.0914	1	0	5	0
6	1	4	1	22.1227	0.9505	5	0	1	0
7	1	4	2	5.0477	0.9758	5	0	2	0
8	1	4	3	1.4331	0.9829	5	0	3	0
9	1	4	4	15.6278	0.9505	5	0	4	0
10	1	4	5	7.4508	0.9625	5	0	5	0

11	2	1	1	27.5245	0.9775	2	0	1	0
12	2	1	2	5.5288	1.0052	2	0	2	0
13	2	1	3	0.3278	1.0215	2	0	3	0
14	2	1	4	18.6368	0.9770	2	0	4	0
15	2	1	5	4.6640	1.0036	2	0	5	0
16	2	4	1	17.3336	1.8532	8	10	1	4
17	2	4	2	4.2424	1.8956	8	10	2	5
18	2	4	3	1.7712	1.9133	8	10	3	6
19	2	4	3	2.9309	1.9133	2	5	3	7
20	2	4	4	9.6935	1.8369	8	10	4	7
21	2	4	5	6.6940	1.8614	8	10	5	8
22	2	4	5	0.6525	1.8615	2	5	5	9
23	4	1	1	22.4341	0.9521	6	0	1	0
24	4	1	2	4.1702	0.9729	6	0	2	0
25	4	1	3	0.9271	0.9776	6	0	3	0
26	4	1	4	17.2186	0.9515	6	0	4	0
27	4	1	5	3.5675	0.9944	6	0	5	0
28	4	2	1	11.7157	2.7569	9	7	1	7
29	4	2	2	5.8245	2.8152	9	7	2	8
30	4	2	3	5.0560	2.8657	9	7	3	9
31	4	2	4	6.6075	2.8732	9	7	4	10

32	4	2	5	3.2918	2.8770	9	7	5	11
33	4	2	5	9.1872	2.8770	6	1	5	8

Appendix 10: Output for Example 7.3

Table 7.17g The resultant path flow and path travel time for example 7.3.

Path number	r	s	k	$f_p^{rs}(k)$	$c_p^{rs}(k)$	link on the path		Arrival time for each link on the path	
1	1	2	1	25.4398	1.9270	1	0	1	0
2	1	2	2	4.4281	1.9492	1	0	2	0
3	1	2	3	2.5812	1.9621	1	0	3	0
4	1	2	4	1.1148	1.9677	1	0	4	0
5	1	2	5	0	1.9677	1	0	5	0
6	1	4	1	13.1576	0.9057	5	0	1	0
7	1	4	2	4.0896	0.9262	5	0	2	0
8	1	4	3	2.3817	0.9381	5	0	3	0
9	1	4	4	6.6096	0.9053	5	0	4	0
10	1	4	5	4.8053	0.9089	5	0	5	0
11	2	1	1	14.9995	0.9149	2	0	1	0
12	2	1	2	4.4385	0.9371	2	0	2	0

13	2	1	3	2.5688	0.9500	2	0	3	0
14	2	1	4	7.4614	0.9123	2	0	4	0
15	2	1	5	4.5636	0.9249	2	0	5	0
16	2	4	1	9.7103	1.7770	8	10	1	4
17	2	4	2	3.3090	1.8100	8	10	2	5
18	2	4	3	1.9276	1.8293	8	10	3	6
19	2	4	4	4.4152	1.7764	8	10	4	7
20	2	4	4	0.0548	1.7764	2	5	4	7
21	2	4	5	3.3496	1.7768	8	10	5	8
22	2	4	5	2.3806	1.7768	2	5	5	8
23	4	1	1	13.9979	0.9099	6	0	1	0
24	4	1	2	3.8265	0.9290	6	0	2	0
25	4	1	3	2.2298	0.9402	6	0	3	0
26	4	1	4	5.2461	0.9100	6	0	4	0
27	4	1	5	3.7171	0.9221	6	0	5	0
28	4	2	1	6.7284	2.7070	9	7	1	7
29	4	2	2	3.6524	2.7436	9	7	2	8
30	4	2	3	3.0557	2.7741	9	7	3	9
31	4	2	4	2.3488	2.7640	9	7	4	10
32	4	2	4	2.7075	2.7640	6	1	4	7
33	4	2	5	2.5401	2.7667	6	1	5	8

34	4	2	5	2.0933	2.7666	9	7	5	11

Appendix 11: Output for Example 8.1

Table 8.3g The resultant path flow and path travel time for Example 8.1.

