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Linear Prediction Approach for Blind Multiuser Detection in Multicarrier CDMA Systems

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Linear Prediction Approach for Blind Multiuser Detection in Multicarrier CDMA Systems

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Abstract

Multicarrier CDMA has emerged recently as a promising candidate for the next generation broad-band mobile networks. Since multicarrier CDMA combines the multicarrier technique and the spread-spectrum CDMA technique, many multiuser detection methods used in DS-CDMA can be applied to multicarrier CDMA system. In this thesis, we propose a direct blind multiuser detection method for multicarrier CDMA based on linear prediction. Using only the spreading code of the desired user, we extract the column vector subspace corresponding to the signal of interest from the channel matrix of the received complex signal. And then zero-forcing (ZF) and minimum mean square error (MMSE) detectors are constructed. Our algorithms do not require channel estimation and avoid the channel estimation error. Simulations show that in most conditions, our algorithms outperform the typical subspace-based algorithm and the eigen-method used in a multicarrier system.

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Chapter 1

Introduction

1.1 CDMA

CDMA (Code Division Multiple Access) is a multiplexing technique where a number of users simultaneously and asynchronously access a channel by modulating their information-bearing signals with preassigned code sequences in the time domain and spreading the signals over the entire band in the frequency domain. The receiver, knowing the code sequences of the user, decodes the received signal and recovers the original data. This is possible because the cross correlations between the code of the desired user and the codes of the other users are small. The capability of suppressing multiuser interference is determined by the cross-correlation characteristics of the spreading codes. Since the bandwidth of the code signal is chosen to be much larger than the bandwidth of the information-bearing signal, the encoding process enlarges (spreads) the spectrum of the signal and is therefore also known as spread-spectrum modulation. The resulting signal is also called a spread-spectrum signal, and CDMA is often denoted as spread-spectrum multiple access (SSMA).

Because of the advantageous properties of CDMA signals: large traffic capacity, multipath interference rejection, narrowband interference rejection, lower probability to interception (LPI), privacy and anti-jamming capability, etc., CDMA technique has been investigated extensively and been considered the strongest candidate for the third-generation wireless personal communication systems.

The remainder of this section is concentrated on the direct sequence CDMA (DS-CDMA), one of the major CDMA techniques.

During the 1980s Qualcomm investigated DS-CDMA techniques, which finally led to the commercialization of cellular spread spectrum communications in the form of the CDMA IS-95 standard in July 1993.

In a DS-CDMA system, the information-bearing signal (the original data signal) is directly modulated by a digital, discrete-time, discrete-valued code sequence. The original data signal can be either analog or digital; in most cases it is digital. In the case of a digital signal the data modulation is often omitted and the data signal is directly multiplied by the code sequence and the resulting signal modulates the wideband carrier. It is from this direct multiplication that the direct sequence CDMA gets its name.

In Figure 1.1, a block diagram of BPSK modulated DS-CDMA transmitter is given. The DS-SS signal resulting from this transmitter is shown in Figure 1.2.

After transmission of the signal, the receiver (Figure 1.3) uses coherent demodulation to despread the SS signal, using a locally generated code sequence. To be able to perform the despreading operation, the receiver must not only know the code sequence used to spread the signal, but the codes of the received signal and the locally generated code must

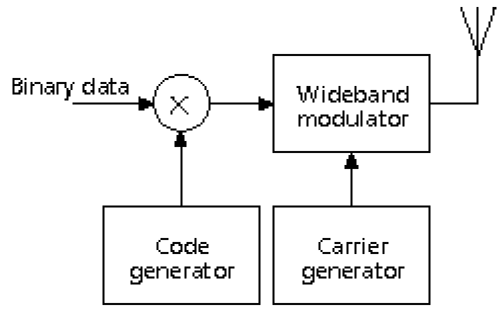


Figure 1.1: *Block Diagram of a DS-CDMA transmitter.*

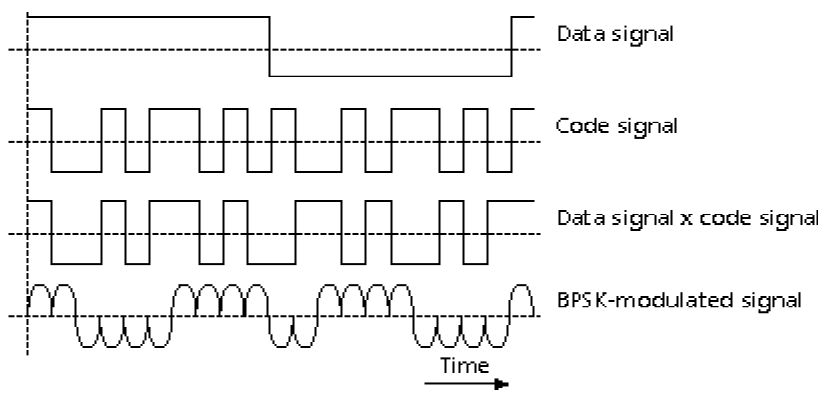


Figure 1.2: *Generation of a BPSK-modulated SS signal.*

also be synchronized. This synchronization must be accomplished at the beginning of the reception and maintained until the entire signal has been received. The code synchronization/tracking block performs this operation. After despreading, a data modulated signal results, and after demodulation the original data can be recovered.

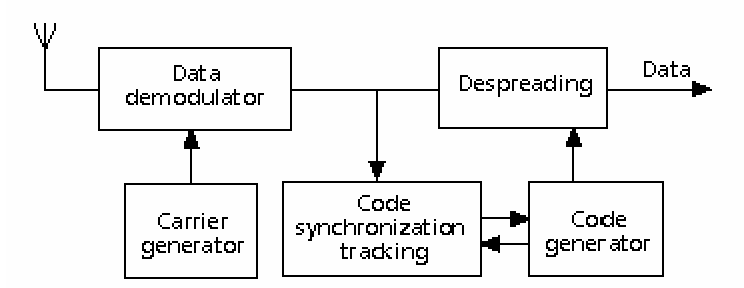


Figure 1.3: *Block Diagram of a DS-CDMA receiver.*

Since direct sequence (DS) waveforms have a wide bandwidth, anytime the bandwidth exceeds the coherence bandwidth of the channel, the channel fading tends to be frequency selective. In the time domain, it means multipath phenomenon. Thus, the composite signals transmitted can only be recovered through sophisticated receiver side processing, e.g. RAKE receiver [5], [6].

1.2 OFDM

In a classical parallel data transmission system, channel frequency band is divided into N nonoverlapping subchannels. Each subchannel is modulated with a separate symbol. Then the N subchannels are frequency-multiplexed. It seems good to avoid spectral overlapping of channels to eliminate interchannel interference. However, this leads to inefficient use of the available spectrum. To cope with this inefficiency, the ideas using parallel data transmission and FDM with overlapping subchannels to avoid the use of high-speed equalization and to combat impulsive noise and multipath distortion, as well as to fully use the available bandwidth, were published from the mid-1960s [4].

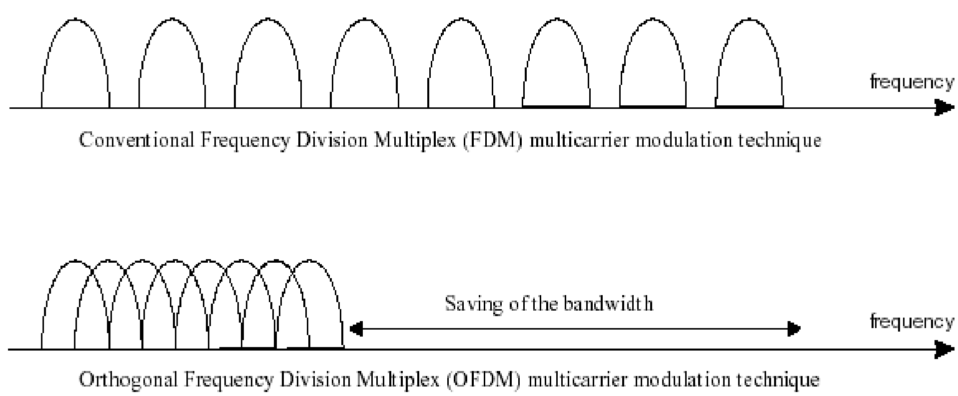


Figure 1.4: C*omparison of the bandwidth utilization.*

Figure 1.4 illustrates the difference of bandwidths between the conventional nonoverlapping multicarrier technique and the overlapping multicarrier technique (e.g. OFDM). It is obvious that the latter is much more frequency efficient with almost 50% bandwidth saving. To realize the overlapping multicarrier technique, however, we need to reduce the crosstalk between subcarriers, which means that we want orthogonality between the subcarriers.

Orthogonal frequency division multiplexing (OFDM) is a communications technique that divides the communications channel into a number of overlapping frequency subchannels. A subcarrier carrying a portion of the user information is transmitted in each subchannel. Each subcarrier is orthogonal (linearly independent of each other) to every other subcarrier, differentiating OFDM from the commonly used frequency division multiplexing (FDM).

Following is the basic principle of OFDM.

Original data symbols with duration T_s are serial-to-parallel converted into N parallel symbol streams with symbol duration $T = NT_s$ and modulate N orthogonal subcarriers respectively. The number of subcarriers N is chosen such that $NT_s >> s_t$ where s_t is the rms delay spread of the channel.

The envelope of an OFDM signal is described by:

$$x(t) = \mathrm{Re}\left\{ A \sum_{k} \sum_{n=0}^{N-1} d_{k,n} f_n(t-kT) \right\} \tag{1.1}$$

where the orthogonal waveforms $f_n(t)$ are chosen as

$$f_n(t) = h_a(t) \exp\{ j \frac{2p(n - \frac{N-1}{2})t}{T} \}, \qquad n = 1, \cdots, N-1 \tag{1.2}$$

Here, $h_a(t)$ is the pulse shaping filter. If it is rectangular, then the frequency separation of the carriers is $\frac{1}{T}$ and the carriers are orthogonal (Figure 1.5).

$$\int_0^T \exp\{-j2p \frac{(m-\frac{N-1}{2})t}{T}\} \exp\{j2p \frac{(n-\frac{N-1}{2})t}{T}\} \mathrm{d}t$$

$$= \int_0^T \exp\{j2p \frac{(n-m)}{T}\} \mathrm{d}t = \left\langle \begin{array}{ll} T & (m=n) \\ 0 & (m \neq n) \end{array} \right. , \quad n, m = 1, \cdots, N-1 \qquad (1.3)$$

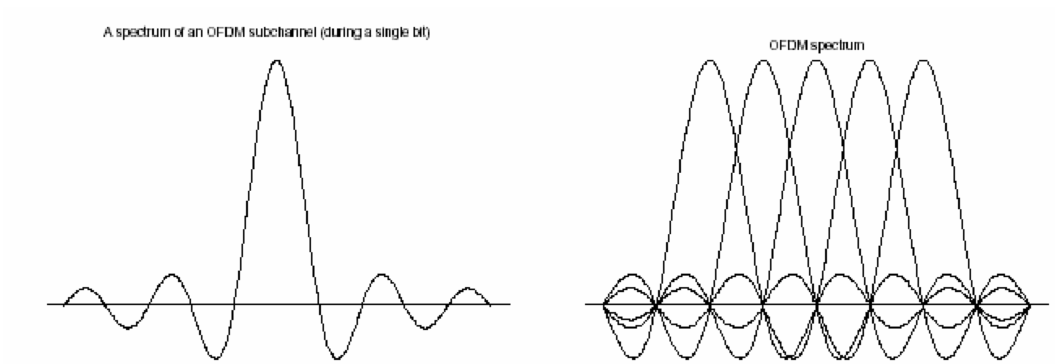


Figure 1.5: *Example of OFDM spectrum (a) a single subchannel (b) 5 subcarriers, at the central frequency of each subchannel, there is no crosstalk from other subchannels.*

Considering the equivalent complex baseband notation, and assuming $h_a(t)$ is rectangular, one OFDM symbol at $k = 0$ has the form

$$x(t) = A \sum_{n}^{N-1} d_{0,n} \exp\{j2p \frac{nt}{NT_s}\}, \quad 0 < t < T \qquad (1.4)$$

Sampling at $t = kT_s$, this yields:

$$X_0(k) = A \sum_{n}^{N-1} d_{0,n} \exp\{j2pnk/N\}, \quad k = 0,1,\cdots,N-1 \qquad (1.5)$$

Note that the samples $\{X_0(k)\}_{k=0}^{N-1}$ are just the Inverse Discrete Fourier Transform (IDFT) of the block $\{d_{0,n}\}_{n=0}^{N-1}$. Thus, OFDM can be simply implemented as IDFT or the faster IFFT.

In the receiver, the DFT provides an efficient demodulation:

$$d_{0,n} = DFT\{X_0(k)\} = \frac{1}{N}\sum_{k=0}^{N-1} X_0(k)\exp\{-j2pnk/N\}, \quad n = 0,1,\cdots,N-1 \qquad (1.6)$$

In order to avoid intersymbol interference (ISI) arising due to channel memory, conventional OFDM systems insert redundancy in the form of a cyclic prefix (CP) of length larger than the FIR channel order [2] after IFFT transformation. CP is discarded at the receiver and the remaining part of the OFDM symbol is FFT processed. A combination of IFFT and CP at the transmitter with the FFT at the receiver converts a frequency-selective channel to separate flat-fading subchannels [1], [2], [7]. Frequency domain equalization is then applied by dividing the FFT output by the corresponding channel frequency gains. But sometimes, the channel has nulls on (or close to) some subcarriers and the CP OFDM does not work in this case. If Channel State Information (CSI) is available at the transmitter, channel nulls can be avoided by not transmitting symbols on those subcarriers. But CSI is not always available or it may be costly to acquire at the transmitter when the channel is time-varying. Thus, zero padding (ZP) has been proposed to replace the CP, in order to guarantee symbol recovery regardless of channel zeros at the cost of modifying the transmitter and complicating slightly the equalizer.

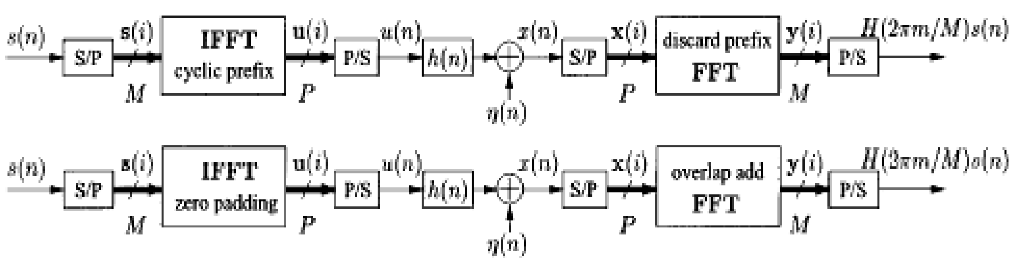


Figure 1.6: *Block diagram for CP-OFDM (top) and ZP-OFDM (bottom).*

1.3 Multicarrier CDMA

Since 1993, Multicarrier CDMA based on a combination of DS-CDMA and OFDM techniques has drawn much attention and been deemed as a promising candidate for the next generation broadband mobile networks. This combination allows one to benefit from the advantages of both techniques. In a Multicarrier CDMA system, data sequence multiplied by a spreading sequence modulates N_c carriers, rather than a single carrier. By transmitting high-speed data through multiple low-rate subcarrier streams, the symbol duration in each substream increases, leading to higher immunity against multipath dispersion. In other words, a multicarrier system requires a lower speed, parallel-type of signal processing, in contrast to a fast, serial-type of signal processing in a single-carrier RAKE receiver. This greatly reduces the complexity of the receiver.

There are three main multicarrier schemes: "multicarrier (MC-) CDMA" [8]-[10], "multicarrier DS-CDMA" [6], [12] and "multitone (MT-) CDMA"[1].

1.3.1 MC-CDMA

In a MC-CDMA system, the original data stream is copied onto all the subcarriers at first, and then on each subcarrier, the data stream is multiplied by just one code from the spreading sequence (in other words, the spreading operation is in the frequency domain). So each carrier conveys a narrowband waveform, rather than a spread spectrum waveform in a single carrier system.

