THE INITIATION AND GROWTH OP THE NUMBER CONCEPT IN PREPARATION FOR ALGEBRA--GRADES K-8

A Thesis

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by

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PREFACE

The writer has a twofold purpose in writing this thesis. First, he feels it is necessary that a program be developed whereby students may begin their preparation for algebra in kindergarten and then build on this foundation naturally and consistently through the first eight grades. Only through deliberate planning can the pupil develop the readiness which facilitates his exploration of formal algebra.

Second, it is hoped that this thesis will provide some degree of enlightenment for teachers whose pupils are mere manipulators. Those teachers who train students to operate mechanically on a multitude of symbols may be persuaded to examine the reasons for reorienting their teaching purposes and procedures.

It is not the writer's intention to glorify algebra or to degrade arithmetic. Both are essential and honorable parts of mathematics.

ACKNOWLEDGMENTS

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CHAPTER 1. INTRODUCTION

1.1 Background

As a teacher of ninth grade algebra, the writer has encountered two major obstacles which thwart his teaching efforts. First, there is a critical need for more time to be devoted to the mastery of algebra. One school term is not adequate for a thorough coverage of all of the concepts which should be mastered by the end of the first course in formal algebra. As a result, the ninth grader, who usually is handicapped by a lack of readiness for algebra, finds an avalanche of totally unfamiliar material heaped upon him faster than he is able to handle it. The exceptional student may respond positively to this challenge, but the ordinary student is frustrated by it. Such an attempt to teach algebra, or any other subject, "all at once" is wholly inconsistent with our current philosophy of education.

Secondly, the student's responses are frequently misdirected by the "compartment" type of mathematical experiences to which he has been exposed previonsly. A difficulty in operating with the positive and negative integers, for example, may stem from the impression of a second grade teacher's shortsighted direction: "We <u>always</u> subtract the smaller number from the larger." The student may struggle unnecessarily with algebra as a result of having been confused about the difference between numbers and symbols for numbers and the relationships that exist between numbers and physical entities. He may have been taught to perform operations without ever having been stimulated to examine the relationships which exist in the number operations. The arithmetic lessons planned without long-range goals are potential stumbling blocks in the algebra classroom.

1.2 Statement of the Problem

How can the teachers of Grades K-8 initiate and promote a continuous growth of the number concept in order to insure an adequate preparation for algebra? This is the question which the writer has attempted to answer through this thesis.

The answer, in general terms, is implied in the philosophy of the Twenty-Pourth Yearbook of the National Council of Teachers of Mathematics:

> Teachers in all grades should view their tasks in the light of the idea that the understandings of mathematics is a continuum, that understandings grow within children throughout their school career. . . . Teachers should find what ideas have been presented earlier and deliberately use them as much as possible as the basis for teaching new ideas. . . . Teachers should look to the future and teach some concepts and understandings even if complete mastery cannot be expected.[1]

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[1]Phillip S. Jones, "The Growth and Development of Mathematical Ideas in Children," <u>The Growth of Mathematical Ideas: Grades K-12</u>, Twenty-Fourth Yearbook of the National Council of Teachers of Mathematics (Washington, D. C.: by the Council, 1959), pp. 3-4.

The problem then is the application of this "open end" philosophy to the opening question.

It should be realized that the proposals of this thesis, although directed toward preparation for algebra, support the basic procedures inherent in any sound and meaningful development of the number concept throughout the study of arithmetic even if preparation for algebra is not a major objective.

1.3 Scope

The historical development of the number concept from an algebraic point of view is remarkably similar to the development of the child's understanding of that same concept. An understanding of one broadens an understanding of the other. For this reason, the writer has delved first into the history of the development of the number concept, including those phases usually associated with the study of algebra.

In discussing the initiation and growth of the child's concept of number, only that material is included which is pertinent to the school years which precede the student's formal algebra training. In most schools formal algebra is encountered by the pupil for the first time in the ninth grade.

I.4 Procedures

The writer has made a survey of the available literature pertaining to the stated problem. Courses of study, curriculum guides, methods books, textbooks, periodicals, pamphlets, yearbooks, and mathematics books were explored. The materials most pertinent to the problem were selected for reading. The information collected from this body of literature was developed into that part of the arithmetic program which provides for the growth of the number concept during the Grades K-8 and thus prepares the student for his formal algebra training.

1.5 Limitations

To the writer's knowledge there are no available results of research conducted in connection with the initiation and growth of the number concept in preparation for algebra. However, at the present time several groups and individuals are making definite proposals that are closely related to the problem of this thesis.[2,3,4,5,6,7,8,9]

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[2]Lowry W. Harding, *Arithmetic for Child Development* (Dubuque, Iowa: William C. Brown Company, 1959).

[3]National Council of Teachers of Mathematics, *The Growth of Mathematical Ideas: Grades K-12*, Twenty-Fourth Yearbook.

[4]W. W. Sawyer, "Algebra in Grade Five," *The Arithmetic Teacher*, VII (January, 1960), 25-27.

[5]Francis J. Mueller, "Building Algebra Readiness in Grades Seven and Eight," *The Arithmetic Teacher*, VI (November, 1959), 269-273.

[6]M. L. Hartung, H. Van Engen, and Lois Knowles, *Teaching Guide for Seeing Through Arithmetic 6* (Chicago: Scott, Foresman, and Company, 1959).

[7]School Mathematics Study Group, *Mathematics for Junior High School* (2 vols; New Haven, Connecticut: Yale University, 1959).

[8]Eugene and State of Oregon Arithmetic Scope and Sequence Committee, "Arithmetic," edited by Oscar Schaaf (Eugene, Oregon, 1958).

[9]B. H. Gundlach, *G. C. M. P. Principals Notebook*:

The writer himself has not had an opportunity to put Into practice in the classroom the suggestions proposed in this thesis for the introduction of the number concept.

1.6 Meanings of Important Words

In order to avoid ambiguity, it is necessary to provide meanings for the important words used throughout this thesis: (1) arithmetic, (2) algebra, and (3) number concept.

Arithmetic.--"Arithmetic," according to McDowell, "is the science which treats of numbers and of the method or way of calculating with them."[10] James and James give a more precise definition of arithmetic. They define arithmetic to be "the study of the integers 1, 2, 3, 4, 5 . . . under the operations of addition, subtraction, multiplication, division, raising to powers, and extracting roots, and the use of these results in everyday life."[11] Buswell's definition is quite comprehensive:

> The meanings of arithmetic can be roughly grouped under a number of categories. I am suggesting four:
> 1. One group consists of a large list of basic concepts. Here, for example, are the meanings of whole numbers, of common fractions, of per cents, and, as most people would say, of ratio and proportion. Here belong, also, the denominate numbers, on which there is only slight disagreement concerning the particular units to be taught.

Elementary K-6 (Cleveland, Ohio: Educational Research Council of Greater Cleveland, 1960).

[10] C. H. McDowell, A Short Dictionary of Mathematics (New York: Philosophical Library, 1957), p. 3.

[11] Glenn James and Robert C. James, Mathematics Dictionary (New York: D. Van Nostrand Company, Inc., 1949), p. 20.

Here, too, are the technical terms of arithmetic--addend, divisor, common denominator, and the like--and, again, there is some difference of opinion as to which terms are essential and which terms are not.

2. A second group of arithmetical meanings includes understandings of the fundamental processes. Children must know when to add, when to subtract, when to multiply, and when to divide. They must possess this knowledge, and they must also know what happens to the numbers used when a given operation is employed.
3. A third group of meanings is composed of the more important principles, relationships, and generalizations of arithmetic, of which the following are typical: When 0 is added to a number, the value of that number is unchanged. The product of two abstract factors remains the same regardless of which factor is used as a multiplier. The numerator and denominator of a fraction may be divided by the same number without changing the value of the fraction.
4. A fourth group of meanings relates to the understanding of our decimal number system and its use in rationalizing our computational procedures and our algorisms.[12]

Algebra.--Buswell's statement leads naturally into the meaning of elementary algebra, for as Nunn says:

> The field of common algebra is that of numbers and their relations. . . .
>
> The algebra with which we are all familiar is only one of an infinite number of possible algebras. Wherever there is a field of inquiry of a certain type, an algebra may be invented to facilitate that inquiry.[13]

James and James emphasize the connection between arithmetic and algebra by defining algebra in the following words:

> A generalization of arithmetic; e. g., the arithmetic facts that 2 + 2 + 2 = 3 x 2, 4 + 4 + 4 = 3 x 4 . . . are all special cases of the (general) algebraic statement

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[12] G. T. Buswell (ed.), Improving the Program in Arithmetic (Chicago: University of Chicago Press, 1946), pp. 18-19.

[13] T. Percy Nunn, The Teaching of Algebra (New York: Longmans, Green and Company, 1931), pp. 5-6.

that x + x + x = 3x, where x is any number. Letters denoting any number, or any one of a certain set of numbers, such as all real numbers, are related by laws that hold for any numbers in the set; e.g., x + x = 2x, for all x (all numbers). On the other hand, conditions may be imposed upon a letter, representing any one of a set, so that it can take but one value, as in the study of equations; e.g., if 2x + 1 = 9, then x is restricted to 4. Equations are met in arithmetic, although not so named. For instance, in percentage one has to find one of the unknowns in the equation, interest = principal times rate, or I = p x r, when the other two are given.[14]

It may be said that a person must understand arithmetic before he can understand algebra. A knowledge of the principles of arithmetic make the mastery of elementary algebra easier since in algebra arithmetic is encountered in a generalized and shorthand form.[15]

Number Concept.--What is a person's mental image of number; that is, what is a person's number concept? Number has been spoken of as probably the oldest unifying theme of mathematics.[16] An individual's mental image of number develops as he acquires knowledge of the number system and how it operates.[17] Originally the number concept included only the counting numbers (1, 2, 3, . . .).[18] As time passed the

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[14]James and James, *op. cit.*, p. 7.

[15]"Preview of Algebra, Prepared by the Division of Instruction" (Columbus Public Schools, 1958), p. 1.