Path number	*r*	*s*	*k*	$f_p^{rs}(k)$	$c_p^{rs}(k)$	Links on the path				Arrival time for each link on the path			
1	1	9	1	3.4259	4.8829	5	15	23	24	1	5	9	13
2	1	9	1	1.8491	4.8829	3	7	14	19	1	5	9	13
3	1	9	1	3.2407	4.8829	3	4	9	19	1	5	9	13
4	1	9	1	1.4787	4.8829	5	13	17	24	1	5	9	13
5	1	9	1	0.0028	4.8829	3	7	17	24	1	5	9	13
6	1	9	1	0.0028	4.8829	5	13	14	19	1	5	9	13
7	1	9	2	3.4258	4.9662	5	15	23	24	2	6	10	14
8	1	9	2	1.8518	4.9662	3	7	14	19	2	6	10	14
9	1	9	2	3.2408	4.9662	3	4	9	19	2	6	10	14
10	1	9	2	1.4816	4.9662	5	13	17	24	2	6	10	14
11	1	9	3	3.4262	5.0496	5	15	23	24	3	7	11	15
12	1	9	3	1.8418	5.0496	3	7	14	19	3	7	11	15
13	1	9	3	3.2406	5.0496	3	4	9	19	3	7	11	15

14	1	9	3	1.4708	5.0496	5	13	17	24	3	7	11	15
15	1	9	3	0.0103	5.0496	5	13	14	19	3	7	11	15
16	1	9	3	0.0103	5.0496	3	7	17	24	3	7	11	15
17	1	9	4	3.4256	5.1329	5	15	23	24	4	8	12	16
18	1	9	4	1.8518	5.1329	3	7	14	19	4	8	12	16
19	1	9	4	3.2407	5.1329	3	4	9	19	4	8	12	16
20	1	9	4	1.4819	5.1329	5	13	17	24	4	8	12	16
21	1	9	4	0	5.1329	3	7	17	24	4	8	12	16
22	1	9	5	3.1741	5.1339	5	15	23	24	5	9	13	17
23	1	9	5	0.9394	5.1339	3	7	14	19	5	9	13	17
24	1	9	5	1.9726	5.1339	5	13	17	24	5	9	13	17
25	1	9	5	3.6899	5.1339	3	4	9	19	5	9	13	17
26	1	9	5	0.2113	5.1339	5	13	14	19	5	9	13	17
27	1	9	5	0.0127	5.1339	3	7	17	24	5	9	13	17
28	9	1	1	4.1667	4.8875	22	21	16	6	1	5	9	13
29	9	1	1	1.4814	4.8875	20	12	8	1	1	5	9	13
30	9	1	1	3.4259	4.8875	20	10	2	1	1	5	9	13
31	9	1	1	0.9259	4.8875	22	18	8	1	1	5	9	13
32	9	1	2	4.1667	4.9755	22	21	16	6	2	6	10	14
33	9	1	2	1.4818	4.9755	20	12	8	1	2	6	10	14
34	9	1	2	3.4257	4.9755	20	10	2	1	2	6	10	14

35	9	1	2	0.9258	4.9755	22	18	8	1	2	6	10	14
36	9	1	3	4.1667	5.0635	22	21	16	6	3	7	11	15
37	9	1	3	1.4809	5.0635	20	12	8	1	3	7	11	15
38	9	1	3	3.4263	5.0635	20	10	2	1	3	7	11	15
39	9	1	3	0.9261	5.0635	22	18	8	1	3	7	11	15
40	9	1	4	4.1667	5.1514	22	21	16	6	4	8	12	16
41	9	1	4	1.4821	5.1514	20	12	8	1	4	8	12	16
42	9	1	4	3.4255	5.1514	20	10	2	1	4	8	12	16
43	9	1	4	0.9257	5.1514	22	18	8	1	4	8	12	16
44	9	1	5	0.7143	5.1309	22	21	16	6	5	9	13	17
45	9	1	5	2.3811	5.1309	20	12	11	6	5	9	13	17
46	9	1	5	4.1264	5.1309	22	18	11	6	5	9	13	17
47	9	1	5	0	5.1369	20	12	8	1	5	9	13	17
48	9	1	5	2.7781	5.1309	20	10	2	1	5	9	13	17
49	3	7	1	3.4259	4.8875	9	19	22	21	1	5	9	13
50	3	7	1	4.1667	4.8875	2	1	5	15	1	5	9	13
51	3	7	1	0.9259	4.8875	2	7	17	21	1	5	9	13
52	3	7	1	1.4815	4.8875	9	12	17	21	1	5	9	13
53	3	7	2	3.4261	4.9755	9	19	22	21	2	6	10	14
54	3	7	2	4.1667	4.9755	2	1	5	15	2	6	10	14
55	3	7	2	0.926	4.9755	2	7	17	21	2	6	10	14