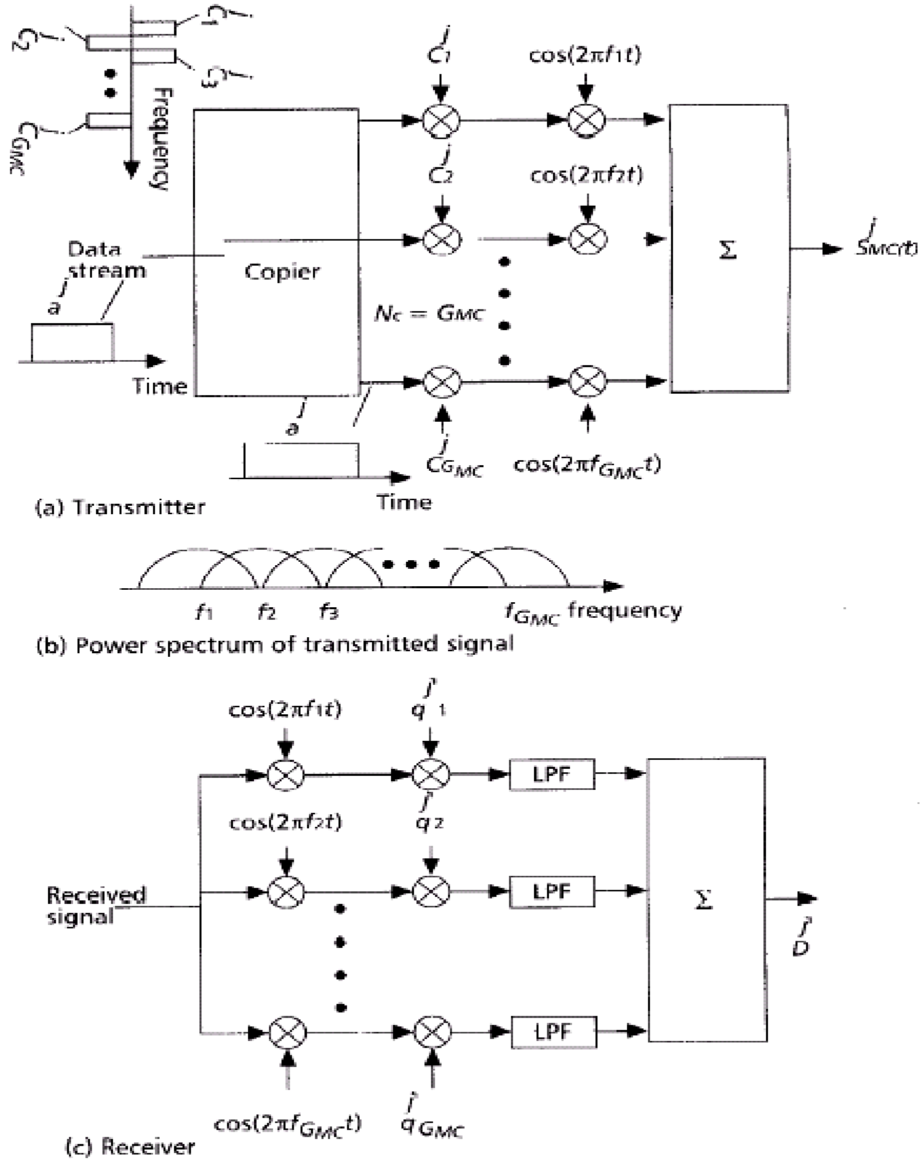


Figure 1.7: *MC-CDMA scheme: a) transmitter; b) power spectrum of transmitted signal; c) receiver.*

Figure 1.7 is a block diagram of this discretized MC-CDMA scheme. Here we assume that the number of carriers (N_c) is equal to the processing gain (G_{MC}). The proper choice of the number of subcarriers and the guard interval (CP) is important in order to increase the robustness against frequency selective fading.

However, the number of carriers does not have to be equal to the processing gain. We know that it is crucial for muticarrier transmission to have frequency non-selective fading over each subcarrier. So, when the original data rate is high enough to become subject to frequency selective fading, the data needs to be first serial-to-parallel converted before spreading over the frequency domain. Figure 1.8 shows this modified scheme.

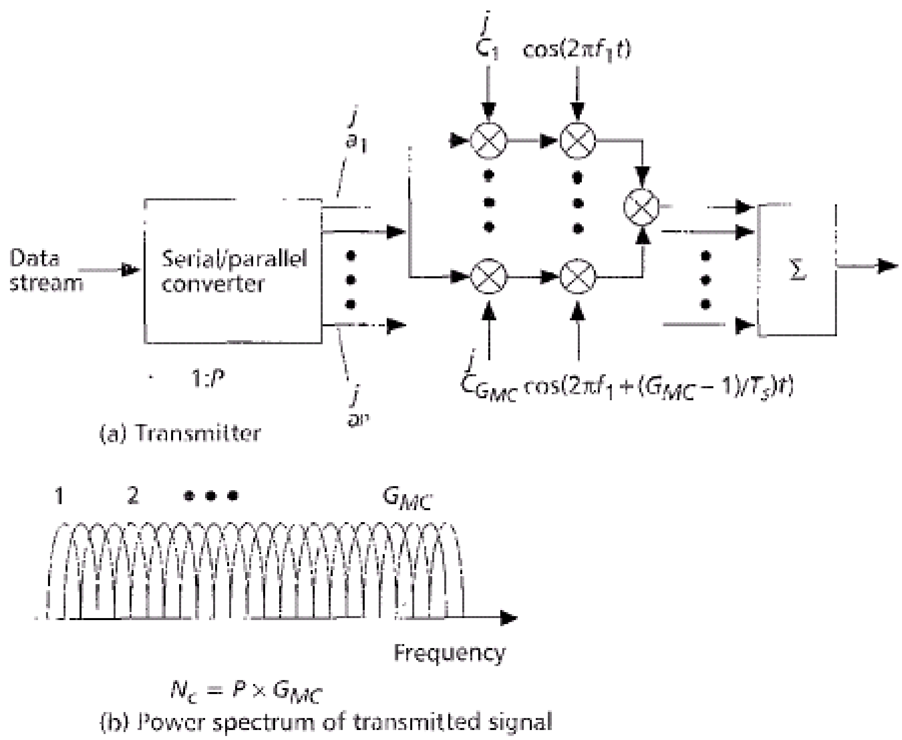


Figure 1.8: *Modification of MC-CDMA scheme.*

1.3.2 Multicarrier DS-CDMA

Multicarrier DS-CDMA (MC-DS-CDMA) is an enhanced version of MC-CDMA where additional time-domain spreading is injected at the transmitter. In a MC-DS-CDMA system, the original data stream is serial-to-parallel converted before being spread by a given spreading code in the time domain, and then each of the data streams modulates a different carrier frequency respectively, similar to a normal DS-CDMA scheme. The available frequency spectrum is divided into N_c equi-width frequency bands, typically much less than the processing gain G_{MC}, and each frequency band is used to transmit a narrow-band DS waveform. This scheme is originally proposed for an uplink communication channel, because the introduction of OFDM signaling into DS-CDMA scheme is effective for the establishment of a quasi-synchronous channel. Figure 1.9 shows the MC-DS-CDMA scheme.

Multicarrier DS-CDMA can be viewed as “a collection of synchronous narrowband DS-CDMA” signals. Indeed, within each subchannel the received signal is exactly the same as that in a synchronous narrow-band CDMA system. The difference, which is critical for narrowband fading/interference resistance, is that signals in Multicarrier DS-CDMA are spread across all subchannels.

Comparing Multicarrier DS-CDMA with MC-CDMA and DS-CDMA, it is obvious that MC-DS-CDMA serves as a compromise between the frequency- and time-domain spreading. When the bandwidth of the subchannel is fixed, Multicarrier DS-CDMA does offer stronger fading resistance than regular narrowband DS-CDMA and higher degree flexibility than MC-CDMA. The tradeoff, as expected, is a higher computational cost.

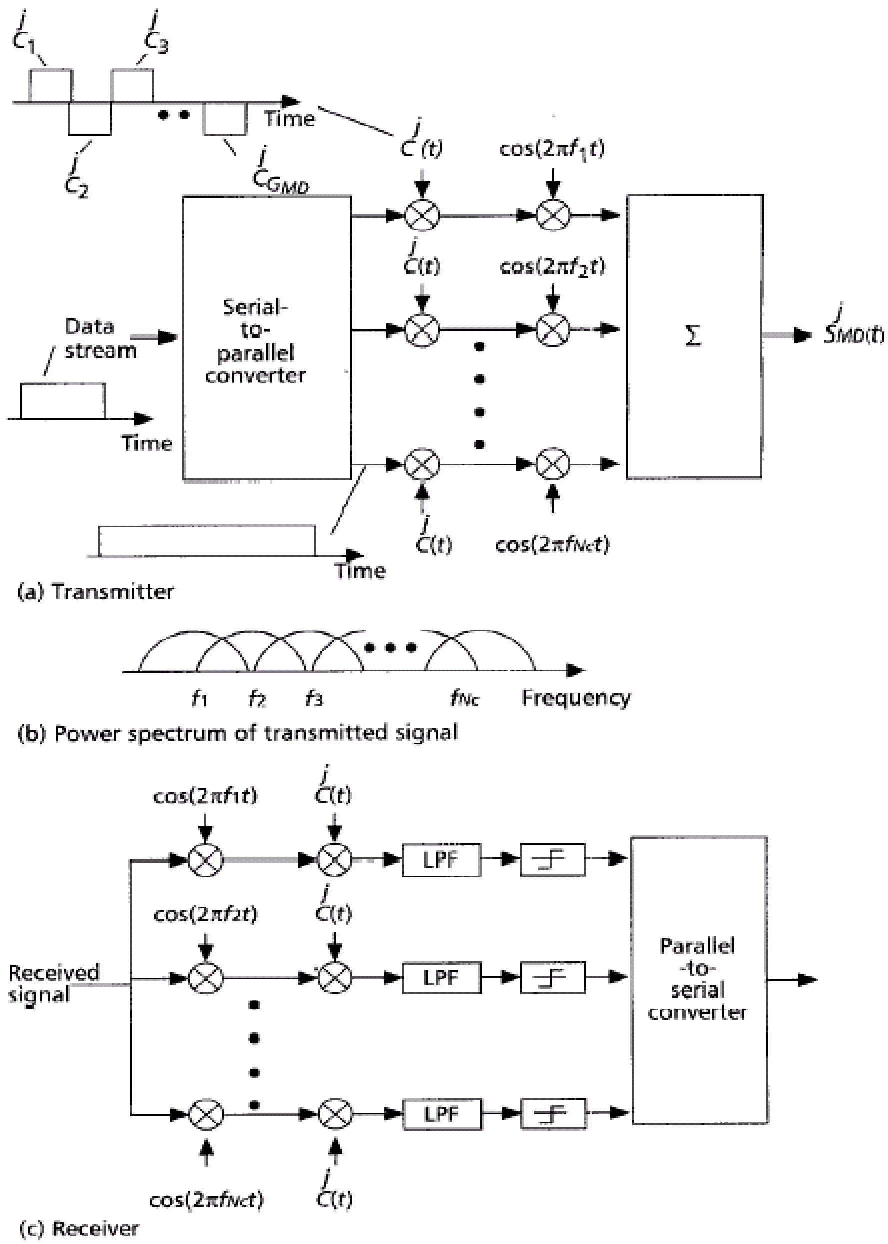


Figure 1.9: *Multicarrier DS-CDMA scheme: a) transmitter; b) power spectrum of transmitted signal; c) receiver.*

1.3.3 MT-CDMA

Figure 1.10 shows the MT-CDMA scheme. The MT-CDMA transmitter spreads the S/P converted data streams using a given spreading code in the time domain so that the spectrum of each subcarrier *prior to* spreading operation can satisfy the orthogonality condition with minimum frequency separation. Therefore, the resulting spectrum of each subcarrier no longer satisfies the orthogonality condition. (In a MC-DS-CDMA system, the resulting spectrum of each subcarrier *after* spreading satisfies the orthogonality condition with the minimum frequency separation.) The MT-CDMA scheme uses longer spreading codes in proportion to the number of subcarriers, as compared with a normal (single carrier) DS-CDMA scheme, therefore, the system accommodate more users than the DS-CDMA scheme.

1.4 Receiver for Multicarrier CDMA Systems

The receiver design for multicarrier CDMA has been studied to achieve the full potential from the combination of multicarrier modulation technique and DS spread-spectrum technique. According to whether using cyclic prefix in transmission and whether using training bits to get the channel information during the detection, normally, the receivers for multicarrier CDMA systems can be classified into three categories.

The first category is both using cyclic prefix and training.

Actually, in multicarrier systems, we only use CP on MC-CDMA system. In such a system, because of the combination of IFFT and CP at the transmitter with the FFT at the receiver, the frequency-selective fading channel is converted into flat-fading channels on each subcarrier respectively. Thus, we just need one-tap equalizer for each subcarrier to get the estimations of the transmitted data [32]. In this system, the channel information is essential to the coefficients of the equalizers, so training processes are performed to estimate the channel before the communication is set up. Note that in a MC-CDMA system using CP, all users need to be synchronous to keep the code orthogonality among users. So the MC-CDMA system using CP is usually used in the downlink channel, where all user signals are synchronous by construction. While in the uplink channel, where all users are potentially asynchronous, the orthogonality among users is distorted, causing inter-symbol interference (ISI) and multiuser interference (MUI). In the receiver, the received signal is combined, in a sense, in the frequency domain [37]; therefore, the receiver can always employ all the received signal energy scattered in the frequency domain. We believe that this is the main advantage of the MC-CDMA scheme over DS-CDMA schemes.

Although using CP greatly reduces the complexity of channel equalization, it is not transmission efficient because of the redundancy part (CP). So recently, most research is focused on multicarrier systems without using CP. The other two categories of receivers we mentioned above come from these systems.

The second category of receivers does not use CP, but use training bits to get the channel fading information. In [22], the simple Despreading and Combing (DC) receiver is used because it is computational attractive. While the DC receiver is simple, the MMSE receiver offers better performance. In [13], Miller and Rainbolt investigated two different design strategies for MMSE detection in MC-DS-CDMA system. In one case, the MMSE filters are designed separately for each carrier with the knowledge of the fading coefficient of the desired user, while in the other case the optimization of the filter is done jointly with the knowledge of the fading coefficient of all the users on all the subcarriers.

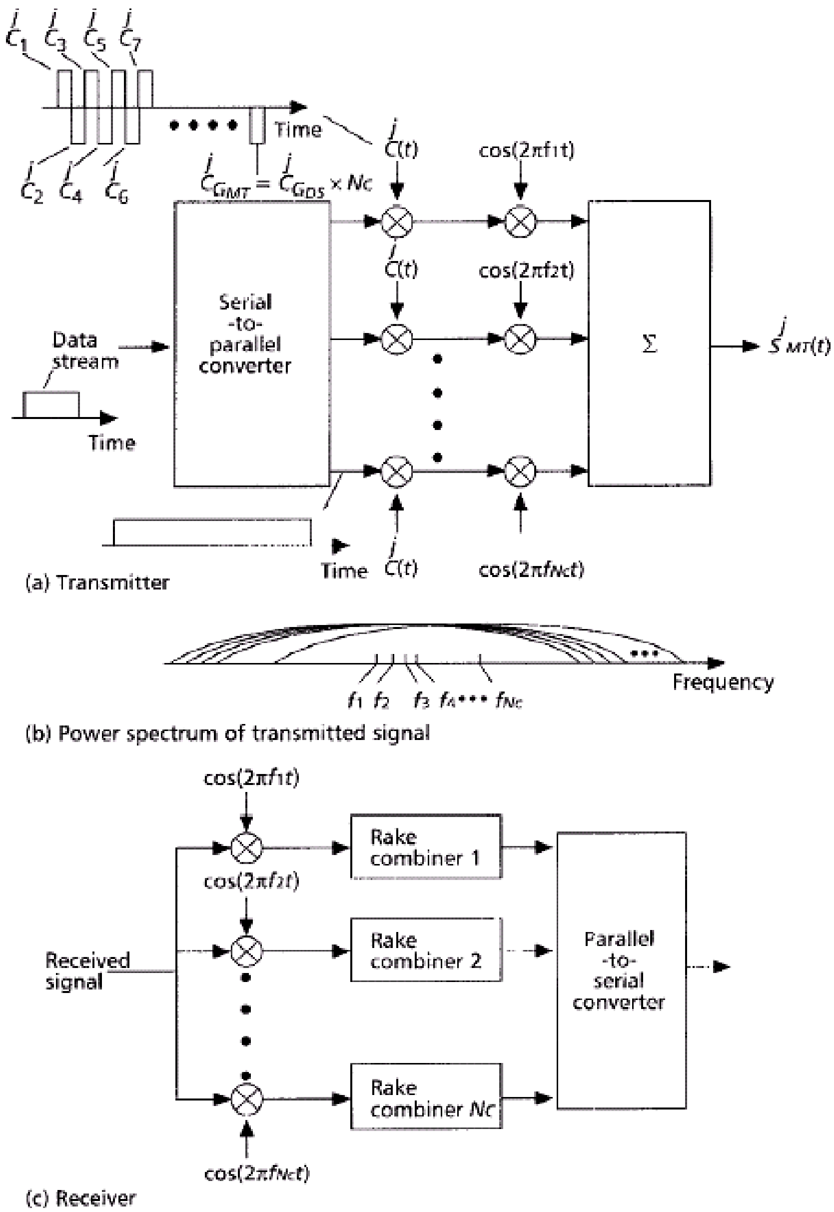


Figure 1.10: *MT-CDMA scheme: a) transmitter; b) power spectrum of transmitted signal; c) receiver.*

The third category of receivers uses neither CP nor training. They use blind detection methods.

For MC-CDMA systems, subspace-based blind MMSE receivers are proposed in [21]. For MC-DS-CDMA systems, a blind batch DC receiver called eigen-method receiver is proposed in

[19], and a corresponding blind adaptive algorithm is presented to determine the weight vector, which is used to combine the received signals from different carriers optimally to maximize the signal-to-noise ratio (SNR).

Subspace-based estimation techniques [15]-[18], which have already been explored for single carrier DS-CDMA system, have also been explored in MC-DS-CDMA systems to get the channel coefficients and timing of the desired user required to construct the MMSE detector [20].

Because subspace methods usually require singular value decomposition (SVD) or eigenvalue decomposition (EVD) of some form of the data correlation matrix, so they are computationally costly. Also, it is difficult to determine the accurate rank of the signal subspace (or the noise subspace) in a practically noisy environment.

Note that there is another kind of blind multiuser detection and channel estimation schemes, namely, linear prediction method that have already been used on DS-CDMA systems [28]-[31]. This method is to bypass the channel estimation and directly estimate a linear filter that can remove the ISI and MAI and /or suppress the additive noise. It is promising because linear prediction method avoids the channel estimation error and is more robust than those subspace methods.

In our thesis, we try to extend the use of linear prediction method from DS-CDMA systems to multicarrier CDMA system receiver design. And we focus on those systems without using CP or training, thus our receivers belong to the third category.

1.5 Overview

The organization of the research work described in this thesis is as follows.

Chapter 2 contains an asynchronous MC-CDMA system model that will be used throughout this thesis.

