A Dissertation

entitled

The Alignment between Teaching Mathematics Through Problem Solving and Recent Mathematical Process Standards and Teaching Practices

by

Awsaf A. Alwarsh

Submitted to the Graduate Faculty as partial fulfillment of the requirements for the Doctor of Philosophy Degree in Curriculum and Instruction

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An Abstract of

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This research examined how three middle school mathematics teachers who were supported by their district to use the Teaching Through Problem Solving approach interpreted and implemented the Standards for Mathematical practices (SMPs) developed by the Common Core State Standards (2010) and the Mathematics Teaching Practices (MTPs) developed by the National Council of Teachers of Mathematics (2014). Data sources included a pre and post-interview with each participant, one lesson plan from each participant, and one lesson observation for that lesson plan. Data analysis involved descriptive and interpretive components of qualitative methods to understand teachers' interpretation and implementation of SMPs and MTPs.

Four themes emerged from this analysis: (1) Supporting teachers to use Teaching Through Problem Solving may help them in their implementation of the SMPs more than in their interpretations, (2) Teachers who use Teaching Through Problem Solving may understand and fully implement the MTPs, (3) Teachers who are supported to use Teaching Through Problem Solving may use Teaching For Problem Solving, and (4) Using Teaching For Problem Solving may result in partial implementation of the SMPs and MTPs.

I dedicate this dissertation to my lovely family, including my mother, my father, my siblings, and my husband.

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I especially thank my parents for supporting me, encouraging me, and always being proud of me. I owe a huge thank you to my sisters and friends for always being tremendously supportive and understanding. I am grateful to my husband and my children for their patience and unwavering love and support.

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List of Abbreviations

CCSSM ....Common Core State Standards for Mathematics

CMP ....Connected Mathematics Program

MTPs ....Mathematics Teaching Practices

NCTM .... National Council of Teachers of Mathematics

SMPs ....Standards for Mathematical Practices

TTPS ....Teaching Through Problem Solving

Chapter 1: Background of the Problem

Introduction

The current focus in mathematics education in the United States is on using problem solving to reinforce students' understanding of mathematical concepts, procedures, reasoning, and discourse (e g., Fi & Degner, 2012; NCTM, 2014; A. Schoenfeld, 2013; Stein, Smith, Henningsen, & Silver, 2000). This focus moves educators to be interested in problem-based instruction to seek ways to develop students' thinking, rather than focusing on practicing procedures and memorization skills. Although problem-based instruction is not a new approach to teaching and learning mathematics, the focus on how to use mathematical problems in problem-based instruction by teachers has changed over time (Schroeder & Lester, 1989). The goal of this dissertation is to have an understanding of how the current focus on problem-based instruction aligns with the most recent state standards and recommended teaching practices.

Schroeder and Lester (1989) clarified this change in focus on problem-based instruction by distinguishing between three approaches: teaching *about*, *for*, and *through* problem solving. Teaching *about* problem solving focuses on teaching students the problem-solving process and/or a number of heuristics. In this approach, educators emphasize Polya's (1945) work on problem-solving heuristics. Heuristics means "serving to discover" and aims to "study the methods and rules of discovery and invention" (Polya, 1945, p. 102). Polya's method includes four phases to solve problems: (1) understanding the problem, (2) making a plan, (3) carrying out the plan, and (4) looking back at the completed solution. I used this approach during my initial years of teaching mathematics. My students were successfully able to solve many *routine* problems. A routine problem is a task that students are familiar with solution procedures (or what is entitled *Exercises*). Based on my personal experience, my students got high grades on their unit tests when I used teaching *about* problem solving during my first years of teaching middle school mathematics. However, my students had difficulty in using learned mathematical rules and concepts in their daily life even though they were able to solve word problems at the classroom. This limitation was tackled when I allowed my students to realize how to utilize mathematical concepts in real-life situations. That is, my focus had shifted to on how to teach *for* problem solving approach.

Teaching *for* problem solving focuses on teaching students mathematics procedures to learn new mathematical concepts and use this knowledge to solve real-life problems (Schroeder & Lester, 1989). When I use this approach to problem-solving instruction, my students filled their curiosities about the reasons behind learning mathematical concepts and how they can use the learned concepts in their lives. However, when my students faced non-routine problems (unfamiliar problems that students did not know how to solve), they were unable to solve them. They couldn't figure out which process and concepts of mathematics they should use. This failure to solve non-routine problem revealed a lack of my student's understanding of mathematical concepts. This difficulty in solving non-routine problems led me to the third approach to problem-solving instruction.

The third type of problem-solving instruction is teaching *through* problem solving (TTPS), which is the current emphasis in mathematics education (e. g., Fi & Degner, 2012; Schoen & Charles, 2003; A. Schoenfeld, 2013). In TTPS, teachers shift their focus beyond teaching word problems' procedures or teaching mathematical concepts to applying them in real-life mathematical problems. In TTPS, teachers use real-life mathematical problems as a mean to understand mathematical concepts and ideas. With TTPS, teachers start their lessons with a problem that allows students to discover new mathematical concepts (Schroeder & Lester, 1989). TTPS provides learning environments for students to explore non-routine problems by themselves and develop solution strategies from their own experiences (Schroeder & Lester, 1989). When students are exposed to daily non-routine problems, they may develop the ability to transform non-routine problems into routine ones (Schroeder & Lester, 1989).

TTPS is more student-centered and productive approach than the other two approaches because students learn mathematics concepts and skills in the context of solving problems, develop higher-level thinking process, and are engaged in an inquiry-oriented environment (Schroeder & Lester, 1989). The main difference between TTPS and the other two approaches is highlighted in Table 1.1.

Table 1.1

Differences of the Focus in Problem Solving Approaches

Teaching *about* problem solving	**Teaching *for* problem solving**	**Teaching *through* problem solving**
Focuses on teaching heuristics and process of problem solving	Focuses on applying new learned mathematical concepts to solve real-life problems	Focuses on solving problems to develop new mathematical concepts and ideas

This table shows the differences between the main focus of the three approaches to problem-based instruction. The focus on teaching *about* problem solving is on teaching students the procedures of solving problem, and the focus on teaching *for* problem solving is on application (how to use mathematical concept in real-life problems).

Whereas the focus on TTPS is on using mathematical problems to develop students' understanding of new mathematical knowledge.

Theory and Research Basis for TTPS

The theoretical foundation of TTPS emphasizes a constructivist view of learning (Cobb, 1994). The main idea of constructivism is that learners learn when they construct new knowledge from their own experiences (Battista, 2003). Learning in a constructivist view is an active process that aims to change learners. TTPS involves a constructivist theory that evolves from Dewey's progressive model for teaching and learning. Dewey (1938) argued that worthy educational experience relates to students' prior knowledge, has an effect on their current understanding, and reflects on how it may influence their future actions. He also recommended the use of authentic problems that require reasoning and critical thinking in school curriculum, and inquiry-learning environments where teachers work as guides to students' thinking. The teaching and learning practices that evolved from Dewey's recommendations are utilized by researchers to map out certain vital pedagogical moves of TTPS (Stanic & Kilpatrick, 1988).

Beyond Recommendations to Strategies

Although constructivism has provided useful foundation of mathematics learning and learner in TTPS, the task of reconstructing pedagogy of TTPS is a considerable challenge. Many researchers in mathematics education (e. g., Fi & Degner, 2012; Lappan, Phillips, Fey, & Friel, 2014; Smith & Stein, 2011) begun to tackle this challenge by framing the pedagogy of TTPS. Fi and Degner (2012) provided a professional development vignette that describes five pedagogical moves of TTPS. These moves focus on: (1) posing a real-life problem with consideration of students' prior experiences, (2) allowing time for students to explore the problem, (3) focusing on the big ideas and reasoning about them, (4) making ideas visible by capturing and recording students' own contributions, and (5) providing time for students to discuss and reflect on what they have learned. Fi and Degner noted that TTPS is a means of teaching mathematics because it gives every student a chance to do mathematics. The authors asserted that "the idea of letting each student struggle productively— through mathematics and engage in mathematical practices is the meaning of the Equity Principle … [and] a responsible way to ensure that students experience the joy, complexity, and beauty of mathematics" (p. 458). That is, TTPS is a way to learn mathematics for understanding as well as a way to achieve equity in mathematics classrooms.

Smith and Stein (2011) described useful pedagogical moves that can support teachers to orchestrate productive mathematical discussions in TTPS. Orchestrating productive mathematical discussion involves five phases that have the promise to gradually improve classroom discussions to be viewed as a reliable process to learn mathematics (see Table 1.2).

Table 1.2

The Five Phases to conduct a Productive Discussion

The phase	Example
Anticipating	The teacher anticipates students' solution strategies of the problem at hand, such as concrete model, logical argument, algebraic proof, and other solutions.
Monitoring	The teacher makes a table to document what each group did and which representations the students used.
Selecting	The teacher selects students' solution strategies purposefully to represent these strategies in the class discussion.
Sequencing	The teacher sequences students' solution strategies coherently, such as representing concrete model, followed by algorithm solution, and then algebraic proof.
Connecting	The teacher makes connections between students' solution strategies and between these solution strategies and a new mathematical concept or idea (such as connecting concrete model, algorithm solution, and algebraic proof that are represented in the groups together and then connecting these solution strategies to the new mathematical concept.

These five phases are: Anticipating, Monitoring, Selecting, Sequencing, and Connecting. Anticipating involves consideration about how students may interpret a problem, use strategies, and how these interpretations and strategies may relate to the mathematical concepts. Anticipating also involves planning to ask effective questions. This phase should occur before conducting the lesson. Monitoring involves making sense of students' mathematical thinking and ideas. Initial planning of how students may respond to a problem will prepare the teacher to notice students' strategies during monitoring and then try to interpret students' understanding. Monitoring should occur during the explore phase of a lesson. Selecting involves the purposeful selection from students' work to share in the class and allow for discussions about important mathematical ideas and strategies. Sequencing involves ordering the students' responses to maximize the chances to address discussion goals in a coherent manner and allow students to build a deep understanding of the mathematical concepts. Finally, connecting involves helping students to draw connections between their own mathematical ideas and other students' ideas. This supports develop connections among the main mathematical ideas in the lesson. These five phases build on each other to support teachers to orchestrate productive mathematical discussions in TTPS.

In addition, the development of the the Connected Mathematics Program's (CMP) series used TTPS as the foundation of this curriculum. CMP has an adoption across the U.S and was funded by the National Science Foundation during the 1990s and then supported from both the University of Maryland and Michigan State University in 2010 (Lappan et al., 2014a). This program is a coherent problem-based curriculum that reflects the understanding of what to teach and how to teach middle-grade mathematics. In the most recent publication of CMP, *A guide to Connected Mathematics 3* (CMP3), Lappan et al. (2014) described five pedagogical actions which are: "(a) (appropriate) scaffolding of students' thinking; (b) a sustained press for students' explanations; (c) thoughtful probing of students' strategies and solutions; (d) helping students accept responsibility for, and gain facility with, learning in a more open way; and (e) attending to issues of equity in the classroom" (Lappan et al., 2014a, p. 17). The pedagogy of TTPS outlines what teachers should do in the classroom.

Organizations Working to Reform Mathematics Education in Schools

Problem solving has been a major focus of mathematics education reform in the last 30 years. This emphasis on problem solving aligns TTPS with the reform agenda of mathematics education set forth by organizations such as the National Council of Teachers of Mathematics (NCTM) in 1989, 2000, and 2014, the National Research Council in 2001, and the National Governors Association in 2010. These organizations intended to ensure a high quality of learning and teaching mathematics and increase the focus on how students should learn and engage in mathematics through discourse, reasoning, and problem solving to develop mathematical proficiency and conceptual understanding. These organizations also work as a guide for teacher to understand the level of thinking and depth of knowledge that students should be engaged to learn mathematical content with understanding by presenting a set of standards and practices that aim to shift the focus of mathematics education from traditional methods toward teaching mathematics for understanding and learning mathematics with understanding. Supporters of these organizations believe that problem solving is a preeminent way to learn mathematics, and is vital because it converges all the strands of mathematics proficiency (e.g., National Research Council, 2001, p. 421). Herein, the term "problem solving" is defined as “mathematical tasks that have the potential to provide intellectual challenges for enhancing students’ mathematical understanding and development” (Jinfa Cai & Lester, 2010, p. 1). When a mathematical problem is not challenging or can be solved by a familiar process, it may be considered as an exercise (Schoenfeld, 2011). The organizations and their attempts to change mathematics education were described hereafter.

NCTM’s (1989-2000) Process Standards

In 1989, NCTM released the document *Curriculum and Evaluation Standards for School Mathematics* that called for more emphasis on conceptual understanding, reasoning, and problem solving, which were actually grounded in the central tenets of constructivism (Pugalee, 2001). The development of NCTM’s (1989) standards replaced the 1970s movement *Back-to-Basics*, which emphasized drill and memorization, toward problem solving, which emphasized the development of students mathematical thinking (NCTM, 1989). The NCTM’s (1989) standards aim to improve the quality of mathematics education in grades K-12 and are guided by five main goals for students to: (a) become mathematical problem solvers, (b) learn to value mathematics, (c) become confident in their mathematical ability, (d) learn to communicate mathematically, and (e) learn to reason mathematically (NCTM, 1989).

In 2000, NCTM revised their (1989) standards with the release of the document *Principles and Standards for School Mathematics*. The NCTM (2000) offered five process standards: problem solving, reasoning and proof, communication, connections, and representation. The problem-solving standard is not the application of previously learned concepts, instead, it is the development of new mathematical knowledge by encouraging students to use different strategies to solve a problem and reflecting on what they learned. The reasoning and proof standard encourages students to investigate and discover mathematical conjectures, and evaluate solutions to a problem and mathematical arguments. The communication standard encourages students to organize their mathematical thinking and ideas to peers and teachers using mathematics language precisely. The connection standard encourages students to connect mathematical ideas and applications to the context of real-life situations. The representation standard encourages students to organize and record mathematical ideas in different ways (e.g., physical, contextual, verbal, symbolic, and visual mathematical representations) and to model real-life problems. These five process standards aim to engage students in meaningful learning of mathematics. According to Draper (2002), NCTM (2000) "called for a more student-centered math classroom that deemphasizes rote memorization of isolated skills and facts and emphasizes problem solving and communication" (p. 520).

It is safe to say that NCTM's (2000) process standards represent a push to use TTPS. Each one of the NCTM's (2000) process standards represent a vital aspect in TTPS. For example, the problem solving standards can be successfully understood and implemented using TTPS because, in TTPS, teachers pose a real-life problem that allow for multiple solution strategies in an inquiry-based environment (Fi & Degner, 2012). The NCTM's (2000) process standards appear in TTPS when teachers conduct discussions to allow students share solution strategies, develop the ability to reason logically, and evaluate what makes sense mathematically. There is also an emphasis on the importance of helping students connect and compare their solution strategies with the solution strategies of their peers and with the main goals of the lesson.

National Research Council's (2001) Strands of Mathematical Proficiency

The National Research Council (2001) developed five strands of mathematical proficiency that are necessary for students to learn mathematics successfully. These five strands are conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. These strands are interwoven and interdependent (National Research Council, 2001). The conceptual understanding strand encourages students towards comprehension and connection of mathematical concepts, operations, and relations to establish the development of procedural fluency. The procedural fluency strand focuses on students' abilities to carry out procedures flexibly, accurately, and appropriately to solve problems. Strategic competence reflects the need for students to formulate, represent, and solve mathematics problems in real life and other disciplines. Adaptive reasoning reflects students' capacity to think logically as well as explain and justify their thinking. In the productive disposition strand, students see that mathematics makes sense, is useful and worthwhile, and also see themselves as performers of mathematics. These five strands should work together in order for students to effectively acquire mathematical proficiency.

Theoretical and empirical evidence in the literature founded that TTPS can provide a context for students to develop mathematical proficiency (e.g., J. Cai, 2003; Goldenberg, Shteingold, & Feurzig, 2003; Rigelman, 2015). Goldenberg et al. (2003) explained that TTPS offers opportunities for students to develop mathematical proficiency because TTPS focuses on students' thinking strategies such as problem solving and reasoning. “When given instruction that emphasizes thinking strategies, children are able to develop the strands of proficiency” (National Research Council, 2001, p. 7). Rigelman (2015) indicated that the type of problems used in TTPS is significant to develop students’ mathematical proficiency described in the National Research Council (2001). That is, the five strands of mathematical proficiency are closely related to TTPS.

CCSSM (2010) Standards for Mathematical Practices

The National Governors Association developed the Common Core State Standards for Mathematics (CCSSM) in 2010 to ensure uniformity in quality of mathematics learning across the United States (CCSSM, 2010). The CCSSM are built on the best of existing standards from top-performing countries and reflect the skills and knowledge students need to be prepared for mathematics in college, career, and life (CCSSM, 2010). As of August 2015, 42 states, the District of Columbia, four territories, and the Department of Defense Education Activity (DoDEA) had adopted the CCSSM. With the adoption of the CCSSM, mathematics teachers are required to implement these standards in a way that represents a real shift in instructional focus (Gurl, Artzt, & Sultan, 2013). This shift in focus entails profound changes in the way students learn and are engaged in their classrooms by teachers and curriculum.

The CCSSM are composed of the Standards for Mathematical Content and the Standards for Mathematical Practices. The major focus of the Standards for Mathematical Content is on what mathematical knowledge students should know to develop their understanding of mathematics. The Standards for Mathematical Content are characterized as a “balanced combination of procedure and understanding” (CCSSM, 2010, p. 8) and include concepts, skills, and attitudes that students should acquire through the context of solving problems. The Standards for Mathematical Content also provide a set of mathematical content areas to be covered at each grade level. The major focus of the Standards for Mathematical Practices (SMPs) is to engage students with tasks that promote reasoning and problem solving. The SMPs are applied across grade levels and capture the processes and proficiencies students should develop, such as thinking skills and mental habits that are specific to mathematics. Table 1.3 shows the eight SMPs that describe varieties of expertise that teachers should seek to develop in their students across all grade levels.

Table 1.3

The CCSSM-Standards for Mathematical Practice (CCSSM, 2010)

Standards for Mathematical Practice (SMPs)	Students' Expertise
SMP1. Make sense of problems and persevere in solving them	Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution.
SMP2. Reason abstractly and quantitatively	Mathematically proficient students make sense of quantities and their relationships in problem situations.
SMP3. Construct viable arguments and critique the reasoning of others	Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments.
SMP4. Model with mathematics	Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.
SMP5. Use appropriate tools strategically	Mathematically proficient students consider the available tools when solving a mathematical problem.
SMP6. Attend to precision	Mathematically proficient students try to communicate precisely to others.
SMP7. Look for and make use of structure	Mathematically proficient students look closely to discern a pattern or structure.
SMP8. Look for and express regularity in repeated reasoning	Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts.

The SMPs are challenging educators to bring authentic, real-world problems into the classroom. Using TTPS allows students to extend their experiences beyond routine problems toward nonroutine problems in real-life context. There is no doubt that TTPS aligns with SMPs, because the main goal of SMPs is engaging students in mathematics by focusing on problem solving. This is also a main goal in TTPS. Moreover, as discussed above, TTPS aligns with both NCTM's (2000) process standards and the National Research Council 's (2001) strands of mathematical proficiency. Since both sets of these standards were used as foundational elements to build SMPs, they align with SMPs. Thus, logically, TTPS aligns with SMPs. This alignment between TTPS and SMPs was not just a logical conception, it has been confirmed by many researchers in the mathematics education literature (e.g., J. Bostic, Pape, & Jacobbe, 2016; Bullock, 2017; Gurl et al., 2013).

NCTM's (2014) Mathematics Teaching Practices

While the SMPs focus on what students should be able to do in the classroom, the NCTM (2014) developed a research-informed framework of teaching practices that focus on what teachers should do to engage students in mathematics. This framework includes eight Mathematics Teaching Practices (MTPs) to support teachers to understand SMPs or any other mathematical standards. The main goal of MTPs is "to fill the gap between the development and adoption of CCSSM and other standards and the enactment of practices… required for their widespread and successful implementation" (NCTM, 2014, p. 4). NCTM (2014) asserted that MTPs need to be consistent components of every mathematics lesson (See Table 1.4).

These MTPs are aligned with SMPs and represent a focus on core high-leverage teaching practices necessary to promote deep understanding of mathematics through an emphasis on reasoning and problem solving (Gurl, Artzt, & Sultan, 2013; NCTM, 2014).

These MTPs were recommended as a means to frame teachers' mathematics instructions and represent effective teaching practices to learn mathematics for understanding. Each one of MTPs described in the book *Principles to Actions* (2014) was supported by discussion of the meaning and rationale behind each practice, providing illustration of instructional strategies relating to a particular topic, and suggesting examples of what teachers and students are doing in the classroom to implement that particular practice. These MTPs are necessary to promote deep learning of mathematics.

Table 1.4

The Eight Mathematics Teaching Practice (NCTM, 2014).

Mathematics Teaching Practices	The goal of this teaching practice is
MTP1. Establish mathematics goals to focus learning.	to guide instructional decisions.
MTP2. Implement tasks that promote reasoning and problem solving.	to engage students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
MTP3. Use and connect mathematical representations	to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
MTP4. Facilitate meaningful mathematical discourse	to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
MTP5. Pose purposeful questions.	to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
MTP6. Build procedural fluency from conceptual understanding	that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
MTP7. Support productive struggle in learning	to engage in productive struggle as they grapple with mathematical ideas and relationships.
MTP8. Elicit and use evidence of student thinking	to assess progress toward mathematical understanding and to adjust instruction continually in ways that support learning.

The MTPs has offered valuable ideas to help teachers engage students in solving problems. TTPS can contribute to these efforts clarifying how teachers would use TTPS to engage students in mathematics through problem solving. TTPS can also be used to heed the NCTM's (2014) call for supporting mathematics learning and problem solving. More information is provided about how these MTPs along with SMPs can be understood and implemented using TTPS, in the next chapter.

Problem Statement

In Ohio, where the current study will be conducted, a full adoption of the Standards for Mathematical Content and SMPs was in place prior to 2017. Full adoption means that the state of Ohio expects mathematics teachers in grades K-12 to incorporate the standards into classroom instruction. Since 2017, Ohio revised the Standards for Mathematical Content but kept the adoption of the SMPs because SMPs "describe varieties of expertise that mathematics educators at all levels should seek to develop in their students" (CCSSM, 2010, p. 6).

The problem is that effective implementation of the SMPs is not obvious for all K-12 mathematics teachers. Many teachers struggle to understand and apply the SMPs accurately (J. Bostic & Matney, 2014; Gurl et al., 2013; NCTM, 2014). This is because the SMPs focus on students' mathematical practices, skills, and habits that students are expected to acquire while learning mathematics, rather than focusing on what teachers should do in the classroom (NCTM, 2014). In other words, teachers need to have support in understanding the SMPs and developing a pedagogy that align with these standards. Teaching practices that uses a traditional approach is limited because they primarily focus on practicing procedures and memorizing information. In contrast, a major focus of the SMPs is on using problem solving to reinforce students' understanding of mathematics (NCTM, 2014). According to the NCTM (1989), "Much of the failure in school mathematics is due to tradition of teaching that is inappropriate to the way most students learn" (p. 6). Although a number of research and professional development recommendations have indicated the failure of traditional methods of teaching mathematics, such practices abound.

Whereas MTPs were developed to facilitate the implementation of SMPs, many teachers still need support to implementing these two sets of standards and practices (Smith, Steele, & Raith, 2017). One of the major reasons for this perpetuity is that implementing the SMPs and MTPs into classroom instructions actually has its roots in the use of constructivism as a guiding set of views about teaching and learning of mathematics. Constructivism is centered on promoting students' reasoning and problem solving rather than limiting their learning of mathematical content and practices (Prawat, 1992). These views influence teachers' decisions on the selection of tasks, implementation of classroom environment, use of varied strategies, and the manner in which they lead classroom discussions (NCTM, 2014). That is, to successfully implement the SMPs and MTPs, many teachers (especially who teach in "traditional" ways) need to change their views of how students learn.

A major obstacle in implementing new standards and teaching practices that focus on problem solving appropriately is that "many teachers, having experienced more traditional classroom cultures and more conventional approaches to problem solving during their education, will need to change their conceptions of the subject in fundamental ways" (Hiebert et al., 1996, p. 19). Authentic changes in teaching practices cannot be achieved when problem solving is a minor part of ongoing classroom activities. Teaching students mathematical concepts and procedures first, then assigning a collection of problems as exercises, is not supported by research.

Hiebert et al. (1996) argued, "Reform in curriculum and instruction should be based on allowing students to problematize the subject. Rather than mastering skills and applying them, students should be engaged in resolving problem" (p. 12). Problematizing the subject means selecting problems that elicit students' curiosities and sense-making skills, while these problems are drawn from their outside-of-school experiences. The role of teachers in problematizing the subject is to provide an inquiry-based environment and allow time for students to find and explain alternative methods to solve the problem. Thereafter, teachers should enable classroom conversations where students talk about the methods used to achieve solutions. Thus, from the perspective of problematizing the subject, teachers need to change their classroom practices in order to implement new standards effectively.

When we look closely, we can see that these descriptions and actions of problematizing the subject are essential pedagogical moves of TTPS. Thus, it is vital for educators and leaders of mathematics education to have an increased interest in using TTPS to bring about definitive changes in classroom practices. Although there are many obstacles in bringing about an authentic change, the pedagogy of TTPS can be used to aid teachers in understanding and implementing reforms in state standards for curriculum as well as for teaching practices. In other words, TTPS offers many advantages and presents a promising teaching approach to understand and implement SMPs and MTPs effectively.

In this dissertation, I will explore teachers' experiences as they interpret and implement SMPs and MTPs to understand how the pedagogy of TTPS foster successful implementation. When a district supports its teachers in using TTPS, it may offer insights into how these teachers interpret and implement SMPs and MTPs. Such information is necessary in order to perceive the effectiveness of using TTPS to change teaching practices. Although the literature has shown the advantages of using TTPS (J. Bostic et al., 2016; Fi & Degner, 2012; Hiebert et al., 1996; Lambdin, 2003), details and models of how TTPS is used to implement SMPs and MTPs, as well as how SMPs and MTPs can be connected to each other is necessary to the success of mathematics education locally as well as nationally.

Purpose of Study and Research Questions

The purposes of this dissertation are: (a) to investigate teachers' interpretations and implementations of SMPs and MTPs when using TTPS; and (b) to investigate models of classroom instructions that focus on implementing SMPs and MTPs using TTPS. Cognizance is taken of the fact that interpretation and implementation of SMPs and MTPs cannot be separated from mathematical content because these standards and practices are embedded within such content. However, this dissertation will not focus on the entire school curriculum; instead, it will focus on selected topics in mathematics to illustrate the interpretations and implementations of SMPs and MTPs using TTPS. To that end, two research questions are posited. The first research question focuses on the interpretation and implementation of SMPs when using TTPS, while the second research question focuses on interpretation and implementation of MTPs. These research questions are as follows:

1) What is middle school teachers' interpretation and implementation of the CCSSM (2010) Standards for Mathematical Practice when their districts support TTPS?

2) What is middle school teachers' interpretation and implementation of the NCTM (2014) Mathematics Teaching Practices when their districts support TTPS?

Chapter 1 Summary

TTPS is a type of problem-based instruction that focus on using real-life problems to learn new mathematical concepts and ideas. With the recent emphasis by mathematical organizations on problem solving, the pedagogy of TTPS would be vital to effectively understand and implement the standards and practices were provided by these organizations. Understanding and implementing the recommendations of these organizations necessitates shifting focus from traditional teaching to TTPS that involves a constructivist perspective of how students learn.

The remainder of this dissertation will include the following: (a) a review of the literature in Chapter Two, which provides more detailes of the theoretical foundation of TTPS, pedagogy of TTPS, and how MTPs and SMPs can be seen in TTPS instruction, (b) a discussion of the methodology utilized in Chapter Three, which employ case study features, (c) research findings in Chapter Four, that focus on analyzing teachers' interpretation and implementation of SMPs and MTPs, and (d) discussion about the findings and suggestions for further research in Chapter Five.

Chapter 2: Review of Literature

Introduction

Promising research findings asserted that TTPS can be used as a base for reform curriculum and instruction (Bailey, 2015; Hiebert et al., 1996). Hence, I resonate with this view and an interested in studying how TTPS is a useful way for teachers to understand and implement the recent recommendations and guidelines by mathematical organizations. Further, some districts in the United States are already supporting their teachers' adaption of TTPS as a means to teach mathematics for understanding. Thus, with a ready site, this dissertation aims to investigate middle-school mathematics teachers' interpretation and implementation of the recent practice standards (e.g., SMPs) and aligned teaching practices to these standards (e.g., MTPs) when using TTPS.

Constructivism will be discussed as the theoretical foundation for this dissertation because it has guided most of the development of the constructivist pedagogy of TTPS (Richardson, 2003). Constructivist pedagogy in TTPS will be used as a framework to guide the interpretation and implementation of SMPs and MTPs. The following section begins with a description of the theoretical foundation, the constructivist pedagogy of TTPS as a framework for this dissertation, and a discussion of how teachers may interpret and implement SMPs and MTPs when using the constructivist pedagogy of TTPS.

Theoretical Foundation

The theory that contributes to framing an explanation of TTPS in mathematics classrooms is constructivism. To understand what kind of learning should be developed using the TTPS approach, it is important to first understand the meaning of learning.

Then, information about what forms of constructivism influence TTPS will be provided to understand the underlying pedagogy of TTPS. Third, explaining how teaching and learning of mathematics under the theory of constructivism informs TTPS is provided to understand how constructivism aids the learning of abstract notions in mathematics. In the following sections, more details about the metaphors for learning, constructivism, and constructivist pedagogy will be presented.

Kind of Learning in TTPS

The history of learning is closely connected to the history of psychology. Learning is defined as "a lasting change in behaviors or beliefs that results from experience" (Halpern & Donaghey, 2003, p. 1458). Learning provides valuable insights and guides our work as educators (Sfard, 1998). Sfard (1998) described two metaphors for learning that guide our thinking about the meaning of learning: The participation metaphor and the acquisition metaphor. The participation metaphor is learning as participation in certain kinds of activities, such as a learning as legitimate peripheral participation or as an apprenticeship in thinking. This metaphor focuses on terms that indicate action such as knowing, participating, communicating, and doing (Sfard, 1998). The goal of learning here is for community building—to be part of the whole. The role of teachers in the participation metaphor pertains to discourse, preserver of the continuity of practice, expert participant, etc.

In other words, the participation metaphor contributes to the meaning of learning in TTPS. It provides guidelines in what students should do to learn mathematics. Students can learn by doing, communicating, and interacting with each other and with their teacher. The participation metaphor also provides guidelines for teachers in how to facilitate students' learning. Teachers can conduct classroom discussions and be an expert participant by asking purposeful questions to develop students' understanding (Sfard, 1998).

The acquisition metaphor conceives human learning as an "acquisition and accumulation of some goods" (Sfard, 1998, p. 5). This metaphor focuses on the acquisition of knowledge regardless of the mechanisms of concept development. Students in the acquisition metaphor can develop concepts as passive receptions of knowledge or as being active constructions of knowledge. The acquisition metaphor stresses the importance of the learner's mind, and the development or constructing of knowledge in the process of learning. The role of teachers in the acquisition metaphor is to help students acquire knowledge by facilitating, mediating, conveying, etc. (Sfard, 1998). In other words, the acquisition metaphor contributes to the meaning of learning in TTPS. TTPS involves concepts development through construction of knowledge. That is, the process of constructing knowledge in TTPS can be conceptualized in term of the acquisition metaphor.

Both metaphors contribute in shaping out the meaning of learning when using the TTPS approach. As Sfard had pointed out, the both metaphors lead to different ways of thinking and different kinds of activities and the devotion to only one metaphor can lead to undesirable practical consequences. The contribution of participation metaphor in the TTPS approach clearly appears when students learn by working in groups and participate in mathematics discussions to solve mathematical problems. The acquisition metaphor appears also in TTPS by focusing on how meanings of mathematical concepts and ideas are created within the individual mind. Both metaphors together contribute to the type of constructivism as learning in TTPS.

Constructivism

Constructivism is an epistemological view used to explain how learners construct knowledge (Lamon, 2003), and it can be defined as “a philosophical theory or position about knowledge and knowledge acquisition” (Janvier, 1996, p. 449). In the constructivist perspective, learning occurs when constructing new knowledge from one’s own prior experiences (Lamon, 2003). The central idea of constructivism is that learners build their own conceptual structure. (Sfard, 1998). Constructivism represents departure from treating learners as passive receivers of information into active participants in the learning process.