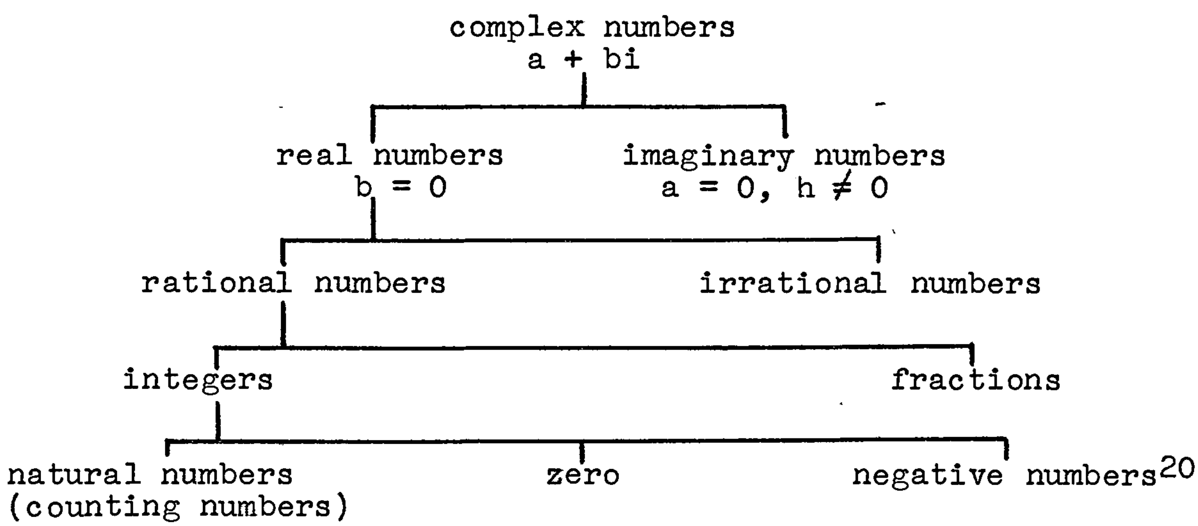
[16]E. Glenadine Gibb, Phillip S. Jones, and Charlotte W. Junge, "Number and Operation," *The Growth of Mathematical Ideas: Grades K-12*, Twenty-Fourth Yearbook of the National Council of Teachers of Mathematics, p. 7.

[17]George E. Hollister and Agnes G. Gunderson, *Teaching Arithmetic in Grades One and Two* (Chicago: D. C. Heath and Company, 1954), p. 6.

[18]Richard Courant and Herbert Robbins, *What is Mathematics?* (New York: Oxford University Press, 1941), p. 52.

concept of number was broadened to include new numbers--zero, the fractions, the negative integers, and the irrational numbers.[19]

Allendoerfer and Oakley give the following representation of the number systems and their relationships:



<u>Summary</u>.--Arithmetic is the study of the positive rational numbers. Algebra is the study of the real number system. The number concept is a person's mental image of number. The study of arithmetic with a view to preparation for algebra, in this thesis, refers to a continually expanding concept of number. The study of number as an expanding and growing concept is fundamental to any effective arithmetic program whether or not preparation for algebra is intended.

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[19]Gibb, <u>loc</u>. <u>cit</u>., pp. 7-8.

[20]C. B. Allendoerfer and C. O. Oakley, <u>Principles of Mathematics</u> (New York: McGraw-Hill Book Company, Inc., 1955), p. 65.

CHAPTER 2. A BRIEF HISTORY OF THE NUMBER CONCEPT FROM AN ALGEBRAIC POINT OF VIEW

The scope of the historical development of the number concept is indeed tremendous. Of necessity, the writer has limited his study to those facts which best illustrate the following points: (1) the similarity between the historical development of the number concept and the student's growth in understanding that same concept and (2) the development of algebra as an outgrowth of man's struggle to understand numbers and the processes that he performs with them.

The number concept had its beginning in the obscure period before recorded history. In 1938 in Moravia, a prehistoric bone was found, which is marked with groups of five notches, each group of five being followed by a larger notch.[21] From evidence such as this, one surmises that early man was interested in number. It is probable that early man felt a need for a system of keeping track of his meager possessions. Perhaps he viewed his belongings as sets to be matched by a one-to-one correspondence to pebbles or notches in a bone. In such a simple, practical manner he would have been better able to comprehend the quantitative aspect of his environment.

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[21]Gibb, Jones, and Junge, _op_. _cit_., p. 10.

The system of matching one object to another--a one-to-one correspondence--is one of the earliest stages of a child's developing number concept.[22] Pre-school children frequently hold up four fingers to indicate how many pieces of candy they want or how old they are.

Just as this period of holding up the fingers to indicate a number is but <u>one</u> phase in the developmental process, so the use of sets of elements and their corresponding representations is only <u>one</u> phase in man's development of the number concept. It required thousands of years for man to progress from this early stage to a stage in which he envisioned an abstract symbol to facilitate his quantitative thinking. Again it required thousands of years for man to learn to generalize the abstract symbols into representations, such as the following: X represents any integer.[23]

Thus it is apparent that algebra did not begin as a polished body of generalizations about number processes. First, man thought in terms of sets of objects. Next, he used abstract symbols, such as //// and eventually "4" to represent the number quality of a set. Finally, he developed processes of comparing and combining these abstract symbols to arrive at new conclusions--the processes of arithmetic.

This stage represents another plateau of growth in the child's developing concept of number--the study of number

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[22]Bruce E. Meserve, <u>Foundations of Algebra for High School Mathematics Teachers</u> (Stillwater, Oklahoma: Oklahoma Agricultural and Mechanical College, 1955), p. 3.

[23]<u>Ibid</u>., p. 6.

operations of arithmetic. Hollister and Gunderson emphasize the step by step development of the child's number concept from the concrete stage of sets to the abstract stage of symbols as follows:

1. The concrete stage involving counting or manipulating tangible objects to discover the ideas and meanings of numbers.
2. The semiconcrete stage in which the pupils work with pictures of real objects and then with dot diagrams, lines, and geometric designs. The latter representations are a further step toward the abstract because, although they are used to represent the number of objects, they do not resemble the objects.
3. The abstract stage in which number symbols and written words are used to represent quantities.[24]

It is of significance to the educator that these three steps apply to both the history of the number concept and the development of a child's understanding of number.

One should not feel that the study of numbers from an algebraic point of view blossomed quickly at this point into a fully developed concept. Only gradually did algebra pass through various stages until it emerged in its present form. These stages are sometimes referred to as the following: (1) the rhetorical phase, (2) the syncopated phase, and (3) the symbolic phase.

In the beginning of the rhetorical phase man had developed a concept of number and abstract symbols to indicate numbers. He was now ready to study these numbers as they combined in operations. His difficulties in this study lay in his method of stating and writing out solutions. Everything

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[24] Hollister and Gunderson, *op. cit.*, p. 19.

was written out in words. It was because of this cumbersome method that the phase was so named.

The earliest records of the activities of the rhetorical period have been preserved in the writings of the Egyptian scribe Ahmes. The Ahmes Papyrus, written about 1650 B. C., was a copy of earlier works.[25] Ahmes introduced it as "the entrance into knowledge of all existing things and all obscure secrets."[26]

In general this "knowledge of all existing things and all obscure secrets" was concerned with word problems and their solution by arithmetic rules and procedures. The Egyptians had a thorough understanding of the four fundamental number operations. Number, by this time, had been extended to include the system of rationals in the form of unit fractions. It is important to note that although the Egyptians solved their problems arithmetically, these same problems would today be solved by algebraic processes.[27] This is an indication of the relationship which exists between arithmetic and algebra.

The early Greeks also played a role in the rhetorical phase. At this time they extended the number system to include the irrationals. Moreover, they interpreted the

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[25]Daoud S. Kasir, The Algebra of Omar Khayyam (New York: Bureau of Publications of Columbia University, 1931), p. 12.

[26]Meserve, op. cit., pp. 7-8.

[27]Ibid., p. 10.

fundamental number operations in terms of geometric constructions.[28] The Greek geometric interpretation of algebra placed restrictive notions upon algebra which lasted until the beginning of the seventeenth century.

The development of algebra was noticeably advanced at about 50 A. D. when Diophantus introduced the use of abbreviations into number problems.[29] This event ushered in the syncopated phase. It should not be inferred that Diophantus introduced the generalized number. His abbreviations were merely a means of eliminating the burden of writing out each word and each idea surrounding the solution of a problem--one step towards man's conception of the generalized number X. By the end of the syncopated phase in the sixteenth century, it could be said that elementary algebra was fairly well perfected. The only remaining task was the development of a good symbolism.[30]

When Descartes recognized X^2 as a number rather than as an area (as had been the custom since the days of the early Greeks), the symbolic phase was begun. Students of elementary algebra were now able to understand the concept of a general member of a specified set.[31]

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[28] Ibid., pp. 17-18.

[29] Ibid., p. 32.

[30] David Eugene Smith, History of Mathematics, II (New York: Dover Publications, Inc., 1953), p. 386.

[31] Meserve, op. cit., pp. 38-39.

The symbolic phase represents the stage of thinking that is the goal of elementary algebra students today. They have previously mastered an understanding of symbols indicating a set of objects, such as 6 or 9. They are now gradually to develop an understanding of the generalized symbol, such as X, which, for example, might stand for the set of rational numbers.

CHAPTER 3. OPPORTUNITIES FOR INITIATING AND DEVELOPING THE NUMBER CONCEPT IN GRADES K-8

Chapter 3 presents and discusses opportunities for initiating and developing various phases of the number concept in Grades K-8. The material has been planned with a consideration of the child's social and mental stage of development. For a concise resume of this chapter the reader may refer to the Grade Placement Summary provided in Chapter 4.

3.1 Kindergarten

How important is the kindergarten presentation of the mathematical ideas that can later be used in algebra? According to one source, the beginnings of mathematical ideas are the very cornerstone of all such concepts; and kindergarten is the time to lay this important building block.[32] This is the viewpoint to which the writer of this thesis adheres. He visualizes the child's development of the number concept as a work of creative art--the weaving of a tapestry. Just as the weaver conceives his goal before he starts to work, so the kindergarten teacher establishes her goal before she starts to teach. Unless she deliberately intends to build

[32]Harding, *op. cit.*, p. 85.

readiness for algebra, her program is likely to fall short of meeting this necessity. In starting his tapestry, the weaver selects with care threads that he knows will enhance his finished product. Likewise, the teacher selects with care ideas which will provide a firm foundation for future learning.

Of the mathematical concepts which are to be initiated in kindergarten and later used in algebra, the number concept is the one which has beeh selected for consideration in this study. Throughout the child's schooling one of the unifying themes of the mathematics program is the familiar and fundamental idea of number.[33]

The kindergarten child is ready for number.[34] It is the task of the teacher to promote her young pupil's number consciousness. This task she accomplishes by means of conscientious planning with respect to the developmental stage of the child and the goal of building algebra readiness.

A brief examination of the developmental stage of the kindergarten student enables one to understand and appreciate the activities planned by the teacher for developing the number concept. According to Harding, the kindergarten child requires activities which involve the use of the large muscles and little manipulative skill. His program must be flexible because the child is impulsive. One of his noticeable characteristics is his use of imagination. He enjoys tales and

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[33]Gibb, Jones, and Junge, loc. cit., p. 7.