In Chapter 3, linear transformation and prediction of the received signal is developed.

We develop batch type algorithms for both zero-forcing (ZF) and minimum mean square error (MMSE) receiver in Chapter 4. Then the algorithms are modified to adaptive versions.

In Chapter 5, we apply the linear prediction method discussed in the former chapters on MC-DS-CDMA system.

Simulation result and comparison of our algorithms with a subspace method are presented in Chapter 6.

Chapter 7 contains the conclusion.

Chapter II

MC-CDMA Formulation

2.1 Notation

As a general notational convention, matrices and vectors will be in boldface. The symbols $E\{\cdot\}, (\cdot)^H, (\cdot)^*, (\cdot)^T, (\cdot)^+$ stand for expectation, Hermitian transposition, conjugatation, transposition, and pseudoinverse, respectively. The symbol **I** (**0**) stands for the identity matrix (the zero matrix or the zero vector) with a proper dimension.

2.2 MC-CDMA Model

Fig 2.1 is a block diagram of a discretized MC-CDMA system.

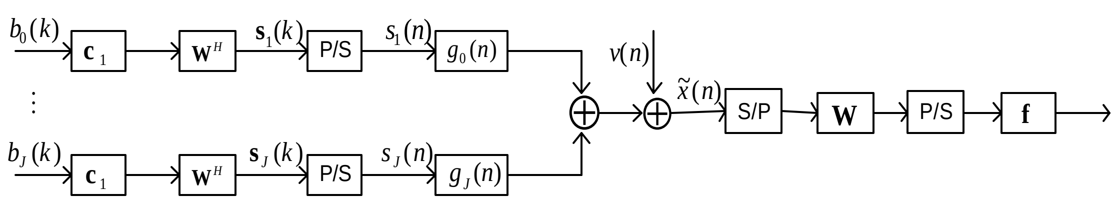


Figure 2.1: *Block diagram of MC-CDMA system.*

Assuming N_c equally-spaced subbands within the given spectrum, we consider an asynchronous MC-CDMA system with J simultaneous users, each of which modulates its signals onto the N_c subcarriers. $b_j(k)$ is the jth user's original data sequence. $\mathbf{c}_j = [c_j(0) \cdots c_j(L_c - 1)]^T$ is the jth user's spreading sequence, where L_c denotes the processing gain. In our system, we assume that the number of subcarriers and the processing gain are equal, i.e. $N_c = L_c$.

The $N_c \times 1$ symbol block for the jth user

$$\mathbf{s}_j(k) = b_j(k)\mathbf{W}^H \mathbf{c}_j \tag{2.1}$$

where $\mathbf{W}^H$ denotes the IFFT transform matrix.

Define $\tilde{\mathbf{c}}_j = \mathbf{W}^H \mathbf{c}_j$. After Parallel-to-Serial transformation, the symbol stream can be expressed as

$$s_j(n) = \sum_k b_j(k)\tilde{c}_j(n - kL_c) \tag{2.2}$$

where $b_j(k)$ is at the symbol rate $1/T_s$, and $\tilde{c}_j(k)$ and $s_j(n)$ are at the chip rate $1/T_c = L_c/T_s$. Furthermore, it is assumed that $\tilde{c}_j(k) = 0$ for $k < 0$ and $k > L_c - 1$.

The received signal before FFT transformation is

$$\tilde{x}(t) = \sum_{j=1}^{J}\sum_i \sum_k b_j(k)\tilde{c}_j(i - kL_c)g_j(t - iT_c - d_j T_c) + v(t) \tag{2.3}$$

where d_j is the random chip delay of user j. $v(t)$ is the Additive White Gaussian Noise with zero mean. $g_j(t)$ represents the multipath channel.

Sample at the chip rate, we get the discretized expression

$$\begin{aligned}
\tilde{x}(n) &= \sum_{j=1}^{J}\sum_i \sum_k b_j(k)\tilde{c}_j(i - kL_c)g_j(n - i - d_j) + v(n) \\
&= \sum_{j=1}^{J}\sum_k b_j(k)\sum_{i=0}^{Lc-1}\tilde{c}_j(i)g_j(n - kL_c - d_j - i) + v(n) \\
&= \sum_{j=1}^{J}\sum_k b_j(k)h_j(n - kL_c - d_j) + v(n)
\end{aligned} \tag{2.4}$$

$$h_j(n) = \sum_{i=0}^{Lc-1} \tilde{c}_j(i) g_j(n-i) \tag{2.5}$$

Define

$$h_{j,i}(n) = h_j(nL_c + L_c - i - 1), \qquad i = 0,1,\cdots, L_c - 1 \tag{2.6}$$

$$\tilde{x}_i(n) = \tilde{x}(nL_c + L_c - i - 1), \qquad i = 0,1,\cdots, L_c - 1 \tag{2.7}$$

Then,

$$\tilde{x}_i(n) = \sum_{j=1}^{J} \sum_k b_j(k) h_{j,i+d_j}(n-k) + v(n), \qquad i = 0,1,\cdots, L_c - 1 \tag{2.8}$$

where the subscript $i + d_j$ is in the modular L_c sense. The S/P operation is to stack up $\tilde{x}_i(n)$ for $i = 0,1,\cdots, L_c - 1$. This gives

$$\begin{bmatrix} \tilde{x}_{Lc-1}(n) \\ \vdots \\ \tilde{x}_1(n) \\ \tilde{x}_0(n) \end{bmatrix} = \sum_{j=1}^{J} \sum_k b_j(k) \begin{bmatrix} h_{j,d_j+L_c-1}(n-k) \\ \vdots \\ h_{j,d_j+1}(n-k) \\ h_{j,d_j}(n-k) \end{bmatrix} + \mathbf{v}(n) \tag{2.9}$$

Define

$$\tilde{\mathbf{x}}(n) = \begin{bmatrix} \tilde{x}_{Lc-1}(n) \\ \vdots \\ \tilde{x}_1(n) \\ \tilde{x}_0(n) \end{bmatrix} \tag{2.10}$$

$$\mathbf{h}_j(n) = \begin{bmatrix} h_{j,d_j+L_c-1}(n) \\ \vdots \\ h_{j,d_j+1}(n) \\ h_{j,d_j}(n) \end{bmatrix} \tag{2.11}$$

Then (2.9) can be written as

$$\tilde{\mathbf{x}}(n) = \sum_{j=1}^{J} \sum_{k} b_j(k) \mathbf{h}_j(n-k) + \mathbf{v}(n)$$

$$= \sum_{j=1}^{J} \sum_{k=0}^{L_h-1} b_j(n-k) \mathbf{h}_j(k) + \mathbf{v}(n) \quad (2.12)$$

where L_h is related to the length of $h_{j,i}(n)$ and the multiuser delay d_j.

Suppose the actual channel coefficients are from $g_j(0)$ to $g_j(L_g - 1)$, where L_g is the maximum length of all J channels. By (2.5) we have

$$L_h = \max \left\lfloor \frac{L_c + L_g - 1 + d_j}{L_c} \right\rfloor \quad , \quad \forall \; j \quad (2.12)$$

where $\left\lfloor \frac{a}{b} \right\rfloor$ is the quotient of a divided by b.

After the FFT transformation, the received $L_c \times 1$ signal vector is

$$\mathbf{x}(n) = \mathbf{W}\tilde{\mathbf{x}}(n) = \sum_{j=1}^{J} \sum_{k=0}^{L_h-1} b_j(n-k) \mathbf{W}\mathbf{h}_j(k) + \mathbf{W}\mathbf{v}(n) \quad (2.13)$$

Assume the smoothing factor is N, then define

$$\mathbf{X}(n) = \begin{bmatrix} \mathbf{x}(n) \\ \mathbf{x}(n-1) \\ \vdots \\ \mathbf{x}(n-N+1) \end{bmatrix} = \sum_{j}^{J} \mathbf{H}_j \mathbf{b}_j(n) + \mathbf{v}'(n) \quad (2.14)$$

where

$$\mathbf{H}_j = \begin{bmatrix} \mathbf{W}\mathbf{h}_j(0) & \cdots & \mathbf{W}\mathbf{h}_j(L_h-1) & & & \mathbf{0} \\ & \ddots & & \ddots & & \\ & & \ddots & & \ddots & \\ \mathbf{0} & & & \mathbf{W}\mathbf{h}_j(0) & \cdots & \mathbf{W}\mathbf{h}_j(L_h-1) \end{bmatrix} \quad (2.15)$$

$$\mathbf{b}_j(n) = \begin{bmatrix} b_j(n) \\ b_j(n-1) \\ \vdots \\ b_j(n-L_h-N+2) \end{bmatrix} \tag{2.16}$$

For simplicity, here we ignore noise at first. Then rewrite

$$\mathbf{X}(n) = \begin{bmatrix} \mathbf{H}_1 & \cdots & \mathbf{H}_J \end{bmatrix} \begin{bmatrix} \mathbf{b}_1(n) \\ \vdots \\ \mathbf{b}_J(n) \end{bmatrix} \tag{2.17}$$

Define

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_1 & \cdots & \mathbf{H}_J \end{bmatrix} \tag{2.18}$$

$\mathbf{H}$ is the channel matrix corresponding to $\mathbf{X}(n)$ of dimension $NL_c \times J(L_h + N - 1)$. Note that $\mathbf{H}$ should be of full column rank, for which a necessary condition is choosing N such that

$$NL_c \geq J(L_h + N - 1) \tag{2.19}$$

From (2.5), we have

$$\begin{bmatrix} h_j(0) \\ \vdots \\ h_j(L_c + L_g - 2) \end{bmatrix} = \begin{bmatrix} \tilde{c}_j(0) & & \\ \vdots & \ddots & \\ \vdots & & \tilde{c}_j(0) \\ \tilde{c}_j(L_c - 1) & & \vdots \\ & \ddots & \vdots \\ & & \tilde{c}_j(L_c - 1) \end{bmatrix} \begin{bmatrix} g_j(0) \\ \vdots \\ g_j(L_g - 1) \end{bmatrix}$$

$$= \tilde{\mathbf{C}}_j \mathbf{g}_j \tag{2.20}$$

Therefore

$$\mathbf{h}_j(k) = \begin{bmatrix} h_{j,d_j+lc-1}(k) \\ \vdots \\ h_{j,d_j+1}(k) \\ h_{j,d_j}(k) \end{bmatrix} = \begin{bmatrix} h_j(kL_c - d_j) \\ \\ h_j(kL_c + L_c - d_j - 2) \\ h_j(kL_c + L_c - d_j - 1) \end{bmatrix}$$

$$= \tilde{\mathbf{C}}_j(kL_c + 1 - d_j : kL_c + L_c - d_j, :) \quad \mathbf{g}_j$$

$$= \tilde{\mathbf{C}}_j(k)\mathbf{g}_j \qquad (2.21)$$

Here we use MATLAB representation to denote $\tilde{\mathbf{C}}_j(k)$ as the submatrix of $\tilde{\mathbf{C}}_j$ from row $kL_c + 1 - d_j$ to $kL_c + L_c - d_j$.

Then, the channel matrix $\mathbf{H}_j$ corresponding to user j can be written as

$$\mathbf{H}_j = \begin{bmatrix} \mathbf{W}\tilde{\mathbf{C}}_j(0) & \cdots & \mathbf{W}\tilde{\mathbf{C}}_j(L_h - 1) & \ddots & & \mathbf{0} \\ & \ddots & & & \ddots & \\ & & \ddots & & & \\ \mathbf{0} & & & \mathbf{W}\tilde{\mathbf{C}}_j(0) & \cdots & \mathbf{W}\tilde{\mathbf{C}}_j(L_h - 1) \end{bmatrix} \begin{bmatrix} \mathbf{g}_j & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \mathbf{g}_j \end{bmatrix}$$

(2.22)

Chapter III

Channel Vector Space Separation for MC-CDMA

The linear prediction method had already been used in joint blind multiuser detection and blind channel estimation/equalization [28]-[31]. In this chapter, we develop the new algorithm for MC-CDMA detector based on the approaches of [28]-[31]. By linear transformation and then linear prediction on the defined data vector $\mathbf{X}(n)$, we aim at extracting the desired signal from the mixed signal received. Based on that, we can achieve ZF and MMSE detectors in the next chapter.

3.1 Linear Transformation

Consider the asynchronous MC-CDMA system formulated in Chapter II. Without loss of generality, assume user 1 is the desired user and we want to detect $b_1(n-d_f)$ from $\mathbf{X}(n)$. The timing of user 1 is assumed to be known through timing recovery. Thus, let $d_1 = 0$ for simplicity, while other $d_j, j \neq 1$ are unknown and are randomly distributed between 0 and $L_c - 1$. If the delays are larger than L_c, it will have the same effect as delaying the symbol streams by an integer multiple of the symbol duration plus a d_j that is less than $L_c - 1$. Delays of integer multiple of the symbol duration will have no effect on the receiver performance. Also, the equalizer delay d_f is between 0 and $L_h + N - 1$. Since choosing the middle value usually has better performance, we can, for example, let $d_f = \lfloor (L_h + N)/2 \rfloor$.

We construct a data vector as

$$\mathbf{Y}_1(n) = \begin{bmatrix} \mathbf{x}(n - d_f + L_h - 1) \\ \vdots \\ \mathbf{x}(n - d_f) \end{bmatrix} \tag{3.1}$$

Proposition 1: There exits a full row rank $L_c \times L_h L_c$ matrix $\mathbf{T}$ such that $\mathbf{TY}_1(n)$ does not contain $b_1(n - d_f)$.

Proof: From (2.15) (2.16) and (3.1), $\mathbf{Y}_1(n)$ can be written as

$$\mathbf{Y}_1(n) = \sum_{j=1}^{J} \begin{bmatrix} \mathbf{Wh}_j(0) & \cdots & \mathbf{Wh}_j(L_h - 1) & & \mathbf{0} \\ & \ddots & & \ddots & \\ \mathbf{0} & & \mathbf{Wh}_j(0) & \cdots & \mathbf{Wh}_j(L_h - 1) \end{bmatrix} \begin{bmatrix} b_j(n - d_f + L_h - 1) \\ \vdots \\ b_j(n - d_f) \\ \vdots \\ b_j(n - d_f - L_h - 1) \end{bmatrix} \tag{3.2}$$

In the channel matrix corresponding to user 1, the column vector corresponding to $b_1(n - d_f)$ in $\mathbf{Y}_1(n)$ is

$$\begin{bmatrix} \mathbf{Wh}_1(L_h - 1) \\ \vdots \\ \mathbf{Wh}_1(0) \end{bmatrix} = \begin{bmatrix} \mathbf{W}\tilde{\mathbf{C}}_1(L_h - 1)\mathbf{g}_1 \\ \vdots \\ \mathbf{W}\tilde{\mathbf{C}}_1(0)\mathbf{g}_1 \end{bmatrix} = \mathbf{C}_1\mathbf{g}_1 \tag{3.3}$$

where the matrix $\mathbf{C}_1$ has dimension $L_h L_c \times L_g$. Choosing $\mathbf{T}$ as the left singular vectors of $\mathbf{C}_1$ corresponding to its zero singular values, we get

$$\mathbf{T}\begin{bmatrix}\mathbf{Wh}_1(L_h-1)\\ \vdots \\ \mathbf{Wh}_1(0)\end{bmatrix}=\mathbf{TC}_1\mathbf{g}_1=\mathbf{0} \tag{3.4}$$

From (2.12), we have $L_hL_c-L_g>L_c$, which guarantees the full row rank of $\mathbf{T}$.

?

Since in our algorithm the desired user's spreading code is known, $\mathbf{T}$ can be computed off-line. Furthermore, $\mathbf{TY}_1(n)$ does not completely cancel other symbol components because different code matrices do not have identical null space.

Using the transformed data vector $\mathbf{TY}_1(n)$, we construct a new data vector

$$\mathbf{Y}_2(n)=\begin{bmatrix}\mathbf{x}(n-d_f+L_h-1+M_1)\\ \mathbf{x}(n-d_f+L_h-1+1)\\ \mathbf{TY}_1(n)\\ \mathbf{x}(n-d_f-1)\\ \mathbf{x}(n-d_f-M_2)\end{bmatrix} \tag{3.5}$$

where M_1 and M_2 satisfy

$$M_1L_c\geq J(M_1+L_h-1)\qquad \text{and} \qquad M_2L_c\geq J(M_2+L_h-1) \tag{3.6}$$

Similar to $\mathbf{X}(n)$, there is a channel matrix corresponding to $\mathbf{Y}_2(n)$, i.e.

$$\begin{bmatrix}\mathbf{H}_{M1} & & \\ \mathbf{H}_{T1} & \mathbf{0} & \mathbf{H}_{T2}\\ & & \mathbf{H}_{M2}\end{bmatrix} \tag{3.7}$$

where $\mathbf{H}_{M1}$ and $\mathbf{H}_{M2}$ are channel matrices corresponding to $\left[\mathbf{x}^{\mathrm{H}}(n-d_f+L_h-1+M_1)\ \cdots\ \mathbf{x}^{\mathrm{H}}(n-d_f+L_h-1+1)\right]^H$ and $\left[\mathbf{x}^{\mathrm{H}}(n-d_f-1)\ \cdots\ \mathbf{x}^{\mathrm{H}}(n-d_f-M_2)\right]^H$ with dimension $M_1L_c\times J(M_1+L_h-1)$ and $M_1L_c\times J(M_1+L_h-1)$. $[\mathbf{H}_{T1}\quad \mathbf{0}\quad \mathbf{H}_{T2}]$ is the channel matrix corresponding to $\mathbf{TY}_1(n)$, with $\mathbf{0}$ being a zero column vector.