According to Janvier (1996), constructivism is not a theory of teaching; instead, constructivism can be used by educators to plan, organize, and support their students’ learning process and construction of knowledge. This means the constructivist pedagogy can be considered here as the creation of teaching practices that are based on constructivism. The writings of both, Vygotsky and Dewey, has guided most of the development of the constructivist pedagogy (Popkewitz, 1998). Popkewitz (1998) stated: “Dewey and Vygotsky are drawn into the pedagogical discussion to argue a relationship between mind and environment” (p. 549). That is, both theorists contribute in how to form students’ thinking in educational settings.

The main characteristics of the constructivist pedagogy are: (a) facilitating discussions to create shared understanding, (b) focusing on student-centered environment to pay attention to individual and respect students’ background, (c) promoting metacognition to develop students' awareness of their own learning and self-regulation during problem solving, and (d) engaging students in tasks that are structured to challenge and add to existing understandings (Richardson, 2003).

Vygotsky's theories feed into the constructivist pedagogy in the fundamental role of social interaction in the development of students' cognition. Vygotsky asserted (in the zone of proximal development) that the level of potential development of students can be maximized when working with adults and/or more capable peers. He stated that "the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers" (Vygotsky, 1978, p. 86). Social interaction with peers creates opportunities for students to talk about their thinking and reflect on the given problems to develop or devise conceptual strategies. In other words, cognitive conflict stimulates learning and determines how students accommodate and organize new knowledge. Interacting with surrounding environment—that involves discussions, making sense of others' views, and comparing personal meaning with those formed by others—can stimulate the cognitive processes that are essential to learning. Thus, the constructivist pedagogy emphasizes that each student interacts with peers in small groups as an effective way of developing skills and strategies, while the role of teachers is to facilitate and guide students throughout their investigations.

In addition, Dewey's work has an influence in the constructivist pedagogy. Dewey (1938) emphasized that the consideration of students' prior knowledge is fundamental to the construction of classroom activities. When teachers utilize students' previous knowledge on a topic, they can plan their instructions adequately, providing proper resource materials, and select effective instructional strategies that develop students' understanding and promote their thinking. Further, involving prior knowledge is the most important factor that influences learning, because such know-how can be reconstructed or expanded to understand new concepts correctly.

Dewey informed the idea that school curriculum must bring real-life problems to classroom learning (Lamon, 2003). He advocated the selection of problems that are pertinent with the lived experiences of students; he added that such problems would stimulate new ways of observation and judgment. Dewey (1938), too, provided a comprehensive view of students' experiences as a continuum, which starts from awareness of past experiences to engaging in present situations to ensure worthwhile future encounters. This idea is utilized in selecting mathematical problems. The type of problems in mathematics should be authentic, related to students' experiences, and focus on developing students' conceptual understanding that help them to generalize concepts for future applications. For example, when a sixth-grade mathematics teacher gives students a mathematical abstraction about adding and subtracting Integers and relates it to a real-life problem such the air temperature, students may relate this abstraction to their experience of the change in temperature and represent negative numbers to the quantities that have values below zero. This process would provide a potential for generalizing the idea of addition and subtraction Integers in the future.

Further, in the constructivist pedagogy teachers pose problems that require reasoning and critical thinking and work as facilitators of students' thinking. This is similar to what has been described by Dewey (1938) as students' intelligent acting and teachers' role in providing guidance. Dewey noted that "intelligent observation and judgment by which a purpose is developed, guidance given by the teacher to the exercise of the pupils' intelligence is an aid to freedom, not a restriction upon it" (p. 71). Likewise, teachers who adopt the constructivist pedagogy provide an environment of proper inquiry and guide classroom discourse. Thus, the students have the flexibility to think of a path solution and analyze tasks on their own; and when they make mistakes, teachers help clear the misconceptions in their thinking processes.

Constructivism provides solid foundation about how students learn mathematics. However, it does not tell us how to teach mathematics. Thus, constructivist pedagogy in mathematics that is derived from a constructivist view of learning forms the pedagogy of TTPS.

The Pedagogy of Teaching Through Problem Solving

The most important aspect that align TTPS with the recent mathematical reform is the pedagogy of TTPS. In the mathematics education literature, many researchers (e.g., Fi & Degner, 2012; Schoen & Charles, 2003; Smith & Stein, 2011) described the main pedagogy of TTPS as the following: (1) The teacher begins a lesson with the goal of using a high-cognitive demand problem to connect students' solutions with a new mathematical concept or idea; and (2) The teacher allows time for students to explore the problem at hand by themselves while the teacher's role is to facilitate students thinking, and to orchestrate meaningful mathematical discussion (Kazemi & Stipek, 2001; Smith & Stein, 2011). Lappan et al. (2014) outlined the pedagogy of TTPS in three instructional phases: **Launch**, **Explore**, **Summarize**. I will use these phases here to explain the pedagogy of TTPS because the participants of this dissertation are using these three phases from the textbook CMP3 to teach middle-school mathematics. This would add more clarity during the discussion of teachers' interpretation and implementation of the SMPs and MTPs in each phase of TTPS.

Launch. In the Launch phase, the teacher launches a high-cognitive demand problem that is connected to students' prior knowledge. Cognitive demands refer to "the kind and level of thinking required of students in order to successfully engage with and solve the task" (Stein et al., 2000, p. 11). Stein et al. (2000) listed the characteristics of mathematical problems at four levels of cognitive demand: *memorization*, *procedures without connection*, *procedures with connections*, and *doing mathematics*. They classified problems that required *memorization* and *procedures without connection* as having a lower-level of cognitive demand. Problems that required *procedures with connections* and *doing mathematics* were as having a higher-level of cognitive demand. Although mathematical problems with a higher-level of cognitive demand are the most difficult problems to implement, research shows that students gain greater learning of mathematics in classrooms that teachers consistently use tasks in a higher-level of cognitive demand (NCTM, 2014; Stein et al., 2000).

The level of cognitive demand required in TTPS is *doing mathematics* (Lappan et al., 2014). Doing mathematics "requires complex and nonalgorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example)" (Stein et al., 2000, p. 16). That is, teachers who use TTPS should launch a problem in *doing mathematics* level without lowering the cognitive demand of the problems. Teachers can focus students' attention on looking for the underling mathematical structure of the problem without suggesting a pathway to solve the problem. A mathematical problem such as the Building Storm Shelters problem (See Appendix A) is an example of a high cognitive demand problem. The focus question of this problem is “what are the shape and perimeter of the rectangles with the greatest and least perimeter?” (Lappan et al., 2014b, p. 55). The main goals of implementing this problem are to deepen students understanding of the area and perimeter of rectangular shapes, explore the relationships between the perimeter and area, and visually represent relationships between the perimeter and area on a graph. This problem presents different challenges for both students and teachers because it requires *doing mathematics*. There is no pathway suggested by the problem and students are required to engage in a complex form of thinking, reasoning, and problem solving. Teachers can support students during the launch phase in TTPS by connecting the problem with students’ prior knowledge and by presenting the challenge of the problem.

Connecting to prior knowledge. In constructivist classrooms, teachers believe that prior knowledge impact students’ learning. When new knowledge is not connected with students’ prior experiences, it will be easily forgotten because this new knowledge does not make meaning to their existing mental framework. In contrast, when teachers allow students to show that there are elements in their prior experience that can be construed differently from how they construe them, this may develop new desirable conceptions. That is, teachers should position a mathematical problem within students’ prior knowledge or experiences as a starting point to build new knowledge. In the Building Storm Shelters Problem, the teacher can review graphing on coordinate planes to connect the problem to the students’ prior knowledge. The teacher can also ask students questions that lead them to be able to plot correctly on a graph, such as “What does the first number in an ordered pair tell us about a point? (How far right or left the point is from the vertical (y) axis)", "What does the second number in an ordered pair tell us about a point? (The distance up or down that the point is from the horizontal (x) axis)", and "How do you plot the point (5, 6)? (Begin at the original (0, 0), and move to the right 5 spaces, then move up 6 spaces. Plot the point there.)" (Lappan et al., 2014a, p. 57). Then, the teacher can introduce the problem by presenting the goal of the problem and asking students to use a fixed number of tiles (e.g., 12 tiles) to build a rectangle.

Presenting the challenge. After posing a problem, the teacher helps students understand the context and the challenge of the problem. This should allow students to have a clear picture of what is expected without lowering the challenge or the cognitive demands of the problem. In the Building Storm Shelters Problem, the teacher can ask students: *How do you know that you found all the rectangles that can be made using 12 tiles?* The teacher can also ask students to pay attention to what happens to the perimeter and shape of the rectangle as they look for all rectangles with area of 24 square units.

Explore. During the explore phase, the teacher allows time for students to solve the problem, asks appropriate questions to help students persevere in their work, and provides scaffolding of students' thinking and individual needs while they work in small groups. The teacher also plans for the summary phase by monitoring students' thinking and ideas as they solve the problem, selecting students' solution strategies to share in the whole class discussion, and sequencing or determining an order for presentation of students' solution strategies.

Providing for individual needs. During the explore phase, the teacher observes individual performance, interacts with individual and small groups through asking purposeful questions, and provids materials that students need. The teacher may also encourage students to explain their thinking and understanding as well as focus their thinking if they are struggling or become off-task without giving away the solution. These questions should be tied to the lesson goals to support students' mathematical thinking, reasoning, and problem solving. Encouraging students to talk about mathematics is essential to help them become better mathematical thinkers and learn from each other.

In the Building Storm Shelters Problem, the teacher observes students while they are making sketches and completing the table, summarizes questions if students need help with graphing, and helps students plot points on their graphs and understand how plotting a single point represents both the length and perimeter of a rectangle (Lappan et al., 2014b, p. 58). The teacher may ask questions such as: "Can you describe the relationship between different length and perimeters of rectangles with an area of 24 square units? (As the length increases, the perimeter decreases to certain point, and then the perimeter increases)", "How is this relationship represented in the table and graph? (When the table is organized from least to greatest length, the perimeter decreases until the middle factor pairs are reached, and then the perimeter increases again. The perimeters repeat when factor pairs are repeated. In the graph, when moving from left to right, the line curves down to a certain point, and then it rises up again)", and "How can you find the maximum perimeter from the table? (Look for the entries where 1 is a factor, which will be the top and the bottom of the table.)" (Lappan et al., 2014b, p. 59).

Planning for the summary. Carefully attending to students' work helps teachers manage the degree of improvisation during a classroom discussion. The five practices to orchestrate productive mathematics discussions that are suggested by Smith and Stein (2011) can be utilized in the explore phase. These five practices are: anticipating, monitoring, selecting, sequencing, and connecting (See Table 1.2). The teacher can anticipate how students might solve the problem and what strategies they might use. In the explore phase when implementing the Building Storm Shelters, the teacher may anticipate that students might use tiles, coordinate graph, tables, or drawing to see the patterns of the relationship between the shape and perimeter of the rectangles. The teacher may then monitor her or his students' progress and solution strategies as they work on the problem. Then, the teacher may decide which students' strategies to focus on and how to sequence the discussion of students' strategies during the whole class. For example, the teacher may plan to present students' physical representation strategy such as tiles or visual representation strategy such as drawing before presenting symbolic representation strategy such as tables. Connection practice can be implemented during the next phase (summarize phase) to help students make connections between their solution strategies and other students' solution strategies as well as the lesson goals.

During the explore phase of a TTPS lesson, one must be careful not to reduce the level of cognitive demand of the problem. A problem that is setup by the teacher as *doing mathematics* level can be reduced by the teacher or students to a different level during its implementation (Stein et al., 2000). Reducing the level of cognitive demand of a problem results in defferent students' learning outcomes (Stein et al., 2000).

Summarize. During the summarize phase, the teacher has students share their solution strategies. The teacher can also help students to connect and compare the different strategies they have shared. In the summarize phase of the Building Storm

Shelters Problem, the teacher might encourage students to connect different mathematical representations of the relationships between perimeter and shapes of rectangles that have the same area. The teacher may begin with displaying a table produced by one of the groups and ask purposeful questions to help students connect the information in the table with the sketches of different rectangles. In this way, students can visualize what the shelters with the largest and smallest perimeters look like. The teachers also might help students to connect the data in the table with the graph of the length and perimeter for rectangles with an area of 24 m^2. To do so, the teacher may ask “Which of the shelters with an area of 24 m^2 has the least perimeter? (4m by 6m)”, “What does it look like? (It’s more square-like)”, “Which of the shelters with an area of 24 m^2 has the greatest perimeter? (1m by 24m)”, “What does it look like? (It is long and skinny)”, “Which floor plan would make a good shelter? Explain your reasoning. (The 4m-by-6m shelter will give a lot of open space)”, “How should we display this on the graph?”, and “How can we tell from the graph that this is the design with the least [or greatest] perimeter?” (Lappan et al., 2014b, p. 60-61). In this way, students may draw connections between different solution strategies and reflect on what they have learned.

Mathematical discussions in TTPS encourage students to think, talk, reason, and inquire (NCTM, 2014). Encouraging whole-class discussions give students opportunity to share their mathematical ideas, solutions, and rationales for selecting their strategies. During mathematical discussions, teachers also can reveal and address students’ misconceptions and ideas and build toward the mathematical goals of the lesson. Connection involves connection among different students’ representations, connection between students’ representations and the key mathematical ideas, and connection between the new learned mathematical ideas with previous ones (NCTM, 2014; Smith & Stein, 2011). Representations include contextual, visual, verbal, physical, and symbolic representational forms (NCTM, 2014). Connecting these forms of representations through meaningful mathematical discussions allows students to deepen their understanding of new mathematics concepts. This is because students' conceptual understanding is correlated to the strength of connections among these representations (NCTM, 2014).

Moreover, having students making connections between varied mathematical representations and connect these representations to the key mathematical ideas in the lesson gives the opportunities to promote conceptual as well as procedural understanding. Tripathi (2008) stated that using "different representations is like examining the concept through a variety of lenses, with each lens providing a different perspective that makes the picture (concept) rich and deeper" (p. 439). When teachers facilitate discussions of the connection between different representations and the new mathematical concepts and ideas, students can understand underling mathematical structures of mathematical concepts and ideas regardless of the form (NCTM, 2014). This can develop conceptual understanding and fluency.

In addition, connection phase includes encouraging students to develop understanding of how mathematical ideas connected with one another to produce a coherent whole. Cai and Lester (2010) stated that "Empirically, teaching through problem solving helps students go beyond acquiring isolated ideas toward developing increasingly connected and complex system of knowledge" (p. 3). Thus, allowing students to connect varied mathematical representations and build mathematical ideas on one another is an important teaching practice in TTPS.

Reflecting on student learning. At the end of the lesson, the teacher assesses student learning through eliciting evidence about students' understanding of the learned mathematical concepts. The kinds of strategies that students used can be used to assess their understanding. The teacher might also reflect on the difficulties that students encountered to plan for the next lesson and reinforce these ideas and concepts throughout the unit.

Mathematical discussions in TTPS. Chapin, O'Connor, and Anderson (2009) outlined five important "talk moves" that help teachers to ask meaningful questions during the explore and summarize phases. These questions can promote mathematical discussions and encourage students to make connections between students' solution strategies and new mathematical concepts (see Table 2.1).

Table 2.1

Talk Moves

Talk moves	Example
Revoicing	You used prim factorization. Is that what you said?
Repeating	Can you repeat what he said in your own words?
Reasoning	Do you agree or disagree and why?
Adding On	Would someone like to add something more to this?
Waiting	Take your time (or we will wait).

Revoicing is when the teacher repeats what a student says to make sure that the teacher understands what the student intended. Repeating is when the teacher asks a student to restate what another student said; thus, students are encouraged to listen to other students' ideas and strategies. Reasoning is when the teacher asks students to apply their own reasoning to other students' reasoning by using questions such as "What do you think about that?" and "Do you agree or disagree and why?" Adding on is when the teacher asks students to listen to what other students are saying so that they are able to add more input to other students' idea. Adding on would help students have responsibility for making sense of one another's reasoning and reaching a general agreement together. Waiting is when the teacher asks a question, the teacher should wait at least few second before calling on someone to answer. Waiting gives students time to think of the posed question. Waiting can provide an opportunity for all students including English Language Learners or students who need extra time, to process the question and formulate a response. These talk moves can be used in TTPS to encourage students to think, talk, and learn. Using these talk moves may also help students to organize their ideas and strategies to present them during the whole class discussion in TTPS lessons.

In addition, productive mathematical discussion needs sociomathematical norms that govern mathematical discussions. Sociomathematical norms refer to specific students' mathematical activities that promote students' engagement in conceptual mathematical thinking and discourse (see Table 2.2) (Kazemi & Stipek, 2001). Four norms were highlighted in this table. Explanations norm consist of mathematical arguments (and not merely procedural descriptions) and summaries of the steps to solve the problem. Errors offer opportunities to reconceptualize a problem and explore contradictions in solutions and alternative strategies. Mathematical thinking involves understanding connections among several strategies. Collaborative work involves individual accountability and getting consensus through mathematical argumentation. These four sociomathematical norms are significant in TTPS for classroom practices to promote deeper understandings of mathematical concepts and ideas. Stephan and

Whitenack (2003) asserted that “sociomathematical norms is very important if genuine problem solving to take place” (p. 154). This kind of mathematical discussions in TTPS make problem solving accessible to all students (Stephan & Whitenack, 2003).

Table 2.2

Sociomathematical Norms

Sociomathematical Norms	Examples
Explanations	- Could you explain why you did it in this way? - Can you show us what you mean by that?
Errors	- So, you’re saying that’s not all possible solutions of grouping the number. Or, what are you saying? - Why don’t you agree with that?
Mathematical thinking	-Well, how could you show me this in a different way? -What did they use or do that was different than what you might have done?
Collaborative work	- What did your team come up with? - Did you come to a consensus with your partner or partners about that solution?

In sum, the **launch** phase includes posing an authentic problem in a high-cognitive demand and presenting the challenge of the problem. **Explore** phase includes allowing time for students to explore and solve the problem by themselves while the teacher pose purposeful questions to facilitate students’ thinking. The teacher has an opportunity to interact with individual and groups to provide needed scaffolding. Planning for the whole class discussion is also an important strategy that teachers can use in the exploring phase. While in the **summarize** phases, teachers can conduct a whole class discussion to allow students to connect their solution strategies to other strategies and to the new learned mathematical concepts or ideas. Talk moves and sociomathematical norms are important teaching practices in TTPS to make problem solving accessible for all students. These three phases (launch, explore, summarize) encompass shared pedagogical moves of TTPS that are essential to reinforce teaching mathematics for understanding.

Empirical Research on TTPS

TTPS is an effective teaching approach to develop students' understanding of and performance in problem solving. J. Bostic (2011) examined the effect of TTPS on students' performance by comparing two sixth grade classrooms in which one was taught using teaching through and the other for problem solving, finding a positive effect on the performances and use of representations among the TTPS group. In addition, Bostic and Jacobbe (2010) found that TTPS significantly promote students' mathematical problem-solving discourse. Similarly, another study conducted by Bostic, Pape, and Jacobbe (2016) compared TTPS and a traditional teacher-led explicit instruction, and found that daily TTPS instruction significantly impacted students' problem-solving performance and content knowledge.

TTPS is a complex approach to teaching. It is challenging because teachers need to have adequate knowledge of the mathematics that is relevant to student solution strategies. Teachers need to be able to recognize whether and how students' solution strategies are connected to the key mathematical concepts that need to be learned. Thus, an increasing number of studies examined the facilitation of TTPS. For example, Sakshaug and Wohlhuter (2010) designed a *Teaching Elementary School Mathematics* graduate course. This course was designed to facilitate elementary teachers' learning to teach mathematics through problem solving. The teachers engaged in problem solving activities during class meeting and they were required to do problem solving research in their classrooms. The results of Sakshaug and Wohlhuter's (2010) study revealed that some teachers established a strong foundation to teach through problem solving. For others, using TTPS was more tentative. However, all the teachers in Sakshaug and Wohlhuter's (2010) study understand the importance of allowing students to work in groups to solve problems. The teachers also became more confident in their students' abilities to be successful problem solvers.

Another study was conducted by a group of elementary school teachers who were trained to be mathematics teacher leaders in their district (Burmeister et al., 2018). This group of teachers discussed some main challenges that they hear from other teachers that may prevent them to implement TTPS. The group provided suggestions about how teachers can tackle these challenges and shift their instruction to TTPS. These challenges were: (a) I do not have the time to spend in problem-solving tasks, (b) I can use problem-solving tasks only at the end of a unit, (c) Not all learner are capable of engaging in the same problem-solving task, (d) My English Language Learners will not understand the problem-solving tasks, (e) I cannot find any problem-solving tasks for my grade level, (f) I cannot find problem-solving tasks that are culturally relevant to my students, and (g) I cannot let them struggle-I might make them cry. The results of Burmeister et al.'s (2018) study identified two key aspects to successfully implement TTPS. They were collaborating with other educators and building a set of high-cognitive demand problems.

For middle and high school grade levels, Selling (2016) investigated how teachers facilitate a mathematical discourse when using TTPS. Selling indicated that teachers could make mathematical discourse explicit without lowering the level of cognitive demands of problems or reducing students' opportunities to engage in problems. The results of Selling's study presented eight talk moves in TTPS that should be enacted after students had participated in solving a problem. Five of these talk moves support mathematical noticing. These five talk moves were highlighting, naming, making evaluative statements, explaining the goal or rationale, and connecting student engagement. The other three talk moves support situated mathematical practices. These three talk moves were framing expansively, eliciting self-assessment, and making the teaching narrative explicit (Selling, 2016). In addition, Wilhelm (2014) investigated 213 middle school mathematics teachers' enactment of high cognitive-demand problems as developed by Stein et al. (2000), when using curriculum that supports the TTPS approach (e.g., Connected Mathematics Project 2 [CMP2]). The results indicated that the mathematical content knowledge and visions of high-quality mathematics instruction of teachers who use TTPS curriculum were significantly and positively related to teachers' selection, enactment, and maintaining of the higher-level of cognitive demand problems.

Interpreting and Implementing SMPs

The eight SMPs that accompany the CCSSM (2010) are meant to capture the processes and proficiency that students should have such as thinking skills and habits of mined that are specific to mathematics. These standards describe ways to engage students with mathematical content and are carried out across all grade levels. In the mathematics education literature, several studies have focused on facilitation, interpretation, and implementation of SMPs. In the following paragraphs, I provide research from the mathematics education literature that focuses on interpreting and implementing each SMP.

A group of middle school teachers found four strategies to effectively understand and implement SMP1 (Make sense of problems and persevere in solving them)

(Wilburne, Wildmann, Morret, & Stipanovic, 2014). These four strategies are useful in changing students' behavior and their thinking to effectively implement SMP1. The first strategy is called *Does it make sense*. For this strategy, the teacher introduces problems with incorrect answers and then challenges students to check the reasonableness of the solutions. Strategy two, *The Process of Elimination*, involves the teacher presents multiple choice problems with both reasonable and unreasonable solutions while students think about the problem and determine reasonable solution. For the strategy three, *Perseverance Log*, the teacher discusses the meaning of perseverance and asks students to keep a perseverance log that includes the problem and how they persevered in solving them. The last strategy is called *Analyzing Incorrect Responses*. For this strategy, the teacher highlights the common errors in solving problems and asks students why the strategy or solution does not make sense. Research by Keazer and Gerberry (2017) clarified a common misunderstanding of SMP1 among preservice teachers who see SMP1 as a process of relying on a procedure instead of relying on thinking. However, Keazer and Gerberry (2017) asserted that the intent of SMP1 is to shift from traditional emphases on procedures toward a focus on thinking. This shift can be facilitated by selecting worthwhile, open-ended, non-routine, and high-cognitive demand problems. Teachers also need to change classroom norms to involve accepting every student's idea and posing students' ideas to the class for consideration and evaluation.

Like SMP1, mathematics teachers have different interpretations of SMP2 (Reason abstractly and quantitatively) which affect their implementation of this practice. As stated by Bleiler, Baxter, Stephens, and Barlow (2015), "Teacher participants in a summer institute unpacked the meaning of the SMPs during intense conversations" (p. 338).

Bleiler et al. conducted a summer institute program for 57 mathematics teachers (grade 3-6) that focused on implementing the CCSSM. This program aimed to describe the prominent interpretations of the SMPs, provide clarifying examples of SMPs that are aligned with the constructed meanings of the teachers, and stimulate discussions that focus on teachers' interpretations of SMPs across the U.S. A large majority of the participants identified SMP2 as the standard that makes the least sense to them. The teachers identified the term *abstract* as one of the most difficult terms to interpret. Some teachers interpreted abstract as "using formulas to represent mathematical ideas in general" (p. 338), while other teachers defined it as "think outside the box or by referring to abstract artists, such as Picasso" (p. 338).

To understand the meaning of SMP2, teachers in Bleiler et al. (2015) program looked for an explanation of how to engage students in the processes of contextulization and decontextualization through problem solving. To clarify the meaning of these processes, Bleiler et al. asked the teachers to solve a mathematical problem and then describe how the engagement processes in the contextulization and decontextualization. Bleiler et al.(2015) concluded that teachers engaged in decontextulizing through problem solving when "they ignored the contextual details of the problem and moved to a more abstract setting" (p. 338). In other words, the teachers characterized the process of decontextualization by finding all possible solutions of the presented problem without necessary attending to the details in the context of the given problem. The process of contextualization was characterized when the teachers recognized which one of the solutions was considered a contextually good solution (that make sense for the context of the problem).

To interpret and implement SMP3 (Construct viable arguments and critique the reasoning of others), Singletary and Conner (2015) applied a research-based framework to describe how SMP3 is interpreted by researchers and suggest a way to implement this standard. They stated that collective argumentation occurs when students work together as groups to arrive at an evidence-based conclusion that progresses from intuition toward deductive reasoning, while the role of the teacher is to ask questions that support collective argumentation. These questions focus on requesting a factual answer (e.g., what is the square root of 25?), method (e.g., how did you get this solution?), idea (e.g., by looking at this graph, what concluding do you arrive?), elaboration (e.g., how do you arrive at the answer?), and evaluation (e.g., do you agree or disagree and why?). These questions are helpful in implementing SMP3.

Research indicates that there is confusion between how to *model the mathematics* and how to *model with mathematics*. Bleiler et al. (2015) clarified that the intent for SMP4 is *model with mathematics*. Some teachers interpret SMP4 as *model the mathematics* which is defined as "using concrete materials or other visual representations to clarify or give meaning to mathematics" (Bleiler et al., 2015, p. 339). With *model the mathematics* teachers usually encourage students to use multiple representations (e.g., manipulatives, graphs, and diagrams) to provide different ways to model a mathematical concept (NCTM, 2000). SMP4 should focus on "using mathematics to describe and/or explain a real-world context" (Bleiler et al., 2015, p. 339). This leads to use knowledge of mathematics to analyze or solve real-world problems.

SMP5 (Use appropriate tools strategically) involves students using available tools such as paper and pencils, protractors, rulers, number line, graphing calculators, and software that is grade appropriate aid in solving a problem (CCSSM, 2010). Students should be able to use technology or any other resources to visualize their assumptions and deepen their understanding to solve problems. Sherman and Cayton (2015) developed a research-based framework to facilitate an implementation of SMP5. This framework linked the use of technology with the goals of instruction. This framework can be used as a tool for assessing how the use of technology supports the goals of instruction. They stated that technology can be used as *Amplifier* (the mathematical goal of the instruction would be similarly achieved without the technology) or *Reorganizer* (the mathematical goal of the instruction would be difficult to achieve without technology). The goals of instruction can focus on three aspects: (a) making mathematically meaningful observations (includes looking for invariant relationships), (b) making mathematical exploration (includes using appropriate tools strategically), and (c) making and testing conjectures (includes modifying thinking and fostering curiosity).

SMP6 (Attend to precision) means that “mathematically proficient students try to communicate precisely to others.” (CCSSI, 2010, p. 6). The goal of precision is to create clarity in mathematical communication with others. Students should work to be precise and clear in both speech and writing. Koestler, Felton, Bieda, and Otten (2013) facilitated an understanding of SMP6 by linking it with both the communication and representation standard in the NCTM (2000) Principles and Standards for School Mathematics. They provided examples of how to implement the practice of attending to precision in the elementary and secondary levels. By linking SMP6 with the communication standard, it can be implemented when teachers have a goal for students to develop a set of mathematical skills and knowledge and become capable to communicate their mathematical ideas. Linking SMP6 with the representation standard directs teachers' attention towards students' facility with expression of numbers and calculations that have implications for precision. Koestler et al. (2013) also noted that attending to precision is "a process of building on students' informal knowledge, not a replacement of it" (p. 73) and "it is possible for informal language to be perfectly precise" (75). In other words, to be precision, students do not necessarily have to memorize or increase vocabulary assignments; instead, precision can be attended to in different contexts of communication including formal and informal language.

Research by Bleiler et al. (2015) indicates that there is a difficulty to distinguish between SMP7 (Look for and make use of structure) and SMP8 (Look for and express regularity in repeated reasoning) among teachers. Some teachers perceive both SMP7 and SMP8 as engaging students in mathematics through identifying structures and patterns in a problem, without a clear cut between them. Bleiler et al. (2015) provide a clear distinction between these two standards. They offer that SMP7 "refers to the exploitation of structure within a given problem and … [SMP8] refers to the exploitation of structure across different problems" (p. 342). This nuanced difference means that students need the opportunity to "look for and make use of structure" within a problem (goal for SMP7), and then use their observations from that problem to "look for and express regularity in reasoning" across other similar problems (goal for SMP8).

The Alignment between SMPs and TTPS

Both the content and process standards in CCSSM (2010) focus on using problem solving to learn mathematics with understanding and require major shifts in classroom practice (Takahashi, Lewis, & Perry, 2013). Thus, a number of researchers have investigated how to facilitate an implement of the CCSSM through problem solving. For example, Takahashi et al. (2013) designed a teaching through problem solving project to facilitate an implementation of the CCSSM. This project incorporates high cognitive demand problems from a Japanies curriculum matereals that support TTPS with a lesson study. The results of Takahashi et al.'s (2013) study revealed that TTPS targets a central problem when implementing the SMPs. That is, using the TTPS approach supports mathematics teachers to change their teaching practices in a way that aligns with changes envisioned in the SMPs.

Foote et al. (2014) published a book intitled *Implementing the CCSSM through Problem Solving* . This book supports a connections between CCSSM and TTPS by helping teachers to interpret the CCSSM using examples of how TTPS supports the interpretation of CCSSM. The authors of this book believe that TTPS is the most effective way of implementing the CCSSM. I agree with these authors that there is a strong alignment between TTPS and the CCSSM because TTPS has a potintial to facilitate mathematics teachers' understanding of how to implement the CCSSM through problem solving.

Interpreting and Implementing MTPs

The SMPs focus on what students should do to learn mathematics, but they do not provide direction about what teachers should do to teach mathematics. Consequently, the NCTM (2014) published the book *Principles to Action: Ensuring Mathematics Success for All* to "fill the gap between the development and adoption of CCSSM and other standards and the enactment of practices . . . required for their widespread and successful implementation" (p. 4). In *Principles to Action*, a set of eight MTPs provide direction on what teachers should do to engage students in the SMPs in an effort to promote a deeper understanding of mathematics.

Using limited modules in *Principles to Action* provided information of what the MTPs would look like at each grade level. Thus, an increasing number of books and studies were published about the MTPs. For example, the set of grade-band books *Taking Action: Implementing Effective Mathematics Teaching Practices* (in grades Pre-K-5, 6–8, and 9-12) were published to foster teachers' development of the MTPs in each grade level. Another book, *Enhancing Classroom Practice with Research behind Principles to Actions*, was published by Spangler and Wanko (2017) to summarize and synthesize the extensive body of research behind the big ideas in *Principles to Action* and to also provide examples of what MTPs would look like at each grade level (Pre-K-2, 3-5, 6-8, and 9-12). Many studies also were published to help teachers to understand MTPs in the context of each grade level by explaining how to implement one or more MTPs through classroom vignettes or using mathematical problem scenarios related to specific grades (e.g., Nabb, Hofacker, Ernie, & Ahrendt, 2018; Thomas, 2017).