[34]Harding, op. cit., p. 74.

dramatic plays. He has little self-consciousness and will participate willingly in suggested activities. His ability to communicate in number is not as well developed as his other language abilities. The arithmetic principles that mean the most to him are those which can be illustrated through the use of <u>concrete materials</u>.[35] He has not acquired an ability for the sequential mastery and use of number.[36]

The Twenty-Fourth Yearbook mentions the fact that the kindergarten child has had some pre-school number experiences. As stated previously, he has made use of the one-to-one correspondence, as in holding up four fingers to indicate his age.[37] He Is now ready for a program of Introductory and developmental activities.

At a quick glance one might assume that the program of the kindergarten teacher lacks planning and co-ordination because of its informal nature. However, a closer examination of her activities reveals the developing number concept in its first stages. In contributing to the development of the complete number concept as required in algebra, the kindergarten teacher develops ideas about the following: (1) the number system, (2) counting, (3) zero, (4) number symbols, (5) rationals, and (6) the fundamental operations.

Ideas about number originate in the child's immediate surroundings, both at home and in school. Through discussion

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[35] <u>Ibid</u>., pp. 5-6.

[36] <u>Ibid</u>., p. 74.

[37] Gibb, Jones, and Junge, <u>loc</u>. <u>cit</u>., p. 10.

periods the teacher can draw out thoughts such as the following:

> Every time I set the table for breakfast, I put one bowl for Mommy, one bowl for Daddy, one bowl for Steve, one bowl for me, and one bowl for Sister. I put out five bowls every day.[38]

Discussions about the numbers other than five can proceed in a similar, realistic fashion. In this natural manner the child progresses in his comprehension of the cardinal numbers.

At this stage the child is making a connection between the cardinal numbers and the counting numbers. The imaginative teacher can plan numerous "exploration" activities for assisting the child in learning to count. For example, she may suggest these: (1) determining the number of windows in the classroom, (2) counting the number of students present, (3) counting cookies into small groups for distribution during snack time.[39]

Another idea to be presented to the kindergarten students is that of zero and its uses. The class may observe that as the teacher fills in an attendance chart, she uses the zero to show no one is absent from school that day. Discussions of temperature can lead to the understanding that zero is a reference point. The students may observe that the use of zero on a price tag makes the difference between an article which costs ten cents and one which costs one cent.

The number symbols may be approached, in the same manner, through discussions. For example, the teacher may

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[38]Harding, op. cit., p. 79.

[39]Ibid., p. 76.

draw attention to the meaning of the number above the classroom door. It can be observed that such a symbol distinguishes the kindergarten classroom from the others. The children are probably already familiar with the use of number symbols in the form of street numbers, residence numbers, locker numbers, and birth dates.[40]

The introduction of rational numbers is another segment of the kindergarten teacher's number concept program. She may present the rationals in simple statements about halves and quarters. For example, she may instruct the students to cut sheets of paper into <u>halves</u> so that each sheet will be enough for two pupils. A fractional idea may be planted with as simple a statement as this: "A lollipop costs only <u>half</u> as much as a balloon; that means that everyone may have either two lollipops or one balloon."[41]

The number concept grows as the pupils think out operations with numbers. Again there is an opportunity for rich operational experiences on a very informal and introductory basis. Comments and questions such as the following may stimulate thoughts about the fundamental operations:

> How many more bottles of milk are needed at that table?
>
> How many more wheels does John need for his cart if he has three?
>
> Mary gave away three sticks of gum from her new pack, so she has two left.
>
> Two sheets for each of you four. That makes eight sheets.

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[40] <u>Ibid</u>., pp. 81-82.

[41] <u>Ibid</u>., p. 77.

> If each of you three boys would bring three blocks, we would have nine.
> This long stick of candy can be divided into three pieces.
> If that group separates equally, they can have two teams of four each.[42]

The kindergarten teacher can lead her pupils to realize the commutativity of addition by helping them to observe that if they first place three blocks and then two blocks on the table the result is the same as when they place first two blocks and then three blocks.

Thus it is apparent that the kindergarten child can benefit from the teacher's efforts to introduce number. His tremendous thirst for activity can be utilized to lead him into a frame of mind ripe for the more formal number program of the first grade. Certainly it is not too early to commence the journey towards algebra.

3.2 Grade I

The first grade number concept program may be viewed as a continuation of the weaving of the imaginary tapestry begun by the kindergarten teacher. At this stage the picture in the tapestry is not readily apparent. One can see vaguely the concept of counting numbers, rationals, zero, and number operations taking form. These are threads which the teacher of Grade I must pick up and continue to weave. Moreover, she will have many new threads to insert. Her work must be done with a clear mental view of her goal--extending the wholesome growth of the number concept.

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[42] *Ibid.*, pp. 79-80.

The task of the first grade teacher is significant. She must guide the student from the manipulative and pictorial level of number thinking, which is typical of the six-year-old, to a higher level of symbolic thinking.[43]

What kind of person is the first grade teacher's pupil? Gesell and Ilg describe him as one who spends most of his day in active play. He throws himself into tasks in a somewhat clumsy fashion and does not do a thorough job of most tasks. He is a curious person with a desire to touch, handle, and explore everything. The typical first grader is the center of his universe. He wants--and needs--to be first and to win. The activities of the previous year are still pursued but with greater intensity of feeling. In most instances he approaches first grade with positive anticipation, a desire to do well, and an eagerness to learn.[44]

How does the number program of Grade I compare with that of kindergarten? Traditionally Grade I is considered the initial stage of the child's formal education. Therefore, it seems natural that the teaching procedures followed in the first grade program are more clearly defined.

According to Harding, the typical first grade program is designed to develop the following number ideas:

1. Counting by one's and ten's to one hundred in

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[43]John R. Clark _et al._, _Outline for Teaching Arithmetic in Grades One to Eight_ (Chicago: World Book Company, 1957) p. 2.

[44]Arnold Gesell and Frances L. Ilg, _The Child from Five to Ten_ (New York: Harper and Brothers, 1946), pp. 99-101, 114, 121-22.

order to build an understanding of the structure of the number system.

2. Using the ordinals.
3. Grouping concrete objects through ten, including putting together and taking away.
4. Handling the rationals one-half and one-fourth in appropriate situations.
5. Interpreting number meanings with an enlarged vocabulary.
6. Understanding number story problems and how to solve them, either with concrete objects or mentally according to the individual's level of development.[45]

To the above number ideas the Eugene Public Schools and State of Oregon Arithmetic Scope and Sequence Committee adds the following two:

1. Addition with sums to 6.
2. Subtraction with minuends to 6.[46]

To the student in Grade I the number system is a limited quantity of whole or natural numbers. His experiences with them have been through a cardinal or ordinal approach. Using the cardinal approach, the child studies the meaning of number by becoming familiar with the natural

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[45] Harding, op. cit., p. 89.

[46] Eugene and State of Oregon Arithmetic Scope and Sequence Committee, op, cit., "Course of Study for First Grade Arithmetic," p. 92.

numbers as names for sets of objects. These cardinal numbers can be ordered by ordering the representative sets. By the ordinal approach the student simply learns the number names in sequence.[47]

The sequence of the number system takes on meaning for the first grader as his teacher leads him through concrete activities. She may suggest examining the page numbers in a story book and stimulate a discussion of the idea that the first page is numbered with the numeral "1". In a follow-up activity the student may make a notebook of his own and use the numerals as he ascribes a number to each page. The pupil may be given an opportunity to analyze the meaning of "sixteen girls in attendance" in class, with emphasis on the idea that sixteen means "ten girls and six more." The list of activities should include active games involving cardinal and ordinal ideas. For example, the fourth child in the third row may be directed to run up and place five crayons on the teacher's desk. Such simple instructions may be incorporated into a variety of game experiences that is limited only by the imagination of the teacher.[48]

In Grade I the number concept is broadened also through counting experiences. The student may be assigned useful tasks, such as checking to see if all the books are in the classroom library and counting materials for distribution. A discussion

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[47]Gibb, Jones, and Junge, loc. cit., pp. 15-16.

[48]Harding, op. cit., p. 94.

of the social plans for the month might include a consideration of the number of students' birthdays to be celebrated in a particular month.[49]

Another vital segment of the developing number concept is the zero. There are numerous ways in which the teacher may present to the first grader this representative of the empty set. The notation "40¢" on the lunch ticket may be explained as "four groups of ten pennies and no extra pennies." The idea of zero may be applied to the fact that there are no students sitting at the sixth table. The child can, through guidance, learn to use zero in representing the fact that he has no money left in his piggy bank.[50]

Further mastery of number symbols may be developed in conversations about telephone numbers and numbers in a table of contents. The student's curiosity may be stimulated to a comparison of the Arabic and Roman numerals as they appear on the faces of clocks. The class may develop a map of the immediate neighborhood surrounding the school. A short field trip will enable them to obtain street numbers and house numbers for the map.[51]

The number concept in Grade I is extended to include more of the rational numbers than were experienced in the kindergarten. The teacher may again use directions to further

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[49] _Ibid._, p. 92.

[50] _Ibid._, p. 97.

[51] _Ibid._, p. 96.

the concept; for example, "One-half of the class will go now, and the other half will go after lunch." Mixing poster paints may be a lesson in the use of fractional parts. The students may share candy and fruit by dividing it into halves and quarters.[52]

In addition to the components of the growing number concept already discussed, the fundamental operations should be presented. In teaching the operation of addition, it is advisable for the teacher to note that addition is commutative and to point this fact out to the pupils. She does not, however, use the word "commutative" with the pupils. She should lead the children to observe that subtraction can undo what has been accomplished by addition and, thereby, begin an understanding of the inverse operations. The following ideas, suggested by Harding, are taken from everyday comments and experiences that offer opportunities for the student to make use of the number operations:

> Sam has five pennies in one hand and three in the other. He has how many pennies In all?
> Three girls and four boys are on this team. How many do we need to have another team?
> If David uses his ten cents to buy an ice cream cone that costs six cents, how much will he have left?
> We need two strips for each window of our "store". There are four windows, so we need eight strips.
> If each of you two bring three books to the circle, there will be enough. We need just six.
> If the lollipops cost two cents each and Mary has five cents, how many can she buy?
> If we cut the large sheets of paper in two, we will use only half as many.[53]

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[52] Ibid., p. 92.

[53] Ibid., pp. 94-95.

Through such activities the teacher reinforces what the pupils have been learning about addition and subtraction; and she also builds readiness for multiplication and division.

Thus through concrete experiences the first grader is encouraged to apply his natural curiosity and boundless energy to learning more about the number concept in preparation for algebra. At the end of this first year of formal education he should have experienced a measure of success in mastering the simple process of counting, the use of zero to indicate absence of quantity, one-digit addition and subtraction, and the meaning of simple fractions. To a great extent his number achievements are dependent upon the enthusiasm, initiative, and wise planning of his teacher.