3.2 Linear Prediction

In this section, we extract $b_1(n-d_f)$ part of $\mathbf{X}(n)$ using linear prediction. Considering that $\mathbf{X}(n)$ contains $b_1(n-d_f)$ while $\mathbf{Y}_2(n)$ does not, we define the following linear prediction problem:

$$e_1(n) = \mathbf{X}(n) - \mathbf{P}\mathbf{Y}_2(n) \tag{3.8}$$

where $\mathbf{P}$ has dimension $NL_c \times ML_c$ with $M = M_1 + M_2 + 1$. Assume that the original data symbols $b_j(n)$ are uncorrelated in time and that $\mathbf{b}_1(n), \ \cdots, \ \mathbf{b}_J(n)$ are mutually uncorrelated with variances (powers) $s_1^2, \ \cdots, \ s_J^2$. Define

$$\mathbf{X}(n) = \overline{\mathbf{h}}_1 b_1(n-d_f) + \overline{\mathbf{H}}_1 \overline{\mathbf{b}}_1(n) \tag{3.9}$$

where $\overline{\mathbf{b}}_1(n)$ contains all symbol components in $\begin{bmatrix}\mathbf{b}_1^H(n) & \cdots & \mathbf{b}_J^H(n)\end{bmatrix}^H$ except for $b_1(n-d_f)$. $\overline{\mathbf{h}}_1$ is the column vector in the channel matrix $\mathbf{H}$ corresponding to $b_1(n-d_f)$, whereas all the other columns of $\mathbf{H}$ comprise $\overline{\mathbf{H}}_1$.

Proposition 2: the optimal linear prediction matrix $\mathbf{P}$ gives

$$e_1(n) = \overline{\mathbf{h}}_1 b_1(n-d_f) \tag{3.10}$$

Proof: Let $\tilde{\mathbf{H}}$ denote the channel matrix obtained by deleting the zero column in the channel matrix of $\mathbf{Y}_2(n)$ in (3.7), i.e.

$$\tilde{\mathbf{H}} = \begin{bmatrix} \mathbf{H}_{M1} & \\ \mathbf{H}_{T1} & \mathbf{H}_{T2} \\ & \mathbf{H}_{M2} \end{bmatrix} \tag{3.11}$$

The assumption that $\mathbf{H}$ in (2.18) is full column rank guarantees that $\mathbf{H}_{M1}$ and $\mathbf{H}_{M2}$ are both full column rank under (3.6). Therefore, $\tilde{\mathbf{H}}$ is also full column rank. Rewrite

$\mathbf{Y}_2(n) = \tilde{\mathbf{H}}\tilde{\mathbf{b}}_2(n)$, where $\tilde{\mathbf{b}}_2(n)$ does not contain $b_1(n-d_f)$ due to the structure of $\mathbf{Y}_2(n)$ and $\tilde{\mathbf{H}}$. Then $e_1(n)$ can be written as

$$e_1(n) = \bar{\mathbf{h}}_1 b_1(n-d_f) + ([\mathbf{0} \quad \bar{\mathbf{H}}_1 \quad \mathbf{0}] - \mathbf{P}\tilde{\mathbf{H}})\tilde{\mathbf{b}}_2(n) \qquad (3.12)$$

where the $\mathbf{0}$ matrices are with proper dimensions due to the fact that $\bar{\mathbf{b}}_1(n) \in \tilde{\mathbf{b}}_2(n)$. Then

$$E\{e_1(n)e_1^H(n)\} = \bar{\mathbf{h}}_1 s_1^2 \bar{\mathbf{h}}_1^H + ([\mathbf{0} \quad \bar{\mathbf{H}}_\mathbf{1} \quad \mathbf{0}] - \mathbf{P}\tilde{\mathbf{H}}) diag\{s_j^2\}([\mathbf{0} \quad \bar{\mathbf{H}}_\mathbf{1} \quad \mathbf{0}] - \mathbf{P}\tilde{\mathbf{H}})^H$$

(3.13)

Minimizing $E\{e_1(n)e_1^H(n)\}$ over $\mathbf{P}$ then gives

$$\tilde{\mathbf{H}} diag\{s_j^2\}([\mathbf{0} \quad \bar{\mathbf{H}}_1 \quad \mathbf{0}] - \mathbf{P}\tilde{\mathbf{H}})^H = \mathbf{0} \qquad (3.14)$$

Because $\tilde{\mathbf{H}}$ is full column rank, we have

$$([\mathbf{0} \quad \bar{\mathbf{H}}_1 \quad \mathbf{0}] - \mathbf{P}\tilde{\mathbf{H}}) = \mathbf{0} \qquad (3.15)$$

Hence, from (3.11) we obtain $e_1(n) = \bar{\mathbf{h}}_1 b_1(n-d_f)$.

?

Define

$$e_2(n) = \mathbf{X}(n) - e_1(n) \qquad (3.16)$$

Then, from (3.9) and (3.10), we have

$$e_2(n) = \bar{\mathbf{H}}_1 \bar{\mathbf{b}}_1(n) \qquad (3.17)$$

Hence, we separate the column vector subspace of the desired signal from the channel matrix vector space of the data vector $\mathbf{X}(n)$.

Also, we will see that the solution of $\mathbf{P}$ and linear prediction errors can be explicitly represented by the data correlations. Recall the linear prediction problem (3.8), it can be rewritten as:

$$e_1(n) = [\mathbf{I} \quad -\mathbf{P}] \begin{bmatrix} \mathbf{X}(n) \\ \mathbf{Y}_2(n) \end{bmatrix} \tag{3.18}$$

Define

$$\mathbf{R} = E\{\begin{bmatrix} \mathbf{X}(n) \\ \mathbf{Y}_2(n) \end{bmatrix} [\mathbf{X}^H(n) \quad \mathbf{Y}_2^H(n)]\}$$

$$= \begin{bmatrix} \mathbf{R}_{11} & \mathbf{R}_{12} \\ \mathbf{R}_{21} & \mathbf{R}_{22} \end{bmatrix} \tag{3.19}$$

It is well known that the least squares solution for the linear prediction problem (3.8) [29] is

$$\mathbf{P} = \mathbf{R}_{12}\mathbf{R}_{22}^{+} \tag{3.20}$$

Also, we can have

$$E\{e_1(n)e_1^H(n)\} = \overline{\mathbf{h}}_1 s_1^2 \overline{\mathbf{h}}_1^H = \mathbf{R}_{11} - \mathbf{R}_{12}\mathbf{R}_{22}^{+}\mathbf{R}_{21} \tag{3.21}$$

$$E\{e_2(n)e_2^H(n)\} = \overline{\mathbf{H}}_1 diag\{s_j^2\}\overline{\mathbf{H}}_1^H = \mathbf{R}_{12}\mathbf{R}_{22}^{+}\mathbf{R}_{21} \tag{3.22}$$

Chapter IV

MC-CDMA Receiver Design

In this chapter, we are concerned with receiver design for the MC-CDMA system based on the result of the last chapter. We focus on the blind multiuser zero-forcing and MMSE detectors without explicit channel estimation.

4.1 Batch Zero-Forcing Detector

A zero-forcing detector $\mathbf{f}$ with delay d_f and dimension $NL_c \times 1$ must satisfy

$$\mathbf{f}^H \mathbf{X}(n) = \mathbf{f}^H \overline{\mathbf{h}}_1 b_1(n-d_f) + \mathbf{f}^H \overline{\mathbf{H}}_1 \overline{\mathbf{b}}_1(n) = b_1(n-d_f) \tag{4.1}$$

Therefore, we need

$$\mathbf{f}^H [\overline{\mathbf{h}}_1 \quad \overline{\mathbf{H}}_1] = [1 \quad \mathbf{0}^H] \tag{4.2}$$

Note again that for (4.2) to have an exact solution, it is necessary that $[\overline{\mathbf{h}}_1 \quad \overline{\mathbf{H}}_1] = \mathbf{H}$ should be full column rank, i.e., (2.l9) should be satisfied when choosing the detector length.

According to (3.21) and (3.22), $\mathbf{f}^H \overline{\mathbf{h}}_1 \neq 0$ iff

$$s_1^2 \mathbf{f}^H \overline{\mathbf{h}}_1 \overline{\mathbf{h}}_1^H = \mathbf{f}^H (\mathbf{R}_{11} - \mathbf{R}_{12} \mathbf{R}_{22}^+ \mathbf{R}_{21}) \neq \mathbf{0} \tag{4.3}$$

and $\mathbf{f}^H \overline{\mathbf{H}}_1 = \mathbf{0}$ iff

$$\mathbf{f}^H \overline{\mathbf{H}}_1 diag\{s_j^2\} \overline{\mathbf{H}}_1^H = \mathbf{f}^H \mathbf{R}_{12} \mathbf{R}_{22}^+ \mathbf{R}_{21} = \mathbf{0} \tag{4.4}$$

Theoretically, $\mathbf{R}_{11} - \mathbf{R}_{12}\mathbf{R}_{22}^{+}\mathbf{R}_{21}$ is with rank 1 due to (3.21). Let $\mathbf{u}$ be its left singular vector corresponding to its nonzero singular value; then, $\mathbf{u}$ is an estimation of $\bar{\mathbf{h}}_1$ and $\mathbf{f}^H\bar{\mathbf{h}}_1 \neq 0$ iff $\mathbf{f}^H\mathbf{u} \neq 0$.

Define

$$\mathbf{B} = [\mathbf{u} \quad \mathbf{R}_{12}\mathbf{R}_{22}^{+}\mathbf{R}_{21}] \tag{4.5}$$

Then we require

$$\mathbf{f}^H\mathbf{B} = [1 \quad \mathbf{0}] \tag{4.6}$$

Hence

$$\mathbf{f}^H = [1 \quad \mathbf{0}]\mathbf{B}^{+} \tag{4.7}$$

The discussion above is under the noiseless assumption. In the noise case, we just need to modify the correlation matrix estimation as

$$\hat{\mathbf{R}} = \mathbf{R} - s_v^2\mathbf{I} \tag{4.8}$$

In case when the noise power is not known, an SVD of $\mathbf{R}$ can be used to find the signal subspace and noise subspace to estimate the noise power.

4.2 Batch MMSE Detector

It is well known that zero-forcing detector may enhance the noise part while performing equalization. Thus, if noise is large, we prefer linear optimum MMSE detector. In this section, we develop a batch algorithm for the MMSE detector.

The MMSE detector $\mathbf{m}$ with delay d_f and dimension $NL_c \times 1$ are designed to minimize the mean square error

$$E\{\left|b_1(n-d_f)-\mathbf{m}^H\mathbf{X}(n)\right|^2\} \tag{4.9}$$

The Wiener solution [24] is

$$\mathbf{m}=\mathbf{R}_{11}^{-1}\mathbf{p} \tag{4.10}$$

where $\mathbf{p}=E\{\hat{b}_1(n-d_f)\mathbf{X}(n)\}=s_1^2\overline{\mathbf{h}}_1$ due to (3.9), thus

$$\mathbf{m}=s_1^2\mathbf{R}_{11}^{-1}\overline{\mathbf{h}}_1 \tag{4.11}$$

Since $\mathbf{u}$ can be an estimation of $\overline{\mathbf{h}}_1$, the detector is

$$\mathbf{m}=s_1^2\mathbf{R}_{11}^{-1}\mathbf{u} \tag{4.12}$$

4.3 Adaptive Detectors

In case the channel is time varying, we can develop an adaptive algorithm.

First, we consider the Zero-forcing detector. We can solve the following minimization problem to find $\mathbf{f}$

$$\min_{\mathbf{f}} J(n)=\left\|\mathbf{f}^H e_2(n)\right\|^2 \qquad \text{subject to} \quad \mathbf{f}^H e_1(n)\neq 0 \tag{4.13}$$

From (3.17), $\mathbf{u}$ is the left singular vector of $\mathbf{R}_{e_1}=E\{e_1(n)e_1^H(n)\}$ corresponding to the nonzero singular value. Thus, $\mathbf{f}$ can be estimated by optimizing

$$\min_{\mathbf{f}} J(n)=\left\|\mathbf{f}^H e_2(n)\right\|^2 \qquad \text{subject to} \quad \mathbf{f}^H\mathbf{u}=1 \tag{4.14}$$

Applying the Frost algorithm in the array signal processing literature [23] on this constrained adaptive optimization, the adaptation of $\mathbf{f}$ is

$$\mathbf{f}(n+1)=\mathbf{u}(\mathbf{u}^H\mathbf{u})^{-1}+[\mathbf{I}-\mathbf{u}(\mathbf{u}^H\mathbf{u})^{-1}\mathbf{u}^H][\mathbf{f}(n)-me_2(n)e_2^H(n)\mathbf{f}(n)] \tag{4.15}$$

where $\mathbf{u}$ can be estimated adaptively as the column of $\mathbf{R}_{e_1}$ with the largest norm and $\mathbf{R}_{e_1}(n+1)=(1-l)\mathbf{R}_{e_1}(n)+l e_1(n)e_1^H(n)$ with some forgetting factor l.

Now, consider the MMSE detector. Note that in solving the minimization problem of (4.9), there are no constraints imposed on the solution. Now we consider minimizing the average output power of the detector while the desired signal part in the output of the detector is constrained to a constant, that is

$$\min_{\mathbf{m}} J(n) = \left\| \mathbf{m}^H \mathbf{X}(n) \right\|^2 = \mathbf{m}^H \mathbf{R}_{11} \mathbf{m} \quad \text{subject to} \quad \mathbf{m}^H \mathbf{u} = 1 \tag{4.16}$$

Then, using the method of Lagrange optimization [24], we get

$$\mathbf{m} = (\mathbf{u}^H \mathbf{R}_{11}^{-1} \mathbf{u})^{-1} \mathbf{R}_{11}^{-1} \mathbf{u} \tag{4.17}$$

Comparing (4.17) with the MMSE detetor (4.12), we find that the only difference is a scalar factor. Thus, we can also obtain MMSE detector through the optimization problem (4.16).

Therefore, we consider MMSE adaptive detector/equalizer by solving the constrained optimization of (4.16). Just as the zero-forcing detector, the adaptation of $\mathbf{m}$ by the Frost algorithm is

$$\mathbf{m}(n+1) = \mathbf{u}(\mathbf{u}^H \mathbf{u})^{-1} + [\mathbf{I} - \mathbf{u}(\mathbf{u}^H \mathbf{u})^{-1} \mathbf{u}^H][\mathbf{m}(n) - m\mathbf{X}(n)\mathbf{X}^H(n)\mathbf{m}(n)] \tag{4.18}$$

4.4 Computational Complexity

In the batch algorithm, the expectation of the data correlation matrix (3.18) can be implemented by average in time. Because of Hermitian symmetry of the correlation matrix, its computational complexity can be reduced to $O(20N_d NL_c^2)$ real operations, where N_d is the total number of original data symbols used in simulation and N is the smoothing factor. In order to estimate $\mathbf{u}$, we need to calculate $\mathbf{R}_{11} - \mathbf{R}_{12}\mathbf{R}_{22}^{+}\mathbf{R}_{21}$ first, where the computation of $\mathbf{R}_{22}^{+}$ can be implemented by recursively estimating $\mathbf{R}_{22}^{+}$ directly from data based on matrix inversion lemma ($O(16N^2L_c^2)$ per iteration) instead of computing the pseudoinverse ($O(4N^3L_c^3)$). The matrix multiplications of $\mathbf{R}_{12}\mathbf{R}_{22}^{+}\mathbf{R}_{21}$ are in the order of $O(8N^3L_c^3)$. Because matrix $(\mathbf{R}_{11}-\mathbf{R}_{12}\mathbf{R}_{22}^{+}\mathbf{R}_{21})$ is rank 1, each column of it is an estimation of $\bar{\mathbf{h}}_1$ up to a constant factor. Therefore, we can simply choose $\mathbf{u}$ as the average of all columns or as the column with the largest norm, instead of performing the SVD. Therefore, the computation of our batch algorithm is in the order of $O(20N_dNL_c^2+12N^3L_c^3)$.

In terms of the same smoothing factor N, we compare our batch algorithms with the subspace algorithm, in which the computation of the expectation of the data correlation matrix is in the order of $O(4N_dNL_c^2)$. And because of the eigen decompositions used to find the noise subspace and to find the estimation of the channel, there are computations in the order of $O(8N^3L_c^3)$. Thus, the total computation of the subspace method is in the order of $O(4N_dNL_c^2+8N^3L_c^3)$.

Batch Algorithm	Correlation Matrix Calculation	Other Computations
Linear Prediction Algorithm	$O(20N_dNL_c^2)$	$O(12N^3L_c^3)$
Subspace Algorithm	$O(4N_dNL_c^2)$	$O(8N^3L_c^3)$

TABLE 1: *Computations of the batch algorithm and the subspace algorithm.*

Table I lists the computations used in the linear prediction batch algorithm and the subspace algorithm.

Since N_d is much larger than N or L_c, which is in the order of thousand, even tens of thousand, we can see that our batch algorithm requires slightly more computation than the subspace algorithm. But it is well known that the subspace algorithm cannot be implemented adaptively, while our algorithms can.

In our adaptive algorithm, in each recursion

$$\mathbf{R}_{e_1}(n+1) = (1-l)\mathbf{R}_{e_1}(n) + l e_1(n) e_1^H(n) \tag{4.19}$$

the updating of $e_1(n)$ (3.8) needs $O(4MNL_c^2)$ computations. And because we need only one column of $\mathbf{R}_{e_1}(n+1)$ with the largest norm to estimate $\mathbf{u}(n)$, updating just one column can save the whole matrix updating of $\mathbf{R}_{e_1}(n+1)$. So the computation is reduced to $O(4NL_c)$. The updating of detectors (4.15) (or (4.19)) needs computation of $O(4N^2L_c^2)$. Thus the total computation of the adaptive algorithm is on the order of $O((4N^2+4MN)L_c^2)$ per iteration.