There has been an increasing focus in the mathematics education literature on MTP7 (supporting productive struggle in learning mathematics) (e.g., Hiebert & Grouws, 2007; Townsend, Slavit, & McDuffie, 2018; Warshauer, 2015). The NCTM (2014) defined productive struggle as students delving "more deeply into understanding the mathematical structure of problems and relationships among mathematical ideas, instead of simply seeking correct solutions" (p. 48). Struggling can be productive if teachers consider posing mathematical problems within students' zone of proximal development (Townsend, Slavit & McDuffie, 2018). In order to have students struggle productively, the given mathematical problems should not be overly difficult nor needlessly frustration. The proper nature of mathematical problems and the support level needed for students should be highly considered. In contrast, unproductive struggle is defined by Hiebert and Grouws (2007) as students who “make no progress towards sense-making, explaining, or proceeding with a problem or task at hand” (as cited in NCTM, 2014). Unproductive struggle occurs when teachers see their students frustrated or having lack of immediate success in solving a problem. As a result, teachers often guide their students step by step through the difficulties. This guidance lowers the cognitive demands of the problem or task.

In addition, Warshauer (2015) found that productive struggle can support the role of doing mathematics and has implications on student learning with understanding. According to Townsend, Slavit, and McDuffie (2018), teachers can use four components to support all students in productive struggling: *Task scaffolding*, *Norms for supportive interactions with peers*, *Teacher motivation*, and *Reassuring classroom environment*. With the first component, *Task scaffolding*, teachers should create tasks that required students to use multiple representation and make mathematical connections within real-world contexts to encourage productive struggle. To support students’ engagement in productive struggle, teachers may bold relevant vocabulary, define key terms, or provide sentence starters to build students’ confidence with mathematical writing. They may also add structures such as ‘y =’ and (____, ______) to clarify what the questions in task expects (Townsend, Slavit & McDuffie, 2018).

The second component is *Norms for supportive interactions with peers*. Support in the classroom does not always come from the teacher. Students may also receive support from their peers when working together in groups. Townsend, Slavit, and McDuffie (2018), Cohen, 1994, and Swing and Peterson (1982) found that students had higher achievement from participation in heterogeneous groups than participation in homogeneous groups. Grouping students heterogeneously not only benefits students with lower ability when they receive explanations, it also benefits students with higher ability because they give explanations to others. Indeed, “the student who does the explaining is the student who benefits” (Cohen, 1994, p. 9). Teachers can also set norms to develop supportive interactions with peers. These norms are a set of agreements statements such as: “You have the right to ask anyone in your group for help,” “You have the duty to assist anyone who asks for help,” and “No one is as smart as all of us together” (Townsend, Slavit & McDuffie, 2018, p. 222). By using norms such as these students would be apt to listen to each other when working in their groups.

The third component is *Teacher motivation*. Teachers can motivate students when they care *for* students, not just care *about* them. Caring for students includes "being available to offer assistance throughout the day, showing interest in their [students'] life experiences and opinions, and giving [students] ... undivided attention during class time” (Townsend, Slavit & McDuffie, 2018, p. 223). That is, building caring relationships with students allows teachers to give their students feedback through encouragement and praise on their unique strategies and willingness to help their peers.

The fourth component is *Reassuring classroom environment*. Developing a classroom environment lead to classroom norms for respect. When students respect each other, they will believe that classroom is a safe place for them to talk and make mistakes. For example, norms such as “listen to others’ ideas [and] disagree with ideas, not people," (Townsend, Slavit & McDuffie, 2018, p. 223) can lead students to engage and participate in mathematics. Moreover, teachers can provide feedback when students are in the explore or summarize phase of a lesson and allow students to revise their work. These opportunities ensure that students persevere in solving tasks through productive struggle (Townsend, Slavit & McDuffie, 2018).

The Alignment between MTPs and TTPS

Although there is an extensive focus on MTPs in the mathematics education literature, only a small number of empirical studies have been conducted to investigate the alignment between MTPs and TTPS. Bailey and Taylor (2015) found that TTPS benefits teachers' enactment of effective teaching practices such as MTPs. They found that having an experience in TTPS is an important first step towards novice teachers' learning how to implement the MTPs. They asserted that using TTPS promote teachers' understanding of the importance of teaching practices such as justifying mathematical reasoning and emphasizing conceptual understanding through problem solving.

Another study was conducted by Prince (2016) to investigate how lesson study can be used to aid eighth-grade mathematics teachers in perceiving and implementing MTPs when using TTPS. The findings from Prince's study revealed that reading MTPs can support the development of teachers' initial conceptions of MTPs. When teachers observe their peers teaching and discuss MTPs during a lesson study about TTPS, these actions enhanced their conceptions and implementations of MTPs.

However, in these two studies, the focus was on teachers who were novice in using TTPS and examined how the lesson study develop their understanding and implementation of the MTPs. They did not focus on how teachers who have years of teaching experience using TTPS interpret and implement both the SMPs and MTPs. It is important to address how teachers experienced in TTPS implement the SMPs and MTPs in order to understand the impact on teachers' interpretation and implementation during classroom instruction.

Significance of Study

To my knowledge, there are no studies that focus on implementing both SMPs and MTPs using the TTPS approach. This dissertation has the potential to provide teachers with resources to understand and implement SMPs and MTPs using TTPS that blend problem solving into daily lessons to learn mathematics with understanding. Instructional models stemming from research conducted in classrooms may support mathematics teachers to effectively implement the SMPs and MTPs through problem solving.

This dissertation will provide direction and insight for middle school mathematics teachers on how to use problem solving instruction as a mean to teach mathematics, which will serve as an instructional model of implementing the SMPs and MTPs using TTPS. This dissertation may also provide valuable information for district leaders and those who work in professional development programs in mathematics education to use problem solving as a way to aid teachers in implementing the SMPs and MTPs.

Chapter 3: Methodology

Introduction

This dissertation argues that a focus on problem solving aligns TTPS with the recent reform in mathematics education such as SMPs and MTPs. Therefore, the purpose of this dissertation is to understand middle-school mathematics teachers' interpretation and implementation of SMPs and MTPs when using TTPS within a district in north-east Ohio. The following two research questions guide this research:

1) How do middle school teachers whose districts support TTPS, interpret and implement the CCSSM (2010) Standards for Mathematical Practice when teaching?

2) How do middle school teachers whose districts support TTPS, interpret and implement the NCTM (2014) Mathematics Teaching Practices when teaching?

At the outset, I will start with the research design and method of inquiry of this dissertation, a brief description of the site of study which will be complemented by descriptions of participants. I will then move on to the data collection section, which describe the methods and procedures for collecting data as well as the process of analyzing data, followed by the sections of ethical considerations, validation, and limitations and delimitations.

Research Design and Methodology

This dissertation aims to understand and describe teachers' interpretation and implementation of SMPs and MTPs when using TTPS. Because the selection of the research approach depends on the purpose of the study, a qualitative research approach is appropriate based on the questions I am asking. A qualitative approach can be used for "understanding the meaning individuals or groups ascribe to a social or human problem"

(Creswell, 2014, p. 32). The descriptive and interpretive components of a qualitative approach are useful in this dissertation to provide descriptions and insights about how mathematics teachers understand and implement SMPs and MTPs.

Moreover, because of the nature of the qualitative approach as inductive reasoning (Creswell, 2007), qualitative research can guide the analysis process of this dissertation. Thus, the constructivist views of learning and pedagogy of TTPS helped me analyze and understand the themes emerged from the teachers' interpretation and implementation of SMPs and MTPs. Qualitative research can be also used as a justification and interpretation for educational reform and change (Creswell, 2014). As a result, the participants and leaders of school districts may be interested in the findings of this dissertation. Therefore, the use of a qualitative study is beneficial.

More specifically, the method of inquiry that frames how I examined teachers' interpretation and implementation of SMPs and MTPs in TTPS was the case study approach. Case study involves "a qualitative design in which the researcher explores in depth a program, event, activity, process, or one or more individuals" (Creswell, 2014, p. 290). With case study, the data collection is extensive. Using multiple sources including interview, observation, and documentation allowed me to have detailed descriptions and examine the participants' experience of the phenomena in more depth.

Role of the Researcher

As a mathematics teacher with a disciplinary background in pure mathematics, I taught upper elementary and middle school levels in Saudi Arabia. Although I used several approaches to teach mathematics, I never adopted the TTPS approach as I did not know how to use it properly. However, during my master's degree, I read about, observed, and watched several videos of teachers who use TTPS.

At present, as a doctoral candidate in a curriculum and instruction program, my research interest lies in discovering more details about TTPS and how TTPS facilitates the interpretation and implementation of mathematics standards. From my experience in teaching mathematics in my country (Saudi Arabia), there was minimal focus on developing students' thinking and problem-solving skills. The focus was on merely transferring information to students and ensuring they got high tests scores. Thus, my interest in TTPS guides my beliefs and understanding in pointing out the limitations in mathematics education in Saudi Arabia and providing guidance in order to develop a potentially productive educational reform.

Selection of Site and Participants

This dissertation focuses on middle-school mathematics teachers from one school. This school was selected because their district supports its teachers to implement TTPS. The district has used a problem-based curriculum since the 1990s and conducts professional development programs to support teachers using the curriculum. Through personal communication with a person who works in the district and organized professional development programs, I gathered information about how teachers were supported to understand and implement TTPS throughout the years. The mathematics educators and professional development leaders of the selected district believe in the potential of TTPS to develop students thinking, conceptual understanding, and procedural fluency for all students. Thus, they selected the recognizable problem-based curriculum series Connected Math Program (CMP1, CMP2, and currently CMP3) and supported teachers to implement this curriculum via many professional development programs during across last 30 years.

When the district adopted CMP1, the teachers were provided either a half or a full day of grade-level professional development for sixth-grade, seventh-grade, and eighth-grade teachers. They also have had math professional development meetings throughout the year where the teachers would study and discuss the curriculum, work mathematics problems posed to students, and discuss teaching strategies. When the district adopted CMP2, the teachers were provided a weeklong summer training for each grade level. The teachers also were supported when the district adopted the current series (CMP3) by attending a two-day workshop at Michigan State. They also sent two teachers from each building to the weeklong training at Michigan State. Through teacher-led professional development planning, the teachers at each grade level work together, talk together, and plan together. Although the district has three middle schools at the time of this study, only six teachers in two schools still use the CMP3 curriculum and a TTPS approach. This is because currently teachers have the choice to select their curriculum and teaching approach. Two of the schools chose to use the curriculum exclusively while one school not so exclusively.

Participants

The number of participants were three teachers. I chose one participant in sixth grade, one in seventh grade, and one in eighth grade because I wanted to look at TTPS across middle-grade levels (6-8 grade). Creswell (2007) recommended that the number of participants in a single case study would not be more than 4 or 5 cases. Analyzing data for a large number of participants and transcribing data from observations take time because proper analysis is time consuming. Consequently, to have the research done in a relatively short period of time, I have to adjust the number of participants to three.

The participants were teachers who have experience in using TTPS. They were selected purposefully to "learn a great deal about issues of central importance to the purpose of the research" (Patton, 1990, p. 169). One of these participants— a teacher in the selected district who uses TTPS—was part of a previous study conducted. The selection of the other two participants was done using the Snowball sampling method from this teacher, who knows other teachers who use TTPS within the same district.

Data Collection

The study took place during one semester (two quarters) of a school year. The methods of collecting data for this dissertation were interview, lesson plan, and observation. These methods are useful to collect data about teachers' experiences from different viewpoints. The data from each one of these methods was used to deepen my understanding of participants' experiences as well as minimize errors in describing and interpreting the data. These multiple sources of data collection also support triangulation.

Interview

Using an interview method strengthened my study because it afforded the opportunity to probe for a deeper understanding of the phenomenon. Face-to-face, in-depth, and semi-structure interviews were conducted with each participant. Using interview as an instrument to collect data offered access to the views and experiences of the participants. I used a semi-structured interview style because the semi-structured nature allows participants to have "a fair degree of freedom in what to talk about, how much to say, and how to express it" (Drever, 1995). Adequate recording procedures were considered when conducting the interviews. The interviews were last 45 minutes at minimum and were conducted in a quiet location that was free from any distraction. The interview questions were developed by the researcher (See Appendix B). The data from interviews was transcribed and then analyzed.

Documents

"In making work and other phenomena visible, documents play a key role" (Prior, 2003). In this dissertation, I used teachers' lesson plans as documents to collect data about teachers' lesson details and process. Each participant was asked to submit a lesson plan. The data from lesson plans was used to make sense of the lesson's tasks, confirm that the lessons are focused on TTPS, and whether appropriate for this dissertation. I kept a copy of lesson plans during the research study to have concrete materials during the data analysis stage.

Observation

To gain access to the selected site to be observed, I first obtained the required permissions. At the site, I identified for how long and when to observe, depending on the participants schedule and convenience. My role at the site was an outsider observer because I wanted to maintain a critical outlook. During observations, I recorded descriptive and reflective notes as well as my own reactions. Creswell (2007) asserted the importance of gathering fieldnotes such as notes about researcher's experiences, hunches, and learnings when conducting an observation. The participants were also videotaped during the observations because videotapes "give us the facility through which to re-visit the aspect of the classroom that we have recorded" (Pirie, 1996, p. 3).

The data from video recording formed my data and not merely facilitate its collection. This is because I looked to the videos several times with different focus such as mathematical discourse in the classroom, teacher's pedagogical moves, and the details for implementing of SMPs and MTPs. I also used these videos to look for commonalities and differences in teaching practices among the participants. For the purpose of this dissertation, videotapes captured the teachers' moves and interactions with their students in the classroom, and not students working alone. Thus, the classroom discussions between the teacher and their students were videotaped during the observation because I focused on the manner in which the teacher leads classroom discussions. In addition, I videotaped the students' solution strategies (such as a concrete model, logical argument, algebraic proof, and other solutions) of a given mathematical problem. This is because I wanted to know how the teacher would monitor, select, sequence, and connect students' solution strategies to present them in a whole class discussion.

Data Analysis

In the data analysis, I provided a detailed description of each case, called a *within-case analysis*, followed by a thematic analysis across the cases, called a *cross-case analysis*. These two steps of analysis within and across the case added more depth and detailed for my study. Then I scanned all of the database that I gathered from interviews, observations, and lesson plans to identify main organizing ideas and to reflect on the major thoughts and actions represented by the participants. This helped to form initial themes and then to move to the process of detailed describing, classifying, and interpreting the data.

Specifically, for case studies, "analysis consists of making a detailed description of the case and its setting" (Creswell, 2007, p. 163). Two main phases were represented by Creswell (2007) were involved in analyzing this current case study, direct interpretation and naturalistic generalization. In the direct interpretation, I looked at each participant alone and drew meaning from her interpretations and practices without looking for the other participants. Then, I looked for similarities and differences among the participants. This is what Creswell described as "a process of pulling the data apart and putting them back together in more meaningful ways" (p. 163). In the naturalistic generalizations, I generalized what people can learn from these cases to apply it for themselves or to use the information in these cases to apply it to other cases.

Interview Analysis

After transcribing the responses of interview questions using the website transcribeme.com, I went through transcriptions to make sure that the transcriptions were correct. Then, I described in detail the context of the setting of the participants and their schools and classrooms. Creswell (2007) asserted that "description becomes a good place to start in a qualitative study (after reading and managing data), and it plays a central role in ethnographic and case studies" (p. 151). Then, I developed short list of categories to winnow the data (not all information will be used).

I developed 6 categories to organize the data from each participant. Then, I stepped back and formed larger meanings of what was going on to look for the big picture. I engaged in interpreting the data by making sense of the phenomena and seeking to elucidate meaning that is implicit in the data. Then I looked for themes across the participants' responses. I worked to reduce and combine these themes into four themes as recommended by Creswell (2007). These themes ranged from *predetermined* themes from the literature to *emergent* themes during analysis.

Lesson Plan Analysis

Prior (2003) asserted that "for ease of analysis, it often makes sense to focus on documents in which written words serve as the mastercode" (p. 5). Thus, I used the lesson plans' headings to code the data from lesson plans as well as teachers' moves in the classrooms. These lesson plans helped me to make sense of the flow of lesson and teacher's actions during the instruction.

Observation Analysis

The videos and field notes were analyzed using two protocols. The first protocol had a focus on the implementation of SMPs, which is the Mathematics Classroom Observation Protocol for Practices (MCOP^2) (See Appendix C). This protocol was adapted from (Gleason, Livers, & Zelkowski, 2017) that "measures the degree to which actions of teachers and students in mathematics classrooms align with practices recommended by national organizations and initiatives" (p. 111). Validation of (MCOP^2) involved feedback from 164 professionals in mathematics education, and reliability involves inter-rater, internal reliability, structure analyses via scree plot, and exploratory factor analysis. Using MCOP^2 protocol allowed me to assess teachers' implementation of SMPs.

The second protocol had a focus on the implementation of MTPs, which is the Mathematics Teaching Practices Observation Protocol (MTP-OP). This protocol was adapted from (Prince, 2016) and was intended to examine the alignment of middle school mathematics lesson with the MTPs (See Appendix D). The MTP-OP was pilot tested with three middle school lessons and then further aided before using it in the Prince's (2016) study. I used the MTP-OP to assess teachers' implementation of MTPs.

Ethical Considerations

Complying with ethical guidelines, I obtained informed consents from the teachers and guardians of all students in the participants' classes. Informed consent is "a mechanism for ensuring that people understand what it means to participate in a particular research study so they can decide in a conscious, deliberate way whether they want to participate" (Mack, Woodsong, MacQueen, Guest, & Namey, 2005, p. 9). I informed the participants about the study (including the purpose of research, how confidentiality will be protected, and expected risks and benefits) in a simple way to make certain they understand the processes using an informed consent letter (See Appendix E). I ensured confidentiality by keeping the videos, audio records, and documents in a safe place (in a portable external hard drive in a locked drawer in my office) and giving the participants pseudonyms in this research. The final ethical issue was I did not share my own personal experiences with participants during an interview. Such sharing reduces the "bracketing" (Creswell, 2007). Further, I did not start the research until I obtained an IRB approval.

Validity

I used two strategies to validate the findings—triangulation and peer debriefing:

- Triangulation—To triangulate the data collection, I conducted an in-depth interview with the participants, collect some of the participants lesson plans documents, and observed the participants in their classroom while they teach.

Then I tried to make sense of the all database from the three sources to check for consistency between these sources of data.

- Peer debriefing— "Peer review or debriefing provides an external check of the research process" (Creswell, 2007, p. 208). I shared the research process and findings with one of my colleagues in the Curriculum and Instruction program. The role of this peer debriefer is to keep the researcher unbiased and ask questions about methods and interpretations. This process can validate the findings of this dissertation.

Limitations and Delimitations

There are some limitations to this dissertation. The first limitation falls in the use of videos. Some teachers may have anxiety from videotaping them during teaching. Another limitation was that teachers may have different ways to engage students with mathematics depending on the mathematical content. Engaging students with mathematics in geometric topics may appear differently than engaging them in algebraic topics. It is time consuming to cover all middle school mathematical content. Thus, I focused on just few topics in mathematics. In addition, there are two delimitations of this dissertation. The first one is that I focused on middle school levels even though TTPS can be used in K-12 grade levels. My interest in middle school levels is derived from my previous experience in teaching these levels. The second delimitation is that my participants were from one school in the selected district.

Chapter 3 Summary

The purpose of this dissertation is to investigate teachers' interpretation and implementation of Standards for Mathematical Practices (CCSSM, 2010) and

Mathematics Teaching Practices (NCTM, 2014) when using Teaching Through Problem Solving. A qualitative research approach was appropriate for this dissertation because I wanted to understand the meaning of teachers' experience. More specifically, the use of the case study approach was appropriate to collect data about teachers who had unique experiences when their district supports them to use TTPS.

I used interviews, documentation, and observation to collect data from the participants. The interview tool was used to collect data about how teachers interpret the SMPs and MTPs. The interviews were recorded, transcribed, and then analyzed using open coding and thematic analysis. In addition, each participant was asked to submit a lesson plan. The data from lesson plans was used to make sense of the lessons' tasks. I observed the teachers in their classrooms to collect data about their teaching moves when using TTPS along with how these moves aligned with the implementation of SMPs and MTPs. These observations were videotaped (with a focus on teachers' actions) and field notes were token. I analyzed these videos using two protocols: $MCOP^2$ to assess teachers' implementation of SMPs and MTP-OP to assess teachers' implementation of MTPs.

Chapter Four: Findings

Introduction

Teachers across the United States are encouraged to implement the Standard for Mathematical Practice (SMPs) and the Mathematics Teaching Practice (MTPs) to improve their instructions (NCTM, 2014). However, implementing these standards and teaching practices is not obvious for many teachers. Many teachers need help to understand and implement SMPs and MTPs. Thus, I focused on how Teaching Through Problem Solving may facilitate a successful interpretation and implementation of both SMPs and MTPs.

This case study attempted to investigate how three teachers interpret and implement SMPs and MTPs when their district supports them to use TTPS. I used data from the three teacher participants to examine the following two research questions:

3) What is middle school teachers' interpretation and implementation of the CCSSM (2010) Standards for Mathematical Practice when their districts support TTPS?

4) What is middle school teachers' interpretation and implementation of the NCTM (2014) Mathematics Teaching Practices when their districts support TTPS?

With these guiding research questions in mind, I carefully studied the data from teacher participants that included one lesson plan, a pre- and post-interview about the lesson, and one to two days of teaching observation for that lesson plan. My goal was to describe how the teacher participants interpret and implement SMPs and MTPs individually and comparatively.

I used six categories to organize the data of each participant. These categories were: (1) typical lesson as described by teacher, (2) interpretation of SMPs as described by teacher, (3) interpretation of MTPs as described by teacher, (4) lesson as observed by researcher, (5) implementation of SMPs observed by researcher, and (6) implementation of MTPs observed by researcher. In the first part of this chapter, I present individual case (within-case analysis) for Hana, Teresa, and Grace. Then, in the second part, I performed a cross-case analysis of the three teacher participants. The results from within- and cross-case analysis revealed that the teachers understood and implemented the SMPs and MTPs when using TTPS. However, they did not implement all SMPs and MTPs when using Teaching For Problem Solving (TFPS). More explanation of these results will be provided within each case.

Within-Case Analysis

Hana's Case

Hana is a sixth-grade female teacher with 19 years of teaching experience in reading and mathematics. She has a bachelor's degree in education for grades 1 to 8, as well as a master's degree in education. The teacher is currently teaching two levels of mathematics classes: Math 1 and Math 2. Math 1 is the lower level for students who follow an Individualized Education Program (IEP) and their learning is at a slower pace. Math 2 is for students who have different abilities in mathematics. Some students in Math 2 have IEP, but need extended time or preferential seating. Hana only used the TTPS approach with Math 2 students because it is hard for Math 1 students to make sense of problems or answer questions that require high-level thinking. This case study used classroom data collected from one of Hana's Math 2 classes.

Typical Lesson as described by Hana. Hana's responses to the pre-interview questions revealed that she used the TTPS approach to help students learn mathematics with understanding. Before using the TTPS approach, she always was trying to get students to understand what they were learning but it was so hard for her to do that. When she used the TTPS approach, it helped her to get students to make sense of mathematics in real-world contexts and also remember what they learned. Hana appreciated how using the TTPS approach helped in facilitating the learning of mathematics for different types of learners (e.g., visual, auditory, and kinesthetic) by allowing students to solve problems in different ways.

During the pre-interview with Hana, I asked her to describe her daily lessons, for Math 2), so I could have an idea about what I might see when I observe her lesson. Hana described her typical lesson according to the following sequence: Introduction, launch phase, explore phase, summarize phase, conclusion. During the introduction, she began her daily lesson with a warm-up activity or going over homework. The purposes of warm-up activities are to get students to start brainstorming about the target goal for the day's lesson, to ask for students' background information, and to have students understand why they are learning about a topic. Next, in the launch phase, she presents the challenge in a problem without telling students how to solve it. In the explore phase, she gives students 30 to 45 minutes to investigate a pathway to solve the given problem. The students work in groups, with a partner, or on their own while the teacher provides scaffolding for students when they struggle. In the summarize phase, the students share their ideas and strategies as a whole class discussion, discuss what strategies work, and why their answers make sense. At conclusion, the students are asked to solve some more practices using identified strategies. Then homework may be assigned.

Interpretation of SMPs as described by Hana. During Hana's pre-interview, she indicated that the SMPs are something that students do on a daily basis. She is very familiar with SMPs because their district is keeping the teachers informed about the state standards (both the content and practice standards). The teachers in this district have department math meetings every month and also district meetings every other month. During these meetings, teachers discuss these standards. Moreover, Hana said it is very helpful that the SMPs are embodied into the Connected Mathematics books and explained how she uses the SMPs in her daily lessons. Based on the pre-interview with Hana, the following table illustrates her interpretation of the SMPs (See Table 4.1).

The interpretation presented revealed that Hana understood the meaning of the SMPs because she described the processes and proficiencies that students should have in a way that aligned with what was intended for the SMPs. She focused on the connection process to encourage students to connect the mathematical problems to real-life situations (SMP1); reasoning process to make sense of the answers and evaluate solution strategies (SMP2 and SMP3); communication process to encourage students to organize their thinking in verbal and written forms (SMP6); and problem solving process to encourage the use of different strategies to solve a problem (SMP7 and SMP8). Moreover, Hana also described SMPs in a way that develops students' proficiencies in mathematics. For example, she seeks to develop students' conceptual understanding by allowing them to show their work, use visual tools, or manipulatives to understand and solve problems (SMP4 and SMP5). It is important to note that Hana understood SMP4 (Model with mathematics) as a combination of both *model the mathematics* and *model with mathematics*. Modeling the mathematics means "using concrete materials or other visual representations to clarify or give meaning to mathematics" (Bleiler et al., 2015). Modeling with mathematics means "apply the mathematics they know to solve problems arising in everyday life, society, and the workplace" (CCSSM, 2010, p. 7).

Table 4.1

Hana's Interpretation of SMPs

SMPs	Interpretation	Sample Response
SMP1. Make sense of problems and persevere in solving them	Students constantly try to solve problems and make sense of their answers by connecting them to real-life situations.	"We tried to train them, does your answer make sense? Going back and talking about does your answer make sense and relate it to real-life situations is something that we teach students to do on their own." (Hana, Pre-Interview, Feb 19, 2019)
SMP2. Reason abstractly and quantitatively	Students make sense of abstract and quantitative answers by linking them to real-life situations.	"We tried to train them about does your answer make sense and relate it to real-life situations." (Hana, Pre-Interview, Feb 19, 2019)
SMP3. Construct viable arguments and critique the reasoning of others	Students critique the reasoning of others by respectfully explaining why they agree or disagree, adding-on to the other's work, telling their strategies, and providing suggestions.	"We talk about being respectful when you are critiquing somebody else's work, explaining why you disagree with them, showing your strategy, telling what you did, adding on to what so-and-so said, [and] giving suggestions to the other person's work." (Hana, Pre-Interview, Feb 19, 2019)
SMP4. Model with mathematics	Students model with mathematics by showing and explaining their work using visual representations or manipulatives.	"We try to use visuals and manipulatives whenever we can; showing your work explaining your work is something we do every day." (Hana, Pre-Interview, Feb 19, 2019)
SMP5. Use appropriate tools strategically	Students use manipulatives and play games as tools to understand and solve problems.	"We have all kinds of hands on. We also play games that can help with surface area and volume." (Hana, Pre-Interview, Feb 19, 2019)
SMP6. Attend to precision	Students attend to precision in mathematical verbal and written forms.	"We had to talk about labelling like centimeters, meters squared, and cubes … we did work with angles so using your protractor correctly and your answer should be within 1 or 2 degree not 5 degrees off." (Hana, Pre-Interview, Feb 19, 2019)
SMP7. Look for and make use of structure & SMP8. Look for and express regularity in repeated reasoning	Both standards aim to encourage students to explore ways to solve problems and understand repeated reasoning in these problems.	"I would say SMP7 and SMP8 is something that a good teacher should be able to do with their students to have that reasoning to make sure it's repeated to make sure the students understand to make sure that the students are doing that exploring. So that they understand instead of you just telling them this is the one way that you need to do the problem." (Hana, Pre-Interview, Feb 19, 2019)

Moreover, Hana did not differentiate between the last two standards in the table (SMP7 and SMP8). She described them as one practice that aims to encourage students' exploration of the patterns or structures that help in solving the problems. It was not clear if Hana knows any difference between these two standards. The interpretation presented in Table 4.1 aligns with the TTPS approach. In other words, using TTPS helped Hana to understand how to implement the SMPs in daily lessons. She said:

> I love teaching through problem solving. I think that it brings so much more meaning to what they're learning. The standards [SMPs] just falling into place while I teach; I know when I'm making my lesson plan that I'm in the back of my head, I'm following all of those. I'm already doing all of those and implementing those. (Hana, Pre-Interview, Feb 19, 2019)

This means when Hana plans her daily lessons, she does not need to search for ways to include these standards in her lesson. This is because the CMP curriculum was designed to focus on encouraging students to develop the expertise that are intended for each one of the SMPs.

Interpretation of MTPs as described by Hana. During the pre-interview with Hana, she exhibited familiarity with MTPs and found a connection between MTPs and SMPs. However, MTPs were more helpful to her. She said, "NCTM I think follows right along with the Common Core; but to me, these are more helpful than Common Core" (Hana, Pre-Interview, Feb 19, 2019). This means, Hana perceived that the MTPs correlated with the SMPs, but the MTPs focused more on helping teacher to use TTPS approach. Hana went one by one through MTPs to interpret how each one of these teaching practices were implemented in her classroom (See Table 4.2).

Table 4.2

Hana's Interpretation of MTPs

MTPs	Interpretation	Sample Responses from Hana
MTP1. Establish mathematics goals to focus learning	The teacher specifies mathematical goals for each day to focus students' learning.	"I really try to specify what our targeted goals for the day are. Like the unit I'm teaching right now is all about adding subtracting multiplying and dividing with decimals. So, I am today just doing subtracting multi digit decimals." (Hana, Pre-Interview, Feb 19, 2019)
MTP2. Implement tasks that promote reasoning and problem solving	The teacher implements a variety of tasks that involve problem solving and reasoning. -The teacher enhances students reasoning by making a discussion about their mistakes.	"I gave them big variety of tasks and they are working through problem solving on that tasks" (Hana, Pre-Interview, Feb 19, 2019) "We did some problems where students in the back had solved the problems incorrectly. So, we talked about what they did." (Hana, Pre-Interview, Feb 19, 2019)
MTP3. Use and connect mathematical representations	The teacher connects a task with the real world using different types of representations.	"This [MTP3] would be to me more of connecting to the real world … If there were any mathematical tools that I had, any visuals that help students understand the problem [I would use them]." (Hana, Pre-Interview, Feb 19, 2019)
MTP4. Facilitate meaningful mathematical discourse	The teacher discusses if the new learning of mathematics make sense and where students would use them in the real world.	"Just talking about what we're learning does it make sense. Where would you use it in the real world?" (Hana, Pre-Interview, Feb 19, 2019)
MTP5. Pose purposeful questions	The teacher asks students purposeful questions to guide them through problem solving.	"This is exactly how we as educators get the students to do the problem solving by asking them what do you think? Who agrees with this who disagrees? Things like that." (Hana, Pre-Interview, Feb 19, 2019)
MTP6. Build procedural fluency from conceptual understanding	The teacher focuses on posing extension problems to building students' procedural fluency from conceptual understanding.	"Once they understood what they were doing, can I build off that? So once I knew that they could successfully subtract a decimal, I started throwing harder problems at them" (Hana, Pre-Interview, Feb 19, 2019)
MTP7. Support productive struggle in learning	The teacher provides individual help, allows students to choose their partners, or assigns partners who have higher levels of mathematical thinking.	"[I] go to them and see what's happening. Do they need more time? Do they need more explanation? Do they need me to sit down one on one? … I think working with partners helps because they have that comfort level … sometimes I have the high work with the low if I know that they're going to be a productive partner with each other." (Hana, Pre-Interview, Feb 19, 2019)
MTP8. Elicit and use evidence of student thinking	The teacher assesses students' understanding when they share their ideas, answers questions, and written work.	"That is definitely where I have students share their ideas and their strategies up at the smart board with each other during problem solving. Sometimes I'll even take their papers and scan them into the smartboard to share their work." (Hana, Pre-Interview, Feb 19, 2019)

In MTP3 (use and connect mathematical representations), Hana described this standard as connecting mathematical representations with real-world situations. She did not add to her description that students should also understand the connections between mathematical representations. Interestingly, Hana described a unique way to develop student's procedural fluency from conceptual understanding (MTP6). She gives her students extension problems that aim to extend students' knowledge of the new learning concepts rather than simply giving them some exercises to practice what they learned. She said, "I did not want to spend too much time simply subtracting with decimals once they grasped that concept" (Hana, Pre-Interview, Feb 19, 2019).