3.3 Grade II

As the second grade teacher looks at the number concept tapestry, she sees that the picture is starting to take shape. There are more counting numbers than there were last year. There are the ordinals and the unit fractions one-half and one-fourth. The number operations are assuming vague forms. Addition appears as a commutative operation. These are loose threads which the second grade teacher will pick up and continue to weave. There are new threads also that she has selected to insert into the tapestry.

The teacher of Grade II must be aware of the development stage of her pupils. The typical second grader is becoming cautious about making new performances. He likes to repeat an activity over and over until he can do it well. His behavior has taken on a more serious and thoughtful tone. He is still interested in himself primarily. The second grader's reading ability is improving.[54] He has a mental ability that limits him almost entirely to thinking in concrete terms. For this reason, the second grade teacher must rely heavily on activities with real, familiar objects as a means of expanding the child's number concept.

What does the number program of the second grade teacher include? According to the Eugene Public Schools and State of Oregon Arithmetic Scope and Sequence Committee her program should include the following:

1. Teaching that number is everywhere in the student's surroundings.
2. Teaching counting to a thousand.
3. Teaching place value: numerals as place holders; grouping; one's, ten's, and hundred's in three-digit numbers.
4. Teaching ordinals through thirty-first.
5. Teaching rationals: one-half, one-third, one-fourth.
6. Teaching a variety of number operation problems solved by concrete devices and dramatizations.
7. Teaching addition: all combinations of one-digit addends; three one-digit numbers; three two-digit

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[54]Gesell and Ilg, <u>op. cit.</u>, pp. 139, 143, 147, 154.

numbers (no carrying); three two-digit numbers, with devices (carrying).

8. Teaching subtraction: two two-digit numbers (no borrowing); two two-digit numbers, with devices (borrowing).
9. Teaching multiplication: finding products with concrete materials.
10. Teaching division: finding quotients with concrete materials.[55]

The second grade teacher who looks beyond her program finds many opportunities to prepare her students for their future study of more general number systems. She encourages them to see relationships that exist among numbers as they are involved in the fundamental operations. The commutativity of addition is again observed. Readiness is developed for the commutativity of multiplication. The inverse nature of the addition and subtraction operations is pointed out and made meaningful. The special behavior of zero in addition and subtraction is noticed.

The teacher of Grade II utilizes the associativity of addition in teaching her pupils to find the sum of three one-digit numbers. Addition is defined to apply to only two numbers. Hence, to add 3, 7, and 8, for example, one of two procedures may be followed:

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[55]Eugene and State of Oregon Arithmetic Scope and Sequence Committee, *op. cit.*, "Course of Study for Second Grade Arithmetic," p. 94.

$$\left.\begin{array}{r} 3 \\ 7 \end{array}\right\} \begin{array}{r} 10 \end{array} \quad \begin{array}{r} +\ 8 \\ \hline \end{array} \ \begin{array}{r} +\ 8 \\ \hline 18 \end{array} \qquad \text{or} \qquad \begin{array}{r} 3 \\ \left.\begin{array}{r} 7 \\ +\ 8 \\ \hline \end{array}\right\} \end{array} \begin{array}{r} 3 \\ +15 \\ \hline 18 \end{array}$$

Although these procedures involve experience with the associative principle of addition, the name of the principle would not be mentioned in second grade.

For developing an understanding of the decimal number system, Harding suggests activities such as keeping records involving money and helping the teacher arrange supplies in groups of ten.[56]

In order to keep the concept of zero before the class, the teacher may encourage the students to use zero instead of leaving a blank space in recording game scores. When correcting the students' papers, she may mark the number of items missed as well as the number correct. Keeping a daily record of the outdoor temperature shows zero as a reference point. The two preceding teaching methods may be utilized also to develop readiness for the introduction of the negative integers.

The number symbols become more meaningful for the students as they participate in activities such as the following: reading their own weight on the scales, reading the numerals on rulers and yardsticks as they measure art paper, and reading Arabic and Roman numerals in telling time.[57]

The students extend their understanding of the rationals as they decide whether one-half, one-third, or one-fourth of the class will be able to play at one time. They

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[56] Harding, _op_. _cit_., p. 109.

[57] _Ibid_., p. 111.

may be asked to carry a fractional part of a stack of books to the storage room. While making covers for books, they may be instructed to fold the rectangular sheet of paper in half. They may be encouraged to estimate how much is a half, a third, or a fourth in dividing the available chalkboard space.[58] This is an opportunity for the pupils to notice the "ordering property" of rational numbers.

Natural physical activities provide opportunities to make the number operations more meaningful. A garden project would afford learning experiences such as these: adding the number of seeds planted by each child in the classroom garden, subtracting the number of germinated seeds from the total number planted, finding the total cost of the seeds planted, and distributing an equal number of seeds to each child for planting.[59]

It is through concrete activities that the second grade teacher is able to work most effectively. The majority of ideas which she stresses are the ones that the students encountered in Grade I. The pupil's familiarity with these recurring ideas gives him a better vantage point for evaluating them. Thus he is able to view the various aspects of the number concept with a greater depth of understanding.

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[58] *Ibid.*, p. 107.

[59] *Ibid.*, p. 109.

3.4 Grade III

As the third grade teacher examines the tapestry and prepares to make her contribution, she thinks of the typical student with whom she is going to work. He is increasing in his ability to use the fine muscles. He is less sensitive than he was last year and more interested in the world outside himself. His reading ability has improved to the extent that he can read to entertain himself. He enjoys variety in his school program.[60]

The task of the third grade teacher is a significant one. According to Hartung, Van Engen, Knowles, and Mahoney, the third grade student should learn to : (1) comprehend our place value system through thousand's, (2) regroup place value numbers as required In carrying and borrowing, (3) understand the ordinals as a system, (4) use fractions with denominators of two through nine, (5) develop a readiness for the reduction of fractions, and (6) Increase his ability to perform the fundamental number operations. In doing so, he will master the basic addition facts and use the commutative principle to change the order of the addends. He will be able to perform addition with two three-digit numbers.[61]

In his development of the subtraction concept, the student will master the basic subtraction facts. At this time the teacher should bring out the important idea of the

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[60]Gesell and llg, *op. cit.*, pp. 169, 173, 183.

[61]Maurice L. Hartung *et al.*, *Teaching Guide for Seeing Through Arithmetic 3* (Chicago: Scott, Foresman and Company, 1956), p. 346.

inverse relationship between the operations of addition and subtraction, which will be recalled later when the pupils encounter the inverse elements of real numbers. The student's understanding will be broadened to the subtraction process with three-digit numbers.

In the third grade the child will further develop his concept of multiplication and division. He should master the basic multiplication facts through products of thirty-six and the basic division facts through dividends of thirty-six. He will learn to use the commutative principle to change the order of the factors in the multiplication process and to see the inverse nature of the relationship between multiplication and division.[62] This relationship will be encountered later in algebra as the inverse property of real numbers.

To these objectives Harding adds one more. He says that the third grader should be exposed to the history of number.[63] This aim is suitable in the third grade program; for, as was mentioned previously, the student is at this point beginning to take an interest in the world outside of himself. The history of number contains the story of man's development of a number system to the base ten. This is the stage at which the third grader finds himself in his path toward mastery of the number concept. His study of the history of number at this time builds readiness for the historical reasons for the development of the real number system.

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[62] Ibid.

[63] Harding, op. cit., p. 118.

In carrying out her objectives, the third grade teacher may rely on numerous real activities in life situations from which her students will learn number ideas. To make the pupils aware of the number system, they may be asked to assist her in tallying attendance by using a bead abacus. The students may be led through an analysis of adding three-digit numbers in the form of dollars and cents.[64]

The meaning of the number symbols can be developed by keeping scores and adding scores, by considering the three-digit number enrollment, and by setting the thermostat.[65]

Fractions, both common and decimal, become meaningful as the pupils participate in learning experiences such as these: noting and comparing symbols for the first quarter, half, third quarter, and full moon on the calendar and in the sky; illustrating seventy-five cents as three-fourths of a dollar; and deciding by the use of a ruler how deep the rainfall report of 2.4 inches would be.[66]

The pupil's ability to operate with numbers increases as he learns various combinations and groupings through the handling of concrete objects.[67] Situations within his environment contain sources of objects which he can visually study and physically manipulate. The following ideas suggest operations with numbers:

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[64]*Ibid*., p. 124.

[65]*Ibid*., p. 126.

[66]*Ibid*., p. 123.

[67]Clark *et al*, *op*. *cit*., p. 5.

> If the show costs ten cents and there are thirty-seven of us, how much money will there be when it is all collected?
> If there were ten punches on your lunch ticket and three are gone, how many do you have left?
> Susie practices on the piano forty-five minutes each day. How long does she practice each week?
> I bought a popsickle and a tablet. The popsickle cost six cents. The tablet was five cents. I spent eleven cents altogether.[68]

In performing number operations, the third grader can, through experience, develop a readiness for algebra. Addition and multiplication, he discovers, are commutative. The associative property of the addition operation makes the adding of numbers involving carrying meaningful. For example, $27 + 6 = (20 + 7) + 6 = 20 + (7 + 6) = 20 + 13 = 20 + (10 + 3) = (20 + 10) + 3 = 30 + 3 = 33$. He learns the inverse nature of the relationship that exists between addition and subtraction. This discovery prepares him for the inverse property of the real numbers of algebra.[69]

His experiences with the number operations reveal the special properties of zero and one: (1) any number minus itself is zero, (2) one times any number is that number, and (3) any number plus zero is that number. Such observations as these prepare the child for an understanding of the existence of the identity and inverse elements of the real number system. Thus, in the third grade the pupil's number concept is further developed.

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[68]Harding, op. cit., p. 125.

[69]Clark et al., op. cit., pp. 5-8.

3.5 Grade IV

What does the imaginary tapestry look like and how does it change through Grade IV? There is an enlarging understanding of the place value nature of our number system; the operations of addition and subtraction; the commutative and associative principles as they occur in the rational number operations; a partially developed ability to perform multiplication and division; the idea of inverse operations, later developed as the inverse properties of real numbers; the concept of unit and non-unit fractions with single-digit denominators; and a recognition of the special properties associated with one and zero in their relation td the fundamental number operations, later recalled as the identity elements of the real numbers.