56	3	7	2	1.4812	4.9755	9	12	17	21	2	6	10	14
57	3	7	3	3.4256	5.0634	9	19	22	21	3	7	11	15
58	3	7	3	4.1667	5.0635	2	1	5	15	3	7	11	15
59	3	7	3	1.4821	5.0635	9	12	17	21	3	7	11	15
60	3	7	3	0.9256	5.0635	2	7	17	21	3	7	11	15
61	3	7	4	3.4263	5.1514	9	19	22	21	4	8	12	16
62	3	7	4	4.1668	5.1514	2	1	5	15	4	8	12	16
63	3	7	4	1.4809	5.1514	9	12	17	21	4	8	12	16
64	3	7	4	0.926	5.1514	2	7	17	21	4	8	12	16
65	3	7	5	2.7781	5.1309	9	19	22	21	5	9	13	17
66	3	7	5	4.1276	5.1309	2	7	11	15	5	9	13	17
67	3	7	5	2.3803	5.1309	9	12	11	15	5	9	13	17
68	3	7	5	0.714	5.1309	2	1	5	15	5	9	13	17
69	7	3	1	3.2407	4.8829	23	24	20	10	1	5	9	13
70	7	3	1	1.4815	4.8829	16	13	8	4	1	5	9	13
71	7	3	1	3.4259	4.8829	16	6	3	4	1	5	9	13
72	7	3	1	1.8518	4.8829	23	18	14	10	1	5	9	13
73	7	3	2	3.2407	4.9662	23	24	20	10	2	6	10	14
74	7	3	2	1.4702	4.9662	16	13	8	4	2	6	10	14
75	7	3	2	3.426	4.9662	16	6	3	4	2	6	10	14
76	7	3	2	1.8408	4.9662	23	18	14	10	2	6	10	14

77	7	3	2	0.0112	4.9662	23	18	8	4	2	6	10	14
78	7	3	2	0.0111	4.9662	16	13	14	10	2	6	10	14
79	7	3	3	3.2408	5.0496	23	24	20	10	3	7	11	15
80	7	3	3	1.4819	5.0496	16	13	8	4	3	7	11	15
81	7	3	3	3.4257	5.0496	16	6	3	4	3	7	11	15
82	7	3	3	1.8516	5.0496	23	18	14	10	3	7	11	15
83	7	3	4	3.2413	5.1329	23	24	20	10	4	8	12	16
84	7	3	4	1.4758	5.1329	16	13	8	4	4	8	12	16
85	7	3	4	3.4264	5.1329	16	6	3	4	4	8	12	16
86	7	3	4	1.8463	5.1329	23	18	14	10	4	8	12	16
87	7	3	4	0.0051	5.1329	23	18	8	4	4	8	12	16
88	7	3	4	0.005	5.1329	16	13	14	10	4	8	12	16
89	7	3	5	3.6894	5.1339	23	24	20	10	5	9	13	17
90	7	3	5	1.9855	5.1339	16	13	8	4	5	9	13	17
91	7	3	5	0.9539	5.1339	23	18	14	10	5	9	13	17
92	7	3	5	3.1744	5.1339	16	6	3	4	5	9	13	17
93	7	3	5	0.1968	5.1339	16	13	14	10	5	9	13	17
94	7	3	5	0	5.1339	23	18	8	4	5	9	13	17

Appendix 12: Output for Example 8.2

Table 8.6i The resultant path flow and path travel time for Example 8.2.