Using TTPS helped Hana in understanding the MTPs. She said, "I feel like NCTM [the MTPs] correlates so much with what we're teaching" (Hana, Pre-Interview, Feb 19, 2019). That is, she perceives the MTPs as teaching practices that are linked and connected with her teaching practices when using the TTPS approach. She described the MTPs in Table 4.2 as she is describing her teaching practices in everyday TTPS lessons.

Lesson as observed by researcher. After conducting the pre-interview with Hana, I observed and videotaped one 95-minute lesson. The title of the observed lesson was *Computing Tips*. The targeted I-can statements were:

(1) I can fluently add, subtract, multiply, and divide multi-digit decimals, and

(2) I can find the percent of a quantity.

The sequence of this lesson started with a warm-up sheet that was intended to gather students' background information about rounding decimals to the hundredth place and how to find a percent of decimals. Next, the teacher discussed why they are learning about how to find tips and its importance. Hana connected the lesson with students' lives by asking them to think of a real-world situation in which they would have to tip someone for a service they have provided. Then she asked these questions: “Who would you tip? How much do you tip? Why would you tip? Do you have a method for finding a tip?”

When students understood the context of finding a tip, Hana began the launch phase with a story problem in which students went to a restaurant and their bill was $963.87. Hana asked students to find 10% of their bill. In the explore phase, Hana allowed time for students to look for entry points and solution paths to compute 10% of $963.87. The students had their choice to work individually, with partners, or in small groups to solve the problem while Hana provided scaffolding for students who were struggling. Most of the students used division to solve the problem (963.87 ÷10 = 96.387), some students used multiplication (963.87 x 0.10 = 96.387), and some students use multiplication incorrectly (963.87 x 10 = 9638.7). Hana monitored students’ strategies and their common mistakes to plan for a whole class discussion.

In the summarize phase, Hana presented the dividing strategy first and asked students to explain it. Then, she presented the incorrect multiplying strategy and asked students if they agree or disagree with this solution. When students explained that it didn’t make sense to pay a tip ($9638.7) more than their bill ($963.87), a student suggested that they should multiply by 0.10 because 10% is 10 out of a hundred. Then, Hana presented the other correct strategy which was multiplying 963.87 by 0.10 and round up the answer to the nearest hundredth. The students concluded that multiplying strategy was faster than dividing strategy and discovered a rule that when multiplying a number by 0.10, they can move the decimal to the left one place value.

Next, Hana presented the challenge of a second problem which was finding 5%, 10%, 15%, and 20% of $26.34 mentally. She first asked students to find 10% of $26.34. Thus, the students used the rule that they discovered in the first problem (moving the decimal to the left one place value when multiplying by 0.10) to find the answer and round up ($2.63). Then, to find 20% of $26.34, the students duplicated the number ($2.63) and found the answer ($5.26). At that point, students figured out very quickly how to compute 5% and 15% of $26.34 mentally and justified their answers. In addition, Hana presented a third problem, *Larry's Lunch Place*. The students modeled this problem by imagining they were eating in a restaurant, ordering food, and calculating their total bill with 6% tax and 20% tip.

In the exploring phase, the students worked with partners and in small groups while Hana walked around them to see their solutions. She provided individual help for students who struggled by asking some questions that got them to start solving the problem. She also asked students to show their strategies in solving the problem and to discuss their strategies with their peers.

In the summarize phase, the teacher wrote her order from the restaurant on the board and asked students to explain how to calculate the total bill with the 6% tax and 20% tip. At the end of the class period, the teacher assigned one problem as homework. By comparing what Hana did in this observed lesson with literature, Hana did use TTPS in the observed lesson. This is because she allowed students to think of entry points and solution paths by themselves to solve problems as well as engaged students in an inquiry-oriented environment. Her focus was on using problem solving to understand the percent of a quantity.

Implementation of SMPs observed by researcher. I used the MCOP² protocol (See Appendix C) to analyze the teachers' implementations of the SMPs. The MCOP² protocol was useful to identify if a teacher implements each one of the SMPs. It includes16 items that specify students' actions, teacher's actions, and the lesson tasks to indicate if a teacher was scored high in implementing SMPs. For example, to be scored high in implementing SMP1, items number 1, 2, 3, 4, 5, 9, 11, 14, and 16 should be attained; and to be scored high in implementing SMP2, items number 5 and 7 should be attained. Missing one or more of these items indicates that a teacher does not have a full implementation of that related standards. This means, if a teacher did not attain item number 1, she would not have a full implementation of SMP1, SMP7, and SMP8.

I used Hana's classroom-teaching observation to describe her implementation of the SMPs. I watched Hana's videotape several times while I was using the MCOP² protocol. Then, I wrote down samples of teacher's actions and students' actions from Hana's observed lesson to describe how she attained each one of the 16 items (See Table 4.3). Finally, I looked at each one of the SMPs in Table 4.3 and navigated (in a vertical direction) to see if the teacher attained all related items.

This table shows that Hana implemented all the SMPs because she attained all the16 items and scored high in this implementation. Using TTPS was a significant factor that helped Hana in this implementation of the SMPs because the students' and teacher's actions in Hana's lesson align with each item in the MCOP² protocol. Interestingly, when Hana used TTPS in the observed lesson, the SMPs just fall into place without force. This means that Hana's implementation of the SMPs and TTPS were connected with each other. Moreover, the 95-minutes class time was a significant factor that gave Hana enough time to implement all SMPs in a one-day lesson.

Table 4.3

Hana's Implementation of SMPs

Items/SMPs	SMP1 Make sense of problems and persevere in solving them	SMP2 Reason abstractly and quantitatively	SMP3 Construct viable arguments and critique the reasoning of others	SMP4 Model with mathematics	SMP5 Use appropriate tools strategically	SMP6 Attend to precision	SMP7 Look for and make use of structure	SMP8 Look for and express regularity in repeated reasoning	Samples of Implementation of SMPs from Hana's Observed Lesson
1= Students engaged in exploration/ investigation/ problem solving	x						x	x	The students were engaged in exploring how to compute 10% of $963.87
2= Students used or generated two or more representations	x				x				The students used and generated two representations to find 10% of $963.87, which were: Symbolic form (963.87 × 0.10 = 96.387 or 963.87 ÷ 10 = 96.387) and verbal form (students pretended that they were going to a restaurant, ordering food, and paying their $963.87 bill with a 10% tip, and they found that tip should be less than what they pay for their bill).
3= Students were engaged in one or more activities	x								The students were engaged in three activities: (1) finding 10% of $963.87, (2) mentally find 5%, 15%, and 20% of 26.34, and (3) solving Larry's Lunch Place problem.

(table continues)

Table 4.3

Hana's Implementation of SMPs (continued)

Items/SMPs	SMP1	SMP2	SMP3	SMP4	SMP5	SMP6	SMP7	SMP8	Samples of Implementation of SMPs from Hana's Observed Lesson
4= Students assessed strategies	x		x					x	The students assessed the two strategies (division and multiplication) that they used to find 10% of 963.87 and found that both strategies gave the same answer (96.387) but using multiplication was faster.
5= Students persevere in problem solving.	x	x	x		x				The majority of students exhibited a strong amount of perseverance and looked for entry point and solution paths to find 10% of 963.87 and to solve Larry's Lunch Place problem
6= The lesson involved fundamental concepts to promote conceptual understanding.							x	x	The lesson included fundamental concepts which were the percent of a quantity, add, subtract, multiply, and divide multi-digit decimals.
7= The lesson promoted modeling with mathematics.		x		x					The students modeled the Larry's Lunch Place problem. They pretended that they were going to and ordering from a restaurant. Then they found their total bill including 20% tip and 6% tax by using what they knew about adding, multiplying, and dividing decimals. Then, they organized their findings in a restaurant receipt template to make sense of their answers.
8= The lesson provided opportunities to examine mathematical structure.							x	x	The students had time to look for the structure of multiplying numbers by 10%. They used this structure to find the following rule: When multiplying a number by 0.10, move the decimal to the left.

(table continues)

Table 4.3

Hana's Implementation of SMPs (continued)

Items/SMPs	SMP1	SMP2	SMP3	SMP4	SMP5	SMP6	SMP7	SMP8	Samples of Implementation of SMPs from Hana's Observed Lesson
9= The lesson included tasks that have multiple paths to a solution or multiple solutions.	x								The problem (finding a 10% tip of $963.87) had multiple paths to a solution. Some students used multiplication (963.87 × 0.10), some of them used division (963.87 ÷ 10), and some of them tried to use fractions (963.87 × 10/100) to solve the problem.
10= The lesson promoted precision of mathematical language.			x			x			The teacher and students attended to precision when reading and writing decimal numbers, doing operation with decimals, and rounding up to the hundredths place.
11= The teacher's talk encouraged student thinking.	x								Most of the time Hana asked questions that required high levels of mathematical thinking such as: "If I go to take this answer [096.387] and change it into money for a tip what would my tip be? Why did you multiply with 0.10? Did anybody solve this a different way? Anybody noticing anything about the way we solved the problem?" (Hana, Observation, Feb 27, 2019)
12= There were a high proportion of students Talking related to mathematics.			x						More than 75% of the students were talking related to mathematics. They answered the teacher's questions, talked with each other to solve the problems, explained their strategies to solve the problems, and made comment in others' work.
13= There was a climate of respect for what others had to say.			x						The students respectfully explained why they were agreeing or disagreeing with other's ideas. For example, when a student said that 20% tip is smaller than 10% tip, another student said, "I disagree because 20% is twice as much as 10%" (Hana, Observation, Feb 27, 2019)

(table continues)

Table 4.3

Hana's Implementation of SMPs (continued)

Items/SMPs	SMP1	SMP2	SMP3	SMP4	SMP5	SMP6	SMP7	SMP8	Samples of Implementation of SMPs from Hana's Observed Lesson
14= The teacher provided wait-time.	x								Hana gave her students thinking time, approximately 10 minutes, for the warmup activity, 5 minutes for how to find a 10% tip of $963.87, and 20 minutes for Larry's Lunch Place problem. Hana also gave wait-time when she posed questions.
15= Students were involved in the communication of their ideas to others.			x						The students communicated with each other in small groups (to solve the lesson's tasks) and in whole class discussions (to share their ideas and strategies).
16= The teacher used student questions/comment to enhance conceptual understanding	x								When a student commented that they should multiply 963.87 by 0.10 to find the %10, the teacher asked Why. The teacher wanted to enhance students conceptual understanding of the fact that 10% = 10/100 = 0.10

Note: x = Implemented; θ = Not Implemented; [shaded] = Not Applicable

Implementation of MTPs observed by researcher. I used the MTP-OP protocol to assess Hana's implementation of the MTPs (See Appendix D). This protocol includes 51 items of teacher's actions (27 items) and students' actions (24 items) in the classrooms that indicate if a teacher achieves a full implementation of the MTPs. A full implementation of the MTPs means that a teacher attains all the items that relate to each one of the MTPs. For example, to fully implement MTP1 (establish mathematics goals to focus learning), a teacher should attain 4 items: (1) the teacher discusses and referres to the mathematical purpose and goal of the lesson during instruction, (2) the teacher uses the mathematics goal to make in-the-moment decisions during instruction, (3) students engage in discussing the mathematical purpose and goals related to their work in the classroom, and (4) students connect their current work with the mathematics that they studied previously. Missing more than one item indicates that a teacher has a partial implementation of the related teaching practices.

I watched the videotape of Hana's observed lesson several times while using the MTP-OP protocol and went over the Hana's post-interview transcript to understand the purposes of her actions in the observed lesson. Then I recorded her actions and her students' actions that were specified in the protocol (See Table 4.4). In this table, I provide an ordered pair of the number of teacher's actions and students' actions that should be attained to indicate a full implementation of each one of the MTPs. For example, in SMP1, (2T, 2S) means there are two of teacher's actions and two of students' actions that should be attained in order to fully implement SMP1.

The teacher and students' actions in this table showed that Hana implemented all the MTPs because Hana attained 50 out of 51 items. It indicates a full implementation of each one of the MTPs. Hana missed one item in MTP3 which was "introducing forms of representations that can be useful to students". She did not introduce a visual form of representations (e.g., a tape diagram) to represent the idea of a percent of quantities. In the MTP-OP protocol, there were three teacher's actions and five students' actions (3T, 5S) that needed to be observed to fully implement SMP3. However, Hana attained a full implementation of MTP3 because this missing teacher's action did not affect students' actions in MTP3. In other words, Hana did not introduce a visual representation for students to use but the students still used different forms of representations which were verbal and symbolic representations. That is, missing this action left an area for improving Hana's implementation of MTP3 while it did not reduce the number of students' actions to implement MTP3.

Table 4.4

Hana's Implementation of MTPs

MTPs	Teacher's Actions	Students' Actions
MTP1. Establish mathematics goals to focus learning (2T,2S)	1-The teacher posted the lesson's goals on the smart board at the beginning of the class time without formally reading them, but she talked about them several times during the lesson to focus students learning. 2-The teacher used the mathematical goals to make in-the-moment decisions during instruction. Throughout the lesson Hana responded to students' thinking and questions to understand the percent of a quantity.	1-The students engaged in discussions about how to compute tips and what was the percent mean. 2-The students were connecting their current understanding of that percent was out of a hundred to their previous understanding of fractions and decimals.
MTP2. Implement tasks that promote reasoning and problem solving (4T, 3S)	1-The teacher provided opportunities for exploring problems when she asked them to solve the problems without telling them how. 2-The teacher posed tasks (e.g., find a 10% tip of $963.87 and Larry's Lunch Place) that required a high level of cognitive demand. 3-The teacher supported students in exploring tasks by asking probe questions. She said, "How did you get that? This is a tip remember, so when you get your answer how many numbers should you have past that decimal?" (Hana, Observation, Feb 27, 2019) 4-The teacher encouraged students to continue using varied approaches and strategies (multiplication, division, and fraction) to solve the task (finding a 10% tip of $963.87).	1-The students persevered in exploring and reasoning through tasks. They explored how to find a 10% tip of $963.87 using multiplication and division and explained their strategies. 2-The students used tools (e.g., paper and pencils and calculators) as needed to support their thinking. 3-The students accepted and expected that their classmates will use a variety of solution approaches and will justify their strategies.

(table continues)

Table 4.4

Hana's Implementation of MTPs (continued)

MTPs	Teacher's Actions	Students' Actions
MTP3. Use and connect mathematical representations (4T, 5S)	1-The teacher allocated substantial instructional time for students to use and discuss mathematical representation (e.g., using multiplication 963.87 × 0.10 = 96.387 and using division 963.87 ÷ 10 = 96.387). She connected the two mathematical representations with the lesson's goal (finding the percent of a quantity). However, she did not provide time to make connections among these two representations. *2-The teacher did not introduce a visual form of representation. 3-The teacher asked students to use paper and pencil or markers to support them to explain and justify their reasoning. 4-The teacher focused students' attention on the structure of the percent.	1-The students used two forms of representations to understand how to find tips, which were verbal representation (pretending they were going to a restaurant and paying for their bill including tips) and symbolic representation (e.g., using dividing or multiplication to find tips). 2-The students described and justified their mathematical understanding with verbal representations (e.g., describing that tips should be less than the bill) and symbolic representation (e.g., the equation 10%=10/100) 3-The students made choices about which forms of representations to use as tools for solving problems. 4-The students contextualized mathematical ideas by connecting them to real-world situations (e.g., finding tips when they were eating or paying for hotel services). 5-The students considered the suitability of using representations when solving problems.
MTP4. Facilitate meaningful mathematical discourse (4T, 4S)	1-The teacher engaged students in purposeful sharing of mathematical approaches. Students worked together to find tips of quantities and then shared their strategies in a whole class discussion. 2-The teacher selected and sequenced students' strategies for a whole class discussion. She presented division first because it takes longer time than multiplication. 3-The teacher facilitated discourse among students by positing them as authors of ides. For example, the teacher said, "[Sam] was trying to tell you that if you change 10% to a decimal, that is equal ten hundredths" (Hana, Observation, Feb 27, 2019) 4-The teacher ensured progress toward mathematical goals by making explicit connections between how using multiplication and division are ways to find tips.	1-The students presenting and explaining ideas and representations to one another in pair, small-groups, and whole-class discussion. 2-The students listened carefully to and critiqued the reasoning of peers. For example, when a student said that he multiplied 963.87 × 10, another student said that we should multiply with 0.10 because 10 is out of a hundred. 3-The students sought to understand the approaches used by peers by trying to use other's strategies. For example, some students switched from using division to multiplication to find the percent of a quantity faster. 4-The students identified how different approaches to solve a problem are the same and how they are different. They found that using multiplication and division strategies gave the same answer but differ in the speediness of computations.

(table continues)

Table 4.4

Hana's Implementation of MTPs (continued)

MTPs	Teacher's Actions	Students' Actions
MTP5. Pose purposeful questions (4T, 3S)	1-The teacher advanced students' understanding by asking questions that build on students' thinking (e.g., "Did anybody notice anything about the way we solved the problem?" and "Did anybody solve this a different way?" (Hana, Observation, Feb 27, 2019) 2-The teacher asked questions that required explanation and justification (e.g., "Why did you multiply with 0.10") (Hana, Observation, Feb 27, 2019). 3-The teacher asked questions that make the mathematics more visible (e.g., "If I go to take this answer [096.387] and change it into money for a tip what would my tip be?") (Hana, Observation, Feb 27, 2019) 4-The teacher frequently allowed sufficient wait time for students to formulate and offer responses.	1-The students were thinking carefully about how to present their responses without rushing. 2-The students justified their reasoning (e.g., a student said that we should multiply with 0.10 because 10 is out of a hundred). 3-The students listened to and commenting on the contributions of their classmates. For example, the following conversation happened: *Student 1: I did 0.10 because for a 10% we have to move the percent sign over two times to change it to a decimal.* *Student 2: But there is no decimal in 10%.* *Student 3: We can change a percent into a decimal, 10% is like a 10 out of a hundred.* (Hana, Observation, Feb 27, 2019)
MTP6. Build procedural fluency from conceptual understanding (4T, 3S)	1-The teacher provided students with opportunities to use their own strategies and methods (multiplication and division) for solving problems. 2-The teacher asked students to discuss and explain their two strategies (multiplication and division). 3-The teacher connected student-generated strategies to more efficient procedures. For example, when students used division and got the answer (096.387), the teacher discussed how to complete their answer and represent it as money (%96.39). 4-The teacher allowed students to model the Larry's Lunch Place problem to support their understanding of the percent.	1-The students made sure that they understand and can explain the mathematical basis for the procedures that they used. They explained why they multiplied by 0.10 and not by 10 to find 10% of a quantity. 2-The students demonstrated flexible use of strategies. They were allowed to use any strategy that worked to solve problems. 3-The students determined whether specific approaches generalize to a broad class of problem. For example, the students determined that they can always move the decimal point one place when multiplying by 0.10 and generalized it to a broad class of problems.

(table continues)

Table 4.4

Hana's Implementation of MTPs (continued)

MTPs	Teacher's Actions	Students' Actions
MTP7. Support productive struggle in learning (3T,2S)	1-The teacher gave students time to struggle with each one of the three problems in the lesson. 2-The teacher helped students realize that confusion and errors are a natural part of learning by facilitating discussions on students' mistakes and misconceptions (e.g., finding 10% by multiplying the quantity by 0.10 and not by 10). 3-The teacher praised students for their efforts in making sense of mathematical ideas (e.g., she said "Yes", "Great discovery", and "Excellent"). (Hana, Observation, Feb 27, 2019)	1-The students asked questions that were related to the sources of their struggles. For example, some students found the tip 9.6387 but they struggled to interpret it as money. So, they asked, "If I round up, is that right?" (Hana, Observation, Feb 27, 2019) 2-The students helped one another without telling their classmates what the answer is because the teacher did not tell them right away if they got the right answer.
MTP8. Elicit and use evidence of student thinking (2T, 2S)	1-The teacher elicited and gathered evidence of student understanding during their attempt to solve the problems. For example, some students multiplied by 10 or 100 to find a 10% of 96.387. 2-The teacher made in-the-moment decisions on how to respond to students with questions. For example, when a student found that 10% of 963.87 is $9638.7 the teacher said that this is a great tip and asked the students if we should give tip more than our bill.	1-The students revealed their mathematical understanding in written work (when they were trying to solve problems) and classroom discussion (when they explained their strategies in a whole-class discussion). 2-The students asked questions and gave suggestions to support the learning of their classmates. For example, student 1 said that a 20% of $26.34 is smaller than a 10% of $26.34. Student 2 replied that he didn't agree because a 20% is twice as much as 10 %.

T = Teacher; S = Student; * = Not Implemented; (#T, #S) = the number of teacher's actions and the number of students' actions

Summary of Hana's Case

Hana explained how she used TTPS in her daily lessons to promote students' learning of mathematics with understanding. She explained how she uses TTPS in a way that align with the literature on TTPS approach. She also understood the meaning of the most SMPs and clearly described the expertise that she should seek to develop in her students for each one in these standards. Hana had a misconception in understanding

SMP4 (Model with mathematics) because she understood it as both model the mathematics and model with mathematics. However, she considered the SMPs as basic things that students should do every day to learn mathematics with understanding. Moreover, Hana showed familiarity with the MTPs during the pre-interview. She understood and described the teaching practices that a teacher needed to successfully implement for each one of the MTPs.

Hana's implementation of the SMPs in the observed lesson was consistent with her explanation of SMPs. She scored high in implementing all these standards in one lesson. The use of TTPS and the long class period were significant factors that facilitate her implementation of the SMPs. Furthermore, Hana's implementation of the MTPs in the observed lesson was consistent with her description of the MTPs in the pre-interview. That is, Hana interpreted and implemented MTPs as teaching practices that help in teaching mathematics content with understanding. She had a full implementation of the MTPs in one lesson.

Teresa's Case

Teresa is a seventh-grade female mathematics teacher with around 22 years of experience in teaching middle school mathematics. She has a bachelor's degree in education. During the first years of her teaching, she used traditional teaching methods, in which a teacher gives direct instructions and students listen and follow their teacher's directions. When her district adopted the Connected Mathematics Program (CMP) and supported them to use the TTPS approach, she took three to four years to improve her teaching practices to be able to use the TTPS approach successfully. She said:

> I was doing this program for three or four years before I finally felt like I was getting good at it and now that I've been doing it for 15 years. I still keep thinking of and learning ways to get better with it. (Teresa, Pre-Interview, Feb 12, 2019)

This means, for Teresa changing teaching practices to use the TTPS approach was not easy. It took her several years to be able to successfully use it. In addition, Teresa is still learning ways to improve her teaching practices when using the TTPS approach.

Typical Lesson as described by Teresa. During the pre-interview with Teresa, she described how she usually implements the TTPS lessons. She said that her TTPS lesson usually starts with a discussion about homework. Then, in the launch phase, she poses a problem with a real-life scenario and presents its challenges. In the explore phase, she provides thinking time for students to think of how to solve the problem independently and then work with partners or groups to share their ideas and critique each other's work. At the same time, Teresa circulates among the groups, asks scaffolding questions, and plans how to discuss students' ideas and misconceptions. In the summarize phase, Teresa asks students to share their solutions and explain their strategies in a whole class discussion. At the end of the class period, if there is enough time, Teresa launches a new problem, gives students time to investigate entry points and solution paths for the problem, and then asks students to bring their ideas about how to solve it next day.

Teresa's description of her TTPS lesson was consistent with the literature on TTPS. She used TTPS for one lesson in a 45-minute class period over two days. This means that the teacher can launch a problem in a day, students explore the problem in class and continue to solve it at home. On the next day, the teacher asks students to share their solution strategies and summarizes their findings.

Interpretation of SMPs as described by Teresa. During Teresa's pre-interview, she indicated that she is familiar with the SMPs. She usually posts the SMPs on a bulletin board in the back of her classroom to peruse them all the time while she is teaching. She also keeps the SMPs in her mind as she is thinking and planning the daily lessons. Her students know these standards because she reminds them with these eight standards at the beginning of each unit to keep them in their minds. She wants the students to know that these practices are designed to go from kindergarten all the way through 12^{th} grade. She sometimes refers to one of these practices during each lesson. I asked Teresa to explain how she uses these standards in her daily lessons. She interpreted each one of the SMPs by explaining what expertise she seeks to develop in her students (See Table 4.5).

When looking at table 4.5, Teresa's interpretation of each one of the SMPs revealed that she understood most of these standards because her interpretation aligned with the literature on the SMPs. However, she did not fully understand the meaning of SMP4. She described SMP4 as *modeling the mathematics* rather than *modeling with mathematics*. This is because Teresa described how she implements SMP4 as encouraging students to use tools such as the number line, chips, and boxes to understand the new mathematical concepts. However, modeling with mathematics means that students used the mathematics they know such as tables, drawings, graphs to understand and solve real-life mathematical problems (CCSSM, 2010). Moreover, Teresa did not differentiate between SMP7 and SMP8 even though she described what these two standards were intended for. She said that SMP7 "goes together" (Teresa, Pre-Interview, Feb 12, 2019) with SMP8. This means, Teresa did not know the difference between these two standards.

Table 4.5

Teresa's Interpretation of SMPs

SMPs	Interpretation	Sample Responses from Teresa
SMP1. Make sense of problems and persevere in solving them	Students make sense of real-life problems and persevere when they understand why they need to persevere.	"I believe that the [CMP] series that we use is designed to help [students to make sense] … I [also] teach the students that we're going to be practicing a growth mindset and that means we will struggle and we're going to make mistakes but we're going to persevere." (Teresa, Pre-Interview, Feb 12, 2019)
SMP2. Reason abstractly and quantitatively	Students develop their reasoning skills and understanding of abstracts and quantities by doing hands on activities and using manipulatives.	"We do a lot of hands on things or the manipulative so that they understand the why." (Teresa, Pre-Interview, Feb 12, 2019)
SMP3. Construct viable arguments and critique the reasoning of others	Students defend their ideas, agree or disagree with their peers, and analyze others' ideas and strategies.	"We embedded into our daily work that we are trying to defend ourselves, argue, critique, and think about the reasoning of others." (Teresa, Pre-Interview, Feb 12, 2019)
SMP4. Model with mathematics	Students model with mathematics using visual materials and manipulatives to understand new mathematical concepts and ideas.	"The modeling with mathematics happens as well whether it's the number lines and chip models, whether it's using a big huge set of boxes that I have to demonstrate." (Teresa, Pre-Interview, Feb 12, 2019)
SMP5. Use appropriate tools strategically	Students use any appropriate tool that can help to understand mathematical concepts and solve mathematical problems.	"We begin our stretching and shrinking unit with rubber band stretching, and we talk about them being math tools … the other day, we had meter sticks and then a 2.5 constructed deck to model walking rates." (Teresa, Pre-Interview, Feb 12, 2019)
SMP6. Attend to precision	Students attend to precision in mathematical speech and written forms.	"This one [SMP6] is either related to measurement or related to money and interpretations" (Teresa, Pre-Interview, Feb 12, 2019)
SMP7. Look for and make use of structure & SMP8. Look for and express regularity in repeated reasoning	Both SMP7 and SMP8 have the same meaning in which students look for patterns in process within and across problems.	"I think that one [SMP7] kind of in my mind really goes together with number eight [SMP8] that you're just trying to look at tables to see if a pattern emerges, look at things that are happening again and again as they come up in different contexts." (Teresa, Pre-Interview, Feb 12, 2019)

Interpretation of MTP as described by Teresa. In the pre-interview with Teresa, I asked her what she knows about the MTPs that were developed by NCTM in 2010. I gave her a copy of the practices written on a sheet of paper. She seemed not attentive to MTPs and said, "I have been focusing on the mathematical practices and not so much from the NCTM stuff" (Teresa, Pre-Interview, Feb 12, 2019). However, she looked closely at them and explained how each one of these practices might appear in her classroom. Table 4.6 showed her interpretation of the MTPs with a sample from her own words.

Teresa's interpretation in Table 4.6 revealed that she understood all the MTPs because her description aligned with the literature on the MTPs. Teresa's description in this table showed that she relied on the CMP curriculum, that support TTPS, to implement MTP2 (Implement tasks that promote reasoning and problem solving), MTP3 (Use and connect mathematical representations), and MTP5 (Pose purposeful questions). In other words, she poses high-cognitive demand problems from CMP book (that ask students to use multiple representations) and uses the suggested questions in the CMP teacher's guide book to effectively implement those problems.

Moreover, she described an interesting idea that helps to implement MTP7 (support productive struggle in learning). She focuses in developing a classroom culture in which students' mistakes are something normal. That is, Teresa builds discussions that encourage and praise students' unique strategies and attempted to solve a problem. So, students do not feel embarrassed when they make mistakes in solving mathematical problems or when they present their ideas and thoughts. These norms encourage students to respect each other and believe that the classroom is a safe place for them to talk and make mistakes. In this way students can gain proficiency and persevere in solving tasks through productive struggle.

Table 4.6

Teresa's Interpretation of MTPs

MTPs	Interpretation	Sample Responses from Teresa
MTP1. Establish mathematics goals to focus learning	The teacher establishes the lesson by posting mathematical goals on the board and talking about them at the beginning or later.	"The math goals are on the board … I can talk about them first and then we delve in but if not, I think it's better to almost wait until after words." (Teresa, Pre-Interview, Feb 12, 2019)
MTP2. Implement tasks that promote reasoning and problem solving	The teacher relies on the CMP curriculum to post and implement tasks that promote reasoning and problem solving.	"[MTP2] is just happening on a daily basis because of the Connected Math Program ... and you follow their suggestive comments and questions that you are able to do" (Teresa, Pre-Interview, Feb 12, 2019)
	Discussing students' mistakes and if their answers make sense assists students' reasoning.	"There were ways to make the problem assists in their understanding. When someone mistakenly would say that the unit rate was 49 [in the equation $C_{Mighty} = 49 + n$], we could talk about what does a unit rate mean? And then when they discovered or thought about the fact that was the price for every T-shirt, they begin to understand that wasn't reasonable." (Teresa, Pre-Interview, Feb 12, 2019)
MTP3. Use and connect mathematical representations	The teacher encourages students to use and connect different types of representations.	"The lesson that you will see, tables, graphs, and equations are just contextual representations of those [real-life problems] and absolutely intertwined with each other." (Teresa, Pre-Interview, Feb 12, 2019)
MTP4. Facilitate meaningful mathematical discourse	The teacher plans for mathematical discourse between pairs, groups, and the whole class.	"[I] think very carefully about the questions that I ask [during] conversations, whether it's one on one with me, whether it's with partners, whether it's with a team." (Teresa, Pre-Interview, Feb 12, 2019)
MTP5. Pose purposeful questions	The teacher plans for questions and uses questions from the CMP teacher's guide book.	"I had in my mind a list of questions. I'm going to make sure I ask this, ask this, ask this; our actual text does a very nice job of giving us ideas as a teacher of the questions that we should be asking." (Teresa, Pre-Interview, Feb 12, 2019)

(table continues)

Table 4.6

Teresa's Interpretation of MTPs (continued)

MTPs	Interpretation	Sample Responses from Teresa
MTP6. Build procedural fluency from conceptual understanding	The teacher poses some exercises after students understand the new concepts to develop procedural fluence from conceptual understanding.	"We start with the concepts and try to have it as meaningful to real-life situations as possible before we just then practice." (Teresa, Pre-Interview, Feb 12, 2019)
MTP7. Support productive struggle in learning	The teacher supports productive struggle by normalizing the errors and discussing the benefits of struggling.	"There are days when I'll say, today I'm looking for my favorite mistake … we are trying to normalize the errors … you have to struggle in order to get stronger, I build that into our discussions all the time." (Teresa, Pre-Interview, Feb 12, 2019)
MTP8. Elicit and use evidence of student thinking	The teacher assesses students' understanding when they share their ideas and critique the reasoning of others.	"[I'm] letting the students talk as much as possible, giving them the opportunity to share ideas, and [encouraging them to] critique the reasoning of others." (Teresa, Pre-Interview, Feb 12, 2019)

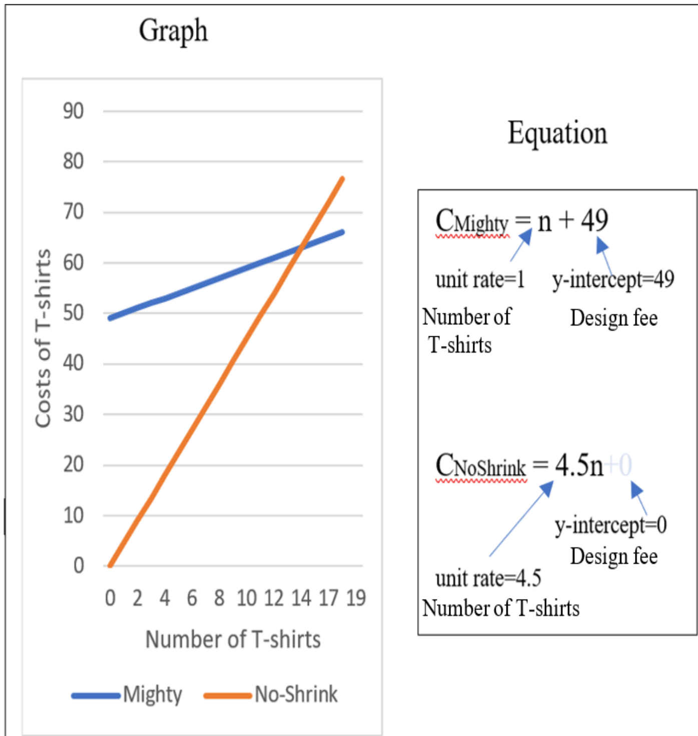
Lesson as observed by researcher. I observed one two-day lesson when Teresa was using the TTPS approach. Each day was a 45-minute class period. The title of the observed lesson was Comparing Costs. The Comparing Costs problem for the lesson was asking for comparing the costs of walkathon t-shirts between two and three companies. Each company represented the cost for its t-shirts as a linear equation or as a set of values in a table. Therefore, students need to understand the meaning of each part of an equation (constant rate and y-intercept) and make decisions about linear relationships using information given in tables and equations.