Before planning her contribution to the tapestry, the teacher must take inventory of the characteristics of the type of student with which she shall work. She finds that the fourth grader is capable of applying himself diligently to his work. His hands and eyes are well co-ordinated. He is able to accept responsibility and work independently. The fourth grader is impressed with what he is told. He prefers written work to oral exercises. Often he has more spontaneous interest in problem solving than his school work offers him.[70]

Keeping in mind the picture of the unfinished tapestry and the developmental characteristics of her students, the teacher of Grade IV plans her number concept program for the

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[70]Gesell and Ilg, _op. cit._, pp. 197, 208-209.

year. According to Hartung, Van Engen, and Knowles, in the fourth year of school the student should learn to understand our place value number system through million's, to perform regrouping as required in carrying and borrowing, to round off numbers, and to understand the numerals for the rational numbers.[71]

The fourth grade program provides more experience in the addition and subtraction of whole numbers. Addition, the inverse operation of subtraction, should be considered as a check on subtraction. The basic facts of multiplication are to be increased through products of eighty-one. It should be emphasized to the student that changing the order of the factors has no effect upon the product and there are special results always obtained when zero and one are factors in multiplication. He should acquire the ability to multiply three-digit numbers by two-digit multipliers. In teaching the multiplication of two- or more-digit numbers, the teacher can develop a readiness for algebra by using the distributive principle. For example, $7 \times 32 = 7(30 + 2) = 7 \times 30 + 7 \times 2 = 210 + 14 = 224$. This principle will be encountered in algebra as the distributive property of the real numbers. Division, the inverse operation of multiplication, should be used as a check on the result of multiplication.[72]

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[71]Maurice L. Hartung, Henry Van Engen, and Lois Knowles, _Teaching Guide for Seeing Through Arithmetic 4_ (Chicago: Scott, Foresman and Company, 1956), p. 365.

[72]_Ibid._

In division Hartung, Van Engen, and Knowles say that the fourth grader should master the basic facts through dividends of eighty-one. He is to observe the special role of zero and one in division. He should notice that division with whole numbers does not always result in another whole number. This idea will be encountered in algebra as the closure property of real numbers. It should be pointed out that the quotients in division are rational numbers and multiplication, the inverse operation of division, is a check on the division operation.

It is advisable to allow time for the reteaching and enlarging of the concept of rational numbers. The program of the fourth grade provides the pupil with a concept of unit and non-unit fractions with denominators up to sixteen. The pupil should observe the use and meaning of improper fractions and mixed numbers.[73]

The Eugene Public Schools and State of Oregon Arithmetic Scope and Sequence Committee add to the above list of aims for the fourth grade teacher the following two: (1) to teach the number Ideas of primitive people and (2) to teach zero as a point of reference.[74]

The goal of the fourth grade teacher is not the teaching of numbers as individuals but the teaching of numbers as members of a system. In this manner the pupils will be enabled

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[73] _ibid_.

[74] Eugene and State of Oregon Arithmetic Scope and Sequence Committee, _op. cit._, "Tentative Fourth Grade Arithmetic Course of Study," p. 7.

to understand and deal with any number by using the principles of the system.[75] The number system involves many relationships. When these relationships are clear to the pupil, he is able to see many facts that would otherwise seem to be isolated elements as related parts of a few general principles. The student need not be burdened at this time with the <u>names</u> of the relationships that he observes. The importance of the commutative and associative principles, for example, lies in the understanding they lend to the number operations. The associative principle adds insight to the operations of addition and multiplication. The associative principle makes "29 + 3" meaningful as it becomes (20 + 9) + 3 = 20 + 12 = 20 + (10 + 2) = (20 + 10) + 2 = 30 + 2 = 32. The associative and distributive principles add meaning to "34 x 8." For example, 34 x 8 = 8 x 34 = 8(30 + 4) = 8 x 30 + 8 x 4 = 240 + 32 = 272. The generalization that the sum and the product of two whole numbers are both whole numbers is called the closure principle. An understanding of generalizations such as these results in a meaningful arithmetic which leads naturally into algebra.[76]

Although the fourth grader is progressing towards maturity in his thinking, the concrete situations that are inherent in his surroundings are still the most fertile source of growth in the number concept. The nature of the number

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[75]Hartung, Van Engen, and Knowles, <u>Teaching Guide for Seeing Through Arithmetic 4</u>, p. 239.

[76]<u>Ibid</u>., p. 6.

system becomes apparent as the teacher discusses with the class the meaning of "212." Some in the class may have seen such a number on a telephone repair truck. As a pupil writes the number on the chalkboard, the teacher explains that the first two on the right side means two units or, in this case, two trucks. The one means ten trucks, and the second two means two hundred trucks. The class, therefore, concludes that the particular repair truck which they saw is the two hundred and twelfth truck bought by the telephone company.[77]

The child's counting ability increases as he engages in activities such as counting sales tax stamps and putting them into hundred-unit bundles of each denomination. Serving as class librarian, he may be made responsible for counting the books and the students to see if there is an adequate number of books for the class.[78]

The zero develops more meaning as the pupil examines its use in the world about him. He may be asked to calculate the difference in the total cost of a book priced at a dollar and eighty cents and one priced at a dollar and eight cents for thirty-seven pupils. The zero may be stressed as a reference point in discussing the distance traveled by a vehicle whose speedometer was set at zero at the beginning of a journey.[79] The concept of zero as a reference point is a fundamental

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[77]Harding, op. cit., p. 140.

[78]Ibid., p. 138.

[79]Ibid., p. 142.

element to understanding the number line. The use of zero in this manner is frequently encountered in algebra.

In studying the number symbols, the students may compare the Roman numerals on monuments with the Arabic counterparts. They may compare the number of digits in hundred's, thousand's, and million's as they are found in the geography and history texts and the encyclopedia.[80]

Class projects may be utilized for a consideration of the rational numbers. For example, the teacher may say: "Three pipe cleaners make one lamb for favors for our spring party. One pipe cleaner is then what part of the total number needed for making one lamb?"[81]

The number operations may, likewise, be drawn from class activities. The students may decide--and at the same time develop insight into the fundamental operations--the total cost of wieners, buns, pickles, paper plates, and napkins for the class picnic. Or they may determine the cost of new desks for their classroom.[82]

By the end of the fourth year of school the pupil has advanced a long way toward the understanding of the number concept. The concept will continue to grow and to develop greater depth of meaning for him as he moves closer to algebra.

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[80]*Ibid.*, p. I41.

[81]*Ibid.*, pp. 138-139.

[82]*Ibid.*, p. 140.

3.6 Grade V

What techniques have prevailed in the weaving of the tapestry up to this point? The teachers have worked primarily with real, physical experiences from the child's natural environment. Therefore, the tapestry is a naturalistic form of art as opposed to the abstract nature of many creations of modern art. However, it is only the technique which gives the tapestry its realism. One must not conclude that numbers are tangible. Numbers are abstract concepts and purely mental construets.[83] As the fifth grade teacher takes her place at the frame of the tapestry, she is aware that she has the Herculean task of bridging the gap from a real world to an abstract world in order that her students may progress from the realm of concrete thinking to that of abstract thinking.

Before the fifth grade teacher plans her program, she too reviews the characteristics of her typical student. The fifth grader is starting to evaluate his faults and assets.[84] For this reason, his learning may be inhibited by an increasing sensitivity to his own inadequacies unless he is given frequent encouragement.[85] He is an active individual and play

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[83]Maurice L. Hartung, Henry Van Engen, Lois Knowles, Teaching Guide for Seeing Through Arithmetic 5 (Chicago: Scott, Foresman and Company, 1957), p. 254.

[84]Arnold Gesell, Frances L. Ilg, and Louis B. Ames, Youth: The Years from Ten to Sixteen (New York: Harper and Brothers, 1956), p. 53.

[85]Board of Education of the City of New York, Curriculum Development in the Elementary Schools, A Curriculum Bulletin Prepared by the Curriculum Council of the New York City Schools (New York: by the Board of Education, 1946), p. 206.

is important to him. At school he is less teacher-centered than he has been previously. It is essential that he be kept interested and motivated. He is alert in seeing and hearing. There is a hint of the restlessness which accompanies the gradual transition from the child's world to the adult's world.[86]

The number concept program planned for the fifth grader includes the following:

1. Extending the understanding of decimal fraction places to tenths and hundredths.
2. Understanding the symbol zero as a reference point, as an indicator of absence of quantity, and as a number having all the properties of numbers and also some special qualities.
3. Recognizing fractions as symbols of division.
4. Becoming aware that the whole numbers are reciprocals of unit fractions and that whole numbers and fractions are a part of the rational system of numbers.
5. Exploring the story behind the existence of the two fractions, common and decimal.
6. Extending the operations of addition and subtraction to include the operations with whole numbers, common fractions, and decimal fractions.

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[86] Gesell, Ilg, and Ames, op. cit., pp. 44, 56, 60, 62.

7. Extending the operations of multiplication and division to include the operations with whole numbers and common fractions.[87]

Among the aims of the fifth grade teacher is the development of the operations with rational numbers. In dealing with the fraction concept, it is the task of the teacher to guide the pupil from the concrete level to the abstract level. It is not too difficult for the student to observe that one-sixth of a cookie and one-third of a cookie make a half of the cookie. This is a concrete fact.[88] However, a giant stride must be taken before the student can say, "One-sixth plus one-third equals one-half." Perhaps in taking this stride, the teacher will begin with exercises using drawings, rulers, and folded paper. The class may be encouraged to examine the ideas that they have developed previously about the operations with fractions. Gradually they will be led to arrive at generalizations and to formulate <u>by themselves</u> rules for adding, subtracting, multiplying, and dividing which they can apply in future encounters with rational numbers.

In teaching the pupils the addition and subtraction of unlike fractions, the teacher uses the idea that there are many names for a number. This idea is helpful in overcoming the difficulties encountered in finding common denominators.

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[87]Eugene and State of Oregon Arithmetic Scope and Sequence Committee, *op. cit.*, "Tentative Fifth Grade Arithmetic Course of Study," pp. 7-8.

[88]William A. Brownell, "Psychological Considerations in the Learning and Teaching of Arithmetic," Tenth Yearbook of the National Council of Teachers of Mathematics (New York: Bureau of Publications of Columbia University, 1935), p. 22.

The approach is particularly worthwhile, for it develops a readiness for the introduction to the number line.[89] Each rational point on the line has a set of names, all of the names being equivalent. For example, the point corresponding to one and one-half has the additional names three-halves, nine-sixths, twenty-one-fourteenths, and many others.

In his article, "Algebra in Grade Five," Sawyer suggests to the teacher that she frequently ask the students how they picture arithmetical expressions. For example, how do they picture "three plus two"? Do they visualize three objects and two objects put together to form a group of five objects? Similarly, they should learn to think of "three minus two" as three objects and then two removed. "Three times two" may be pictured as three rows of two objects. This emphasis upon what the pupils are doing when they perform a number operation, Sawyer says, is the important part of algebra--that is, seeing clearly the processes involved. The answer is important, too, but less so.[90]

Readiness for further extension of the number concept is developed in the fifth grade program as the student observes the relationships that exist in number operations. The addition of rational numbers, the child may be led to observe, is a commutative and associative operation that always results in another rational number. Subtraction is not a commutative

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[89]Henry Van Engen, "Twentieth Century Mathematics for the Elementary Schools," _The Arithmetic Teacher_, VI (March, 1959), 74.