Path number	r	s	k	$f_p^{rs}(k)$	$c_p^{rs}(k)$	Links on the path				Arrival time for each link on the path			
1	1	9	1	5	5.6412	3	7	17	24	1	4	7	11
2	1	9	1	5	5.6412	3	7	14	19	1	4	7	11
3	1	9	2	5.7435	6.1815	3	7	17	24	2	6	9	13
4	1	9	2	1.3338	6.1815	5	15	23	24	2	6	10	14
5	1	9	2	2.9227	6.1815	3	7	14	19	2	6	9	13
6	1	9	3	1.0898	5.7023	3	7	17	24	3	6	9	13
7	1	9	3	4.9995	5.7023	5	15	23	24	3	6	10	14
8	1	9	3	3.9107	5.7023	3	7	14	19	3	6	9	13
9	1	9	4	5.7435	6.1815	3	7	17	24	4	8	11	15
10	1	9	4	1.3338	6.1815	5	15	23	24	4	8	12	16
11	1	9	4	2.9227	6.1815	3	7	14	19	4	8	11	15
12	1	9	5	1.0898	5.7023	3	7	17	24	5	8	11	15
13	1	9	5	4.9995	5.7023	5	15	23	24	5	8	12	16
14	1	9	5	3.9107	5.7023	3	7	14	19	5	8	11	15
15	9	1	1	5	5.6412	22	18	11	6	1	4	7	11
16	9	1	1	5	5.6412	22	18	8	1	1	4	7	11
17	9	1	2	4.8651	6.1815	22	18	11	6	2	6	9	13
18	9	1	2	1.3333	6.1815	20	10	2	1	2	6	10	14

19	9	1	2	3.8015	6.1815	22	18	8	1	2	6	9	13
20	9	1	3	1.9682	5.7023	22	18	11	6	3	6	9	13
21	9	1	3	5	5.7023	20	10	2	1	3	6	10	14
22	9	1	3	3.0318	5.7023	22	18	8	1	3	6	9	13
23	9	1	4	4.8651	6.1815	22	18	11	6	4	8	11	15
24	9	1	4	1.3333	6.1815	20	10	2	1	4	8	12	16
25	9	1	4	3.8015	6.1815	22	18	8	1	4	8	11	15
26	9	1	5	1.9682	5.7023	22	18	11	6	5	8	11	15
27	9	1	5	5	5.7023	20	10	2	1	5	8	12	16
28	9	1	5	3.0318	5.7023	22	18	8	1	5	8	11	15
29	3	7	1	5	5.6412	2	7	17	21	1	4	7	11
30	3	7	1	5	5.6412	2	7	11	15	1	4	7	11
31	3	7	2	5.7435	6.1815	2	7	17	21	2	6	9	13
32	3	7	2	1.3338	6.1815	9	19	22	21	2	6	10	14
33	3	7	2	2.9227	6.1815	2	7	11	15	2	6	9	13
34	3	7	3	1.0898	5.7023	2	7	17	21	3	6	9	13
35	3	7	3	4.9995	5.7023	9	19	22	21	3	6	10	14
36	3	7	3	3.9107	5.7023	2	7	11	15	3	6	9	13
37	3	7	4	5.7435	6.1815	2	7	17	21	4	8	11	15
38	3	7	4	1.3338	6.1815	9	19	22	21	4	8	12	16
39	3	7	4	2.9227	6.1815	2	7	11	15	4	8	11	15

40	3	7	5	1.0898	5.7023	2	7	17	21	5	8	11	15
41	3	7	5	4.9995	5.7023	9	19	22	21	5	8	12	16
42	3	7	5	3.9107	5.7023	2	7	11	15	5	8	11	15
43	7	3	1	5	5.6412	23	18	14	10	1	4	7	11
44	7	3	1	5	5.6412	23	18	8	4	1	4	7	11
45	7	3	2	4.8651	6.1815	23	18	14	10	2	6	9	13
46	7	3	2	1.3333	6.1815	16	6	3	4	2	6	10	14
47	7	3	2	3.8015	6.1815	23	18	8	4	2	6	9	13
48	7	3	3	1.9682	5.7023	23	18	14	10	3	6	9	13
49	7	3	3	5	5.7023	16	6	3	4	3	6	10	14
50	7	3	3	3.0318	5.7023	23	18	8	4	3	6	9	13
51	7	3	4	4.8651	6.1815	23	18	14	10	4	8	11	15
52	7	3	4	1.3333	6.1815	16	6	3	4	4	8	12	16
53	7	3	4	3.8015	6.1815	23	18	8	4	4	8	11	15
54	7	3	5	1.9682	5.7023	23	18	14	10	5	8	11	15
55	7	3	5	5	5.7023	16	6	3	4	5	8	12	16
56	7	3	5	3.0318	5.7023	23	18	8	4	5	8	11	15