On the first day, Teresa started the lesson by posting and discussing the following: the plan of what students will do during the lesson, homework, and the targeted I-can statement. The plan of the lesson was sharing students' solution strategies and summarizing the findings of the previous problem (Problem 2.2: Henri and Emile's Race) and begin working on Problem 2.3: Comparing Costs. The homework was to complete working on Comparing Costs problem, part A. The targeted I-can statement was: I can determine whether a table or equation is linear.

Next, Teresa reminded students about the challenge in the prior problem (Problem 2.2) and asked them to share their solutions and explain their strategies to solve it in a whole class discussion. The teacher launched part A of the Comparing Costs problem by connecting it with real-life situations and explaining its challenges. She hung two t-shirts on the front of the class-wall for all students to see, and an equation was hanging on each T-shirt. The teacher asked students to compare the equations for the two situations and find out which company provided a better deal.

During the explore phase, Teresa gave the students worksheets to record their work and answers to the problem. The students worked with partners to solve the problem while the teacher circulated among the groups and asked scaffolding questions. At the end of the class, Teresa asked the students to continue working on the problem at home.

The next day, the teacher started the class by reminding the students of the challenges in Comparing Costs problem and posted the two equations for the problem that represented the cost of t-shirts in each company: $C_{Mighty} = 49 + n$ and $C_{NoShrink} = 4.5n$. During whole class discussion, the teacher asked students to share their answers, strategies, and thoughts about the problem. She asked students to explain the meaning for each part of each equation, present how the parts of the equations show up in a table and in a graph, find the cost per t-shirt under each plan, find the number of t-shirts that need to be sold so each of the two companies would be equal, and explain why the relationship between the cost and the number of t-shirts for each company was linear (See Figure 1). The teacher recorded the findings on the board and told students to correct any mistakes on their worksheet.



Table

Number of T-shirts	Cost in Mighty	Cost in No-Shrink
0	49	0
1	50	4.5
2	51	9
3	52	13.5
4	53	18
5	54	22.5
6	55	27
7	56	31.5
8	57	36
9	58	40.5
10	59	45
11	60	49.5
12	61	54
13	62	58.5
14	63	63

Figure 1. Different Representations for Compare Costs Problem

Figure 1 shows the various strategies students used to compare the costs of t-shirts between Mighty Company and No-Shrink Company. The students drew a table, substituted the *n* in the equations, and sketched a graph of the intersection point between the two companies. They found that if they bought 14 t-shirts, the cost would be the same for each company.

During the last 15 minutes of the class, the teacher presented the challenge of the problem Comparing Costs – part B. This part of problem provided table of values, that represented the cost of t-shirts for a third company (Big T), with the two equations that represented the cost of t-shirts at Mighty and No-Shrink. Teresa asked the students to work with their partners to compare the cost for the Big T company to the cost for the two companies in part A. She walked around the students and asked scaffolding questions. Then, she asked students to share their equation ($C_{BigT} = 2.5n + 34$) for the problem and to continue working on part B of the problem at home.

By comparing how Teresa taught the observed lesson with the literature on TTPS, it was determined that she used the TTPS approach to teach the observed lesson. She engaged students in an inquiry-oriented environment by allowing them to think about how to solve problems on their own to develop their understanding of new mathematical concepts and ideas. Moreover, Teresa's description of her typical lesson was consistent with her actions in the lesson that I observed.

Implementation of SMPs observed by researcher. I watched Teresa's videotape of the Computing Costs lesson while using the $MCOP^2$ protocol (See Appendix C). This protocol was useful to identify the 16 items of student's and teacher's actions that indicate if the teacher implemented each one of the SMPs. If the teacher attains all the 16 items in the $MCOP^2$ protocol, this indicates a full implementation of the SMPs, and also indicates that the teacher scored high on implementing the SMPs. Missing one or more of these items indicates either partial or no implementation of the related standards. For example, if a teacher missed one item in SMP2, this indicates a partial implementation of SMP2 while if a teacher missed the two items in SMP2, this indicates that the teacher did not implement SMP2. Table 4.7 shows how Teresa's implemented the 16 items in the observed lesson.

Table 4.7

Teresa's Implementation of SMPs

Items/SMPs	SMP1 Make sense of problems and persevere in solving them	SMP2 Reason abstractly and	SMP3 Construct viable arguments and critique the reasoning of others	SMP4 Model with mathematics	SMP5 Use appropriate tools strategically	SMP6 Attend to precision	SMP7 Look for and make use of structure	SMP8 Look for and express regularity in repeated reasoning	Samples of Implementation of SMPs from Teresa's Observed Lesson
1= Students engaged in exploration/ investigation/ problem solving	x						x	x	The majority of the students regularly engaged in exploration, investigation, and problem solving to solve the Comparing Costs problem. They explored unit rate, y-intercept, and solutions of an equation in a different context. They also investigated how to compare costs using equations and tables.
2= Students used or generated two or more representations	x				x				The students used a variety of representations (equations, graphs, and tables) to compare the costs of t-shirts between two different companies.
3= Students were engaged in activities	x								Most of the students spent more than two-third of the lesson engaged in solving the Comparing Costs problem.
4= Students assessed strategies	x		x					x	The students assessed the two student-generated strategies (using a table and guess and check) to find the number of t-shirts in which the two companies were equal.

(table continues)

Table 4.7

Teresa's Implementation of SMPs (continued)

Items/SMPs	SMP1	SMP2	SMP3	SMP4	SMP5	SMP6	SMP7	SMP8	Samples of Implementation of SMPs from Teresa's Observed Lesson
5= Students persevere in problem solving.	x	x	x		x				Most students persevered by looking for entry points to solve the problems. They found unit rates, wrote equations, and compared the costs between two companies using equations and between three companies using a table and equations.
6= The lesson involved fundamental concepts to promote conceptual understanding.							x	x	The lesson included the concepts: Coefficient, Solution of an equation, and Y-intercept. The teacher used real-life problems (Comparing Costs problem) to develop students' conceptual understanding of these concepts.
7= The lesson promoted modeling with mathematics.		x		x					The students used the mathematics they knew (e.g., a table, equation, or graph) to model the costs of t-shirts in each company and compare the costs.
8= The lesson provided opportunities to examine mathematical structure.							x	x	Within a problem, the students examined a table to find the pattern when the same calculation steps recur and used this pattern to write an equation. Across problems, the students found the similarity between the today problem and the two previous problems to generalize the concept of a linear relationship.
9= The lesson included tasks that have multiple paths to a solution or multiple solutions.	x								The lesson included multiple paths to a solution. Some students used tables and the others used a guess and check strategy to find the intersection point between the two equations ($C_{Mighty} = 49 + n$ and $C_{NoShrink} = 4.5n$)

(table continues)

Table 4.7

Teresa's Implementation of SMPs (continued)

Items/SMPs	SMP1	SMP2	SMP3	SMP4	SMP5	SMP6	SMP7	SMP8	Samples of Implementation of SMPs from Teresa's Observed Lesson
10= The lesson promoted precision of mathematical language.			x			x			The teacher asked students to interpret symbols and calculate them precisely. She asked, What is the y-intercept in the equation [CMighty = 49 + n], what do you think 49 really means?", "How much is it going to cost you to get 12 shirts if you go to Mighty, if you go to No-shrink?", "Do you agree with the math that I got from your classmate?" (Teresa, Observation, Feb 14, 2019)
11= The teacher's talk encouraged student thinking.	x								The teacher asked, "Does anybody have a different equation they would like to offer? What does the coefficient mean? What that 1 mean? Why do you think they did not put 1 in the equation? How do I know that t is independent variable?" (Teresa, Observation, Feb 14, 2019)
12= There were a high proportion of students Talking related to mathematics.			x						Most of the class time, Teresa asked all students to talk to each other and discuss their ideas before answering her questions.
13= There was a climate of respect for what others had to say.			x						The students respectfully listened and commented on what others had to say. For example, a student was working with a partner and said, "I'm saying it's $2.5 per T-shirt but he doesn't agree with me" (Teresa, Observation, Feb 15, 2019). So, the student took a pencil and proved that the cost per T-shirt is $2.5 by finding the unit rate from a given table.
14= The teacher provided wait-time.	x								The teacher gave 5 minutes to think about problem 2.3 – part A and 10 minutes to think about problem 2.3 – part B. She also gave time after asking questions; she said, "Stand to tell me when you know" (Teresa, Observation, Feb 14, 2019).

(table continues)

Table 4.7

Teresa's Implementation of SMPs (continued)

Items/SMPs	SMP1	SMP2	SMP3	SMP4	SMP5	SMP6	SMP7	SMP8	Samples of Implementation of SMPs from Teresa's Observed Lesson
15= Students were involved in the communication of their ideas to others.			x						The teacher engaged students in communication by asking, "See if your partner knows", "Would you and your partner see if you agree on which of these variables is independent or dependent?", "See if you and your partner come up with a defense?" (Teresa, Observation, Feb 14, 2019).
16= The teacher used student questions/comment to enhance conceptual understanding	x								The following conversation happened when a student said that the coefficient of the equation (C_{Mighty} = n + 49) is 1: *Teresa: I don't see any 1 in that equation.* *Student: There is 1 in front of n.* *Teresa: Why didn't they put it in that equation?* *Student: it's a short cut because algebra.* (Teresa, Observation, Feb 15, 2019)

x = Implemented; θ = Not Implemented; [grey box] = Not Applicable

Table 4.7 shows that Teresa attained all 16 items of the MCOP² and scored high on all indicators on the MCOP² protocol. In other words, she implemented all the SMPs in the observed lesson. In the pre-interview, Teresa explained SMP4 (Model with mathematics) differently than was intended by the CCSSM. Nonetheless she was able to implement this standard successfully in the observed lesson. She encouraged the students to use tables, graphs, and equations to model the problem. Using the TTPS approach was a significant factor that facilitated Teresa's implementation of the SMPs. Her focus on using high cognitive demand problems from the CMP curriculum to learn new mathematical concepts with understanding encouraged her students to develop important processes and proficiencies specified in the SMPs.

Implementation of MTPs observed by researcher. I used the MTP-OP protocol (See Appendix D) while I was watching the videotape of Teresa's lesson. This protocol helped me to specify Teresa's actions and her students' actions in the observed lesson that indicated a successful implementation of each one of the MTPs. The MTP-OP protocol includes 51 items (27 items of the teacher's actions and 24 items of the students' actions) that indicate if a teacher has a full implementation of the MTPs. Missing more than one item indicates a partial implementation of the related teaching practices. For example, if the teacher missed two items when implementing MTP1, this means that the teacher had a partial implementation of MTP1. Noticing that each one of the MTPs includes between four to nine of these items. I recorded and organized these items of Teresa's actions and her students' actions for each one of the MTPs in Table 4.8.

Table 4.8

Teresa's Implementation of MTPs

MTPs	Teacher Actions	Students Actions
MTP1. Establish mathematics goals to focus learning (2T,2S)	1-The teacher posted the lesson's goal on the smart board at the beginning of the class period, which were: I can determine whether a table or equation is linear. She also discussed and referred to the mathematical goal during instruction by asking students to compare the costs of the two companies using the equations ($C_{Mighty} = 49 + n$ and $C_{NoShrink} = 4.5n$). Then she asked them to compare these two equations with a table value that represent the cost of a T-shirt for a third company. 2-The teacher used the mathematical goal to make in-the-moment decisions during instruction by focusing throughout the lesson on students understanding of what a unit rate and constant are in an equation, what a linear relationship is, and how to compare costs using tables and equations.	1-Most of the class time, the students were engaging in (peer-to-peer, groups, and whole class) discussions during instruction. 2-The students connected their current work (Comparing Costs problem) with mathematics that they studied previously (problem 2.2 and problem 2.1) by finding the similarity between these problems.

(table continues)

Table 4.8

Teresa's Implementation of MTPs (continued)

MTPs	Teacher Actions	Students Actions
MTP2. Implement tasks that promote reasoning and problem solving (4T, 3S)	1-The teacher provided opportunities for exploring and solving the Comparing Costs problem that extended students' understanding. 2-The teacher used the CMP book to pose tasks (Comparing Costs problem) that require a high level of cognitive demand. 3-The teacher provided scaffolding for students to explore path solutions to solve problems without telling them how to solve them. 4-The teacher encouraged students to use varied strategies to solve a problem. For example, she asked students about their strategies to solve the problem: "For what number of t-shirts is the cost of the two companies equal?" (Teresa, Observation, Feb 15, 2019)	1-The students persevered in exploration. They explored what a unit rate, y-intercept, and solution of an equation were in a different context. They also investigated how to compare costs using equations, tables, and graphs. 2-The students used different representations such as tables, graphs, and equations to understand how to compare costs. 3-The students accepted a variety of solutions from their classmates. For example, some students solve a task using tables while others used a guess and check strategy.
MTP3. Use and connect mathematical representations (4T, 5S)	1-The teacher regularly encouraged students to discuss and make connections among representations. For example, she asked students, "How would a unit rate, y-intercept, and solution of an equation show up in a table? In a graph?" (Teresa, Observation, Feb 15, 2019) 2- The teacher introduced graphs as a useful way for students to use in the comparison when students used tables and equations to compare costs. 3- The teacher hung two t-shirts on the front of the class-wall for all students to see and an equation was hanging on each T-shirt to visualizes the costs of the T-shirt in each company. 4- The teacher focused students' attention on what each part in a linear equation represented, what information that a line in the graph tells us, and how can we find needed information from a table.	1-The students used multiple forms of representations (table, graph, and equation). 2-The students justified their mathematical understanding and reasoning with drawing. For example, when the teacher posted the first quadrant in the smartboard and pointed at y-intercept of the equation ($C_{Mighty} = 49 + n$), she asked, "Why did the mathematician decided to call this a y-intercept?" (Teresa, Observation, Feb 15, 2019). A student drew the other three quadrants and said that the line in the graph hits y-axis and continues to the other quadrants. So, it's called y-intercept. 3-The students made choices about which strategy to use (tables or guess and check). 4-The students contextualized mathematical ideas by connecting equations and tables with the Comparing Costs problem. 5-The students were allowed to use any representation that can help in their understanding.

(table continues)

Table 4.8

Teresa's Implementation of MTPs (continued)

MTPs	Teacher Actions	Students Actions
MTP4. Facilitate meaningful mathematical discourse (4T, 4S)	1-The teacher engaged students in purposeful sharing of mathematical ideas and reasoning. For example, she said, "See if you and your partner come up with a defense as to whether [26 is] the write answer or whether I should have a 27." (Teresa, Observation, Feb 15, 2019) 2-The teacher selected student solution strategies for a whole-class discussion, but she did not sequence them. She asked students about their strategies and discussed all strategies that students used. 3-The teacher facilitated discourse among students by asking students to justify their solutions and answers. For example, when a student guessed the answer, Teresa asked the student to explain this strategy. 4-Teresa made explicit connections between the lesson goals and different representations (table, graph, and equation).	1-The students explained their ideas and reasoning to one another in pair, small-group, and whole class discourse. For example, during a peer-to-peer discussion to solve Comparing Costs problem – part B, the teacher asked the pairs of students, "What is the discussion here?" A student said, "We are talking about how the design fee is better for Mighty, and I think that is worst for No-shrink because the design fee ($34) is less than Mighty ($49) and more than No-shrink ($0)." (Teresa, Observation, Feb 15, 2019) 2-The students listened carefully and critiqued the reasoning of their peers by saying why they agree or disagree. For example, during solving the Comparing Costs problem – part B, a student said, "I'm saying it's $2.5 per T-shirt but he doesn't agree with me". The teacher asked, "How could you prove it to him?" (Teresa, Observation, Feb 15, 2019). The student took a pencil and found the interval between the number of t-shirts and the costs of them. 3-The students seek to understand the approaches used by peers. 4-The students identified how different approaches to solve a problem are the same and how they are different. They identified how comparing costs using equations was different from comparing costs using tables.
MTP5. Pose purposeful questions (4T, 3S)	1-The teacher asked questions that build on student thinking. For example, she asked, "Would you and your partner see if you agree on which of these variables is independent or dependent variable?", "Did anybody solve this a different way?", "What this table tells you about Big T?" (Teresa,	1-The student had time to think carefully about how to present their responses without rushing. For example, the teacher asked students to stand when they now the answer. 2-The students justify their reasoning not simply provided the

(table continues)

Table 4.8

Teresa's Implementation of MTPs (continued)

MTPs	Teacher Actions	Students Actions
	Observation, Feb 15, 2019) 2-The teacher asked questions that require explanation and justification. For example, she asked, "Why was it better at No-Shrink ($C_{NoShrink}$ = 4.5n) for 12 shirts but then it is way better to go to Mighty (C_{Mighty} = 49 + n) for 20 shirts?" (Teresa, Observation, Feb 15, 2019) 3-The teacher asked questions that make mathematics more visible. For example, when a student said that the design fee (y-intercept) in the equation ($C_{NoShrink}$ = 4.5n) is 0 but he did not know way, Teresa asked, "What could this equation really look like?" (Teresa, Observation, Feb 15, 2019). Another student said that $C_{NoShrink}$ = 4.5n + 0, but they did not need to write it in that equation. 4-Most of the time, the teacher allowed wait time that more students can formulate and offer responses.	answer. For example, during a peer-to-peer discussion, the teacher asked pair of students, "What is the discussion here?" (Teresa, Observation, Feb 15, 2019). A student said, "We are talking about how the design fee better for Mighty, and I think that is worst for No-shrink because the design fee ($34) is less than Mighty ($49) and more than No-shrink ($0)." (Teresa, Observation, Feb 15, 2019) 3-The students listened and commented the contribution of their classmates. For example, when a student said that 1 is the coefficient of the equation C_{Mighty} = 49 + n, another student made a comment that 1 means one doller per T-shirt.
MTP6. Build procedural fluency from conceptual understanding (4T, 3S)	1-The teacher provided students with opportunities to use their own strategies for solving problems. She asked students to solve the problems at home and bring their ideas next day. 2-Teresa explained why the procedures that they were using worked to solve particular problems. For example, when a student made a comment that y-intercept is at the start of the table, Teresa said, "Although this table started at zero, we are going to have tables that start with negatives. It's important to say it is y value when x equals zero." (Teresa, Observation, Feb 15, 2019) 3-Teresa connected students' strategies to more efficient procedures. For example, when a student found that the costs of 12 shirts at Mighty is $61, Teresa asked, "How did you get that?" The student replied he added 12 to 49. Teresa repeated what the student said in more efficient procedures and said, "you substitute n	1-The students were able to explain the mathematical basis for the procedures that they are using. For example, a student took a pencil and found the interval between the number of t-shirts and the costs of them to find the unit rate. 2-The students demonstrated flexible use of strategies and methods. They used two strategies to solve the problem (For what number of t-shirts is the cost of the two companies equal?). 3-The students determined whether specific approaches generalize to a broad class of problems. For example, when students discussed the similarity between Comparing Costs problem and the previous problems (problem 2.1 and problem 2.2), they were able to generalize how to find a coefficient, y-intercept, and solution of an equation for any linear equation.

(table continues)

Table 4.8

Teresa's Implementation of MTPs (continued)

MTPs	Teacher Actions	Students Actions
	with 12 in the equation (C_{Mighty} = 49 + n) and add that together to get $61". 4-The teacher used visual models (a graph) to support students' understanding of comparing costs between companies.	
MTP7. Support productive struggle in learning (3T,2S)	1-The teacher gave students time to struggle after launching each problem. 2-The teacher helped students realize that making mistakes is a natural part of learning. She asked questions to reveal any misconception that may occur. For example, she pointed at the equation (d = 45 + t) and asked, "Do you think d is the dependent variable because it starts with d?" (Teresa, Observation, Feb 14, 2019) 3-The teacher praised students for their ideas and persevering in solving problems. For example, when a student used a table to solve a problem, she said, "Check it out, what a great idea." (Teresa, Observation, Feb 15, 2019).	*1-The students did not ask questions that are related to the sources of their struggles neither that may help them make progress in understanding. 2-The students worked together to solve the Comparing Costs problem.
MTP8. Elicit and use evidence of student thinking (2T, 2S)	1-The teacher more frequently elicited and gathered evidence of student understanding during instruction. Sometimes she asked students to give a quiet thump to show if they understand or not. Other times, she asked questions such as "Does this equation (C_{Mighty} = 49 + n) have a coefficient? What that 1 means in the equation? How do I know that t is the independent variable?" (Teresa, Observation, Feb 15, 2019) 2-The teacher made in-the-moment decision on how to respond to students. For example, when Teresa asked if any student has a different equation than d = 45 + t, a student answered d = t + 45. Thus, Teresa made a discussion on why these two equations are equivalent.	1-The students reveled their mathematical understanding and reasoning by writing the solutions in the given sheet and answering their teacher's questions. 2-The students shared and discussed their ideas and reasoning with their partners to support the learning of their classmates.

* = Not Implemented; T = Teacher; S = Student; (#T, #S) = #T number of the teacher's actions, #S number of students' actions

The teacher's actions and the students' actions in this table revel that Teresa implemented all MTPs in the observed lesson. She implemented all the 27 teacher items and her class implemented 23 out of 24 of the student items. In MTP7 (Support productive struggle in learning), the students missed one item which was asking questions that were related to the sources of their struggles and may help them make progress in understanding. However, missing one item did not affect the overall implementation of MTP7 because the students worked with peers and asked questions to each other. Moreover, using TTPS approach with a curriculum that sports this approach of teaching facilitated Teresa's implementation of MTPs. She said that in a TTPS lesson, "it's not about telling them, it's about them discovering it and recognizing that they can solve a problem in different ways" (Teresa, Post-Interview, Feb 26, 2019). That is, using the pedagogy of TTPS approach (such as allowing students to use different strategies and representations to solve a problem) helped Teresa in understanding and implementing the MTPs. Noticing that, Teresa's interpretation of MTPs during the pre-interview was consistent with her implementation of these practices in the observed lesson.

Summary of Teresa's Case

Teresa used the TTPS approach and the CMP curriculum to understand and implement the SMPs and MTPs. Using TTPS helped Teresa to understand all the SMPs except SMP4 (modeling with mathematics) and the difference between SMP7 (Look for and make use of structure) and SMP8 (Look for and express regularity in repeated reasoning). However, she was able to successfully implement all SMPs in her observed lesson. This mean, she understands all the SMPs, but she couldn't provide a concise description of these standards. In addition, Teresa understands all the MTPs even though she was unfamiliar with them. She also had a full implementation of all the MTPs in the observed lesson. Teresa felt very fortunate that her district supported them to use TTPS and provided a helpful curriculum to use.

Grace's Case

Grace is an eighth-grade female mathematics teacher with around 20 years of experience in teaching middle school mathematics. She has a bachelor's degree in education, and she is the coach of the school's mathematics team. Grace is currently teaching two levels of mathematics classes: algebra 1 and the regular eighth grade math. Grace used the CMP curriculum and the TTPS approach to teach both levels of mathematics classes. This case study used classroom data collected from one of Grace's algebra 1 classes.

Typical Lesson as Described by Grace. In the pre-interview, Grace described how she uses the TTPS approach in her typical lessons. Grace said that she usually starts a lesson with posting the homework assignment on the board, grading homework, putting answers up on the smart board, discussing any difficult areas or questions, and then collecting the homework. Her purpose for collecting homework is to visually see what specific things the students struggle with and to provide individual comments. In the launch phase, Grace pre-teaches some vocabulary or reviews some basic ideas that students might need for the new lesson, then she presents the challenge of the problem. In the explore phase, the teacher gives students time to write or start thinking about the problem independently. Next, students discuss their ideas and strategies to solve the problem with partners while the teacher circulates among the groups asking scaffolding questions and determining what their strategies and ideas are. In the summarize phase, Grace said, “We'll get together as a class and say OK. So, what did people think? Do I have to do this? We'll kind of go through it that way” (Grace, Pre-Interview, Feb 19, 2019). Her description of the summarize phase was brief, but I believe she meant that she discusses students’ ideas and strategies in a whole class discussion. Grace’s description of her typical lesson aligns with the literature on the TTPS approach. She seems to fully understand how to use TTPS. She also relies on the CMP book to plan and implement her daily lessons when using the TTPS approach.

Interpretation of SMPs as described by Grace. During the pre-interview with Grace, she showed familiarity with the SMPs. She said, “This is how they're supposed to be solving their problems in all of their concepts that we are working through” (Grace, Pre-Interview, Feb 19, 2019). In other words, she described the SMPs as the process and expertise that students should use to solve mathematical problems. Grace also shared, “[we] don't use all of them [SMPs] daily. There's not enough time. And it just kind of depends on what the lesson is” (Grace, Pre-Interview, Feb 19, 2019). Therefore, Grace does not implement all the SMPs in one-day lessons because a 45-minute class period is not long enough to implement all the SMPs. Grace also indicated that she does not implement all the SMPs in some types of lessons, but she did not provide a specification of these types of lessons. I asked Grace to go one by one through these standards and explain how she usually uses them in her TTPS lessons (See Table 4.9).

Grace’s interpretation in this table revealed that she understood seven of the eight SMPs. She did not interpret SMP4 in a way that aligns with the literature. She described SMP4 as *modeling the mathematics* in which students are encouraged to use concrete tools to understand new concepts. However, what was intended by the CCSSM for SMP4

Table 4.9

Grace's Interpretation of SMPs

SMPs	Interpretation	Sample Responses from Grace
SMP1. Make sense of problems and persevere in solving them	Students persevere by applying all strategies they know.	"Keep trying, what else can we do to make sure that you are applying all the strategies that you know." (Grace, Pre-Interview, Feb 19, 2019)
SMP2. Reason abstractly and quantitatively	Students explain how their answers make sense.	"Can you give me a reasonable answer that you think that might happen? Is it going to be greater than zero less than zero or something like that? Do you think this is make sense?" (Grace, Pre-Interview, Feb 19, 2019)
SMP3. Construct viable arguments and critique the reasoning of others	Students justify their answers with their partners first, and then in a whole class discussion.	**"**They're doing their partners, they kind of justify to each other why did I do what I do. When we discuss it in a group then I'll say ok, so why did this happen? Give me the justification for this." (Grace, Pre-Interview, Feb 19, 2019)
SMP4. Model with mathematics	Students use tools to represent abstracts as concrete forms to understand mathematics.	"We're going to get the concrete for that abstract. We're working on transformations right now. So, they have a transparency with that figure on it. We are going to physically rotate, reflect or translate that object then reason out from there. So, what's the algebra rule that goes with it." (Grace, Pre-Interview, Feb 19, 2019)
SMP5. Use appropriate tools strategically	Students use mathematical tools to solve problems and practice some exercises.	"Sometimes we're using calculators, graphing calculators, or transparencies to help us with that. They all use math IXL on their computer to help them reinforce those skills." (Grace, Pre-Interview, Feb 19, 2019)
SMP6. Attend to precision	Students should be precise in mathematical verbal and written forms.	"We talk about being precision a lot especially with algebra. Pay attention to the details, losing negative signs, adding numbers correctly, being careful in your mathematics using your order of operations, those kinds of things." (Grace, Pre-Interview, Feb 19, 2019)
SMP7. Look for and make use of structure	Students look for structures to solve a problem.	"Would it be helpful to make a table for this? Would it be helpful to make a graph for this? What is that structure that's going to help us solve that problem?" (Grace, Pre-Interview, Feb 19, 2019)
SMP8. Look for and express regularity in repeated reasoning	Students look for repeated reasoning across problems.	"What I've done in the past, what can I do use it now. Is this something that's going to be happening again and again and again?" (Grace, Pre-Interview, Feb 19, 2019)

is *modeling with mathematics* where students use the mathematics they know to solve real-life problems. Furthermore, Grace's interpretation presented in this table aligns with the TTPS approach. In other words, using TTPS helped Grace to understand how to implement the SMPs in her lessons. Grace said, “[TTPS] helps students for the process Standards because you're given that problem and you're trying to do some of those things to help you solve that problem” (Grace, Pre-Interview, Feb 19, 2019). That is, Grace considered the TTPS approach as a way to facilitate implementation of the SMPs.

Interpretation of MTPs as described by Grace. During the pre-interview with Grace, I asked her about what she knows about the MTPs. She said, “I rely more on this [SMPs] than I think about this [MTPs]” (Grace, Pre-Interview, Feb 19, 2019). This means, she focuses on the SMPs more than the MTPs when she plans and implements her daily lessons. However, she gave the impression that she tries to implement most of the MTPs every class period when she said, “I think this is kind of what we strive to do as much as we can every class period. Does all of this happen every class period? No” (Grace, Pre-Interview, Feb 19, 2019). This means, Grace does not implement all MTPs in some classes, but she tries to implement MTPs in most of them. I asked Grace to explain how she implements the MTPs in her typical TTPS lessons. I recorded in Table 4.10 Grace’s interpretation of the MTPs and included sample responses to the pre-interview questions from her own words.

Grace’s interpretation of the MTPs in Table 4.10 revealed that she fully understood each one of the MTPs because her description of these practices aligned with the literature on MTPs. Grace’s interpretation also illustrated that she relies on using the CMP curriculum to understand and implement some teaching practices in her TTPS lessons. That is, Grace uses high-cognitive demand problems from the CMP curriculum to promote students reasoning and problem solving (MTP2), and open-ended approaches that support students’ use of multiple representations (MTP3). She also uses the suggested purposeful questions (from the CMP curriculum) that encourage a task to be investigated in multiple ways (SMP5).