[90]W. W. Sawyer, _loc. cit._, p. 26.

nor an associative operation; and it does not always result in another rational number with which the fifth grader is familiar. The teacher may suggest that subtraction may yield numbers with which he is now unfamiliar but with which he will work in later arithmetic lessons. Through such ideas as these the teacher lays the groundwork for the closure property of real numbers and the negative integers. She develops the inverse property of real numbers by illustrating that the operation of addition can undo the results of the operation of subtraction. The multiplication of rational numbers is another operation which is commutative and associative; it results in another rational number.

In order to be sure that the students are digesting the above generalizations, the teacher may ask thought-provoking questions, such as: "Does the division of whole numbers always result in another whole number?" In this manner, she enlarges the students' understanding of the system of numbers and builds readiness for future extensions of the number system.

The special properties of zero and one are reinforced at the fifth grade level. The class should be led to make generalizations such as these:

1. Any number plus zero is that number.
2. Any number minus zero is that number.
3. Any number minus itself is zero.
4. Zero times any number is zero.
5. Zero divided by any number is zero.

6. Any number times one is that number.

7. Any number divided by itself is one.

The first andsixth generalizations above build readiness for the identity element, while the third and seventh build readiness for the existence of the inverse element.

As the fifth grade teacher plans her program for the year, she too relies on real experiences. The decimal nature of our number system is studied through the school health report. The children read in the report that nine per cent of the fifth grade students are underweight. By direct examination of their weights, they determine whether nine per cent written in decimal form is "0.90" or "0.09."[91]

A deeper understanding of number symbols is likely to result from discussions of such topics as the bird population in different states and the markings on railroad cars. In finding the average cost per child of the class picnic, the symbols for fractional remainders in division become significant.[92]

The fractions, common and decimal, are a part of the fifth grader's number program. In a health project, he may prepare toothpaste using a recipe that lists the fractional parts of ths ingredients. Fractions are used when the class determines how to divide a dozen peppermint candy bars among the twenty-eight class members. Decimal fractions are needed when the class keeps account of the money collected by the

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[91]Harding, <u>op</u>. <u>cit</u>., p. 157.

[92]<u>Ibid</u>., p. 159.

room for the Red Feather Drive. During the scrap paper drive, the students may compare the progress of various home room teams; one may have collected six-tenths of its goal and another nine-tenths of its goal.[93]

The task of the fifth grade teacher is indeed a large one. Her success in building algebra readiness depends largely on her professional insight into the number concept.

3.7 Grade VI

As the teacher of Grade VI takes her place at the frame of the imaginary tapestry, she sees that, in spite of the tremendous amount of work that has been done, the picture is far from complete. There is still an assortment of new bobbins beside the frame. From these she shall select new threads for new concepts and bring the students closer to the open door of algebra.

In making her contribution, with what type of student will she work? Many of her pupils are experiencing a sudden spurt in physical growth. The sixth grader no longer regards play as paramount in his life, and he prefers more adult projects. In mental and social activities he shows increased facility. At school he prefers a "tough" teacher who challenges him. Although he is eager to learn, he may become fatigued with even the best planned teaching routine. He is far more aware of his faults than of his assets. It is not

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[93] _Ibid._, p. 156.

uncommon for the sixth grader to speak of school as one of his "problems."[94]

The Eugene Public Schools and State of Oregon Arithmetic Scope and Sequence Committee cover the sixth grade number operations program as follows:

1. In the study of addition to learn (a) to estimate sums of whole numbers and decimals, (b) to add four to six addends with as many as six digits, (c) to mentally add three or more two-digit addends, (d) to add fractions and mixed numbers without common denominators, and (e) to add decimal fractions and mixed decimal fractions.
2. In the study of subtraction to learn (a) to estimate the difference between three- and four-digit whole and decimal fraction numbers, (b) to subtract using a six-digit subtrahend, (c) to subtract fractions and mixed numbers without common denominators, and (d) to subtract decimal fractions and mixed decimal fractions.
3. In the study of multiplication to learn (a) to estimate the product of a four-digit number and a two-digit number; (b) to multiply mentally by ten, one hundred, and one thousand; (c) to mentally multiply a four-digit number by a one-digit number; (d) to multiply with common and mixed

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[94] Gesell, Ilg, and Ames, op. cit., pp. 77, 88, 93-94, 96, 100.

fractions; (e) to multiply decimal fractions with products to ten-thousandths; and (f) to estimate the products of decimal fractions.

4. In the study of division to learn (a) the impossibility of dividing by zero; (b) to divide mentally by ten, one hundred, and one thousand; (c) to divide a five-digit number by a two-digit number; (d) to estimate quotients by a one- or two-digit divisor; (e) to divide with common and mixed fractions; (f) to find decimal quotients resulting from the division of a whole number by a larger whole number; (g) to divide with a decimal in the dividend; (h) to divide a mixed decimal by a decimal; and (i) to estimate decimal quotients.[95]

Hartung, Van Engen, and Knowles include as objectives in their number concept program for the fifth grade (1) to introduce the positive and negative numbers as members of a number system and (2) to introduce the number line emphasizing the fact that there are sets of names for every rational point along the line.[96]

Brown and Coffman add one important new objective that brings the student closer to algebra--to introduce the generalized number.[97]

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[95]Eugene and State of Oregon Arithmetic Scope and Sequence Committee, _op. cit._, "Tentative Sixth Grade Arithmetic Course of Study," p. 7.

[96]Hartung, Van Engen, and Knowles, _Teaching Guide for Seeing Through Arithmetic 6_, p. 407.

[97]Joseph C. Brown and Lotus D. Coffman, _The Teaching of Arithmetic_ (Chicago: Row, Peterson and Company, 1924), p. 359.

In order to take care of all objectives, the number program for the sixth grade requires serious planning. The need for challenging the sixth grader may be satiated in new ideas, such as the introduction of the negative numbers. The first brush with the more "grown-up" subject, algebra, is especially appealing to the student at this time, and it may be used to develop a wholesome readiness for future work with algebraic concepts. However, the operations with signed numbers are not studied at this time. There are various tangible means of making signed numbers real to the sixth grader; for example, (1) the scale on the thermometer, (2) the scores of different games, (3) directed distances, and (4) the stock market and weather reports in newspapers and magazines.[98] Through such concrete devices the introduction of the negative numbers doubles the scope of the number field for the student.

The study of positive and negative numbers is naturally followed by the introduction to the number line. Zero is indicated as the reference point that is neither negative nor positive.[99] The line shows geometrically that there is a one-to-one correspondence between the points on a line and the rational system of numbers. It becomes evident to the student that the familiar natural numbers studied previously are a part of the rational number system as they are placed

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[98]Hartung, Van Engen, and Knowles, Teaching Guide for Seeing Through Arithmetic 6, pp. 252-53.

[99]ibid.

on points to the right of zero.[100] Familiarity with the negative numbers is acquired as they are assigned to points on the line to the left of zero. The study of the number line should stress the fact that there are many names for each of the points on the line. The student should learn that he may extend the line in either direction to include any desired number.[101]

Another concept that may be introduced to challenge the sixth grader is that of the generalized number, usually represented by a letter of the alphabet. Although this is a new idea for the sixth grader, he will accept it naturally. Just as he is accustomed to use N. Y. to represent New York, so he will learn to use C to represent cost. However, there is a necessary distinction to be emphasized. In the former case "N. Y." is an abbreviation which represents an unvarying quantity; whereas, in the latter case the letter "C" is a generalized number which represents any number that is an answer to the problem.

The student has reached a maturity level at which inconsistencies in the number system that may have confused him previously may be clarified. For instance, in a discussion of population the teacher may point out "ten one's are called ten, ten ten's are called a hundred, and ten hundred's are

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[100] Ibid., p. 343.

[101] Ibid., p. 164.

[102] Brown and Coffman, loc. cit.

called a thousand, but ten thousand's are not called a million."[103]

In sixth grade the pupil may be led to observe some general properties of numbers. His observations, however, must be made at his developmental level. He should be encouraged to express in his own language his observations of the commutative principle, the associative principle, the distributive principle, the additive and multiplicative inverse elements, and the additive and multiplicative identity elements.

As the sixth school year comes to a close, the student prepares to leave the elementary building. His success from kindergarten to Grade VI is proportional to the alertness of his teachers, who have cued their teaching to the many changing facets of his personality.

3.8 Grade VII

The seventh grade teacher now approaches the frame of the imaginary tapestry. The picture has progressed from the muted colors and vague forms representing the meager number experiences of the pre-primary child to the bold colors and sharp outlines representing the more sharply defined number concepts acquired by the sixth grader. However, the tapestry is not yet complete. There are many threads left loose which the teacher of Grade VII must pick up. In addition, she will weave her share using a subtle, new technique. As a result, her students will discover new and different ways of looking

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[103]Harding, op. cit., p. 176.

at the number story thus depicted. They will develop the ability to view number in the perspective of algebra students.

The typical student with whom this teacher will work loses his own identity in his "gang." He displays a wide variety of interests. At times he is buoyed away from reality by his unleashed enthusiasm. He is capable of bringing his group to a high pitch of excitement leaving an unwary teacher in doubt of how to restore quiet. It is the task of the teacher to utilize his enthusiasm and vast range of interests in productive activities.[104]

As the teacher of Grade VII builds her curriculum, she has two major ideas which she plans to emphasize: (1) how the number system grew out of primitive man's attempts to keep track of the quantities in his possession and in his environment and (2) how number, as a fundamental concept, is related to the total mathematics program and to the world.

The following idea outline suggested for the seventh grade number program is condensed from the proposed curriculum of the School Mathematics Study Group:[105]

1. The System for Naming and Writing Numbers
   - 1.1 The history of our numeration system began in ancient times.
   - 1.2 Reading and writing numerals in the decimal system follows a pattern.

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[104] Gesell, Ilg, and Ames, op. cit., pp. 131-33.

[105] School Mathematics Study Group, op. cit., I.

1.3 Our knowledge of the decimal system of numeration may be increased by studying the numerals as used in a system to another base.

2. The Whole Numbers

2.1 The counting numbers plus zero comprise the whole number system.

2.2 The counting numbers are known as "natural" numbers and are man's answer to the question "How many?"

2.3 Each of the fundamental operations has properties which are observed in the elementary grades but which are not named directly.

2.4 The inverse of the addition operation is the subtraction operation, and the inverse of the multiplication operation is the division operation.