Chapter V

MC-DS-CDMA Formulation

Our new algorithm based on linear prediction can also be applied on the MC-DS-CDMA system.

5.1 System Model

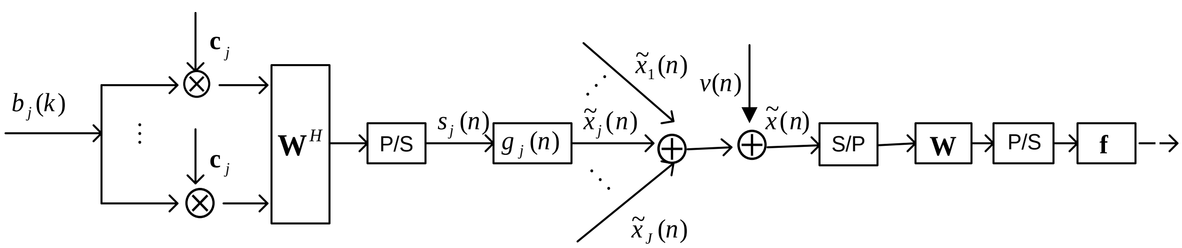


Figure 5.1: *Block diagram of MC-DS-CDMA system*.

Fig 5.1 is a discretized MC-DS-CDMA system.

Different from the MC-DS-CDMA system model we mentioned in section 1.3.2, the jth user's original data sequence $b_j(k)$ is copied onto N_c subcarriers (instead of being serial-to-parallel converted) before being spread in the time domain. The number of subcarriers N_c here no longer needs to equal to the processing gain L_c, and actually it is much less than L_c.

Actually, our model is a modified version of that we mentioned in section 1.3.2, by adding a copier on each of the data streams after the serial-to-parallel converter [6]. Note that in our model, the same data symbol modulates N_c subcarriers, so even when the received signals on some of the subcarriers are badly destroyed, as the system in Section 1.3.2 might be, we still can detect out the right one according to the received signals on other subcarriers. For simplicity, we omitted the serial-to-parallel converter here.

Define the MC-DS-CDMA signal after the IFFT block ($\mathbf{W}^H$) of Figure 5.1 as

$$\mathbf{s}_j(k,l_c)=\begin{bmatrix} s_j(kL_cN_c+0+l_cN_c)\\ s_j(kL_cN_c+1+l_cN_c)\\ \vdots \\ s_j(kL_cN_c+N_c-1+l_cN_c)\end{bmatrix}$$

$$=b_j(k)\mathbf{W}^H\begin{bmatrix} c_j(l_c)\\ \vdots \\ c_j(l_c)\end{bmatrix}, \quad l_c=0,1,\cdots,L_c-1 \tag{5.1}$$

After parallel-to-serial conversion, the symbol stream now is

$$s_j(kL_cN_c+p+l_cN_c)=b_j(k)c_j(l_c)\sum_{l=0}^{N_c-1}\exp\{j2p\frac{lp}{N_c}\},\quad p=0,1,\cdots N_c-1 \tag{5.2}$$

Let $n=kL_cN_c+p+l_cN_c$, then

$$s_j(n)=\sum_k c_j(\left\lfloor\frac{n-kL_cN_c}{N_c}\right\rfloor)\sum_{l=0}^{N_c-1}\exp\{j2p\frac{l\bmod_{N_c}(n-kL_cN_c)}{N_c}\}\cdot b_j(k) \tag{5.3}$$

The received signal before FFT transformation is

$$\tilde{x}(n)=\sum_{j=1}^{J}\sum_i\sum_k c_j(\left\lfloor\frac{i-kL_cN_c}{N_c}\right\rfloor)\sum_{l=0}^{N_c-1}\exp\{j2p\frac{l\bmod_{N_c}(i-kL_cN_c)}{N_c}\}b_j(k)$$

$$\cdot g_j(n-i-d_j)+v(n)$$

$$=\sum_{j=1}^{J}\sum_k b_j(k)\sum_{i=0}^{L_cN_c-1}c_j(\left\lfloor\frac{i}{N_c}\right\rfloor)\sum_{l=0}^{N_c-1}\exp\{j2p\frac{l\bmod_{N_c}(i)}{N_c}\}$$

$$\cdot g_j(n-i-kL_cN_c-d_j)+v(n)$$

$$= \sum_{j=1}^{J} \sum_{k} b_j(k) \sum_{l=0}^{N_c-1} \sum_{i=0}^{L_c N_c-1} c_j\left(\left\lfloor \frac{i}{N_c} \right\rfloor\right) \exp\{ j2p \frac{l \bmod_{N_c}(i)}{N_c} \}$$

$$\cdot g_j(n - i - kL_c N_c - d_j) + v(n)$$

$$= \sum_{j=1}^{J} \sum_{k} b_j(k) \sum_{l=0}^{N_c-1} h_j(n - kL_c N_c - d_j, l) + v(n)$$

$$= \sum_{j=1}^{J} \sum_{k} \hat{\mathbf{h}}_j^T (n - kL_c N_c - d_j) \hat{\mathbf{b}}_j(k) + v(n) \tag{5.4}$$

where

$$h_j(n,l) = \sum_{i=0}^{L_c N_c-1} c_j\left(\left\lfloor \frac{i}{N_c} \right\rfloor\right) \exp\{ j2p \frac{l \bmod_{N_c}(i)}{N_c} \} g_j(n-i) \tag{5.5}$$

$$\hat{\mathbf{h}}_j(n) = [h_j(n,0) \quad h_j(n,1) \quad \cdots \quad h_j(n, N_c - 1)]^T$$

$$= \sum_{i=0}^{L_c N_c-1} [c_j\left(\left\lfloor \frac{i}{N_c} \right\rfloor\right) \exp\{ j2p \frac{0 \bmod_{N_c}(i)}{N_c} \} \quad \cdots$$

$$c_j\left(\left\lfloor \frac{i}{N_c} \right\rfloor\right) \exp\{ j2p \frac{(N_c - 1) \bmod_{N_c}(i)}{N_c} \}]^T g_j(n-i)$$

$$\overset{def.}{=} \sum_{i=0}^{L_c N_c-1} \tilde{\mathbf{c}}_j(i) g_j(n-i) \tag{5.6}$$

$$\hat{\mathbf{b}}_j(k) = [b_j(k) \quad b_j(k) \quad \cdots \quad b_j(k)]^T \tag{5.7}$$

Define

$$\hat{\mathbf{h}}_{j,i}(n) = \hat{\mathbf{h}}_j(nL_c N_c + L_c N_c - i + 1), \qquad i = 0,1,\cdots, L_c N_c - 1 \tag{5.8}$$

$$\tilde{x}_i(n) = \tilde{x}(nL_c N_c + L_c N_c - i + 1), \qquad i = 0,1,\cdots, L_c N_c - 1 \tag{5.9}$$

Then

$$\tilde{x}_i(n) = \sum_{j=1}^{J} \sum_{k} \hat{\mathbf{h}}_{j,i+d_j}^T (n-k) \hat{\mathbf{b}}_j(k) + v(n) \qquad i = 0,1,\cdots, L_c N_c - 1 \tag{5.10}$$

where the subscript $i+d_j$ is in the modular $L_c N_c$ sense. Stack up $\tilde{x}_i(n)$ for $i = 0,1,\cdots, L_c N_c - 1$ to obtain

$$\begin{bmatrix} \tilde{x}_{LcN_c-1}(n) \\ \vdots \\ \tilde{x}_1(n) \\ \tilde{x}_0(n) \end{bmatrix} = \sum_{j=1}^{J} \sum_{k} \begin{bmatrix} \hat{\mathbf{h}}^T_{j,d_j+L_cN_c-1}(n-k) \\ \vdots \\ \hat{\mathbf{h}}^T_{j,d_j+1}(n-k) \\ \hat{\mathbf{h}}^T_{j,d_j}(n-k) \end{bmatrix} \hat{\mathbf{b}}_j(k) + \mathbf{v}(n) \tag{5.11}$$

Define

$$\tilde{\mathbf{x}}(n) = \begin{bmatrix} \tilde{x}_{L_cN_c-1}(n) \\ \vdots \\ \tilde{x}_1(n) \\ \tilde{x}_0(n) \end{bmatrix} \tag{5.12}$$

$$\hat{\mathbf{H}}_j(n) = \begin{bmatrix} \hat{\mathbf{h}}^T_{j,d_j+L_cN_c-1}(n) \\ \vdots \\ \hat{\mathbf{h}}^T_{j,d_j+1}(n) \\ \hat{\mathbf{h}}^T_{j,d_j}(n) \end{bmatrix} \tag{5.13}$$

Then (5.11) can be written as

$$\begin{aligned} \tilde{\mathbf{x}}(n) &= \sum_{j=1}^{J} \sum_{k} \hat{\mathbf{H}}_j(n-k)\hat{\mathbf{b}}_j(k) + \mathbf{v}(n) \\ &= \sum_{j=1}^{J} \sum_{k=0}^{L_h-1} \hat{\mathbf{H}}_j(k)\hat{\mathbf{b}}_j(n-k) + \mathbf{v}(n) \end{aligned} \tag{5.14}$$

where L_h is related to the length of $\hat{\mathbf{h}}^T_{j,i}(n)$ and the multiuser delay d_j.

Suppose the actual channel coefficients are from $g_j(0)$ to $g_j(L_g - 1)$, where L_g is the maximum length of all J channels. By (5.3), we have

$$L_h = \max\left\lfloor \frac{L_c N_c + L_g - 1 + d_j}{L_c N_c} \right\rfloor, \quad \forall \ \mathrm{j} \tag{5.15}$$

After FFT transformation, the received $L_c N_c \times 1$ signal vector is

$$\mathbf{x}(n) = \tilde{\mathbf{W}}\tilde{\mathbf{x}}(n) = \sum_{j=1}^{J}\sum_{k=0}^{L_h-1} \tilde{\mathbf{W}}\hat{\mathbf{H}}_j(k)\hat{\mathbf{b}}_j(n-k) + \tilde{\mathbf{W}}\mathbf{v}(n) \tag{5.16}$$

where we define $L_c N_c \times L_c N_c$ matrix

$$\tilde{\mathbf{W}} = \begin{bmatrix} \mathbf{W} & & 0 \\ & \ddots & \\ 0 & & \mathbf{W} \end{bmatrix} \tag{5.17}$$

$\mathbf{W}$ is the $N_c \times N_c$ FFT transform matrix.

Assume the smoothing factor is N, then define

$$\mathbf{X}(n) = \begin{bmatrix} \mathbf{x}(n) \\ \mathbf{x}(n-1) \\ \vdots \\ \mathbf{x}(n-N+1) \end{bmatrix} = \sum_{j=1}^{J} \mathbf{H}_j \mathbf{b}_j(n) + \mathbf{v}'(n) \tag{5.18}$$

where

$$\mathbf{H}_j = \begin{bmatrix} \tilde{\mathbf{W}}\hat{\mathbf{H}}_j(0) & \cdots & \tilde{\mathbf{W}}\hat{\mathbf{H}}_j(L_h-1) & & & \mathbf{0} \\ & \ddots & & \ddots & & \\ & & \ddots & & \ddots & \\ \mathbf{0} & & & \tilde{\mathbf{W}}\hat{\mathbf{H}}_j(0) & \cdots & \tilde{\mathbf{W}}\hat{\mathbf{H}}_j(L_h-1) \end{bmatrix} \tag{5.19}$$

$$\mathbf{b}_j(n) = \begin{bmatrix} \hat{\mathbf{b}}_j(n) \\ \hat{\mathbf{b}}_j(n-1) \\ \vdots \\ \hat{\mathbf{b}}_j(n-L_h-N+2) \end{bmatrix} \tag{5.20}$$

For simplicity, here we ignore noise at first. Then rewrite

$$\mathbf{X}(n) = \begin{bmatrix} \mathbf{H}_1 & \cdots & \mathbf{H}_J \end{bmatrix} \begin{bmatrix} \mathbf{b}_1(n) \\ \vdots \\ \mathbf{b}_J(n) \end{bmatrix} \tag{5.21}$$

Define

$$\mathbf{H} = [\mathbf{H}_1 \quad \cdots \quad \mathbf{H}_J] \tag{5.22}$$

$\mathbf{H}$ is the channel matrix corresponding to $\mathbf{X}(n)$ of dimension $NL_cN_c \times JN_c(L_h + N - 1)$.

It is the same as in MC-CDMA system that $\mathbf{H}$ should be full column rank, for which a necessary condition is choosing N such that

$$NL_cN_c \geq JN_c(L_h + N - 1) \tag{5.23}$$

From (5.6), we have

$$\begin{bmatrix} \hat{\mathbf{h}}_j^T(0) \\ \vdots \\ \hat{\mathbf{h}}_j^T(L_cN_c + L_g - 2) \end{bmatrix} = \begin{bmatrix} \tilde{\mathbf{c}}_j^T(0) & & \\ \vdots & \ddots & \\ \vdots & & \tilde{\mathbf{c}}_j^T(0) \\ \tilde{\mathbf{c}}_j^T(L_cN_c - 1) & & \vdots \\ & \ddots & \vdots \\ & & \tilde{\mathbf{c}}_j^T(N_c - 1) \end{bmatrix} \begin{bmatrix} diag\{g_j(0)\}_{N_c \times N_c} \\ \vdots \\ diag\{g_j(L_g - 1)\}_{N_c \times N_c} \end{bmatrix}$$

$$= \tilde{\mathbf{C}}_j \mathbf{G}_j \tag{5.24}$$

Then

$$\hat{\mathbf{H}}_j(k) = \begin{bmatrix} \hat{\mathbf{h}}_{j,d_j+L_cN_c-1}^T(k) \\ \vdots \\ \hat{\mathbf{h}}_{j,d_j+1}^T(k) \\ \hat{\mathbf{h}}_{j,d_j}^T(k) \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{h}}_j^T(kL_cN_c - d_j) \\ \vdots \\ \hat{\mathbf{h}}_j^T(kL_cN_c + L_cN_c - d_j - 2) \\ \hat{\mathbf{h}}_j^T(kL_cN_c + L_cN_c - d_j - 1) \end{bmatrix}$$

$$= \tilde{\mathbf{C}}_j(kL_cN_c + 1 - d_j : kL_cN_c + L_cN_c - d_j, :) \quad \mathbf{G}_j$$

$$= \tilde{\mathbf{C}}_j(k)\mathbf{G}_j \tag{5.25}$$

Here we use MATLAB representation to denote $\tilde{\mathbf{C}}_j(k)$ as the submatrix of $\tilde{\mathbf{C}}_j$ from row $kL_cN_c + 1 - d_j$ to $kL_cN_c + L_c - d_j$.

5.2 Linear Transformation

As in chapter III, we construct a data vector $\mathbf{Y}_1(n)$

$$\mathbf{Y}_1(n)=\begin{bmatrix}\mathbf{x}(n-d_f+L_h-1)\\ \vdots \\ \mathbf{x}(n-d_f)\end{bmatrix}$$

$$=\sum_{j=1}^{J}\begin{bmatrix}\tilde{\mathbf{W}}\hat{\mathbf{H}}_j(0) & \cdots & \tilde{\mathbf{W}}\hat{\mathbf{H}}_j(L_h-1) & & \mathbf{0}\\ & \ddots & & \ddots & \\ \mathbf{0} & & \tilde{\mathbf{W}}\hat{\mathbf{H}}_j(0) & \cdots & \tilde{\mathbf{W}}\hat{\mathbf{H}}_j(L_h-1)\end{bmatrix}\begin{bmatrix}\hat{\mathbf{b}}_j(n-d_f+L_h-1)\\ \vdots \\ \hat{\mathbf{b}}_j(n-d_f)\\ \vdots \\ \hat{\mathbf{b}}_j(n-d_f-L_h-1)\end{bmatrix}$$

(5.26)

In the channel matrix corresponding to user 1, the matrix corresponding to $\hat{\mathbf{b}}_1(n-d_f)$ is

$$\begin{bmatrix}\tilde{\mathbf{W}}\hat{\mathbf{H}}_1(L_h-1)\\ \vdots \\ \tilde{\mathbf{W}}\hat{\mathbf{H}}_1(0)\end{bmatrix}=\begin{bmatrix}\mathbf{W}\tilde{\mathbf{C}}_1(L_h-1)\mathbf{G}_1\\ \vdots \\ \mathbf{W}\tilde{\mathbf{C}}_1(0)\mathbf{G}_1\end{bmatrix}=\mathbf{C}_1\mathbf{G}_1 \quad (5.27)$$

Thus, due to proposition 1, we can find a full row rank $L_cN_c \times L_hL_cN_c$ matrix $\mathbf{T}$ such that $\mathbf{TY}_1$ does not contain $\hat{\mathbf{b}}_1(n-d_f)$. Choosing $\mathbf{T}$ as the left singular vectors of $\mathbf{C}_1$ corresponding to its zero singular values, we get

$$\mathbf{T}\begin{bmatrix}\tilde{\mathbf{W}}\hat{\mathbf{H}}_1(L_h-1)\\ \vdots \\ \tilde{\mathbf{W}}\hat{\mathbf{H}}_1(0)\end{bmatrix}=\mathbf{TC}_1\mathbf{G}_1=0 \quad (5.28)$$

The construction of $\mathbf{Y}_2(n)$ and the linear prediction part are almost the same as those in the MC-CDMA system, except for using channel vector corresponding to $\hat{\mathbf{b}}_1(n-d_f)$ instead of that corresponding to $b_1(n-d_f)$ in the MC-CDMA system. Also note that the corresponding vectors or matrices need to be modified to the proper dimensions.