Table 4.10

Grace's Interpretation of MTPs

MTPs	Interpretation	Sample Responses from Grace
MTP1. Establish mathematics goals to focus learning	The teacher posts and talks about learning goals for each lesson without formally reading them.	"We always have learning targets or focused questions for every lesson up on the smart board … we don't necessarily formally read through that, but I'll say something about them." (Grace, Pre-interview, Feb 19, 2019)
MTP2. Implement tasks that promote reasoning and problem solving	The type of problems in the CMP curriculum do promote reasoning skills.	"Our task themselves from CMP does this, it does promote the reasoning." (Grace, Pre-interview, Feb 19, 2019)
MTP3. Use and connect mathematical representations	The teacher focuses on using and understanding the connections between concrete, abstract, and visual representations using the CMP curriculum.	"The whole point of it [CMP curriculum] is understanding the connection between the concrete, the abstract, and the visual versus the equations … I think we're constantly showing all the different kinds of representations especially tables, graphs, and equations in this transformation unit." (Grace, Pre-interview, Feb 19, 2019)
MTP4. Facilitate meaningful mathematical discourse	The teacher encourages discussions (pairs, groups, and whole class) about how to solve a problem.	"That's the talking of the problem, kids talk to each other, they talk to me, we talk to the group." (Grace, Pre-interview, Feb 19, 2019)
MTP5. Pose purposeful questions	The teacher poses questions from the CMP book.	"Those questions that are embedded in the CMP curriculum are already there. I don't necessarily have to seek them on my own." (Grace, Pre-interview, Feb 19, 2019)
MTP6. Build procedural fluency from conceptual understanding	The teacher gives students some exercises after solving problems.	"Now we're understanding that mathematics and how do we get to that rule. We use the math IXL to help us. Now let's apply it and use it." (Grace, Pre-interview, Feb 19, 2019)
MTP7. Support productive struggle in learning	The teacher supports students during solving problems gradually.	"They've got to struggle with that problem a little bit to think about it on their own, and then talk about it with their partners. So that they're struggling but maybe now so I can give them a hint, then OK now I'm really stuck, raise my hand, then I'll come over and give them another question." (Grace, Pre-interview, Feb 19, 2019)
MTP8. Elicit and use evidence of student thinking	The teacher assesses students' understanding when they share their ideas.	"They have to describe why did you do what you did? and that question happens every day." (Grace, Pre-interview, Feb 19, 2019)

Lesson as Observed by Researcher. I observed and videotaped Grace while she taught one lesson during a one 45-minute class period. I conducted a post-interview with Grace to understand her purposes and actions when she was teaching the observed lesson. The results from the observed lesson revealed that Grace did not use the TTPS approach. The main focus of TTPS should involve letting students solve real-life problems by themselves to learn new mathematical concepts. However, Grace did not allow students to think of entry points to solve the problems. She taught students to use one strategy to understand the new mathematical concepts and then use this strategy to solve the problems in the lesson. That is to say, Grace used *Teaching For Problem Solving* (TFPS) in the observed lesson. The main focus in TFPS is on teaching students some mathematics procedures to learn new mathematical concept and apply this knowledge to solve problems. Moreover, the teacher did not use the suggested questions from the CMP teacher's guide book. These questions were designed to help teachers to use TTPS to implement the high-cognitive demand problems in each lesson.

The title of the Grace's observed lesson was Spinning on a Grid: Coordinate Rules for Rotations. The Spinning on a Grid problem asked students to rotate a flag on a grid and find coordinate rules for rotations that would rotate a flag 90° or 180° counterclockwise about point A. The targeted I-can statement was: I can apply the coordinate rule (x,y) —> (□ , □) that tells how to move any point on a grid to its image under turns of 90° and 180°. Grace started her class by posting homework and the learning goal for the lesson on the smart board. Then she asked students to grade and correct their homework assignment while she explained and wrote the right answers on the board. Next, she collected the homework sheets. In the launch phase, Grace reviewed algebra rules for reflections and translations that students studied before, then she introduced a third type of transformation which was rotation. She posted the problem 3.3: Spinning on a Grid – part A on the board. She gave her students worksheets to record their work and answers to the problem and transparencies to use to rotate the flag. In the explore phase, Grace asked them to use the transparency and rotate the flag. She said, “Take your transparency, lined it on top of the figure, and rotate it 90° counterclockwise around the origin”. Then, Grace asked students to write the new ordered pairs for the flag’s image on their worksheets, and label the flag’s image using A′, B′, C′, D′, E′. Next, the students talked with their partners to check their answers, find a rule for rotating a flag 90° counterclockwise, and justify their rule to each other. At the same time, Grace circulated among the groups and asked them to explain their rule for the 90° rotation. In the summarize phase, Grace asked students to give her the new ordered pairs for the flag’s image, the rule for rotating a flag 90° counterclockwise, and a justification for this rule. She recorded the answers on the smart board (See figure 2).

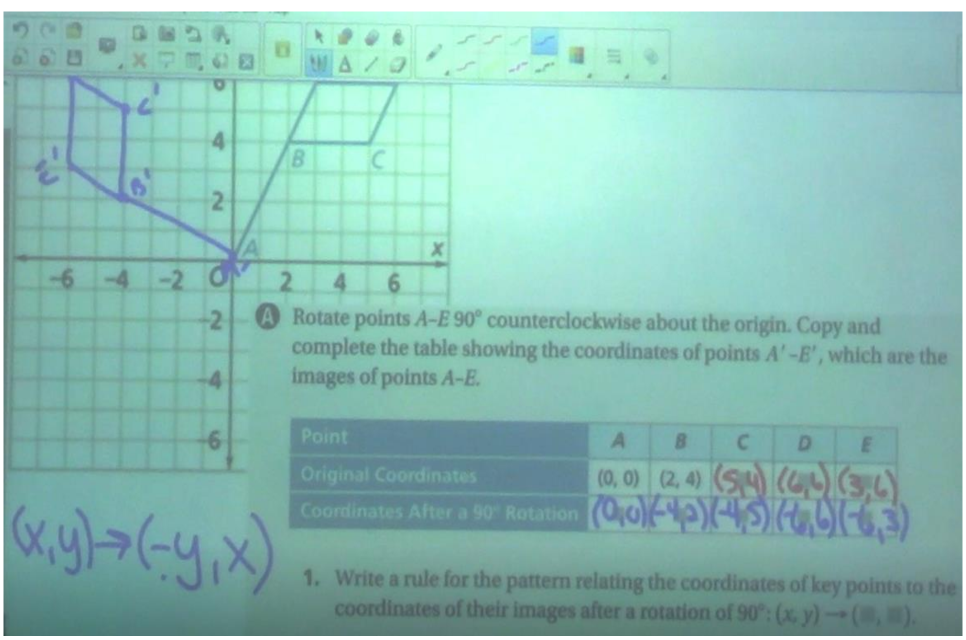


Figure 2. Rotating a flag 90° Counterclockwise

Figure 2 shows the solution to the Spinning on a Grid problem when using the transparency. It shows the flag (A, B, C, D, E) and its image after 90° rotation (A´, B´, C´, D´, E´), a table of the flag's order pairs and the flag image's order pairs after rotation, and the rule for this rotation which was (x,y) —> (-y , x).

Next, Grace asked the students if this rule can work in any quadrant. When students were not sure about the answer, she chose a point in the second quadrant and asked the students to use the transparency to rotate that point. Thus, students found that this rule works in any quadrant.

Afterward, Grace launched part B of the Spinning on a Grid problem. She started the explore phase when she asked students to work with their partners and use the transparency to rotate the flag 180° counterclockwise. She asked them to record their new ordered pairs in the table, label the new image using A´, B´, C´, D´, E´, and find a rule for rotating a figure 180° counterclockwise. Grace walked around the room as students worked and asked them scaffolding questions. In the summarize phase, as a whole class discussion, the teacher asked students about their new ordered pairs and their rule for rotating a figure 180° counterclockwise. She wrote down the results and remined the students of the targeted I-can statement of the lesson. Before the end of the class, she gave the students two tasks to practice the new learned rules for rotations and discussed how to solve the homework problems using these rules.

It was clear that Grace did not know the difference between the TTPS and TFPS approach because she described her observed lesson as a TTPS lesson. Although both approaches are similar in using problem solving to learn mathematics, the difference is in the purpose of using problems. If she used the TTPS approach, she would have allowed students to develop different strategies to solve the problem by themselves to understand the concept of rotation.

Implementation of SMPs Observed by Researcher. I watched the videotape of Grace's observed lesson several times while I was using the MCOP² protocol (See Appendix C) to assess her implementation of the SMPs in the observed lesson. I recorded samples of the students' and the teacher's actions from the observed lesson that indicated how the teacher attained each item in the MCOP² protocol (See Table 4.11). The MCOP² protocol has 16 items that assess a teacher's implementation of the SMPs. If the teacher attains all 16 items in a lesson, she would score high in implementing the SMPs. Missing one or more of these items indicate that the teacher does not have a full implementation of the associated standards.

Table 4.11

Grace's Implementation of SMPs

Items/SMPs	SMP1	SMP2	SMP3	SMP4	SMP5	SMP6	SMP7	SMP8	Samples of Implementation of SMPs from Grace's Observed Lesson
	Make sense of problems and persevere in solving them	Reason abstractly and quantitatively	Construct viable arguments and critique the reasoning of others	Model with mathematics	Use appropriate tools strategically	Attend to precision	Look for and make use of structure	Look for and express regularity in repeated reasoning	
1= Students engaged in exploration	x						x	x	The students had the opportunity to explore rules for rotating a flag 90° or 180°.

(table continues)

Table 4.11

Grace's Implementation of SMPs (continued)

Items/SMPs	SMP1	SMP2	SMP3	SMP4	SMP5	SMP6	SMP7	SMP8	Samples of Implementation of SMPs from Grace's Observed Lesson
2= Students used or generated two or more representations	θ				θ				The students used one representation (physical representation), which is using the transparency to rotate a figure on a grid.
3= Students were engaged in activities	x								All students engaged in mathematical activities which were rotating a figure 90° and 180° counterclockwise and finding a rule for each rotation.
4= Students assessed strategies	θ		θ					θ	The students did not assess the strategy they used (rotating a figure using a transparency). They just followed the teacher's instructions.
5= Students persevere in problem solving.	x	x	x		x				The majority of students used mental reasoning to explain the rules for rotating a figure 90° and 180° counterclockwise.
6= The lesson involved fundamental concepts to promote conceptual understanding.							x	x	The lesson included the concept of rotation, origin, and ordered pairs after a rotation to promote students' conceptual understanding of rotating a figure on a grid 90° and 180°.
7= The lesson promoted modeling with mathematics.		x		x					The students used the transparency and a table of ordered pairs to solve the Spinning on a Grid problem.
8= The lesson provided opportunities to examine mathematical structure.							x	x	The students examined the pattern in a table that contained both the ordered pairs of a figure and the new ordered pairs of that figure after rotation to find a rule for rotation.

(table continued)

Table 4.11

Grace's Implementation of SMPs (continued)

Items/SMPs	SMP1	SMP2	SMP3	SMP4	SMP5	SMP6	SMP7	SMP8	Samples of Implementation of SMPs from Grace's Observed Lesson
9= The lesson included tasks that have multiple paths to a solution or multiple solutions.	θ								The lesson included one path (using transparency) to a solution (the rule of rotation).
10= The lesson promoted precision of mathematical language.			x			x			The teacher said, "Be careful, ordered pairs should have parentheses around them, you should have a negative sign in the right position … remember to label your image A´, B´, C´, D´, E´." (Grace, Observation, Feb 21, 2019)
11= The teacher's talk encouraged student thinking.	x								The teacher asked students to justify their rules of rotation. She asked, "What happened? What algebra rule that're going to let me rotate a figure counterclockwise 90°? Look at your transparency, is there something that happened with that transparency?" (Grace, Observation, Feb 21, 2019)
12= There were a high proportion of students Talking related to mathematics.			x						Most of the students were talking related to mathematics of the lesson. The students checked the accuracy of their rotated image and the new ordered pairs with each other and find a rule for rotation.
13= There was a climate of respect for what others had to say.			x						Many students were respectfully sharing their work with partners to find rules for rotations and commenting in small groups when the teacher asked them to explain the rule of rotation.
14= The teacher provided wait-time.	x								The teacher provided time for the students to find rules for rotating a flag 90° and 180° counterclockwise and to justify their answers.

(table continues)

Table 4.11

Grace's Implementation of SMPs (continued)

Items/SMPs	SMP1	SMP2	SMP3	SMP4	SMP5	SMP6	SMP7	SMP8	Samples of Implementation of SMPs from Grace's Observed Lesson
15= Students were involved in the communication of their ideas to others.			x						The students shared their rotated image of the flag, the new ordered pairs, and their ideas about the rules of rotations.
16= The teacher used student questions/comment to enhance conceptual understanding	θ								The teacher did not use students' questions to enhance their understanding. For example, when a student asked that "we have been doing a counterclockwise rotation, is it a different thing if we do it clockwise? [the teacher replied] if we are going clockwise on the 90°, it's going to give us something different. For our air tests' purposes, we only have to know the counterclockwise" (Grace, Observation, Feb 21, 2019)

x = Implemented; θ = not Implemented; [shaded] = Not Applicable

This table showed that 4 items (item 2, 4, 9, and 16) were not implemented in Grace's observed lesson. This indicated that Grace did not have a full implementation of the standards: SMP1 (Make sense of problems and persevere in solving them), SMP3 (Construct viable arguments and critique the reasoning of others), SMP5 (Use appropriate tools strategically), and SMP8 (Look for and express regularity in repeated reasoning). In other words, Grace did not focus on implementing all SMPs in the observed lesson even though she explained their meaning during the pre-interview. The time was Grace's main reason for not implementing all the SMPs in a one-day lesson.

Moreover, in item 11, the Spinning on a Grid problem was supposed to have multiple paths for a solution (e.g., using transparency, finding slopes, drawing perpendiculars, etc.) as suggested in the CMP book, but Grace asked the students to use one path which was using the transparency. Grace changed the cognitive demand of the problem during its implementation from *doing mathematics* to *procedures with connection*. The problem in the CMP curriculum was designed as a *doing mathematics* problem, which required complex thinking and required students to explore and understand the nature of the concept of rotation. However, the teacher changed the cognitive demand of the problem to *procedures with connection*, which required using procedures to develop a deeper level of understanding of mathematical concepts and suggesting pathways to follow to solve the problems. Both ways (*doing mathematics* and *procedures with connection*) required high-cognitive demand, but *procedures with connection* is a level lower than *doing mathematics*.

In addition, during the post-interview with Grace, I asked her about the reason for teaching students to use only the transparency to do rotations. She said, “I liked the transparencies … because then they can get at more of a visual idea of rotation, I mean they can physically see what's happening when they're doing that rotation” (Grace, Post-Interview, Mar 4, 2019). This revealed that Grace focused on using one representation (physical representation) to understand the concept of rotation and to solve the Spinning on a Grid problem.

Implementation of MTPs Observed by Researcher. I used Grace’s videotape of the observed lesson with the MTP-OP protocol (See Appendix D) to identify the teacher’s actions and the students’ actions that indicate a full implementation of the MTPs. The MTP-OP protocol includes 51 items of teacher’s actions and students’ actions that indicate if a teacher has a full, partial, or no implementation of the MTPs. Each one of the MTPs includes between four and nine items of the teacher and student actions. If a teacher implemented all the items, then she has a full implementation of the MTPs. Missing more than one item indicates a partial implementation while missing all the items indicates no implementation of the associated teaching practices. For example, if a teacher implements all the four items in MTP1, then she has a full implementation of the MTP1. If a teacher missed 2-3 items in MTP1, then the teacher partially implements MTP1; and if a teacher missed all the items in MTP1, this indicates that the teacher did not implement MTP1. I also used Grace's responses in the post-interview to understand her actions in the observed lesson. Table 4.12 showed Grace's actions and her students' actions to implement the MTPs.

Table 4.12

Grace's Implementation of MTPs

MTPs	Teacher's Actions	Students' Actions
MTP1. Establish mathematics goals to focus learning (2T,2S)	1-The teacher posted the lesson's goal on the smart board. She also referred to the lesson's purposes throughout instruction, which were rotating a flag on a grid, finding the ordered pairs of the flag's image after rotation, and finding rules for rotations. 2-The teacher used the mathematical goal to make in-the moment decisions during instruction. That is, she engaged students in activates to be able to apply coordinate rules for rotations to move any point on a grid to its image under turns of 90° and 180°.	1-The students were engaging in peer-to-peer and whole class discussions during instruction. 2-The students were connecting their current work of rotation with translations and reflections that they studied previously.
MTP2. Implement tasks that promote reasoning and problem solving (4T, 3S)	1-The teacher provided opportunities for exploring and solving the Spinning on a Grid problem that extend students' understanding. 2-The teacher posed the Spinning on a Grid problem that required a high level of cognitive demand. 3-The teacher supported students in exploring the Spinning on a Grid problem. *4-The teacher did not encourage students to use varied approaches and strategies to solve the problems.	1-The students persevered in exploring the Spinning on a Grid problem. 2-The students used transparencies to support their thinking and problem solving. *3-The students did not expect that their classmates will use a variety of solutions.

(table continues)

Table 4.12

Grace's Implementation of MTPs (continued)

MTPs	Teacher's Actions	Students' Actions
MTP3. Use and connect mathematical representations (4T, 5S)	*1-The teacher did not allocate instructional time for students to use, discuss, and make connections among representations (students were taught to use a physical representation, which was the transparency, to rotate the flag in the problem 3.3). *2-The teacher did not introduce other forms of representations that can be useful to students (e.g., abstract representations such as using slopes, or visual representations such as drawing perpendiculars). 3-The teacher asked students to use a visual support (which was the image of the flag in their transparencies) to justify their reasoning of the rotations' rules. 4-The teacher focused students' attention on the structure of rotating a figure 90°and 180°.	*1-The students did not use multiple forms of representations. *2-The students did not justify their reasoning using other representations. *3-The students did not make choices about which forms of representations to use as tools for solving the problem. *4-The students did not contextualize mathematical ideas by connecting them to real-world situations. *5-The students did not consider the advantages or suitability of using various representations when solving problems.
MTP4. Facilitate meaningful mathematical discourse (4T, 4S)	*1-The teacher did not engage students in purposeful sharing of mathematical ideas and approaches using varied representations. *2-The teacher did not select and sequence students' approaches. *3-The teacher did not facilitate discourse among students by positioning them as authors of ideas, who defend their approaches. *4-The teacher did not make explicit connections to student approaches.	1-The students presented and explained their ideas and reasoning to one another in pairs and whole-class discussion. 2-The students critiqued the reasoning of peers when they justified the rules of rotations. *3-The students did not seek to understand the approaches used by peers. *4-The students did not identify how different approaches to solving a problem are the same or different.
MTP5. Pose purposeful questions (4T, 3S)	1-The teacher advanced students' understanding by asking questions that build on students' thinking (e.g., when students rotated the flag, the teacher asked them to explain what was happening with the ordered pairs of the flag).	1-The students were thinking carefully to present their responses. For example, they had time to think and present their responses after these questions, "What algebra rule is going to let me rotate a figure counterclockwise 90°? What's happening?" and "Where is your new point A, B, C, D, and E?"

(table continues)

Table 4.12

Grace's Implementation of MTPs (continued)

MTPs	Teacher's Actions	Students' Actions
	2-The teacher asked questions that require explanation. For example, the following scenario happened: *T: What algebra rule is going to let me rotate a figure counterclockwise 90°? What's happening?* *S: The original y turns to the negative and the x stays the same.* *T: y turned negative where?* *S: To the left.* *T: In what position?* *S: In the x-axis.* *T: So, did you see in your transparency that x and y flip flopped positions?* *S: Yes.* (Grace, Observation, Feb 21, 2019) 3-The teacher asked questions that make mathematics more visible. For example, she asks, "Look to the clock, what direction is counterclockwise?" 4-The teacher provided wait time for more depth of the questions. She provided time for the students to think of the question: "What algebra rule is going to let me rotate a figure counterclockwise 90°? What's happening?" (Grace, Observation, Feb 21, 2019).	2-The students were justifying their reasoning. For example, when the teacher was asking to find a rule for rotating a figure counterclockwise 180°, the following scenario happened: *S: minus x, minus y* *T: why?* *S: we're basically flipping it.* *T: What do you mean?* *S: Negative quadrant it's like opposite quadrant.* *T: Did x and y go back to what they are supposed to be?* *S: yes.* *T: y is pointing to the positive but it's pointing down, and y is negative, and x was pointing into the positive but know it's pointing into negative.* (Grace, Observation, Feb 21, 2019) 3-The students listened to, commented on, and questioned the contributions of their classmates when they were working with partners.
MTP6. Build procedural fluency from conceptual understanding (4T, 3S)	*1-The teacher did not provide students with opportunities to use their own reasoning strategies and methods for solving problems. *2-The teacher did not ask students to discuss and explain why the procedures that they were using worked to solve certain problems. *3-The teacher did not connect student-generated strategies and methods to more efficient procedures. 4-The teacher used visual models (transparency) to support students' understanding of general methods.	*1-The students did not demonstrate flexible use of strategies and methods. 2-The students made sure that they understood and can explain the mathematical basis for the procedures that they were using (e.g., students explained how the ordered pairs "flip flopped" (Grace, Observation, Feb 21, 2019) when using the transparency for rotation 90°). 3-The students determined whether specific approach can be generalized to a broad class of problems (e.g., the students discussed if the rules of rotations can work for any point in the quadrants).

(table continues)

Table 4.12

Grace's Implementation of MTPs (continued)

MTPs	Teacher's Actions	Students' Actions
MTP7. Support productive struggle in learning (3T,2S)	*1-The teacher did not give students time to struggle with tasks. *2-The teacher did not help students realize that confusion and errors are a natural part of learning. 3-The teacher praised students for their efforts in making sense of mathematical ideas (e.g., she said "Good"),	1-The students asked questions that were related to the sources of their struggles. For example, a student asked, "When do you put Ȁ?" (Grace, Observation, Feb 21, 2019) *2-The students did not help one another without telling their classmate what the answer is.
MTP8. Elicit and use evidence of student thinking (2T, 2S)	1-The teacher elicited and gathered evidence of students' understanding at strategic points during instruction. For example, she circulated among the groups while they were trying to find a rule for a 90° rotation and asked them questions to reveal their understanding. Moreover, in a whole class discussion, Grace asked questions to explain their reasoning of the rules of rotations. 2-The teacher made in-the-moment decisions on how to respond to students with questions that probe and scaffold. For example, when two students could not figure out how to find a rule for rotating a figure 90°, the following scenario happened: T: look at your order pairs to start with, do you see anything about the numbers? *S1: It's like (x , y) turned to the opposite (y , x).* *T: So, they flip flopped and what?* *S1: Oh! It's negative y, x.* (Grace, Observation, Feb 21, 2019)	1-The students revealed their mathematical understanding in written work and classroom discourse. For example, students were able to draw an image of a flag under 90° and 180° rotation, found the new ordered pairs of rotating a figure, and applied rules for rotations. 2-The students asked questions and responded to their partners when the teacher asked them to check their solutions.

* = Not Implemented; T = Teacher; S = Student; (#T, #S) = #T number of the teacher's actions, #S number of students' actions

The data in Table 4.12 shows that Grace had a full implementation of three teaching practices: MTP1 (Establish mathematics goals to focus learning), MTP5 (Pose purposeful questions), and MTP8 (Elicit and use evidence of student thinking). Grace had a partial implementation of MTP2 (Implement tasks that promote reasoning and problem solving), MTP3 (Use and connect mathematical representations), MTP4 (Facilitate meaningful mathematical discourse), MTP6 (Build procedural fluency from conceptual understanding), and MTP7 (Support productive struggle in learning). For each there were some missing teacher actions and student actions. Grace missed 12 out of 27 actions, and her students missed 10 out of 24 actions. In total, Grace missed 22 out of 51 items. Noticing that, the teacher's missing actions may affect the students' missing actions. For example, in MTP2, Grace missed one item which was encouraging students to use varied strategies to solve the problems. This missing item suggests that Grace's students did not expect that their classmates would use a variety of solutions.

Grace's interpretation of the MTPs was not consistent with her implementation of these practices. This is because she described how she would implement the MTPs in a TTPS lesson, while I observed her implementation of the MTPs in a TFPS lesson. This revealed that Grace did not implement all the MTPs when using the TFPS approach.

Summary of Grace's Case

In the pre-interview with Grace, she described her typical TTPS lessons in a way that aligned with the literature on the TTPS approach. She also described how she typically implements the SMPs in her TTPS lessons in a way that align with what was intended by the CCSSM process standards (except for the SMP4). She had a misconception in understanding the SMP4 (Model with mathematics). For the MTPs, Grace's description of the MTPs revealed that she understood all these teaching practices. However, she firmly noted that, in a 45-minutes class period, "There's not enough time" (Grace, Pre-Interview, Feb 19, 2019) to implement all the SMPs and MTPs". In the observed lesson, Grace used the TFPS approach, which indicated that she did not know the difference between TTPS and TFPS. The observed lesson was a 45-minute class period, and she did not have a full implementation of the SMPs and MTPs in that lesson.

Cross-Case Analysis

In order to address the two research questions in this case study, I looked across the cases (Hana, Teresa, and Grace) to explore the commonalities and differences between the teacher participants' interpretations and implementations of the SMPs and MTPs. For the comparison, I again used the six categories to organize the data: (1) typical lesson as described by teacher, (2) interpretation of SMPs by teacher, (3) interpretation of MTPs by teacher, (4) lesson as observed by researcher, (5) implementation of SMPs observed by researcher, and (6) implementation of MTPs observed by researcher. By looking at the first three categories, the commonalities and differences between teachers' interpretations provided three different models on how to use TTPS to understand the SMPs and MTPs. The other three categories illustrated some differences between the TTPS and TFPS approaches when implementating the SMPs and MTPs.

Typical Lesson as Described by Teacher

I compared teachers' responses to the pre-interview questions with what their typical TTPS lessons look like. I aligned these responses with the pre-interview questions and their lesson plans (launch, explore, summarize). I added two phases (introduction and conclusion) to illustrate how the teachers usually start and end their TTPS lessons. Table 4.13 shows the commonalities and differences between the teachers' descriptions of their TTPS lessons and samples of their responses to the pre-interview questions.

The commonalities in Table 4.13 showed that Hana, Teresa, and Grace provided a similar description of their TTPS lessons in the four phases (introduction, launch, explore, summarize). These commonalities revealed that the teachers understood how to

Table 4.13: *Across Teacher Comparison of Teachers' Descriptions of Typical Lessons They Teach*

Phase	Commonalities	Differences	Hana's Description	Teresa's Description	Grace's Description
Introduction	Starting the lesson with homework	-Hana: warm up activities -Teresa: Taking time to discuss homework -Grace: Grading and correcting homework	"We will start with some type of warm up. It may possibly be going over homework, but I really try not to do that often." (Hana, Pre-Interview, Feb 19, 2019)	"[We will] take time to discuss homework problems that I've given students. There's a lot of posing questions and partner discussions at times." (Teresa, Pre-Interview, Feb 12, 2019)	"We always start with the copy of the assignment down on the smart board. Then we grade the homework, put answers up on the smart board we'll discuss what things that they have questions on" (Grace, Pre-interview, Feb 19, 2019)
Launch	Presenting the challenge in a problem	-Hana: Discussing the importance of the new concepts in real world -Teresa: Posing a problem with a real life scenario -Grace: Reviewing some concepts that students learned	"I really want them to understand why you're learning that. Where would you use it in the real world? Then we'll go into the main problem of the lesson." (Hana, Pre-Interview, Feb 19, 2019)	"I will pose a problem with a real life scenario" (Teresa, Pre-Interview, Feb 12, 2019)	"Sometimes we have to do some review of some basic things that we've talked about previously that we might need for this lesson. Then we kind of read through the lesson." Grace, Pre-interview, Feb 19, 2019)
Explore	-Providing time for students to think about the problem while students work with partners -Gathering information about students' ideas, strategies, and misconceptions	Not indicated in their descriptions	"We spend a good portion of class from 30 to 45 minutes to actually find the main target goal. They're working in groups or partners or on their own … I don't tell them right away if they're right or not." (Hana, Pre-Interview, Feb 19, 2019)	"I give independent think time before I allow them to share ideas … partners share ideas first and then share ideas [with groups] to see if they're in agreement … during this time, I'm eavesdropping trying to see what misconceptions or what interesting ideas they might have of solving a problem trying to then bring that together at the end of that lesson" (Teresa, Pre-Interview, Feb 12, 2019)	"I let them write or start thinking about this. See what ideas. OK. Now discuss it with your partners. What do you need to talk about or what are your strategies to solve this? What are we going to do? … I talk to individual groups or partners and see what you give, any questions. What are your strategies?" Grace, Pre-interview, Feb 19, 2019)

(table continues)

Table 4.13: *Across Teacher Comparison of Teachers' Descriptions of Typical Lessons They Teach (continued)*

Phase	Commonalities	Differences	Hana's Response	Teresa's Response	Grace's Response
Summarize	Discussing students' ideas and strategies in a whole class discussion	Not indicated in their descriptions	" Then we pull back together as a class and we talk about what they discovered. They share ideas [and] strategies. I have them locked to the smart board and share." (Hana, Pre-Interview, Feb 19, 2019)	" Then bring that [misconceptions and ideas] together at the end of that lesson … students might give their ideas about something and then other students are asked to analyze who solved this correctly." (Teresa, Pre-Interview, Feb 12, 2019)	" Then we'll get together as a class and say OK. What did people think? Do I have to do this? We'll kind of go through it." Grace, Pre-interview, Feb 19, 2019)
Conclusion	Not indicated in their descriptions	Hana: Homework maybe assigned Teresa: Homework maybe continue solving the problem Grace: Homework is assigned, and students are giving an overview of how to apply the new learned concept to solve homework tasks	" If I think they really understand it, I'll give them some homework on that [or] we'll just hold homework off and I might use it as a warm-up the next day." (Hana, Pre-Interview, Feb 19, 2019)	If there is no time for summarize phase I will " tell them to continue to think about it [the problem] that evening and bring ideas." Teresa, Pre-Interview, Feb 12, 2019)	" We kind of give them the homework and give them an overview of that." Grace, Pre-interview, Feb 19, 2019)

use the TTPS approach because their descriptions align with the literature on TTPS. In addition, this table showed the differences between the teachers' descriptions of their TTPS lessons. They differed in how they began their lessons (introduction phase), how they connected the problems with students' prior knowledge (launch phase), and how they assigned homework (conclusion phase). These differences revealed that the teachers had different preferences in how to conduct TTPS lessons.

Interpretation of SMPs as Described by Teacher

I compared teachers' interpretations of the SMPs using Table 4.1 (Hana's interpretation of SMPs), Table 4.5 (Teresa's Interpretation of SMPs), and Table 4.9 (Grace's interpretation of SMPs). Table 4.14 illustrates similarities and differences between the three teachers' interpretations of how to implement each one of the SMPs. Again, the teachers explained the SMPs by focusing on what students should do in the classroom to learn mathematics. The comparisons presented in Table 4.14 showed some commonalities and some differences between the teachers' interpretations of the SMPs. The commonalities between the three teachers' interpretation revealed that the teachers understand all the SMPs except SMP4 (Model with mathematics). There was a common misconception among the teachers in interpreting SMP4 as *modeling the mathematics*, which is different than *modeling with mathematics*. Modeling the mathematics means using concrete materials or other visual representations to understand new mathematical concepts or ideas while modeling with mathematics means applying the mathematics that students know to solve real-life problems. Noticing that, Hana added to SMP4 that students model with mathematics by showing their work in solving problems which revealed that she understood this standard as both *modeling with mathematics* and *modeling the mathematics*.

Table 4.14

A Comparison Between Teachers' Interpretations of SMPs

SMPs	Hana's Interpretation	Teresa's Interpretation	Grace's Interpretation
SMP1. Make sense of problems and persevere in solving them	Connecting the problems with real-life situations.	Making sense of real-life problems and understanding why they need to persevere in solving them	Applying all strategies that they learned previously
SMP2. Reason abstractly and quantitatively	Linking answers to real-life situations	Using hands on activities and manipulatives to understand abstracts and quantities	Explaining how their answers make sense.
SMP3. Construct viable arguments and critique the reasoning of others	Explaining their ideas or strategies and why they agree or disagree with their peers.		
SMP4. Model with mathematics	Using visual and concrete materials to understand and solve problems		
	Showing and explaining their work	N/A	N/A
SMP5. Use appropriate tools strategically	Using tools to understand and solve problems		
SMP6. Attend to precision	Attending to precision in mathematical verbal and written forms.		
SMP7. Look for and make use of structure	Exploring ways to solve problems and understanding repeated reasoning	Looking for patterns and repeated reasoning within and across problems.	Looking for structures to solve a problem.
SMP8. Look for and express regularity in repeated reasoning	Exploring ways to solve problems and understanding repeated reasoning	Looking for patterns and repeated reasoning within and across problems.	Looking for repeated reasoning across problems.

N/A= Not Applicable

In addition, there were differences between the teachers' interpretations of (SMP1, SMP2, SMP7, and SMP8). These differences revealed that the teachers had different preferences in implementing the SMPs. For example, for SMP1 (Make sense of problems and persevere in solving them), the teachers focused on what students should do to make sense and persevere using different motivations. Hana encouraged students to make sense of problem and persevere by connecting them to real-life situations because it's hard for students to make sense of problems if these problems are not in real-life situations (e.g., compute the unknown value n in the equation n -7.23 = 6.90). Thus, Hana's students are constantly trying to solve problems and go back to check if their answers make sense. Teresa encouraged her students to make sense and persevere by using real-life problems and discussing with the students that "we're going to be practicing a growth mindset and that means we will struggle and we're going to make mistakes but we're going to persevere" (Teresa, Pre-Interview, Feb 12, 2019). Grace encouraged her students to make sense and persevere by trying different strategies and ideas. That is, although the teachers had differences in their interpretations of the SMPs, they had the same goals for encouraging students to do what was intended for the SMPs.