2.5 One and zero are special numbers.

3. Factoring and Primes

3.1 A prime is any counting number other than one that is divisible only by itself and one.

3.2 A number is a factor of a second number if there is a third number such that the product of the first number and the third number is the second number.

3.3 A composite number is one which can be expressed as the product of two smaller whole numbers.

3.4 The greatest common factor of two counting numbers is the largest counting number that is a factor of each of them.

3.5 The least common multiple of two counting numbers is the smallest counting number which is a multiple of each of them.

4. The Rational Number System

4.1 In general, a rational number is a whole number over a counting number.

4.2 The number line may be used to find geometric solutions for the four fundamental operations with rational numbers.

4.3 The rational numbers can be ordered.

4.4 The multiplication of rational numbers is commutative and associative.

4.5 The addition of rational numbers is commutative and associative.

4.6 A rational number can be expressed in decimal form by dividing the numerator by the denominator.

4.7 Every rational number may be named by a decimal which either terminates or repeats.

The above program as suggested by the School Mathematics Study Group would be a meaningless effort if it did not have roots in the physical world surrounding the seventh grader. The Study Group has included many activities built around concrete materials and observable phenomena which the teacher may use to clarify ideas for her students.

Although the content of the seventh grade program may at first appear complex, the alert teacher can create vivid scenes to simplify the material. For example, the class may momentarily go back in history and visualize a group of shepherds gathered to talk around an ancient well as they water their sheep. Most of the students have been impressed elsewhere with the comfort and security of the old quotation: "I know mine and mine know me." As one of the shepherds moves away from his companions, he calls to his flock. And surely enough his own know his voice and leave the other flocks to join him. However, lest there be one wayward sheep that should not respond, the shepherd takes count of them before leaving. His method of counting is most unusual. He compares each of his sheep with a notch cut into his crook. From a scene as simple as this the meaning of one-to-one correspondence is reinforced.

As a sequel to the lesson the teacher may suggest a problem such as the following:

> An old deck of cards contains all the necessary red cards, but it is not known whether any of the black cards are missing. Can you suggest a way, without counting, to find whether or not all the black cards are in the deck?[106]

The nature of our numerals comes to light as the students play a game of largest number and decide which would be easier--to write or to read a numeral larger than that of the opponent.

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[106] *Ibid.*, p. 29.

The commutative principle may be made clear by leading the pupils to an examination of common situations which are examples of the principle, such as the following: (1) washing your face and washing your hair, (2) going one block north and going one block west, and (3) counting to one hundred and writing the alphabet. There are other real situations which are not examples of the principle: (1) putting the cat out and going to bed, (2) eating dinner and getting up from the table, and (3) raking the leaves and burning them.

The inverse operations may also be clarified through common situations. The students may be led to discover that an inverse operation "undoes" what has been done. As an example of an inverse operation, one might suggest getting into a car and, the inverse, getting out of the car. The class will observe that not all activities have inverse operations--for example, smelling a rose and talking.

The concept of rational numbers as ratios may be explored through the actual measurement of the height of buildings, trees, and fences and the lengths of the shadows of these objects. The resulting measurements are next arranged into a table. In this manner the ratio of the height of the object to the length of the shadow becomes apparent.

The aspects of the Grade VII number program mentioned here do not encompass all of the objectives of the seventh grade teacher. As mentioned at the beginning of the chapter, there are numerous loose threads left by the sixth grade teacher that are to be picked up during this school term.

The negative numbers, especially, will need reinforcement. However, to avoid repetition, tbe writer has included here only those ideas which shed a new light on the number concept. With this new perspective, the seventh grader is becoming equipped to view number not only as an algebra student but also as a student of mathematics.

3.9 Grade VIII

Standing before the frame of the imaginary tapestry, the teacher of Grade VIII sees a well-developed picture--the efforts of the previous teachers who have worked diligently to promote the growth of number concepts in their students. The eighth grade teacher will pick up the loose threads and continue to weave them as the other teachers have done. She will select new bobbins to introduce new ideas--such as, the real numbers and the operations with positive and negative integers. She has just one short school term in which to finish the task of making her students ready for their entrance into a more formal study of the generalized number system.

It is fortunate that in this final, busy year the eighth grade teacher has a co-operative student with whom to work. The typical eighth grader appears quiet and withdrawn, but he is mentally active. He is able to organize and to carry out plans. His sense of responsibility is well-developed. He knows his capabilities. He controls his enthusiasm and is selective in his interests.[107]

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[107]Gesell, Ilg, and Ames, *op. cit.*, p. 165.

The following idea outline prepared for the eighth grader's number program is condensed from the proposed eighth grade textbook of the School Mathematics Study Group:[108]

1. Studying the Negative Numbers

    1.1 The number line, a geometrical representation of our number system, is again encountered as a helpful tool.

        1.11 There is a one-to-one correspondence between some of the points on the number line and all the numbers with which the pupils are familiar.

        1.12 The point corresponding to zero is the point of intersection of the left and right rays of the number line.

        1.13 The negative numbers correspond to the points to the left of zero on the number line and are defined as follows: If _a_ is any rational number associated with a point on the right half of the number line, then we define _negative a_, (-a), by saying (-a) is the number which added to _a_ gives the sum 0.

    1.2 The operations with negative numbers are developed by the teacher in a meaningful pattern. It is necessary that the operations satisfy the commutative, associative,

[108]School Mathematics Study Group, _op. cit._, II.

and distributive properties. Following such a development, these definitions are made:

1.21 The addition of positive and negative numbers may be defined in the following manner:

1.211 $a + (-b) = a - b$ if a, b are positive rational numbers and a is greater than b.

1.212 $a + (-b) = -(b-a)$ if a, b are positive rational numbers and b is greater than a.

1.213 $(-a) + (-b) = -(a + b)$ if a, b are positive rational numbers.

1.22 The multiplication of negative numbers may be defined in the following manner:

1.221 $(-a)b = -ab$ if a, b are any two positive rational numbers.

1.222 $(-a)(-b) = ab$ if a, b are any two positive rational numbers.

1.23 The division of negative numbers may be defined in ths following manner:

1.231 If $bx = -a$, where b is a positive rational number and $-a$ is a negative rational number, then x must represent a negative number from what we know about multiplication. Hence, $x = \frac{-a}{b}$

is a negative number and we conclude that a negative number divided by a positive number is a negative number.

1.232 If $(-b)x = a$, where a is positive, then x must be a negative number from our knowledge of multiplication. Hence, $x = \frac{a}{-b}$ is a negative number and we conclude that a positive number divided by a negative number gives a negative number.

1.233 If $(-b)x = (-a)$, x must be a positive number since -b must be multiplied by a postive number to give (-a). Hence, $x = \frac{-a}{-b}$ is a positive number and we must conclude that a negative number divided by a negative number is a positive number.

1.24 The subtraction of negative numbers may be defined in the following manner:

1.241 $a - b = a + (-b) = -(b - a)$ if a, b are any positive rational numbers and b is greater than a.

1.242 $a - (-b) = a + b$ if a, b are any positive rational numbers.

I.243 (-a) - b = -(a + b) if a, b are any positive rational numbers.

I.244 (-a) - (-b) = (-a) + b If a, b are any positive rational numbers.

2. The System of Real Numbers

2.1 The set of integers contains the set of positive integers, negative integers, and zero.

2.2 The set of integers is contained in the set of rational numbers, which enables man to measure. The rationals satisfy the following properties:

2.21 Closure: If a, b are rational numbers, then a + b is a rational number, a x b is a rational number, a - b is a rational number, and $\frac{a}{b}$ is a rational number if b does not equal 0.

2.22 Commutativity: If a, b are rational numbers, then a + b = b + a and ab = ba.

2.23 Associativity: If a, b, c are rational numbers, then a + (b + c) = (a + b) + c and a(bc) = (ab)c.

2.24 Identities: If a is a rational number, then a + 0 = a and a x 1 = a.

2.25 Distributivity: If a, b, c are rational numbers, then a(b + c) = ab + ac.

2.26 Additive inverse: If a is a rational number, then there is a number (-a) such that a + (-a) = 0.

2.27 Multiplicative inverse: If a is a rational number and a does not equal zero, then there is a number b such that a x b = 1.

2.28 Order: If a and b are different rational numbers, then either $a < b$, or $a > b$.

2.3 The rational numbers are dense: that is, there is a third rational number between any two distinct rational numbers.

2.4 In addition to the set of rational numbers there exists a set of irrational numbers.

2.5 Any rational number has a periodic non-terminating decimal representation and the rational numbers can be defined as the set of all periodic decimals.

2.6 An irrational number can be defined as any number with a non-terminating, non-periodic decimal representation.

2.7 The rational numbers and the irrational numbers considered together are known as the set of real numbers.

2.8 The real number system has the following properties:

2.81 The real number system is closed under addition, subtraction, multiplication, and division.

2.82 The real numbers are commutative under addition and multiplication.

2.83 The real numbers are associative under addition and multiplication.

2.84 The real numbers have an additive and a multiplicative identity element.

2.85 Multiplication is distributive over addition of real numbers.

2.86 For every real number there is another real number which is its additive inverse; for every real number, except zero, there is another real number which is its multiplicative inverse.

2.87 The real numbers are ordered.

2.88 The real numbers are dense.

2.89 The real number line is complete; that is, to each point on the number line there corresponds a real number.

2.9 In addition to the irrational numbers which arise from extracting roots of rational numbers, there are many more irrational numbers, such as pi, which are called transcendental numbers. (These numbers the pupils will encounter in high school mathematics.)

3. Very Large and Very Small Numbers; Scientific Notation

   3.1 Extremely large numbers are becoming increasingly useful to man.

   3.2 A very large number may be written in scientific notation; that is, it may be written as a product of a number between one and ten with the appropriate power of ten. For example, 9,100,000 may be written as 9.1×10^6.

   3.3 Very small numbers may be written in scientific notation by making use of negative exponents. For example, 0.000056 may be written as 5.6×10^{-5}.

   3.4 In performing calculations with numbers written in scientific notation, the teacher builds readiness for operations with exponents.

In presenting the number program to her students, the eighth grade teacher relies heavily on one particular technique--she vividly constructs a "need." In the study of rational numbers, she describes the background of ancient man in which there were no methods for measuring fractional parts of units of distance, weight and so forth. Lacking these conveniences, man recognizes a need for something that he can use in dealing with fractional parts of units. The solution to his problem is found in the rational numbers.

The need for the irrational numbers may be illustrated as the students construct triangles using the straight edge and compass. The triangle of which the hypotenuse can be shown to be not a rational number makes apparent the need for a system of numbers which includes some numbers that are not rational.