5.3 Batch Detectors

Define

$$\mathbf{X}(n) = \overline{\mathbf{H}}_0 \hat{\mathbf{b}}_1(n-d_f) + \overline{\mathbf{H}}_1 \overline{\mathbf{b}}_1(n) \tag{5.29}$$

where $\overline{\mathbf{b}}_1(n)$ contains all symbol components in $\left[\mathbf{b}_1^H(n) \quad \cdots \quad \mathbf{b}_J^H(n)\right]^H$ except for $\hat{\mathbf{b}}_1(n-d_f)$. $\overline{\mathbf{H}}_0$ is the matrix in the channel matrix $\mathbf{H}$ corresponding to $\hat{\mathbf{b}}_1(n-d_f)$, whereas all the other columns of $\mathbf{H}$ comprise $\overline{\mathbf{H}}_1$.

So, according to Section 4.1 and Section 4.2, a zero-forcing detector $\mathbf{f}$ with delay d_f and dimension $NL_cN_c \times 1$ for a MC-DS-CDMA system must satisfy

$$\mathbf{f}^H \mathbf{X}(n) = \mathbf{f}^H \overline{\mathbf{H}}_0 \hat{\mathbf{b}}_1(n-d_f) + \mathbf{f}^H \overline{\mathbf{H}}_1 \overline{\mathbf{b}}_1(n) = b_1(n-d_f) \tag{5.30}$$

Therefore, we need

$$\mathbf{f}^H \begin{bmatrix} \overline{\mathbf{H}}_0 \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}_{N_c \times 1} & \overline{\mathbf{H}}_1 \end{bmatrix} = \begin{bmatrix} 1 & \mathbf{0}^H \end{bmatrix} \tag{5.31}$$

Define

$$\mathbf{R} = E\left\{ \begin{bmatrix} \mathbf{X}(n) \\ \mathbf{Y}_2(n) \end{bmatrix} \begin{bmatrix} \mathbf{X}^H(n) & \mathbf{Y}_2^H(n) \end{bmatrix} \right\}$$

$$= \begin{bmatrix} \mathbf{R}_{11} & \mathbf{R}_{12} \\ \mathbf{R}_{21} & \mathbf{R}_{22} \end{bmatrix} \tag{5.32}$$

According to (4.3) and (4.4),

$$\mathbf{f}^H = \begin{bmatrix} 1 & \mathbf{0}^H \end{bmatrix} \mathbf{B}^+ \tag{5.33}$$

where

$$\mathbf{B} = \begin{bmatrix} \mathbf{U}\begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}_{N_c \times 1} & \mathbf{R}_{12}\mathbf{R}_{22}^{+}\mathbf{R}_{21} \end{bmatrix} \tag{5.34}$$

Note that $\mathbf{R}_{11} - \mathbf{R}_{12}\mathbf{R}_{22}^{+}\mathbf{R}_{21}$ here is no longer rank 1 but rank N_c, so $\mathbf{U}$ is its left singular vectors corresponding to its nonzero singular values.

The MMSE detector $\mathbf{m}$ with delay d_f and dimension $NL_cN_c \times 1$ is

$$\mathbf{m} = \mathbf{R}_{11}^{-1}\mathbf{p} \tag{5.35}$$

where $\mathbf{p} = E\{\mathbf{X}(n)\hat{b}_1(n-d_f)\} = \overline{\mathbf{H}}_0 \begin{bmatrix} s_1^2 \\ \vdots \\ s_1^2 \end{bmatrix}_{N_c \times 1}$, thus

$$\mathbf{m} = \mathbf{R}_{11}^{-1}\overline{\mathbf{H}}_0 \begin{bmatrix} s_1^2 \\ \vdots \\ s_1^2 \end{bmatrix}_{N_c \times 1} \tag{5.36}$$

Since $\mathbf{U}$ can be an estimation of $\overline{\mathbf{H}}_0$, the detector is

$$\mathbf{m} = \mathbf{R}_{11}^{-1}\mathbf{U} \begin{bmatrix} s_1^2 \\ \vdots \\ s_1^2 \end{bmatrix}_{N_c \times 1} \tag{5.37}$$

Chapter VI

Simulation Results

Under different channel conditions, simulation results are presented in this chapter for both the MC-CDMA system and the MC-DS-CDMA system to investigate the performance of the linear prediction algorithms proposed in this thesis.

6.1 MC-CDMA system

In this section, simulations are focused on the MC-CDMA system described in Section 2.2. Comparisons of the performance between the proposed method and the commonly used subspace method [21] are made.

We use Gold codes with length $L_c = 31$ as the spreading sequences. There are altogether $J = 10$ users, unless otherwise stated. Assume user 1 as the desired user. For simplicity, we use random binary sequences (+1, -1) with equal probability as users' original data sequences. The noise is zero-mean Additive White Gaussian Noise (AWGN) with variance s_v^2.

The multipath fading channel is randomly generated [15] by

$$g(t) = \sum_{q=1}^{L_d} a_q p(t - t_q) \tag{6.1}$$

where

L_d : total number of multipaths;

t_q : associated delay of the qth path;

a_q : attenuation of the qth path;

p(t): pulse function (e.g. raised-cosine pulse with roll-off factor of 0.5).

In our asynchronous system, the user delay d_j, the number of multipath components L_d and the multipath delay t_q are uniformly distributed within $[1 \quad L_c]$, $[1 \quad L_g]$ and $[0 \quad T_d]$ (where T_d is the maximum delay spread of the channel) respectively.

Define the maximum number of resolvable paths of the channel as L_g, i.e.

$$L_g = \left\lfloor \frac{T_d}{T_c} \right\rfloor + 1 \tag{6.2}$$

In order to keep the flat fading on each subcarrier, the bandwidth of each subcarrier has to be smaller than the coherence bandwidth of the channel, i.e. $\frac{1}{T_s} < \frac{1}{L_g T_c}$. Usually, we choose $L_g \approx \frac{L_c}{4}$.

Following are some definitions:

1. Signal-to-Noise Ratio (SNR)

Signal-to-noise ratio (SNR) is defined as the ratio of the power of the desired user s_1^2 (assume $s_1^2 = 1$) to noise.

$$SNR = 10 \log_{10} \left(\frac{s_1^2}{s_v^2} \right) \quad \text{(dB)} \tag{6.3}$$

2. Near-Far Ratio (NFR)

We assume that user 1 is the desired user and all other users have the same signal power s_j^2. Near-far ratio is defined as

$$NFR = 10\log_{10}(\frac{s_j^2}{s_1^2}) \quad \text{(dB)}, \qquad j \neq 1 \tag{6.4}$$

3. Signal-to-Interference -and-Noise Ratio (SINR)

Signal-to-interference -and-noise ratio is defined as the ratio of the power of the signal of interest to interference, including inter-symbol interference (ISI) and interference from other users, and noise.

Form (3.9), for any linear detector $\mathbf{f}$, the output SINR is

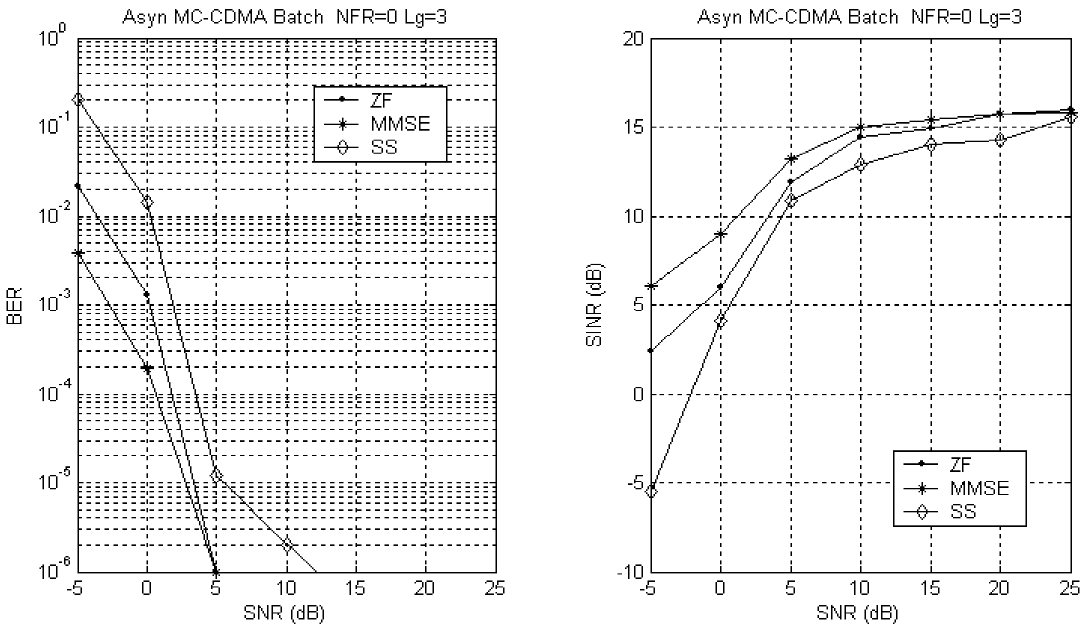
$$SINR = \frac{\mathbf{f}^H \overline{\mathbf{h}}_1 \overline{\mathbf{h}}_1^H \mathbf{f}}{\mathbf{f}^H (\mathbf{R}_{11} - \overline{\mathbf{h}}_1 \overline{\mathbf{h}}_1^H)\mathbf{f}} \tag{6.5}$$

As performance measures, we estimate the SINR and the BER at the output of the detector. All results are average of 100 Monte Carlo simulations.

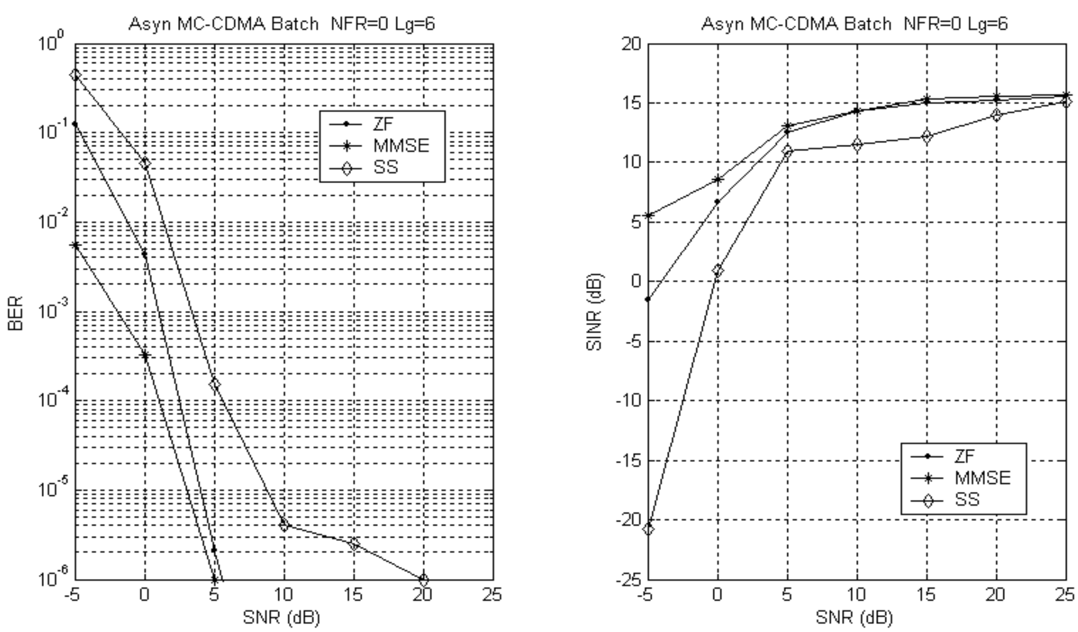
Simulation 1: For the asynchronous MC-CDMA system, performance of batch algorithm versus SNR was tested with different channel length L_g. We implemented the zero-forcing batch algorithm (ZF) and the minimum mean square error batch algorithm (MMSE), and compare with the subspace-based algorithm (SS) [21] with the same detector length $2L_c$. There are $J = 10$ users, $NFR = 0dB$ (Figure 6.1) or $NFR = 10dB$ (Figure 6.2) and the detection delay is $d_f = 0$. We use 1,000 data symbols to estimate the ZF and the MMSE detectors in our algorithm and the desired user's channel in the subspace algorithm. Up to 10,000 data symbols are used to obtain all the stochastic results.

In the subspace algorithm, the rank of the signal or noise subspace was determined by taking empirically selected threshold. Specifically, the threshold is obtained by finding the largest eigenvalue of the received data correlation matrix, and multiplying it with e. e is a parameter determined by trying many test rounds and finding one that gives the best result.

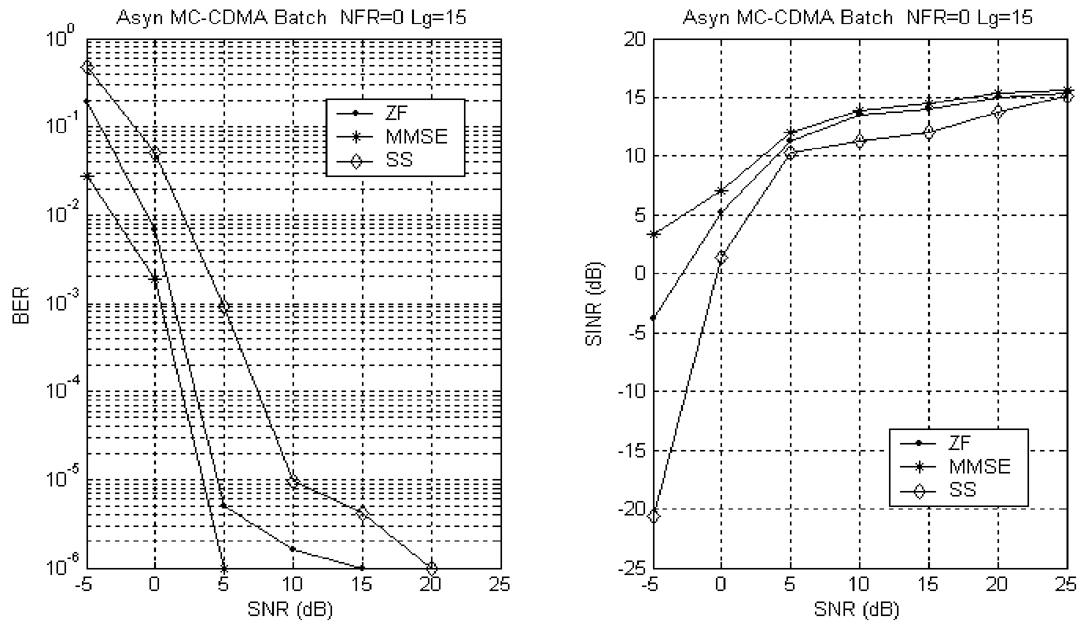
Simulation results show that our linear prediction algorithm outperforms the subspace-based algorithm as expected. As channel length increases, i.e. the number of multipath components increases, the SINR at the output of the ZF and MMSE detectors for our algorithms decreases a little bit, while the subspace-based algorithm deceases much more. Hence, the BER of the ZF and MMSE detectors for our algorithms increases a little bit, while the subspace-based algorithm increases much more. Also, we can see when SNR is larger than 10dB, the MMSE detector performs excellently no matter how long the channel is.