For the last two standards in Table 4.14 (SMP7 and SMP8), both Hana and Teresa did not differentiate between them. They explained them as one standard. However, Grace differentiated between these two standards and explained what each one of them was intended for. This revealed that not all teachers understand the difference between SMP7 and SMP8 when using the TTPS approach.

Interpretation of MTPs as Described by Teacher

I used Table 4.2 (Hana's interpretation of the MTPs), Table 4.6 (Teresa's interpretation of the MTPs), and Table 4.10 (Grace's interpretation of the MTPs) to compare teachers' interpretations of the MTPs. The purpose of this comparison was to identify commonalities and differences between the teachers' interpretations of the MTPs. I organized these comparisons in Table 4.15.

Table 4.15

A Comparison Between Teachers' Interpretations of MTPs

MTPs	Hana's Interpretation	Teresa's Interpretation	Grace's Interpretation
MTP1. Establish mathematics goals to focus learning	Specifying mathematical goals for each day to focus students learning	Posting mathematical goals without formally reading them.	
MTP2. Implement tasks that promote reasoning and problem solving	Implementing variety of tasks that involves problem solving process and reasoning.	Implementing a mathematical problem from CMP book	
	Discussing students' mistakes		N/A
MTP3. Use and connect mathematical representations	Connecting a task with real world using different representations.	Encouraging students to use and connect representations.	
MTP4. Facilitate meaningful mathematical discourse	Discussing if the new learning of mathematics make sense and where students can use them in real life.	Planning for mathematical discourse (pairs, groups, whole class).	Discussing how to solve a problem (pairs, groups, and whole class)
MTP5. Pose purposeful questions	Asking purposeful questions that guide students through problem solving process	Using questions from the CMP book.	
MTP6. Build procedural fluency from conceptual understanding	Posing extension problems	Posing some exercises	
MTP7. Support productive struggle in learning	Providing individual help, allowing students to choose their partner or assigning partners with higher mathematical thinking level to work with	Normalizing the errors and discussing the benefits of struggling	Supporting students during solving problems gradually
MTP8. Elicit and use evidence of student thinking	Assessing students understanding when they share their ideas		

N/A= Not indicated in the teacher's description

The teachers' interpretations of the MTPs presented in Table 4.15 revealed that the teachers understood all the MTPs because their interpretations aligned with what was intended by the NCTM. Moreover, this table illustrated some commonalties and differences in how the teachers implemented the MTPs in their classrooms. For the commonalities, the three teachers similarly interpreted the MTP8 (elicit and use evidence of student thinking). They all assess students' understanding when they share their ideas and explain their reasoning. Teresa and Grace provided similar interpretations of the MTP1, MTP2, MTP3, MTP5, and MTP6 as shown in the table while Hana interpreted them differently.

The differences between Hana's interpretation and the two other teachers' interpretations refer to different factors such as the length of the class time. For example, in the MTP2 (Implement tasks that promote reasoning and problem solving), Hana gives her students a variety of high cognitive demand problems while Teresa and Grace give their students one high cognitive demand problem from the CMP curriculum. The main reason of this difference about the number of problems is the length of class time. Hana has 95-minute class periods while Teresa and Grace have 45-minute class periods. The 95-minute class period gives Hana plenty of time to implement a variety of high cognitive demand problems in one lesson.

The three teachers have different explanations of how they usually implement the MTP4 and MTP7 in their TTPS lessons. These differences illustrate three different models to understand and implement MTP4 and MTP7. For example, for MTP7 (Support productive struggle in learning), each of the three teachers provided a different explanation for how they support productive struggle. Hana indicated that she supports students who are struggling by providing extra time, scaffolding questions, or more explanation. Sometimes she allows students to choose partners they are comfortable with and sometimes she assigns partners with different level of mathematical thinking. Teresa supports productive struggle by discussing with students how struggling in solving high cognitive demand problems makes students stronger because making errors and mistakes is something normal with these types of problems. Grace supports productive struggle by giving students a problem and allowing them to struggle and think on their own. Then she allows students to work with partners and discuss their thinking. When the partners still are struggling, she gives them a hint or asks them scaffolding questions. That is, the differences between teachers' interpretations of the MTPs provide varied models and a deeper understanding of how to implement these teaching practices when using TTPS.

Lesson as Observed by Researcher

I used the videotapes of the teachers' observed lessons along with post-interviews to compare teachers' data. I organized the teachers' practices from observed lessons into five phases: Introduction, launch, explore, summarize, and conclusion. I added the introduction and conclusion phases to illustrate any commonalities or differences in how a teacher starts or ends their classes. I also used the post-interviews to help me understand teachers' actions in the observed lessons.

Table 4.16 shows that Hana and Teresa used the TTPS approach while Grace used the TFPS approach. That is, the comparison in this table revealed that most of Grace's teaching practices were similar to the two other teachers (Hana and Teresa), but the big differences occurred in the exploring and summarizing phase. These differences classify Grace's lesson as a TFPS lesson.

Table 4.16. *A Comparison between the Teachers' Observed Lessons by Researcher*

Phases	Commonalities	Differences	Hana	Teresa	Grace
Introduction	Connecting the lesson with previous knowledge	N/A	Warm-up activities	Discussing and solving homework	Grading and correcting homework
Launch	Presenting the challenge in a problem	N/A	Discussing the importance of finding tips, and then creating a real-life scenario in which students asked to find 10% tip of $963.87	Discussing the previous problem, then -presenting part A of the Comparing Costs problem	Making a revision about what students learned in the transformation unit. Then clarifying the meaning of the Spinning on a Grid problem
Explore	Hana and Teresa: Providing time for students to think of path solutions	Grace: teaching one path solution	Providing time for students to think of path solutions, without telling them if their answers right or wrong. -Students worked with partners to find an entry point to the problem	Providing time for students to think of path solutions. -Students worked with partners to find an entry point to the problem -Asking students to continue working on the problem at home	Leading students to use the transparency to rotate a flag 90° - providing time for students to finding a rule for rotating a flag 90° -Students worked with partners to find rotations' rules
Summarize	Hana and Teresa: Discussing students' ideas, strategies, and mistakes	Grace: asking about the answers and justification of the answers	Discussing the strategies that students discovered (dividing: and multiplying) -Discussing students' mistakes and asked students if they agree to this solution or not and why	Next day: Asking students to share their solutions as a whole class discussion and -Discussed students mistakes in solving the problem. --presenting part B of the Comparing Costs problem	Asking students what the rotation's rules are and why, in a whole class discussion
Conclusion	Assigning homework	N/A	Assigning homework	Assigning a problem for homework	Asking students to solve some exercises and assigning homework

The TTPS approach involves allowing students to use multiple solution strategies to understand new mathematical concepts. Hana used the TTPS approach when she allowed students to use different strategies to understand the percent of a quantity. Teresa, also, used the TTPS approach in the observed lesson when she allowed students to use multiple strategies to understand linear relationships. Grace, however, used the TFPS approach in the observed lesson. Her decision to demonstrate the single strategy of using rotation with a transparency changed the focus of the lesson from allowing students to use different strategies to using a single strategy to make sense of and solve problems.

Implementation of SMPs Observed by Researcher

To compare teachers' implementations of SMPs, I compared Table 4.3 (for Hana)**,** Table 4.7 (for Teresa), and Table 4.11 (fore Grace). These tables have 16 items that indicate if a teacher has full, partial, or no implementation of the SMPs. To have a full implementation of the SMPs, a teacher should attain all 16 items. Missing more than one item indicates a partial implementation of the related standard. The results of this comparison revealed that Hana and Teresa attained all 16 items and scored high in their implementations. That is, Hana and Teresa had a full implementation of the SMPs in the observed lessons.

Grace attained 12 items out of 16. She did not implement item2 (Students used or generated 2 or more representations), item4 (Students assessed strategies), item9 (The lesson included tasks that have multiple paths to a solution or multiple solutions), and item16 (The teacher used student questions/comment to enhance conceptual understanding). In other words, Grace had a partial implementation of SMP1 (Make sense of problems and persevere in solving them), SMP3 (Construct viable arguments and critique the reasoning of others), SMP5 (Use appropriate tools strategically), and SMP8 (Look for and express regularity in repeated reasoning). The results of this comparison revealed that the teachers who use TTPS (Hana and Teresa) had a full implementation of all the SMPs while the teacher who used TFPS (Grace) had a partial implementation of the SMPs.

Implementation of MTPS Observed by Researcher

I compared each teacher's implementation of MTPs using Table 4.4 (for Hana), Table 4.8 (for Teresa), and Table 4.12 (for Grace). The results of this comparison revealed that the teachers who use TTPS (Hana and Teresa) had a full implementation of all MTPs while the teacher who used TFPS (Grace) had a partial implementation of some MTPs in the observed lessons. Grace did not have a full implementation of five teaching practices: MTP2 (Implement tasks that promote reasoning and problem solving), MTP3 (Use and connect mathematical representations), MTP4 (Facilitate meaningful mathematical discourse), MTP6 (Build procedural fluency from conceptual understanding), and MTP7 (Support productive struggle in learning).

Summary of Cross-Case Analysis

I compared the teacher participants' data to identify commonalities and differences among them. The comparison revealed that all the teacher participants explained how they used the TTPS approach in their lessons in a way that aligns with the literature on TTPS. The comparison of their explanations of the SMPs revealed a common misconception in understanding SMP4 (Model with mathematics). The comparison of the teachers' explanations of the MTPs revealed that all of them understood each one of the MTPs in a way that aligned with wat was intended by the NCTM (2014).

The comparison between the teachers' observed lessons illustrated that Hana and Teresa used the TTPS approach while Grace used the TFPS approach. In the implementation of the SMPs, Hana and Teresa had a full implementation of all the SMPs in one lesson while Grace had a partial implementation of some SMPs. In the implementation of the MTPs, Hana and Teresa had a full implementation of all MTPs while Grace had a partial implementation of the MTPs in the observed lesson.

Summary of Chapter 4

In this chapter, I used four data sources, which were one lesson plan, one pre- and post-interview, and an observation for that lesson plan, to collect data from three teacher participants about their interpretations and implementations of the SMPs and MTPs. The goal for collecting the data was to understand how teachers who are supported by their district to use TTPS interpret and implement the SMPs and MTPs. I performed a within-case analysis for each teacher participant. Then I performed a cross-case analysis. I organized the results using these six categories: (1) typical lesson as described by teacher, (2) interpretation of SMPs by teacher, (3) interpretation of MTPs by teacher, (4) lesson as observed by researcher, (5) implementation of SMPs observed by researcher, and (6) implementation of MTPs observed by researcher.

The results from the analysis revealed that the teachers interpreted the SMPs as intended by the CCSSM process standards but had a common misconception in understanding SMP4 (Model with mathematics). In addition, the teachers had commonalities and differences in interpreting the MTPs while their descriptions of these teaching practices aligned with what was intended by the NCTM (2014). Based on observing one lesson, it was found that the teachers who use TTPS (Hana and Teresa) had a full implementation of all SMPs and MTPs while teachers who use TFPS (Grace) had a partial implementation of some SMPs and MTPs. From these findings, four themes emerged:

1) **Supporting teachers to use TTPS helps them in their implementation of the SMPs more than in their interpretations.**
2) **Teachers who use TTPS understand and fully implement the MTPs.**
3) **Teachers who are supported to use TTPS may use TFPS.**
4) **Using TFPS may result in a partial implementation of the SMPs and MTPs.**

More discussion about these themes will be provided in the following chapter.

Chapter 5: Discussion

In this chapter, I present a summary of the study and answers to the two research questions based on the findings presented in Chapter 4. I also discuss theoretical and practical implications of this study in the field of mathematics education and teachers' understanding, and implications of the recent mathematical process standards and teaching practices. I conclude with a discussion of the limitations of this study and recommendations for future research.

Summary of Study

Throughout the U.S., an increasing number of states are adopting the Standards for Mathematical Practices (SMPs) that were released by the Common Core State Standards for Mathematics in 2010. These SMPs focus on what students should do to learn mathematics with understanding. With the widespread adaptation of SMPs across the country, mathematics teachers are expected to implement these in their classrooms. However, many teachers struggle to understand and implement them (Bostic & Matney, 2014; Mateas, 2016; Smith, Steele, & Raith, 2017). Although the National Council of Teachers of Mathematics released the Mathematics Teaching Practices (MTPs) in 2014, in order to support an implementation of the SMPs, many teachers still struggle to incorporate the SMPs into their practice.

A key reason for this struggle is that SMPs and MTPs are consistent with a constructivist view of learning, in which students construct new knowledge from their own understanding. When practice is based on constructivist views of learning, teachers “no longer present the content through clear demonstrations; they must instead create the conditions that will allow students to take their own effective mathematical actions”

(Smith, 1996, p. 393). This view requires teachers to shift their teaching away from direct instruction. Although constructivism provides insight into students' learning, it does not tell us how to teach. Thus, constructivist pedagogy provides us with a set of teacher actions that are based on a foundation of constructivism. Both the SMPs and MTPs recognize the need to invoke a type of teaching pedagogy that focuses on using problem solving to learn mathematics with understanding. This type of teaching should guide and support students' construction of knowledge through questioning, investigating, testing, and refining their own ideas rather than following procedures. An important type of teaching pedagogy that is recommended and discussed at length by NCTM (1980, 1989, 2000, 2014) is Teaching Through Problem Solving (TTPS).

The pedagogy of TTPS approach offers opportunities for mathematics teachers to understand and implement SMPs and MTPs (Gurl et al., 2013; Takahashi et al., 2013). This is because the theoretical foundation of the TTPS approach aligns with the recent focus on problem solving by CCSSM. Therefore, conducting an empirical study to investigate the alignment between TTPS and both SMPs and MTPs offers insight into how TTPS facilitates teachers' understanding and implementations of SMPs and MTPs.

With a ready site (a middle school that supports mathematics teachers' usage of the TTPS approach), I conducted a qualitative case study to investigate teachers' interpretations and implementations of SMPs and MTPs when using TTPS. I used a pre-interview, one lesson plan, a classroom teaching observation of that lesson plan, and a post-interview as instruments for collecting my data from three mathematics teachers. I performed within-case analysis that involved an in-depth exploration of individual cases, and a cross-case analysis that involved an exploration of similarities and differences between the cases. This analysis was guided by the two research questions:

RQ1 What is middle school teachers' interpretation and implementation of the CCSSM (2010) Standards for Mathematical Practice when their districts support TTPS?

RQ2 What is middle school teachers' interpretation and implementation of the NCTM (2014) Mathematics Teaching Practices when their districts support TTPS?

The finding from this analysis revealed that the teacher participants interpreted all the SMPs and MTPs (except SMP4. Model with mathematics). Moreover, when the teacher participants used TTPS, they implemented all the SMPs and MTPs. When they used Teaching For Problem Solving (TFPS), they implemented some of the SMPs and MTPs. This finding contributed to answering the two research questions. I will answer these research questions then discuss four themes that emerged from this finding at the following sections.

Answering Research Question 1: Teachers' Interpretation and Implementation of SMPs

Middle school mathematics teachers who use TTPS interpret and implement all the CCSSM (2010) Standards for Mathematical Practice in a way that aligns with what was intended by the CCSSM. SMP4 (Model with mathematics) was the only standard that revealed a misconception among teachers when interpreting it. In addition, when middle school mathematics teachers use the TTPS approach, they may have a full implementation of SMPs. When they use a different approach of teaching such as TFPS, they may have a partial implementation of SMPs.

Answering Research Question 2: Teachers' Interpretation and Implementation of the MTPs

Middle school mathematics teachers who use TTPS interpret and implement all the NCTM (2014) Mathematics Teaching Practices in a way that aligns with what was intended by the NCTM. In addition, when middle school mathematics teachers use TTPS, they may fully implement MTPs. When they use TFPS, they may partially implement MTPs.

Emerging Themes

Four themes emerged from the findings of the current study. These themes provided a deeper explanation of the answers of the two research questions. These themes also illustrated the significance of supporting teachers to use TTPS in their interpretation and implementation of SMPs and MTPs. These themes were:

(1) Using the TTPS approach may help teachers in their implementations of SMPs more than in their interpretations.

(2) Teachers who are supported to use TTPS may understand and fully implement MTPs.

(3) Teachers who are supported to use TTPS may use TFPS.

(4) Using TFPS may result in a partial implementation of SMPs and MTPs.

Discussion of Theme One: Using the TTPS Approach May Help Teachers in their Implementations of SMPs More than in their Interpretations

This theme illustrates that when teachers use TTPS, it may lead to a full implementation of the SMPs. However, they may not be able to fully explain the meaning of all eight SMPs. For example, data in my study showed the teachers had a common misconception in interpreting SMP4 (Model with mathematics). They interpreted SMP4 as *modeling the mathematics*, which is using visual and concrete materials to understand and solve problems, rather than *modeling with mathematics,* which comprises using mathematics that students know to solve real-life problems. This is common among many mathematics teachers (Bleiler et al., 2015). Two of the teacher participants (Hana and Teresa) were able to implement SMP4 in the observed lessons when they used TTPS. They encouraged students to use their mathematical knowledge to solve real-life problems. This suggests that teachers understand how to encourage students to model with mathematics, but they may not know the precise name for this action.

The important factors that support teachers to have a full implementation of SMPs when using TTPS may be curriculum materials and professional development programs. Takahashi et al., (2014) asserted that providing curriculum materials that support TTPS as well as conducting professional development programs that focus on how to use TTPS, is an effective way to facilitate teachers' implementation of the SMPs. Similarly, the participants in my study are supported by their district which provides curriculum materials such as the CMP curriculum that employs TTPS. This curriculum is a problem-centered curriculum focusing on promoting an inquiry-based teaching-learning classroom environment. It has a separate, extensive teacher’s guide book. One lesson in the CMP teacher’s guide book may have two to four pages of text and includes suggestions for starting and running conversations during Launch, Explore, and Summarize phases in each TTPS lesson. The SMPs are embedded within each problem in the most recent version of the CMP curriculum.

This district also provided ongoing professional development programs that focused on implementing current standards such as SMPs in the context of TTPS. Such professional development programs are offered every other month during the year. This district adopted the CMP curriculum back in the 1990s before the development of CCSSM. The teachers have been learning to implement the CMP curriculum since that time. As different standards were developed, the authors of CMP revamped the curriculum. The teachers now use version 3, which is CMP3. They continue to receive professional development programs to effectively understand and implement the CCSSM (content and process). The professional development programs in this district provide mathematics teachers a great opportunity to understand and fully implement SMPs when using TTPS.

Discussion of Theme Two: Teachers Who are Supported to Use TTPS May Understand and Fully Implement MTPs

This theme illustrates that using TTPS offers opportunity for teachers to understand and be able to fully implement the eight MTPs developed by the NCTM in 2014. That is, using TTPS facilitates teachers' understanding of what they should do to implement each one of the MTPs in a way that aligns with what was intended by the NCTM.

Interpretation. The data shows that Hana, Teresa, and Grace understand all the MTPs in the context of TTPS phases. The teachers explained that in the **Launch** phase, they establish their typical TTPS lessons with clear goals by posting the lesson's goals on the board, talking about them to help students know what they need to accomplish, and guiding instructional decisions toward those set goals (MTP1). The teachers also implement high cognitive-demand problems from the CMP curriculum to help students build new mathematical knowledge and thus promote students' reasoning (MTP2).

In the **Explore** phase, the teachers support productive struggle. They provide students with individual help, allow them to work in heterogeneous groups (students with different abilities), and discuss students' mistakes (MTP7). The teachers use purposeful questions from the CMP curriculum to guide students through problem solving and encourage them to explain their thinking (MTP5).

In the **Summarize** phase, teachers engage students in meaningful discussions to advance the latter's learning and build shared understanding with the whole class (MTP4). They engage students to use and connect mathematical representations to deepen their understanding of newly learned mathematical concepts (MTP3). They also elicit and use evidence of students' thinking when they share their ideas to assess understanding (MTP8). Finally, the teachers build procedural fluency from conceptual understanding by posing some extension problems and exercises from the CMP curriculum after solving a problem. This helps students become skillful in using procedures (MTP6). It is important to note that some MTPs can be implemented in more than one phase in TTPS. For example, the teachers indicated for MTP1 (Establish mathematical goals to focus learning), they construct their lesson with a clear goal of what students will learn. It could also be seen in the explore and summarize phase when teachers make in-the-moment decisions to connect students' learning with lesson goals.

These interpretations are evident in teachers' understanding of each one of the MTPs because they align with the literature on MTPs. For example, NCTM (2014) recommended that teachers should encourage students to use and make connections among different representations such as physical, contextual, verbal, symbolic, and visual mathematical representations (MTP3). This study's data shows teachers explained how they encourage their students to use different representations to understand and solve problems. Hana said she provided mathematical tools and visuals to encourage students to use different representations. Teresa revealed that the types of problems she uses from the CMP curriculum encourage students to use different representations such as graphs, tables, and equations; these representations are connected with each other. As Grace noted, "The whole point of it [the CMP curriculum] is understanding the connection between the concrete, the abstract, and the visual" (Grace, Pre-interview, Feb 19, 2019). These interpretations by the teachers suggest that teacher understand MTP3 as encouraging students to use and connect different representations. Such understanding aligns with NCTM tenets. The TTPS approach appears when teachers consistently implement the CMP curriculum.

Implementation. Teachers have a full implementation of MTPs when using the TTPS approach. A full implementation of MTPs means that all actions by teachers and students that are required to implement each individual MTP are attained in one lesson. Thus, when teachers use the TTPS approach, their lessons become an example of how to implement all MTPs in one lesson. For example, Hana and Teresa used TTPS in the observed lessons and had a full implementation of MTPs in that time.

The important factors that support teachers to have a full implementation of MTPs when using TTPS may be curriculum materials such as the CMP and teaching experience in using the TTPS approach. Using a curriculum that supports TTPS can be an important factor if there is to be a full implementation of the MTPs. The CMP teacher guides provide detailed suggestions on how to successfully implement a TTPS lesson in a way that aligns with MTPs. Each mathematical problem in the CMP curriculum revolves around specific mathematical concepts or ideas. Thus, teachers develop their lessons with the goal of teaching specific mathematical concepts through solving problems. For example, in Hana's observed lesson, the goals were understanding the percent of a quantity and developing fluency in adding, subtracting, multiplying, and dividing decimals through solving the *Larry's Lunch Place* problem from the CMP3 book. This problem necessitated asking students to imagine they are eating in a restaurant, ordering food, and calculating their total bill with 6% tax and 20% tip. Thus, the problem provided an opportunity to focus students' learning on predetermined goals throughout the lesson. That is, these types of problems in the CMP curriculum facilitate an implementation of MTP1 (Establish mathematics goals to focus learning).

Moreover, the high cognitive demand problems in the CMP curriculum facilitate teachers' implementation of MTP2 (Implement tasks that promote reasoning and problem solving) and MTP3 (Use and connect mathematical representations). To help teachers in implementing these high cognitive demand problems successfully, detailed explanation on how to implement these problems in the classroom were suggested in the CMP teacher's guide book. That is, using these problems provides opportunities for promoting students' reasoning and problem solving, which simultaneously facilitates implementation of MTP2. Problems from the CMP curriculum also require students to use different representations (such as tables, equations, graphs) to solve them, which promotes implementation of MTP3.

The CMP teacher's guide books also suggest in detail ways for teachers to conduct classroom discussions and to ask purposeful questions, which facilitates the implementation of MTP4 (Facilitate meaningful mathematical discourse) and MTP5 (Pose purposeful questions). The curriculum also provides suggestions on how to discuss and connect students' solution strategies with each other, and with the newly learned concepts during the summarize phase in a TTPS lesson. This facilitates implementation of MTP6 (Build procedural fluency from conceptual understanding).

To implement MTP7 (Support productive struggle in learning), the CMP curriculum provides suggested questions that help teachers to look out for students with discrete individual needs. To assess students' understanding and progress toward mathematical learning goals and reflect on students' learning, the CMP curriculum suggests questions for teachers to ask, which promote an implementation of MTP8 (Elicit and use evidence of student thinking). That is, the CMP curriculum which supports the TTPS approach, facilitates teachers' understanding and implementation of MTPs.

In addition, teachers' experience in TTPS can be considered a factor to have a full implementation of the MTPs. Prince (2016) found that teachers who participate in a TTPS lesson study were able to successfully change their teaching practices to implement most MTPs. The participants in Prince's study were novices in using TTPS and trained for three months to use this approach of teaching. In comparison, the teacher participants in my study had more than 15 years' experience in using TTPS. This suggests that teachers' experience in using TTPS may contribute to the finding that teachers have a full implementation of MTPs.

Discussion of Theme Three: Teachers Who are Supported to Use TTPS May Use TFPS

The research findings suggest that teachers who are supported by their district to use TTPS may use a Teaching For Problem Solving approach. Although the theoretical foundation of both approaches TTPS and TFPS is constructivism (Lester, 2013), the focus on using problems to learn mathematics is different (See Table 1.1). TTPS focuses on development. Students in TTPS develop new mathematical concepts through solving problems. TFPS focuses on application in which a teacher concentrates on teaching students mathematical procedures to learn new mathematical concepts and then apply the newly learned concepts to solve real-life problems (Schroeder & Lester, 1989). For example, during Grace's observed lesson, *Spinning on a Grid: Coordinate Rules for Rotations*, she decided to guide her students to use a transparency to rotate a figure 90° and find rules for rotations. Her decision to guide students in using the transparency to understand the concept of rotation changed the lesson's direction from TTPS to TFPS. The focal point was learning the concept of rotation first and then using the new knowledge to solve the problem, rather than learning the concept of rotation through problem solving. That is, Grace used TFPS in the observed lesson.

The nature of problem solving lessons and class time may have an effect on teacher's decision to change their teaching approach. Some lessons can be implemented in one class period. However, TTPS requires allowing sufficient time for students to discover multiple solution strategies to solve a problem. That is, more than one class period may be needed to learn new mathematical concepts. Thus, a teacher may decide to guide students to use one strategy to solve a problem. The pace of the *Spinning on a Grid* lesson as one 45-minute class period led Grace to develop a single strategy to learn rotation and use this understanding to solve the problem in one class. Thus, the class time affected her decision to change her teaching approach from TTPS to TFPS. Burmeister et al. (2018) indicated that the time required to implement a problem-solving task can be a reason that prevent some teachers from using TTPS. Therefore, Burmeister et al. (2018) suggested that when teachers use TTPS, "students have time to conceptualize the mathematics while still acquiring the expected procedural fluency" (p. 2). That is, using TTPS can give students more exposure to CCSSM content and practices than using other approaches to teaching (Burmeister et al., 2018).

Wilhelm (2014) indicated that using TTPS successfully is significantly related to teachers' selection, enactment, and maintaining of the higher-level of cognitive demand in problems. Although both TTPS and TFPS lessons require using high cognitive demand problems, the cognitive demand differs between these two approaches of teaching. TTPS requires doing mathematics. Doing mathematics requires complex thinking (i.e., there is no pathway suggested or a work-out example), requires students to explore possible strategies and solutions of a problem, and requires students to analyze the nature of mathematical concepts and processes (Stein et al., 2000). On the other hand, TFPS requires *procedures with understanding*. *Procedures with understanding* is a high cognitive demand level that "require[s] some degree of cognitive effort … [and] suggest[s] pathways to follow" (Stein et al., 2000, p. 16). Tasks at this level require students to go beyond recalling or classifying, to reasoning and justifying solutions of a problem. Using TTPS offers more opportunities for students to learn by doing mathematics than using TFPS.

Moreover, there are different outcomes between using TTPS and TFPS in this mathematical context. For example, when Grace implemented a TFPS lesson to teach rotation, students learned the concept of rotation, origin, and ordered pairs after rotating a figure 90° and 180°. However, if the teacher implements a TTPS lesson, students would learn more mathematical content, such as the angle of rotation and the relation between slope and rotation. Students' learning would also be different between using TTPS and TFPS. The focus in TTPS is on developing new mathematical concepts and ideas while the focus in TFPS is on applying new mathematical concepts and ideas (See Table 1.1). If Grace used TTPS, students would have to consider multiple approaches and when and how each one is useful. Yet, Grace used a TFPS approach. That is, her students did not have an opportunity to develop new ideas to find rules for rotation. They were presented with a new mathematical idea (which was using the transparency to rotate a figure) and then applied this idea to find rules for rotation. In addition, all the SMPs and MTPs can be addressed when using TTPS. On the other hand, some of the SMPs and MTPs can be addressed when using TFPS (as discussed in the next section).

Discussion of Theme Four: Using TFPS May Result in a Partial Implementation of SMPs and MTPs

Using the TFPS approach may result in a partial implementation of SMPs because teachers may suggest one pathway to follow to solve a problem. This action results in missing important expertise that teachers should develop in their students to fully implement SMPs. For example, when Grace used TFPS, four items from the MCOP² protocol were missing in Grace's observed lesson: Item 2 (Students used or generated two or more representations), Item 4 (Students assessed strategies), Item 9 (The lesson included tasks that have multiple paths to a solution or multiple solution), and Item 16 (The teacher used student questions/comment to enhance conceptual understanding). According to the MCOP² protocol, these missing items resulted in a partial implementation of SMP1 (Make sense of problems and persevere in solving them), SMP3 (Construct viable arguments and critique the reasoning of others), SMP5 (Use appropriate tools strategically), and SMP8 (Look for and express regularity in repeated reasoning). Thus, using TFPS can result in a partial implementation of SMPs.

Using TFPS may also result in a partial implementation of MTPs. Partial implementation means that some items are missing during the process of implementation. The data shows that when Grace used TFPS, she missed 22 out of 51 items from the MTP-OP protocol. This results in a partial implementation of MTP2 (Implement tasks that promote reasoning and problem solving), MTP3 (Use and connect mathematical representations), MTP4 (Facilitate meaningful mathematical discourse), MTP6 (Build procedural fluency from conceptual understanding), and MTP7 (Support productive struggle in learning). For example, four missing actions resulted in a partial implementation of SMP3 in Grace’s observed lesson. These missing action were: (1) the teacher did not allocate instructional time for students to use, discuss, and make connections among representations; (2) the teacher did not introduce other form of representations; (3) they did not use multiple forms of representations; and (4) they did not make choices about which form of representations to use as tools for solving the problem. That is, using TFPS may result in a partial implementation of SMPs and MTPs.

In sum, the theme one illustrates how supporting teachers to use TTPS can result in understanding and a full implementation of SMPs. Theme two highlights how using

TTPS facilitates teachers' interpretation and implementation of MTPs. Theme three focuses on certain obstacles that may affect teachers' decision to change their teaching approach from TTPS to TFPS. Theme four notes how teachers' decision to use TFPS affects their implementation of SMPs and MTPs.

Theoretical Implications

The results of this study show that when teachers change the cognitive demands of a mathematical problem, it may result in changing the teaching approach form TTPS to TFPS. Changing the cognitive demands of a problem confirms previous findings in extant literature about mathematical problems and their enactment in classrooms. Stein et al. (2000) discussed how mathematical problems with high cognitive demands are often being transformed into less-demanding problems during instruction. In other words, the kind of thinking needed to solve problems can change during a lesson. Stein and Smith (1998) illustrated how problems pass through three phases during a lesson (See Figure 3).