The junior high school student is especially "science-minded." His imagination is easily captured in the small world of atoms and the large world of celestial bodies. Discussions of the weight of atoms in grams, the distance between planets, the factors involved in space travel, and the speed of light readily illustrate the need for scientific notation in writing and speaking of very large and very small numbers.

At the end of eighth grade the student stands at the door of his formal algebra training, the stepping stone to a further study of mathematics, if such is his wish. His background in the number concept is the result of his own efforts and the efforts of <u>every one</u> of his arithmetic teachers from kindergarten through Grade VIII. For this student there will be no frustrations resulting from too much and too unfamiliar material being heaped upon him all at once. He has been guided naturally through a number concept continuum. His training in kindergarten and the first eight grades has been aimed at priming him for future growth in mathematics. It is only natural that this beginning algebra student slides into his new role with eagerness and self-confidence.

At the same time, the pupil who does not wish to study mathematics any further has benefited from such a program. His background, as a result of this work with the number concept, will enable him to comprehend the number situations which he will encounter every day.

CHAPTER 4. SUMMARY

4.1 General Summary

In Chapter III of this thesis the writer compares the development of the number concept to the weaving of a tapestry. Why does he select for his comparison this almost forgotten art? The primary reason is that this ancient skill, tapestry weaving, illustrates vividly the need to establish a definite goal before starting the actual work. Before the weaver takes a single thread in hand, he studies a finished painting--his goal. This painting, called a "cartoon," he places directly under his frame. Now he is ready to duplicate the picture in cloth. As he weaves, he frequently separates the threads stretched on the frame and studies the cartoon beneath to be certain that he is not deviating from his goal.

In planning for the elementary study of number, it is, likewise, essential that clear goals be established before any teaching is done. Readiness for algebra is not likely to result from a number program unless one of the goals of the program is the building of readiness for algebra. After the goals are clearly stated, it is necessary that each teacher from kindergarten to the eighth grade "separate the threads and study the cartoon beneath" frequently during the school term.

Each lesson must be treated not as a complete segment with immediate goals only, but as an interlocking unit in a continuum aimed toward the objectives of the total program.

The second similarity between the number concept and the tapestry is that both result from gradual, continuous processes. The weaver is a patient man; he knows that he cannot create a masterpiece at one sitting. Day after day he weaves, back and forth he moves the bobbins, single thread by single thread he progresses.

The same is true of the history of the number concept. The step-by-step process was ever so gradual. The sequence of phases shows a natural and consistent development.

Likewise, the student's growth in the development of the number concept is gradual and continuous. Efforts must be consistently applied from kindergarten to the eighth grade if the student is to develop a readiness for algebra. Each idea should be presented as many times as possible to the pupil while he is in school. Ideas which are taught once and then dropped permanently are like loose threads left hanging on the tapestry; they weaken the finished product. There must be a continual "picking up of the loose threads" as the student progresses from kindergarten to the eighth grade.

The outstanding weaver is one who loves his work. For him there is no monotony in weaving. He enjoys the variety in the colors and textures of the threads that he selects, in the intricate patterns that he creates, and in the subtle techniques that he devises. Most important, his enjoyment of these variations stems not merely from single perceptions but rather from the knowledge that each variation will enhance the completed work.

The outstanding teacher, too, is one who has enthusiasm for her work. In her classes there is no monotonous reteaching or tiresome drill. She makes clever use of visual materials and physical experiences. She accumulates a wealth of varied teaching techniques. Most important, she evaluates each learning activity in the light of its contribution to the goals of the program.

In this thesis the writer has included suggestions for the teacher of each level from kindergarten to the eighth grade. However, students are not all moulded in the same pattern. Consequently, when presenting the material, the teacher must take into consideration not only the grade level but also the individual--<u>his</u> developmental stage and <u>his</u> readiness.

Once again the problem is stated: How can the teachers of Grades K-8 initiate and promote the growth of the number concept? This study yields a threefold answer:

(1) The teachers of Grades K-8 must establish as one of their goals the deliberate building of a systematic concept of number. Moreover, they must evaluate each lesson in the light of whether or not it leads toward the goal.

(2) The teachers must have an awareness that the child's concept of number grows <u>gradually</u> by means of <u>continuous</u> efforts from kindergarten to the eighth grade. It is the task of each

teacher to find out what the child has been exposed to previously so that she may reinforce the material and build upon it. In addition, she must find out what is to follow her teaching in order that she may develop readiness rather than erect obstacles for future learning.

(3) A readiness for further study of mathematics is promoted through an informed selection of content, concrete materials, physical experiences, and varied teaching techniques suited to the maturity level of the student.

4.2 Grade Placement Summary

The following summary is a condensed, over-all view of Chapter 3. It is intended to be used as a general guide to the approach to the teaching of the number concept that is advocated in this thesis. The program proposed here includes for each grade level (1) topics which the pupil should master at the present time, (2) topics which the teacher should build readiness for in anticipation of future mastery by the student, and (3) topics studied in the past which should be reinforced for more permanent retention by the student.

Kindergarten

Student learns:
- One-to-one correspondence
- Cardinal numbers
- Counting numbers

Teacher builds readiness for:
- Ordinals
- Place value
- Zero to indicate absence of quantity
- Rational numbers
- Fundamental operations
- Commutative property of addition

Grade I

Student learns:
- Cardinal numbers
- Ordinals
- Counting numbers
- Zero to indicate absence of quantity
- Rational numbers (one-half, one-fourth)
- Fundamental operations (addition, subtraction)
- Gommutative property of addition

Teacher builds readiness for:
- Place value
- Zero as a reference point
- Fundamental operations (multiplication, division)
- Inverse operations (addition, subtraction)

Program reinforces:
- One-to-one correspondence

Grade II

Student learns:
- Cardinal numbers
- Ordinals
- Counting numbers
- Rational numbers (one-half, one-fourth, one-third)
- Fundamental operations (addition, subtraction)
- Commutative property of addition
- Associative property of addition
- Inverse operations (addition, subtraction)
- Place value

Teacher builds readiness for:
- Zero as a reference point
- Fundamental operations (multiplication, division)
- Commutative property of multiplication
- Regrouping for carrying and borrowing
- Zero in addition and subtraction

Program reinforces:
- One-to-one correspondence
- Zero to indicate absence of quantity

Grade III

Student learns:
- Place value to 1000
- Rational numbers (denominators of 2, 3, 4, 5, 6, 7, 8, 9)
- Fundamental operations (multiplication, division)
- Inverse operations (multiplication, division)

History of numbers
Regrouping for carrying and borrowing

Teacher builds readiness for:
Properties of zero and one in fundamental operations
Zero as a reference point
Decimals

Program reinforces:
Zero to indicate the absence of quantity
Cardinal numbers
Ordinals
Counting numbers
Commutative property of addition
Associative property of addition
Inverse operations (addition, subtraction)

Grade IV

Student learns:
Place value through million's
Fundamental operations (multiplication, division)
Rational numbers (denominators to sixteen)
Zero as a reference point

Teacher builds readiness for:
Associative property of multiplication
Decimals
Properties of zero and one in fundamental operations
Fundamental operations with rational numbers
Closure

Program reinforces:
Inverse operations (all fundamental operations)
Associative property of addition
Regrouping for carrying and borrowing
Commutative operations (addition, multiplication)
Zero to indicate the absence of quantity

Grade V

Student learns:
Decimals to hundredths
Fundamental operations with rational numbers
Associative property of multiplication

Teacher builds readiness for:
Closure
Many names for a number
Number line
Rational number system

Commutative, associative, and distributive principles
Properties of zero and one in fundamental operations
Operations with decimals
Negative integers

Program reinforces:
Zero (point of reference, absence of quantity)
Inverse operations (addition and subtraction, multiplication and division
Commutative and associative property of addition
Commutative property of multiplication

Grade VI

Student learns:
Number line
All operations with decimals and rational numbers

Teacher builds readiness for:
Rational number system
Negative integers
Generalized number
Commutative, associative, and distributive principles
Closure
Properties of zero and one in fundamental operations

Program reinforces:
Fundamental operations with rational numbers
Zero (point of reference, absence of quantity)
Commutative and associative properties of addition
Commutative and associative properties of multiplication
Inverse operations (addition and subtraction, multiplication and division)

Grade VII

Student learns:
Ancient system of numeration
Number system to other bases
Commutative, associative, and distributive principles
Closure
Identity elements
Density
Transitivity

Order
Primes
Composites
Multiples
Pactors
Powers
Rational number system

Teacher builds readiness for:
Negative integers
Completeness of real number line
Generalized number

Program reinforces:
Fundamental operations with rational numbers
Number line
One-to-one correspondence

<u>Grade VIII</u>

Student learns:
Negative integers
Operations with the rational numbers
Set of irrational numbers
Real number system
Complete number line
Inverse elements

Teacher builds readiness for:
Infinity
Transcendentals
Generalized number

Program reinforces:
Closure
Commutative, associative, and distributive principles
Identity element
Order
Density
Transitivity

CHAPTER 5. CONCLUSIONS

How can the teachers of Grades K-8 initiate and promote a continuous growth of the number concept in order to insure an adequate preparation for algebra? In studying this question, the writer has arrived at the following conclusions:

(1) Currently in the world of mathematics there is developing a sentiment that the number concept should be taught in such a way in Grades K-8 that: (a) adequate preparation for algebra and the courses beyond algebra is ensured and (b) numbers and operations with numbers have more meaning. Individuals and groups are producing recommendations, curriculum guides, and textbooks advocating the systematic inclusion of the number concept in Grades K-8.

(2) In history may be found many of the answers to the problems of motivation and development of the number concept. Vivid descriptions serve to enliven mathematics and make it more palatable for the student. Furthermore, the historic development of sets, cardinal numbers as names for the number quality of sets, counting numbers, and rational numbers closely parallels their development as mental images in children.

(3) Mathematics, If it's abstract content is to have meaning, must be derived, in Grades K-8, from the physical entities which it counts, measures, and describes. In Grades

K-8 the teacher should guide the pupil to find meaning for the number concept in his surroundings. As the child matures, the amount of abstraction which he can fathom increases. Therefore, the teacher must not bind her program in the physical environment; she should proceed to the abstract when her pupils are ready to leave the concrete. Her program must, however, continually revert to entities which are familiar to the children.

(4) The number system should be built from the counting numbers. All numbers may be presented, although not directly, as natural and needed outgrowths of the counting numbers.

(5) Because of the psychological factors involved in elementary and junior high school education, number cannot be made rigorously logical. However, it can be presented consistently and taught in such a manner that it may be easily extended to the more abstract mathematics of algebra.

(6) Mastering the number concept requires that the student develop a sense of structure and an ability to make generalizations. Otherwise, for him numbers are meaningless symbols involved in misleading operations that degenerate during the first period of disuse.

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