Asyn MC-CDMA Batch NFR=0 Lg=3
BER
SNR (dB)
ZF
MMSE
SS
Asyn MC-CDMA Batch NFR=0 Lg=3
SINR (dB)
SNR (dB)
ZF
MMSE
SS

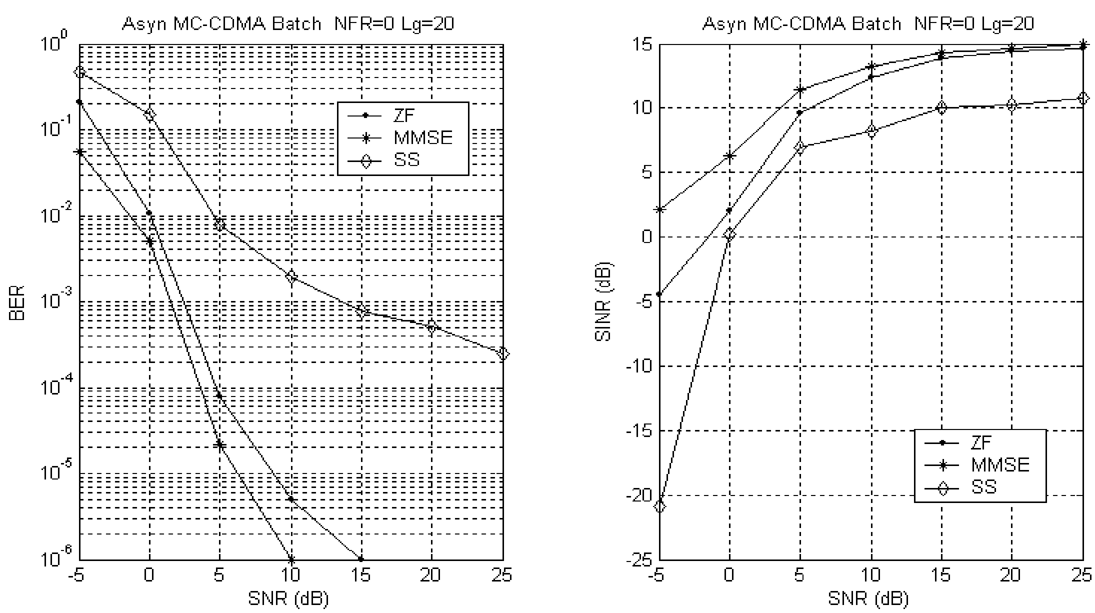
(a)


Asyn MC-CDMA Batch NFR=0 Lg=6
BER
SNR (dB)
ZF
MMSE
SS
Asyn MC-CDMA Batch NFR=0 Lg=6
SINR (dB)
SNR (dB)
ZF
MMSE
SS

(b)



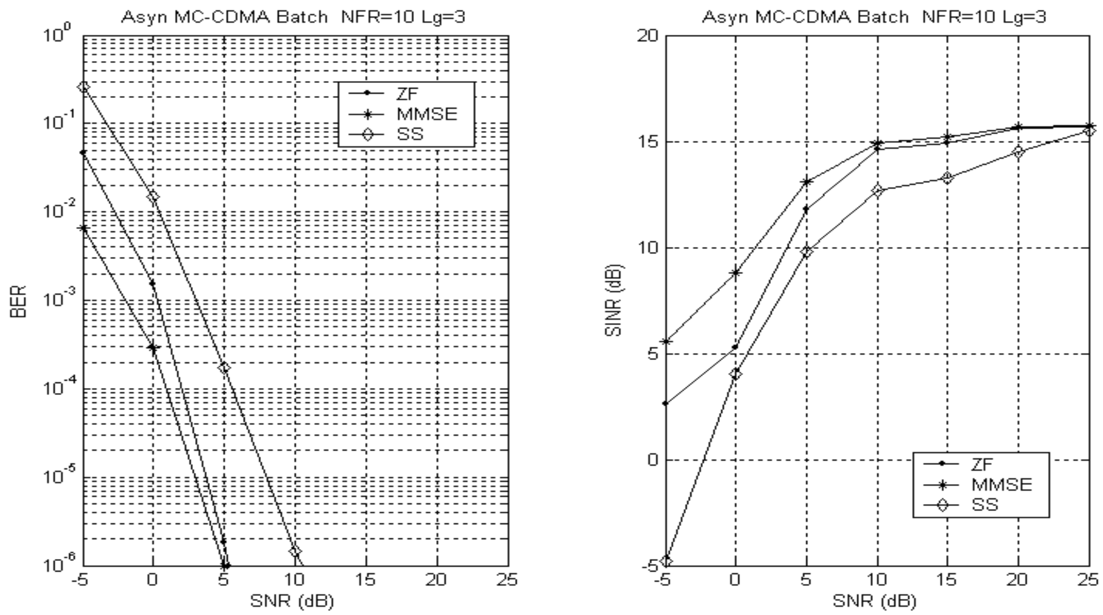
(c)



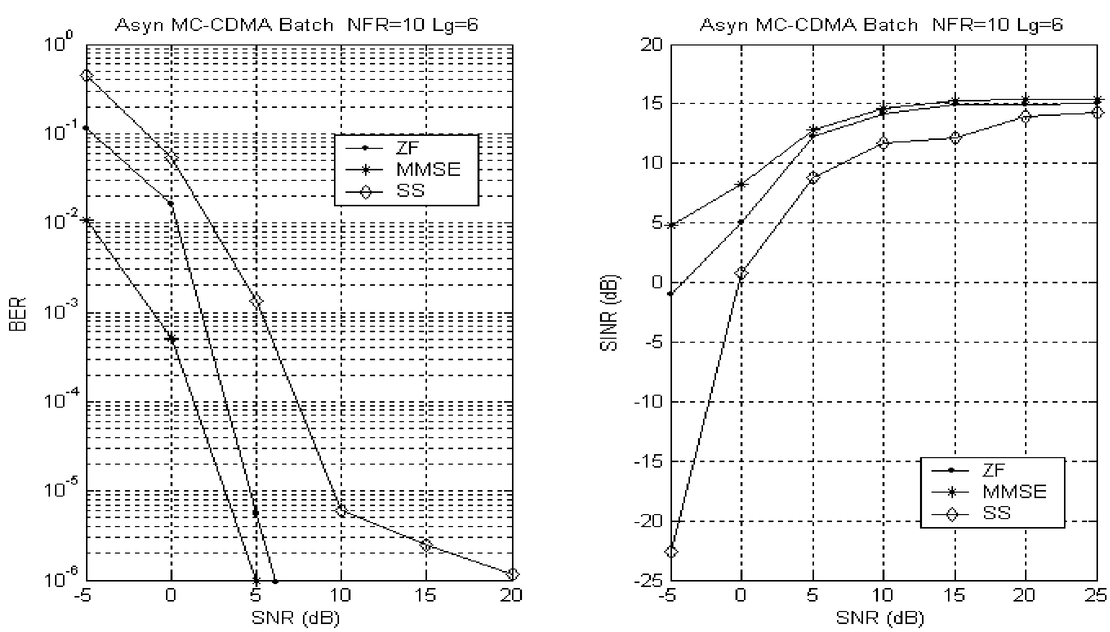
(d)

Figure 6.1: *Performance of the batch algorithms versus SNR (dB) for different channel length of asynchronous MC-CDMA system.* $J = 10$, *near-far ratio 0dB.*

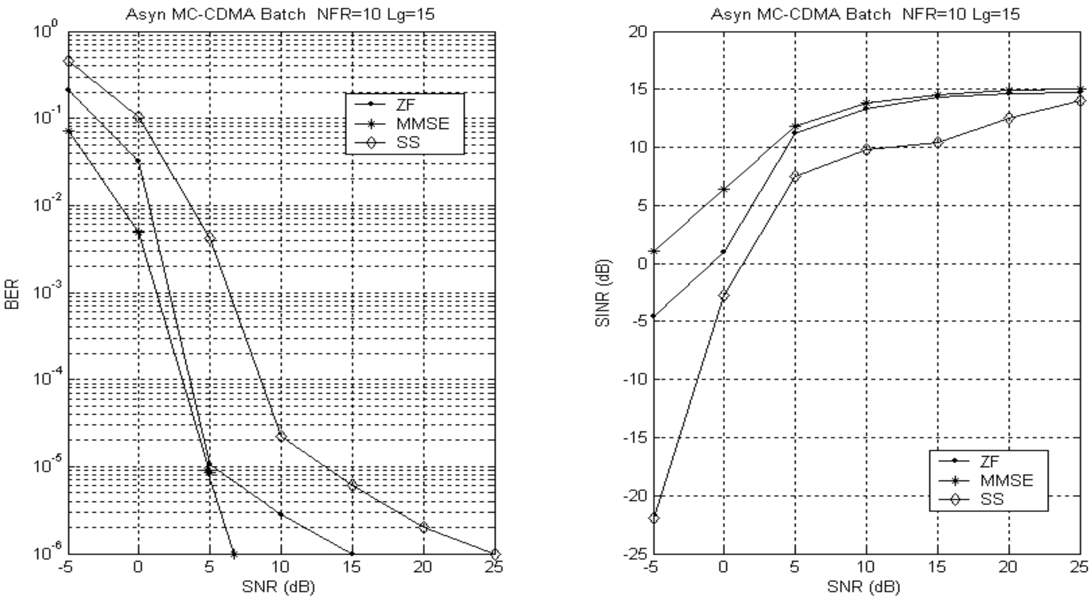
(a) $L_g = 3$ *(b)* $L_g = 6$ *(c)* $L_g = 15$ *(d)* $L_g = 20$.


Asyn MC-CDMA Batch NFR=10 Lg=3
BER
SNR (dB)
ZF
MMSE
SS
Asyn MC-CDMA Batch NFR=10 Lg=3
SINR (dB)
SNR (dB)
ZF
MMSE
SS

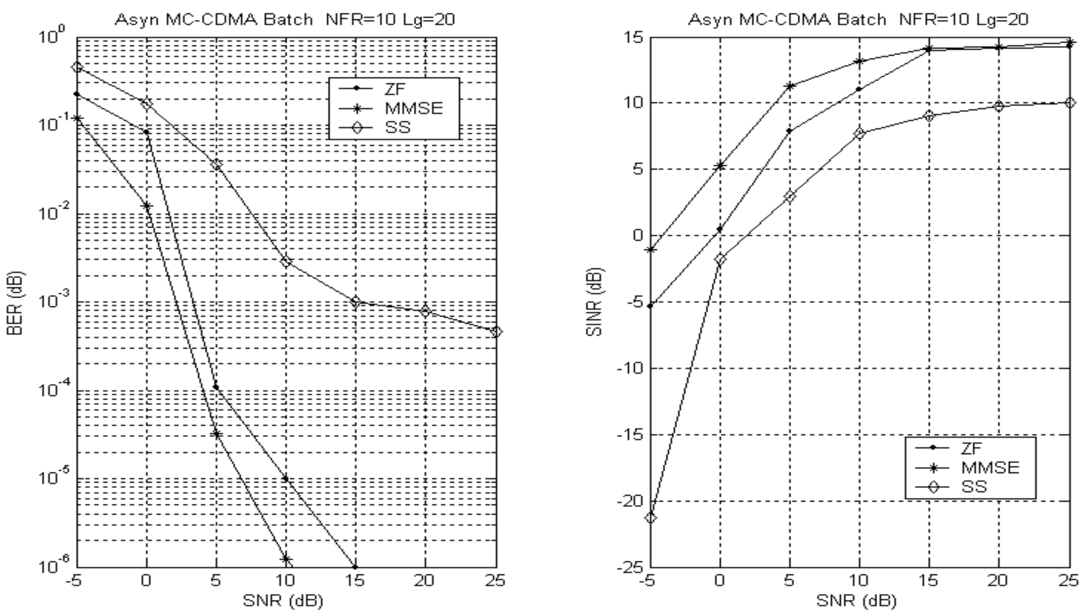
(a)


Asyn MC-CDMA Batch NFR=10 Lg=6
BER
SNR (dB)
ZF
MMSE
SS
Asyn MC-CDMA Batch NFR=10 Lg=6
SINR (dB)
SNR (dB)
ZF
MMSE
SS

(b)



(c)



(d)

Figure 6.2: *Performance of the batch algorithm versus SNR (dB) for different channel length of asynchronous MC-CDMA system.* $J = 10$, *near-far ratio 10dB.*

(a) $L_g = 3$ *(b)* $L_g = 6$ *(c)* $L_g = 15$ *(d)* $L_g = 20$.

Simulation 2: For the asynchronous MC-CDMA system, performance of the batch algorithm versus NFR was tested. The SINR of the subspace-based algorithm is 3dB below the SINR of the ZF and MMSE detectors of our algorithms when the NFR is less than 15dB. As the near-far ratio (NFR) increases, the SINR of the ZF and MMSE detectors of our algorithms remain the same, but the SINR of the subspace-based algorithm drops greatly when the NFR is larger than 20dB, which implies that our algorithm is much more near-far resistant than the subspace-based algorithm.

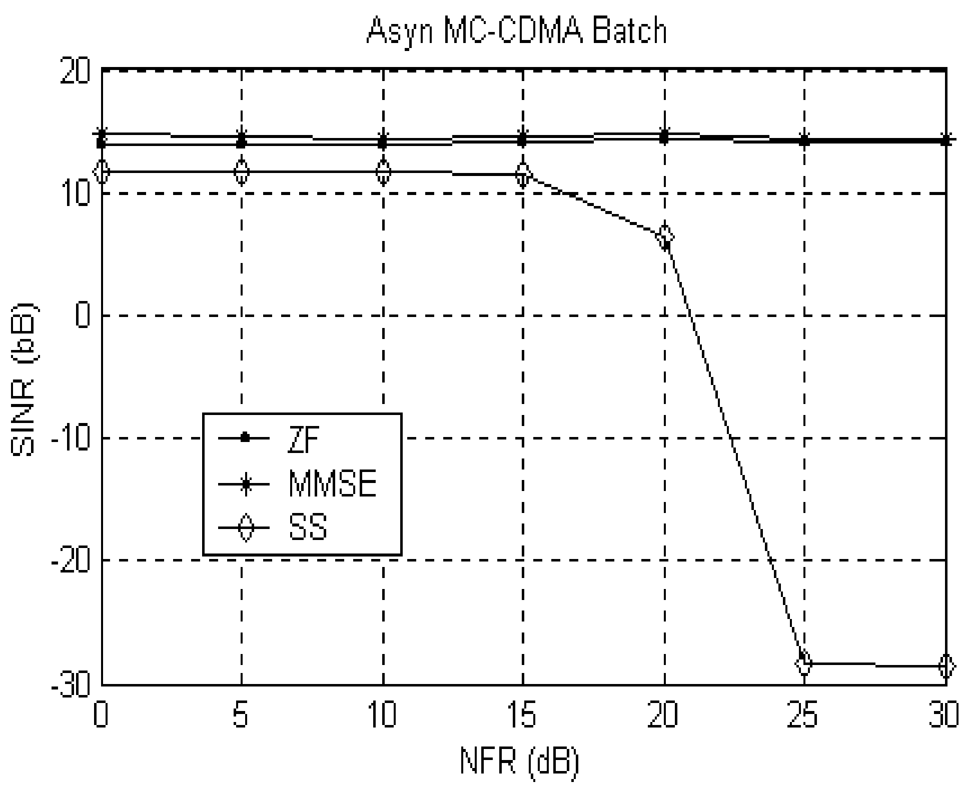


Figure 6.3: *Performance of the batch algorithm versus NFR of asynchronous MC-CDMA system.* $J = 10$, $L_g = 6$ *and* $SNR = 10dB$.

Simulation 3: For asynchronous MC-CDMA system, performance of batch algorithm versus number of users was tested. Results show that out algorithms perform better than the subspace algorithm.

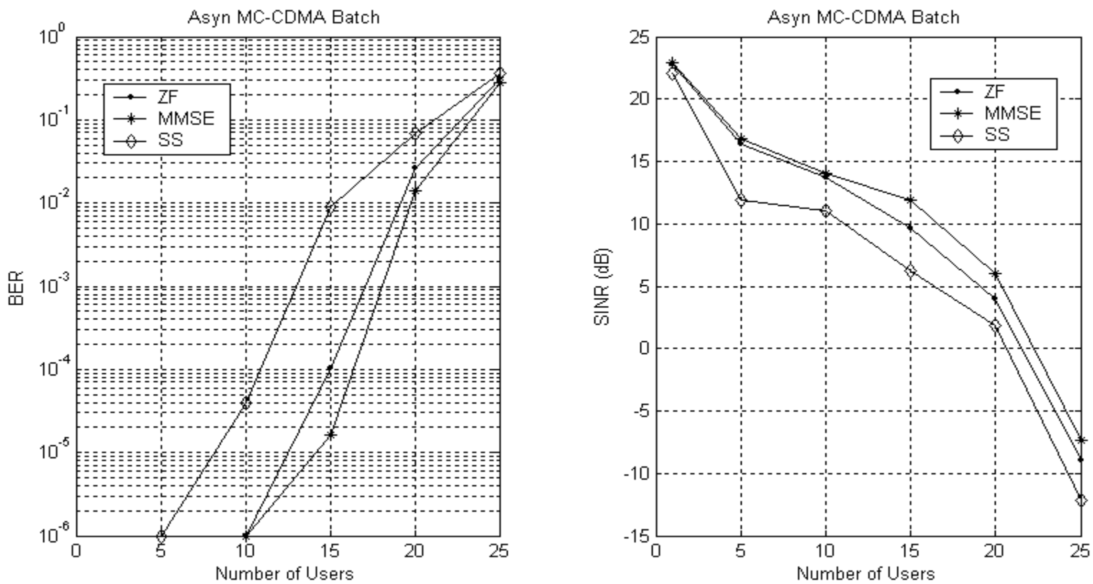


Figure 6.4: *Performance of the batch algorithm versus number of users of asynchronous MC-CDMA system.* $L_g = 6$, *near-far ratio* $10dB$ *and* $SNR = 10dB$.

Simulation 4: We test the BER performance of the adaptive algorithm versus SNR here (Figure 6.5) with different channel lengths. The detector length of the zero-forcing adaptive algorithm (ZF) and MMSE adaptive algorithm (MMSE) is both $2L_c$. There are $J = 10$ users, $NFR = 10dB$ and the detection delay is $d_f = 0$. We use up to 3,000 data symbols for iteration. Simulation results show that as the channel length increases, the performance of both algorithms decrease because of the multipath interference. Also, the output SINR versus the recursion number in symbols is shown in Figure 6.6. It is shown that after 500 to 1,000 iterations, the algorithms converge well.