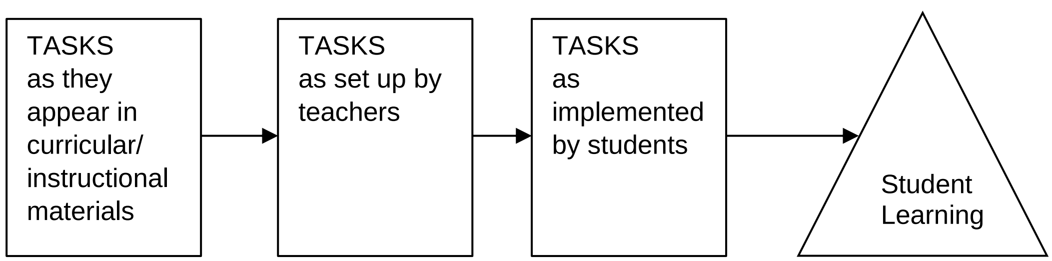


Figure 3. Mathematical Task Framework (Stein & Smith, 1998)

The first phase is the problem as it appears in curricular or instructional lesson. The second phase comprises the problem as it is set up by teachers during a lesson. The third phase is the problem as it is implemented and worked on by students. These three phases determine opportunities students have to learn mathematics. Simply selecting a high cognitive demand problem does not ensure students engage with it and think in cognitively complex ways. Teachers may change the cognitive demand of the problem as it is set up during classroom instruction (second phase), and both students and teachers contribute to how the problem is being implemented (third phase). The results of my study fit in Stein and Smith's (1998) Mathematical Task Framework. In the CMP curriculum that supports TTPS most mathematical problems are designed for *doing mathematics*. Teachers may change the level of cognitive demand from *doing mathematics* as it appears in curricular or instructional materials (first phase) to *procedures with connections* as set up by teachers and implemented by students (in the second and third phase). This means problems that are intended for the *doing mathematics* level can be set up and implemented by teachers as *procedures with connections*. Thus, changing the cognitive demand level (doing mathematics to procedures with connections) may result in changing the teaching approach from TTPS to TFPS.

In addition, the results from this study illustrate that teachers can maintain the cognitive demand of problems when using TFPS. Stein et. al (2000) indicated that when a teacher sets up a problem as *procedures with connections* and implements it as *procedures with connections*, this indicates the teacher maintains the high cognitive demand of the problem. That is, teachers can maintain high cognitive demand levels of problems while using the TFPS approach if they set up and implement problems as procedures with connections.

Practical Implications

This study contributes to a better understanding of how supporting teachers to use the TTPS approach facilitates their understanding and implementation of SMPs and

MTPs. Training teachers to use TTPS is not an easy task. It may take years for teachers to fully understand how to effectively use this approach to teaching mathematics because TTPS requires a shift in thinking about instructional pedagogy. Thus, ongoing professional development programs are recommended to successfully help teachers use the TTPS approach. This study also illustrates the importance of providing a curriculum like the CMP curriculum to support teachers to use the TTPS approach. This curriculum provides high cognitive demand problems in real-life situations, provides suggestions on how to implement them successfully, and suggests purposeful questions that teachers can use to conduct productive discussions. These types of problems also encourage using different representations of a concept to deepen students' conceptual understanding. That is, a well-designed curriculum offers opportunities for teachers to successfully use TTPS approach.

In addition, the finding that teachers who use TTPS fully implement SMPs and MTPs in one lesson may inspire professional development programs that encourage other teachers to use TTPS. For example, leaders of professional development programs may take the following actions to encourage teachers to use TTPS: They can ask teachers to collaborate to interpret SMPs in their own words and using a language their students will understand. Teachers can then reflect on those interpretations and see which of them align with what is intended by the CCSSM. When they discuss which approach to teaching each one uses and which of these teaching approach facilitates a full implementation of SMPs and MTPs—in this case—they may be able to move their practices toward TTPS. Although teachers have a choice in whether or not to use this approach of teaching, all of them need to implement SMPs. When they see how using the

TTPS approach facilitates a full implementation of all SMPs, they may be convinced to use this approach. Teachers can read how the teacher participants in this study used TTPS to deepen their understanding of how all SMPs can be implemented in one lesson. It is to be noted that it is hard to implement all SMPs in one lesson (Mateas, 2016). Yet, this study confirms it is possible to implement all these standards in one lesson when using the TTPS approach.

Another practical implication of the findings highlights the importance of collaboration among teachers within and across schools. A key aspect to successfully implement TTPS is collaboration among teachers (Burmeister et al., 2018). The data in my study showed that the teachers at each grade level talk together and plan together through teacher-led professional development programs. This offered opportunities for the teachers to successfully use TTPS approach and to implement the SMPs and MTPs using the CMP curriculum. The findings also raise a concern about schools with different implications. Students' learning outcomes from schools whose teachers use TTPS and implement all the SMPs and MTPs would be different than schools whose teachers use different approaches to teaching mathematics. Thus, schools should collaborate to achieve common goals.

Limitations of Study

I acknowledge certain limitations of this study. First, my assessment of teachers' implementations of SMPs and MTPs was based on one lesson for each teacher. It is possible more information about how teachers implement SMPs and MTPs may shed additional light on how these implementations relate to the CCSSM's content standards being taught in each lesson. Further, it is noted that SMPs are not meant to be stand-alone elements because they are embedded within the mathematical content in that lesson's instructions. The SMPs are used to increase opportunities to deepen students' understanding of mathematical content. That is, teachers may implement SMPs differently to facilitate students' understanding of mathematical content. Second, the researcher conducted this study with only three teachers from one middle school. Notice that there were two middle schools in the district that use the CMP curriculum, and the teachers from one of these schools agreed to participate in this study. Failure to study the other school reduces generalizability of the findings. Moreover, the participating school has only one mathematics teacher for each grade level. This number reduces the opportunity to study commonalities and differences between teachers' interpretations and implementations of SMPs and MTPs within each grade level. Last, I do not have teaching experience in using TTPS. This may influence my interpretation of the data. However, I read about TTPS and observed many teachers while they teach using TTPS. I also developed a protocol to differentiate between TTPS and other problem-based instructions during my master's degree. This provides me with rich information about and understanding of the TTPS approach.

Future Research

This research is essential for filling the gap in literature related to the alignment between TTPS and the SMPs and MTPs. Given the limited research base on the influence the TTPS approach has on teachers' understanding and implementation of SMPs and MTPs, this research could be enhanced and extended in several important areas. First, conducting this study across middle school levels provided a unique opportunity to investigate the influence of TTPS on mathematics teachers' implementations of SMPs and MTPs in each middle school grade. Extending this research beyond the middle school levels would enable future researchers to observe changes in teachers' implementations of SMPs and MTPs at the elementary or high school levels. Second, extending the research to other schools that use the CMP curriculum may also enhance and extend the findings of this study. Third, the teachers in this study had a full implementation of SMPs and MTPs in the observed TTPS lessons. This study may be augmented by investigating if teachers have a full implementation of SMPs and MTPs in every TTPS lesson. Fourth, this study illustrated how the use of the CMP curriculum provided a significant support for teachers in their interpretation and implementation of SMPs and the MTPs. It would be interesting to strengthen this finding by comparing it with how teachers who chose not to adopt the CMP curriculum interpret and implement SMPs and the MTPs. Finally, the finding that teachers who are supported by their district to use TTPS —by providing professional development programs—understand and implement all SMPs and MTPs in one lesson, may fuel future research that investigate in-depth the nature of such professional development programs.

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Appendix A: The Building Storm Shelters Problem

1.2 Building Storm Shelters

Constant Area, Changing Perimeter

Sometimes, during fierce winter storms, people are stranded in the snow. To prepare for this kind of emergency, parks often provide shelters at points along hiking trails.

When you make a floor plan for anything from a bumper-car ride to a storm shelter, you need to consider the use of space to find the best possible plan. Sometimes you want the greatest, or *maximum*, possible area or perimeter. At other times, you want the least, or *minimum*, area or perimeter.

If a rectangular floor space has a fixed area, what rectangle will have the greatest perimeter? The least perimeter?

Problem 1.2

The rangers in a national park want to build several storm shelters. The shelters must have 24 square meters of rectangular floor space.

A Experiment with different rectangles that have whole-number dimensions. Sketch each possible floor plan on grid paper. Record your data in a table such as the one started below. Look for patterns, and describe the data.

Shelter Floor Plans

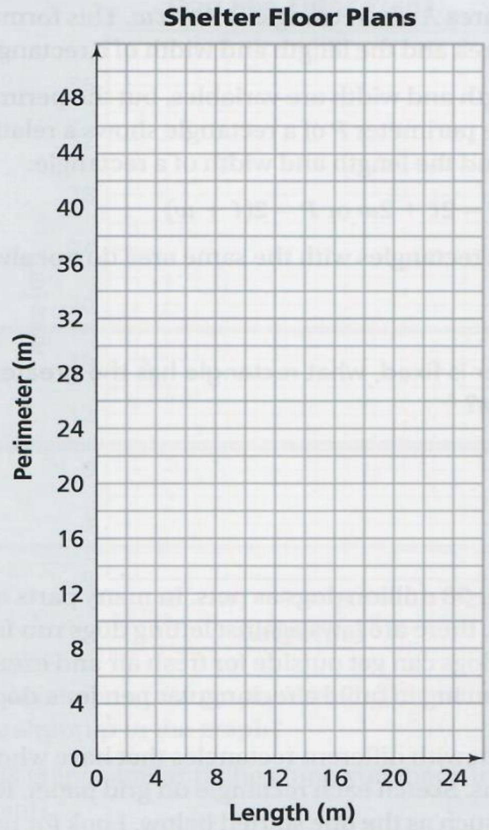
Rectangle	Length	Width	Perimeter	Area
1 m × 24 m	1 m	24 m	50 m	24 m^2

B Suppose the walls are made of flat rectangular panels that are 1 meter wide and have the needed height.

1. What determines how many wall panels are needed, area or perimeter? Explain your reasoning.
2. Which design would require the most panels? Explain.
3. Which design would require the fewest panels? Explain.

continued on the next page >

C **1.** Use your table to make a graph, such as the one below, to compare lengths and perimeters of various rectangles with an area of 24 square meters.



2. Describe the shape of the graph. How do the patterns that you saw in your table show up in the graph?

D **1.** Suppose you build a storm shelter with 36 square meters of rectangular floor space. Which design has the least perimeter? Which has the greatest perimeter? Explain your reasoning.

2. In general, describe the rectangle that has the greatest perimeter for a *fixed*, or unchanging, area. Describe the rectangle that has the least perimeter for a fixed area.

Adopted from Connected Mathematics 3 (Lappan et al., 2014a).

Appendix B: Interview Questions

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12/11/2018 -

The Pre-Teaching Interview

Thanks for giving your time to be interviewed for this research study. The purpose of this study is to better understand middle-school mathematics teachers' interpretation and implementation of Standards for Mathematical Practices (CCSSM, 2010) and Mathematics Teaching Practices (NCTM, 2014) when using a curriculum that supports teaching through problem solving. You will notice that this interview is being audio-recorded for later transcription. It is likely to take around 45 minutes of your time. This is a confidential interview and you will not be personally identified in any part of the study. If you have any questions as we proceed, please ask me. If there is anything you do not wish to respond to, please say so. Do you have any questions before we start?

1. What does a typical lesson look like in your classroom?
2. What do you know about the Standards for Mathematical Practices released by CCSSM in 2010? (I will hand a copy of these standards to the participant)
3. Would you mind going one-by-one through these standards to explain how you use these standards in your daily lessons?
4. What are the most challenging of these standards?
5. What are your thoughts about whether TTPS facilitates or hinders the implementation of the Standards?
6. What do you know about the Mathematics Teaching Practices released by NCTM in 2014? (I will hand a copy of these teaching practices to the participant)

If the participant appears familiar with these teaching practices I will ask the following question:

   a. Would you mind going one-by-one through these teaching practices to explain how do you use them in your daily lessons?

If the participant appears not familiar with these teaching practices I will proceed to the next question.

7. Do you have further comments or questions about SMPs and/or MTPs?

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12/11/2018 -

The Post-Teaching Interview

Thank you for participating in this study about teachers' interpretations and implementations of SMPs and MTPs. I would like you to clarify your purposes of some teaching moves in your videotape. You will notice that this interview is being audio-recorded for later transcription. It is likely to take around 45 minutes of your time. This is a confidential interview and you will not be personally identified in any part of the study. If you have any questions as we proceed, please ask me. If there is anything you do not wish to respond to, please say so. Do you have any questions before we start?

1. What was your purpose when implementing this problem?
2. How did the learning goals fit with this lesson?
3. Describe where problem solving and reasoning were developed in this lesson.
4. How did you help students make connection among mathematical representation during this lesson?
5. Did you plan for the mathematical discourse? How and why?
6. What is your goal when posing questions during the lesson?
7. How was conceptual understanding and procedural fluency developed in this lesson?
8. How did you engage students in productive struggle as they grappled with mathematical ideas and relationships?
9. How did student's thinking help you move through the lesson?
10. I have ____ video clips from your teaching. The 1st video is ____ minutes long and the 2nd video is ___ minutes long. For each video clip, can you explain what was your purpose and what things were you thinking about at this point in the lesson?

Appendix C: The Mathematics Classroom Observation Protocol for Practices (MCOP²)

Standards for Mathematical Practice

MCOP² Item	Make sense of problems and persevere in solving them	Reason abstractly and quantitatively	Construct viable arguments and critique the reasoning of others	Model with mathematics	Use appropriate tools strategically	Attend to precision	Look for and make use of structure	Look for and express regularity in repeated reasoning
1	X						X	X
2	X				X			
3	X							
4	X		X					X
5	X	X	X		X			
6							X	X
7		X		X				
8							X	X
9	X							
10			X			X		
11	X							
12			X					
13			X					
14	X							
15			X					
16	X							

Short Descriptors of the MCOP² Items:

Factor	MCOP² Items	Description
SE	**1)** Students engaged in exploration/investigation/problem solving.	Students regularly engaged in exploration, investigation, or problem solving. Over the course of the lesson, the majority of the students engaged in exploration/investigation/problem solving.
SE	**2)** Students used a variety of means (models, drawings, graphs, concrete materials, manipulatives, etc.) to represent concepts.	The students manipulated or generated two or more representations to represent the same concept, and the connections across the various representations, relationships of the representations to the underlying concept, and applicability or the efficiency of the representations were explicitly discussed by the teacher or students, as appropriate.
SE	**3)** Students were engaged in mathematical activities.	Most of the students spend two-thirds or more of the lesson engaged in mathematical activity at the appropriate level for the class. It does not matter if it is one prolonged activity or several

		shorter activities. (Note that listening and taking notes does not qualify as a mathematical activity unless the students are filling in the notes and interacting with the lesson mathematically.)
SE, TF	**4)** Students critically assessed mathematical strategies.	More than half of the students critically assessed mathematical strategies. This could have happened in a variety of scenarios, including in the context of partner work, small group work, or a student making a comment during direct instruction or individually to the teacher.
SE	**5)** Students persevered in problem solving.	Students exhibited a strong amount of perseverance in problem solving. The majority of students looked for entry points and solution paths, monitored and evaluated progress, and changed course if necessary. When confronted with an obstacle (such as how to begin or what to do next), the majority of students continued to use resources (physical tools as well as mental reasoning) to continue to work on the problem
TF	**6)** The lesson involved fundamental concepts of the subject to promote relational/conceptual understanding.	The lesson includes fundamental concepts or critical areas of the course, as described by the appropriate standards, and the teacher/lesson uses these concepts to build relational/conceptual understanding of the students with a focus on the "why" behind any procedures included.
TF	**7)** The lesson promoted modeling with mathematics.	Modeling (using a mathematical model to describe a real-world situation) is an integral component of the lesson with students engaged in the modeling cycle (as described in the Common Core State Standards).
TF	**8)** The lesson provided opportunities to examine mathematical structure. (symbolic notation, patterns, generalizations, conjectures, etc.)	The students have a sufficient amount of time and opportunity to look for and make use of mathematical structure or patterns.

TF	**9)** The lesson included tasks that have multiple paths to a solution or multiple solutions.	A lesson which includes several tasks throughout; or a single task that takes up a large portion of the lesson; with multiple solutions and/or multiple paths to a solution and which increases the cognitive level of the task for different students.
TF	**10)** The lesson promoted precision of mathematical language.	The teacher "attends to precision" in regards to communication during the lesson. The students also "attend to precision" in communication, or the teacher guides students to modify or adapt non-precise communication to improve precision.
TF	**11)** The teacher's talk encouraged student thinking.	The teacher's talk focused on **high levels** of mathematical thinking. The teacher may ask lower level questions within the lesson, but this is not the focus of the practice. There are three possibilities for high levels of thinking: analysis, synthesis, and evaluation. ***Analysis****:* examines/ interprets the pattern, order or relationship of the mathematics; parts of the form of thinking. **Synthesis**: requires original, creative thinking. **Evaluation**: makes a judgment of good or bad, right or wrong, according to the standards he/she values.
SE	**12)** There were a high proportion of students talking related to mathematics.	More than three quarters of the students were talking related to the mathematics of the lesson at some point during the lesson.
SE, TF	**13)** There was a climate of respect for what others had to say.	**Many** students are sharing, questioning, and commenting during the lesson, including their struggles. Students are also listening (active), clarifying, and recognizing the ideas of others.
SE	**14)** In general, the teacher provided wait-time.	The teacher **frequently** provided ample amount of "think time" for depth and complexity of a task or question posed by the teacher or a student.

SE	**15)** Students were involved in the communication of their ideas to others (peer-to-peer).	**Considerable** time **(more than half)** was spent with peer to peer dialog (pairs, groups, whole class) related to the communication of ideas, strategies and solution.
SE	**16)** The teacher uses student questions/comments to enhance conceptual mathematical understanding.	The teacher **frequently** uses student questions/ comments to coach students, to facilitate conceptual understanding, and boost the conversation. The teacher sequences the student responses that will be displayed in an intentional order, and/or connects different students' responses to key mathematical ideas.

* SE= Students Engagement, TF= Teacher Facilitation, Adapted from (Gleason, Livers, & Zelkowski, 2017)

Appendix D: Mathematics Teaching Practices Observation Protocol (MTP-OP).

Mathematics Teaching Practices Adapted from (Prince, 2016):

1. Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.

- Goals should describe what mathematical concepts, ideas, or methods students will understand more deeply as a result of instruction and identify the mathematical practices that students are learning to use more proficiently.
- Evidence:
  - Teacher: Discussing and referring to the mathematical purpose and goal of a lesson during instruction
  - Teacher: Using the mathematics goal to make in-the-moment decisions during instruction
  - Students: Engaging in discussions of the mathematical purpose and goals related to their current work in the mathematics classroom
  - Students: Connecting their current work with the mathematics that they studied previously and seeing where the mathematics is going.

2. Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.

- These tasks encourage reasoning and access to the mathematics through multiple entry points, including the use of different representations and tools, and they foster the solving of problems through varied solution strategies.
- Evidence:
  - Teacher: Providing opportunities for exploring and solving problems that extend students' current mathematical understanding
  - Teacher: Posing tasks that require a high level of cognitive demand
  - Teacher: Supporting students in exploring tasks without taking over student thinking
  - Teacher: Encouraging students to use varied approaches and strategies to make sense of and solve tasks
  - Students: Persevering in exploring and reasoning through tasks
  - Students: Using tools and representations as needed to support their thinking and problem solving
  - Students: Accepting and expecting that their classmates will use a variety of solution approaches and that they will discuss and justify their strategies to one another

3. Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.

- The general classification scheme for types of representations includes important connections among contextual, visual, verbal, physical, and symbolic representational forms.
- Evidence:

- Teacher: Allocating substantial instructional time for students to use, discuss, and make connections among representations
- Teacher: Introducing forms of representations that can be useful to students
- Teacher: Asking students to make math drawings or use other visual supports to explain and justify their reasoning
- Teacher: Focusing students' attention on the structure or essential features of mathematical ideas that appear regardless of representation
- Students: Using multiple forms of representations to make sense of and understand mathematics
- Students: Describing and justifying their mathematical understanding and reasoning with drawings, diagrams, and other representations
- Students: Making choices about which forms of representations to use as tools for solving problems
- Students: Contextualizing mathematical ideas by connecting them to real-world situations
- Students: Considering the advantages or suitability of using various representations when solving problems

4. Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.

- Mathematical discourse includes the purposeful exchange of ideas through classroom discussion, as well as through other forms of verbal, visual, and written communication.
- Evidence:
  - Teacher: Engaging students in purposeful sharing of mathematical ideas, reasoning, and approaches, using varied representations
  - Teacher: Selecting and sequencing student approaches and solution strategies for whole-class analysis and discussion
  - Teacher: Facilitating discourse among students by positioning them as authors of ideas, who explain and defend their approaches
  - Teacher: Ensuring progress toward mathematical goals by making explicit connections to student approaches and reasoning
  - Students: Presenting and explaining ideas, reasoning and representations to one another in pair, small-group, and whole-class discourse
  - Students: Listening carefully to and critiquing the reasoning of peers, using examples to support and counterexamples to refute arguments
  - Students: Seeking to understand the approaches used by peers by asking clarifying questions, trying out others' strategies, and describing the approaches used by others
  - Students: Identifying how different approaches to solving a task are the same and how they are different

5. Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.

- Purposeful questions allow teachers to discern what students know and adapt lessons to meet varied levels of understanding, help students make important mathematical connections, and support students in posing their own questions.

 Evidence:

o Teacher: Advancing student understanding by asking questions that build on, but do not take over or funnel, student thinking

o Teacher: Making certain to ask questions that go beyond gathering information to probing thinking and requiring explanation and justification

o Teacher: Asking intentional questions that make the mathematics more visible and accessible for student examination and discussion

o Teacher: Allowing sufficient wait time so that more students can formulate and offer responses

o Students: Thinking carefully about how to present their responses without rushing to respond quickly

o Students: Reflecting on and justifying their reasoning, not simply providing answers

o Students: Listening to, commenting on, and questioning the contributions of their classmates

6. Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.

o Teacher: Providing students with opportunities to use their own reasoning strategies and methods for solving problems

o Teacher: Asking students to discuss and explain why the procedures that they are using work to solve particular problems

o Teacher: Connecting student-generated strategies and methods to more efficient procedures as appropriate

o Teacher: Using visual models to support students' understanding of general methods

o Students: Making sure that they understand and can explain the mathematical basis for the procedures that they are using

o Students: Demonstrating flexible use of strategies and methods while reflecting on which procedures seem to work best for specific types of problems

o Students: Determining whether specific approaches generalize to a broad class of problems

7. Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.

 Such instruction embraces a view of students' struggles as opportunities for delving more deeply into understanding the mathematical structure of problems and relationships among mathematical ideas, instead of simply seeking correct solutions.

 Evidence:

o Teacher: Giving students time to struggle with tasks, and asking questions that scaffold students' thinking without stepping in to do the work for them

o Teacher: Helping students realize that confusion and errors are a natural part of learning, by facilitating discussions on mistakes, misconceptions, and struggles

o Teacher: Praising students for their efforts in making sense of mathematical ideas and perseverance in reasoning through problems o Students: Asking questions that are related to the sources of their struggles and will help them make progress in understanding and solving tasks

o Students: Helping one another without telling their classmates what the answer is or how to solve the problem

8. Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

 Evidence:

o Teacher: Eliciting and gathering evidence of student understanding at strategic points during instruction

o Teacher: Making in-the-moment decisions on how to respond to students with questions and prompts that probe, scaffold, and extend

o Students: Revealing their mathematical understanding, reasoning, and methods in written work and classroom discourse

o Students: Asking questions, responding to, and giving suggestions to support the learning of their classmates.

Appendix E: Informed Consent

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12/11/2018 -

ICF Version Date: 12/11/2018

Dept. of Curriculum and Instruction
2801 W. Bancroft St. (MS 924)
Toledo, Ohio 43606
Phone # 419-530-5371
Fax # 419-530-2466

ADULT RESEARCH SUBJECT - INFORMED CONSENT FORM
(The Alignment Between Teaching Mathematics Through Problem Solving and Recent Mathematical Process Standards and Teaching Practices)

Principal Investigator: *Dr. Debra Johanning, Associate Professor, 419-530-5275*
Awsaf Alwarsh, Doctoral Student, 419-9732990

Purpose: You are invited to participate in the research project entitled, *The Alignment Between Teaching Mathematics Through Problem Solving and Recent Mathematical Process Standards and Teaching Practices*, which is being conducted at the University of Toledo by Awsaf Alwarsh under the direction of Dr. Debra Johanning.

The purpose of this study is to explore middle school teachers' interpretation and implementation of the standards for mathematical practices (CCSSM, 2010) and mathematics teaching practices (NCTM, 2014) when using Teaching Trough Problem Solving.

Description of Procedures: This research study will involve a pre-teaching interview, observation and videotaping of one full lesson, and a post-teaching interview related to your teaching. Interviews will be audio recorded. The pre-teaching interview session will last approximately 45 minutes. The participant will be asked to provide a lesson plan for the lesson to be observed. The classroom teaching of the lesson will be observed and videotaped. The post-teaching interview will last approximately 45 minutes
and used to clarify the instructional intensions of teacher's moves during the observed lesson.

Permission to record: Will you permit the researcher to *{audio record/video record}* during this research procedure?

YES	☐	_______ Initial Here	NO	☐	Initial Here	________________________

After you have completed your participation, the researcher will debrief you about the data, theory and research area under study and answer any questions you may have about the research.

Potential Risks: There are minimal risks to participation in this dissertation, including loss of confidentiality.

Potential Benefits: If you participate in this research, others may benefit by learning about the results of this research.

University of Toledo IRB Approved

Approval Date: 12/11/2018

Confidentiality: The researchers will make every effort to prevent anyone who is not on the research team from knowing that you provided this information, or what that information is. The consent forms with signatures will be kept separate from responses, which will not include names and which will be presented to others only when combined with other responses. Although we will make every effort to protect your confidentiality, there is a low risk that this might be breached.

Voluntary Participation: Your refusal to participate in this dissertation will involve no penalty or loss of benefits to which you are otherwise entitled. In addition, you may discontinue participation at any time without any penalty or loss of benefits.

Contact Information: Before you decide to accept this invitation to take part in this dissertation research, you may ask any questions that you might have. If you have any questions at any time before, during or after your participation you should contact a member of the researcher: Dr. Debra Johanning debra.johanning@utoledo.edu or Awsaf Alwarsh, aalwars@rockets.utoledo.edu
If you have questions beyond those answered by the researcher or your rights as a research subject or research-related injuries, the Chairperson of the SBE Institutional Review Board may be contacted through the Office of Research on the main campus at (419) 530-2844.
Before you sign this form, please ask any questions on any aspect of this dissertation that is unclear to you. You may take as much time as necessary to think it over.

SIGNATURE SECTION – Please read carefully

You are making a decision whether or not to participate in this research study. Your signature indicates that you have read the information provided above, you have had all your questions answered, and you have decided to take part in this research.
The date you sign this document to enroll in this study, that is, today's date must fall between the dates indicated at the bottom of the page.

Name of Subject (please print)	Signature	Date
Name of Person Obtaining Consent	Signature	Date

This Adult Research Informed Consent document has been reviewed and approved by the University of Toledo Social, Behavioral and Educational IRB for the period of time specified in the box below.

Approved Number of Subjects:____________________

University of Toledo IRB Approved

Approval Date: 12/11/2018

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Dept. of Curriculum and Instruction
2801 W. Bancroft St. (MS 924)
Toledo, Ohio 43606
Phone # 419-530-5371
Fax # 419-530-2466

MINOR CHILD RESEARCH SUBJECT- PARENT/GUARDIAN INFORMED CONSENT FORM

(The Alignment Between Teaching Mathematics Through Problem Solving and Recent Mathematical Process Standards and Teaching Practices)

Principal Investigator: *Dr. Debra Johanning, Associate Professor, 419-530-5275*
Awsaf Alwarsh, Doctoral Student, 419-9732990

Purpose: Your child is invited to participate in the research project entitled, *The Alignment Between Teaching Mathematics Through Problem Solving and Recent Mathematical Process Standards and Teaching Practices*, which is being conducted at the University of Toledo by Awsaf Alwarsh under the direction of Dr. Debra Johanning.

The purpose of this study is to explore how middle school teachers use Teaching Trough Problem Solving to help students learn mathematics.

Description of Procedures: This research will take place in your child's classroom as part of regular classroom instruction. I will also be observing and videotaping your child's class and teacher in order to study teaching through problem solving. Data collection will take place daily across the estimated 1-3 days it takes to observe the teaching of a full lesson. If you consent, understand that the following data will be collected as part of the research

- Videotaped data of the lesson that your child's math teacher teaches as part of regular instruction.

If it acceptable to have your child participate please complete the checkboxes and initial below and provide signatures on the last page and return this form to your child's classroom teacher. A child cannot participate without parent's/guardian's written consent.

Permission to record: Will you permit the researcher to video record your child as part of regular classroom mathematics instruction of mathematics instruction during this research?

YES ☐ _______ Initial Here NO ☐ Initial Here ________________________

My signature on this consent form also indicates that I understand the extent and nature of my child's role in the data collection process. My child's involvement in data collection as part of this project will extend from the date of signing this consent through when the lesson ends. I understand that I can withdraw my child from participation in data collection for this research at any time without prejudice or penalty. If you give your child consent to participate, at the time data is collected from your child, the researcher will explain why the data is being collected and ask the child if they are willing to participate. If your child is uncomfortable or not willing, the data will not be collected. If your child is not involved in data collection, the video camera will be placed so they are not sitting in the area where the camera is aimed.

University of Toledo IRB Approved

Approval Date: 12/11/2018

Potential Risks: There are minimal risks to participation in this study, including loss of confidentiality. If being videotaped during regular class instruction causes your child to feel upset or anxious, you may stop at any time.

Potential Benefits: Future teachers and students will benefit from the data collected by learning about the results of this research.

Confidentiality: The researchers will make every effort to prevent anyone who is not on the research team from knowing that your child provided videotaped information. The consent forms with signatures will be kept separate from responses. This data will be viewed by the research team only and will not be shared with others. Although we will make every effort to protect confidentiality, there is a low risk that this might be breached.

Voluntary Participation: Your refusal to allow your child to participate in this research will involve no penalty or loss of benefits to which you or your child are otherwise entitled and will not affect your relationship with The University of Toledo, your child's school or your child's teacher. In addition, you may discontinue participation at any time without any penalty or loss of benefits.

Contact Information: Before you decide to accept this invitation to take part in this study, you may ask any questions that you might have. If you have any questions at any time before, during or after your participation you should contact a member of the researcher: Dr. Debra Johanning debra.johanning@utoledo.edu or Awsaf Alwarsh, aalwars@rockets.utoledo.edu

If you have questions beyond those answered by the researcher or your rights as a research subject or research-related injuries, the Chairperson of the SBE Institutional Review Board may be contacted through the Office of Research on the main campus at (419) 530-2844.

Before you sign this form, please ask any questions on any aspect of this study that is unclear to you. You may take as much time as necessary to think it over.

University of Toledo IRB Approved

Approval Date: 12/11/2018

SIGNATURE SECTION – Please read carefully

You are making a decision whether or not to participate in this research study. Your signature indicates that you have read the information provided above, you have had all your questions answered, and you have decided to take part in this research.

The date you sign this document to enroll in this dissertation, that is, today's date must fall between the dates indicated at the bottom of the page.

____
Name of Child (please print)

______________________________ ____________________ ______________
Name of Person Obtaining Consent Signature Date

This Adult Research Informed Consent document has been reviewed and approved by the University of Toledo Social, Behavioral and Educational IRB for the period of time specified in the box below.

Approved Number of Subjects: ________________________

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Dept. of Curriculum and Instruction
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Phone # 419-530-5371
Fax # 419-530-2466

CHILD RESEARCH SUBJECT ASSENT FORM

(The Alignment Between Teaching Mathematics Through Problem Solving and Recent Mathematical Process Standards and Teaching Practices)

Principal Investigator: *Dr. Debra Johanning, Associate Professor, 419-530-5275*
Awsaf Alwarsh, Doctoral Student, 419-9732990

- You are being asked to be in a study that researches how Teaching Through Problem Solving help students to learn mathematics.
- You should ask any questions you have before making up your mind. You can think about it
  and discuss it with your family or friends before you decide.
- It is okay to say “No” if you don’t want to be in the study. If you say “Yes” you can change your mind and then quit the study at any time without getting in trouble.

A research study is a way to learn more about people. If you decide that you want to be part of this study, you will be asked to do what you normally do in math class and to let us video tape you and your class.

We will show the videotaped lessons to your math teacher. We will not show the video to anyone else. I will also be assessing your responses to teaching practice. If you do not feel comfortable being videotaped, that is okay. If you decide to be videotaped and then change your mind, that is also okay. If you change your mind we will not use the video with you in it. If you change your mind, let us know.

Not everyone who takes part in this study will benefit. A benefit means that something good happens to you. We think the benefits might be that we learn ways to help teachers help students learn mathematics.

If you have any questions about the study, you can ask Dr. Debra Johanning or Awsaf Alwarsh. You can call the investigator listed at the top of this page if you have a question later.

If you decide to be in this study, please print and sign your name below.

I, ______________________________________, want to be in this research study.
(Print your name here)

Sign your Name: ________________________________ Date: _____________________

University of Toledo IRB Approved

Approval Date: 12/11/2018