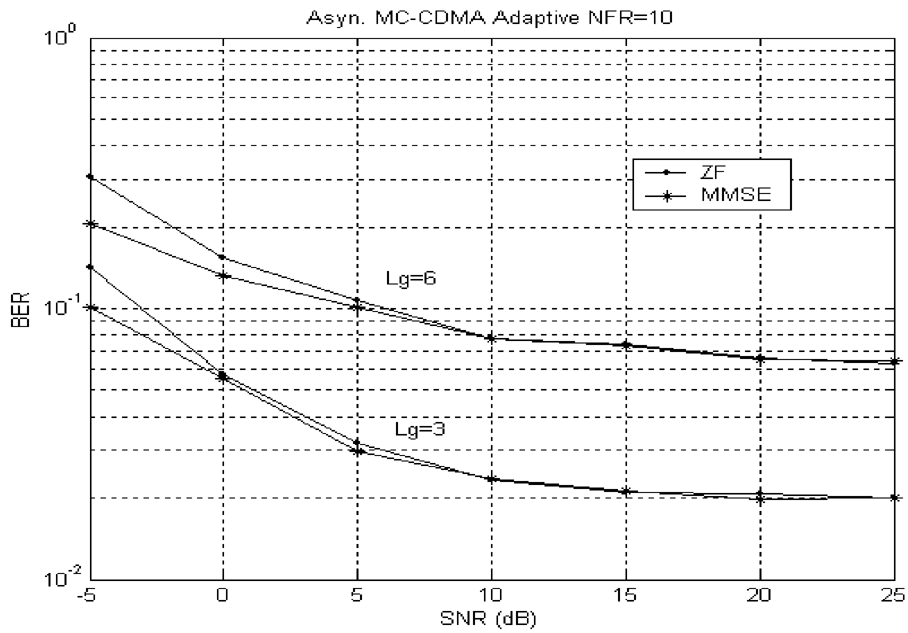


Figure 6.5: *Performance of the adaptive algorithm versus SNR (dB) for different channel length of asynchronous MC-CDMA system.* $J = 10$, *near-far ratio 10dB,* $L_g = 3$ *and 6.*

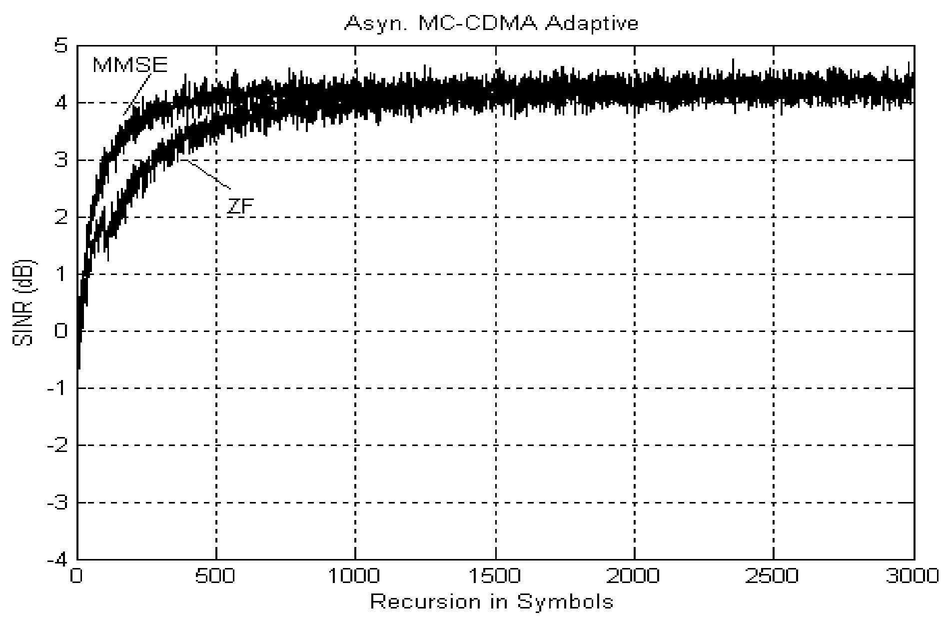


Figure 6.6: *Performance of the adaptive algorithm versus recursion in symbols of asynchronous MC-CDMA system. J=10,* $L_g = 6$, *NFR* $= 10dB$ *and* $SNR = 15dB$.

6.2 MC-DS-CDMA system

In this section, simulations are focused on the MC-DS-CDMA system described in Section 5.1. We compare the performance of the proposed method with the subspace method [20] and the eigen method [19].

We keep the total available system bandwidth $\frac{1+a}{T_c}$ constant, i.e., T_c is constant. Since in the MC-DS-CDMA system, $T_s = N_c L_c T_c$ and T_s is usually kept constant, thus we have $N_c L_c = const.$.

We choose the number of subcarriers N_c so as to meet the following conditions:

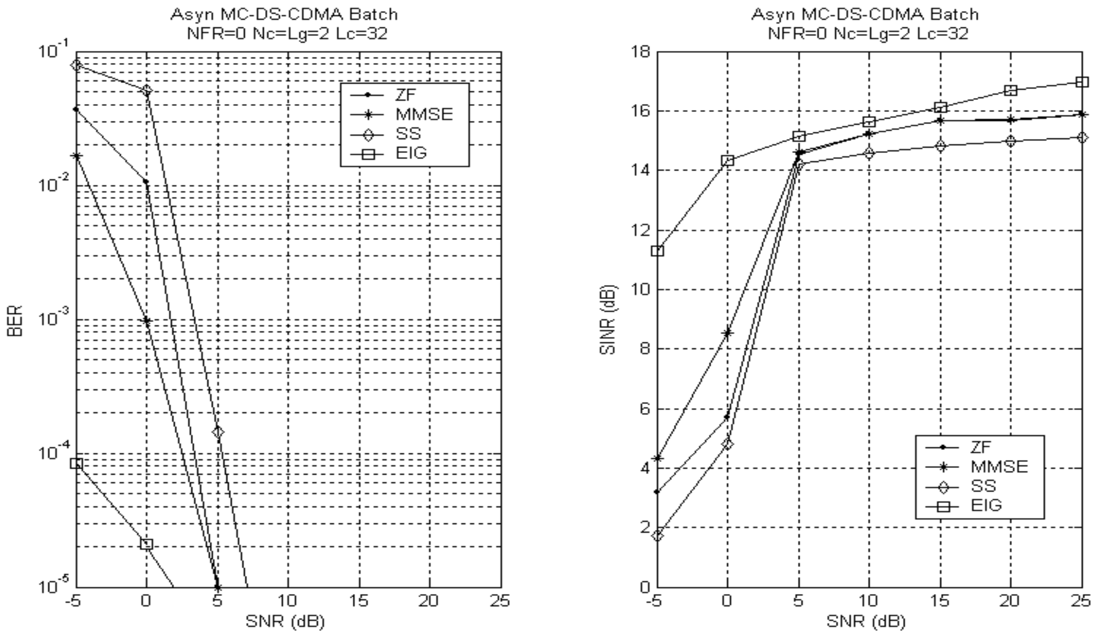
- Each sub-band of the MC-DS-CDMA system has only flat fading, i.e., $T_d \le N_c T_c$.
- All sub-bands are subject to independent fading, i.e. $\frac{1+a}{N_c T_c} \ge \frac{1}{T_d}$, where $0 < a \le 1$ and $\frac{1}{N_c T_c}$ is the bandwidth of each sub-band.

From above, we have $\frac{T_d}{T_c} \le N_c \le (1+a)\frac{T_d}{T_c}$, due to (6.2), we choose $N_c = L_g$.

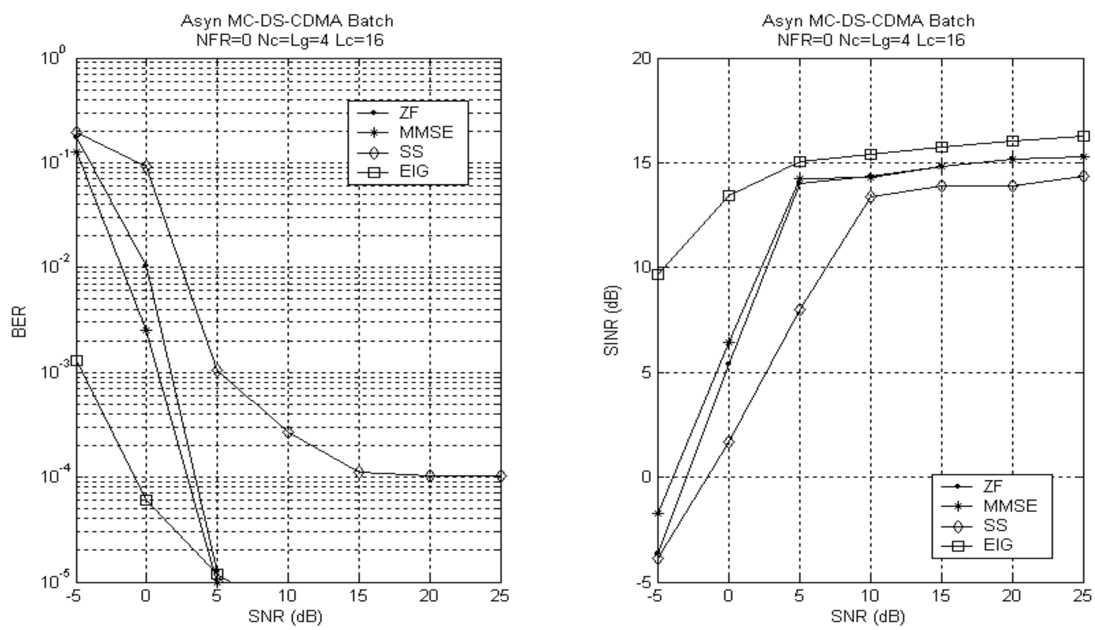
Furthermore, we use Walsh codes as the spreading sequences instead of Gold codes and kept the number of users as 10. Other conditions and definitions are the same as in the MC-CDMA system.

Simulation 1: For the asynchronous MC-DS-CDMA system, performance of batch algorithms versus SNR was tested with different channel length L_g and number of subcarriers N_c. We implement the zero-forcing batch algorithm (ZF) and MMSE batch algorithm (MMSE) with the detector length $2L_c$, and compare with the subspace-based algorithm (SS) and the eigen-method (EIGEN). $NFR = 0dB$ or $10dB$ and the detection delay is $d_f = 0$.

Simulation results show that when the NFR is low, our algorithms work better than the subspaced-based algorithm, but not so well as the eigen-method. When the NFR increases, the performance of the eigen-method becomes worse much more quickly than the other three algorithms. When the SNR is low, e.g. below $0dB$, the eigen-method is better, as the SNR increases, our linear prediction algorithms out perform others.



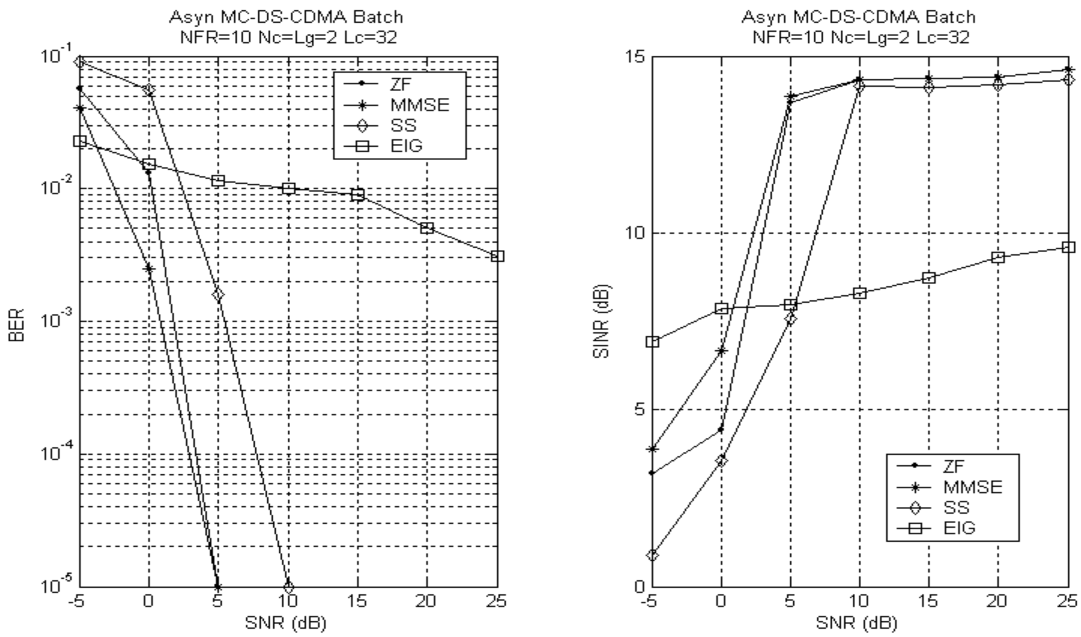
(a)



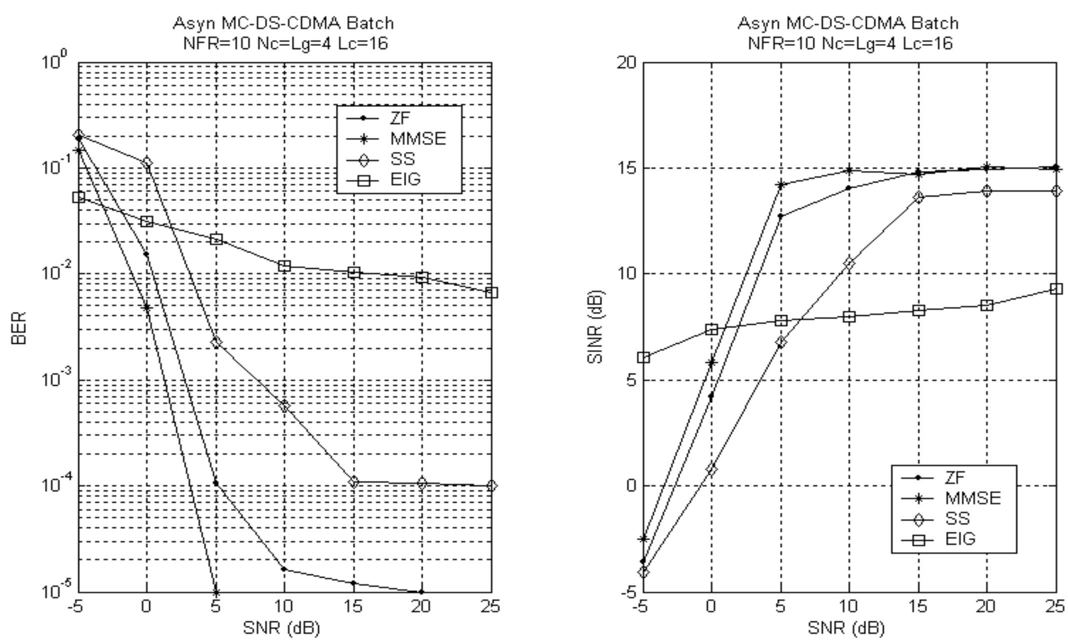
(b)

Figure 6.7: *Performance of the batch algorithm versus SNR (dB) for different channel length and number of subcarriers of asynchronous MC-DS-CDMA system.* $J = 10$, *near-far ratio 0dB.*

(a) $N_c = L_g = 2$ *and* $L_c = 32$ *(b)* $N_c = L_g = 4$ *and* $L_c = 16$.



(a)

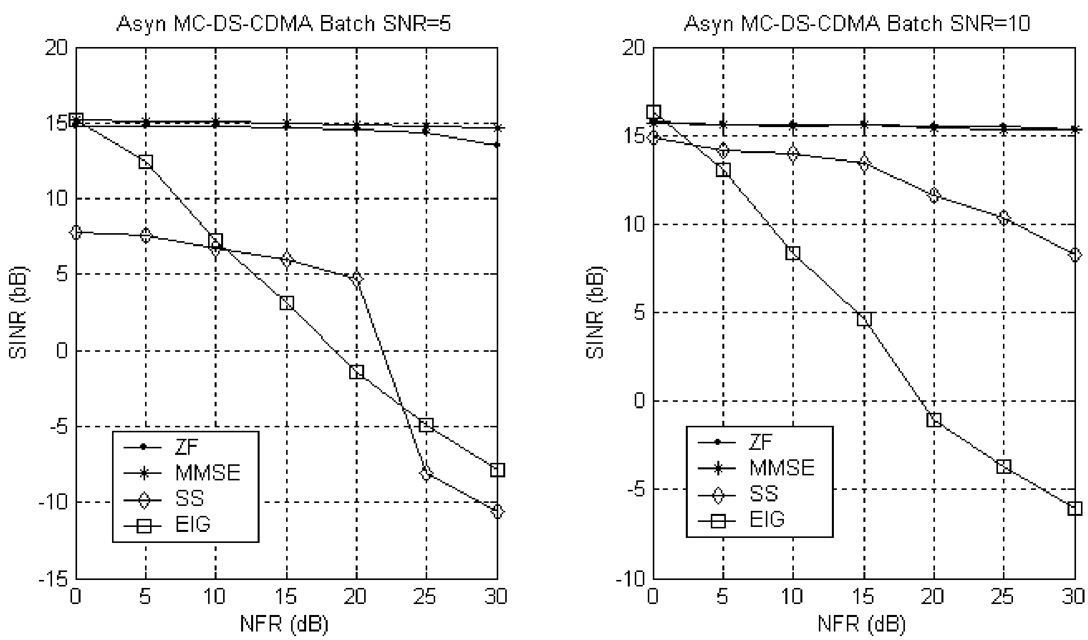


(b)

Figure 6.8: *Performance of the batch algorithm versus SNR (dB) for different channel length and number of subcarriers of asynchronous MC-DS-CDMA system.* $J = 10$, *near-far ratio 10dB.*

(a) $N_c = L_g = 2$ *and* $L_c = 32$ *(b)* $N_c = L_g = 4$ *and* $L_c = 16$.

Simulation 2: For the asynchronous MC-DS-CDMA system, performance of batch algorithm versus NFR was tested. As the near-far ratio (NFR) increases, the SINR of the ZF and MMSE detectors of our algorithms decreases little, but the SINR of the subspace-based algorithm and the eigen-method drop greatly, which implies that our algorithm is much more near-far resistant than other two algorithms. When the SNR increases, the near-far resistance of the subspace algorithm improves, while the eigen-method still performs poorly.



(a) (b)

Figure 6.9: *Performance versus NFR of asynchronous MC-DS-CDMA system.* $J = 10$, $N_c = L_g = 4$ *and* $L_c = 16$. *(a)* $SNR = 5dB$, (b) $SNR = 10dB$.

Chapter VII

Conclusion

In this thesis, we have proposed linear prediction based blind multiuser detection algorithms for multicarrier systems, including the MC-CDMA and the MC-DS-CDMA system. We focus the research on an asynchronous MC-CDMA system at first. With some modifications, our algorithms are then directly applied to the asynchronous MC-DS-CDMA system. With the knowledge of only the desired user's spreading sequence, the zero-forcing (ZF) and MMSE detector can be obtained without explicit channel estimation. This avoids the channel estimation error, which exists in the subspace-based blind algorithms. Comparisons are made between our algorithms and the subspace-based algorithm (one more comparison is made in the MC-DS-CDMA system with the eigen-method). Simulation results show that our algorithms outperform other algorithms because our algorithms are more robust and accurate and more near-far resistant.

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